SELECTIVE FECUNDABILITY AND CONTRACEPTIVE EFFECTIVENESS

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Fecundable women—that is, women not pregnant or in a period of postpartum anovulation—typically vary in their monthly chance of conception or fecundability. This variation sets the stage for certain automatic selections that complicate the measurement of contraceptive effectiveness. To illustrate these matters, Tietze,[1] in 1959, used a three-point distribution of fecundabilities, taking as his values of fecundability .50, .10 and .01 and assigning them weights of .60, .38 and .02, respectively. Despite its crudity the model yielded a set of times required for conception that approximated reasonably well those of two large samples of American women. By means of this model Tietze documented that in the absence of contraception, the fecundability composition, or "pregnancy potential" as he termed it, of a cohort of exposed women changes rapidly by virtue of the tendency of pregnancy to select out the most fecund and to leave behind an increasingly subfecund residual group. In the presence of contraception this selective process is slowed. The more effective is the contraception, the slower becomes the selection. In his discussion, he called attention to the fact that a group's contraceptive performance will be affected depending on whether they initiate contraception during amenorrhea when their fecundability is temporarily low, or at a time such as just before their first postpartum ovulation when their pregnancy potential is at its maximum, or after several fecundable months without protection when that potential has been reduced by the selective removal of some of the more fecund.

It appears worthwhile to rehearse these important selections on the basis of a more detailed model than was available to Tietze. The model to be used is an extension of the Type I-geometric distribution. Recently Jain successfully fitted this distribution to the intervals between marriage and first conception of a sample of 2,190 Taichung women.[2] The model differs from Tietze's earlier one in three respects. It is fitted to a non-Western population rather than a Western population. It yields a continuous distribution of fecundabilities, meaning that any fecundability from 0 to 1 may be assigned a relative frequency. It exhibits right skewness with the mode of fecundability less than the mean in contrast to Tietze's distribution, which has 60 per cent, and therefore the mode, at the very high fecundability of .50. Right skewness was also obtained by Potter and Parker[3] when they fitted a Type I-geometric distribution to the conceptive delays of wives from the Princeton Fertility Study.

This paper has three aims. First, Jain's fitted Type I-geo-

TABLE I. MEAN, MODE AND STANDARD DEVIATION OF FECUNDABILITY ACCORDING TO NUMBER OF FECUNDABLE MONTHS WITHOUT PREGNANCY BY PRESENCE OR ABSENCE OF CONTRACEPTION

Months of Fecundable Exposure	*Absence of Contraception* *Mean*	*Mode*	*Standard Deviation*	*.99 Effective Contraception* *Mean*	*Standard Deviation*
0	.163	.128	.078	.163	.078
1	.156	.122	.075	.163	.078
2	.149	.116	.072	.163	.078
3	.143	.111	.069	.163	.078
6	.127	.098	.062	.162	.078
12	.104	.079	.052	.162	.078
24	.077	.057	.039	.161	.078
48	.050	.037	.026	.160	.077

metric distribution is used to corroborate that change in fecundability composition is most rapid in the absence of contraception and becomes progressively slower in the presence of more effective contraception. Second, an extension of this model is presented and then applied by studying the proportions of women accidentally conceiving within two years of contraceptive practice as a function of contraceptive effectiveness and length of preceding period of nonprotection. Finally, discussion centers on the current relevance of the implications drawn by Tietze for measuring contraceptive effectiveness.

MODEL

The main assumptions of the model to be used are (1) that for any given woman fecundability is constant and, therefore, her number of fecundable months before conception behaves as a geometrically distributed random variable; and (2) among women fecundability varies according to a Pearson Type I density. This curve, also known as the Beta distribution, has two parameters (call them a and b) and is useful for representing a density of fecundability because subject to the restriction $b > a > 1$, it generates a wide family of unimodal curves defined over the range 0 to 1 and yielding means below 0.5. The mathematical details relating to the model are given in the appendix.

