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Cost - Benefit Analysis for Inland Navigation Improvements

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Brigadier General, USA
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COST-BENEFIT ANALYSIS FOR INLAND NAVIGATION IMPROVEMENTS

A Report Submitted to the

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Edited by

Leon N. Mosses & Lester B. Lave
Econometrics Center
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AN ECONOMETRIC MODEL OF THE DEMAND FOR TRANSPORTATION

A DISSERTATION

SUBMITTED TO THE GRADUATE SCHOOL

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

for the degree

DOCTOR OF PHILOSOPHY

Field of Economics

By

JAMES P. STUCKER

Evanston, Illinois
August 1969

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CHAPTER I

INTRODUCTION

Transportation planners and analysts are often faced with the problem of estimating the demand for a particular mode of transportation. Prospective transportation investments may take many forms. Small or large alterations in the characteristics of an operating mode may be under consideration. The introduction of an alternative, but proven mode into the present system may be contemplated. Or, a completely new and novel mode may have been proposed. In all of these cases the planners must be concerned with the effects of the structural change upon (1) the operation of the mode in question, (2) the operations of all other modes in the system, and (3) the producers and consumers of the products which are transported.

Implicit in the above questions are the assumptions that there is some overall demand for transportation and that the various modes compete for shares of the total traffic, i.e., that the modes are to some extent substitutes for one another. At the same time it is recognized that the modes may differ significantly in the service which they offer to the shipper. Somehow we believe these differences can be compared and we can speak of better services and, indeed, of better modes. In fact, we may even seek to determine which mode, or conceivable future mode, might constitute the optimum supply for certain transport needs.

This paper presents an economic model of a simple transportation market and attempts to deal with the above questions. We consider the complete interdependence of all elements of the system and attempt to answer the two fundamental questions: How much will be transported, and how this amount will be allocated among the competing modes.

Since the time of Alfred Marshall's analysis of the demand for knife handles economists have been aware that the demand for freight transportation is a derived demand. It is not desired for its own sake but rather because it contributes to the consumption of something which is desirable.[1] Freight transport is desirable only because it moves goods from one market to another and, in general, the people in both markets are made better off by the transfer. All evaluations of the "benefits" of transportation improvements must focus upon this flow of goods and the effects of changes in the flow on producers and consumers in the product markets. Beginning with an article by P. A. Samuelson[2] an impressive body of literature has developed dealing with these market flows. This literature focuses upon the demand side of the transportation market to almost the complete neglect of the supply side. Intermarket transport

1. In a recent article, Kelvin J. Lancaster has moved the point of ultimate demand back another step. He postulates that goods are consumed only because they possess "properties or characteristics" which yield utility. Thus, the demand for consumption goods itself is a derived demand. As we shall show this has far reaching applications in the theory of substitute goods. See K. J. Lancaster, "A New Approach to Consumer Theory," *Journal of Political Economy*, Vol. 14 (1966), 132-157.

2. Paul A. Samuelson, "Spatial Price Equilibrium and Linear Programming," *American Economic Review*, Vol. XLII, No. 2 (May 1962), 283-303.

demand is seen as being dependent upon the product supply and demand in markets and the supply of transportation between each pair of markets. However, to allocate the flows of goods among the markets it has been necessary to deal with transport supply in a very restricted manner.

Our approach will be the converse of the above. By considering only a simplified demand structure it will be possible to probe deeper into the supply characteristics of a single transport market. The former approach may be said to result in a multi-market single mode transportation model whereas ours will lead to a "two-market multi-mode transport model."

In the multi-market models the demand for transportation between any two product markets is dependent upon the product supply and demands in all markets and the supply of transportation between each pair of markets. These transport supplies are viewed as independent of the rest of the model. In the multi-mode model we will restrict ourselves to two product markets so that the demand for transportation between them is uniquely determined and we are able to investigate the interactions of transport supply when several modes are present.

Given the derived demand for transportation all we need is a "supply of transportation" to determine equilibrium in the transport market. However, this supply of transport is rather difficult to define. Supply will usually be composed of several operating modes each offering a slightly different service to the shipper. Only by considering all modes which are present, or contemplated, and all aspects of their service is it possible to determine how the demand will be met, i.e., the modal split and the resulting rates, quantities and costs.

All transport modes are designed to move products from one market to another. However, each moves the products in a slightly different manner resulting in different service characteristics. We usually say they offer differing "qualities of service." These service attributes are of concern to the shipper. He must consider all the implications of shipping by the various available modes before it is possible for him to make an intelligent choice of mode decision.

Some of the more commonly listed quality attributes are: the time required for transport; schedules and the convenience of shipping times; the reliability of schedules; breakage, spoilage and deterioration of the product enroute; packaging requirements; and interface costs on joint hauls. The main assumption of this paper is that conceptually each of these quality attributes can, and should, be expressed as costs associated with shipment by an individual mode. Each shipper must then take *all* costs into account in making his choice of mode decision, that is he must place quality costs on an equal basis with transport rates. With this assumption we shall be able to develop an analytical model of the transportation market and, subject to the form of the individual equations and the specification of parameters, derive (1) the total quantity shipped and unit costs borne by the shippers, (2) the modal split, should one occur, and the conditions under which it will occur, (3) the conditions under which a resulting modal split is an efficient, cost minimizing, allocation of traffic, and (4) the resulting transport rates and associated costs for each participating mode of transport.[3]

3. A similar approach has recently been advocated for passenger transport as well. See R. E. Quandt, and W. J. Baumol, "The Demand

With the two-market multi-mode transportation model we shall also be able to investigate how changes in the structure of the transport market will affect market solutions and modal splits whether these structural changes arise from changes in product demands or supplies, changes in transport supply by any mode, changes in the quality aspects of any mode, or the introduction of another mode into the system.

The next chapter contains a short review of the literature on the demand for freight transportation. It traces the development of the concepts contained in the multi-market single mode model and then reviews some recent attempts to estimate freight transport parameters. In Chapter III we develop the two-market multi-mode transportation model and investigate the implications of our assumptions. The model is then expanded to include one large, monopolistic-acting mode sharing a market with several competitive modes, and touches upon the role of federal regulatory agencies. In Chapter IV we convert the analytical model into an empirical model and present the results of parameter estimations. Chapter V contains a summary of our work and concluding remarks.

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for Abstract Transport Modes: Theory and Measurement," *Journal of Regional Science*, Vol. 6, No. 2, 13-26. Their approach recognizes the multidimensional nature of passenger travel and treats it as such, rather than attempting to collapse the various attributes into a single, cost, dimension. We feel that, for freight transport at least, the transformation is feasible. See also W. J. Baumol, "Calculation of Optimal Product and Retailer Characteristics: The Abstract Product Approach," *Journal of Political Economy*, Vol. 75, No. 5 (Oct. 1967), 674-685.

CHAPTER II

REVIEW OF THE LITERATURE ON FREIGHT TRANSPORTATION

In this chapter we shall discuss some of the principal articles published on the demand for freight transportation. Empirical studies will be covered as well as papers of a purely theoretical nature. While we have attempted a wide survey and believe we have covered most of the important contributions to transport demand, this review is not meant to be exhaustive. The selected theoretical works represent steps in the advancement of understanding of basic concepts and their implications; the empirical studies represent the state of the art and illustrate quite well present data limitations. We shall begin with the theoretical studies.

Theoretical Studies of Transportation

One of the earliest statements of trade equilibrium between two regions was by A. A. Cournot in 1838.[1] He asserted that when two markets or regions both produce and consume a commodity the market clearing conditions when the regions were isolated would be

(2.1) $$D_a(P_a) - S_a(P_a) = 0, \text{ and}$$
$$D_b(P_b) - S_b(P_b) = 0.$$

1. A. A. Cournot, *The Mathematical Principles of the Theory of Wealth*, (1838), Ch. X; citations are from the Irwin edition, Homewood, Ill., 1963.

Whereas if "communication" were present between the regions, and we assume that in isolation the price in Market A was lower than that of Market B by more than the transport costs, the equilibrium conditions became

(2.2) $$D_a(P_a') + D_b(P_a' + T^c) = S_a(P_a') + S_b(P_a' + T^c).$$

In the notation we are using:

A and B designate the two product markets;

P_a and P_b are product prices in the two markets;

D_a and D_b are the regional demand functions;

S_a and S_b are the regional supply functions;

P_a' is the after trade price in the exporting market; and

T^c is the unit transport cost.

Although he was not concerned with explicitly deriving the demand for transportation, Cournot's trade conditions formed, and still represent, the basic equilibrium conditions for all transportation models. These conditions are formed from the assumptions of a homogeneous commodity which is demanded and supplied under conditions of perfect competition in two separate geographic regions. When he speaks of transportation costs he is referring to the costs to the shipper, which he defines as costs to the transport merchant or industry plus some normal profit. It thus appears that the transportation industry itself is purely competitive and somehow instantaneously moves goods from market to market. This implicit treatment, or neglect, of the transport industry is a condition which we shall find present in almost all of the "transportation" models.

The first significant generalization of the Cournot two-market model seems to have been the initial development of the linear programming transportation model by Koopmans and Hitchcock.[2] In this specification there are many markets among which the commodity can flow. However, conditions of production and consumption within the product markets are now ignored. Each market is considered as either a supply point or a consumption point and the exact amount which it exports or imports is specified.[3] Unit transport costs between each pair of regions are specified and the problem is to find the transport cost minimizing flows. Thus the model generalizes the number of markets but specializes all other relationships. While Cournot said very little about transportation costs, he did not express them as being completely independent of quantity as the Koopmans-Hitchcock specification does. This specification ignores the effects of trade on the producers and consumers of the commodity being traded and concerns itself simply with determining the least cost pattern of flows which will satisfy the market requirements.[4]

2. The standard presentation of this model is found in T. Koopmans, *Activity Analysis of Production and Allocation*, Cowles Commission Monograph No. 13, John Wiley, New York, 1951.

3. This requires a balance condition relating total exports to total imports. This condition takes the place of any equilibrating actions which might be present within the product markets.

4. Another broad class of transportation-type problems has arisen in the programming field. These are mainly network problems but may appear under many different names and may focus upon commodities, markets and/or transport modes. However, in all cases the quantity to be exported or imported is specified for each region, and the transport charge for each possible link is given and fixed. Transport times may be included in the analysis and capacity constraints may or may not be imposed on each link. The problem is to minimize the total

At about this same time S. Enke was also grappling with the transportation problem. His model as it evolved was also for a world composed of many markets and one commodity. However, he hypothesized that if each region was conceived to have linear product supply and demand functions, and unit transport charges between regions were independent of volume, the (transport cost minimizing and market clearing) flows could be determined by analogy with the electrical network studies of Clerk Maxwell and Kirchhoff.[5] While his multi-market model was a maze of resistors, power sources and current flows, he was able to present a verbal discussion of the economic solution for the three-market case based upon excess supply and demand functions.[6] Thus, while Koopmans and Hitchcock were able to develop the fixed quantities case to a point where a solution was possible, Enke's contribution was to suggest an

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transport costs. These are actually allocation problems which ignore production and consumption conditions both in the product market and, essentially, in the transportation market.

In the multi-mode models it is possible for a modal split to occur. However, it must be of the "all or nothing" variety, that is either all of the traffic between two markets will be carried by one mode, or one mode will carry all it can handle before another will be brought into operation.

See, for example: K. B. Haley, "The Multi-Index Problem," *Operations Research,* Vol. 11 No. 3 (May, June), 1963, 368-379, and R. W. Lewis, E. F. Rosholdt, and W. L. Wilkinson, "A Multi-Mode Transportation Network Model," *Naval Research Logistics Quarterly,* Vol. 12, Nos. 3 & 4 (Sept., Dec.), 1965, 261-274.

5. S. Enke, "Equilibrium Among Spatially Separated Markets: Solution by Electric Analogue," *Econometrica,* Vol. 19 (Jan. 1951), 40-47.

6. For an excellent discussion of this three-market case in geometric form see: Eugene Silberberg, *The Demand for Inland Waterway Transportation,* unpublished Ph.D. dissertation, Purdue University, 1964.

artificially contrived solution method for the variable export and import case. This path was also followed by Samuelson several years later.

In 1962 Samuelson was able to take Enke's electric analogue transportation model and express it in terms of an economic maximization model.[7] He created the concept of "social pay-off" which was defined as the algebraic area under a region's excess-demand curve and showed that the act of maximizing "net social pay-off" over all regions in the model would result in trade equilibrium. By formulating his artificial maximization problem in a programming framework and heuristically sketching its solution, he was able to show that the Koopmans-Hitchcock problem was logically wholly contained within the larger problem.

Although the title of Samuelson's article is "Spatial Price Equilibrium and Linear Programming," the model which he presents is decidedly non-linear, even assuming linear product supply and demand curves. This problem specification issue is formulated very clearly in Vernon Smith's paper of 1963.[8] Smith reformulates Samuelson's problem and shows it as the dual of a general programming problem of rent minimization.

We have now seen the transportation problem presented as an electric analogue problem, a social pay-off maximization problem and a rent minimization problem. Smith seems to feel that it is intuitively clearer or purer to think of a competitive market system as one which minimizes the sum of producer and consumer rents, and this is probably true. While Samuelson's "net social pay-off" was novel, the concept of rent has been

7. Samuelson, p. 283.

8. V. L. Smith, "Minimization of Economic Rent in Spatial Price Equilibrium," *Review of Economic Studies*, Vol. XXX, No. 1 (1963), 24-31.

with us for a long time and has become a satisfying analytic abstraction. Smith's contribution was to formulate once again the transportation problem and to explicitly derive Cournot's equilibrium relations and the conditions under which they would hold.

The latest step in the development of the many-market, variable export and import, fixed transport charge model was the development of a solution algorithm in 1964 by Takayama and Judge.[9] They were able to formulate the Samuelson problem in a quadratic programming context and develop a specialization of the simplex linear programming algorithm for its solution.

We have traced the development of what we shall refer to as the many-market single-mode transportation model. It is an analysis of the production, flow and consumption of a single commodity in and between many regions or markets. Each region may both produce and consume the commodity and transportation between all pairs of regions is feasible. All transport charges are given and are independent of volume. Capacity constraints are never binding for the transporters. This model must be considered as a system served by a single mode of transport although the actual means of transportation are usually not specified, as all regions are connected by some means of transport whose only distinguishing attribute is its per unit charge.

While it is somewhat paradoxical that the analytical models of transportation flows have almost completely disregarded the supply of

9. T. Takayama, and G. Judge, "Equilibrium Among Spatially Separated Markets: A Reformulation," *Econometrica,* Vol. 32, No. 6 (Oct. 1964), 510-524.

transportation, this neglect has certainly not been present in the institutional studies of transport systems. The structures of the railroad, trucking and shipping industries have been studies in great depth and detail. Volumes have been written on the conditions of supply in these industries and concerning the nature and quality of their product. However, only recently have serious attempts been made to quantify these production relationships.[10] These quantifications are useful in their own right since they allow comparisons of transport rates and costs both independently and across modes; however, their most important application would be in the determination of transport flows if they could be integrated into the many-market transportation model to provide a complete statement of the transportation process. This paper presents an initial effort at this integration. In Chapter III we develop a two-market model which admits several modes of transport and allows each mode to exhibit unique service attributes.

Empirical Studies of Freight Transportation

We now turn from theoretical studies of the demand for freight transportation to some recent attempts to estimate this demand. The models were developed in terms of point-to-point flows of particular

10. J. S. DeSalvo, *Linehaul Process Functions for Rail and Inland Waterway Transportation*, unpublished Ph.D. dissertation, Northwestern University, 1968.

C. W. Howe, "Methods for Equipment Selection and Benefit Evaluation in Inland Waterway Transportation," *Water Resources Research*, Vol. 1 (1965), 25-39.

J. R. Meyer, M. J. Peck, J. Stenanson, and C. Zwick, *The Economics of Competition in the Transportation Industries*, Cambridge, Harvard University Press, 1960.

commodities and, conceptually, transport demands could be estimated either directly or indirectly through estimates of regional product demands and supplies. Three of the four empirical studies we reviewed were forced to abandon the point-to-point flow concept completely. Due basically to data limitations they were able to operate only with aggregate shipments, composed of many point-to-point flows of many commodities. The degree of necessary aggregation varied in the different applications; and we shall discuss them in the order of the grossness of their input data, from more aggregate to less. These studies all considered several transport modes, and two of them examined cross modal effects. The fourth study to be reviewed, by E. Silberberg, differs fundamentally from the others. He first attempts to estimate regional supplies and demands for several commodities. These regional estimates are then entered as inputs into a linear programming model to predict the flows of trade between the regions under the assumption that the competitive environment actually minimizes transport costs. Discussion of this study which, in effect, is an application of the Koopmans-Hitchcock model will conclude the chapter.

The first paper to be considered is by Benishay and Whitaker, who were concerned with estimation of national demand functions for rail, motor and water transport.[11] In particular they desired to estimate the direct or "own" price elasticity of demand for transport of each mode. By estimating a separate demand function for each of the modes and entering only one price or rate variable in each equation, that pertaining

11. H. Benishay, and G. R. Whitaker, Jr., *Demand and Supply in Freight Transportation,* The Transportation Center, Northwestern Univ., 1965.

to the mode under consideration, cross demand effects were not explicitly taken into account.

Benishay and Whitaker begin with the basic, two-market spatial competition model and suggest that the logic of this model can be extended to cover the case of many commodities and regions. This is a correct view as Takayama and Judge have shown that a multi-commodity, multi-region spatial competition model can be solved through the use of quadratic programming techniques. The possibility of this extension, however, does not justify, as Benishay and Whitaker apparently believe, the derivation of transportation demand functions by mode that aggregate all commodities and regions.

Another assumption underlying their empirical investigations appears questionable. They assert that the price elasticity of total transportation demand is probably zero in the short run, where the short run is defined as that period of time within which locational adjustments to across-the-board transportation price changes do not occur. This statement does not appear to be justified. If product demand and supply curves display their usual forms, the derived demand for transportation will slope downward more sharply than the demand curve for the product, but will seldom be a vertical line.

Equations of the following sort were estimated for each of the three modes.

(2.3) $$T_t^i = b_0 + b_1 P_t^i + b_2 I_t + b_3 U_t + b_4 Y_t$$

where

T_t^i is ton-miles per capita carried by mode i in year t,

P_t^i is total freight revenue divided by total ton-miles carried for mode i in year t, deflated by the C.P.I.,

I_t is the Index of Industrial Production for year t,

U_t is the percent of urban concentration in year t, and

Y_t represents the year of the observation, 1946 = 1.

Some of the variables and underlying data, as well as the methodology, require comment. The index of industrial production is included for obvious reasons: the larger the output of the economy the greater are likely to be the shipments of commodities by all modes as well as for each individually. The time trend variable is included as a kind of catch-all and designed to pick up the influence of omitted variables that are systematically related to time. The reason for the inclusion of a variable on urban concentration requires somewhat more explanation.

There is a broad tendency toward the regional equalization of population in the United States. That is, many of the less densely populated areas of the nation have been growing more rapidly than the already densely settled areas. At the same time there appears to be a continuing tendency towards market orientation of manufacturing industry. On the basis of these two tendencies some location specialists have concluded that there has also been a reduction in the relative importance of interregional trade in manufactured commodities and therefore their transport. Benishay and Whitaker's inclusion of the percent urbanization variable appears to be an outgrowth of this reasoning. The actual effect of this variable in the estimating equations will be commented upon shortly. At this point it should, however, be mentioned that even if the reasoning is correct so far as manufactured goods are concerned a tendency towards urbanization should have the opposite effect on shipments of primary products. These products are locationally bound and tend to occur in relatively few places. Hence, if urbanization and broad dispersal of manufacturing activity do indeed go together one might expect that the relative importance of interregional trade in primary products, and therefore the transport of these products, would increase. The authors did not carry out separate regressions for manufactured and primary products, and therefore did not determine whether the urbanization variable has such opposite effects.

The variables considered so far bear more directly on total transport demand than demand for the individual modes. We turn now to the variable used to represent the price of transport, P^i. In the authors' scheme this is the sole variable that is used to differentiate the demand for one mode of transport from the demand for another. They have data on total freight revenue and revenue ton-miles for each mode on an annual basis. By dividing the former by the latter value for each year they derive an average price or rate per ton-mile. This figure was then deflated by the Consumer Price Index to remove the purely monetary affects of overall price changes and trends. These average rates per ton-mile reflect, of course, real changes in actual freight rates in each year. Such changes are not, however, the only things that may cause the average to change. A greater increase, for example, in the shipment of goods with high freight rates in any year than in other goods will cause the average to change. Similarly, a change in the distance profile of shipments will also cause the average revenue per ton-mile to change unless total transport changes increase proportionately with distance. In short, it is not at all clear what the price figure they use in their regressions really measures. Let us comment on some variables that they did not include.

Since they had average freight rates for each of their three modes, for each of sixteen years, Benishay and Whitaker might have included all three rate variables in each demand equation. This would have provided estimates not only of the own or direct elasticity but also of the cross elasticities of transport demand. They considered this idea but rejected it as they felt the rates would be highly correlated. However, failure to include the rates for competing modes does not solve this problem. They claim that they have estimated the direct elasticity of demand for each mode since they have included in each estimating equation only the price of the mode involved. The point is, however, that the rates of the other modes were changing over the entire period of their investigation so that the changes in tons carried which they observe reflect these changes as well as changes in the rate of that particular mode. The elasticities they estimate are joint elasticities, some unknown combination of direct and cross elasticities. By experimenting with various ways of including the rate of the other modes they might have been able to separate these into direct and cross components.

Benishay and Whitaker experimented with the inclusion of terms to represent quality differences between the modes. They argued that if changes in quality are always reflected in changes in rates the quality variable need not be considered explicitly. Clearly the more important condition, as they recognize, is one where the modes engage in quality competition, that is where the quality is improved but rates remain constant and thus do not reflect the higher quality service.

Speed was the only quality variable that the authors felt they could quantify in a meaningful way. To ascertain if the modes vary the speed of their service in response to demand conditions they constructed average speed variables for both rail and motor transport. The figure for rail was derived from ICC data by dividing train-miles by train-hours to give a mile-per-hour figure. The motor averages were derived from highway checks of actual truck operating speeds. These averages were each regressed on the Index of Industrial Production and a time trend. These fits were poor so the averages were not included in the demand equations.[12]

Benishay and Whitaker present the results of nine separate regressions. For each of three modes they use (1) the actual values of the variables, (2) logarithms of the values, and (3) the first difference of the logarithmic values. Overall the results were quite good and yielded parameter estimates which were in accord with their expectations. The price coefficients in the rail and motor equations were with one exception all of negative sign and highly significant. The price coefficients for water transport were negative in two of the three cases but never significant.

The income coefficients all displayed positive signs and were all significant except for the motor first difference equation. This

12. This was a rather strange test. These regressions showed the correlations between speed and both the Index of Industrial Production and time to be low. Thus, to include the speed variables in the demand regressions would have caused no problems. The "tests" revealed little concerning the effects of speed on tons shipped whereas the later inclusion of the speed variables could have.

equation has by far the worst fit of the nine regressions with an R^2 value of .460, down from .986 in the logarithmic equation, with all three parameter estimates insignificant.

The urban concentration coefficients came out negative as expected in all but one case. However, only three of the nine estimates were statistically significant.

All in all Benishay and Whitaker's parameter estimates appear quite reasonable, most of them confirming our *a priori* expectations as to sign and size. Total ton-miles for each transport mode tend to increase with national production and to decrease with average rates as we would expect. Since their interest was in national demands individual flows were not considered; however, their reluctance to include the quality variable, speed, and rates of the competing modes is regrettable. This latter point, the investigation of the cross modal rate effects, is considered in the remaining studies.

The next article to be discussed is by A. Hurter.[13] He begins by observing that the demand for the services of inland waterway carriers depends primarily on two things: (1) the demand for the kinds of goods which are shipped on the waterways; and (2) the share of this demand carried by waterway carriers. Hurter sees his efforts as explaining the aggregate imports and exports for all regions of the United States and for the region covered by Mississippi River carriers that take place by barge.

13. A. P. Hurter, Jr., "Some Aspects of the Demand for Inland Waterway Transportation," *The Economics of Inland Waterway Transportation*, The Transportation Center, Northwestern University, 1965.

His estimating process is divided into two parts. In the first section he attempts to relate ton-miles carried by barge to rate per ton-mile and other explanatory variables. The second section deals with tons carried by barge and rail carriers. The first estimating equation relates ton-miles carried by all waterway carriers to average freight revenues per ton-mile, G.N.P. and a time trend for the period 1946 through 1962. The G.N.P. variable, measured in constant dollars, plays the role of an indicator of the real level of economic activity for the country as a whole. This relation is estimated first in log-linear form and then using the first differences of the logarithms. For both regressions the coefficients of the average revenue term, the price elasticities, are negative as expected. However, only in the log-linear equation is it significantly different from zero at -0.672. Both estimates of the income elasticity term are positive and significant at 2.360 and 3.095. In the log-linear regression the time coefficient is significant and converts into a yearly growth rate of 3.7 percent. The coefficient of determination of this equation is 0.924.

In the equations relating tons to average revenue per ton Hurter distinguishes between three types of carriers: (1) all waterway carriers; (2) rail carriers; and (3) Mississippi River carriers. For each of these he runs a series of regressions. First, tons carried are regressed on average revenue per ton and a time trend; then an income variable, G.N.P. in constant dollars, is entered. This second set of equations also contains a price variable for the competing mode. This is the rail average revenue variable in both water equations and the "all waterways price" in the rail equations. Each equation is run for each mode first in log-linear form and then in terms of first differences of the logs. Separate regressions are also run for each of the five principal AAR commodity classes.[14]

Some of the results Hurter obtains, as will be indicated below, are quite good; some, however, are peculiar. The estimates of price elasticities for the "all waterway carriers" group come out with wrong, i.e., positive, signs. Such a result suggests that as the price of barge service increases more tonnage is carried by barge. In the rail equations a large number of the cross price elasticities have the wrong, in this case negative, sign. These estimates, however, are never statistically significant. By far the best results obtained were for water carriers operating on the Mississippi River system. These will be discussed in somewhat greater detail.

For this river system he begins by determining the main commodities carried by barge. This information is summarized in Table 2.1. We see that Groups I, III and V comprise more than 98 percent of the total tons transported and total revenue earned by the system carriers. Table 2.2 summarizes Hurter's price elasticity estimates for Mississippi River carriers, and for the rail carriers for the three main commodity groups. None of the price elasticity estimates for Products of Agriculture carried by water are significantly different from zero. They do, however, all have the right sign. The parameter estimates for Products of Mines and Manufactures all have the proper sign and all but one are statistically significant.

14. The Association of American Railroads' commodity classification approved by the Interstate Commerce Commission for reporting purposes during these years recognized five principal commodity classes:

I. Products of Agriculture
II. Animals and Products
III. Products of Mines
IV. Products of Forests
V. Manufactures and Miscellaneous Products

Table 2.1

COMMODITIES CARRIED ON THE MISSISSIPPI RIVER SYSTEM
(1962)

Commodity Class		Percent Contribution to	
		Total Tons	Total Revenue
I.	Products of Agriculture	13.5	17.6
II.	Animals and Products	(0.0)	(0.0)
III.	Products of Mines	64.5	42.0
IV.	Products of Forests	(0.0)	(0.0)
V.	Manufacturers	21.5	41.0

Source: A. P. Hurter, *Some Aspects of the Demand for Inland Waterway Transportation*, p. 15.

The estimates obtained from the rail carrier equations are in a somewhat different category. Significant estimates are obtained for commodity classes I and V, but not for Products of Mines. While significant, the direct elasticity for Group V has the wrong, or a positive, sign when the regression is run in terms of logs. It does, however, come out negative when first differences of the logs are used. Half of the cross elasticities have improper signs and there appears to be no correspondence between these cross elasticities and those appearing in the water equations.

These results indicate that Hurter was able to include the price of substitute goods, the rates of the competing mode, in his demand equations without incurring the dire complications predicted by Benishay and Whitaker. In fact, his estimates were quite good. All estimated own price elasticities were negative and most cross elasticities came out positive as we would expect. Considering the grossness of his data these results are about as consistent as he could have expected. The next article we review carries these cross price investigations slightly further.

Table 2.2

HURTER'S REGRESSION PRICE COEFFICIENTS
(Estimated Elasticities)

Commodity Class	Mississippi River Carriers						Rail Carriers			
	Set One[1]		Set Two[2]				Set Two[2]			
	Own Price		Own Price		Rail Price		Own Price		Water Price	
	log	d log	log	d log	log	d log	log	d log	log	d log
Products of Agriculture	-0.341	-0.800	-0.891	-0.745	0.811	0.501	-0.528*	-0.536*	0.054	-0.273
Products of Mines	-3.957*	-3.442*	-3.402*	-3.538*	3.149*	3.702*	-0.602	-1.431	0.700	0.119
Manufactures	-2.093*	-3.138*	-0.865*	-3.023*	2.434*	0.171	1.577*	-2.162*	-1.008*	0.752

*Denotes an estimate statistically different from zero at the .05 confidence level.

1. Equation set one regresses tons carried by each mode on average revenue per ton, G.N.P. and time trend.

2. Equation set two adds the rate of the competing mode to the list of independent variables.

Source: A. P. Hurter, *Some Aspects of the Demand for Inland Waterway Transportation*, op. cit., Table 8, p. 25; Table 12, p. 32; Table 13, p. 34; Table 10, p. 29, and Table 11, p. 30.

In a recent volume on the demand for transportation,[15] E. Perle concerned himself with the relation between motor carrier and rail service and whether these are substitutable goods. He estimated the own and cross price elasticities of demand for the two modes as well as the "elasticity of substitution" between them. The main theoretical model that Perle employs to justify his empirical work is the two-market spatial competition model where two modes of transport are employed. The assumption is made that these two modes produce transport with different production functions but offer an identical quality of service. As we shall see in the next chapter this allows the solution for the transport market and, therefore, the solution for the product market, to be readily determined. A total demand for transportation is derived from the product market demand and supply functions. Then, under the assumption that each of the two-transport sectors behaves as a perfectly competitive industry, the separate supply functions are summed and an aggregate supply of transportation is obtained. The intersection of the transport demand function and the aggregate transport supply functions determines the equilibrium price of transportation and the equilibrium quantity transported from the market with the lower price to the higher priced market. Perle then contends that many of these two-market models can be aggregated to obtain a regional or national demand for transportation. We have considered this aggregation process in discussing a similar statement by Benishay and Whitaker and do not believe it to be meaningful.

15. E. D. Perle, *The Demand for Transportation: Regional and Commodity Studies in the United States*, Department of Geography, University of Chicago, (Planographed), 1964.

Perle's empirical work is concerned with the relation between the quantities carried by rail and motor carriers and their rates. He employs time series data on tons carried and freight revenues by regions of the United States and by commodity class.[16] The estimating equations take the following form.

$$\log T^m = a^m + b^m \log P^m + b^{mr} \log P^r \tag{2.4}$$

where

T^m is tons carried by motor carriers,

P^m is the motor rate, and

P^r is the rail rate.

Since the estimation is in terms of logs, b^m is the own price elasticity for motor transport and b^{mr} is the cross price elasticity. A similar estimation is carried out for rail.

$$\log T^r = a^r + b^r \log P^r + b^{rm} \log P^m \tag{2.5}$$

Perle also converts his data into ratios and estimates what he calls his modified model.

$$\log (T^m/T^r) = a + b \log (P^m/P^r) \tag{2.6}$$

In equation (2.6) b is considered to be the elasticity of substitution between motor and rail transport.

Perle estimates these equations for each commodity and region. This gives him 45 estimates for each parameter. He then performs an

16. Perle uses published ICC data grouped according to the AAR classifications discussed previously. See footnote 14.

aggregation over regions for each commodity, and over commodities for each region. Finally, he pools all commodities and all regions into one "macro" estimate. Wherever possible in the aggregation dummy variables are employed for commodity, region and year to see if they improve the equations. Since 585 separate elasticities were estimated, it is not possible to comment on individual results. Instead some general observations will be made, and the results of the three macro equations will be presented. Some of the other results are presented later and related to similar equations estimated by Hurter.

In general the motor carrier equations gave better fits than the rail equations. In addition, the modified model, equation (2.6) consistently appears to give better results than either of the other two.[17] The estimates obtained when the data for commodity groups and regions were aggregated are presented below.

	R^2
$\log T^m = 6.450 - 2.023 \log P^m + 1.554 \log P^r$	.359
$\log T^r = 6.543 - 0.979 \log P^m - 0.723 \log P^r$	.344
$\log (T^m/T^r) = 1.098 - 1.872 \log (P^m/P^r)$	.428

17. Note the relationship between Perle's two models. Dividing equation (4) by equation (5) we have

$$T^m/T^r = a^m/a^r \, P^{m\,(b^m - b^{rm})} \, P^{r\,(b^{mr} - b^r)}$$

which is equivalent to equation (6) only if

$$b = (b^m - b^{rm}) = (b^r - b^{mr}).$$

Thus, it seems that adding an additional restriction to the model increases the goodness of fit. This is only possible due to the transformation of the dependent variable. The coefficients of determination are not comparable over different dependent variables.

The estimates in the motor equation are reasonable with tonnage sensitive to changes in both the motor rate and the rail rate. Both elasticity estimates have the right sign. The rail equation, however, did not turn out quite as expected. The cross price elasticity came out negative which implies that if the rate for shipping goods by truck is raised the amount shipped by rail will decline. Such a result suggests a complementary rather than a substitution relationship between the modes.

The set of elasticity estimates presented in Table 2.3 were obtained when Perle utilized data aggregated in a manner similar to the data used by Hurter. Thus, Tables 2.2 and 2.3 can be used to crudely compare the works of Hurter and Perle.

Table 2.3

REGRESSION PRICE COEFFICIENTS
(Estimated Elasticities)

Commodity Class	Rail		Motor	
	Own Price	Motor Price	Own Price	Rail Price
Products of Agriculture	-2.187*	0.989	0.378	1.417
Products of Mines	-0.955	0.191	-2.254*	0.727
Manufactures	-1.578*	-0.583	-1.214*	0.136

*Denotes an estimate statistically significant at the .05 level.
Source: Perle, Table 8, p. 59, and Table 9, p. 59.

Silberberg states that his objective is to construct a model of the demand for barge transportation on the Mississippi River system.[18] He begins with a formulation of the two-region spatial competition model, introduces alternative modes of transport and derives a demand

18. Silberberg, *op. cit.*

for each of the modes under the assumption that they offer an identical quality of service. He also presents a geometric solution for the three-market case that is interesting and original. It, however, considers only one mode of transport.

Silberberg's empirical investigations are more closely related to the theoretical model he espouses than any of the others we have reviewed. The theoretical model treats transport of particular goods between specific supply and demand regions and his empirical work is set in the same framework. That is, he attempts to predict the cost-minimizing flows for the barge transportation of three commodities, coal, grain, and iron and steel products, between twelve districts of the Mississippi River System. This empirical procedure consists of two parts, a regression model and a linear programming transportation model. The object of the former is to estimate the annual quantity of each of the three commodities shipped by barge into or out of each of the twelve districts into which the Mississippi River System is divided. These shipments and the barge rates for the commodities between the twelve districts are then entered as inputs into three separate programming problems. These yield the region-to-region flows which satisfy the fixed demands from the available supplies at minimum total transportation cost. Then, since his model is constructed from historical data, he is able to compare (1) his predicted district barge exports and imports with the actual, (2) the predicted district to district flows based on actual exports and imports with the actual flows, and (3) the results of his complete model, predicted flows based on predicted exports and imports, with the actual flows. Each of these two parts will now be considered in somewhat greater detail.

In the regression portion of his analysis Silberberg employs equations of the following sort to estimate barge shipments from each of the twelve regions.

$$(2.7) \qquad {}_qS_i^w = b_0 + b_1\, {}_qI_i + b_2\, ({}_qR_i^r - {}_qR_i^w).$$

Here

${}_qS_i^w$ is total tons of commodity i shipped by barge from region q,

${}_qI_i$ is a measure of some economic activity to which barge shipments of commodity i are related,

${}_qR_i^r$ is the weighted average rail rate for commodity i from region q, and

${}_qR_i^w$ is the weighted average barge rate for commodity i from region q.

Similar equations were used to estimate quantities shipped by barge into each region. The major difference between the two sets of equations lies in I, the measure of related economic activity. The measures selected for the export equations were such things as production of the commodity in the region. The import equations, on the other hand, employed indices that were thought to be more revealing of consumption levels.

Both sets of equations required data on total shipments of each commodity into and out of each region. This information was obtained from statistics published by the U. S. Corps of Army Engineers for the period 1955 to 1961.[19] The equations also required data on barge and

19. Corps of Engineers, United States Department of the Army, *Waterborne Commerce of the United States*, Supplement to Part 5, 1956-1961, Domestic Inland Traffic Areas of Origin and Destination of Principal Commodities.

rail rates, and here he encountered difficulty since a large part of water traffic is exempt and need not move at published rates. Silberberg indicates, however, that talks he conducted with representatives of barge companies revealed that certain commodities which need not move at published rates did, in fact, travel at rates close to them. Coal and grains were in this category. Since iron and steel products came under ICC regulation and were required to travel at the published rates, he claims that rate data were available for all his commodities. The available rates were, however, for point-to-point shipments whereas his regions were often composed of several states and all exports from each region are aggregated.

Silberberg's solution to this problem was to select one large central city in each region as the supply or demand point for the entire region. For example, in region Twelve, the region in which the Illinois River is found, Chicago was chosen as the demand point for each of the three commodities and the supply point for iron and steel products. Peoria was chosen as the supply point for coal and grain. Published rates on shipments between main cities were used to represent region to region rates. These rates were combined with actual tonnages of each commodity shipped between the regions to obtain an average commodity rate for all shipments into or out of each region.[20] A comparable

20. For example, in constructing an export "rate" for one commodity and one year for region One he had observations on shipments from region One to each of the regions ($S^w_{1\,1}$, $S^w_{1\,2}$, ..., $S^w_{1\,12}$) and on the rates for these shipments, ($R^w_{1\,1}$, ..., $R^w_{1\,12}$). His "average rate" was then

$$R^w_i = (\sum_{j=1}^{12} S^w_{ij} R^w_{ij})/(\sum_{j=1}^{12} S^w_{ij}).$$

calculation was carried out for rail, and these weighted average barge and rail rates were then entered into the estimating equations in an attempt to isolate the partial demand for barge transportation. Some comments follow on the regression results for the individual commodity groups.

In the coal export equations, emphasizing once more that in the present context exports and imports refer only to shipments by barge, coal production was used as the index of economic activity. Coal consumption was used in the import equations. The results obtained from the export equations were rather mixed. The R^2's ranged from .56 to .99 with six of the eight being .90 or better. If, however, we focus on the structure and meaningfulness of the estimates the results are somewhat less encouraging. Five of the eight b_1 estimates are significant at the five percent level, but one of them is of the wrong or negative sign, meaning that the higher the index of related economic activity the lower the barge shipments. Somewhat more disturbing is the fact that only one of the eight b_2 estimates is significant. In other words, the variable expressing the difference between the rail and barge rates has no significant explanatory power, a result that goes contrary to theoretical expectations and industry belief. The use of average regional rates based on rates between cities is undoubtedly one source of the difficulty. Some additional comments on this latter point will be made shortly.

The coal import results were similar. The R^2's were in general somewhat lower except for region Three where it was much lower, having a value of .27. Again the estimates were quite disappointing so far as the b_2 values are concerned. It is true that five of the b_2 estimates had statistical significance; however, they were all much lower in value than in the supply equations and three of the significant estimates had the wrong, or a negative, sign.

The results for grain and iron and steel products were much the same. In the grain supply equations only two of the b_1 estimates were significant, and one of these was of negative sign. Only one of the b_2 estimates turned out significant in the grain import equations. Much the same results were found in the iron and steel equations. The only overall pattern that emerged from the equations was that the b_1 estimates fell between plus one and minus one and were not significant. The four significant b_2 estimates ranged in value from -0.069 to -0.837. Three significant b_2 estimates were obtained in the iron and steel import equations. Again, however, the problem of signs was encountered with two of the estimates being positive and the other negative.

The main impression obtained from an overall view of Silberberg's regression results is that they represent somewhat encouraging first approximations. The poorness of some of the results is surprising. One would think, for example, that the larger the output of coal in a region the greater would be the barge shipments of coal from that region. No clearcut findings of this type came out of the analysis. Still more disturbing are the mixed results obtained for the rate variable. On *a priori* grounds one would have expected that the greater the positive difference between the rail rate and the barge rate, the more tonnage would move by barge. As indicated, however, the coefficient pertaining to this variable often appeared with the wrong sign and, even when of the correct sign, was most often not statistically different from zero. We suspect that many of these difficulties arose from the use of

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flows with actual shipments.[21] The absolute difference between actual and predicted region-to-region shipments was taken, summed, and then expressed as a percent of actual total tonnage. In terms of this measure of error, the results for coal shipments were quite good, approximately 25 percent. The errors for the two remaining commodity groups were high, however, many of them approaching 100 percent.

Earlier we indicated that one of the strong arguments in favor of the Silberberg study is that the empirical formulation grew directly out of a well-defined theoretical model. Even here, however, one important shortcoming must be noted which pertains to the use of three independent programming formulations, one for each commodity group. This criticism is given despite the fact that it seems clear he could not have resolved the difficulty.

The typical transportation programming model treats transport cost between regions as constant and unresponsive to changes in the volume of shipments. Such an approach is valid under certain circumstances. If, for example, the sector of the transport industry being studied is largely regulated and rates are therefore sticky, there is no difficulty. If the industry is largely unregulated but the commodity being studied constitutes a small part of the total tonnage being moved by the mode, the use of a fixed set of rates is again not likely to introduce serious error. However, inland water transport is basically a competitive industry and, at least in the short run, probably has a rising rather than a perfectly elastic supply function. Moreover, the three commodities that Silberberg studied constitute a very large part of the total

21. This is comparison (3) as described on page 28 above.

tonnage moving by barge. Thus, it would seem that the use of constant rates is open to serious question and might account for the large errors found in the predicted interregional shipments.

This completes our review of the literature on the estimation of the demand for freight transportation. Generalizing from the specific comments we have made concerning each of the studies there appear to be three major problem areas, not counting the data limitations. These are: (1) usually an improper method of aggregating markets must be resorted to before estimation is possible; (2) modal demand curves are estimated with no direct reference to the total demand for transportation; and (3) quality differences between modes are usually completely ignored. In the following chapters we present an alternative estimation procedure which attempts to deal with these problems.

CHAPTER III

THE TWO-MARKET MULTI-MODE TRANSPORTATION MODEL

In this chapter we develop our two-market multi-mode transportation model. The concepts utilized in the model are introduced first. The next section contains a discussion of the quality-associated costs of transportation and our method of integrating these costs into the model. Finally, we present the full model in algebraic form and investigate the effects of structural changes. The appendices following the chapter are concerned with the elasticities of transport demand and the examination of a monopolistic model. In Appendix A we derive the elasticity of the total demand for transportation whereas Appendix B presents the elasticity of a modal demand. Appendix C then examines the behavior of a monopolistic transport mode facing this modal demand curve.

Geometrical Development of the Concepts

We shall begin by introducing the concepts utilized in our transportation model. The starting point is the derivation of the demand for the transportation of some commodity between two markets. A supply of transportation function is introduced to close the system and determine the equilibrium quantities and prices. The model is then expanded to include shipper incurred transportation costs other than the direct intermarket transport rate and several modes of transport. Finally, these concepts are combined in a discussion of the two-mode transportation market with quality differentials and the conditions necessary for the occurrence of a modal split.

Figures 3.1 through 3.4 illustrate the classical derivation of the demand curve for transportation for the special case in which all curves are linear.[1] In Fig. 3.1 we have the demand and supply conditions in two markets, Market A on the left and Market B on the right side, for some commodity. These markets are assumed to be isolated from each other. Under this restriction the equilibrium prices and quantities are observed as P_a and Q_a in Market A and P_b and Q_b in Market B. Since the market price in B is higher, in isolated equilibrium, than

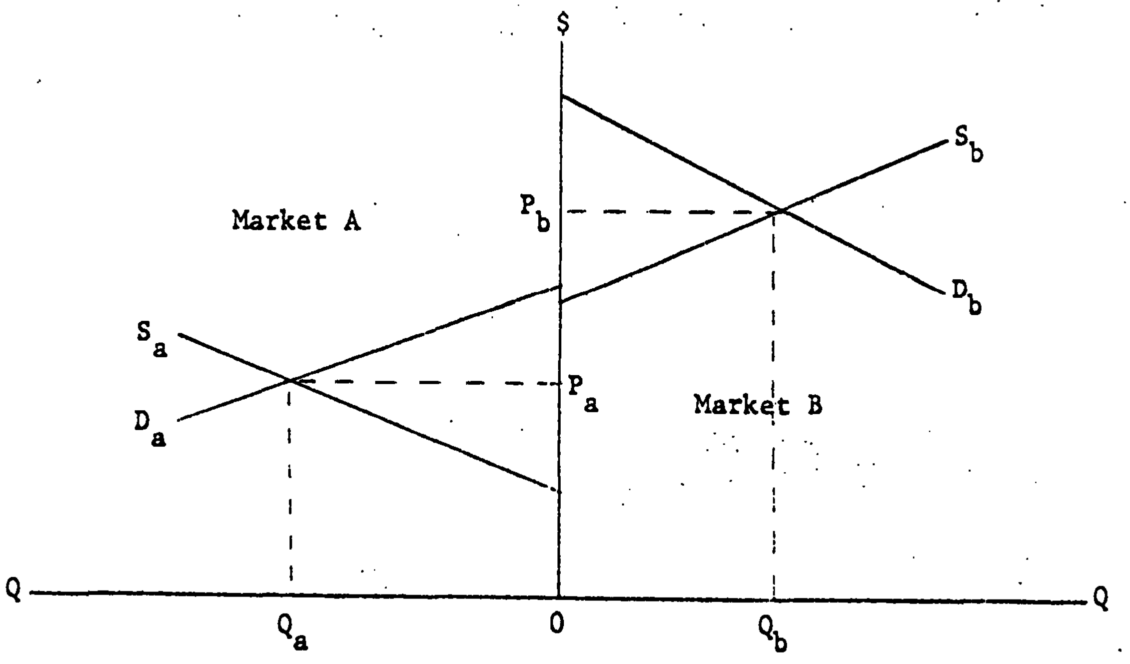

Fig. 3.1 -- Two-Market Isolated Equilibrium

1. This first set of diagrams may be found in many sources. See, for example: Samuelson, *op. cit.*, or S. Kobe, "Elasticity of Derived Demand for Transport Service," *Waseda Economic Papers*, Vol. III (1959), 48-59.

the market price in A it is obvious that if trade between the markets were allowed any movement of the commodity which occurred would be from Market A to Market B. Consequently we shall designate Market A as the exporting region and Market B as the importing region.

Figure 3.2 shows the demand in Market B for the importation of the product from Market A, Market B's excess demand curve. As before, P_b and Q_b are the equilibrium price and quantity under conditions of isolation. At prices above P_b local supply is in excess of local demand. Hence excess demand is undefined above this price. At price P_b local demand and supply are equal and excess demand is zero, as at point X. For prices below P_b, but above P_e, local demand exceeds local supply and excess demand is positive and is given by the line segment $\overline{XY}$. At prices below P_e the local supply is non-existent and the original

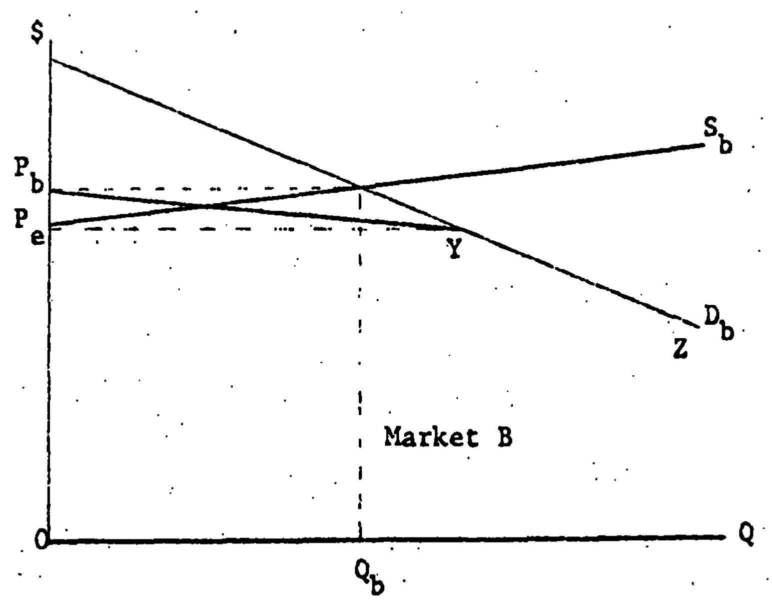

Fig. 3.2 -- Derivation of the Excess Demand Curve

demand curve is also the excess demand curve. Thus, the demand in B for A's product is given by $\overline{XYZ}$.

Figure 3.3 then illustrates the derivation of the excess supply curve for Market A. This is accomplished in a similar fashion. Subtracting the demand curve, D_a, horizontally from the supply curve, S_a, we are left with the curve $\overline{LMN}$, the excess supply curve.

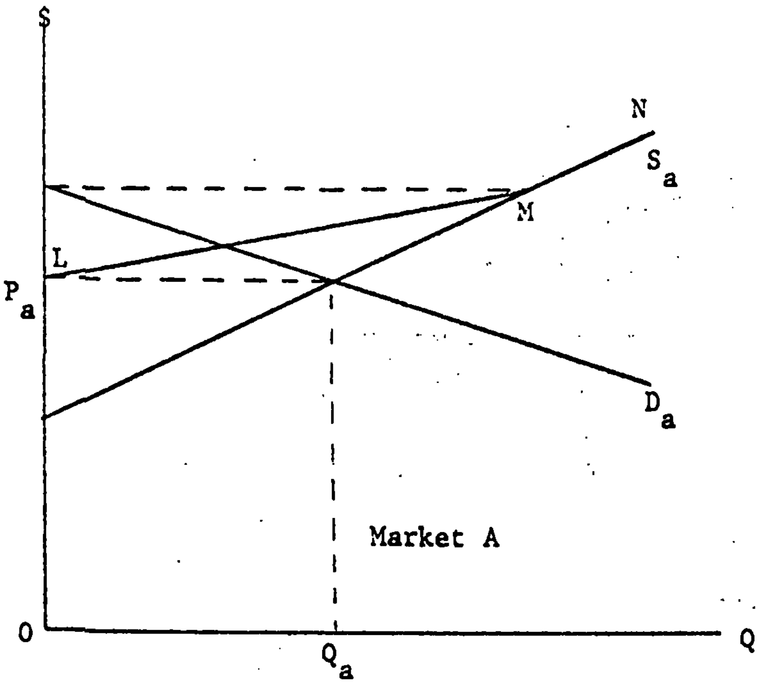

Fig. 3.3 -- Derivation of the Excess Supply Curve

Since the excess supply and demand curves we have derived represent prices and quantities of the same product we can plot them on the same diagram, as in Fig. 3.4. Here ES_a represents the excess supply curve for Market A and ED_b represents the excess demand curve for Market B. If we would now allow the commodity to flow between the markets and, by an Act of God, transportation was completely costless, we would find

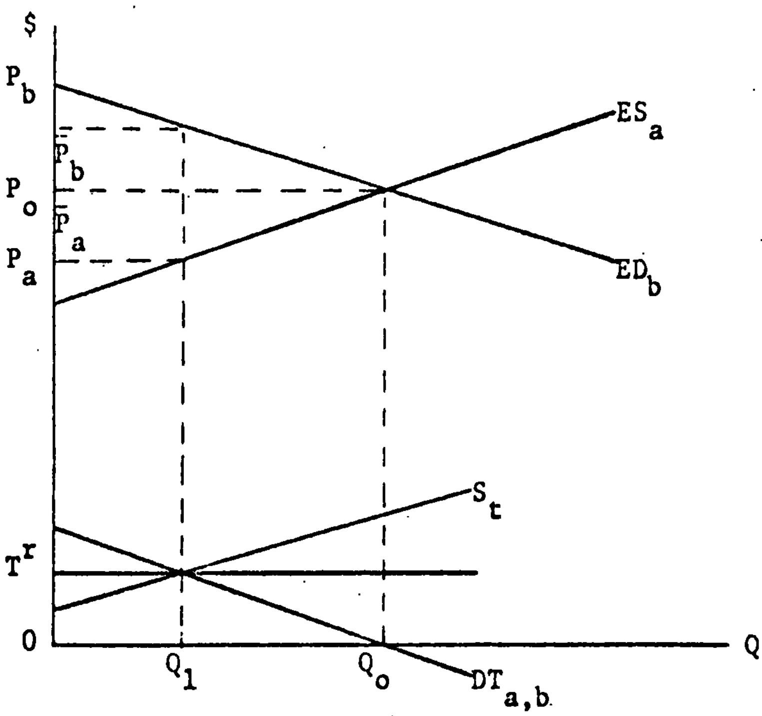

Fig. 3.4 -- Equilibrium in the Transport and Product Markets

Q_o units flowing from Market A to Market B and the prices in the two markets equalized at P_o.

However, if transport costs are not zero, how can we ascertain how much, if any, of the product will flow from Market A to Market B? Suppose that the transport cost of a unit is equal to $\bar{T}_o$, which is equal to the difference between the isolation price in Market B, P_b, and the isolation price in Market A, P_a. Then, a unit of A's product delivered in B will cost a buyer in B the supply price of the product in A plus the transport cost from A to B, T_o. Now, in the derivation of the excess supply curve for Market A we saw that for product prices up to P_a domestic demand was sufficient to dispose of the domestic supply in Market A. Hence, only to sell quantities greater than Q_a need the producers look for outside markets. However, these producers will only desire to sell more than Q_a if they receive a price higher than P_a. With a transport cost of T_o they will desire to sell in Market B only if the price they receive there is greater than $P_a + T_o = P_b$. But for prices greater than P_b, excess demand in Market B is zero. Therefore, for transport costs greater than T_o there can be no flow of product from A to B, that is, the demand for transportation from A to B must be zero. Analogous reasoning will show that for transportation costs between zero and T_o there would be a desire to ship quantities ranging from Q_o to zero. This relationship is shown in Fig. 3.4 by the curve $DT_{a,b}$. It results from the vertical subtraction of the excess supply curve from the excess demand curve and is the derived demand curve for the transportation of the product from Market A to Market B.

After having derived the demand for transportation, it is only necessary to designate a transport rate or to specify a transport supply function to close the system and determine the post-trade prices and quantities. The solutions for the transportation market, the exporting market and the importing market are achieved simultaneously. This is also shown in Fig. 3.4. Our first assumption concerning the supply side of the transportation market is that there is a single transport industry which stands ready to carry any and all quantities of the product from A to B at some constant price. This rate is represented on the diagram as T^r. The intersection of the rate line and the transport demand curve determines the quantity which will be shipped between the markets, Q_1. Returning to the upper portion of the diagram we can observe the new, post-trade, product prices in the two markets, $\bar{P}_a$ in Market A and $\bar{P}_b$ in Market B. If, instead of charging a fixed rate for all quantities, we conceive of the transport industry as having an industry supply curve, S_t, the intersection of this supply curve and the transport demand curve will determine the solution prices and quantities.

Before introducing costs associated with service attributes into the analysis it will help to take a closer look at Fig. 3.4. The excess demand curve gives the "price," the total money outlay, which consumers in B would be willing to give in order to obtain additional units of the product. The excess supply curve represents the additional quantities of the product which producers in A would be willing to supply at various "prices," dollar amounts realized by these producers. If transport were completely costless these two curves could be set equal. However, if there are any costs of transportation whatever, excess demand price cannot be equated with excess supply price, since the amount paid by the consumer is no longer the amount realized by the producer. Thus far we have been assuming that the only cost of transportation is the rate charged by the transport industry. As long as this is the case, the price to the consumer is equal to the price received by the producer plus the transport rate. This is the situation illustrated in Fig. 3.4. However, if there are costs associated with the transportation of goods from market to market other than those covered by the transport rate, this treatment also becomes inadequate. Excess demand price must then cover excess supply price plus the transport rate plus the other costs.

This new situation is illustrated in Fig. 3.5. Here the curve T^a represents the quality associated costs which the shipper must consider along with the transport rate. Equilibrium in the transportation is achieved at the intersection of the total cost of transport curve, $T^c = T^r + T^a$, with the transport demand curve. This occurs at point Z in the diagram. The quantity Q_t is then shipped from Market A to Market B at a transport *rate* of T^r and a transport *cost* to the shipper of T^c. The post-trade product prices may again be observed from the upper portion of the diagram.

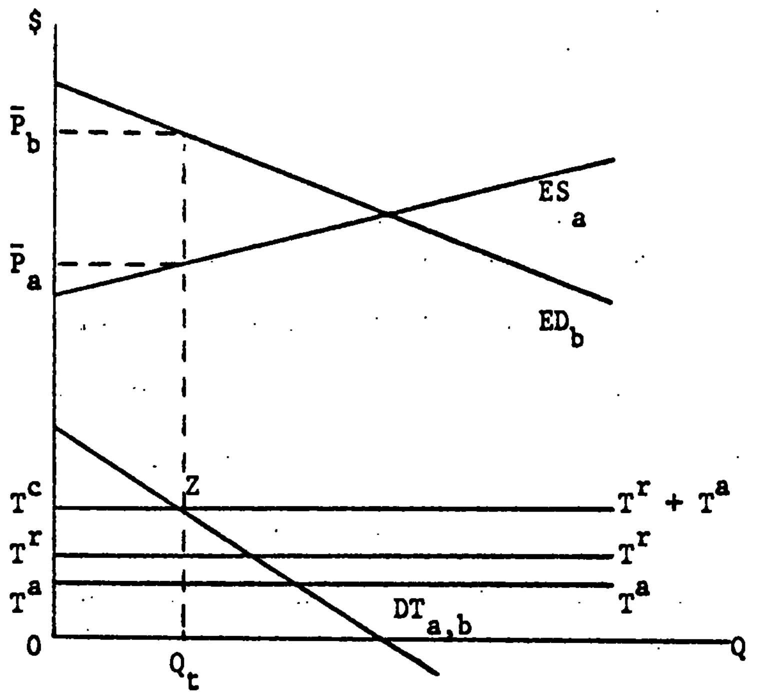

Fig. 3.5 -- One Mode Model with Quality Associated Costs

The next step in the development of our model is the introduction of an alternative mode of transportation. We first assume that quality costs are zero, i.e., perfect and instantaneous transport. This case is illustrated in Fig. 3.6 for both a constant rate situation and an increasing supply price situation. In 3.6.A Mode One charges a lower rate than Mode Two but encounters a capacity constraint at Q_1. A competitive, efficient, solution is achieved with Q_o units of the product

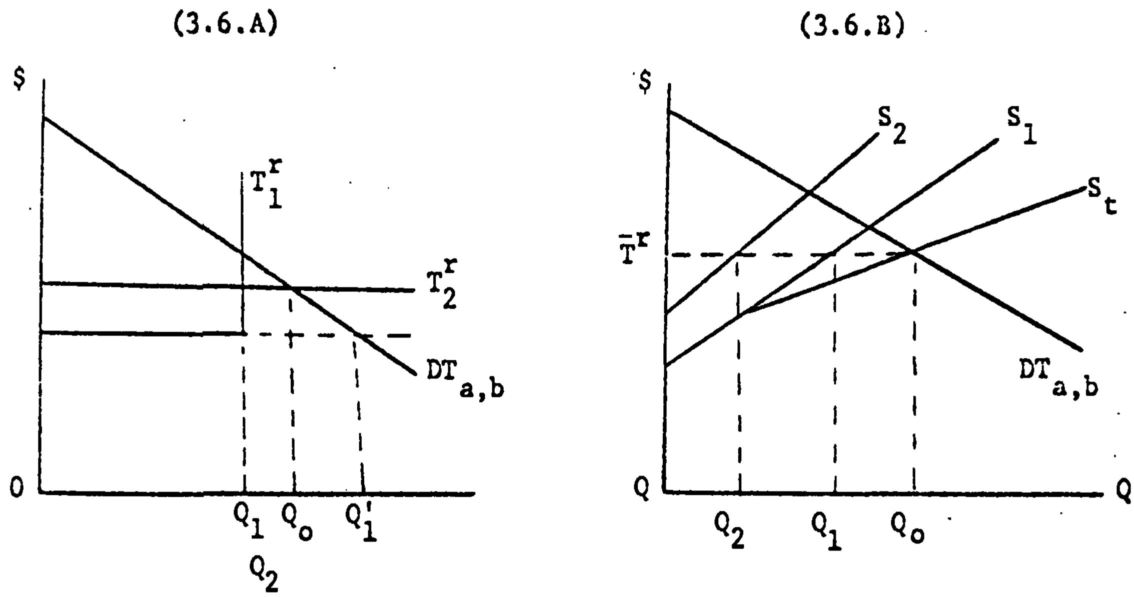

Fig. 3.6 -- Two Mode Models

being transported from Market A to Market B; Q_1 traveling by Mode One and Q_2 by Mode Two. As in most market situations this solution may be viewed as the result of an arbitrage process with each consumer (shipper) constantly striving for the least cost source of supply. It should be noted that if a capacity limit had not been imposed upon Mode One it would have carried all the traffic, in this case Q_1' and no modal-split would have occurred.

Figure 3.6.B represents a situation where transport rates are no longer assumed constant. The industry supply curves for the two modes of transportation are represented by S_1 and S_2. These curves are added horizontally to obtain a total supply of transportation which is then set equal to the demand for transportation. The competitive solution results in a traffic split of Q_1 and $Q_o - Q_1 = Q_2$, with transport rates equalized at $\bar{T}^r$. The diagram shows that a competitive modal-split will occur only if S_t is upward sloping and, at its point of intersection

with the transport demand curve, is not identical with one of the supply curves.

The culmination of our geometrical model is presented in Fig. 3.7 where we have two modes supplying transport services with differing levels of quality costs. The diagram has become rather cluttered but the method of solution should now be obvious. As the shipper is interested in all transport costs, we first sum the supply curve and the quality cost curve vertically for each mode to obtain the unit transport cost curves, T_1^c and T_2^c. Then, to find the competitive market solution, we sum these transport cost curves horizontally to obtain a "transport supply curve" and find the point where this curve intersects the transport demand curve. As Fig. 3.7 is constructed, this is at a quantity of Q_o and transport costs of $\bar{T}_c$. This yields a modal-split of Q_1 and Q_2. Then, working back to the modal supply curves, we see that the rate charged by Mode One will be $\bar{T}_1^r$ and that of Mode Two $\bar{T}_2^r$. In general these rates will not be equal. The conditions for a modal-split are, again, that S_t be rising and that its intersection with the demand for transportation occur at a point where both modes are represented.

These simple diagrams have served nicely in illustrating the development of our analysis. However, when the full model is presented, as in Fig. 3.7, the diagram becomes rather confused, even when dealing only with linear curves. After a discussion of the measure and form of our associated costs, we shall utilize mathematical methods in discussing the model and tracing the effects of shifting the various curves.

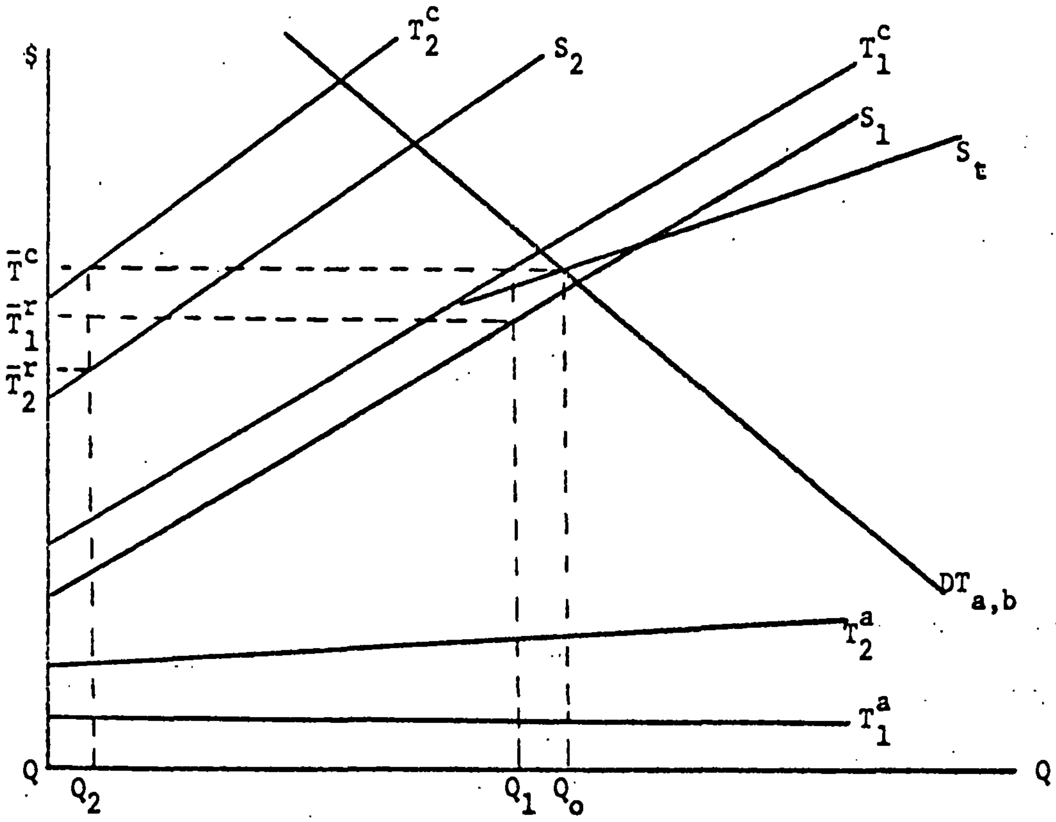

Fig. 3.7 -- Two-Mode Model with Quality Associated Costs

Transportation Associated Costs

We now take a closer look at the associated costs of transportation and the way we have entered them in our model. Markets A and B are separate points and the physical movement of goods from one market to the other requires time and effort. The effort is supplied, over time, by the transportation industry. The transport rate is reimbursement received by this industry for the effort it expends. During the time the goods are in transit from A to B they are unavailable for use. This unavailability may incur several alternative forms of economic costs, all of which we shall call simply "time costs." Another category of costs may be termed "inconvenience costs." This category is tied in very closely with the definition of the product of the transportation industry. Since both these types of associated costs must be borne by someone, we shall investigate their effects on our model. Then we shall discuss the nature of the product of the transportation industry and the interdependence between the definition of this product and the nature of the quality-associated costs. Finally, we shall discuss a third group of associated costs which might be termed "feeder and interface costs." These costs arise when transport of the product involves more than one "haul" and, perhaps, more than one mode of transportation.

