Endogenous Job Contact Networks

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Abstract

We use the UK Quarterly Labour Force Survey to document the presence of two phenomena. First, a positive correlation between unemployment rate and the proportion of job seekers who use social networks to find jobs–*network use*. Second, a non-monotone relation between unemployment rate and the proportion of recently employed workers who found their job through friends or relatives–*network productivity*. This second relation is positive for low levels of unemployment rate, while it becomes negative for high levels of unemployment rate. Existing models of social networks and labor market generally assume that social networks are exogenous. This assumption implies a negative correlation between unemployment rate and network productivity. We develop a model where social networks are used to collect job offers, but workers decide strategically how much to invest in their network. We show that equilibrium job contact networks are dense and more productive in transmitting information when labor market turnover is moderate, while they are less productive and segmented into clusters for either high or low turnover. The equilibrium response of job contact networks to changes in labor market conditions is sufficient to generate the empirical patterns which we document for the UK.

JEL classifications: A14; J64; J63; D85; E24

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1 Introduction

It is well established that job contact networks play a prominent role in the matching between workers and vacant jobs. Empirical work, starting from Rees (1966) and Granovetter (1973), and generalized over the years, shows that between 30% and 50% of jobs are filled through social exchange of information.[1] This evidence has led to a number of theoretical studies in economics which explore the importance of social networks in labor markets, see, e.g., Boorman (1975), Montgomery (1991), Mortensen and Vishwanath (1994), Arrow and Borzekowski (2004), Calvó-Armengol (2004), Calvó-Armengol and Jackson (2004, 2007), Calvó-Armengol and Zenou (2005), Fontaine (2007) and Cahuc and Fontaine (2007). Most of this work (with some exception we shall describe below in this introduction) takes the network as given and focuses on its implications for aggregate and individual labor market outcomes.

A direct implication of the assumption that job contact networks are exogenous is a negative correlation between the probability that connections match job seekers with vacancy jobs (*network productivity*) and aggregate unemployment rate. In fact, in periods of high unemployment, most job seekers will be linked with other job seekers and therefore information flow of job offers in the network will be limited. We use the UK Quarterly Labor Force Survey (QLFS) over the period from 1995 to 2005 to explore whether this correlation is present in the data.[2] As a proxy of network productivity we consider the proportion of newly employed workers that found a job through a friend or acquaintance that worked at the same employee. We also consider total network productivity: the number of newly employed workers that found a job through a friend or acquaintance that worked at the same employee. Figure 1(a) and figure 1(b) depicts the network productivity and the total network productivity against unemployment rate in Great Britain over the period 1995 to 2005, respectively. Conflicting with the theoretical prediction, we find that for low levels of unemployment

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[1]The findings of Granovetter (1973) and Rees (1966) have been generalized across countries, industrial sectors and demographic characteristics, see e.g., Blau and Robins (1990), Topa (2000), Munshi (2003), Bayer et al. (2005), Cingano and Rosolia (2006), Loury (2006). See Ioannides and Loury (2004) for an exhaustive survey on social networks and labor market.

[2]Only males of working age (aged 16 to 64) resident in Great Britain are considered in our analysis. The sample is representative of the UK population. Appendix A provides a detailed account of the data and of the derivation of variables.
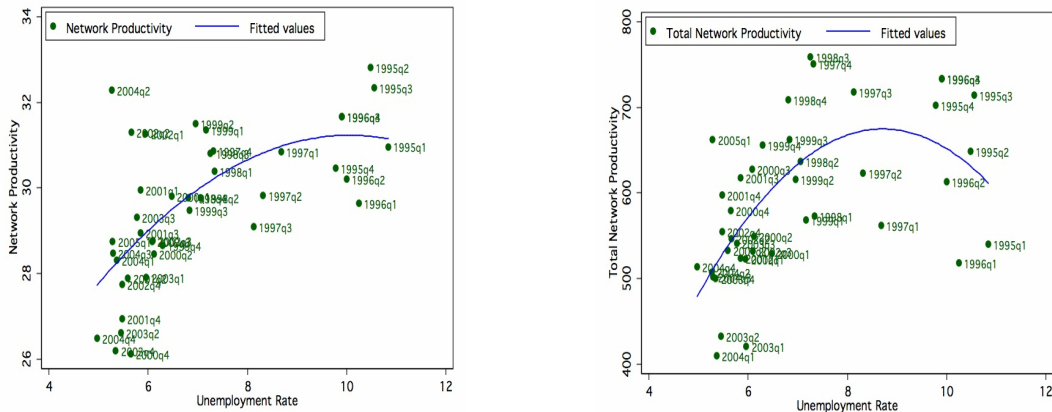
(a) Network productivity and unemployment rate in Great Britain (b) Total network productivity and unemployment rate in Great Britain

Figure 1: Productivity of networks and unemployment.

rate there is a strong positive relation between unemployment rate and network productivity. This relation eventually becomes negative for high levels of unemployment rate. A possible explanation for this pattern is that the extent to which individuals use or invest in their social networks depend on labor market conditions. As a proxy of network use we consider the proportion among job seekers that reported to use friends and acquaintances as the main job search method; total network use is the total number of job seekers that reported to use friends and acquaintances as the main job search method. Figure 2(a) and figure 2(b) depict network use and total network use against unemployment rate and it shows the presence of a strong positive correlation between the two.[3]

In view of these findings, we develop a simple model where the role of social networks as a manner of obtaining information is taken as given, but the actual job contact network is endogenously determined by strategic interaction. We will show that, by allowing individuals to adjust their network investment to changes in labor market conditions, we can replicate the empirical patterns documented in Figures 1-2.

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[3]We would like to make three remarks about these empirical patterns. First, Appendix A contains the estimates of the correlations and of the quadratic fit displayed in figures 1-2. Second, in the same Appendix, we show that these patterns still obtain when we disaggregate the data at a regional level. Finally, we have considered the relation between network productivity and network use at some period t with respect to lagged unemployment (unemployment at some period $t - \Delta$) and we found qualitatively similar patterns.
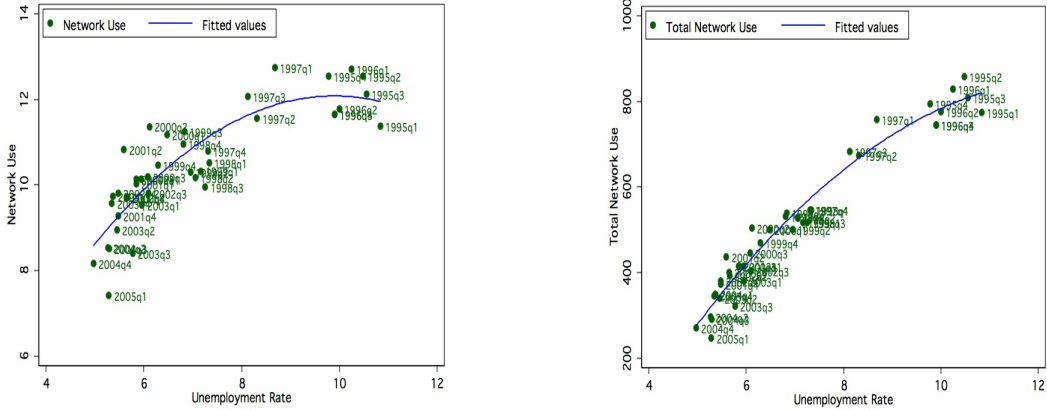
(a) Network use and unemployment in Great Britain

(b) Total network use and unemployment rate in Great Britain

Figure 2: The use of networks and unemployment.

In our basic model, workers lose their job with some probability–*job destruction rate*–, while with some other probability they access direct information about new available jobs–*vacancy rate*. Workers, anticipating the risk of becoming unemployed, invest in connections with the view of accessing information that other workers may have. The protocol of job contact network formation follows Cabrales et al. (2007). In particular, each worker chooses a level of socialization and the profile of socialization determines the meeting possibilities across pair of workers. When unemployed Paul meets Helen, he receives information about a vacant job only if Helen has it and she does not need it (i.e., Helen has a *needless offer*), and she has chosen Paul among all her unemployed friends. The incentives to form connections therefore depend on the riskiness of the labor market and on the value of connections in transmitting information, which, in turn, is related to the state of the labor market.

We first clarify the role that job contact networks play in matching workers with vacant jobs. For a given level of socialization, the likelihood of a match resulting from social interaction–*the network matching rate*–is described by a urn-ball matching function of the kind traditionally used in search models of the labor market, augmented by the socialization dimension.[4] The network

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[4]Urn-ball models have been extensively studied in probability theory. In labor market contexts, urns play the role

matching rate is increasing and concave in socialization effort, it depends positively on the ratio between the proportion of workers who have a needless offer–*job network supply*–and the proportion of unemployed–*job network demand*, and it is increasing in the average number of connections that workers form with unemployed. The network matching rate is a measure of the job contact network productivity: it corresponds to the proportional decrease of unemployment rate associated to a given socialization effort relative to unemployment rate when socialization effort is zero (there are not connections linking up workers).

Secondly, we characterize job contact equilibrium networks. Our model predicts that job contact networks are dense and more productive when job destruction rate and vacancy rate are moderate, while they are less productive and segmented into clusters for either high or low labor job destruction and vacancy rate. The equilibrium response of job contact networks to labor market conditions generates different patterns of unemployment rate and network productivity as compared to a model where the job contact network is fixed. We document a variety of new empirical predictions in the paper and we hope that this will stimulate further empirical research in this field. In this introduction, we focus on the equilibrium prediction of our model with regard to the relation between network productivity and unemployment rate and network use and unemployment rate. To illustrate this, we derive for different levels of job destruction rate the equilibrium socialization effort (network use) and the corresponding equilibrium level of network productivity and unemployment rate. Figure 3(a) and figure 3(b) plot network use and network productivity against unemployment rate, respectively. These equilibrium relations are non-monotonic and they are consistent with the patterns observed in the UK labor market (see figures 1-2).

The basic model assumes that a worker accesses a direct offer with some exogenous probability. In actual practice, the probability that a worker accesses directly information of a vacant job depends both on his direct search activity and on the number of new jobs that firms are willing to create. In the second part of the paper we examine how each of these two aspects interacts with workers'

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of vacancies, which has to be filled by balls, which are the workers. Each ball reaches a urn, chosen at random. From each urn a ball is extracted and a match between a vacancy and a worker is formed. Because of the randomness in the placement of balls, coordination frictions emerge. We refer to Mortensen and Pissarides (1999) and Petrongolo and Pissarides (2001) for surveys on the matching processes used in the labor market literature, and to Albrecht et al. (2004) for an extensions to the case of multiple applications.
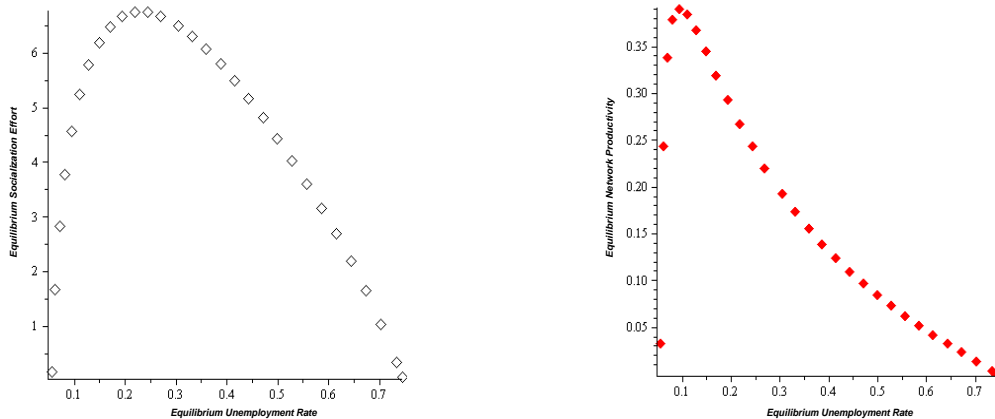
(a) Equilibrium network use and equilibrium unemployment rate

(b) Equilibrium network productivity and equilibrium unemployment rate

Figure 3: The use of networks and unemployment rate when vacancy rate equals 0.2, cost of socialization equals 0.01, job destruction rate varies from 0 to 1.

incentives to invest in connections and how this interplay shapes labor market outcomes.

