Part IV
Updraught calculation

The routine accvud computes the prognostic updraught mass flux scheme. Main controls are in the namelists namluc, with tunable parameters in namluc0. In addition to the convective diffusion fluxes, it also outputs convective condensations fluxes, and updates the updraught prognostic mass flux variables as well as the 4 microphysical variables, noted here T_{mp}, q, q_i, q_ℓ. The routine also receives the mean grid box temperature, noted $\overline{T}$.

1 Initial calculations

The environment condensates are summed into the array ZQC. If LCONRES=.F., this array is set to zero instead.
The ice fraction is initialized to $\alpha_i = \mathsf{fonice}(\mathsf{T_{mp}})$. The local latent heat $L_v((T_{mp}, \alpha_i)$ of the environment is put in ZLHE. The dry static energy of the environment is $\mathsf{ZDSE} = s_e = c_p T_{mp} + \phi$.
The budget latent heat is

$$L_v(T_{\rm surf}, \mathsf{fonice}(T_{\rm surf})) - (c_{pv} - c_{pa})(1 - \delta_m q_{\rm surf}) T_{\rm surf}$$

The moist static energy of the environment is

$$\mathsf{ZHSE}^l \equiv h_e = s_e^l + L_{\rm bud} \cdot q^l$$

The moist static energy along a moist adiabat starting at the surface (accounting for slanted convection) is calculated with

$$\mathsf{ZHSEAD} \equiv h^l_{e_a d} = \max(h^l_e, h^{l+1}_{e_a d} + (\phi^l - \phi^{l+1}) \frac{\mathsf{zatslc}}{1 + \mathsf{zatslc}}$$

The integrated buoyancy of a non diluted ascent is stored in

$$\mathsf{ZS17} \equiv I_b = \sum \frac{(h_{e\_ad} - h_e)^l + (h_{e\_ad} - h_e)^{l+1}}{2} (\phi^l - \phi^{l+1})$$

The advected prognostic variables are PUDOM and PUDAL. The advected prognostic velocity is actually $\omega_u - \mathsf{GCVADMW} \cdot \overline{\omega}$, but we now set GCVADMW=0. $\omega_u \triangle t$ is put into PUDOM, the values of PUDAL lower than 10^-11 are reset to zero, and the other ones are averaged over the vertical to produce the mean advected updraught mesh fraction $\mathsf{ZSIG9} \equiv \sigma_u^-$. The key LLSIGPROP=.T. allows to use a varying mesh fraction over the vertical, proportional to ω_u. Otherwise (default case), we copy ZSIG9 at all levels into PUDAL.

2 Triggering

While not excluding further enhancements, we keep at this stage the earlier approach for the triggering.
The layer activity is estimated during the construction of the cloudy profile, based on local buoyancy and if appropriate, large scale moisture convergence.
We assume that the base of any cloud has the properties of the blue point associated to the large scale mean.

This way the starting point of the profile is saturated, and no dry adiabatic path is considered in order to reach this point. At the LCL, we have some chance to have

$$\overline{T_d} \leq T_u \leq \overline{T}$$

(T_u resulting from a dry adiabatic ascent), unless the environment saturates at a lower level or the instability is very pronounced.
The wet bulb temperature shares the same property:

$$\overline{T_d} \leq \overline{T_w} \leq \overline{T}$$

Now, considering the cloud base (the LCL), if the properties immediately below are equal to the mean environment (with no condensate), the condensate at the LCL provides the water to bring these properties to the

blue point: (T_w, q_w) (isobaric isenthalpic transformation, $dh = 0 = c_p dT + L dq$). Along the profile construction, a test is done to be sure that the cloudy air is warmer than its environment: when it gets colder than the local $\overline{T_w}$, a new profile should be started from the local blue point, and considering that the condensate obtained on the earlier profile should provide the water to reach local $\overline{q_w}$. It does not represent an evaporation process but a correction of the vapour content, using moisture that the calculation has turned too promptly into condensate.
Using this condensate is thus a numerical trick, to limit the risk of some parasite feedbacks.

3 The updraught profile

3.1 Starting point

The microphysical mean values q, $q_c \equiv q_\ell + q_i$, T_{mp}, with the surface conditions T_s, q_s, define the input state. The corresponding values of the saturation water vapour $q_{\rm sat}$, the wet bulb q_w, T_w are also passed as arguments.

$$T_u^L = T_w^L \qquad q_u^L = q_w^L \qquad q_{cu}^L = \max(0, q_c^l + (q^l - q_w^l)) \qquad s_u^L = (c_{pa} + (c_{pv} - c_{pa}) q_u^L) T_u^L + \phi^L$$

$$T_{vu}^L = (1 - q_{cu}^L + \frac{R_v - R_a}{R_a} q_u^L) T_u^L \qquad T_{ve}^L = (1 - q_c^L + \frac{R_v - R_a}{R_a} q^L) T_{mp}^L \qquad \alpha_i^L = 0 \qquad \omega_u^L = 0$$

The moist pseudo-adiabat is, as in the earlier scheme, obtained by

- starting from a saturated state,
- applying an isobaric mixing with the environment,
- following an upward trajectory conserving both moist static energy and total moisture.

3.2 Mixing

The mixing follows the relations

$$\frac{\partial q_u}{\partial \phi} = \lambda^u \left(q - q_u\right), \quad \frac{\partial T_u}{\partial \phi} = \lambda^u \left(T_{mp} - T_u\right), \quad \frac{\partial q_{cu}}{\partial \phi} = \lambda^u \left(q_c - q_{cu}\right). \tag{1}$$

where the entrainment rate is given by

$$\lambda_u^{l+1} = \lambda_{\min} + (\lambda_{\max} - \lambda_{\min}) e^{-\lambda_{\max}^{3/4}\lambda_{\min}^{1/4}(\phi^l - \phi^b)} + \beta_E \left[\max(0, \mathsf{ZS10})\right]^{\gamma_E}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ are functions of the integrated buoyancy of a non diluted ascent I_b. The parameters GCVBEE $\equiv \beta_E$ and GCVEEX $\equiv \gamma_E$ allow a feedback of the buoyancy on the entrainment rate.
Mixing with the environment is proportional to $(\psi_e - \psi_u)$ and to the updraught mass flux. In the original scheme, the difference between ψ_e and $\overline{\psi}$ was neglected. In the case of the SCMF scheme, we proposed to express $\psi_u - \psi_e$ in function of $\psi_u - \overline{\psi}$ and σ_u — under the assumption that we could neglect the part occupied by the downdraught.
The properties of the environment entrained into the updraught are given by

$$\psi_u - \psi_e = \frac{1}{1 - \sigma_u}(\psi_u - \overline{\psi})$$

supposing that we get a reasonable guess of the updraught mesh fraction σ_u, using the value advected from the previous time step.
The same treatment applies to the condensed water, with the environment condensate q_c as chosen above — normally the grid box mean, $\overline{q_c} = \overline{q_\ell} + \overline{q_i}$.
Under LSCMF=.T. *the entrainment rate* ZMIX $\equiv \xi = \lambda_u \triangle\phi$ *is divided by* $(1 - \sigma_u^-)$, *using the mean advected updraught mesh fraction* σ_u^-. See further in this text more details about the additional test key LLDIVENT and the division of ZENTR $= \lambda_u$ used in the updraught vertical velocity equation.
The mixing is applied actually at constant pressure. The updraught values ψ_u^l are mixed with the environment into the values ψ_b, which represent the departure point of the subsequent ascent:

$$T_b = T_u^{l+1} + \xi(T_{mp}^{l+1} - T_u^{l+1}) \qquad q_b = q_u^{l+1} + \xi(q^{l+1} - q_u^{l+1}) \qquad q_{cb} = q_{cu}^{l+1} + \xi(q_c^{l+1} - q_{cu}^{l+1})$$

The entrainment of dryer air into the updraught implies a reduction of the specific moisture and the temperature. Thereafter (T_b, q_b) could well depart from the blue point. If the mixture is no longer saturated, we should re-evaporate part of the condensate, while if it is over saturated, we could re-condense it.
This was not done in the earlier scheme, because the state b is an intermediate internal state, while the goal is to move to a saturated point at the end of the ascent, when reaching level l. The return to saturation in b conserves the moist static energy and the total water, so there is no difference starting the moist adiabat from b.

3.3 Prognostic Mixing

The key LENTCH activates the historic mixing. The expression of the entrainment rate λ is then expressed by

$$\lambda = \left\{ \lambda_{tx} + \frac{\beta_E}{\triangle\phi^{l+1}} \max\left(0, \frac{\omega_u^{l-1} - \omega_u^{l+1}}{2\omega_u^l}\right)^{\gamma_E} \right\} (1 - \zeta^{l+1}) + \lambda_{tn}\zeta^{l+1} \tag{2}$$

The mixing is reduced in presence of downdraught activity, because the subgrid downdraughts are confining the updraughts. Wide downdraught are unstable and split in several narrower ones. So when the downdraught mesh fraction increases the air mixed to the updraught no longer has the properties of the mean grid box, but comes from a narrower confined circulation: the difference $|\psi_u - \psi_e|$ between the mixing air and the updraught air decreases. To represent this effect, instead of reducing $(\psi_u - \psi_e)$, we reduce the coefficient λ_u in Eq. 1, when the downdraught prognostic mesh fraction increases. Beware we are speaking about mixing, not entrainment: in reality the entrained flow is not affected, but well its properties.
The reduction of λ is obtained through the prognostic variable $\zeta \equiv$ PENTCH, such that $0 \leq \zeta \leq 1$, following

$$\frac{\partial \zeta}{\partial t} = \alpha_E \sigma_d - \frac{\zeta}{\tau_E} = \frac{1}{\tau_E}(\kappa_E \sigma_D - \zeta) \tag{3}$$

i.e. if the downdraught disappears, ζ returns to zero with a relaxation time τ_E. This is discretized as

$$\zeta^+ = \frac{\zeta^- + \frac{\kappa_E \triangle t}{\tau_E}\hat{\sigma_d}}{1 + \frac{\triangle t}{\tau_E}} \tag{4}$$

In this expression, we use a smoothed downdraught mesh fraction, defined as

$$\hat{\sigma_d}^l = \frac{\hat{\sigma_d}^{l+1} + \sigma_d^l + \sigma_d^{l-1}}{3}$$

When there are many downdraughts, in steady state, $\zeta \to 1$ and the entrainment rate is reduced to a minimum turbulent value λ_{tn}. When there are no downdraughts, $\zeta \to 0$ and the entrainment takes bigger value. In this case, it includes a turbulent part represented by the parameter λ_{tx} and a more organized part.