Time Costs

We have defined the costs arising from the unavailability of the product during transit as time costs. These time costs may take any of several forms. We shall look at several situations which may occur in the transportation market and investigate the time costs associated with each. The framework of the single mode model is used: Market A is the exporting market; Market B is the importing area; and one transport industry serves the pair of markets.

Situation I: Consumer Ownership. We assume that consumers from Market B, in order to obtain additional units of the product, purchase these units in Market A and contract with the transportation industry to carry them to Market B where they are consumed. The sales take place in A and the purchasers are forced to await the delivery of the product to B. Time costs are thus a function of the consumer's time preference. He pays money today for goods which cannot be consumed until some time in the future.

If we assume that all consumers possess the same discount rate, i, the time costs may be expressed as iPt, where P is the price of the product and t is the number of time periods it takes to move the product from market to market. Considering the discount rate and transit time to be fixed, we see that time costs are an increasing function of the quantity of goods shipped from A to B. Additional goods can be purchased in A only at increasing prices. Hence, time costs per unit increase. Transport rates may or may not be included in the time cost calculation depending upon when they are considered to be paid. Since we have thus far considered only positively sloped supply curves for transportation, their inclusion would result in a further increase in per unit time costs as the quantity of transport increases.

In Fig. 3.8 we have illustrated these time costs, both for the case where the transport rate is included, TC_1, and where they are ignored, TC_2. Fig. 3.9 then integrates these costs into our analysis of the transportation market. Since the time costs are borne by the consumers in B, we adjust their excess demand curve accordingly. The original excess demand curve minus the time costs necessary to obtain the product yields $ED_{b,a}$, the demand in B for the product from A. This adjusted excess demand curve and the excess supply curve then determine the demand for transportation curve, D. As illustrated in Fig. 3.9, $\bar{Q}$ units of the product will be exported from A to B at a transport rate of $\bar{T}^r$. The equilibrium product price will be $\bar{P}_a$ in Market A and $\bar{P}_b$ in Market B. Importers of the good in B will pay $\bar{P}_a$ for the product in Market A, $\bar{T}^r$ for the transportation of the product from A to B; and incur waiting costs of $\overline{TC}$.

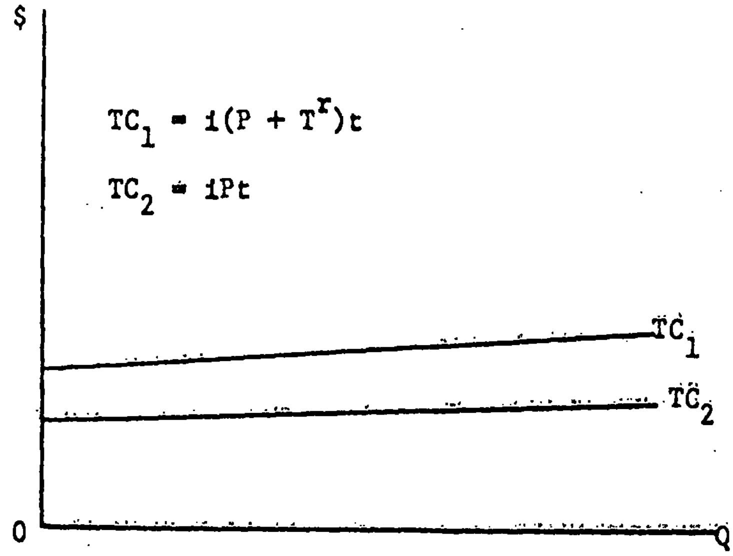

Fig. 3.8 -- Time Preference Cost Curves

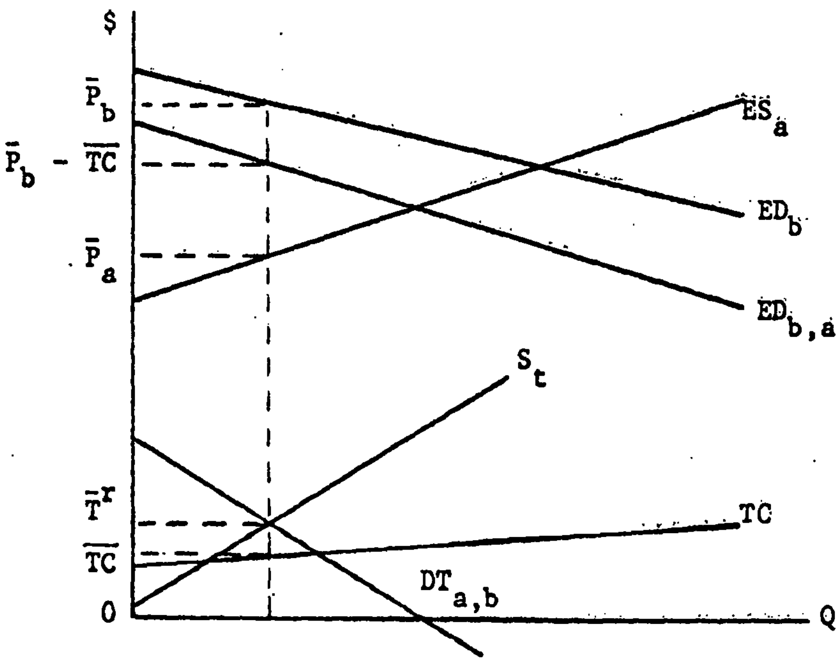

Fig. 3.9 -- One-Mode Model with Consumers' Time Preference Costs

Situation II: Producer Ownership. This situation assumes the producers retain ownership of the product until it is sold in Market B. Again time costs are incurred, but here they are borne initially by the producers and may be considered as a form of inventory costs. The suppliers invest in their product and do not receive a return until some later date. As production costs increase with quantity produced, these inventory costs will be viewed as increasing with quantity shipped. Figure 3.8 may again be used to illustrate these costs per unit if we now consider i to be the producer's interest rate or capital cost. Given a perfect capital market these costs will be identical with the time preference costs of Situation I.

Figure 3.10 integrates these inventory costs into the analysis of the transportation market. In this case the time costs are added to

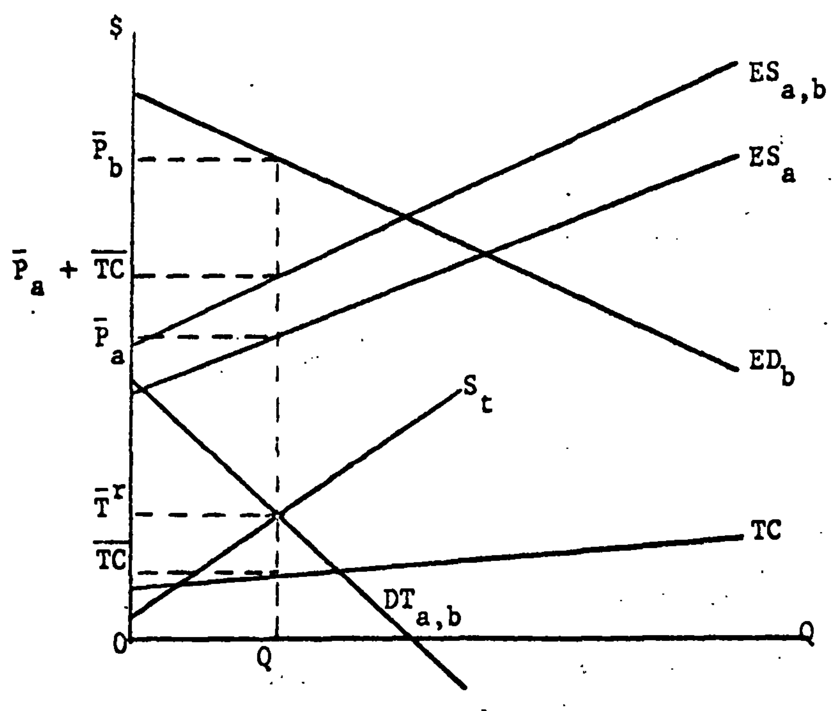

Fig. 3.10 -- Two-Market Model with Producers' Inventory Costs

supply costs to obtain the supply of A's product in B, $ES_{a,b}$. If we assume this time cost function is identical with the one utilized in Fig. 3.9, the solutions for the two cases will be the same. All that differs is the initial bearing of the time costs, and this does not affect the solution values of Q, P_a, P_b, T^r or TC.

Other Situations. At this point we could go on to discuss some other situations. Two which readily come to mind are (III) Carrier Ownership and (IV) Fourth Party Ownership. However, by now it should be obvious that no matter who the shipping agent is, he must incur a time cost during the transportation of the product. The only differences in the analysis are concerned with how we initially handle the time cost functions. In cases III and IV we would probably begin by adjusting the transportation supply curve. However, no matter how we proceed, the market solutions would be identical with Situations I and II.

Other Associated Costs and the Transportation Supply Curve

The costs which we wish to group under the term of transportation associated costs mostly stem from aspects of transportation which are usually termed the "quality of the service." In addition to transit time, two other main categories are usually considered in discussions of freight transportation. These are (1) losses due to breakage, spoilage or other deterioration of the product, and (2) uncertainty costs which may arise with respect to either category (1) or transit time.

The question we wish to address here is whether or not these associated costs will increase with the quantity shipped. Offhand we might think that some of them will and some will not. However, a little consideration will reveal that their behavior is tied in quite closely with what we consider the output of the transportation industry to be and with the shape of the transportation supply curve.

If we believe that a transportation industry must render a quite specific service for its output to be meaningfully measured, i.e., it must adhere rigidly to schedules and maintain losses and uncertainties at specified (minimum) levels no matter what level of traffic is being carried, then our associated costs will probably remain constant or perhaps even decrease as risks are spread. However, any industry which must maintain this level of quality control will almost certainly experience increasing costs and display a rising supply curve. On the other hand, if we specify the output of our industry to be only the physical movement of goods from market to market within a reasonable period of time, it is reasonable to expect the supply curve to be relatively flat but for the quality of service to decrease as greater demands are placed upon the industry.

We should also mention another phenomenon quite often encountered in transportation activities. This is the congestion which may be encountered in almost any phase of transport activities. When congestion occurs it causes both costs and transit times to increase and often can be alleviated only by quite sizeable investment projects.

Feeder Line and Interface Costs

In our analyses we have been considering the individual markets to be points in space. We now relax this assumption and attempt to move closer to a real world situation. While there may be a single railhead or truck terminal where interregional shipments originate and terminate, there are usually many other points in the region where production and consumption occur. It is common practice to have a system of intra-regional transport which links these points and performs the pick-up and delivery functions. At the exporting terminals, then, it is necessary to transfer the commodities from the intra-regional system to the interregional system; this process is then reversed at the receiving terminal.

It is to the advantage of producers and consumers of goods which are traded interregionally to be located near the transport terminals and minimize intra-regional transport costs. However, all of the economic activities cannot be located alongside the terminals; nor is it likely that all producers and consumers could be located equidistant from the terminals. Hence, in most situations where interregional trade occurs we would imagine the first units would originate from producers close to the exporting terminals and be consumed by households near the importing terminals. Then, as more product units are drawn into trade, they would be drawn from, and consumed at, points farther away.

When we attempt to measure the freight rates between regions, we are usually forced to consider only the rates for the interregional hauls. However, to fully reflect the total costs of transport, the feeder and interface costs must also be considered. In this paper we include these as another category of associated costs.

In this section we have introduced our concept of the associated costs of transportation and argued that they, like transport supply curves, would increase with quantities shipped. Analytically we have considered these costs in several fashions and demonstrated that they all result in the same solutions. However, in the next section, where we again consider two modes of transport with differing associated costs, it will be found necessary to handle these costs in a manner analogous to the transport rates. Hence, we now adopt that approach and define the total costs of transportation as the sum of the transport rate and the associated costs. This will allow us to retain a single and well-defined total demand for transportation function.

Qualitative Analysis of the Two-Mode Two-Market Model

In this section we shall present our transportation model in a more formal manner and investigate the effects of controlled shifts in the functions. We will find that we are able to place an unambiguous sign upon all displacements of equilibrium and that in many instances it is possible to make statements regarding the relative sizes of these displacements.

Assuming that a modal-split does occur our two-mode model may be formulated as

(3.1)	$Q^d = D(T^c, A, B)$	the derived total demand for transportation,
(3.2)	$Q_1 = S^1(T_1^r)$	the supply of Mode One,
(3.3)	$Q_2 = S^2(T_2^r)$	the supply of Mode Two,
(3.4)	$T_1^a = Z^1(Q_1)$	associated costs for shipment by Mode One,
(3.5)	$T_2^a = Z^2(Q_2)$	associated costs for shipment by Mode Two,
(3.6)	$T_1^c = T_1^r + T_1^a$	definitions of total transport costs,
(3.7)	$T_2^c = T_2^r + T_2^a$	

$$(3.8) \qquad Q^d = Q_1 + Q_2$$

$$(3.9) \qquad T^c = T_1^c \qquad \text{equilibrium conditions.}$$

$$(3.10) \qquad T^c = T_2^c$$

Since we are primarily interested in investigating the effects of rate and associated cost changes upon quantities and shipper costs, we first invert (3.2) and (3.3) to express rates as functions of quantities. Then the definitions and equilibrium conditions can be utilized to reduce the model to a set of five equilibrium relations in five endogenous variables.

$$(3.11) \qquad Q_1 + Q_2 = D(T^c, A, B)$$

$$(3.12) \qquad T_1^r = \xi^1(Q_1)$$

$$(3.13) \qquad T_2^r = \xi^2(Q_2)$$

$$(3.14) \qquad T^c - T_1^r = z^1(Q_1)$$

$$(3.15) \qquad T^c - T_2^r = z^2(Q_2)$$

Equations (3.11) through (3.15) can be solved for Q_1, Q_2, T^c, T_1^r and T_2^r. Knowing the values assumed by these variables, we can then utilize equations (3.6) through (3.10) to find Q, T_1^c, T_2^c, T_1^a and T_2^a.

Now, to perform our qualitative analysis we rewrite (3.11) through (3.15) adding a shift parameter to each function.

$$(3.11.1) \qquad Q_1 + Q_2 - D(T^c, \alpha_1) = 0$$

$$(3.12.1) \qquad -\xi^1(Q_1, \alpha_2) + T_1^r = 0$$

$$(3.13.1) \qquad -\xi^2(Q_2, \alpha_3) + T_2^r = 0$$

$$(3.14.1) \qquad -z^1(Q_1, \alpha_4) + T^c - T_1^r = 0$$

$$(3.15.1) \qquad -z^2(Q_2, \alpha_5) + T^c - T_2^r = 0$$

Then, to calculate the rate of change of any system variable, call it X_i, associated with a change in any parameter, call it α_j, we differentiate the system totally with respect to α_j and solve the resulting system of linear equations for $\partial X_i/\partial \alpha_j$. Performing this calculation for all X_i over all α_j and utilizing the assumptions contained in inequalities (3.16) through (3.25) it is found that we can unambiguously evaluate the signs of the resulting expressions. Table 3.1 contains the results of this analysis.

(3.16) $\dfrac{\partial D(T^c,\alpha_1)}{\partial T^c} < 0$ the demand curve is downward sloping

(3.17) $\dfrac{\partial \xi^1(Q_1,\alpha_2)}{\partial Q_1} > 0$

the supply curves are upward sloping

(3.18) $\dfrac{\partial \xi^2(Q_2,\alpha_3)}{\partial Q_2} > 0$

(3.19) $\dfrac{\partial Z^1(Q_1,\alpha_4)}{\partial Q_1} > 0$

the assoc. cost curves are upward sloping

(3.20) $\dfrac{\partial Z^2(Q_2,\alpha_5)}{\partial Q_2} > 0$

(3.21) $\dfrac{\partial D(T_1^c,\alpha_1)}{\partial \alpha_1} > 0$ the demand curve shifts to the right

(3.22) $\dfrac{\partial \xi^1(Q_1,\alpha_2)}{\partial \alpha_2} > 0$

the supply curve shifts upward

(3.23) $\dfrac{\partial \xi^2(Q_2,\alpha_3)}{\partial \alpha_3} > 0$

$$\text{(3.24)} \quad \frac{\partial Z^1(Q_1,\alpha_4)}{\partial \alpha_4} > 0$$

the assoc. cost curve shifts upward

$$\text{(3.25)} \quad \frac{\partial Z^2(Q_2,\alpha_b)}{\partial \alpha_5} > 0$$

Table 3.1

QUALITATIVE ANALYSIS OF THE TWO-MODE MODEL

Resulting From a Shift of	Direction of the Change in							
	Q	Q_1	Q_2	T^c	T_1^r	T_2^r	T_1^a	T_2^a
The demand curve, rightward	+	+	+	+	+	+	+	+
Mode One's supply, upward	-	-	+	+	+	+	-	+
Mode Two's supply, upward	-	+	-	+	+	+	+	-
Quality costs, Mode One, upward	-	-	+	+	-	+	+	+
Quality costs, Mode Two, upward	-	+	-	+	+	-	+	+

Table 3.1 shows: (1) an increase in demand will cause an increase in the solution values of all endogenous variables; (2) a shift upwards in a modal supply curve will result in a decrease in the quantity carried by that mode, total quantity shipped and quality costs for that mode, all other variables increasing in value; (3) an increase in quality costs for a mode will cause a decrease in the quantity carried by that mode, its rate and the total quantity shipped, with all other variables again increasing in value.

Furthermore, if we examine shifts of the same nature for the supply curve and quality cost curve of each mode, e.g., we compare uniform upward shifts of five cents per ton of the supply curve and the quality cost curve, we can derive the relationships expressed in Table 3.2. These are, for uniform shifts:

(T_1) The effect upon Q of an increase in the rate structure of Mode One is the same as that brought about by an increase in quality costs.

(T_2) The same is true for Mode Two's supply and associated cost curves.

(T_3) and T_4) The effect upon the quantity carried by a mode is the same whether it is brought about by a change in its supply or its quality costs.

(T_5) Two theorems are contained here. (a) Changes in the quantity carried by a mode due to changes in the structure of the other mode are symmetric, (b) whether the structural changes are concerned with rates or associated costs.

(T_6) and (T_7) Changes in shipper costs resulting from rate changes are identical with those resulting from associated cost changes.

(T_8) and (T_9) The effects of changes in rates and costs for one mode upon the solution rates and costs of the other are symmetric.

Table 3.2

FURTHER QUALITATIVE ANALYSIS OF THE TWO-MODE MODEL

Resulting From a Unit Shift of	Size of the Change in					
	Q	Q_1	Q_2	T^c	T_1^r	T_2^r
Mode One's supply, upward	T_1	T_3	T_5	T_6		T_9
Mode Two's supply, upward	T_2	T_5	T_4	T_7	T_8	
Assoc. costs, Mode One, upward	T_1	T_3	T_5	T_6		T_9
Assoc. costs, Mode Two, upward	T_2	T_5	T_4	T_7	T_8	

This completes the development of our two-mode transportation model. In Appendix A to this chapter we algebraically derive the demand for transportation curve from the product market supply and demand curves. The elasticity of this curve is then presented in terms of the original functions. Appendix B utilizes our model of the transportation market to derive the demand for the services of one of the transport modes. The elasticity of this "modal average revenue function" is then derived and expressed in terms of the other market functions. In Appendix C the model is altered slightly to allow one of the transport modes to be composed of a single firm, a monopolist. The behavior of this monopolist is then examined as he confronts the modal demand curve derived in Appendix B.

APPENDIX A TO CHAPTER THREE

The Elasticity of the Derived Demand for Transportation

We now present a more formal derivation of the demand curve for transportation. This will allow us to explicitly derive the elasticity of this transport demand and provide a firm foundation for what follows.[1]

Our basic conditions are given as:

(3.A.1) $$q_a^d = f_a^d (P_a)$$ demand in A,

(3.A.2) $$q_a^s = f_a^s (P_a)$$ supply in A,

(3.A.3) $$q_b^d = f_b^d (P_b)$$ demand in B, and

(3.A.4) $$q_b^s = f_b^s (P_b)$$ supply in B.

Then, under the assumption that the isolation price in A is less than the isolation price in B, we derive

(3.A.5) $$Q_a^s = f_a^s (P_a) - f_a^d (P_a) = F_a^s (P_a)$$ excess supply in A, and

(3.A.6) $$Q_b^d = f_b^d (P_b) - f_b^s (P_b) = F_b^d (P_b)$$ excess demand in B.

In (3.A.5) and (3.A.6) quantities are expressed as functions of price. We invert them to obtain (3.A.7) and (3.A.8)

(3.A.7) $$P_a = \varphi_a (Q_a^s)$$

(3.A.8) $$P_b = \varphi_b (Q_b^d)$$

Now, we integrate the importing and exporting markets by dropping the superscript from the Qs and subtract excess demand price from excess

1. This appendix is basically the same as Kobe's article. However, as he adds an extraneous minus sign to the final expression (our equation (3.A.25)) our conclusions will differ somewhat.

supply price to obtain the demand for transportation from A to B.

(3.A.9) $$T^c = \varphi_b(Q) - \varphi_a(Q) = \Psi(Q)$$

(3.A.10) $$Q = \Psi^{-1}(T^c) = D(T^c)$$

where T^c is the transportation cost per unit of the product and Q is the quantity transported. We desire to find the price (transport cost) elasticity of equation (3.A.10).

We define

(3.A.11) $$E_a^d = \frac{P\ f_a^{d\prime}(P)}{f_a^d(P)}$$ Elasticity of demand in A.

(3.A.12) $$E_a^s = \frac{P\ f_a^{s\prime}(P)}{f_a^s(P)}$$ Elasticity of supply in A.

(3.A.13) $$E_b^d = \frac{P\ f_b^{d\prime}(P)}{f_b^d(P)}$$ Elasticity of demand in B.

(3.A.14) $$E_b^s = \frac{P\ f_b^{s\prime}(P)}{f_b^s(P)}$$ Elasticity of supply in B.

(3.A.15) $$\eta_a^s = \frac{P\ F_a^{s\prime}(P)}{F_a^s(P)}$$ Elasticity of excess supply in A.

(3.A.16) $$\eta_b^d = \frac{P\ F_b^{d\prime}(P)}{F_b^d(P)}$$ Elasticity of excess demand in B.

These last two elasticities may also be expressed as:

(3.A.17) $$\eta_a^s = \left[\frac{Q\ \varphi_a'(Q)}{\varphi_a(Q)}\right]^{-1} \quad \text{and}$$

(3.A.18) $$\eta_b^d = \left[\frac{Q\ \varphi_b'(Q)}{\varphi_b(Q)}\right]^{-1}$$

From (3.A.9) we can express the transport rate elasticity of transport demand as

(3.A.19) $$E^d = \left[\frac{Q\ \Psi'(Q)}{\Psi(Q)}\right]^{-1} .$$

Expanding (3.A.19) by (3.A.9) it becomes

(3.A.20) $$E^d = \left[\frac{Q\ \varphi_b'(Q)}{\Psi(Q)} - \frac{Q\ \varphi_a'(Q)}{\Psi(Q)}\right]^{-1}$$

$$= \left[\frac{Q\ \varphi_b'(Q)}{\varphi_b(Q)}\ \frac{\varphi_b(Q)}{\Psi(Q)} - \frac{Q\ \varphi_a'(Q)}{\varphi_a(Q)}\ \frac{\varphi_a(Q)}{\Psi(Q)}\right]^{-1} .$$

Now, from (3.A.15) through (3.A.18) we see that

(3.A.21) $$\frac{Q\ \varphi_b'(Q)}{\varphi_b(Q)} = \left(\eta_b^d\right)^{-1} = \left[\frac{P\ F_b^{d\prime}(P)}{F_b^d(P)}\right]^{-1} \quad \text{and}$$

(3.A.22) $$\frac{Q\ \varphi_a'(Q)}{\varphi_a(Q)} = \left(\eta_a^s\right)^{-1} = \left[\frac{P\ F_a^{s\prime}(P)}{F_a^s(P)}\right]^{-1} .$$

Further expansion of the right-hand sides of (3.A.21) and (3.A.22) by (3.A.6) and (3.A.5) respectively and substitution into (3.A.20) yields

$$(3.A.23)\quad E^d = \left\{\left[\frac{P\, f_b^{d\prime}(P) - P\, f_b^{s\prime}(P)}{F_b^d(P)}\right]^{-1} \frac{\varphi_b(Q)}{\Psi(Q)} - \left[\frac{P\, f_a^{s\prime}(P) - P\, f_a^{d\prime}(P)}{F_a^s(P)}\right]^{-1} \frac{\varphi_a(Q)}{\Psi(Q)}\right\}^{-1}$$

$$= \left\{\left[\frac{P\, f_b^{d\prime}(P)}{f_b^d(P)}\, \frac{f_b^d(P)}{F_b^d(P)} - \frac{P\, f_b^{s\prime}(P)}{f_b^s(P)}\, \frac{f_b^s(P)}{F_b^d(P)}\right]^{-1} \frac{\varphi_b(Q)}{\Psi(Q)}\right.$$

$$\left. - \left[\frac{P\, f_a^{s\prime}(P)}{f_a^s(P)}\, \frac{f_a^s(P)}{F_a^s(P)} - \frac{P\, f_a^{d\prime}(P)}{f_a^d(P)}\, \frac{f_a^d(P)}{F_a^s(P)}\right]^{-1} \frac{\varphi_a(Q)}{\Psi(Q)}\right\}^{-1} .$$

Finally, using (3.A.11) through (3.A.14), (3.A.23) may be expressed as

$$(3.A.24)\quad E^d = \left\{\left[E_b^d\, \frac{f_b^d(P)}{F_b^d(P)} - E_b^s\, \frac{f_b^s(P)}{F_b^d(P)}\right]^{-1} \frac{\varphi_b(Q)}{\Psi(Q)}\right.$$

$$\left. - \left[E_a^s\, \frac{f_a^s(P)}{F_a^s(P)} - E_a^d\, \frac{f_a^d(P)}{F_a^s(P)}\right]^{-1} \frac{\varphi_a(Q)}{\Psi(Q)}\right\}^{-1}$$

or as

$$(3.A.25)\quad E^d = \left\{\left[E_b^d\, \frac{q_b^d}{Q} - E_b^s\, \frac{q_b^s}{Q}\right]^{-1} \frac{P_b}{T^c} - \left[E_a^s\, \frac{q_a^s}{Q} - E_a^d\, \frac{q_a^d}{Q}\right]^{-1} \frac{P_a}{T^c}\right\}^{-1} .$$

When expressed as in equation (3.A.25) the elasticity of transport demand is not as complex as it may at first appear. The first set of brackets, [ ], enclose the demand and supply elasticities for the importing region. Since demand elasticity is negative the two elasticities are additive. An increase in either elasticity will make the bracketed term more negative and, since it is twice inverted, increase the elasticity of transport demand. The demand elasticity is weighted by the ratio of domestic consumption in B to imports and the supply elasticity is weighted by the ratio of domestic supply to imports. Since Market B is the importing region, domestic consumption is greater than domestic supply and the demand elasticity is given the greater weight.

Within the second pair of brackets the situation is reversed. Market A is the exporting region so the supply elasticity is more heavily weighted. Again the elasticities are additive; however, here their sum is positive. The negative sign in front of the brackets causes the elasticities to affect the transport elasticity positively, i.e., an increase in the demand or supply elasticity of the exporting region will result in a higher (more negative) elasticity of transport demand.

The remaining two terms in equation (3.A.25) are the weights for the bracketed expressions. Since B will be the importing region, we know by definition that $P_b > P_a$ and the importing region's elasticities are weighted heavier than the exporting region's.

APPENDIX B TO CHAPTER THREE

The Elasticity of Mode Two's Average Revenue Curve

One way to derive the average revenue curve for Mode Two is to begin with equations (3.11) through (3.15). From this set we remove equation (3.13), the transport rate relation for Mode Two. This leaves four equations containing five variables. The solution of this reduced set for Q_2 in terms of T_2^r is the average revenue curve for Mode Two.

We now proceed to derive this average revenue curve in a cruder manner. This is necessary to provide a basis for the derivation of the elasticity of this function.

Given equations (3.1) through (3.10) we invert equations (3.2) and (3.3) and then reduce the set to the following five equations.

(3.B.1)	$Q_1 + Q_2 = D(T^c)$	demand for transportation.
(3.B.2)	$T^c = \xi^1(Q_1) + Z^1(Q_1) = \Gamma(Q_1)$	total costs of shipping by Mode 1.
(3.B.3)	$T_2^r = \xi^2(Q_2)$	supply curve for Mode 2.
(3.B.4)	$T_2^a = Z^2(Q_2)$	associated costs for Mode 2.
(3.B.5)	$T^c = T_2^r + T_2^a$	definition of total costs of shipping by Mode 2.

First, we define the inverse of (3.B.2) to be (3.B.6)

$$Q_1 = L(T^c). \tag{3.B.6}$$

Equation (3.B.6) is then subtracted from equation (3.B.1) to yield

$$Q_2 = D(T^c) - L(T^c) = \Psi(T^c) \tag{3.B.7}$$

and its inverse

(3.B.8) $$T^c = C(Q_2).$$

Now from (3.B.5) we see that $T_2^r = T^c - T_1^a$, so that

(3.B.9) $$T_2^r = C(Q_2) - Z^2(Q_2) = F(Q_2)$$

is the average revenue curve for Mode Two.

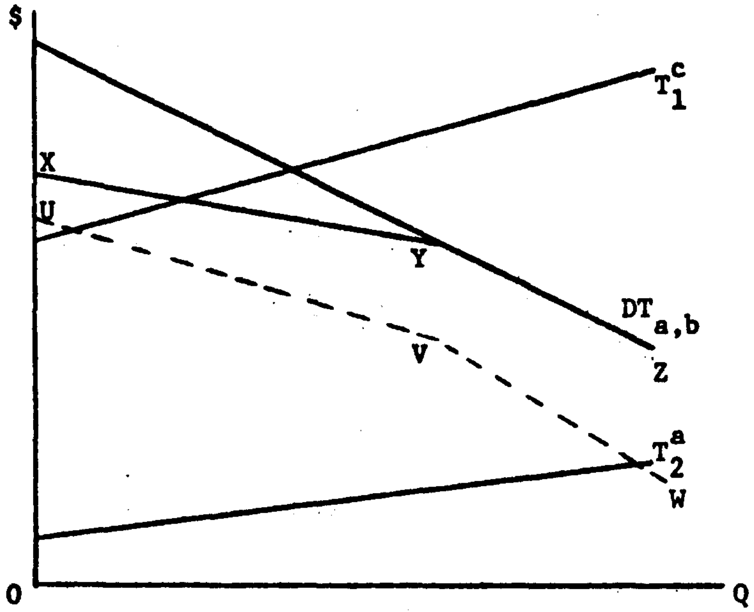

Fig. 3.B.1 -- Derivation of the Average Revenue Curve for Mode Two

This procedure is illustrated graphically in Fig. 3.B.1. Here we have the demand for transportation curve, $DT_{a,b}$, the total cost curve for Mode One, T_1^c, and the associated cost curve for Mode Two, T_2^a. The total cost function for Mode One is first subtracted horizontally from the demand curve to yield $\overline{XYZ}$. This derived curve represents the shipper's demand for transport by Mode Two. *However, since* *not all shipper costs are received as revenue by the transport industry, it is not the average revenue curve for Mode Two.* That latter curve is derived by subtracting the associated cost function vertically from the derived curve $\overline{XYZ}$ to yield $\overline{UVW}$.

In Fig. 3.B.2 we have included a supply curve for Mode Two and illustrated the market solution in the same manner as in Fig. 3.7. The average revenue curve for Mode Two is also included to illustrate that it does, indeed, show the revenue received by that mode.

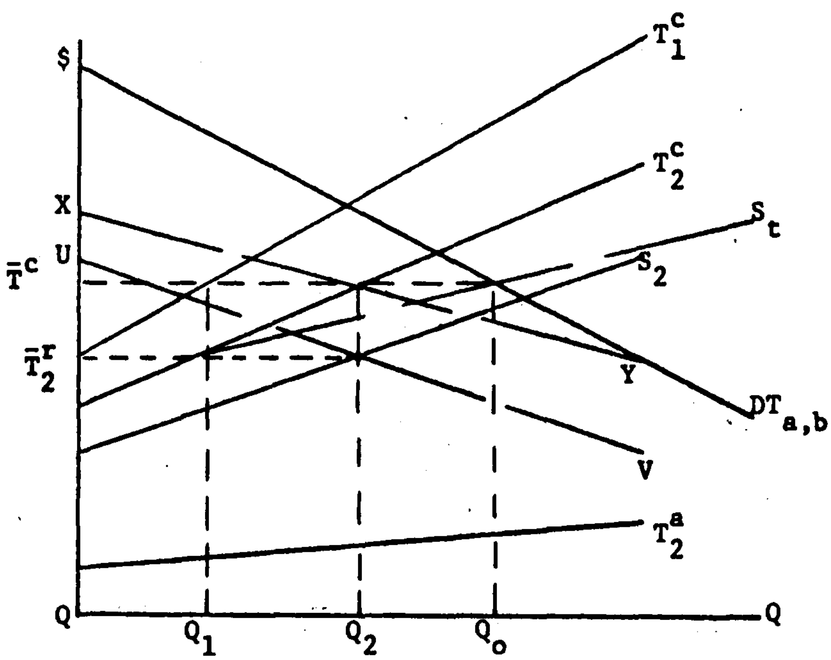

Fig. 3.B.2 -- The Two Mode Associated Cost Model

We now continue with the derivation of the elasticity of Mode Two's average revenue curve. Given the average revenue curve as defined by equation (3.B.9), its elasticity will be

$$(3.B.10)\quad E_2^d = \left|\frac{Q_2\ F'(Q_2)}{F(Q_2)}\right|^{-1} = \left|\frac{Q_2\ C'(Q_2)}{C(Q_2)}\frac{C(Q_2)}{F(Q_2)} - \frac{Q_2\ Z^{2\prime}(Q_2)}{Z^2(Q_2)}\frac{Z^2(Q_2)}{F(Q_2)}\right|^{-1}$$

Now, going back to equation (3.B.7), if we define the transport cost elasticity of this curve to be E_Ψ we see that

(3.B.11) $$E_\Psi = \frac{T^c\, \Psi'(T^c)}{\Psi(T^c)} = \left[\frac{Q_2\, C'(Q_2)}{C(Q_2)}\right]^{-1}$$ from (3.B.8).

Hence

(3.B.12) $$E_2^d = \left[(E_\Psi)^{-1}\frac{C(Q_2)}{F(Q_2)} - (E_2^Z)^{-1}\frac{Z^2(Q_2)}{F(Q_2)}\right]^{-1}$$

where E_2^Z is the elasticity of the Z^2 curve.

Now,

(3.B.13) $$E_\Psi = \frac{T^c\, \Psi'(T^c)}{\Psi(T^c)}$$

$$= \frac{T^c\, D'(T^c)}{D(T^c)}\frac{D(T^c)}{\Psi(T^c)} - \frac{T^c\, L'(T^c)}{L(T^c)}\frac{L(T^c)}{\Psi(T^c)}$$

$$= E^d\frac{D(T^c)}{\Psi(T^c)} - \frac{T^c\, L'(T^c)}{L(T^c)}\frac{L(T^c)}{\Psi(T^c)}\,.$$

Further, using (3.B.2)

(3.B.14) $$\frac{T^c\, L'(T^c)}{L(T^c)} = \left[\frac{Q\,\Gamma'(Q_1)}{\Gamma(Q_1)}\right]^{-1}$$

$$= \left[\frac{Q_1\xi^{1\prime}(Q_1)}{\xi^1(Q_1)}\frac{\xi^1(Q_1)}{\Gamma(Q_1)} + \frac{Q_1\, Z^{1\prime}(Q_1)}{Z^1(Q_1)}\frac{Z^1(Q_1)}{\Gamma(Q_1)}\right]^{-1}$$

$$= \left[(E_1^s)^{-1}\frac{\xi^1(Q_1)}{\Gamma(Q_1)} + (E_1^Z)^{-1}\frac{Z^1(Q_1)}{\Gamma(Q_1)}\right]^{-1}$$

Substituting this expression back into (3.B.13) we obtain

$$(3.B.15)\quad E_{\Psi} = E^{d}\,\frac{D(T^{c})}{\Psi(T^{c})} - \left|(E_{1}^{s})^{-1}\frac{\xi^{1}(Q_{1})}{(Q_{1})} + (E_{1}^{Z})^{-1}\,\frac{Z^{1}(Q_{1})}{\Gamma(Q_{1})}\right|^{-1}\frac{L(T^{c})}{\Psi(T^{c})}\,.$$

And, the substitution of (3.B.15) into (3.B.12) yields our result

$$(3.B.16)\quad D_{2}^{d} = \left[\left\{E^{d}\,\frac{D(T^{c})}{\Psi(T^{c})} - \left|(E_{1}^{s})^{-1}\,\frac{\xi^{1}(Q_{1})}{\Gamma(Q_{1})} + (E_{1}^{Z})^{-1}\,\frac{Z^{1}(Q_{1})}{\Gamma(Q_{1})}\right|^{-1}\frac{L(T^{c})}{\Psi(T^{c})}\right\}^{-1}\frac{C(Q_{2})}{F(Q_{2})} - (E_{2}^{z})^{-1}\,\frac{Z^{2}(Q_{2})}{F(Q_{2})}\right]^{-1}$$

$$(3.B.17)\quad E_{2}^{d} = \left[\left\{E^{d}\,\frac{Q}{Q_{2}} - \left|(E_{1}^{s})^{-1}\,\frac{T_{1}^{r}}{T^{c}} + (E_{1}^{Z})^{-1}\,\frac{T_{1}^{a}}{T^{c}}\right|^{-1}\frac{Q_{1}}{Q_{2}}\right\}^{-1}\frac{T^{c}}{T_{2}^{r}} - (E_{2}^{Z})^{-1}\,\frac{T_{2}^{a}}{T_{2}^{r}}\right]^{-1}.$$

Equation (3.B.17) relates the elasticity of Mode Two's average revenue curve to the elasticity of the demand for transportation, the elasticity of the supply and associated cost functions for Mode One, the elasticity of Mode Two's associated cost curve, the proportion of total traffic carried by Mode Two, and the proportion of total shipper costs attributable to transport rates for each mode. It is easily generalized to include more transport modes. If we assume n modes are sharing the traffic, the elasticity of Mode i's average revenue curve is

$$(3.B.18)\quad E_{i}^{d} = \left(\left[E^{d}\,\frac{Q}{Q_{i}} - \sum_{\substack{j=1\\ j\neq i}}^{n}\left\{\left[(E_{j}^{s})^{-1}\,\frac{T_{j}^{r}}{T^{c}} + (E_{j}^{Z})^{-1}\,\frac{T_{j}^{a}}{T^{c}}\right]^{-1}\frac{Q_{j}}{Q_{i}}\right\}\right]^{-1}\frac{T^{c}}{T_{i}^{r}} - (E_{i}^{Z})^{-1}\,\frac{T_{i}^{a}}{T_{i}^{r}}\right)^{-1}.$$

APPENDIX C TO CHAPTER THREE

The Two-Market Multi-Mode Transportation Model: The Monopolistic Case

Thus far our analysis of the transportation market has been limited to the study of competitive transport modes. The models were developed under the assumption of perfect intra-mode competition as well as inter-modal competition. In this section the former assumption is partially relaxed. One of the modes is allowed to consist of a single firm, a monopolist. All other modes continue to be thought of as separate competitive industries.

We begin with the simple two-mode case with no associated costs. The stability of the market is examined under various cost conditions for the monopolist, the non-optimality of the modal-split is seen, and the monopolist's reaction to changes in market conditions is studied. Transportation associated costs are then allowed in the model and, finally, we examine the behavior of a monopolist who exercises some degree of control over the associated costs of his mode.

The Two-Mode Model

We begin with the case of two supplying modes of transport. Mode One is a competitive transportation industry while Mode Two is a "monopolist." He is a monopolist in that he is the only producer of his particular type of transportation. Nevertheless, he is in direct competition with the other transportation industry. This monopolist is fully aware of all market conditions and we assume that he is large enough to greatly affect the market by his actions. Probably his scale of operations is at least as large as that of the entire industry comprising Mode One.

In Fig. 3.C.1 we have a demand for transportation curve, DD, and a derived demand for the services of Mode Two, D_2D_2.[1] Since there are no associated costs, shippers base their mode choice decision on supply price alone. Mode One is composed of price takers whereas the monopolistic Mode Two is a price setter. He attempts to maximize his profits and is assumed to act in a straightforward manner, employing no devious strategies.

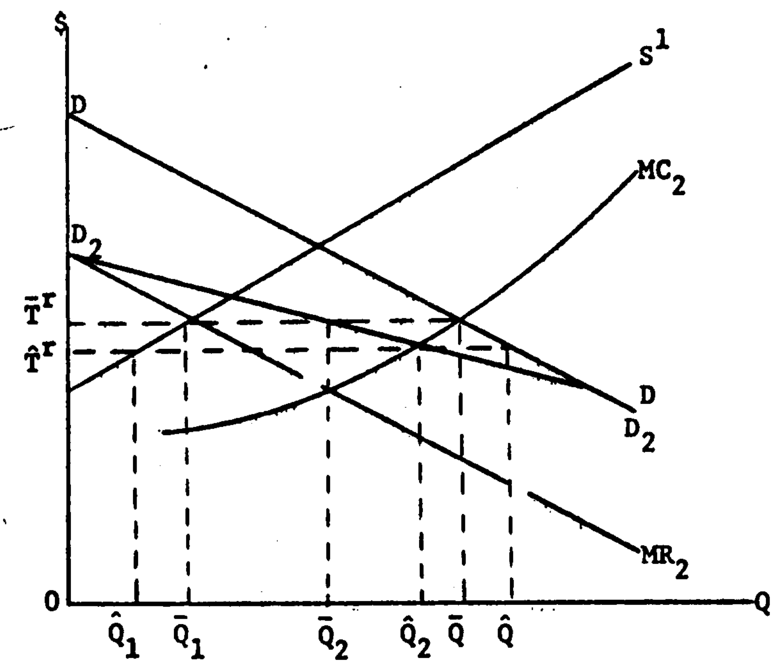

Fig. 3.C.1 -- The Simple Monopolistic Model

Monopolistic profits will be at a maximum when marginal revenue is equated with marginal cost. Hence the monopolist will set his transport rate at $\bar{T}^r$. Charging this price he will attract $\bar{Q}_2$ units of

1. See Appendix B for the derivation of this curve, especially, Fig. 3.B.1, p. 65.

product to be transported, $\bar{Q}_1$ units will travel by Mode One and demand will be satisfied.

Figure 3.C.1 also brings out the social non-optimality of this system. This is evident in two respects. First, the existing modal split does not minimize total transport costs; second, total output in the transportation industry(s) is restricted. Both of these conditions arise from the profit maximizing behavior of the monopolist. If he were forced to be a marginal cost pricer they would be eliminated. Under marginal cost pricing the monopolist Mode Two would carry $\hat{Q}_2$, charging a rate of $\hat{T}^r$. This would allow Mode One to capture only $\hat{Q}_1$ although total traffic would increase to $\hat{Q}$ units. Thus, if he priced according to marginal cost the monopolist would capture a larger share of the market, even though his total profit would be reduced.

In Fig. 3.C.2 we have illustrated an even more interesting case. Here the monopolist is assumed to display falling long run marginal costs. Profit maximization requires that he set a rate of $\bar{T}^r$ and bring about the modal split $\bar{Q}_1$, $\bar{Q}_2$. However, it is in the best interests of society to disband Mode One and revert to a true monopolistic situation. Enforcement of marginal cost pricing would then result in the monopolist carrying all of $\hat{Q}$ at a rate of $\hat{T}^r$. The monopolistic solutions are stable if marginal cost cuts marginal revenue from below and profits are positive. The social solutions require regulation, enforcement and, if profits are negative, subsidies.

Parametric Associated Costs

Here we examine the two-mode model where one mode is composed of a single firm and there are costs other than the transport rate

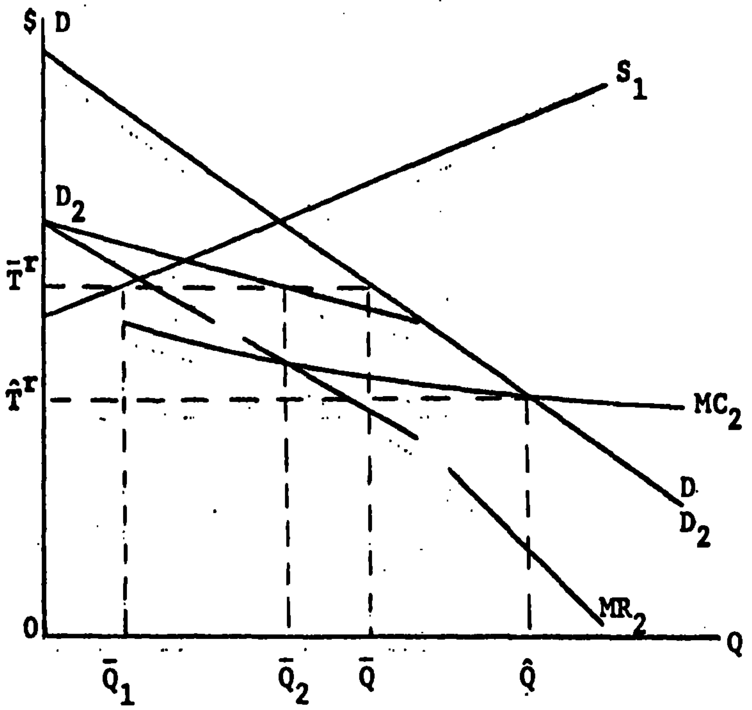

Fig. 3.C.2 -- The Monopolistic Model with Falling Marginal Costs

associated with shipment by each mode. The carriers are assumed to exercise no control over these other costs.

The average revenue curve facing Mode Two was derived in Appendix B as

$$T_2^r = C(Q_2) - Z^2(Q_2) = F(Q_2). \tag{3.C.1}$$

Instead of an industry supply curve for Mode Two we now have the monopolist's cost curve which we specify as

$$Y = \theta(Q_2) \tag{3.C.2}$$

where Y is Mode Two's total cost of carrying Q_2 units of the product from Market A to Market B.

The monopolist's profit function is then

(3.C.3) $$\Pi = Q_2 T_2^r - Y = Q_2 F(Q_2) - \theta(Q_2),$$

and his profit maximizing quantity, $\bar{Q}_2$, is given by the solution of

(3.C.4) $$\frac{d\Pi}{dQ_2} = F(Q_2) + Q_2 F'(Q_2) - \theta'(Q_2),$$

(marginal revenue equals marginal cost), so long as

(3.C.5) $$\frac{d^2\Pi}{dQ_2^2} = 2 F'(Q_2) + Q_2 F''(Q_2) - \theta''(Q_2) < 0.$$

The rate which he must charge to carry this quantity is then given by

(3.C.6) $$\bar{T}_2^r = F(\bar{Q}_2).$$

To investigate how we would expect the monopolist to react to a change in any of the underlying functions we again add our shift parameter, α, to equation (3.C.1). The equilibrium relationship (3.C.4) then becomes

$$F(Q_2,\alpha) + Q_2 F_{Q_2}(Q_2,\alpha) - \theta'(Q_2) = 0.$$

Differentiating (3.C.7) totally with respect to α yields

(3.C.8) $$[2 F_{Q_2}(Q_2,\alpha) + Q_2 F_{Q_2,Q_2}(Q_2,\alpha) - \theta''(Q_2)] \frac{dQ_2}{d\alpha}$$

$$= -[F_\alpha(Q_2,\alpha) + Q_2 F_{Q_2,\alpha}(Q_2,\alpha)].$$

Or, defining the first bracketed expression in (3.C.8) as W

(3.C.9) $$\frac{dQ_2}{d\alpha} = -\frac{1}{W}[F_\alpha(Q_2,\alpha) + Q_2 F_{Q_2,\alpha}(Q_2,\alpha)].$$

Now, W was assumed to be negative in (3.C.5) so that

$$\text{(3.C.10)} \qquad \frac{dQ_2}{d\alpha} \gtrless 0 \quad \text{as} \quad F\,(Q_2,\alpha) + Q_2\, F_{Q_2,\alpha}\,(Q_2,\alpha) \gtrless 0.$$

Looking at (3.C.10), we would require $F_\alpha(Q_2,\alpha)$ to be positive by definition. Q_2 is also required to be non-negative. Thus the sign of $\frac{dQ_2}{d\alpha}$ will depend upon the sign of $F_{Q_2,\alpha}(Q_2,\alpha)$ and the weight attached to it. Only if this term is negative and $Q_2\, F_{Q_2,\alpha}(Q_2,\alpha) > -F_\alpha(Q_2,\alpha)$ will $\frac{dQ_2}{d\alpha}$ be negative. That is, if the shift of the average revenue curve does not affect the slope of the curve or if the slope is increased by the shift, the equilibrium quantity transported will increase. If the slope is decreased, and if the reduction in revenue due to the slope shift outweights the increase in revenue due to the upward shift of the curve, quantity transported will fall. As the average revenue curve must be downward sloping, $\frac{dT^r}{d\alpha}$ will always differ in sign from $\frac{dQ_2}{d\alpha}$.

Endogenous Associated Costs for the Monopolistic Mode

Finally, we assume that the monopolistic mode has some control over its associated cost function. There is a quality of service variable, λ, which costs the firm money. However, as the value of this variable is increased, the quality of service of the monopolistic mode increases, and the associated costs decrease. We have

(3.C.11) $T_2^r = C(Q_2,\alpha) - Z^2(Q_2,\lambda)$ Mode Two's average revenue function

(3.C.12) $Y = \theta(Q_2,\lambda)$ Mode Two's total cost function

The firm's profit function is thus

$$\text{(3.C.13)} \qquad \Pi = Q_2\,[C(Q_2,\alpha) - Z^2(Q_2,\lambda)] - \theta(Q_2,\lambda).$$

To obtain the greatest profits, the monopolist will maximize this function with respect to both Q_2 and λ. His profit maximizing conditions are

$$\text{(3.C.14)} \qquad \frac{\partial \Pi}{\partial Q_2} = C(Q_2,\alpha) + Q_2\, C_{Q_2}(Q_2,\alpha) - Z^2(Q_2,\lambda) - Q_2\, Z^2_{Q_2}(Q_2,\lambda) - \theta_{Q_2}(Q_2,\lambda) = 0$$

and

$$\text{(3.C.15)} \qquad \frac{\partial Q}{\partial \lambda} = -Q_2\, Z^2_{\lambda}(Q_2,\lambda) - \theta_{\lambda}(Q_2,\lambda) = 0.$$

Defining

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial^2 \Pi}{\partial Q_2^2} & \dfrac{\partial^2 \Pi}{\partial Q_2 \partial \lambda} \\ \dfrac{\partial^2 \Pi}{\partial \lambda \partial Q_2} & \dfrac{\partial^2 \Pi}{\partial \lambda^2} \end{bmatrix},$$

the sufficiency conditions for profit maximization are that

$$\text{(3.C.16)} \qquad B_{11},\ B_{22} > 0 \quad \text{and} \quad |B| < 0.$$

To investigate the effects of a shift in the average revenue curve upon the profit maximizing values of Q_2 and λ, we differentiate (3.C.14) and (3.C.15) totally with respect to α to obtain

$$\text{(3.C.17)} \qquad \left[2C_{Q_2} - 2Z^2_{Q_2} + Q_2\, C_{Q_2,Q_2} - Q_2\, Z^2_{Q_2,Q_2} - \theta_{Q_2,Q_2}\right]\frac{\partial Q_2}{\partial \alpha}$$

$$\left[- Z^2_{\lambda} + Q_2\, Z^2_{Q_2,\lambda} + \theta_{Q_2,\lambda}\right]\frac{\partial \lambda}{\partial \alpha} = -\left[C_{\alpha} + C_{Q_2,\alpha}\right]$$

(3.C.18)
$$- \left| Z^2_{\lambda} + Q_2 \, Z^2_{\lambda,Q_2} + \theta_{\lambda,Q_2} \right| \frac{\partial Q_2}{\partial \alpha}$$

$$- \left| Q_2 \, Z^2_{\lambda,\lambda} + \theta_{\lambda,\lambda} \right| \frac{\partial \lambda}{\partial \alpha} = 0$$

Equations (3.C.17) and (3.C.18) are linear in the two unknowns and may be solved for $\frac{\partial Q_2}{\partial \alpha}$ and $\frac{\partial \lambda}{\partial \alpha}$. It can be shown that the first bracketed expression in (3.C.17) is equivalent to B_{11}; the second bracketed expression in (3.C.17) and the first bracketed expression in (3.C.18) are equivalent to $B_{12} \equiv B_{21}$; and the second bracketed expression in (3.C.18) is equivalent to B_{22}. Hence, we may rewrite these two equations as

(3.C.19)
$$B_{11} \frac{\partial Q_2}{\partial \alpha} + B_{12} \frac{\partial \lambda}{\partial \alpha} = - \left| C_{\alpha} + Q_2 \, C_{Q_2,\alpha} \right|$$

(3.C.20)
$$B_{21} \frac{\partial Q_2}{\partial \alpha} + B_{22} \frac{\partial \lambda}{\partial \alpha} = 0.$$

Solution by Cramer's Rule yields

(3.C.21)
$$\frac{\partial Q_2}{\partial \alpha} = - \frac{B_{22} \left| C_{\alpha} + Q_2 \, C_{Q_2,\alpha} \right|}{|B|}$$

(3.C.22)
$$\frac{\partial \lambda}{\partial \alpha} = \frac{B_{21} \left| C_{\alpha} + Q_2 \, C_{Q_2,\alpha} \right|}{|B|}$$

The expression $\left| C_{\alpha} + Q_2 \, C_{Q_2,\alpha} \right|$ is analogous to the expression $\left| F_{\alpha} + Q_2 \, F_{Q_2,\alpha} \right|$ found in (3.C.9). Here we are shifting only the "C" curve, whereas before the "$C - Z^2 = F$" curve was shifted. This expression will again directly determine the sign of $\frac{\partial Q_2}{\partial \alpha}$, as $B_{22} > 0$ and $|B| < 0$.

The sign of $\frac{\partial \lambda}{\partial \alpha}$ is only slightly more difficult to determine. The sign of $B_{21} \equiv B_{12} = -\left| Z_{\lambda}^{2} + Q_2 \; Z_{Q_2,\lambda}^{2} + \theta_{Q_2,\lambda} \right|$ may be either positive or negative. The first two terms of this expression represent the marginal increment to profits which could be brought about by increasing the quality of service. The final term represents the marginal cost of this prospective increase. (Assuming for the moment that $\left| C_{\alpha} + Q_2 \; C_{Q_2,\alpha} \right|$ is positive) if MR > MC, λ will be increased; whereas if MR < MC, λ will be decreased.

CHAPTER IV

ESTIMATION OF THE DEMAND FOR TRANSPORTATION

This chapter contains the results of our empirical investigations. As we are primarily interested in transport by rail, barge and motor, we first expand the work of Chapter III into a three-mode model. This model is then converted into suitable form for estimation and and estimating equations are derived. The data are then described and the empirical results discussed.

The Estimating Model

The three-mode model may be expressed as

(4.1)	$Q^d = D(T^c)$	Demand for transportation;
(4.2)	$T_1^r = \xi_1(Q_1)$	Transport rate by Mode One;
(4.3)	$T_2^r = \xi_2(Q_2)$	Transport rate by Mode Two;
(4.4)	$T_3^r = \xi_3(Q_3)$	Transport rate by Mode Three;
(4.5)	$T_1^a = Z_1(Q_1)$	Associated costs for Mode One;
(4.6)	$T_2^a = Z_2(Q_2)$	Associated costs for Mode Two;
(4.7)	$T_3^a = Z_3(Q_3)$	Associated costs for Mode Three;
(4.8)	$T_1^1 = T_1^r + T_1^a$	Total costs of shipping by Mode One;
(4.9)	$T_2^c = T_2^r + T_2^a$	Total costs of shipping by Mode Two;
(4.10)	$T_3^c = T_3^r + T_3^a$	Total costs of shipping by Mode Three;

$$(4.11)\quad T^c = T_1^c$$

$$(4.12)\quad T^c = T_2^c$$

$$(4.13)\quad T_c = T_3^c$$

$$(4.14)\quad Q^d = Q_1 + Q_2 + Q_3$$

Equilibrium conditions

The following definitions continue to apply:

$Q^d \equiv$ quantity of transport demanded;

$T^c \equiv$ total costs, to the shipper, of transportation;

$T_i^c \equiv$ total costs of shipping by Mode i, (i = 1, 2, 3);

$Q_i \equiv$ quantity transported by Mode i, (i = 1, 2, 3);

$T_i^r \equiv$ transport rate, per ton, charged by Mode i, (i = 1, 2, 3);

$T_i^a \equiv$ other shipping costs, borne by the shipper, associated with shipping by Mode i, (i = 1, 2, 3).

Now, the theoretical model has been developed for the case where one specific market ships to only one other market. However, data to fit such a model are unavailable at the present time. Any estimation attempts must utilize data of a cross-sectional nature, i.e., shipments between many pairs of markets during the same time period. To use data of this type it is necessary to add several new variables to the structural equations which explicitly allow for the differences in market size and the distances between the markets. We define

A ≡ an indicator of the economic size of a city (O-D) pair, and

M ≡ the distance, in miles, between a city pair.

The size indicator, A, is inserted in equation (4.1) as an exogenous variable and the distance variable, M, is included in equations (4.2) through (4.7) in a like manner.

We now substitute equations (4.8) through (4.14) into equations (4.1) through (4.7) to eliminate Q^d, T_i^c and T_i^a (i = 1, 2, 3). The reduced equilibrium system then appears as

$$(4.15) \qquad Q_1 + Q_2 + Q_3 = D(T^c, A)$$

$$(4.16) \qquad T_1^r = \xi_1(Q_1, M)$$

$$(4.17) \qquad T_2^r = \xi_2(Q_2, M)$$

$$(4.18) \qquad T_3^r = \xi_3(Q_3, M)$$

$$(4.19) \qquad T^c - T_1^r = Z_1(Q_1, M)$$

$$(4.20) \qquad T^c - T_2^r = Z_2(Q_2, M)$$

$$(4.21) \qquad T^c - T_3^r = Z_3(Q_3, M)$$

Two specifications are now imposed upon the system.

1. The equations are linear in all included variables.
2. The Q_i do not appear in equations (4.16), (4.17) and (4.18)

Specification 1 is necessary due to the nature of the model and the estimation problems involved. The second specification recognizes that rates for the commodities we are concerned with are fixed by law and that the regulatory process may cause the process of rate adjustment to be both long and complex. As will be seen below, it also allows the empirical work to be separated into two distinct parts.

With these specifications the model may be written as

$$(4.22) \qquad Q_1 + Q_2 + Q_3 = \alpha + \beta\, T^c + \gamma\, A$$

$$(4.23) \qquad T^c - T_1^r = a_1 + b_1\, Q_1 + c_1\, M$$

$$(4.24) \qquad T^c - T_2^r = a_2 + b_2\, Q_2 + c_2\, M$$

$$(4.25) \qquad T^c - T_3^r = a_3 + b_3\, Q_3 + c_3\, M$$

(4.26) $$T_1^r = g_1 + h_1 M$$

(4.27) $$T_2^r = g_2 + h_2 M$$

(4.28) $$T_3^r = g_3 + h_3 M$$

In this formulation we see that equations (4.26), (4.27) and (4.28) are each a self-contained sub-set of order zero and may be estimated separately and independently.[1]

Equations (4.22) through (4.25) then constitute a derived structure of the first order. Using (4.26), (4.27) and (4.28) to eliminate the rate variables, they become

(4.29) $$Q_1 + Q_2 + Q_3 - \beta T^c = \alpha + \gamma A$$

(4.30) $$-b_1 Q_1 + T^c = (a_1 + g_1) + (c_1 + h_1) M$$

(4.31) $$-b_2 Q_2 + T^c = (a_2 + g_2) + (c_2 + h_2) M$$

(4.32) $$-b_3 Q_3 + T^c = (a_3 + g_3) + (c_3 + h_3) M$$

The following sections deal with our empirical investigations of these two sets of estimating equations.

Empirical Results -- The Rate Equations

Having derived the estimating equations, we now turn to our empirical work. In this section we shall deal with equations (4.26), (4.27) and (4.28), the transport rate equations. The following section contains our discussion of the demand equations. As stated previously,

1. The terminology and procedure is taken from H. A. Simon, "Causal Ordering and Identifiability," Chapter III of W. C. Hood and T. C. Koopmans, *Studies in Econometric Method,* Cowles Commission Monograph No. 14, Wiley, New York, 1952.

we are interested in three modes of transport, rail, water and motor. The modes are discussed in that order.

Rail Transport

We begin with our investigation of point-to-point rail rates. Utilizing cross-sectional data obtained from the 1963 Carload Waybill Statistics published by the Interstate Commerce Commission, we estimate regression equations linking the rail rate, for commodity classes and commodity groups, to a small number of variables representing the characteristics of the commodities and shipments.

The Data. The Carload Waybill Statistics are published annually and contain data derived from a one-percent sample of audited revenue waybills terminated by Class I railroads. This information is processed and presented in a number of formats. The data utilized in this study come primarily from the State-to-State Distribution Series.[2] In this series data are presented on total annual shipments by commodity and by state of origin and state of destination. For each shipment information is given on carloads, tons, revenue, short-line ton-miles, short-line car-miles, average tons per car, average short-line haul per ton and haul per car, and average revenue per 100 pounds, car, short-line car-mile and short-line ton-miles.

From the above list we selected the average revenue per 100 pounds, which was later converted to a per ton figure, as our rail rate variable. For explanatory variables we chose tons, average short-line haul per ton and average tons per car.

2. "State-to-State Distribution(s)," *Carload Waybill Statistics*, 1963, Statements SS-1 through SS-6, Bureau of Economics, Interstate Commerce Commission, Washington, D.C., Jan. 1966.

The tons figure is the total tonnage of the commodity carried from the state of origin to the state of destination during 1963. This should not be confused with the tons included in an individual shipment. It is an annual total. Information on the size of individual shipments would be valuable in that quantity rates and discounts are usually figured in these terms. Such information is not available however. The annual tonnage data were selected because we believed it would prove even more important in determining the level of rail rates. This annual tonnage reveals how important the product is to both the shipper and the railroads. We believed that as more of a product is shipped annually the rate charged per ton would tend to decrease.

The average short-line haul per ton became our distance variable. Clearly, in constructing the freight rate for a ton of any commodity, the distance it is to be carried is the primary determinant. This belief was verified time and time again.

Average tons per car was used as a surrogate for the density of the product. The rate charged for transporting commodities of different densities should vary, since undoubtedly the cost of transporting them varies. This variable was only utilized when more than one commodity was being examined at the same time. That is, we selected the average tons per car figure for *all* shipments of a commodity. Our explanatory variable density then changed in value only when a new commodity was considered. This was also true of the price variable.

So much is heard of value of service pricing in transportation that we felt it necessary to include a variable to attempt to capture this influence, if it was in fact present. For the vast majority of the commodities we were going to investigate, no demand studies have ever been undertaken and no measures of elasticity, however crude, were available. However, we were able to find data on wholesale prices for the ICC commodities and commodity groups.[3] These prices were for the year 1959 and, although most of the commodity prices had undoubtedly changed between 1959 and 1963, it was felt that *relative* prices probably had not changed too much.

Before presenting the results of our analysis, it is necessary to discuss the levels of aggregation of the ICC data. At the lowest level we have state-to-state information for 262 "commodity classes." These "commodity classes" are small groups of commodities. These commodity classes are then distributed into five "commodity groups," the data are aggregated and state-to-state shipments of the commodity groups are presented. Finally, for the highest level of aggregation, all of the data are grouped together and tables giving the state-to-state shipments of Group 960, "All Commodities," are presented. Our empirical work included all three levels of data. We present our analysis of Group 960 first, then the commodity group totals, and, lastly, the commodity classes.