Here, we discuss the effect of allowing workers to choose how much to invest in collecting information personally and how much to invest in their social network. These two decisions are strategically linked: marginal returns from personal search depend on network productivity, which, in turn, depends on the extent to which individuals collect information personally. This interplay leads to multiple equilibria. In one equilibrium personal search is pervasive and job contact networks are small, while in the other equilibrium the reverse holds. We can draw two conclusions from the characterization. First, network productivity is constant across equilibria, and the model predicts that groups with lower costs of acquiring information personally experience a higher network productivity, regardless of their network size[5] Second, even if network productivity is constant across equilibria, the high personal search equilibrium Pareto dominates the low personal search equilibrium and it has a lower unemployment rate. Hence, two distinctive groups, facing identical labor market conditions, may experience a very different unemployment rate, depending on the equilibrium they coordinate. Empirical work has widely documented the persistence of labor market inequalities in

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[5]This prediction is consistent with the findings of Battu et al. (2008) that immigrant groups with a higher level of country assimilation–which might be a proxy for higher hosting country language proficiency–have more productive social networks.

wages, unemployment rates and participation in the labor force across groups such as whites, blacks and hispanic whites that cannot be explained by individual characteristics.[6] Our findings show that the interplay between formal and informal search generates different unemployment outcomes across groups, where these differences are entirely driven by strategic interaction.

Summarizing, this paper illustrates the importance of considering the possible endogenous nature of job contact networks to labor market conditions. The relationship between job contact networks and labor market conditions should then be taken seriously in future empirical analysis.[7] It also clear that understanding how social network usage responds to changes in labor market conditions is important for the design of effective policy intervention which try to reduce labor market inequalities.

We have focused on labor markets, but the question of how the state of the economy shapes informal institutions is much broader. For example, there is a large empirical work on the effects of social capital on economic growth. Often this work struggles with the fact that social capital is itself an endogenous variable, arising concerns of identification of the models. Referring to this literature, Durlauf (2002, p. F474) noted: "...it seems clear that researchers need to provide explicit models of the codetermination of individual outcomes and social capital, so that the identification problems (...) may be rigorously assessed.". Our paper aims to contribute to this line of research providing a tractable model where the interplay among aggregate variables and individual investment in informal organizations is mapped into equilibrium correlations among these variables that cannot be accounted for in a model where informal institutions are taken as given.

As we have anticipated early in this introduction, most of the literature on labor market and social networks assumes that social networks are exogenous and are not affected by the labor market. Notable exceptions are Boorman (1975) and Calvó-Armengol (2004).[8] Boorman (1975) is the first

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[6]Trejo (1997) found that in the US, taking into account individual characteristics, one can explain only one third of the wage gap for Blacks and three quarters for Hispanics. Blackaby et al. (1998) reports similar findings for ethnic minorities in the UK. As for network usage across groups, Holzer (1987), using US data, finds that, while black and whites do not differ significantly in network usage (21.3% and 23.9% respectively), hispanics rely much more on social networks (32.8%).

[7]Most of the empirical work on labor market and social networks does not analyze the impact of labor market conditions on the use and productivity of social networks. Some exceptions are Battu et al. (2008), Topa (2001), Bayer et al. (2005) and Elliott (1999). In particular, Elliott (1999) studied the use of information job search channels by less educated urban workers and found that job seekers are more likely to use their social contacts when they live in high poverty neighborhoods.

[8]The theory of network formation is a very active field of study. For a survey of this research, see Goyal (2007) and

to provide a model of communication process among social contacts for job seeking purposes and his focus is on the incentives of workers to form weak *versus* strong ties. Calvó-Armengol (2004) provides a characterization of stable job contact networks in a two-sided link formation model of the type introduced by Jackson and Wolinsky (1996). Our model is complement to this early work. We do not focus on the specific details of the architecture of equilibrium job contact networks because our aim is to explore how labor market conditions affect social networks usage and productivity taking an explicit account of their feedbacks on workers' incentives to form connections. To the best of our knowledge, our paper is the first to analyze these questions systematically.

Our work is also related to the literature of labor search pioneered by Diamond (1982). For recent surveys see Pissarides (2000) and Mortensen and Pissarides (1999). We borrow from this literature the idea that search units are chosen optimally to maximize the net returns from search and we apply it to the formation of job contact networks. Our extension on Section 5 combines individual search, which has been the focus of this literature, with informal search in networks. In the conclusion we elaborate on how to extend our framework to a dynamic labor search model.

The rest of the paper is organized as follows. Section 2 develops the basic model. Section 3 derives the network matching rate. Section 4 characterizes equilibria and discusses the predictions of our model. Section 5 extends the basic model to allow for individual search and endogenous vacancy rate. Section 6 concludes. All proofs are in Appendix B.

2 Model

The model has three building blocks: labor market turnover, the formation of job contact networks and information diffusion within the network. There is a large set of risk neutral workers $\mathcal{N} = \{1, ..., n\}$. Initially all workers are employed and earn an exogenous wage that, without loss of generality, is normalized to 1.

Labor Market Turnover. Two exogenous parameters govern the labor market turnover.

Job destruction. A randomly selected sample of size $B = bn > 1$ of workers become unemployed,

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Jackson (2008). Complex random networks have also received large attention in economics, e.g., Galeotti et al. (2007) and Cabrales, Calvó-Armengol and Zenou (2007); for a survey see Vega-Redondo (2007).

where $b \in (0,1) \cap \mathbb{Q}$. We denote by $\mathcal{B} \subset \mathcal{N}$ the set of workers who lose their job and we call them *job seekers*.

Job opening. A number $V = an$ of new vacancies opens in the market, where $a \in (0,1] \cap \mathbb{Q}$. These vacant jobs are distributed to workers in the following way: bV vacancies reach a randomly selected sample of job seekers, while the remaining vacancies reach a randomly selected sample of workers who did not lose their job. Let $\mathcal{A} \subset \mathcal{N}$ be the set of workers who receive a direct job offer.

Under this protocol, nobody receives more than one direct job offer. The set $\mathcal{U} = \mathcal{B} \cap \{\mathcal{N} \setminus \mathcal{A}\}$ is the set of workers who have lost their job and did not receive a direct offer; note that $|\mathcal{U}| = b(1-a)n$. The set $\mathcal{O} = \mathcal{A} \cap \{\mathcal{N} \setminus \mathcal{B}\}$ contains workers who have not lost their job and received a direct offer. We say that a worker $i \in \mathcal{O}$ has a *needless offer* and we note that there are $|\mathcal{O}| = a(1-b)n$ needless offers.

Ex-ante, a representative worker anticipates that with probability $b(1-a)$ he will be unemployed and without a new offer, in which case he will earn an unemployment benefit which, without loss of generality, is normalized to 0. In order to insure themselves against this risk, workers invest in social connections with the view of accessing needless offers.

Job Contact Network. We specify the network formation game below. For the moment let's assume that workers are located in an undirected network g. A link between workers i and j is denoted by $g_{ij} = 1$, while $g_{ij} = 0$ means that i and j are not linked. The set of all possible undirected networks is $\mathcal{G}$. With some abuse of notation, we denote the set of i's neighbors belonging to $\mathcal{V} \subset \mathcal{N}$ in network g as $\mathcal{N}_i(\mathcal{V}) = \{j \in \mathcal{V} \setminus \{i\} : g_{ij} = 1\}$; $\eta_i(\mathcal{V}) = |\mathcal{N}_i(\mathcal{V})|$ is the number of links that i has with workers belonging to $\mathcal{V}$.

Job transmission in the network. We assume that information about jobs flow only from workers with a needless offer to job seekers. Formally, each $i \in \mathcal{O}$ passes the information to one and only one $j \in \mathcal{N}_i(\mathcal{B})$, chosen at random. If i has not links with job seekers, i.e., $\mathcal{N}_i(\mathcal{B})$ is the empty set, the offer is lost.[9] We are assuming that information only flows one-step in the network. In actual

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[9]We are assuming that a worker with a needless offer passes the information only to one of his social contacts, chosen at random. This assumption greatly simplifies the analysis, but it may be not realistic. The implication of this assumption is that two job seekers both connected with a worker with a needless offer will "compete" for such offer. Suppose, we relax this assumption and let the worker with a needless offer to give it to both job seekers. Then the two job seekers will have an identical offer and therefore competition for the job will still be present in the hiring process

practice, we may receive information from social contacts, which in turn they have received from their acquaintances, e.g., Granovetter (1973, 1974). Indirect information flow can be easily accommodated in our framework as we show in Appendix C.

Formation of job contact networks. The protocol of network formation follows Cabrales et al. (2007). We consider the following simultaneous network formation game. Each worker i chooses a costly networking effort $s_i \geq 0$; the cost of effort s_i is cs_i.[10] The set of pure strategies available to worker i is $\mathcal{S}_i = \mathbb{R}_+$. A pure strategy profile is $\mathbf{s} = (s_1, ..., s_n) \in \mathcal{S} = \mathbb{R}_+^n$, and $\mathbf{s}_{-i}$ indicates the strategies of all workers other than worker i. We denote by $y(\mathbf{s}) = \sum_{i \in \mathcal{N}} s_i$ the aggregate workers' socialization effort. For a profile $\mathbf{s}$, we assume that a link between an arbitrary pair of workers i and j forms with probability [11]

$$\Pr(g_{ij} = 1|\mathbf{s}) = \begin{cases} \min\left\{\frac{s_i s_j}{y(\mathbf{s})}, 1\right\} & \text{if } y(\mathbf{s}) > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

A profile $\mathbf{s}$ generates a multinomial random graph. When workers choose the same level of effort, say s, the induced random graph is binomial, the probability that two workers are connected equals the per-capita socialization effort, $\min\{s/n, 1\}$, and the average connectivity of the random graph (the expected number of neighbors of a node) is $\min\{s/n, 1\}(n-1)$.[12]

There are two complementary interpretations of this process of network formation. The first interpretation is that socialization effort reflects the time that an individual spends in organizations, clubs, conferences, churches, etc. An individual who participates in many organizations has greater chances to meet other people and form connections with them. In turn, these connections may provide valuable information about job opportunities. The second interpretation is that individuals are connected in a pre-existing network. For example, we may think of a group of immigrants leaving

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(given that one job offer corresponds to one vacancy).

[10]All the results we present can be derived for arbitrary cost functions $C(s)$ which are increasing and convex in s.

[11]Expression (1) can be derived by requiring three axioms on network formation. These axioms are: *one*, undirected links, i.e., $\Pr(g_{ij} = 1|\mathbf{s}) = \Pr(g_{ji} = 1|\mathbf{s})$; *two*, aggregate constant returns to scale, i.e., for all $i \in \mathcal{N}$, $\sum_{j=1}^{n} \Pr(g_{ij} = 1|\mathbf{s}) = s_i$; and *three*, anonymous link formation, i.e., for all $j, l \in \mathcal{N}$, $\Pr(g_{ji} = 1|\mathbf{s})/s_j = \Pr(g_{li} = 1|\mathbf{s})/s_l$, for all $i \in \mathcal{N} \setminus \{j, l\}$. See Cabrales et al. (2007) for details.