RESULTS

According to the model, based on the parameters $a = 3.48$ and $b = 17.89$, at the start of marriage the hypothetic population of Taiwan women have fecundabilities averaging .16 together with a mode of .13 and a standard deviation of .08. Table 1 contrasts the changes of fecundability composition to be expected over a 48-month period when contraception is absent and when it is present and 99 per cent effective. As might be anticipated from Tietze's analysis, change is rapid in the

TABLE 2. MEAN FECUNDABILITY ACCORDING TO NUMBER OF FECUNDABLE MONTHS WITHOUT PREGNANCY, BY EFFECTIVENESS OF CONTRACEPTION

Months of Fecundable Exposure	*Effectiveness of Contraception*							
	.00	*.50*	*.75*	*.90*	*.95*	*.97*	*.98*	*.99*
0	.163	.163	.163	.163	.163	.163	.163	.163
1	.156	.160	.161	.162	.163	.163	.163	.163
2	.149	.156	.160	.162	.162	.162	.163	.163
3	.143	.153	.158	.161	.162	.162	.162	.163
6	.127	.145	.154	.159	.161	.162	.162	.162
12	.104	.130	.145	.156	.159	.161	.161	.162
24	.077	.107	.131	.149	.156	.159	.160	.161
48	.050	.079	.108	.137	.149	.154	.157	.160

TABLE 3. RELATIVE FREQUENCIES OF WOMEN HAVING SPECIFIED FECUNDABILITIES AMONG WOMEN WHO HAVE NOT CONCEIVED AFTER STATED NUMBER OF MONTHS

Number of Months Without Conceiving	*Relative Frequencies of Women With Fecundabilities of*					
	.01	*.05*	*.13*	*.20*	*.26*	*.50*
0	1.000	26.972	65.280	46.075	23.669	.160
6	1.000	21.060	30.067	12.829	4.128	.003
12	1.000	16.443	13.848	3.572	.720	0.0
24	1.000	10.024	2.938	.277	.022	0.0
48	1.000	3.725	.132	.002	0.0	0.0

former case, but very slow in the latter. When contraception is absent, not only do the mean and mode of fecundabilities decline steeply, but so does the standard deviation, owing to the fact that the women still not pregnant become more and more homogeneously subfecund. In contrast, when contraception is 99 per cent effective, very little change takes place over the 48-month period.

Table 2 documents the increasingly rapid decline of mean fecundability over the span of 48 months when the effectiveness of contraception is taken at progressively lower values.

The mechanism whereby such rapid changes of fecundability composition occur in the absence of contraception is illustrated in Table 3. Here the relative frequencies of women having six different fecundability values are given for specified durations of exposure. The frequency of women with a fecundability of .01 is taken as standard. At the start of exposure, these subfecund women are far outnumbered by women having fecundabilities near the mode (i.e., .13, .20 and .26). However, by month 48, women with fecundabilities of .13 or more are nearly all pregnant, and those with fecundabilities of .01 and .05 are now predominant among the six groups chosen. In the presence of 99 per cent effective contraception, pregnancy remains as selective as ever with respect to fecundability. However, so few women are becoming pregnant, even women of high natural fecundability, that the relative frequencies of different fecundability values are changing only very gradually, with commensurately slow changes in the mean and standard deviation of fecundability.

The proportion accidentally conceiving within a given duration of contraceptive practice depends on the effectiveness of the contraceptive and the users' distribution of natural fecundabilities. For a given population of women and for a given effectiveness of contraception, the rate of accidental conception will be highest if the women are initiating the contraceptive right after the period of postpartum anovulation so that no opportunity exists for pregnancy to remove the more fecund.

TABLE 4. PROPORTIONS ACCIDENTALLY CONCEIVING WITHIN TWO YEARS ACCORDING TO EFFECTIVENESS OF CONTRACEPTION AND NUMBER OF PREVIOUS MONTHS WITHOUT PROTECTION

Previous Months Without Protection	*Accidental Pregnancies During Two Years of Contraception*					
	Contraception Effectiveness					
	.90	*.95*	*.96*	*.97*	*.98*	*.99*
0	.314	.175	.143	.110	.075	.038
6	.256	.140	.114	.087	.059	.030
12	.217	.116	.094	.072	.048	.025
24	.165	.087	.070	.053	.036	.018
48	.112	.058	.047	.035	.024	.012

If the women come to the contraceptive as a selectively subfecund group who have survived a period of nonprotection, then because of their lower fecundabilities the proportion conceiving accidentally will be lower. This relation is illustrated by Table 4, which gives proportions accidentally conceiving within two years of contraceptive practice as a function of contraceptive effectiveness and length of preceding period of nonprotection. The selection that takes place during an unprotected period of four years is enough to reduce the cumulative rate of accidental pregnancy by a factor of roughly three.