Group 960: All Commodities. In the analysis of shipments of Group 960 between pairs of states, we were able to use information only on rate, distance, annual tonnage and the geographical location of the states. The other variables, price and density, are, at this level of analysis, the average price and the average density of "all commodities and thus are constant.

3. *Freight Revenue and Wholesale Value at Destination of Commodities Transported by Class I Line-Haul Railroads, 1959,* Bureau of Transport Economics and Statistics, Interstate Commerce Commission, Statement No. 6112, File No. 18-C-23, Washington, D.C., Oct. 1961.

Since the aim of this study is to examine how the selected variables, individually and jointly, help to determine or to explain the level of rail rates, we have entered these variables into our regression equations in a stepwise fashion. This allows us to observe, and test, the increase in explanatory power of the equation as each additional variable is entered, and also to observe the stability or robustness of the coefficients of previously entered variables. The sequence in which the variables enter the equation was determined beforehand and no variable, once having entered the equation, was allowed to drop out. The entering sequence was, in almost all cases, the same as would have occurred had we chosen the variable with the largest "F to enter" statistic at each step.

In earlier studies, we had found linear equations usually provided a better fit than semi-log or log-linear equations. For this case our regression equations were

(4.33) $$R_{ij} = \beta_{10} + \beta_{11} M_{ij}\ e_{1ij}, \text{ and}$$

(4.34) $$R_{ij} = \beta_{20} + \beta_{21} M_{ij} + \beta_{22} Q_{ij} + e_{2ij}$$

where

R_{ij} = average rate in cents-per-ton, for shipments from state i to state j;

M_{ij} = average distance, in miles, traveled by shipments from state i to state j;

Q_{ij} = annual tonnage in thousands of tons, shipped from state i to state j;

β_{mn} = the n^{th} coefficient to be estimated in the m^{th} equation; and

e_{kij} = the disturbance terms, (k = 1, 2).

Table 4.1 contains the results of this estimation process. This table contains three sub-tables. The upper table contains the simple correlation coefficients for this set of data. The center table contains the estimated coefficients, the standard errors for the coefficients of the third equation and the coefficients of determination. At the bottom we have an analysis of variance table. The various F statistics show whether the explanatory power of the basic regression equation is increased significantly as each new variable is forced to enter.

From the center table we observe that distance alone is able to account for 68 percent of the variance of rail rates. The distance coefficient estimate of 1.42 tells us that for each mile of haul, the rate increases 1.42 cents above the base rate of $3.1937. This $3.19, the intercept estimate, could possibly be interpreted as the average terminal charges over all commodities. The distance coefficient shifts very little as the other variables are added to the equation.

When the quantity variable is first entered its coefficient is estimated as -2.01 -- for every one thousand extra tons carried per year, the rate per ton decreases by 2.01 cents. Although the coefficient of determination increases by only .01, from .68 to .69, we see from the analysis of variance table that the coefficient is significant, i.e., that annual tonnage does contribute significantly to the explanation of rail rates.

<u>Commodity Group Totals</u>. We now turn to the analysis of rail rates when the data are broken down into commodity groups. This is one step less in the aggregation procedure than the data utilized in the last section. There we were working with state-to-state totals for all

Table 4.1

RAIL RATE REGRESSIONS

ALL COMMODITIES

SIMPLE CORRELATION COEFFICIENTS

	Distance	Quantity	Rate
Distance	1.000	-0.223	0.826
Quantity		1.000	-0.231
Rate			1.000

ESTIMATED COEFFICIENTS

Eq.	Intercept	Distance	Quantity	R^2
(4.33)	319.37	1.42		.68
(4.34)	354.15	1.40	-2.01	.69

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	1609	7,830,784			
Reduction for mean	1	5,329,003			
Remainder	1608	2,501,780	1,553	3425	S.
Reduction for distance	1	1,708,606			
Remainder	1607	793,174	493	3461	S.
Reduction for quantity	1	5,823			
Remainder	1606	787,350	490	12	S.

commodities; here we are using state-to-state shipments for the five commodity groups. However, the nature of the data is the same as before and the same independent variables are considered. Our discussion will consist primarily of a comparison of the estimates across the five groups.

Tables 4.2 through 4.6 contain the information needed for this discussion. They are set up in exactly the same format as Table 4.1 and should appear familiar.

A comparison of the distance coefficients reveals that they are all positive and highly significant. They are very stable over the different equations dealing with the same set of data. However, for different data sets the coefficients differ greatly. They range from a high of 2.1 cents per mile for Group II Animals and Products, down to about .78 cents per mile for Group III, Products of Mines. Such large differences in line-haul charges deserve comment. Animals and Products would probably travel in specialized equipment: stock cars and refrigerator cars. This type of equipment is not readily adaptable to carry other types of products. The annual volume of traffic for this commodity group is much less than the volume of the other groups. On the other hand, Group III, Products of Mines, has by far the greatest annual volume of traffic and the products would mostly travel in open hopper cars. The structure of the competitive market is also much different for mineral products. Here we have a small number of relatively powerful shippers who are probably able to bargain quite effectively with the transport industry.

The tonnage coefficients also appear quite robust. They are all negative as expected, but for Groups I and II are not significantly

Table 4.2

RAIL RATE REGRESSIONS

GROUP I: PRODUCTS OF AGRICULTURE

SIMPLE CORRELATION COEFFICIENTS

	Distance	Quantity	Rate
Distance	1.000	-0.232	0.939
Quantity		1.000	-0.229
Rate			1.000

ESTIMATED COEFFICIENTS

Eq.	Intercept	Distance	Quantity	R^2
(4.33)	97.65	1.41		.88
(4.34)	106.78	1.41	-2.23	.88

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	628	2,055,201			
Reduction for mean	1	1,268,282			
Remainder	627	786,918	1,255	1011	S.
Reduction for distance	1	693,407			
Remainder	626	93,511	149	4642	S.
Reduction for quantity	1	118			
Remainder	625	93,393	149	1	N.S.

Table 4.3

RAIL RATE REGRESSIONS

GROUP II: ANIMALS AND PRODUCTS

SIMPLE CORRELATION COEFFICIENTS

	Distance	Quantity	Rate
Distance	1.000	0.007	0.905
Quantity		1.000	-0.016
Rate			1.000

ESTIMATED COEFFICIENTS

Eq.	Intercept	Distance	Quantity	R^2
(4.33)	425.18	2.08		.82
(4.34)	441.69	2.08	-51.88	.82

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	227	1,288,423			
Reduction for mean	1	1,044,351			
Remainder	226	244,072	1,079	967	S.
Reduction for distance	1	200,042			
Remainder	225	44,029	195	1022	S.
Reduction for quantity	1	123			
Remainder	224	43,906	196	1	N.S.

Table 4.4

RAIL RATE REGRESSIONS

GROUP III: PRODUCTS OF MINES

SIMPLE CORRELATION COEFFICIENTS

	Distance	Quantity	Rate
Distance	1.000	-0.222	0.877
Quantity		1.000	-0.262
Rate			1.000

ESTIMATED COEFFICIENTS

Eq.	Intercept	Distance	Quantity	R^2
(4.33)	185.36	0.79		.77
(4.34)	203.80	0.78	-0.85	.77

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	543	360,742			
Reduction for mean	1	247,462			
Remainder	542	113,280	209	1184	S.
Reduction for distance	1	87,180			
Remainder	541	26,099	48	1807	S.
Reduction for quantity	1	541			
Remainder	540	25,557	47	11	S.

Table 4.5

RAIL RATE REGRESSIONS

GROUP IV: PRODUCTS OF FORESTS

SIMPLE CORRELATION COEFFICIENTS

	Distance	Quantity	Rate
Distance	1.000	-0.228	0.955
Quantity		1.000	-0.290
Rate			1.000

ESTIMATED COEFFICIENTS

Eq.	Intercept	Distance	Quantity	R^2
(4.33)	355.51	0.96		.91
(4.34)	406.14	0.94	-15.86	.92

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	353	1,127,350			
Reduction for mean	1	766,353			
Remainder	352	360,896	1,025	747	S.
Reduction for distance	1	325,893			
Remainder	351	32,002	91	3607	S.
Reduction for quantity	1	2,014			
Remainder	350	29,988	85	24	S.

Table 4.6

RAIL RATE REGRESSIONS

GROUP V: MANUFACTURES AND MISCELLANEOUS

SIMPLE CORRELATION COEFFICIENTS

	Distance	Quantity	Rate
Distance	1.000	-0.239	0.833
Quantity		1.000	-0.227
Rate			1.000

ESTIMATED COEFFICIENTS

Eq.	Intercept	Distance	Quantity	R^2
(4.33)	359.77	1.58		.69
(4.34)	387.21	1.57	-5.86	.70

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	1361	7,219,579			
Reduction for mean	1	4,894,562			
Remainder	1360	2,325,017	1,709	2863	S.
Reduction for distance	1	1,614,009			
Remainder	1359	711,007	523	3084	S.
Reduction for quantity	1	1,914			
Remainder	1358	709,092	522	4	N.S.

different from zero. Between groups they vary widely in size. The greatest effect is again for Group II and the smallest for Group III. Thus the comments made in the last paragraph are also pertinent here. The commodity group with a small annual volume and requiring specialized equipment receives large quantity discounts; whereas the large volume group probably charging rates close to costs, receives almost no discount. This quantity effect should not be taken too seriously, however, as we shall later see that it is probably mostly due to equation error.[4]

Commodity Groups Utilizing Data on Commodity Classes. Again, we are attempting to explain the level of rail rates for commodity groups. However, we are now considering data on the individual commodity classes comprising the groups. We have state-to-state shipment information for each of the commodity classes. In estimating the equations for each commodity group, we now utilize the data for all of the commodity classes included within that group. Thus we are able to include the price and density variables in the estimating equations. These equations now become, for each commodity group,

$$(4.35)\quad R_{ijk} = \beta_{10} + \beta_{11} M_{ijk} + e_{1ijk}$$

$$(4.36)\quad R_{ijk} = \beta_{20} + \beta_{21} M_{ijk} + \beta_{22} D_k + e_{2ijk}$$

$$(4.37)\quad R_{ijk} = \beta_{30} + \beta_{31} M_{ijk} + \beta_{32} D_k + \beta_{33} P_k + e_{3ijk}$$

$$(4.38)\quad R_{ijk} = \beta_{40} + \beta_{41} M_{ijk} + \beta_{42} D_k + \beta_{43} P_k + \beta_{44} Q_{ijk} + e_{4ijk}$$

4. See especially Appendix A to this Chapter.

where

R_{ijk} ≡ average revenue received, in cents per ton, for transporting the k^{th} commodity class from state i to state j;

M_{ijk} ≡ average distance, in miles, traveled by shipments of commodity k from state i to state j;

Q_{ijk} ≡ annual tonnage, in thousands of tons, of the k^{th} commodity class shipped from state i to state j;

D_k ≡ the density, in average tons per car, of the k^{th} commodity class shipped from state i to state j;

P_k ≡ the average wholesale price, in dollars per ton, of the k^{th} commodity class;

β_{mn} ≡ the n^{th} coefficient to be estimated in the m^{th} equation; and

e_{lijk} ≡ the disturbance terms, (l = 1, 2, 3, 4).

Tables 4.7 through 4.11 present the results of this estimation process. These tables differ slightly from the previous tables in that they do not contain the correlation coefficients.

To begin with, we note that the distance coefficients are again all positive and highly significant. They also retain their intra-group stability. Comparison of these with the distance coefficient estimates contained in Tables 4.2 through 4.6 reveals that the coefficients retain their intergroup standings in magnitude, and that coefficients of the same commodity group are roughly of the same magnitude. However, they are usually slightly higher for the less aggregate data.

The density variables, which we are using for the first time, all enter with negative, and significant coefficients -- the greater the average weight per carload the less the rate per ton. This effect is greatest for manufactured products where it is approximately 30 cents

Table 4.7

RAIL RATE REGRESSIONS
COMMODITY CLASSES

GROUP I: PRODUCTS OF AGRICULTURE

ESTIMATED COEFFICIENTS

Intercept	Distance	Density	Price	Quantity	R^2
48.62	1.47				.87
665.51	1.34	-13.57			.89
565.99	1.35	-12.01	0.23		.89
575.92	1.35	-12.71	0.23	14.89	.90

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	1159	3,214,932			
Reduction for mean	1	1,566,880			
Remainder	1158	1,648,051	1,423	1101	S.
Reduction for distance	1	1,433,202			
Remainder	1157	214,848	185	7718	S.
Reduction for density	1	38,901			
Remainder	1156	175,946	152	256	S.
Reduction for price	1	1,926			
Remainder	1155	174,020	150	13	S.
Reduction for quantity	1	1,706			
Remainder	1154	172,313	149	11	S.

Table 4.8

RAIL RATE REGRESSIONS
COMMODITY CLASSES

GROUP II: ANIMALS AND PRODUCTS

ESTIMATED COEFFICIENTS

Intercept	Distance	Density	Price	Quantity	R^2
332.42	2.23				.82
808.54	2.11	-20.68			.83
478.58	2.02	-17.23	0.54		.85
477.43	2.02	-17.25	0.54	7.70	.85

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	197	1,182,713			
Reduction for mean	1	948,006			
Remainder	196	233,707	1,192	796	S.
Reduction for distance	1	190,764			
Remainder	195	42,942	220	866	S.
Reduction for density	1	3,760			
Remainder	194	39,182	201	19	S.
Reduction for price	1	4,420			
Remainder	193	34,761	180	25	S.
Reduction for quantity	1				
Remainder	192	34,761	181	0	N.S.

Table 4.9

RAIL RATE REGRESSIONS
COMMODITY CLASSES

GROUP III: PRODUCTS OF MINES

ESTIMATED COEFFICIENTS

Intercept	Distance	Density	Price	Quantity	R^2
145.00	0.87				.78
609.67	0.83	-7.57			.82
496.61	0.82	-6.46	3.05		.83
494.81	0.82	-6.37	2.99	-0.39	.83

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	935	377,326			
Reduction for mean	1	231,628			
Remainder	934	145,697	155	1485	S.
Reduction for distance	1	114,038			
Remainder	933	31,658	33	3360	S.
Reduction for density	1	5,164			
Remainder	932	26,494	28	182	S.
Reduction for price	1	1,893			
Remainder	931	24,600	26	72	S.
Reduction for quantity	1	71			
Remainder	930	24,529	26	3	N.S.

Table 4.10

RAIL RATE REGRESSIONS
COMMODITY CLASSES

GROUP IV: PRODUCTS OF FORESTS

ESTIMATED COEFFICIENTS

Intercept	Distance	Density	Price	Quantity	R^2
269.02	0.98				.94
931.44	0.98	-16.98			.94
900.75	0.97	-16.46	0.21		.94
870.73	0.96	-15.47	0.20	-3.85	.94

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	444	1,360,657			
Reduction for mean	1	830,917			
Remainder	443	529,739	1,195	695	S.
Reduction for distance	1	495,356			
Remainder	442	34,383	77	6368	S.
Reduction for density	1	4,047			
Remainder	441	30,336	68	59	S.
Reduction for price	1	143			
Remainder	440	30,192	68	2	N.S.
Reduction for quantity	1	71			
Remainder	439	30,121	68	1	N.S.

Table 4.11

RAIL RATE REGRESSIONS
COMMODITY CLASSES

GROUP V: MANUFACTURES AND MISCELLANEOUS

ESTIMATED COEFFICIENTS

Intercept	Distance	Density	Price	Quantity	R^2
286.69	1.91				.60
1654.07	1.75	-36.13			.71
1392.87	1.70	-30.95	0.25		.73
1294.47	1.71	-31.29	0.25	15.11	.73

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	4200	18,216,183			
Reduction for mean	1	9,097,443			
Remainder	4199	9,118,740	2,171	4189	S.
Reduction for distance	1	5,428,736			
Remainder	4198	3,690,003	878	6167	S.
Reduction for density	1	1,035,729			
Remainder	4197	2,654,274	632	1638	S.
Reduction for price	1	179,886			
Remainder	4196	2,474,387	589	305	S.
Reduction for quantity	1	1,087			
Remainder	4195	2,473,300	589	2	N.S.

Table 4.12

RAIL RATE REGRESSIONS

FINAL COEFFICIENTS ESTIMATES

Group	Intercept	Miles	Quantity	Density	Price	R^2	n
"All Commodities"							
	354.15	1.40*	-2.01*			.69	1609
Commodity Groups							
I	106.78	1.41*	-2.23			.88	628
II	441.69	2.08*	-51.88			.82	227
III	203.80	0.78*	-0.85*			.77	543
IV	406.14	0.94*	-15.86*			.92	353
V	387.21	1.57*	-5.86			.70	1361
Individual Commodities by Commodity Groups							
I	575.92	1.35*	14.89*	-12.71*	0.23*	.90	1159
II	477.43	2.02*	7.70	-17.25*	0.54*	.85	197
III	494.81	0.82*	-0.39	-6.37*	2.99*	.83	935
IV	870.73	0.96*	-3.85	-15.47*	0.20	.94	444
V	1294.47	1.71*	15.11	-31.29*	0.25*	.73	4200
All Commodity Groups							
	1226.41	1.36*	2.77*	-23.92*	0.35*	.73	6935

*Denotes an estimate significantly different from zero, in the direction indicated by its sign, at the $\alpha = .025$ level.

per extra ton per car. It falls off to about 16 cents for Products of Forests and Animals and Products, to 12 cents for Products of Agriculture and finally to 6 cents for Products of Mines.

The wholesale price of the commodity was the third variable entered. We reasoned that if the railroads engaged in value of service pricing, this variable should contribute significantly to the equation. The estimated coefficients are quite stable and significant with the exception of Group IV, Products of Forests. The estimated effect is greatest for Products of Mines where a one-dollar increase in the wholesale price of the product is associated with a 3.1-cent increase in the rate. However, the most significant effect is achieved for Group V, Manufactures. These results are in accord with our *a priori* beliefs.

Due to the strange behavior of the tonnage coefficient estimates, we shall discuss those effects further in Appendix A to this chapter. Table (4.12) summarizes our final coefficient estimates for the rail rate equations.

Barge Transport

The data utilized in our investigations of water and motor freight rates were of a different nature than the rail data. The rail data were taken from public documents -- annual statistics collected and published by the Interstate Commerce Commission. No such statistics existed for water or truck transport. For these modes it was necessary to collect and process revenue and tonnage information from private sources. This was accomplished quite satisfactorily for motor carriage, less so for barge.

The truck data will be seen to include some forty-six large carriers and to cover at least the central United States quite well. The water data are for only one carrier, A. L. Mechling Barge Lines, Inc., (MBL). However, MBL is a large, and hopefully representative, water carrier. During 1963, MBL operated on at least seventeen different river systems and was one of the six largest water common carriers in the United States.

The Data. From A. L. Mechling Barge Lines, Inc., we have the following usable information derived from freight bills for the year 1963.

1. date of the freight bill.
2. origin river.
3. destination river.
4. net weight of the shipment.
5. barge owner and number.
6. commodity by AAR commodity code.
7. ICC revenue account number.
8. revenue.
9. ton-miles.

An inspection of the data revealed that the ICC account 301, freight revenue, was the only revenue account appropriate for our use. The other revenue accounts were for special operations, e.g., revenue from charters, demurrage, special services, etc., and contained relatively few observations. It was also evident that although operations on seventeen river districts were included in the data, the coverage was not uniform enough to allow a full analysis of shipments by originating and receiving river districts. The decision was made to simply aggregate all the river district observations rather than to analyze perhaps three or four common shipment patterns.

We were then left with 4220 observations on tonnage, commodity, revenue and ton-miles. With this information it was possible to handle the water rate investigations in almost the same manner as the rail rate analysis was carried out. A major dissimilarity was that while the rail observations were annual totals, a water observation was a single shipment.

Average revenue per ton was again taken as our dependent variable. It was obtained by simply dividing the revenue for each shipment by the tons included in that shipment. To derive our distance figure it was only necessary to divide ton-miles by tons.

Since the commodity groups for the water data are the same as those utilized for the rail data we were able to carry over the density and price variables from the rail analysis. Average tons per rail car is again used as a surrogate for the density of the commodity class and wholesale price at destination is the value variable.

The quantity variable is given directly. However, this quantity variable is *not* analogous to the rail quantity variable. In the rail data, quantity represented the total annual tonnage carried between two states. Here, the variable represents the size of an individual shipment. In the truck data to be discussed later, the quantity variable also represents the size of a specific shipment. While it would have been possible to derive an annual tonnage figure, analogous to the rail quantity variable for the water data it was not possible to make the conversion for the truck data. Since creation of a similar variable for truck shipments was impossible, and because of the small contribution of the rail quantity variable to the rail equations, we decided to neglect the conversion for water. Hence, the water and truck quantity variables represent only the size of individual shipments. As it turns out, this is a much better, more important in terms of explanatory power, variable.

<u>All Commodity Classes</u>. To begin our analysis of water rates we pooled all of the observations and ran the following regressions:

(4.39) $$R_{ik} = \beta_{10} + \beta_{11} M_{ik} + e_{1ik}$$

(4.40) $$R_{ik} = \beta_{20} + \beta_{21} M_{ik} + \beta_{22} Q_{ik} + e_{2ik}$$

$$(4.41) \quad R_{ik} = \beta_{30} + \beta_{31} M_{ik} + \beta_{32} Q_{ik} + \beta_{33} D_k + e_{3ik}$$

$$(4.42) \quad R_{ik} = \beta_{40} + \beta_{41} M_{ik} + \beta_{42} Q_{ik} + \beta_{43} D_k + \beta_{44} P_k + e_{4ik}$$

where

R_{ik} ≡ average revenue received, in cents per ton, for transporting the i^{th} shipment of commodity class k;

M_{ik} ≡ distance, in miles, of the i^{th} shipment of commodity k;

Q_{ik} ≡ tons carried in the i^{th} shipment of commodity k;

D_k ≡ the density, in average tons per car, of the k^{th} commodity class;

P_k ≡ the average wholesale price, in dollars per ton, of the k^{th} commodity class;

β_{mn} ≡ the n^{th} coefficient to be estimated in the m^{th} equation; and

e_{lik} ≡ the disturbance terms, (l = 1, 2, 3).

Table 4.13 contains the estimated coefficients and an analysis of variance table. We note first of all that all of the coefficients are significantly different from zero. The distance coefficients are positive and quite robust. Each 100-mile increase in distance will increase the water rate by about thirty cents. Later, we will compare these coefficients with the rail coefficients.

The quantity coefficients are negative as expected and quite significant. The greater the amount shipped at any one time, the less is the rate per ton. The density coefficient also appears negative and highly significant. Its effect is by far the largest of any examined here. A one hundred ton increase in the density variable would lower the water rate by perhaps $3.00.

The price coefficient is very interesting. Its value is relatively small but it is positive and highly significant. Value of service rate

Table 4.13

WATER RATE REGRESSIONS

ALL COMMODITIES

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
58.769	0.296				.519
230.607	0.278	-0.171			.640
394,535	0.298	-0.158	-3.794		.670
322.947	0.288	-0.148	-2.785	0.164	.685
	(0.004)	(0.004)	(0.203)	(0.012)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	4220	90,268			
Reduction due to mean	1	55,336			
Remainder	4219	34,933	8.28	6683	S.
Reduction due to distance	1	18,116			
Remainder	4218	16,817	4.22	4293	S.
Reduction due to quantity	1	4,240			
Remainder	4217	12,576	2.98	1423	S.
Reduction due to density	1	1,046			
Remainder	4216	11,530	2.74	382	S.
Reduction due to price	1	518			
Unaccounted for	4215	11,012	2.61	198	S.

making is usually taken to refer to rail pricing. Here we have some evidence of it in association with water rates. We usually refer to water transport as a competitive industry and assume that rates should be based solely on costs. However, only if we are willing to ignore all demand considerations can we expect the price variable to have no effect. If we allow for any demand conditions, we have a situation where price affects quantity and quantity affects costs and thus rates. While this may or may not be (or be considered to be) value of service rate making it certainly allows product price a place in rate determination.

Commodity Classes by Commodity Groups. We next divided the data up into the five major AAR commodity groups and ran separate regressions for each group. The results of this work are presented in Tables 4.14 through 4.17. Commodity Group II, Animals and Products, was not represented in the water data and Commodity Group IV, Products of Forests, was represented by only two commodity classes. Hence we have no regression results for the former group and, due to degrees of freedom considerations, have only two meaningful regressions for the latter.

Comparing these tables, we see that all of the distance coefficients are positive and significant. They range in value from about 0.20 for Group III, Products of Mines, through 0.24 for Group I, Products of Agriculture, and 0.34 for Group V, Manufactures and Miscellaneous, up to about 0.40 for Group IV, Products of Forests. These coefficients appear robust and reasonable. The great explanatory power of the distance variable for Groups I and IV should be noted.

For Groups I and V the quantity coefficients came out with the expected, negative sign and were all significant. For Group IV the

Table 4.14

WATER RATE REGRESSIONS

GROUP I: PRODUCTS OF AGRICULTURE

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
63.680	0.242				.919
89.078	0.243	-0.027			.922
174.529	0.243	-0.015	-1.781		.924
140.831	0.244	-0.017	-1.361	0.206	.925
	(0.002)	(0.005)	(0.333)	(0.046)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	1110	89,944			
Reduction due to mean	1	61,399			
Remainder	1109	28,544	25.74	2,385	S.
Reduction due to distance	1	26,240			
Remainder	1108	2,303	2.08	12,621	S
Reduction due to quantity	1	75			
Remainder	1107	2,228	2.01	37	S.
Reduction due to density	1	59			
Remainder	1106	2,168	1.96	30	S.
Reduction due to price	1	38			
Unaccounted for	1105	2,130	1.93	19	S.

Table 4.15

WATER RATE REGRESSIONS

GROUP III: PRODUCTS OF MINES

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
-9.949	0.210				.542
-34.383	0.202	0.028			.547
-183,774	0.196	0.005	2.629		.609
-147.429	0.197	0.005	1.905	0.410	.612
	(0.007)	(0.010)	(0.373)	(0.159)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	728	79,603			
Reduction due to mean	1	70,242			
Remainder	727	9,360	12.88	5455	S.
Reduction due to distance	1	5,075			
Remainder	726	4,285	5.90	860	S.
Reduction due to quantity	1	46			
Remainder	725	4,239	5.85	7	S.
Reduction due to density	1	575			
Remainder	724	3,664	5.06	113	S.
Reduction due to price	1	33			
Unaccounted for	723	3,630	5.02	6	S.

Table 4.16

WATER RATE REGRESSIONS

GROUP IV: PRODUCTS OF FORESTS

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
67.936	0.443				.973
139.490	0.428	-0.081			.973
238.571	0.389	-0.035	-2.390		.974
-8.186	0.389	-0.035	2.250	0.922	.968
	(0.039)	(0.087)	(0.000)	(0.000)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	59	8,917			
Reduction due to mean	1	7,183			
Remainder	58	1,734	29.90	240	S.
Reduction due to distance	1	1,686			
Remainder	57	48	0.84	201	S.
Reduction due to quantity	1	1			
Remainder	56	47	0.83	1	N.S.
Reduction due to density					
Remainder					
Reduction due to price					
Unaccounted for					

Table 4.17

WATER RATE REGRESSIONS

GROUP V: MANUFACTURES AND MISCELLANEOUS

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
51.784	0.377				.612
181.301	0.337	-0.121			.665
244.027	0.338	-0.129	-1.320		.666
120.965	0.328	-0.110	0.713	0.159	.682
	(0.006)	(0.007)	(0.469)	(0.015)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	2322	724,161			
Reduction due to mean	1	448,152			
Remainder	2321	276,009	118.92	3769	S.
Reduction due to distance	1	169,033			
Remainder	2320	106,977	46.11	3666	S.
Reduction due to quantity	1	14,433			
Remainder	2319	92,544	39.91	362	S.
Reduction due to density	1	360			
Remainder	2318	92,184	39.77	9	S.
Reduction due to price	1	4,331			
Unaccounted for	2317	87,853	37.92	114	S.

quantity effect was estimated to be negative but never significant. It appears that for this group distance may well be the determining factor.

In the regressions dealing with Products of Mines, the quantity coefficient appears positive, but significant only if both density and price are excluded from the equation. As in the rail section, we are led to infer that this is due to the market structure of the Mining Industry. Its market power vis-a-vis the transportation industry's must be very great. Rates for comparable distances for Mining products are much lower than for any of the other commodity classes and quantity discounts do not exist.

The density coefficients are even less consistent. For Group I, they are negative and significant. The coefficients of Groups IV and V change their sign from negative to positive as the price variable is entered and lose their significance, while the density coefficients for Products of Mines are positive and significant. Perhaps we may attribute this behavior to rational action on the part of the carriers. Table 4.18 indicates that the average density of the commodities contained in Group I is about the same as the overall average density but

Table 4.18

ANALYSIS OF WATER RATE REGRESSION DENSITY COEFFICIENTS

Commodity Group	Average Density (Tons per car)	Standard Deviation (Tons per car)	Estimated Coefficients Eq. (4.41)	(Eq. 4.42)
I	54.1	29.9	(-)*	(-)*
III	71.9	11.2	(+)*	(+)*
IV	45.7	8.1	(-)	(+)
V	44.4	10.3	(-)*	(+)
All	51.7	13.7	(-)*	(-)

*Indicates a coefficient statistically different from zero, in the indicated direction, at the 0.05 level of significance.

there is a wide difference between commodities. The negative density coefficients for Group I may then be an attempt to allocate costs among the commodities in a proper manner. Groups IV and V have a lower average density but variance is much lower than for Group I so that as strict a cost allocation is perhaps not viewed as a necessity. Group III meanwhile illustrates a very high average density and a below average standard deviation. Hence for the commodities included in this group, any further increase in density may be thought to cause carriage costs to rise rather than to fall.

The price coefficients are, happily, all positive and significant except for the Group IV coefficient which is meaningless.

We summarize our results in Table 4.19. This table is analogous to Table 4.12 which summarized the rail results.

Table 4.19

WATER RATE REGRESSIONS

FINAL COEFFICIENT ESTIMATES

Group	Intercept	Miles	Quantity	Density	Price	R^2	n
Individual Commodities by Commodity Groups							
I	140.831	0.244*	-0.017*	-1.361*	0.206*	.925	1110
II							
III	-147.429	0.197*	0.005	1.905*	0.410	.612	728
IV	139.490	0.428*	-0.081			.973	59
V	120.965	0.328*	-0.110*	0.713	0.159*	.682	2320
All Commodities							
	322.947	0.288*	-0.148*	-2.785*	0.164*	.685	4220

*Denotes an estimate significantly different from zero, in the direction indicated by its sign, at the $\alpha = .025$ level.

Motor Transport

The Data. We were extremely fortunate in accumulating data for use in our analysis of freight rates for motor carriers. Officials of the Central States Motor Freight Bureau, Inc. kindly made available for our use data derived from a 1964 revenue study they had conducted. They had sampled the freight bills of 46 of the larger motor carriers operating out of the Central States region. Gross revenue of these 46 carriers was 60 percent of the total region's gross revenue. The sample was not limited to intra-regional shipments, but covered all shipments regardless of origin or destination.[5]

For analysis here we selected data on volume and truck-load interstate shipments. It was felt that of all the available truck shipments this was the category which was most likely to be in competition with rail and water carriage. For this category, we again had individual shipment observations on revenue, size of shipment, distance, AAR commodity class,[6] and, from our rail study, measures of the density and price of each commodity class.

Our analysis of truck rates was carried out in the same fashion as the analysis of water rates so there is no need to rewrite the equations on the definitions of the variables.

5. Commodities in the truck data were coded according to the National Freight Classification numbers. With the aid of the *Table of National Motor Freight Classification Numbers with Applicable Standard Transportation Commodity Code Numbers*, (6/22/65), published by the National Motor Freight Traffic Association, Washington, D.C., we were able to convert these into the AAR classifications. A computer tape matching these classifications is now on file at the Econometric Research Center, Northwestern University.

6. Their procedure, developed by a consultant statistician, was, in essence, to record each 50th waybill for truck-load shipments and each 200th waybill for less-than-truck-load shipments.

All Commodity Classes. Table 4.20 contains the estimated coefficients for the first regression run. Here all of the interstate volume and truck-load data were utilized. The most noticeable of the coefficients are those associated with the quantity variable. We must remember that this variable is now a measure of the size of an individual shipment. The coefficients are large in value, highly significant and very robust. For every one-ton increase in shipment size, the rate charged per ton will decrease by about $1.30. This variable assumes an importance in the truck regressions it did not display in the water results. We now wish that a comparable variable had been available for the rail analysis.

The distance coefficients are also highly stable and significant. On the average each extra mile of haul will result in about 2.8 cents added to the rate per ton. The density coefficients are negative and significantly different from zero, but the price coefficient, while significant, is so small as to be almost meaningless. We should note here that these data are dominated by observations on Group V, Manufactures and Miscellaneous, almost completely. The table gives useful summary statistics but it must be remembered that very little weight, justifiably, from the data, is given to Groups I through IV.

Commodity Classes by Commodity Groups. Tables 4.21 through 4.25 contain the results of the ragression analysis applied to the individual commodity groups. We note first that the overall explanatory power of these regressions is relatively small as compared to the rail or water regressions. Our four independent variables leave a substantial proportion of the variance of truck rates unexplained.

Table 4.20

MOTOR RATE REGRESSIONS

ALL COMMODITY GROUPS

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
921.748	2.860				.233
2912.837	2.833	-134.329			.454
3046.173	2.772	-130.370	-5.488		.456
3144.157	2.814	-129.696	-7.642	-0.070	.456
	(0.049)	(2.363)	(1.337)	(0.019)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000,000)	M.S. (1,000,000)	F.	Pr. α=.025
Total	9267	70,310			
Reduction due to mean	1	36,967			
Remainder	9266	33,343	4	9241	S.
Reduction due to distance	1	7,776			
Remainder	9265	25,568	3	2818	S.
Reduction due to quantity	1	7,373			
Remainder	9264	18,195	2	3686	S.
Reduction due to density	1	40			
Remainder	9263	18,154	2	20	S.
Reduction due to price	1	28			
Unaccounted for	9262	18,126	2	14	S.

Table 4.21

MOTOR RATE REGRESSIONS

GROUP I: PRODUCTS OF AGRICULTURE

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
1329.031	1.626				.234
2764.884	2.296	-131.772			.588
4220.643	2.324	-121.928	-60.433		.634
3949.472	2.350	-118.209	-48.046	-0.283	.637
	(0.221)	(14.476)	(21.458)	(0.287)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. $\alpha = .025$
Total	100	599,255			
Reduction due to mean	1	462,279			
Remainder	99	136,976	1384	334	S.
Reduction due to distance	1	32,000			
Remainder	98	104,976	1071	30	S.
Reduction due to quantity	1	48,477			
Remainder	97	56,499	582	83	S.
Reduction due to density	1	6,309			
Remainder	96	50,190	523	12	S.
Reduction due to price	1	509			
Unaccounted for	95	49,681	523	1	N.S.

Table 4.22

MOTOR RATE REGRESSIONS

GROUP II: ANIMALS AND PRODUCTS

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
1190.845	1.282				.665
1894.572	1.390	-51.744			.712
2140.549	1.418	-51.401	-10.623		.715
1916.308	1.410	-51.603	-4.668	0.128	.716
	(0.109)	(15.009)	(15.383)	(0.209)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	77	367,478			
Reduction due to mean	1	289,115			
Remainder	76	78,363	1031	280	S.
Reduction due to distance	1	52,108			
Remainder	75	26,255	350	149	S.
Reduction due to quantity	1	3,673			
Remainder	74	22,582	305	12	S.
Reduction due to density	1	245			
Remainder	73	22,338	306	1	N.S.
Reduction due to price	1	116			
Unaccounted for	72	22,222	309	0.5	N.S.

Table 4.23

MOTOR RATE REGRESSIONS

GROUP III: PRODUCTS OF MINES

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
819.050	2.648				.596
1401.683	2.373	-30.111			.644
901.188	2.347	-35.006	9.981		.669
1063.097	2.252	-41.148	2.393	12.305	.692
	(0.256)	(10.170)	(5.536)	(5.582)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	70	223,475			
Reduction due to mean	1	175,806			
Remainder	69	47,669	691	254	
Reduction due to distance	1	28,431			
Remainder	68	19,238	283	100	S.
Reduction due to quantity	1	2,248			
Remainder	67	16,990	254	8	S.
Reduction due to density	1	1,198			
Remainder	66	15,792	239	5	S.
Reduction due to price	1	1,098			
Unaccounted for	65	14,693	226	4	S.

Table 4.24

MOTOR RATE REGRESSIONS

GROUP IV: PRODUCTS OF FORESTS

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
1129.459	2.137				.335
2137.502	2.504	-91.098			.647
2264.707	2.486	-91.515	-3.137		.647
1570.596	2.583	-93.904	21.420	-1.050	.661
	(0.375)	(15.382)	(28.523)	(0.839)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000)	M.S. (1,000)	F.	Pr. α=.025
Total	45	200,771			
Reduction due to mean	1	179,118			
Remainder	44	21,653	492	364	S.
Reduction due to distance	1	7,243			
Remainder	43	14,409	335	22	S.
Reduction due to quantity	1	6,766			
Remainder	42	7,643	182	37	S.
Reduction due to density	1	4			
Remainder	41	7,639	186	0	N.S.
Reduction due to price	1	288			
Unaccounted for	40	7,351	184	1.6	N.S.

TABLE 4.25

MOTOR RATE REGRESSIONS

GROUP V: MANUFACTURES AND MISCELLANEOUS

ESTIMATED COEFFICIENTS
(and final standard errors)

Intercept	Distance	Quantity	Density	Price	R^2
898.326	2.948				.236
2910.595	2.910	-135.387			.457
3040.786	2.848	-131.518	-5.368		.459
3162.747	2.904	-130.683	-8.068	-0.087	.460
	(0.051)	(2.415)	(1.387)	(0.019)	

ANALYSIS OF VARIANCE

Source of Variance	D.F.	S.S. (1,000,000)	M.S. (1,000,000)	F.	Pr. α=.025
Total	8975	68,919			
Reduction due to mean	1	35,875			
Remainder	8974	33,044	4	8968	S.
Reduction due to distance	1	7,787			
Remainder	8973	25,257	3	2766	S.
Reduction due to quantity	1	7,326			
Remainder	8972	17,931	2	3663	S.
Reduction due to density	1	36			
Remainder	8971	17,895	2	18	S.
Reduction due to price	1	42			
Unaccounted for	8970	17,853	2	21	S.

The distance coefficients are all of the corect sign, significant and generally robust. However, the mileage variable alone does not display nearly the power it had in the rail and water data. For Groups II and III the first equations are quite respectable but certainly not overwhelming.

The quantity variables, however, display more importance than previously. Their coefficients are large and powerful. It appears that motor carriers give very large discounts for increases in shipment size. Groups II and III whose results, in general, were the most satisfactory and which gave relatively more emphasis to the distance variable in turn showed less reliance on the quantity variable. The quantity coefficients were smaller in value and slightly less significant for these two groups.[7]

The density coefficients came out negative except for Group III, Products of Mines, and the final coefficient for Group IV, which was not significant. In general, these coefficients displayed very little

7. It should be noted that since our data were limited to volume and interstate shipments the usual high correlation between distance and shipment size was not present. See: W. Y. Oi and A. P. Hurter, Jr., *Economics of Private Truck Transportation*, Wm. C. Brown, Dubuque, Iowa, 1965. The simple correlations between distance and shipment size for our data were

Group	Correlation
I	0.317
II	0.302
III	-0.346
IV	0.175
V	-0.014

In only one instance was this correlation greater than the correlations between these variables and the dependent variable, rate. That was for Group II where the simple correlation between shipment size and rate was only 0.04.

stability and density showed very little explanatory power as compared to the distance and quantity variables.

The price variable also showed very little explanatory power. The coefficients were significant only for Groups III and V and these differed in sign and relative size. In three of the five cases the coefficient was estimated to be negative indicating higher value commodities travel at lower rates. This is hardly in accord with our expectations or with the results of the rail and water analysis.

In general, the results of the truck analysis were not on a par with the rail and water results. The overall explanatory power of the equations was less and the coefficients of the density and price variables came out rather poorly. On the other hand, the distance coefficients, while weaker in overall explanatory power, were all of the right sign, robust and highly significant. The really impressive performers were the size of shipment variables. These displayed an importance never approached in the rail or water data.

The final coefficient estimates for the motor rate equations are summarized in Table 4.26. Following our discussion of the demand equations, all of our empirical results will be compared and summarized.

Empirical Results -- The Demand Equations

Comparable shipment data for several modes of freight transport, even of a cross-sectional nature, are very scarce at the present time. We were forced, therefore, to utilize shipment information from the 1963 Census of Transportation.[8] The Commodity Transportation Survey

8. For a critical appraisal of this census see: J. P. Crecine, L. N. Moses, and J. Stucker, "The Census of Transportation: An

Table 4.26

MOTOR RATE REGRESSIONS

FINAL COEFFICIENT ESTIMATES

Group	Intercept	Miles	Quantity	Density	Price	R^2	n
Individual Commodities by Commodity Groups							
I	3949.47	2.35*	-118.21*	-48.05*	-0.28	.64	100
II	1916.31	1.41*	-51.60*	-4.67	0.13	.72	77
III	1063.10	2.25*	-41.15*	2.39	12.30*	.69	70
IV	1570.60	2.58*	-93.90*	21.42	-1.05	.66	45
V	3162.72	2.90*	-130.68*	-8.07*	-0.09*	.46	8975
All Commodities							
	3144.16	2.81*	-129.70*	-7.64*	-0.07*	.46	9267

*Denotes an estimate significantly different from zero, in the direction indicated by its sign, at the α=.025 level.

of this census contains data on the shipments of commodities, by Transportation Commodity Classification (TCC) groups and by major mode of transportation between twenty-five standard metropolitan statistical areas (SMSAs).[9,10]

We considered it necessary to utilize the same commodity groupings for both the rate equations and the demand equations. Since this required the difficult transformation of the rate data from AAR commodity

Evaluation," published in *Papers--Seventh Annual Meeting,* Transportation Research Forum, Richard B. Cross, Oxford, Ind., 1966.

9. "Commodity Transportation Survey," *1963 Census of Transportation* Parts 1 & 2, GPO, Washington, D.C. 1966.

10. The Transportation Commodity Classification is composed of the first five digits of the Standard Transportation Commodity Code developed by the Association of American Railroads.

groups into STCC commodity groups, we restricted our analysis to four STCC groups. These were

STCC 26 -- Pulp, paper and allied products,

STCC 29 -- Petroleum and coal products,

STCC 30 -- Rubber and miscellaneous plastics products, and

STCC 32 -- Stone, clay and glass products.

Group 29 was selected as it was the only commodity group which contained a usable number of barge shipments. The other groups were selected on the basis of number of usable rail and motor observations and their compatibility with the rate commodity groupings. Our findings are reported in the following sections. Section 1 deals with the estimation of the two-mode model for STCC Groups 26, 29, 30 and 32. Section 2 discusses the application of the three-mode model to Group 29, Petroleum and coal products.

The Two-Mode Model

Using the subscripts r and m to denote rail and motor transport, our two-mode model is

$$Q_r + Q_m = \alpha + \beta T^c + \gamma A \tag{4.43}$$

$$T^c - T_r^r = a_r + b_r Q_r + c_r M \tag{4.44}$$

$$T^c - T_m^r = a_m + b_m Q_m + c_m M \tag{4.45}$$

$$T_r^r = f_r + g_r M \tag{4.46}$$

$$T_m^r = f_m + g_m M. \tag{4.47}$$

A two-step procedure is again utilized in estimating this system. First the rate equations are estimated; then the reduced form of equations (4.43), (4.44) and (4.46) is derived and estimated.

Since the census data did not include freight rates, it was necessary to use our other data sources for the rate equations.[11,12] This necessitated conversion of those data from their original AAR groupings into the STCC groups.[13] Freight rates were then regressed on distance, quantity, density and price, as before. These estimates are presented in Tables 4.35 through 4.38 following this section. Table 4.36 also contains the estimated coefficients for water transport of STCC group 29. As these rate results are quite comparable with those presented previously by AAR group, we shall concentrate on the demand equations.

Substituting equations (4.46) and (4.47) into (4.43), (4.44) and (4.45), we obtain

$$(4.48) \qquad Q_r + Q_m - \beta T^c = \alpha + \gamma A$$

$$(4.49) \qquad -b_r Q_r + T^c = (a_r + f_r) + (c_r + g_r) M$$

$$(4.50) \qquad -b_m Q_m + T^c = (a_m + f_m) + (c_m + g_m) M.$$

11. See footnote 8.

12. An investigation was made of the possibility of aggregating published freight tariffs to obtain rates for the selected commodity groups. It was found that due to the large number of commodities contained in each STCC group and the many possible rates for each commodity, from a single city to another city, because of quantity discounts, packaging specifications, etc., a very expensive effort would be required to produce any reasonable results.

13. This was a very subjective procedure as the codes were largely incompatible. An attempt was made to match each classification to the other, individually. The following groupings appeared to contain a minimum of misclassification.

STCC 26 -- AAR 653, 657, 659, 661, 663, 665, 669 and 671
STCC 29 -- AAR 501, 503, 505 and 507
STCC 30 -- AAR 525 and 549
STCC 32 -- AAR 663, 637, 639, 641, 693, 695, 701 and 721.

The reduced form of this system is then seen to be

$$(4.51)\quad Q_r = (b_r+b_m-\beta b_r b_m)^{-1} \Big\{\alpha b_m + (a_r+f_r)(\beta b_m-1) + a_m+f_m)$$
$$+ [(c_r+g_r)(\beta b_m-1) + (c_m+g_m)] M + \gamma b_m A\Big\}$$

$$(4.52)\quad Q_m = (b_r+b_m-\beta b_r b_m)^{-1} \Big\{\alpha b_r + (a_r+f_r) + (a_m+f_m)(\beta b_r-1)$$
$$+ [(c_r+g_r) + (c_m+g_m)(\beta b_r-1)] M + \gamma b_r A\Big\}$$

$$(4.53)\quad T^c = (b_r+b_m-\beta b_r b_m)^{-1} \Big\{\alpha b_r b_m + (a_r+f_r)b_m + (a_m+f_m)b_r$$
$$+ [(c_r+g_r)b_m + (c_m+g_m)b_r] M + \gamma b_r b_m A\Big\}$$

where the endogenous variables are presented as functions of the exogenous variables.[14]

This is the system we wish to estimate. If data on all of the variables were available, estimates of all the parameters could be obtained. However, data on the third dependent variables, total transport costs, are not available. At the present time freight rates can be observed or constructed, but associated costs cannot. Until microeconomic data on individual commodity shipments and their freight rates and transit times are gathered over modes and over time, values for

14. This transformation was performed in the traditional manner. Utilizing matrix notation equations (4.48), (4.49), and (4.50) can be written as

$$Bx = Cy.$$

Premultiplication by B^{-1} then yields

$$x = B^{-1} Cy,$$

the reduced form equations.

this variable will remain unknown. We are restricted, then, to estimating the first two equations of the reduced form.

To illustrate the useful information to be gained from a quantitative analysis of these "demand equations," we digress for a moment. In Chapter III we were able to derive qualitative statements concerning changes in the equilibrium values of the endogenous variables brought about by changes in system parameters.[15] Applying that analysis to the two-mode linear model specified above, we can construct Table 4.27. This table relates unit changes in parameters and exogenous variables to the induced changes in transport (endogenous) variables. Given empirical estimates of these parameters, the table and the relation $\Delta Y = \partial Y/\partial X \; \Delta X$ can be utilized to make predictions of system effects arising from parametric changes.

For example, suppose the ICC should grant a five cent per mile increase in all rail rates. Between two markets M miles apart the rail rate would increase by (5¢·M·Z/Z) or by 5M¢. The truck rate, assumed independent of the rail rate, would remain constant. The 5M¢ increase in the rail rate would shift the T^c function upward by a like amount. This would result in a decrease in the total quantity shipped between the two markets of $\frac{M\beta b_m}{Z}$ (since β is negative). This decrease in the total quantity is composed of a reduction of $\frac{M(\beta b_m - 1)}{Z}$ in the quantity carried by rail and an increase of $\frac{M}{Z}$ in the quantity transported by truck. The reduction in quantity shipped also lowers the value of T^c and the T^as by $\frac{M(\beta b_r b_m - b_r)}{Z}$. Shifts in all other system parameters can be traced through the table in a similar manner.

15. See Chapter III, pp. 54-58.

Table 4.27

QUALITATIVE ANALYSIS FOR THE RESTRICTED TWO-MODE LINEAR MODEL

[∂Y/∂X]

Exogenous Changes (X)	Induced Changes in Endogenous Variables (Y)							
	Q	Q_r	Q_m	T^c	T_r^r	T_m^r	T_r^a	T_m^a
Transport demand (α)	b_r+b_m	b_m	b_r	$b_r b_m$	0	0	$b_r b_m$	$b_r b_m$
Transport demand (β)	$T^c(b_r+b_m)$	$T^c b_m$	$T^c b_r$	$T^c b_r b_m$	0	0	$T^c b_r b_m$	$T^c b_r b_m$
Transport demand (γ)	$A\ (b_r+b_m)$	$A\ b_m$	$A\ b_r$	$A\ b_r b_m$	0	0	$A\ b_r b_m$	$A\ b_r b_m$
Transport demand (A)	$\gamma\ (b_r+b_m)$	$\gamma\ b_m$	$\gamma\ b_r$	$\gamma\ b_r b_m$	0	0	$\gamma\ b_r b_m$	$\gamma\ b_r b_m$
Rail Rate (f_r)	βb_m	$\beta\ b_m-1$	1	b_m	Z	0	$\beta b_r b_m-b_r$	$\beta b_r b_m-b_r$
Rail Rate (g_r)	$M\ \beta b_m$	$M(\beta b_m-1)$	M	Mb_m	M Z	0	$M(\beta b_r b_m-b_r)$	$M(\beta b_r b_m-b_r)$
Motor Rate (f_m)	βb_r	1	$\beta\ b_r-1$	b_r	0	Z	$\beta b_r b_m-b_m$	$\beta b_r b_m-b_m$
Motor Rate (g_m)	$M\ \beta b_r$	M	$M(\beta b_r-1)$	Mb_r	0	M Z	$M(\beta b_r b_m-b_m)$	$M(\beta b_r b_m-b_m)$
Assoc. costs, rail (a_r)	βb_m	βb_m-1	1	b_m	0	0	b_m	b_m
Assoc. costs, rail (b_r)	$Q_r\beta b_m$	$Q_r(\beta b_m-1)$	Q_r	$Q_r b_m$	0	0	$Q_r b_m$	$Q_r b_m$
Assoc. costs, rail (c_r)	$M\ \beta\ b_m$	$M(\beta b_m-1)$	M	Mb_m	0	0	Mb_m	Mb_m
Assoc. costs, motor(a_m)	βb_r	1	βb_r-1	b_r	0	0	b_r	b_r
Assoc. costs, motor(b_m)	$Q_m\beta b_r$	Q_m	$Q_m(\beta b_r-1)$	$Q_m b_r$	0	0	$Q_m b_r$	$Q_m b_r$
Assoc. costs, motor(c_m)	$M\ \beta b_r$	m	$M(\beta b_r-1)$	Mb_r	0	0	Mb_r	Mb_r

NOTE: All expressions have a common denominator of $Z = b_r+b_m-\beta b_r b_m$

Having investigated the information which would result if complete estimation of the reduced form were possible, we are now able to address the implications of partial estimation. It turns out that

(1) as seen previously, complete estimation of the rate equations is possible;

(2) estimation of only the first two equations of the reduced form while, in general, ruling out quantitative (cardinal) estimates of the system effects will allow qualitative comparisons to be made.

For example, we may be able to derive an estimate of the ratio b_m/b_r as, say, three. This would allow us to estimate the decrease in total quantity shipped and the increase in shipper's cost resulting from an "x" cent increase in the rail rate to be three times that which would result from a similar increase in the motor rate. The same relation holds for changes in the associated costs. The relative total elasticities of transport demand with respect to the transport rates can also be derived.

After extensive preliminary investigations we selected the following information from the available Census of Transportation data for our reduced form estimation.[16]

16. These preliminary investigations were presented in two documents. The relation between quantities shipped, by mode, and distance was reported upon in "Demand and Interface," *1967 Report* to the Corps of Engineers.

The relationship between total exports and imports, by commodity, and various market indicators was presented in C. Letta, M.S. thesis, Transportation Center, Northwestern University.

Briefly we may summarize the conclusions as: (1) abstractly there is a very close relationship between distance and quantities shipped; but (2) none of the common indicators of economic activity (value added, employment in manufacturing, personal income, etc.) were found to be more than slightly correlated with a region's imports or exports.

$_{ij}Q_l^k$ the quantity of commodity k transported from SMSA i to SMSA j by mode l,

$_{ij}M_l^k$ the distance in miles traveled by goods shipped between area i and area j,

$_iP^k$ the production of commodity k in area i, and

$_iC^k$ the consumption of commodity k in area i.

$$(i, j = 1, 2, \ldots 25)$$
$$(k = 26, 29, 30, 32)$$
$$(l = \text{rail, truck})$$

The quantity figures were available directly from the census tape, whereas some processing of the data was necessary to obtain the other variables. The mileage values were obtained by dividing reported ton-miles by reported tons. Area production is the sum of an area's total exports and its reported internal shipments. Consumption is an area's total imports plus its reported internal shipments.

The data were then divided into two groups according to whether the shipments traveled less than or greater than two hundred miles. This was to ascertain whether motor transport actually was dominant in the shorter shipments with rail transport becoming competitive at approximately this distance.[17] For the aggregate regressions a dummy variable was utilized to identify the two groups. A second-degree term in distance was also added to the equation. This was an attempt to restrict the dummy variable to reporting definite breaks in the function as opposed to reflecting mild nonlinearities.

Quantities by mode were first regressed on M, M^2, $_iP$ and $_jC$ for each distance group and commodity. The groups were then combined and

17. This is a common assertion. See, for example, Meyer, *op. cit.*

an aggregate equation estimated, for each commodity group, with only the dummy variable differentiating between the distance groups. Finally, another set of aggregate regressions were estimated containing two additional independent variables. These were $_jP$ and $_iC$ and represented an attempt to move behind the transport demand function to the area product supply and demand functions.[18] Tables 4.28 through 4.31 contain the parameter estimates and related statistics.

In general, the overall fit of the equations may perhaps be said to range from poor to fair. However, only one of the twenty-four equations is not statistically significant. Since the dummy variable was set up to assume a value of one for distances less than two hundred miles and zero for greater distances, we would expect the coefficient to display a negative sign for rail and a positive sign for motor transport. This expectation is realized in four of the eight equations. Group 26 yielded the expected results for both specifications of the model. Groups 29 and 32, however, yielded the "correct" signs for the third set of regressions. When the additional C and P variables were entered into the equation the coefficient of the dummy variable for rail transport became positive.

STCC 30's dummy variable coefficient came out positive for both equations. However, the peculiarity here appears to be not the absence of a shift between the distance groups but the behavior of the greater distance group itself. This is reflected in the mileage coefficient. For rail transport they are all positive, indicating that shipments

18. In terms of the reduced form all of these Ps and Cs are elements of the vector A.

Table 4.28

STCC 26 - PAPER, PULP AND ALLIED PRODUCTS

DEMAND EQUATIONS

Regression	Sample Size	Intercept	D	M	M^2	P_i	C_j	C_i	P_j	F	R^2
Distance < 200 mi.											
Rail	42	23,148.0		-389.910*	1.398*	0.001	0.006*			4.98*	.35
Motor	42	13,945.0		-932.090	1.666	0.085*	0.077*			19.77*	.68
Distance ≥ 200 mi.											
Rail	203	2,311.0		-3.800	0.001	0.003	0.007*			2.88*	.05
Motor	203	8,391.0		-20.010*	0.005*	0.006*	0.013*			22.91*	.32
All Distances											
Rail	245	2,667.0	-4,489.0	-4.550	0.001	0.003*	0.007*			3.27*	.06
Motor	245	-13,048.0	31,266.0*	-17.321	0.005	0.021*	0.032*			32.03*	.40
Rail	245	6.2	-2.3	-0.007	0.002	0.006*	0.010*	-0.006*	-0.004	3.08*	.28
Motor	245	-13.4	-59.3*	-0.037	0.011	0.038*	0.058*	0.003	-0.006	21.19*	.61

*Statistically significant at the $\alpha = 0.05$ level.

Table 4.29

STCC 29 - PETROLEUM AND COAL PRODUCTS

DEMAND EQUATIONS

Regression	Sample Size	Intercept	D	M	M^2	P_i	C_j	C_i	P_j	F	R^2
Distance < 200 mi.											
Rail	21	4,696.0		327.240	-2.316	0.002*	-0.001			2.94*	.42
Motor	21	507,544.0		-7284.280	23.584	0.007*	0.004			4.52*	.53
Distance ≥ 200 mi.											
Rail	144	39,075.0		-49.857*	0.015*	0.001*	-0.001			2.78*	.07
Motor	144	65,932.0		-110.299*	0.036*	0.001*	-0.001			12.66*	.27
All Distances											
Rail	165	39,942.0	-2,722.0	-57.220*	0.018*	0.001*	0.001			3.11*	.09
Motor	165	68,722.0	110,096.0*	-158.570*	0.054*	0.001*	0.001*			14.69*	.32
Rail	165	29.4	29.6*	-0.052*	0.014	0.018*	**	-0.016*	0.005	8.53*	.48
Motor	165	-192.8	474.8*	-0.154	-0.011	0.062	0.110*	0.179*	**	13.84*	.57

*Statistically significant at the α = 0.05 level.
**This coefficient was not strong enough to enter the final regression.

Table 4.30

STCC 30 - RUBBER AND MISCELLANEOUS PLASTICS PRODUCTS

DEMAND EQUATIONS

Regression	Sample Size	Intercept	D	M	M^2	P_i	C_j	C_i	P_j	F	R^2
Distance < 200 mi.											
Rail	39	-14,017.0		83.558	-0.268	0.008*	0.046*			7.02*	.45
Motor	39	8,758.0		2.064	-0.176	0.030*	0.092*			14.78*	.63
Distance ≥ 200 mi.											
Rail	238	-1,391.0		1.545	-0.001	0.002*	0.010*			3.28*	.05
Motor	238	-1,183.0		-7.542*	0.002	0.019	0.037*			47.79*	.45
All Distances											
Rail	277	-2,257.0	395.4	1.997	-0.001	0.002*	0.013*			4.63*	.08
Motor	277	-2,394.0	2,448.0	-7.673*	0.002	0.020*	0.043*			49.50*	.48
Rail	277	-4.3	1.0	0.002	-0.001	0.004*	0.022*	0.010	-0.004*	5.98*	.36
Motor	277	-1.4	5.9	-0.009*	0.003	0.040*	0.044*	-0.013	0.003	38.12*	.70

*Statistically significant at the $\alpha = 0.05$ level.

Table 4.31

STCC 32 — STONE, CLAY AND GLASS PRODUCTS

DEMAND EQUATIONS

Regression	Sample Size	Intercept	D	M	M^2	P_i	C_j	C_i	P_j	F	R^2
Distance < 200 mi.											
Rail	21	-52,260.0		776.570	-2.626	0.008*	0.164			1.15	.22
Motor	21	106,362.0		-2857.534	11.425*	0.015	0.284*			6.82*	.63
Distance ≥ 200 mi.											
Rail	156	13,816.0		-12.374*	0.003	0.001	0.001			2.71*	.07
Motor	156	38,665.0		-59.608*	0.015*	0.002	0.003			20.55*	.36
All Distances											
Rail	177	13,364.0	-1,104.0	-12.235	0.002	0.001	0.001			2.25*	.06
Motor	177	35,174.0	60,372.0	-56.751*	0.013*	0.002*	0.004*			17.93*	.34
Rail	177	2.7	137.4*	-0.108*	0.008	-0.006	0.054*	0.031	-0.014	9.61*	.52
Motor	177	-16.1	403.1*	-0.289*	0.023	0.024	0.141*	0.042	-0.037	11.83*	.56

*Statistically significant at the α = 0.05 level.

increase with distances. This is rather strange. Due to the nature of the reported data, we might expect shipments to increase with distance for the first two hundred miles or so. Such behavior is in fact displayed by Group 30. More to be expected, due to the comparative advantage argument, is the behavior exhibited by Groups 29 and 32. For these groups the rail coefficient is positive for the shorter distances while the truck coefficient is negative. For the greater distances, and in the aggregate regressions, both coefficients are negative.

With one exception the signs of the estimated coefficients of the square of distance were the opposite of the sign of the distance coefficients. However, this coefficient was never large enough in value to completely override the influence of the first degree distance term. That is, if the distance coefficient was negative and the distance squared coefficient positive (for the relevant range) the net effect of distance on quantity was negative.

Our model assumes that ${}_iP$ and ${}_jC$ will exert a positive influence on quantities transported whereas ${}_iC$ and ${}_jP$ will display a negative influence. Sixty-six of the seventy-eight estimated coefficients had the "proper" sign. Fifty of these estimates were significantly different from zero. Of these fifty, only one displayed an "improper" sign. STCC Group 29 was the biggest offender in this regard. This is probably due to the large quantity of barge shipments for these products. These water shipments are ignored in this section but will be investigated later with the three-mode model.

The Ps and Cs entered in our estimating equations are to be considered as elements of the vector A. That is

(4.54) $A' = ({}_iP \ {}_jC \ {}_iC \ {}_jP)$ and

(4.55) $\gamma' = (\gamma_1 \ \gamma_2 \ \gamma_3 \ \gamma_4)$.

Then referring to equations (4.51) and (4.52) the reduced form parameters of the Ps and Cs are seen to be the elements of Table 4.32.

Table 4.32

SELECTED REDUCED FORM PARAMETERS

Mode	Variable			
	${}_iP$	${}_jC$	${}_iC$	${}_jP$
Rail	$\frac{\gamma_1 b_m}{Z}$	$\frac{\gamma_2 b_m}{Z}$	$\frac{\gamma_3 b_m}{Z}$	$\frac{\gamma_4 b_m}{Z}$
Motor	$\frac{\gamma_1 b_r}{Z}$	$\frac{\gamma_2 b_r}{Z}$	$\frac{\gamma_3 b_r}{Z}$	$\frac{\gamma_4 b_r}{Z}$

From this table we can see that the reduced form estimates can be utilized to derive estimates of the ratio of b_m and b_r, and the ratios of the γ_is. It also illustrates the hazards of overidentification. For each commodity group it is possible to derive four estimates of b_m/b_r from the final set of regression equations. For our purposes we shall use the simple average of the ${}_iP$ and ${}_jC$ coefficients obtained from the third set of estimating equations as our "best" estimate of this ratio. As indicated previously, these ratios can also be used to estimate the ratios of the elasticities of transport demand with respect to the transport rates.[19]

19. From Table 4.27 we see that

$$\frac{\partial Q}{\partial f_r} = \frac{\beta b_m}{Z} \text{ and } \frac{\partial Q}{\partial f_m} = \frac{\beta b_r}{Z}.$$

Table 4.33 contains the derived ratio estimates. Reduced form estimation was also carried out for STCC groups 23, 35, 36 and 37. Table 4.33 also contains the ratio estimates for these groups. Their reduced form parameter estimates are contained in Tables 4.39 through 4.42.

The Ratios b_m/b_r may be used to compare elasticities in the following manner. Suppose that the two markets of interest are five hundred miles apart. From Tables 4.35 through 4.38 we derive estimates of rate ratios, T_r^r/T_m^r, for the commodity groups. These turn out to be 0.475, 0.525, 0.643 and 0.872 for groups 26, 29, 30 and 32 respectively. Multiplying these by the ratios of b_m/b_r then yields the ratio of the elasticities. These are 0.086, 0.525, 0.129 and 0.150. Thus, for markets five hundred miles apart, we estimate the elasticity of transport demand with respect to the truck rate to range from twice the elasticity with respect to the rail rate for Petroleum and Coal Products to twenty times the elasticity with respect to the rail rate for Stone, Clay and Glass Products.

The elasticity of Q with respect to the rail rate is

$$E_{Q,T_r^r} = \frac{T_r^r}{Q}\frac{\partial Q}{\partial T_r^r} = \frac{T_r^r}{Q}\frac{\beta b_m}{Z} .$$

Similarly, the elasticity of Q with respect to the motor rate is

$$E_{Q,T_m^r} = \frac{T_m^r}{Q}\frac{\partial Q}{\partial T_m^r} = \frac{T_m^r}{Q}\frac{\beta b_r}{Z} .$$

Hence

$$\frac{E_{Q,T_r^r}}{E_{Q,T_m^r}} = \frac{T_r^r}{T_m^r}\frac{b_m}{b_r} .$$

Table 4.33

RATIO ESTIMATES

Commodity Group	b_m/b_r	γ_2/γ_1
STCC 26 Pulp, Paper and Allied Products	.182	1.929
STCC 29 Petroleum and Coal Products	1.000	1.000
STCC 30 Rubber and Miscellaneous Plastics Products	.201	4.325
STCC 32 Stone Clay and Glass Products	.375	.750
STCC 23 Apparel and Related Products	.029	1.494
STCC 35 Machinery, except Electrical	1.258	1.107
STCC 36 Electrical Machinery and Equip Equipment	.282	1.489
STCC 37 Transportation Equipment	1.989	2.194

The Three-Mode Model

The three-mode model, relating rail, motor and water transport, was then estimated for STCC Group 29, Petroleum and Coal Products. This model has been developed previously and was presented in equations (4.22) through (4.28). The rate equations were first estimated. Parameter estimates and related statistics for these equations are presented in Table 4.36. The reduced form of the remaining structural equations was then derived and the demand equations were estimated. These estimates are contained in Table 4.34.

