DOCUMENT RESUME

ED 053 166 TM 000 684

AUTHOR	Lord, Frederic M.; Wingersky, Marilyn S.
TITLE	Efficiency of Estimation When There is Only One Common Factor.
INSTITUTION	Educational Testing Service, Princeton, N.J.
SPONS AGENCY	Office of Naval Research, Washington, D.C. Personnel and Training Research Programs Office.; Office of Naval Research, Washington, D.C. Psychological Sciences Div.
REPORT NO	RB-71-16
PUB DATE	Mar 71
NOTE	17p.
EDRS PRICE	EDRS Price MF-$0.65 HC-$3.29
DESCRIPTORS	*Factor Analysis, *Factor Structure, *Mathematical Models, Mathematics, Sampling, *Statistical Analysis, Statistics

ABSTRACT

Explicit formulas are derived for the asymptotic sampling variances and covariances of the maximum likelihood estimators for factor-analysis parameters in the special case where there is just one common factor. The effect of the number of variables on these variances and covariances is indicated. A formula is given showing to what extent the usual covariance between two of a set of variables can be estimated more efficiently when there is known to be just one common factor. (Author)

RB-71-16

EFFICIENCY OF ESTIMATION WHEN THERE IS ONLY ONE COMMON FACTOR

Frederic M. Lord

and

Marilyn S. Wingersky

This research was sponsored in part by the
Personnel and Training Research Programs
Psychological Sciences Division
Office of Naval Research, under
Contract No. N00014-69-C-0017

Contract Authority Identification Number
NR No. 150-303

Frederic M. Lord, Principal Investigator

Educational Testing Service

Princeton, New Jersey

March 1971

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Educational Testing Service Princeton, New Jersey 08540	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

EFFICIENCY OF ESTIMATION WHEN THERE IS ONLY ONE COMMON FACTOR

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report

5. AUTHOR(S) (First name, middle initial, last name)

Frederic M. Lord and Marilyn S. Wingersky

6. REPORT DATE March 1971	7a. TOTAL NO. OF PAGES 9	7b. NO. OF REFS 4
8a. CONTRACT OR GRANT NO. N00014-69-C-0017 b. PROJECT NO. NR No. 150-303 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) RB-71-16 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Personnel and Training Research Programs Office of Naval Research Arlington, Virginia 22217

13. ABSTRACT

Explicit formulas are derived for the asymptotic sampling variances and covariances of the maximum likelihood estimators for factor-analysis parameters in the special case where there is just one common factor. The effect of the number of variables on these variances and covariances is indicated. A formula is given showing to what extent the usual covariance between two of a set of variables can be estimated more efficiently when there is known to be just one common factor.

DD FORM 1473 1 NOV 65 (PAGE 1)

S/N 0101-807-6801

Security Classification

3ND PPSO 13152

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Factor analysis						
Mathematical statistics						

DD FORM 1 NOV 65 1473 (BACK)
(PAGE 2)

Security Classification

EFFICIENCY OF ESTIMATION WHEN THERE IS ONLY ONE COMMON FACTOR

Abstract

Explicit formulas are derived for the asymptotic sampling variances and covariances of the maximum likelihood estimators for factor-analysis parameters in the special case where there is just one common factor. The effect of the number of variables on these variances and covariances is indicated. A formula is given showing to what extent the usual covariance between two of a set of variables can be estimated more efficiently when there is known to be just one common factor.

EFFICIENCY OF ESTIMATION WHEN THERE IS ONLY ONE COMMON FACTOR*

1. Introduction

Under multinormality assumptions, Lawley (1967) and Lockhart (1967) give formulas for the inverse of the asymptotic variance-covariance matrix of the maximum likelihood estimators for factor analysis parameters. Anderson & Rubin (1956, eq. 12.25) give a complicated formula for the asymptotic covariance between two estimated loadings on the same factor. Explicit formulas for other asymptotic sampling variances and covariances do not seem to be readily available. The present paper gives reasonably convenient explicit formulas for these for the special case where there is just one common factor. (In this paper, the terms asymptotic and large-sample imply a large number of observations, not a large number of variables.) It is shown how n, the number of random variables, affects the efficiency with which factor loadings can be estimated.

When the one-factor model holds for a set of n variables, the covariance σ_{ij} between any two variables x_i and x_j can be estimated more efficiently than by using the ordinary sample covariance s_{ij}. An expression for the increase in efficiency is given. A practical test-theory problem that motivated the derivation is briefly considered in the final section.

