A Quantum Phase Transition of the Distorted Kagome Lattice Antiferromagnet in Magnetic Field

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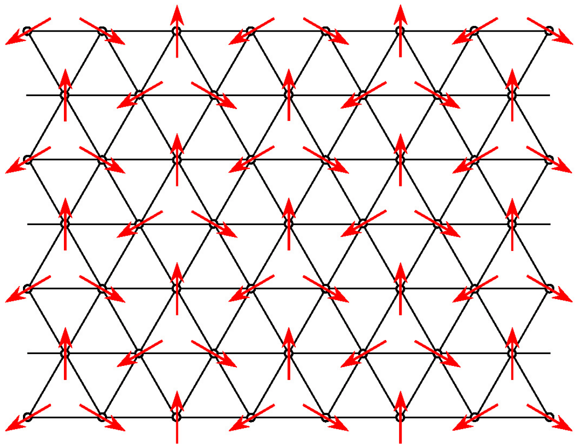
H. Nakano and TS: JPSJ 79 (2010) 053707 (arXiv:1004.2528)
TS and H. Nakano: PRB 83 (2011) 100405(R) (arXiv:1102.3486)
H. Nakano and TS: JPSJ 80 (2011) 053704 (arXiv: 1103.5829)
TS and H. Nakano: physica status solidi B 250 (2013) 579
H. Nakano and TS: JPSJ 82 (2013) 083709
H. Nakano and TS: JPSJ 83 (2014) 104710

2D frustrated systems

- Heisenberg antiferromagnets

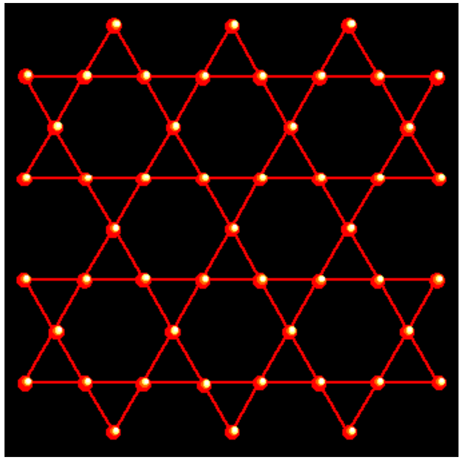
$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

Triangular lattice



Classical ground state
120 degree structure

Kagome lattice

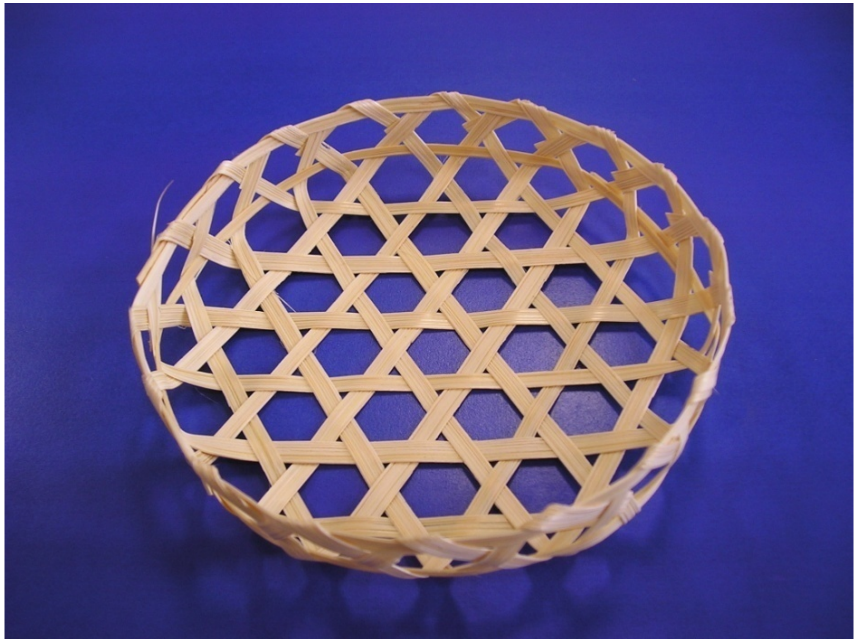


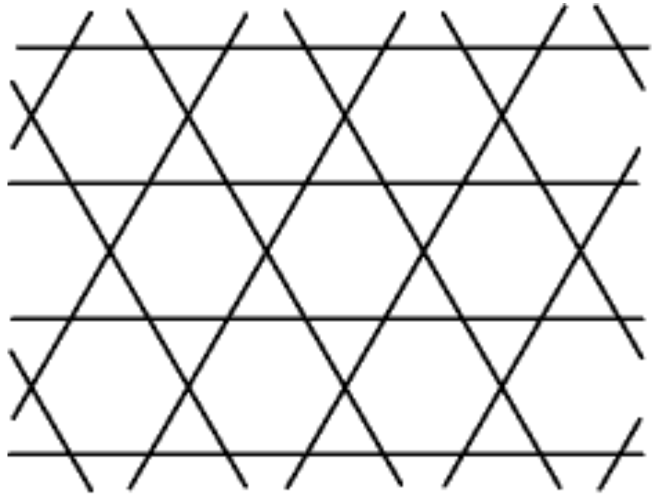
Macroscopic degeneracy
(a global plane is not fixed)

Kagome lattice

Itiro Syôzi: Statistics of Kagomé Lattice, PTP **6** (1951)306

kagome





Corner sharing triangles

S=1/2 Kagome Lattice AF

- Herbertsmithite $ZnCu_3(OH)_6Cl_2$ impurities

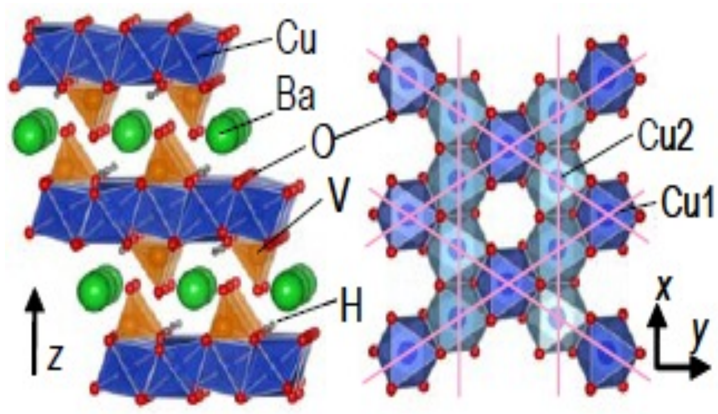
  Shores et al. J. Am. Chem. Soc. 127 (2005) 13426

- Volborthite $CuV_2O_7(OH)_2 \cdot 2H_2O$ lattice distortion

  Hiroi et al. J. Phys. Soc. Jpn. 70 (2001) 3377

- Vesignieite $BaCu_3V_2O_8(OH)_2$ ideal ?

  Okamoto et al. J. Phys. Soc. Jpn. 78 (2009) 033701



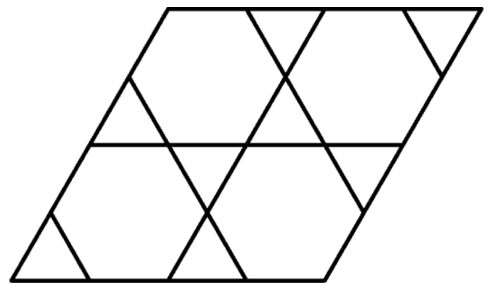
Methods

Frustration ⇨ Exotic phenomena

Kagome lattice

Triangular lattice

Pyrochlore lattice



Numerical approach

Numerical diagonalization

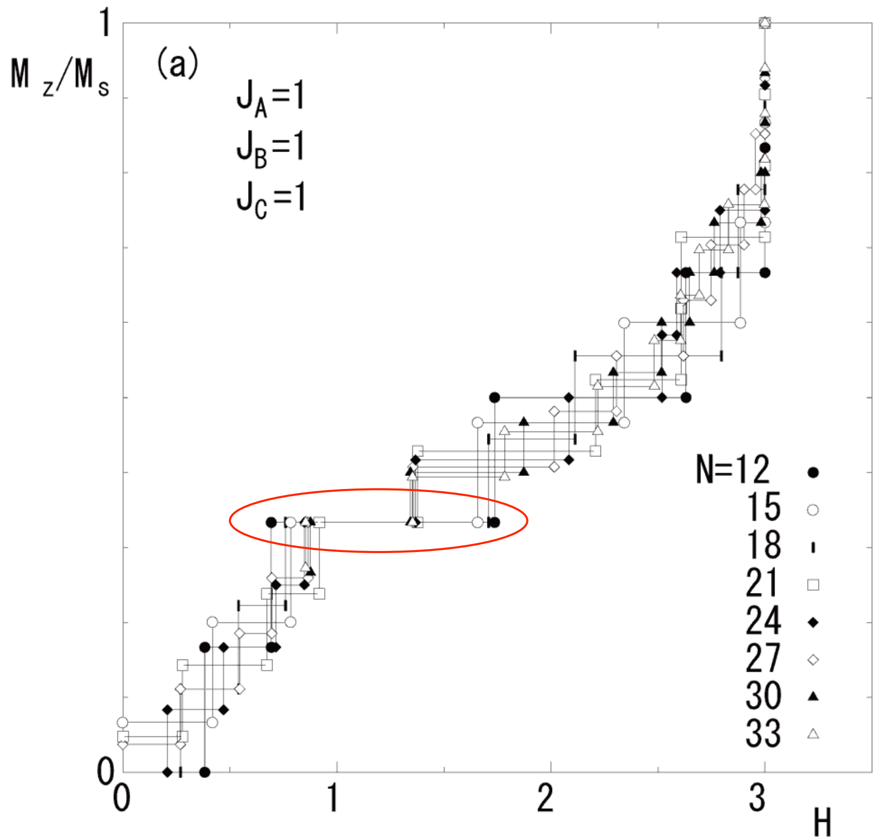
Quantum Monte Carlo (negative sign problem)

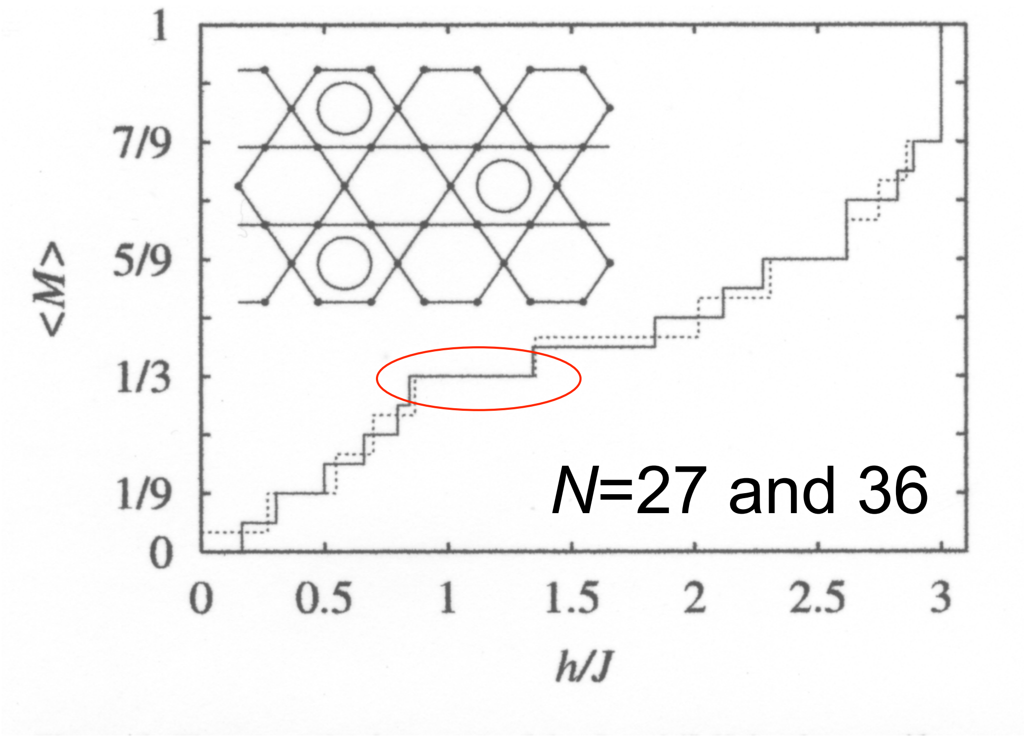
Density Matrix Renormalization Group
(not good for dimensions larger than one)

Magnetization process of S=1/2 kagome lattice AF

Hida: JPSJ **70** (2001) 3673

Honecker et al: JPCM **16**(2004)S749





1/3 plateau ?