Table 4.34

STCC 29 - PETROLEUM AND COAL PRODUCTS

DEMAND EQUATIONS, THREE MODE MODEL

Regression	Sample Size	Intercept	D	M	M^2	i^P	j^C	i^C	j^P	F	R^2
Distance < 200 mi.											
Rail	32	169,030		-2,324.800*	8.092	3.098*	-1.669			11.01	.62
Motor	32	2,478,300		-41,772.000*	160.790	3.505	1.124			2.53	.27
Water	32	648,940		-21,008.000	49.994	59.114*	104.140*			3.11	.32
Distance ≥ 200 mi.											
Rail	144	38,892		-50.846*	0.015*	0.177*	-0.074			3.03	.08
Motor	144	65,151		-108.060*	0.036*	0.244*	-0.242			12.18	.26
Water	144	-1,441,100		1,254.500	-0.425	31.723*	78.590*			13.35	.28
All Distances											
Rail	176	51,399	24,869	-89.137*	0.038*	0.457*	-0.230			9.17	.21
Motor	176	172,260	463,210	-406.400	0.148	1.159	0.728			6.69	.16
Water	176	-1,320,100	1,149,700	658.600	-0.204	35.255*	87.850*			13.19	.28
Rail	176	59,604	20,849	-109.800*	0.037*	0.483*	-0.322	-0.354	0.563*	8.56	.26
Motor	176	232,560	453,030*	-510.150	0.188	1.285	0.472	-2.912	1.721	5.03	.17
Water	176	-1,256,200	1,146,200	566.160	-0.168	35.362*	87.740*	-3.203	0.910	9.32	.28

*Statistically significant at the $\alpha = 0.05$ level.

Table 4.35

STCC 26 - PULP, PAPER AND ALLIED PRODUCTS

RATE EQUATIONS

Mode	Int.	Distance		Quantity		Density		Price		R^2
		Coef.	"t"	Coef.	"t"	Coef.	"t"	Coef.	"t"	
Rail	405.61	1.281	39.25							.75
Motor	734.71	2.918	16.99							.45
Rail	484.18	1.267	39.75	-0.156	5.40					.76
Motor	1950.36	2.966	20.84	-92.300	12.73					.62
Rail	1205.79	1.275	48.67	-0.067	2.73	-23.113	15.68			.84
Motor	2196.31	2.980	21.01	-92.826	12.86	-8.731	2.04			.63
Rail	1260.07	1.276	48.31	-0.068	2.76	-24.221	7.71	-0.073	0.40	.84
Motor	4096.52	3.101	20.61	-94.123	13.07	-49.542	2.72	-2.260	2.30	.68

Average Values of the Variables

Mode	Sample Size	Rate	Distance	Quantity	Density	Price
Rail	512	1283.98	685.76	441.82	33.15	251.43
Motor	355	1984.80	428.45	13.39	28.05	349.54

Table 4.36

STCC 29 - PETROLEUM AND COAL PRODUCTS

RATE EQUATIONS

Mode	Int.	Distance		Quantity		Density		Price		R^2
		Coef.	"t"	Coef.	"t"	Coef.	"t"	Coef.	"t"	
Rail	221.05	1.431	46.08							.87
Motor	904.43	1.735	8.72							.74
Water	0.32	0.178	59.08							.87
Rail	277.83	1.407	45.63	-0.087	4.23					.87
Motor	1107.48	1.784	9.07	-15.371	1.54					.76
Water	0.28	0.178	59.10	0.002	1.34					.87
Rail	1211.69	1.370	44.69	-0.078	3.93	-28.980	4.95			.88
Motor	3942.61	1.792	8.97	-15.226	1.51	-88.908	0.60			.76
Water	0.25	0.178	58.59	0.002	1.23	0.093	0.11			.87
Rail	448.57	1.312	39.53	-0.077	3.97	-9.954	1.34	3.300	4.04	.89
Motor	5709.15	1.775	8.61	-13.749	1.28	-148.966	0.77	2.185	0.49	.77
Water	1.17	0.178	58.86	-0.001	0.26	-2.158	1.78	-0.384	2.56	.87

Average Values of the Variables

Mode	Sample Size	Rate	Distance	Quantity	Density	Price
Rail	327	890.97	468.17	524.13	31.79	56.02
Motor	29	1656.04	433.24	14.60	31.95	63.38
Water	522	116.99	478.35	1566.34	32.65	35.97

Table 4.37

STCC 30 - RUBBER AND MISCELLANEOUS PLASTIC PRODUCTS

RATE EQUATIONS

Mode	Int.	Distance		Quantity		Density		Price		R^2
		Coef.	"t"	Coef.	"t"	Coef.	"t"	Coef.	"t"	
Rail	1134.06	1.417	4.74							.43
Motor	1149.42	3.444	9.41							.22
Rail	1531.16	1.440	5.16	-1.533	2.35					.52
Motor	3890.97	3.461	11.24	-226.584	11.44					.45
Rail	2487.97	1.476	5.29	-1.316	1.95	-38.573	1.16*			.54
Motor	3196.36	3.487	11.28	-227.259	11.46	30.857	0.93*			.45
Rail	-1994.74	1.400	4.95	-1.772	2.59	36.828	-0-*	2.149	-0-*	.55
Motor	2250.26	3.487	11.26	-227.259	11.44	52.711	-0-*	0.392	-0-*	.45

Average Values of the Variables

Mode	Sample Size	Rate	Distance	Quantity	Density	Price
Rail	32	2280.44	808.75	270.84	27.09	1221.78
Motor	315	2506.36	393.96	12.13	22.44	1162.52

*Since this STCC group contained only two AAR groups there was insufficient. variance in their variables to produce reliable estimates.

Table 4.38

STCC 32 - STONE, CLAY AND GLASS PRODUCTS

RATE EQUATIONS

Mode	Int.	Distance		Quantity		Density		Price		R^2
		Coef.	"t"	Coef.	"t"	Coef.	"t"	Coef.	"t"	
Rail	-7.83	1.883	22.15							.73
Motor	1387.12	1.958	5.77							.19
Rail	-7.24	1.882	21.29	-0.001	0.02					.73
Motor	2238.58	2.534	8.93	-82.451	8.63					.47
Rail	551.50	1.729	19.19	0.027	1.61	-10.486	4.64			.76
Motor	2566.70	2.527	9.15	-75.476	7.87	-12.740	2.98			.50
Rail	-296.65	1.660	21.74	0.007	0.47	4.258	1.66	1.525	8.64	.83
Motor	1671.28	2.531	9.25	-75.760	7.98	4.132	0.43	0.927	1.98	.51

Average Values of the Variables

Mode	Sample Size	Rate	Distance	Quantity	Density	Price
Rail	185	628.69	338.11	1234.69	51.55	89.34
Motor	145	2131.34	380.06	12.98	32.65	374.40

Table 4.39

STCC 23 - APPAREL

DEMAND EQUATIONS

Regression	Sample Size	Intercept	D	M	M^2	i^P	j^C	i^C	j^P	F	R^2
Distance < 200 mi.											
Rail	33	4.922		0.017	0.001	0.002*	-0.001			2.19	.23
Motor	33	-7843.700		74.651	-0.291	0.067*	0.188*			8.40	.54
Distance ≥ 200 mi.											
Rail	143	-91.530		-0.001	0.001	0.002*	0.002			6.25	.15
Motor	143	-262.740		-0.441	-0.001	0.029*	0.033*			18.92	.35
All Distances											
Rail	176	-44.880	-87.130	-0.012	0.001	0.002*	0.001			5.42	.14
Motor	176	2350.400	2486.300*	0.409	-0.001	0.041*	0.102*			25.59	.43
Rail	176	-100.290	92.190	-0.050	0.001	0.001	0.003	0.004*	-0.001	5.39	.18
Motor	176	-2346.300	2552.200	0.425	-0.001	0.042*	0.107*	-0.004	-0.003	18.10	.43

*Statistically significant at the $\alpha = 0.05$ level.

Table 4.40

STCC 35 - MACHINERY, EXCEPT ELECTRICAL

DEMAND EQUATIONS

Regression	Sample Size	Intercept	D	M	M^2	i^P	j^C	i^C	j^P	F	R^2
Distance < 200 mi.											
Rail	67	-23018.000		306.800	-2.092	0.090*	0.034			26.26	.63
Truck	67	14565.000		-337.930*	0.804	0.032*	0.072*			24.50	.61
Distance ≥ 200 mi.											
Rail	391	1084.700		-4.783	0.001	0.009*	0.008*			18.26	.16
Truck	391	5396.100		-10.333*	0.003*	0.012*	0.011*			40.35	.29
All Distances											
Rail	458	-11005.000	12650*	2.413	-0.000	0.025*	0.023*			30.32	.25
Truck	458	-14.227	11031*	-8.626*	0.002	0.017*	0.022*			54.20	.37
Rail	458	-9424.900	12497*	1.526	-0.000	0.027*	0.020*	-0.007	0.002	21.92	.25
Truck	458	-1236.400	11219*	-7.877*	0.002	0.016*	0.024*	0.005	-0.000	38.95	.38

*Statistically significant at the $\alpha = 0.05$ level.

Table 4.41

STCC 36 - ELECTRICAL MACHINERY AND EQUIPMENT

DEMAND EQUATIONS

Regression	Sample Size	Intercept	D	M	M^2	i^P	j^C	i^C	j^P	F	R^2
Distance < 200 mi.											
Rail	60	-352.52		-16.208	0.109	0.000	0.008*			3.39	.20
Truck	60	-575.27		-69.000	0.054	0.055*	0.075*			9.36	.40
Distance ≥ 200 mi.											
Rail	347	-2790.50		2.244	-0.001	0.009*	0.014*			9.73	.10
Truck	347	-589.51		-5.123*	0.001*	0.029*	0.027*			46.10	.35
All Distances											
Rail	407	-2316.70	-419.09	2.111	-0.001	0.007*	0.013*			8.53	.10
Truck	407	-2378.20	6121.70*	-5.220*	0.001	0.033*	0.037*			56.73	.41
Rail	407	-2096.30	-540.65	1.687	-0.000	0.007*	0.015*	0.002	-0.003	6.49	.10
Truck	407	-2130.20	6135.40*	-5.293*	0.001	0.034*	0.037*	-0.002	-0.000	40.37	.41

*Statistically significant at the $\alpha = 0.05$ level.

Table 4.42

STCC 37 - TRANSPORTATION EQUIPMENT

DEMAND EQUATIONS

Regression	Sample Size	Intercept	D	M	M^2	i^P	j^C	i^C	j^P	F	R^2
Distance < 200 mi.											
Rail	36	-70108		771.160	-3.819	0.021*	0.079*			15.83	.67
Truck	36	48082		-205.550*	7.871*	0.062*	0.080*			55.07	.88
Distance ≥ 200											
Rail	200	-48061		25.523	-0.010	0.030*	0.059*			36.09	.43
Truck	200	17744		-35.157*	0.011*	0.006*	0.005*			33.57	.41
All Distances											
Rail	236	-46581	2945.3	22.988	-0.010	0.029*	0.061*			37.25	.45
Truck	236	-18261	65116.0*	-15.879	0.004	0.014*	0.032*			30.88	.40
Rail	236	-47979	2803.5	23.510	-0.011	0.031*	0.081*	-0.006	-0.010*	27.43	.46
Truck	236	-21297	65316.0*	-18.744	0.005	0.011*	0.015	0.016*	0.008*	24.11	.43

*Statistically significant at the $\alpha = 0.05$ level.

The overall fit of the demand equations is, again, not very good, although all but one are statistically significant. All of the dummy variable coefficients were positive, indicating that all modes shared in the shorter hauls. The very large barge coefficient reflects what we felt to be a rather unexpected aspect of the data. The census reported many large water shipments of less than fifty miles, indicating that, for this commodity group at least, barge was competitive with rail and truck for short hauls as well as long hauls. Its long haul advantage can be seen from the estimated mileage coefficients. Except for the short distance data group, water shipments actually increased with distance. The rail and motor coefficients, on the other hand, display the expected negative sign. The miles squared coefficients are, as before, all of opposite sign to the miles coefficients, and of smaller influence.

The production and consumption parameters are again a mixed group. The rail and water production coefficients are all of the correct sign and statistically significant. The motor coefficients are all positive but only one is significant. The consumption coefficients do not behave as well. For water shipments they all are of the proper sign and significant. However, for rail and motor they are mostly negative and are all insignificant.

The two additional market variables seem to add very little to the explanatory power of the equations. In each case the equation "F" value decreased, and the estimated coefficient was never both of the proper sign and statistically significant.

The reduced form parameters of the two-mode model allowed us to estimate comparative market responses. This was also possible with the three-mode model. Table 4.43 displays both the reduced form parameters and the response parameters. The average of the two-market coefficients for the third regression set was again utilized as our best estimate of the ratios. Normalizing on motor transport, this procedure yielded estimates of

$$\frac{b_m b_w}{b_r b_w} = 0.039 \text{ and } \frac{b_r b_m}{b_r b_w} = 75.55.$$

It appears that the transportation market for STCC group 29 is much more responsive (seventy-five times) to changes in barge costs than to changes in truck costs. At the same time, it is much more responsive (twenty-five times) to changes in truck costs than to changes in rail costs.

As rate elasticities depend upon the levels of the rates, and as barge rates were much lower than rail or truck rates, we estimated relative rate elasticities for two sets of markets, one pair 500 miles apart and the other 1,000 miles. For both sets of markets the estimated ratio of the rail rate elasticity was 0.18 for the near markets and 2.64 for the more distant market pair. This difference was due entirely to the relative rate structures.

The rather large difference between the value of the market response ratio and the value of the elasticity ratio for motor and water transport is due to the much higher rates charged by trucking firms. For *equal* changes in rates, the market is much more responsive to the water rate than to the motor rate. However, for *proportional* changes, much greater in absolute value for motor than for barge, market responses are about equal.

Table 4.43

RATIO ESTIMATES FOR THE THREE-MODE MODEL

A

Selected Market Response Parameters

Exogenous Change	Induced Change ∂Q	∂T^c
∂ (Rail Rate or Assoc. Cost)	$\frac{\beta b_m b_w}{B}$	$\frac{b_m b_w}{B}$
∂ (Motor Rate or Assoc. Cost)	$\frac{\beta b_r b_w}{B}$	$\frac{b_r b_w}{B}$
∂ (Water Rate or Assoc. Cost)	$\frac{\beta b_r b_m}{B}$	$\frac{b_r b_m}{B}$

B

Reduced Form Parameters

Equation	Coefficient of i^P	j^C
Rail Equation	$\frac{\gamma_1 b_m b_w}{B}$	$\frac{\gamma_2 b_m b_w}{B}$
Motor Equation	$\frac{\gamma_1 b_r b_w}{B}$	$\frac{\gamma_2 b_r b_w}{B}$
Water Equation	$\frac{\gamma_1 b_r b_m}{B}$	$\frac{\gamma_2 b_r b_m}{B}$

$$[B \equiv b_r b_m + b_r b_w + b_m b_w - \beta b_r b_m b_w]$$

Summary of the Empirical Results

In the preceding pages we have discussed our empirical results individually and in some detail. Here we wish to summarize those results and to compare the separate findings. We begin again with the rate equations.

The distribution of observations by commodity class for each mode should be noted first of all. Although there is a substantially larger number of truck shipments contained in our data, these truck shipments are mostly Group V products, Manufactures and Miscellaneous. Rail and water shipments, while also predominately of Group V products, also contain a large number of Group I and Group III observations. Relatively few shipments of Group II and Group IV products were observed. Again we must discuss the difference between the rail data and the truck and water data. A rail observation is for a total annual tonnage, whereas a truck or water observation is for a single shipment. Thus the number of observations for the rail data denotes the number of origin-destination pairs observed for the year and may bear little relation to the number of separate shipments of the various commodity groups. There were no water shipments of Group II, Animals and Products, present in our data, and only two separate commodity classes were present in the water data for Group IV, Products of Forests. To summarize, over 95 percent of the truck shipments in our sample were of Group V products. The water data is spread mostly over Groups I, III and V, whereas the rail data seems to cover most products except those contained in Group II, Animals and Products. Table 4.44 presents the percentage of shipments observed, by commodity group and by mode of transport.

Table 4.44

RATE DATA
DISTRIBUTION OF OBSERVATIONS BY COMMODITY GROUP
AND MODE OF TRANSPORT
(percent)*

Commodity Group	Mode of Transport			
	Truck	Rail	Water	All Modes
Group I	0.49 (1.08)	5.68 (16.71)	5.44 (26.30)	11.60
Group II	0.38 (0.83)	0.96 (2.84)	- -	1.34
Group III	0.34 (0.76)	4.58 (13.48)	3.56 (17.25)	8.49
Group IV	0.22 (0.49	2.17 (6.40)	0.29 (1.40)	2.68
Group V	43.95 (96.85)	20.57 (60.56)	11.37 (55.05)	75.89
Total	45.38 (100.00)	33.96 (100.00)	20.66 (100.00)	100.00

*The figures in parenthesis are the intra-mode percentages.

Table 4.45 contains the averages and standard deviations for the variables by commodity group and mode of transport. The first two independent variables, distance and quantity, deal with the nature of the shipment while the latter two, density and price, are concerned with the type of individual products which are included in the commodity groups.

Average freight rates are much higher for truck shipments than for rail or water shipments except for Group II. Here, the rail rate is surprisingly high. In all other cases the average truck rate is substantially above average rail rates and about five times the average water rate.

Table 4.45

FREIGHT RATE DATA ANALYSIS

Group		Avg. Revenue Per Ton(¢)		Distance (Miles)		Quantity (Tons)		Density (Tons/Car)		Price ($/Ton)	
		Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.
All Commodities	Truck	1997	1896	376	320	14.75	6.64	30.78	13.42	817	921
	Rail	1302	1347	653	661	(1472.00)	(8794.00)	38.71	15.89	316	731
	Water	362	287	1024	700	896.02	590.70	51.71	13.72	120	235
Group I: Products of Agriculture	Truck	2150	1176	504	349	13.46	5.60	26.52	4.29	426	339
	Rail	1162	1192	760	759	(1029.00)	(2690.00)	38.49	15.20	144	200
	Water	235	160	707	634	990.89	310.16	54.11	4.60	57	29
Group II: Animals and Products	Truck	1937	1015	582	645	14.81	4.46	25.19	5.63	640	394
	Rail	2194	1091	837	443	(248.00)	(294.00)	18.51	7.11	627	293
	Water	-	-	-	-	-	-	-	-	-	-
Group III: Products of Mines	Truck	1584	831	289	242	16.71	6.39	59.09	13.53	33	13
	Rail	497	394	406	402	(6286.00)	(22,930.00)	59.59	9.99	17	15
	Water	310	113	1528	398	1294.58	303.75	71.93	11.16	30	25
Group IV: Products of Forests	Truck	1995	701	405	189	12.70	4.37	36.48	3.40	200	109
	Rail	1368	1093	1117	1075	(1412.00)	(3682.00)	38.57	5.65	107	112
	Water	348	172	633	384	755.22	92.87	45.65	8.05	37	39
Group V: Manufactures and Misc.	Truck	1999	1918	373	315	14.76	6.67	30.63	13.31	832	930
	Rail	1471	1473	620	595	(586.00)	(1118.00)	35.09	14.00	437	903
	Water	439	344	1029	716	729.32	690.47	44.38	10.28	181	302

The distance variables display the reverse ranking in most cases. Barge shipments are usually the longest, followed by rail with truck hauls always the shortest. Exceptions to this are for Groups I and IV where the average distance for rail is greater than for barge. This appears to be due more to the relative shortness of the water shipments than to the length of rail shipments for these groups.

Comparison of the quantity variables is possible only for truck and water, and here the two extremes are evident. The average interstate volume truck shipment is about fifteen tons whereas the size of the average water shipment is approximately nine hundred tons. This helps to explain the size of the quantity discounts offered by the trucking industry.

Looking at the last two variables, we see that as a rule the less dense, higher-priced commodities travel by truck. Rail ranks second and water shipments are composed mainly of denser, lower-valued products. This is in accord with our *a priori* expectations. However, the inter-mode densities do not display as great a spread as we had imagined; and while the general statement is certainly true for Groups I and V it is much weaker as regards the other groups. Indeed, for Group II it appears that denser products travel by truck more than by rail. The very low value of the commodities classified as Group III, Products of Mines, should also be observed. The average value of these products is only about thirty dollars per ton. This is about one-twenty-fifth of the average value of all the products carried by motor transport and about one-fourth of the average value of those carried by barge.

Utilizing the data contained in Table 4.45, Tables 4.46 and 4.47 have been constructed. These tables illustrate the importance of

Table 4.46

RATE DATA
DISTRIBUTION OF TONNAGE BY COMMODITY GROUP
(percent)

Commodity Group	Mode of Transport		
	Truck	Rail	Water
I	0.98	11.68	29.09
II	0.83	0.48	0
III	0.86	57.57	24.92
IV	0.42	6.14	1.18
V	96.91	24.11	44.81
Total	100.00	100.00	100.00

Table 4.47

RATE DATA
DISTRIBUTION OF REVENUE BY COMMODITY GROUP
(percent)

Commodity Group	Mode of Transport		
	Truck	Rail	Water
I	1.06	15.58	19.73
II	0.81	1.20	0
III	0.70	32.87	22.33
IV	0.42	9.64	1.19
V	97.01	40.70	56.76
Total	100.00	100.00	100.00

various commodity groups to each mode of transport. Table 4.46 shows the percentage distribution of tonnage carried by each transport mode, whereas Table 4.47 shows the percentage of total revenue attributable to the commodity groups for each mode of transport. It is seen that for motor carriers over 95 percent of their tonnage is of Group V products and these products also account for over 95 percent of total revenue. For rail transport the biggest tonnage is for Group III, Products of Mines; but, because of the low rates charged for these products, the major source of revenue is Group V, where less tonnage is carried but higher rates are charged. Water transport carries mostly products included in Groups I, III and V, and mostly in proportion to their contribution to total revenue. There is a slight bias in favor of lower than average rates for Group I, Products of Agriculture, and higher rates for Group V, Manufactures and Miscellaneous.

Table 4.48 contains our final coefficient estimates for the rate equations arranged by commodity group and by mode of transport. This table allows comparisons directly across modes.

While most of the equations fit quite well, we see that the fit of the rail and water equations is always significantly better than the fit of the truck equations. This is specially true for Group V which contained the greatest number of observations as well as the greatest number of individual products. Here the truck equation was able to explain less than one-half of the total variance of the truck rates, whereas the rail and water equations could account for almost three-quarters of the variance of their respective rates.

The intercept terms are, all except one, positive as we would expect, and without exception greatest for truck and smallest for water transport. Group III shipments by water display a negative intercept. Shipments of this group of products by barge were on the average much longer and much larger than any other water shipments, and much more so than any of the rail or truck shipments. This longer average length of haul was accompanied by a small variance so that very few short shipments of this group of products were observed. These products were also heavier than the average and of much less value per ton. As we

Table 4.48

RATE REGRESSIONS

FINAL COEFFICIENT ESTIMATES

Group	Mode	Intercept	Miles	Quantity	Density	Price	R^2	n
I	Truck	3949.47	2.35*	-118.21*	-48.05*	-0.28	.64	100
	Rail	575.92	1.35*	(14.89*)	-12.71*	0.23*	.90	1159
	Water	140.83	0.24*	-0.02*	-1.36*	0.21*	.93	1110
II	Truck	1916.31	1.41*	-51.60*	-4.67	0.13	.72	77
	Rail	477.43	2.02*	(7.70)	-17.25*	0.54*	.85	197
	Water							
III	Truck	1063.10	2.25*	-41.15*	2.39	12.31*	.69	70
	Rail	494.81	0.82*	(-0.39)	-6.37*	2.99*	.83	935
	Water	-147.43	0.20*	0.01	1.91*	0.41*	.61	728
IV	Truck	1570.60	2.58*	-93.90*	21.42	-1.05	.66	45
	Rail	870.73	0.96*	(-3.85)	-15.47*	0.20	.94	444
	Water	139.49	0.43*	-0.08	-	-	.97	59
V	Truck	3162.75	2.90*	-130.68*	-8.07*	-0.09*	.46	8975
	Rail	1294.47	1.71*	(15.11)	-31.29*	0.25*	.73	4200
	Water	120.97	0.33*	-0.11*	0.71	0.16*	.68	2323
All Commodities								
	Truck	3144.16	2.81*	-129.70*	-7.64*	-0.07*	.46	9267
	Rail	1266.41	1.36*	(2.77*)	-23.92*	0.35*	.73	6935
	Water	322.95	0.29*	-0.15*	-2.79*	0.16*	.69	4220

*Denotes an estimate significantly different from zero, in the direction indicated by its sign, at the $\alpha = .025$ level.

shall see later, it appears that all modes allowed these low-valued commodities to move at lower rates than other products. Also, as noted previously, it appears that the mining industry displays much power in bargaining with the transport industry and is able to negotiate rates which are probably quite close to (marginal) costs. These factors cause the barge rate, when extrapolated back to zero quantity and zero distance in a linear manner, to yield the negative intercept.

Every distance coefficient was estimated to be positive and significantly different from zero at the $\alpha = 0.025$ level. Again the larger estimates were for motor shipments and the smaller for barge with only one exception. There were no observations on water transport of Group II products. The rail distance coefficient for this commodity group was estimated to be greater than the motor distance coefficient. This can probably be explained by examining the nature of the products contained in this commodity group and the characteristics of the typical shipments. These products are, on the average, quite low in weight and high in value. For the railroads they represent longer than average shipments and quite small total annual tonnages. These characteristics point to a product group which is probably rather inefficiently carried by rail and whose value can apparently stand a rather large freight rate. This is the one group of products which stand out in our data as being discriminated against by one of the transport modes.

As we have remarked previously the quantity data, and hence the estimated coefficients, are not comparable between rail and the other modes. The rail observations were for annual total tonnage of the commodity class, whereas the truck and water data are on individual shipments and their tonnage figures represent the size of the individual shipments. After a lengthy analysis of the rail quantity coefficients, reported on both within this chapter and in Appendix A following this chapter, we have to conclude that the overall affect of the size of annual tonnage upon rates appears to be positive. That is, as more of a product class is shipped between two points during a year the average shipping rate increases. Neither the characteristics of the shipments or the attributes of the products appear to have any connection with the intergroup behavior of these coefficients. Groups I, II and V have a positive coefficient whereas Groups III and IV yield a negative estimate. However, we do observe that the average value of the products included in Groups III and IV is substantially lower than the average. Perhaps the railroads favor these groups with annual tonnage discounts simply for this reason.

The truck and water tonnage coefficient estimates are much more consistent. In all cases the statistically significant estimates are negative in sign, indicating lower rates for larger shipments. In only one case, water shipments of Group III products, does an estimate come out positive, and here it is very small as well as being statistically insignificant. The discounts are always much greater for truck than for water, reflecting the fact that most of the truck shipments are rather small whereas the average water shipment is very large and displays a large variance. For both modes the greatest discounts are given on Group V products, their major source of revenue.

Most of the density coefficients came out negative, indicating "heavier" commodities received reduced rates per ton, and implying that, over the range of densities carried by the modes, weight is less a constraint than space to the transport industries. All of the rail estimates turned out negative and statistically significant. The coefficient for Group V products appears to be appreciably larger than the others, probably because of the large number of individual products contained in this group. Table 4.45 indicates that there is quite a large variance in the densities of these products, allowing a weight discount to be utilized effectively. The coefficients for truck and barge are a mixed group and the summary statistics are not of much help in interpreting them. There appear to be factors at work here which are not revealed by our data.

The price coefficients are mostly positive indicating some element of value of service pricing. All of the rail estimates are positive and only one is not significant. The coefficient for Group III, Products of Mines, displays by far the greatest value, indicating a pronounced effort to limit the rate to only what "the traffic will bear" for these very low-value products. The water coefficients are also all positive and significant with the Group III coefficient significantly larger than the others. The truck coefficient estimates are much more complex. It appears that for most products the trucking industry charges lower fares for higher valued products. This within group tendency is, however, slightly modified by the between group rate behavior. It appears that the higher valued products within the highest valued group, Group V, are favored less by the rate making behavior of the trucking industry than are the higher valued products within the lower valued groups. This observation is valid for Groups I, IV and V where the price effect decreases in size as the average value of the products in the groups decreases. The price coefficient for Group II is positive but small and not statistically significant. However, the coefficient for Group III is positive, large and significant, illustrating once again the special treatment offered this group of products by all the modes.

In summary, our rate results appear highly significant and mostly in accord with our *a priori* expectations. On the whole rates are higher (and increase faster with distance) for truck transport than for rail, are greater for rail than for water. Significant size of shipment discounts are present for, at least, truck and barge transport. Railroads appear to increase their rates between points with more than an average amount of traffic, or, at least, when rate increases occur they are concentrated primarily on these hauls. Heavier products travel at lower rates per ton than less dense products, although there are some oddities in the truck and water estimates here. And, finally, it appears that some element of value of service pricing is present in the rail and barge industries for all products, and also for some products carried by the trucking industry. This price effect is especially evident among the very low valued products comprising Commodity Group III, Products of Mines, where all modes offer large discounts to the lower valued products These results appear sufficiently valid for many uses. However, any potential user should note the large differences in rate behavior between the different commodity groups as well as between modes of transport. The aggregate estimates, while useful as summary statistics, should be viewed with caution.

Before presenting the summary of results of our quantity equation estimation we shall review the manner in which these equations differ from the "transport demand equations" which have been estimated by others. The usual estimating equation is a simple linear or log-linear equation which expresses the quantity carried by a certain mode of transportation as a function of the rate charged by that mode, perhaps the rates charged by competing modes of transport and, usually, several exogenous variables. This simple procedure is beset with several technical difficulties. First of all, the estimation of a single demand equation is a touchy matter. Unless the estimating equation is derived from a sound, and valid, theory we cannot be certain that the demand relation will actually be captured. A supply curve may be estimated or we may estimate some combination of the two curves. This indeterminacy is compounded when we desire to estimate two or more separate but interrelated demand functions. We must first weed out the supply influences and then separate the demand influences by mode. These are very difficult problems. In short, simple estimation procedures which deal with single demand functions for several modes of transport may (1) not capture the demand influences at all, and/or (2) not discriminate between the modal demands in a meaningful manner.

We have attempted to overcome these difficulties by constructing an economic model of the transportation market which takes account of the influences of several separate modal supply curves and then deriving our estimating equations directly from this model. This procedure has been only partially successful. In order to construct a model which at least partially reflects the real world we have had to introduce the associated costs of transportation. These costs differ for each mode of transport and have not been captured in any of the data collected thus far. Therefore, it has been impossible to estimate our entire model and, in general, we have not been able to derive estimates of the underlying structural parameters. That is, we also are unable to estimate meaningful "modal demand curves." We have been able, however, to estimate the reduced form equations relating the quantity carried, by mode, to the exogenous variables contained in the model. These estimates allow us to predict the quantities which will be carried by the various modes of transport given specified levels of the exogenous variables and also to predict the changes in quantities as these variables change in value. That is, if more of a product is produced in some shipping area we shall be able to predict the increase in the transport of that commodity. If desired, we can construct elasticities of demand, by mode, with respect to these exogenous variables. However, from knowledge of the reduced form parameters alone, we are not able to derive relationships between the endogenous variables. That is, if some structural change should take place, say a change in the cost structure of one mode, we shall not be able to predict how this will affect the quantities carried by that mode or by the other modes.

Our estimates do allow us to make qualitative comparisons of several structural parameters and their effects upon selected endogenous variables. We are able to compare the sensitivity of the transport market in terms of total quantity shipped and total shipping costs borne by the shipper for changes in the rate structure of the various modes. These estimates can also be converted into estimates of elasticity ratios. These results, while falling far short of the full range of estimates we had wished to obtain, were all that could be derived from the available data. In fact, it may be contended that the data base is insufficiently structured and not statistically sound enough to produce reliable estimates of even this limited number of parameters.

Our estimates should be viewed, therefore, as first approximations, as the best that can be achieved at the present time, but as only fragmentary evidence of what may be found when a thorough, detailed analysis of the transport market becomes possible. We hope that our theoretical development, the specification of the variables which must be considered, will assist and encourage the gathering of more complete and comprehensive data. We shall first discuss the reduced form parameter estimates and then comment upon the derived sensitivity ratios.

Table 4.49 contains our final parameter estimates for the quantity equations. Although the overall fit of the equations is not overly impressive it should be noted that all of the equation F values are significant and approximately half of the R^2 values are of respectable size. Since the dummy variable was set up to assume a value of one for distances less than two hundred miles and zero for greater distances, the assumption of motor transport being dominant for the shorter shipments with rail becoming competitive only after several hundred miles yields the expectation that this coefficient would display a negative sign for rail and a positive sign for motor transport. This expectation is realized for only one of the commodity groups. Group 26, Pulp, Paper and Allied Products, yielded the expected results. For the other commodity groups all coefficients were positive. This implies that, for the shipments covered by our data at least, all modes of transport share in the short hauls.

Five of the nine estimated intercept terms turned out negative. For the usual demand equation this would be a disastrous result. However, it is rather to be expected in our reduced form equations. A glance at these equations, (4.51) and (4.52), reveals that each intercept

Table 4.49

DEMAND REGRESSIONS

FINAL COEFFICIENT ESTIMATES
(All Distances)

Group and Mode	Intercept	D	M	M^2	P_i	C_j	C_i	P_j	F	R^2	n
STCC Group 26											
Rail	6	-2	-0.007	0.002	0.006*	0.010*	-0.006*	-0.004	3.08*	.28	245
Motor	-13	59*	-0.037	0.011	0.038*	0.050*	0.003	-0.006	21.19*	.61	245
STCC Group 29											
Rail	59,604	20,849	-109.800*	0.037*	0.483*	-0.322	-0.354	0.563*	8.56*	.26	176
Motor	232,560	453,030*	-510.150	0.188	1.285	0.472	-2.912	1.721	5.03*	.17	176
Water	-1,256,200	1,146,200	566.160	-0.168	35.362	87.740	-3.203	0.910	9.32*	.28	176
STCC Group 30											
Rail	-4	1	0.002	-0.001	0.004*	0.022*	0.010	-0.004*	5.98*	.36	277
Motor	-1	6	-0.009*	0.003	0.040*	0.044*	-0.013	0.003	38.12*	.70	277
STCC Group 32											
Rail	3	137*	-0.108*	0.008	-0.006	-0.054*	0.031*	-0.014	-9.61*	.52	177
Motor	-16	403*	-0.289*	0.023	0.024	0.141*	0.042	-0.037	11.83*	.56	177

*Denotes an estimate statistically different from zero at the $\alpha = 0.05$ level.

term contains one bracketed expression, (βb_1-1), which is negative by assumption. If this expression dominates, the overall intercept term will be negative. It should also be noted that whenever the intercept is estimated as negative the dummy variable coefficient is estimated to be positive.

In general we expected the mileage coefficients to be negative. Our theory assumes that for each commodity group the total amount transported would decrease with distance. This assumption was valid. However, if one mode enjoys a great advantage in the shorter hauls and another in the longer ones, the latter mode may display a positive distance coefficient and remain consistent with our theory so long as this positive effect is more than compensated for by the reduction in tonnage for the former mode. Two of the nine estimated coefficients came out positive and both of these were offset by negative intercept terms. Water shipments of Group 29, Petroleum and Coal Products, increased rather largely with distance and rail shipments of Group 30, Rubber and Miscellaneous Plastics Products, increased a very small amount. However, all of the statistically significant estimates showed a negative mileage effect.

Without exception the signs of the estimated coefficients of the square of distance were the opposite of the sign of the distance coefficients. This coefficient, however, was never large enough in value to completely override the influence of the first degree term. That is, if the distance coefficient was negative and the distance squared coefficient positive, for the relevant range, the net effect of distance on quantity was negative.

The model assumes that the P_i and C_j will exert a positive influence on quantities transported whereas C_i and P_j will display a negative influence. This expectation was realized in most cases. All of the P_i coefficients displayed the proper sign, and only one C_j coefficient came out negative. All statistically significant estimates were of the proper sign. The C_i and P_j coefficients are less satisfactory. Two C_i coefficients displayed positive signs and four of the P_j coefficients, one statistically different from zero, are also positive. This necessitated the basing of the sensitivity ratios solely on the P_i and C_j coefficients.

The derived sensitivity ratio and elasticity ratio estimates are summarized in Table 4.50. In all cases the transportation market appears much more sensitive to changes in motor rates than to changes in rail rates. This was an implicit assumption of our theoretical model and is here verified empirically. The condition is most easily described by referring to Fig. 4.1. If a modal split is to occur between two markets, and if we expect the railroad to carry most of the shipments with truck becoming competitive only as more of the product is transported so that the railroad can no longer easily carry all of the tonnage, the modal total cost curves must cross in the manner illustrated. Under these conditions it is easy to demonstrate that the supply of transportation curve is affected more by changes in truck supply than by changes in rail supply. This implies that the total quantity market will be influenced in a similar manner.[20]

20. If we define the modal cost curves as

$$C_r = a_r + b_r Q_r$$
$$C_m = a_m + b_m Q_m$$

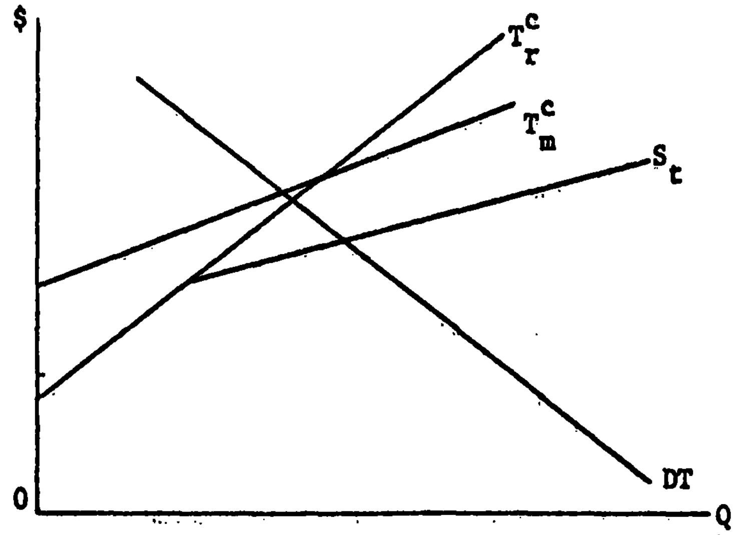

Fig. 4.1 -- A Typical Modal Split

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then

$$Q_r = \frac{C_r}{b_r} - \frac{a_r}{b_r}$$

$$Q_m = \frac{C_m}{b_m} - \frac{a_m}{b_m}$$

and, setting total costs equal,

$$Q = Q_r + Q_m = C\left(\frac{1}{b_r} + \frac{1}{b_m}\right) - \frac{a_r}{b_r} - \frac{a_m}{b_m}$$

so that

$$\frac{\partial Q}{\partial a_r} = -\frac{1}{b_r} \text{ and } \frac{\partial Q}{\partial a_m} = -\frac{1}{b_m} .$$

Hence

$$\left|\frac{\partial Q}{\partial a_r}\right| < \left|\frac{\partial Q}{\partial a_m}\right| \text{ if and only if } b_r > b_m,$$

which is the way we have constructed the functions in Fig. 4.1.

Table 4.50

DEMAND ANALYSIS

DERIVED ESTIMATES

Commodity Group	Market Sensitivity Ratios		Elasticity Ratios			
			500 miles		1,000 miles	
	Sens. to Δ rail rate / Sens. to Δ truck rate	Sens. to Δ water rate / Sens. to Δ truck rate	$\frac{E_{Q,T^r}}{E_{Q,T^m}}$	$\frac{E_{Q,T^w}}{E_{Q,T^m}}$	$\frac{E_{Q,T^r}}{E_{Q,T^m}}$	$\frac{E_{Q,T^w}}{E_{Q,T^m}}$
Group 26 Pulp, paper and allied products	0.18		0.09		0.08	
Group 29 Petroleum and coal products	0.04	75.55	0.21	3.78	0.03	5.14
Group 30 Rubber and miscellaneous plastics products	0.20		0.13		0.12	
Group 32 Stone, clay and glass products	0.38		0.15		0.21	

This situation is also in accord with the rate analysis where we found that rail rates were, as a rule, much lower than motor rates. Thus, unless there is a quality of service differential favoring truck equal to the rate differential, rail will have the initial advantage. The quality of service differential for these product groups is probably very small as they are non-perishable, bulky, and except for Group 30, heavy and low or medium priced products. Group 32, Stone, Clay and Glass Products, displays the largest ratio indicating the least change in comparative advantage over the observed tonnage range. Group 29 indicates the most change.

Applying the same reasoning to the three-mode model, we see that water transport clearly dominates the market when large quantities are to be moved. This is again an obvious result.

The elasticity ratio estimates are included in the table for completeness and to allow us to again comment that as freight rates increase with distance, rate or associated cost elasticities of demand will, in general, increase as the markets under consideration are farther apart.

APPENDIX A TO CHAPTER FOUR

The Effects of Annual Tonnages Upon Rail Rates

The behavior of the annual tonnage coefficient estimates in the rail regressions requires special consideration. In the first two sections of that analysis (those dealing with all commodities and with commodity group totals) the tonnage coefficients were *always* estimated with a negative sign and all but four were statistically different from zero. The estimates did not behave so well in the final set of regressions. With the less aggregate data, and including variables representing price and density, six of the ten tonnage coefficients were estimated as positive. Only two of these were significantly positive but *none* of the four negative coefficients was significant.

Upon observing these estimates it was decided to investigate the effect of the quantity variable further. A new set of regressions was run for each commodity group using the same data but forcing the tonnage variable to enter the estimating equation at the second step, distance being retained as the first independent variable. Tables 4.A.1 through 4.A.5 contain the results of these runs.

The tonnage coefficient estimates definitely are not stable. They shift greatly in size and frequently change sign. For Groups I and V, the estimates are at first negative and then, as other variables enter the equation, become positive. For Groups III and IV, the estimates are first significantly negative and then move in the direction of positiveness, losing their significance in the process. The tonnage coefficient for Group II first appears positive, then goes negative, and finally returns positive, never achieving significance.

Table 4.A.1

RATE REGRESSION RAIL QUANTITY ANALYSIS

GROUP I: PRODUCTS OF AGRICULTURE

ESTIMATED COEFFICIENTS

Intercept	Distance	Quantity	Price	Density	R^2
48.62	1.47*				.87
51.10	1.47*	-1.66			.87
-30.88	1.45*	2.56	0.61*		.88
575.92	1.35*	14.89*	0.23*	-12.71*	.90

*Indicates an estimate significantly different from zero, in the direction indicated by its sign, at the $\alpha = .025$ level.

SIMPLE CORRELATION COEFFICIENTS

	Quantity	Distance	Price	Density	Rate
Quantity	1.000	-0.172	-0.107	0.281	-0.164
Distance		1.000	0.096	-0.459	0.933
Price			1.000	-0.450	0.190
Density				1.000	-0.564
Rate					1.000

Table 4.A.2

RATE REGRESSION RAIL QUANTITY ANALYSIS

GROUP II: ANIMALS AND PRODUCTS

ESTIMATED COEFFICIENTS

Intercept	Distance	Quantity	Price	Density	R^2
332.42	2.23*				.82
326.10	2.23*	28.66			.82
54.36	2.10*	-7.37	0.61*		.84
477.43	2.02*	7.70	0.54*	-17.25*	.85

*Indicates an estimate significantly different from zero, in the direction indicated by its sign, at the α = .025 level.

SIMPLE CORRELATION COEFFICIENTS

	Quantity	Distance	Price	Density	Rate
Quantity	1.000	0.050	0.074	0.010	0.053
Distance		1.000	0.301	-0.336	0.903
Price			1.000	-0.238	0.420
Density				1.000	-0.423
Rate					1.000

Table 4.A.3

RATE REGRESSION RAIL QUANTITY ANALYSIS

GROUP III: PRODUCTS OF MINES

ESTIMATED COEFFICIENTS

Intercept	Distance	Quantity	Price	Density	R^2
145.00	0.87*				.78
153.93	0.86*	-0.99*			.79
93.69	0.84*	-0.68*	3.91*		.81
494.81	0.82*	-0.39	2.99*	-6.36*	.83

*Indicates an estimate significantly different from zero, in the direction indicated by its sign, at the α = .025 level.

SIMPLE CORRELATION COEFFICIENTS

	Quantity	Distance	Price	Density	Rate
Quantity	1.000	-0.117	-0.140	0.150	-0.160
Distance		1.000	0.190	-0.179	0.885
Price			1.000	-0.262	0.321
Density				1.000	-0.344
Rate					1.000

Table 4.A.4

RATE REGRESSION RAIL QUANTITY ANALYSIS

GROUP IV: PRODUCTS OF FORESTS

ESTIMATED COEFFICIENTS

Intercept	Distance	Quantity	Price	Density	R^2
269.02	0.98*				.94
299.90	0.97*	-14.26*			.94
281.94	0.96*	-13.42*	0.32*		.94
870.73	0.96*	-3.85	0.20	-15.47*	.94

*Indicates an estimate significantly different from zero, in the direction indicated by its sign, at the $\alpha = .025$ level.

SIMPLE CORRELATION COEFFICIENTS

	Quantity	Distance	Price	Density	Rate
Quantity	1.00000	-0.19660	-0.19615	0.41537	-0.23630
Distance		1.00000	0.53079	-0.07798	0.96700
Price			1.00000	-0.17551	0.53879
Density				1.00000	-0.16256
Rate					1.00000

Table 4.A.5

RATE REGRESSION RAIL QUANTITY ANALYSIS

GROUP V: MANUFACTURES AND MISCELLANEOUS

ESTIMATED COEFFICIENTS

Intercept	Distance	Quantity	Price	Density	R^2
286.69	1.91*				.60
363.58	1.88*	-102.28*			.60
243.68	1.78*	-82.16*	0.40*		.66
1394.47	1.71*	15.11	0.25*	-31.29*	.73

*Indicates an estimate significantly different from zero, in the direction indicated by its sign, at the $\alpha = .025$ level.

SIMPLE CORRELATION COEFFICIENTS

	Quantity	Distance	Price	Density	Rate
Quantity	1.00000	-0.14261	-0.08787	0.28913	-0.18602
Distance		1.00000	0.18421	-0.18916	0.77158
Price			1.00000	-0.34869	0.38040
Density				1.00000	-0.47689
Rate					1.00000

The overall tendency appears to be that the tonnage variable exerts a negative influence on rate when tonnage and distance are the only explanatory variables. Then, the negative influence becomes less, sometimes changing into a positive influence, as other variables are entered into the equation. This finding is confirmed in Table 4.A.6 where we have again utilized the pooled data for all five commodity groups. Here the coefficient is first significantly negative and then proceeds to become positive, never losing its significance.

This table also shows the simple correlation between the tonnage and rate data as -0.098. Further processing of these data show that the partial correlation of tonnage and rate, given the distance data, and the partial correlation of tonnage and rate, given distance and density, is positive. Hence, if we believe that mileage and density belong in our regression equations along with annual tonnage we must conclude that the true effect of tonnage on rate is positive. When we have excluded one of the above variables from our analysis, we have misspecified the equation and the effect of the excluded variable upon both rate and tonnage forces the tonnage coefficient to be biased negatively.[1]

1. In some earlier work with Group V, Manufactures, we tried different tonnage data in less complex equations. In these equations we entered only distance and tonnage and found that the present definition of Q, as annual tonnage of the k-th commodity, gave more significant results than defining it as either the state-to-state annual tonnage of (1) all manufactured products or (2) all commodities. We believe this conclusion would hold for the other groups as well.

Table 4.A.6

RATE REGRESSION RAIL QUANTITY ANALYSIS

ALL COMMODITY GROUPS

ESTIMATED COEFFICIENTS

Intercept	Distance	Quantity	Price	Density	R^2
284.41	1.56*				.59
296.59	1.55*	-5.61*			.59
180.85	1.48*	-3.86*	0.52*		.67
1226.41	1.36*	2.77*	0.35*	-23.92*	.73

*Indicates an estimate significantly different from zero, in the direction indicated by its sign, at the $\alpha = .025$ level.

SIMPLE CORRELATION COEFFICIENTS

	Quantity	Distance	Price	Density	Rate
Quantity	1.00000	-0.08072	-0.05146	0.18655	-0.09819
Distance		1.00000	0.13907	-0.26000	0.76585
Price			1.00000	-0.36151	0.38354
Density				1.00000	-0.52092
Rate					1.00000

CHAPTER V

SUMMARY AND CONCLUSIONS

Researchers engaged in the estimation of demand relationships recognize that goods and services usually exist which are close substitutes for those they are directly investigating. However, this recognition is seldom transformed into action. Ideally, these substitute goods should be dealt with in the estimation process. One common and easy method is to enter the prices of these goods in the demand equations. Usually this is done in a very ad hoc manner with little consideration given to what the estimated coefficients may actually represent.

This approach is perhaps most common in the field of transportation. Observing many transportation links and networks which are served by two or more modes of transport, most people believe these modes are substitutes for one another, more or less perfect, depending upon the level of their "quality of service differentials." However, most attempts to estimate the demand for these transport modes are hampered by an inability, or a lack of desire, to explicitly deal with this substitutability.

This paper has attempted to construct an economic model of a transportation market. We employed traditional economic methods and used as our starting point the derived demand for transportation. By quantifying the variables usually considered as determinants of quality of service, and expressing these as costs associated with shipment by the respective modes of transport, we were able to treat these different modes as perfect substitutes. This allowed us to view the shipper's choice of mode decision as a simple problem of cost minimization. Emphasizing at all times the interdependencies within the market, we were able to investigate the conditions under which a modal split may occur and to derive the "demand curves" confronting the individual modes.

In Chapter I we reviewed the structure and functioning of a transportation market. Possible reasons for investigating the workings of such a market were also discussed. The best of the current economic models of the transportation process were classified as basically "multi-market single-mode transport models." These models have probed deeply into the relationships between product demands and supplies and transport demands. However, these investigations have been possible only by almost completely ignoring the supply side of the transportation market. Our "two-market multi-mode transportation model" was then proposed as the next step in the development of a full multi-market multi-mode model of transportation.

Chapter II contained a review of the transportation literature. In a rather thorough review of the theoretical literature we traced the major steps in the development of transportation models from 1838 through 1964. It was found that these studies have become increasingly concerned with the case where many markets are served by a single mode of transportation, or several modes displaying quite simple supply attributes. Selected recent empirical works were then reviewed. These works, in contrast with the theoretical studies, were concerned with determining quantities carried by different modes of transport. In constructing a theoretical basis for their empirical studies the authors were forced to utilize the traditional two-market model, adapting it to consider several different modes of transport. An exception to this approach was E. Silberberg. He followed the path of the theoreticians and estimated a multi-market single-mode model adapted to recognize the existence of a competing mode of transport.

In Chapter III we developed our two-market multi-mode transportation model. The construction of the transportation market began with the classical derivation of the demand for transportation for a single commodity between two markets. Introduction of a single supply of transport then brought this model up to date. Our first generalization was the introduction of several competing sources of supply, each of which was considered as a perfectly competitive industry. The concept of shipper cost was then introduced. All former models of transportation seem to operate on the assumption that the direct transport rate is the only cost of direct concern to the shipper. We put forth the assumption that all of the components of quality service can and should be considered also as costs which the shipper must take into account. Time preference costs, inventory costs, schedule and convenience costs, feeder-line costs and interface and congestion costs were then considered, and the connection of these costs with the definition of the supply of transportation was discussed.

The transportation model was then extended to include all these components of shipper costs. This was accomplished in two steps. First, transportation associated costs were considered in a market with only a single supply of transportation. Then, several supplying modes, each with individual associated cost characteristics, were included in the model.

The complete model was then cast in terms of a simultaneous equation system and a qualitative analysis was undertaken. This analysis allowed us to investigate the effects of changes in system parameters upon the equilibrium values of all the endogenous variables. It was found that by making quite plausible assumptions concerning the signs of simple partial derivatives we were able to unambiguously derive the signs of the resulting changes in solution values of the variables. It was also shown that in many cases statements could be made concerning the relative sizes of the effects. That is, we were able to show that a five-cent increase in the transport rate for a certain mode would have the same effect upon the quantities carried by the various modes and shipper's costs as a five-cent increase in associated costs for that mode. A complete listing of these theorems is contained in Table 3.2

In the three appendices to Chapter III we derived measures of response for the transportation market. Appendix A contained a derivation of the elasticity of transport demand in terms of the elasticities of the product supply and demand curves. Appendix B showed the derivation of the elasticity of the "demand" for a particular mode of transport. This elasticity was presented in terms of the elasticity of the demand for transportation and the elasticities of the transport supply and associated cost functions.

In Appendix C to Chapter III we investigated the implications of allowing one of the transport modes to be composed of a single large firm. This model, with its obvious reference to rail transport, views this one, monopolistic, industry as competing directly with one or more competitive industries. Applying this model first to a transportation market with no associated costs, the conditions under which a stable solution will arise were determined, and the welfare implications of this solution investigated. The model was then expanded to include associated costs for both the monopolist and the other modes. Finally, we investigated the behavior of the monopolist when he is able to exercise some control over the quality of his service. Now, varying the quality of a transport product is actually only a special case of product differentiation, so that this analysis is quite analogous to that of a firm considering the effects of, say, advertising expenditures. Both categories of expenditures allow the firm some control over its "demand curve," and in each case the firm must consider both the cost of the control and its effects upon the market.

Chapter IV reports on our empirical investigations. To allow the theoretical model to be estimated certain specializations were required. First, the equations were assumed linear in all variables. This allowed the application of standard estimating techniques. Second, since all available data was of a cross-sectional variety, adjustment variables were added to each equation to compensate for the differences in market size and shipment distance. Finally, in recognition of the fact that transport rates are closely regulated by the Interstate Commerce Commission and, therefore, at least slow in responding to market forces, the rates were specified as being exogenous to the remainder of the system.

These specifications resulted in a simultaneous linear equation system. Since the transport rates were assumed to be exogenously determined, they were estimated independently of each other and of the rest of the system. The rate equations were then substituted into the system to derive a linear structure of the first order. The reduced form of this structure was then obtained and utilized in estimating demand equations for the individual transport models.

Prior to our study a few simple estimating equations for rail and motor rates existed. These usually expressed the rates as linear functions of distance and, perhaps, distance squared for individual commodities. By utilizing published sources of rail data, and obtaining comprehensive shipment data from private sources for motor and water transport, we were able to conduct a thorough analysis of transport rates for these three modes. Our approach was to relate the average revenue per ton of the commodity shipped to the distance it was shipped, a quantity variable and several product attributes. For rail shipments the quantity variable was the total annual tonnage shipped between the two markets. For motor and water transport the size of the individual shipments was available. The carload waybill statistics yielded observations on the average density and estimated prices of the commodities. These were used as product attributes.

Rate regressions were estimated for all five of the major AAR commodity groupings. Then, after conversion of the commodity groups, rate parameters were estimated for the four STCC groups selected for reduced form estimation.

Distance proved to be the best single explanatory variable for rail and water transport, whereas shipment size was the most important for motor transport. The quantity variable also proved significant in explaining water rates, whereas its effects were quite mixed for rail. These quantity relationships for rail transport were explored further in the appendix to that chapter. The product attributes, while contributing to the explanatory power of the equations, did not exhibit as strong an influence as distance or quantity.

The reduced form equations expressed the endogenous variables as functions of the exogenous variables. Ideally, we would have estimated these equations and derived estimates of all structural parameters. However, no data existed on transport associated costs, nor was it possible to estimate this variable from the available data. Consequently, we could estimate only the quantity equations and were unable to derive estimates of individual structural parameters. However, in the qualitative analysis performed in Chapter III it was shown that most changes in the system are represented by groups of parameters. This indicated that useful knowledge may perhaps be gained without complete identification. We then performed a complete qualitative analysis for the estimating model. This analysis, the results of which were presented in Table 4.27 indicated that, with only partial estimation of the reduced form, *comparative* statements could be made concerning changes in shipper's cost and quantities shipped brought about by changes in freight rates or quality associated costs.

For the two-digit Standard Transportation Commodity Classification groups 26, 29, 30 and 32 we regressed quantity shipped, by mode, on distance, a dummy variable indicating short or long haul, distance squared, and a vector of market attributes. In one set of regressions the market attributes were production of the commodity in the exporting region and consumption of the commodity in the importing region. For the other set of regressions, we added consumption in the exporting market and production in the importing market.

These parameter estimates indicated that transportation market responses, in terms of shipper's cost and quantities shipped, were about five times greater for a unit change in motor rates or associated costs than for unit changes in rail cost parameters for Groups 26 and 30. For Commodity Group 32 the response to changes in motor costs was almost three times greater than for rail cost changes, whereas for Group 29 the responses were approximately equal. Estimates of the comparative rate elasticities of transport demand were also derived.

In general the overall fits of these reduced form regressions were not too good. It is felt that this was due primarily to the nature of the model and the available shipment data. Attempting to estimate a derived demand from cross-sectional data is difficult, even with the best possible data. A comprehensive analysis of the individual markets over time is required if reliable parameter estimates are to be obtained.

We have attempted to demonstrate that several modes of transport can be handled in a simultaneous equation model, analogous to the manner in which several markets have been handled previously. We believe that this has been demonstrated satisfactorily, and hope that it will contribute to the eventual development of a complete multi-market multi-mode model of production, transportation and consumption.

BIBLIOGRAPHY

Baumol, W. J., "Calculation of Optimal Product and Retailer Characteristics: The Abstract Product Approach," *Journal of Political Economy*, Vol. 75, No. 5 (Oct. 1967) 674-685.

Benishay, H., and G. R. Whitaker, Jr., *Demand and Supply in Freight Transportation*, The Transportation Center, Northwestern University, 1965.

Cournot, A. A., *The Mathematical Principles of the Theory of Wealth* (1838), Irwin, Homewood, Ill., 1963.

Crecine, J. P., L. N. Moses, and J. Stucker, "The Census of Transportation: An Evaluation," *Papers -- Seventh Annual Meeting*, Transportation Research Forum, Richard B. Cross, Oxford, Ind., 1966.

DeSalvo, J. S., *Linehaul Process Functions for Rail and Inland Waterway Transportation*, unpublished Ph.D. dissertation, Northwestern University, 1968.

Enke, S., "Equilibrium Among Spatially Separated Markets: Solution by Electric Analogue," *Econometrica*, Vol. 19 (1951), 40-47.

Haley, K. B., "The Multi-Index Problem," *Operations Research*, Vol. 11, No. 3 (1963), 368-379.

Howe, C. W., "Methods for Equipment Selection and Benefit Evaluation in Inland Waterway Transportation," *Water Resources Research*, Vol. 1 (1965), 25-39.

Hurter, A. P., Jr., "Some Aspects of the Demand for Inland Waterway Transportation," *The Economics of Inland Waterway Transportation*, The Transportation Center, Northwestern University, 1965.

Kobe, S., "Elasticity of Derived Demand for Transport Service," *Waseda Economic Papers*, Vol. III (1959), 48-59.

Koopmans, T., *Activity Analysis of Production and Allocation*, Cowles Commission Monograph No. 13, John Wiley, New York, 1951.

Lancaster, K. J., "A New Approach to Consumer Theory," *Journal of Political Economy*, Vol. 14 (1966), 132-157.

Lewis, R. W., E. F. Rosholdt, and W. L. Wilkinson, "A Multi-Mode Transportation Network Model," *Naval Research Logistics Quarterly*, Vol. 12 (1965), 261-274.

Meyer, J. R., M. J. Peck, J. Stenanson, and C. Zwich, *The Economics of Competition in the Transportation Industries*, Harvard University Press, Cambridge, 1960.

National Motor Freight Traffic Association, *Table of National Motor Freight Classification Numbers with Applicable Standard Transportation Commodity Code Numbers*, Washington, D. C., June 1965.

Oi, W. Y., and A. P. Hurter, Jr., *Economics of Private Truck Transportation*, Wm. C. Brown, Dubuque, Iowa, 1965.

Perle, E. D., *The Demand for Transportation; Regional and Commodity Studies in the United States*, Department of Geography, University of Chicago, (Planographed), 1964.

Quandt, R. E., and W. J. Baumol, "The Demand for Abstract Transport Modes: Theory and Measurement," *Journal of Regional Science*, Vol. 6, No. 2, 13-26.

Samuelson, Paul A., "Spatial Price Equilibrium and Linear Programming," *American Economic Review*, Vol. XLII, No. 2 (May 1962), 283-303.

Silberberg, E., *The Demand for Inland Waterway Transportation*, unpublished Ph.D. dissertation, Purdue University, 1964.

Simon, H. A., "Causal Ordering and Identifiability," Chapter III of W. C. Hood and T. C. Koopmans, *Studies in Econometric Method*, Cowles Commission Monograph No. 14, Wiley, New York, 1952.

Smith, V. L., "Minimization of Economic Rent in Spatial Price Equilibrium," *Review of Economic Studies*, Vol. XXX (1963), 24-31.

Takayama, T., and G. Judge, "Equilibrium Among Spatially Separated Markets: A Reformulation," *Econometrica*, Vol. 32, No. 6 (Oct. 1964), 510-524.

United States Department of the Army, Corps of Engineers, "Domestic Inland Traffic Areas of Origin and Destination of Principal Commodities," Supplement to Part 5, *Waterborne Commerce of the United States*, Washington, D. C., 1956-1961.

United States Department of Commerce, Bureau of the Census, "Commodity Transportation Survey," *1963 Census of Transportation*, Parts 1 & 2, GPO, Washington, D. C., 1966.

United States Department of Commerce, Interstate Commerce Commission, Bureau of Economics, "State-to-State Distribution(s)," *Carload Waybill Statistics, 1963*, Statements SS-1 through SS-6, Washington, D. C., Jan. 1966.

United States Department of Commerce, Interstate Commerce Commission, Bureau of Transport Economics and Statistics, *Freight Revenue and Wholesale Value at Destination of Commodities Transported by Class I Line-Haul Railroads, 1959,* Statement No. 6112, File No. 18-C-23, Washington, D. C., Oct. 1961.

VITA

James P. Stucker was born on August 27, 1936, in Cullom, Illinois. He attended primary and secondary schools there. After graduation from secondary school in 1954, he attended Bradley University for one year. He then spent four years as an enlisted man in the United States Navy. Following his release from active service in 1959, he resumed his studies at Illinois State University and was awarded a bachelor's degree in Business Education. After a short period of service with Henning, Strouse and Jordan, a public accounting firm, he entered the Northwestern graduate program in 1963, from which he received a Ph.D. in Economics in August, 1969. Mr. Stucker is at present a member of the professional staff of The RAND Corporation.

NORTHWESTERN UNIVERSITY

A MODEL OF THE DEMAND FOR TRANSPORTATION: THE CASE OF AIR FREIGHT

A DISSERTATION

SUBMITTED TO THE GRADUATE SCHOOL
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
for the degree

DOCTOR OF PHILOSOPHY

Field of ECONOMICS

By

WILLIAM BRUCE ALLEN

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INTRODUCTION

Our goal is to study the demand for air freight. What type of firms ship by air? What kinds of goods travel by air? What are the relevant factors determining the demand for air freight?

In the process of answering these questions a theory of the profit maximizing firm, with transportation costs included, will be considered. A probability model of modal choice, based on the relevant factors of transport demand, will be developed. The model uses a statistical technique known as discriminant analysis, and we are looking for variables which enable us to discriminate air users from users of other transport modes.

The concept of elasticity of choice will be developed from the probability of choice analysis. That is, how sensitive is a shipper's choice of mode to changes in the explanatory variables? In particular, from the airline's point of view, how sensitive is a shipper's choice of mode to changes in the explanatory variables - such as rate, frequency of service, etc. - which are under the control of the air carrier?

Finally, we include the results of the tests of the model.

CHAPTER I

HISTORICAL REVIEW

Although goods were carried in flight almost coincidentally with the invention of the airplane, the air cargo industry really developed after World War II.[1] Military experience showed shippers that goods could physically be moved by air - both in terms of volume and in terms of reliability. After the War, some shippers, studying their transportation needs, found that certain goods or situations could economically command air use. That is, studies indicated that air cargo had become the cheapest way of sending these goods.

Recently air cargo has been growing at the rate of twenty percent per year. Such a growth rate is somewhat deceiving because of the small base involved. Air freight only amounts to one tenth of one percent of all cargo ton-miles shipped in this country.[2]

Many studies have been done by the industry to study the potential for air freight.[3] The goal was to determine investment procedures and allocation of resources between passenger services and freight services.

[1]For an exhaustive survey of air freight happenings from 1918 to 1945 see John Frederick, Commercial Air Transportation, fourth edition, (Homewood, Illinois, Richard D. Irwin Inc.,1955).