2. Formulas for Sampling Variances and Covariances

The factor analysis model for the one-factor case is

$$\Sigma = \lambda\lambda' + \Psi \tag{1}$$

---

*This research was sponsored in part by the Personnel and Training Research Programs, Psychological Sciences Division, Office of Naval Research, under Contract No. N00014-69-C-0017, Contract Authority Identification Number, NR No. 150-303, and Educational Testing Service.

where $\Sigma \equiv \|\sigma_{ij}\|$ is the (population) variance-covariance matrix for $n \geq 3$ observed variables x_i, $\lambda \equiv \{\lambda_i\}$ is the vector of loadings on their common factor, and $\Psi \equiv \|\psi_{ij}\|$ is a diagonal matrix. It will be assumed that $\psi_{ii} > e > 0$, $i = 1,2,\ldots,n$, where e is some small quantity.

For the one-factor case, ignoring terms of order $1/N$, the expected values of the second derivatives of the logarithm of the likelihood function may be summarized in matrix form as follows

$$A \equiv -N^{-1}\|\mathcal{E}(\partial^2 \log L/\partial\lambda_i\partial\lambda_j)\| = c\Sigma^{-1} + \Sigma^{-1}\lambda\lambda'\Sigma^{-1} \quad , \tag{2}$$

$$B \equiv -N^{-1}\|\mathcal{E}(\partial^2 \log L/\partial\lambda_i\partial\psi_{jj})\| = \Sigma^{-1}\Delta \quad , \tag{3}$$

$$D \equiv -N^{-1}\|\mathcal{E}(\partial^2 \log L/\partial\psi_{ii}\partial\psi_{jj})\| = \frac{1}{2}\|(\sigma^{ij})^2\| \quad , \tag{4}$$

where N is the number of observations on which each σ_{ij} is based,

$$c \equiv \lambda'\Sigma^{-1}\lambda \quad , \tag{5}$$

Δ is the diagonal matrix whose nonzero elements are $(1 - c)\psi^{ii}\lambda_i$, and σ^{ij} and ψ^{ij} are the elements of Σ^{-1} and Ψ^{-1} respectively (in this section, except for N, the number of observations, upper-case letters are used for matrices, lower-case unsubscripted Greek letters for vectors, and lower-case Roman or subscripted Greek letters for scalars). These three equations are readily found from Jöreskog's convenient summary (1969, eqs. 17, 19, 22).

Denote the maximum likelihood estimators of the parameters by $\hat{\lambda}_1$, $\hat{\lambda}_2, \ldots, \hat{\lambda}_n, \hat{\psi}_{11}, \hat{\psi}_{22}, \ldots, \hat{\psi}_{nn}$. The asymptotic variance-covariance matrix of these estimators is given by the inverse of the partitioned matrix

$$N\begin{bmatrix} A & B \\ B' & D \end{bmatrix} \quad .$$

This inverse can be found from the standard formula

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1}BM^{-1}CA^{-1} & - A^{-1}BM^{-1} \\ - M^{-1}CA^{-1} & M^{-1} \end{bmatrix} \quad , \tag{6}$$

where $M \equiv D - CA^{-1}B$.

A standard formula applied to (1) gives

$$\Sigma^{-1} = \Psi^{-1} - (1 - c)\kappa\kappa' \quad , \tag{7}$$

where $\kappa \equiv \Psi^{-1}\lambda$. Similarly, from (2),

$$A^{-1} = c^{-1}(\Sigma - \tfrac{1}{2} c^{-1}\lambda\lambda') \quad . \tag{8}$$

From (7), we find that

$$\Sigma^{-1}\lambda = (1 - c)\kappa \quad . \tag{9}$$

From (3) through (9),

$$M \equiv D - B'A^{-1}B = \tfrac{1}{2} \|\chi^{ii} + f^{-2}\kappa_i^2\kappa_j^2\| \tag{10}$$

where

$$f \equiv \sum_{i=1}^{n} \psi^{ii}\lambda_i^2 = \frac{c}{1 - c} \quad , \tag{11}$$

$$\chi^{ii} \equiv (f\psi^2_{ii}g_i)^{-1} \quad , \tag{12}$$

$$g_i \equiv (f - 2\psi^{ii}\lambda^2_{\ i})^{-1} \quad . \tag{13}$$

Then,

$$M^{-1} = 2(X - dX\gamma\gamma'X) \tag{14}$$

where $X \equiv \|X_{ii}\| \equiv \|\chi^{ii}\|^{-1} = \|f\psi^2_{ii}g_i\|$ is a diagonal matrix, $\gamma \equiv \{\kappa_i^2\}$, and $d \equiv (f^2 + \gamma'X\gamma)^{-1}$.