Reexamination

from the viewpoint of

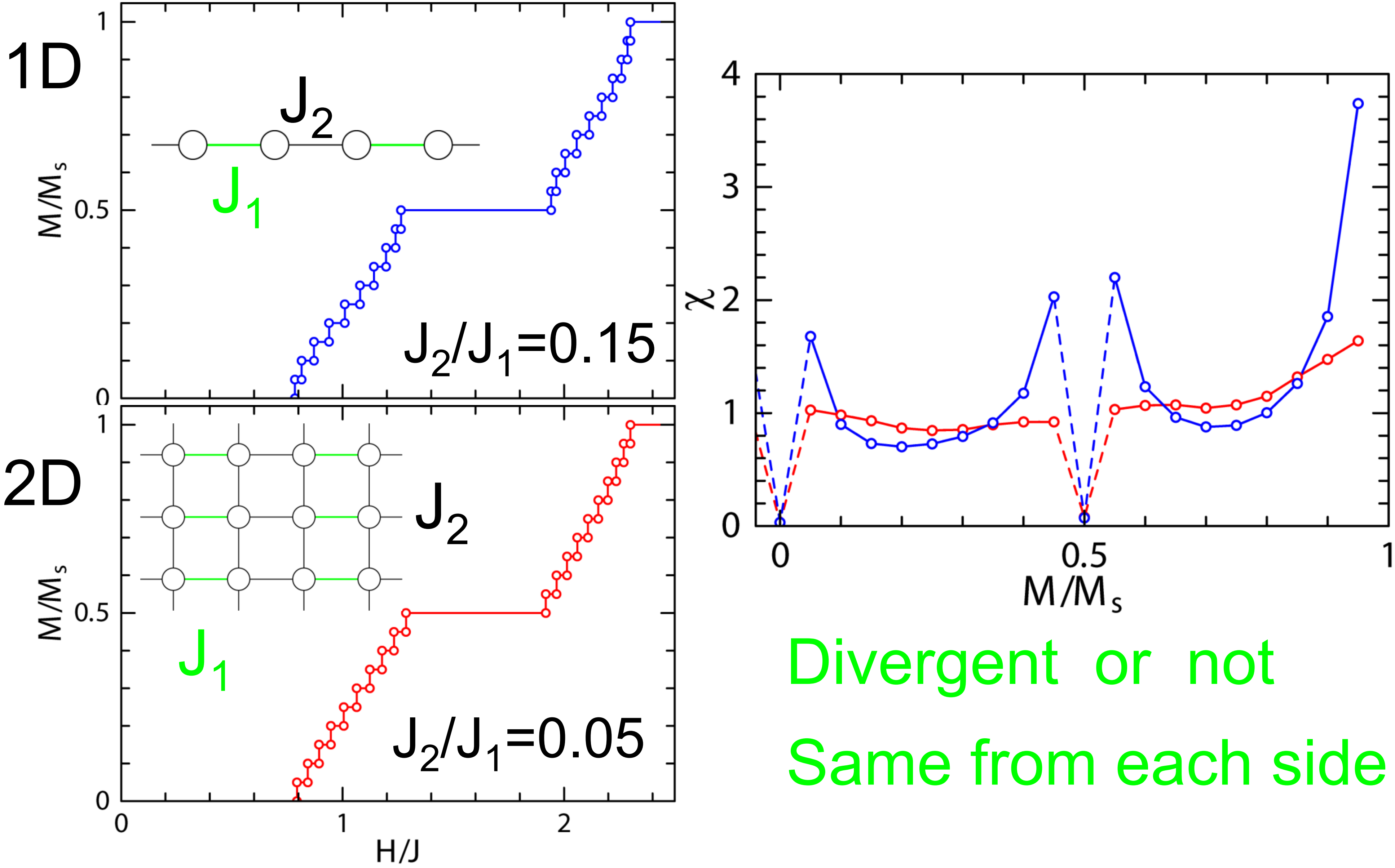
Field derivative of magnetization

$$\chi \propto \frac{\partial M}{\partial H}$$

as a function of $m = \frac{M}{M_{\mathrm{s}}}$

HN and T.Sakai: JPSJ **79** (2010) 053707 (Letter)
T.Sakai and HN: PRB **83** (2011) 100405(Rapid comm.)

Interacting S=1 Dimer Systems



Divergent or not

Same from each side

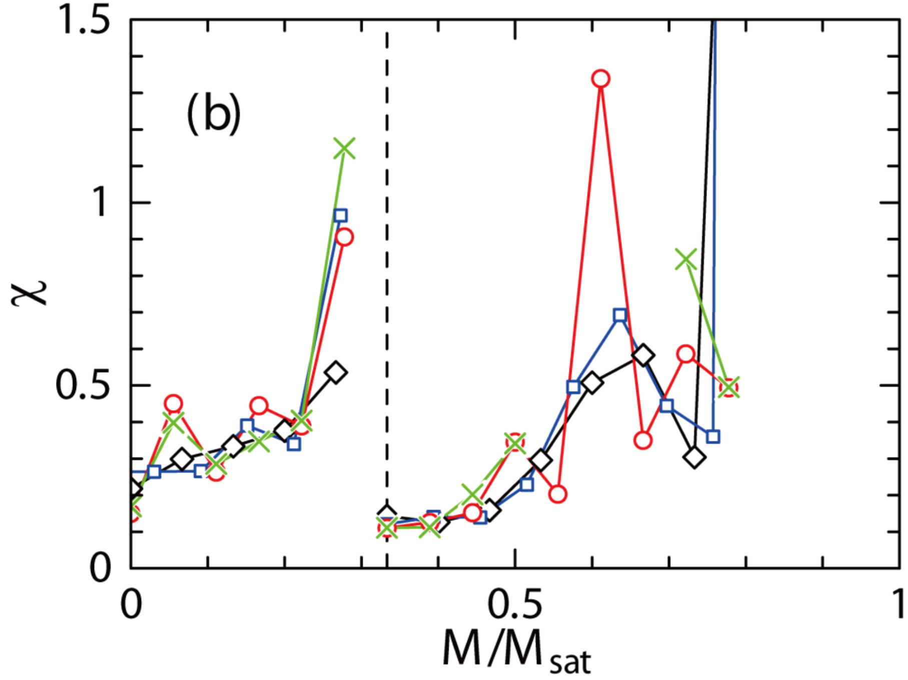
S=1/2 Kagome lattice AF

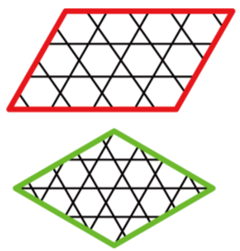
H. Nakano and TS: JPSJ **79** (2010) 053707

Reexamination from the viewpoint of

Field derivative of magnetization

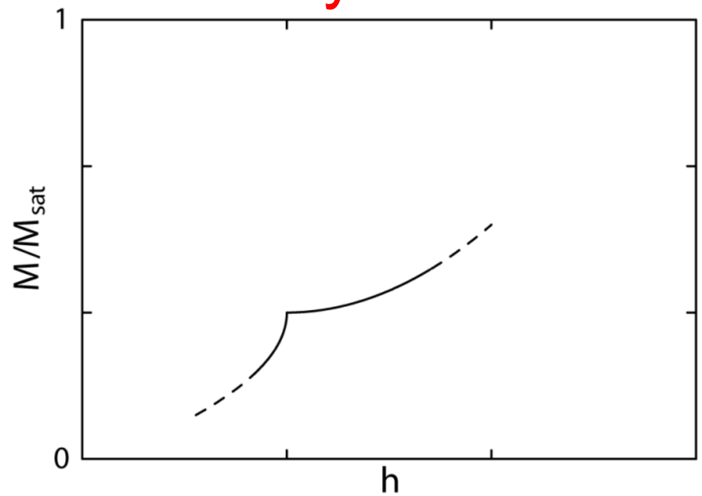
$$\chi \propto \frac{\partial M}{\partial H} \quad \text{as a function of} \quad m = \frac{M}{M_s}$$





