Mathematical Commentary on Le Corbusier's Modulor

October 24, 2019

Abstract

This contribution presents mathematical comments on Le Corbusier's scale of proportions the Modulor. The analysis covers the structure of the scale, approximation routines, errors of the geometric deduction, and the evaluation of the postulates of harmonious design.

1 Introduction

The Modulor is a famous scale of proportions created by French-Swiss architect Le Corbusier.[1] The initial excitement about the Modulor was in no small part due to its timing and the outstanding promotional skills of its creator. The proposal arose exactly when Europe was facing the challenge of recovery from the destructions of World War II. Luckily, reconstruction programs were fully supported by new technologies and the industry of prefabricated materials. The new methods of construction gave birth to new architecture. Designers, architects, engineers, and special committees were working on a wide assortment of questions of standardization. The new architecture sought a new aesthetic; the pre-war decorative traditions did not fit into the new uniform building process.

Le Corbusier was among the professionals engaged in the study of new architectural regulations. He had been earnestly interested in the norms of architecture since the 1920s (Turner 1971; Fischler 1979; Loach 1998; Evans 1995: 281; Cohen 2014). In the late 1940s Le Corbusier clearly understood the importance of the moment and made an effort to gain a leading position on the frontiers of standardization. The architect's ability to address the most timely questions was one of his undisputed talents; even though many of his projects were never commissioned, they often responded to critical demands of the society. The architect announced that he found a solution for harmonious standardization of mass production that is based on mathematical foundations and human scale. He called his invention, a reference tool in designing new buildings, the Modulor. In 1950 and 1955 Le Corbusier published two volumes under the same name: *Le Modulor I* and *Modulor 2*. They described the

[1] A short review of the book The Modulor can be found in (Ostwald 2001), the account on the development of the Modulor is given in (Matteoni 1986).

path that led to his new invention and referred to the feedback of colleagues, representatives of industry, authorities, and scholars.

Today the Modulor is evaluated along the same lines as a number of his other projects: it is considered by many to be an original, bold, but not quite practical idea (Evans 1995: 275). Le Corbusier and his followers realized only a few constructions with certain reference to the Modulor scale. Both the impact and the defects of the scale are widely discussed in the publications on architecture history.[2] The Modulor occupies a distinguished place in the history of art and architecture: '... the affective economy of the Modulor – and, indeed, of modernity – has been reified by a large swath of academic literature and popular consumption' (Tell 2019). The system of proportions is a part of many courses for future architects and designers. While not being practical, it is considered by many scholars as a compelling marriage of mathematics and art. Thus, it is important to have a rigorous self-contained commentary on the Modulor's mathematics.

The book *The Modulor* contains an abundance of calculations and geometric constructions. However, Le Corbuiser's notations and the style of exposition require some effort on the part of the reader, even though mathematics of the project is not very complicated; if properly rephrased, it is accessible to anyone with a good middle-school geometry course background. My goal here is to provide a detailed self-sufficient analysis of the Modulor's mathematics. Unfortunately, my conclusions contribute to the growing criticism of the architect's professional practices.

Many observations of this paper are certainly well known (see e.g. Evans 1995, Linton 2004, Loach 1998, Tell 2019). However, in the exception of (Linton 2004), the analysis of Modulor is rarely accompanied by a systematic mathematical argument, while the interpretation of mathematical statements and the role of Le Corbusier's assistants vary significantly through the literature.

The first volume can be divided into three parts: geometric constructions, the description of the final scale, and speculations on possible applications. The second volume collects feedback on the project. In this present paper I first comment on the final proportions of the Modulor. Second, I discuss the geometric deduction of the scale. For our purposes I do not find it necessary to comment on applications.

2 Sequences of the Scale

According to Le Corbusier, the Modulor is a tool for designers, architects, and constructors. The architect stated that this tool would help professionals to design buildings of beautiful proportions from prefabricated materials. Mathematically, the Modulor scale is simply a pair of sequences of measurements, called the 'red sequence' and the 'blue sequence'. Numbers in these sequences are represented by partitions of a rectangular diagram. To emphasize the derivation of the Modulor scale from human proportions, the diagram features a man with a raised hand (Fig.1).

[2]For one of the earliest discussions see (Pevsner 1957).

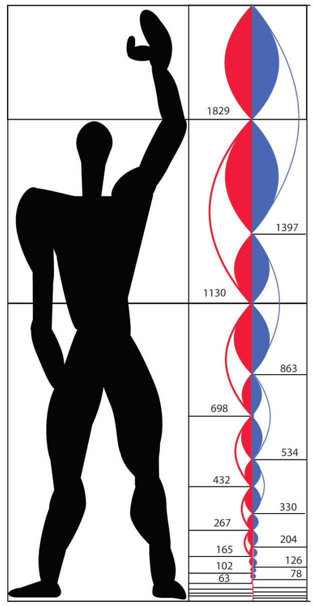

Fig 1. Scale of proportions of the Modulor, vector graphics by the author after Le Corbusier.