We have

$$X_{ii}\gamma_i = f\lambda_i^2 g_i \tag{15}$$

so that

$$d = [f^2 + f \sum_{i=1}^{n} g_i(\psi^{ii}\lambda_i^2)^2]^{-1} \quad . \tag{16}$$

Finally, after substituting (15) and (16) into (14), we have by (6) the asymptotic variances and covariances of the ψ_{ii} :

$$\mathrm{Cov}(\hat{\psi}_{ii},\hat{\psi}_{jj}) = \frac{2}{N}\,(f\psi^2_{ij}g_i - df^2\lambda_i^2\lambda_j^2 g_i g_j) \quad . \tag{17}$$

Similarly, after some algebra,

$$\mathrm{Cov}(\hat{\lambda}_i,\hat{\psi}_{jj}) = \frac{1}{N}\,(-2\lambda_i\psi_{ij}g_i + df^2\lambda_i\lambda_j^2 g_i g_j) \quad , \tag{18}$$

$$\mathrm{Cov}(\hat{\lambda}_i,\hat{\lambda}_j) = \frac{1}{N}\,[\psi_{ij}(1 + g_i) + \frac{1}{2}\,\lambda_i\lambda_j(1 - df^2 g_i g_j)] \quad . \tag{19}$$

$$\chi^{ii} \equiv (f\psi_{ii}^2 g_i)^{-1} \quad , \tag{12}$$

$$g_i \equiv (f - 2\psi^{ii}\lambda^2_{\ i})^{-1} \quad . \tag{13}$$

Then,

$$M^{-1} = 2(\chi - d\chi\gamma\gamma'\chi) \tag{14}$$

where $\chi \equiv \|\chi_{ii}\| \equiv \|\chi^{ii}\|^{-1} = \|f\psi_{ii}^2 g_i\|$ is a diagonal matrix, $\gamma \equiv \{\kappa_i^2\}$, and $d \equiv (f^2 + \gamma'\chi\gamma)^{-1}$.

We have

$$\chi_{ii}\gamma_i = f\lambda_i^2 g_i \tag{15}$$

so that

$$d = [f^2 + f \sum_{i=1}^{n} g_i(\psi^{ii}\lambda_i^2)^2]^{-1} \quad . \tag{16}$$

Finally, after substituting (15) and (16) into (14), we have by (6) the asymptotic variances and covariances of the ψ_{ii} :

$$\mathrm{Cov}(\hat{\psi}_{ii},\hat{\psi}_{jj}) = \frac{2}{N}\,(f\psi_{ij}^2 g_i - df^2\lambda_i^2\lambda_j^2 g_i g_j) \quad . \tag{17}$$

Similarly, after some algebra,

$$\mathrm{Cov}(\hat{\lambda}_i,\hat{\psi}_{jj}) = \frac{1}{N}\,(-2\lambda_i\psi_{ij}g_i + df^2\lambda_i\lambda_j^2 g_i g_j) \quad , \tag{18}$$

$$\mathrm{Cov}(\hat{\lambda}_i,\hat{\lambda}_j) = \frac{1}{N}\,[\psi_{ij}(1 + g_i) + \frac{1}{2}\,\lambda_i\lambda_j(1 - df^2 g_i g_j)] \quad . \tag{19}$$

3. The Role of n

Now that these formulas are available, we can examine the role of n, the number of variables, in determining the sampling variances and covariances. Let us assume that there is a uniform upper bound to all the σ_{ii}. Then it appears from (11), (13), (16) that

f is of order n,

g_i is of order n^{-1},

d is of order n^{-1},

$fg_i = 1$ plus terms of order n^{-1}.

