

Tema 10:
CUADRIPOLOS

Cuadripolos

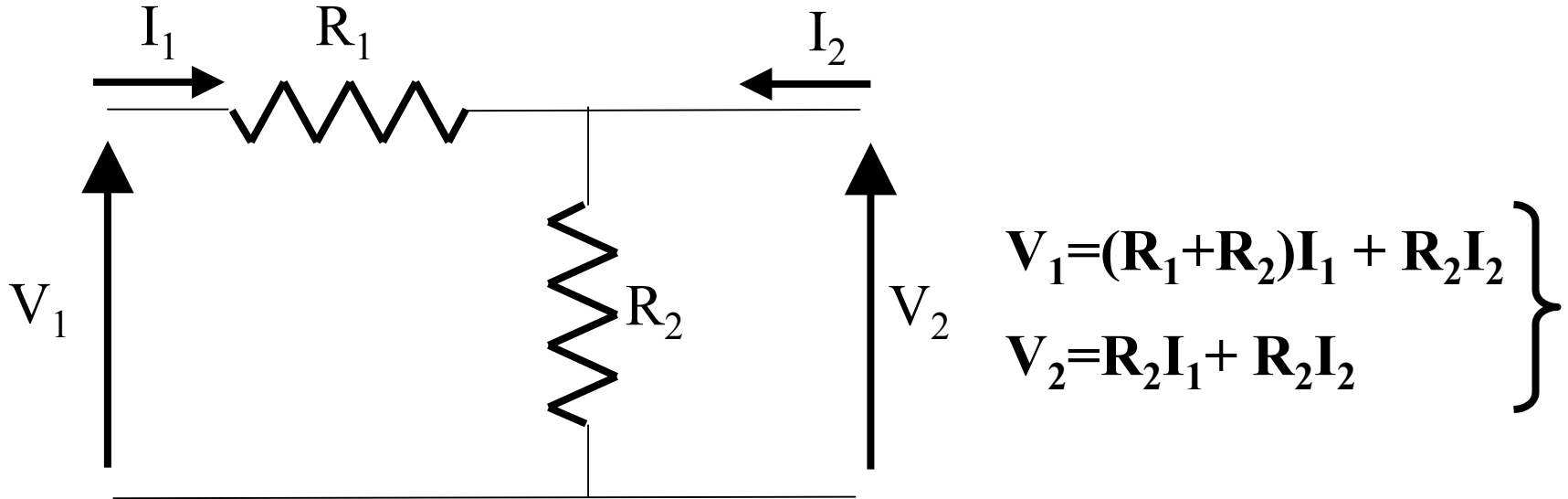
- **Descripción general**
- **Parámetros de cuadripolos**
- **Asociación de cuadripolos**

Cuadripolos

- **Relación de una red con los circuitos exteriores**
- **Red = caja negra con puertas de acceso**
- **Aplicaciones:**
 - **Modelo de componentes electrónicos**
 - **Circuitos equivalentes de la red inicial**

Cuadripolos

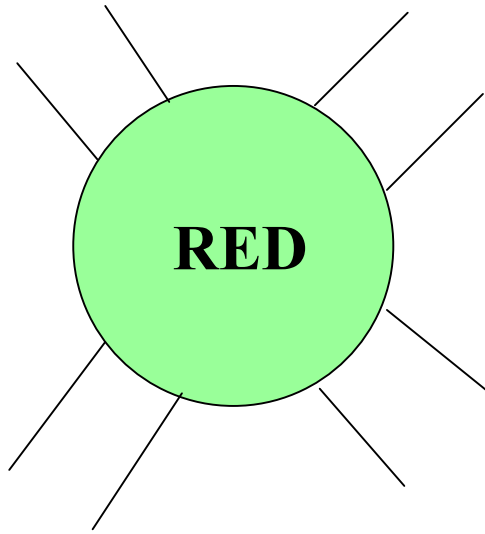
- Puertas de acceso con V e I asociados
- Sistemas de ecuaciones relacionan las V e I de las puertas



Cuadripolos

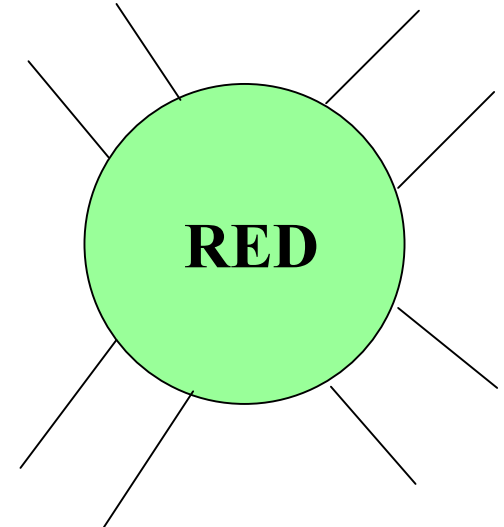
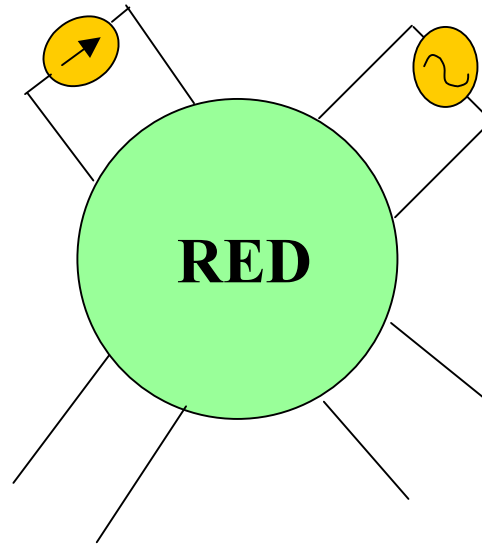
- Se describe el circuito a partir de las relaciones entre las V e I de las puertas
- Circuitos lineales \Rightarrow ecuaciones lineales
 - Coeficientes = parámetros de la red
 - Permiten conocer V e I cualquier puerta conocidas las otras
 - Red sin generadores independientes

Cuadripolos



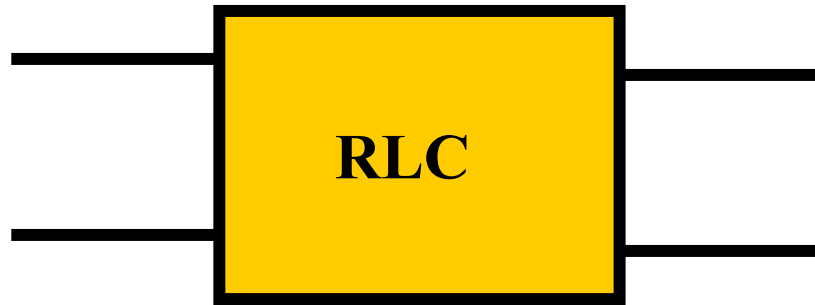
$$\left. \begin{aligned} Y_1 &= P_{11} X_1 + \dots + P_{1N} X_N \\ Y_2 &= P_{21} X_1 + \dots + P_{2N} X_N \\ &\dots \\ Y_N &= P_{N1} X_1 + \dots + P_{NN} X_N \end{aligned} \right\}$$

Red Multipuerta



Extracción de fuentes

Cuadripolos



Red bipuerta o cuadripolo

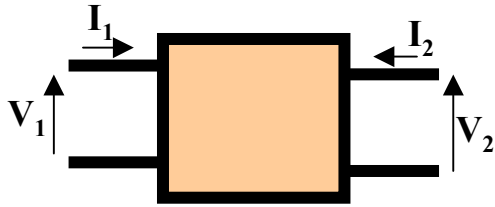
- **Definido por dos ecuaciones**
- **Parámetros dependen de variables controladas y controladoras**

$$\begin{pmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix}$$

Y: variables controladas

X: variables controladoras

Parámetros de Impedancia Z



• Variables controladas V

• Variables controladoras I

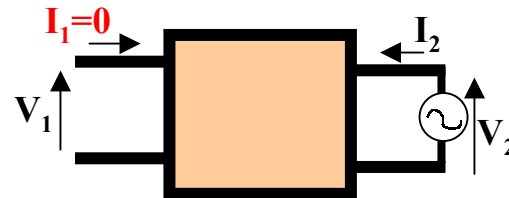
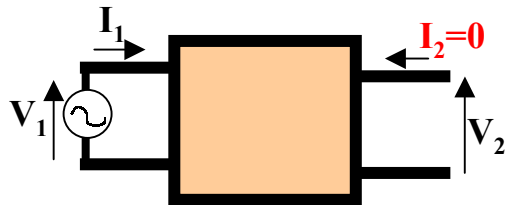
$$\left. \begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned} \right\} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$Z_{11} = V_1 / I_1 | I_2 = 0$ Z entrada circ. abierto a la salida

