ALTERNATIVE CONCEPTIONS ON THE TEACHING
OF THE CONCEPT OF MASS

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From an educational perspective, one of the claims of constructivism is this: learning is an activity of personal exploration that implies an active reorganisation of a framework of meanings about the world. The pre-conceptions in the mind of a student about some segment of science have an important role in the way of learning that segment. This is the main reason why there have been many investigations trying to detect the students' ideas in a great range of subjects. On the other hand, teachers and schoolbooks' conceptions have been poorly investigated, although the social influence of a teacher on his students is well known. Some of the students misconceptions are induced by their teachers.
Consequently, research based on the teachers' ideas is very important. There are teachers' alternative conceptions in fundamental subjects such as mass, weight, energy, heat, and so on. Some of them influence decisively the students' learning.

The author of this communication has made an exhaustive research into conceptions of mass both in the history of Physics and in Physics teaching. He found some historical and epistemological reasons for the multiple answers to questions like these:

- What is the scientific meaning of mass?
- Does the mass of a particle depend on its velocity?
- Is the mass of a particle equivalent to its total energy?
- In an isolated system, could there be conversion of mass into energy or energy into mass?
- Is the total mass of a system the sum of their particles' mass?
- Is the energy of a particle zero when its mass is zero?
- Etc.

The objective of this communication is to give the results of the research developed on the teachers' ideas about mass, and to discuss the historical and epistemological reasons why some of them are surely misconceptions, using a methodology based on heuristic tools.

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Abstract

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1. Two different conceptions about the relativistic momentum

A well known current university Physics book[1] writes:

> "As a result, if the conservation of linear momentum is to be valid in any inertial frame (as demanded by the principle of relativity), a new definition of linear momentum is required. The relativistic definition of linear momentum is
>
> $$\vec{p} = m\vec{v}$$
>
> where the relativistic mass, m, is
>
> $$m = \gamma m_0 = \frac{m_0}{\sqrt{1 - v^2 / c^2}}$$
>
> The quantity m_0 is the rest mass of the particle - the mass measured in its rest frame"

Another book[2], about the definition of momentum, writes:

> "Consider a particle of mass m with velocity $\vec{v}$ in some reference frame. The modified definition of the momentum $\vec{p}$ of the particle is
>
> $$p = \frac{mv}{\sqrt{1 - v^2 / c^2}} = \gamma m\vec{v}$$
>
> With this definition, momentum is conserved in every inertial frame."

These diferente conceptions are disseminated in many other books. After all, what is the relativistic momentum, $m\vec{v}$ or $\gamma m\vec{v}$?

2. What is the mass of a particle?

One concept is a «construct». It depends on a structure of other concepts. The concept of mass in the early Newtonian Physics is different of the concept of mass in the Physics of the last century. Mach's concept of mass is certainly different of mine.

I'm going to speak of «my concepts». Certainly they are not mine, because they have resulted from a «negotiation» with the concepts of others.

When I speak of mass of a particle, I'm thinking of a property of the particle. If it is a property, it depends on the particle itself, on its internal state (if it´s important to consider the structure of the particle). It must be an invariant if the particle is isolated, when the internal state of the particle doesn't change. The mass of a particle is the magnitude of its momenergy 4-vector. It is a magnitude characteristic of the particle and totally independent of its state of motion. The only concept of mass that I know with these characteristics corresponds to the following definition:

$$momenergy = mass \times \frac{spacetime\ displacement}{proper\ time\ for\ that\ displacement}$$

Adopting the same units for time and space (whether meter or second or year) and a given inertial frame, the momenergy components of a particle are:

$$E = m\frac{dt}{d\tau} \qquad p_x = m\frac{dx}{d\tau} \qquad p_y = m\frac{dy}{d\tau} \qquad p_z = m\frac{dz}{d\tau}$$

The square of the magnitude of momenergy is, then,

$$(magnitude\ of\ momenergy) = E^2 - (p_x)^2 - (p_y)^2 - (p_z)^2 =$$

$$= m^2 \frac{(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2}{(d\tau)^2} = m^2 \frac{(d\tau)^2}{(d\tau)^2} = m^2$$