The numbers of the red and blue sequences vary in different versions of the Modulor.[3] For example one can find diagrams created by Le Corbusier with the following sequences:

$$\text{red:} \quad 2, 7, 9, 16, 25, 41, 66, 108, 175. \tag{2.1}$$

$$\text{blue:} \quad 2, 9, 11, 20, 31, 51, 82, 216. \tag{2.2}$$

$$\text{red:} \quad 27, 43, 70, 113, 183. \tag{2.3}$$

$$\text{blue:} \quad 86, 140, 226. \tag{2.4}$$

$$\text{red:} \quad 43.2, 69.8, 113, 183. \tag{2.5}$$

$$\text{blue:} \quad 53, 86, 140, 226. \tag{2.6}$$

$$\text{red:} \quad 39, 63, 102, 165, 267, 432, 698, 1130, 1829. \tag{2.7}$$

$$\text{blue:} \quad 30, 48, 78, 126, 204, 330, 534, 863, 1397, 2260. \tag{2.8}$$

The first example (2.1)-(2.2) is an earlier version based on a human of height 175 cm. The later versions are based on the height 6 feet (182.88 $\simeq$ 183 cm). According to Le Corbusier, this scale was found geometrically. He claimed that the scale has the following characteristics:

[3] Sequence (2.1) – (2.2) can be found in (Le Corbusier 2000: I, 51); Sequence (2.3) – (2.4) in (Le Corbusier 2000: I, 67); Sequence (2.5) – (2.6) in Le Modulor étude 1945, Document 32285, FLC; Sequence (2.7)–(2.8) in Document 21007, FLC.

- it is based on human proportions;
- it resolves a mismatch between the Anglo-Saxon and the French metric systems;
- it provides guidelines to build aesthetically from prefabricated materials;
- it is based on rigorous calculations derived from the so-called 'right angle rule' and the rule of the golden ratio.

The sequences of the Modulor have some curious mathematical properties. For example, in the sequences (2.1) - (2.8) above it is possible to identify groups of three values that mimic Fibonacci numbers, highly praised by Le Corbusier. In a Fibonacci series each successive number in the sequence is the sum of the preceding two:

$$9 + 16 = 25, \quad 48 + 78 = 126, \quad 102 + 165 = 267, \quad 43.2 + 69.8 = 113, \quad \dots$$

However, a watchful eye immediately notices that there are deviations from this pattern:

$$330 + 534 \neq 863, \quad 698 + 1130 \neq 1829, \quad \dots \quad .$$

Another noteworthy property is that the numbers in the red sequence are very close to double values of the blue sequence:

$$330 = 2 \cdot 165, \quad 534 = 2 \cdot 267, \quad 863 \simeq 2 \cdot 432, \quad \dots \quad .$$

These phenomena, together with deviations from the pattern, are easily explained by the mathematical meaning of these numbers.

3 Construction of Red and Blue Sequences

Mathematics knows many important sequences that follow different patterns. For example, let's fix a non-zero number a_0 (initial value) and another non-zero number q (common ratio). Starting from a_0, one multiplies or divides it by q over and over again to get new elements of the sequence called a geometric progression:

$$\dots, \quad \frac{a_0}{q^2}, \quad \frac{a_0}{q}, \quad a_0, \quad a_0 q, \quad a_0 q^2, \quad \dots \quad .$$

Also one may consider *a Fibonacci type sequence* defined by a linear recurrence relation. Starting with two initial values a_0 and a_1, each following term is calculated as the sum of the previous two:

$$a_n = a_{n-1} + a_{n-2}, \qquad \text{for} \quad n = 2, 3, \dots . \tag{3.1}$$

For example, with $a_0 = a_1 = 1$ one gets the classical sequence of Fibonacci numbers,

$$1, \quad 1, \quad 2, \quad 3, \quad 5, \quad 8, \quad 13, \quad \dots .$$

Different initial values produce different sequences. For example, with $a_0 = 5, a_1 = 12$,

$$5, \quad 12, \quad 17, \quad 29, \quad 46, \quad 75, \quad 111, \quad \ldots .$$

One may ask whether there exists a sequence that is at once a geometric progression and at the same time a Fibonacci type sequence. In other words, this sequence should have a form $a_n = a_0 q^n$ and, at the same time, enjoy the property $a_n = a_{n-1} + a_{n-2}$. It is not difficult to prove that a geometric progression with a special common ratio $q = \frac{1+\sqrt{5}}{2}$ or $q = \frac{1-\sqrt{5}}{2}$ would possess both properties.[4] Note that $(1+\sqrt{5})/2$ is the famous golden ratio, denoted from now on as φ.