[2]1963 Census of Transportation, Commodity Transportation Survey - Production Area Series - Area 2, Advance Report TC63 (A)C2-2, (Washington D.C., U.S. Department of Commerce, Bureau of the Census, 1963), p. 4, table C.

[3]See [4], [5], [6], [7], [8], [11], [12], [13], [15], [16], [19], [23], [27], [29], [34], [36], [40], [44], [45], [46], [47], [48], [52], [53], [54], [64], [66], [67], [72], [77], [78], [84], [85], [87], [88], [93], [94], [98], [103], [110], [112], [117], [119], [122], [128].

Other studies have been done by the academic community.[4] A few dissertations have been written in the field.[5] We will review the above mentioned material, but our conclusion will be that no study has provided a satisfactory explanation for why shippers will ship by air or of how much shippers will ship by air. This is shown by the facts that all past forecasts have been overly optimistic and that the airlines enthusiastically responded to our offer to explore this area.

Some goods like industrial diamonds, hormones, pearls, platinum and certain furs move virtually 100% by air. Other goods such as oil, coal and basic foodstuffs never move by air. Still other goods like fresh onions, cigarettes, perfumes, resins, projectors and typewriters move by air and also by other modes. More specifically, in the latter case, the same good is shipped by different modes between locations i and j. This reflects demand conditions, or the firm's cost structure or both. What types of goods are "air types" will be mentioned in the empirical section.

Observation of the types of goods carried by air indicate that two characteristics of shipments tend to predominate - those of high value and those of low density. These two characteristics are modified by the geographic concentration of the source of supply or demand (e.g. if an area is only accessible by air); the average length of haul (which attempts to measure the market are of the good); perishability (both physically-related and obsolescence-related); and seasonality (which attempts to measure a demand for speed).

[4]See [14], [20], [21], [22], [24], [25], [26], [28], [30], [31], [32], [33], [37], [38], [41], [43], [56], [61], [63], [68], [69], [80], [83], [102], [103], [104], [107], [113], [114], [116], [127].

[5]See [96], [100], [123].

Why high value goods should be carried by air or are considered "air candidates" can be better appreciated when the theoretical section on transport costs and transport-associated costs and the theory of the firm are viewed. Here we cite empirical evidence by Brewer [26] that more than sixty percent of the commodities now being transported by air have values in excess of three dollars per pound. It is argued by the proponents of the high value thesis that high rates can be "absorbed" better by high value goods.[6]

That low density (pounds per cubic foot) prevails can be witnessed in air managerial statements that planes "bulk" out before they "weight" out. That is, the weight capability of the planes is seldom met because all the volume is filled.

Shipments also tend to be small. More than seventy percent are less than one hundred pounds.[7]

Initial analysis shows that pick-up and delivery service is basically limited to areas within a twenty-five mile radius of the airports. Of course, within this area (and out of it also), the shipper can take the good to the terminal himself, hire a common carrier to do so, or give his business to a freight forwarder.

[6]It should be pointed out that if rates are stated in ton-mile terms, air may not be at as great a disadvantage as first appears. For if the shipper is traditionally a surface shipper, he will neglect the fact that great circle distances are shorter than surface point-to-point distances. Since he thinks of distance between i and j in terms of surface miles, air cost will always be overstated. See Stanley Brewer, Vision of Air Cargo, (Seattle, Bureau of Business Research, University of Washington, 1957), p. 27.

[7]Stanley Brewer, John Thompson, and William Boore, The Aircraft Industry - A Study of the Possibilities for Use of Air Freight, (Seattle, University of Washington, 1960).

Recently air freight has grown away from its previous joint-product status. While all-cargo carriers exist (e.g. Flying Tiger, Slick), the bulk of the business is carried by airlines best known for their passenger business. A change has occurred in the operation of these carriers. Approximately sixty percent of their cargo business is carried on all-cargo flight. Some of this is done by all-cargo planes (in some cases old propeller driven passenger planes converted for freight use, but in more and more cases new jet aircraft specifically designed for freight). The rest is done by quick-change (Q-C) aircraft; aircraft which serve passengers during the day and cargo during the night. Forty percent (and an always decreasing percentage) is carried in the belly of planes on passenger runs. At one time 100% was in this category. In this last case, freight is the low priority item ranking behind the passenger, his baggage, and mail.

Air's natural advantage is in the speed it has to offer. It also "sells" other time related services such as reduced warehousing, reduced inventory, ability to take advantage of a time-limited situation, less spoilage, etc. In addition, other services are sold in the form of less packaging, less pilferage, less damage, etc. These services, the <u>more</u> of the desirable items like speed and the <u>less</u> of the undesirable items like damage, are compensated for at a rate which is higher than other modes charge. Thus, for the same amount shipped, the transportation bill will be higher for air than for other modes, i.e. $r_a\bar{Q} > r_s\bar{Q}$ where r_i is the rate by a (air) or s (sea) and $\bar{Q}$ is the amount shipped.

However, the transportation bill is not the only cost of a firm which is affected by the mode of transport used. Mode of transport also influences the firm's costs of inventories, warehousing, packaging, insurance, etc.

If the firm's goal is to maximize profits, then it must analyze the effect of a mode choice on its profit picture. Again we stress that this entails analysis beyond the transport sector. The firm must consider a profit analysis which surveys the entire company.[8]

Many of the difficulties that the air industry has had, and is having, in selling the use of air services stem from a lack of understanding of the above type of analysis. Many firms have autonomous transportation divisions which render the firm's transportation decisions so as to minimize transport costs for shipping outputs, the levels of which have been determined by another division.

Such suboptimization will seldom lead to an overall profit maximization by the firm. In this case, the whole is not equal to the sum of its parts, because some sectors are interdependent upon other sectors. Thus optimizing each individual sector will not necessarily optimize for the total firm (unless one finds the overall optimum position and then defines its constituent parts the suboptima). Therefore, absolutely minimizing transportation costs, for which, under much of current industry thinking, the transportation manager is rewarded, may really result in a profit position which is unfavorable relative to the profit position which is available if a broader, all-firm scope is taken.

[8] See American Management Association, A New Approach to Physical Distribution, (New York, 1967).

For instance, suppose a firm has two sectors x and y. Sector x has a cost function $a^2 - a+5$. Sector y has a cost function $2a^2 - 4a+10$. Together, as a firm, the cost equation is $Z=3a^2 - 5a+15$. Sector x and sector y are inter-related through the additive function Z. Suppose that the goal is to minimize Z. Suppose that sector y must accept the decision of section x. Sector x is obviously minimized at a value of $a=\frac{1}{2}$. However, when both are combined into an operating unit (the firm), the firm achieves an overall minimum at $a=(5/6)$.

To capture this notion of interdependence of sectors, the concept of full distribution costs has arisen[9]. Full distribution costs refer to all costs which are directly or indirectly related to the transportation decision being considered. This concept, in relation to air freight, has evolved quite recently from H.T. Lewis' and J.W. Cullaton's book The Role of Air Freight in Physical Distribution [83]. Prior to this work, air freight was not considered a candidate for the job except for emergencies and perishables. All thinking seemed to gravitate around this limited number of shipping situations. The point to be made from the full distribution cost argument is that higher costs in the transport sector may be offset by lower costs in other sectors to such a degree that it becomes more profitable to ship by the higher price mode.

[9]See [20] p. 5; [25] pp. 46, 61, 62; [26] p. 39; [29] p. xiv; [42]; [56] pp. 16, 26; [64] pp. 11, 13, 54, 67, 71; [65] p. 98; [73] p. 20; [81] pp. 32-33; [82] p. 39; [100] Chapter 4; [102] p. 6; [110] pp. 1, 35, 42, 121; [113] pp. 163, 164, 217, 231; [116]; [123] pp. 44-45; [103] p. A-13.

Most distribution arguments stress the interdependence of the transportation sector with the warehousing, inventory, insurance, etc., sectors. One contribution of this dissertation will be to show an important interdependence effect with the production sector. That is, it will be shown that use of different modes of transport implies different levels of output for profit maximization.

Most of the distribution studies were verbalizations of the problem. Their important function was to suggest variables, which our theoretical analysis attempts to incorporate. If shippers were interested in these costs, it was felt that the analysis undertaken should attempt to answer their questions.

Two basic approaches have been used in past air freight investigations: a macro approach and a micro approach. The macro approach ties air freight with growth in population, Gross National Product, industrial production, and other aggregate indicators.[10] It is hypothesized that a certain relationship, e.g. log linear, exists between the dependent variable (generally air tonnage) and the independent variables. Such procedures do not deal explicitly with the determinants of demand. They cannot, therefore, be used to reveal the influence of changes that are expected to take place in the cost and quality of air freight on tonnage. In a similar vein are some recent air cargo forecasts which are obtained by simple extrapolation or by fitting a growth curve to past data on air cargo tonnages.[11] These forecasts

[10] See [23], [27], [35], [44], [52], [53], [93], [94], [100].

[11] See Northrop Corporation, Air Freight Statistics, (Hawthorne, California, Norair Division, NB 60-265, 1960).

receive the same criticisms as above.

Most micro approaches to the problem were attempts to implement full distribution analysis. Very few studies involved empiricalwork, and most empirical work found was relatively poor in quality and in method. A pioneering work is found in Umpleby's [123] masters dissertation. Although his work is basically verbal, he stresses the importance of the costs of production, market prices, the nature of the demand, substitute modes available, and en-route delays as they affect selling prices.[12] These items play a large role in the analysis contained in the present paper.

Case studies of distribution costs for individual firms are also found in the literature.[13]

Wein [127] attempts to construct a point demand for air freight. Air freight demand is made up of a normal component (what is traveling at today's rates) plus air cargo potential (which is the penetration rate times the tonnages in categories of goods which are air candidates). Prime categories for air are general and household freight traveling by class I, II, and III common and contract carriers, LCL rail freight, LTL truck freight, rail freight forwarder traffic, REA traffic, and surface mail. The penetration rate is calculated by comparing the current air rate versus the competing mode's rate. Multiplying the penetration rate times the tonnage in each category, then adding this to the normal component, yields the demand point for today's rate. Wein generally neglects all other costs

[12] Arthur Umpleby, A Study of the Economics of Air Freight in Production and Distribution, (Columbus, Ohio State University, Masters Thesis, 1957), pp. 23-24.

[13] See [3], [6], [64], [86], [102].

or services in his analysis and concentrates only on rates.

Sealy and Herdon [102] break down the productive processes of machinery and textile firms into four components: (1) transport, (2) inventory, (3) warehousing, and (4) packing. The authors then calculated total distribution costs by air and sea. They found that only two percent of the consignments studied were cheaper by air when assessed on freight charges alone; but that eighteen percent were cheaper when assessed on total distribution costs.[14] Total cost by surface transport was, on the average, the surface freight charge plus 254% (of the surface freight charge).

On the same basis, the total cost by air is the air rate charge plus 45% (of the air freight charge).[15]

From the above a demand curve for air freight was derived. Given the current levels of traffic for good A and rates of a (for air) and s (for sea), the percentage drop in the air rate could be calculated which would make $1.45a < 3.54s$. At that, and all lower rates, all of commodity A would go by air. This was done for all commodities and yielded the graph AA. (Currently 18.6% of the consignments are going by air).

[14] Kenneth Sealy and Peter Herdon, Air Freight and Anglo-European Trade, (London, ATS Ltd., 1961), p. 8.

[15] Ibid, p. 39.

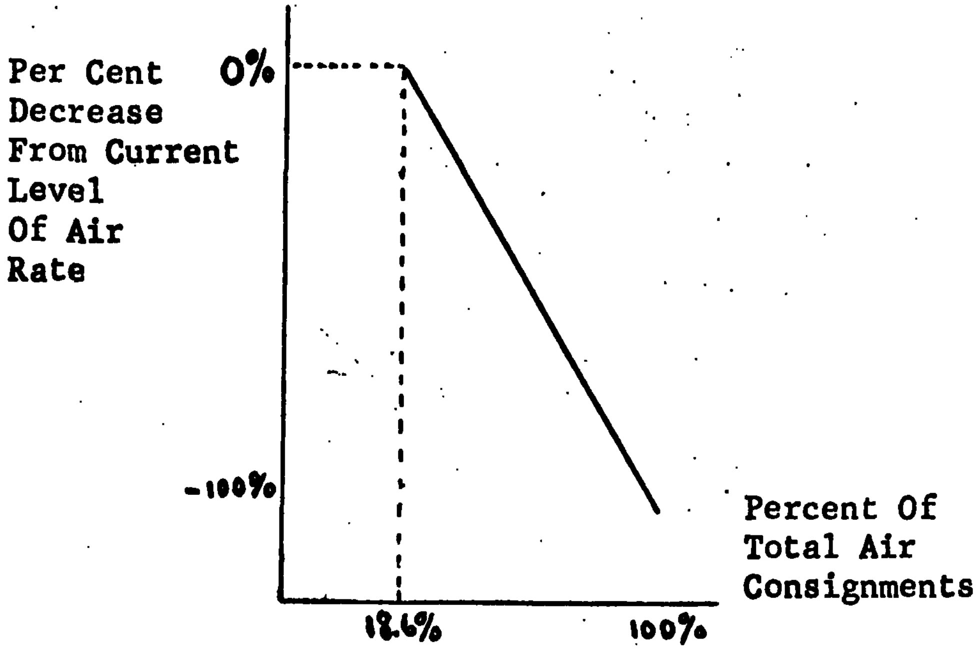

Diagram I-1

Similar types of studies are described in Stratford [113], and [114]. More commodities are surveyed (forty) and more sectors of the firm are considered (six). The type of analysis is referred to as substitution analysis. Unfortunately, none of the empirical results are presented nor were production costs taken into account.

Brewer [27] derives the demand curve for air freight between the United States and Europe for 1957. Using Bureau of the Census data, he chooses to view all commodity groups having an average value per pound greater than $.40. The rationale for this choice is that an average of $.40 per pound means an appreciable proportion of the goods in that category have value of greater than $1.00 per pound, which is his true criterion for air candidacy. Brewer subscribes to a "well developed thesis that high value items have a greater need to use the fastest means of transportation and can withstand higher transport costs than can low value items."[16]

He assumes each distribution of prices in each commodity group is the same, i.e. if the average value per pound is $\bar{x}$, the percentage of goods above $1.00 per pound in that group is easily calculated. Multiplying these percentages times the tonnages actually moved in that commodity group yields a schedule of value per pound versus total tonnage. Assuming that commodities can generally withstand transportation costs between 10-20% of their value, taking 15% of their values yields a schedule of rates versus tonnage which would just move at that rate. Obviously all tonnages moving at a high rate would also move at lower rates. By such a method Brewer traces out a demand curve for 1957 as shown below:

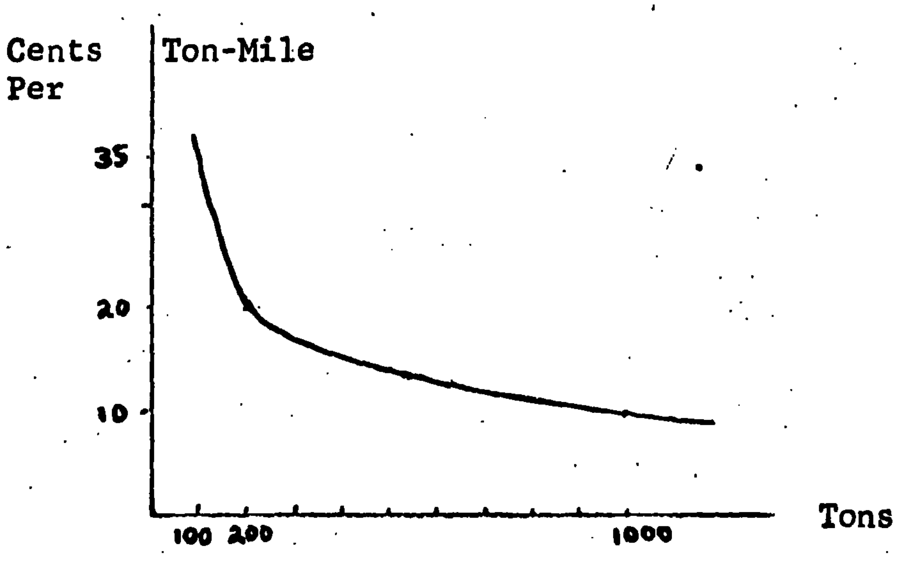

Diagram I-2

[16]Stanley Brewer, The North Atlantic Market for Air Freight, (Renton, Washington, The Boeing Co., 1962), p. 17.

Brewer's analysis depends solely on the value per pound of the good. Such strong allegiance to that single variable has had many followers.[17] However, dissidents have recently attacked the sole use of value per pound as the criterion and purpose that it must be considered along with other transport variables.[18]

Two important theoretical contributions remain to be mentioned. The first is <u>The Demand for Air Cargo Transportation: A Pilot Study</u> by Ferguson and Schnabel [56]. There is a lower limit to the demand for air freight which is set by the volume of traffic currently moving at existing rates of the competing modes. "The extent to which the actual demand curve differs from such a lower limit estimate depends on the difference to the shippers of the value of air as opposed to surface transportation. This difference in the value of air and other forms of transportation is intimately tied to the value of speed in making possible savings in inventory costs."[19]

Output is assumed exogenous. The authors then attempt to design an optimal inventory policy co-ordinated with an optimal transportation policy. The end result states the air rate as an additive function of the surface rate and the change in the expected shortage cost when air is used. No empirical tests are reported.

[17]See [36], [58], [65], [106], [116], [123].

[18]See [48], [59], [113].

[19]Allen Ferguson and Constance Schnabel, <u>The Demand for Air Cargo Transportation: A Pilot Study</u>, (Evanston, Northwestern University Transportation Center, 1960). p. v and p. 17.

The second theoretical work is Baumol and Vinod's Studies in the Demand for Freight Transportation [14]. Both a micro and a macro model are presented. Only the micro model is of interest here. Output is assumed exogenous (although Baumol later states that profit maximization and not transport and inventory cost minimization should be the goal). An inventory type of approach is taken. Quantity transported is a non-linear function of price difference between regions, shipping cost per unit (freight, insurance, etc.), carrying costs per unit, and the level of safety stock. The above relationship is derived from a marginal revenue equals marginal cost framework. This is a form of profit maximization. However, production costs are not included in the cost calculations. Such a suboptimization may not lead to the overall goal of profit maximization.

Both the Ferguson and Schnabel and the Baumol and Vinod studies come under criticism due to the assumption of exogenous output. In addition, the failure to put their maximization or minimization schemes into a proper time-order is criticized. Revenues and costs are being received and incurred throughout their analyses without regard to their sequencing and, hence, without regard to capital costs on tied up funds. Although the model presented in Chapter II does not totally rectify this problem, an attempt is made to incorporate capital costs.

A few studies suggest a model of overall profit maximization, but no formalizations of the problem are presented.[20]

[20] See [48], [52], [77], [113].

A different type of approach is taken by the Stanford Research Institute (SRI) [110] which attempted not to provide an estimate of the demand curve for air freight, but rather to provide a check list for shippers and airlines to attempt to identify characteristics of the <u>firm</u> as a potential user of air freight. SRI advocated a firm approach rather than a commodity approach.

SRI concluded that businesses could have three major reasons for using air freight:

(1). Use of speed to decrease time in transit

(2). Use of speed to decrease the costs of holding goods in inventory

(3). Use of superior conditions of carriage by air to decrease risks and costs associated with alternative transportation

Specifically, speed may enable the firm to increase sales in a time limited situation, increase utilization of productive facilities and equipment, decrease investment in goods in transit, and meet unpredictable demand and emergencies.

Inventory investment can be decreased with increased speed. Also increased speed can decrease the risk of inventory obsolescence, decrease the investment and operating expenses associated with inventory facilities and services, and decrease inventories held by jobbers or wholesalers.

The service characteristics of air freight are such as to decrease the risk associated with having goods lost, stolen, damaged, or spoiled in transit; to decrease costs and time over which provisions for protecting or preserving goods are required; to enhance control or management over goods in transit; to decrease duties in international movements, and to expedite handling of small lots.

A large check list is used, asking the shipper about his shipment policies and desires. If a certain number of questions are answered in the affirmative, a more thorough investigation of air freight possibilities should be undertaken.

Essentially the SRI report was a surrender to the non-quantifiers. All attempts to have an empirical analysis had been fruitless and will continue to be so, concluded SRI. Researchers in the area criticized the report for this conclusion and suggested that its value was in pointing out the parameters to be considered in future empirical analyses.[21]

While transport economics has been citing the <u>in</u>elasticity of transport demand,[22] all writers in air freight have emphasized the elasticity of the demand for air freight.[23] Brewer is typical when he states, "The presence of substantial elasticity in the demand curve for air transportation is well recognized."[24] Sealy and Herdon estimate the elasticity to be 1.7.[25] Echard estimates an elasticity of 3.0.[26]

[21] See [103], [113].

[22] See [95] pp. 82-97, and [99] pp. 174-176.

[23] See [127] p. 6; [29] p. 82; [94] p. 31;[73] pp. 106-107.

[24] Stanley Brewer, <u>Vision in Air Cargo</u>, (Seattle, Bureau of Business Research, University of Washington, 1960), p. 13.

[25] Quoted from Alan Stratford <u>Air Transport Economics in the Supersonic Era</u>, (New York, St. Martin's Press, 1967), p. 221.

[26] E.W. Echard, <u>Free World Air Cargo: 1965-1980 Rate Elasticity Forecast</u>, (Commercial Marketing Research Department, CMRS 77, Lockheed Georgia Co., June, 1967), p. 3.

These substantial estimates of elasticity suggest that reducing air rates will overwhelm the carriers with business. Elasticity is an important planning tool for the carriers and also rate-making bodies. It shall, therefore, be investigated in the context of the model of the dissertation.

As can be witnessed, quite a bit of work has been done in the field of air cargo. Unfortunately, much of it has involved considerable resources but little appreciable output. The goal of this disseratation is to rectify this state and give a study solidly based on economic theory. The outline will be as follows: Chapter II is the micro development of the theory of transport demand. Here demand functions for transportation with respect to rate, time in shipment, damage rate, etc., will be derived for both inputs and for output under all market conditions. Chapter III is the air freight demand model in its statistical form. Herein the discriminant model is developed which will test the theory of the previous chapter. Chapter IV is the adaption of the model to statistical testing and the empirical results of the model. Herein that part of the theoretical model tested is described, as is the way in which the model was adapted to conform to available data. Finally, the empirical findings are presented and discussed.

CHAPTER II

THE MICRO THEORY OF TRANSPORT DEMAND

I

Dupuit, in 1844, said, "The ultimate purpose of a means of transportation ought not to be to reduce the expenses of transportation, but to reduce the expenses of production."[1] Transportation rates are but one member of a transportation package, which includes the costs of production. Since 1844 researchers and theorists have attacked problems calling for the minimization of transport costs or the minimization of transport and transport associated costs (e.g. packaging, inventory costs, etc.), but few have paid attention to Dupuit's emphasis on the relationship of production and transportation. The costs of transportation and the costs associated with transportation (of both inputs and outputs) will be viewed to see how they fit into the conventional spaceless and timeless theoretical analysis of the theory of the firm.

In order to develop a theory of freight transport demand a situation is postulated which closely resembles the scenario in which the theory of the firm is studied. The model below adds nothing but a limited concept of space and a limited concept of time to the conventional analysis.

Implicit in the classroom teachings of price theory is the coincidence of resource sources, the production point,

[1]Howard T. Lewis, "The Economics of Air Freight", in Nicholas A. Glaskowsky ed., Management for Tomorrow, (Stanford, California, Graduate School of Business, Stanford University, 1957), quoting Dupuit, p. 71.

and the consumption points. These assumptions will now be altered by supposing that consumption takes place away from the production point. It will still be assumed, as does conventional analysis, that consumption takes place at a point (as opposed to having consumers located over a plane). Raw materials will also be located away from the production point. Thus the theory of the demand for transport will be developed in the context of having the profit maximizing firm producing its product at point B; using resources on hand at B, but also resources from other points W; and selling the final product at C points. Hence distance enters the analysis.

Time also enters into the analysis. Goods must move from the production point to the consumption points in order to produce revenue. Resources must move from W points to the production points in order for goods to be produced. Both these movements entail a cost - that of capital tied up on goods or materials in shipment. The profit maximizing producer must account for these time costs when making his production decision. Demand, however, will be considered as instantaneous once the goods reach market, i.e. all goods are sold on the day that they arrive in the market. Hence time enters the analysis.

Since the theory of the demand for transport is analogous to the theory of the firm, a thorough analysis would parallel a price theory textbook with all the various market cases, e.g. monopoly in the product market with competition in the factor market, etc. Rather than burden the analysis with many similar arguments, a few cases will be presented. The procedure will become obvious and the interested reader may trace out particular market schemes which interest him.

Initially a case will be considered where W and B coincide. Only one C exists and pure competition prevails in all markets. Only one mode exists to carry the finished product from B to C. This mode is characterized by a certain transport rate per unit carried, a certain time for transporting goods from B to C, and a certain damage, loss, theft, or perishability rate. The producer has a variable cost of production function which is assumed to yield the typical U shaped average and marginal cost curves. Costs of production are incurred instantaneously. The inventory problem will be disregarded for the time being, i.e. it is assumed that the demand horizon is for one time period. Demand will be considered as instantaneous once the goods reach the market, i.e. all the goods are sold on the day they arrive at the market. To insure the relevence of transport time it is assumed that payment for the final product is on a C.O.D. basis. There is a separation in time between the incidence of production of the goods and the realization of the revenues from their sale. This is measured by the cost of the tied up capital referred to above. Later multimodes, multimarkets, imperfect competition, transport of inputs, and inventory holdings will be considered.

II

Under the assumptions of perfect competition, the firm can sell all the output it chooses to produce at C for the market price. How does the transport rate, the time in transit, the interest rate, and the damage rate influence the firm's decision of what output to produce and ship?

Initially, the determinants of a firm's demand for a single type of transportation will be examined. The following notation is employed:

π	the producing firm's profit
Q	the quantity the firm produces and ships
P	the market price of the firm's product
α	the time required to ship the goods from point of production to the point of sale
T	the transport charge per unit of product
i	the interest rate
β	the damage, pilferage, loss, or perishability rate

The firm's profit is:

(1). $$\pi = \frac{[(1-\beta)(P-T)]Q}{(1+i)^{\alpha}} - f(Q)$$

In this equation, the second term on the right side is the total cost of production. The numerator of the first term is net revenue. This is defined as total revenue, PQ, times (1- β) minus total transportation cost, TQ. times (1- β). The total revenue is multiplied by (1- β) because only (1- β) of the goods produced ever reach the market, i.e. only $\tilde{Q}$ = (1- β)Q sellable goods actually arrive at C, although Q left from B.[2] Transport rates are payable for services rendered and for only non-damaged goods. Since this net revenue is received only after α days, it is divided by $(1+i)^{\alpha}$ to determine its present value.

[2]It is assumed here that the firm bears the cost of damage, etc. If the transportation firm "made good" on all losses, the producing firm would behave as if β =0 and end up selling β % of Q to the transport company.

Profit in equation (1) is then differentiated with respect to Q and the result set equal to zero:

(2). $$\frac{\partial \pi}{\partial Q} = \frac{(1-\beta)(P-T)}{(1+i)^{\alpha}} - f'(Q) = 0$$

(3). $$f'(Q) = \frac{(1-\beta)(P-T)}{(1+i)^{\alpha}}$$

with (4). $f''(Q) > 0$ holding.

Equations (3) and (4) state the usual profit maximizing conditions; that marginal cost, f'(Q), equal marginal revenue,

$$\frac{(1-\beta)(P-T)}{(1+i)^{\alpha}}$$

and that the marginal cost curve is rising. Marginal revenue will be referred to as $\hat{P}$ and called the net discounted price. With i, α, β, and P constant, and T variable, equation (3) also yields the firm's demand function for transportation with respect to rate. If its marginal cost curve is U shaped, this function is truncated but has the usual negative slope.[3] The truncation occurs because the firm will stop production if the transport cost rises to the point where

$$\frac{(1-\beta)(P-T)}{(1+i)^{\alpha}} = \hat{P}$$

is less than minimum average variable cost.

The situation is illustrated in diagrams II-1-a and II-1-b. Cost of production and net prices are measured on the vertical

[3]Variations in i, α, β, and P as well as T, have the expected results, i.e. $\frac{\partial Q}{\partial i} < 0$, $\frac{\partial Q}{\partial \alpha} < 0$, $\frac{\partial Q}{\partial \beta} < 0$, $\frac{\partial Q}{\partial P} > 0$ and $\frac{\partial Q}{\partial T} < 0$.

Quantity produced and shipped increases as T, i, α and β fall. It falls as P increases.

axis. Q is the quantity the firm would produce and ship if the market price were P and the transport rate were zero. T_m, T_2, T_1, represent successively higher transport rates. At rate T_1 the firm will produce and ship Q_1. Any further increase in the transport rate will cause the firm to cease operation.

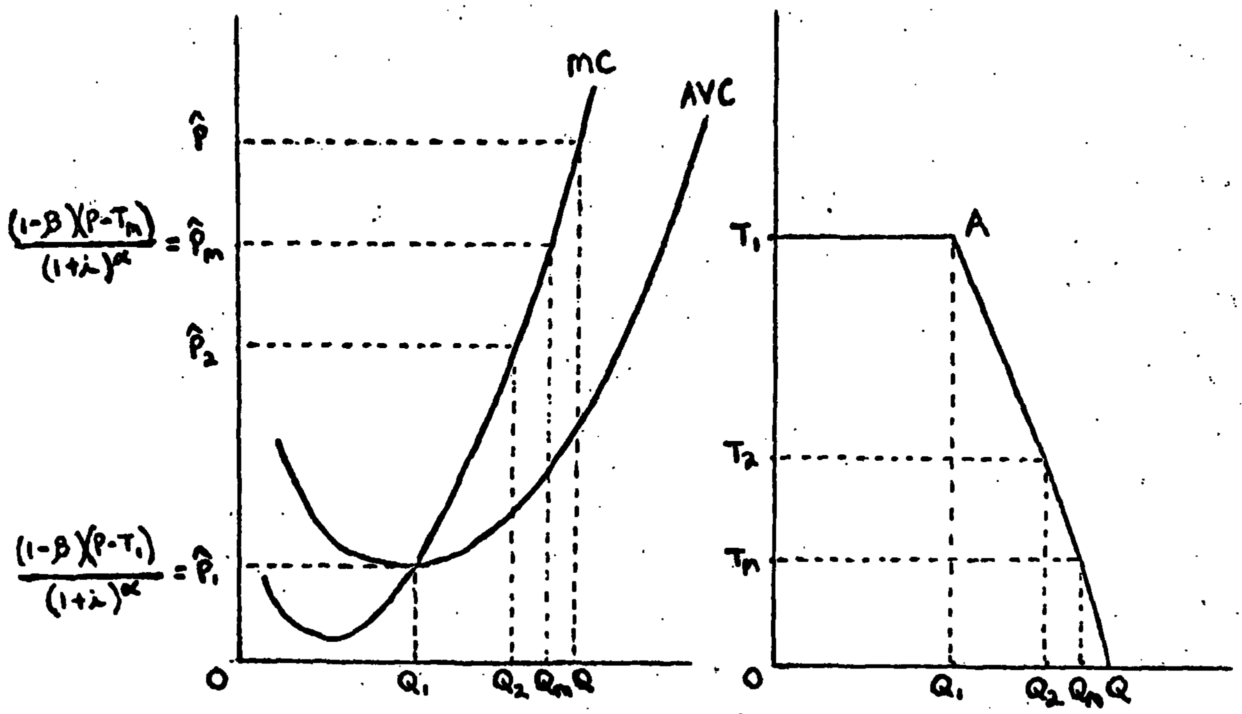

Diagram II-1-a Diagram II-1-b

The demand function is shown in diagram II-1-b, where rates are measured on the vertical and quantity produced and shipped on the horizontal axis.[4, 5, 6]

[4] See Appendix to Chapter II, note 1.

[5] See Appendix to Chapter II, note 2.

[6] See Appendix to Chapter II, note 3.

It is still rather commonplace to hear the view expressed that the demand for transportation is not very responsive to changes in rates, i.e. that transport demand is relatively inelastic.[7] Such a position receives little support from the above derivation. As has been shown, if the transport rate rises above a certain level, shipments will fall to zero. Even if the horizontal portion of the demand function is ignored, the remaining portion, AQ, can exhibit conventional elasticity properties, having a portion where elasticity exceeds unity and a portion where it is less. Whether or not this is the case depends on the marginal cost curve. The assertion that transport demand functions are inelastic is, therefore, one that must be justified by reference to empirical rather than theoretical arguments and these have not been forthcoming.[8]

The relationship between the elasticity of transport demand and the elasticity of supply (the marginal cost of production curve) is shown below.

Arc elasticity of supply is approximated by:

$$\frac{\%\Delta Q}{\%\Delta(P-T)} = \frac{\dfrac{Q_1-Q_2}{Q_1+Q_2}}{\dfrac{(P-T_1)-(P-T_2)}{(P-T_1)+(P-T_2)}} = \frac{\dfrac{Q_1-Q_2}{Q_1+Q_2}}{\dfrac{T_2-T_1}{2P-(T_1+T_2)}} = \frac{\dfrac{A}{B}}{\dfrac{C}{2P-(T_1+T_2)}} = \frac{A[2P-(T_1+T_2)]}{BC} = \eta$$

[7] For discussions around the issue see Hugh S. Norton, Modern Transportation Economics, (Columbus, Ohio, Charles E. Merrill Books, Inc., 1963), p. 297 or Dudley F. Pegram, Transportation: Economics, and Public Policy, revised edition, (Homewood, Illinois, Richard D. Irwin, Inc., 1968), pp. 174-76.

[8] As an extreme case, consider a firm operating under conditions of constant cost with some short run capacity restriction on the output. Its transport demand function will still have a perfectly elastic portion, but the remainder of it will be perfectly inelastic.

Arc elasticity of the associated transport demand (adjusted to be positive) is approximated by:

$$\frac{\%\Delta Q}{\%\Delta T} = \frac{\frac{Q_1 - Q_2}{Q_1 + Q_2}}{\frac{T_2 - T_1}{T_1 + T_2}} = \frac{\frac{A}{B}}{\frac{C}{T_1 + T_2}} = \frac{A(T_1 + T_2)}{BC} = \epsilon$$

If the elasticity of supply is multiplied by $\frac{T_1 + T_2}{2P - (T_1 + T_2)}$ it will yield the elasticity of demand for transport. With certain values of T_1, T_2, and P, ϵ will exceed 1.

Or alternatively, consider point elasticity. If the cost function, T, and P are such that the kink in the demand curve occurs at a quantity less than half way to the quantity where T=0, the curve will have elastic sections. It is seen, however, that the kink increases the chances of the transport demand curve being inelastic.

It should be obvious from the above diagram that a marginal cost curve, with a gentler slope than the one presented, will yield a demand curve for transportation with a gentler slope than the one derived above.

The above analysis treated the transport and the transport associated costs from the demand side, i.e. as net prices. The same analysis could have been treated just as easily from the cost side, yielding of course, the same results.[9]

[9] See Appendix to Chapter II, note 4.

III

Suppose now that uncertainty is allowed to enter the analysis. In the real world the precise market price in the future is unknown. Likewise the exact delivery time is not known with perfect certainty; nor is the damage rate. Businessmen expect certain market prices, delivery times, and damage rates to prevail, i.e. the variables have probability distributions associated with them; and, in the absence of any a priori knowledge, the businessman expects the average of the variable to occur. We will assume all distributions are symmetrical (which is essential for generality of the conclusions). T and i are fixed.

The handling of the P and β distributions is straightforward (if they are the only variable distributions, i.e. if α is fixed). To maximize profits the firm should assume that the average market price $\bar{P}$, $\bar{P} = \sum_{k=1}^{n} P_k x_k$ where $\{x_k\}$ is the probability distribution of prices, and the average damage rate $\bar{\beta}$, $\bar{\beta} = \sum_{h=1}^{n} \beta_h \varphi_h$ where $\{\varphi_h\}$ is the probability distribution of damage rates, will prevail. Placing these values in equation (3) above will yield the firm's expected demand for transportation.

Variable delivery times change the analysis. It is not known whether the goods will reach the market in $\alpha_1, \alpha_2, \cdots$ or α_m days. To maximize profits the firm will not do best to assume that the expected delivery time prevails. The true criterion is the produce and ship the output dictated by the expected net discounted price $\hat{\bar{P}}$, i.e.

(5). $$\hat{\bar{P}} = \sum_{k=1}^{m} \frac{(1-\beta)(P-T)}{(1+\lambda)^{\alpha_k}} \gamma_k$$

where $\{\gamma_k\}$ is the probability distribution of days. When only P and β were considered, $\hat{\bar{P}}$, $\bar{P}$, and $\bar{\beta}$ were one in the same criteria. However, when variable time costs enter, the discounting changes the analysis.

$\hat{\bar{P}}$ is not equal to $\dfrac{(1-\bar{\beta})(\bar{P}-T)}{(1+\lambda)^{\bar{\alpha}}}$. In fact [10]

$$\hat{\bar{P}} > \frac{(1-\bar{\beta})(\bar{P}-T)}{(1+\lambda)^{\bar{\alpha}}}.$$

With all three variables random, the net discounted price is:

(6). $$\hat{\bar{P}} = \sum_{m}\sum_{n}\sum_{\eta} \frac{\gamma_m \varphi_\eta x_n [(1-\beta_\eta)(P_n-T)]}{(1+\lambda)^{\alpha_m}}$$

The demand curve for transport is solved for as above.

The second moment of $\{\alpha_k\}$ also yields an interesting conclusion in this model. The variance of α may be viewed as a measure of reliability or dependability of service. Given the symmetry of the α distribution, increasing or decreasing the variance of α will not change $\bar{\alpha}$. Will the shipper prefer dependability, ceteris paribus?

[10] See Appendix to Chapter II, note 5.

Under the assumptions of the model (a one period demand model with no inventory holding), the firm will prefer undependability in delivery time. That is, given a choice between two types of delivery, both having the same mean, the shipper will prefer the one with the greatest variance of delivery time - for it will yield the highest expected net discounted price (which implies the greatest profit in the competitive model).

The demonstration of this risk preference is shown below:

Taking the first two partials of $\hat{P} = \frac{(1-\beta)(P-T)}{(1+i)^{\alpha}}$ with respect to α yields:

(7). $\frac{\partial \hat{P}}{\partial \alpha} = \frac{-[(1-\beta)(P-T)] \log_e(1+i)}{(1+i)^{\alpha}} < 0$

(8). $\frac{\partial^2 \hat{P}}{\partial \alpha^2} = \frac{+[(1-\beta)(P-T)][\log_e(1+i)]^2}{(1+i)^{\alpha}} > 0$

Thus as α increases, $\hat{P}$ decreases but does so at a decreasing rate, i.e. the curve is concave upward as is shown below.

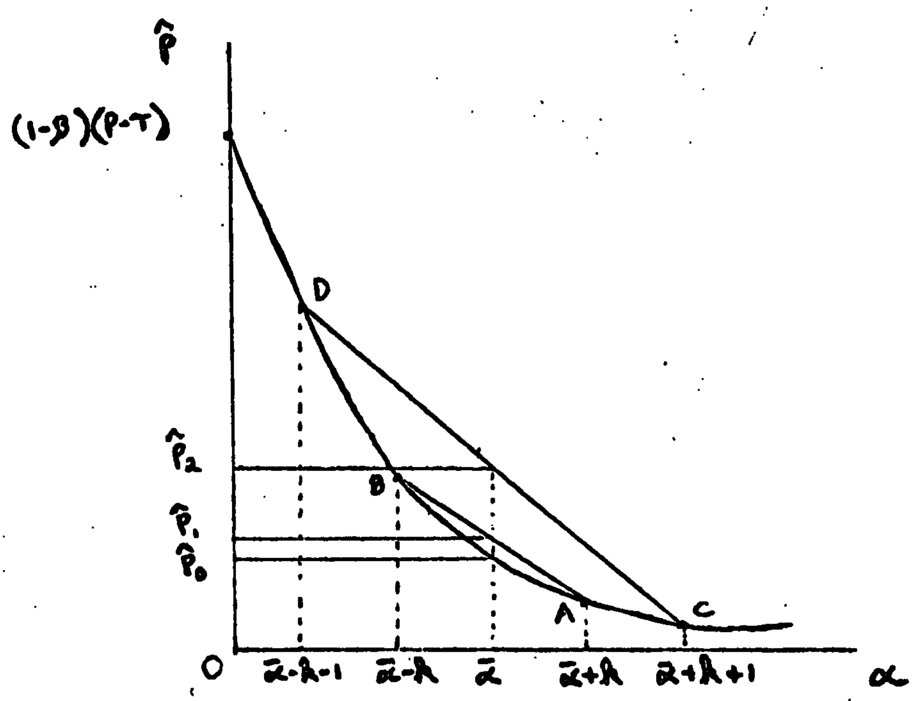

Diagram II-2

Suppose that the range of one choice is from $\bar{\alpha}$-k to $\bar{\alpha}$ +k days. The second choice takes one day longer or shorter than the other choice, i.e. $\bar{\alpha}$-k-1 and $\bar{\alpha}$+k+1. The chords BA and DC, drawn show that the expected price is always higher when the variance of delivery time is greater. Consider $\bar{\alpha}$. If α were not variable, the net discounted price is $\hat{P}_0$. If, however, α has probability distribution one, expected net discounted price is $\hat{P}_1$. Probability distribution two (with a greater variance) yields a still higher price $\hat{P}_2$. This also ties into the analysis of why $\bar{\alpha}$ does not yield the expected net discounted price.

IV

The analysis can be extended by allowing for speculative inventory holding. The desire to hold inventory comes about as the result of variable market prices $\{P_k\}$. Production (and hence shipping decisions) must take into account the possibility of inventory holdings. Under what conditions will a firm wish to hold inventory - obviously under conditions which increase their expected profits. Inventory cost knowledge is thus necessary to determine the firm's profit maximizing output, Q. Once the goods are produced, the firm may act differently than was postulated in determining the Q because Q is now fixed and the production costs become fixed costs. At that time the firm will only compare inventory costs versus expected revenues.

A very simple case of the above model will be dealt with. The transport rate is T; the interest rate is i; the time in transit is α ; the firm's cost function C=f(Q);

and the probability distribution $\{x_m\}$ of market prices $\{P_m\}$. Inventory cost per unit per day is q. Perfect competition exists in all markets.

When goods had to be sold on arrival in the market, as was assumed previously, the profit maximizing output was determined by assuming that $\hat{\bar{P}}$ would be the net discounted price that would prevail (where $\hat{\bar{P}} = \frac{\bar{P} - T}{(1+i)^{\alpha}}$).

With inventory holding allowed, the shipper now has the option of shipping his goods, having them arrive α days from the date of shipment, viewing the market price at that time and deciding whether he wishes to sell or to hold the goods in inventory, waiting for a more attractive market price. Although tomorrow's price may be higher, this must be weighed against the inventory cost and the extra time cost.

The analysis becomes more complicated as the number of days a good can be held in inventory increases. It also is more complicated the greater the variance of $\{P_m\}$. To avoid such problems yet showing the point of the analysis, we constrain the inventory holding period to one day. That is, the shipper either sells his good in the market on the α th or the $\alpha+1$ st day. After the $\alpha+1$ st day, the good spoils so that it is unsellable. The price probability distribution is symmetrical, $\{x_k\}$ where k=1,2,3. The mean price will be $\bar{P} = P_2$.

It is clear that if the market price is P_3 on the α th day, the shipper will sell all goods on that day - for he never can do better. If the price is P_2 on the α th day, the shipper will also choose to sell all his goods on that day. Without any a priori knowledge, the price he expects to prevail on the $\alpha+1$ st day is $\bar{P} = P_2$. Getting P_2 today is better than getting P_2 tomorrow. However, if the market price is P_1 α days from now, the shipper's decision depends on the level of inventory costs and the interest rate.

The profit maximizing condition when no inventories were allowed is shown as:

(9). $\hat{P}_1 x_1 + \hat{P}_2 x_2 + \hat{P}_3 x_3 = f'(Q) = \hat{\bar{P}}$

The firm should account for inventories in its output decision if $\frac{\hat{\bar{P}} - \hat{q}}{(1+i)} > \hat{P}_1$

(The $\hat{\bar{P}}$-$\hat{q}$ is divided by (i+1) because this is the expected price received one day after $\hat{P}_1$ would be received. $\hat{q}$ are discounted inventory costs). This says that the expected net price after inventory costs for the sales of the goods on the α +1 st day exceed what the goods could have sold for on the α th day.

If $\frac{\hat{\bar{P}} - \hat{q}}{(1+i)} > \hat{P}_1$, profit maximizing output is determined as follows:

(10). $\pi = \left[\frac{\hat{\bar{P}} - \hat{q}}{1+i} x_1 + \hat{P}_2 x_2 + \hat{P}_3 x_3 \right] Q - f(Q)$

(11). $\frac{\partial \pi}{\partial Q} = \frac{\hat{\bar{P}} - \hat{q}}{1+i} x_1 + \hat{P}_2 x_2 + \hat{P}_3 x_3 - f'(Q) = 0$

(12). $\frac{\hat{\bar{P}} - \hat{q}}{1+i} x_1 + \hat{P}_2 x_2 + \hat{P}_3 x_3 = f'(Q)$

Solving (12) for Q yields the optimum output produced and transported to maximize expected profits. Obviously shippers must account for inventory costs in determining output to be produced and transported. Also obvious is that the level of inventory costs influence transport demand.

When inventory holding becomes profitable more will be shipped. This is so because the right hand side of (12) exceeds the right hand side of (9). If $\frac{\hat{p} - \hat{q}}{1+i} > \hat{P}_1$

output produced and shipped will be greater when inventory holding becomes more profitable. The higher the inventory costs, the lower the differential of $\frac{\hat{p} - \hat{q}}{1+i}$

and P_1 and thus the less shipped.

A graph of q versus Q would appear as below:

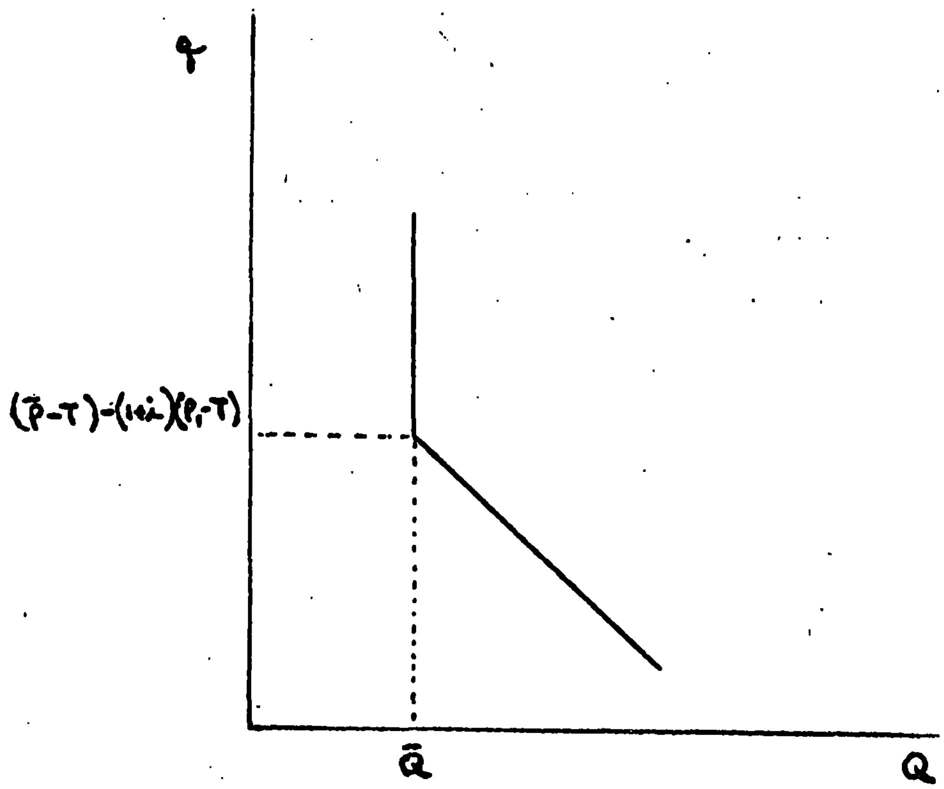

Diagram II-3

At the kink $\hat{P}_1 = \frac{\hat{\bar{P}} - \hat{q}}{1+i}$. Any further raising of q will make inventory holding unprofitable and thus the output produced and shipped will be that dictated by $\bar{P}$ (say $\bar{Q}$).[11]

To show how the analysis becomes more complicated as the number of possible prices below the mean price increases. Consider the following complication: vector of prices $\{P_k\}$ and probabilities $\{x_k\}$, k=1,5 and symmetrical. Thus $\bar{P}=P_3$ and $\hat{\bar{P}}=\hat{P}_3$. However, three possibilities must be allowed for.

It should be mentioned that if P_3, P_4, or P_5 prevails, the goods will all be sold on the α th day (when they arrive at the market).

Case 1. $(\bar{P}-T) - (1 + i)(P_1-T) < q$

$(\bar{P}-T) - (1 + i)(P_2-T) < q$

Inventory holding is too expensive, so that output implied by $\bar{P} = f'(Q)$ will be produced and shipped and sold when it arrived in the market place.

Case 2. $(\bar{P}-T) - (1 + i)(P_1-T) > q$

$(\bar{P}-T) - (1 + i)(P_2-T) < q$

Inventory holding is too expensive is the market price on the α th day is P_2 but not if the market price is P_1. To maximize profits the firm should produce and ship the quantity dictated by the following:

(13). $\pi = \left[\frac{\hat{\bar{P}} - \hat{q}}{1+i} x_1 + \hat{P}_2 x_2 + \hat{P}_3 x_3 + \hat{P}_4 x_4 + \hat{P}_5 x_5\right] Q - f(Q)$

(14). $\frac{\partial \pi}{\partial Q} = \frac{\hat{\bar{P}} - \hat{q}}{1+i} x_1 + \hat{P}_2 x_2 + \hat{P}_3 x_3 + \hat{P}_4 x_4 + \hat{P}_5 x_5 - f'(Q) = 0$

(15). $\frac{\hat{\bar{P}} - \hat{q}}{1+i} x_1 + \hat{P}_2 x_2 + \hat{P}_3 x_3 + \hat{P}_4 x_4 + \hat{P}_5 x_5 = f'(Q)$

[11]See Appendix to Chapter II, note 6

Case 3. $(\bar{P}-T) - (1 + i)(P_1-T) > q$

$(\bar{P}-T) - (1 + i)(P_2-T) > q$

Inventory holding is profitable for the discounted market prices $\hat{P}_1$ and $\hat{P}_2$. To maximize expected profits the firm should produce and ship the quantity dictated by the following:

$$(16).\quad \pi = \left[\frac{\hat{\bar{P}} - \hat{q}}{1+i}(x_1+x_2) + \hat{P}_3 x_3 + \hat{P}_4 x_4 + \hat{P}_5 x_5\right] Q - f(Q)$$

$$(17).\quad \frac{\partial \pi}{\partial Q} = \frac{\hat{\bar{P}} - \hat{q}}{1+i}(x_1+x_2) + \hat{P}_3 x_3 + \hat{P}_4 x_4 + \hat{P}_5 x_5 - f'(Q) = 0$$

$$(18).\quad \frac{\hat{\bar{P}} - \hat{q}}{1+i}(x_1+x_2) + \hat{P}_3 x_3 + \hat{P}_4 x_4 + \hat{P}_5 x_5 = f'(Q)$$

As an example of how the analysis complicates as the number of inventory days allowed increases, suppose the goods can be sold on the α th, $\alpha+1$ st, and $\alpha+2$ nd days after production. (The latter two sales the result of inventory holding for one or two days). Suppose the price $\{P_k\}$ probability distribution is $\{x_k\}$, k=1,3.

Firstly, if the market price is P_3 or P_2 on the α th day, the goods will be sold on delivery.

Three things may happen. Inventory costs may be so high that the most profitable decision for the firm to make is to sell all goods on the α th day, regardless of the market price; or inventory costs may be such as to make one day inventory profitable - in such case, if the market price is P_1 on the α th day, the firm will hold the goods until the next day (when the expected market price is $\bar{P}$) and then sell the goods no matter what market price prevails; or inventory costs may be such as to make two day inventory holding profitable - in such a case, if the market price is P_1 on the α th day, the firm will hold the goods until the α +1 st day. If the market price is P_2 or P_3 on the α +1 st day, the goods will be sold then. If the price is P_1, the goods will be held until the α+2 nd day (when the expected market price is $\bar{P}$) and then sold at whatever the market price may be.

Optimal outputs are found as follows:

For case one above, it is where $\bar{P}=f'(Q)$.

For case two, $\frac{\hat{P}-\hat{q}}{1+i}x_1+\hat{P}_2x_2+\hat{P}_3x_3=f'(Q)$

For case three (assuming that the storage costs are due when the goods leave the warehouse),

$$\frac{\hat{P}-2q}{(1+i)^2}x_1^2+\hat{P}_2(x_2+x_1x_2)+\hat{P}_3(x_3+x_1x_3)=f'(Q)$$

As the variance of the probability distribution of prices and the number of days of possible inventory holding increases, the analysis becomes complicated at an increasing rate.

Inventory holding in a multi-time period analysis has been developed elsewhere.[12]

[12] William Baumol et al, Studies on the Demand for Freight Transportation, (Princeton, New Jersey, Mathematica, August, 1967).

V

Imperfect competition can also be added to the analysis. Transport rates and time costs can be viewed as tax analogues - the rate T affects the demand curve for the product as would a per unit tax and the time costs affecting the demand curve as would an _ad valorem_ tax. Damage costs must be viewed slightly differently. Net discounted demand curves now exist and yield net discounted marginal revenue curves. Changing the transport variables changes the position of the marginal revenue curve and these changes, with the marginal cost of production curve, trace out demand curves for the various variables as was done in the case of perfect competition.

Assume a linear demand curve for the final good, $P=a-bQ$. What demand curve will the shipper "see" after accounting for the costs of transport and those associated with transport? The price that the firm will receive in the market place depends on the number of goods which actually reach the market place in a salable condition. If Q are produced and shipped, $(1-\beta)Q$ will reach the market. Market price will be $P=a-(1-\beta)bQ$. Total revenue will be $TR=a(1-\beta)Q - (1-\beta)^2bQ^2$. Since this revenue will not be received until α days, it must be discounted to be made comparable to the production costs which are incurred instantaneously. The present value of total revenue is thus:

(19). $$\hat{TR} = \frac{a(1-\beta)Q-(1-\beta)^2bQ^2}{(1+i)^\alpha}$$

The shipper must pay a transport bill of $(1-\beta)TQ$ in α days. This is also discounted to make comparable all elements of the analysis:

$$\frac{(1-\beta)TQ}{(1+i)^{\alpha}}$$

Discounted net revenue will be:

$$(20).\quad \hat{TR}_{net} = \frac{a(1-\beta)Q-(1-\beta)^2 bQ^2-(1-\beta)TQ}{(1+i)^{\alpha}}$$

Profits are:

$$(21).\quad \pi = \frac{a(1-\beta)Q-(1-\beta)^2 bQ^2-(1-\beta)TQ}{(1+i)^{\alpha}} - f(Q)$$

The net discounted marginal revenue function is thus:

$$(22).\quad \hat{MR}_{net} = \frac{a(1-\beta)-2(1-\beta)^2 bQ-(1-\beta)T}{(1+i)^{\alpha}}$$

Equating (22) to f'(Q) and solving for Q and varying one of β, α, i, T, and a (changes in a will measure parallel shifts in the market demand curve), while holding the other constant, will yield the various demand curves for transport.[13]

Probability distributions for the variables T, i, a, b, α, and β could also be considered. These would not alter the analysis seriously and would affect the model as they did under pure competition.

[13]See Appendix to Chapter II, note 7.

VI

Another line of expansion involves allowing for the transport of an input. The demand for transport of a factor of production can be viewed as a derived demand curve twice removed. That is, the demand for a resource itself is said to be derived from the demand for the product. Then the demand for transport is derived from the demand for the resource. Only the transport rate on the input will be viewed. The procedure for changes in the other variables should now be obvious.

Assume that perfect competition exists in all markets. The firm's production function is $Q=f(L,Z)$; where L is located at B selling for P_L, but Z is found at W (away from B) selling for P_Z. Z is subject to a per unit transport charge of t_1 for the trip from W to B. Therefore, $P_Z + t_1$ is the price of a unit of Z at B.

The firm will produce any given output where the marginal conditions $MRS = (P_Z + t_1) / P_L = MPP_Z / MPP_L$ hold. The profit maximizing output is easily determined. This yields an output Q_1 which will subsequently be transported to C and a Z_1 which needs to be transported from W to B. Thus one point (t_1,Z_1) on the demand curve for transport of an input is determined.

The theory of adjustments by the firm to changes in input prices has been well developed.[14] Changes in t will change the price of resource Z at B. Increases of t will increase Z's price and hence decrease its use; therefore,

[14] See for example, Charles Ferguson, MicroEconomic Theory, (Homewood, Illinois, Richard D. Irwin, Inc., 1967), pp

quantity transported of Z decreases. Decreases in t will decrease Z's price and hence increase its use; therefore, the quantity transported of Z increases.

To show that this must be true consider the initial optimum position (Q_1, Z_1, t_1). Call it point one. Suppose transport rates decrease to t_2, i.e. $(t_1 - t_2) > 0$. A new profit maximizing position results, i.e. (Q_2, Z_2, t_2). Call it point two.

At point two Z is now cheaper because $t_1 > t_2$. If point one's profits are evaluated at point two's resource prices, they will be less than those profits evaluated at point one's resource prices (since this latter position is the revealed "best", the profit maximizing situation).

Thus,

$$(23).\quad \frac{(1-\beta)(\rho-\tau)}{(1+i)^{\alpha}} Q_1 - P_L L_1 - (P_z + \tau_1) Z_1 > \frac{(1-\beta)(\rho-\tau)}{(1+i)^{\alpha}} Q_1 - P_L L_1 - (P_z + \tau_2) Z_1$$

or

$$(23').\quad \hat{P} Q_1 - P_L L_1 - (P_z + \tau_1) Z_1 > \hat{P} Q_1 - P_L L_1 - (P_z + \tau_2) Z_1$$

The same analysis is made of point two. Point two's profits evaluated at point two's resource costs will exceed point two's profits evaluated at point one's resource costs (since point two is a point of maximum profit).

Thus,

$$(24).\quad \hat{P} Q_2 - P_L L_2 - (P_z + \tau_2) Z_2 > \hat{P} Q_2 - P_L L_2 - (P_z + \tau_1) Z_2$$

Adding (23') and (24),

$$\hat{p}Q_1 + \hat{p}Q_2 - p_L L_1 - p_L L_2 - p_Z Z_1 - p_Z Z_2 - t_1 Z_1 - t_2 Z_2 > \hat{p}Q_1 + \hat{p}Q_2 - p_L L_1 - p_L L_2 - p_Z Z_1 - p_Z Z_2 - t_1 Z_2 - t_2 Z_1 \tag{25}$$

Substracting common terms,

$$-t_1 Z_1 - t_2 Z_2 > -t_2 Z_1 - t_1 Z_2 \tag{26}$$

Multiplying by -1,

$$t_1 Z_1 + t_2 Z_2 < t_2 Z_1 + t_1 Z_2 \tag{27}$$

Subtracting common terms from both sides,

$$t_1(Z_1 - Z_2) - t_2(Z_1 - Z_2) < 0 \tag{28}$$

or

$$(t_1 - t_2)(Z_1 - Z_2) < 0 \tag{29}$$

It was assumed that $(t_1-t_2) > 0$. This implies that $(Z_1-Z_2) < 0$ or $Z_1 < Z_2$. Thus if the transport rate on inputs is reduced, making Z cheaper to the firm, the firm will use more Z and thus demand more transport. Likewise, if the rate was increased from t_1 to t_3, then $(t_1-t_3) < 0$ implying $(Z_1-Z_3) > 0$ or $Z_1 > Z_3$ -- a decrease in the quantity demanded of Z and thus of the quantity demanded of transport of Z.

The analysis yields a downward sloping demand curve for transport of an input.[15]

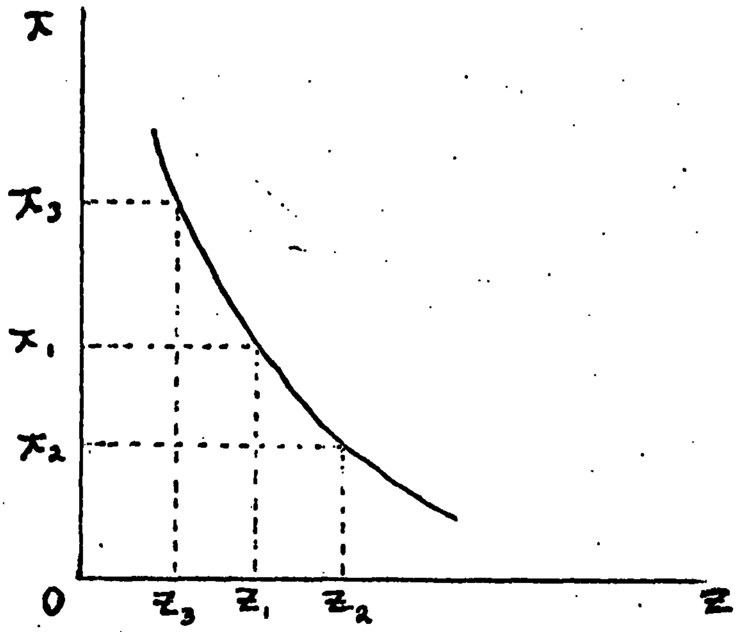

Diagram II-4

The above analysis also holds if the market for the final product is imperfect.

There is a lesson here for transport companies. It stems from the interdependence of transport rates with profit maximizing output. Certain rate changes on inputs influence the firm's cost of product curves and hence the demand for transport of final goods. Likewise, certain rate changes on outputs influence profit maximizing output, and hence the demand for transportation of inputs. Changes in travel time and damage rate can have the same effect.

[15]Time costs (α and i) and damage rates (β) will have negative effects on quantity demanded and shipped, i.e.

$\frac{\partial Z}{\partial i} < 0$, $\frac{\partial Z}{\partial \alpha} < 0$, and $\frac{\partial Z}{\partial \beta} < 0$ as expected.

Presumably the transportation firm wishes to maximize profits. Thus, when contemplating rate or service changes, it should be aware of the important intertwining of transportation and production in the theory of the firm. For instance, decreasing α in the transport of the final good will increase $\hat{P}$, increase Q, which will (unless the resource is inferior) increase the use of Z. Thus on two fronts, more final product transported and more input transported, the transportation company has increased its business both quantity wise and revenue wise. When the rates are changed, the shipping firm will change Q and Z, but whether the transport company's revenues change favorably depends on the elasticity of transport demand - both input and output.[16] If imperfect competition in the product market is added, the analysis is complicated further by the elasticity of demand for the product.

VII

Two additional complications remain to be added to the analysis - that of multimarkets and multimodes. For simplicity view only the demand for transport of the final product.

Above the theory was developed in terms of a single mode and a single market. Its extension to the case of choice between m modes or n markets, with perfect competition in the product market, is now a simple matter. The firm takes the α , T, and β values of each mode and the P's

[16]See Appendix to Chapter II, note 8.

of each market and calculates each combination of market and mode's $\hat{P}$'s (or net discounted prices). It chooses the combination of mode and market with the highest $\hat{P}$ and then equates marginal cost of production to this value to determine what quantity it will produce and ship. If marginal costs are constant or rising, this two stage decision process will automatically assure profit maximization. If, in other words, $\hat{P}_m$ exceeds $\hat{P}_j$ for $j=1,2,\ldots,m-1$, then Q_m will exceed Q_j and π_m will exceed π_j. Under these conditions, it will ship all of its product to one market via one mode.

It should be pointed out that different transport modes generally imply different final outputs produced and shipped. Thus, if the shipper is talking solely of using air freight, he is talking of output produced and shipped Q_A; if he is talking solely of using sea freight, he is talking of output produced and shipped Q_S; where Q_A in general does not equal Q_S. A similar statement holds for different markets. Of course, the shipper should only be interested in the highest output.

Supposing one market and two modes, A and S, we have graphically:

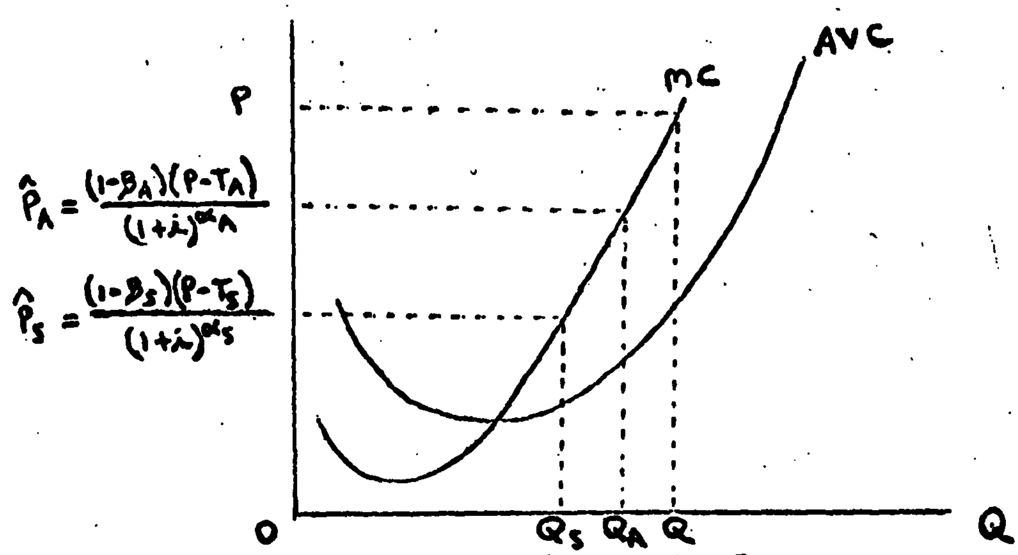

Diagram II-5

The only time that modal splitting would occur would be when there is a tie for the highest net discounted price. Under those circumstances the shipper could ship all by air, all by sea, or split the shipment. The theory developed here can not explain the magnitude of splitting when the shipper is indifferent as to modes.

