

Übungen zur Vorlesung “Program Extraction from Proofs”

It suffices to indicate derivations in E-ID $^\omega$ informally. Also disregard the difference between \forall^U , \rightarrow^U and \forall , \rightarrow .

Aufgabe 25. Consider pointwise equality $=_T$ for the algebra T (cf. 2.1.2). Denote the constructors of the algebra T by 0 and Sup. Prove that

$$\forall_f^U(0 =_T \text{Sup} f \rightarrow F) \quad \text{and} \quad \forall_{f_1, f_2}^U(\text{Sup} f_1 =_T \text{Sup} f_2 \rightarrow f_1 =_{N \rightarrow T} f_2).$$

Aufgabe 26. Let C_0, C_1, C_2 denote the constructors of the witnessing algebra $\mu_{=T}$. The introduction axioms for the witnessing predicate $=_T^r$ are

$$\begin{aligned} (=_T^r)_0^+ &: \forall_{x_1, x_2}(F \rightarrow =_T^r(C_0, x_1, x_2)), \\ (=_T^r)_1^+ &: =_T^r(C_1, 0, 0), \\ (=_T^r)_2^+ &: \forall_{f, f_1, f_2}(\forall_n =_T^r(f n, f_1 n, f_2 n) \rightarrow =_T^r(C_2 f, \text{Sup} f_1, \text{Sup} f_2)) \end{aligned}$$

State the elimination axiom $(=_T^r)^-$, and prove the inversion lemma:

$$\begin{aligned} =_T^r(C_s f, z_1, z_2) &\rightarrow \exists_{f_i} \text{Eq}(z_i, \text{Sup} f_i) \quad (i = 1, 2), \\ =_T^r(C_2 f, \text{Sup} f_1, \text{Sup} f_2) &\rightarrow \forall_n =_T^r(f n, f_1 n, f_2 n). \end{aligned}$$

Aufgabe 27. Following the file `tut07.scm`, inductively define $\text{Even}(n)$, and print its introduction and elimination axioms. Do the same for

```
(add-ids (list (list "Ev" (make-arity (py "nat")) "algEv")
                (list "Od" (make-arity (py "nat")) "algOd"))
              '("allnc n(F -> Ev n)" "EfqEv")
              '("Ev 0" "InitEv")
              '("allnc n.Od n -> Ev(n+1)" "GenEv")
              '("allnc n(F -> Od n)" "EfqOd")
              '("allnc n.Ev n -> Od(n+1)" "GenOd"))
```

Aufgabe 28. Following the file `tut07.scm`, prove the existence of Fibonacci numbers. Check and display the proof using `(cdp)`, and display the proof expression (a lambda term) using `(dpe)`. Indicate the corresponding derivation tree.

Abgabetermin. Montag, 25. Juni 2007, 11:15 Uhr