The concept of the Modulor outlined by Le Corbusier purports that any two consecutive terms of the red or blue sequence should be in the relation of the golden ratio φ:

$$a_n/a_{n+1} = \varphi.$$

According to Le Corbusier's theory, the presence of the golden ratio connects the scale with the rules of harmonic design. Hence by definition the terms of the Modulor should form a geometric progression:

$$\ldots, \quad \frac{a_0}{\varphi^2}, \quad \frac{a_0}{\varphi}, \quad a_0, \quad a_0\varphi, \quad a_0\varphi^2, \quad \ldots \quad . \tag{3.2}$$

In that case the Fibonacci property (3.1) would be guaranteed for all entries of the sequence.

However, there is a serious inconvenience hidden in sequence (3.2): its values are irrational numbers. Irrational numbers are impractical for design or construction; irrational numbers must unavoidably be substituted by values that approximate them.[5] For example, whether the golden ratio is written as 1.6, or 1.618034, or even 1.61803398874989484882, or with any other higher level of accuracy, these are nevertheless just approximate values of the golden ratio, since the exact value of φ represented in the decimal form is infinite and nonperiodic.

Even a very high level approximation comes at a cost: by switching to approximate values of the sequence, one cannot satisfy both properties $a_n = a_0\varphi^n$ and $a_n = a_{n-1} + a_{n-2}$ for all elements of the sequence. For example, consider the red sequence in one of the most elaborated versions (Le Corbusier 2000: I, 82), shown here in the first column of Table 1. In the second column of the table the ratios of two consecutive terms of the sequence are calculated:

$$95\,280.7/58\,886.7 \simeq 1.6180, \quad 58\,886.7/36\,394.0 \simeq 1.6180, \ldots .$$

Conceptually, all these ratios should be close to the value of φ. The third column contains the differences of two consecutive terms:

$$95\,280.7 - 58\,886.7 \simeq 36\,394.0, \quad 58\,886.7 - 36\,394.0 \simeq 22\,492.7, \ldots .$$

[4] Substitute $a_n = a_0 q^n$ into $a_n = a_{n-1} + a_{n-2}$ to obtain $a_0 q^n = a_0 q^{n-1} + a_0 q^{n-2}$. Divide both sides by $a_0 q^{n-2}$ to get the quadratic equation $q^2 = q + 1$, which has two irrational roots $q = \frac{1 \mp \sqrt{5}}{2}$. Thus, if a geometric progression has the Fibonacci recursion property, the common ratio is necessarily $q = \frac{1 \mp \sqrt{5}}{2}$. Following the argument in the other way, it is clear that this condition is also sufficient.

[5] See this observation also in (Evans 1995: 275)

The differences should match the corresponding elements of the first column in accordance with the Fibonacci rule. This simple exercise offers an insight into architect's approach to calculations of the Modulor sequences. The smallest values of Table 1 clearly indicate that Le Corbusier preferred to use the Fibonacci rule over the golden ratio relation for calculation of these numbers.[6] For small values this approach accumulates a significant error. The fundamental concept of 'golden ratio rule' is violated, the ratios are not sufficiently close to φ anymore.

Let us summarize the comments on the blue and red sequences:

- The original concept of the Modulor scale is based on the requirement that its consecutive measures should be in the golden ratio relation.

- The red sequence consists of the *approximate* values of elements of a geometric progression

$$\ldots, \quad \frac{a_0}{\varphi^4}, \quad \frac{a_0}{\varphi^3}, \quad \frac{a_0}{\varphi^2}, \quad \frac{a_0}{\varphi}, \quad a_0, \tag{3.3}$$

  where, in the earlier version, the initial value $a_0 = 175$, and in the later version $a_0 = 183$. The common ratio is the inverse of the famous golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$.

- The blue sequence consists of *approximate* values in a geometric progression which is the double of the red sequence:

$$\ldots, \quad \frac{2a_0}{\varphi^4}, \quad \frac{2a_0}{\varphi^3}, \quad \frac{2a_0}{\varphi^2}, \quad \frac{2a_0}{\varphi}. \tag{3.4}$$

- All elements of the true geometric progressions (3.3), (3.4) naturally satisfy the Fibonacci property. The approximate numbers of the Modulor scale inherit the Fibonacci property up to errors caused by approximations. The primary reason for deviations in the Modulor scale from the Fibbonaci rule is the approximation.

- The true values of the geometric progressions (3.3), (3.4) are irrational numbers, multiples of $\frac{1}{(1+\sqrt{5})^k}$. Their decimal form is infinite without periodic pattern. These are difficult to use in practice. In particular, none of the values operated by Le Corbusier are exact, all of them are approximations of the geometric series at different levels of accuracy. Le Corbusier permitted himself a very loose interpretation of approximation rules of irrational numbers and rounding values. This is discussed further in Section 4 below.