DISCUSSION

Tietze's earlier article[4] called attention to potential sources of bias facing any comparison of contraceptive methods. First, if method A is initiated more immediately after a birth than is method B, the amount of overlap between its practice and amenorrhea will be greater than for the other method. This means an unfair advantage since the risk of pregnancy is so low during amenorrhea. This source of bias has continued to receive too little attention in many studies.

A second source of bias is unequal length of observation. As seen from Table 2, in the presence of contraception, the average monthly risk of accidental pregnancy decreases with the passage of time, the decrease being more rapid the less effective is the method. Given a longer observation period, the low-risk women can contribute longer experiences to average down the overall rate of pregnancy, conventionally expressed as the number of accidental pregnancies per 1,200 women-months of contraceptive exposure. Accordingly, a pregnancy rate of this type is influenced by the extraneous factor of observation length.[5] In the ten years since Tietze's article, this defect of the Pearl pregnancy rate has come to be more generally recognized and life table procedures more often substituted as a way around this problem.[6]

However, Table 4 makes it clear that when comparing two contraceptives it is not enough to be using life table measures and to control for the factor of amenorrhea overlap. It is also necessary to assure similar distributions of the open fecundable intervals between end of amenorrhea and initiation of the contraceptive. If method A is started much later after a birth than is method B, then a possibility exists that users of A are more selected for low pregnancy risk and for that reason alone might give an appearance of greater effectiveness than method B.

Two contexts in particular are conducive to unequal open intervals. First is the case of a family planning program that is replacing indigenous contraception with a new method or at least a method new to the community. Practice of a method that has been around for a while is likely to be started soon after a birth when the need and disposition to practice contraception is greatest. In contrast, the abrupt introduction of a new method catches women at widely varying intervals from their last birth. In this situation the old method may be benefited by more overlap with amenorrhea, but the new method may draw an even larger advantage from the long intervals since last birth that tend to select for subfecund women or even women unknowingly sterile.

The second situation applies to a family planning program in which patients are regularly started on one method and then, upon their discontinuing it for some reason other than pregnancy, are shifted to a second method, sometimes with a gap of nonprotection intervening. Especially if these gaps are common, the women eligible for the second method would be selectively subfecund and, therefore, an apparent showing of greater protection against pregnancy by the second method could be spurious. Of course, if the initial method is highly effective and most switches to the second method are immediate—e.g., after removal of the device, the patient goes home with a supply of pills—then little selection toward subfecundity would be seen. Indeed, in this case the net bias might be against the second method for having less overlap with postpartum amenorrhea than does the first.

Obviously too, merely measuring the lengths of the two sets of open intervals is not enough to determine the direction and magnitude of bias. A 30-month open interval implies considerable selection toward subfecundity if it consists wholly of fecundable months unaccompanied by contraception; it implies somewhat less selection if it includes a year of postpartum amenorrhea or a pregnancy ending in abortion; it implies scarcely any selection at all if throughout its length another highly efficient contracetpive has been used. As a practical matter it is not feasible to standardize for all these contingencies. The most favorable condition for a comparison of two methods —always short of the ideal of a true experiment with random allocation of methods to patients—is when use of the two methods commences soon after a birth and the investigator is able to either subtract out woman-months of amenorrhea or assume confidently that overlap with amenorrhea averages out about the same in the two groups.

As a last consideration, it is interesting to compare the results of Table 4 with the cumulative two-year "gross" pregnancy rates obtained by life tables procedures for the sample of 7,295 Taichung women representing the Taichung IUCD Medical Follow-Up Study.[7] By "gross" pregnancy rates is meant estimates of the pregnancy rates that would have been observed in the absence of competing causes of IUD termination such as removals and expulsions. These two-year cumulative pregnancy rates per 100 acceptors of IUD were 14.9 (3.1), 15.0 (1.5), 12.3 (1.2) and 6.2 (0.8) for acceptors aged under 25 years, 25–29, 30–34 and 35–39, respectively. The parentheses enclose standard errors. Mean intervals from last birth to insertion of the device were 8, 10, 14 and 18 months respectively. Of course, some of these intervals included months of amenorrhea, practice of contraception and perhaps even a few pregnancies ending in wastage.

