Carrier Frequency Offset Estimation in OFDM Systems as a Quadratic Eigenvalue Problem

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Abstract. *Carrier frequency offset (CFO) in orthogonal frequency division multiplexing (OFDM) systems is a major problem in achieving orthogonality between subcarriers. In this paper, we propose an estimator for CFO in OFDM systems using the subspace method. We used the linear prediction property of a complex sinusoidal signal. The estimator was obtained by solving a quadratic eigenvalue problem. Its performance was examined and compared with a Maximum-Likelihood (ML), Cyclic-Prefix based and Pilot-based estimators by the measures of mean square error (MSE), and Cramér-Rao Lower Bound (CRLB). Numerical results are presented to show the visible advantages obtained by using this estimator.*

Keywords

OFDM, Carrier Frequency Offset estimation, Quadratic Eigenvalue problem, subspace method, Yule-Walker

1. Introduction

The OFDM modulation has been widely used in modern communication systems because of its robustness against the frequency selectivity in wireless channel. OFDM systems can convert a frequency selective channel into a parallel collection of frequency flat channels, leading to a great simplified equalizer which increases the robustness of OFDM systems against inter symbol interference (ISI). This can be accomplished with the utilization of advanced Peak to Average Power Ratio (PAPR) techniques, see in [1]. OFDM systems are very sensitive to frequency synchronization due to the Doppler shift caused by relative motion [2], [3], or a mismatch between carrier frequency in local oscillators or between the transmitter and the receiver. The CFO corrupts the mutual orthogonality between subcarriers and introduces inter-carrier interference (ICI) between subcarriers [4]. Imprecise CFO estimation causes both a severe performance degradation because of the reduction of the signal amplitude at the output of each matched filter and an interference between adjacent subchannels [5]. In the most recent years, many works have been directed toward the advancement of CFO synchronization methods for OFDM systems acting in both data-aided and non-data-aided (or blind) methods [6,7]. Additionally, a hybrid of data-aided and blind methods has been recently advised [8], which is referred to as the semi-blind approach. For example, blind methods which use the orthogonality between null subcarriers, which are referred to zero padding [9], [10], are used in the transmitted OFDM symbols and the information-bearing subcarriers (see in [11], [12]). In [13], redundant information in the cyclic prefix (CP) before the OFDM symbols made possible estimation without additional pilots, joint symbol timing. The CFO maximum likelihood (ML) estimator has thus been derived. In [14], frequency offsets are estimated by inserting null subcarriers into a single OFDM block and a deterministic maximum likelihood (ML) approach for the CFO estimation has been derived. In [15], an iterative frequency domain maximum likelihood CFO algorithm leads to ML and LS iterative estimators using the available pilot grid that is used for channel state information estimation.

In this paper, we propose a method for estimating the CFO via exploiting a linear prediction approach from a finite number of noisy measurements of the received signal. The method utilizes a modified set of Yule−Walker (YW) equations that leads to a quadratic eigenvalue problem which gives estimate of the linear predictor (LP) parameter when it is solved. Then we can estimate the CFO from the phase of the LP parameter.

This paper is organized as follows. In Sec. 2, we describe the signal model. Section 3 describes the proposed CFO estimation method. In Sec. 4, simulation results are presented and discussed. The conclusions are drawn in Sec. 5.

2. Signal Model

In OFDM systems which are depicted in Fig. 1, data are sent block by block. A series of complex data is divided into blocks and transmitted through subcarriers.

We can easily perform OFDM with IDFT, which can be efficiently implemented by low complexity inverse fast fourier transform (IFFT), where the most common tactics in producing OFDM are based on the use of IFFT as the final samples are produced in time domain

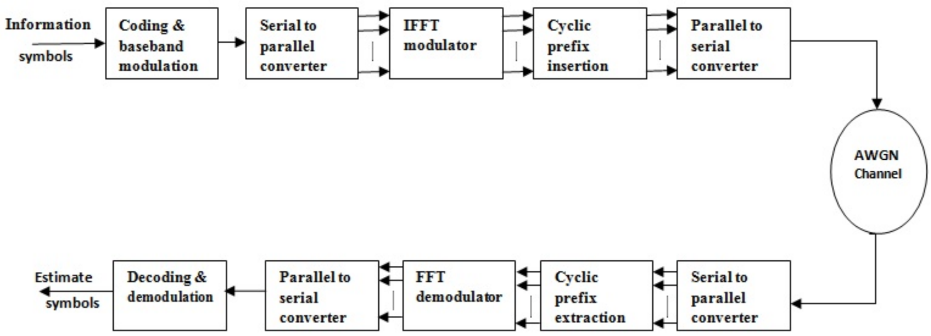

Fig. 1. Block diagram of an OFDM system.

