A Science Metric for RV Exoplanet Characterization

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Abstract

Known RV exoplanets are prime targets for a high-contrast imaging mission, which could detect and characterize these enigmatic objects in reflected starlight. To help define and differentiate candidate concepts, and to clarify the expectations for such a mission, we study a science metric for RV exoplanets (N_{RV}), defined as the estimated number of planets detected and characterized by the mission. In this report, we develop and compute N_{RV}, estimating its value as $N_{RV} \approx 3.45$ for a reasonable set of engineering parameters for a probe-class mission.

Introduction

We compute the science metric N_{RV} as the number of known RV exoplanets that satisfy four criteria:

#1 *permitted pointing*: for a coronagraph mission, the angle between the host star and the sun (γ) must be greater than the solar avoidance angle, $\gamma > \gamma_1$ and, for a star-shade mission, it must also be less than the angle at which the star shade appears illuminated, $\gamma < \gamma_2$

#2 *contrast*: the ratio of the central surface brightness of the planet image to the surface brightness of scattered starlight at the position of the planet—expressed in delta magnitudes (Δmag)—must be better than the systematic limit (Δmag_0), i.e. $\Delta mag < \Delta mag_0$

#3 *wavelength*: criteria #1–2 must be satisfied at all wavelengths deemed critical for planet characterization.

#4 *timeliness*: criteria #1–3 must be satisfied during the time span of the mission

The input catalog of RV exoplanets provides the *natural* parameters for computing criteria #1–4. The mission design provides the *engineering* parameters. In this report, we develop the methods to compute N_{RV} from the input catalog and mission design. Many of the algorithms and much of the code used here was developed earlier, for completeness estimates and design reference missions, by Brown (2004a, 2004b, 2005) and Brown & Soummer (2010).

Natural parameters, from the input catalog

On May 10, 2013, we drew 423 known RV exoplanets from www.exoplanets.org, comprising all objects satisfying the search term "PLANETDISCMETH == 'RV'." We reduced the list to 419 planets by dropping 3 host stars with unknown distances (HD 13189, BD+48 738, and HD 240237) and 1 star with missing orbital information (Kepler-68). The final sample includes 406 planets with unknown orbital inclination angle (i) and 13 planets with *known* i. The input catalog includes 346 unique host stars.

The input catalog contains the following parameters for each RV exoplanet:

(i) stellar right ascension (α in hours) and declination (δ in degrees)
(ii) stellar distance (d) in pc
(iii) mass of the star (m_s) in solar masses
(iv) minimum planetary mass ($m_p \sin i$) in Jupiter masses
(v) semimajor axis (a) in AU
(vi) orbital eccentricity (ε)
(vii) argument of periapsis of the star (ω_s) in degrees. ($\omega_p = \omega_s + 180°$)
(viii) orbital period (T) in days
(ix) epoch of periapsis (T_0) in JD

In addition, we introduce the following natural photometric parameters:

(x) geometric albedo of the planet (p)
(xi) planetary phase function ($\Phi(\beta)$)
(xii) planetary radius (R_p)

In this report, we choose $p = 0.5$, $\Phi = \Phi_L$, the Lambert phase function, and $R_p = R_{Jupiter}$.

Engineering parameters from the mission design

We adopt the following values for four engineering parameters that are noncontroversial:

(a) $\gamma_1 = 45°$ (coronagraph), $\gamma_1 = 45°$ and $\gamma_2 = 80°$ (star shade)
(b) mission timing: January 1, 2020, to December 31, 2022 (duration 3 years)
(c) $\Delta mag_0 = 25$
(d) $\lambda_{lc} = 800$ nm, the longest critical wavelength.