N=36
N=36
N=33
N=30

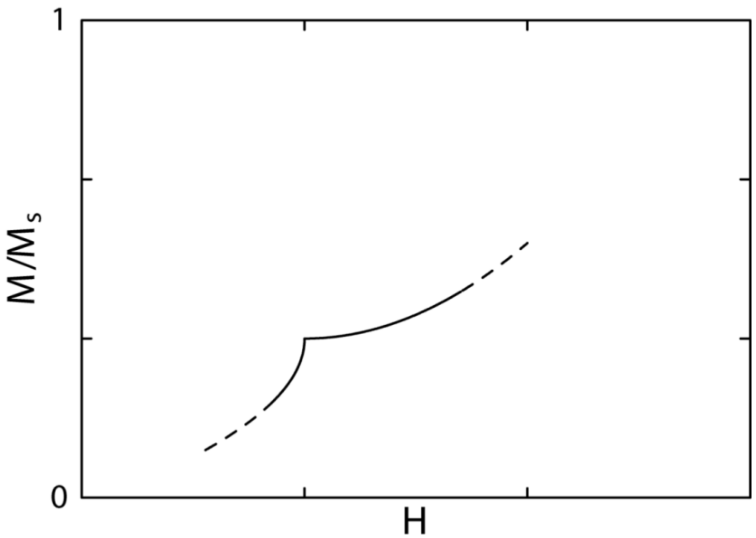
Anomaly at m=1/3



Magnetization ramp

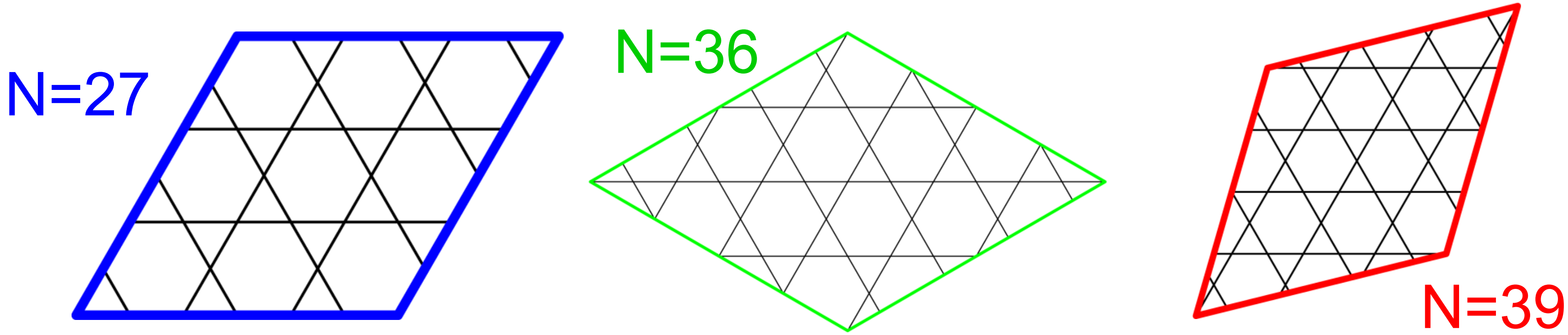
Ski jump

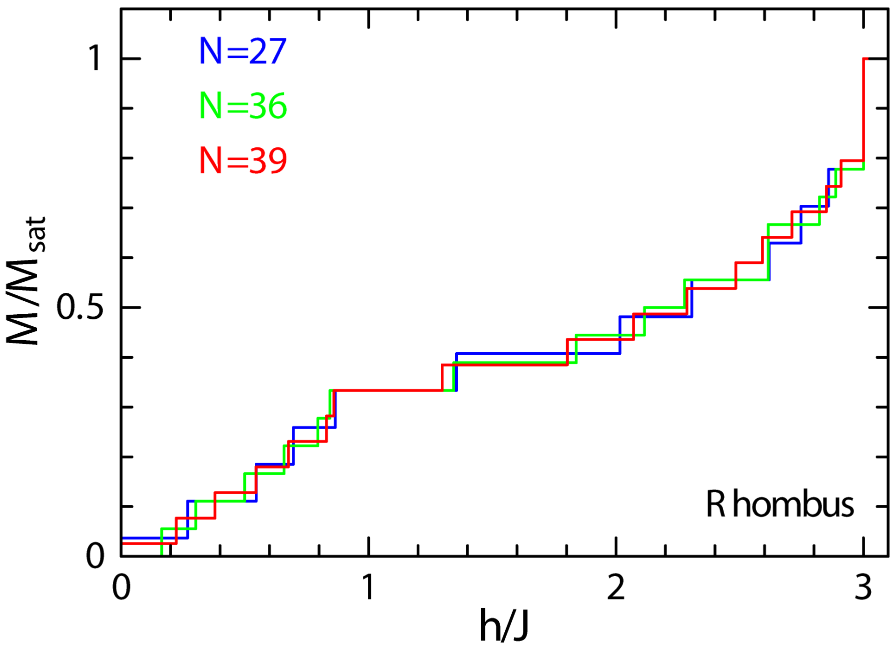
Jump ramp

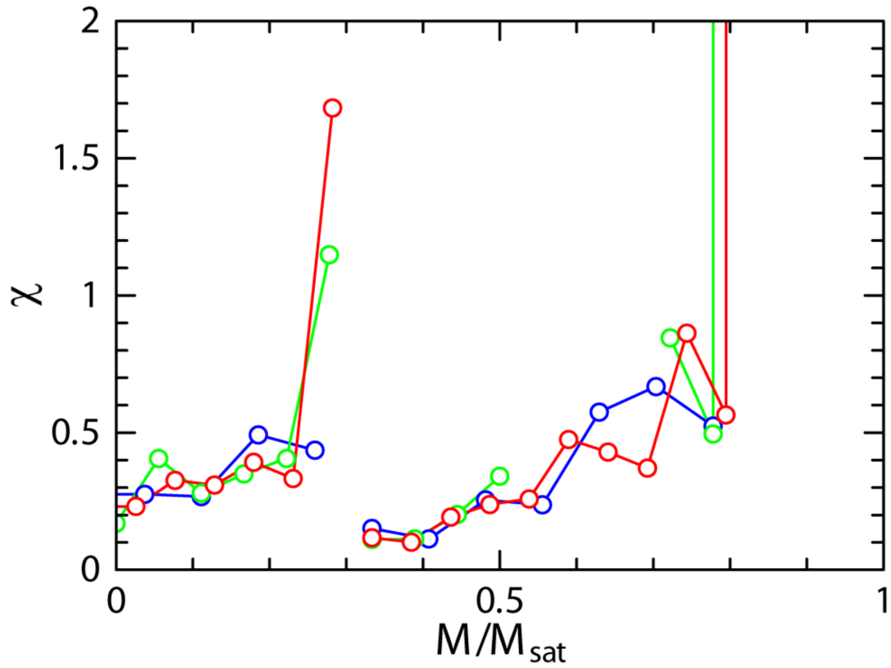


Magnetization curve
of Kagome lattice AF

Results for Rhombic Clusters



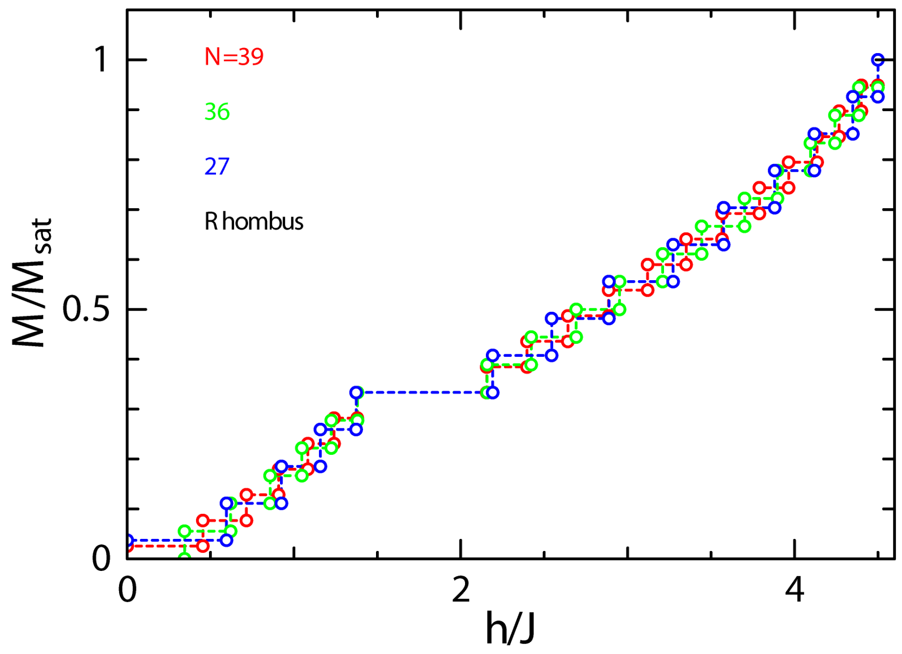


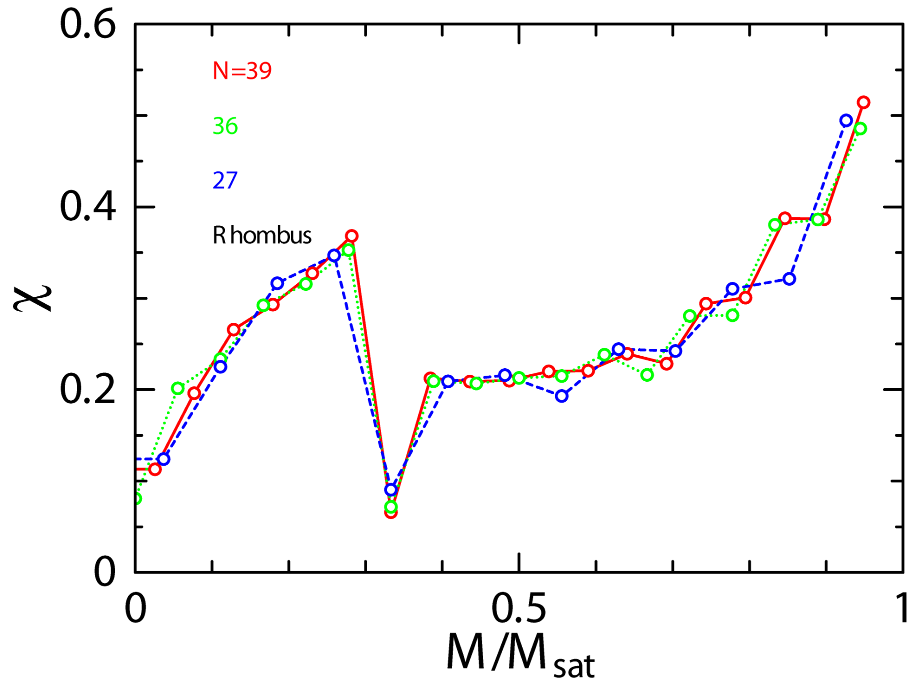


Characteristics of the ramp appear clearly for N=39.