Now, the second term in (17), also both terms in (18), are of order n^{-1}. Thus the sampling covariances between $\hat{\psi}_i$ and $\hat{\psi}_j$, $i \neq j$, also between $\hat{\psi}_i$ and $\hat{\lambda}_i$ or $\hat{\lambda}_j$, vanish for large n. If we neglect terms of order n^{-1},

$$\mathrm{Var}\ \hat{\psi}_{ii} \doteq 2\psi_{ii}^2/N \quad , \tag{20}$$

$$\mathrm{Var}\ \hat{\lambda}_i \doteq (\psi_{ii} + \lambda_i^2/2)/N \quad , \tag{21}$$

$$\mathrm{Cov}(\hat{\lambda}_i, \hat{\lambda}_j) \doteq \lambda_i \lambda_j / 2N \qquad (i \neq j) \quad . \tag{22}$$

4. Estimating the Covariances between Variables

Under the one-factor model, the maximum likelihood estimators of the population variances and covariances are given by

$$\hat{\sigma}_{ii} \equiv \hat{\lambda}_i^2 + \hat{\psi}_{ii} = \frac{1}{N} \sum_{a=1}^{N} (x_{ia} - \bar{x}_i)^2 \quad , \tag{23}$$

$$\hat{\sigma}_{ij} \equiv \hat{\lambda}_i \hat{\lambda}_j \quad \text{if} \quad i \neq j \quad . \tag{24}$$

Only the second formula gives a different estimator than would be appropriate without the single-factor assumption.

The asymptotic variance of $\hat{\sigma}_{ij}$ is given by

$$\text{Var } \hat{\sigma}_{ij} \doteq \lambda_i^2 \text{ Var } \hat{\lambda}_j^2 + \hat{\lambda}_j^2 \text{ Var } \hat{\lambda}_i^2 + 2\lambda_i\lambda_j \text{ Cov}(\hat{\lambda}_i, \hat{\lambda}_j) \quad .$$

By (21) and (22), omitting terms of order $1/n$, when $i \neq j$

$$\begin{aligned} \text{Var } \hat{\sigma}_{ij} &\doteq \frac{1}{N} (\lambda_i^2 \psi_{jj} + \lambda_j^2 \psi_{ii} + 2\lambda_i^2 \lambda_j^2) \\ &\doteq \frac{1}{N} (\sigma_i^2 \sigma_j^2 + \sigma_{ij}^2 - \psi_{ii} \psi_{jj}) \quad . \end{aligned} \tag{25}$$

Without the one-factor assumption, one would estimate σ_{ij} by the second bivariate sample moment

$$s_{ij} \equiv \frac{1}{N} \sum_{a=1}^{N} (x_{ia} - \bar{x}_i)(x_{ja} - \bar{x}_j) \quad , \tag{26}$$

which has an asymptotic variance of $(\sigma_i^2 \sigma_j^2 + \sigma_{ij}^2)/N$. Thus, for large n and N the use of the one-factor assumption decreases the sampling variance of our estimate of σ_{ij} by the amount $\psi_{ii}\psi_{jj}/N$.

5. Estimating a Validity Coefficient of a Composite

Suppose x_n is a criterion variable of interest, and suppose that we are interested in using the "total score"

$$X \equiv \sum_{i=1}^{n-1} x_i$$

to predict x_n . The "validity coefficient" for the effectiveness of the total score for this purpose is the correlation

$$\rho(X,x_n) = \sum_{i=1}^{n-1} \sigma_{in}/\sigma_X\sigma_{nn} \quad .$$

When the single-factor model holds, every term in the numerator can be estimated more accurately by (24) than by (26). This seems at first sight to guarantee a reduction in the sampling errors of the estimated validity coefficient, an important consideration when choosing among several predictors.

Some algebra (not given here) shows, however, that the sampling variance of the estimated numerator is the same whether the one-factor model is assumed or not. The same is true for the estimated $\rho(X,x_n)$. The reason is that although Var $\hat{\sigma}_{ij}$ is smaller under (24) than under (26), $\mathrm{Cov}(\hat{\sigma}_{ij},\hat{\sigma}_{gh})$ is larger under (24). Both Var $\hat{\sigma}_{ij}$ and $\mathrm{Cov}(\hat{\sigma}_{ij},\hat{\sigma}_{gh})$ are involved in the formulas for $\mathrm{Var}(\sum_{i=1}^{n-1} \hat{\sigma}_{in})$ and Var $\hat{\rho}(X,x_n)$.