According to Table 4, the two-year cumulative pregnancy rate of 15 per 100 acceptors of IUD, calculated for women under 30 years of age, corresponds to a contraceptive effectiveness of .95 or .96 depending on whether one ignores the intervals averaging eight months between previous birth and insertion. The slightly lower pregnancy rate of 12 for women aged 30–34 years may reflect the increase in preceding open interval (from eight and ten to 14 months) as well as a slight decrease in fecundability. The much lower pregnancy rate of six for women aged 35–39 years can only partly be explained by the longer open intervals (18 months instead of eight) even if it is assumed that these intervals consisted entirely of fecundable months without the protection of contraception. Accordingly one has to stipulate either an increase in IUD effectiveness to .97 or higher, or, more plausibly, a lower fecundability combined with a larger minority of women who are unknowingly sterile.

APPENDIX

Derivations relating to Type I-geometric distribution are given elsewhere and need not be repeated here.[8] However, it is convenient to recall some of the formulae.

At the start of exposure, the women's fecundabilities are characterized by the following Type I density:

$$f(p) = \frac{1}{B(a,b)} p^{a-1}(1-p)^{b-1} \quad 0 \leq p \leq 1 \text{ where the Beta function}$$

$$B(a,b) = \int_0^1 p^{a-1}(1-p)^{b-1}\, dp.$$

The mean, mode and variance of fecundability have simple expressions, namely,

$$\bar{p} = \frac{a}{a+b}\,;\ \dot{p} = \frac{a-1}{a+b-2}\,;\ \text{and } \delta^2 = \frac{a\,b}{(a+b)^2\,(a+b+1)}\,.$$

It can be shown that the women who have been exposed t months without conceiving have a density of fecundabilities that conforms to a Type I distribution with parameters a' and b', related to the original parameters a and b by $a' = a$ and $b' = b + t$. Hence, the mean, mode and variance of fecundabilities of these women may be obtained from the formulae above simply by substituting b' for b. Likewise, it can be proved that women who conceive during the jth month have a Type I density of fecundabilities with parameters $a'' = a + 1$ and $b'' = b + j - 1$.

A contraceptive has effectiveness, e, $0 \leqq e \leqq 1$, if the user's "natural fecundability," p, is reduced by a proportionate factor, $1-e$, that is, is reduced to a "residual fecundability" of $(1-e)p$. Consider a population of women whose density of natural fecundabilities initially follows a Type I curve with parameters a and b and who are practicing contraception of effectiveness e. For convenience let $c = 1 - e$. The proportions who are expected to remain nonpregnant for 1, 2 and, more generally, j months are

$$\begin{aligned} Q(1|c) &= \int_0^1 f(p)\,(1-c\,p)\,d\,p \\ &= \int_0^1 \frac{p^{a-1}(1-p)^{b-1}}{B(a,b)}\,(1-c\,p)\,d\,p \\ &= \frac{B(a,b)}{B(a,b)} - c\,\frac{B(a+1,b)}{B(a,b)}; \\ Q(2|c) &= \int_0^1 f(p)\,(1-c\,p)^2\,d\,p \\ &= \frac{B(a,b)}{B(a,b)} - 2\,c\,\frac{B(a+1,b)}{B(a,b)} + c^2\,\frac{B(a+2,b)}{B(a,b)}; \text{ and} \\ Q(j|c) &= \int_0^1 f(p)\,(1-c\,p)^j\,d\,p \\ &= \sum_{i=0}^{j} (-1)^i \binom{j}{i} c^i\,\frac{B(a+i,b)}{B(a,b)}. \end{aligned}$$

The proportion of women expected to conceive during the kth month, $k = 1, 2, \ldots$, is

$$\begin{aligned} P(k|c) &= \int_0^1 f(p)\,(1-c\,p)^{k-1}\,c\,p\,d\,p \\ &= \sum_{i=0}^{j} (-1)^i \binom{k-1}{i} c^{i+1}\,\frac{B(a+i+1,b)}{B(a,b)}. \end{aligned}$$