Triangular lattice

N=39, 36, and 27 Rhombus

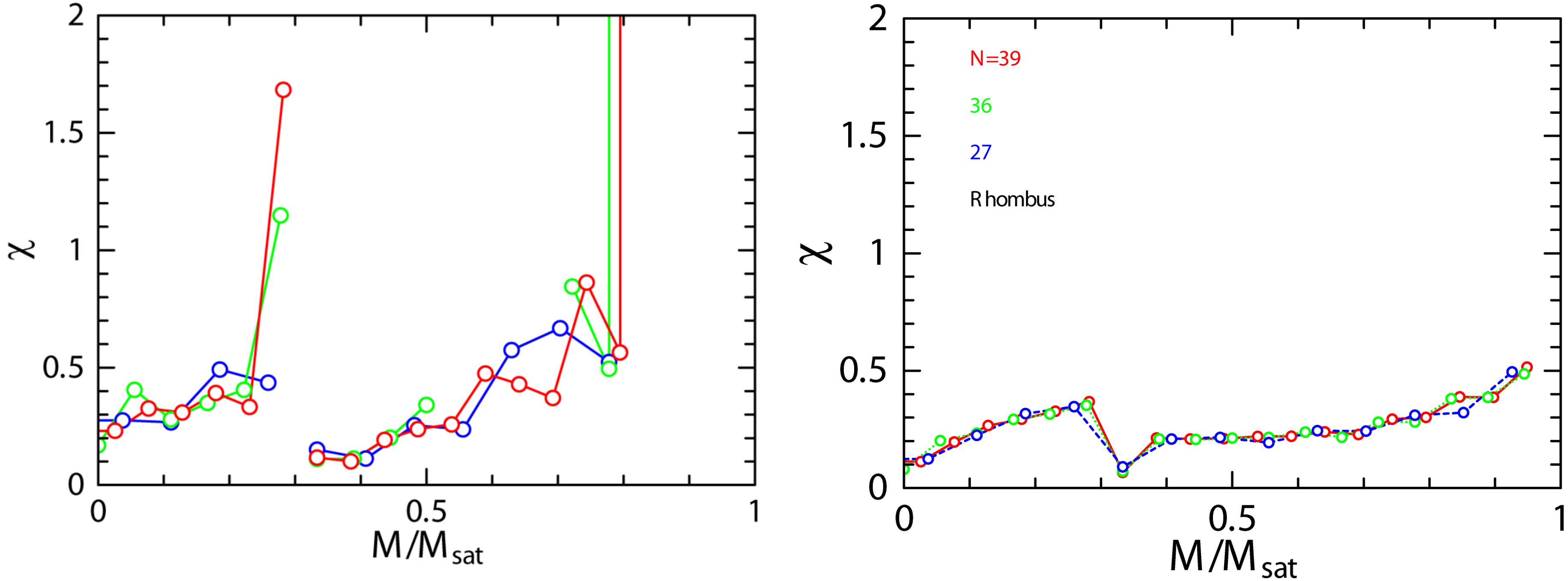




Typical magnetization plateau at M/M_{sat}=1/3

Comparison of χ

Kagome　　　　Triangular

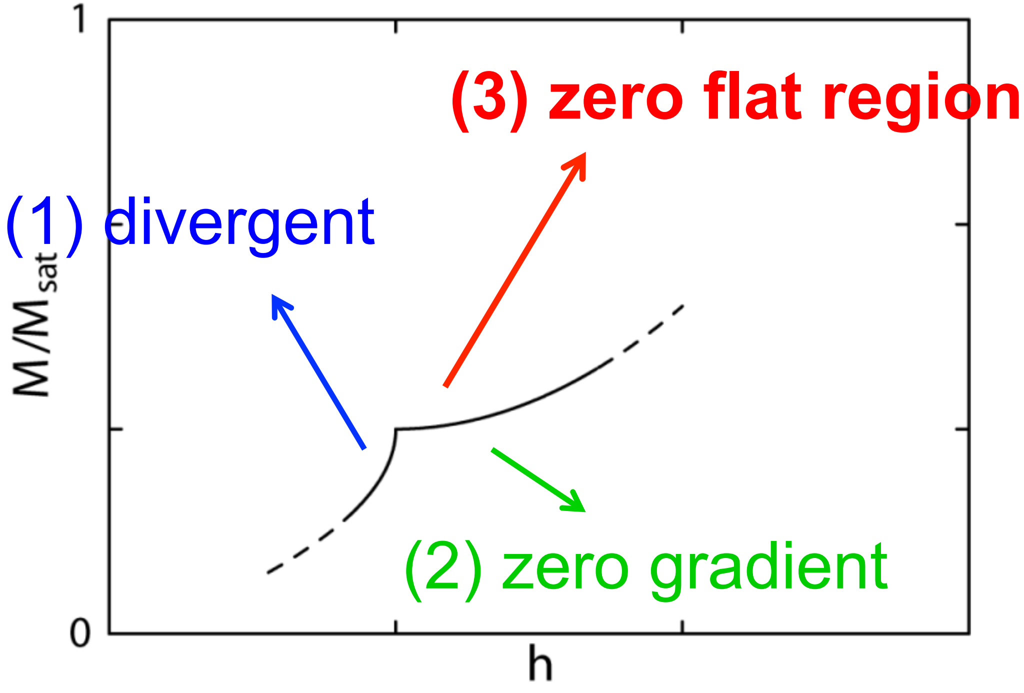


Clear difference at M/M_{sat}=1/3

Ramp　　　　Plateau

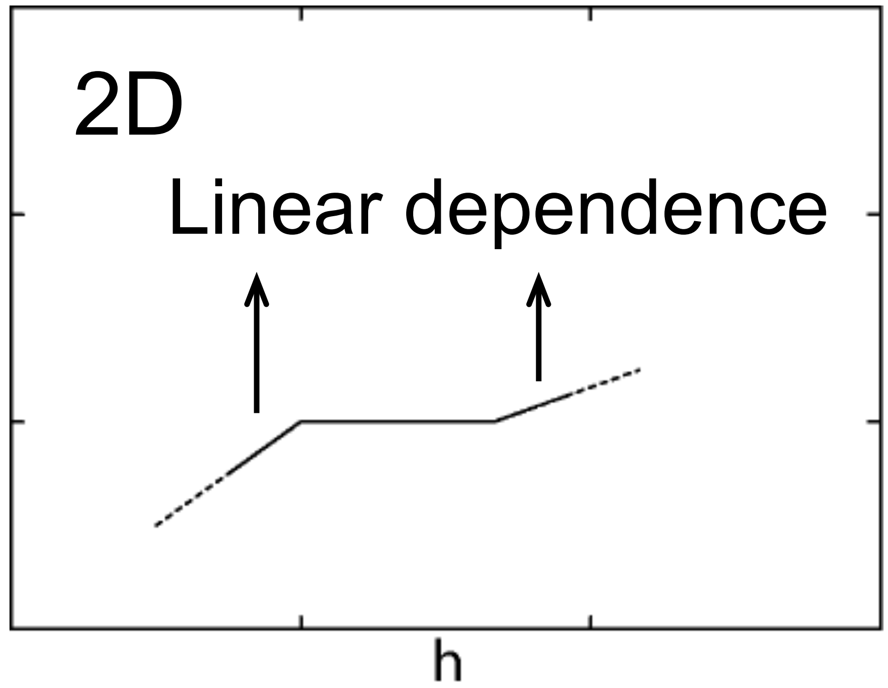
Features of Magnetization Ramp

Magnetization ramp



Kagome lattice

Magnetization plateau



Triangular lattice

Critical exponent

$|m\text{-}mc|=|H\text{-}Hc|^{1/\delta}$

$\delta=2$ 1D

Affleck 1990, Tsvelik 1990, TS-Takahashi 1991

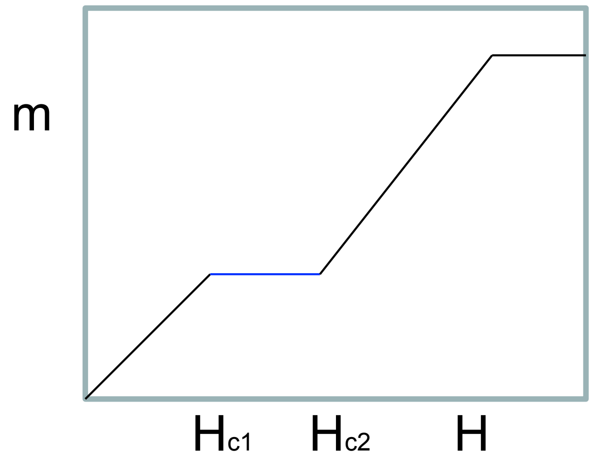
$\delta=1$ 2D

Katoh-Imada 1994

1/3 magnetization plateau

$$m-\frac{1}{3}\sim(H-H_{c2})^{1/\delta_+},$$
$$\frac{1}{3}-m\sim(H_{c1}-H)^{1/\delta_-}.$$

$H_{c1}=H_{c2}$?



Estimation of δ

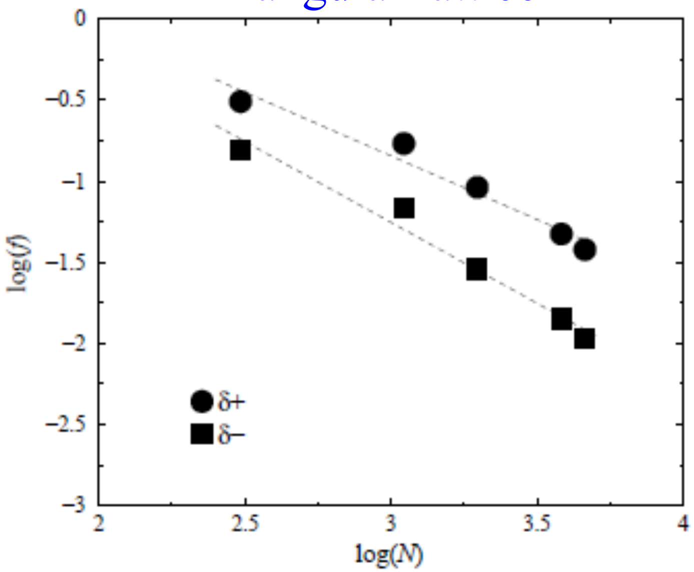
cf. TS and M. Takahashi: PRB 57 (1998) R8091

$$f_{\pm}(N) \equiv \pm[E(N, \frac{N}{3} \pm 2) + E(N, \frac{N}{3}) - 2E(N, \frac{N}{3} \pm 1)],$$

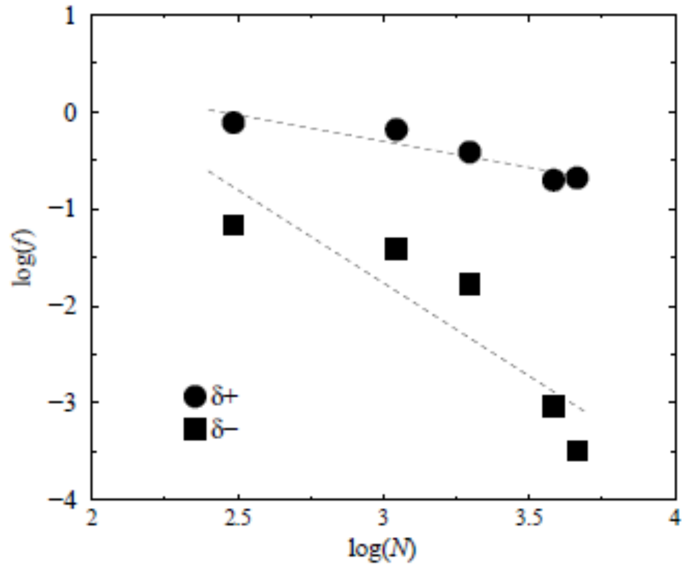
$$f_{\pm}(N) \sim \frac{1}{N^{\delta_{\pm}}}$$

Numerical diagonalization of rhombic clusters for N=12, 21, 27, 36, 39

Triangular lattice



Kagome lattice



$\delta_{-} = 1.0 \pm 0.2, \quad \delta_{+} = 0.8 \pm 0.2,$

$\delta_{-} = 1.9 \pm 1.0, \quad \delta_{+} = 0.5 \pm 0.2,$

$\delta_{-}=\delta_{+}=1$ Conventional (2D)

$\delta_{-}=2$ $\chi \to \infty$ (1D like)

$\delta_{+}=1/2$ $\chi=0$

$H_{c1}=H_{c2}$? (Plateau vs Ramp)

Triangular lattice

$$H_{c2}-H_{c1}=0.3\pm0.2$$

$H_{c1}\neq H_{c2}$

1/3 plateau

Kagome lattice

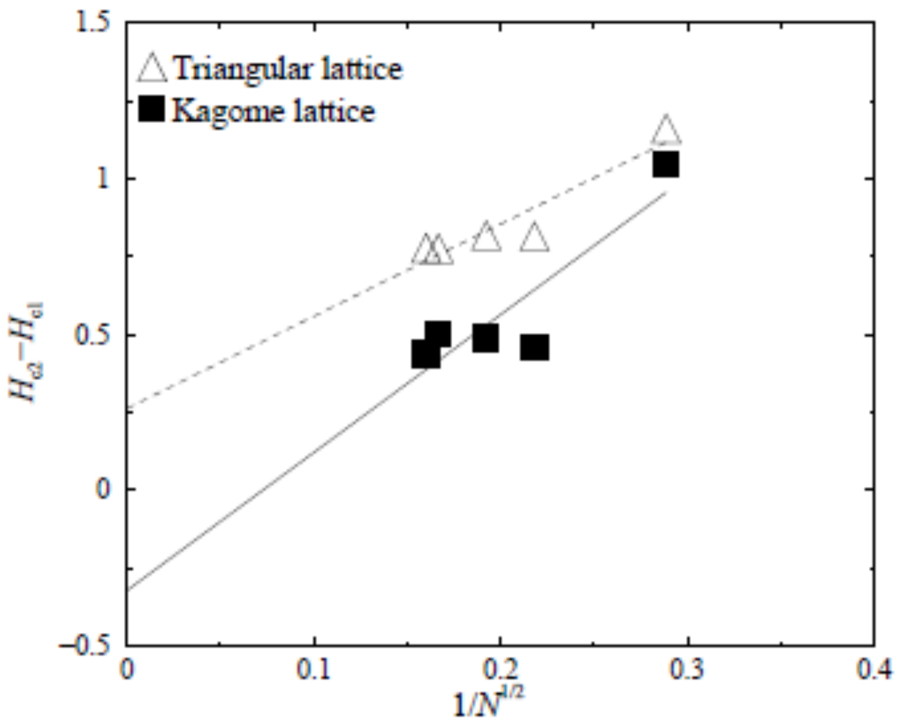
$$H_{c2}-H_{c1}=-0.3\pm0.5$$

$H_{c1}=H_{c2}$

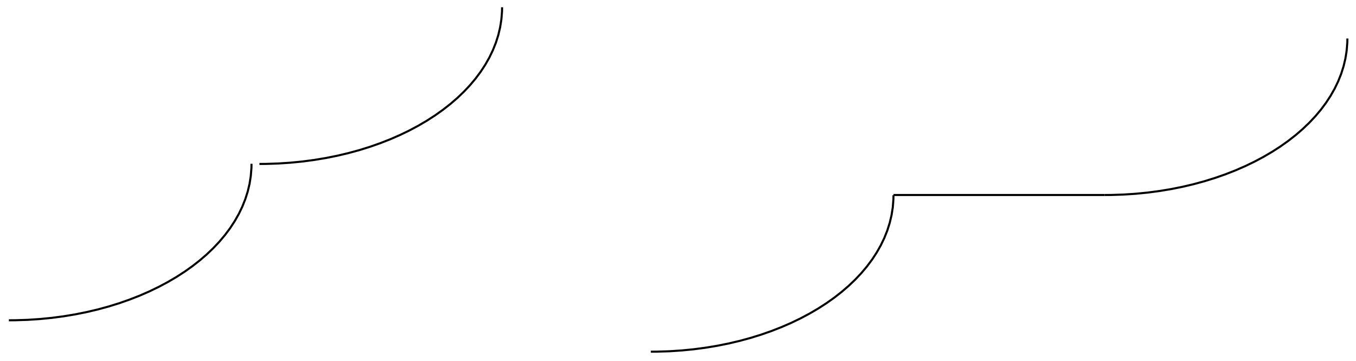
No plateau

$\Delta \sim k \Rightarrow \Delta \to 1/N^{1/2}\ (N\to\infty)$

if gapless



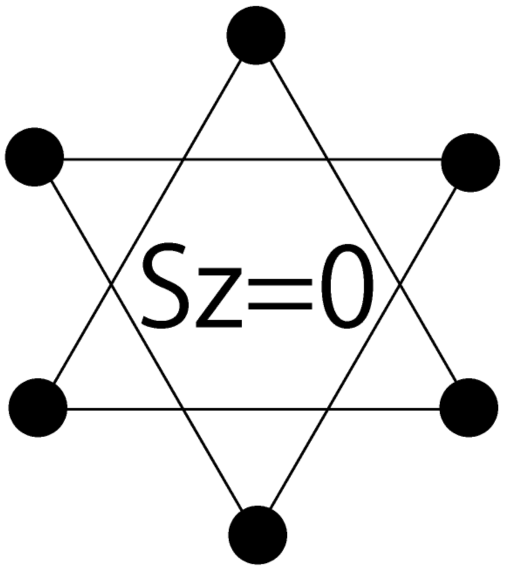
Magnetization ramp ?

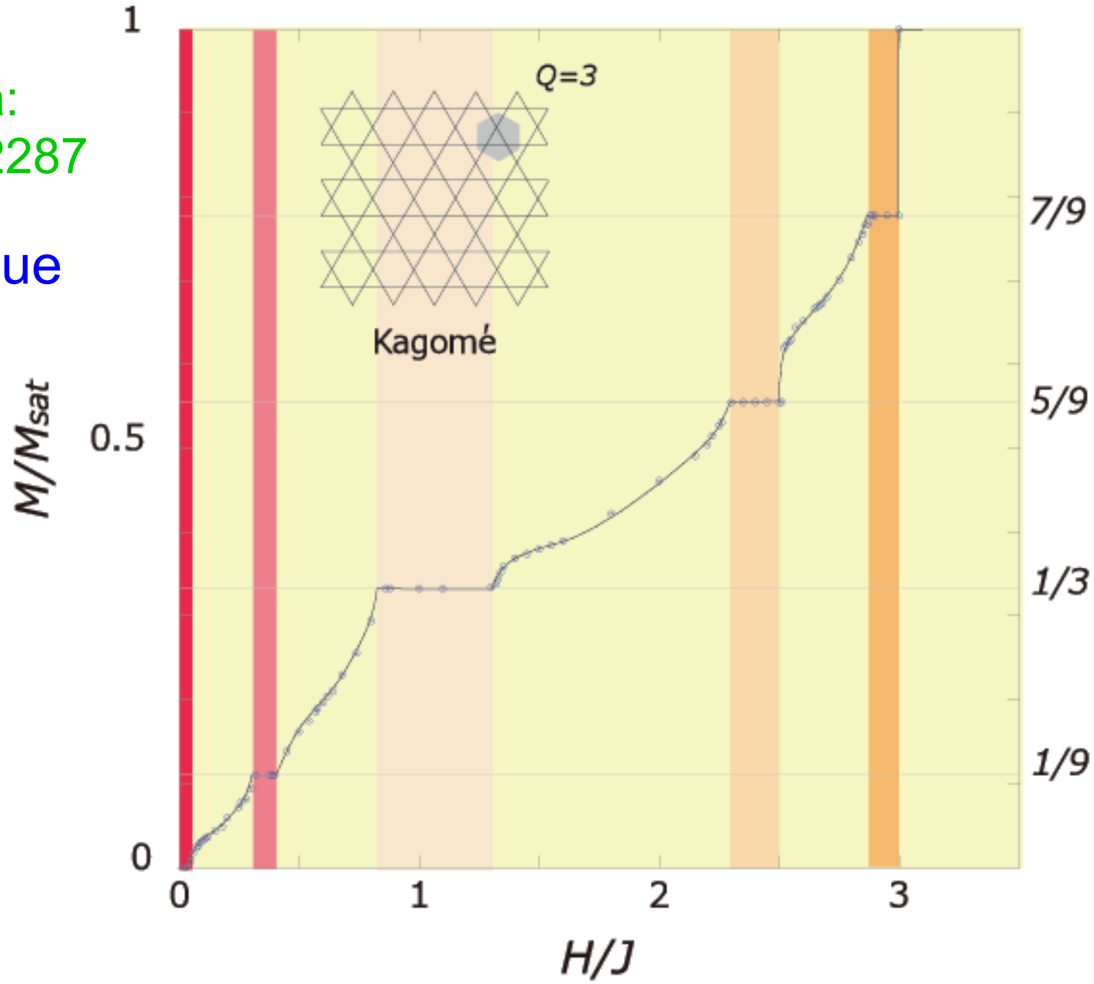


Grand Canonical Analysis

Nishimoto, Shibata, Hotta:
Nature Comm. **4** (2013) 2287

Deformation technique





cf. Diagonalization up to 63 spins

Capponi et al. PRB 88 (2013) 144416

Plateaux at 1/3, 5/9, 7/9

Purpose of this study

to know the true behavior around 1/3 height of the magnetization process of the S=1/2 Heisenberg kagome-lattice antiferromagnet from an unbiased meth