References

Anderson, T. W. & Rubin, H. (1956). Statistical inference in factor analysis. In J. Neyman (ed.), Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability, Vol. V. Berkeley: University of California Press. Pp. 111-150.

Jöreskog, K. G. (1969). A general approach to confirmatory maximum likelihood factor analysis. Psychometrika 34, 183-201.

Lawley, D. N. (1967). Some new results in maximum likelihood factor analysis. Proc. Roy. Soc. Edinburgh Sect. A 67, 256-264.

Lockhart, R. S. (1967). Asymptotic sampling variances for factor analytic models identified by specified zero parameters. Psychometrika 32, 265-277.

NR 150-303, N-00014-69-C-0017

ONR Distribution List

4 Chief of Naval Research
Code 458
Department of the Navy
Arlington, Virginia 22217

1 Director
ONR Branch Office
495 Summer Street
Boston, Massachusetts 02210

1 Office of Naval Research
Branch Office Chicago
536 South Clark Street
Chicago, Illinois 60605

1 Director
ONR Branch Office
1030 East Green Street
Pasadena, California 91101

6 Director, Naval Research Laboratory
Washington, D. C. 20390
ATTN: Library, Code 2029 (ONRL)

6 Director
Naval Research Laboratory
Washington, D. C. 20390
ATTN: Technical Information Division

12 Defense Documentation Center
Cameron Station, Building 5
5010 Duke Street
Alexandria, Virginia 22314

1 Commanding Officer
Service School Command
U. S. Naval Training Center
San Diego, California 92133

3 Commanding Officer
Naval Personnel and Training
Research Laboratory
San Diego, California 92152

1 Commanding Officer
Naval Medical Neuropsychiatric
Research Unit
San Diego, California 92152

1 Commanding Officer
Naval Air Technical Training Center
Jacksonville, Florida 32213

1 Dr. James J. Regan, Code 55
Naval Training Device Center
Orlando, Florida 32813

1 Technical Library
U. S. Naval Weapons Laboratory
Dahlgren, Virginia 22448

1 Research Director, Code 06
Research and Evaluation Department
U. S. Naval Examining Center
Building 2711 - Green Bay Area
Great Lakes, Illinois 60088
ATTN: C. S. Winiewicz

1 Dr. A. L. Slafkosky
Scientific Advisor (Code AX)
Commandant of the Marine Corps
Washington, D. C. 20380

1 Behavioral Sciences Department
Naval Medical Research Institute
National Naval Medical Center
Bethesda, Maryland 20014

1 Commanding Officer
Naval Medical Field Research Laboratory
Camp Lejeune, North Carolina 28542

1 Director
Aerospace Crew Equipment Department
Naval Air Development Center
Johnsville
Warminster, Pennsylvania 18974

1 Chief
Naval Air Technical Training
Naval Air Station
Memphis, Tennessee 38115

1 Director
Education and Training Sciences Dept.
Naval Medical Research Institute
National Naval Medical Center
Building 142
Bethesda, Maryland 20014

1 Commander
Submarine Development Group TWO
Fleet Post Office
New York, New York 09501

1 Commander
Operational Test & Evaluation Force
U. S. Naval Base
Norfolk, Virginia 23511

1 Mr. S. Friedman
Special Assistant for Research & Studies
OASN (M&RA)
The Pentagon, Room 4E794
Washington, D. C. 20350

1 Chief of Naval Operations, (Op-07TL)
Department of the Navy
Washington, D. C. 20350

1 Chief of Naval Material
NMAT-03424, Dept. of the Navy
Washington, D. C. 20360

1 Mr. George N. Graine
Naval Ship Systems Command (SHIP 03H)
Department of the Navy
Washington, D. C. 20360

1 Chief
Bureau of Medicine and Surgery
Code 513
Washington, D. C. 20390

1 Chief
Bureau of Medicine and Surgery
Research Division (Code 713)
Department of the Navy
Washington, D. C. 20390

6 Technical Library (Pers-11b)
Bureau of Naval Personnel
Department of the Navy
Washington, D. C. 20370

3 Personnel Research and Development
Laboratory
Washington Navy Yard, Building 200
Washington, D. C. 20390
ATTN: Library, Room 3307