Comments. These values of γ are being used in current NASA studies. A duration of three years is deemed about right for a probe-class mission, and this absolute timing is deemed to be reasonable for implementation. In recent years, NASA studies, laboratory tests, and experience with space instruments have built confidence that $\Delta mag_0 = 25$ can be achieved at usefully small angular separations (s) at visible and near-infrared wavelengths. Δmag_0 is ultimately determined by the brightest unstable speckles of starlight. Brown (2005). Spectral features near but below 800 nm are needed to characterize exoplanets in terms of methane and molecular oxygen.

Because typical separations between RV exoplanets and host stars are known to be smaller than a few Airy rings at 800 nm, N_{RV} is exquisitely sensitive to the minimum angle at which the performance Δmag_0 is achieved. This angle is called the inner working angle (IWA), and its value *is* controversial.

In this report, we adopt the value

(e) $IWA = 0.34$ arcsec .

Comments. IWA is commonly expressed

$$IWA = n\ \lambda_{lc} / D\ , \qquad (1)$$

where n is a number of Airy rings and D is the diameter of the telescope aperture. Equation (1) is simply a degenerate translation between two operational parameters, IWA and λ_{lc}, and two descriptive parameters,

(e_1') n
(e_2') D.

We can use Equation (1) to explore combinations of values of e_{1-2}' that produce the adopted values of IWA and λ_{lc} (see Table 1).

n	λ_{lc} (nm)	D (m)	IWA (arcsec)
4	800	1.94	0.34
3	800	1.46	0.34
2	800	0.97	0.34

Table 1. Some values of e_{1-2}' for which Equation (1) produces the value $IWA = 0.34$ arcsec for $\lambda_{lc} = 800$ nm.

By recognizing that $IWA = 0.34$ arcsec is compatible with various reasonable and accepted combinations of n and D at $\lambda_{lc} = 800$ nm, we implicitly satisfy criterion #3.

Criterion #1: permitted pointing

We compute γ from α and δ using the fact that the dot product of the unit vectors from the telescope to the star and the telescope to the sun is equal to $\cos\gamma$. We calculate the unit vectors in the rectangular ecliptic coordinates for any given time.

Figure 1 shows the permitted pointing on a day selected at random (June 20, 2010).