The organized part is linked to the vertical acceleration in the updraught: to keep the same mesh fraction than below with a higher updraught velocity, the mass flux must increase, which is obtained by entraining more air from the environment. Again, the mixing does not increase in proportion of the entrainment, so that instead of affecting the difference $(\psi_u - \psi_e)$, we limit the increase of the entrainment rate λ with an exponent γ_E and a factor β_E. The formulation assumes that the mesh fraction is the same at levels l, $l-1$, $l+1$. This is not valid at the bottom of the updraught, where the totality of the updraught mass flux has to be created through entrainment. But it would'nt be realistic to take for it the air at the base level of the cloud: several levels below should intervene. The limit case $\beta_E = 1 = \alpha_E$ represents a complete entrainment of mean gridbox air from the same level mixing in the updraught. The acceleration effect is limited by the exponent $\beta_E < 1$, which allows a smoother profile.
The parameter GCVACHI is used here to put a threshold ω_u, so that the organized part disappears when ω_u is smaller than this threshold.

For the cloud ensemble we get a new difficulty, because the relaxation with a non entraining ascent required to know in advance the fraction of the ascent which remained to complete: this is not possible while building the prognostic mixing profile. The uncompleted fraction of the ascent is now roughly evaluated with

$$\mathsf{ZFRAA} = 1 - \max\Big\{0, \min\big(1, \frac{1}{\phi}_x \sum_{k=L-1}^{l} \delta_{\mathrm{act}}^k(\phi^k - \phi^{k+1})\big)\Big\}$$

Finally, we limit the mixing coefficient λ to an extreme value GPEMAX.
The tunable parameters (and possible typical values) are:

$\lambda_{tx} \equiv$ TENTRX (5.E-06 s^2m^{-2})	$\lambda_{tn} \equiv$ TENTR (2.5E-6 s^2m^{-2})
$\beta_E \equiv$ GCVBEE (0.1)	$\gamma_E \equiv$ GCVEEX (0.5)
$\kappa_E \equiv$ GPEFDC (10.0)	$\tau_E \equiv$ GPETAU (600. s)
$1/\phi_x \equiv$ GPEIPHI (1.1E-5 s^2m^{-2})	
GPEMAX (4.E-3 s^2m^{-2})	GCVACHI (1.0 Pa/s)

3.4 Pseudo-adiabatic ascent

We want to construct the moist pseudo-adiabat from the model level b to the next level $h \equiv b-1$, immediately above it.
The moist static energy of state 'b' is

$$\mathsf{ZHS\_REE} \equiv h_b = c_p^{l+1} T_b + \phi^{l+1} + L_{\text{bud}}\, q_b$$

After this mixing step, a saturated pseudo adiabatic ascent is followed, *but assuming total water conservation*:

$$q + q_c \equiv q + q_i + q_\ell = q_b + q_{\ell b} + q_{ib}$$

i.e. that all moisture stays in the system, but only for the *crossing* of the layer. The total water for a layer is re-evaluated at the base of each layer (at the same time as mixing), but the result is then assumed constant along the whole layer height while calculating the moist adiabat segment crossing it.
Doing this, the earlier scheme was some way off from the theoretical moist adiabat, for which all condensate should be evacuated immediately.
For the new scheme, keeping the condensed water is unavoidable.
The local latent heats $L(T_b, \alpha_{ib})$ depend on the ice fraction, α_{ib}. We set LLNEWM=.F. to compute $\alpha_{ib} =$ fonice(T_b); otherwise a step transition at the triple point would be used. The saturation moisture is also function of T_b and α_{ib}.
To compute the saturated values T_h, q_h, we must follow the saturated pseudo-adiabat $q = q_s(T)$ which is non linear. To solve this, we use a *Newton algorithm* which linearizes q_s in the neighbourhood of the preceding iteration:

$$q^{k+1} = q_s\left(T^k\right) + \frac{\partial q_s}{\partial T^k}\left(T^{k+1} - T^k\right) \tag{5}$$

Of course, we must have saturation at the level h: $q_h = q_s$.

The heat absorbed by the raising condensate is assumed much smaller than the heat absorbed by the moist air, and we still follow the moist pseudo adiabat, along which h is conserved:

$$dh = c_p dT + L dq + d\phi = 0 \tag{6}$$

$$c_p = c_{pb} + \left(c_{pv} - c_{w|i}\right)(q - q_b) = c_{pb} + \gamma\, \triangle q \tag{7}$$

$$L = L_b + \left(c_{pv} - c_{w|i}\right)(T - T_b) = L_b + \gamma\, \triangle T \tag{8}$$

where we defined $\gamma \equiv \left(c_{pv} - c_{w|i}\right)$, and used the conservation of total water.

$$dc_p = \gamma dq\ , \qquad dL = \gamma dT \qquad \Longrightarrow \gamma dh = c_p\, dL + L\, dc_p + \gamma\, d\phi = d(Lc_p) + \gamma d\phi = 0$$

$$L\, c_p - L_b\, c_{pb} + \gamma\, \triangle\phi = 0 \tag{9}$$

Multiplying (7) by (8), and combining with (9) yields

$$Lc_p = L_b c_{pb} + \gamma\, \triangle T\, c_{pb} + \gamma\, \triangle q\, L_b + \gamma^2 \triangle T\, \triangle q \qquad \Longrightarrow \quad \triangle T\, c_{pb} + \triangle q\, L_b + \gamma \triangle T\, \triangle q + \triangle\phi = 0$$

hence

$$\begin{array}{rcl} c_{pb}\,(T - T_b) + L_h\,(q - q_b) + \triangle\phi &=& 0 \\ c_{ph}\,(T - T_b) + L_b\,(q - q_b) + \triangle\phi &=& 0 \end{array} \tag{10}$$

if we neglect the second order term (i.e. $(T-T_b)(q-q_b)$ is taken either as $(T_h-T_b)(q-q_b)$ or $(T-T_b)(q_h-q_b)$).

To estimate $\triangle\phi$ we have, for the *"equi-pressure cloud"* approach (GCVADS $= 0$):

$$d\phi = -\frac{dp}{\rho} = -RT\frac{dp}{p}$$

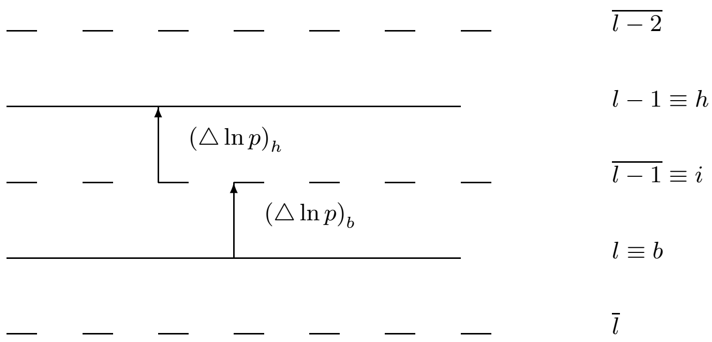
Noting $i \equiv \bar{h}$ the interface between the two full model levels b and $h \equiv b-1$:

$$\begin{array}{rcl} (\triangle \ln p)_b &=& \ln \dfrac{p_b}{p_i} \qquad (\triangle \ln p)_h = \ln \dfrac{p_i}{p_h} \\ \triangle\phi &=& R_b\, T_b\, \left(\triangle \ln p\right)_b + R_h\, T_h\, \left(\triangle \ln p\right)_h \\ &=& R_b\, T_b\, \left(\triangle \ln p\right)_b + \left(R_b + R_v (q_h - q_b)\right) T_h \left(\triangle \ln p\right)_h \\ &\equiv& \widetilde{R}_b^-\, T_b + \widetilde{R}_b^+\, T_h + \widetilde{R}_v^+\, T_h\,(q_h - q_b) \end{array} \tag{11}$$

where we used again the conservation of total water

$$R_h - R_b = R_v(q_h - q_b) + Ra(q_{cb}q_b - q_{ch} - q_h) = R_v(q_h - q_b)$$

The three coefficients $\widetilde{R}_b^-$, $\widetilde{R}_b^+$, $\widetilde{R}_v^+$ are independent of the subsequent computations of q_h and T_h. Practically in the routine, we have:



$$\begin{aligned}
\mathsf{PALPH(KLON,KLEV)} &\equiv \ln \frac{p_{\overline{l-1}}}{p_{l-1}} = (\triangle \ln p)_h \quad \text{and} \quad \ln \frac{p_l}{p_{\overline{l-1}}} = (\triangle \ln p)_b \\
\mathsf{PLNPR(KLON,KLEV)} &\equiv \ln \frac{p_{\bar{l}}}{p_{\overline{l-1}}} \\
\mathsf{PLNPR(KLON,KLEV)} &\equiv \ln \frac{p_{\bar{l}}}{p_{\overline{l-1}}} = (\triangle \ln p)_b - (\triangle \ln p)_{h+1} \\
\mathsf{ZRBB(KLON)} &\equiv \widetilde{R}_b^- = R_b\,(\triangle \ln p)_b = R_b \ln \frac{p_l}{p_{\overline{l-1}}} \\
\mathsf{ZRBH(KLON)} &\equiv \widetilde{R}_b^+ = R_b\,(\triangle \ln p)_h = R_b \ln \frac{p_{\overline{l-1}}}{p_{l-1}} \\
\mathsf{ZRVH(KLON)} &\equiv \widetilde{R}_v^+ = R_v\,(\triangle \ln p)_h = R_v \ln \frac{p_{\overline{l-1}}}{p_{l-1}} \\
\text{with } R_b &= R_a\,(1 - l_b - q_b) + R_v\, q_b \\
&= R_a\,(1 - l_b) + (R_v - R_a)\, q_b
\end{aligned}$$