[12]We refer to Erdos and Reny (1959) for a study of binomial random graphs, and to Chung and Lu (2002) for a study of multinomial random graphs. Vega-Redondo (2007) and Jackson (2008) provide a detailed overview of the rapidly growing literature on complex networks.

in the same neighbor and define the pre-existing network based on their geographical proximity. While the existence of such a link (e.g., leaving in closed proximity) is a necessary condition for information exchange, it is not sufficient: for the information to flow from one individual to another their communication links must be active, which requires effort from both workers. In this case, workers' socialization effort determines the strength of each pre-existing link in the community– reliability of connections.[13]

Utilities and Equilibrium. For a strategy profile $\mathbf{s} = (s_i, \mathbf{s}_{-i})$, let $\Psi_i(\mathbf{s})$ be the probability that worker $i \in \mathcal{B}$ accesses at least one offer from the network, which we shall refer as to i's network matching rate and that we will derive in the next section. The expected utility to a worker $i \in \mathcal{N}$ is:

$$EU_i(s_i, \mathbf{s}_{-i}) = 1 - b(1-a)[1 - \Psi_i(\mathbf{s})] - cs_i.$$

The last term represents the cost of socialization, while the first part is the probability that worker i will be employed and therefore earning a wage 1. This is the complement of the probability that worker i is a job seeker and neither he accesses a direct offer nor he accesses information from the network. A pure strategy equilibrium is $\mathbf{s}$ such that, for all $i \in \mathcal{N}$,

$$EU_i(s_i, \mathbf{s}_{-i}) \geq EU_i(s_i', \mathbf{s}_{-i}), \forall s_i' \in \mathcal{S}_i.$$

We focus on pure strategy symmetric equilibrium in large labor markets, hereafter equilibrium. A large labor market is a labor market in which $n, V, B \rightarrow \infty$ such that $\lim_{n \to \infty} B/n = b$ and $\lim_{n \to \infty} V/n = a$.[14]

3   Unemployment and network productivity in exogenous networks

This section clarifies the role that job contact networks play in matching job seekers with vacant positions and the relation between unemployment rate and network productivity in a model where

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[13]Following up on this interpretation, we note that in this paper we are assuming that the pre-existing network is complete. It may be interesting to extend the framework to general topology of pre-existing networks.

[14]For sake of clarity, we present the analyses for large labor markets. However, our results can be easily extended to an environment with finite and sufficiently large n.

job contact networks are exogenous. We shall see that the likelihood of a match resulting from social exchange of information is described by a urn-ball matching function of the kind traditionally used in search models.[15] The arguments of this function are the ratio between supply and demand of job information in the network and the average connectivity of the job contact network. Furthermore, as is standard in models with exogenously fixed social networks, we find a negative correlation between unemployment rate and the productivity of job contact networks.

There are two ways in which vacant positions and job seekers may be matched: job seekers may receive a direct offer and/or they may access an offer from their social contacts. Select a job seeker $i \in \mathcal{B}$, who chooses s_i. For simplicity, let's assume that $s_j = s$ for all $j \neq i$, for the time being, and let $\mathbf{s} = (s_i, \mathbf{s}_{-i})$. Since all offers are identical, the probability that worker i finds a job–the *matching rate*–is given by:

$$m_i(\mathbf{s}) = a + (1 - a)\Psi_i(\mathbf{s}).$$

We now derive the network matching rate of a job seeker i, i.e., $\Psi_i(\mathbf{s})$. With some abuse of notation, let $p_i = \Pr(g_{ij} = 1|\mathbf{s})$, for all $j \neq i$, and $p = \Pr(g_{jj'} = 1|\mathbf{s})$, for all $j, j' \neq i$. The probability that $i \in \mathcal{B}$ has η links with workers who have a needless offer is:

$$\Pr\left(\eta_i(\mathcal{O}) = \eta\right) = B\left(\eta|p_i, |\mathcal{O}|\right),$$

where $B(\cdot|p_i, |\mathcal{O}|)$ is the binomial distribution, p_i is the probability of a success and $|\mathcal{O}|$ is the number of trials. When determining the probability that $i \in \mathcal{B}$ gets a job from one of his acquaintances, we must consider the number of links that an arbitrary i's neighbor with a needless offer, say $j \in \mathcal{N}_i(\mathcal{O})$, has with other job seekers, i.e., $\Pr(\eta_j(\mathcal{B}) = \omega|g_{ij} = 1)$. This is clearly not the same as the probability that j has η links with job seekers. In fact, we need to take into account that j has already a link

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with job seeker i. Using Bayes' rule, we have that for all $\omega \in 1, ..., |\mathcal{B}|$: [16]

$$\Pr\left(\eta_j(\mathcal{B}) = \omega | g_{ij} = 1\right) = \frac{\frac{\omega}{|\mathcal{B}|} \Pr\left(\eta_j(\mathcal{B}) = \omega\right)}{\sum_{v=0}^{|\mathcal{B}|} \frac{v}{|\mathcal{B}|} \Pr\left(\eta_j(\mathcal{B}) = v\right)} = \frac{\omega \Pr\left(\eta_j(\mathcal{B}) = \omega\right)}{E[\eta_j(\mathcal{B})|\mathbf{s}]}, \tag{2}$$

where $E[\eta_j(\mathcal{B})|\mathbf{s}] = p(nb - 1) + p_i$ is the expected number of links that j has with job seekers under profile $\mathbf{s}$. Consider now such a worker $j \in \mathcal{N}_i(\mathcal{O})$. If j is connected with ω job seekers (including i), the probability that j passes information to i is $1/\omega$. So, the probability that j gives information to i is:

$$\sum_{\omega=1}^{|\mathcal{B}|} \Pr(\eta_j(\mathcal{B})) = \omega | g_{ij} = 1) \frac{1}{\omega}.$$

We now observe that the probability that each i's neighbors with a needless offer passes the information to i is independent. Hence, if worker i has η links with workers like j, the probability that i does not hear about a new job is:

$$\left[1 - \sum_{\omega=1}^{|\mathcal{B}|} \Pr(\eta_j(\mathcal{B}) = \omega | g_{ij} = 1) \frac{1}{\omega}\right]^{\eta}.$$

Thus, the expected probability that $i \in \mathcal{B}$ does not get an offer via the network is:

$$\phi_i(\mathbf{s}) = \sum_{\eta=0}^{|\mathcal{O}|} \left\{ \Pr(\eta_i(\mathcal{O}) = \eta) \left[1 - \sum_{\omega=1}^{\mathcal{B}} \Pr(\eta_j(|\mathcal{B}|) = \omega | g_{ij} = 1) \frac{1}{\omega}\right]^{\eta} \right\}. \tag{3}$$

Using the binomial identity, $|\mathcal{B}| = bn$ and $|\mathcal{O}| = na(1 - b)$, we can rewrite (3) as follows:

$$\phi_i(\mathbf{s}) = \left[1 - p_i \frac{1 - (1 - p_i)(1 - p)^{nb-1}}{p(nb - 1) + p_i}\right]^{na(1-b)}. \tag{4}$$

Since $p = s^2 / [s(n - 1) + s_i]$ and $p_i = s_i s / [s(n - 1) + s_i]$, in large labor markets we obtain:

$$\lim_{n \to \infty} \phi_i(\mathbf{s}) = e^{-\frac{a(1-b)}{b} \frac{1 - e^{-sb}}{s} s_i}.$$

Therefore, when $s_j = s$ for all $j \neq i$ and worker i chooses s_i, the probability that $i \in \mathcal{B}$ gets at least

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[16]In general, suppose we have a binomial random graph with n workers. Let p be the probability that an arbitrary link is formed so that $z = p(n - 1)$ is the average connectivity of the random graph. Then $\Pr(\eta_j(\mathcal{N}) = \omega | g_{ij} = 1) = \frac{\Pr(g_{ij}=1|\eta_j(g)=\omega) \Pr(\eta_j(g)=\omega)}{\sum_{l=0}^{n-1} \Pr(g_{ij}=1|\eta_j(g)=l) \Pr(\eta_j(g)=l)}$. Since each link is formed independently with probability p, we have that $\Pr(g_{ij} = 1 | \eta_j(g) = l) = l/(n - 1)$, which implies that $\Pr(\eta_j(g) = \omega | g_{ij} = 1) = [\omega \Pr(\eta_j(g) = \omega)]/z$.

one offer via the network is

$$\Psi_i(s_i, s) = 1 - e^{-\frac{a(1-b)}{b}\frac{1-e^{-sb}}{s}s_i}. \tag{5}$$

Remark 1 *Suppose $s_j = s$ for all $j \neq i$ and worker i chooses s_i. The network matching rate for worker i is increasing in his own socialization effort while it is decreasing in others' socialization effort.*

This remark captures two simple ideas. The first idea is that connections lead to personal advantages: the greater the connectivity of an individual, the higher is the chance of accessing job information.[17] The second idea is that job seekers compete for job offers in the network: the value of a job seeker's connections decreases with the number of connections formed by other job seekers.

The following lemma illustrates the properties of the network matching rate, the matching rate and overall unemployment rate, when workers choose the same socialization effort s. In that case, each job seeker has the same network matching rate, which we denote by $\Psi_i(s, s) = \Psi(s)$; we can then write the overall matching rate as $m(s) = a + (1 - a)\Psi(s)$. Unemployment rate is denoted by $u(s) = b(1-a)(1 - \Psi(s))$. We also define the *productivity* of the network as the proportional decrease in unemployment relative to a situation in which workers are isolated, i.e., $\Delta(s) = [u(0) - u(s)]/u(0)$.

Lemma 1 *Consider a large labor market and suppose that $s_i = s$ for all $i \in \mathcal{N}$. The network matching rate equals the productivity of the network, namely*

$$\Psi(s) = \Delta(s) = 1 - e^{-\frac{a(1-b)}{b}\left(1 - e^{-sb}\right)}. \tag{6}$$

The network matching rate and the matching rate are increasing and concave in a and in s, while they are decreasing in b. Unemployment rate is decreasing and convex in a and s, while it is increasing in b.

To understand how network information transmission alleviates labor market frictions and determines the level of unemployment rate, it is useful to start considering the extreme case in which

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[17]The idea that social connections create personal advantages is a fundamental premise of the influential work of Granovetter (1973,1974) on job contact networks. A number of recent empirical studies document the role of connections in providing personal advantages in the labor market, e.g., Loury (2006).

the social network is complete, i.e., $s \to \infty$. In this case the network matching rate equals

$$1 - e^{-\frac{a(1-b)}{b}}.$$

In a complete network the rate at which a match results from the social exchange of information is described by a standard urn-ball matching function of the kind traditionally employed in search models of the labor market. In our context, each job seeker is an urn, and every employed worker with a needless offer places a ball (the job offer) in one randomly selected urn. Not all the needless offers result in a match because some job seekers will receive multiple offers, while others none. These frictions depend negatively on the balls-urns ratio, which is the ratio between the proportion of needless offers–*the job network supply*–and the proportion of job seekers–*the job network demand*, i.e., $a(1-b)/b$.

When job contact networks are not complete, the productivity of the network is lower because with positive probability an employed worker with a needless offer only connects with other employed workers, a fact which prevents information to flow.[18] In particular, the probability that a worker with a needless offer has only employed friends is $(1 - s/n)^{nb}$. When n goes to infinity this converges to e^{-sb}, where sb is the average number of links that a worker has with job seekers. Thus, the higher is workers' average connectivity with job seekers, the more productive is the network in transmitting information.

Overall, Lemma 1 shows that unemployment rate and network productivity are negatively correlated: an increase (decrease) in job destruction rate (vacancy rate) decreases network productivity and it increases unemployment rate. This is a prediction which is obtained in all existing models with exogenous social networks. Indeed, for a given network of contacts, changes in labor market conditions which decrease job network supply relatively to job network demand will unambiguously increase unemployment rate and decrease network productivity. As we documented in the introduction these predictions are not consistent with empirical patterns of unemployment and network productivity in the UK labor market. In the next section, we shall see that a key element to explain

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[18]Note that this inefficiency is not an artifact of our restriction on one-step information flow. Suppose, for example, that information flow two steps. Then a needless offer is lost whenever the owner of this offer is isolated and, otherwise, when he passes the information to an employed worker, who in turn is only connected with other employed workers.

these empirical correlations is to allow individuals to adjust their network investment to changes in the underlying state of the labor market.

4   Analysis

We now study the implication of allowing individuals to choose strategically how much they wish to invest in the job contact network. We first show that the equilibrium level of network investment depends non-monotonically on the level of labor market turnover: job contact equilibrium networks are denser when job-destruction rate and/or vacancy rate are neither too low nor too high. Secondly, we provide examples to illustrate the implication of endogenous job contact networks on the equilibrium relation between unemployment rate and both network productivity and network investment. We also illustrate how connections generate substantial inequalities in the hazard rate of *ex-ante* identical workers and how the level of these inequalities depend on labor market conditions. We finally show that individuals over-invest in social connections relatively to what is socially desirable.