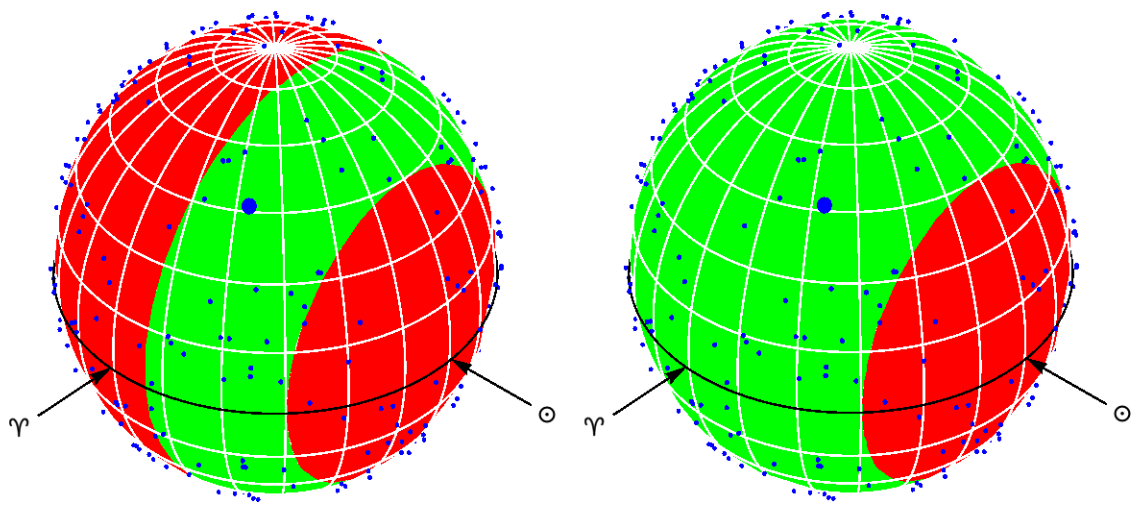

Figure 1. Celestial spheres on June 20, 2010, showing the positions of the ecliptic equator (black line), the vernal equinox, the sun, and the host stars of the 419 RV planets in our input catalog (blue dots). Left: for a star-shade mission ($\gamma_1 = 45°$, $\gamma_2 = 80°$); right: for a coronagraph mission ($\gamma_1 = 45°$, $\gamma_2 = 180°$). To provide a concrete example, a random host star, HD 2952, is shown as a large dot. If a star lies in the green region, it can be observed, but not if it lies in the red. As time passes, the sun, the coordinate grid, and the red/green zones remain fixed, while the vernal equinox and host stars revolve at constant ecliptic latitude, counterclockwise in this view, at the rate of one revolution per year.

For this study, we implement criterion #1 by creating 346 "validity" lists, one for each host star, of the days during the mission when the star is observable. By evaluating the other criteria only on "valid" days, we implicitly satisfy criterion #4.

Criterion #2: contrast

The following four steps are the master computations for N_{RV}. The steps evaluate the contrast criterion by asking this question:

$$s > IWA \cap \Delta mag < \Delta mag_0 \,?$$

Our computations answer this question for all 419 planets, for every day on the host star's validity list, and for two values of i: "face-on" (0.9°) and "edge-on" (89.1°).

In the first of the four steps, we compute the three-dimensional position the planet relative to the star from parameters ii–ix, for both the face-on and edge-on inclinations. (The 13 planets with known i are not in play for this mission due to their small separations.) Second, from the planet's position, we compute the apparent separation (s), the radial distance (r) from star to the planet, and the phase angle (β), which is the planetocentric angle between the star and earth. Third, we compute Δmag from parameters x–xii using Eq. (19) in Brown (2004b). Fourth, for each planet, we select the values of the maximum apparent separation (s_{max}) and the associated delta magnitude (Δmag_{max}).

We assume the planet is detectable and characterizable if $s_{max} > IWA$ and $\Delta mag_{\max} < \Delta mag_0$ for any valid time.

We distinguish three possible cases for each planet:

Case 1: $s_{max} > IWA$ and $\Delta mag_{\max} < \Delta mag_0$ at no valid time, for neither the face-on nor the edge-on inclinations. This is true for 415/419 planets in the current treatment.
Case 2: $s_{max} > IWA$ and $\Delta mag_{\max} < \Delta mag_0$ at valid times, for either the face-on or the edge-on inclinations, but not both. This is true for 1/419 planets.
Case 3: $s_{max} > IWA$ and $\Delta mag_{\max} < \Delta mag_0$ at valid times, for both the face-on and edge-on inclinations. This is true for 3/419 planets.

In the current treatment, the value of the metric is $N_{RV} = 3 + P$, which is the sum of the case-3 planets plus the probability (P_1) of the case-2 planet satisfies the contrast criterion for a random value of i. We know that $0 < P_1 < 1$, and explain how to compute it, below.

We found no meaningful differences in s_{max} and Δmag_{max} for the star-shade and coronagraph mission architectures.

Incorrect computation of contrast

It is tempting—but incorrect for the mission under study—to compare IWA with the *maximum possible* apparent separation

$$s_x \equiv a(1+\varepsilon)/d \qquad (2)$$

and to compare Δmag_0 with the delta magnitude at quadrature

$$\Delta mag_x = -2.5\log\left(p\,\Phi\left(\frac{\pi}{2}\right)\left(\frac{R_p}{a(1+\varepsilon)}\right)^2\right). \qquad (3)$$

Table 2 compares values of (s_x, Δmag_x) and (s_{max}, Δmag_{max}), the latter for both the face on and edge on inclinations.