The density of fecundabilities of women practicing contraception who either conceive the kth month or remain protected for j months, $j, k = 1, 2, \ldots$, no longer conforms to a Type I distribution. However, the first and second moments around the origin of their natural fecundabilities have straightforward expressions. The mean natural fecundability of those still not pregnant at the end of j months of contractive exposure is

$$\begin{aligned} M(p_{j|c}) &= \frac{1}{Q(j|c)} \int_0^1 p\,f(p)\,(1-c\,p)^j\,d\,p \\ &= \frac{1}{Q(j|c)} \sum_{i=0}^{j} (-1)^i \binom{i}{j} c^i\,\frac{B(a+i+1,b)}{B(a,b)}. \end{aligned}$$

The corresponding second moment is

$$\begin{aligned} \mu_2(p_{j|c}) &= \frac{1}{Q(j|c)} \int_0^1 p^2\,f(p)\,(1-c\,p)^j\,d\,p \\ &= \frac{1}{Q(j|c)} \sum_{i=0}^{j} (-1)^i \binom{j}{i} c^i\,\frac{B(a+2+i,b)}{B(a,b)}. \end{aligned}$$

The variance of fecundability among women successfully practicing contraception j months is then available from

$$V(p_{j|c}) = \mu_2(p_{j|c}) - [M(p_{j|c})]^2.$$

The mean and second moment around the origin of women who accidentally conceive during the kth month of contraception practice is

$$M(p_{k|c}) = \frac{1}{P(k|c)} \int_0^1 p\, f(p)\, (1 - c\,p)^{k-1}\, c\, p\, d\, p$$

$$= \frac{1}{P(k|c)} \sum_{i=0}^{k-1} (-1)^i \binom{k-1}{i} c^{i+1} \frac{B(a+2+i, b)}{B(a, b)}; \text{ and}$$

$$\mu_2(p_{k|c}) = \frac{1}{P(k|c)} \sum_{i=0}^{k-1} (-1)^i \binom{k-1}{i} c^{i+1} \frac{B(a+3+i, b)}{B(a, b)}.$$

A last result needed is the proportion $S(n,k,c)$ who may expect to practice successfully contraception of effectiveness e for n additional months if they went unprotected without experiencing pregnancy for k months before adopting contraception. At the start of contraceptive practice these women have fecundabilities conforming to a Type I distribution with parameters a and $b+k$. Hence the proportion of these women whose contraception will be successful for n months is:

$$S(n, k, c) = \frac{1}{Q(k|c)} \int_0^1 \frac{p^{a-1}(1-p)^{b+k-1}}{B(a, b)} (1 - c\,p)^n\, d\, p$$

$$= \frac{1}{Q(k|c)} \sum_{i=0}^{n} (-1)^i \binom{n}{i} c^i \frac{B(a+i, b+k)}{B(a, b)}.$$

To evaluate numerically ratios of *Beta* coefficients, use is made of the basic identity

$$B(a, b) = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)} = B(b, a),$$

where $\Gamma(n)$ denotes the *gamma* function with real parameter n. This identity leads to the convenient calculating formulae

$$\frac{B(a+k, b+j)}{B(a, b)} = \frac{\Gamma(a+k)\,\Gamma(b+j)}{\Gamma(a+b+j+k)} \cdot \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)}$$

$$= \frac{a(a+1)\ldots(a+k-1)(b)(b+1)\ldots(b+j-1)}{(a+b)(a+b+1)\ldots(a+b+j+k-1)}; \text{ and}$$

$$\frac{B(a+k, b)}{B(a, b)} = \frac{a(a+1)\ldots(a+k-1)}{(a+b)(a+b+1)\ldots(a+b+k-1)}.$$

It is believed that rounding errors does not affect the values given in the tables below, which are taken to only three places. At any rate, with use of double precision, proportions and moments were calculated by two separate computational routines—one utilizing logarithms and one not—and the differences found did not affect the first three decimal places.

Jain employed the method of moments to derive his parameter values of $a=3.48$ and $b=17.89$, used in the applications below. M. C. Sheps has called the writers' attention to unpublished procedures by Majumdar for estimating these same parameters by maximum likelihood. In general, maximum likelihood estimators are to be preferred over those obtained by the methods of moments. However, in the present case, the method of moments appears to have worked well.

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