4.1   Job contact equilibrium networks

The following proposition characterizes interior equilibria.[19]

Proposition 1 *Consider a large labor market. An interior equilibrium s^* exists if and only if $c < ab(1-a)(1-b)$, and s^* is the unique solution to:*

$$b(1-a)\frac{\partial \Psi_i}{\partial s_i}(s^*, s^*) = c \tag{7}$$

In the unique interior equilibrium, socialization effort balances worker's marginal returns with marginal costs. The marginal returns are the marginal increase in network productivity, weighted by the likelihood that the worker needs the network to find a job, $b(1-a)$. In particular, the marginal

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[19]We note that there always exists a zero socialization equilibrium, i.e., $s_i = 0$ for all $i \in \mathcal{N}$. However, analyzing the best response dynamic after a perturbation around the equilibrium, it is possible to show that the zero socialization equilibrium is unstable whenever it coexists with an interior equilibrium. A formal proof of this statement is available upon request to the authors.

increase in network productivity is:

$$\frac{\partial \Psi_i}{\partial s_i}(s^*, s^*) = \frac{a(1-b)}{bs^*}\left(1 - e^{-s^*b}\right)e^{-\frac{a(1-b)}{b}\left(1-e^{-s^*b}\right)}.$$

This simple characterization allows us to investigate how labor market turnover shapes job contact networks. Are job contact networks more connected when job-destruction rate (respectively, vacancy rate) is high, or rather when job-destruction rate (respectively, vacancy rate) is low?

Let us look at the economic forces at work here. An increase in the job destruction rate b makes investment in social networks by individuals more attractive as the labor market becomes more risky (complement effect). On the other hand, by decreasing job network supply and increasing job network demand, an increase in job destruction rate decreases the marginal value of connections, which makes investment in networks less attractive (substitute effect). We now observe that when job-destruction rate is low to begin with, the job contact network is not very connected because the labor market is not very risky. In this case, an increase in job destruction rate decreases the marginal value of connections only slightly, and, overall, the complement effect dominates the substitute effect. As jobs are destroyed at an higher rate, the network becomes denser and this amplifies the substitute effect. Thus, when b is sufficiently high, its increase decreases socialization effort. The following proposition confirms these intuitions.

Proposition 2 *Consider a large labor market and suppose that $c < ab(1-a)(1-b)$. 1) For every $b \in (0,1)$, there exists $\bar{a}(b) > 0$ such that if $a < \bar{a}(b)$, then s^* increases in a, otherwise, it decreases in a. 2) For every $a \in (0,1)$, there exists $\bar{b}(a) > 0$ such that if $b < \bar{b}(a)$, then s^* is increasing in b, otherwise, it decreases in b.*

In view of Proposition 2, our model predicts that job contact networks are more likely to be segmented into clusters when the labor market turnover is extremely high or low. Indeed, from the theory of random graphs we know that in a Poisson graph when the average connectivity is lower than 1 the resulting network comprises different segregated components. Otherwise, the majority of workers are, almost surely, connected directly or indirectly (the network has a giant component).[20]

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[20]We refer to Vega-Redondo (2007) and Jackson (2008) for a formal treatment of random graphs.
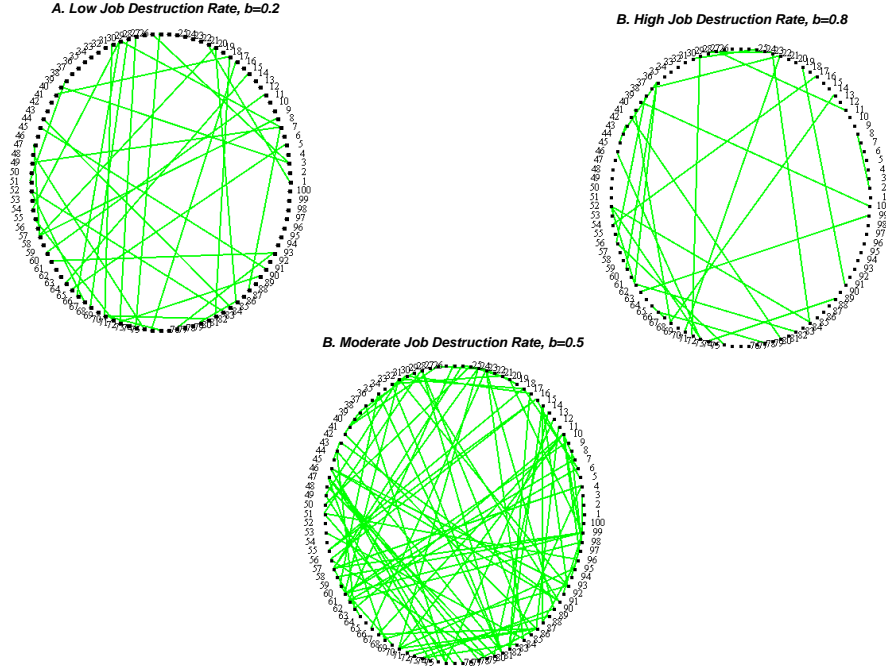
Figure 4: Equilibrium job contact networks and job destruction rate, when $a = 0.4$, $c = 0.03$, and $n = 100$.

Here, the average connectivity of the network is s^* and Proposition 2 suggests that a giant component will emerge when the level of labor market turnover is at intermediate situations. Figure 4 illustrates these findings. Panel A and panel B of figure 4 depict a realization of the equilibrium (random) network for low and high job destruction rates, respectively, while panel C of figure 4 depicts a realization of the equilibrium network in a labor market with moderate job destruction rate.

While Proposition 2 provides clear cut results on how labor market conditions shapes job contact networks, it is silent about how labor market conditions affect network productivity and unemployment rate both at the aggregate and at individual level. We now elaborate on these issues.

We first observe that the non-monotonic relation between job contact networks and job destruction rate leads to a similar relation between network productivity and job destruction rate. In fact, when job destruction rate is low, job contact networks are sparse which implies that network productivity is low. In this case, an increase in job destruction rate increases the connectivity of the network and, consequently, its productivity also increases. However, as job destruction rate increases further, the network becomes crowded of job seekers. This creates a congestion effect which even-
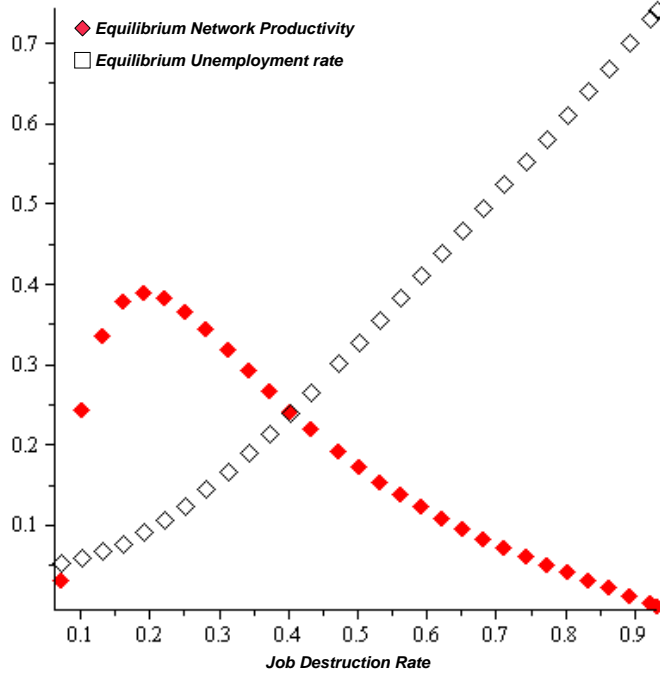
17

Figure 5: Equilibrium network productivity, unemployment rate and job destruction rate, when vacancy rate equals 0.2, cost of socialization equals 0.01, job destruction rate varies from 0 to 1.

tually reduces network productivity. Figure 5 shows how the equilibrium network productivity and the equilibrium unemployment rate are affected by job destruction rate.

Our second observation, is that the equilibrium relation between unemployment rate and both socialization effort and network productivity is consistent with the empirical patterns documented in the UK labor market. To see this, we fix vacancy rate to $a = 0.2$ and costs of socialization to $c = 0.01$. For different values of job destruction rate we derive the equilibrium level of network investment and the corresponding network productivity and unemployment rate. Figure 3(a) plots unemployment rate against socialization effort, while figure 3(b) plots unemployment rate against network productivity.

We conclude by illustrating how labor market conditions affect inequalities across individuals located in different position in the network.

Example: Ex-Post Inequalities across job seekers. Table 6.1 reports the equilibrium productivity of a connection[21], Δ_i^*, and the unemployment rate of job seekers conditioning on the number

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[21]Formally, this is the proportional decrease in the unemployment rate of a job seeker as we add him and additional connection, i.e., $\Delta_i^* = [u_i^*(\eta) - u_i^*(\eta + 1)]/u_i^*(\eta)$

of connections they have established with workers with a needless offer, $u_i(s^*|\eta_i(\mathcal{O} = \eta))$. The value of a connection increases in a, while it decreases in b, and it changes substantially with changes in the labor market conditions. For example, when $a = 0.1$ and $b = 0.01$ an additional connection leads to a reduction in individual's hazard rate of 90 percent, while when $b = 0.08$ the reduction is of roughly 41 percent. Hence, individuals' unemployment rates vary with their network location, and the extent to which networks induce inequalities across workers very much depends on labor market conditions. The role of social networks in creating persisting inequality in a model where the network is exogenous will therefore be generally biased, because it does not take into account how workers react to the incentives to socialization.

Table 6.1: Network location and unemployment rate, $c = 0.001$.

$i \in \mathcal{B}$	$u_i(s^*|\mu_i(\mathcal{O}) = 0)$	$u_i(s^*|\mu_i(\mathcal{O}) = 1)$	$u_i(s^*|\mu_i(\mathcal{O}) = 2)$	$u_i(s^*|\mu_i(\mathcal{O}) = 3)$	Δ_i^*
			$a = 0.1$		
$b = 0.02$	0.9	0.048	0.0025	0.0001	0.946
$b = 0.05$	0.9	0.287	0.0917	0.029	0.68
$b = 0.08$	0.9	0.526	0.307	0.179	0.415
			$a = 0.25$		
$b = 0.02$	0.75	0.038	0.002	0.0001	0.948
$b = 0.05$	0.75	0.165	0.036	0.008	0.78
$b = 0.08$	0.75	0.323	0.139	0.06	0.568
			$a = 0.4$		
$b = 0.02$	0.6	0.023	0.0009	0.000035	0.96
$b = 0.05$	0.6	0.093	0.014	0.002	0.843
$b = 0.08$	0.6	0.184	0.056	0.017	0.69

$u_i(s^*|\eta_i(\mathcal{O}) = x)$ is unemployment rate of job seeker with x relevant connections. Δ_i^* is equilibrium value of a connection.

4.2 Job contact efficient networks.

Given the salience of job contact networks, it is important to understand their welfare properties. We consider the problem of a social planner who chooses a symmetric profile $s \in [0, \infty)$ to maximize the expected utility of a randomly selected worker. The objective function of the social planner is

$$SW(s) = 1 - b(1-a)[1 - \Psi(s)] - cs.$$

We note that increasing socialization effort has two effects on the employment rate of a worker. One the one hand, it increases the expected number of links of a worker, which, in turn, increases his chances of employment. On the other hand, it increases the average connectivity of the neighbors

of a worker and this decreases his hazard rate. While the social planner balances these two effects, individual workers only take into account the first effect. Since workers do not internalize the negative externalities that their links produce on others, the job contact network in the interior equilibrium is over connected relatively to what is socially desirable. The following proposition formalizes these intuitions.

Proposition 3 *Consider a large labor market and let $\tilde{s}$ be the solution of the planner problem. If $c \geq ab(1-a)(1-b)$, then $\tilde{s} = 0$. Otherwise, $\tilde{s} < s^*$ and it is the unique solution to*

$$ab(1-a)(1-b)e^{-\tilde{s}b}[1 - \Psi(\tilde{s})] = c. \tag{8}$$

5 Extensions

The basic model assumes that a worker accesses a direct job offer with some exogenous probability. In actual practice, the probability that a worker accesses direct information of a vacant job depends both on his search activity and on the number of vacant jobs that firms are willing to open. In this section we examine how these two aspects interact with workers' incentives to built up a job contact network and how this interplay shapes labor market outcomes.