For the *"equi-geopotential cloud"* approach (GCVADS $= 1$), we take directly $\triangle\phi$ from the environment. Modulation between both cases with parameter GCVADS, is obtained by

$$\begin{aligned}
\mathsf{ZRBB} &= (1 - \mathsf{GCVADS}) \cdot \mathsf{ZRBB} + \mathsf{GCVADS} \cdot \frac{\phi^l - \phi^{l+1}}{T_b} \\
\mathsf{ZRBH} &= (1 - \mathsf{GCVADS}) \cdot \mathsf{ZRBH} \\
\mathsf{ZRVH} &= (1 - \mathsf{GCVADS}) \cdot \mathsf{ZRVH}
\end{aligned}$$

For the *"ensemblist"* formulation, the relaxation to the not entraining profile is performed by multiplying by the fraction of buoyancy-excess with respect to the not entraining, undiluted plume:

$$\mathsf{ZFFAND} = \frac{1}{1 + \mathsf{GCVNU}(1 - \mathsf{ZFRAA}) \max(0, h_{\mathrm{ad}} - h_u)} \tag{12}$$

$$\mathsf{ZRBB} = \mathsf{ZRBB} \cdot \mathsf{ZFFAND} \quad , \qquad \mathsf{ZRBH} = \mathsf{ZRBH} \cdot \mathsf{ZFFAND} \quad , \qquad \mathsf{ZRVH} = \mathsf{ZRVH} \cdot \mathsf{ZFFAND}$$

The updraught condensate now results from the condensation along the ascent, and the subsequent mixing, as no precipitation may be considered. For this reason, the mixing and re-evaporation of condensate becomes important. Nevertheless we may consider a similar limitation of the condensate as in the earlier scheme, in relation to the detrainment of the condensate excess.
Combining (10) and (11):

$$\begin{aligned}
c_{pb}\,(T - T_b) + L\,(q - q_b) + \widetilde{R}_b^-\, T_b + \widetilde{R}_b^+\, T + \widetilde{R}_v^+\, T\,(q - q_b) &= 0 \\
\left(c_{pb} + \widetilde{R}_b^+\right)(T - T_b) + \left[\widetilde{R}_v^+\, T_b + \widetilde{R}_v^+\,(T - T_b) + L\right](q - q_b) + \left(\widetilde{R}_b^+ + \widetilde{R}_b^-\right) T_b &= 0 \qquad (13)
\end{aligned}$$

Let be

$$
\begin{array}{lcrcl}
\mathsf{ZCP} & \equiv & \widetilde{C}_p & \equiv & c_p + \widetilde{R}_b^+ + \widetilde{R}_v^+ (q - q_b) \\
\mathsf{ZLH} & \equiv & \widetilde{L} & \equiv & L + \widetilde{R}_v^+ \; T_b + \widetilde{R}_v^+ (T - T_b) = L + \widetilde{R}_v^+ T
\end{array}
\qquad (14)
$$

The last term of $\widetilde{C}_p$ makes that we still have:

$$
\frac{\partial \widetilde{C}_p}{\partial q} = \frac{\partial \widetilde{L}}{\partial T} = \gamma + \widetilde{R}_v^+
$$

(while the double use of the non linear term is avoided by using $\widetilde{C}_{pb}$ and not $\widetilde{C}_p$ in the next equations). Introducing an iterative process (Newton's loop), represented by control variable k:

$$
\begin{array}{lclclcl}
\widetilde{C}_{pb} \left(T^k - T_b\right) & + & \widetilde{L}^k \left(q^k - q_b\right) & + & \left(\widetilde{R}_b^+ + \widetilde{R}_b^-\right) T_b & = & 0 \\
\widetilde{C}_{pb} \left(T^{k+1} - T_b\right) & + & \widetilde{L}^{k+1} \left(q^{k+1} - q_b\right) & + & \left(\widetilde{R}_b^+ + \widetilde{R}_b^-\right) T_b & = & 0 \\
\hline
\widetilde{C}_{pb} \left(T^{k+1} - T^k\right) & + & \widetilde{L}^{k+1} \left(q^{k+1} - q_b\right) & - & \widetilde{L}^k \left(q^k - q_b\right) & = & 0
\end{array}
\qquad (15)
$$

For the iterative process, we make a first guess with:

$$
\begin{array}{lcl}
T^{k=0} & = & T_b + \left(T^l - T^{l+1}\right) \\
q^{k=0} & = & q_b - \frac{1}{\widetilde{L}} \left\{ \widetilde{C}_{pb} \left(T^{k=0} - T_b\right) + \left(\widetilde{R}_b^+ + \widetilde{R}_b^-\right) T_b \right\}
\end{array}
\qquad (16)
$$

This way we include the term $\widetilde{R}_b^+ + \widetilde{R}_b^-$ (and the first equation (15)) in the jump from b to $k = 0$. The subsequent iterations provide adjustments starting from this level $k = 0$. Replacing b by $k = 0$ in the previous set and then setting $q = q^{k=0}$, $T = T^{k=0}$ in the first equation, yields:

$$
\widetilde{R}_{k=0}^+ + \widetilde{R}_{k=0}^- = 0 \qquad (17)
$$

(it corresponds to setting $q = q_b$ and $T = T_b$ in equation (10) which implies $\triangle\phi = 0$ for a moist adiabat).
We also have

$$
\widetilde{L}^{k+1} = L^{k+1} + \widetilde{R}_v^+ \; T^{k+1} = L^k + \gamma \left(T^{k+1} - T^k\right) + \widetilde{R}_v^+ \; T^{k+1} = \widetilde{L}^k + \left(\gamma + \widetilde{R}_v^+\right) \left(T^{k+1} - T^k\right) \qquad (18)
$$

$$
\begin{array}{lcl}
\widetilde{C}_p^{k+1} & = & c_p^{k+1} + \widetilde{R}_b^+ + \widetilde{R}_v^+ \; \left(q^{k+1} - q^k\right) = c_p^k + \gamma \left(q^{k+1} - q^k\right) + \widetilde{R}_b^+ + \widetilde{R}_v^+ \; \left(q^{k+1} - q^k\right) \\
 & = & \widetilde{C}_p^k + \left(\gamma + \widetilde{R}_v^+\right) \left(q^{k+1} - q^k\right)
\end{array}
\qquad (19)
$$

Replacing b successively by k and $k + 1$ in the third equation (15) yields

$$
\widetilde{C}_p^k \left(T^{k+1} - T^k\right) + \widetilde{L}^{k+1} \left(q^{k+1} - q^k\right) = 0 \qquad (20)
$$

$$
\widetilde{C}_p^{k+1} \left(T^{k+1} - T^k\right) + \widetilde{L}^k \left(q^{k+1} - q^k\right) = 0 \qquad (21)
$$

Transforming equation (13) for iteration $k + 1$, with b=k and (17):

$$
\begin{array}{l}
\left(c_p^k + \widetilde{R}_b^+\right) \left(T^{k+1} - T^k\right) \\
\quad + \left[\widetilde{R}_v^+ \; T^k + \widetilde{R}_v^+ \left(T^{k+1} - T^k\right) + L^k + \gamma \left(T^{k+1} - T^k\right)\right] \left(q_s(T^k) + \frac{\partial q_s}{\partial T^k} \left(T^{k+1} - T^k\right) - q^k\right) = 0
\end{array}
$$

Dismissing the second degree terms in $\left(T^{k+1} - T^k\right)$

$$
\begin{array}{cl}
 & \left(T^{k+1} - T^k\right) \left[\widetilde{R}_b^+ + c_p^k + \left(L^k + \widetilde{R}_v^+ \; T^k\right) \frac{\partial q_s}{\partial T^k} + \left(\widetilde{R}_v^+ + \gamma\right) \left(q_s(T^k) - q^k\right)\right] \\
 & \quad + \left(L^k + \widetilde{R}_v^+ \; T^k\right) \left(q_s(T^k) - q^k\right) = 0 \\
\Longleftrightarrow & \left(T^{k+1} - T^k\right) \left[\widetilde{C}_p^k + \widetilde{L}^k \frac{\partial q_s}{\partial T^k} + \left(\widetilde{R}_v^+ + \gamma\right) \left(q_s(T^k) - q^k\right)\right] + \widetilde{L}^k \left(q_s(T^k) - q^k\right) = 0
\end{array}
$$

Using (18):

$$
\left(T^{k+1} - T^k\right) \left[\widetilde{C}_p^k + \widetilde{L}^k \frac{\partial q_s}{\partial T^k}\right] + \widetilde{L}^{k+1} \left(q_s(T^k) - q^k\right) = 0 \qquad (22)
$$

Using (20) to replace $\widetilde{L}^{k+1}$ yields

$$\begin{aligned} \left(T^{k+1}-T^k\right)\left[\widetilde{C}_p^k+\widetilde{L}^k\frac{\partial q_s}{\partial T^k}-\widetilde{C}_p^k\frac{q_s(T^k)-q^k}{q^{k+1}-q^k}\right] &= 0 \\ \Rightarrow \qquad q^{k+1}\left(\widetilde{C}_p^k+\widetilde{L}^k\frac{\partial q_s}{\partial T^k}\right)-\widetilde{C}_p^k q_s(T^k)-\widetilde{L}^k\frac{\partial q_s}{\partial T^k}q^k &= 0 \end{aligned}$$

hence

$$\mathsf{ZDELQ}\equiv\left(q^{k+1}-q^k\right)=\frac{q_s(T^k)-q^k}{1+\frac{\widetilde{L}^k}{\widetilde{C}_p^k}\frac{\partial q_s}{\partial T^k}} \tag{23}$$

(21) gives

$$\mathsf{ZDELT}\equiv\left(T^{k+1}-T^k\right)=-\frac{\widetilde{L}^k}{\widetilde{C}_p^{k+1}}\left(q^{k+1}-q^k\right) \tag{24}$$

We use also equations (18) and (19):

$$\widetilde{C}_p^{k+1}-\widetilde{C}_p^k = \left(\gamma+\widetilde{R}_v^+\right)\left(q^{k+1}-q^k\right) \tag{25}$$

$$\widetilde{L}^{k+1}-\widetilde{L}^k = \left(\gamma+\widetilde{R}_v^+\right)\left(T^{k+1}-T^k\right) \tag{26}$$

$\left(\gamma+\widetilde{R}_v^+\right)$ is the variable ZDCP in the code.
The condensate and the vapour are all the time linked by the conservation of total water:

$$q^{k+1}=q^k+\mathsf{ZDELQ} \quad , \qquad q_c^{k+1}=q_c^k-\mathsf{ZDELQ}$$

The Newton algorithm uses successively (23), (25), (24), (26) in NBITER iterations.
The above somehow complex development has the big advantage of its precision, so that NBITER may be kept as low as 2, as it was for the calculation of condensation without vertical motion.