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k \exp\left(\mathrm{j}\frac{2\pi nk}{N}\right), \quad n = 0, 1, \ldots, N-1 \quad (1)$$

where s_n is the IFFT of the data blocks. After removing the cyclic prefix and conducting FFT, the complex envelope of the received baseband signal in an OFDM block, in the absence of timing offset and channel distortion, can be expressed as

$$y(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} H_k s_k \exp\left(\mathrm{j}\frac{2\pi n(k+\epsilon)}{N}\right) + w(n), \quad n = 0, 1, \ldots, N-1 \quad (2)$$

where N is the number of subcarriers, ϵ is the relative frequency offset of the channel (CFO), H_k is the channel transfer function at the kth subcarrier frequency, $w(n)$ is the complex envelope of additive white Gaussian noise (AWGN), which is assumed to be zero-mean, uncorrelated, circular random vector [16], with variance $\sigma_w^2 = E\{|w(n)|^2\}$.

We can write (2) as

$$y(n) = \alpha(n) \exp\left(\frac{\mathrm{j}2\pi n\epsilon}{N}\right) + w(n), \quad n = 0, 1, \ldots, N-1 \quad (3)$$

where

$$\alpha(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} H_k s_k \exp\left(\mathrm{j}\frac{2\pi nk}{N}\right). \quad (4)$$

If we use known training symbols for transmitting data (or we can use $\alpha^*(n) \cdot y(n)$) in a non-dispersive channel, we can assume that $\alpha(n)$ becomes α, now we can easily write (3) as follows

$$y(n) = \alpha \exp\left(\frac{\mathrm{j}2\pi n\epsilon}{N}\right) + w(n), \quad n = 0, 1, \ldots, N-1. \quad (5)$$

3. The Proposed CFO Estimation Method

We are going to estimate ϵ using linear prediction property of complex sinusoidal signal. The estimator can be obtained by solving a quadratic eigenvalue problem. Consider the received signal in (5) without the noise component.

$$u(n) = \alpha \exp\left(\frac{\mathrm{j}2\pi n\epsilon}{N}\right) \quad (6)$$

Now we define $\omega \triangleq \frac{2\pi\epsilon}{N}$, then shifting $u(n)$ by one, we get

$$u(n-1) = \alpha \mathrm{e}^{\mathrm{j}\omega n} \mathrm{e}^{-j\omega}, \quad (7)$$

equation (7) yields

$$u(n) = \beta u(n-1), \; \beta = \mathrm{e}^{\mathrm{j}\omega}, \quad (8)$$

multiplying (8) by $u^*(n-k)$ and taking expectation gives us the linear equations of order one as follows

$$r_u(k) = \beta r_u(k-1), \quad k = 0, 1, \ldots, q. \quad (9)$$

Using (5) we will have

$$r_y(k) = r_u(k) + \sigma_w^2 \delta(k), \quad k = 0, 1, \ldots, q. \quad (10)$$

Substituting (10) to (9) yields

$$r_y(k) - \sigma_w^2 \delta(k) = \beta r_y(k-1) - \sigma_w^2 \,\beta\, \delta(k-1), \quad k = 0, 1, \ldots, q. \quad (11)$$

Rearranging (11) we will have

$$r_y(k) - \beta r_y(k-1) = \sigma_w^2 \delta(k) - \sigma_w^2 \,\beta\, \delta(k-1), \quad k = 0, 1, \ldots, q. \quad (12)$$

Now we can arrange equations in (12) from lag = 0 to lag = q in matrix form

$$\begin{bmatrix} r_y(0) & r_y(-1) \\ r_y(1) & r_y(0) \\ \vdots & \vdots \\ r_y(q) & r_y(q-1) \end{bmatrix} \begin{bmatrix} 1 \\ -\beta \end{bmatrix} = \sigma_w^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -\beta \end{bmatrix}. \quad (13)$$