$Z_{21} = V_2 / I_1 | I_2 = 0$ Z transf. sal.-entr. circ. abierto a la salida

$Z_{12} = V_1 / I_2 | I_1 = 0$ Z transf. entr.-sal. circ. abierto a la entrada

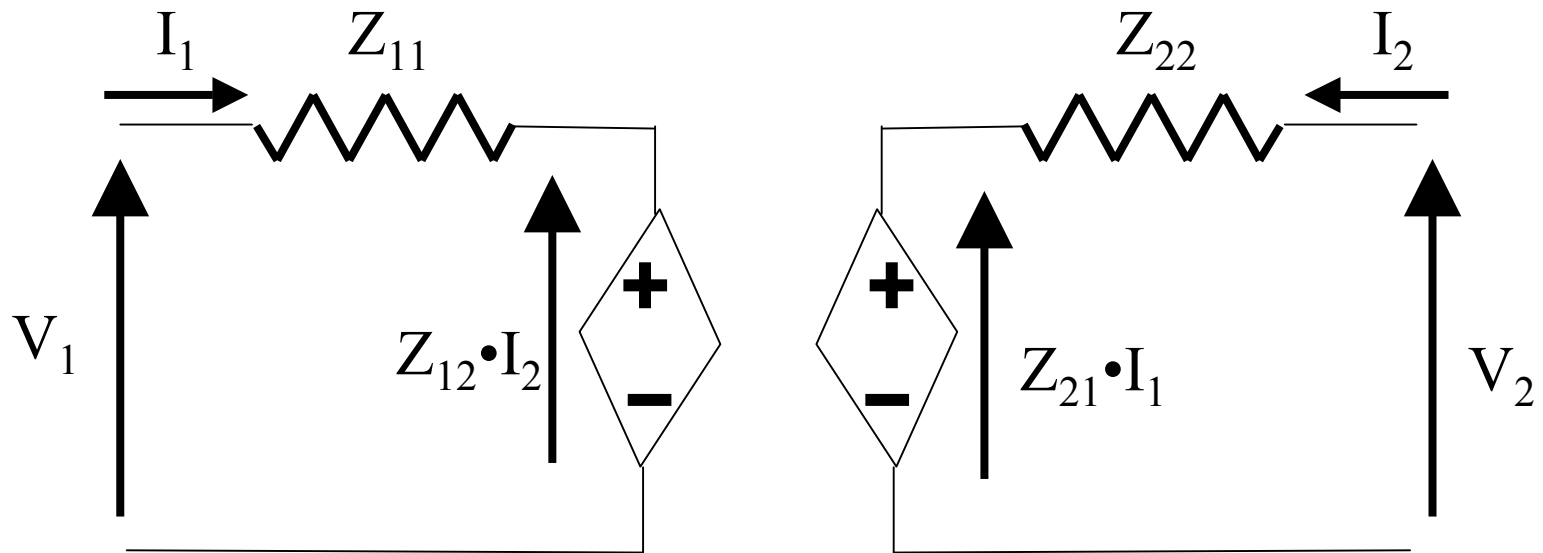
$Z_{22} = V_2 / I_2 | I_1 = 0$ Z salida circ. abierto a la entrada



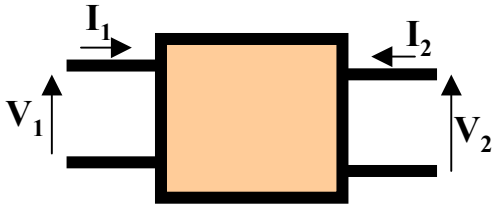
PARAMETROS DE IMPEDANCIA EN CIRC. ABIERTO

Parámetros de Impedancia Z

Circuito equivalente



Parámetros de admitancia Y



- Variables controladas I
- Variables controladoras V

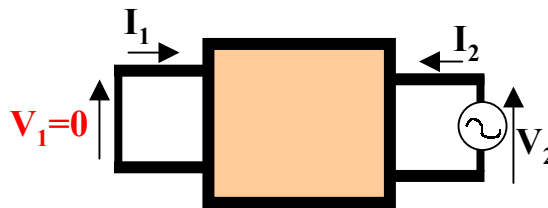
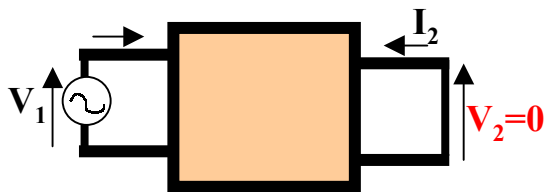
$$\left. \begin{aligned} I_1 &= Y_{11} V_1 + Y_{12} V_2 \\ I_2 &= Y_{21} V_1 + Y_{22} V_2 \end{aligned} \right\} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$Y_{11} = I_1 / V_1 | V_2 = 0$ Y entrada cortocirc. a la salida

$Y_{21} = I_2 / V_1 | V_2 = 0$ Y transf. sal.-entr. cortocirc. a la salida

$Y_{12} = I_1 / V_2 | V_1 = 0$ Y transf. entr.-sal. cortocirc. a la entrada

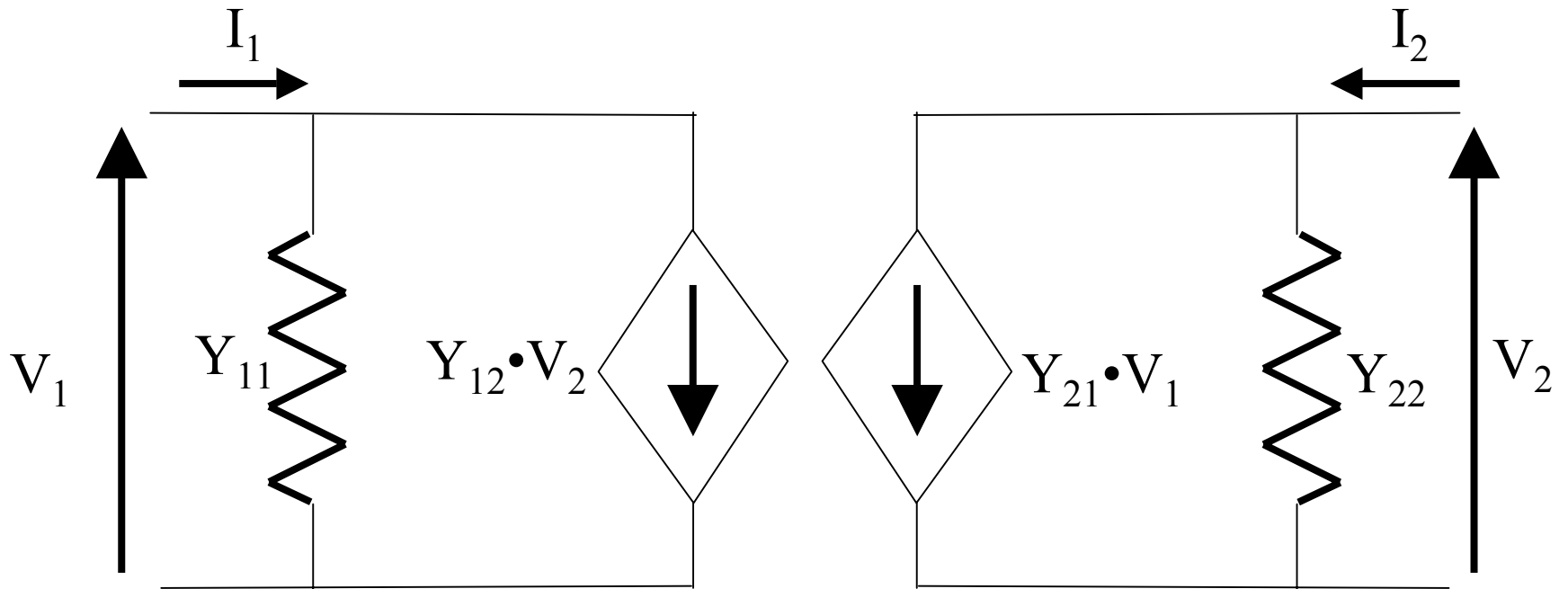
$Y_{22} = I_2 / V_2 | V_1 = 0$ Y salida cortocirc. a la entrada



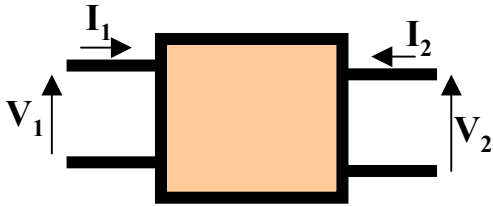
PARAMETROS DE ADMITANCIA EN CORTOCIRC.