The risen parcel has to be warmer than the environment. we compute an index

$$\mathsf{ZBLUE}\equiv\delta_B=\begin{cases}0 & \text{if } T_u^l>T_w^l,\\ 1 & \text{otherwise.}\end{cases}$$

which imposes to go back to the blue point when it is equal to 1. This return to the wet bulb conserves the total water, so the condensate absorbs the water vapour increment (this is a correction, of the final state, corresponding to no actual condensation process):

$$q_{cu}^l=(1-\delta_B)\left\{q_{cb}+\max(0,q_b-q_u^l)\right\}+\delta_B(q_c^l+q_u^l-q_w^l) \quad T_u^l=(1-\delta_B)T_u^l+\delta_B T_w^l \quad q_u^l=(1-\delta_B)q_u^l+\delta_B q_w^l$$

3.5 Limitation of the updraught condensate

The total condensation during the ascent is obtained by the decrement of q_u:

$$\mathsf{ZDQL}^{\bar{l}}=\max(0,q_b-q_u^l)(1-\delta_B)$$

(when returning to the blue point, no real condensation is considered).
No precipitation may occur within the updraught: the condensate generated along the ascent can

- either, stay in the updraught, while the feedback of it occurs through the reduction of buoyancy it induces (which plays when checking the convective ability, as well as in the prognostic vertical motion equation);
- or, be detrained (and enter the micro physical scheme, which may precipitate it).

The danger of keeping all the condensate in the updraught is that we will reach much higher values than in the earlier scheme, where the increment we compute here was not added completely to what came from below, but it was considered that a part vanished though precipitation. If moreover we entrain some environment condensate, we end with much higher values of updraught condensate, which might seem unrealistic. But a negative feedback exists from the mass flux calculation, where the condensate reduces the buoyancy and lowers the level of organized detrainment.

Considering that anyway no condensate could stay in the updraught above the limit of precipitation, we can keep the earlier scheme limitation, now decreeing that the detrainment process has to move out the excess condensate, as did the precipitation before:

$$\begin{aligned} \mathsf{ZLN} &\equiv q^l_{cu} = q_{cb}e^{-1/\chi} - \left(q^l_u - q_b\right)\chi(1-e^{-1/\chi}) \\ \text{with } \mathsf{ZLIQ} &\equiv \chi \equiv \frac{\phi_0}{\triangle\phi^l_u} = \frac{\phi_0}{\widetilde{R}^-_b T_b + \left(\widetilde{R}^+_b + \widetilde{R}^+_v(q^l_u - q_b)\right)T^l_u} \end{aligned} \qquad (27)$$

This expression, based on a critical cloud thickness inducing precipitation $\phi_0 \equiv \mathsf{ECMNP}$, is now justified by the fact that a significant part of the condensate has to be detrained, so that only a fraction of it is moved upwards to the next layer.
Making ECMNP $\to \infty$ brings back the (unrealistic) case of all produced condensate being carried on in the updraught:

$$\lim_{x\to\infty} x(e^{-\frac{1}{x}} - 1) = -1 \quad\Longrightarrow\quad q^l_{cucarried} \to q_{cb} - (q^l_u - q^b_u) = q^l_{cugross}$$

This detrainment process may differ for liquid water and ice: therefore, we introduced a second critical thickness ECMNPI for ice cloud, and the actual ϕ_0 is a weighted mean of ECMNPand ECMNPI function of the ice fraction.

$$\mathsf{ZECMNP} = \alpha_i\mathsf{ECMNPI} + (1-\alpha_i)\mathsf{ECMNP}$$

3.6 The buoyancy force

The local buoyancy force (per unit mass) is given by

$$F_b = g\frac{T_{vu} - \overline{T}_v}{\overline{T}_v} \quad , \qquad \text{with } T_v = T(1 - q_c + \frac{R_v - R_a}{R_a}q)$$

With no condensate, the virtual temperature within the cloud is defined by:

$$RT = R_aT_v \quad\Longrightarrow\quad T_v = T(1 + \frac{R_v - R_a}{R_a}q)$$

while in presence of a single condensed phase, it becomes [Dufour and Van Mieghem, 1975]

$$\begin{aligned} T_v - T_{vw} \approx \frac{q_c}{1-q}T_v \Longrightarrow T_{vw} &\approx T_v(1 - \frac{q_c}{1-q}) = T(1 + \frac{R_v - R_a}{R_a}q)(1 - \frac{q_c}{1-q}) \\ &= T(1 + \frac{R_v - R_a}{R_a}q - \frac{q_c}{1-q} - \frac{R_v - R_a}{R_a}\frac{q_cq}{1-q}) \\ &\approx T(1 - q_c + \frac{R_v - R_a}{R_a}q) \end{aligned}$$

which is the approximation chosen up to now in Aladin.
In the code we now introduce two full arrays:

$$\mathsf{ZTVN(JLON, JLEV)} \equiv T^l_{vu} \qquad \text{and} \qquad \mathsf{ZTVE(JLON, JLEV)} \equiv \overline{T}^l_v$$

3.7 Activity test

The convective ability of the raised parcel is then checked:

- buoyancy of the updraught air with respect to the environment:

$$\mathsf{ZKUO1} = T_{vu} - T_{ve} \quad , \qquad \mathsf{ZKUO1} > 0 \Longrightarrow \mathsf{INBU} = 1$$

- Large scale moisture convergence, if the closure is based on it (LCAPE=.FALSE.):

$$\mathsf{ZKUO2} = \sum_L^l L^l_e \cdot \mathsf{PCVGQ}^l \triangle p^l \quad , \qquad \mathsf{ZKUO2} < 0 \Longrightarrow \mathsf{INBU} = 0$$

- Development of a less local criterion allowing inertial effects to cross local counter-gradients: checking $\omega_u^* < \overline{\omega} < 0$ will do, as the prognostic velocity equation contains such inertial terms. This is controlled by the namelist parameter GCVACHI (the criterion is not used when GCVACHI ≤ 0):

$$-\omega_u \geq \mathsf{GCVACHI} > 0 \Longrightarrow \mathsf{INBU} = 1$$

- When a layer is *diagnosed* buoyant, the layer below it is a posteriori *declared* buoyant:

$$\mathsf{INBU}^{l+1} = \max(\mathsf{INBU}^{l+1}, \mathsf{INBU}^{l})$$

4 Prognostic updraught velocity

4.1 Vertical motion equation

We chose for the prognostic model variable the relative updraught velocity $\omega_u^* = (\omega_u - \omega_e)$.
The complete evolution equation for this variable is then

$$\frac{\partial \omega_u^*}{\partial t} + (\mathbf{V}\cdot\nabla)_\eta \omega_u^* + \dot{\eta}_u \frac{\partial \pi}{\partial \eta}\frac{\partial \omega_u^*}{\partial \pi} = \text{source}(\omega_u^*)$$

$$\frac{\partial \omega_u^*}{\partial t} + (\mathbf{V}\cdot\nabla)_\eta \omega_u^* + \dot{\eta} \frac{\partial \pi}{\partial \eta}\frac{\partial \omega_u^*}{\partial \pi} + \left(\dot{\eta}_u \frac{\partial \pi}{\partial \eta} - \dot{\eta}\frac{\partial \pi}{\partial \eta}\right)\frac{\partial \omega_u^*}{\partial \pi} = \text{source}(\omega_u^*) \tag{28}$$

From this equation, the model dynamics sees only

$$\left.\frac{\partial \omega_u^*}{\partial t}\right|_{\text{dyn}} + (\mathbf{V}\cdot\nabla)_\eta \omega_u^* + \dot{\eta} \frac{\partial \pi}{\partial \eta}\frac{\partial \omega_u^*}{\partial \pi} = 0 \tag{29}$$

and treats it by the semi-lagrangian scheme. It represents a simple passive advection, without any source term (same as for humidity, hydrometeors, ozone, etc.).
The physics then solves the remaning part locally, at fixed vertical coordinate:

$$\left.\frac{\partial \omega_u^*}{\partial t}\right|_{\Phi} + (\dot{\eta}_u - \dot{\eta})\frac{\partial \pi}{\partial \eta}\frac{\partial \omega_u^*}{\partial \pi} = \text{source}(\omega_u^*) \tag{30}$$

For this, we must express the source term at the right-hand side. If we neglect the horizontal pressure gradient between updraught and environment, $\pi_u \approx \pi$. We have

$$\overline{\omega} = \sigma_u \omega_u + (1-\sigma_u)\omega_e \qquad \Longrightarrow \omega_u - \overline{\omega} = (1-\sigma_u)(\omega_u - \omega_e) = (1-\sigma_u)\omega_u^*$$

For the mean grid box and for the updraught:

$$\overline{\omega} \equiv \dot{\pi} = \frac{\partial \pi}{\partial t} + (\mathbf{V}\cdot\nabla)_\eta \pi + \dot{\eta}\frac{\partial \pi}{\partial \eta}$$

$$\omega_u \equiv \dot{\pi}_u = \frac{\partial \pi_u}{\partial t} + (\mathbf{V}\cdot\nabla)_\eta \pi_u + \dot{\eta}_u\frac{\partial \pi_u}{\partial \eta_u}$$

$$\omega_u - \overline{\omega} = (1-\sigma_u)\omega_u^* \approx (\dot{\eta}_u - \dot{\eta})\frac{\partial \pi}{\partial \eta} \tag{31}$$

We may normally assume that $\omega_e \ll \omega_u$. If we neglect all the derivatives of ω_e, we may consider

$$\left.\frac{\partial \omega_u}{\partial t}\right|_{\Phi} + (\dot{\eta}_u - \dot{\eta})\frac{\partial \pi}{\partial \eta}\frac{\partial \omega_u}{\partial \pi} = \text{source}(\omega_u) \tag{32}$$

Considering $\omega \approx -\rho g w$, this becomes:

$$\left.\frac{\partial \rho_u w_u}{\partial t}\right|_{\Phi} + \left(\dot{\eta}_u \frac{\partial \pi}{\partial \eta} - \dot{\eta}\frac{\partial \pi}{\partial \eta}\right)\frac{\partial \rho_u w_u}{\partial \pi} = -\frac{1}{g}\text{source}(\omega_u)$$

