

Entropy Inequalities:

Note: σ below means the “ σ -algebra generated by”

1. $H(\alpha \vee \beta) = H(\alpha) + H(\beta|\alpha)$
2. $H(\alpha|\mathcal{B}) \leq H(\alpha)$ with equality iff $\alpha \perp \mathcal{B}$
3. $H(\alpha|\mathcal{B}) \geq 0$ with equality iff $\alpha \subseteq \mathcal{B}$
4. $H(\alpha \vee \beta) = H(\alpha)$ iff $\beta \preceq \alpha$
5. $H(\alpha_1 \vee \dots \vee \alpha_n) = \sum_{i=1}^n H(\alpha_i|\alpha_1 \vee \dots \vee \alpha_{i-1})$
6. $H(\alpha_1 \vee \dots \vee \alpha_n) \leq \sum_i H(\alpha_i)$ with equality iff α_i are independent.
7. Assume $\beta \preceq \alpha$. Then $H(\beta) \leq H(\alpha)$ and equality holds iff $\beta = \alpha$.
8. Assume $\mathcal{B} \preceq \mathcal{C}$. Then $H(\alpha|\mathcal{C}) \leq H(\alpha|\mathcal{B})$
9. $H(\alpha \vee \beta|\mathcal{C}) = H(\alpha|\mathcal{C}) + H(\beta|\sigma(\alpha \cup \mathcal{C})) \leq H(\alpha|\mathcal{C}) + H(\beta|\mathcal{C})$
10. $H(\alpha|\sigma(\mathcal{B} \cup \mathcal{C})) \leq H(\alpha|\mathcal{B})$ with equality iff $\alpha \perp_{\mathcal{B}} \mathcal{C}$
11. If $\mathcal{B} \perp_{\mathcal{C}} (\sigma(\beta \cup \mathcal{D}))$, then $H(\beta|\sigma(\mathcal{D} \cup \mathcal{C} \cup \mathcal{B})) = H(\beta|\sigma(\mathcal{D} \cup \mathcal{C}))$.
12. If $\gamma \preceq \alpha$, then $H(\gamma|\mathcal{B}) \leq H(\alpha|\mathcal{B})$
13. If T is an MPT, then for all i, j, k ,

$$H(T^{-i}\alpha \vee \dots \vee T^{-j}\alpha) = H(T^{-i-k}\alpha \vee \dots \vee T^{-j-k}\alpha)$$
14. If T is an MPT, then for all i, j, k ,

$$H(T^{-i}\alpha \vee \dots \vee T^{-j}\alpha|\mathcal{B}) = H(T^{-i-k}\alpha \vee \dots \vee T^{-j-k}\alpha|T^{-k}(\mathcal{B}))$$