- Some values of the Modulor do not comply with the initial concept that the scale grows proportionally to the golden ratio.

[6] Robin Evans (1995: 395, remark 7) mentions that doubling of series was Le Corbusier's idea, while introduction of Fibonacci numbers could be Jerzy Soltan's contribution.

red sequence	ratios	differences	continued→	red sequence	ratios	differences
95 280.7	1.6180	36 394.0		182.9	1.6186	69.9
58 886.7	1.6180	22 492.7		113.0	1.6189	43.2
36 394.0	1.6180	13 901.3		69.8	1.6157	$26.\underline{6}$
22 492.7	1.6180	8 591.4		43.2	1.6180	16.5
13 901.3	1.6180	$5\,309.\underline{9}$		26.7	1.6182	10.2
8 591.4	1.6180	3 281.6		16.5	1.6176	6.3
5 309.8	1.6181	2 028.2		10.2	1.6190	3.9
3 281.6	1.6180	$1\,253.\underline{4}$		6.3	1.6154	2.4
2 028.2	1.6180	774.7		3.9	1.625	1.5
1 253.5	1.6180	478.8		2.4	1.600	0.9
774.7	1.6180	295.9		1.5	1.667	0.6
478.8	1.6180	182.9		0.9	1.5	
295.9	1.6178	113		0.6		

Table 1. Analysis of one of variations of the red sequence of the Modulor.

The observation that the Modulor is just a pair of geometric progressions may illuminate the comment made by Jerzy Soltan, the architect's assistant and collaborator:

> 'After the first few days he had a strong reaction against the whole thing, saying 'It seems to me that your invention is not based on a two-dimensional phenomenon but on a linear one. Your "Grid" is merely a fragment of a linear system, a series of golden sections moving towards zero on the one side and towards infinity on the other.' 'All right,' I replied, 'let us call it henceforth a rule of proportions.' (Le Corbusier 2000: I, 47).

This comment is usually interpreted by scholars exactly as Le Corbusier orders us to understand it: Soltan objected that the rules are one-dimensional rather than two-dimensional. However, it is also quite possible that the main point of Soltan's objection was that the sophisticated number games played by Le Corbusier produced a trivial mathematical object – a geometric progression.

4 Some Notes on Approximation

As was observed in Section 3, approximation became an unavoidable part of the construction of the Modulor: without rounding values, a user of the scale would be forced to deal with irrational numbers. The procedure of approximation deserves some additional comments.

Any professional working in science, engineering, applied mathematics, or computer science is aware that throwing away few insignificant digits is not without consequences. Further operations with rounded numbers may accumulate errors with a notable impact on the final result (see e.g. Chartier 2006). A set of well-known rules helps professionals to avoid hazards of rounding procedures.

Clearly, Le Corbusier did not realize or did not find important that all of his arithmetic manipulations, including the most elaborate, involved approximate values. He never did any calculations involving the formal expression of irrational number $(1+\sqrt{5})/2$, and he often talked about 'exact values' versus rounded 'practical values', even though all of his numbers were approximate.

The architect obviously did not care about any consistency of his rounding-off procedures. He constantly switched between different approximations of the Modulor measures, trying to fit them into various statements. In particular, his pivotal claim that the Modulor produces convenient numbers in switching between metric and Anglo-Saxon systems is false.[7] The claim is based on manipulated approximations of a few values of the scale and does not hold for all measures of the Modulor (see the tables of (Le Corbusier 2000: I, 57)).

It is easy to find numerous examples in the book that illustrate the architect's looseness in rounding of values. On one occasion he refused to round up his measures (Le Corbusier 2000: I, 56), but on another he was very flexible about their values (Le Corbusier 2000: I, 234).

5 Two Rules of Composition

We understand that the numbers of the Modulor diagram are rounded elements of geometric progressions

$$183, \quad \frac{183}{\varphi}, \quad \frac{183}{\varphi^2}, \quad \frac{183}{\varphi^3}, \quad \dots$$

and

$$2\cdot\frac{183}{\varphi}, \quad 2\cdot\frac{183}{\varphi^2}, \quad 2\cdot\frac{183}{\varphi^3}, \quad \dots$$

with $\varphi = \frac{1+\sqrt{5}}{2}$. The architect declared that these sequences create a measuring tool for harmonious design. Let us follow the justifications of the statement.

Le Corbusier claimed that the proportions were deduced geometrically from some postulates of harmonious composition applied in design, art, and architecture. Specifically, for the Modulor the architect focused on two principles: the right angle rule and the golden ratio rule.

The right angle rule suggests that a well-balanced composition should contain a collection of naturally inscribed right angles. In his book, Le Corbusier provided a number of his own observations and experiments, not only as a supporting evidence of the rule, but also as a proof of his long-standing interest and expertise in regulating lines.[8]

The golden ratio rule is a very popular idea that the number φ plays important role in art and nature. I postpone a separate comment on this concept until Section 9.