We neglect the tendency of the density, and the perfect gas law yields

$$\rho = \frac{p}{R_a T_v} \qquad \Longrightarrow \frac{\partial \rho}{\partial \pi} = \frac{\rho}{p}\frac{\partial p}{\partial \pi} - \rho\frac{\partial \ln T_v}{\partial \pi} \qquad \approx \frac{\rho}{\pi} - \rho\frac{\partial \ln T_v}{\partial \pi}$$

Hence

$$\underline{\left.\frac{\partial w_u}{\partial t}\right|_{\Phi} + (\dot{\eta}_u - \dot{\eta})\frac{\partial \pi}{\partial \eta}\frac{\partial w_u}{\partial \pi}} + (\dot{\eta}_u - \dot{\eta})\frac{\partial \pi}{\partial \eta}\left(\frac{w_u}{\pi} - w_u\frac{\partial \ln T_v}{\partial \pi}\right) = -\frac{1}{\rho g}\text{source}(\omega_u)$$

The underlined terms represent the evolution of w_u in the physics and are equal to the source term of w_u:

$$\left.\frac{\partial w_u}{\partial t}\right|_{\Phi} + (\dot{\eta}_u - \dot{\eta})\frac{\partial \pi}{\partial \eta}\frac{\partial w_u}{\partial \pi} = \text{source}(w_u)$$

Hence the source of ω_u^* is given by

$$\text{source}(\omega_u) = -\rho g \cdot \text{source}(w_u) + (\dot{\eta}_u - \dot{\eta}) \frac{\partial \pi}{\partial \eta} \left(\frac{\omega_u}{\pi} - \omega_u \frac{\partial \ln T_v}{\partial \pi} \right) \tag{33}$$

and Eq. 30 becomes

$$\left. \frac{\partial \omega_u^*}{\partial t} \right|_\Phi + (\dot{\eta}_u - \dot{\eta}) \frac{\partial \pi}{\partial \eta} \left(\frac{\partial \omega_u^*}{\partial \pi} - \frac{\omega_u}{\pi} + \omega_u \frac{\partial \ln T_v}{\partial \pi} \right) = -\rho g \cdot \text{source}(w_u) \tag{34}$$

or, introducing Eq. 31, and neglecting $\omega_u^* \omega_e$:

$$\left. \frac{\partial \omega_u^*}{\partial t} \right|_\Phi + (1 - \sigma_u) \omega_u^* \left(\frac{\partial \omega_u^*}{\partial \pi} - \frac{\omega_u^*}{\pi} + \omega_u^* \frac{\partial \ln T_v}{\partial \pi} \right) = -\rho g \cdot \text{source}(w_u) \tag{35}$$

We now have to express the source term for w_u.
The source of momentum results from the budget of the external forces:

- Buoyancy, due to the difference between the parcel virtual temperature and its environment virtual temperature:

$$\mathbf{F_b} = \frac{gB}{1+\gamma} \qquad \text{with} \qquad B = \frac{T_{vu} - \overline{T_v}}{\overline{T_v}}$$

  Where the virtual temperature is reduced to account for the weight of the condensates. This requires to get estimates of both the in-draught properties, and its direct environment at a given level.

- Braking associated to the entrainment of air from the environment, which has to be accelerated up to the updraught velocity: this depends on the *difference* between the updraught velocity and the vertical velocity of the air in its neighbourhood, i.e.

$$-\frac{1}{M_u} \frac{dM_u}{dz} W^2 = -g \lambda_u W^2 = -\frac{1}{\rho^2 g} \lambda_u {\omega_u^*}^2$$

  Remark: the entrainment coefficient is ARPÈGE-ALADIN is expressed as

$$\frac{\triangle M_u}{M_u} = \lambda_u \triangle \phi = \frac{E_u \triangle p}{M_u} \tag{36}$$

  and a profile for λ_u is chosen a priori, in relation to the integrated buoyancy.

- Aerodynamic braking, also proportional to ω_u^*

$$-\mathcal{K}_{du} W^2 \longrightarrow -\mathcal{K}_{du} \frac{1}{g^2 \rho^2} {\omega_u^*}^2$$

To estimate the dissipative terms as well as the buoyancy, we need to assess the updraught properties at different levels, and the corresponding environment properties, while we are only given the large scale average properties $\overline{\omega}$, $\overline{T}$, $\overline{q}$, $\overline{\ell}$,...

Eq. 35 becomes finally

$$\left. \frac{\partial \omega_u^*}{\partial t} \right|_\Phi + (1 - \sigma_u) \omega_u^* \left(\frac{\partial \omega_u^*}{\partial \pi} - \frac{\omega_u^*}{\pi} + \omega_u^* \frac{\partial \ln T_v}{\partial \pi} \right) = -\frac{g^2}{1+\gamma'} \frac{p}{R_a} \frac{T_{vu} - \overline{T_v}}{\overline{T_v} T_{vu}} + \frac{{\omega_u^*}^2}{p} R_a T_{vu} (\lambda_u + \mathcal{K}_{du}/g)$$

or, neglecting the term in $\ln T_v$ and the departure from hydrostatic pressure,

$$\left. \frac{\partial \omega_u^*}{\partial t} \right|_\Phi = -\frac{g^2}{1+\gamma'} \frac{p}{R_a} \frac{T_{vu} - \overline{T_v}}{\overline{T_v} T_{vu}} + \frac{{\omega_u^*}^2}{p} \left\{ (1 - \sigma_u) + R_a T_{vu} (\lambda_u + \mathcal{K}_{du}/g) \right\} - \frac{(1 - \sigma_u)}{2} \frac{\partial {\omega_u^*}^2}{\partial p} \tag{37}$$

4.2 Numerical stability issues

4.2.1 Discretization

Now let us study closer the motion equation (37). Schematically it writes:

$$\frac{\partial \omega_u^*}{\partial t} \quad = \quad A\omega_u^{*2} - \sigma_e \omega_u^* \frac{\partial \omega_u^*}{\partial p} - B'$$

where

$$A^l = \frac{\sigma_e + (\lambda_u^l + \mathcal{K}_{du}/g)R_a T_{vu}^l}{p^l} > 0, \qquad B'^l = \frac{g^2}{1+\gamma'}\frac{p^l}{R_a}\frac{T_{vu}^l - \overline{T_v}^l}{\overline{T_v}^l T_{vu}^l} > 0, \qquad \sigma_e = (1-\sigma_u)$$

We define

$$f^l = \omega_u^{*l}\triangle t \le 0 \quad , \quad c^{\bar{l}} = \frac{\sigma_e^l \omega_u^{*l} + \sigma_e^{l+1}\omega_u^{*l+1}}{2}\triangle t \quad , \quad B = B'(\triangle t)^2 \quad , \quad \triangle p^l = p^{\overline{la}} - p^{\overline{l-1}}$$

and note f_n^l the value at level l and time step n.
The big problem is the auto-advection of ω_u by itself. The idea is to apply a discretization similar to that proposed by Geleyn et al. [1982].

$$f_{n+1}^l - f_n^l = A^l\left(f_{n+1}^l\right)^2 - \frac{1}{\triangle p^l}\left\{c_n^{\bar{l}}\left(f_{n+1}^{l+1} - \frac{f_n^{l+1}-f_n^l}{2}\right) - c_n^{\overline{l-1}}\left(f_{n+1}^l - \frac{f_n^l - f_n^{l-1}}{2}\right)\right\} - B^l$$

This yields the second degree equation

$$\underbrace{A^l}_{\mathcal{A}^l}\left(f_{n+1}^l\right)^2 \underbrace{-\left(1-\frac{c_n^{\overline{l-1}}}{\triangle p^l}\right)}_{\mathcal{B}^l} f_{n+1}^l + \underbrace{f_n^l - B^l - \frac{1}{\triangle p^l}\left\{c_n^{\bar{l}}\left(f_{n+1}^{l+1} - \frac{f_n^{l+1}-f_n^l}{2}\right) + c_n^{\overline{l-1}}\left(\frac{f_n^l - f_n^{l-1}}{2}\right)\right\}}_{\mathcal{C}^l}$$

Considering the auto-advection term only, diagonal dominance requires:

$$\left|1 - \frac{c_n^{\overline{l-1}}}{\triangle p^l}\right| \ge \left|\frac{c_n^{\bar{l}}}{\triangle p^l}\right|$$

$$\text{or, since } \omega_u \le 0 \quad , \qquad (\omega_u\sigma_e)^{\overline{l-1}} - (\omega_u\sigma_e)^{\bar{l}} \le \frac{\triangle p^l}{\triangle t}$$

Which means that the decrease of the vertical velocity between to adjacent levels times the time step must be less than the corresponding decrease of pressure: this condition is easily fulfilled.

4.2.2 Practical implementation

The calculation is imbedded into the first vertical loop which computes the updraught profile. The entrainment is known at level ILEV = JLEV+1. *In the code,* $f_n^l \equiv$ PUDOM^l, *while the updated value is temporarily put in* PUO$^l = f_{n+1}^l$:

- *put the advecting velocity* $c_n^{\bar{l}}$ *into* ZFORM, *based on the updraught velocity and mesh fraction advected from the previous time step. For testing, a coefficient* NLUC(3) *was introduced allowing to suppress auto-advection with* NLUC(3)=0. *To have auto-advection, it is necessary to set* NLUC(3)=1.
- *the buoyancy term*

$$-B^{l+1} = \mathsf{ZBL} = \frac{-p^{l+1}\cdot \mathsf{ZBU}\cdot \mathsf{ZKUO1}^{l+1}}{T_{vu}^{l+1}T_{ve}^{l+1}}$$

- *the 3 coefficients of the quadratic equation are*

$$\mathcal{A}^{l+1} = \frac{R_a T_{vu}^{l+1}(\mathsf{ZKSG} + \lambda_u^{l+1}) + \sigma_e^{l+1}}{p^{l+1}} \qquad \ge 0$$

$$\mathcal{B}^{l+1} = 1 - \frac{c_n^{\bar{l}}}{\triangle p^{l+1}} \qquad \ge 0$$

$$d_n^l \equiv \frac{f_n^{l+1} - f_n^l}{2} \quad , \qquad \text{level } l+1 \text{ stored in } \mathsf{zs14}$$

$$\mathcal{C}^{l+1} = \mathsf{ZBL} + f_n^{l+1} - \frac{c_n^{\bar{l}} d_n^l + c_n^{\overline{l+1}}(f_{n+1}^{l+2} - d_n^{l+1})}{\triangle p^{l+1}} \qquad \le 0$$

- *the updated value is*

$$f^l_{n+1} = \frac{\mathcal{B} - \sqrt{\mathcal{B}^2 - 4\mathcal{A}\mathcal{C}}}{2\mathcal{A}}$$

It is required that $\mathcal{C} \leq 0$ to get a negative value of f.