Parámetros de admitancia Y

Circuito equivalente



Parámetros Híbridos h



• Variables controladas $V_1 I_2$

• Variables controladoras $V_2 I_1$

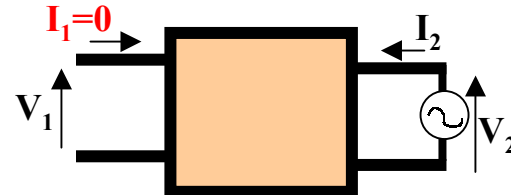
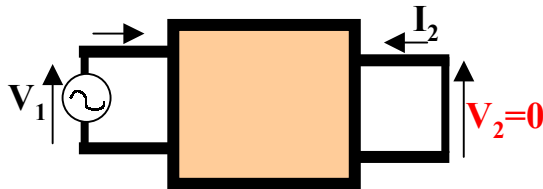
$$\left. \begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned} \right\} \begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

$h_{11} = V_1 / I_1 | V_2 = 0$ Z entrada cortocirc. a la salida

$h_{21} = I_2 / I_1 | V_2 = 0$ G corriente cortocirc. a la salida

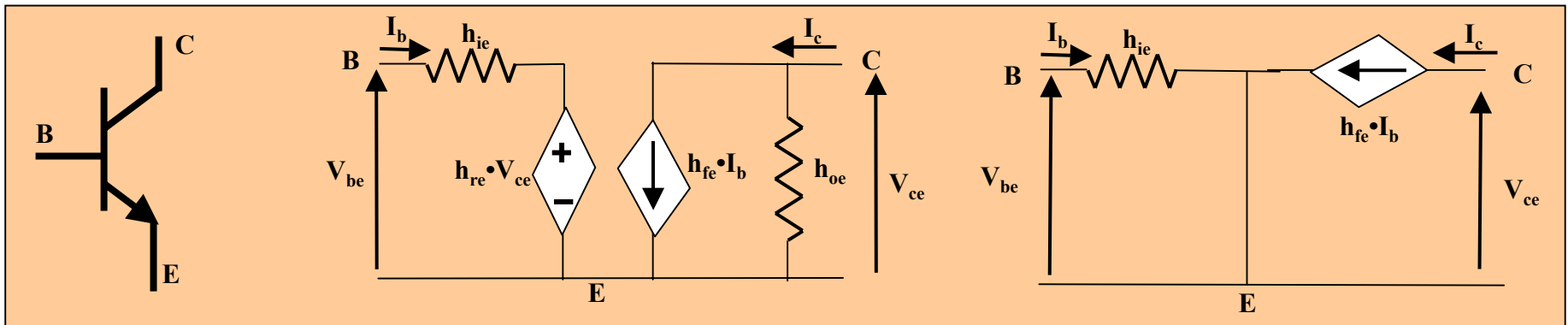
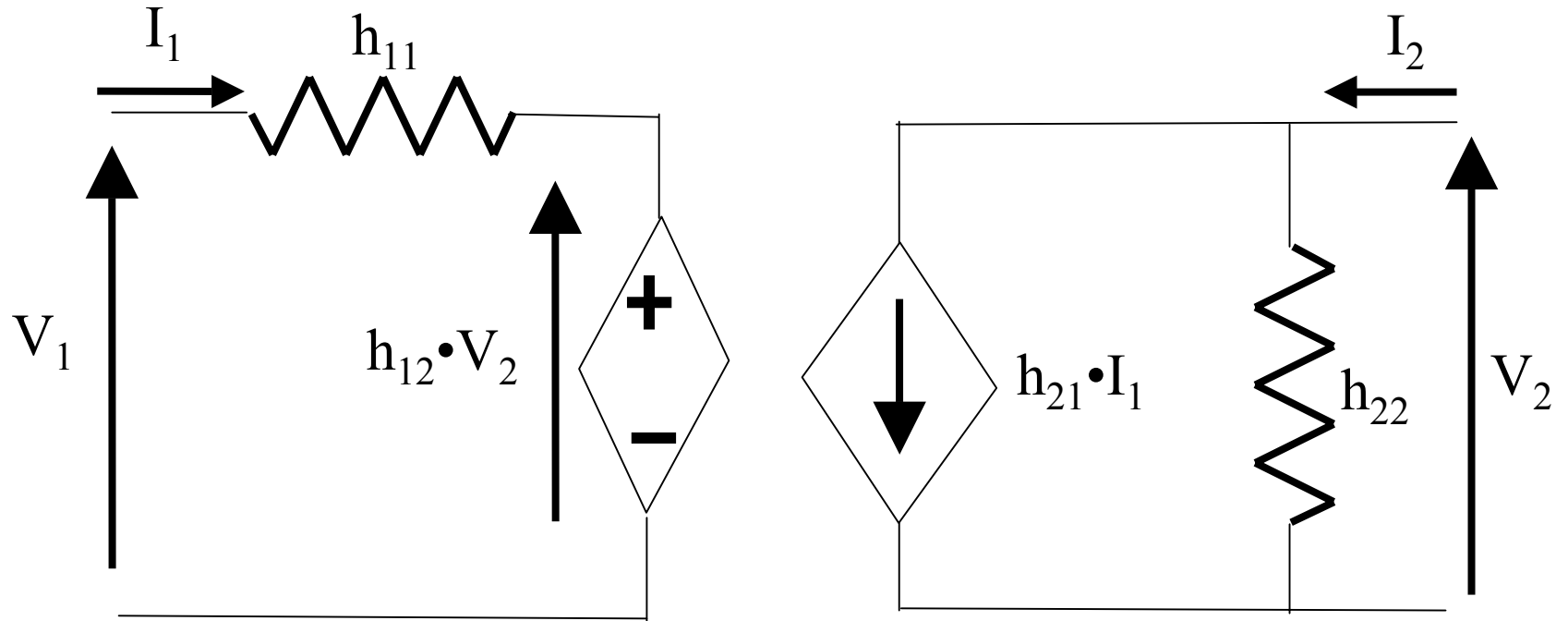
$h_{12} = V_1 / V_2 | I_1 = 0$ G tensión circ. abierto a la entrada

$h_{22} = I_2 / V_2 | I_1 = 0$ Y salida circ. abierto a la entrada

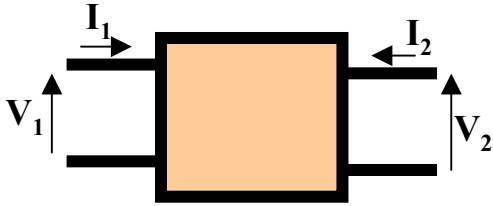


Parámetros Híbridos h

Circuito equivalente



Parámetros Híbridos g



- Variables controladas I_1 V_2
- Variables controladoras I_2 V_1

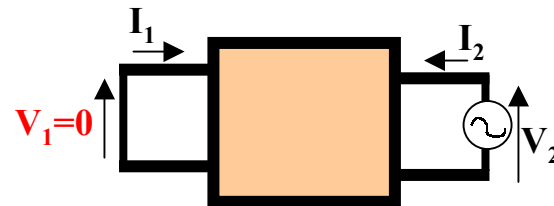
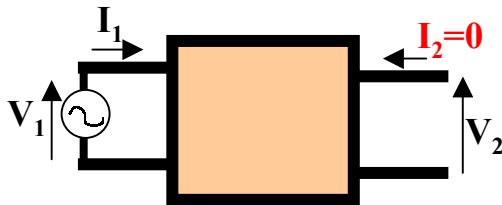
$$\left. \begin{aligned} I_1 &= g_{11} V_1 + g_{12} I_2 \\ V_2 &= g_{21} V_1 + g_{22} I_2 \end{aligned} \right\} \begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$$