Lanczos diagonalization

- **We treat system sizes as large as possible.**

N_s**=42** (WR within the S=1/2 systems)

Parallel calculation in K computer

⇒ anomalous critical exponents

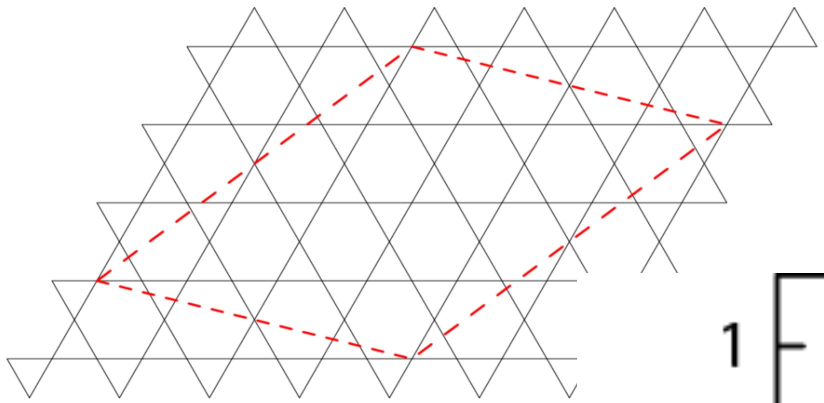
- **We observe the behavior when a distortion is switched on.**

The $\sqrt{3}\times\sqrt{3}$-Type

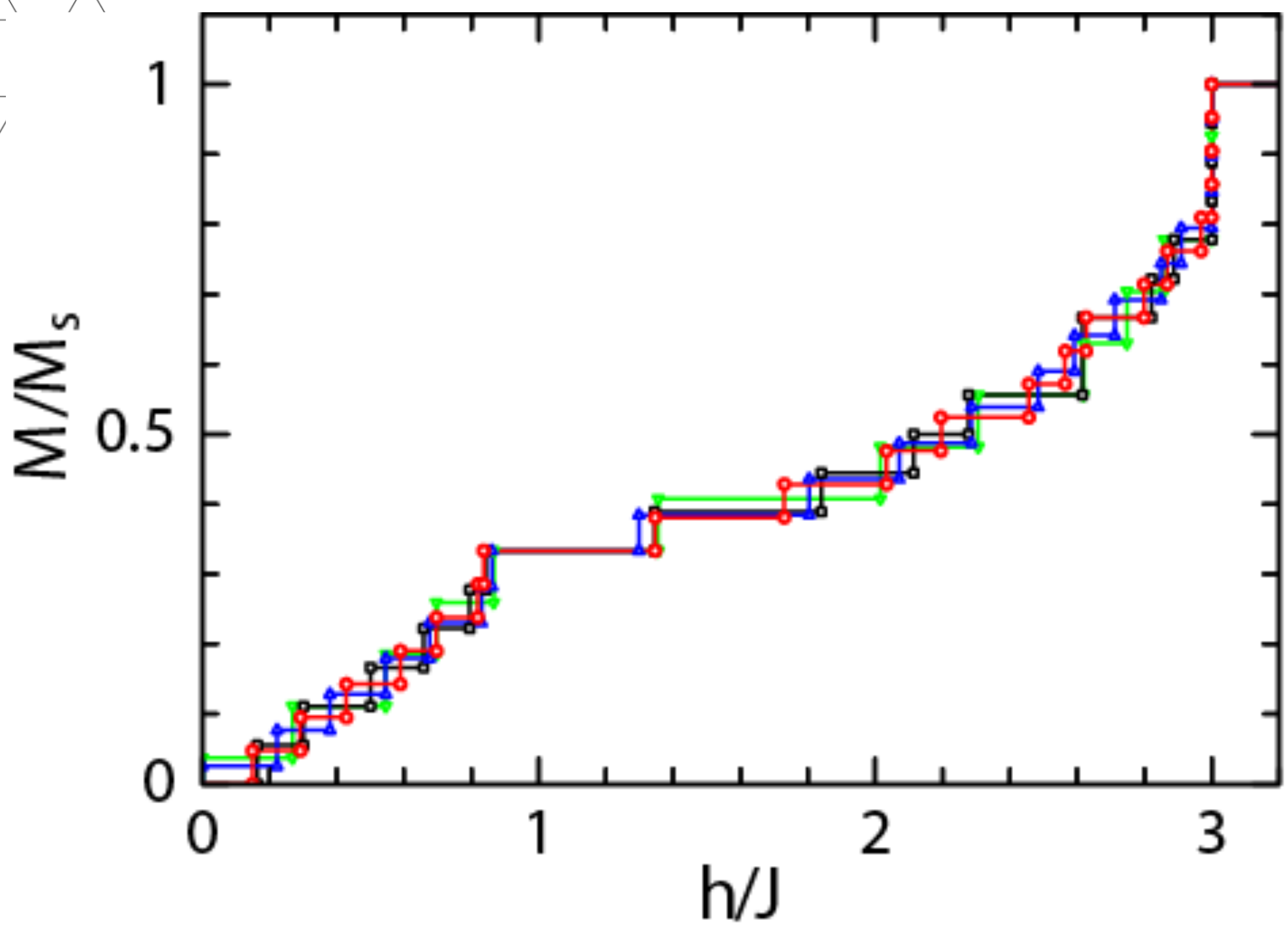
⇒ boundary between two different phases

Magnetization Process of N_s=42

WR within the S=1/2 systems



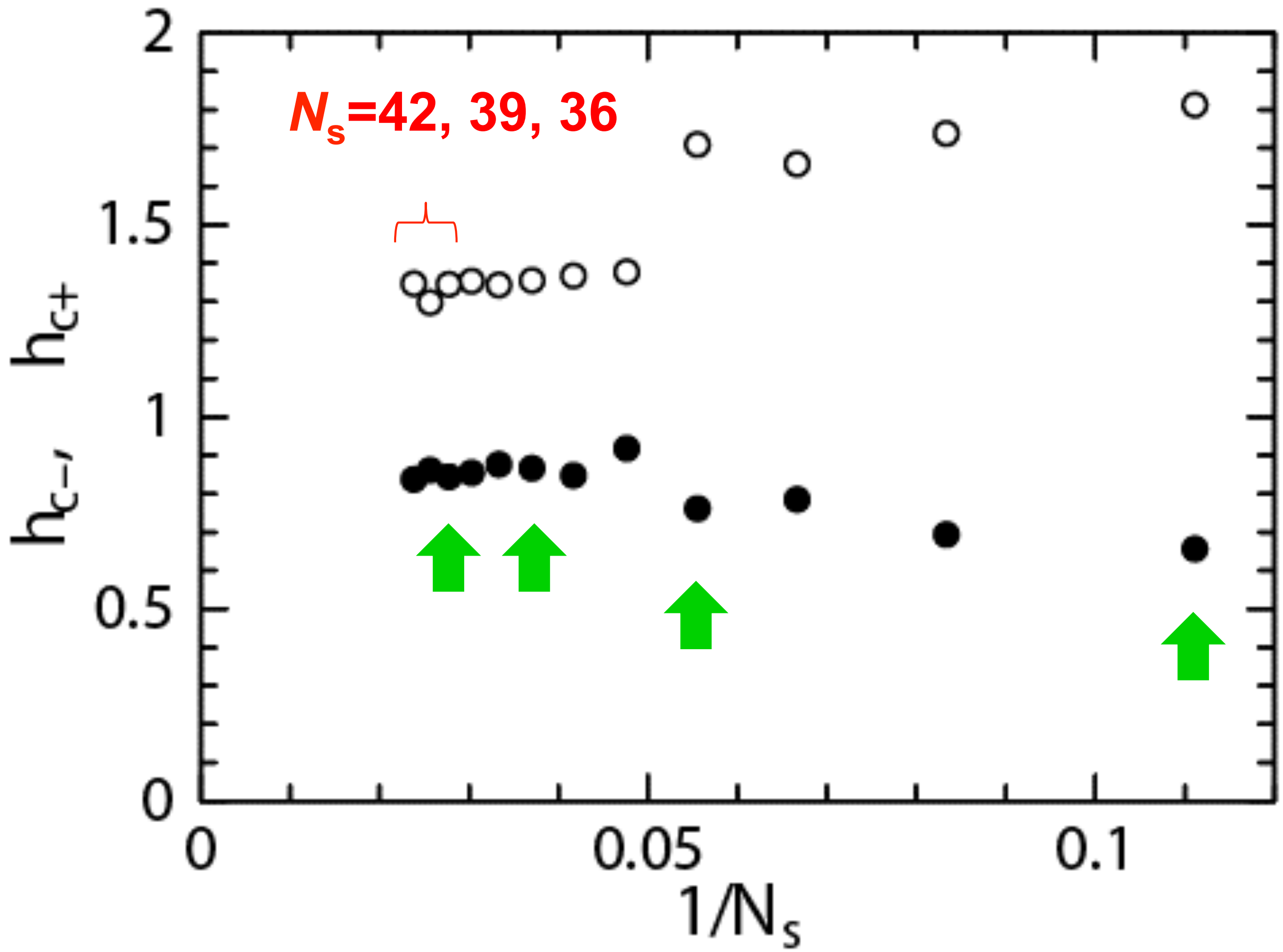
nonrhombic



N_s=39, 36, 27
rhombic

Width of the state at 1/3 height

Up to N_s=33 (Hida: JPSJ **70** (2001) 3673)