The transport firm must now make its rate and service decisions with the recognition that certain changes in his policy may make another mode more attractive to the shipper. Certain policy changes may make the transport firm's revenue function take great jumps, i.e. it may be discontinuous because of shipper's switching modes.

When imperfect competition in the product market enters the multimarket, multimode analysis, the possibility of the firm splitting modes, becomes a reality. However, this is really an aggregation problem. On any given route the firm will only ship by one mode. Over separate routes different modes may be used.

If there is one product market and two modes, the firm has two net discounted marginal revenue functions to view. The firm must now choose the <u>maximum maximorem</u>. Each marginal revenue curve implies a particular output. The firm chooses that output and hence that mode which is the greatest of the great (in yielding the largest profit). No clear cut net price rule can be stated here. Profits will change depending on the elasticity of the product demand curve. Depending on the demand assumption made, modal splitting will only occur if the net discounted prices are equal[17] or net discounted marginal revenues are equal.[18]

[17] See Appendix to Chapter II, note 9.

[18] See Appendix to Chapter II, note 10.

If more consumption points are added, the model becomes one of profit maximization when separate markets exist, i.e. the standard model of third degree price discrimination. The rule in the standard model is to produce where $\sum MR=MC$ to maximize profits. In the context of the above analysis, the shipper must compare all possible $\sum MR=MC$ relationships and pick the <u>maximum maximorem</u>.[19] Results here depend on the elasticity of demand for the product.

VIII

Modal choice of inputs is much simpler. Under assumptions of perfect competition in the factor markets, the firm merely chooses the mode which yields the lowest net price of the resource.

[19]See Appendix to Chapter II, note 11.

APPENDIX TO CHAPTER II

Note 1. Most rate structures have some amount of taper built into them. Instead of being constant, as was assumed in the text, transport charges per unit decline as quantity shipped increases. The adjustment the model requires to take such economies into account is a bit complicated. There is now a marginal revenue function unequal to $\hat{P}$ which must be equated to marginal cost of production to yield a profit maximization.

Suppose that the transport function is T=T(Q).

Equation (1) in the text now becomes:

$$(1'). \quad \pi = \left[\frac{(1-\beta)[\rho - T(Q)]}{(1+\lambda)^{\alpha}}\right] Q - f(Q)$$

Profit in equation (1') is then differentiated with respect to Q and the result set equal to zero:

$$(2'). \quad \frac{\partial \pi}{\partial Q} = \frac{(1-\beta)\rho}{(1+\lambda)^{\alpha}} - \frac{(1-\beta)[T(Q) + QT'(Q)]}{(1+\lambda)^{\alpha}} - f'(Q) = 0$$

or

$$(3'). \quad f'(Q) = \frac{(1-\beta)\rho}{(1+\lambda)^{\alpha}} - \frac{(1-\beta)[T(Q) + QT'(Q)]}{(1+\lambda)^{\alpha}}$$

which states that marginal cost equals marginal revenue.

To insure profit maximization we need:

$$(4') \quad \frac{\partial^2 \pi}{\partial Q^2} = -\frac{(1-\beta)}{(1+\lambda)^{\alpha}}\left[2T'(Q) + QT''(Q)\right] - f''(Q) < 0$$

or

$$(5'). \quad \frac{-(1-\beta)[2T'(Q) + QT''(Q)]}{(1+i)^{\alpha}} < f''(Q)$$

which states that the slope of the marginal cost curve is greater in value than the slope of the marginal cost of transport curve.

If we require that the firm's transport bill always be increasing, i. e. $\frac{\partial[T(Q)\cdot Q]}{\partial Q} = T'(Q)\cdot Q + T(Q) > 0$

then the net marginal revenue curve must always intersect marginal cost at less than $\frac{(1-\beta)P}{(1+i)^{\alpha}}$.

If, in addition, we require $T''(Q) > 0$, i.e. the curve is concave upward, then the following possibilities exist for changes in the level of the taper:

1.

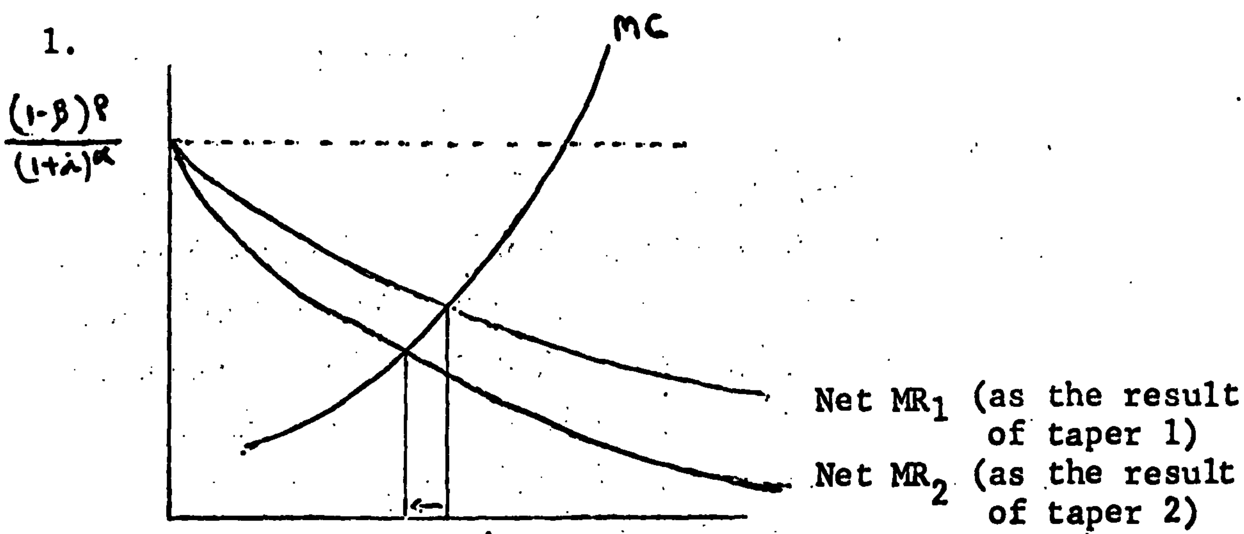

Diagram II-1

where taper 2 involves higher transport costs than taper 1.

2.

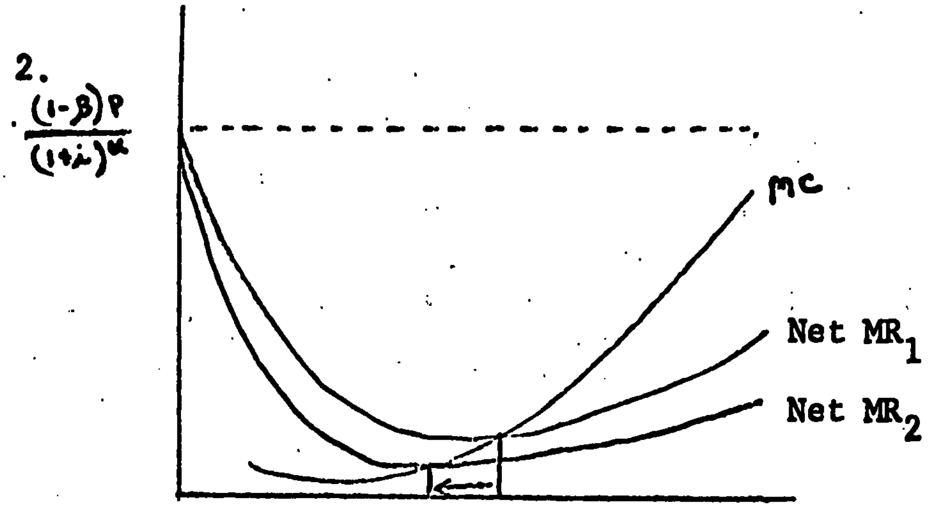

Diagram II'-2

3.

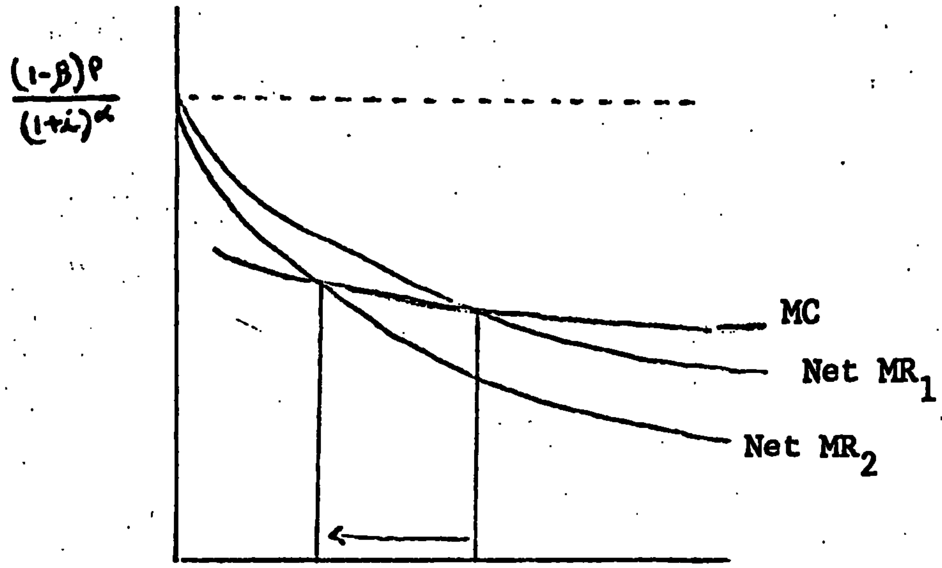

Diagram II'-3

Thus, with the conditions given, no changes occur in the analysis. A higher rate structure will inhibit shipments. A change in the analysis does occur in the sense that industries of increasing returns can now be viewed.

Without the second order condition, it is possible to have reversals. Consider case 4.

4.

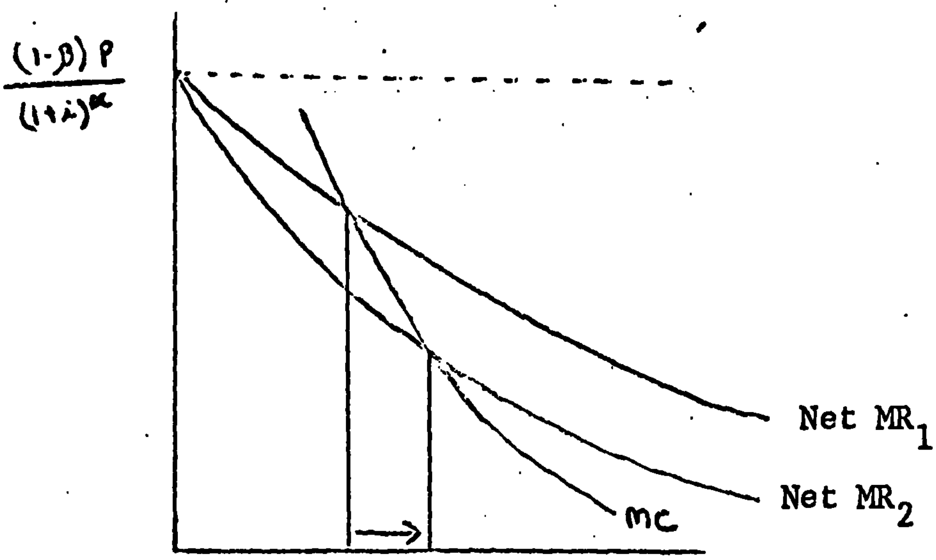

Diagram II'-4

Note 2. Other demand for transport functions can be constructed, e.g. the demand for transport as a function of travel time, etc. All will have kinks in them at the critical values of α, β, i, and P (α_1, β_1, i_1, and P_1) where P_1 equals minimum average variable cost. These are shown below.

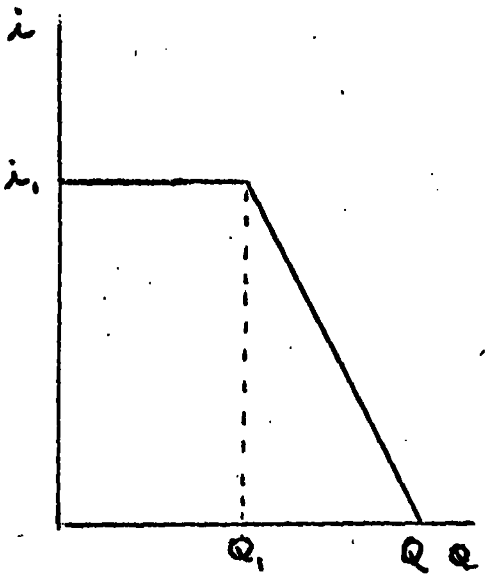

Diagram II'-5

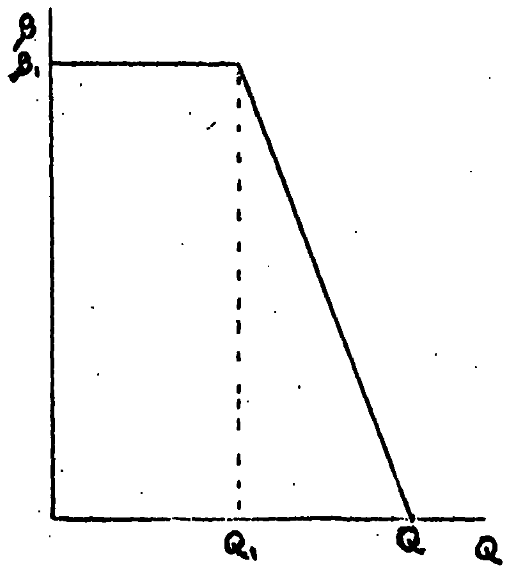

Diagram II'-6

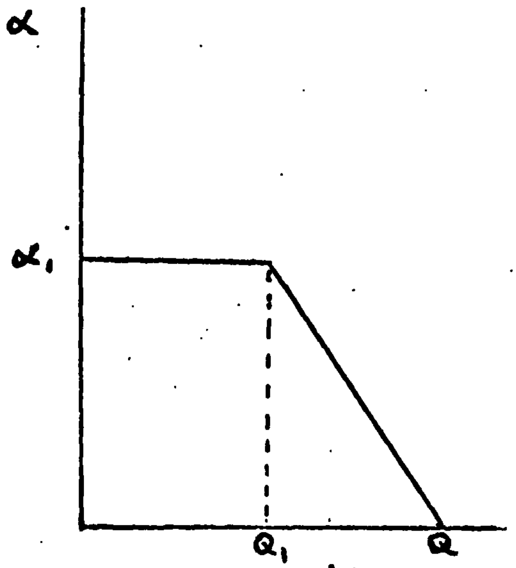

Diagram II'-7

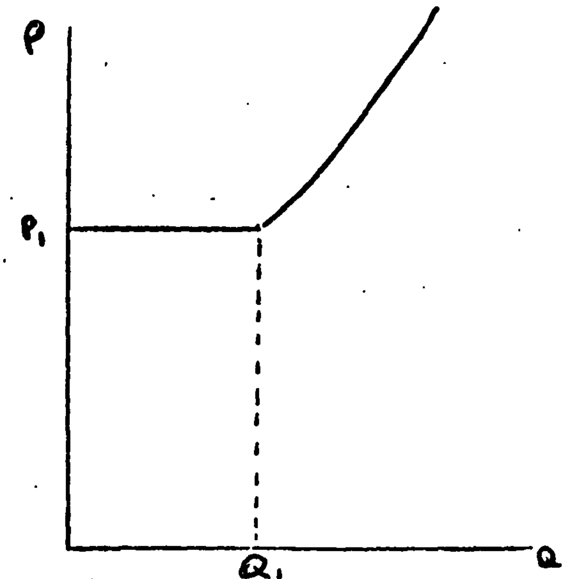

Diagram II'-8

Note 3. The reader may be interested in an experiment carried out by the author based partly on reality and partly on assumption. It is designed to view the process in the text of deriving a transport demand curve for a firm from information on market price, time of transport, the interest rate, the transport rate, and the firm's cost function.

From the Bureau of the Census information on Schedule B commodity 6933120 - wire cloth and other woven wire products of iron and steel - was obtained. This commodity group had an average value per pound of $4.757. The commodity moved solely by air during January, 1967 at a transport rate of $.72 per pound. (Such a commodity was chosen because the transport rate lies between 10-20% of the value per pound - a criterion Brewer used for his demand study in <u>The North Atlantic Market for Air Freight</u> mentioned in Chapter I). The interest rate is 6.5% per annum. Transport time is nine days (based on a single firm's experience of time required when air shipping from the production point to the final destination point (North Atlantic Route - New York to Europe). Thus $P=4.757$, $T=.72$, $i=.065/365$, and $\alpha=9$ represents the reality.

To determine the demand curve for transportation the cost function of the firm must be known. Not having any a priori knowledge as to its form, it is assumed to be TC= $(1/3)Q^3-(1/2)Q^2+Q+5$. Such a form yields the conventional U shaped average and marginal cost curves.

The firm's demand for transport function is derived by setting marginal revenue (net discounted price) equal to marginal cost. Doing so and solving for Q by the quadratic method yields the desired function.

$$(1). \quad Q = \frac{1}{2} + \sqrt{\frac{P-T}{(1+i)^{\alpha}} - \frac{3}{4}}$$

This function is subject to the constraint that $\frac{P-T}{(1+i)^{\alpha}} \geq$ MIN AVC = (13/16). The firm will produce and ship at least (3/4) of a unit or none at all.

Taking partials of (1) with respect to the transport variables (T, i, α) yields:

$$(2). \quad \frac{\partial Q}{\partial T} = \frac{-1}{2(1+i)^{\alpha}(Q-\frac{1}{2})} < 0 \qquad \text{since } Q > (3/4)$$

$$(3). \quad \frac{\partial Q}{\partial i} = \frac{-\alpha(P-T)}{2(1+i)^{\alpha+1}(Q-\frac{1}{2})} < 0$$

$$(4). \quad \frac{\partial Q}{\partial \alpha} = \frac{-[\log_e(1+i)](P-T)}{2(1+i)^{\alpha}(Q-\frac{1}{2})} < 0$$

All relationships have the expected sign, i.e. if the transport rate increase, ceteris paribus, quantity produced and shipped falls; likewise for increases in time and the interest rate.

Second derivatives of (1) will tell about the shapes of the demand for transport functions with respect to changes in T, i, and α . Taking these derivatives:

$$(2'). \quad \frac{\partial^2 Q}{\partial T^2} = \frac{-1}{4(1+i)^{2\alpha}(Q-\frac{1}{2})^3} < 0$$

$$(3'). \quad \frac{\partial^2 Q}{\partial i^2} = \frac{\alpha(\alpha+1)(P-T)}{2(Q-\frac{1}{2})^3(1+i)^{\alpha+2}} - \frac{\alpha^2(P-T)^2}{4(Q-\frac{1}{2})^3(1+i)^{2\alpha+2}} \gtrless 0$$

$$(4'). \quad \frac{\partial^2 Q}{\partial \alpha^2} = \frac{(P-T)(Q-\frac{1}{2})^2(1+i)^{\alpha}[\log_e(1+i)]^2}{2(Q-\frac{1}{2})^3(1+i)^{2\alpha}} - \frac{(P-T)[\log_e(1+i)]^2}{4(Q-\frac{1}{2})^3(1+i)^{2\alpha}} \gtrless 0$$

Thus the transport demand function with respect to rate is bowed out as shown in diagram 1-b in the text. The functions with respect to i and α depend on the absolute values of transport demand assumed. Thus while it is known that the curves are downward sloping, there is no a priori or theoretical expectation of them being concave, convex, or some mixture of both.

It is also interesting to see the effect of market price changes on quantities produced and shipped. Taking the first and second partials of (1) with respect to P yields:

$$(5). \quad \frac{\partial Q}{\partial P} = \frac{1}{2(1+i)^{\alpha}(Q-\frac{1}{2})} > 0$$

$$(5'). \quad \frac{\partial^2 Q}{\partial P^2} = \frac{-1}{4(1+i)^{2\alpha}(Q-\frac{1}{2})^3} < 0$$

The above equations (1-4) were solved for in the example for the values of the variables given. Experiments were then conducted by changing the numerical values of the variables. The i and α functions were found to be very inelastic. Rates were allowed to range between \$.50 and \$1.00 per pound to correspond to Brewer's method cited above. The function was highly inelastic. (Recall, however, that these results are a function of the cost curve assumed).

Note 4. Consider, for example, a model where β =0 and α =0. Define total cost as the sum of the total cost of production and the total cost of transport, i.e. TC=f(Q) + TQ. Total revenue (TR) equals PQ, where P is the <u>market</u> price. Profits (π) are π = TR-TC=PQ-f(Q)-TQ. The profit maximizing conditions read:

(1). $$\frac{\partial \pi}{\partial Q} = P - f'(Q) - T = 0$$

or

(2). $$P = f'(Q) + T$$

This says equate market price to the marginal cost of production plus the marginal cost of transport. Subtracting T from both sides gives the same results as the net discounted price argument would give with α =0 and β =0. Similar, but slightly more complex, proofs show the same for α , i, and β .

Note 5. Suppose the following:

α_k	Prob $\alpha_k = \gamma_k$
1	.2
2	.6
3	.2

Thus $\tilde{\alpha} = 2$. If P=5, T=1, i = .5, and β =0,

then:

α_k	$\hat{P}_k = \dfrac{(1-\beta)(P-T)}{(1+i)^{\alpha_k}}$	Prob $\hat{P}_k = \gamma_k$
1	2.67	.2
2	1.78	.6
3	1.19	.2

So $\hat{\hat{P}} = 1.84$ and not 1.78.

Note 6. Consider the above example when $f(Q) = (1/2)Q^2$. Marginal cost of production is Q. Setting net marginal revenue equal to marginal cost yields:

(1). $\dfrac{\hat{\hat{P}} - \hat{q}}{1+i} x_1 + \hat{P}_2 x_2 + \hat{P}_3 x_3 = Q$

Differentiating Q with respect to $\hat{q}$ yields:

(2). $\dfrac{\partial Q}{\partial \hat{q}} = \dfrac{-x_1}{1+i} < 0$

which is less than zero and hence a negative slope.

Note 7. The reader may be interested in an experiment carried out by the author. Suppose the U shaped cost function (Min AVC at the origin) $C=f(Q) = (1/2)Q^2$. Marginal cost = Q. Setting MR=MC and solving for Q yields the profit maximizing output.

(1). $$MR = \frac{a(1-\beta) - 2b(1-\beta)^2 Q - T(1-\beta)}{(1+i)^{\alpha}} = Q = MC$$

(2). $$Q = \frac{a(1-\beta) - T(1-\beta)}{(1+i)^{\alpha} + 2b(1-\beta)^2}$$

The amount produced and shipped as a function of T, α, i, β, and a is easily shown.

The slopes of the functions are found by taking the first partials:

(3). $$\frac{\partial Q}{\partial T} = \frac{-(1-\beta)}{(1+i)^{\alpha} + 2b(1-\beta)^2} < 0$$ as expected

(4). $$\frac{\partial Q}{\partial a} = \frac{1-\beta}{(1+i)^{\alpha} + 2b(1-\beta)^2} > 0$$ as expected

(5). $$\frac{\partial Q}{\partial \beta} = \frac{(a-T)(1+i)^{\alpha} - 2b(a-T)(1-\beta)^2}{\left[(1+i)^{\alpha} + 2b(1-\beta)^2\right]^2} \gtreqless 0$$ because of elasticity of product demand possibilities

(6). $$\frac{\partial Q}{\partial i} = \frac{\alpha(1+i)^{\alpha-1}\left[(T-a)(1-\beta)\right]}{\left[(1+i)^{\alpha} + 2b(1-\beta)^2\right]^2} < 0$$

as expected because $[(T-a)(1-\beta)] < 0$; for if not; the marginal revenue function will be plotted in the negative quadrant - which is impossible.

(7). $$\frac{\partial Q}{\partial \alpha} = \frac{\alpha \log_e(1+i)\left[(T-a)(1-\beta)\right]}{\left[(1+i)^{\alpha} + 2b(1-\beta)^2\right]^2} < 0$$ as expected

(8). $$\frac{\partial Q}{\partial b} = \frac{2(a-T)(1-\beta)^3}{\left[(1+i)^{\alpha} + 2b(1-\beta)^2\right]^2} < 0$$ since $a-T < 0$

Note 8. As one of the many possible examples, consider an increase in t. Such an increase will increase the marginal cost of producing any given output. ($MC = P_Z + t/MPP_Z = P_L/MPP_L$. If t increases to t_1, Z becomes more expensive and less is used; whereas L becomes relatively less expensive, so more is used. Given diminishing marginal productivity, MPP_L decreases. Since P_L is constant, MC must increase). The increase in MC will decrease profit maximizing Q. The result will be less Q transported. Less Z will also be transported. However, the transportation company may have gained, lost, or had constant revenues. If the demand for transport of the resources is elastic or of unitary elasticity, total revenue of the transport firm will decrease. However, if the demand for transport of the resource is inelastic enough, the increase in revenue for the input transportation may be more than enough to offset the loss of revenues in the transport of the final product. The interested reader can investigate cases. The important point here is that the transport company should recognize the domain of impact of their decisions.

Note 9. The question of modal splitting in this context is a difficult one because it entails changing an assumption made previously about the demand for the product. If modal splitting is allowed for, certain goods will get to the market very rapidly whereas other goods will arrive after a longer period of time. Assume, for simplicity, the existence of an expensive, instantaneous mode (called air) and a free, slow mode (called sea). How is the equilibrium output of goods decided? What will be their prices? Speed loses its advantage if the A goods must wait until the S goods arrive (so that Q_A+Q_S goods can confront the market). However, if the A goods are sold when they arrive in the market place now and the S goods are sold when they arrive α days from now, the instantaneous demand function concept is lost.

One could perhaps assume that the demand period was for a certain length of time and that the market equilibrium is determined by the end of that period, but that instantaneously arriving goods are sold right now at the equilibrium market price. In such a case $P = f(Q_A+Q_S) = \tilde{P} = a-b(Q_A+Q_S)$. Revenues are attributed when the goods arrive at the market place. Hence, if modal splits exist, one set of total revenues is received immediately and the other set is received in α days.

The overall profit function reads (after proper discounting);

$$(1).\quad \hat{\pi} = (\tilde{P}-T)Q_A + \frac{\tilde{P}}{(1+\lambda)^\alpha}Q_S - f(Q_S+Q_A)$$

or

$$(2).\quad \hat{\pi} = aQ_A - bQ_SQ_A - bQ_A^2 - TQ_A + \frac{aQ_S}{(1+\lambda)^\alpha} - \frac{bQ_S^2}{(1+\lambda)^\alpha} - \frac{bQ_SQ_A}{(1+\lambda)^\alpha} - f(Q_S+Q_A)$$

The optimal amount of goods shipped by air is found by:

$$(3).\quad \frac{\partial\hat{\pi}}{\partial Q_A} = a - bQ_S - 2bQ_A - T - \frac{bQ_S}{(1+\lambda)^\alpha} - f'(Q_S+Q_A) = 0$$

The optimal amount of goods shipped by sea is found by:

$$(4).\quad \frac{\partial\hat{\pi}}{\partial Q_S} = -bQ_A + \frac{a}{(1+\lambda)^\alpha} - \frac{2bQ_S}{(1+\lambda)^\alpha} - \frac{bQ_A}{(1+\lambda)^\alpha} - f'(Q_S+Q_A) = 0$$

Setting equation (3) equal to equation (4) gives:

$$(5).\quad a-bQ_S-2bQ_A-T-\frac{bQ_S}{(1+i)^{\alpha}}=-bQ_A+\frac{a}{(1+i)^{\alpha}}-\frac{2bQ_S}{(1+i)^{\alpha}}-\frac{bQ_A}{(1+i)^{\alpha}}.$$

Adding and subtracting the proper terms to both sides yields:

$$(6).\quad a-bQ_S-bQ_A-T=\frac{a}{(1+i)^{\alpha}}-\frac{bQ_S}{(1+i)^{\alpha}}-\frac{bQ_A}{(1+i)^{\alpha}}$$

which says

$$(7).\quad \tilde{P}-T=\frac{\tilde{P}}{(1+i)^{\alpha}}$$

Only when net prices are equal can modal splitting occur and if $\tilde{\Pi} > \Pi_A$ and Π_S . This may not be a relevant profit position if the net discounted demand curves intersect after the respective net discounted marginal revenue curves become negative.

Note 10. Consider the case where the demand curve exists daily and the firm is considering splitting modes. The firm will be satisfying a certain amount of demand today with Q_A units, for which they receive a price $P_A = a-bQ_A$ and a net price $\tilde{P}_A = a-bQ_A-T$. In α days Q_S reaches the market for which they receive a price $P_S = a-bQ_S$ and a net discounted price $\tilde{P}_S = \frac{a-bQ_S}{(1+i)^{\alpha}}$

Profits are:

$$(1).\quad \Pi = aQ_A-bQ_A^2-TQ_A+\frac{aQ_S-bQ_S^2}{(1+i)^{\alpha}}-f(Q_S+Q_A)$$

The optimal amounts of Q_A and Q_S are found by taking the partials:

(2). $\frac{\partial \pi}{\partial Q_A} = a - 2bQ_A - T - f'(Q_S + Q_A) = 0$

(3). $\frac{\partial \pi}{\partial Q_S} = \frac{a - 2bQ_S}{(1+i)^{\alpha}} - f'(Q_S + Q_A) = 0$

Equating $\frac{\partial \pi}{\partial Q_A}$ and $\frac{\partial \pi}{\partial Q_S}$ yields:

(4). $a - 2bQ_A - T = \frac{a - 2bQ_S}{(1+i)^{\alpha}}$

which says only split modes when net discounted marginal revenues are equal and $\tilde{\pi} > \pi_A$ and π_S.

Note 11. Consider two markets and two modes. The firm must look at,

(1). $\widetilde{MR}_A^1 + \widetilde{MR}_S^2 = MC$

(2). $\widetilde{MR}_A^1 + \widetilde{MR}_A^2 = MC$

(3). $\widetilde{MR}_A^2 + \widetilde{MR}_S^1 = MC$

(4). $\widetilde{MR}_S^2 + \widetilde{MR}_S^1 = MC$

In cases (1) and (3) it is possible to notice a firm shipping by different modes. However, this is an aggregation problem because of the two markets involved.

CHAPTER III

AIR FREIGHT DEMAND MODEL: DISCRIMINANT ANALYSIS

The firm's demand curve for transport with respect to several variables has been developed above. Also shown was a model for modal choice. It is time to put the theoretical model of transport choice and transport demand into a testable statistical model. The variables suggested to us by the theoretical analysis will be used to see if they explain empirical shipments and yield the expected direction of influence.

A statistical model will now be developed which will form the framework for the empirical tests of the theoretical model. A model, expressing the probability that a certain shipment identified by values of the explanatory variables goes by a particular mode, is presented below.

At this stage in the analysis, it is not important to explicitly denote the explanatory variables in the analysis. Identify the explanatory variables by a vector X [where $X = P, T_A, T_S, i, \alpha_A, \alpha_S, \beta_A, \beta_S)$]. Each shipment has a vector of explanatory variables associated with it, i.e. each shipment will be sold at a certain price, incur a certain transport cost, take a certain number of days to get from origin to destination, etc.

The model is based on a statistical technique known as discriminant analysis. Discriminant analysis has been used quite often in the biological sciences. Recently it has been used in economics.[1]

Basically it is a method which separates entities (as measured by the vector of explanatory variables) into members of certain populations. It accomplishes this by a linear function (if there are two populations; or by n-1 linear equations if there are n populations), the coefficients of which are determined by maximizing the distance between the means of the two populations divided by the variance within the populations.[2]

For the two population case, any vector of explanatory variables, X, which yields a value of the discriminant function higher than a certain value (a critical value) will be said to be a member of one population; any X yielding a value of the discriminant less than the critical value will be said to be a member of the other population.

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[1]See for example:

D. Durand, "Risk Elements in Consumer Installment Financing", Financial Research Program, Studies in Consumer Installment Financing 8, (New York, National Bureau of Economic Research, 1941), p. 125.

G. Tintner, "Some Applications of Multivariate Analysis to Economic Data", Journal of the American Statistical Association, Volume 46, 1946, p. 476.

Stanley Warner, Stochastic Choice of Mode in Urban Travel: A Study in Binary Choice, (Evanston, Northwestern Press, 1962).

D. Blood and C. Baker, "Problems of Linear Discrimination", Journal of Farm Economics, Volume XL, 1958, pp. 674-683.

[2]T.W. Anderson, Introduction to Multivariate Statistical Analysis, (New York, John Wiley and Sons, 1958), Chapter 6.

The basis for this analysis is T.W. Anderson's Introduction to Multivariate Statistical Analysis.[3] Let A and S be two populations (air and sea) with density functions $z_A(X)$ and $z_S(X)$ respectively. The goal is to divide the universe of observations into two mutually exclusive and exhaustive regions R_A and R_S. If an observation falls into region R_A, it is said that it comes from A.

The design is to determine the probability that a given observation comes from A based on discriminant analysis. If the probability is of a certain level, that observation will be assigned to population A. The goal is to choose the R_A and R_S so that the costs of misclassification (i.e. the cost associated with classifying an actual air shipment as a sea shipment - placing an actual resident of R_A into R_S - and vice versa) are minimized.

If the *a priori* probabilities are known (q_h is the *a priori* probability for the h^{th} population) for the populations A and S, the conditional probability of an observation coming from a population given the values of the components of the vector of explanatory variables can be defined. The *a priori* probabilities may be based on previous studies done or may be based on the assumption of equal ignorance, i.e. $q_A = q_S = .5$.

The conditional probability of the observation (with explanatory vector X) coming from A is:

(1). $$\frac{q_A \; z_A(X)}{\sum_{h=A,S} q_h \; z_h(X)}$$ (Anderson, p. 143)

Call this $P_{r_A}(X)$.

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[3] Ibid

An analogous relationship exists for $P_{r_S}(X)$, i.e.

(2). $$\frac{q_S\ z_S(X)}{\sum_{h=A,S} q_h\ z_h(X)}$$

Now form $P_{r_S}(X)/P_{r_A}(X)$ which is:

(3). $$\frac{P_{r_S}(X)}{P_{r_A}(X)} = \frac{q_S\ z_S(X)}{q_A\ z_A(X)}$$

From Anderson it is known that if the below are multivariate normal populations with equal covariance matrices i.e. $N(M^1, V)$ and $N(M^2, V)$ where $M^h = (M_1^h, \ldots, M_w^h)$ is the vector of means of the h^{th} population (h=A,S) and V is the matrix of variances and covariances of each population (the assumption that the V's are equal for all populations is not crucial - unequal V's merely add computational burden but leave the analysis intact), then the h^{th} density is:

(4). $$z_h(X) = \frac{1}{(2\pi)^{\frac{1}{2}z}\ |V|^{\frac{1}{2}}}\ e^{-\frac{1}{2}(X-M^h)'\ V^{-1}\ (X-M^h)}$$

(Anderson, p. 133)

Thus,

(5). $$z_A(X) = \frac{1}{(2\pi)^{\frac{1}{2}z}\ |V|^{\frac{1}{2}}}\ e^{-\frac{1}{2}(X-M_A)'\ V^{-1}\ (X-M_A)}$$

(6). $$z_S(X) = \frac{1}{(2\pi)^{\frac{1}{2}z}\ |V|^{\frac{1}{2}}}\ e^{-\frac{1}{2}(X-M_S)'V^{-1}(X-M_S)}$$

Then form $z_S(X)/\ z_A(X)$

which is

$$(7)\quad \frac{z_S(X)}{z_A(X)} = e^{X'V^{-1}(M_S-M_A)-\frac{1}{2}(M_S+M_A)'V^{-1}(M_S-M_A)}$$

By substitution

$$(8).\quad \frac{P_{r_S}(X)}{P_{r_A}(X)} = \frac{q_S\ z_S(X)}{q_A\ z_A(X)} = \frac{q_S}{q_A}\ e^{(M_S-M_A)'V^{-1}X-\frac{1}{2}(M_S+M_A)'V^{-1}(M_S-M_A)}$$

where: $P_{r_h}(X)$ is the predicted probability that mode h is chosen

q_h is the a priori probability that mode h is chosen

M_h is the vector of means of the explanatory variables

$(M_S-M_A)'V^{-1}X$ is the discriminant function[4]

V^{-1} is the inverse of the variance covariance matrix

It can be shown that:

$$(9).\quad P_{r_A}(X) = \frac{1}{1+\frac{P_{r_S}(X)}{P_{r_A}(X)}}$$

The derivation is as follows:

$$(10).\quad \frac{P_{r_S}(X)}{P_{r_A}(X)} + \frac{P_{r_A}(X)}{P_{r_A}(X)} = \frac{\sum_{h=A,S} P_{r_h}(X)}{P_{r_A}(X)}$$

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[4]Ibid, p. 143.

Now a probability distribution must sum to one, so that:

$$(11).\quad \sum_{h=A,S} P_{r_h}(X) = P_{r_A}(X) + P_{r_S}(X) = 1$$

Thus,

$$(12).\quad \frac{P_{r_S}(X)}{P_{r_A}(X)} + \frac{P_{r_A}(X)}{P_{r_A}(X)} = \frac{\sum_{h=A,S} P_{r_h}(X)}{P_{r_A}(X)} = \frac{1}{P_{r_A}(X)}$$

Therefore,

$$(13).\quad P_{r_A}(X) = \frac{1}{\frac{P_{r_S}(X)}{P_{r_A}(X)} + \frac{P_{r_A}(X)}{P_{r_A}(X)}} = \frac{1}{1 + \frac{P_{r_S}(X)}{P_{r_A}(X)}}$$

From the above is determined the value of $P_{r_S}(X)/P_{r_A}(X)$

Calling

$$(14).\quad D_{SA}(X) = [(M_S - M_A)'V^{-1}X - \tfrac{1}{2}(M_S + M_A)'V^{-1}(M_S - M_A)] + \log (q_S/q_A)$$

By substitution,

$$(15).\quad P_{r_A}(X) = \frac{1}{1 + e^{D_{SA}(X)}}$$

$D_{SA}(X)$ is the discriminant function $[(M_S-M_A)'V^{-1}X]$ and its associated constants $[-\tfrac{1}{2}(M_S+M_A)'V^{-1}(M_S-M_A) + \log (q_S/q_A)]$ and is obtained by a straight forward method from the data as shown by Anderson pp. 150-151.

The analysis can easily be generalized to more than two modes. If n modes are available, there are n regions, and n-1 relevant $P_{r_h}(X)/ P_{r_A}(X)$ relations.

In general,

$$(16). \quad P_{r_A}(X) = \frac{1}{1 + \sum_{h \neq A} e^{D_{hA}(X)}}$$

It is of interest to determine the sensitivity of choice to a change in the decision variables e.g. $\partial P_{r_A}(X)/ \partial T_A$.

Multiplying the above by $T_A/P_{r_A}(X)$ yields:

$$(17). \quad \frac{\Delta P_{r_A}(X)}{P_{r_A}(X)} \Big/ \frac{\Delta T_A}{T_A}$$ which is termed

as the own price elasticity of choice. Taking $\partial P_{r_A}(X)/\partial T_S$ and multiplying by $T_S/ P_{r_S}(X)$ yields:

$$(18). \quad \frac{\Delta P_{r_A}(X)}{P_{r_A}(X)} \Big/ \frac{\Delta T_S}{T_S}$$ which is termed

as the cross price elasticity of choice.

Viewing

$$(19). \quad P_{r_A}(X) = \frac{1}{1 + e^{D_{SA}(X)}}$$

$D_{SA}(X)$ will be of the form

$$D_{SA}(X) = -\alpha_1 - \beta_1 x_1 - \beta_2 x_2 - \cdots - \beta_w x_w$$

where x_h is one of the explanatory variables.

(20). $$\frac{\partial P_{r_A}(X)}{\partial x_1} = \frac{\dfrac{e^{D_{SA}(X)}}{x_1}}{\left[1+e^{D_{SA}(X)}\right]^2} =$$

$$= \frac{-\partial e^{-\alpha - \beta_1 x_1 - \beta_2 x_2 - \cdots}}{\partial x_1}\left[P_{r_A}(X)\right]^2$$

$$= \beta_1 e^{D_{SA}(X)}\left[P_{r_A}(X)\right]^2$$

So,

(21). $$\frac{\Delta P_{r_A}(X)}{P_{r_A}(X)} \Big/ \frac{\Delta x_1}{x_1} = \beta_1 x_1 e^{D_{SA}(X)} P_{r_A}(X)$$

If $x_1 = T_A$, it is expected that

(22). $$\frac{\partial P_{r_A}(X)}{\partial T_A} < 0 \text{ or } \beta_1 e^{D_{SA}(X)}\left[P_{r_A}(X)\right]^2 < 0$$

Since $e^{D_{SA}(X)} > 0$ and $\left[P_{r_A}(X)\right]^2 > 0$, $\beta_1 < 0$ is needed to insure $\partial P_{r_A}(X)/\partial T_A < 0$. This is as expected from the theoretical section, i.e. the sign of the own transport rate to be negative.

Cross rate elasticities and elasticities with respect to changes in the other explanatory variables may also be calculated.

The probability function can be graphed as shown below:

$P_{r_A}(X)$	$e^{D_{SA}(X)}$	$D_{SA}(X)$
1/2	1	0
0	∞	+ ∞
1	0	- ∞
3/4	1/3	-1.10
4/5	1/4	-1.39
1/3	2	.69
1/4	3	1.10
1/5	4	1.39

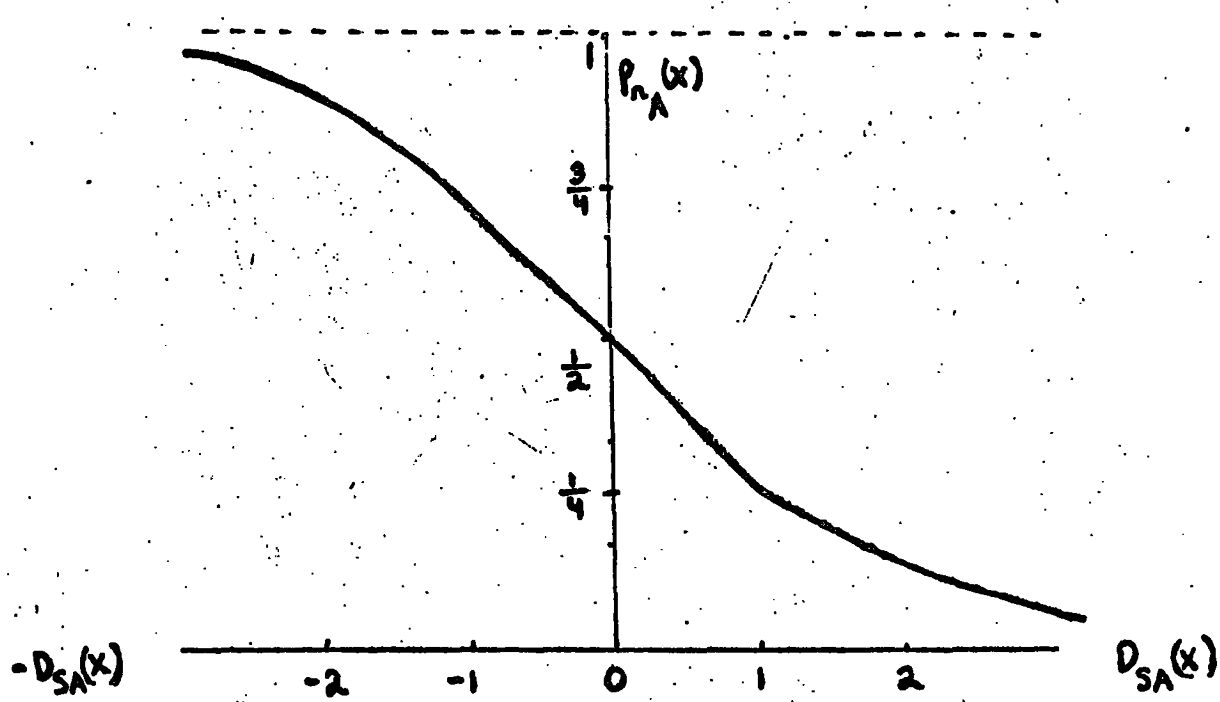

DIAGRAM III-1

APPENDIX TO CHAPTER III

SENSITIVITY ANALYSIS OF A DISCRIMINANT MODEL

It has been shown that:

$$(1).\quad P_{r_A}(X) = \frac{1}{1+\dfrac{q_S}{q_A}\, e^{(M_S-M_A)'V^{-1}X-\frac{1}{2}(M_S+M_A)'V^{-1}(M_S-M_A)}}$$

Now perform a sensitivity analysis on $P_{r_A}(X)$ for various values of q_A. The purpose of this sensitivity analysis is to see how various choices of q_A (e.g. equal ignorance, past study results, educated guesses, etc.) influence the end results and how great the range of possible end results is.

Assume possible values for

$$(2).\quad W = (M_S-M_A)'V^{-1}X - \tfrac{1}{2}(M_S+M_A)'V^{-1}(M_S-M_A).$$

Suppose W takes on values of -1, 0, 1, and 2 which says e^W takes on values .37, 1, 2.7, and 7.4.

The effects of different *a priori* probability choices under the assumptions given above are shown below:

q_A	q_S	(W=-1) $P_{r_A}(X)$	(W= 0) $P_{r_A}(X)$	(W= 1) $P_{r_A}(X)$	(W= 2) $P_{r_A}(X)$
.5	.5	.73	.50	.27	.119
.1	.9	.23	.10	.04	.013
.2	.8	.40	.20	.08	.033
.3	.7	.54	.30	.14	.054
.4	.6	.65	.40	.20	.083
.6	.4	.80	.60	.36	.170
.7	.3	.86	.70	.45	.240
.8	.2	.92	.80	.60	.350
.9	.1	.96	.90	.77	.550

At first glance it appears that the _a priori_ probabilities are very important for the model to function. For instance, a vector of independent variables yielding a W of 1 will yield a $P_{r_A}(X)$ of .04 if the _a priori_ probability for air is .1; whereas the same observation will yield a $P_{r_A}(X)$ of .77 if q_A = .9. However, these are not the necessary probabilities for proper classification.

It can be recognized that the correct value of the _a priori_ probabilities are only needed for "true" predicted probabilities of air movement -- but is not necessary for classification purposes. This can be seen by looking across the above table. For any given _a priori_ probability the value of the discriminant (the $\tilde{P}_{r_A}(X)$) will always give higher numerical probabilities as W decreases. Proper choice of $\tilde{P}_{r_A}(X)$ given a q_A will insure the same classification of air and sea under any q_A chosen. Thus, although only one *a priori* probability is truely correct and will give the precise numerical probability that a shipment will go by air, any *a priori* probability will classify a given observation in the same manner relative to any other observation.

As an example, suppose the true *a priori* probability is $q_A = .5$ (implying $q_S = .5$). If W equals -1, the true probability that that particular shipment would go by air is .73. However, suppose that $q_A = .1$ was chosen. The W of -1 would yield $P_{r_A}(X) = .23$. This would not be the numerical probability that this good goes by air, but the choice of $\tilde{P}_{r_A}(X)$ insures that this shipment is classified as air. If $\tilde{P}_{r_A}(X) = .5$ when $q_A = .5$, then $\tilde{P}_{r_A}(X) = .1$ when $q_A = .1$. Analogous statements hold for other values of q_A.

CHAPTER IV

EMPIRICAL RESULTS

It is time to turn to the actual empirical testing of the theoretical model. The theory presented previously can incorporate the influence of competition among many modes.

Rate structures can either be known, as they are under regulation, or subject to variation, as they would be in a competitive situation. A variety of quality differences can be introduced, i.e. differences in such things as average travel time and average damage and loss rates pose no problem as was shown above. The same is true of the probability distributions of these quality variables.

In order to simplify exposition, however, the analysis will be conducted under the atmosphere of perfect competition and perfect knowledge. There will only be five determinants of transport demand considered: the shipper's cost of production, the interest rate, transport rates, average travel times, and the demand for the shipper's product. Inventory costs, aside from those that are directly due to transport time, are ignored. Production and the costs it entails are treated as instantaneous, but revenue is received only after goods reach market.

The analysis developed in the theoretical body of the thesis suggests that what is popularly called modal splitting, i.e. the division of shipments between several modes, is due to various kinds of aggregation, i.e. to the grouping of data over time, groupings of firms who produce different products, have different cost functions, and/or ship to different markets.

Previous work by Beuthe and Moses[1] clearly demonstrates that individual shippers of relatively homogeneous products tend to choose one mode or another. For that study the authors were able to obtain excellent data on shipments of grain from hundreds of individual elevators in Illinois to the Chicago market and to Southern markets, primarily to New Orleans. Data on transport costs from each elevator to these markets by rail, truck, and water were obtained directly. Excellent price data was available on a daily basis. It was found that more than eighty-five per cent of all shipments involved a single mode.

Phenomena that involve exclusive choice lend themselves to a statistical technique known as discriminant analysis in which the dependent variable takes on values of zero or one. In the grain study there was a vector of independent variables for each shipping point, these variables representing time, cost, etc., of shipping. The model was able to separate the various mode populations successfully, which is to say that it tended to correctly allocate shipments to the mode actually used. For the study of air cargo demand detailed data of the above sort was not available. It was, therefore, necessary to adjust the basic statistical procedure from that described in Chapter III above. The data available and the required adjustment are explained below.

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[1] M.V. Beuthe and L.N. Moses, "The Demand for Transportation: The Influence of Time", in Transportation: A Service, (New York Academy of Sciences, New York, 1968), pp. 61-65.

The most comprehensive data on overseas shipments of commodities is published by the Bureau of the Census.[2] This source contains information on tonnages shipped by air and sea from various customs districts to different countries. Information on total value and value per pound are also found in this source. The quantity and value data are in terms of a seven digit classification of commodities. Information on rates by air and sea was obtained from the International Air Transport Association[3] and the Federal Maritime Commission.[4] The model was applied to the North Atlantic Trade Route, and in particular to shipments from the United States to Great Britain, because it involves the largest number of commodity groups. A sample of 459 Schedule B commodities was available for January, 1967. Obviously many more than 459 commodities are traded. The data availability is constrained because of the Maritime commodity-rate code, which is not the same as Schedule B. Only the Schedule B codes that have a one to one correspondence with the Maritime code were used. This gave unique rates to each Schedule B movement.

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[2]U.S., Department of Commerce, Bureau of the Census, U.S. Exports of Domestic and Foreign Merchandise, District of Exportation by Country of Destination by Schedule B Commodity and Method of Transportation, January, 1967, microfilm I.D. number EM 565, Suitland, Maryland.

[3]A.C.T. (Air Cargo Tariff) Worldwide, a publication of Scandinavian Airlines System and Swissair, provided by IATA.

[4]Rates are based on North Atlantic-United Kingdom Freight Conference Tariff 46, (FMC-1), Washington, D.C., 1967.

The above data falls short of the quality of the information that was available for the grain study. As government publications go, a seven digit classification is relatively detailed. It, nevertheless, represents an aggregation of shippers with different costs of production, who are located differently and who face different transport costs. The empirical work had, for example, to proceed as if all producers were located in the New York customs district and paid transport from there to Great Britain.

Almost all of the seven digit groups involve more than one commodity, and some more than a hundred. There is every reason to believe that the individual commodities within any group have different values per pound, but the average of the entire group had to be used. It is known that several different rates are charged for many of the individual commodities within a seven digit group. This cut the potential sample size down as was explained above. Rate structures are, in addition, tapered. Since quantities for individual commodities within a group, and the frequency distribution of sizes of shipments were unknown, simple average rates were used.

Finally, data on total transport time, as against schedule time which is considered misleading because of days in loading, unloading, queuing, etc., were only available for a single firm which estimated that sea and air delivery required thirty-eight and nine days respectively. These times were used in a test run of a subsample of our 459 sample with results that are mentioned below. In summary, the statistical analysis of competition between air and sea had to be cross sectional, each seven digit group being treated as the homogeneous output of a single shipper or a group of identical shippers.

Discriminant analysis can be applied to data of the above kind. The rationale for such application is summarized in the diagram below where commodity groups 1,2,...,n are arranged on the horizontal axis from the lowest value per pound, measured at the origin, to the highest value per pound.

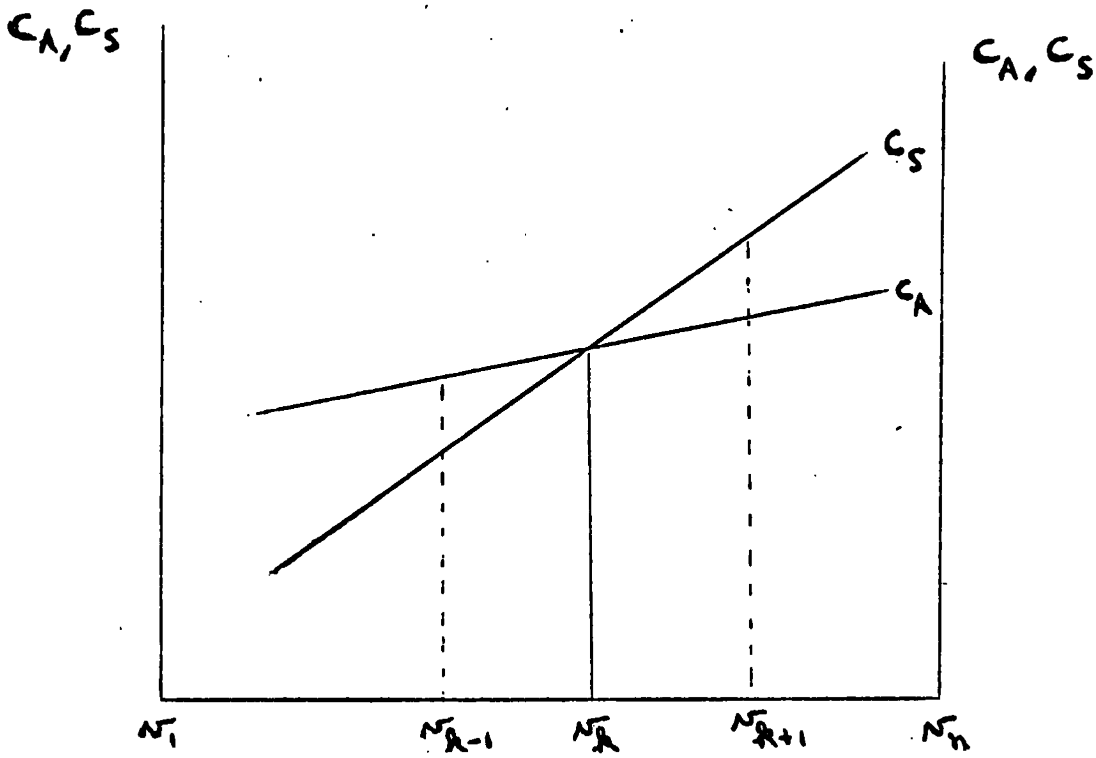

DIAGRAM IV-1

The costs, including the direct transport charge and the value of time, of shipping these by air and sea are measured on the vertical axis and are shown in the two functions C_A and C_S. The latter starts out lower because ship rates are lower than air rates. It rises more rapidly because the value of time or cost of capital tied up in shipment is based on a fixed percentage of the value of the commodities. (Also other costs such as warehousing and damage which are higher by sea than air have a positive relationship to value). According to this appraoch, shippers of the k th commodity would be indifferent between the two modes. The k + 1,...,n commodities would go exclusively by air, and the remaining ones exclusively by sea.

A discriminant model based on the above logic was applied to the sample of 459 commodities with results that were neither very good nor very bad. That such an application could even be contemplated was due to the fact that 343 of the commodity groups actually involved exclusive mode choice, 66 going entirely by air and 277 going entirely by sea. This is a surprising result, given the degree of heterogeneity within most seven digit groups. It is a result that supports the basic approach to transport demand which the theoretical section of the dissertation developed.

A second experiment, involving an application of ordinary regression techniques to the entire sample was also performed. Here the dependent variable was percent of total tonnage of a given commodity group that went by air. The results were poor. The diagram below explains why this is the case.

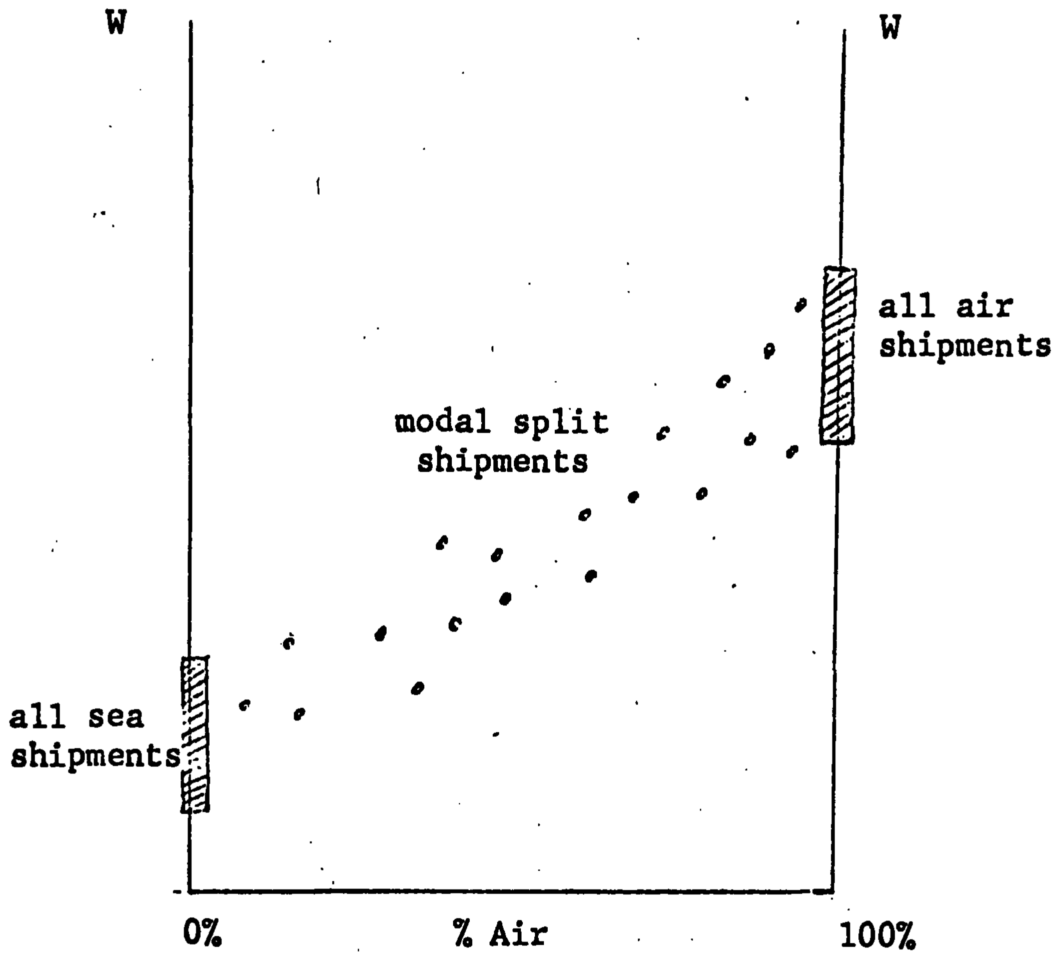

DIAGRAM IV-2

Here per cent air is measured on the horizontal axis, and W, a vector of explanatory variables is measured on the vertical axis. The two end points represent the commodity groups which exhibit exclusive mode choice. For each of these groups, wide variations in the independent variables, W, are associated with no variation in the dependent variable. A least square "line" involves substantial deviations and therefore a low R^2.

One of the objectives of the study was to estimate a meaningful demand function for air freight. The results of the above two experiments suggested that, given the data available, it would be best to adopt a compromise approach that would involve three stages: (1). application of discriminant analysis to the commodity groups that exhibited exclusive modal choice; (2). use of ordinary regression techniques to analyze the remainder; (3). the drawing together of the results from the first two stages and the derivation of an overall demand function. The entire procedure implies that modal split groups are more heterogenous than those that move exclusively by one mode. The data necessary to carry out a satisfactory test of this hypothesis are not available; had they been it would not have been necessary to adopt the compromise approach. The number of separate commodities listed in each of the commodity groups were, however, compared. There were, on the average, fewer separate commodities in the exclusive mode groups.

As explained in Chapter III, discriminant analysis enables observations to be separated into two or more classifications (in this case, two, sea and air) on the basis of a vector of independent variables. The coefficients of the discriminant function are determined by maximizing the variance between populations over the variance within populations. Once the function is determined, a certain amount of leeway is allowed the researcher in determining a cutoff score for values of the discriminant (the $\tilde{P}_{r_A}(X)$ in Chapter III).

This score is the criterion for classification. If a value yielded by the discriminant function for the independent variables associated with a given commodity is above the critical value of the discriminant, the commodity is classified in one population; if below, it is classified in the other population. The decision rule is to minimize the cost of misclassifications. In the present study the $\widetilde{P}_{r_A}(X)$ was chosen which minimized the squared error of tonnage misclassified.

When there are only two regions of classification, discriminant analysis is equivalent to a linear probability model.[5] Our model in Chapter III will thus be simplified by using this equivalence. The linear regression was carried out with the result shown in Table IV-1.

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[5]Dwight Blood and C.F. Baker, "Some Problems in Linear Discrimination", Journal of Farm Economics, XL, (November, 1958), pp. 674-683.

TABLE IV-1

a. Regression equation

$$r = 45.06606 - \underset{(2.89083)}{9.94313} \log Q - \underset{(3.19658)}{26.34271} \log (\Delta T)/V$$

b. Correlation table

	log Q	log (Δ T)/V	r
log Q	1	.478	-.384
log (Δ T)/V		1	-.515
r			1

c. Proportion of variance explained

log Q	.14759
log (Δ T)/V	.14192
R^2 =	.2895 [6]

d. F = 69.2695

Part (a) of this table is the regression equation with r the probability that a shipment goes by air; Q the total tonnage of each commodity shipped from New York to Great Britain in January, 1967; Δ T the air rate minus the sea rate; and V the price or value per pound of the good.

[6] To see how well the discriminant model works, construct a "corrected" R^2 by regressing predicted on actual values of r. The "corrected" R^2 was .932 on the above run indicating that the discriminant function is a good one.

The variables have the correct signs. Let us first examine the logic behind the negative sign of (Δ T)/V holding V constant. If T_A, the rate by air, rises and T_S, the ship rate, is constant, falls, or rises less, Δ T increases in absolute amount. In this case, the probability that a commodity will move by air should fall. The implied sign for (Δ T)/V is therefore negative. Holding Δ T constant, a change in V has the same impact. For example, as V increases, greater savings in capital can be achieved by faster transport, and the possibility is greater that the commodity will go by air. Since an increase in V reduces (Δ T)/V and increases r, the effect is negative. Thus the negative coefficient seems reasonable though the impact of simultaneous changes in Δ T and V is less clear.[7]

The price variable has, the reader will note, not been discounted as in the theoretical analysis because adequate data on transport time was not available. As mentioned earlier, information from a large firm estimated that its sea and air shipments required thirty eight and nine days. These values were used in a regression on a sample of 69 commodities, a difference in capital costs between the modes being derived for each of the commodities. The inclusion of this variable slightly increased the R^2. (The increase being based on other runs of the 69 sample size). The coefficient had the proper sign but was not statistically

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[7]As further substantiation a result shown later shows that when run separately, V has a positive sign and Δ T a negative sign, thus accounting for the negative sign of log (Δ T)/V.

significant.[8] It was, therefore, decided to leave the time variable out of the analysis until better data could be obtained.

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[8]For regression on modal split goods:

$$r = 1.43781 - \underset{(.09106)}{.25788} \log Q - \underset{(238.5)}{181.2} \Delta C$$

Proportion of variance explained

log Q	.25825
Δ C	.01555
R^2 =	.2738

where $\Delta C = C_A - C_S$

$C_A = T_A + i_A$

$i_A = (1+i)^{\alpha_A} V_A - V_A$

and Δ C is in cents per pound

Here the Δ C coefficient is not statistically significant.

For the discriminant function:

$$r = 1.36882 - \underset{(.10344)}{.08522} \log Q - \underset{(283.4)}{1340.2} \Delta C$$

Proportion of variance explained

log Q	.13352
Δ C	.33254
R^2 =	.4661

Here the Δ C coefficient is vastly significant but the coefficient of log Q is insignificant. Because of the insignificance of the coefficients, it was felt that the time cost runs should be dropped until better data is available.