$g_{11} = I_1 / V_1 | I_2 = 0$ Y entrada circ. abierto a la salida

$g_{21} = V_2 / V_1 | I_2 = 0$ G tensión circ. abierto a la salida

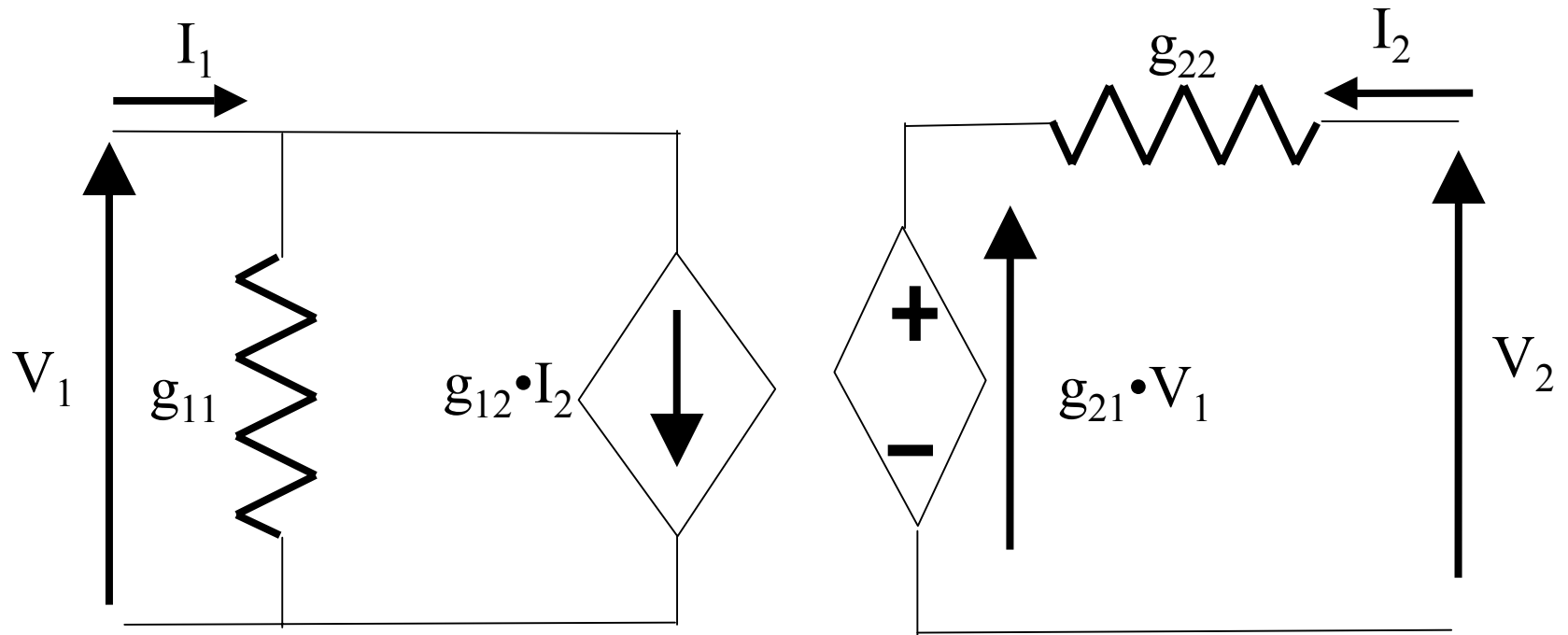
$g_{12} = I_1 / I_2 | V_1 = 0$ G corriente cortocirc. a la entrada

$g_{22} = V_2 / I_2 | V_1 = 0$ Z salida cortocirc. a la entrada

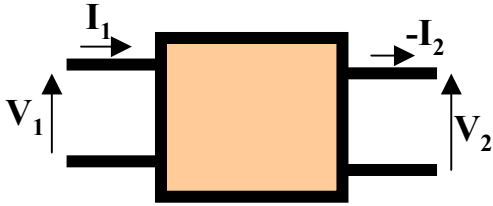


Parámetros Híbridos g

Circuito equivalente



Parámetros Transmisión ABCD

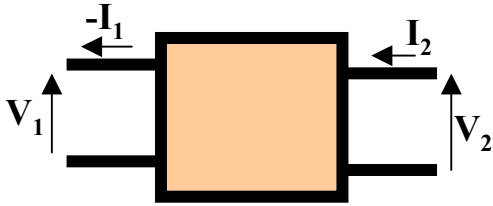


- Variables controladas I_1 V_1 (entrada)
- Variables controladoras I_2 V_2 (salida)

$$\left. \begin{array}{l} V_1 = AV_2 + BI_2 \\ I_1 = CV_2 + DI_2 \end{array} \right\} \begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

$A = V_1 / V_2 | I_2 = 0$ G inversa tensión circ. abierto a la salida
 $C = I_1 / V_2 | I_2 = 0$ Y transf. entr.-salida circ. abierto a la salida
 $B = V_1 / -I_2 | V_2 = 0$ Z transf. entr.-salida cortocirc. a la salida
 $D = I_1 / -I_2 | V_2 = 0$ G inversa corriente cortocirc. a la salida

Parámetros Transmisión Inversa A'B'C'D'



- Variables controladas I_2 V_2 (salida)
- Variables controladoras I_1 V_1 (entrada)

$$\left. \begin{aligned} V_2 &= A'V_1 + B'I_1 \\ I_2 &= C'V_1 + D'I_1 \end{aligned} \right\} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \begin{pmatrix} V_1 \\ -I_1 \end{pmatrix}$$

$A' = V_2 / V_1 | I_1 = 0$ G inversa tensión circ. abierto a la entrada
 $C' = I_2 / V_1 | I_1 = 0$ Y transf. salida-entr. circ. abierto a la entrada
 $B' = V_2 / -I_1 | V_1 = 0$ Z transf. salida-entr. cortocirc. a la entrada
 $D' = I_2 / -I_1 | V_2 = 0$ G inversa corriente cortocirc. a la entrada

Relación entre parámetros

- **Distintos parámetros \Rightarrow distintas combinaciones
mismas variables**

- **Relaciones entre las matrices de parámetros**

- **Simplificaciones:**

- **Pasivo (RLC):**

$$Z_{12}=Z_{21}; Y_{12}=Y_{21}; AD-BC=1; h_{12}=-h_{21}; g_{12}=-g_{21}$$

- **Pasivo y eléctricamente simétrico:**

$$Z_{11}=Z_{22}; Y_{11}=Y_{22}; A=D; |h|=1; |g|=1$$

Relación entre parámetros

$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	Y	H	G	F
	$\begin{bmatrix} \frac{z_{22}}{ z } - \frac{z_{12}}{ z } \\ -\frac{z_{21}}{ z } \frac{z_{11}}{ z } \end{bmatrix}$	$\begin{bmatrix} \frac{ z }{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{z_{11}} - \frac{z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} \frac{ z }{z_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{ z }{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$
$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	Z	H	G	F
	$\begin{bmatrix} \frac{y_{22}}{ y } - \frac{y_{12}}{ y } \\ -\frac{y_{21}}{ y } \frac{y_{11}}{ y } \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} - \frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} \frac{ y }{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{ y }{y_{22}} & \frac{y_{12}}{y_{22}} \\ -\frac{y_{21}}{y_{22}} & \frac{1}{y_{22}} \end{bmatrix}$	$\begin{bmatrix} -\frac{y_{22}}{y_{21}} & \frac{1}{y_{21}} \\ -\frac{ y }{y_{21}} & -\frac{y_{11}}{y_{21}} \end{bmatrix}$
$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	Z	Y	G	F
	$\begin{bmatrix} \frac{ h }{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} - \frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} \frac{ h }{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{h_{22}}{ h } - \frac{h_{12}}{ h } \\ -\frac{h_{21}}{ h } & \frac{h_{11}}{ h } \end{bmatrix}$	$\begin{bmatrix} -\frac{ h }{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$
$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$	Z	Y	H	F
	$\begin{bmatrix} \frac{1}{g_{11}} - \frac{g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} \frac{ g }{g_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{ g }{g_{22}} - \frac{g_{12}}{g_{22}} \\ -\frac{g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{g_{22}}{ g } - \frac{g_{12}}{ g } \\ -\frac{g_{21}}{ g } & \frac{g_{11}}{ g } \end{bmatrix}$	$\begin{bmatrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{ g }{g_{21}} \end{bmatrix}$
$F = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$	Z	Y	H	G
	$\begin{bmatrix} \frac{A}{C} & \frac{ F }{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} - \frac{ F }{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{ F }{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} \frac{C}{A} & -\frac{ F }{A} \\ \frac{1}{A} & \frac{B}{A} \end{bmatrix}$

Asociación de Cuadripolos

- **Asociando cuadripolos se obtiene un nuevo cuadripolo**
- **Los parámetros del nuevo cuadripolo se pueden obtener de los parámetros de los cuadripolos componentes**
- **Cuadripolos complejos se descomponen en otros sencillos asociados para obtener los parámetros globales**

Asociación de Cuadripolos

• Asociaciones y parámetros que facilitan el cálculo de los parámetros totales:

• Serie-serie $\Rightarrow Z$

• Paralelo-paralelo $\Rightarrow Y$

• Serie-paralelo $\Rightarrow h$

• Paralelo-serie $\Rightarrow g$

• Cascada \Rightarrow Transmisión

• Serie

• Variable controlada V

• Variable controladora I

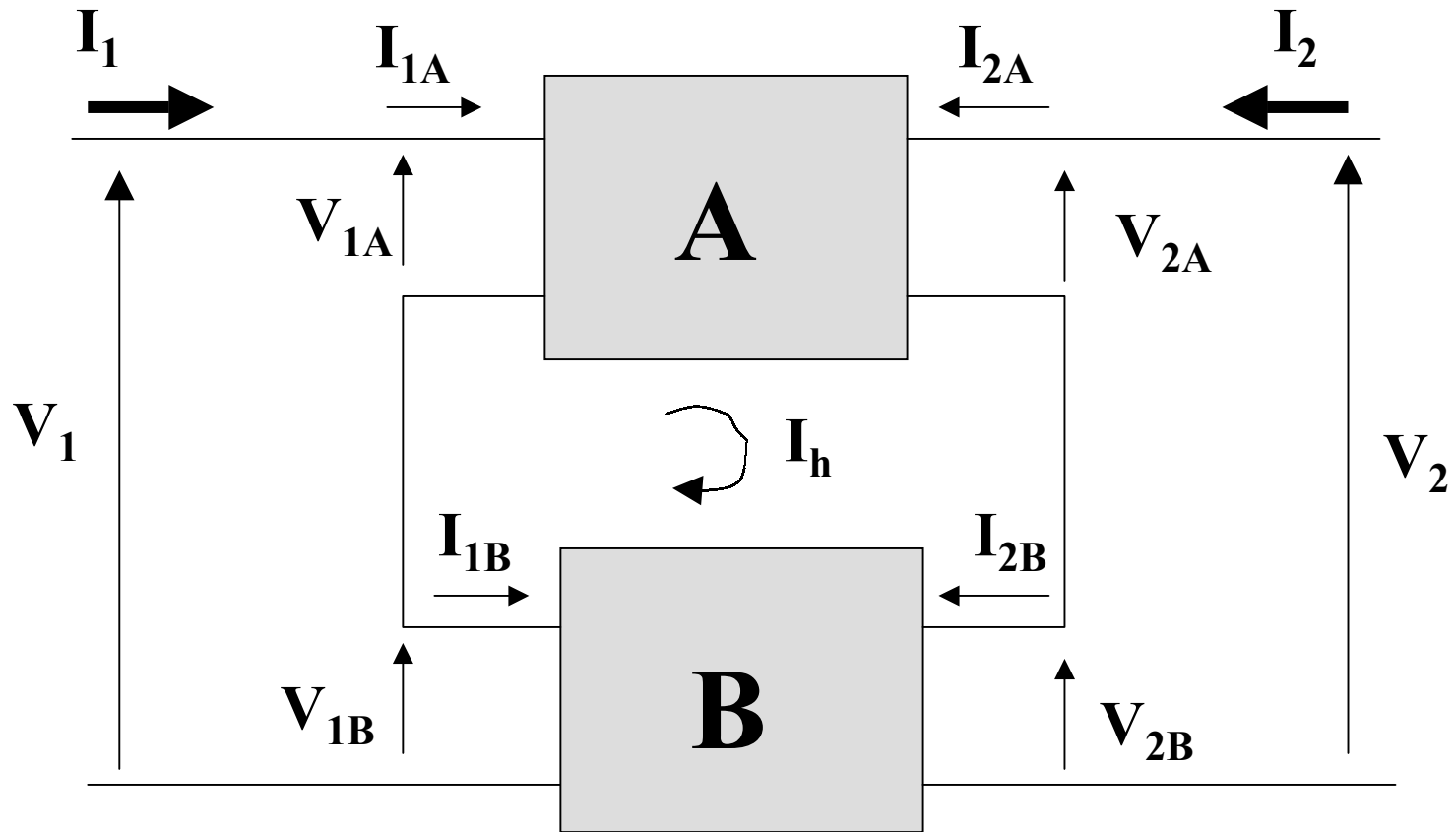
• Paralelo

• Variable controlada V

• Variable controladora I

Asociación Serie-Serie

Entradas y salidas en serie



Asociación Serie-Serie

$$V_1 = V_{1A} + V_{1B}$$

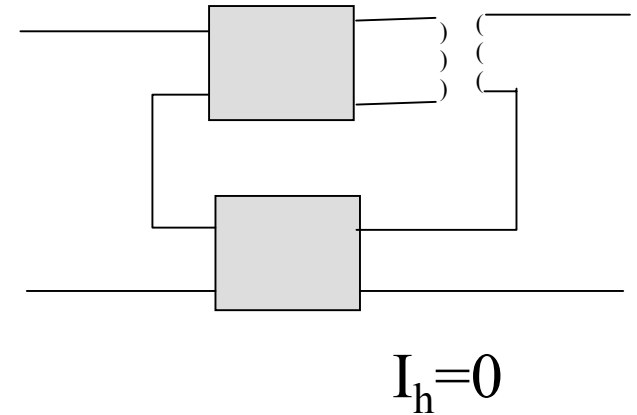
$$V_2 = V_{2A} + V_{2B}$$

$$I_1 = I_{1A} = I_{1B}$$

$$I_2 = I_{2A} = I_{2B}$$

$$\left. \begin{aligned} V_{1A} &= I_{1A} Z_{11A} + I_{2A} Z_{12A} \\ V_{2A} &= I_{1A} Z_{21A} + I_{2A} Z_{22A} \end{aligned} \right\}$$

$$\left. \begin{aligned} V_{1B} &= I_{1B} Z_{11B} + I_{2B} Z_{12B} \\ V_{2B} &= I_{1B} Z_{21B} + I_{2B} Z_{22B} \end{aligned} \right\}$$



$$\left. \begin{aligned} V_1 &= V_{1A} + V_{1B} = I_1 (Z_{11A} + Z_{11B}) + I_2 (Z_{12A} + Z_{12B}) \\ V_2 &= V_{2A} + V_{2B} = I_1 (Z_{21A} + Z_{21B}) + I_2 (Z_{22A} + Z_{22B}) \end{aligned} \right\} \left. \begin{aligned} V_1 &= I_1 Z_{11} + I_2 Z_{12} \\ V_2 &= I_1 Z_{21} + I_2 Z_{22} \end{aligned} \right\}$$