4.2.3 Stability of the non advective part

This equation is not related to space.

$$\begin{aligned}
\frac{\partial f}{\partial t} &= Af^2 - B' \quad , \qquad A, B' > 0 \\
\Longleftrightarrow \int \frac{d(\sqrt{\frac{A}{B'}} f)}{\left(\sqrt{\frac{A}{B'}} f\right)^2 - 1} &= \sqrt{AB'} \int dt \\
\Longrightarrow \sqrt{AB'} t &= -\mathrm{arc\,tanh} \sqrt{\frac{A}{B'}} f + \mathrm{arc\,tanh} \sqrt{\frac{A}{B'}} f(0) \\
\text{if } f(0) = 0 \Longrightarrow f(t) &= \sqrt{\frac{B'}{A}} \tanh(-\sqrt{AB'}\, t) = -\sqrt{\frac{B'}{A}} \underbrace{\frac{e^{\sqrt{AB'} t} - e^{-\sqrt{AB'} t}}{e^{\sqrt{AB'} t} + e^{-\sqrt{AB'} t}}}_{<1}
\end{aligned}$$

The analytical solution tends asymptotically to the steady value $f = -\sqrt{B'/A}$, while staying all the time smaller in absolute value.

A quick numerical test showed us that an explicit formulation

$$F_{n+1} = F_n - AF_n \cdot |F_n| - B$$

converges only if $A \cdot B \leq 1$, while the implicit version

$$F_{n+1} = F_n + AF^2_{n+1} - B \Longrightarrow F_{n+1} = \frac{1 - \sqrt{1 - 4A(F_n - B)}}{2A}$$

always converges to the correct value $\sqrt{B/A}$.
So this part should not cause trouble in our scheme.
In the implementation we prevent downward draught velocities, resetting them to zero, so we never allow the dissipative term to induce downward motion.

5 Convective activity and layers classification

The convective activity index is equal to 1 in all layers that could be part of the cloud profile (saturation and buoyancy).

5.1 Buoyancy indicator

Whichever be the triggering method, we get an index $\delta_{\mathrm{stab}} \equiv \mathsf{INBU}(\mathsf{JLON}, \mathsf{JLEV})$, gathering all layers which could potentially be part of the present cloud.

5.2 Cloud base

The base of a cloud is easily detected by the index

$$\mathsf{INBAS}^l = \mathsf{INBU}^l \cdot (1 - \mathsf{INBU}^{l+1})$$

and it is also valid for multiple cloud bases along the vertical.

5.3 Actual cloud layers

The upper layers of the cloudy profile might not be reached within a time step. If we want to address this behaviour, we need to define additional indexes:

- INDET is 1 where the advected mesh fraction from previous time step σ_u^- is greater than zero (or greater than a threshold). This is a way to estimate the current position of the top of the cloud resulting from previous time step.
- KNACT $\equiv \delta_{\text{act}}$ is 1 in the layers with actual activity.
  These layers are buoyant (INBU $= 1$) and either were active at previous time step (INDET $= 1$), or are at the base of a cloud (INBAS $= 1$), or are reachable within the time step by the top of the cloud. The latter is estimated by constructing

  $$\Sigma^L = 0 \quad , \qquad \Sigma^l = \mathsf{INBU} \cdot \left(\Sigma^{l+1} + (1 - \mathsf{INDET}^l) \frac{\triangle p^l}{{\omega_u^*}^l} \right) \quad , \quad l = L - 1, \dots 1$$

  and the layer l is reachable within one time step if

  $$\Sigma^l \leq \triangle t$$

  The practical coding writes:

  $$\begin{aligned} (1 - \mathsf{KNACT}) &= (1 - \mathsf{INDET}) \cdot (1 - \mathsf{INBAS}) \cdot (1 - \max(0, \mathrm{sign}(1, \Sigma^l - \triangle t))) \\ \mathsf{KNACT} &= \mathsf{INBU} \cdot \mathsf{KNACT} \end{aligned}$$

6 Mesh fraction and mass flux

6.1 CAPE closure

The CAPE closure yields a diagnostic value of the mesh fraction:

$$\sigma_u \int_{p_t}^{p_b} \omega_u^* \left[(1 + \mu q)(\frac{1}{c_p} \frac{\partial s}{\partial p}) + \mu T \frac{\partial q}{\partial p} \right] R_a \frac{dp}{p} \quad = \quad \frac{1}{\tau} \int_{p_t}^{p_b} R_a (T_{vu} - T_v) \frac{dp}{p}$$

In the code, ZFORM *contains at this stage* $(-{\omega_u^*}^{\bar{l}} \triangle t \cdot \mathsf{INBU})$. *The advection term is calculated as*

$$\left(-\omega_u^* \triangle t \frac{\partial \psi}{\partial p} \triangle p \right)^l \quad = \quad \frac{\mathsf{ZFORM}^{\overline{l-1}} (\psi^l - \psi^{l-1}) + \mathsf{ZFORM}^{\bar{l}} (\psi^{l+1} - \psi^l)}{2}$$

let be

$$\begin{aligned} \mathsf{ZS15} &= \sum_{l=1}^{L} \delta_{\text{act}}^l \frac{\triangle p^l}{p^l} \left[T_u^l (1 - \ell_u^l + \mu q_u^l) - T^l (1 + \mu q^l) \right] = \frac{\mathsf{CAPE}}{R_a} && \geq 0 \\ \mathsf{ZS16} &= \sum_{l=1}^{L} \frac{\delta_{\text{act}}^l}{p^l} \left[\frac{1 + \mu q^l}{c_p^l} \left(-\omega_u^* \triangle t \frac{\partial s}{\partial p} \triangle p \right)^l + \mu T^l \left(-\omega_u^* \triangle t \frac{\partial q}{\partial p} \triangle p \right)^l \right] && \geq 0 \quad \text{normally as } h^{l+1} > h^l \\ \Longrightarrow \quad & \mathsf{ZSIGB} \equiv \sigma_u = \frac{\mathsf{ZS15} \cdot \triangle t}{\mathsf{PTAUX} \cdot \mathsf{ZS16}} && \geq 0 \end{aligned}$$

With the CAPE closure, the mesh fraction is diagnostic and the advected convective mesh fraction is used only to evaluate the mesh fraction tendency in the final budgets (and even that is better avoided, because it can lead to instability).

6.2 Moisture Convergence prognostic closure

The closure on the large scale moisture convergence actually addresses the same kind of mechanism, as it is this convergence which fuels the local CAPE. It has the advantage to yield a prognostic equation for σ_u.

The prognostic closure as expressed by Chen and Bougeault [1990]:

$$\underbrace{\frac{\partial \sigma_u}{\partial t} \cdot \int_{p_t}^{p_b} \left(h_u - \overline{h}\right) \frac{dp}{g}}_{\text{storage}} = \underbrace{L \int_{p_t}^{p_b} \sigma_u \omega_u^* \frac{\partial \overline{q}}{\partial p} \frac{dp}{g}}_{-\text{consumption}} + \underbrace{L \cdot \ TMC}_{\text{input}} \qquad (38)$$

distinguished

- LHS: a storage as moist static energy through the increase of the updraught section: the introduction of cloud water does not change this, as the conversion to condensate occurs only afterwards, when this moisture is consumed by the updraught.
- *TMC*: the total *water vapour* moisture convergence: vapour which is the main fuel for updraught activity, dry air and condensate have a much smaller impact on the heat processes. In the earlier schemes, dry static energy convergence was ignored in this equation: now the convergence of condensate is ignored with the same arguments.
- The first term of the RHS represents the rate of water vapour consumption by the updraught, which
  - converts some of it to condensate; This directly affects the large scale tendency of water vapour (Q_2^{cu});
  - detrains the remaining part within the grid box.
  - in the large scale tendency we included the vertical turbulent diffusion flux divergence, but this was only for compensation and does not participate to the consumption of water vapour by the updraught.

  For these reasons, all what is left is, as earlier, the pseudo-subsidence term, and no change to equation (38) is needed.

Now, unlike the earlier scheme,

- we no longer have a separation of resolved and subgrid precipitation, so we forget about LSRCON/LSRCONT.
- Neither is the modulation of the convergence by mesh size relevant.

Therefore, we now use directly the resolved water vapour moisture convergence, defined as

$$TMC \equiv -\int_{p_t}^{p_b} \left[\overline{\mathbf{V}} \cdot \nabla \overline{q} + \overline{\omega} \frac{\partial \overline{q}}{\partial p}\right] \frac{dp}{g} - \left(J_q(p_b) - J_q(p_t)\right)$$

which we compute in array ZCVGQ in aplpar.
The key LLSIGPROP=.T. allows to use a variable mesh fraction over the vertical, proportional to the updraught vertical velocity. If false, a single value of σ_u is used over each vertical. The practical implementation computes:

$$\mathsf{ZA13}^l = h_u^l - h_w^l = c_p^l (T_u^l - T_w^l) + L(\overline{T}^l) \left(q_u^l - q_w^l\right)$$

$$\mathsf{ZS4} = \sum_{l=1}^{L} L^l \cdot \ TMC^l \cdot \triangle p^l = \sum_{l=1}^{L} \mathsf{ZLHE}^l \cdot \mathsf{PCVGQ}^l \cdot \mathsf{PDELP}^l \cdot \mathsf{INBU}^l$$

$$\mathsf{ZS15} = \begin{cases} \sum_{l=1}^{L} \delta_{\text{act}}^l \triangle p^l \sigma_{u-}^l (h_u - h_w)^l & \text{if not LLSIGPROP} \\ \sum_{l=1}^{L} \delta_{\text{act}}^l (-\omega_u^{*l} \triangle t) \sigma_{u-}^l (h_u - h_w)^l & \text{if LLSIGPROP} \end{cases}$$

$$\mathsf{ZS16} = \begin{cases} \sum_{l=1}^{L} \delta_{\text{act}}^l \triangle p^l (h_u - h_w)^l & \text{if not LLSIGPROP} \\ \sum_{l=1}^{L} \delta_{\text{act}}^l (-\omega_u^{*l} \triangle t)(h_u - h_w)^l & \text{if LLSIGPROP} \end{cases}$$

$$\mathsf{ZS17} = \begin{cases} \sum_{l=1}^{L} \delta_{\text{act}}^l L^l \triangle p^l \left(-\omega_u^* \triangle t \frac{\partial q}{\partial p}\right) & \text{if not LLSIGPROP} \\ \sum_{l=1}^{L} \delta_{\text{act}}^l L^l (-\omega_u^{*l} \triangle t) \left(-\omega_u^* \triangle t \frac{\partial q}{\partial p}\right) & \text{if LLSIGPROP} \end{cases}$$