The logic behind the statement that the quantity variable's coefficient has the correct sign is a bit complex. As will be explained below, it involves a relationship between value per pound and total quantity exported and a further relationship between average size of shipment and total quantity exported.

Air freight capacity is quite limited and cannot be expanded much in the short run. It is therefore not possible to ship by air all, or even a significant percentage, of those commodities for which export tonnages are very large. It would not, for example, be possible to ship all U.S. exports of coal by air, even if all air freight capacity were allocated to this commodity. This reasoning suggests that a variable for total tonnage of each commodity group exported should be included. The (hoped for and realized) negative relationship between total quantity and probability of air shipment is due to several factors.

A high rate structure has been established which tends to maximize the return to fixed capacity and to allocate that capacity to those commodities most able to bear high rates, i.e. the high value commodities.

There tends to be a negative, though not strong, relationship between total tonnage exported and average value per pound.[9] In other words, goods that are exported

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[9]For a sample of 69 commodities, the following was run:

$$r = 134.58040 - \underset{(2.55626)}{15.40050} \log Q + \underset{(.232)}{2.184} V - \underset{(61.839)}{156.126} \Delta T$$

	log Q	V	ΔT	r
log Q	1	-.230	.169	-.384
V		1	-.173	.504
ΔT			1	-.166
r				1

in large tonnages tend to have a low value per pound. Since air rates are above ship rates, it is less likely that goods whose export tonnage is large will go by air.

The negative sign of the coefficient may also reflect the operation of another variable, average size of shipment, for which there was no data. There is probably a positive relationship between total tonnage and average size of shipment. There is definately a relationship between average size of shipment and the cost disadvantage of air because sea rates have more taper built into them. Thus, the larger the tonnage of a commodity that is exported, the larger the average size of shipment tends to be, the greater the relative costliness of air, and the lower the probability of air shipment.

The quantity variable is seen to be very significant.

After completing the analysis in Table IV-1, a value of r equal to .4 was chosen to discriminate between the two transport populations. This value minimized the squared error of tons misclassified, and yielded the classification shown in Table IV-2. Thirty-seven groups -- twelve sea and twenty-five air -- were misclassified.

TABLE IV-2

	Predicted Air	Predicted Sea	Total
Actual Air	41	25	66
Actual Sea	12	265	277
Total	53	290	343

There is some intuitive appeal to using .5 as the cutoff value. It implies each population is equally likely to occur. It is also a "popular" value to choose.[10] The analysis was carried out with this value and a much poorer result was obtained. Fewer sea commodities were misclassified but many more air were misclassified. The explanation for the poorer result is that the empirical modal is biased against air shipments. Since data on transport time was not availabe, the simple difference in transport rates, Δ T, had to be used. This variable overstates the cost advantage of sea freight and overstates the cost disadvantage of air freight. Use of a value of r less than .5 compensates for the bias, increasing the probability of air shipment. Thus even if excellent data on tonnage had not been available, a cutoff value less than .5 would have been used.

The model above predicted the number of sea shipments very well. However, it was disturbing that a model of air freight demand did not predict the number of air shipments very well -- with twenty-five of sixty-six shipments misclassified. It was felt that the cost disadvantage of air (T_A-T_S) may have been too high. This is felt for two reasons. Reason one was stated above -- that time costs are not included in the model. Reason two relates to average shipment size and value per pound. There is a tendency for high value goods to travel in relatively small average shipment size. There is also a tendency for low value goods to travel in relatively large average shipment size. In the analysis above the same average shipment size was assumed for all commodities. This has resulted in an overstating of Δ T. View the diagram below.

[10]Blood and Baker, op. cit, p. 682.

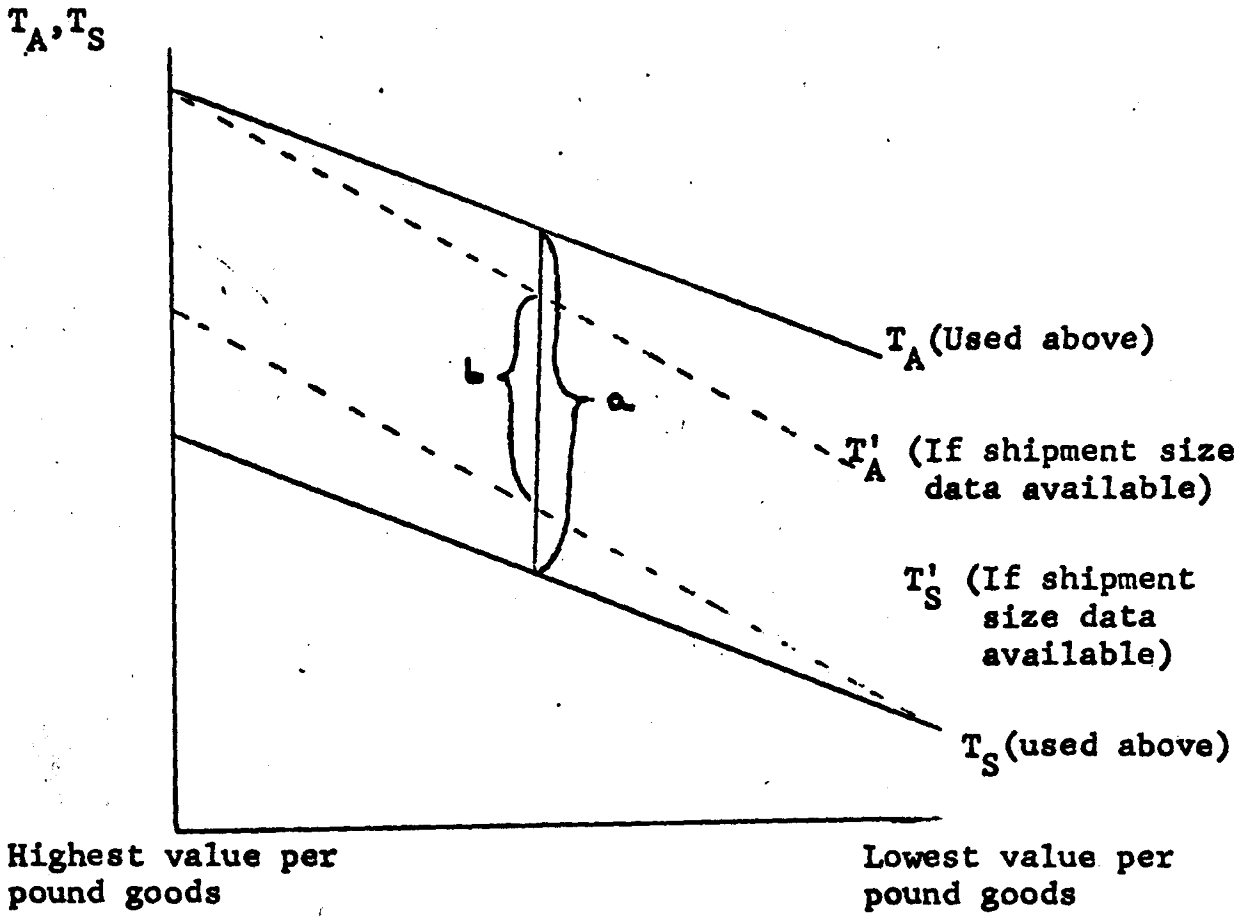

DIAGRAM IV-3

The diagram graphs T_A and T_S versus the goods listed by their value per pound from highest to lowest value. Both transport industries have a value of service rate structure so that high value goods pay higher rates. For the same size shipment, air rates always exceed sea rates. For any given commodity the assumption of equal shipment size means that the variable $T_A - T_S$ is reported as a in the diagram above.

More realistically, the known difference in shipment size should be allowed for (and shipment sizes' effect on rates). (But the answer to how much should be allowed for without data on average shipment size explains why this study did not account for the same in the empirical work). Allowing for the above would change the T_A line as follows. For high value goods it would be the same as assumed. However, for lower value goods, as more weight is shipped, a taper comes into play; and the rate decreases below that which is reported. These rates trace out T'_A above.

A similar type of occurence happens to sea rates. These rates are based on fairly large shipment size. For shipments less than this large size higher rates must be paid. The effect on the ship rates is shown as T'_S above.

If shipment size were available, one would be able to state the "true" ΔT which each shipment faced. This would appear as b in the diagram above.

To correct for this, the ΔT for the twenty-five misclassified groups was lowered to one. (It was realized that this is unrealistically low; however, it was only meant to demonstrate that ΔT was too high previously). The program was then rerun. The cutoff value of $\widetilde{P}_A(X)$ was increased as expected. The classification scheme appeared as below:

TABLE IV-3

	Predicted Air	Predicted Sea	Total
Actual Air	53	13	66
Actual Sea	4	273	277
Total	57	286	343

In addition, only the smallest quantities remain misclassified. A much better classification results by lowering the cost disadvantage of air.

It should be pointed out that the commodities misclassified are members of the lowest value per pound sector which travels by air. Likewise, the sea misclassified are members of the highest value per pound sector which travels by sea. Why do these relatively low value goods travel by air? (Especially when many higher value goods travel by sea and are appropriately classified as such). It is suggested here that these goods may represent emergency shipments. In future runs with different monthly data these commodities will be run with a dummy variable denoting emergency shipment.

The results obtained for the commodity groups that involved modal splits are shown in Table IV-4. The variables are the same as earlier:

TABLE IV-4

a. Regression equation

$$r = 129.28368 - \underset{(2.48905)}{25.80999} \log Q - \underset{(2.62722)}{5.82761} \log (\Delta T)/V$$

b. Correlation table

	log Q	log (Δ T)/V	r
log Q	1	.261	-.722
log (Δ T)/V		1	-.325
r			1

c. Proportion of variance explained

log Q	.52162
log (Δ T)/V	.01996
R^2 =	.5416

d. F = 66.7504

The signs of the coefficients are all correct and significant. Note the difference in the significance of the variables in this equation compared with that of the discriminant equation. Rates and prices are relatively more significant in the discriminant model, as we would have hoped.

Some air freight demand functions will shortly be presented. Their reliability depends on the statistical goodness of the three-stage compromise approach that has been adopted. Some insight into this is obtained by computing an overall R^2. Obviously, if we have discriminated well in stage one, many predicted values will equal actual values.

Here the deviations are zero. The residuals for the misclassified groups, where the errors are 100%, are squared and summed. The second stage yields a percentage of each commodity group that will go by air. These percentages were multiplied by total export tonnages and predicted absolute tonnages were obtained. Again the errors were squared and summed. An overall R^2 was obtained by weighting the R^2's from stages one and two by their relative contributions to total air tonnage. This yielded an overall R^2 of .6616.[11]

In general it is hazardous to separate observations into separate groups, calculating separate relationships, and then determine overall goodness by regressing predicted on actual values. Such a procedure can readily turn a very poor result into what appears to be a good result. In the present case, however, there was sound theoretical and empirical justification for the procedure.

The empirical relations estimated in the preceding two stages can be used to trace out portions of direct and cross demand curves. This can be done in several ways, one of which involves across the board percentage changes in air rates. According to this method, a one percent change, for example, in all air rates is assumed, and a new set of air rates, T_A, computed. Ship rates, T_S, are assumed constant and a new set of Δ T's derived. These are entered into the two equations of stages one and two, everything else is held constant, new r's and thus new Q_A's are estimated. This procedure was followed for changes up to plus and minus ten

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[11] As compared to the procedure on page 78, which yielded a R^2 of .2315.

percent.

The results of the above changes are shown in the graphs below. First the direct demand curve for air freight:

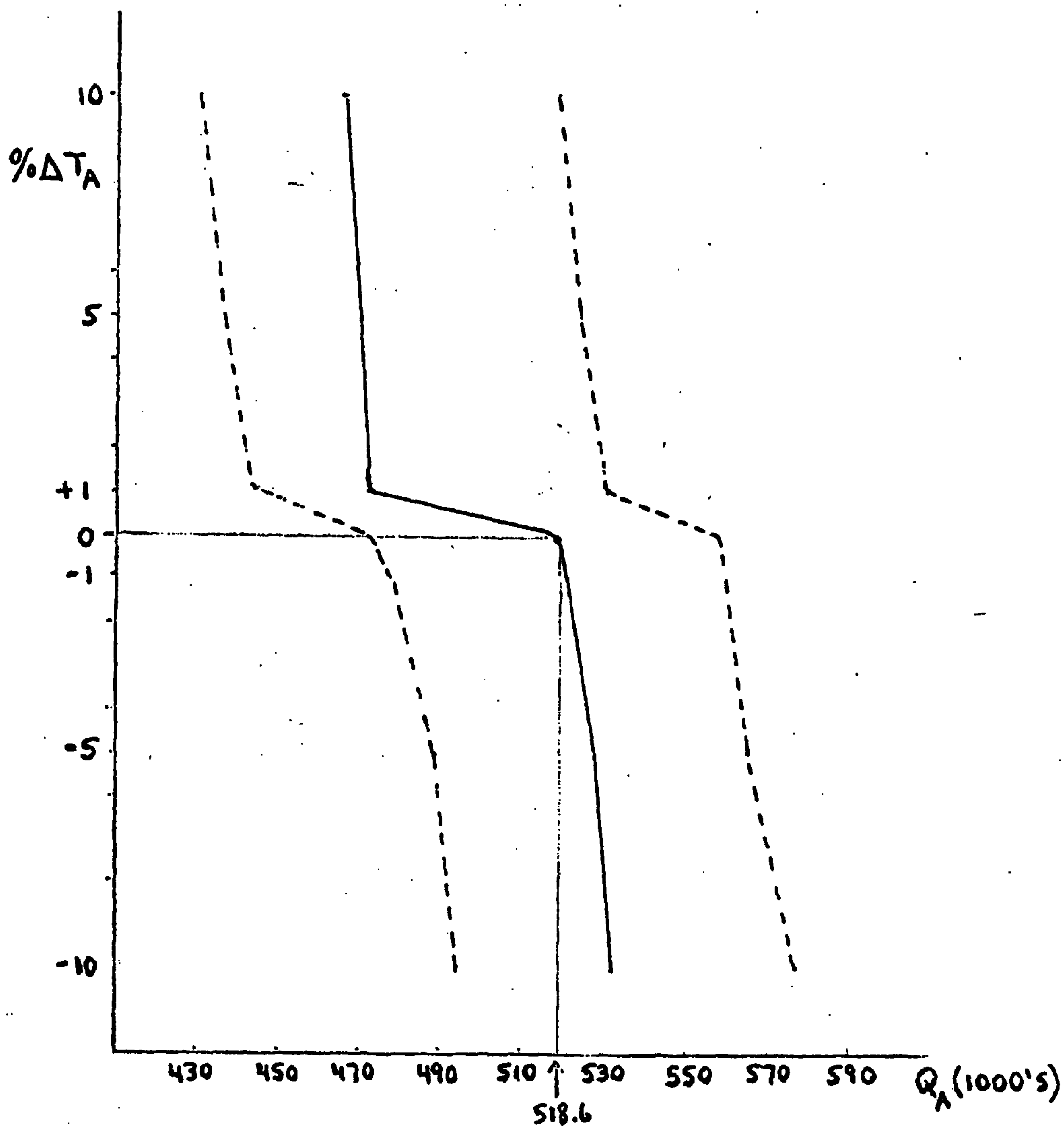

DIAGRAM IV-4

The solid line is the estimated demand curve. The two dotted lines form the 95 percent confidence level demand "band". Since the rate and price coefficient can vary in a certain range, the demand estimates will also have a range of variation. All elasticity statements must be made with this band concept in mind.

The diagram above was found to be relatively inelastic over the range investigated, with .3 being about an average value for decreases in air rates. This is a good deal more inelastic than current industry thinking suggests (in fact, the industry feels it is dealing with an elastic demand curve as is mentioned in Chapter I). In order to get large increases in air freight, percentage changes in excess of 15 percent must be assumed. Such changes would have caused some commodities that are exported in large tonnages by sea to shift to air. The method employed above is, however, not valid for large percentage changes because it cannot be assumed that the coefficients of the regression equations remain unchanged.

The diagram below shows the cross demand relationship. It is also relatively inelastic over the range investigated.

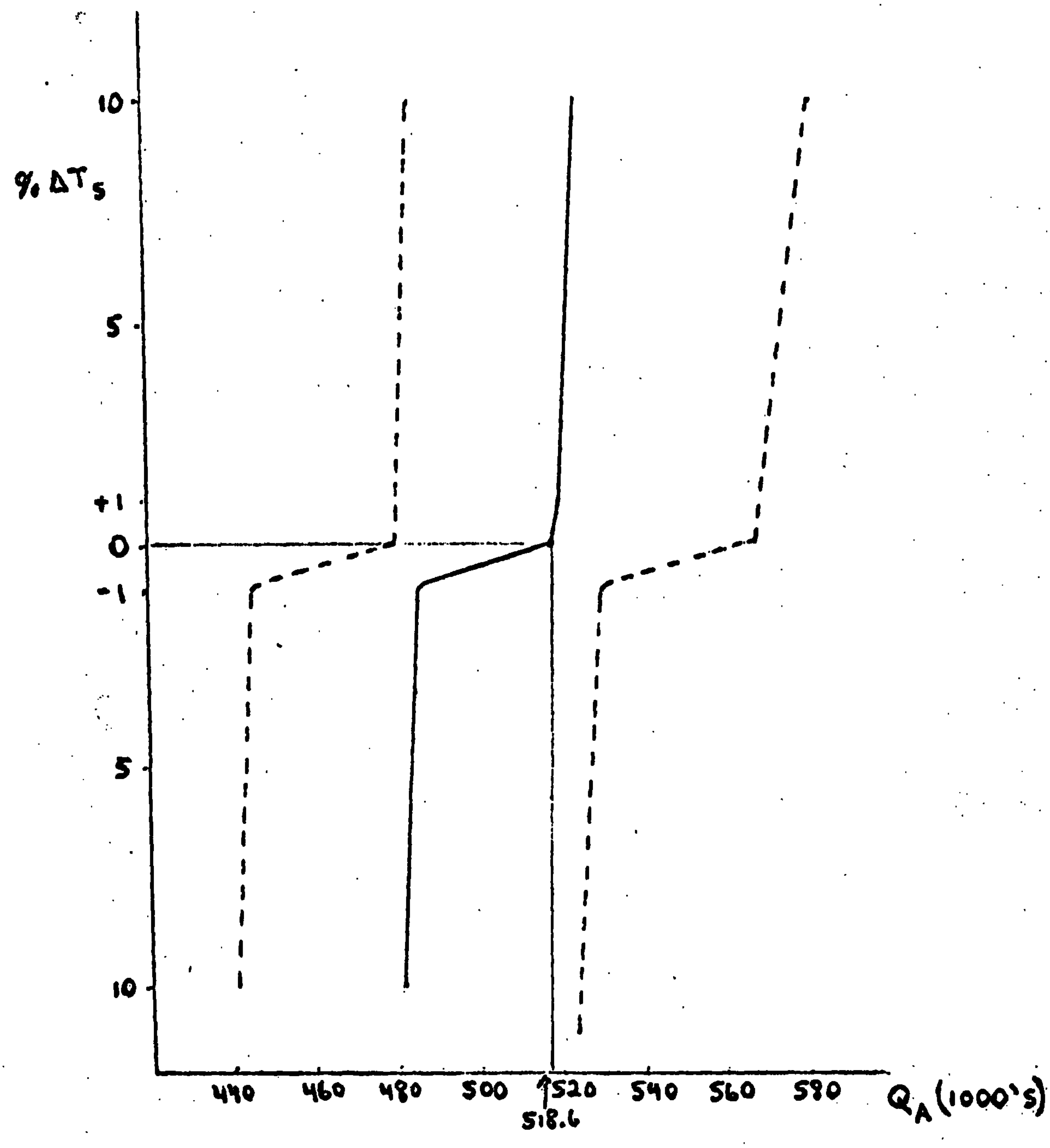

DIAGRAM IV-5

Notice the kinks in both curves. They are caused by goods with r values near the critical value of $\widetilde{P}_A(X)$ switching over from either above it to below it or from below it to above it as the result of the changes in Δ T. If the range of Δ T was increased, more kinks could be expected as goods in the discriminant part of the analysis switch classifications. Obviously as T_A increases, previous air goods are going to switch over to sea. In addition, previous sea goods, misclassified as air goods, will fall back into the sea classification. The opposite occurs as T_A decreases. As T_S increases, previous all sea goods will switch over to air. In addition, some all air goods which had previously been misclassified into sea will now be properly classified back into air. As T_S decreases, vice versa.

It should be recognized that the above method of estimating demand functions is biased downward because total export tonnage of each commodity group is held constant. There are two reasons why these tonnages could change in response to changes in air rates. First, a reduction in these rates should increase the profitability of selling in foreign markets and cause a diversion of sales to them from the domestic market. Second, the assumption that tonnage is constant implies that firms operate subject to constant cost and a short run constraint on capacity. If they have rising marginal cost curves, as was assumed in the theoretical analysis, a reduction in air rates will not only cause some existing tonnage to shift to air, but will also bring about some increase in output, and therefore exports.

This chapter was the empirical culmination of the development of a micro theory of transport. A method of estimating transport demand functions was presented. The method was applied to sea-air competition for shipment between the New York customs district and Great Britain. It is hoped that a domestic application of such a model will be forthcoming. Such functions would be immensely useful to the industries involved and to regulatory bodies. At the present time, these bodies make rate decisions on the basis of evidence on demand elasticity that is obtained by the application of questionable techniques to inadequate data.

BIBLIOGRAPHY

[1] Air Cargo Magazine, Air Freight Survey, 1963.

[2] ________, Air Freight Survey, 1964.

[3] ________, April, 1961.

[4] Air France, Do It Yourself Distribution Cost Comparison Analysis: Surface Freight versus Air Freight, New York, 1962.

[5] Air Research Bureau, Air Freight Marketing Survey, Brussels, ARB 208, no date.

[6] ________, Air Freight Market Study, Brussels, no date.

[7] ________, Intra-European Air Cargo Traffic, Brussels, ARB, 102, 1955.

[8] Alden, V.W., and Echard, E.W., Air Cargo Study for U.S.-Latin America Area, Market Research Department, Lockheed Georgia Co., August, 1965.

[9] American Management Association, A New Approach to Physical Distribution, New York, 1967.

[10] Anderson, T.W., Introduction to Multivariate Statistical Analysis, New York, John Wiley and Sons, 1958.

[11] Aviation Studies Limited, Air Cargo Commodity Survey, London, 1961.

[12] ________, Dimensions of World Traffic: III Freight, No. 87, London, 1960.

[13] ________, Scheduled Airlines Freight Traffic Growth, No. 52, London, 1958.

[14] Baumol, William, and Vinod, H.D., Studies on the Demand for Freight Transportation, Volume 1, Princeton, Mathematica, 1967.

[15] Bellaschi, J.J., An Analysis of International Cargo Flow Between Nine Geographical Areas of the World, Renton, Washington, The Boeing Co., No. TRS 645, 1960.

[16] _________, Summary of the University of Washington International Cargo Study, Renton, Washington, The Boeing Co., No. 5256, 1960.

[17] Beuthe, M.V., and Moses, L.N., "The Demand for Transportation: The Influence of Time", in Transportation: A Service, New York, New York Academy of Sciences, 1968, pp. 61-65.

[18] Blood, Dwight M., and Baker, C.B., "Some Problems of Linear Discrimination", Journal of Farm Economics, XL, 1958, pp. 674-683.

[19] The Boeing Co., Forecast of Free World Passenger and Cargo Air Traffic (1965-70-75), Renton, Washington, Market Research and Operations Analysis Section, 1959 and 1962.

[20] Brewer, Stanley, Air Cargo - The Big Breakthrough, Seattle, Bureau of Business Research, University of Washington, 1959.

[21] _________, Air Cargo Comes of Age, Seattle, Graduate School of Business Administration, University of Washington, 1966.

[22] _________, Air Cargo - The Gold is in the Terminals, Seattle, Management Series No. 5, College of Business Administration, University of Washington, no date.

[23] _________, Air Cargo - The Next Ten Years: 1956-1965, Renton, Washington, The Boeing Co., 1957.

[24] ________, The Aircraft Industry - A Study of the Possibilities for Use of Air Freight, Seattle, University of Washington, November, 1960.

[25] ________, The Environment of International Air Carriers in the Development of Freight Markets, Seattle, Graduate School of Business Administration, University of Washington, 1967.

[26] ________, The Europe-Asia Market for Air Freight, Seattle, College of Business Administration, University of Washington, 1963.

[27] ________, The North Atlantic Market for Air Freight, Renton, Washington, The Boeing Co., 1962.

[28] ________, Rhocrematics: A Scientific Approach for the Management of Material Flows, Seattle, Bureau of Business Research, University of Washington, 1960.

[29] ________, U.S. International Air Cargo Future, Renton, Washington, The Boeing Co., 1958 and 1960.

[30] ________, Vision in Air Cargo, Seattle, Bureau of Business Research, University of Washington, 1957.

[31] ________, and Rosenzweig, James, The Domestic Environment of the Air Cargo Industry, Seattle, Graduate School of Business Administration, University of Washington, 1967.

[32] ________, and ________, Military Airlift and its Relationship to the Commercial Air Cargo Industry, Seattle, Graduate School of Business Administration, University of Washington, 1967.

[33] ________, Thompson, John, and Boore, William, The Aircraft Industry - A Study of the Possibilities for Use of Air Freight, Seattle, University of Washington, 1960.

[34] ________, and Ulvested, Roger, The Cargo Jet in Military Air Transportation - An Economic Evaluation, Renton, Washington, The Boeing Co., 1960.

[35] Busch, G.A., and Mayer, K.M., An Examination of the Economic and Technical Factors Underlying the Future U.S. Domestic Market for Air Cargo Transportation Services, National IAS/ARS Joint Meeting, 1961.

[36] Carter, H.A., and Jackovich, J.C., Trends in Air Cargo, Paper No. 61-88-1782, The Boeing Co., 1961.

[37] Carteri, John, "Air Cargo Economics", Air Cargo, Law and Contemporary Problems, Durham, Duke University, 1950.

[38] Caves, Richard, Air Transport and Its Regulators: An Industry Study, Cambridge, Harvard University Press, 1962.

[39] 1963 Census of Transportation, Commodity Transportation Survey Production Area Series - Area 2, Advance Report TC63(A)C2-2, Washington, U.S. Department of Commerce, Bureau of the Census.

[40] Craig, Thomas R., Forecast of Free World Air Freight Traffic: 1962-1975, Renton, Washington, The Boeing Co., 1964.

[41] Crane, John B., "The Economics of Air Transportation", Harvard Business Review, XXII, Summer, 1944.

[42] Dawson, Michael, "A Technique of Air Cargo Market Research", American Transportation Research Forum 1964, Oxford, Indiana, Richard B. Cross Co., 1965.

[43] DeLoach, D.B., and Miklius, W., "The Effect of Air Freight Rates on Cut Flowers Sales", Journal of Farm Economics, August, 1961.

[44] Douglas Aircraft Co., Air Cargo Analysis and Forecast, Santa Monica, California, 1956.

[45] __________, The Air Cargo Handbook, Santa Monica, California, 1958.

[46] __________, Commercial Air Cargo Transportation: A Survey, Santa Monica, California, 1959.

[47] __________, Preliminary Analysis of Future Air Cargo and Air Parcel Post, Santa Monica, California, 1955.

[48] __________, Survey of Domestic Freight Movement: 1960-61, Long Beach California, 1963.

[49] Douglas, Lord ofKirtleside., "The Economics of Speed, An Examination of the Future Roles of Jet and Turbo Prop Transport Aircraft", Journal of the Institute of Transport, XXVII, May, 1957.

[50] Durand, D., "Risk Elements in Consumer Installment Financing", New York, Financial Research Program, Studies in Consumer Installment Financing 8, National Bureau of Economic Research, 1941.

[51] Durham, J.A., and Feldstein, M.J., "Regulation as a Tool in the Development of the Air Freight Industry", Virginia Law Review, October, 1948.

[52] Echard, E.W., Air Cargo Growth Study, Marketing Research Department, MRS 49, Lockheed Georgia Co., 1965.

[53] ________, Free World Air Cargo: 1965-1980 Rate Elasticity Forecast, Commercial Marketing Research Department, CMRS 77, Lockheed Georgia Co., June, 1967.

[54] ________, Marketing Planning Report - The 1963 Census of Transportation: Air Cargo, Marketing Research Department, MRS 53, Lockheed Georgia Co., no date.

[55] Federal Maritime Commission, North Atlantic-United Kingdom Freight Conference, Tariff 46 (FMC-1), Washington, D.C., 1968.

[56] Ferguson, Allen R., and Schnabel, Constance, The Demand for Air Cargo: A Pilot Study, Evanston, Northwestern University Transportation Center, 1960.

[57] Ferguson, Charles, Microeconomic Theory, Homewood, Illinois, Richard D. Irwin, Inc., 1967.

[58] Frederick, John, Commercial Air Transportation, 4th edition, Homewood, Ill., Richard D. Irwin.

[59] Gilbert, Jerome, "The Potential for International Commercial Air Shipments", International Forum for Air Cargo 1964, New York, Society of Automotive Engineers Inc., 1964.

[60] Geisser, Seymour, Predictive Discrimination in Multivariate Analysis, edited by Paruchuri Krishnaish, New York, Academic Press, 1966.

[61] Gentry, Dwight, "Air Cargo Transportation and Marketing", Journal of Marketing, July, 1952.

[62] Gorham, G., Sealy, K.R., and Stratford, A.H., "Why Companies Use Air Freight", International Forum for Air Cargo, New York, Society of Automotive Engineers, Inc., 1964.

[63] Gorrill, Edgar S., "Rationalization of Air Transport", Journal of Air Law and Commerce, IX, June, 1938.

[64] Groenewege, Adrianus, and Hertmayer, Roderick, Air Freight: A Key to Greater Profit, Southall, Middlesex, England, Aerad Printers and Publishers, 1964.

[65] Grumbridge, Jack, Marketing Management in Air Transport, London, George Allen Unwin Ltd., 1966.

[66] Guth, Herbert, and Waldo, Richard, Domestic Air Cargo Forecast: 1955-1960, Washington, D.C., Program Planning Staff, Office of the Administrator of Civil Aeronautics, October, 1952.

[67] Guy, John L., "Air Cargo Marketing in Europe - Problems and Possibilities", Third International Forum for Air Cargo, New York, American Institute of Aeronautics and Astranautics, 1966.

[68] Henry, Robert S., "The Impact of Air Freight on Surface Transportation", Air Cargo, Law and Contemporary Problems, Durham, Duke University, 1950.

[69] Heskett, J.L., "Macroeconomic Cost of Physical Distribution", Third International Forum for Air Cargo, New York, American Institute of Aeronautics and Astranautics, 1966.

[70] Hill, D.M., and Irwin, N.A., Modal Split Models for Interurban Travel, Washington, D.C., Northeast Corridor Transportation Project Technical Paper No. 7, U.S. Department of Transportation, Feb. 1967.

[71] Hoel, Paul, Introduction to Mathematical Statistics, 3rd edition, New York, John Wiley and Sons Inc., 1962.

[72] International Civil Aeronautics Organization, Air Freight in the Latin American Region, Montreal, June, 1965.

[73] ________, Air Freight Trends and Developments in the World Air Freight Industry - A Preliminary Study and Forecast, Montreal, Document 8235-c/937, circa 1963.

[74] Josephs, J.A., A Discriminant Analysis Approach to Modal Split, Washington, D.C., Northeast Corridor Transportation Project Technical Paper No. 7, U.S. Department of Transportation, Feb., 1967.

[75] Kendall, M.G., A Course in Multivariate Analysis, London, Griffen's Statistical Monographs and Courses, 1965.

[76] ________, And Stuart, A., The Advanced Theory of Statistics, Vol. 3, 1966

[77] King, R.W., "The Outlook for the Future in Air Cargo Transportation", Third International Forum for Air Cargo, New York, American Institute of Aeronautics and Astranautics, 1966.

[78] Konschegg, Lambert, "Air Freight - Its Development and Economic Advantages", Third International Forum for Air Cargo, New York, American Institute of Aeronautics and Astranautics, 1966.

[79] Lansing, John B., and Blood, Dwight M., "A Cross Section Analysis of Nonbusiness Air Travel", Journal of the American Statistical Association, LIII, 1958, pp. 928-947.

[80] Larsen, Spencer Allen, and Reitz, William, Air Cargo Potential in Drugs and Pharmaceuticals, Detroit, Wayne University Press, 1946.

[81] Lemieux, P.M., Wickstrom, L.A., and Pressler, L.L., All Air Freight Distribution - U.S. to Europe, Honeywell, Inc., no date.

[82] Lewis, H.T., "The Economics of Air Freight", in Management for Tomorrow, edited by Nicholas Glaskowsky, Stanford, Graduate School of Business, Stanford University, 1958.

[83] __________, and Cullaton, J.W., The Role of Air Freight in Physical Distribution, Boston, Harvard University Press, 1956.

[84] Lockheed Aircraft Corp., Air Cargo Trends in the U.S., Burbank, California, No. SLR/1005, 1952.

[85] Loeing, Grover, "Air Freight in Competition with Motor Transport", Aeronautical Engineering Review, XVI, June, 1957.

[86] Magnusson, Glen, and Nye, Thomas, International Cargo - Air or Sea, Payne Bruce and Associates, Inc.

[87] McDonald, B.A., and Drew, J.L., Air Transportation in the Immediate Postwar Period, Buffalo, Curtiss Wright, Airplanes Division, 1944.

[88] McIntyre, Douglas P., The Economics of Air Freighting, Melbourne, TAA, 1958.

[89] Miller, Ronald E., Domestic Airline Efficiency - An Application of Linear Programming, Cambridge, The MIT Press, 1963.

[90] Mongini, Arrigo, Some Aspects of Discriminant and Other Interurban Modal Split Models, Washington, D.C., Northeast Corridor Transportation Project Technical Paper No. 7, U.S. Department of Transportation, Feb., 1967.

[91] Mossman, Frank, "Logistics Systems and Total Cost Analysis", Papers of the Transportation Research Forum, Oxford, Indiana, Richard B. Cross Co., 1963.

[92] National Economic Development Office, Exports by Air, London, Her Majesty's Stationary Office, 1967

[93] Northrop Corporation, Air Freight Statistics, Hawthorne, California, Norair Division, NB 60-265, 1960.

[94] ________, Air Freight Trends: 1946-1970, Hawthorne, California, Norair Division, NB 60-263, 1960.

[95] Norton, Hugh S., Modern Transportation Economics, Columbus, Ohio, Charles E. Merrill Books, Inc., 1963.

[96] Orion, Henry, Domestic Air Cargo 1945-1965: A Study of Competition in a Regulated Industry, New York, Columbia University Ph.D. Thesis, 1967.

[97] Orr, Earle W., "A Synthesis of Theories of Location, of Transport Rates, and of Spatial Price Equilibrium", Journal of Regional Science, 1958.

[98] Parrish, Wayne W., "Let the Customer be Heard", Third International Forum for Air Cargo, New York, American Institute of Aeronautics and Astranautics, 1966.

[99] Pegrum, Dudley F., Transportation: Economics and Public Policy, revised edition, Homewood, Illinois, Richard D. Irwin Inc., 1968.

[100] Richards, Anthony, <u>The Role of Aviation in the Domestic Freight Transportation System of the United States</u>, McGill University, Ph.D. Thesis, 1963.

[101] Scandinavian Airlines System and Swissair, <u>A.C.T.</u> (Air Cargo Tariff) <u>Worldwide</u>, New York, 1967.

[102] Sealy, Kenneth, and Herdon, Peter, <u>Air Freight and Anglo-European Trade</u>, London, ATS Ltd., The London School of Economics and Political Science, 1961.

[103] ________, and Stratford, Alan, "Why Companies Use Air Freight", <u>International Forum for Air Cargo</u>, New York, Society of Automotive Engineers Inc., 1964.

[104] ________, and Wheatcroft, S.F., <u>World Air Freight</u>, London, Blackburn and General, 1959.

[105] Short Bros. and Harland Ltd., <u>The Potential of Air Freight</u>, Belfast, 1959.

[106] ________, <u>Survey of World Trade and Air Freighting</u>, Queens Island, Belfast, Design Department Report No. DD44, June, 1960.

[107] Shutz, George, "Air Freight from a Marketing Point of View", <u>Journal of Marketing</u>, October, 1960.

[108] Smykay, Bowersox, and Mossman, <u>Physical Distribution Management</u>, New York, MacMillan, 1961.

[109] Stanford Research Institute, <u>Economic Principles for Pricing Aircraft Services</u>, South California Labs, November, 1961.

[110] ________, <u>How to Identify Potential Users of Air Freight</u>, South Pasadena, California, 1963.

[111] Stephenson, Paul R., "Manufacturer's Physical Distribution Service Knowledge and Penalties: An Experimental Analysis", Papers of the Transportation Research Forum, Oxford, Indiana, Richard B. Cross Inc., 1963.

[112] Stoessel, Robert F., Air Freight - From Acorn to Oak, London, World Air Cargo Marketing Forum, November, 1967.

[113] Stratford, Alan, Air Transport Economics in the Supersonic Era, New York, St. Martin's Press, 1967.

[114] ________, Prospects for the Growth of Intra European Air Cargo, Rome, Centro Per Lo Sviluppo Dei Trasporti Aerie, June, 1962.

[115] Suits, D., "Linear Probability Functions and Discriminations", Ann Arbor, Michigan, discussion paper of the Research Seminar in Quantitative Economics, University of Michigan, October, 1957.

[116] Tapner, H., Air Cargo, London, Cassell and Co. Ltd., 1967.

[117] Thayer, Russell, "Changing Concepts of Air Cargo", Third International Forum for Air Cargo, New York, American Institute of Aeronautics and Astranautics, 1966.

[118] Tintner, G., "Some Applications of Multivariate Analysis to Economic Data", Journal of the American Statistical Association, Vol. 41, 1946.

[119] Tipton, Stuart G., "Beyond the Breakthrough in Air Freight", Third International Forum for Air Cargo, New York, American Institute of Aeronautics and Astranautics, 1966.

[120] Traffic World Marketing Study, An Assessment of the Potential Market for Air Freight, Washington, D.C., Traffic World Publishing Co., 1964.

[121] The Transportation Workshop 1967, Air Transportation 1975 and Beyond: A Systems Approach, Boston, The MIT Press., 1968.

[122] Ulvested, Roger, New Markets for Air Freight, Renton, Washington, The Boeing Co., 1960.

[123] Umpleby, Arthur, A Study of the Economics of Air Freight Production and Distribution, Columbus, Ohio, Ohio State University, Masters Thesis, 1957.

[124] United States Department of Commerce, Bureau of the Census, U.S. Exports of Domestic and Foreign Merchandise, District of Exportation by Country of Destination by Schedule B Commodity and Method of Transportation, Suitland, Maryland, Microfilm identification number EM563, 1967.

[125] Warner, Stanley, "Multivariate Regression of Dummy Variates Under Normality Assumptions", Journal of the American Statistical Association, December, 1963, pp. 1054-1063.

[126] ________, Stochastic Choice of Mode in Urban Travel: A Study in Binary Choice, Evanston, Northwestern University Press, 1958.

[127] Wein, Harold, Domestic Air Cargo: Its Prospects, East Lansing, Michigan, Bureau of Business and Economic Research, Graduate School of Business Administration, Michigan State University, 1962.

[128] Young, Douglas, Current Air Cargo Traffic Statistics, Montreal, Canadair Ltd., November, 1961.

VITA

William Bruce Allen

Born: January 25, 1942

Place: New Brunswick, New Jersey

Education: Brown University AB Economics 1964
Northwestern University Ph.D. Economics 1969

Publications: "Overseas Trade - Competition of Air and Sea" with Leon N. Moses in Papers and Proceedings of the Transportation Research Forum, Oxford, Indiana, Richard B. Cross and Co., 1968.

NORTHWESTERN UNIVERSITY

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FREIGHT TRANSPORTATION MODE CHOICE:
An Application to Corn Transportation

A DISSERTATION

SUBMITTED TO THE GRADUATE SCHOOL

IN PARTIAL FULFILLMENT OF THE REQUIRMENTS

for the degree

DOCTOR OF PHILOSOPHY

Field of Economics

By

MICHEL VINCENT BEUTHE

Evanston, Illinois

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Seiko Higa 860 Hinman

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CONTENT

BIBLIOGRAPHY

Anderson, T.W. "An Introduction to Multivariate Statistical Analysis" New York, John Wiley and Sons, 1968.

Arrow Transportation Company, "Guide to Published Barge Rates on Bulk Grain" Schedule No. 5., April 12, 1966.

Beckmann, Martin, McGuire C.B. and Winsten C.B., Studies in the Economics of Transportation, New Haven, Yale University Press, 1956.

Blood, D.M. and Baker, C.B., "Some Problems of Linear Discrimination", Journal of Farm Economics, XL (1958) 674-683.

Board of Trade of the City of Chicago, "The 109th Annual Report, Statistics 1966", 1967.

Fetter, F.A., "The Economic Law of Market Areas", Quarterly Journal of Economics, May 1924, 520-529.

Hyson, C.D. and Hyson, W.P. "The Economic Law of Market Areas", Quarterly Journal of Economics, May 1950, 319-327.

Illinois Co-operative Crop Report Service, Illinois Department of Agriculture "Illinois Agricultural Statistics, Annual Summary, 1966", Bulletins, 1966, No. 1.

Illinois Motor Carriers, "Agricultural and Materials Tariff", No. 600, issued by D.S. Mullins, Issuing Officer, May 16, 1966.

Josephs J.A. "Discriminant Analysis Approach to Model Split," in Approaches to the Model Split: Intercity Transportation, Northeast Corridor Transportation Project, Technical Paper No. 7, February, 1967 A1-A7.

Kendall, M.G. and Stuart, A., "The Advanced Theory of Statistics," Vol II and III, London: Charles Griffin and Co. Ltd., 1946 and 1966.

Kloth, D.W. "The Transportation of Illinois Corn, Wheat and Soybeans - Volume Movements to Introstate and Interstate Destinations and Factors Influencing such Movements", Unpublished Master Thesis, Southern Illinois University, 1964.

Lansing J.B. and Blood, D.M., "A Cross-Section Analysis of Non-business Air Travel", Journal of American Statistical Association L.III (1958) 928-947.

Mongini A. Some Aspects of Discriminant Function and Other Interurban Model Split Models. in Approaches to the Model Split: Intercity Transportation, Northeast Corridor Transportation Project, Technical Paper No. 7, February 1967, E1 - E18.

Moses, L.N. and Beuthe, M.V. "The Demand for Freight Transportation: The Influence of Time", Transportation: A Service, (New York Academy of Sciences, New York, 1968 pp. 61-65).

Moses, L.N. and Williamson H.F.Jr., Choice of Mode in Urban Transportation, The Transportation Center, Northwestern University, 1965.

Rao, C.R., "Advanced Statistical Methods in Biometric Research," New York, John Wiley and Sons, Inc. 1952.

Rao, C.R., "Linear Statistical Inference and its Applications" New York, John Wiley and Sons, Inc, 1965.

Sakashita, N. "A Microscopic Theory of Traffic Assignment", Papers and Proceedings of the First Far East Conference of the Regional Science Association, Tokyo, 1963, 49-59.

Thomas, G.B. "Calculus and Analytical Geometry," Third Edition, Addison and Wesley, 1960.

Tobin J. "Estimations of Relationships for Limited Dependent Variables", Econometrica, Vol. 26, No. 1. Jan 1968, 24-36.

Thorburn T.H., "Supply and Demand of Water Transport," F.F.I. Report, The Business Research Institute at the Stockholm School of Economics, Stockholm, 1960.

Tintner, G. "Econometrics", New York, John Wiley and Sons, Inc. 1963.

United States Department of Agriculture, Consumer and Marketing Service, Grain Division, Chicago. "Grain Market News Weekly."

Wardrop J.G., "The Distribution of Traffic on a Road System", in Theory of Traffic Flow, edited by R. Herman, 1961.

Warner S.L., "Stochastic Choice of Mode in Urban Travel: A Study in Binary Choice", Northwestern University Press, 1962.

Warner, S.L. "Multivariate Regressions of Dummy Variates under Normality Assumptions". Journal Of the American Statistical Association, December, 1963, 1054-1063.

Wills, W.J. Problems in Grain Transportation, School of Agricultural Publications, No. 22, Southern Illinois University, Carbondale, Illinois, 1966.

VITA

MICHEL VINCENT BEUTHE

Born in Forest (Brussels), Belgium on the 4th December, 1935

DOCTEUR EN DROIT and LICENCIÉ EN SCIENCES ECONOMIQUES (UNIVERSITÉ CATHOLIQUE DE LOUVAIN)

PUBLICATIONS: "Economie en Croissance Lente: Le Cas de la Belgique", O.B.A.P., Brussels, 1964.

"The Demand for Freight Transportation: The Influence of Time", with L.N. Moses, in Transportation: A Service, (New York Academy of Sciences, New York, 1968 pp. 61-65)

FREIGHT TRANSPORTATION MODE CHOICE:

An Application to Corn Transportation

The aim of the present study is first to propose a model of the mode of transportation choice for a particular commodity; second, using this theoretical model, to develop statistical method(s) for forecasting choices by shippers. In a sense it is a model of demand for transportation, although it is not designed to estimate the tonnage to be transported. Instead it answers the question: which mode(s) will be used? (The actual tonnage transported may be found in surveys of production).

The choice of a mode for a specific transportation task is made by comparing the various advantages and disadvantages of the physically possible modes. These characteristics are the rates and other costs, which together comprise the direct outlay costs; the expected time taken to accomplish the transportation; the risk of loss or damage; the dependability of the service from other points of view; and many other conveniences and inconveniences presented by each transportation mode.

It seems safe to assume that shippers as entrepreneurs are essentially profit maximising, and that all the mode characteristics have for them only one relevant and common dimension: the money value. Depending upon whether a characteristic involves a saving or a cost for the shipper relative to other modes, this money value can be positive or negative. Most of the direct outlay costs are readily available whenever public transportation is involved. The time costs can be easily estimated if accurate information exists about journey time. But the other characteristics that one is to call "quality" differences between modes, are far more difficult to convert into money values . They are, however, taken into account in actual mode choice. As such they cannot be ignored in a theoretical model, and any method which aims at forecasting the demand for a mode of transportation should propose some means of estimating their money equivalents. This is one important aspect of the present research.

Chapter I presents the theoretical choice model. The time element of cost is introduced at once along with the direct outlay costs. The "quality" characteristics are then introduced by means of a stochastic element which makes the model suitable to statistical analysis. Chapter II generalizes the stochastic model and discusses the statistical method appropriate for its testing. Chapter III applies and tests the model to a concrete problem of mode choice: the corn shipments in Illinois.

As it stands, this statistical method is able to forecast in probabilistic terms the mode which will be used from a particular origin. Applied systematically to all relevant origins it can predict the regional pattern of demand for each mode. Whenever it is possible to link some important costs of transportation to the space co-ordinates a definite regional pattern should be expected. Chapter IV and V, therefore, attempt to build a spatial model of mode choice along the lines of traditional location theory. It is closely related to the first model and uses its estimates of the "quality" differences money equivalents.

Throughout this study it is assumed that the commodity transported is produced under conditions of perfect competition. No attempt has been made to generalize to other conditions, although such a task appears quite feasible.

CHAPTER I

The Firm's Transport Demand in Perfect Competition.

Consider a firm that operates under conditions of perfect competition. It is located at A and sells all of its output of a single homogeneous product in a market located at B. The firm purchases all of its inputs locally so that the only transport it requires is for shipping its product to market. In order to introduce the two problems of mode choice and quantity the firm will ship in the simplest possible manner, two modes with radically different characterisitcs will be assumed. One of them is very expensive but very fast: it will be treated as instantaneous in the theoretical model. The other mode is very slow but very cheap: the firm's cost function is given. The price of its product in the market (P), the number of days required to ship the product to the market by the slow mode (α), the interest rate per day (i) and the per unit transport cost by the fast mode (g) are also given.

Suppose the firm's total cost (of production) function is of the form:

(1) $C = Z^n$, where Z is output per day and $n > 1$.

The firm's total cost of producing and shipping by the fast mode will then be:

(2) $C_1 = Z^n + gZ$.

Total cost of producing and shipping by the slow mode is:

(3) $C_2 = (1 + i)^\alpha Z^n$, where the term $(1 + i)^\alpha Z^n$ represents the interest charge on the money the firm must tie up in production cost when it ships by the slow mode.

In the present theoretical formulation the price of the product does not depend on the mode by which it is shipped. There is therefore only one revenue function:

(4) $R = PZ.$

The profit by each of the modes is:

(5) $\pi_1 = PZ - C_1 = PZ - Z^n - gZ.$

(6) $\pi_2 = PZ - C_x = PZ - (1 + i)^{\alpha} Z^n.$

π_1 and π_2 are not comparable since the former is realized without delay whereas the latter is realized only after some days. They can be made comparable by discounting back to the present. Let us call this discounted value $\hat{\pi}_2$.

(7) $$\hat{\pi}_2 = \frac{PZ - (1 + i)^{\alpha} Z^n}{(1 + i)^{\alpha}} = \frac{PZ}{(1 + i)^{\alpha}} - Z^n.$$

We now take the difference between equations (5) and (7):

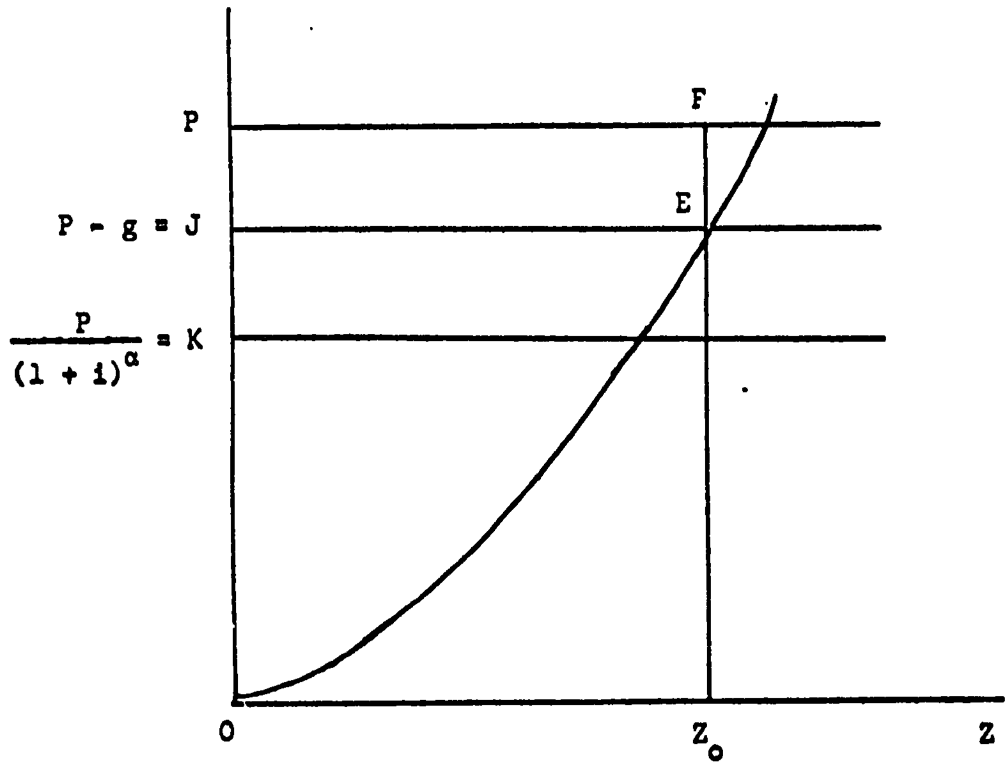
(8) $$\hat{\pi}_2 - \pi_1 = X = \frac{PZ}{(1 + i)^{\alpha}} - Z^n - PZ + Z^n + gZ$$

$$= Z\left[\frac{P}{(1 + i)^{\alpha}} - (P - g)\right].$$

For any given output, in other words, aside from times when transport capacity is strained, choice of mode depends solely on the two net prices in brackets. If X is positive the firm will ship by the slow mode; if X is negative it will ship by the fast mode; if X is equal to zero it will be indifferent. In the latter case the firm may ship all by one mode, all by the other, or engage in modal splitting.

The argument is illustrated in Diagram 1 where OC is the firm's marginal cost (of production) function, the first derivative of equation (1). The firm maximizes profit by equating marginal cost to whichever of the net prices is higher. Under the conditions shown the firm would produce Z_o, ship all of this output by the fast mode, expend OZ_oE on production, JEFP on transport, and realize a profit of OEJ.

Diagram 1



The diagram emphasizes several results: (a) the choice of mode depends on the price of the product, time of transport, the interest rate, and money cost of transport, but not on the production cost; (b) the firm's cost function determines only the output it will ship.

The extreme assumption that one mode is instantaneous and the other one free may now be dropped and a more realistic definition of X adopted.

$$(9) \qquad X = Z \left[\frac{(P - g_2)}{(1 + i)^{\alpha}} - \frac{(P - g_1)}{(i + i)^{\beta}} \right].$$

Here g_1 and g_2 are respectively the money charges for the slow and fast modes; α and β are the trip times.

It is possible to generalize this model in two ways. First, one may wish to analyze a case where the mode choice would exist among three modes or more. With three modes, there would be a third discounted profit function,

$$(10) \qquad \hat{\pi}_3 = \frac{PZ - g_3 Z - (1 + i)^{\gamma} Z^n}{(1 + i)^{\gamma}}$$

Then the choice decision is taken on the basis of the successive binary comparison of the three pair of discounted profit functions. In other words, for any given output, we take the differences

$$\hat{\pi}_2 - \hat{\pi}_1 = X_{21}$$

$$(11) \qquad \hat{\pi}_3 - \hat{\pi}_1 = X_{31}$$

$$\hat{\pi}_3 - \hat{\pi}_2 = X_{32}$$

and, if $X_{31} > 0$

$X_{32} > 0$, mode 3 is chosen;

if $X_{21} < 0$

$X_{31} < 0$, mode 1 is chosen;

if $X_{21} > 0$

$X_{32} < 0$, mode 2 is chosen.

There would be indifference between two modes if one of the X_{ij}'s were equal to zero. This decision procedure can be used as well with any number of modes or routes: for n modes there would be (n - 1): X_{ij}'s, and (n - 1) of these should be used to accept or reject any particular mode.

It is now very easy to generalize to the case where more than one market is involved. There is no need to restrict the choice problem to a choice among transportation modes. The choice might be among various carriers of a unique mode to one market, among several routes and several modes to one market, as well as among all of these to various markets. For each combination of route, carrier, mode, market there would be a particular price of the product on the relevant market. Then one proceeds as above, from the n net prices to the (n - 1): X_{ij}'s, and through successive (n - 1) subsets of these to the final choice.

Thus, we have achieved a model to explain (under conditions of perfect competition) not only the transportation mode choice, but more generally the mode and market choice. The choice is reached through successive binary comparisons of all the possible mode-market combinations. There is no limitation as to the number of these combinations.

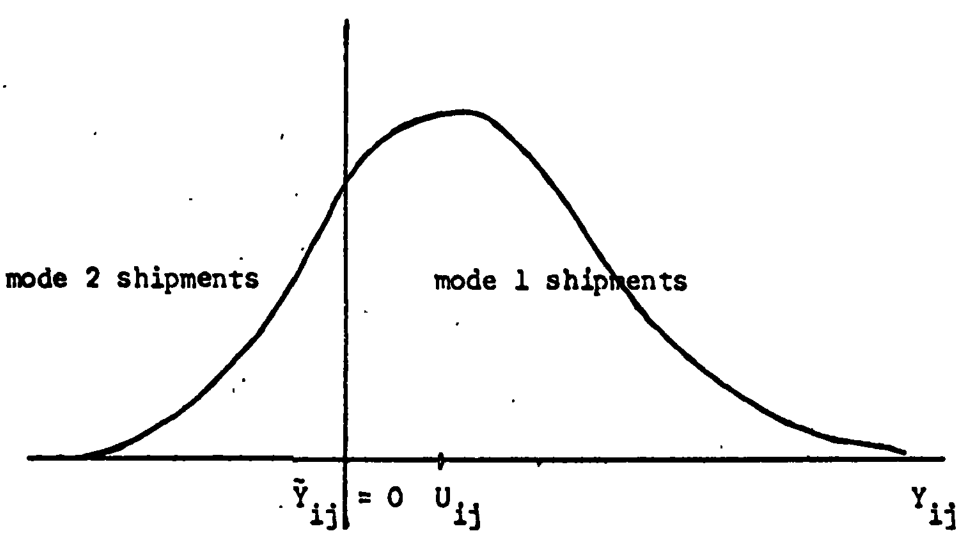
A Stochastic Model.

Throughout the first section of this chapter it was implicitly assumed that the only costs of transportation were the direct money costs and the interest cost arising from the transportation time. However, there are still some other costs incurred by the shipping firm, which may vary with the mode of transportation: the costs resulting from the particular loading equipment of the shipper, the cost of uncertainty, and, in general, the cost of the difference in service quality of the various modes. All these costs are grouped under the heading "additional costs".

We might then consider that the choice between any two modes is not based on their X_{ij}, the difference between their net prices as defined in the first section, but rather on a $Y_{ij} = X_{ij} + E_{ij}$, where E_{ij} is the difference between their additional costs. This Y_{ij} will be called the net income difference. The E_{ij}'s are presumably known or estimated by the shippers and incorporated into the calculation of which mode is most profitable. In reality, the shippers use Y_{ij} instead of X_{ij} and choose the higher net income mode.

However, because of their nature, the E_{ij}'s cannot be observed by the economist[1]. If we were to test this "choice of mode" model on a sample of firms, no information would be available on these "additional costs". Let us examine how this model could be set up for a statistical investigation. Assume that Y_{ij} is distributed with mean U_{ij} and variance σ_u over a population of firms (which produce a good and must choose which of two modes to use.[2] According to this model, the firms which record a positive value of Y_{ij} should choose to ship all of their output by mode 1, those which have a negative value of Y_{ij} should choose mode 2, and those with $Y_{ij} = 0$ should be indifferent between modes. The situation is illustrated by Diagram 2, where $\tilde{Y}_{ij} = 0$ appears as the critical value separating two regions. The region where net income is greater than $\tilde{Y}_{ij}$ should ship only by mode 1 while that where net income is less than $\tilde{Y}_{ij}$ should ship only by mode 2. Note that the mean value U_{ij} may be different from zero, as it is the case in Diagram 2.

Diagram 2



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(1) Note that all these costs are highly particular to each shipper and vary according to their scale of operation and location.

(2) Note that nothing is assumed concerning the form of the distribution.

The choice of mode decision is not so easily solved since the E_{ij}'s are not available, and the parameters of the distribution of Y_{ij} are not known. Let us assume, however, that E_{ij} is distributed independently of the X_{ij}.[1]

If the additional costs were so small that they did not affect mode choice, the distribution of X_{ij} could be cut into two pieces at $\tilde{X}_{ij} = 0$, as it was done for Y_{ij} in Diagram 2. Thus, $\tilde{X}_{ij}$ would define two decision regions for mode i and mode j respectively. The distributions of these two modes origins would correspond to the portions of thedistribution of the X_{ij} to each side of the critical value. If the E_{ij} were this small, we would be back to the deterministic model of section I.

However, if E_{ij} is of significant magnitude, the two distributions should overlap each other. As E_{ij} is a random variable independent of X_{ij}, for any particular value of X_{ij}, say X^o_{ij}, there exists a complete conditional distribution of $X^o_{ij} + E_{ij}$. It follows that among the shippers characterized by X^o_{ij}, those with $X^o_{ij} + E_{ij} > 0$ will choose mode i, while those with $X^o_{ij} + E_{ij} < 0$ will choose mode j. The overlapping distributions presented by Diagram 3 illustrate this influence of E_{ij}.

Had the deterministic model been the correct one, its test would have been an easy one. The theory provides its own decision value, i.e., $\tilde{X}_{ij} = 0$. A simple check of the transportation mode used by firms with, on one hand, a positive value of X_{ij}, and, on the other hand, a negative value of X_{ij} could have solved the question. Since it doesn't take account of other costs, this model is not realistic; the influence

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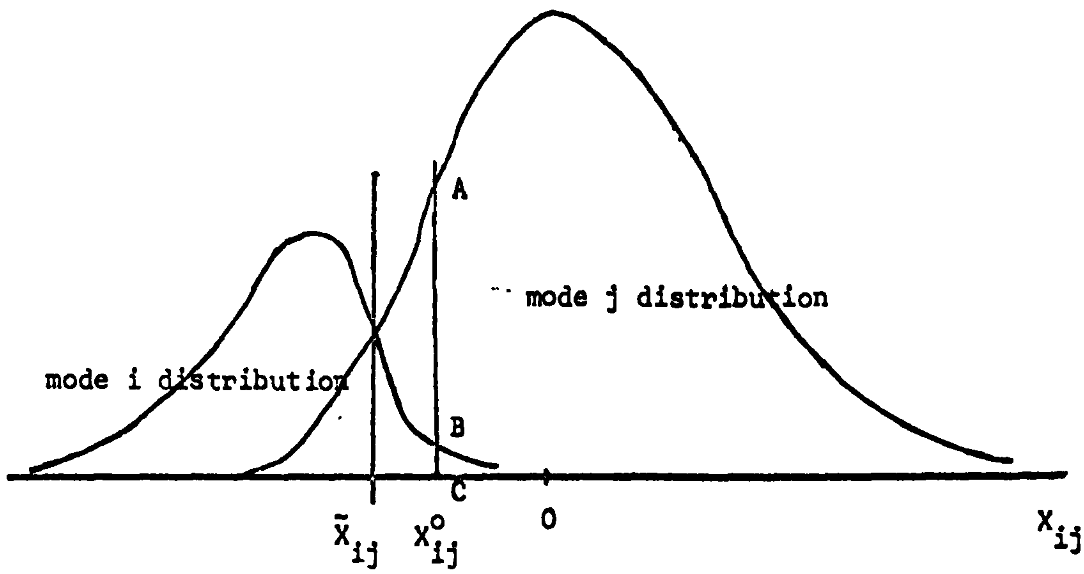
(1) Note that nothing is assumed concerning the form of the distribution.

Diagram 3



of the unknown E_{ij} must be taken into account. In these circumstances, there is no a priori reason to expect that $\tilde{X}_{ij} = 0$, since the mean of E_{ij} can take on any value. Thus, the problem becomes one of finding the appropriate critical value $\tilde{X}_{ij}$ which best separates the two populations. Then, given $\tilde{X}_{ij}$, the chances of correct mode choice based on it can be computed. If these are great, it should be an indication that the shippers choose the higher income mode, and that E_{ij} does not play an important role. If the X_{ij}'s do not predict model choice well, then either the theory is incorrect or the magnitude and the variance of E_{ij} are such that the role of the X_{ij}'s is minimal. In any case, it is worthwhile to insist that, if the present stochastic model is accepted, all the choice criteria which will be reached, however optimal they might be, will only be probability statements. In other words, any rule of classification will be subject to a certain expected probability of error.

This method of origin classification is correct whenever only one shipper at a particular origin has to be classified. However, one might have to classify several shippers at the same origin with the same X_{ij}'s or several origins with an identical X_{ij} value. For any particular shipper characterized by X_{ij}^{o} the probability that he will use the jth mode is greater if X_{ij}^{o} is greater than X_{ij}, (see diagram 3). This is so because it is more likely that its $Y_{ij} = X_{ij}^{o} + E_{ij}$ will be greater than zero. But, given that E_{ij} is a random variable, some of the X_{ij}^{o} shippers may have a Y_{ij} value smaller than zero. These origins would use the ith mode. It follows that a certain proportion of these shippers should be allocated to mode i. The percentage split between the two modes can be read from diagram 3: for X_{ij}^{o}, it corresponds to the ratio of AB to BC[1]. This second approach to mode classification is undoubtedly the correct one whenever several shippers are located at the same origin.

It should also be used whenever one wants to estimate the proportions in which the modes will be used to ship a commodity from a region of production. However, to forecast the choice of a particular shipper - or, as in the case of Chapter III, the choice of each of a series of shippers - one should retain the former "discrimination" approach which generates only exclusive choices of mode. In that respect, the present stochastic model should be clearly distinguished from all the "model-split" models.

Before closing this chapter, an important point must be made concerning the meaning of the critical value $\tilde{X}_{ij}$. For the sake of this argument, let us provisionally agree that $\tilde{X}_{ij}$ is the value indicated in Diagram 3: the X_{ij} at which the two distributions intersect. E_{ij} is a random variable independent of X_{ij}. Therefore, to any particular value of X_{ij}, say X_{ij}^{o}, corresponds a conditional distribution of $X_{ij}^{o} + E_{ij}$, similar to the distribution of E_{ij}, but with median value

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(1) Chapter II gives a method to estimate these percentages.

equal to $X_{ij}^{o} + \overline{E}_{ij}$, where $\overline{E}_{ij}$ is the median of E_{ij}. If the E_{ij} distribution was known, the percentage split between the two modes for any given value of X_{ij} could be determined on the former distribution. In particular, for X_{ij}^{o}, all the observations with sum $X_{ij}^{o} + E_{ij}$ greater than zero should correspond to mode i shipments, and all the observations with such a sum smaller than zero should correspond to mode j shipments. At $\tilde{X}_{ij}$, it is observed that the percentage split between the two modes is: half of the shipments by mode 1, and half of the shipments by mode 2. This may occur only because the median value $\tilde{X}_{ij} + \overline{E}_{ij} = 0$. It follows that $\overline{E}_{ij} = -\tilde{X}_{ij}$. Thus we have a way of estimating what might be called the average quality difference between modes.