$$Z_{11} = Z_{11A} + Z_{11B}$$

$$Z_{21} = Z_{21A} + Z_{21B}$$

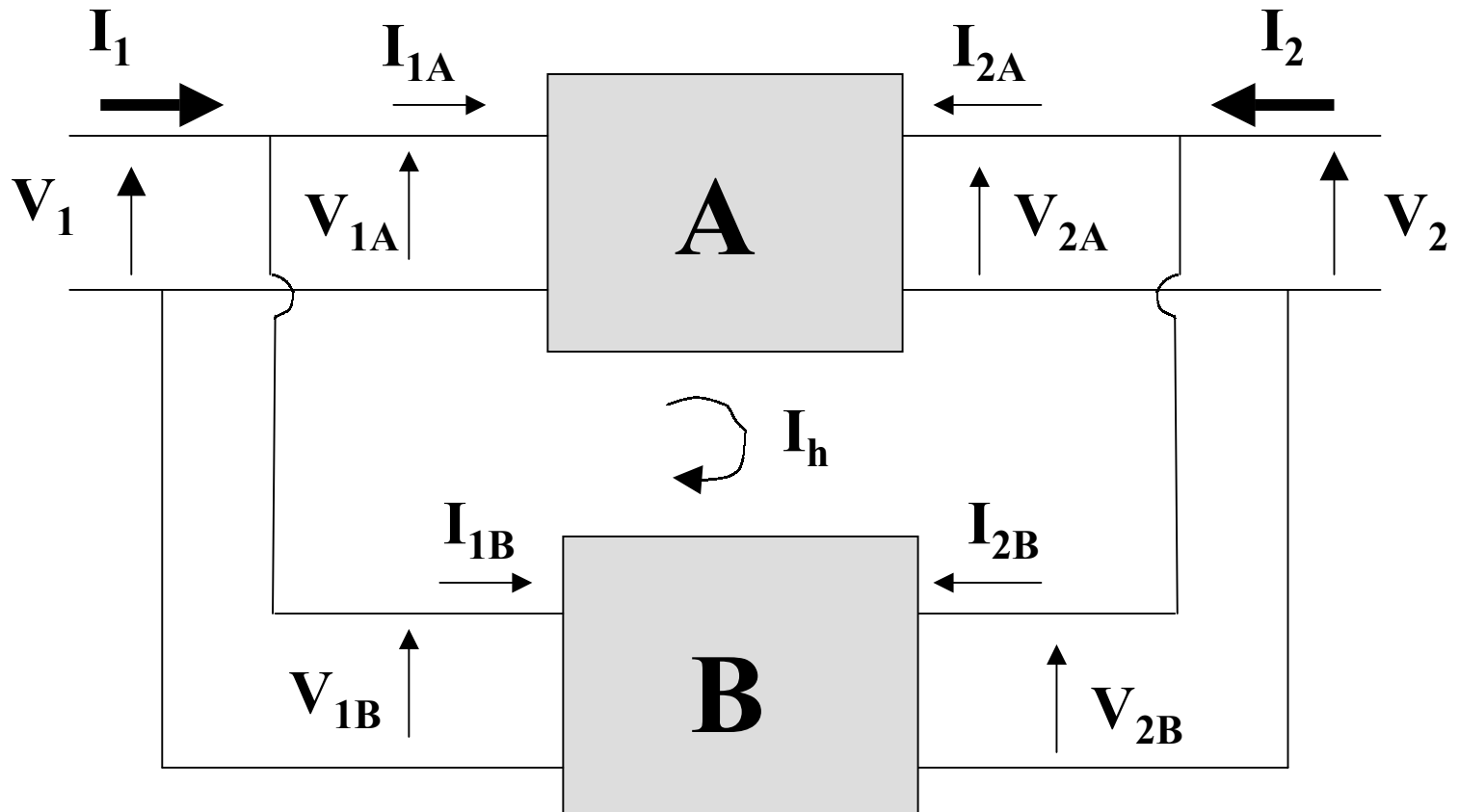
$$Z_{12} = Z_{12A} + Z_{12B}$$

$$Z_{22} = Z_{22A} + Z_{22B}$$

$$(Z) = (Z_A) + (Z_B)$$

Asociación Paralelo-Paralelo

Entradas y salidas en paralelo



Asociación Paralelo-Paralelo

$$V_1 = V_{1A} = V_{1B}$$

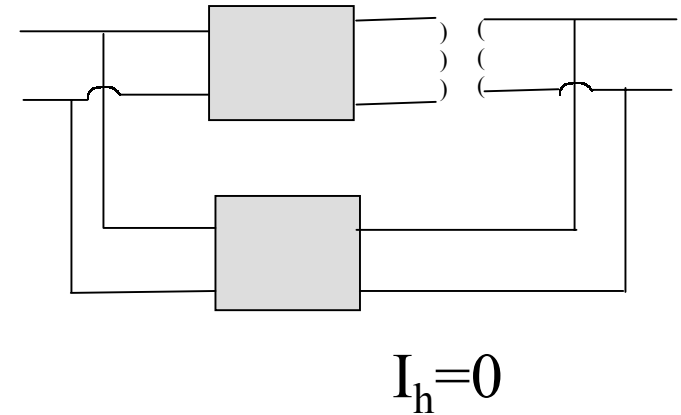
$$V_2 = V_{2A} = V_{2B}$$

$$I_1 = I_{1A} + I_{1B}$$

$$I_2 = I_{2A} + I_{2B}$$

$$\left. \begin{aligned} I_{1A} &= V_{1A} Y_{11A} + V_{2A} Y_{12A} \\ I_{2A} &= V_{1A} Z_{21A} + V_{2A} Y_{22A} \end{aligned} \right\}$$

$$\left. \begin{aligned} I_{1B} &= V_{1B} Y_{11B} + V_{2B} Y_{12B} \\ I_{2B} &= V_{1B} Y_{21B} + V_{2B} Y_{22B} \end{aligned} \right\}$$



$$\left. \begin{aligned} I_1 &= I_{1A} + I_{1B} = V_1 (Y_{11A} + Y_{11B}) + V_2 (Y_{12A} + Y_{12B}) \\ I_2 &= I_{2A} + I_{2B} = V_1 (Y_{21A} + Y_{21B}) + V_2 (Y_{22A} + Y_{22B}) \end{aligned} \right\} \begin{aligned} I_1 &= V_1 Y_{11} + V_2 Y_{12} \\ I_2 &= V_1 Y_{21} + V_2 Y_{22} \end{aligned}$$