The incorrect computation produces the estimate $N_{RV} = 5$, so we must understand why it is wrong. Figure 2 shows the full daily record of the results of the master computation for the six planets with largest value of s_x. In the correct computations, the case-3 planets are epsilon Eri b, GJ 832 b, and 55 Cnc d, and the case-2 planet is mu Ara c.

The first of three problems with the incorrect computation is the mission duration (three years) is short compared with the orbital periods of planets that achieve large s. This means that the largest possible separation may not be achieved during the mission. By inspection, this true for epsilon Eri b, HD 217107 c, and HD 190360 b—and it may be true of others.

The second problem—a concern for planets with elevated values of ε and with values of i and ω_p away from 0° and 180°, the maximum possible apparent separation may be closer to $a\,(1 - \varepsilon) / d$ than to $a\,(1 + \varepsilon) / d$.

The third problem is that Δmag_{max} will be significantly different from Δmag_x—greater or smaller—for the same eccentric orbits oriented to produce significantly reduced separations at apoapsis.

				mission duration: 1/1/20–12/31/2			
				edge-on orbits		face-on orbits	
	RV exoplanet	s_x (arcsec)	Δmag_x (β=90°)	s_{max} (arcsec)	Δmag_{max}	s_{max} (arcsec)	Δmag_{max}
1	epsilon Eri b	1.31	21.73	1.29	21.69	1.23	22.23
2	GJ 832 b	0.77	21.51	0.77	21.70	0.58	23.22
3	55 Cnc d	0.45	22.34	0.45	22.32	0.45	22.34
4	HD 217107 c	0.41	23.15	0.28	22.38	0.13	21.05
5	mu Ara c	0.38	22.45	0.38	22.41	0.32	21.72
6	HD 190360 b	0.33	22.19	0.26	21.67	0.17	20.69

Table 2. Estimates of s_{max} and Δmag_{max} for the six RV exoplanets with greatest values of $a(1+\varepsilon)/d$. Columns 3–4: the incorrect estimates (s_x, Δmag_x) that disregard knowledge of the RV orbit, particularly ε and ω_p, and don't account for the mission duration being significantly shorter than the orbital periods involved, Columns 4–5 and 6–7: mission values of s_{max} and Δmag_{max} for all edge-on orbits (i = 89.1°) and all face-on orbits (i = 0.9°). In red: cases that fail the contrast criterion when the full knowledge of the RV orbit and the mission duration are taken into account.

Table 3 provides the information in the input catalog on the six RV exoplanets with largest values of $a(1+\varepsilon)/d$.