[7]See this observation also in (Tell 2019: 32, 34).

[8]See (Fischler 1979) on Le Corbusier's relations with golden ratio.

6 Three Squares Construction

The central role in the geometric deduction of the Modulor scale is given to the question that was posed by Le Corbuiser to his assistant Gerald Hanning. Le Corbusier recalles in the book that in 1943, due to German occupation, Hanning has had to flee Paris for Savoy. Before the departure of the young collaborator Le Corbursier formulated the following problem:[9]

> Take a man-with-arm-upraised, 2·20 m. in height; put him inside two squares, 1·10 by 1·10 meters each, superimposed on each other; put a third square astride these first two squares. This third square should give you a solution. The *place of the right angle* should help you to decide where to put this third square (Le Corbusier 2000: I, 37).

Presumably, the question should be based on the two postulates outlined above. Strangely, the question does not refer to the golden ratio at all.[10] The words 'superimposed', 'astride', 'give a solution' may have many possible mathematical interpretations. However, the context allows one to reconstruct the rigorous mathematical problem. It can be reformulated in the following words:

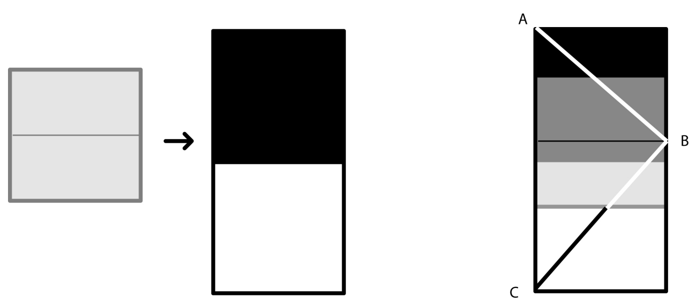

Fig 2. Statement of the Problem 6.1. Image: author.

> **Problem 6.1.** We have two equal squares of the side length 1.10 m. Let them be black and white. Put them together to form a rectangle. We have one more square of the same size with a marked middle line, let it be a gray square. One wants to know, how to place the gray square on the top of the black-white rectangle so that the angle ABC in the picture is a right angle. (Here, A and C are vertices of rectangle, and B is one of endpoints of the middle line of the gray square (Fig. 2).)

[9](Evans 1995: 279) states that these instructions themselves contain a mathematical contradiction, but this is not the case. As we will see below, there exists a solution of this problem. The errors were made in the proposed solutions.

[10]See this observation also in (Linton 2004: 56). My approach in this section has a lot of common points with careful geometric analysis of Linton and I agree with most of his statements excluding few observations that I mention further in the text.

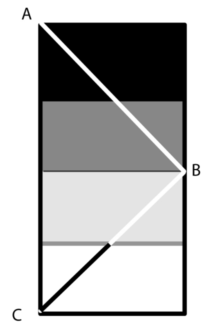

Fig 3. The unique solution to the Problem 6.1. Image: author.

The answer easily follows from some facts of the standard middle school geometry course:[11]

> **Solution to Problem 6.1.** There exists exactly one solution of this problem. It is symmetric: the gray square must be placed right in the middle of the rectangle, its marked middle line must coincide with the border between the black and white squares. Only such positioning of the gray square satisfies the conditions of the problem (Fig. 3).

Proof. Let $\angle ABC$ be a right angle, and let OD be the boundary between the black and white squares. Then the points B and D necessarily coincide.

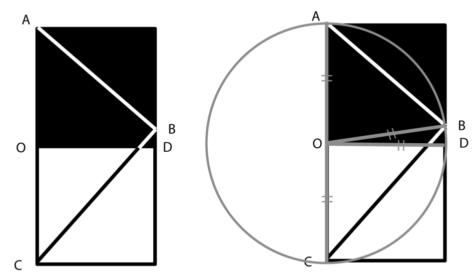

Fig 4. Proof of the solution to the Problem 6.1. Image: author.

Indeed, consider a circle circumscribed around the right triangle ABC. Then AC is the diameter[12] of this circle, and the midpoint O of AC is the center of that circle. Then $AO = BO = CO = r$, the radius of the circle. We have squares, so $AO = CO = DO = r$, hence $DO = BO = r$, but this is possible only if the points D and B coincide: $D = B$ (Fig. 4).

7 The Le Corbusier - Maillard Diagram

Several pages of *The Modulor* describe the solving process, the exchange of ideas with Hanning, and the final construction. However, the final 'solution' to Problem 6.1 is different from the unique symmetric placement described above!

[11] See (Linton 2004: 62) who provides another proof and makes a remark on the proof provided here.

[12] We use the following statement: suppose that $\angle B$ is the right angle of a right triangle ABC inscribed in a circle. Then AC is a diameter of this circle.