$$Y_{11} = Y_{11A} + Y_{11B}$$

$$Y_{21} = Y_{21A} + Y_{21B}$$

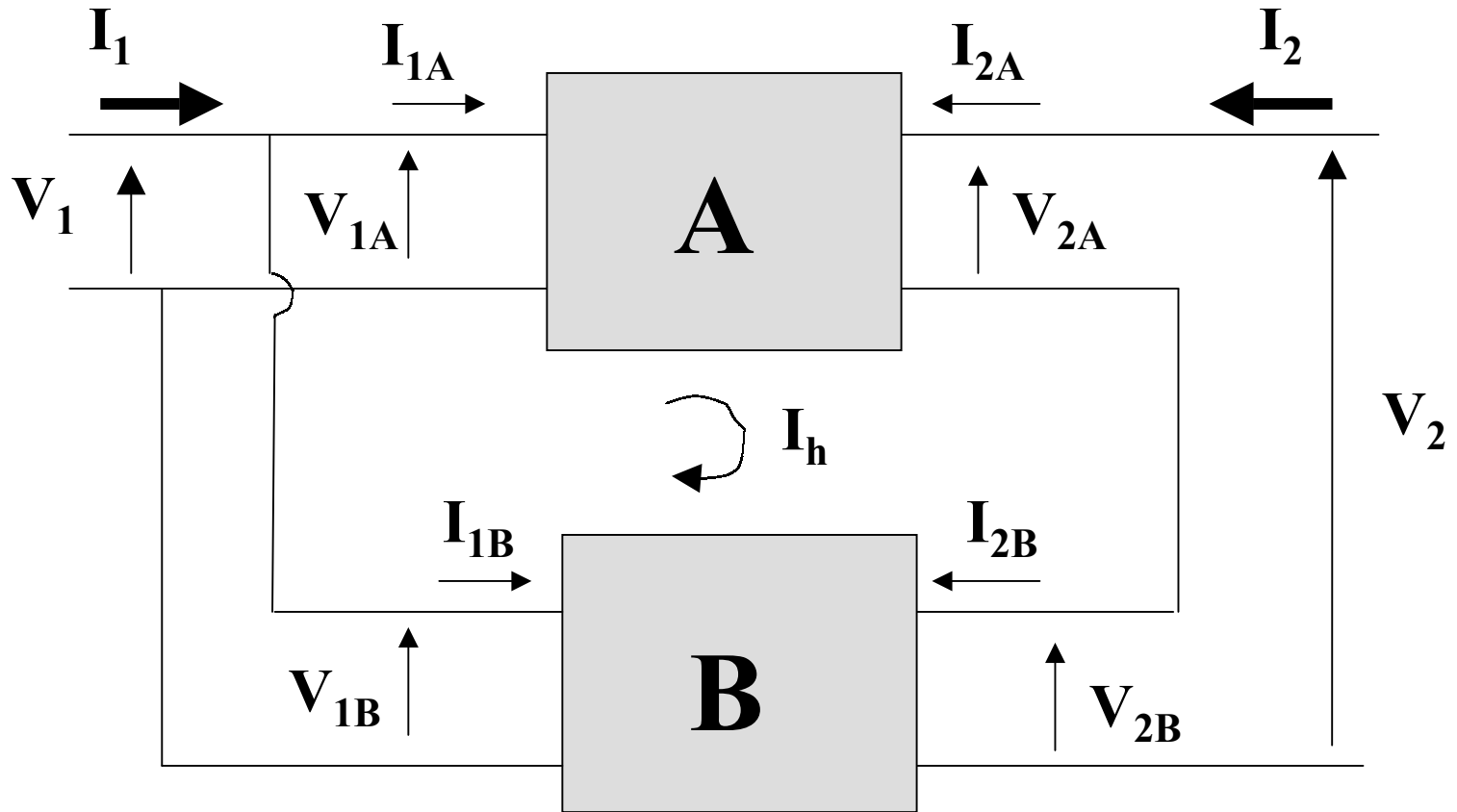
$$(Y) = (Y_A) + (Y_B)$$

$$Y_{12} = Y_{12A} + Y_{12B}$$

$$Y_{22} = Y_{22A} + Y_{22B}$$

Asociación Paralelo-Serie

Entradas paralelo y salida en serie



Asociación Paralelo-Serie

$$V_1 = V_{1A} = V_{1B}$$

$$V_2 = V_{2A} + V_{2B}$$

$$I_1 = I_{1A} + I_{1B}$$

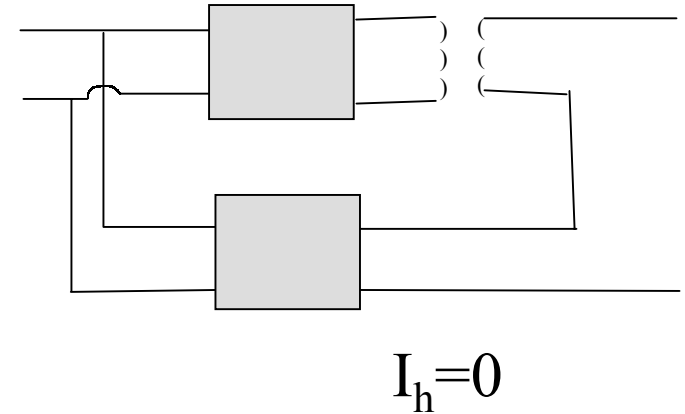
$$I_2 = I_{2A} = I_{2B}$$

$$I_{1A} = V_{1A} g_{11A} + I_{2A} g_{12A}$$

$$V_{2A} = V_{1A} g_{21A} + I_{2A} g_{22A}$$

$$I_{1B} = V_{1B} g_{11B} + I_{2B} g_{12B}$$

$$V_{2B} = V_{1B} g_{21B} + I_{2B} g_{22B}$$



$$I_1 = I_{1A} + I_{1B} = V_1 (g_{11A} + g_{11B}) + I_2 (g_{12A} + g_{12B})$$

$$V_2 = V_{2A} + V_{2B} = V_1 (g_{21A} + g_{21B}) + I_2 (g_{22A} + g_{22B})$$

$$I_1 = V_1 g_{11} + I_2 g_{12}$$

$$V_2 = V_1 g_{21} + I_2 g_{22}$$

$$g_{11} = g_{11A} + g_{11B}$$

$$g_{21} = g_{21A} + g_{21B}$$

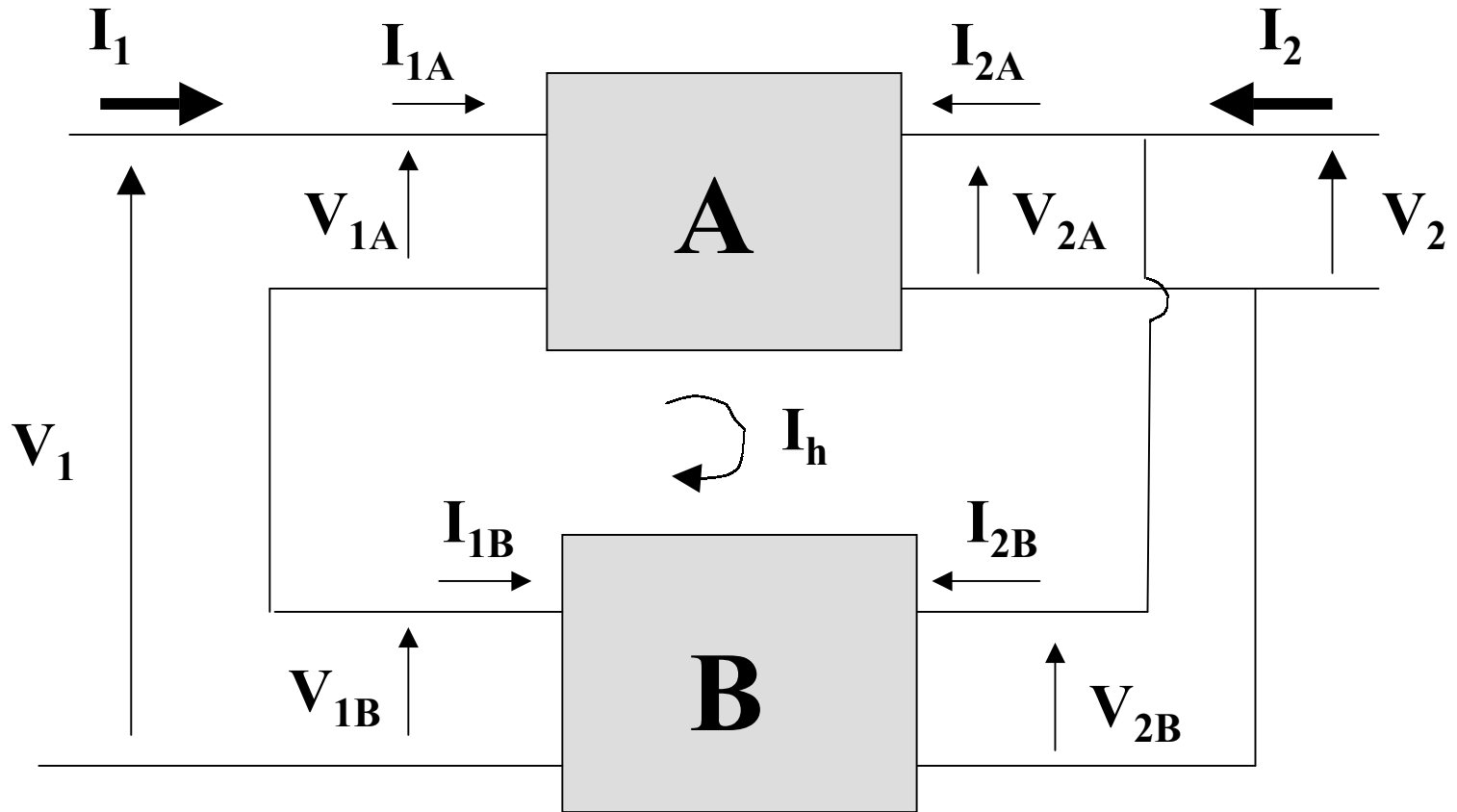
$$g_{12} = g_{12A} + g_{12B}$$

$$g_{22} = g_{22A} + g_{22B}$$

$$\mathbf{(g) = (g_A) + (g_B)}$$

Asociación Serie-Paralelo

Entradas serie y salida en paralelo



Asociación Serie- Paralelo

$$V_1 = V_{1A} + V_{1B}$$

$$V_2 = V_{2A} = V_{2B}$$

$$I_1 = I_{1A} = I_{1B}$$

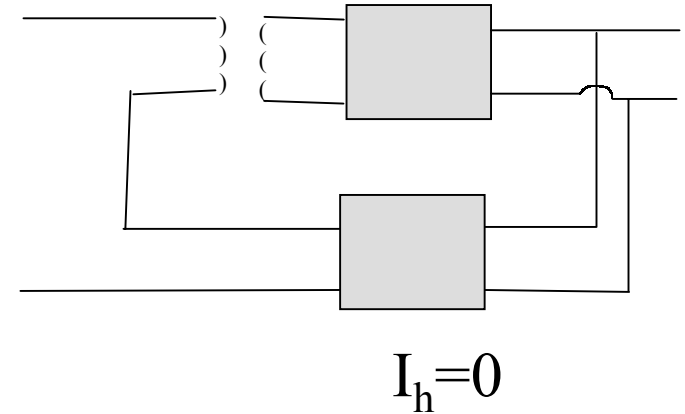
$$I_2 = I_{2A} + I_{2B}$$

$$V_{1A} = I_{1A} h_{11A} + V_{2A} h_{12A}$$

$$I_{2A} = I_{1A} h_{21A} + V_{2A} h_{22A}$$

$$V_{1B} = I_{1B} h_{11B} + V_{2B} h_{12B}$$

$$I_{2B} = I_{1B} h_{21B} + V_{2B} h_{22B}$$



$$V_1 = V_{1A} + V_{1B} = I_1 (h_{11A} + h_{11B}) + V_2 (h_{12A} + h_{12B})$$

$$I_2 = I_{2A} + I_{2B} = I_1 (h_{21A} + h_{21B}) + V_2 (h_{22A} + h_{22B})$$

$$V_1 = I_1 h_{11} + V_2 h_{12}$$

$$I_2 = I_1 h_{21} + V_2 h_{22}$$

$$h_{11} = h_{11A} + h_{11B}$$

$$h_{21} = h_{21A} + h_{21B}$$

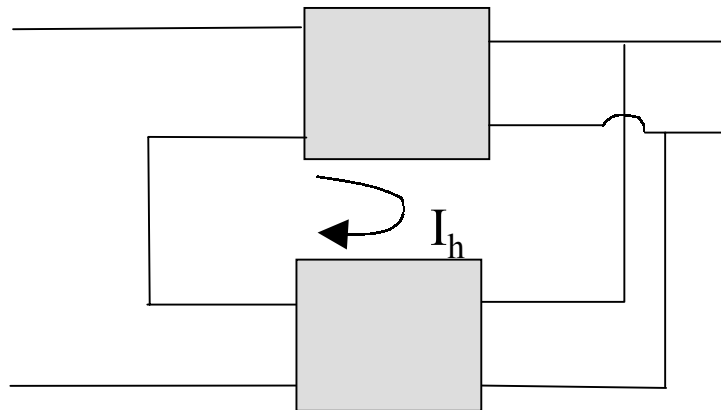
$$h_{12} = h_{12A} + h_{12B}$$

$$h_{22} = h_{22A} + h_{22B}$$

$$\mathbf{(h)} = \mathbf{(h_A)} + \mathbf{(h_B)}$$

Test de Brune

- Permite comprobar si $I_h = 0$
- Calcula la contribución de las variables independientes a I_h por superposición si ambas dan $I_h = 0$
- $I_h = 0 \Rightarrow$ los cuadripolos no interaccionan entre si y no hace falta poner un transformador