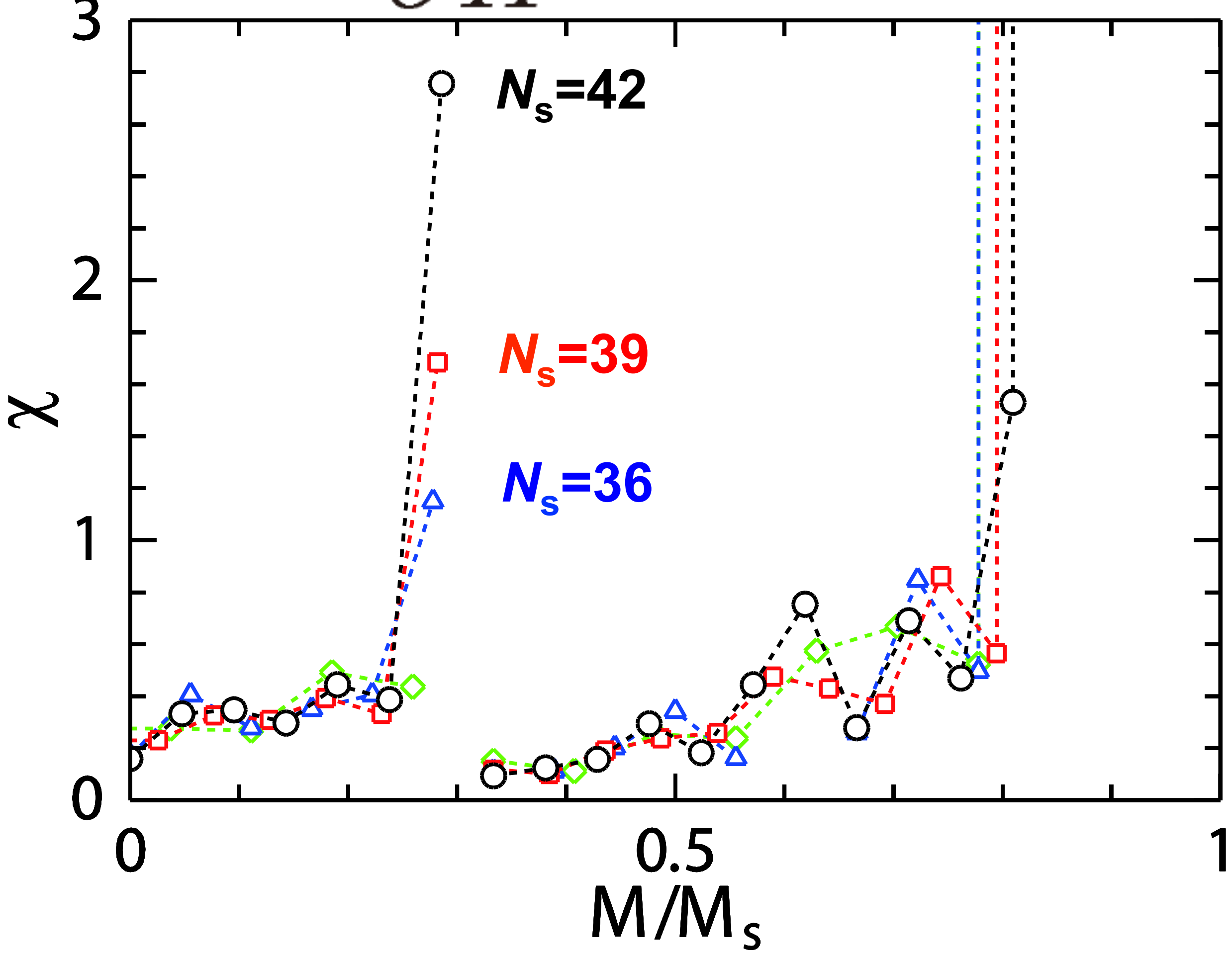


Weak size dependence for $N_s \geqq 21$

No clear evidence for the formation of state with 9-site structure

Differential Susceptibility

$$\chi \propto \frac{\partial M}{\partial H} \text{ as a function of } m = \frac{M}{M_s}$$



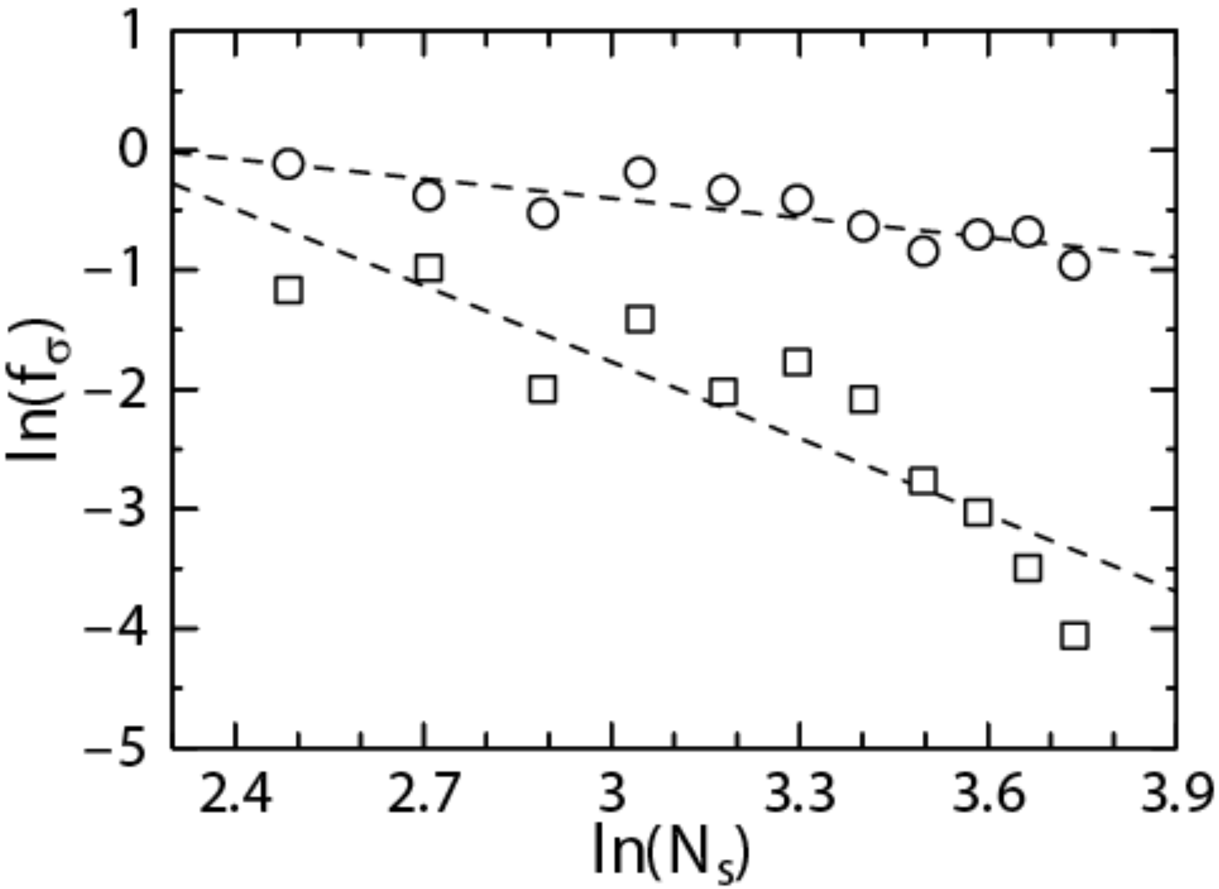
Divergent behavior below m=1/3

Continuous behavio between m=1/3 and m>1/3

Exponent δ

critical behavior $|m-m_c| \propto |H-H_c|^{1/\delta}$

$$f_\pm(N_s) = E(N_s, M = \frac{M_s}{3} \pm 2) + E(N_s, M = \frac{M_s}{3}) - 2E(N_s, M = \frac{M_s}{3} \pm 1)$$



$$f_\pm(N_s) \sim \frac{1}{N_s^{\delta_\pm}}$$

$\delta_+ = 0.54 \pm 0.36$

Different from $\delta=1$ for 2D system

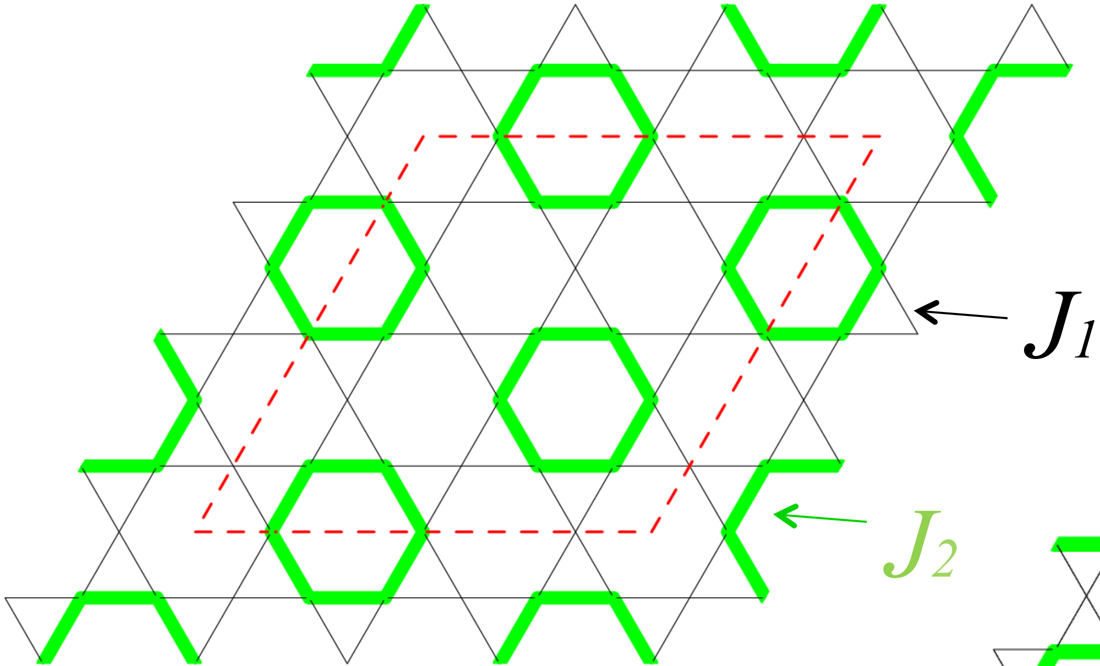
$\delta_- = 2.13 \pm 1.10$

Comparison is necessa with other estimates.