And the prognostic mesh fraction is given by

$$\mathsf{ZSIGB} = \frac{\mathsf{ZS15} + \mathsf{ZS4}\triangle t}{\mathsf{ZS16} + \mathsf{ZS17}}$$
$$\sigma_{u+}^l = \begin{cases} \delta_{\text{act}}^l \cdot \mathsf{ZSIGB} & \text{if not } \mathsf{LLSIGPROP} \\ \delta_{\text{act}}^l \frac{-\omega_u^{*l}\triangle t}{\triangle p^l} \cdot \mathsf{ZSIGB} & \text{if } \mathsf{LLSIGPROP} \end{cases}$$

In this, the arrays ZS15, ZS16, ZS17, ZS4 must be positive:

- $\mathsf{ZS15}, \mathsf{ZS16} \geq 0$, the store of moist static energy inside the updraught,
- $-\sigma_u \cdot \mathsf{ZS17} \leq 0$ its consumption by the updraught. As soon as it tends to zero or a positive value (actually when $\mathsf{ZS17} < \epsilon_7 = 1.E - 11$), we cut the convection by imposing $\sigma_b^+ = 0$.
- $\mathsf{ZS4}\triangle t \geq 0$ is the water vapour quantity brought by the convergence. It must stay non negative, in this context. Along the profile construction, ZKUO2 limited the activity to the layers where the cumulated moisture convergence from below stays positive.

If $\mathsf{ZSIGB} < 0$ or $\mathsf{ZSIGB} > \mathsf{GCVALMX}$, KNND is set to zero so the updraught is disabled.
The effective mass flux at the interfaces $(-\sigma_u\omega_u^*)^{\bar{l}}$ is store into ZFORM. This value is protected against the nonlinear instability:

$$\mathsf{ZFORM}^{\bar{l}} = \max(0, \mathsf{ZFORM}^{\overline{l+1}} + \frac{\mathsf{ZFORM}^{\bar{l}} - \mathsf{ZFORM}^{\overline{l+1}}}{1 + \frac{\max(0, \mathsf{ZFORM}^{\bar{l}} - \mathsf{ZFORM}^{\overline{l+1}})}{\triangle p^{l+1}}}$$

The the following quantities are derived:

$$\mathsf{ZDMF}^l = \mathsf{ZFORM}^{\bar{l}} - \mathsf{ZFORM}^{\overline{l-1}}$$
$$\mathsf{ZDMFQCU}^l = \frac{\mathsf{ZFORM}^{\overline{l-1}}(q_{cu}^l - q_{cu}^{l-1}) + \mathsf{ZFORM}^{\bar{l}}(q_{cu}^{l+1} - q_{cu}^l)}{2}$$
$$\mathsf{ZDAL}^l = \delta_{\text{act}}^l(\mathsf{PUDAL}^l - \mathsf{ZSIG9})$$

7 Horizontal momentum profile

The idea derive the cloud base horizontal velocity by writing a null cumulated vertical budget of momentum was introduced in the first development without deep assessment. We recently discovered major problems around this assumption. Moreover, writing this budget required to know the detrainment profile. Instead, we now set the cloud base velocity equal to the environment.
The updraught momentum budget in an active layer writes:

$$\frac{\partial M_u \mathbf{V}_u}{\partial p} = -\frac{1}{g}\frac{\partial \sigma_u \omega_u^* \mathbf{V}_u}{\partial p} = E\overline{\mathbf{V}} - D\mathbf{V}_u + \frac{\sigma_u}{g}\nabla\overline{\phi}$$

Gregory et al. [1997] proposes to represent the horizontal pressure gradient term as

$$\frac{\sigma_u}{g}\nabla\overline{\phi} = \mathcal{G}_u M_u \frac{\partial \overline{\mathbf{V}}}{\partial p}$$

yielding finally

$$\frac{\partial \mathbf{V}_u}{\partial \phi} = -\lambda_u(\mathbf{V}_u - \overline{\mathbf{V}}) + \mathcal{G}_u \frac{\partial \overline{\mathbf{V}}}{\partial \phi}$$

Like for the thermodynamic variables, the discretization makes a mixing $l + 1 \rightarrow b$ at constant level, followed by an ascent $b \rightarrow l$:

$$\mathbf{V}_u^b - \mathbf{V}_u^{l+1} = \xi^l(\mathbf{V}_u^b - \overline{\mathbf{V}}^{l+1}) = \frac{\xi^l}{1+\xi^l}(\mathbf{V}_u^{l+1} - \overline{\mathbf{V}}^{l+1}) \quad , \qquad \mathbf{V}_u^l - \mathbf{V}_u^b = \mathcal{G}_u(\overline{\mathbf{V}}^l - \overline{\mathbf{V}}^{l+1})$$
$$\text{with} \quad \xi^l = \lambda_u^{l+1}\triangle\phi^{\bar{l}} \quad \text{and} \quad \mathsf{ZRMIX}^l \equiv \xi'^l = \frac{\xi^l}{1+\xi^l}$$

Hence

$$\begin{aligned} \mathbf{V}_u^l - \mathbf{V}_u^{l+1} &= -\xi'^l(\mathbf{V}_u^{l+1} - \overline{\mathbf{V}}^{l+1}) + \mathcal{G}_u(\overline{\mathbf{V}}^l - \overline{\mathbf{V}}^{l+1}) \\ \mathbf{V}_u^l &= \mathbf{V}_u^{l+1}(1-\xi'^l) + \mathcal{G}_u\overline{\mathbf{V}}^l - (\mathcal{G}_u - \xi'^l)\overline{\mathbf{V}}^{l+1} \end{aligned}$$

Expressing the momentum profile as:

$$\mathbf{V}_u^l = \beta^l \mathbf{V}_u^b + (1-\beta^l)\widehat{\mathbf{V}}^l \tag{39}$$

where b represents the base of the active segment, yields

$$\begin{aligned} \beta^{l-1} &= \beta^l(1-\xi'^{l-1}) \quad , \qquad \beta^b = 1 \\ \widehat{\mathbf{V}}^{l-1} &= \widehat{\mathbf{V}}^l + \frac{\mathcal{G}_u(\overline{\mathbf{V}}^{l-1} - \overline{\mathbf{V}}^l) + \xi'^{l-1}(\overline{\mathbf{V}}^l - \widehat{\mathbf{V}}^l)}{(1-\beta^{l-1})} \quad , \qquad \widehat{\mathbf{V}}^b = \overline{\mathbf{V}}^b \end{aligned}$$

On the inactive layers, we must keep $\mathbf{V}_u = \overline{\mathbf{V}}$.
we pose:

$$(\mathsf{ZUM}, \mathsf{ZVM}) \equiv \widehat{\mathbf{V}} \quad , \qquad \mathsf{ZBET} \equiv \beta \quad , \qquad (\mathsf{ZA13}, \mathsf{ZA14}) \equiv \mathbf{V}_u^b$$

- *In the inactive layers:* ZBET $= 0$ *and* (ZUM, ZVM) $=$ (PU, PV)
- *At the base of each connected active segment (*INBAS $= 1$*):* ZBET $= 1$ *and* (ZUM, ZVM) $=$ (PU, PV).
- *In the active layers above the base:*

$$\begin{aligned} \mathsf{ZBET}^l &= \mathsf{ZBET}^{l+1} \cdot (1 - \mathsf{ZRMIX}^l) \\ (\mathsf{ZUM}, \mathsf{ZVM})^l &\equiv \widehat{V}^l = (\widehat{V})^{l+1} + \frac{\xi'^l(\mathbf{V}^{l+1} - \widehat{V}^{l+1}) + \mathcal{G}_u(\mathbf{V}^l - \mathbf{V}^{l+1})}{1-\beta^l} \end{aligned}$$

The velocity at the base of the updraught is taken equal to the value in the environment. Also, in all inactive layers, we must impose $\mathbf{V}_u = \overline{\mathbf{V}}$.
In this case,

$$(\mathsf{ZA13}, \mathsf{ZA14})^l \equiv \mathbf{V}_u^b = \begin{cases} \overline{\mathbf{V}}^l & \text{where } \delta_{\text{act}} = 0 \text{ or } \mathsf{INBAS} = 1 \\ \mathbf{V}_u^{l+1} & \text{elsewhere} \end{cases}$$

This yield to momentum profiles very similar to those presented by Kershaw and Gregory [1997] and Gregory et al. [1997].
We put the final values of the updraught horizontal momentum profile into the arrays ZUM *and* ZVM*, to use in the final budgets.*

$$(\mathsf{ZUM}, \mathsf{ZVM}) = \mathbf{V}_u = (1-\beta)(\mathsf{ZUM}, \mathsf{ZVM}) + \beta(\mathsf{ZA13}, \mathsf{ZA14})$$

8 Output fluxes and variables

8.1 Condensation fluxes

Along the ascent, the total condensation at the interfaces was stored into ZDQL. Now this is re-interpolated so that ZDQL $\equiv \triangle q_{ca}$ contains condensation increments at the full levels.
The condensate production ZDQL is partitioned between ice and liquid water, using α_i. The convective condensation fluxes are obtained from

$$\begin{aligned} \mathsf{PFCCQN}^l \equiv F_{vi\_c}^l &= F_{vi\_c}^{l-1} + \alpha_i \triangle q_{ca}(-\sigma_u\omega_u^*)/g \\ \mathsf{PFCCQL}^l \equiv F_{v\ell\_c}^l &= F_{v\ell\_c}^{l-1} + (1-\alpha_i) \triangle q_{ca}(-\sigma_u\omega_u^*)/g \end{aligned}$$

where $(-\sigma_u\omega_u^*)$ represents the updraught mass flux (ZFORM$/\triangle t$) re-interpolated to the full levels.