Equations in (13) can be considered as an eigenvalue problem

$$(\mathbf{R}_y - \lambda \mathbf{B})\mathbf{V} = \mathbf{0}_{q+1} \quad (14)$$

where $\mathbf{R}_y = \begin{bmatrix} r_y(0) & r_y(-1) \\ r_y(1) & r_y(0) \\ \vdots & \vdots \\ r_y(q) & r_y(q-1) \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \lambda = \sigma_w^2,$

$\mathbf{V} = \begin{bmatrix} 1 \\ -\beta \end{bmatrix}$, and $\mathbf{0}_{q+1}$ is a column vector having $q+1$ zeros and $q \geq 1$. Therefore, a single solution is guaranteed. The dimensions of $\mathbf{R}_y$, $\mathbf{B}$, and $\mathbf{V}$ are $(q+1) \times 2$, $(q+1) \times 2$ and 2×1 respectively. We can estimate $r_y(k)$ for $k = 0, \dots, q$ to form $\mathbf{R}_y$ using the autocorrelation method [17]

$$\hat{r}_y(k) = \frac{1}{N} \sum_{n=1}^{N-1} y(n) y^*(n-k). \tag{15}$$

Multiplying both sides of (14) by $(\mathbf{R}_y - \lambda \mathbf{B})^{\mathrm{H}}$ leads to a quadratic eigenvalue problem.

$$(\mathbf{A}_0 + \lambda \mathbf{A}_1 + \lambda^2 \mathbf{A}_2)\mathbf{V} = \mathbf{0}_2 \tag{16}$$

where

$$\mathbf{A}_0 = \mathbf{R}_{\mathbf{y}}^{\mathrm{H}} \mathbf{R}_y,\ \mathbf{A}_1 = -(\mathbf{R}_y^{\mathrm{H}} \mathbf{B} + \mathbf{B}^{\mathrm{H}} \mathbf{R}_y),\ \mathbf{A}_2 = \mathbf{B}^{\mathrm{H}} \mathbf{B}.$$

Each of the matrices in (3) has a 2×2 dimension. There are different methods to solve the quadratic eigenvalue problem. A solution of the problem is in [18].

Solve the generalized eigenvalue problem, and estimate the AR parameter (β). Then estimate the CFO via

$$\hat{\epsilon} = \left(\frac{N}{2\pi}\right) \angle -\hat{\beta}. \tag{17}$$

4. Simulation Results

To demonstrate the effectiveness of the proposed algorithm, the performance of the proposed method is evaluated and compared with CP-based [19], ML method by Moose [5], and Pilot-based method by Classen [20] in this section.

Performance of the CFO estimator is compared in terms of root mean-squared error (MSE) in the following way [21]:

$$\mathrm{MSE} = \frac{1}{I} \sum_{i=1}^{I} \|\hat{\epsilon}_i - \epsilon\|^2 \tag{18}$$

where $\hat{\epsilon}_i$ is the estimate of ϵ in the ith trial, I is the total number of trials.

In Fig. 2 and 3, the true CFOs are 0.2 and 0.35 subcarrier spacing, respectively. Using QAM modulation with $N_{\mathrm{FFT}} = 32$, $N_{\mathrm{CP}} = 6$, the parameter q in the proposed method is set to 14, the number of simulation runs is $I = 100$, and the SNR is set to be from 0 to 20 dB for both cases. It is clear from the MSE of the estimation in the first case that from 0 to 6 dB, the proposed algorithm has a better performance except the Classen algorithm. However, from 6 to 21 dB, the proposed algorithm has the best performance, which implies that the proposed algorithm leads to increased efficiency in