5.1 Job search and job contact networks

Workers generally use different search methods in their attempt to obtain information about jobs, see e.g. Holzer (1987), Osberg (1993), Elliott (1999) and Wabha and Zenou (2005). The empirical and theoretical literature on social networks in labor markets often focuses on information flow in the network, and it ignores the process under which workers become aware of offers directly. Here, we extend the basic model by relating the probability that a worker accesses a direct offer to his search intensity. Personal search effort and socialization effort are strategically related because the information that is available in the network–the proportion of needless offers–depends on the extend to which workers collect information personally.[22]

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[22]In a recent paper, Galeotti and Goyal (2008) study a model where individuals can search personally and form links to access information that others have acquired personally. In their model individuals invest in specific links, while we assume that investments in links are not individual-specific. Most importantly, their paper focuses on explaining the

The formation of the network and the information diffusion process in the network follow the rules described in the basic model. However, now each worker i chooses not only a socialization effort s_i, but also a costly search effort $a_i \in [0, 1]$. If worker i exerts effort a_i, he accesses a direct offer with probability a_i and incurs a cost αa_i, $\alpha > 0$. A strategy profile is now a socialization effort $\mathbf{s}$ and a profile of search effort $\mathbf{a} \in [0, 1]^n$.

Given a profile $(\mathbf{s}, \mathbf{a})$ the probability that worker i accesses at least one information via the network is $\Psi_i(\mathbf{s}, \mathbf{a}_{-i})$. The network matching rate of worker i depends only on the search effort of other workers. Furthermore, when $s_j = s$ for all $j \in \mathcal{N}$ and $a_j = a$ for all $j \neq i$, then $\Psi_i(\mathbf{s}, \mathbf{a}_{-i})$ is the same as the network matching rate derived in Section 3, i.e., expression $\Psi_i(s_i, s, a)$ is given by (5). The expected utility to worker i under a profile $(\mathbf{s}, \mathbf{a})$ is

$$EU_i(\mathbf{s}, \mathbf{a}) = 1 - b(1 - a_i)[1 - \Psi_i(s_i, s, \mathbf{a}_{-i})] - \alpha a_i - cs_i.$$

The following proposition characterizes interior equilibria. We shall restrict attention to the case in which $\alpha < b$, where we recall that b is the job destruction rate.[23]

Proposition 4 *Consider a large labor market and suppose $\alpha < b$. For every $b \in [0, 1]$, there exists $\alpha(b) > 0$ and $c(b, \alpha) > 0$ such that an interior equilibrium exists if and only if $\alpha \leq \alpha(b)$ and $c \leq c(b, \alpha)$. An interior equilibrium (s^*, a^*) solves*

$$b[1 - \Psi(s^*, s^*, a^*)] = \alpha \tag{9}$$

$$b(1 - a)\frac{\partial \Psi_i}{\partial s_i}(s^*, s^*, a^*) = c \tag{10}$$

When $c = c(b, \alpha)$ there exists only one interior equilibrium, otherwise there exist two interior equilibria, (s^H, a^L) and (s^L, a^H), where $s^H > s^L$ and $a^H > a_L$.

Equilibrium condition (9) equates marginal costs of search effort with its marginal returns. Marginal returns from personal search decrease with the network matching rate. Hence, effort to col-

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specialization in information acquisition and social communication, while our focus is on how labor market conditions affect socialization effort and personal search and how this interplay affects market outcomes.

[23]Two remarks are in order. First, if $\alpha > b$ there exists a unique equilibrium: workers do not search and do not socialize. Second, when $\alpha < b$ there exists only one corner equilibrium: $s_i = 0$ and $a_i = 1$, for all $i \in \mathcal{N}$.

lect information personally and socialization effort are strategic substitutes. Equilibrium condition (10) is analogous to equilibrium condition (7). The strategic relation between individual search effort and socialization effort leads to multiple equilibria. Equilibrium (s^H, a^L) describes a labor market where workers search personally with low intensity while the social network is very dense. Low search effort can be justified only if network productivity is sufficiently high. Since low search effort induces a low level of job network supply, this is attainable only if job contact networks are very dense. The reverse holds in the equilibrium (s^L, a^H). Here, workers are highly engaged in collecting information personally, which is consistent with equilibrium only if the likelihood of forming connections is low, i.e., socialization effort is low.

Equilibrium condition (9) in Proposition 4 implies that network matching rate is constant across the two interior equilibria and it decreases with the costs of collecting information personally. So, our model predicts that groups with lower costs of collecting information personally have also more productive social networks. This is in line with empirical findings of Battu et al. (2008) which show that ethnic minority groups with a higher level of country assimilation (which could be related to lower language proficiency or lower knowledge of the functioning of local labor markets) have more productive social networks.

Even if network productivity is constant across equilibria, the two equilibria lead to a different unemployment rate and are Pareto ranked. In particular, the equilibrium with high individual search activity and small job contact networks has a lower unemployment rate and it Pareto dominates the equilibrium with low individual search and dense networks.

Corollary 1 *Consider the interior equilibria (s^H, a^L) and (s^L, a^H) described in Proposition 4. 1.) (s^L, a^H) Pareto dominates (s^H, a^L), 2.) network matching rate is the same in the two equilibria and 3.) unemployment rate under (s^L, a^H) is lower than unemployment rate under (s^H, a^L).*

Hence, two groups of individuals facing identical labor market conditions may experience very different labor market outcomes depending on the equilibrium they coordinate. This provides a rationale for labor market inequalities across groups which is entirely driven by strategic interaction.

5.2 Endogenous arrival of offers

We now study how workers' incentives to invest in socialization interplay with firms' incentives to open new jobs, and how this interplay shapes overall labor market outcomes. Firms' incentives to create new jobs depend on the probability that vacant jobs are eventually filled. This depends on network productivity and therefore on workers' socialization effort. The main finding of this section is that the decision of firms to create new vacancies and the decision of workers to invest in social connections are strategic complements. This complementarity naturally leads to multiple equilibria with the feature that equilibrium with a high vacancy rate also entails a high socialization effort.[24]

For simplicity we assume that the productivity of each worker equals 2. Since wage is set to 1, the firm's per-worker profit is also 1. To define the expected profits to a firm opening a new job we need to define the probability that the job is filled. This is the vacancy filling rate and it equals $m_f(\mathbf{s}, a) = b + (1 - b)\Psi_f(\mathbf{s}, a)$, where $\Psi_f(\mathbf{s}, a)$ is the probability that a vacant job is filled via the network. Using the condition that the expected number of filled vacancies, $Vm_f(\mathbf{s}, a)$, must equal the expected number of workers who find a new job, $Bm(\mathbf{s}, a)$, we obtain a relation between the network matching filling rate $\Psi_f(\mathbf{s}, a)$ and the network matching rate $\Psi(\mathbf{s}, a)$. Formally,[25]

$$\Psi_f(\mathbf{s}, a) = \frac{b(1 - a)}{a(1 - b)}\Psi(\mathbf{s}, a). \tag{11}$$

The expected profits to a firm opening a vacancy is:

$$\Pi_f(a, \mathbf{s}) = b + (1 - b)\Psi_f(a, \mathbf{s}) - k,$$

where $k \in [0, 1]$ is the cost of opening a vacancy.

An equilibrium is a symmetric profile $\mathbf{s}^*$ and a vacancy rate a^* such that: 1) $\mathbf{s}^*$ is a Nash equilibrium, given a^*, and 2) $\Pi_f(a^*, \mathbf{s}^*) = 0$.

We assume that $k \in (b, 1]$ and we focus on interior equilibria.[26] The following proposition

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[24]The presence of multiple equilibria in the context of endogenous search and strong positive externalities (increasing returns to matching) have been addressed by early papers, e.g., Diamond (1982) and Howitt and McAfee (1987). A novelty of our result is that here externalities are generated by the transmission of information in the network.

[25]We observe that, for a given symmetric profile of socialization s, we can use Lemma 1 to show that the network filling vacancy rate is increasing in s and b, while it is decreasing in a.

[26]Two observations are in order. First, it is easy to verify that if $k < b$ then the only equilibrium is $a^* = 1$ and

characterizes interior equilibria.

Proposition 5 *Consider a large labor market and suppose that $k > b$. There exists a number $\bar{c}(k, b) > 0$ such that an interior equilibrium exists if and only if $c \leq \bar{c}(k, b)$. An interior equilibrium (s^*, a^*) solves:*

$$b(1 - a^*)\frac{\partial \Psi_i}{\partial s_i}(s^*, s^*, a^*) = c \tag{12}$$

$$\frac{b}{a^*}[a^* + (1 - a^*)\Psi(s^*, s^*, a^*)] = k \tag{13}$$

When $c = \bar{c}(k, b)$, there exists only one interior equilibrium, while if $c < \bar{c}(k, b)$ there are two interior equilibria, $(\hat{s}^H, \hat{a}^H)$ and $(\hat{s}^L, \hat{a}^L)$, where $\hat{s}^H > \hat{s}^L$ and $\hat{a}^H > \hat{a}^L$.

When the cost of socialization is sufficiently low, there are two interior equilibria. In one equilibrium few vacant jobs are created and workers choose a low level of socialization. When the vacancy rate is low, the job network supply is also low and so workers have low incentives to form connections. Overall, network productivity is low, which justifies the low equilibrium level of the vacancy rate. In the other equilibrium, quite the opposite occurs. Firms create many new jobs under the expectation of a high network productivity. Since under a high vacancy rate the job network supply is also high, workers heavily invest in connections and this self fulfills firms' expectation of a high network productivity. So, the endogenous formation of job contact networks determine *per se* strategic complementarities between the incentive of firms to open new vacancies and the incentives of workers to invest in job contact networks.

The discussion above suggests that network productivity is higher under equilibrium $(\hat{s}^H, \hat{a}^H)$ than under equilibrium $(\hat{s}^L, \hat{a}^L)$. Since in the former equilibrium it is also more likely that a worker accesses a direct offer than in the latter equilibrium, this suggests that unemployment rate is lower in the equilibrium with dense networks and high vacancy rate. This intuition is confirmed in the next result, which also shows that the two equilibria can be ranked in the Pareto sense.

$s^* = 0$. Two, when $k > b$, there always exists an equilibrium in which $a^* = 0$ and $s^* = 0$. Since firms believe that the job contact network is empty, the vacancy filling rate equals b, which is lower than the costs of the vacancy, so firms will not create new jobs. Given that no new jobs are created, it is optimal for workers to set zero effort.

Corollary 2 *Consider the interior equilibria $(\hat{s}^H, \hat{a}^H)$ and $(\hat{s}^L, \hat{a}^L)$ described in Proposition 5. 1) $(\hat{s}^H, \hat{a}^H)$ Pareto dominates $(\hat{s}^L, \hat{a}^L)$, 2) network marching rate under $(\hat{s}^H, \hat{a}^H)$ is higher than network matching rate under $(\hat{s}^L, \hat{a}^L)$, and, 3) unemployment rate under $(\hat{s}^H, \hat{a}^H)$ is lower than unemployment rate under $(\hat{s}^L, \hat{a}^L)$.*

6   Conclusion

This paper documents specific relations between unemployment rate and both network use and network productivity in UK labor market. These relations are not consistent with the prediction of the existing models of social networks and labor market, which assume that the job contact network is exogenous. We explore how labor market conditions affect the formation of job contact networks and how this interplay shapes labor market outcomes. We have shown that taking into account this form of endogeneity leads to predictions which are in line with the documented empirical pattern.

Our model is static and this is clearly a limitation of the framework. We find the focus on this simple model to be a useful starting point to illustrate how individuals react to changes in macro variables and how this affects the informal institutions present in the economy and economic outcomes. However, we also believe that it is important to extend our framework in a full fledge dynamic model. In such a model, at each point in time, labor market conditions would be different and so will be workers' incentives to invest in social networks. Also, the position of a worker in the wage distribution will affect his incentives to keep investing in connections. This model would allow to provide novel empirical predictions on how, for example, the use of job contact networks changes with job tenure and to what extent social networks can explain durable inequality among groups and individuals.

Appendix A: Data Description and Derived Variables Description

We used data from the United Kingdom Quarterly Labour Force Survey (QLFS) from the First quarter of 1995 to the first quarter of 2005. Each wave of the QLFS covers around 60,000 households incorporating from around 150,000 to around 125,000 individuals depending on the wave. The sample is representative of the UK population. Only males of working age (aged 16 to 64) resident in Great Britain are considered in our analysis, so that we are left with about 40-45,000 individuals per wave, depending on the wave. We exclude Northern Ireland from the sample, because of political instability of the region, that we fear could influence the use of social networks. In any case, the number of observation excluded is limited.