$\sqrt{3}\times\sqrt{3}$-Type Distortion

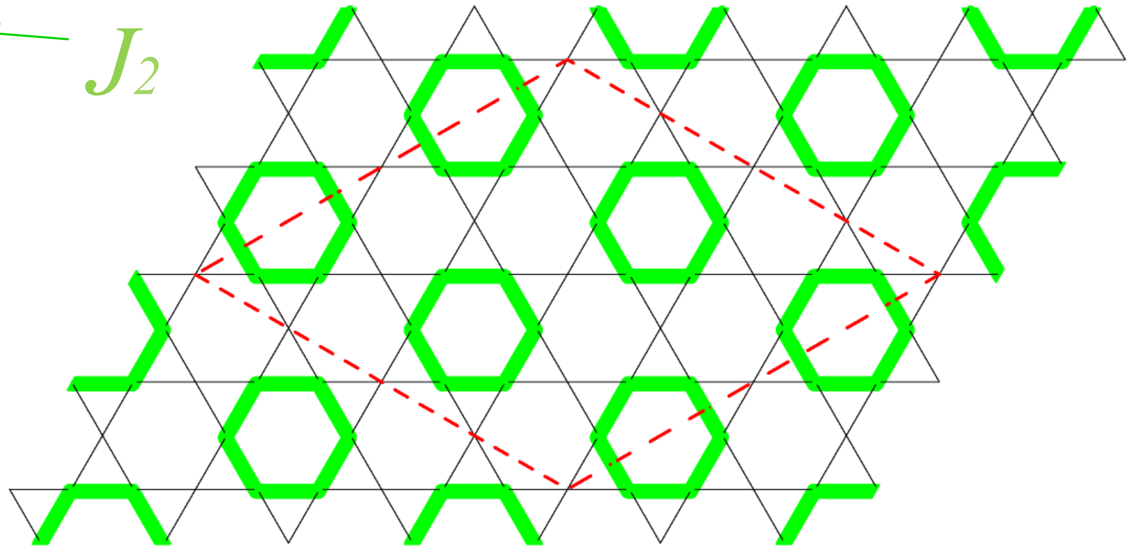
Hida: JPSJ **70** (2001) 3673

HN, Y.Hasegawa and T.Sakai:
JPSJ **83** (2014) 084709



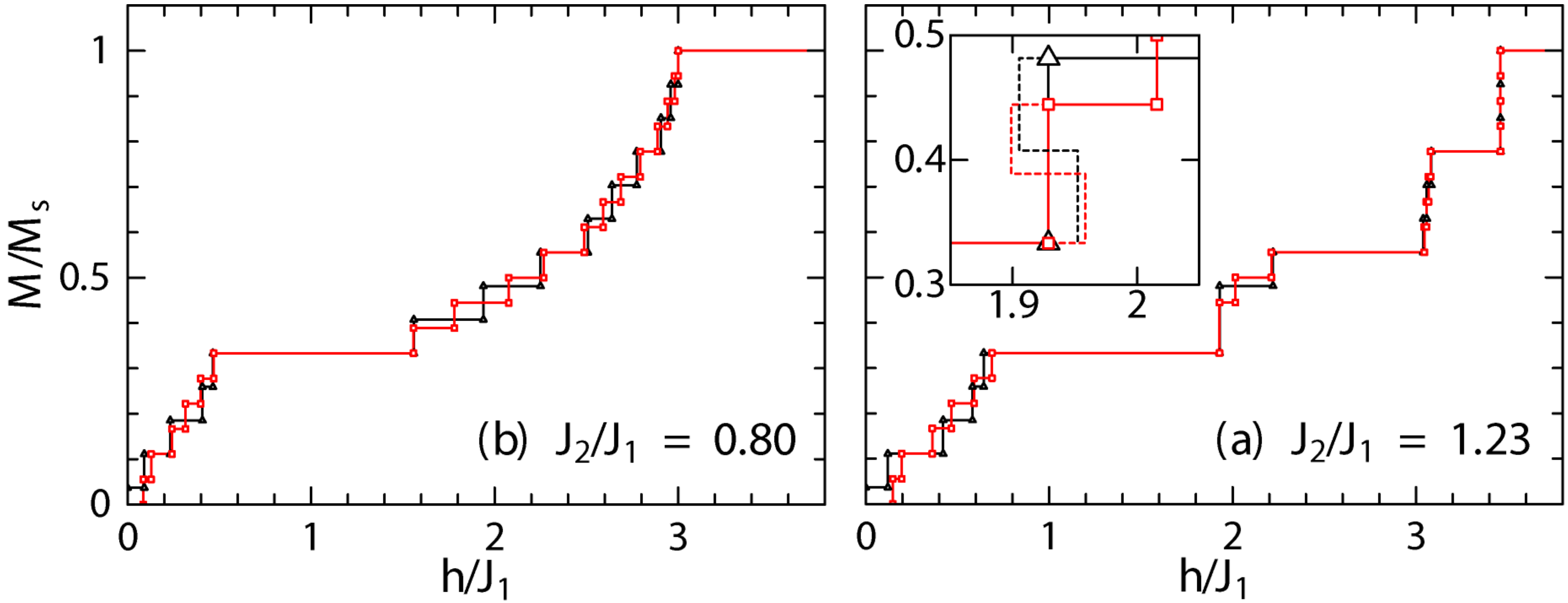
N_s=27

N_s=36



MH Curves with Distortion

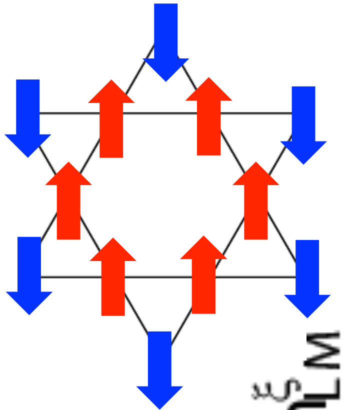
HN, Y.Hasegawa and T.Sakai:
JPSJ **83** (2014) 084709



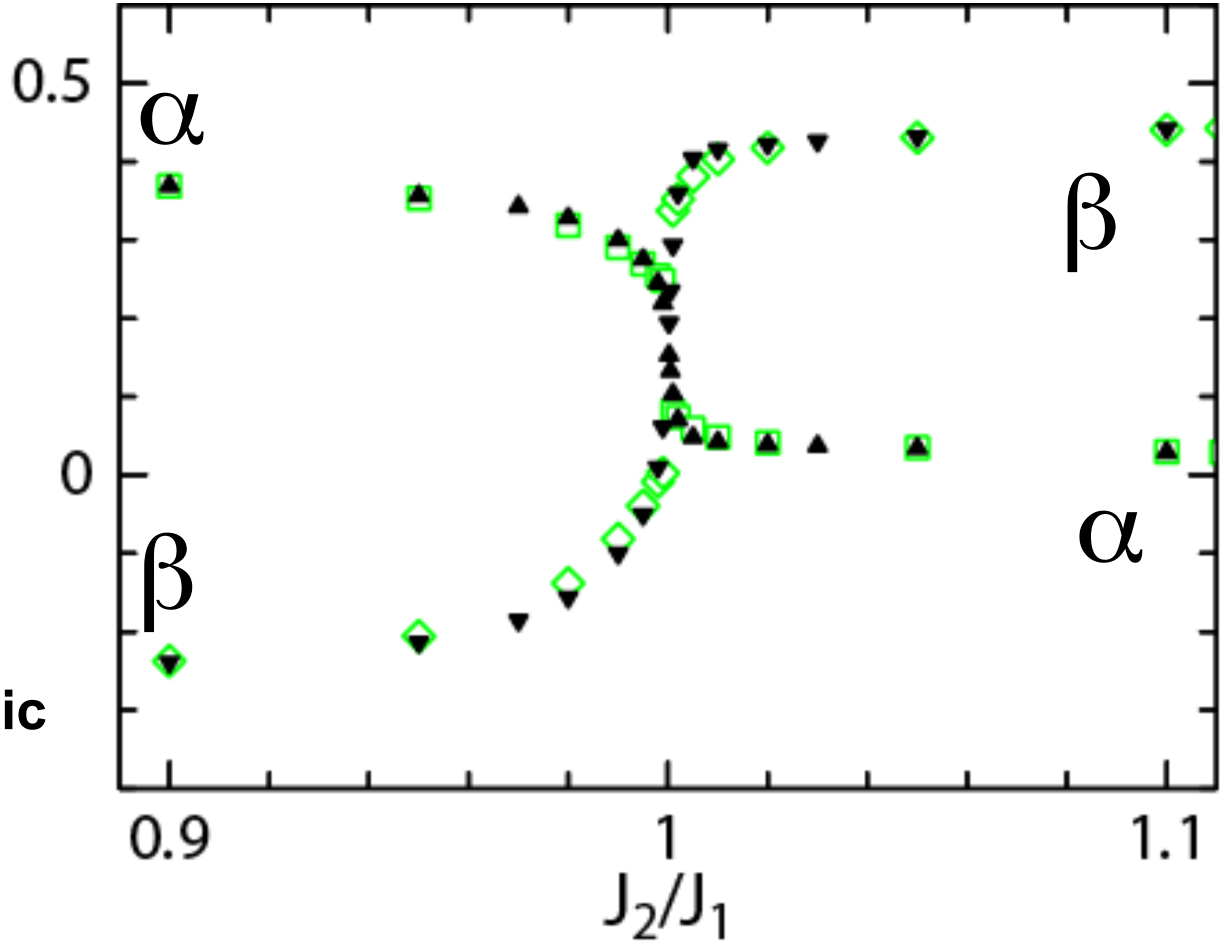
m=1/3 plateau

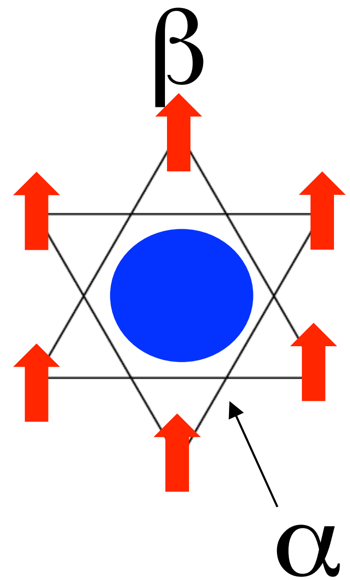
Spin-flop phenomenon even in a spin-isotropic system

Local Magnetization at *m*=1/3



Ferrimagnetic



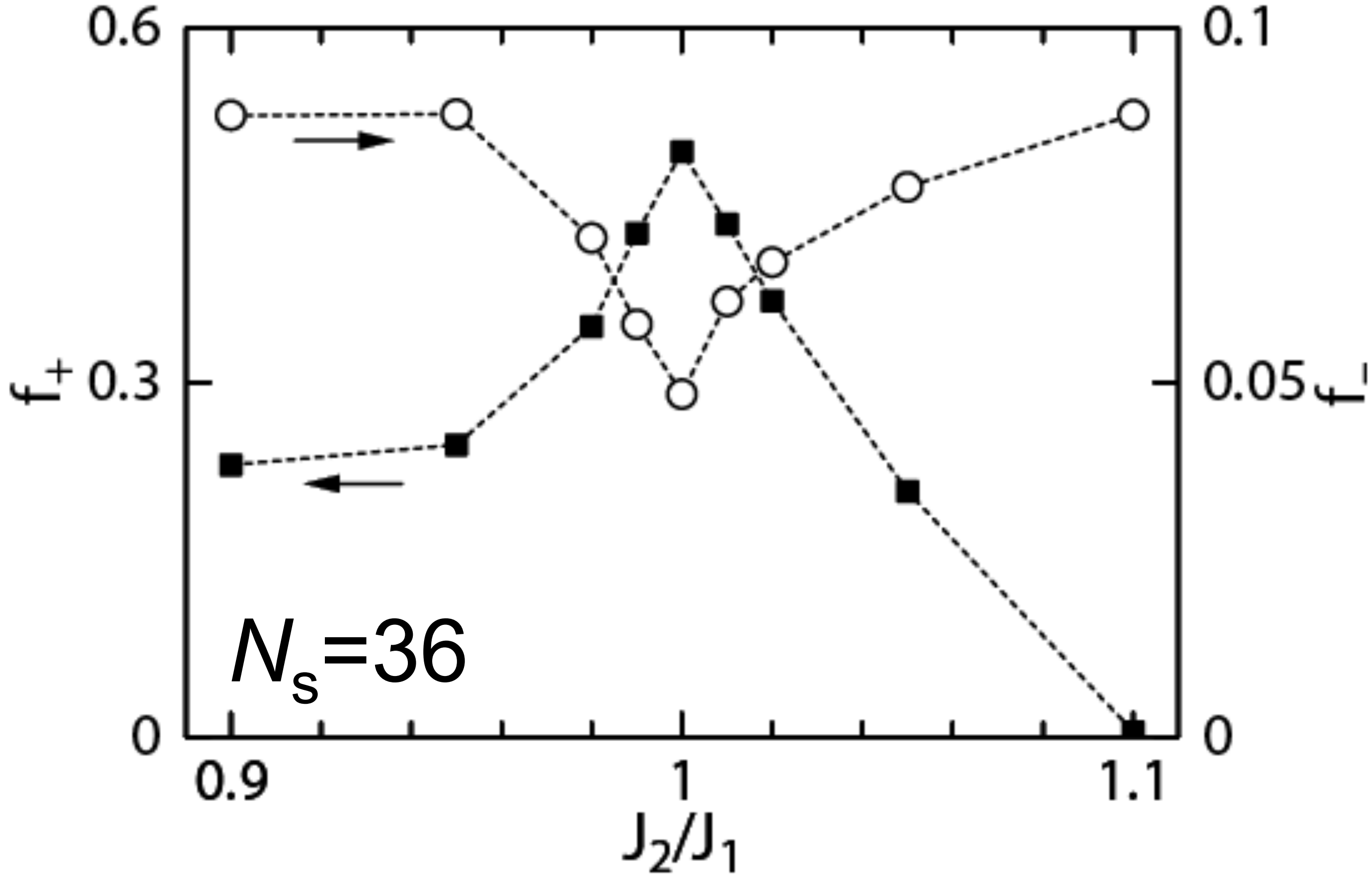


spin-flop

J_2=J_1 is only at a boundary between two different states

Behavior around m=1/3

$$f_{\pm}(N_s) = E(N_s, M = \frac{M_s}{3} \pm 2) + E(N_s, M = \frac{M_s}{3}) - 2E(N_s, M = \frac{M_s}{3} \pm 1)$$



also suggests clearly that J_2=J_1 is a bounc

Summary

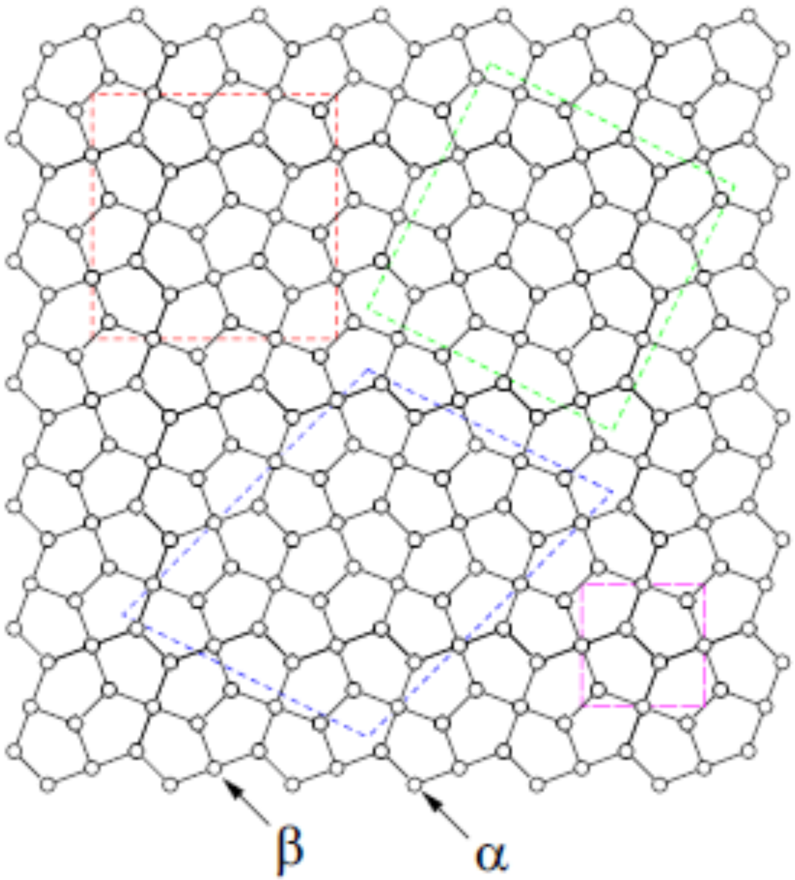
We study the magnetization process of kagome-lattice AF with and without the distortion.