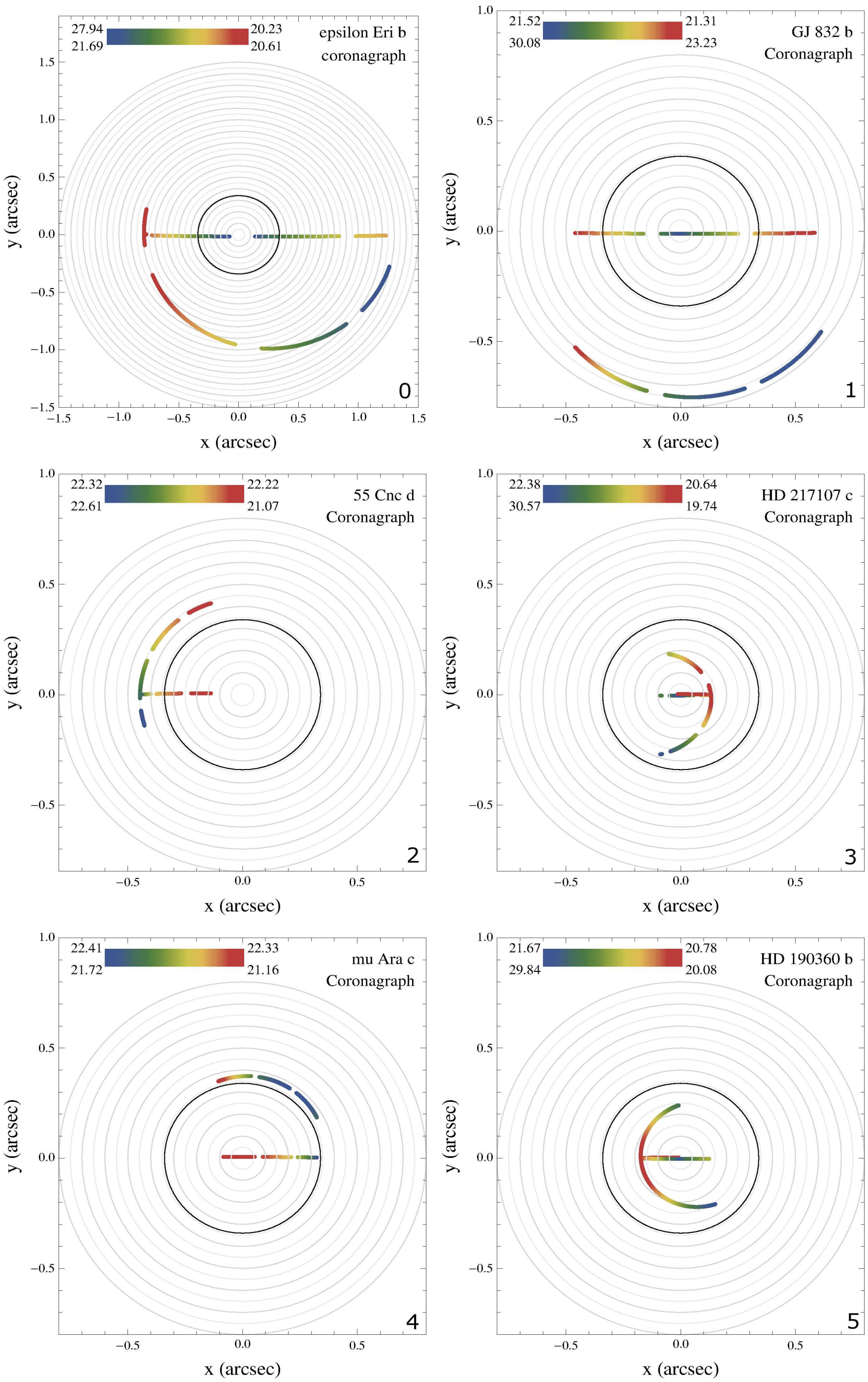

Figure 2. Photo-astrometric plots for the RV exoplanets in Table 2 during the mission. The orbits are curved or linear for the face-on and edge-on cases, respectively. The upper/lower numbers on the color key give the range of Δmag for edge-on/face-on. The dark circle is $IWA = 0.34$ arcsec.

	RV exoplanet	d(pc)	m_s ($m_\odot$)	$m_p \sin i$ ($m_♃$)	a(au)	ϵ	ω_p	period (days)	periapsis (JD -2450000)	$a(1+\epsilon)/d$ (arcsec)
1	epsilon Eri b	3.22	0.82	1.05	3.38	0.25	186.00	2500.	-1060.00	1.31
2	GJ 832 b	4.95	0.45	0.64	3.40	0.12	124.00	3416.	1211.00	0.77
3	55 Cnc d	12.34	0.91	3.54	5.47	0.02	74.00	4909.	3490.00	0.45
4	HD 217107 c	19.86	1.11	2.62	5.33	0.52	18.60	4270.	1106.32	0.41
5	mu Ara c	15.51	1.15	1.89	5.34	0.10	237.60	4206.	2955.20	0.38
6	HD 190360 b	15.86	0.98	1.54	3.97	0.31	192.93	2915.	3541.66	0.33

Table 3. Details on the six RV exoplanets with largest values of $a(1+\varepsilon)/d$.