8.2 Condensate detrainment area

It was assumed that only a fraction of the condensate remained in the ascent, the remainder being detrained. This detrained condensate is obtained by a local budget.
The updraught mass and condensate budgets are expressed basically

$$\begin{aligned}\frac{\partial M_u}{\partial p} &= D_u - E_u + \frac{\partial \sigma_u}{\partial t} \\ \frac{\partial M_u q_{cu}}{\partial p} &= D_{cu}\, q_{cu} - E_u\, q_{ce} + \frac{\partial \sigma_u q_{cu}}{\partial t} - \text{condensation}\end{aligned}$$

The condensation in the ascent is a local source $M_u \triangle q_{ca}$, which is parted between q_{cu} staying in the updraught and q_{cD} which is detrained.
Neglecting the tendency of the mesh fraction,

$$M_u \triangle q_{cu} + q_{cu} \triangle M_u + E_u\, q_{ce} + M_u \triangle q_{ca} = D_{cu}\, q_{cu}$$

If A is the total grid box area, the mass of condensate detraining during a time step results from:

- The mass of condensate generated over $\triangle t$:

$$\triangle q_{ca}^l \frac{(-\sigma_u^l \omega_u^{*l} \triangle t)A}{g}$$

- The mass of condensate carried away by the divergence of the updraught mass flux:

$$\frac{A}{g}\, [(-\sigma_u \omega_u^* \triangle t)\, q_{cu}]_{\overline{l}}^{\overline{l-1}}$$

- The mass of environment condensate which is entrained: we have $E_u \triangle p \equiv \lambda_u \triangle \phi(-\sigma_u \omega_u^*)$, so the entrained mass is

$$-\lambda_u^l \triangle \phi^l\, (-\sigma_u \omega_u^* \triangle t)^l\, \overline{q_c}^l\, \frac{A}{g}$$

- The mass of condensate which is detrained:

$$D_{cu} \triangle t\, q_{cu}^l\, \frac{A}{g} \triangle p^l = \delta \sigma_D^l\, q_{cD}^l \frac{A \triangle p^l}{g}$$

  where $\delta \sigma_D$ is the increase of the detrainment mesh fraction σ_D over the time step $\triangle t$.
  Note that $\triangle p$ comes from the mathematical definition of D, similar to the one of E, while physically both contain a dependence to the updraught mass flux $-\sigma_u \omega_u^*$.

- The local storage:

$$\delta \sigma_u^l\, q_{cu}^l\, \frac{A \triangle p^l}{g}$$

  (the storage volume is $\sigma_u A \cdot z = \sigma_u A \frac{\triangle p}{\rho g}$, and for this term there is no dependence on the updraught flux).

So we may evaluate the actual detrainment coefficient for condensate:

$$\begin{aligned}D_{cu} \triangle t \cdot q_{cu} &= \delta \sigma_D \cdot q_{cD} \\ &= \frac{1}{\triangle p} \left\{ (-\sigma_u \omega_u^* \triangle t)^l \triangle q_{ca}^l + [(-\sigma_u \omega_u^* \triangle t) q_{cu}]_{\overline{l}}^{\overline{l-1}} + \lambda_u^l \triangle \phi^l (-\sigma_u^l \omega_u^{*l} \triangle t) \overline{q_c}^l - \delta \sigma_u^l\, q_{cu}^l\, \triangle p^l \right\}\end{aligned}$$

In the code, the vertical profile of ZDEQC $= \delta \sigma_D \cdot q_{cD} \triangle p$ is stored in the array ZBET.
An experimental treatment consisted to squeeze this vertical detrainment profile (if GCVSQDCX¡1). JLEV1 is the highest level of the original profile, JLEV3 its lowest level, JLEV2 the highest level where ZDEQC is a fraction GCVSQDR of the profile maximum over the vertical. The profile is squeezed from JLEV3-JLEV1 to JLEV2-JLEV1 levels so that the resulting profile between JLEV1 and JLEV3 provides the same flux than the original.
The final condensate detrainment profile is put in ZA14.
Dividing ZA14 by $\triangle p$ yields $\sigma_D q_{cD}$. To obtain the detrainment mesh fraction σ_D, an hypothesis is required on q_{cD}: we assume that $q_{cD} = q_{cu}$, unless it yields $\sigma_D > 1 - \sigma_u$:

$$\text{PSIGDE} = \min\left(\frac{\text{ZA14}}{q_{cu} \triangle p}, (1 - \sigma_u)\right)$$

8.3 Convective diffusion fluxes

These have now simple expressions:
Noting $\mathsf{ZFF}^{\bar{l}} = \mathsf{ZFORM}^{\bar{l}}/(g\triangle t) = (\sigma_u\omega_u^*)^{\bar{l}}/g$,

$$(\mathsf{PSTRCU},\mathsf{PSTRCV}) \equiv J_V^{\mathrm{conv}\,\bar{l}} = \mathsf{ZFF}^{\bar{l}}\frac{(\mathbf{V}_u^l+\mathbf{V}_u^{l+1})-(\mathbf{V}^l+\mathbf{V}^{l+1})}{2}$$

$$(\mathsf{PDIFCQ}) \equiv J_q^{\mathrm{conv}\,\bar{l}} = \mathsf{ZFF}^{\bar{l}}\frac{(q_u^l+q_u^{l+1})-(q^l+q^{l+1})}{2}$$

$$(\mathsf{PDIFCQI}) \equiv J_{qi}^{\mathrm{conv}\,\bar{l}} = \mathsf{ZFF}^{\bar{l}}\frac{(q_{iu}^l+q_{iu}^{l+1})-(q_i^l+q_i^{l+1})}{2}$$

$$(\mathsf{PDIFCQL}) \equiv J_{q\ell}^{\mathrm{conv}\,\bar{l}} = \mathsf{ZFF}^{\bar{l}}\frac{(q_{\ell u}^l+q_{\ell u}^{l+1})-(q_\ell^l+q_\ell^{l+1})}{2}$$

$$(\mathsf{PDIFCS}) \equiv J_s^{\mathrm{conv}\,\bar{l}} = \mathsf{ZFF}^{\bar{l}}\frac{(s_u^l+s_u^{l+1})-(s_e^l+s_e^{l+1})}{2}$$

8.4 Lifting Condensation Level

With $\mathsf{SCO} < 0$ (we take $\mathsf{SCO} = -20$), an additional test may declare that there is after all no convective activity. First, we store

$$\mathsf{ZLCL} \quad = \quad \min(\overline{q_{\mathrm{sat}}} - q_u)$$

There is no problem if it yields a negative value, but sometimes, the cloud existence being based on wet bulb and virtual temperature, we could still have $T_u < \overline{T}$ and $q_u = q_{\mathrm{sat}}(T_u) < q_{\mathrm{sat}}(\overline{T})$.
In this case, ZLCL could yield a positive value, and we add an additional existence criterion: the cloud exists only when

$$\frac{(F_{vi\_c}^L + F_{v\ell\_c}^L)}{|\mathsf{SCO}|} > \mathsf{ZLCL}$$

In the earlier scheme which considered the surface precipitation flux, it meant that the surface precipitation reported to the reference value $|\mathsf{SCO}|$ must be bigger than the saturation deficit the cloud would have if brought to the environment temperature.
Considering now instead the cumulated condensate generation along the profile does not seem to affect the validity of the approach.
The feasibility indicator KNND is reset to zero when the LCL is not reached, inducing finally that

- all fluxes are set to zero;
- the environment properties are set to the mean grid box;
- the detrained properties: $\sigma_D = 0$, $q_{\ell D} = 0 = q_{iD}$; but $q_D = \overline{q}$, $T_D = \overline{T}$.
- $\omega_u^* = 0$, but σ_u not affected (?)

8.5 Output properties

The environment vertical velocity to pass to the downdraught is updated in $\mathsf{POMDT} = \omega\triangle t$:

$$\omega_e = \frac{\overline{\omega} - \sigma_u\omega_u}{1-\sigma_u} = \overline{\omega} - \sigma_u\omega_u^*$$

The microphysical variables are updated as follows:

$$q^l = q^l + \frac{g\triangle t}{\triangle p^l}\left\{(J_q^{\mathrm{conv}\,\overline{l-1}} - J_q^{\mathrm{conv}\,\bar{l}}) + (F_{v\ell\_c}^{\overline{l-1}} - F_{v\ell\_c}^{\bar{l}}) + (F_{vi\_c}^{\overline{l-1}} - F_{vi\_c}^{\bar{l}})\right\}$$

$$q_\ell^l = q_\ell^l + \frac{g\triangle t}{\triangle p^l}\left\{(J_{q\ell}^{\mathrm{conv}\,\overline{l-1}} - J_{q\ell}^{\mathrm{conv}\,\bar{l}}) - (F_{v\ell\_c}^{\overline{l-1}} - F_{v\ell\_c}^{\bar{l}})\right\}$$

$$q_i^l = q_i^l + \frac{g\triangle t}{\triangle p^l}\left\{(J_{qi}^{\mathrm{conv}\,\overline{l-1}} - J_{qi}^{\mathrm{conv}\,\bar{l}}) - (F_{vi\_c}^{\overline{l-1}} - F_{vi\_c}^{\bar{l}})\right\}$$

$$T_{mp}^l = T_{mp}^l + \frac{g\triangle t}{c_p^l\triangle p^l}\left\{(J_s^{\mathrm{conv}\,\overline{l-1}} - J_s^{\mathrm{conv}\,\bar{l}}) - L_v(T_{mp})(F_{v\ell\_c}^{\overline{l-1}} - F_{v\ell\_c}^{\bar{l}}) - L_s(T_{mp})(F_{vi\_c}^{\overline{l-1}} - F_{vi\_c}^{\bar{l}})\right\}$$

If these relations lead to negative specific contents, these are compensated and the correction is added to the turbulent diffusion fluxes, for instance

$$(\triangle J_q^{\rm cor})^l = \frac{\triangle p^l}{g\triangle t}\min(0, q^l) \qquad \text{and} \qquad J_q^{\rm tur} = J_q^{\rm tur} + J_q^{\rm cor}$$

and similarly for q_i and q_ℓ.

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