According to Le Corbusier's account (Le Corbusier 2000: I, 37), Hanning gave the first proposal on 25 August 1943. Interestingly, Hanning's construction does not match the formulated question at all: rather, it looks like a solution to some other problem. At the same time, his construction does refer to the golden ratio, as well as to $\sqrt{2}$ as a diagonal of a square.[13] Le Corbusier responded with an alternative construction, claiming that he himself and his collaborator Elisa Maillard had arrived at a better solution. The proposal of Le Corbusier and Elisa Maillard consists of the following steps (Le Corbusier 2000: I, 38).

Starting from a gray square of the side length 1.10 m, create a golden rectangle (Fig. 5):

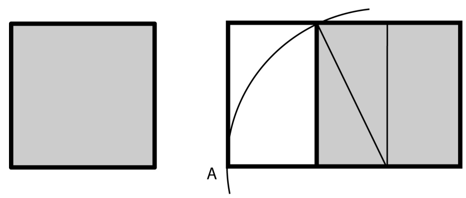

Fig 5. Step one of the Le Corbusier - Maillard solution. Image: author.

Let A be a corner of the golden rectangle, and let B be the midpoint of the gray square on the opposite side. Find a point C so that ABC is a right angle. Reconstruct the rectangle with the side AC (Fig. 6):

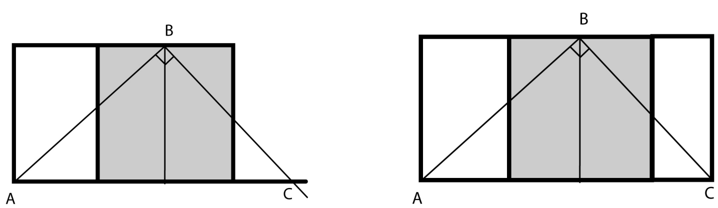

Fig 6. Step two of the Le Corbusier - Maillard solution. Image: author.

Divide this long rectangle into two equal parts by a gray midline. These equal parts will be black and white squares with gray square on the top of them (Fig. 7):

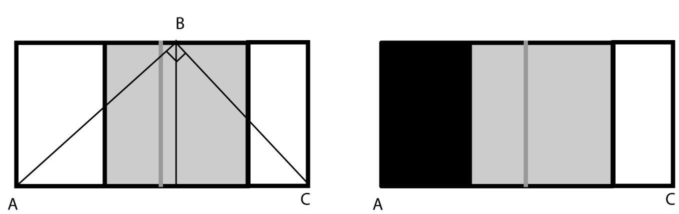

Fig 7. Step three of the Le Corbusier - Maillard solution. Image: author.

The architect concludes: 'Thus we have solved the problem set to us, namely to insert in two contiguous squares containing a man with arm-upraised a third square at the 'place of the right angle' (Le Corbusier 2000: I, 39). Certainly, mathematics proves that the only existing solution of Problem 6.1. is a symmetric one. An 'alternative non-symmetric solution' is an

[13] I conjecture that this mismatch between the problem and the solutions is the result of Le Corbusier's denial of the true extent of an *independent* research of foundations of the norms by his assistant. See also (Loach 1998: 207).

easy puzzle: Le Corbusier's construction is geometrically wrong. The gray midline divides the rectangle into black and white parts that are not squares, but rectangles. The side AC is not exact double, but $9/2\sqrt{5} \simeq 2.01246$ times longer than the side of the initial square. Hence each of the black and white rectangles has a side that is 0.6% longer than the other one. This is explained in the letter of mathematician René Taton, carefully reproduced by Le Corbusier in the book (Le Corbusier 2000: I, 231-234).[14] The deviation of rectangles from squares is so tiny that it could easily be overlooked. It is understood that Le Corbusier did most of constructions experimentally through drawing rather than by logical geometric arguments.

It is important to comment on the transition between geometric manipulations and the final Modulor scale. We have noted that the final scale of proportions consists of two sequences of measures. The architect claimed that the scale was derived geometrically from some postulates of harmonious composition. We have seen that geometrical constructions contained errors. Moreover, from the text of *The Modulor* it is clear that there is no logical connection between the red and blue sequences and the three squares constructions, except that they both use the golden ratio. The announced transition does not exist[15] The final scale was never a result of any mathematical deduction.

An ex post facto version of a diagram appeared on the cover of Modulor 2. It is attributed to Justino Serralta and André Maissonier (Le Corbusier 2000: II, 2). Curious visual resemblance can be noticed between this geometrically correct diagram and the initial (incorrect) drawings of Hanning in his letter to Le Corbusier 25 August 1943 (FLC B317). For the same reasons as above, the retrospectively introduced corrected diagram does not deduce the Modulor proportions mathematically.[16] Moreover, as it is observed in (Linton 2004: 59) the golden ratio does not play any significant role in this construction and can be equally substituted by any other number, such as $\sqrt{2}$, $\sqrt{3}$, etc.