1 Commandant of the Marine Corps
Headquarters, U. S. Marine Corps
Code A01B
Washington, D. C. 20380

1 Technical Library
Naval Ship Systems Command
Main Navy Building, Room 1532
Washington, D. C. 20360

1 Library, Code 0212
Naval Postgraduate School
Monterey, California 93940

1 Technical Reference Library
Naval Medical Research Institute
National Naval Medical Center
Bethesda, Maryland 20014

1 Scientific Advisory Team (Code 71)
Staff, COMASWFORLANT
Norfolk, Virginia 23511

3 Technical Director
Personnel Research Division
Bureau of Naval Personnel
Washington, D. C. 20370

1 Technical Library
Naval Training Device Center
Orlando, Florida 32813

1 Dr. Earl I. Jones
Director
Naval Training Research Institute
Naval Personnel & Training
Research Laboratory
San Diego, California

1 Head, Personnel Measurement Staff
Capital Area Personnel Service Office-
Navy
Ballston Tower #2, Room 1204
801 N. Randolph Street
Arlington, Virginia 22203

1 Director of Research
U. S. Army Armor Human Research Unit
Fort Knox, Kentucky 40121
ATTN: ATSAG-EA

1 Director
Behavioral Sciences Laboratory
U. S. Army Research Institute of
Environmental Medicine
Natick, Massachusetts 01760

1 U. S. Army Behavior and Systems
Research Laboratory
Commonwealth Building, Room 239
1320 Wilson Boulevard
Arlington, Virginia 22209

1 Behavioral Sciences Division
Office of Chief of Research and
Development
Department of the Army
Washington, D. C. 20310

1 Commandant
U. S. Army Adjutant General School
Fort Benjamin Harrison, Indiana 46216
ATTN: ATSAG-EA

1 Dr. George S. Harker, Director
Experimental Psychology Division
U. S. Army Medical Research Laboratory
Fort Knox, Kentucky 40121

1 LTC William C. Cosgrove
USA CDC Personnel & Administrative
Services Agency
Ft. Benjamin Harrison, Indiana 46216

1 Chief, Personnel & Training Reg. Br.
Advanced Systems Division
Air Force Human Resources Lab.
Wright-Patterson AFB
Ohio 45433
(Attn: Dr. Melvin T. Snyder)

1 AFHRL (TR/Dr. G. A. Eckstrand)
Wright-Patterson Air Force Base
Ohio 45433

1 Personnel Research Division (AFHRL)
Lackland Air Force Base
San Antonio, Texas 78236

1 AFOSR (SRLB)
1400 Wilson Boulevard
Arlington, Virginia 22209

1 Headquarters, U. S. Air Force
AFPTRBD
Programs Resources and Technology Div.
Washington, D. C. 20330

1 AFHRL (HRTT/Dr. Ross L. Morgan)
Wright-Patterson Air Force Base
Ohio 45433

1 LTCOL F. R. Ratliff
Office of the Assistant Secretary
of Defense (M&RU)
The Pentagon, Room 3D960
Washington, D. C. 20301

1 Dr. Ralph R. Canter
Military Manpower Research Coordinator
OASD (M&RA) MR&U
The Pentagon, Room 3D960
Washington, D. C. 20301

1 Dr. Andrew R. Molnar
Computer Innovation in Education
Section
Office of Computing Activities
National Science Foundation
Washington, D. C. 20550

1 Dr. Alvin E. Goins, Executive Secretary
Personality and Cognition Research
Review Committee
Behavioral Sciences Research Branch
National Institute of Mental Health
5454 Wisconsin Avenue, Room 10A02
Chevy Chase, Maryland 20015

1 Office of Computer Information
Center for Computer Sciences and
Technology
National Bureau of Standards
Washington, D. C. 20234

1 Director, National Center for
Educational Research & Development
U. S. Office of Education
Dept. of Health, Education & Welfare
Washington, D. C. 20202

1 Mr. Joseph J. Cowan, Chief
Psychological Research Branch (P-1)
U. S. Coast Guard Headquarters
400 Seventh Street, S.W.
Washington, D. C. 20226

1 ERIC Clearinghouse on Vocational
and Technical Education
The Ohio State University
1900 Kenny Road
Columbus, Ohio 43210
ATTN: Acquisition Specialist