In the survey, individuals are asked about their working status and job search behavior. In particular, in case they were looking for a job, they are asked about their main search method.[27] The available options are: direct approach, seeking finance to open own business, seeking permits, reply to adverts, advertise yourself, situation vacant, job centre, careers office, job club, private employment agency, waiting for responses, ask friends and relatives, not looking for work and anything else. The total number of job seeking workers that in each quarter reported to use friends and relatives as the main job search channel is our proxy for total network use, while our proxy for network use is the proportion between these workers and the number of workers that were actively looking for a job, i.e., all respondents but the ones that responded they were not looking for work. While the first measure might be positive related with unemployment just because there is more people looking for job the higher is unemployment, the second reflects the methods workers mostly use to search for a job as unemployment increases.

The survey respondents that found a job in the last three months are asked how they did found the current position. The available options are the following:[28] (1) Replying to a job advertisement (2) Job centre, job market etc; (3) Careers office; (4) Job club; (5) Private employment agency or

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[27]Note that the interviewed cannot give multiple options or the relative weight given to each search channel in his job seeking activity.

[28]There has been some changes in this question throughout the history of the QLFS. Until 1994, there were less available options to answer. Most importantly, from the second quarter of 2005, this question is addressed to everybody who found a new job in the last twelve months or less. For these reasons, we excluded all waves before 1994:Q4 and after 2005:Q1 from our sample.

business; (6) Hearing from someone who worked there; (7) Direct application; (8) Some other way. We used two variables as a proxy of the network productivity. First, we calculated the total number of workers who found a job recently who stated to have found it through a friend that worked there, which we refer to as total network productivity. Second, we calculated the proportion of such workers over the total newly employed workers, which we refer to as network productivity.

As for unemployment rate, we calculated it using the ILO definition, that is, the number unemployed workers looking for a job or waiting to start a job in next two weeks over the whole active population. We obtained an average unemployment over the period of 6.49%, ranging from a minimum of 4.77% to a maximum of 10.67%.

The correlation of unemployment rate and network productivity is 0.6178, while it is higher, namely 0.8273, with network use. This implies that an increase of one percent in unemployment is associated to a 0.381 percent increase in network productivity and a 0.684 percent increase in network use. The correlation of unemployment with aggregate network use is, not surprisingly, higher, namely 0.9715, while it is lower with total network productivity, namely −0.4902. This implies that an increase of one percent in unemployment is associated to a 0.24 percent increase in total network productivity and a 0.943 percent increase in total network use. Since correlations is a measure of linear association between variables and we are interested to see if the relationship between these variables present any concavity, we regress our proxies of network use and productivity against unemployment and unemployment squared, respectively. The first part of Table 7 summarizes the results.

To investigate the relation between these variables at a regional level, we use a disaggregation to 19 regions in Great Britain–Tyne and Wear, Rest of Northern Region, South Yorkshire, West Yorkshire, Rest of Yorkshire and Humberside, East Midlands, East Anglia, Inner London, Outer London, Rest of South East, South West, West Midlands (Metropolitan), Rest of West Midlands, Greater Manchester, Merseyside, Rest of North West, Wales, Strathclyde, Rest of Scotland. In this case the correlations of network use and productivity with unemployment are still positive, but lower: they are 0.4299 and 0.1504, respectively. When we plot these variables against the local unemployment rate we obtain the pattern displayed in Figure 6. To better capture the relation of network use and network

productivity with unemployment rate, we introduce regional fixed effects. In particular, we regress network use and network productivity against unemployment rate, unemployment squared and we introduce region fixed effects. The estimates are reported in the second part of Table 7 and they confirm the results we have obtained at the aggregate level.
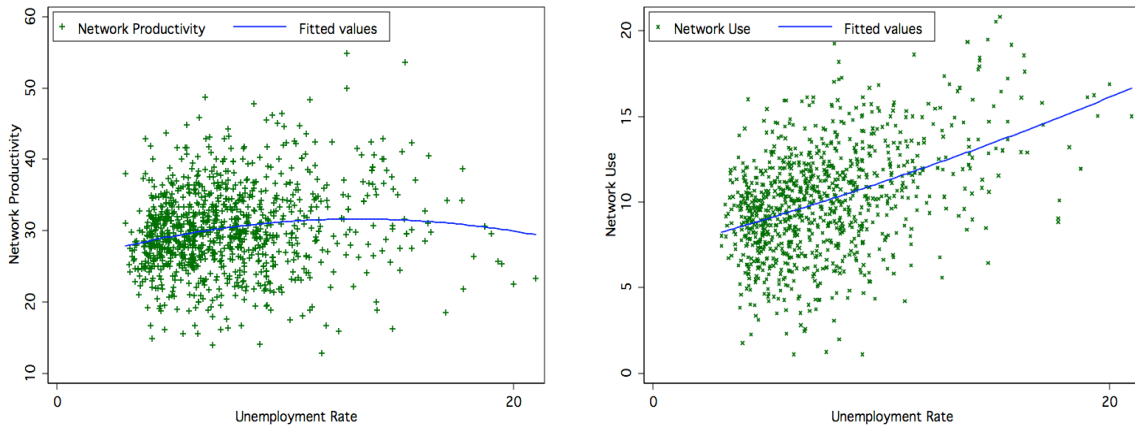
(a) Network productivity and unemployment rate in Great Britain, regional level.

(b) Total network use and unemployment in Great Britain, regional level.

Figure 6: Productivity of networks and unemployment, disaggregated at regional level.

Table 7: Regression of network use and network productivity against unemployment and unemployment squared, at aggregate and regional level.

	Aggregate Level				Regional Level			
	Tot. Network Productivity	Tot. Network Use	Network Productivity	Network Use	Tot. Network Productivity	Tot. Network Use	Network Productivity	Network Use
u	240.9*	242.04*	2.447*	2.809*	2.060*	6.654*	1.218*	0.751*
	(69.24)	(32.95)	(1.310)	(0.623)	(0.455)	(0.439)	(0.338)	(0.143)
u^2	−14.19*	−9.706*	−0.121	−0.145*	−0.0643*	−0.144*	−0.032*	−0.007
	(4.521)	(2.151)	(0.0855)	(0.040)	(0.0208)	(0.0201)	(0.0154)	(0.006)
Const.	−347.4	−656.4*	18.91*	−1.506	18.59*	−17.02*	22.83*	5.161*
	(249.6)	(118.81)	(4.725)	(2.248)	(2.246)	(2.169)	(1.671)	(0.706)
Obs.	41	41	41	41	779	779	779	779
Adj.R^2	0.365	0.961	0.381	0.751	0.888	0.833	0.114	0.316
Regional Dummies	NO	NO	NO	NO	YES	YES	YES	YES

Source: Quarterly Labour Force Survey, male respondents, aged 16-64, residents in Great Britain, waves from 1995:Q1 to 2005:Q4.

* significant at a 5% confidence level. Standard errors in parentheses.

Appendix B: Proofs of Lemmas and Propositions

Proof of Lemma 1 First note that (6) follows immediately from (5) by setting $s_i = s$. It is straightforward to verify that $\frac{\partial \Psi}{\partial s}(s) > 0$ and that $\frac{\partial \Psi}{\partial a}(s) > 0$. Second, $\frac{\partial^2 \Psi}{\partial s^2}(s) = -a(1-b)[\frac{\partial \Psi}{\partial a}(s) + (1-\Psi(s))be^{-sb}] < 0$, where the inequality follows because $\frac{\partial \Psi(s)}{\partial s} > 0$. Similarly, $\frac{\partial^2 \Psi}{\partial a^2}(s) = -\frac{\partial \Psi(s)}{\partial a}(1 - e^{-sb})(1-b)/b < 0$, where the inequality follows because $\frac{\partial \Psi(s)}{\partial a} > 0$. Third, we show that $\Psi(s)$ is decreasing in b. The derivative of $\Psi(s)$ with respect to b has the same sign as the derivative of $\frac{(1-b)}{b}\left(1 - e^{-sb}\right)$ with respect to b, which is given by

$$\frac{e^{-sb}[b(1-b)s+1] - 1}{b^2}. \tag{14}$$

Expression (14) is negative when $e^{-sb}[b(1-b)s+1] < 1$ which is equivalent to $-sb+\ln(1+b(1-b)s) < 0$. This holds because $-sb+\ln(1+b(1-b)s)|_{s=0} = 0$ and $-sb+\ln(1+b(1-b)s)$ is decreasing in s. Hence, $\frac{\partial \Psi}{\partial b}(s) < 0$ for all $s > 0$. We now conclude the proof of Lemma 1. Note that $u(s) = b(1-a)[1-\Psi(s)]$. Since $\Psi(s)$ is increasing and concave in s, it follows that $u(s)$ is decreasing and convex in s. Since $\Psi(s)$ is increasing and concave in a it follows that $\frac{\partial u}{\partial a}(s) = -b[1 - \Psi(s) + (1-a)\frac{\partial \Psi}{\partial a}(s)] < 0$ and that $\frac{\partial^2 u}{\partial a^2}(s) = b[2\frac{\partial \Psi}{\partial a}(s) - (1-a)\frac{\partial^2 \Psi}{\partial a^2}(s)] > 0$. Since $\Psi(s)$ is decreasing in b, it follows that $\frac{\partial u}{\partial b}(s) = (1-a)[1 - \Psi(s) - b\frac{\partial \Psi}{\partial b}(s)] > 0$. ∎

Proof of Proposition 1 Suppose an interior equilibrium exists. Consider a profile $\mathbf{s}$ where $s_j = s$, $\forall j \neq i$. For every $s > 0$, i's best response, say $\hat{s}_i$, has the property that $\hat{s}_i s \leq [\hat{s}_i + (n-1)s]$. Indeed, if $\hat{s}_i s > [\hat{s}_i + (n-1)s]$, then i could decrease his own networking effort and still be linked with any arbitrary worker with probability 1. Hence, an interior equilibrium s^* solves:

$$\frac{\partial EU_i}{\partial s_i}(s^*, s^*) = -b(1-a)\frac{\partial \phi_i}{\partial s_i}(s^*) - c = 0,$$

where

$$\frac{\partial \phi_i}{\partial s_i}(s, s) = -\frac{a(1-b)}{b}\left[1 - \frac{1 - (1-s/n)^{nb}}{nb}\right]^{na(1-b)-1}$$
$$\left[\frac{1-b}{n}\left(1 - \frac{s}{n}\right)^{nb-1} + \left(1 - \left(1 - \frac{s}{n}\right)^{nb}\right)\left(\frac{1}{s} - \frac{1}{snb}\right)\right].$$

In large labor markets we have that:

$$\lim_{n\to\infty} \frac{\partial \phi_i}{\partial s_i}(s,s) = -\frac{a(1-b)}{sb}\left(1-e^{-sb}\right)e^{\frac{a(1-b)}{b}\left(1-e^{-sb}\right)}.$$

Therefore, s^* must solve

$$\frac{a(1-a)(1-b)}{s^*}\left(1-e^{-s^*b}\right)e^{-\frac{a(1-b)}{b}\left(1-e^{-s^*b}\right)} = c,$$

which is equivalent to condition (7) stated in Proposition 1. We now show that there exists a unique solution to this equation and we derive the conditions for existence. We start noticing that the LHS is decreasing in s^* because both $\left(1-e^{-s^*b}\right)/s^*$ and $e^{-\frac{a(1-b)}{b}\left(1-e^{-s^*b}\right)}$ are decreasing in s^*. Furthermore, when s^* goes to 0, the LHS converges to $ab(1-a)(1-b)$, while when s^* goes to infinity the LHS converges to 0. Since marginal returns are continuous in s_i, it follows that an interior symmetric equilibrium exists if and only if $c < ab(1-a)(1-b)$, in which case there is only one symmetric interior equilibrium. This concludes the proof of Proposition 1. ∎

Proof of Proposition 2 To see how equilibrium socialization effort responds to changes in a and b, we derive ds^*/da and ds^*/db by implicit differentiation of (7). The partial derivative of marginal returns with respect to socialization effort is

$$-a(1-a)\phi(s^*)\left[\frac{1-e^{-s^*b}-s^*be^{-s^*b}}{s^*} + (1-e^{-s^*b})a(1-b)e^{-s^*b}\right] < 0,$$

where the inequality follows because $1 - e^{-s^*b} - s^*be^{-s^*b} > 0$ (see proof of Lemma 1). This implies the sign of ds^*/da and ds^*/db equals the sign of the partial derivative of (7) with respect to a and b, respectively. As for a, the partial derivatives of marginal returns with respect to a is

$$(1-b)\frac{1-e^{-s^*b}}{s^*}\left[1-2a-a(1-a)\frac{1-b}{b}(1-e^{-s^*b})\right].$$

Let $A(a,b,s^*) = 1-2a-a(1-a)\frac{1-b}{b}(1-e^{-s^*b})$; this is a continuous function in a, $A(0,b,s^*) = 1$ and $A(1,b,s^*) = -1$. Moreover, whenever $A(a,b,s^*) = 0$, $\frac{\partial A}{\partial a}(a,b,s^*)|_{A(a,b,s^*)} = -2-(1-2a)^2/[a(1-a)] < 0$. Hence, $A(a,b,s^*)$ can cross the x-axis only once. That is, for given s^*, there exists a unique $a(b)$

30

such that $A(a(b), b, s^*) = 0$, and $A(a, b, s^*) > 0$ for all $a < a(b)$, while $A(a, b, s^*) < 0$ for all $a > a(b)$. This concludes the proof of part I of Proposition 2.