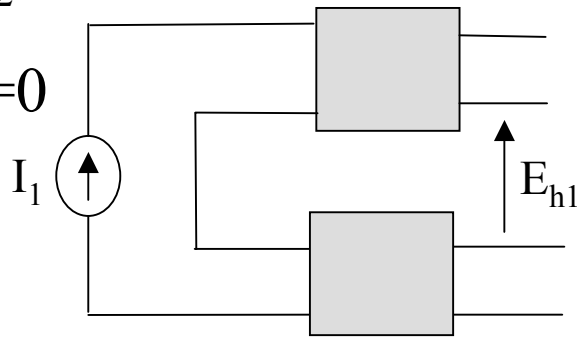
Test de Brune

Serie-Serie

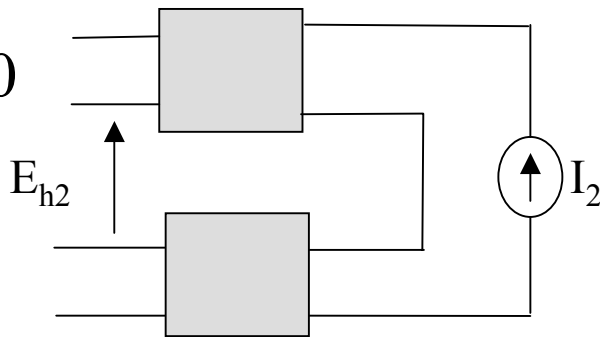
Variables independientes

I_1 e I_2

a) $I_2=0$



b) $I_1=0$



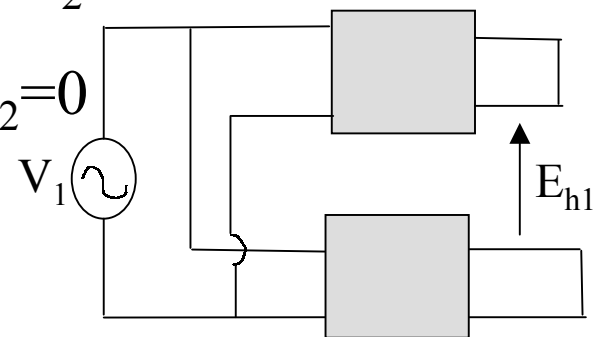
$$E_{h1}=0 \text{ y } E_{h2}=0 \Rightarrow I_h=0$$

Paralelo-Paralelo

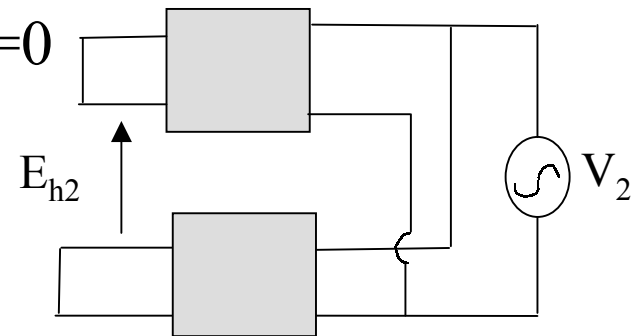
Variables independientes

V_1 y V_2

a) $V_2=0$



b) $V_1=0$



$$E_{h1}=0 \text{ y } E_{h2}=0 \Rightarrow I_h=0$$

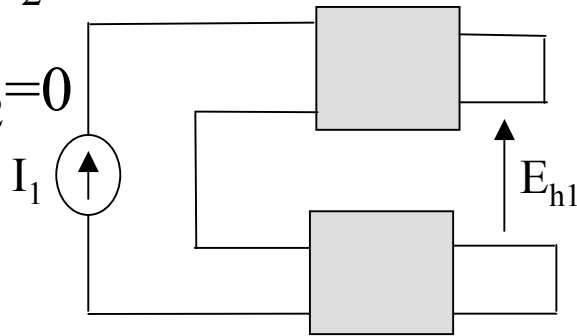
Test de Brune

Serie-Paralelo

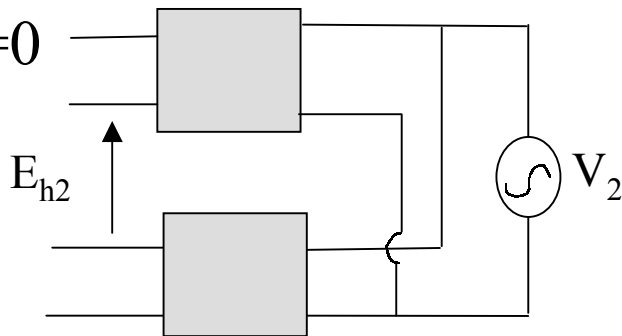
Variables independientes

I_1 y V_2

a) $V_2=0$



b) $I_1=0$



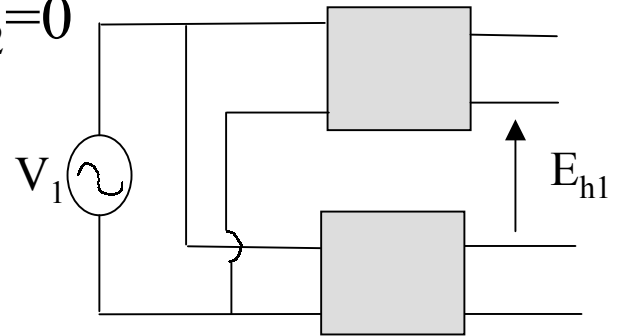
$$E_{h1}=0 \text{ y } E_{h2}=0 \Rightarrow I_h=0$$

Paralelo-Serie

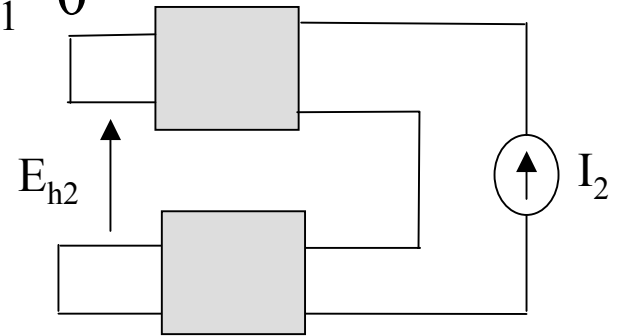
Variables independientes

V_1 e I_2

a) $I_2=0$

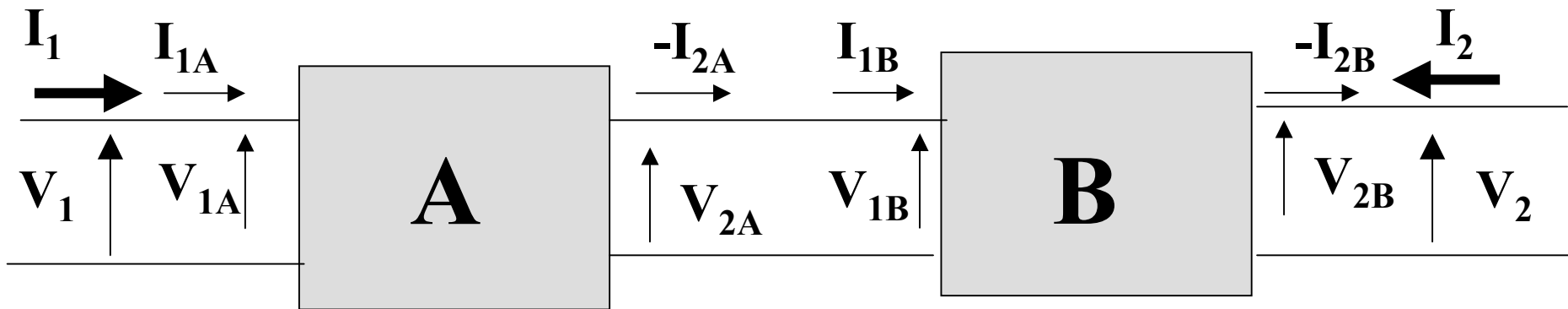


b) $V_1=0$



$$E_{h1}=0 \text{ y } E_{h2}=0 \Rightarrow I_h=0$$

Asociación en Cascada



Asociación en Cascada

$$\left. \begin{array}{l} V_1 = V_{1A} \\ V_2 = V_{2B} \end{array} \right\} \begin{array}{l} V_{1A} = V_{2A} A_A - I_{2A} B_A \\ I_{1A} = V_{2A} C_A - I_{2A} D_A \end{array} \left. \right\} \begin{array}{l} V_{1B} = V_{2B} A_B - I_{2B} B_B \\ I_{1B} = V_{2B} C_B - I_{2B} D_B \end{array} \left. \right\}$$

$$\begin{array}{l} I_1 = I_{1A} \\ I_2 = I_{2B} \end{array} \quad \begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} V_{1A} \\ I_{1A} \end{pmatrix} = \begin{pmatrix} A_A & B_A \\ C_A & D_A \end{pmatrix} \begin{pmatrix} V_{2A} \\ -I_{2A} \end{pmatrix} = \begin{pmatrix} V_{1B} \\ I_{1B} \end{pmatrix}$$

$$\begin{array}{l} I_{1B} = -I_{2A} \\ V_{1B} = V_{2A} \end{array} \quad \begin{pmatrix} V_{1B} \\ I_{1B} \end{pmatrix} = \begin{pmatrix} A_B & B_B \\ C_B & D_B \end{pmatrix} \begin{pmatrix} V_{2B} \\ -I_{2B} \end{pmatrix} = \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A_A & B_A \\ C_A & D_A \end{pmatrix} \begin{pmatrix} A_B & B_B \\ C_B & D_B \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} \quad (F) = (F_A) * (F_B)$$