1 ERIC Clearinghouse on Educational
Media and Technology
Stanford University
Stanford, California 94305

1 Dr. Don H. Coombs, Co-Director
ERIC Clearinghouse
Stanford University
Palo Alto, California 94305

1 Dr. Richard C. Atkinson
Department of Psychology
Stanford University
Stanford, California 94305

1 Dr. Richard S. Hatch
Decision Systems Associates, Inc.
11428 Rockville Pike
Rockville, Maryland 20852

1 Director
Human Resources Research Organization
300 North Washington Street
Alexandria, Virginia 22314

1 Human Resources Research Organization
Division #1, Systems Operations
300 North Washington Street
Alexandria, Virginia 22314

1 Human Resources Research Organization
Division #3, Recruit Training
Post Office Box 5787
Presidio of Monterey, California 93940

1 Human Resources Research Organization
Division #5, Air Defense
Post Office Box 6021
Fort Bliss, Texas 79916

1 Human Resources Research Organization
Division #4, Infantry
Post Office Box 2086
Fort Benning, Georgia 31905

1 Human Resources Research Organization
Division #6, Aviation
Post Office Box 428
Fort Rucker, Alabama 36360

1 Dr. Robert J. Seidel
Human Resources Research Organization
300 N. Washington Street
Alexandria, Virginia 22314

1 Dr. Marvin D. Dunnette
University of Minnesota
Department of Psychology
Elliot Hall
Minneapolis, Minnesota 55455

1 Dr. John C. Flanagan
American Institutes for Research
Post Office Box 1113
Palo Alto, California 94302

1 Dr. Robert Glaser
Learning Research and Development Center
University of Pittsburgh
Pittsburgh, Pennsylvania 15213

1 Dr. Albert S. Glickman
American Institutes for Research
8555 Sixteenth Street
Silver Spring, Maryland 20910

1 Dr. Bert Green
Department of Psychology
Johns Hopkins University
Baltimore, Maryland 21218

1 Dr. Duncan N. Hansen
Center for Computer Assisted Instruction
Florida State University
Tallahassee, Florida 32306

1 Dr. M. D. Havron
Human Sciences Research, Inc.
Westgate Industrial Park
7710 Old Springhouse Road
McLean, Virginia 22101

1 Mr. Harry H. Harman
Division of Analytical Studies and Services
Educational Testing Service
Princeton, New Jersey 08540

1 Dr. Lee J. Cronbach
School of Education
Stanford University
Stanford, California 94305

1 Psychological Abstracts
American Psychological Association
1200 Seventeenth Street, N.W.
Washington, D. C. 20036

1 Dr. Bernard M. Bass
University of Rochester
Management Research Center
Rochester, New York 14627

1 Dr. Lee R. Beach
Department of Psychology
University of Washington
Seattle, Washington 98105

1 Dr. Roger A. Kaufman
Graduate School of Leadership and Human Behavior
U. S. International University
8655 E. Pomerada Road
San Diego, California 92124

1 Dr. George E. Rowland
Rowland and Company, Inc.
Post Office Box 61
Haddonfield, New Jersey 08033

1 Mr. Roy Ference
Room 2311
U. S. Civil Service Commission
Washington, D. C. 20415

1 Dr. Robert R. Mackie
Human Factors Research, Inc.
Santa Barbara Research Park
6780 Cortona Drive
Goleta, California 93017

1 Dr. Stanley M. Nealey
Department of Psychology
Colorado State University
Fort Collins, Colorado 80521

1 Dr. Gabriel D. Ofiesh
Center for Educational Technology
Catholic University
4001 Harewood Road, N.E.
Washington, D. C. 20017

1 Mr. Luigi Petrullo
2431 North Edgewood Street
Arlington, Virginia 22207

1 Dr. Len Rosenbaum
Psychology Department
Montgomery College
Rockville, Maryland 20852

1 Dr. Arthur I. Siegel
Applied Psychological Services
Science Center
404 East Lancaster Avenue
Wayne, Pennsylvania 19087

1 Dr. Paul Slovic
Oregon Research Institute
Post Office Box 3196
Eugene, Oregon 97403

1 Dr. Diane M. Ramsey-Klee
R-K Research & System Design
3947 Ridgemont Drive
Malibu, California 90265

1 Dr. Ledyard R Tucker
Psychology Building
University of Illinois
Urbana, Illinois 61820

1 Dr. John Annett
Department of Psychology
Hull University
Hull
Yorkshire, ENGLAND

1 Dr. Joseph W. Rigney
Behavioral Technology Laboratories
University of Southern California
University Park
Los Angeles, California 90007