We now prove part II. The partial derivative of marginal returns with respect to b is:

$$B(a, b, s^*) = \left[(s^*(1-b) + 1)e^{-s^*b} - 1\right] + a\frac{1-b}{b^2}(1 - e^{-s^*b})[1 - (1 + s^*b(1-b))e^{-s^*b}].$$

Note that $\lim_{b \to 0} B(a, b, s^*) = s^* > 0$ and that $\lim_{b \to 1} B(a, b, s^*) = s^* - (1 - e^{-s^*}) < 0$. Furthermore, when $B(a, b, s^*) = 0$, we have that:

$$a(b, s^*) = \frac{b^2 \left[1 - (s^*(1-b) + 1)e^{-s^*b}\right]}{(1-b)(1 - e^{-s^*b})[1 - (1 + s^*b(1-b))e^{-s^*b}]}.$$

Then,

$$
\begin{aligned}
\frac{\partial B}{\partial b}\Big|_{B(a,b,s^*)=0} &= -e^{-bs^*}s^*(2 + (1-b)s*) + \frac{a(b, s^*)}{b^3}e^{-2bs^*}[(-2(e^{bs^*} - 1)^2 + b^2 s^*(2 + e^{bs^*}(s^* - 2) - 2s^*) \\
&\quad + b^4 s^{*2}(e^{bs^*} - 2) + b(e^{bs^*} - 1)(e^{bs^*} + 3s^* - 1) - b^3 s^*(e^{bs^*}(1 + 2s^*) - 1 - 4s^*)] \\
&= \frac{e^{-bs^*}}{b(1-b)(e^{bs^*} - 1)(e^{bs^*} - 1 - (1-b)bs^*)}[bs^*(2 + (1-b)s^*) + (e^{bs^*} - 1 - (1-b)s^*) \times \\
&\quad \times [-2(e^{bs^*} - 1)^2 + b(e^{bs^*} - 1)(e^{bs^*} - 1 + 3s^*) + b^2(2 + e^{bs^*}(s^* - 2) - 2s^*)s^* \\
&\quad - b^3 s^*(+e^{bs^*}(1 + 2s^*) - 1 - 4s^*) + b^4(e^{bs^*} - 2)s^2]] < 0.
\end{aligned}
$$

Hence, there exists a unique $\bar{b}(a)$ such that $B(a, \bar{b}(a), s^*) = 0$ and $B(a, b, s^*) > 0$ for all $b < \bar{b}(a)$, while $B(a, b, s^*) < 0$ for all $b > b(a)$. This concludes the proof of Proposition 2. ∎

Proof of Proposition 3 The derivative of social welfare with respect to socialization effort is:

$$\frac{dSW}{ds}(s) = b(1-a)a(1-b)\left[1 - \frac{1 - (1 - \frac{s}{n})^{nb}}{nb}\right]^{na(1-b)-1}\left(1 - \frac{s}{n}\right)^{nb-1} - c,$$

and in large labor markets become

$$\lim_{n \to \infty} \frac{dSW}{ds}(s) = b(1-a)a(1-b)e^{-sb}e^{-\frac{a(1-b)}{b}(1-e^{-sb})} - c.$$

Note that $\frac{dSW}{ds}(0) = ab(1-a)(1-b) - c$, $\lim_{s \to \infty} \frac{dSW}{ds}(s) = -c$. Moreover, $\frac{dSW}{ds}(s)$ is strictly decreasing

31

in s, since both $\left(1 - e^{-sb}\right)$ and sb are strictly increasing in $s \in (0, \infty)$. Hence, if $c \geq ab(1-a)(1-b)$, the social planner chooses $\tilde{s} = 0$, while for all $c < ab(1-a)(1-b)$ the optimal solution $\tilde{s}$ is uniquely defined by

$$ab(1-a)(1-b)e^{-\frac{a(1-b)}{b}\left(1-e^{-\tilde{s}b}\right)-\tilde{s}b} = c.$$

Finally, it is easy to verify that at equilibrium s^*, $\frac{dSW}{ds}(s^*) < 0$, and since $\frac{dSW}{ds}(s)$ is decreasing in s, it follows that $\tilde{s} < s^*$ for all $c < ab(1-a)(1-b)$. This concludes the proof of Proposition 3. ∎

Proof Proposition 4 It is easy to see that an interior equilibrium (a^*, s^*) solves conditions (9) and (10) stated in Proposition 4. We now show existence. Using condition (9) we have that $\phi(s^*, a^*) = \alpha/b$, which is equivalent to

$$a^* = -\frac{b}{(1-b)(1-e^{-s^*b})}\ln\left(\frac{\alpha}{b}\right).$$

Since $a^* \in (0, 1)$ it must be the case that $1 + [b\ln(\alpha/b)]/[(1-b)(1-e^{-s^*b})] > 0$. Let $\alpha(b)$ be such that: $1 + [b\ln(\alpha(b)/b)]/[(1-b)] = 0$. Note that if $\alpha \geq \alpha(b)$ then $1 + [b\ln(\alpha/b)]/[(1-b)(1-e^{-s^*b})] \leq 0$ for all s^*, while if $\alpha \leq \alpha(b)$ then $1 + [b\ln(\alpha/b)]/[(1-b)(1-e^{-s^*b})] > 0$ for sufficiently high s^*. So, a necessary condition for an interior equilibrium is that $\alpha < \alpha(b)$. Suppose $\alpha < \alpha(b)$. Next, using the above expression for a^* and $\phi(s^*, a^*) = \alpha/b$, we obtain that condition (10) holds if and only if

$$V(b, c, s^*) = \frac{cs^*}{\alpha} + \ln\left(\frac{\alpha}{b}\right)\left(1 + \frac{b}{(1-b)(1-e^{-s^*b})}\ln\left(\frac{\alpha}{b}\right)\right) = 0.$$

Note that $V(b, c, s^*)$ is the sum of two convex function in s^* and therefore it is convex in s^*. Moreover, $\lim_{s^* \to 0} V(a, b, s^*) = \lim_{s^* \to \infty} V(b, c, s^*) = \infty$. Finally, note that the LHS is strictly decreasing in c and, since $\alpha < \alpha(b)$, $V(b, 0, s^*) < 0$. For every $\alpha < \alpha(b)$, there exists a $c(\alpha, b) > 0$ such that if $c = c(\alpha, b)$, $V(b, c, s^*) = 0$ has a unique solution in s^*, while for all $c < c(\alpha, b)$ the equation has two solutions, s^H and s^L, with $s^H > s^L$. Since a^* is decreasing in s^*, it follows that under s^H, the equilibrium vacancy rate is a^L which is lower than the equilibrium vacancy rate a^H under s^L. This completes the proof of Proposition 4. ∎

Proof of Corollary 1 The equilibrium unemployment rate under equilibrium (s^*, a^*) is $u(s^*, a^*) =$

32

$b(1-a)\phi(s^*, a^*) = (1-a)\alpha$, where the last equality follows from equilibrium condition (9). Hence, equilibrium unemployment rate is decreasing in a^*. So unemployment rate is lower under (s^L, a^H) than under (s^H, a^L). Next, note that in equilibrium (s^*, a^*), $Eu(s^*, a^*) = 1 - b(1-a)\phi(s^*, a^*) - \alpha a^* - cs^* = 1 - \alpha - cs^*$, where the second equality follows from equilibrium condition (9). Hence, expected equilibrium utility is decreasing in s^* and therefore equilibrium (s^L, a^H) Pareto dominates equilibrium (s^H, a^L). Finally, from equilibrium condition (9) it is immediate to see that network matching rate is constant across the two interior equilibria. ∎

Proof of Proposition 5 It is easy to verify that an interior equilibrium (s^*, a^*) solves condition (12) and condition (13) stated in Proposition 5. We now prove existence. First, condition (13) can be rewritten as follows:

$$e^{-\frac{a^*(1-b)}{b}}\left(1-e^{-s^*b}\right) = \frac{b-ka^*}{b(1-a^*)}. \tag{15}$$

Taking the log of expression (15) we obtain

$$(1-e^{-s^*b}) = -\frac{b}{a^*(1-b)}\ln(\frac{b-ka^*}{b(1-a^*)}), \tag{16}$$

and taking again the log we have that:

$$s^* = -\frac{1}{b}\ln\left(1 + \frac{b}{a^*(1-b)}\ln\left[\frac{b-ka^*}{b(1-a^*)}\right]\right). \tag{17}$$

Let $\tilde{s}(a^*, b, k)$ be the s^* which solves 17.

Second, using condition (15) and condition (17), we rewrite condition 12 as follows

$$s^* = -\frac{b-ka^*}{c}\ln\left[\frac{b-ka^*}{b(1-a^*)}\right], \tag{18}$$

and we denote by $\bar{s}(a^*, c, b, k)$ the s^* which solves 18.

An interior equilibrium is (a^*, s^*) such that $\bar{s}(a^*, c, b, k) = \tilde{s}(a^*, c, b, k) = s^*$. Now note that $\bar{s}(a^*, c, b, k)$ is a well defined function when (1.) $b - ka^* > 0$ and (2.) $b - ka^* < b(1-a^*)$. We observe that (2.) holds because $k > b$. So, consider all $a^* \in (0, b/k]$ and note that $\bar{s}(a^*, c, b, k) > 0$, $\bar{s}(0, c, b, k) = \bar{s}(b/k, c, b, k) = 0$ and that $\bar{s}(a^*, c, b, k)$ is concave in a^*, since $\partial^2 \bar{s}(a^*, c, b, k)/\partial a^2 =$

33

$-(b-k)^2/[(1-a^*)^2(b-ka^*)] < 0$.

Next, note that $\tilde{s}(a^*, b, k)$ is a well defined function when (1.) above holds, (1a.) $1 + \frac{b}{a(1-b)}\ln(\frac{b-ka}{b(1-a)}) > 0$ and (2a.) $\ln(1 + \frac{b}{a(1-b)}\ln(\frac{b-ka}{b(1-a)})) < 0$. It is easy to check that condition (2.) above implies condition (2a.). It is also easy to verify that there exists a unique $\bar{a}(b,k) > 0$ such that $1 + \frac{b}{\bar{a}(b,k)(1-b)}\ln\left(\frac{b-k\bar{a}(b,k)}{b(1-\bar{a}(b,k))}\right) = 0$, that $\bar{a}(b,k) < b/k$ and that condition (1a.) holds if and only if $a < \bar{a}(b,k)$. Hence, (1a.) implies (1.). So, a necessary condition for an interior equilibrium is that $a^* \in [0, \bar{a}(b,k)]$, which we now assume. Note here that: $\tilde{s}(0, b, k) = -\frac{1}{b}\ln(\frac{b-ka^*}{b(1-a^*)}) > 0$, $\tilde{s}(\bar{a}(b,k), b, k) = +\infty$ and that $\tilde{s}(a^*, b, k)$ is increasing and convex in $a \in [0, \bar{a}(b,k)]$.