For two inertial frames, S and S' we have the invariant:

$$m^2 = E^2 - p^2 = (E')^2 - (p')^2$$

In conventional units (for example in SI units), this invariant is expressed as follows:

$$m^2c^4 = E^2 - p^2c^2 = (E')^2 - (p')^2 c^2$$

In conventional units the mass of a particle can be defined by the equation

$$m = c^{-2}\left(E^2 - p^2c^2\right)^{1/2} \Leftrightarrow m = \sqrt{\left(\frac{E}{c^2}\right)^2 - \left(\frac{p}{c}\right)^2}$$

We know that the proper time between two events is always less than the laboratory time:

$$d\tau = \frac{dt}{\gamma} = dt\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

As the energy of the particle is, in conventional units,

$$E = mc^2 \frac{dt}{d\tau} = \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

we see that when v = 0, then $E_0 = mc^2$.

Rest energy and mass of a particle are equivalent concepts. The light speed, *c*, is a conversion factor between joules and kilograms. Now we recognize that these units are different only because of historical accident[3].

3. What is the mass of a system of particles?

Let's consider an isolated system of particles. We continue interested in a magnitude that could be considered a property of the system, a characteristic unaffected by collisions among the parts of the system, unaffected by any transformation, decay or annihilation the system may undergo, which has a value independent from the choice of reference frame. The only magnitude I know with these characteristics is the magnitude of the total momenergy. So, I consider as mass of an isolated system the magnitude of its total momenergy. This 4-vector is unaffected by collisions among the parts of the system. It is also unaffected by any transformation, decay or annihilation the system may undergo.

Adopting the same units for time and space, the magnitude of the 4-vector momenergy of the system is

$$m_s = \sqrt{(E_s)^2 - (p_{xs})^2 - (p_{ys})^2 - (p_{zs})^2} = \sqrt{(E_s)^2 - (p_s)^2}$$

In conventional units we have

$$m_s = \sqrt{\left(\frac{E_s}{c^2}\right)^2 - \left(\frac{p_s}{c}\right)^2}$$

An important frame is the zero-total momentum frame (where the momentum of the system is zero). Then, it is, *in a zero-total momentum frame and with the same units for mass and energy*:

$$m_s = E_s = \sum_{i=1}^{n} E_i = \sum_{i=1}^{n} m_i + \sum_{i=1}^{n} K_i$$

As we see, the total mass of a system is not the sum of their particles' mass. The kinetic energy of the particles also contributes to the mass of the system.

When the particles interact, as well as move, the energies of interaction have to be taken into account. They therefore contribute to the total energy of the system, Es , which gives the mass, *in conventional units*:

$$m_s = \sqrt{\left(\frac{E_s}{c^2}\right)^2 - \left(\frac{p_s}{c}\right)^2}$$

4. But...is there such thing as mass?

Recent work by Bernhard Haisch, Alfonso Rueda, H. Puthoff and others appears to offer a radically different insight into the idea of mass. Their work suggests that inertia is a property arising out of a "vast, all-pervasive electromagnetic field" - the zero point field, which exists in the vacuum even at the temperature of absolute zero. The interaction between this background «sea» of electromagnetic radiation, uniform and isotropic, and the massless electric charges immersed in it creates the appearance of mass. Mass does not exist. Only massless electric charges and energy do. A stone is heavy because its great number of electric charges is embedded in the zero point field and is being acted by it[4].

5. Misconceptions about the concept of mass

5.1 The mass of a body is "the amount of matter of the body".

It's an old misconception. It comes from the first definition of Newton in the Principia. Today we know that the amount of matter (better, the amount of substance) may be expressed in a unit named mole (mol). If we have a given amount of a substance (0,5 mol of NaCl, for example) its mass changes with its temperature and with the physical state (atoms tightly bonded in a solid are less massive than the same atoms free).

5.2 The mass of a particle depends on the velocity of the particle

As we see, the mass of a particle can be defined by the equation

$$m = c^{-2}\left(E^2 - p^2c^2\right)^{1/2} \Leftrightarrow m = \sqrt{\left(\frac{E}{c^2}\right)^2 - \left(\frac{p}{c}\right)^2} \quad \text{(in conventional units)}$$

The energy of the particle depends on the inertial frame (then on the velocity). The momentum also depends on the inertial frame. But the mass of the particle is an invariant, doesn't depend on the inertial frame, that is, <u>doesn't depend on its velocity</u>.