Thus, the conclusions about the geometric foundation of the Modulor are very dubious. Let us summarize:

- Le Corbusier's geometric construction contains a mathematical error. The construction does not produce squares, and it does not solve his own Problem 6.1. Manipulations presented in the book do not provide any mathematical deduction of the Modulor, but rather represent a collection of disconnected visual experiments. In some cases these experiments are based on wrong assumptions. Le Corbusier's 'mathematical foundation' of the Modulor scale is faulty and scientifically wrong.

[14](Linton 2004: 59-63) investigates three diagrams, attributing the last one to Taton. However, I doubt that the mathematician created a diagram of his own; he may simply provide an explanation of the diagram of Maillard and Le Corbusier.

[15]See also (Fischler 1979: 100). The phrasing in the citation by André Wogenscky could also be an implicit evidence: 'The research resumed at a brisk pace after the Second World War, and it was at this time that, with the help of collaborators and as a result of slow, tentative process, the worksite grid was abandoned and the Modulor was invented' (Wogenscky 1987: 124). According to Jerzy Soltan (1987: 2), Gerald Hanning left the atelier around this time in 1945, and that could be one of the reasons that geometry was abandoned for the new direction towards anthropomorphic scales.

[16]See also Section 9.

- There is no scientific justification (nor could be any) of the Modulor sequences. It seems that mathematical transition between the three squares and sequences never existed.

8 The Significance of Faulty Mathematics

For a mathematically minded person the verdict that Le Corbusier's geometric manipulations contain mathematical errors would be the end of the story. From the point of view of a mathematician, a faulty justification of a statement crosses out any further consideration of its implications. The value of the Modulor as a mathematical tool is nil since it is based on wrong mathematics. However, it is valid to ask about the severity of this mistake. If Le Corbusier's rectangles are only 0.6% off from being squares, does this small error really matter? Unfortunately, in this case even such a small error can not be ignored. The error, it has to be said, is very basic, and it is also very large (Evans 1995: 396, remark 23).

First of all, the Modulor is said to be based upon the concept of perfect harmony and ideal composition. All of the perfectionism is shaken if from the very beginning were to be declared that a square, considered by many to be an ideal figure, would be substituted by 'almost a square'. Second, Le Corbusier clearly expressed the ambitions for his invention to be of a mathematical, scientific nature. With such commitment, mathematical mistakes are simply unacceptable, since science chooses its methods with fastidious care. Finally, recall that one of the purposes of the Modulor is to serve as a recommended set of measures. In that light, the difference between a square and 'almost a square' may be critical. We have already shown that approximations require a cautious approach. For engineers and designers the hidden error of 0.6% could possibly cause deviations in computations and unexpected obstacles in production.

There is another important question: did Le Corbusier himself understand the absurdity of his mathematical manipulations? Even avid critics of Le Corbusier attribute his mistakes to a lack of mathematical education:

> Like a medieval alchemist fixed on finding a way to turn base metal into gold, Le Corbusier had become engrossed in geometric conundrums and ended up with two Fibonacci sequences. ... Unaware of the pointlessness of what he had done, Le Corbuiser saw this as a huge achievement (Millais 2017: 119).

However, this is not exactly the case. From the very beginning Le Corbusier was aware that his construction could be seriously questioned. By the time of publication of the book he definitely knew about the main flaws of his geometric manipulations, and it was a serious concern for him.

Surprisingly, Le Corbusier honestly wrote this in the book! The architect recalled that right away Hanning had objected to the diagram[17] of his mentor with exactly the same

[17] The first proposal by Hanning in his letter 25 August 1943 (FLC B317) seems to contain a similar error within his own diagram: an inscribed angle is erroneously marked as being right angle. This indicates that Hanning's discovery of the flaws of both diagrams was not immediate.

argument as presented here (Le Corbusier 2000: I, 42; Linton 2004: 56). Later in the book Le Corbusier wrote that in 1948 doubts on mathematical deduction had risen again in his mind (Le Corbusier 2000: I, 63), at which point he asked mathematician René Taton for a consultation. The mathematician responded with a careful argument about unequal lengths of the sides of 'squares'. Le Corbusier reproduced the full letter of Taton (Le Corbusier 2000: I, 231-234; see also Loach 1998: 209), while *Modulor 2* contains the evidence that Le Corbusier received (and mainly ignored) a number of proposals for fixing the faulty logic of his deductions (Le Corbusier 2000: II, 44-48; Evans 1995: 291). The architect clearly understood the implications of Taton's answer, but nevertheless concluded that 'This answer by a mathematician may be interpreted thus: the original hypothesis (1942) is confirmed ...' (Le Corbusier 2000: I, 234).