Computation of *P*

Case-2 planets—those that are obscured for some values of i but detectable for others—are expected always to make a significant contribution to N_{RV}. If the number of class-2 planets is j_2, we recognize that their contribution to N_{RV} is the sum of j_2 Bernoulli random variables with unique probabilities P_k for all $1 \le k \le j_2$. We estimate each value P_k by a Monte Carlo experiment, as follows. First, we create a large sample of random values of i drawn from the appropriate random deviate, which is $\arccos(1-2Q)$, where Q is a uniform random deviate on the interval 0—1. Second, we compute s_{max} and Δmag_{max} for each value of i by the four-step master computation described above. Third, we determine the number of values of i that result in satisfying the contrast criterion on at least one valid day, and therefore are detectable. Fourth, we estimate P_k as the ratio of this number to the total number of values of i in the experiment. When we perform this experiment for mu Ara c, the result is P_1 = using 10,000 random value of i.

Using P_1, we estimate the metric as $N_{RV} = 3.45$.

If $j_2 > 1$, the density of the case-2 contribution to N_{RV} is the convolution of the j_2 individual Bernoulli densities, as explained and illustrated in Brown & Soummer (2010).

Summary

We have defined and evaluated a science metric, N_{RV}, for a mission to characterize known RV exoplanets using spectral features in reflected light. Correct evaluation of N_{RV} calls for computing the planetary position and brightness at times when pointing at the host star is permitted. For a typical probe-class mission, using non-controversial assumptions, we estimate $N_{RV} = 3.45$.

References

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R. A. Brown 2004b, "New information from radial-velocity data sets," *ApJ*, 610, 1079.
R. A. Brown 2005, "Single-visit photometric and obscurational completeness," *ApJ*, 624, 1010
R. A. Brown & R. Soummer 2010, "New completeness methods for estimating exoplanet discoveries by direct detection," *ApJ*, 715, 122

Workpoints & background information

• Assumptions could be relaxed—or new parameters tried—in future studies, which would increase the accuracy of the metric. Many of the current assumptions are optimistic or neutral, which means the current estimate is optimistic.

• No attempt has been made to prioritize these workpoints

• Exposure times are not taken into account—neither for the limiting search nor for the charactering spectroscopy. An exposure time calculator and a set of observational overheads could be introduced.

• Even though exoplanets with mass above Jupiter all have about 1 Jupiter radius, we could introduce a variable planet radius for smaller masses. It could be computed from the planet mass by some acceptable mass-radius relation. The planet mass *m* is well determined from *m* sin *i* for any known or assumed value of *i*.

• Shaklan on solar avoidance and star-shade glint: "For solar avoidance, 45 deg is probably the minimum for a starshade. We are investigating a solar diffraction term that may limit us to larger values and we are considering 50 deg for now. For a coronagraph 45 deg is reasonable." Lisman: "The maximum allowable off-sun angle for observations is 80 or 85 degrees. You cannot let sun strike the telescope facing side of the occulter." Also from Lisman: "I do not agree with the 45 degree number [Shaklan's 45° for the star shade]. We have been using 30 degrees as the lower limit and Stuart has just recently suggested a change but we have not settled on the exact number."

• Shaklan on limiting delta magnitude Δmag_0: We think that a systematic detection limit of delta_mag = 26 is reasonable for a starshade (tolerances aren't that tough to achieve). I suggest the same value for a coronagraph for working angles about >3 l/D, and dmag=25 for < 3 l/D.

• Shaklan on IWA: Since we're talking about probes, a telescope diameter of about 1.5 m is the maximum. A coronagraph at 2 l/D in the visible would have IWA = 165 mas. A 32 m starshade at 36,000 km would have IWA = 90 mas. The starshade and a 1.5 m telescope could be co-launched.

• Shaklan on mission duration: Mission duration is a cost driver. 3 years is reasonable for the baseline mission. The starshade could carry 5 years worth of fuel.

• Is the longest wavelength 1 micron or 800nm, to ensure the ability to detect oxygen and methane.