Let's translate this into space-time language. A particle is at rest in an inertial frame. Its 4-vector of energy and momentum points to the pure timelike direction, it has energy, no momentum. Mass equals the energy. In an accelerator, the particle is put in motion at high speed. The space component of the 4-vector, originally zero, grows to a great value (it corresponds to momentum). The direction of the momenergy 4-vector changes (it tilts from the vertical, the purely timelike direction). Its time component, that is, the energy of the particle, also changes. Nevertheless, the magnitude of the momenergy 4-vector, that is, the mass of particle doesn't change.

5.3 A particle has a relativistic mass: it measures the inertia of the particle when it moves at high velocity.

The acceleration of a particle is

$$\vec{a} = \frac{\vec{F} - \left(\vec{F} \cdot \vec{\beta}\right) \vec{\beta}}{\gamma\ m}$$

Where $\vec{F}$ is the force acting on the particle and $\vec{\beta} = \frac{\vec{v}}{c}$.

We see that in the general case the acceleration doesn't have the direction of the force and we cannot get the expression $m = \frac{\vec{F}}{\vec{a}}$ that gives the inertial mass. There are two exceptions:

1 - When $\vec{F}$ is perpendicular to $\vec{v}$, we can consider a "transverse mass" $m_t = \gamma\ m$

2 - When $\vec{F}$ is parallel to $\vec{v}$, we can consider a "longitudinal mass" $m_l = \gamma^3$ m

We have two, and not only one expression where m depends on v. We cannot speak about a property of the particle.

The so called "relativistic mass" is equal to the "transverse mass". This mass cannot measure the inertia or resistance to the increase of velocity, because the force perpendicular to $\vec{v}$ doesn't increase this velocity .

When we apply a force on a moving particle of a given rest mass, it appears to have more resistance to acceleration when its speed v increases because it takes more time to get a given increase □v. But it is an effect of time dilatation, not of mass increase. Measured by a clock instantaneously travelling with the particle, the time (proper time) is always the same for the same effect of the force. But, from our point of view, the time is dilated (laboratory time). This is a kinematic effect, not a dynamic effect. The rest energy of the particle is the same. Then, the mass is the same.[5]

5.4 The mass of a system is the sum of the masses of their particles

A body is an agglomerate of particles. The mass of the body as being the sum of the mass of its particles it's a spontaneous thinking. All the classical Physics is based on this thinking.

In a well known paper «Does the inertia of a body depend upon its energy-content?»[6], Einstein wrote these famous words:

"The mass of a body is a measure of its energy-content". A consequence of this sentence is: if we have a system, a microwave oven, for example, with some food, we must consider, to the mass of the oven, not only the masses of the particles (of the food and of the oven), but also all the energy it contents, including the electromagnetic radiation used to cook or heat the food. Then, the mass of the oven is more than the sum of the particles mass.

A consequence of our definitions presented before is: energy is additive; momentum is additive; but mass is not additive. An example:

Let's consider two photons with the same energy, hv, moving in opposite directions: one of them has the momentum $\vec{p}$ and the other has the momentum - $\vec{p}$.

For *each photon* we have:

E (energy) = hv

p (magnitude of photon momentum) = hv /c

$$m = \sqrt{\left(\frac{E}{c^2}\right)^2 - \left(\frac{p}{c}\right)^2} = \sqrt{\left(\frac{h\upsilon}{c^2}\right)^2 - \left(\frac{h\upsilon / c}{c}\right)^2} = 0$$

As we see, photon is a massless particle, but it has energy.

For the *system of two photons* we have:

E (energy) = $hv + hv = 2\ hv$

p (magnitude of photon momentum) = $hv/c - hv/c = 0$

$$m_s = \sqrt{\left(\frac{E_s}{c^2}\right)^2 - \left(\frac{p_s}{c}\right)^2} = \sqrt{\left(\frac{2h\upsilon}{c^2}\right)^2} = \frac{2h\upsilon}{c^2} \square\ 0+0$$

Interpretation: a photon has no rest energy, that is, no mass. However, a photon can contribute with energy and momentum to a system of particles, then contribute to the mass of the system. A system of zero-mass photons itself can have nonzero mass[6].