It is often stated that in spite of lack of technical skills, Le Corbusier had a great passion for mathematics: 'Corbu was not strong in mathematics, but he was very much under its spell' (Soltan 1987: 10). I disagree with this common opinion. Le Corbusier did not appreciate mathematics as a science, a discipline that honestly seeks for an absolute truth. He was attracted by the great authority of mathematics in the same manner as he was attracted by political powers. The fact that that the statement certified by a mathematical theorem cannot be argued was more appealing to him than mathematics itself. It is also not uncommon to safeguard the controversial genius of Le Corbusier by attributing the flaws of the Modulor to incompetency of his assistants, or to obscure his errors with heavy philosophical terminology. Neither way serves the academic community well. It is known that Le Corbusier was not lavish in giving credit for successful ideas to his collaborators, many of whom later became world-renown professionals. It is not appropriate to continue to remove Le Corbuiser from the focus of the critique of his own writings. The substitution of an honest discussion of faulty science by ambiguous philosophical speculations does not cross out the existence of errors. This approach, unfortunately, erects the walls in the dialogue between art historians and mathematical scientists, who are always ready to praise the presence of their science in fine arts and architecture, but condemn any attempt to fake it.

9 Golden Ratio and Other Regulating Lines Rules

A commentary on the Modulor would not be complete without a discussion of the postulates of harmonic composition. What if someone were go back to the Modulor's origins, start from the fundamentals, do the math properly, and scientifically deduce harmonious standardization?

Unfortunately, such a project would be hopeless. The main reason for the skepticism is that the whole project stems from the myth of the exceptional role of the golden ratio in harmonious compositions, as well as other arguable statements on the effectiveness of regulating lines.

The number $\varphi = \frac{1+\sqrt{5}}{2}$ has an unusual fate in the history of modern Western civilization. From the point of view of mathematics, φ is certainly an interesting but yet not an outstandingly remarkable number; many other numbers could boast analogous or even more interesting properties. But the epic fame of this ratio lies in the belief that it is the key to achieving the harmony of proportions. Typically, one can find three types of statements on golden ratio (Gamwell 2015: 88-101):

- mathematical properties of φ;
- aesthetic significance of φ and importance in growth processes in nature;
- spiritual significance of φ.

Mathematical statements are usually formulated correctly and can be checked by rigorous mathematical methods. Statements on spiritual value of the golden ratio suggest its divine origin and metaphysical properties, and cannot be proved or disproved by any scientific methods. The main problem is contained in the assertions of the applications of the golden ratio in art and nature. Often heavily mixed with the spiritual part, these statements claim to be part of an objective scientific knowledge. As such, they should be supported by historical evidence, data and experiments, but there is an obvious lack of those. Scattered through correct mathematical statements about φ, they create a 'half-truth/half-lie', which is more difficult to object than just completely erroneous content. As Marcus Frings (2002: 20) pointed out, 'The established scientific art history only incidentally participates in those speculations, but also formulates little contradiction'. However, the problem is not that scholars do not devise careful explanations of the true and false assertions regarding φ. There exist a number of publications where the authors separate valid statements from general misconceptions, but it is hard to hear these individual skeptical voices amid the chorus of 'common facts' that are repeated in the literature without inspection. Decades of shallow writing on this subject has piled up the texts with 'evident and well-known' but wrong information[18].

It is worth mentioning that while Le Corbusier can certainly fault for knowingly publishing inaccurate mathematics of his own, he cannot be blamed for being under the spell of the myth of the golden ratio. Due to active popularization, the legend of the golden ratio has had a great influence on the art of the twentieth century. My main point is that any variation of the Modulor project by definition would constitute a scientific fiasco: regulating lines and golden numberism are unscientific speculations, and as such cannot serve for scientific applications in art and architecture.

10 Conclusions

It is useful to conclude with a few insightful quotes about Le Corbusier and his Modulor:

[18]For the details on the golden ratio myth I refer the reader to these texts written by art historians and mathematicians: (Gamwell 2015; Gardner 1994; Frascari and Volpi Ghirardini 2015; Herz-Fischler 2005; Frings 2002; Markowsky 1992).

- My own contact with Corbu led me always to think of him as a man full of boyish eagerness to try everything to win a commission, a tempting piece of work, an exciting project (Soltan 1987: 3).
- The Modulor had more to do with desire than with math (Tell 2019: 39).
- ...the second volume showed that the Modulor had by then become an instrument with which Le Corbusier tried to maintain his hegemony over postwar production... (Cohen 2014: 9)

I can only concur with the regrets of the members of the Royal Institute of British Architects on the cases when the architectural genius suffers damage from the architect being the journalist and the advertiser of himself (Pevsner 1957: 457).

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Abbreviation

FLC: Fondation Le Corbusier, http://www.fondationlecorbusier.fr.

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