- N_s=42 ⇒ anomalous exponents
- Kagome point is just a boundary during the √3×√3 distortion change.

References

HN and T.Sakai: JPSJ **79** (2010) 053707 (Letter)
T.Sakai and HN: PRB **83** (2011) 100405(Rapid comm.)
HN and T.Sakai: JPSJ **80** (2011) 053704 (Letter)
HN, M.Isoda, and T.Sakai: JPSJ 83 (2014) 053702
HN, Y.Hasegawa and T.Sakai: JPSJ **83** (2014) 084709
HN and T.Sakai: JPSJ 83 (2014) 104710 arXiv.1408.4538

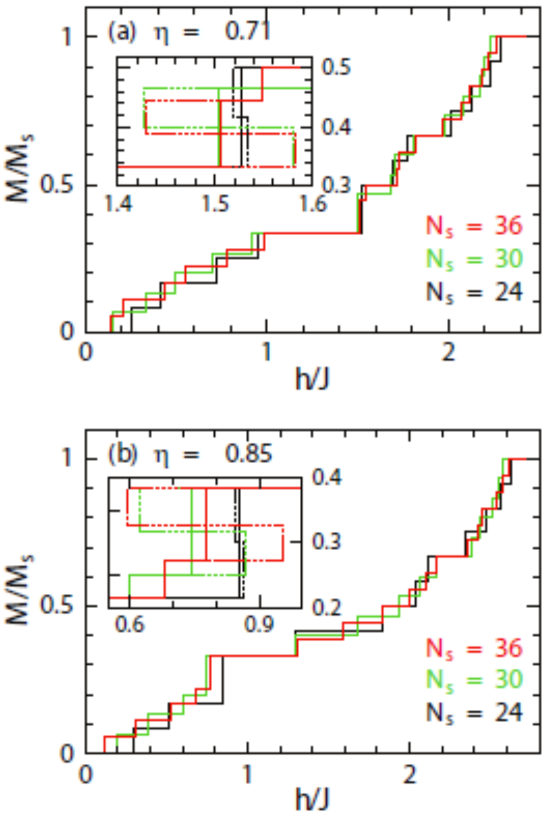
cf. Cairo pentagon lattice



J : α-α bond
J’: α-β bond

η= J’/J

Magnetization jump



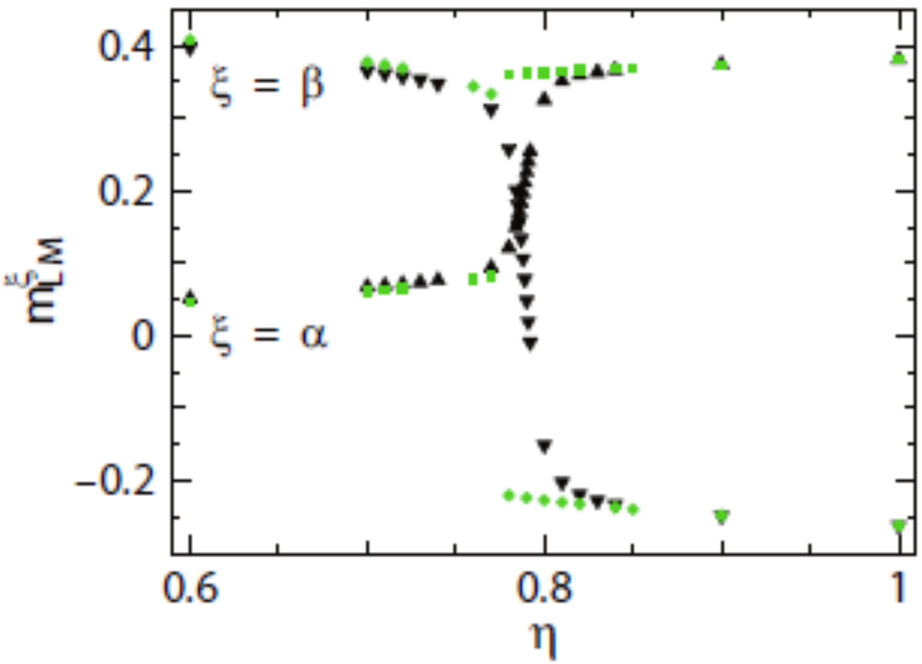
Higher side of 1/3 plateau

Critical point η ∼ 0.8

lower side of 1/3 plateau

Jump ⇔ Classical long-range order

Quantum phase transition



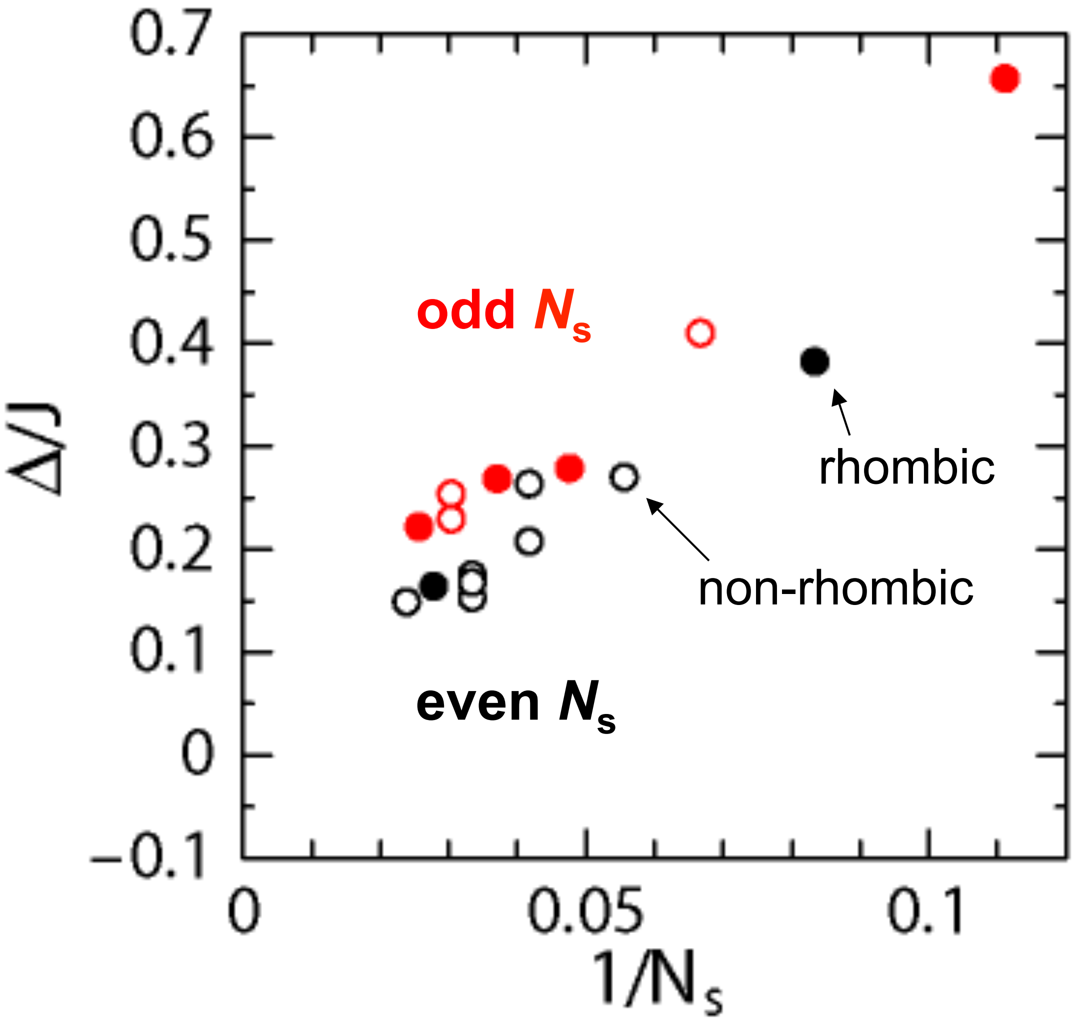
Cairo pentagon lattice AF

Critical ration J'/J ~ 0.8 quantum phase transition

Spin flop after 1/3 plateau for J'/J < 0.8

Spin flop before 1/3 plateau for J'/J >0.8

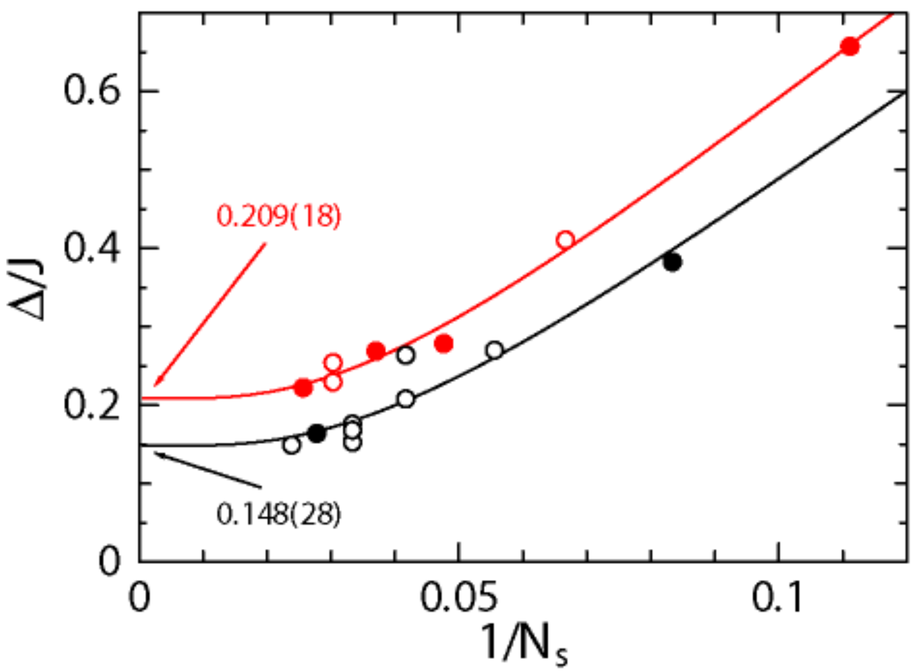
Spin gap up to N=42

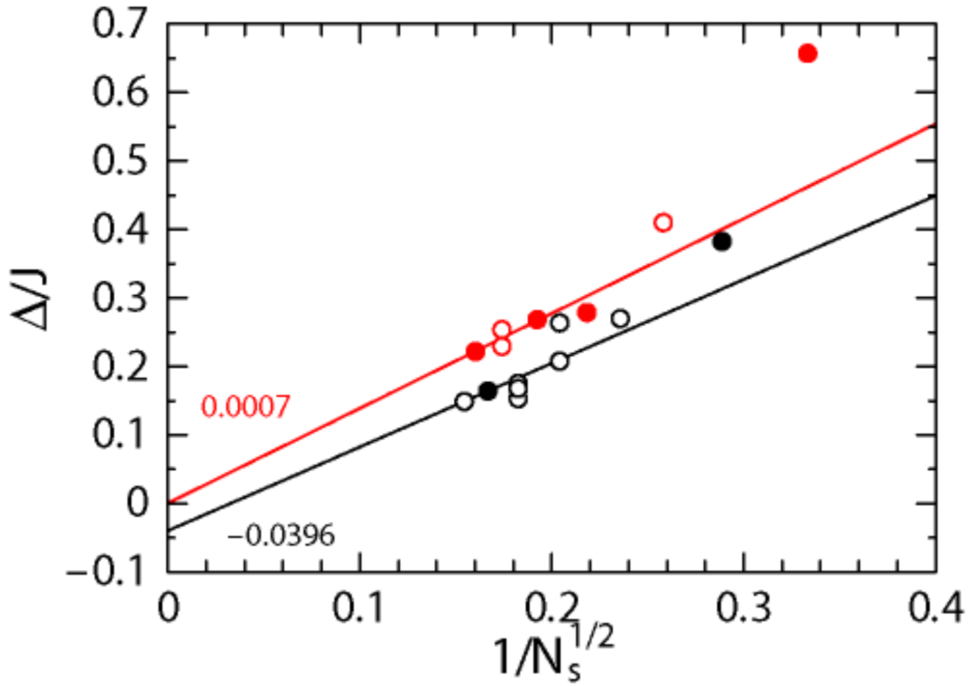


Important to divide data into two groups of even N_s and odd N_s.

Not good to treat all the data together.

Analysis of our finite-size gaps





Two extrapolated results disagree from odd N_s and even N_s sequences.

Feature of a gapless system (U(1) Dirac SL)

H. Nakano and TS: JPSJ 80 (2011) 053704 (arXiv: 1103.5829)