5.5. The equivalence mass-energy means that the mass of a particle and its energy are essentially the same property.

The statement that mass and energy are equivalent doesn't mean that energy and mass are the same. The energy of a particle (or an isolated system) is only the time component of a momenergy 4-vector. Then it depends on the inertial frame from which the particle (or the isolated system) is regarded. In contrast, the mass measures entire magnitude of that 4-vector. Then is an invariant: it is independent of the inertial frame.

The equivalence of mass and energy for a particle refers to the rest energy of the particle, not to its total energy. For a system refers to the energy of the system in the zero-total-momentum frame. The correct relation between mass and energy for a particle is

$$E_0 = mc^2$$

and not

$$E = mc^2 \text{ or } E_0 = m_0c^2 \text{ or } E = m_0c^2$$ [7]

5.6. In an isolated system where the sum of the proper masses of its particles decrease (or increase) there is conversion of mass into energy (or energy into mass)

It is not true! The mass of an isolated system remains unchanged by interactions between the constituents of the system. As a matter of fact, the system mass is the magnitude of the total momenergy. This is unaffected by collisions among the parts of the system or any transformations these parts may undergo. As the mass of an isolated system is constant, it is impossible to be converted in energy. On the other hand, energy of an isolated system is constant. If it is constant, it is impossible to be converted in mass.

5.7 When a system transfers energy to the exterior, its mass is converted into energy

In the paper of Einstein we have refered before, we can read: "if the energy changes by L, the mass changes in the same sense by $L/9 \times 10^{20}$."When a system transfers energy to the exterior, its energy decrease, then its mass also decrease. Any changes in mass and energy occur in parallel. There is no conversion of one into the other[8]. If the energy of a body changes by ΔE_0, then its mass (measures the inertia) changes in the same sense by $\Delta m = \Delta E_0/c^2$.

When, for example, there is an explosion, the total mass of the rest of the bomb, the expanding gases, the fragments of explosion and the radiation has the same value as before the explosion. What happens is merely a change in the makeup of mass or energy contents of the system: less rest energy in the individual constituents - sum of their individual masses has decreased; more kinetic energy, including kinetic energy of the photons and neutrinos produced. In a the zero-total momentum frame and with the same units for mass and energy, given the expression $m_s = E_s = \sum_{i=1}^{n} E_i = \sum_{i=1}^{n} m_i + \sum_{i=1}^{n} K_i$, the first term of the sum has decreased and the second term has increased but m_s is constant.

5.8 The gravitational mass is equal to $m = E / c^2$

An argument in favour of equation $m = E / c^2$ is this: it defines the gravitational mass, then we need it to explain the gravitational attraction. It is not true. The gravitational attraction between two relativistic bodies is determined by their energy-momentum tensors, not just by their energies. On the other hand, the existence of a gravitational mass implies the existence of a gravitational force with the sense of the origin of the gravitational field, what is not valid in general relativity. As a matter of fact, when a relativistic particle such as a photon or an electron travels with energy E and velocity $v = \beta c$ in the gravitational field created by a heavy body of mass M, the force acting on the particle is given by the equation

$$\vec{F}_g = \frac{-GM\left(E / c^2\right)\left[\vec{r}\left(1+\beta^2\right) - \vec{\beta}\left(\vec{\beta}\cdot\vec{r}\right)\right]}{r^3}$$

When $\beta \ll 1$, this equation implies

$$\vec{F}_g = \frac{-GM\left(E_0 / c^2\right)\vec{r}}{r^3}$$

As $m = E_0 / c^2$ we obtain the classical equation.

However, when $\beta \cong 1$, the force is not directed to the origin of gravitational field. It has a component along the velocity.

If, for example, we have a photon falling vertically towards Earth, we have $\beta = 1$ and $\vec{\beta}\left(\vec{B}\cdot\vec{r}\right) = \vec{r}$ and er can think in E / c^2 as m, but this is not anymore than a mathematical variable necessary to use a classical expression. When the photon travels horizontally we have to think in $2 \times E / c^2$ as m. There is not a property of the photon called gravitational mass, exclusively dependent on it.

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