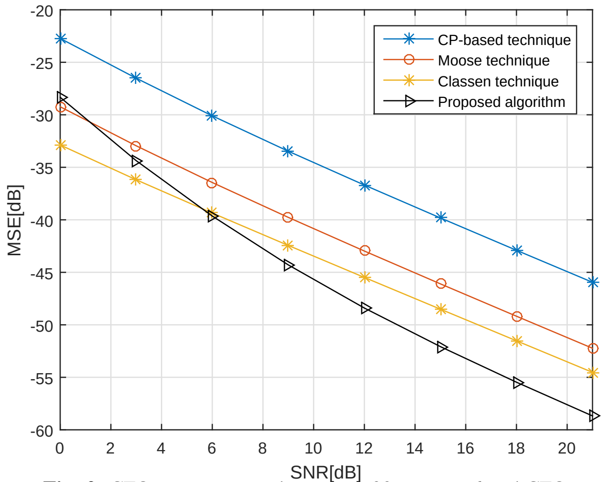

Fig. 2. CFO estimation with $N_{\mathrm{FFT}} = 32$, $N_{\mathrm{CP}} = 6$ and CFO = 0.2.

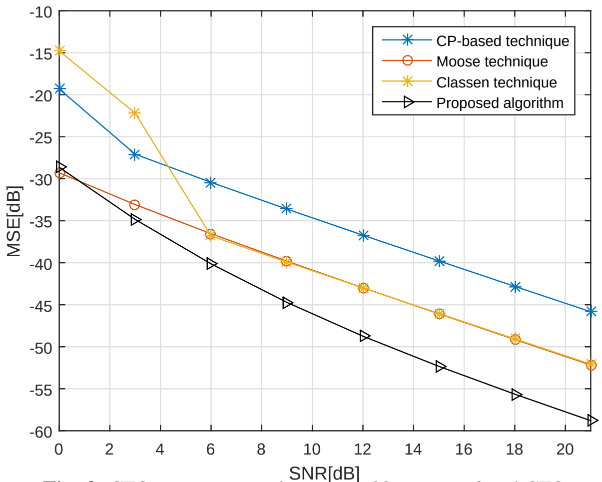

Fig. 3. CFO estimation with $N_{\mathrm{FFT}} = 32$, $N_{\mathrm{CP}} = 6$ and CFO = 0.35.

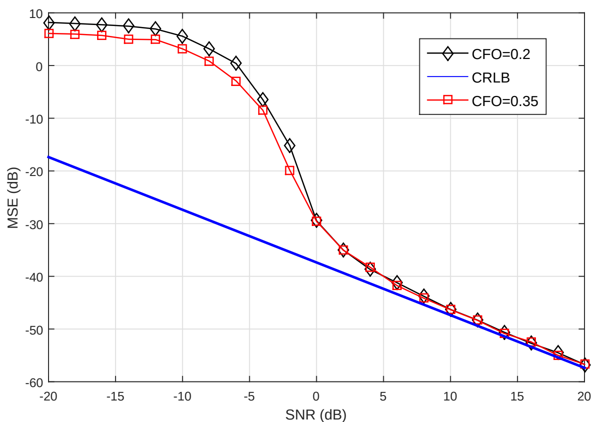

Fig. 4. MSE($\hat{\omega}$) versus SNR, $N_{\mathrm{FFT}} = 32$.

high SNRs. Furthermore, in the case with CFO = 0.35, the proposed method has better performance at the entire range, which implies that the proposed method has better accuracy in high CFOs for most values of SNRs compared to other methods.

In Fig. 4, two cases of estimation have been compared with Cramér-Rao Lower Bond (CRLB) [22] which is calculated from

$$\mathrm{CRLB}(\hat{\omega}) = \frac{6}{\mathrm{SNR}\, N(N^2 - 1)}. \quad (19)$$

From -20 to 0 dB, the proposed method has acceptable performance in very low SNRs. From 0 to 20 dB, the proposed algorithm has very good performance at high SNRs, approaching the CRLB and showing the linearity of the proposed method.

It can be seen from these figures that, the proposed method can estimate the CFO better than the previous methods.

5. Conclusion

A carrier frequency offset estimation in OFDM systems as a quadratic eigenvalue problem has been proposed. It was tested using a predetermined type of modulation, coding, while it only depended on known training data. Its performance has been examined and compared with ML Moose estimator, CP-based estimator, Classen estimator, and Cramér-Rao Lower Bond (CRLB). Numerical results have shown that the proposed estimator has higher accuracy compared with the estimators for most values of SNRs. In future work, the proposed method can be applied to emerging wireless technologies [23], mobile communication networks and digital television broadcasting systems [24].

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