Combining the properties of $\bar{s}(a^*, c, b, k)$ and $\tilde{s}(a^*, b, k)$, we obtain that for all $a^* \in [0, \bar{a}(b,k)]$, the function

$$G(a, c, b, k) = \bar{s}(a, c, b, k) - \tilde{s}(a, b, k)$$

is concave in a^*, $G(0, c, b, k) < 0$ and $G(\bar{a}(b,k), c, b, k) < 0$. Let $\tilde{a} = \arg\max_{a^* \in [0, \bar{a}(b,k)]} G(a^*, c, b, k)$. It is then clear that an interior equilibrium exists if and only if $G(\tilde{a}, c, b, k) \geq 0$. Furthermore, if $G(\tilde{a}, c, b, k) = 0$ then there is only one interior equilibrium, while if $G(\tilde{a}, c, b, k) > 0$ there are two interior equilibria, a low $a^* = a^L$ with corresponding high $s^* = s^H$, and a high a^* a with corresponding low $s^* = s^L$. Since for fix $k > b$ and $a^* \in [0, \bar{a}(b,k)]$, $G(a^*, c, b, k)$ is strictly decreasing in c, $\lim_{c \to 0} G(a^*, c, b, k) = \infty$ and $\lim_{c \to \infty} G(a^*, c, b, k) < 0$, it follows that there exists a $\bar{c}(k, b)$ such that if $c = \bar{c}(k, b)$ there exists one interior equilibrium, while for all $c \leq \bar{c}(k, b)$ there are two interior equilibria. This concludes the proof of Proposition 5. ∎

Proof Corollary 2 The vacancy filling rate under equilibrium (s^*, a^*) is $\Psi_f(s^*, a^*) = [k-b]/[1-b]$. Since $\Psi_f(s^*, a^*) = \Psi(s^*, a^*)b(1-a^*)/[a^*(1-b)]$, it follows that $\Psi(s^*, a^*) = a^*(k-b)/[b(1-a^*)]$, which is increasing in a^*. So the network matching rate under $(\hat{s}^L, \hat{a}^H)$ is higher than the network matching rate under $(\hat{s}^H, \hat{a}^L)$. Second, unemployment rate under equilibrium (s^*, a^*) is $u(s^*, a^*) = b(1-a^*)\phi(s^*, a^*) = b - a^*k$, which is decreasing in a^*. So, unemployment rate under under $(\hat{s}^L, \hat{a}^H)$ is lower than unemployment rate under $(\hat{s}^H, \hat{a}^L)$. Finally, note that expected utility under equilibrium (s^*, a^*) is $EU(s^*, a^*) = 1 - b(1-a^*)\phi(s^*, a^*) - cs^* = 1 - b + a^*k - cs^*$, which is increasing in a and decreasing in s. So, $(\hat{s}^L, \hat{a}^H)$ Pareto dominates $(\hat{s}^H, \hat{a}^L)$. ∎

Appendix C: Indirect information flow

This appendix examines the implications of indirect information flow in matching workers and firms and how it shapes workers' socialization incentives. Information flow in the network now follows the following process. As in the basic model, each worker l with a needless offer gives it to one of his unemployed neighbors, chosen at random. If worker l has only employed friends, then he chooses one of them at random, say j, and gives him the information. For simplicity, we assume that if j had himself a needless offer, then the offer he receives from l is lost. If, on the contrary, worker j did not have a needless offer, then he selects at random one of his unemployed friends, say i, and passes the information to him. We also assume that the information passed from j to i reaches i with probability $\delta \in [0, 1]$, where δ is the decay in the information flow. So, information now may flow two-steps away in the network.

We observe that a job seeker worker does not hear about new jobs from his friends if: 1) he does not access information from his friends who received a needless offer directly and 2) he does not get information from his contacts who are employed, do not have a needless offer directly, but have heard of a job opportunity from at least one of their friends. The probability associated to the event described in 1) is given by (4). We now derive the probability associated to the event described in 2). For concreteness, in what follows, $i \in \mathcal{B}$ and chooses s_i, while all other workers choose effort s. Moreover, worker j is employed and he does not have a needless offer, $j \in \mathcal{N} \setminus \{\mathcal{B} \cup \mathcal{O}\}$, while worker $l \in \mathcal{O}$.

First, suppose j and l are linked, i.e., $g_{jl} = 1$. The probability that j receives information from l is:

$$(1 - p_i)(1 - p)^{nb-1} \sum_{v=1}^{n(1-b)} \Pr(\eta_l(\mathcal{N} \setminus \{\mathcal{B}\}) = v | g_{lj} = 1) \frac{1}{v}.$$

That is, worker l's friends must be all employed, $(1 - p_i)(1 - p)^{nb-1}$, and, conditioning on having v links, worker l gives the information to j with probability $1/v$. So, if worker j is linked with ω

workers such as l, the probability that j does not receive information is:

$$\left[1 - (1-p_i)(1-p)^{nb-1} \sum_{v=1}^{n(1-b)} \Pr(\eta_l(\mathcal{N} \setminus \{\mathcal{B}\}) = v | g_{lj} = 1) \frac{1}{v}\right]^{\omega}.$$

Summing across all possible number of j's neighbors who are employed and with a needless offer, we obtain the probability that j accesses at least an indirect offer:

$$\Theta(\mathbf{s}) = 1 - \sum_{\omega=0}^{|\mathcal{O}|} B(\omega | p, |\mathcal{O}|) \left[1 - (1-p_i)(1-p)^{nb-1} \sum_{v=1}^{n(1-b)} \Pr(\eta_l(\mathcal{N} \setminus \{\mathcal{B}\}) = v | g_{lj} = 1) \frac{1}{v}\right]^{\omega},$$

and in a large labor market it is equal to:

$$\Theta(\mathbf{s}) = 1 - e^{-a\left(e^{-sb} - e^{-s}\right)}. \tag{19}$$

Note that in a complete network worker l has always links with unemployed workers and therefore every worker like j will never receive information. When the network is not complete, it is easy to verify that the probability that j gets information is non-monotonic in s–it first increases when s is low to begin with and then it decreases. Therefore, greater connectivity of workers other than unemployed i may have a positive effect on the probability that i gets a job. This illustrates a novel effect which emerges from indirect information flow. In fact, when the network is not very connected to start with, high socialization investments of other workers have a positive effect on the value of worker i's socialization investment because it makes more likely that i's neighbors have job information to pass along.

Second, consider our original job seeker i and suppose he has η links with workers like j above. The probability that i does not receive an offer from each of these η contacts is:

$$\sum_{d=0}^{\eta} B(d | \Theta(\mathbf{s}), \eta) \left[1 - \sum_{t=1}^{nb} \Pr(\eta_j(\mathcal{B}) = t | g_{ij} = 1) \frac{\delta}{t}\right]^{d}.$$

In words, with probability $B(d | \Theta(\mathbf{s}), \eta)$, d out of the η contacts of i have received an offer from one of their employed friends. Suppose j is one of these individuals; then the probability that i receives information from j depends on the level of the decay in the information flow and the number of

36

unemployed workers connected to j. Summing across all possible number of links that worker i can have with workers like j, we obtain the probability that i does not access an indirect offer:

$$\phi_i^{in}(s_i, \mathbf{s}_{-i}) = \sum_{\eta=0}^{n(1-a)(1-b)} B(\eta|p_i, n(1-a)(1-b)) \sum_{d=0}^{\eta} B(d|\Theta(\mathbf{s}), \eta) \left[1 - \sum_{t=1}^{nb} \Pr(\eta_j(\mathcal{N}^U) = t|g_{ij}=1)\frac{\delta}{t} \right]^d,$$

and clearly the probability that i gets at least an indirect offer is $\Psi_i^{in}(s_i, \mathbf{s}_{-i}) = 1 - \phi_i^{in}(s_i, \mathbf{s}_{-i})$. In a large labor market, this is equal to

$$\Psi^{in}(s_i, \mathbf{s}_{-i}) = 1 - e^{-\frac{\delta(1-b)(1-a)}{b}\frac{s_i}{s}\left(1-e^{-sb}\right)\Theta(\mathbf{s})}. \tag{20}$$

In a symmetric profile where $s_i = s$ for all $i \in \mathcal{N}$, the probability that an unemployed worker hears a job indirectly is non monotonic in socialization effort and the intuition follows from the effect of indirect information flow which we have discussed above.

Finally, the overall probability that an unemployed worker gets at least an offer in a symmetric profile is:

$$\tilde{\Psi}^{in}(s) = 1 - \phi(s)\phi^{in}(s) = 1 - e^{-\frac{1-b}{b}\left(1-e^{-sb}\right)(a+\delta(1-a)\Theta(s))}. \tag{21}$$

The network matching rate is decreasing in the decay of information flow and under full decay we are back to the network matching rate (6) developed in Section 3. Under indirect information flow, the expected utility of a worker i choosing s_i and facing a strategy of others $s_j = s$ for all $j \neq i$ is:

$$EU_i(s_i, \mathbf{s}_{-i}) = 1 - b(1-a)\phi_i(s_i\mathbf{s}_{-i})\phi_i^{in}(s_i, \mathbf{s}_{-i}) - cs_i.$$

Proposition 6 *Consider a large labor market and consider indirect information flow. An interior equilibrium exists if and only if $c < ab(1-a)(1-a)$. In a symmetric interior equilibrium every worker chooses $\hat{s}$ which is the unique solution to:*

$$\frac{(1-b)(1-a)}{\hat{s}}\left(1 - e^{-\hat{s}b}\right)(a + \delta(1-a)\Theta(\hat{s}))\left(1 - \tilde{\Psi}^{in}(\hat{s})\right) = c \tag{22}$$

Proof of Proposition 6. Equilibrium condition (22) is obtained by taking the partial derivatives

of $EU_i(s_i, \mathbf{s}_{-i})$ with respect to s_i and imposing symmetry, i.e, $s_j = s$ for all $j \in \mathcal{N}$. We now show that a solution exists and it is unique if and only if $c < ab(1-a)(1-b)$. To see this note that when $s \to 0$ the LHS of (22) equals $ab(1-a)(1-b)$ and when $s \to \infty$ the LHS of (22) equals 0. So, it is sufficient to show that the LHS of (22) is decreasing in s, which we now prove. We first claim that the following expression is decreasing in s:

$$\frac{1}{s}\left(1 - e^{-sb}\right)(a + \delta(1-a)\Theta(s))$$

Taking the derivatives of the above expression with respect to s we have that

$$\frac{1}{s^2}\left[-(a + \delta(1-a)\Theta(s))(1 - e^{-sb}(1-b)) + (1 - e^{-sb})\delta(1-a)\frac{\partial\Theta(s)}{\partial s}\right].$$

Since $\frac{\partial\Theta}{\partial s}(s) = (1 - \Theta(s))a\left(e^{-s} - be^{-sb}\right)$, then the above derivative is negative if and only if

$$\delta(1-a)a(1-\Theta(s))(1 - e^{-sb})\left(e^{-s} - be^{-sb}\right) < (a + \delta(1-a)\Theta(s))(1 - e^{-sb}(1-b))$$

Since the RHS of the inequality is always positive, if $\left(e^{-s} - be^{-sb}\right) < 0$ the claim follows. So, suppose that $\left(e^{-s} - be^{-sb}\right) > 0$. Here note that

$$1 - e^{-sb}(1-b) > (1 - e^{-sb})\left(e^{-s} - be^{-sb}\right)$$

if and only if, taking the log, $\ln(1-b) + sb > 0$ which is obviously true. Next, note that $a + \delta(1 - a)\Theta(s) > \delta(1-a)a(1 - \Theta(s))$ if and only if $a + \delta[\Theta(s) - a(1 - \Theta(s))] > 0$, which holds because $a + \delta[\Theta(s) - a(1 - \Theta(s))] > a(1 - \delta(1-a)) > 0$. These two observations imply our first claim. Using similar arguments, it is easy to show that $(1 - \tilde{\Psi}^{in}(s))$ is also decreasing in s. Hence, the LHS of expression 22 is decreasing in s. Proposition 6 follows. $\blacksquare$

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