Further, one could also estimate the other parameters of the distribution of E_{ij}. It is possible to compute from Diagram 3, or from the two mode distributions, the percentage split between the two modes: for X_{ij}^{o} it is equal to the ratio of the ordinates of the two distributions at X_{ij}^{o}. But, this percentage corresponds to the relative number of X_{ij}^{o}'s which, in the distribution of $X_{ij}^{o} + E_{ij}$, are greater and smaller than $X_{ij}^{o} + E_{ij} = 0$. It follows that it gives the value of the cumulative distribution of E_{ij} at $E_{ij} = -X_{ij}^{o}$. Proceeding in that fashion for successive values of X_{ij}, it is possible to derive the cumulative distribution of E_{ij}, then its density function.

To estimate in this way the parameters of E_{ij}'s distribution, it is necessary to assume that X_{ij} and E_{ij} are distributed independently of each other. If this were not the case, the conditional distribution of $X_{ij} + E_{ij}$ given X_{ij} would vary with X_{ij}. Then the percentage split between the two modes at various values of X_{ij} would give only some information about the successive different conditional distributions. In principle, this information is not sufficient to estimate with precision the parameters of the distribution of E_{ij}, though it might provide a useful approximation.

Section II dealt with the choice between two modes. Chapter II will generalize the stochastic model to the case of more than two modes. We will also discuss the statistical method of determining the best critical value of X_{ij} along with other statistical problems.

CHAPTER II

Discrimination With One Variable

Since the particular problem relevant to later statistical work involves a choice among three modes, most of this chapter will deal with this particular case. However, it should be noted that the theoretical equations could be readily applied to a general situation for n modes, even though examples are confined to n = 3.

Let us define π_1 as the population of origins which ship by barge, π_2 as the population of origins which ship by rail, and π_3 as the population of truck transportation origins. Each origin is characterized by three variables: X_{21}, the difference between rail transportation net price and barge transportation net price, X_{31} the difference between truck and barge net prices, and X_{32}, the similar difference for truck and rail. $p_1(X)$, $p_2(X)$, $p_3(X)$ are the multivariate density functions of the respective populations, where X is the vector of origin characteristics (X_{21}, X_{31}, X_{32}). Next, q_1, q_2, q_3 are the a priori probabilities of drawing an observation (any observation) from the respective populations: they correspond to the relative frequencies of each population in the universe of the three populations. For the time being, it is assumed that both the $p_i(X)$'s and the $q_i(X)$'s are known.

Given an observation X, the conditional probability that X comes, say, from π_1 (the probability that an origin with characteristic vector X ships by barge) is

$$(1) \qquad P_1(X) = \frac{q_1\, p_1(X)}{\sum_{i=1}^{3} q_i\, p_i\,(X)} \, .$$

$p_1(X)$ is the probability that X will be observed among all the X's belonging to sub-population π_1. To find $p_1(X)$, the probability that a random observation X of π_1 be drawn from the total population of all origins, $p_1(X)$ must be weighed by q_1, the probability that an observation (any observation) will come from π_1. In general, the product $q_i\ p_i\ (X)$ is the probability that X of π_i will be drawn and the sum of these products over the three sub-populations gives the probability of X. It follows that the ratio (1) is the conditional probability that, given observation X, X comes from π_1. The expected loss of classifying that observation as from π_2 is:

(2) $$\sum_{i=1,3} \frac{q_i\ p_i(X)}{\sum_{i=1}^{3} q_j\ p_j(X)}\ C(2/i),$$

where C(2/i) is the cost of classifying an observation from π_1 as from π_2. Then, one chooses the population π_k, or the mode of transportation k, which minimizes the expected loss for X. Assuming here that all the C(i/j)'s are equal, this is equivalent to minimizing

(3) $$\sum_{\substack{i=1 \\ i \neq k}}^{3} q_i\ p_i(X),$$

Proceeding in this fashion for successive observations, it is possible to define decision or classification regions R_1, R_2, R_3. The rule is: assign X to R_k if

(4) $$\sum_{i \neq k} q_i\ p_i(X) < \sum_{i \neq j} q_i\ p_i(X), \text{ for all } j \neq k.$$

This procedure minimizes the expected loss and is unique, if the probability of equality between right-hand and left-hand sides of the equation is zero[1]. Subtracting from each side of (4) $\sum_{i \neq k,j} q_i\ p_i(X)$,

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(1) T.W. Anderson, '<u>Introduction to Multivariate Statistical Analysis</u>' 1958, pp. 143-144.

which is, in this case, the only $q_i\, p_i(X)$, $i \neq j$ and k, remaining common to both sides, the rule becomes: assign X to R_k if

(5) $$q_k\, p_k(X) > q_j\, p_j(X), \text{ for all } j \neq k.$$

In other words, as the probability that the observation comes from π_k is greater, the observation must be assigned to R_k. Finally the criterion may be writtem as: assign X to R_k if

(6) $$\frac{p_k(X)}{p_j(X)} > \frac{q_j}{q_k}, \text{ for all } j \neq k.$$

In our case there are two such equations for each of the three regions of classification. For instance, the X's such that

(7) $$\frac{p_1(X)}{p_2(X)} > \frac{q_2}{q_1}$$
$$\frac{p_1(X)}{p_3(X)} > \frac{q_3}{q_1}\text{, should be assigned to } R_1.$$

As is shown in this example, this statistical approach to the mode choice problem leads to binary comparisons of every pair of modes. In that respect it corresponds to the deterministic model with three modes (which was discussed in Chapter I). Aside from the probabilistic approach, there is however one difference. One of the results of Chapter I was that the only relevant variable for deciding between any two modes was the difference between the net prices provided by these two modes. When comparing truck and barge transportations, one should refer only to X_{31}; when comparing rail and barge one should use only X_{21}; only X_{32} should be used for the truck-rail comparison. But note that until now in this chapter, the vector X containing the three net price differences have been always used. We shall now try to include only the X_{ij} relevant to each of the binary comparisons.

Let us define $n_i(X_{ij})$ as the probability distribution of X_{ij}, over the ith population. Then, say, $n_1(X_{21})$ is the probability distribution of X_{21} over the barge origin population. The $n_i(X_{ij})$ are assumed to be normally distributed with means ${}_iU_{ij}$ and variances ${}_i\sigma_{ij}$. It follows that

$$(8) \qquad \frac{n_i(X_{ij})}{n_j(X_{ij})} = \frac{\frac{1}{{}_i\sigma_{ij}} \cdot (2\pi)^{-\frac{1}{2}} \cdot e^{-(X_{ij} - {}_iU_{ij})^2/2\,{}_i\sigma_{ij}^2}}{\frac{1}{{}_j\sigma_{ij}} \cdot (2\pi)^{-\frac{1}{2}} \cdot e^{-(X_{ij} - {}_jU_{ij})^2/2\,{}_j\sigma_{ij}^2}}$$

$$(9) \qquad \ln \frac{n_i(X_{ij})}{n_j(X_{ij})} = \ln \frac{{}_j\sigma_{ij}}{{}_i\sigma_{ij}} + \frac{(X_{ij} - {}_jU_{ij})^2}{2\,{}_j\sigma_{ij}^2} - \frac{(X_{ij} - {}_iU_{ij})^2}{2\,{}_i\sigma_{ij}^2}$$

$$(10) \qquad = \ln \frac{{}_j\sigma_{ij}}{{}_i\sigma_{ij}} + \frac{({}_i\sigma_{ij}^2 - {}_j\sigma_{ij}^2)X_{ij}^2 + 2\,X_{ij}({}_iU_{ij}\,{}_j\sigma_{ij}^2 - {}_jU_{ij}\,{}_i\sigma_{ij}^2)}{2\,{}_i\sigma_{ij}^2\,{}_j\sigma_{ij}^2} + \frac{{}_jU_{ij}^2\,{}_i\sigma_{ij}^2 - {}_iU_{ij}^2\,{}_j\sigma_{ij}^2}{2\,{}_i\sigma_{ij}^2\,{}_j\sigma_{ij}^2}$$

or

$$(11) \qquad \ln \frac{n_i(X_{ij})}{n_j(X_{ij})} = a_{ij} + b_{ij}X_{ij} + c_{ij}X_{ij}^2$$

where, $$a_{ij} = \ln \frac{{}_j\sigma_{ij}}{{}_i\sigma_{ij}} + \frac{{}_jU_{ij}^2\,{}_i\sigma_{ij}^2 - {}_iU_{ij}^2\,{}_j\sigma_{ij}^2}{2\,{}_i\sigma_{ij}^2\,{}_i\sigma_{ij}^2},$$

$$(12) \qquad \begin{aligned} b_{ij} &= ({}_iU_{ij}\,{}_j\sigma_{ij}^2 - {}_jU_{ij}\,{}_i\sigma_{ij}^2)/{}_i\sigma_{ij}^2\,{}_j\sigma_{ij}^2, \\ c_{ij} &= ({}_i\sigma_{ij}^2 - {}_j\sigma_{ij}^2)/2\,{}_i\sigma_{ij}^2\,{}_i\sigma_{ij}^2. \end{aligned}$$

Note that the first subscript of a_{ij}, b_{ij} and c_{ij} refers to the distribution subscript of the numerator, and the second subscript to the subscript of the distribution in the denominator. If it is possible to assume that ${}_i\sigma^2_{ij} = {}_i\sigma^2_{ij} = \sigma^2_{ij}$, equation (10) becomes,

$$(13) \qquad \ln \frac{n_i(X_{ij})}{n_j(X_{ij})} = a'_{ij} + b'_{ij}\, X_{ij}, \text{ with}$$

$$(14) \qquad \begin{aligned} a'_{ij} &= ({}_jU^2_{ij} - {}_iU^2_{ij})/2\, \sigma^2_{ij}, \\ b'_{ij} &= ({}_iU_{ij} - {}_jU_{ij})/\sigma^2_{ij} \end{aligned}$$

Now the criteria for R_1 given by (7) can be rewritten as

$$(15) \qquad \begin{aligned} \ln \frac{n_1(X_{21})}{n_2(X_{21})} &= a'_{12} + b'_{12}\, X_{21} > \ln {}^{q_2}/q_1 \\ \ln \frac{n_1(X_{31})}{n_3(X_{31})} &= a'_{13} + b'_{13}\, X_{31} > \ln {}^{q_3}/q_1, \text{ if the variances are} \end{aligned}$$

equal. Similar sets of criteria can be derived for R_2 and R_3. The critical values X_{21} and X_{31} which define R_1 are those which make the left-hand side of the inequalities (14) equal to their right-hand side. Each of these critical values can also be used to define regions of classification when the choice is restricted to two modes. It is then easy to compute the chances of misclassifying an origin or firm with characteristic X_{ij} when the choice is restricted to modes i and j. With ${}_iU_{ij} > {}_jU_{ij}$ the probability of misclassifying an observation from π_i as from π_j is

$$(16) \qquad P(j/i) = \int_{-\infty}^{\tilde{X}_{ij}} (2\, {}_i\sigma^2_{ij}\pi)^{-\frac{1}{2}} . e^{-\frac{1}{2}(X_{ij} - {}_iU_{ij})^2 / {}_i\sigma^2_{ij}} . \, dX_{ij}$$

(17) $$= \int_{-\infty}^{(\tilde{X}_{ij} - {}_iU_{ij})/{}_i\sigma_{ij}} (2\, {}_i\sigma_{ij}^2 \pi)^{-\frac{1}{2}} . e^{-\frac{1}{2}(Z_{ij})^2} . dZ_{ij}$$

while the probability of misclassifying an observation from π_j as from π_i is

(18) $$P(i/j) = \int_{(\tilde{X}_{ij} - {}_iU_{ij})/{}_i\sigma_{ij}}^{\infty} (2\, {}_i\sigma_{ij}^2)^{-\frac{1}{2}} . e^{-\frac{1}{2}(W_{ij})^2} . dW_{ij}$$

Furthermore, it is possible to compute the overall chances of misclassifying an observation from π_i as from π_j or π_k. With ${}_iU_{ij} > {}_jU_{ij}$, and ${}_iU_{ik} > {}_kU_{ik}$,

(19) $$P(j,k/i) = \int_{-\infty}^{({}_i\tilde{X}_{ij} - {}_iU_{ij})/{}_i\sigma_{ij}} \int_{-\infty}^{({}_i\tilde{X}_{ik} - {}_iU_{ik})/{}_i\sigma_{ik}} f(Z_{ij}, Z_{ik}) dZ_{ij}\, dZ_{ik}$$

where $f(Z_{ij}, Z_{ik})$ is the standard form of the bivariate normal distribution. The respective probabilities of correct classification are equal to $1 - P(j/i)$, $1 - P(i/j)$, and $1 - P(j,k/i)$.

An alternative and equivalent, though more complicated, method of classification would be to compute the $P_i(X)$'s, the conditional probabilities that given X, the ith mode will be chosen, and assign X to the mode with the greatest $P_i(X)$.(1) Following (1) above,

(20) $$P_i(X) = \frac{q_i\, N_i(X)}{\sum_{j=1}^{3} q_j\, N_j(X)} ,$$

where multivariate normal distributions are assumed. But, as

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(1) See S.L. Warner '<u>Multivariate Regression of Dummy Variates under Normality Assumption</u>' Journal of American Statistic Association, December, 1963, pp. 1054-1063.

(21) $$P_1(X) + P_2(X) + P_3(X) = 1,$$

(22) $$P_1(X) = \frac{1}{1 + \frac{P_2(X)}{P_1(X)} + \frac{P_3(X)}{P_1(X)}} .$$

Then, the conditional probability that the ith mode will be chosen, when the choice is restricted to the ith and jth mode, is

(23) $$P_{ij}(X) = \frac{P_i(X)}{P_i(X) + P_j(X)} ,$$ so that

(24) $$\frac{P_{ij}(X)}{P_{ij}(X)} = \frac{P_i(X)}{P_j(X)} .$$

It follows that

$$P_1(X) = \frac{1}{1 + \frac{P_{21}(X)}{P_{12}(X)} + \frac{P_{31}(X)}{P_{13}(X)}} .$$

According to the theoretical model

(25) $$P_{ij}(X) = P_{ij}(X_{ij}).$$

Therefore,

(26) $$P_1(X) = \frac{1}{1 + \frac{P_{21}(X_{21})}{P_{12}(X_{21})} + \frac{P_{31}(X_{31})}{P_{13}(X_{31})}} ,$$

or,

(27) $$P_1(X) = \frac{1}{1 + \frac{q_2}{q_1} e^{(a'_{21} + b'_{21} X_{21})} + \frac{q_3}{q_1} e^{(a'_{31} + b'_{31} X_{31})}} ,$$

using (12) for the case where the variances are equal, and the fact that,

(28) $$\frac{P_{ij}(X_{ij})}{P_{ji}(X_{ij})} = \frac{q_i \, n_i(X_{ij})}{q_j \, n_j(X_{ij})}$$

Moreover,

$$P_2'(X) = P_1(X).\frac{q_2}{q_1}e^{(a'_{21} + b'_{21} X_{21})},$$

(29)

$$P_3'(X) = P_1(X).\frac{q_3}{q_1}e^{(a'_{31} + b'_{31} X_{31})}.$$

However, these last two equations should not be relied upon to compute $P_2(X)$ and $P_3(X)$, because they do not use all the information available to do so. For instance, $P_2(X)$ as computed by (27), uses only the ratios $P_{21}(X_{21})/P_{12}(X_{21})$ and $P_{31}(X_{31})/P_{13}(X_{31})$, but not $P_{32}(X_{32})/P_{23}(X_{32})$ which evaluates the relation between mode 2 and mode 3. This feature would have been without consequence, had not equation (24) been used. This can be seen in the following way. Let us assume that

(30) $$\frac{P_2(X)}{P_1(X)} = \frac{a}{b} ; \frac{P_3(X)}{P_1(X)} = \frac{c}{b} ; \frac{P_2(X)}{P_3(X)} = \frac{a}{c}$$

Then,

(31) $$\sum_{i=1}^{3} P_i(X) = \frac{1}{1 + \frac{a}{b} + \frac{c}{b}} + \frac{1}{1 + \frac{b}{a} + \frac{c}{a}} + \frac{1}{1 + \frac{b}{c} + \frac{a}{c}} = 1$$

and, say,

(32) $$P_2(X) = \frac{1}{1 + \frac{b}{a} + \frac{c}{a}} = P_1(X).\frac{P_2(X)}{P_1(X)} = \frac{1}{1 + \frac{a}{b} + \frac{c}{b}} . \frac{a}{b} = P_2'(X)$$

Equations (27) give the conditions for having the sum of the $P_i(X)$'s equal to 1 and $P_i(X) = P_i'(X)$. These conditions are guaranteed when the successive ratios of probability are based on the same variable(s). This is what equations (24) preclude. Then if

(33)

$$P_{31}(X_{31})/P_{13}(X_{31}) = k/b \neq c/b, \text{ while}$$

$$\frac{P_{21}(X_{21})}{P_{12}(X_{21})} = \frac{a}{b}, \frac{P_{23}(X_{32})}{P_{32}(X_{32})} = \frac{a}{c},$$

(34) $$P_2(X) = \frac{1}{1 + \frac{b}{a} + \frac{c}{a}}, \text{ and } P_2'(X) = \frac{1}{1 + \frac{a}{b} + \frac{k}{b}} \cdot \frac{a}{b}$$

The two formulas give different results for the same probability. $P_2(X)$ should be taken as the correct result, since it is based on the full available information concerning mode 2. It remains that one cannot be sure that the sum of these $P_i(X)$'s equals 1. In general, they will not add to one, but the divergence should be small. Still using (30),

(35) $$\sum_{i=1}^{3} P_i(X) = \frac{1}{1 + \frac{a}{b} + \frac{k}{b}} + \frac{1}{1 + \frac{b}{a} + \frac{b}{a}} + \frac{1}{1 + \frac{b}{k} + \frac{a}{c}}.$$

Assuming k/b > c/b, the second term of (32) is smaller than the second term of (28). It is exactly smaller by

(36) $$K = \frac{be}{(a + b + c)\ (a + b + c + e)}, \text{ if } k = c + e.$$

Similarly, if k/b c/b the third term of (32) is greater by

(37) $$L = \frac{ce}{(a + b + c)\ (a + b + c - e)}, \text{ if } k = c - e$$

K and L should be small, and as they tend to compensate each other, the sum of the $P_i(X)$ should be close to one. We conclude that the use of $P_i(X)$ would be reasonably safe.

Obviously, no such problem arises in the computation of

(38) $$P_{ij}(X_{ij}) = \frac{1}{1 + \frac{P_{ji}\ (X_{ij})}{P_{ij}\ (X_{ij})}} = \frac{1}{1 + e^{(a'_{ji} + b'_{ji}\ X_{ij})}}$$

Finally, let us note that $P_i(X)$ corresponds to the conditional probability that a given observation within the classficiation region of the ith mode is correctly classified into the ith population. Similarly, $P_{ij}(X_{ij})$ corresponds to the probability of correct classification in the case of restricted mode choice. These probabilities correspond to the percentage split between modes of all the shippers with an identical X_{ij} value.

CHAPTER III

An Application - Corn Transportation in Illinois.

The purpose of this chapter is to apply and test the model proposed in Chapters I and II to a concrete problem of mode choice: the corn shipments in Illinois.

I. Grain Marketing in Illinois.

A. It seems necessary to summarize how grain is marketed from the farms to the final market before discussing the available body of data and its value..(1)

Most of the grain (wheat, corn, soybeans, oats) is delivered by the farmers at the harvest or shortly after, but less so for the corn which is progressively sold during the whole year..(2) The farmer has the choice of several outlets for his grain. He may sell to other farmers, usually feeders who do not raise enough grain to feed their livestock, or to millers established in grain producing areas. In these cases, he uses his own truck or the services of a commercial trucking concern to deliver the grain. The farmers located near a terminal market may truck and sell the grain directly to terminal elevators equipped to unload trucks. They may also sell the grain to an intinerant trucker, who buys the grain on the farm and resells it wherever a suitable market is available. However, the great bulk of the grain is trucked to the country elevator.

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(1) Most of this information on grain marketing was found in '<u>Grain Transportation in the North Central Region</u>', U.S. Department of Agriculture, Transportation and Facilities Research Division, Report No. 490, July 1961; and in stenciled notes prepared by the Grain Exchange Institute, Inc., Board of Trade of the City of Chicago, 1966.

(2) See '<u>Field Crops Production, Sales, Prices</u>' Illinois Coop. Crop Report Service, Illinois Department of Agriculture Bulletin, 65-3, 1965.

Elevators obtain grain from farmers by purchasing it outright at the time of delivery, by contracting in advance of delivery, by accepting grain for storage with the sale to be consummated at a later date, and by handling grain for the farmer's account, without ever taking possession of it. Most of the grain passing through the country elevator is grown no more than 15 miles away from the elevator. The most usual method employed in buying the farmer's grain is outright purchase. The country elevator operator receives continuous market information from commission merchants operating in the terminal market, from interior dealers who purchase grain for resale to interior mills, processing plants, and terminal elevators, and by radio broadcasts. He bases his price on these bids, taking into consideration the freight charges to the terminal market, the terminal market handling costs, and his own costs and profit margin.

The country elevator grain can be sold locally, or it can be shipped to an interior mill, or processing plant. The country elevator also ships to some principal terminal market or to an export point. There, his interests are usually represented by a commission merchant or cash grain receiver. The grain is either consigned to them, or sold for deferred delivery on 'to arrive' or 'on track' bids. When consigning grain, the country elevator ships the grain to its representative to be sold after arrival in the market. In this case, he retains ownership of the grain, pays all costs of shipping the grain, is subject to any loss incurred while the grain is in transit, and takes the risk of adverse price changes. When the grain is sold on a deferred shipment basis, the price is agreed on at the time of the contract, usually before the grain is shipped. Then the buyer assumes the risk of price fluctuation.

Once the grain is loaded, the country elevator operator fills out the 'order' bill of lading, which is signed by the carrier. This kind of bill of lading is negotiable in bank, if endorsed by the shipper. In this way the elevator is able to finance its operations. The bill of lading is sent by the bank to another bank in the consignee's city, and after payment is released to the consignee, who can then dispose of the shipment. The elevator operator pays interest to his bank for the money borrowed up to the time the draft is honored by the consignee, who will eventually borrow against the shipment.

Besides the country elevators, there are some river elevators, which originally were operated as sub-terminals, i.e., to feed the terminal elevators. In recent years, due to the growth of the poultry industry in the South, and, above all, to the demand for barge grain at the Gulf for export, most of the grain originating on the river did not go to the terminal houses. For instance, in 1964, only 15 per cent of all the barge grain shipments from Illinois were sent to Chicago and other terminal places. Even a smaller percentage of the barge corn shipments reached these places. Many of the river elevators do not have rail facilities. They generally rely on grain bought from country elevators which deliver by truck.

The terminal elevators are located in Chicago, Peoria, and St. Louis. They buy from cash grain merchants, river elevators, and country elevators. Most of the terminal elevators can receive grain by barge, rail, and truck. The terminal elevator, in turn, sells to processors, millers, distillers, feed manufacturers, exporters, and sometimes to elevators in other parts of the country. Most of their shipments are by rail or water. They benefit from the rail transit privilege for the grain received by rail or by barge, when the inbound barge shipment is regulated.

B. Some detailed information is available concerning part of the grain movements just described. In some cases these data deserve quite a number of comments. However, the most important data will first be quickly described, in order to indicate the kind of concrete problem that will be dealt with later.

The Board of Trade of the City of Chicago made available to us monthly data covering all the grain shipments from Illinois to Chicago by truck and by rail during the year 1966. These include the shipment origins, the name of the railway carrying from these origins, the rail mileages, the rail rates, and the quantity of each shipment. For the rail shipments, the number of days it took for each shipment to reach Chicago was also given.

Truck rates for grain shipments within Illinois are published.

Truck and rail shipment origins of corn, as provided by this body of data, have been mapped for three months. As can be seen from these maps, most of the shipments originate in the upper third of the State. However, within that region, few shipments arise along the Illinois River. Although abundant quantities of corn are grown in that area[1], this pattern should be expected. If some of that area's production is shipped to a market, it should go by the river.

Additional information could be gathered to complete this body of data. One of the most important Illinois grain dealers gave a list of the country elevators with which its river stations (down to Peoria) were dealing during the crop-year 1966, the estimated quantities of grain these country elevators shipped to various destinations, and the transportation modes they used. Rail and truck rates could be compiled for these barge grain origins. Finally, some barge rates are published, while it was possible to estimate the barge transportation time on data provided by the Corps of Army Engineers.

These last data about complete the information required to apply our statistical model to the corn shipments from the upper part of Illinois.

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(1) 'Illinois Agricultural Statistics, Annual Summary, 1966', Illinois Coop. Crop Report Service, Illinois, Department of Agriculture Bulletin, 66-1, pp. 60-61.

2. Critique of the Data.

We must now examine how well the data fit the theoretical model. Some of their particularities must also be reviewed in more detail.

The theoretical model requires perfect competition in the production of a homogeneous product. The study of the shipments of an agricultural product as widely grown in Illinois and as well defined as corn should reasonnably meet that requirement. Doubtless, there are several qualities of corn according to its moisture and the percentage of damaged kernels. These differences in quality can be of some importance as to the corn destination, and the price it commands on the market. However, we cannot narrow down further than corn in general. Another reason for choosing corn was that the harvest is sold progressively during the entire year, so that it would be possible to compare the results of several months.

Given the nature of our information and according to the dominant pattern of corn shipments, the actual choice among shipments will be restricted in the statistical study to truck shipments to Chicago, rail shipments to Chicago, and the combination truck to the river then barge to New Orleans.

Table I gives the sample sizes per mode for each of the months which have been retained for the statistical analysis. It gives also the number and the percentage of origins common to two samples. As can be readily seen, the large majority of origins use exclusively one mode of transportation.(1) This fact seems to indicate that country elevators make their decisions much in the way that was suggested in Chapter I, and that the discrimination model of inclusive choice is the relevant one(2).

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(1) From now on, as a convenience, we shall not refer any more to mode-market combinations but only to modes whenever this convention will be permissible without confusion.

(2) Note that in practically all cases, there is only one shipper per origin.

Table I. Number of Origins per Mode (i.e., sample sizes), Number and Percentage of Origins common to two Modes.

	January	February	July
Truck Origins	48	53	38
Rail Origins	225	260	193
Barge Origins	159	159	159
Origins common to			
Rail and Truck	26 10%	24 8%	18 8%
Barge and Truck	10 5%	9 4%	6 3%
Barge and Rail	28 8%	45 2%	32 10%

We do not have any information about shipments which could have been made by these same origins to some other destinations, or to the same destinations by other modes. However, the fact, just mentioned, that the country elevators seem to make an exclusive choice, tends to make this lack of information irrelevant. The data as we have them should give reliable information about choices which actually maximized the shippers' profit. Moreover, on principle ground, it would not matter if some relevant, and, for some origins, some best mode-market combination could not be entered in the statistical analysis. In that event, the latter would be restricted to the modes for which some information is available, the decision rules obtained would only be pertinent to the choice among these modes, and the probabilities conditional upon the restriction over the available modes.

However, it does not seem that the present statistical analysis should rest on such a conditional basis. Another fragmentary piece of evidence exists to support that point of view. In 1963, according to a survey made through questionnaires to country elevators,[1] the three modes shipments represented about 60 per cent of all the corn shipped from the five crop districts which enclose the area under study; 21 per cent were shipped to Peoria, Decatur, and other destinations in Illinois; about 5 per cent was shipped by barge to Chicago, and the rest was shipped to more distant markets, mainly to the South and the West, by rail or by truck. Accordingly, if one sets aside the corn shipments to destinations within Illinois other than Chicago, approximately 80 per cent of the remaining corn shipments were made by one of the three modes the present study is taking into account. These statistics should confirm the relevance of these three modes.

As mentioned above, three months have been selected for separate statistical analysis. The month period appeared to be a reasonable compromise among several requirements: the need for sufficient sample size, the wish to take into account seasonal pecularities, and the necessity of using a period short enough to keep the influence of the variations in market prices within reasonable limits. This may be the time to point out that this analysis will deal with three different market prices: the price on the market of New-Orleans, the price for truck grain in Chicago, and the price for rail grain in Chicago. Monthly averages of these prices have to be introduced since we do not know the precise price that each shipment of corn obtained.[2] Variations in the relations among these three prices during a month probably explain a certain amount of the shipments which are made by two modes from a common origin.

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(1) D.W. Kloth, 'The Transportation of Illinois Corn, Wheat, and Soybeans - Volume Movements to Intrastate and Interstate Destinations and Factors Influencing such Movements', Unpublished Master Thesis, Southern Illinois University, 1964, pp. 43 - 58.
As the categories used by the author do not correspond completely to ours, these percentages are only good approximations. The five districts used to reach these percentages are the Northwest, Northeast, West, East, and Central Districts.

(2) Averages computed from the spot prices of No. 2 Yellow Corn published in 'Grain Market News, Weekly', United States Department of Agriculture, Consumer and Marketing Service, Grain Division, Chicago.

The months of January and February were chosen because they are not harvest months, while considerable quantities of corn are still shipped. Presumably they should be months during which there is no capacity constraint on the choice of the shippers. The month of July has been chosen for the opposite reason: according to the statistics of the Chicago Board of Trade, it is the second most active month for the receipt of all grains.(1) Note that the Illinois River is rarely blocked by ice; in January 1966 the river was completely free of ice, while in February the traffic was only slowed at the locks during a few days.(2)

It has been noted above that the truck and rail data were available on a monthly basis for each of the grains. Unhappily, this is not the case for the barge data. On the one hand, they do not make any distinction among different kinds of grain, and it cannot be ascertained whether corn, wheat, or soybeans were shipped to the river station; on the other hand, they are yearly data, which do not tell which origins actually shipped to the river during a particular month. These two weaknesses of the barge data are not as serious as they may appear at first. First, in the area under study, the corn production is so dense and abundant relative to the production of other grain that it is very likely that if some grain has been shipped from a particular origin to the river, some corn has been shipped.(3) Second, yearly data can only be over-extensive. In other words, by including in a month sample all the origins which shipped some grain to the river during the year, one includes all the marginal origins which do not usually ship by barge during that month. They certainly cannot introduce a bias favorable to a strong decision rule. As, at the same time, the information concerning the truck and rail shipment origins is complete and presumably very accurate,

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(1) Board of Trade of the City of Chicago, 'The 109th Annual Report, Statistics 1966', 1967, pp. 4.

(2) 'Grain Market News, Weekly', ibid., issues of January and February.

(3) 'Illinois Agricultural Statistics, Annual Summary, 1966', ibid. pp. 60-61.

we can still hope to obtain satisfactory statistical results. If they are so, they should be really meaningful, since the defects of the barge sample could only worsen the results.

Another weakness of the barge sample is that, if the data indicate to which river point each country elevator is shipping, they do not say to which destination the corn is sent from the river station. A solution to this problem has already been suggested above. As not much grain is sent to Chicago or other points on the river by barge, but most of the grain is sent to the Gulf,(1) we shall assume that all the corn shipped to a river point is thereafter shipped to New Orleans (by far the most important destination). Given the situation of perfect competition, the prices of grain on the various markets should be tightly connected, and the differences between them closely related to the costs of transportation. Therefore, the proposed solution should not be a source of much distortion.

We have now to compute for each of the origins in the three samples the net prices which could be obtained by using each of the three modes during the relevant months. Before doing so, we must still review the quality of some basic components of these net prices. First, the rates. The rail rates, as is well known, are regulated, so that we can trust that for each origin the published rate is the rate which has been used or would have been used. For some origins, several rates were proposed. The usual case was of an origin which had a special rate for shipments to the East through Chicago (I.P. rate), besides the regular rate. Whenever such a special rate was lower, it was retained under the presumption that the grain was then going beyond Chicago.

The Transportation Act of 1940 exempted certain classes of traffic carried by water carriers, and in certain cases grain transportation

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(1) This fact has been checked on the records of barge shipments of the same corporation which gave us the list of country elevators it was dealing with: only 15 per cent of its all grain barge shipments went to Chicago.

falls within this exempted class. This exemption is found in Section 303, paragraph B, Part III, of the Interstate Commerce Act: "Nothing in Part III shall apply to the transportation by water carriers of commodities in bulk the cargo space of the vessel in which such commodities are transported is used for carrying of not more than three such commodities ... For the purpose of this sub-section two or more vessels while navigated as a unit shall be considered to be a single vessel." As a consequence, according to reliable information from grain dealers, the actual rates for grain transportation by barge were 15 per cent below the common carrier published rates during the period of time which is dealt with.(1)

Section 14 and Section 15 of the Illinois Motor Carrier of Property Act set forth the Duties and Practices of common and contract carriers by motor vehicle. Along with other provisions, these sections require every common or contract carrier by motor vehicle to (1) establish and observe just and reasonable rates, charges, and classifications, and (2) to file with the Commission and print and keep open to the public inspection tariffs showing all the rates and charges for transportation and services in connection therewith. Section 18 provides that any motor carrier operating upon the highways of this state who transports commodities for a rate other than the lawful rate on file ... is guilty of a misdemeanor and shall be puhished by a fine of not less than 25.00 dollars nor more than 300.00 dollars or by imprisonment or both. Discussions with officials of the state regulatory body revealed that efforts are made to enforce the law. Since firms can change rates legally by publishing a new tariff, and since the motor carrier industry is very competitive, it is believed that these rates could be used to approximate truck charges from each community to Chicago.(2) They should correspond to the actual rates as far as the

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(1) The precise source of the published rates was 'Guide to Published Barge Rates on Bulk Grain', Schedule No. 5, issued by Arrow Transportation Company, April 12, 1966.

(2) These rates are published through the intermediary of truckers' associations. The general tariff per mileage block appear to be the same for all the associations. It can be found, for instance, in the 'Agricultural and Materials Tariff', No. 600, of the Illinois Motor Carriers' Bureau, issued by Donald S. Mullins, Issuing Officer, May 16, 1966.

common and contract carriers are concerned. When the transportation is performed by the farmer, by the country elevator, or by an itenerant dealer, they can be taken as approximations to the cost or the charge for the transportation.

It is important to point out that for the three modes, the rates do not depend on the quantity shipped. The only restriction in that respect is a requirement of minimum loading which is approximately equal to the average capacity of the mode unit of transportation (a barge, a boxcar, a truck).

The rail shipments data give the number of days each boxcar of grain took to reach Chicago. It is equal to the number of days between the data at which the waybill was completed and the data at which the grain was inspected in Chicago.(1) After a careful examination of these data, it appears that this lapse of time includes some time during which the boxcar was loaded and waiting for a train at the country elevator, and also some time during which the car was waiting to be inspected (for instance, when the train reached Chicago during the weekend). Therefore, this time input is a fairly comprehensive measure of the transportation time. Some simple linear regressions of the number of days as a dependent variable on the rail mileage as an independent variable were computed for each of the railways involved in grain transportation. They were run for the months of January and February together, in order to have sufficient sample sizes, and for the month of July. While the fit of a linear regression line appeared to be the correct one on the diagrams of the plotted data, the regressions' R^2's are ver low. In fact they are so low that the regressions only interest is that they give good estimates of the average time of transportation for all the relevant mileages. Since we do not have any information about the rail transportation time for many origins from which grain was shipped by truck or barge, these regressions will be used to produce the rail time input required to compute the discounted net prices.(2) The regression results are given in

(1) All grains are inspected at their arrival by the Division of Inspection of the State of Illinois Department of Agriculture. They must also be weighed under the control of the Weighing Department of Chicago Board of Trade. These requirements explain how such detailed data were available.

(2) Note that these estimated times were used even in the case of rail origins, for which one could have used as well their particular time averages. Some experimental computations made with these averages, when available, showed that their introduction did not affect the results in any meaningful way whatsoever.

Table II.

No time information was available for grain truck transportation. It was decided to take that time uniformly equal to one day both for the transportation by truck to the river point and for the truck shipments to Chicago. This assumption seems to correspond fairly well to the nature of the rail time information which includes more than the time of moving the grain. It allows some time for the loading, unloading and inspection of the grain.

The time to transport the grain from river points to New Orleans by barge has been estimated from data recording all the barge movements on the Illinois River with the points and the dates of shipment and arrival.[(1)] Again some simple linear regressions have been computed on the basis of the data concerning the movements from points on the Illinois River to New Orleans and its vicinity. The results of these regressions were not much better than the rail time regressions, but give also good estimates of the average time of barge transportation from the various points on the Illinois River to the Gulf. Table III gives the coefficients of these regressions. Note the negative regression coefficient for the month of July. It is obviously wrong but should not matter much here. The coefficient is relatively so small that it produces practically the same time estimates for all the river points which are relevant for this study (from Havana to Lockport).

It is worthwhile to mention that some fees are requested for the inspection and weighing of the grain. As they are proportional to the quantities processed, they have not been introduced in the computation of the differences between net prices.[(2)] For the transportation by barge to New Orleans, five cents per bushel as handling cost at the river station and at New Orleans have been added to the cost of transportation.

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(1) Provided by the U.S. corps of army engineers.

(2) See Board of Trade of the City of Chicago, '<u>The 109th Annual Report, Statistics 1966</u>', 1967, pp. 183-184.

Table II. Rail Time Regressions: Time in days on rail mileage: $T = b_o + b_1 D$ - One regression per railway, per month or group of months.

Railway	Months	Sample size	b_o	b_1	R^2
A	Jan + Feb	528	.995	.020 (.003)	.095
	July	558	1.954	.006 (.001)	.091
B	Jan + Feb	1,119	2.276	.012 (.002)	.036
	July	498	2.799	.007 (.002)	.032
C	Jan + Feb	1,556	2.578	.006 (.001)	.019
	July	527	2.641	.006 (.001)	.036
D	Jan + Feb	361	1.254	.030 (.005)	.082
	July	142	-.775	.045 (.008)	.190
E	Jan + Feb	828	2.473	.008 (.001)	.040
	July	666	2.437	.008 (.001)	.067
F	Jan + Feb	1,149	2.816	.004 (.002)	.005
	July	518	4.535	-.009 (.003)	.016
G	Jan + Feb	141	1.612	.018 (.005)	.086
	July	110	1.157	.024 (.005)	.171
H	Jan + Feb + July	111	3.170	.005 (.003)	.020
I	Jan + Feb + July	267	2.034	.016 (.003)	.130
J	Jan + Feb + July	292	.401	.049 (.008)	.112
K	Jan + Feb + July	134	3.123	.016 (.004)	.116

Note : Only railway H's coefficient of regression is not significant at .05 level.

Table III. Barge Time Regressions : Time in days, as dependent variable, on distance, as an independent variable. $T = a_o + a_1 D$.

Month	Sample Size	a_o	a_1	R^2
January	242	-2.164	.013 (.002)	.345
February	301	-6.883	.015 (.002)	.386
July	258	14.260	-.002 (.003)	.049

3. The Statistical Results.

After this review of all the components of the net prices, it is possible to compute them. As we are concerned with a choice among three modes, there are three such net prices for each origin in the samples. There are also three differences of net prices. They will be denoted by X_{ij}, the difference between the net prices of the ith and jth modes. Then the formula for X_{31} is:

$$(33) \qquad X_{31} = \frac{P_3 - g_3}{(1 + i)^{\alpha}} - \frac{P_1 - g_1}{(1 + i)^{\beta}},$$

where P_1 is the price in New Orleans, and P_3 is the price for truck grain in Chicago; g_1 is the transportation cost (including handling cost) by truck and barge to New Orleans, and g_3 is the transportation cost by truck to Chicago; α is the number of days to transport the corn to Chicago from a particular origin, and β the number of days by barge. Finally i is the rate of interest per day calculated on the basis of an annual rate of interest of 6 1/2 per cent.(1) The formulas for X_{21} and X_{32} are:

$$(34) \qquad X_{21} = \frac{P_2 - g_2}{(1 + i)^{\gamma}} - \frac{P_1 - g_1}{(1 + i)^{\beta}}, \quad \text{and}$$

$$(35) \qquad X_{32} = \frac{P_3 - g_3}{(1 + i)^{\alpha}} - \frac{P_2 - g_2}{(1 + i)^{\gamma}},$$

where P_2 is the price for rail corn in Chicago, g_2 is the transportation cost by rail to Chicago, and γ is the number of days by rail to Chicago for a particular origin. Similar formulas exist for X_{13}, X_{12} and X_{23}.

On the basis of the sample observations, it is possible to estimate the distributions' parameters which are required to compute the discriminant functions and the critical values. First, we need to compute the ${}_i\bar{X}_{ij}$'s, the averages of the differences between net prices; then their variances ${}_i\sigma^2_{ij}$; and,

(1) This was the current interest rate according to the Grain Dealers.

finally the covariance of each pair of X_{ij} and X_{kj}, which will be denoted by ${}_j\bar{\sigma}_{ij}$. The additional subscript in front of the estimated parameters indicates the sample of origins on which it is computed.

The various parameters were estimated in two different ways. No account was taken at first of the quantities shipped by each origin. In other words the same weight was given to each origin. Then, in order to give less weight to some origins which shipped very little corn, and which, therefore, are less indicative of a commitment to a particular mode, each origin was given a weight proportional to the quantity shipped.

In all cases it was assumed that the a priori probabilities were equal. The reason for this assumption is that we want to find out how the mode choice decisions were taken regardless of the geographical circumstances in which they were made. To put the point more concretely, the extension of Chicago and its suburbs are such that not much grain is grown within a short distance of Chicago. As truck transportation is particularly favorable for short haul, there could not be many corn shipments by truck. Our sample sizes for the truck shipments demonstrate this point: they are relatively small.(1) If no correction were made and the a priori probabilities were taken as equal to the relative frequencies of the respective kinds of shipments, the statistical analysis, minimizing correctly the chances of misclassification, would produce rules of classification such that practically no origin would be attributed to truck transportation. This is hardly what is looked for in this analysis.

The important problem of the normality assumption still remains to be discussed. It will be convenient to defer its discussion until after the review of the statistical results.

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(1) See Table I.

Table IV gives the discriminant functions. They are computed according to the formula (11) of Chapter II. The solution of each of these functions, i.e. the value of X_{ij} which makes them equal to zero, gives the required critical values for each of the binary problems. These critical values denoted by $\tilde{X}_{ij}$, are given in Table V, along with the $_i\bar{X}_{ij}$. Note that all the X_{ij}'s are in cents per one hundred pounds.

Let us remember that $\bar{E}_{ij}$, the quality difference, equals $-\tilde{X}_{ij}$, the critical value. It follows that in January truck transportation is preferred to barge transportation, rail transportation is preferred to barge transportation, and truck transportation is preferred to rail transportation. These preferences mean that, at equal cost of transportation, the shippers in average prefer a particular mode. The same results appear for the month of February. In both months the quality differences are very consistent since $\bar{E}_{31}$ is greater than $\bar{E}_{21}$ as the preference of truck over rail requires. In other words, the preference of truck over barge transportation is greater than the preference of rail over barge transportation, which is consistent with the preference of truck over rail transportation.

The situation is different in July. During this month, truck transportation is still preferred to the two other modes, but the preference over rail is stronger. Very consistently, barge transportation is preferred to rail transportation, since $\bar{E}_{21}$ is negative. This result is undoubtedly expressing the rail capacity constraint during the harvesting season.

It is also worthwhile mentioning that the preferences are stronger in February. This could be the result of the relative uncertainty that existed in February 1966 about the weather and the reliability of barge transportation. In these circumstances, rail should be more strongly preferred to barge, and so should truck transportation be. It is not clear, however, why the preference of truck over rail transportation should also be stronger.

Table VI gives the probabilities that an observation will be successfully attributed to the true mode. These probabilities were defined in equations (15), (16), and (17) of Chapter II. They are the probabilities for

Table IV Discriminant Functions (X_{ij}'s are in Cents Per Hundred Pounds)

January weighed

$$\ln \frac{n_3(X_{31})}{n_1(X_{31})} = 2.458 - .273\ X_{31} - .066\ X_{31}^2$$

$$\ln \frac{n_2(X_{21})}{n_1(X_{21})} = 2.215 + .394\ X_{21} - .018\ X_{21}^2$$

$$\ln \frac{n_3(X_{32})}{n_2(X_{32})} = 1.938 + .648\ X_{32} + .033\ X_{32}^2$$

January non-weighed

$$\ln \frac{n_3(X_{31})}{n_1(X_{31})} = 2.037 - .128\ X_{31} - .042\ X_{31}^2$$

$$\ln \frac{n_2(X_{21})}{n_1(X_{21})} = 1.891 + .375\ X_{21} - .011\ X_{21}^2$$

$$\ln \frac{n_3(X_{32})}{n_2(X_{32})} = 2.071 - .767\ X_{32} + .048\ X_{32}^2$$

February weighed

$$\ln \frac{n_3(X_{31})}{n_1(X_{31})} = 2.567 - .261\ X_{31} - .041\ X_{31}^2$$

$$\ln \frac{n_2(X_{21})}{n_1(X_{21})} = 2.498 + .342\ X_{21} - .005\ X_{21}^2$$

$$\ln \frac{n_3(X_{32})}{n_2(X_{32})} = 2.283 + .562\ X_{32} + .024\ X_{32}^2$$

Table IV Discriminant Functions (continued)

February non-weighed

$$\ln \frac{n_3(X_{31})}{n_1(X_{31})} = 2.269 - .024\ X_{31} - .017\ X_{31}^2$$

$$\ln \frac{n_2(X_{21})}{n_1(X_{21})} = 2.174 - .370\ X_{21} + .005\ X_{21}^2$$

$$\ln \frac{n_3(X_{32})}{n_2(X_{32})} = 1.820 - .364\ X_{32} + .009\ X_{32}^2$$

July weighed

$$\ln \frac{n_3(X_{31})}{n_1(X_{31})} = 1.774 + .361\ X_{31} - .038\ X_{31}^2$$

$$\ln \frac{n_2(X_{21})}{n_1(X_{21})} = -1.363 + .597\ X_{21} - .020\ X_{21}^2$$

$$\ln \frac{n_3(X_{32})}{n_2(X_{32})} = 2.505 - .671\ X_{32} + .037\ X_{32}^2$$

July non-weighed

$$\ln \frac{n_3(X_{31})}{n_1(X_{31})} = .955 + .265\ X_{31} - .057\ X_{31}^2$$

$$\ln \frac{n_2(X_{21})}{n_1(X_{21})} = -.712 + .240\ X_{21} + .011\ X_{21}^2$$

$$\ln \frac{n_3(X_{32})}{n_2(X_{32})} = 2.670 + .743\ X_{32} + .042\ X_{32}^2$$

Table V Averages of Sample X_{ij}'s, and Critical Values for the Binary Problems. (Cents Per Hundred Pounds)[1]

Problem	Truck-Barge			Rail-Barge			Truck-Rail		
	B		T	B		R	R		T
Month	$_1\bar{X}_{31}$	$\tilde{X}_{31}$	$_3\bar{X}_{31}$	$_1\bar{X}_{21}$	$\tilde{X}_{21}$	$_2\bar{X}_{21}$	$_2\bar{X}_{32}$	$\tilde{X}_C$	$_3\bar{X}_{32}$
Jan, weighted	-13.18	-8.50	-4.85	-7.33	-4.65	-1.79	-5.02	-3.54	-2.99
non-weighted	-12.60	-8.59	-4.71	-6.69	-4.45	-2.01	-4.99	-3.44	-2.91
Feb, weighted	-16.28	-11.75	-7.82	-8.95	-6.71	-4.33	-6.96	-5.26	-4.69
non-weighted	-15.72	-12.24	-8.24	-8.36	-6.39	-4.66	-7.34	-5.82	4.88
July, weighted	-7.52	-3.57	.16	.34	2.50	5.01	-6.79	-5.29	-5.54
non-weighted	-6.92	-3.39	.53	.99	2.89	4.80	-7.22	-5.02	-5.41

(1)
T for Truck, R for Rail and B for Barge.

the binary problems, when the choice is restricted to two modes. It appears that the classification can be done rather successfully in many cases. However, when rail and truck shipments are compared, a fair percentage of origins which actually use truck would be mis-allocated.

Table VII gives the probabilities that an origin will be correctly assigned to the mode it uses as against the two other modes. They have been computed according to equation (18) of Chapter II. The probabilities of correct overall classification are not quite as good as the probabilities of correct binary classification. It could not be otherwise. The application of a second classification rule to the same set of observations can only reduce the number of origins correctly classified.

Let us note that the discounting of the net prices contributed to a reduction of the distance between the ${}_i\bar{X}_{ij}$ and ${}_j\bar{X}_{ij}$ involved in each binary problem, thereby decreasing the probabilities of correct classification.(1)

It is now appropriate to examine the assumptions, made in Chapter II, that the X_{ij}'s normally distributed can be safely used. To test if the parent distributions of the X_{ij}'s sample distributions were normally distributed, Chi-square tests were made for each of the X_{ij} over the month of January sample. The results were as follows: two of the tests sustained the hypothesis of normality at the .10 and .50 level of significance respectively. The four other tests rejected the hypothesis at the .05 level of significance. When plotted, these four sample distributions were shown to have tails but otherwise were either irregular or closer to some Chi-square distributions. As no general pattern of distribution appeared through that examination, it does not seem that one could find a distribution which could fit the six samples.

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(1) Because only a few days time is involved, the reduction due to discounting is small.

Table VI Probability of a correct classification for the binary problems.[1]

Problem	Truck-Barge		Rail-Barge		Truck-Rail	
Month	B	T	B	R	R	T
January, weighed	.84	.94	.78	.84	.74	.61
non-weighed	.77	.91	.75	.79	.77	.58
February, weighed	.83	.92	.74	.76	.74	.57
non-weighed	.74	.85	.72	.69	.70	.62
July, weighed	.80	.90	.74	.81	.75	.46
non-weighed	.75	.80	.72	.72	.81	.45

(1) T for Truck, R for Rail, and B for Barge.

Table VII Probabilities that an observation will be correctly classified when the choice is not restricted.[(1)]

	January		February		July	
	weighed	non-weighed	weighed	non-weighed	weighed	non-weighed
Barge	.73	.69	.64	.65	.67	.65
Rail	.67	.66	.58	.48	.61	.61
Truck	.57	.57	.56	.59	.45	.35

(1) Computed from the 'Tables of the Bivariate Normal Distribution Function and Related Functions' published by the National Bureau of Standards, Applied Mathematical Series No. 50, 1959.

These results are disappointing, but should have been expected. On the one hand, one might think that each X_{ij} is normally distributed over the total sample of the origins of the three modes. There are no a priori reasons to expect another distribution. Therefore, one might speculate that they are normally distributed as "pure" random variables are. But, if the total sample is normally distributed, each separate sub-sample cannot be so distributed. This can be seen if one considers that the X_{ij}'s which make up the ith mode sub-sample correspond largely to the X_{ij}'s in one of the tails of the total sample distribution. In the extreme case where the E_{ij}'s would be insignificant, the distribution of the X_{ij}'s would correspond to a chunk of the former distribution and could not be normal. On the other hand, these observations are also conditioned by the actual geographic circumstances of the experiment, and some of the possible value of X_{ij}'s within their observed range are not at all represented in the sample. These distributions are essentially reflecting the geographic pattern of corn production.

This situation raises a double question. First, if it is not possible to justify the hypothesis of normality - on empirical or theoretical grounds - is it still possible to use it as a purely ad hoc device? This is the question of the robustness of the proposed statistical procedure. Second, is there not another method, a distribution-free method, which could be used in place of the procedure with normality assumption?

As an answer to the first question, M.G. Kendall and A. Stuart propose to apply the discriminators, or classification rules, to each member of the samples on which they are based and to observe the errors in the samples.(1) This procedure has been applied to the non-weighed January sample, and the results reported in Table VIII.

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(1) M.G. Kendall and A. Stuart '<u>The Advanced Theory of Statistics</u>', Vol. 3, 1966, pp. 324-325.

Table VIII

Problem		Probability of Correct Classification					
		Estimated			Actual		
		B	T	R	B	T	R
Truck-Barge	$\tilde{X}_{31} = -8.59$	.77	.91	-	.79	.85	-
Rail-Barge	$\tilde{X}_{21} = -4.45$	.75	-	.79	.75	-	.77
Truck-Rail	$\tilde{X}_{32} = -3.44$	-	.58	.77	-	.58	.79

Table IX

Problem		Actual Probability of Correct Classification		
		B	T	R
Truck-Barge	$\hat{X}_{31}$ = -6.96 to -7.34	.82	.85	-
Rail-Barge	$\hat{X}_{21}$ = -4.40 or -4.41	.76	-	.77
Truck-Barge	$\hat{X}_{32}$ = -3.31 to -3.38	-	.56	.81

By adding the two percentages for each problem, it can be seen that the global percentage of actual correct classification is hardly different from the estimated one. Moreover, the percentages directly computed on the samples do not differ significantly from the estimated percentages.

It is possible to go further to test the robustness of the "normal" procedure. In answer to the second question, there is a distribution-free method to find classification rules. Thus, the new discriminators can be compared with the former discriminators, and their actual probabilities of correct classification can be computed. This distribution-free discriminator is simply the X_{ij} value which maximizes over the sample the probabilities of correct classification.(1) It depends only on the rank order of the observed X_{ij}'s and can be found by simple counting of the observations. Table IX gives the new discriminators denoted as $\hat{X}_{ij}$.

Since no distribution is assumed and the computation directly made on the samples, the discriminators cannot be estimated more accurately than they are given in Table IX. For instance, there are no observations of X_{31} between -6.95 and -7.35, neither in the truck origins sample nor in the barge origins sample. In this situation one might arbitrarily decide to choose the mid-value -7.15 as discriminator. Obviously, this is a weakness of the distribution-free method. Furthermore, there were no observations of X_{31} between -6.95 and -8.95 in the truck sample. But there were a few observations between -7.35 and -8.95 in the barge sample. In this particular situation, the discriminator $\hat{X}_{31}$ was unduly pulled toward a greater value than would have been the case if the truck sample had been larger and/or the observations more evenly spread over the range of X_{ij} values. Given the relatively small size of the truck sample, this constitutes an important deficiency of the method. To a lesser extent, the same problem arises in the truck-rail case, while the larger sizes of the rail and barge samples minimized the difficulty in the rail-barge case. Given the nature of our information, these 'distribution-free' estimators cannot be considered as very reliable. Yet, they are not

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(1) M.G. Kendall and A. Stuart, ibid. pp. 332-335.

significantly different from the 'normal' discriminators when the differences between the two discriminators are compared to the standard deviations of the sub-samples. This comparison is made in Table X.

Table X[1]

Variates	$\hat{X}_{ij} - \tilde{X}_{ij}$	Actual Probabilities of Correct Classification		
		B	T	R
X_{31}	1.440	.82	.85	-
X_{21}	.045	.76	-	.77
X_{32}	.095	-	.56	.81
		Standard Deviation		
X_{31}		5.39	2.89	-
X_{21}		3.36	-	3.02
X_{32}		-	2.68	2.06

Moreover, the actual probabilities of correct classification are of a similar order.

Considering the deficiencies of the distribution-free method, the nature of our information, and the apparent robustness of the 'normal' method, we think that the latter should be adopted.

Let us note that to improve the accuracy of the results, it was decided not to use the assumption that the variances were equal, even in the cases where an F-test sustained such a hypothesis. Although very convenient in computing the $\tilde{X}_{ij}$'s this hypothesis would decrease the robustness of the procedure. It is also unwarranted by the nature of the samples.

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(1) $\hat{X}_{ij}$ is taken as the mid-value of the intervals given in Table IX.

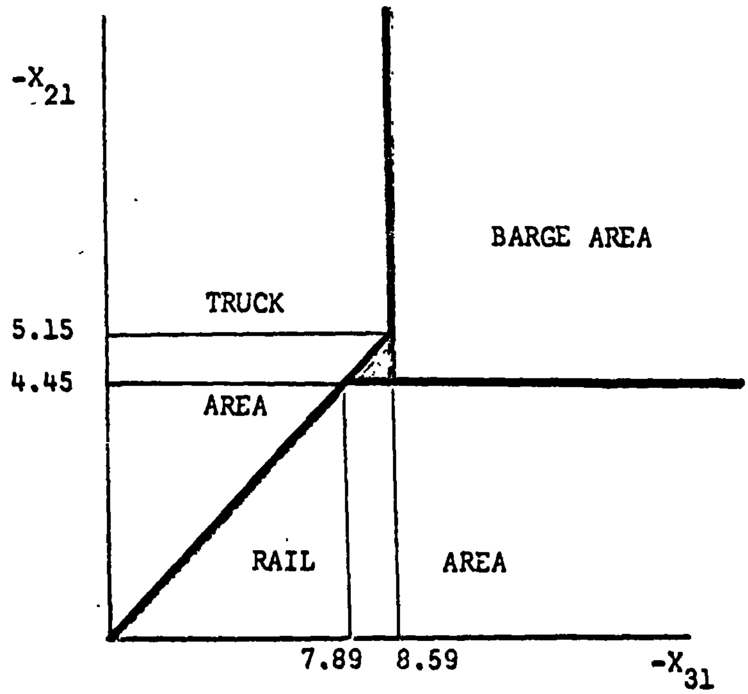
As mentioned above, the critical values are consistent, since the preferences are transitive. But will these critical values as decision rules classify an observation in a transitive way? In other words it is desirable that, if an observation is classified as a truck origin by the barge-truck classification rule, and as a rail origin by the rail-truck classification rule, it should not be classified as a barge origin by the rail-barge rule. Transitivity in classification will not be generally respected, and this can be seen in the following way. From the definition of X_{ij}, we have

(36) $$X_{31} - X_{21} = X_{32} ,$$

for any particular observation. In the case of the January non-weighted critical values: $X_{31} \geq -8.59$ if truck is to be chosen in place of barge,
$X_{21} \leq -4.45$ if barge is to be chosen in place of rail,
$X_{32} \leq -3.44$ if rail is to be chosen in place of truck.
Such an intransitive classification is entirely possible within restriction (36). If X_{32} cannot be greater than -3.44, it follows that on the left-hand side of (36) X_{31} should be small compared to X_{21}. In the present situation X_{31} can be as small as -8.59, and X_{21} as large as -4.45. Subtracting -4.45 from -8.59 gives -.4.14, the corresponding value of X_{32}, which is smaller than -3.44. Therefore, there could be observations classified in an intransitive fashion by this set of decision rules. Diagram 1 maps the area of intransitivity for this case. The shaded area corresponds to the set of X_{31} and X_{21} values such that the classification would be intransitive. It suggests also that to preclude intransitivity, it is necessary that $\tilde{X}_{31} - \tilde{X}_{21} = \tilde{X}_{32}$, in which case the shaded area would be reduced to a point.

None of the sets of decision rules fulfills this condition. However, this problem should not be given much importance. In the case of the January non-weighted data, no observation was within the area of intransitivity. Moreover, the observations falling in that area could only correspond to origins where the shippers are close to indifference. One might wish not to classify them at all. Another solution would be to correct the $\tilde{X}_{ij}$'s in such a way that the transitivity condition $\tilde{X}_{31} - \tilde{X}_{21} = \tilde{X}_{32}$ would be respected. In the case of January non-weighted data, we would then have $\tilde{X}_{31} = -8.36$, $\tilde{X}_{21} = -4.68$, and $\tilde{X}_{32} = -3.66$.

DIAGRAM 1



Given equation (36) it is also possible to draw the boundaries of the three decision regions on the X_{31} X_{21} plane, as was done in Diagram 1.

We can now compute the probabilities that each mode will be chosen at a particular shipment point by using equation (25) for $P_1(X)$, and the corresponding equations for $P_2(X)$, and $P_3(X)$. Let us compute an example, assuming that $X_{31} = -10$, $X_{21} = -6$, and $X_{32} = -4$. Then, for the January weighed set of rules,

$$\ln \frac{n_3(X_{31})}{n_1(X_{31})} = -1.412, \quad \ln \frac{n_2(X_{21})}{n_1(X_{21})} = -.797, \quad \ln \frac{n_3(X_{32})}{n_2(X_{32})} = -.126.$$

The inverse of these ratios are identical except for the sign. It follows that

$$P_1(X) = \frac{1}{1 + e^{-1.412} + e^{-.797}} = .591; \quad P_2(X) = \frac{1}{1 + e^{-.126} + e^{.797}} = .244$$

$$P_3(X) = \frac{1}{1 + e^{1.412} + e^{.126}} = .161.$$

$P_1(X) + P_2(X) + P_3(X) = .591 + .244 + .161 = .996$, which is very close to 1. As the probability that barge will be chosen is .591, one should classify this origin as a barge origin. Note that if the probabilities were used to classify origins, and if the classification was always transitive, one could be satisfied by a probability greater than .333 to attribute a shipment point to a mode of transportation. Since the possibility of intransitivity is not excluded, one might set a level slightly higher before classifying an origin in a mode category.

Both methods reach exactly the same classificatin of origins. If it is then wished to estimate the volume of shipments by each mode, one could for instance compute the sum of the tonnages shipped by all the origins in each class. Alternatively, one might regroup the origins per county and allocate each county production to the three modes in proportion to the number of shippers in each class.

Finally, one could investigate the influence of variation in the market prices, the rates, the interest rate, and the time on the choices of mode.

CHAPTER IV.

A Predictive Model of Regional Demands for Freight Transportation

The model developed in Chapters I and II, and applied in Chapter III can predict the mode choice of a particular origin or firm. Obviously this operation can be repeated systematically for all the origins in a given space. In that way, some pattern of regional demands could eventually be determined. However, this does not constitute a spatial model, since the origins spatial location does not enter explicitly as a variable in the model.

In this chapter, an attempt is made to develop a spatial model to predict which mode of transportation will be used to carry a commodity from a given point defined by its location in the space, and thereon from an entire production region. This model should incorporate some of the results obtained in the preceding chapters. First a well-known spatial competition model is recalled and set up in terms which make it applicable to the present context. Second, its elements are used to build up a model for explaining choice of mode. Then various cases of the model are presented.

I. Spatial Competition Model.(1)

It is assumed that a homogeneous product is produced under conditions of constant cost and perfect competition at several locations. Consumers are distributed over a uniform transport space, meaning that every point is in straight-line connection with every other point. Transportation cost is the sum of loading and unloading the commodity plus the carrying cost, which is proportional to distance. Once transportation cost is known, it is possible to compute the delivered price, the sum of production and transport cost, from each production site to each consumption place. The problem is to determine the market area of each production site.

(1) The first expression of this model was given by F.A. Fetter in "The Economic Law of Market Areas", Quarterly Journal of Economics, May 1924, p. 525.

The solution rests on the derivation of market boundaries along which consumers are indifferent as between alternative suppliers because their delivered prices are equal.

Take a simple case involving two production sites only, the boundary will be the locus of the points in space where the following equation holds:

(1) $$P_a + C_a Z_a + L_a = P_b + C_b Z_b + L_b.$$

where P_a and P_b are the prices or costs of productions at the points A and B respectively,

C_a and C_b are the carrying costs per ton-mile from A and B,

Z_a and Z_b are the distances over which the product is carried from A and B respectively,

L_a and L_b are the costs of loading and unloading a ton of commodity produced at A and B.

If one assumes that $C_a = C_b$, and $L_a = L_b$, equation (1) becomes:

(2) $$C_a(Z_a - Z_b) = P_b = P_a.$$

Let us define $P_b - P_a \equiv C_{ab}Z_{ab}$, where Z_{ab} is the distance between A and B, and C_{ab} is a coefficient such that the identity holds. Equation (2) becomes:

(3) $$(Z_a - Z_b) = C_{ab}/C_a \cdot Z_{ab}.$$

As $P_b - P_a$ is a constant value, so are Z_{ab} and C_{ab}. The cost of transportation C_a is also a constant, while Z_a and Z_b are variables. Equation (3) is therefore the equation of a hyperbola, defined as the locus of the points for which the difference between the distances which separate them from two fixed points is equal to a constant. Here the two fixed points are A and B. The position and the shape of this hyperbola will depend on the right-hand-side of the equation, i.e., on the value of the ratio C_{ab}/C_a and on the distance Z_{ab}. Diagram 1 illustrates the various possibilities: If $P_b - P_a = 0$, i.e., if $C_{ab}Z_{ab} = 0$, then $Z_a = Z_b$, and the boundary is a perpendicular straight line midway between the two production points. If $C_{ab}Z_{ab}$ is greater than zero, and if the ratio

C_{ab}/C_a is smaller than one, the boundary will be a real hyperbola. Its vertex is to the right of the mid-point and it curves around the higher cost site B. The greater the ratio, the closer to B is the vertex and the more sharply curved the hyperbola. The distance between the mid-point and the vertex is readily seen equal to $C_{ab}/2C_a \cdot Z_{ab}$. If $C_{ab}Z_{ab}$ is greater than zero, but the ratio $C_{ab}/C_a = 1$, then $Z_a - Z_b = Z_{ab}$ and the hyperbola degenerates in a simple horizontal line going from B to the right. The points on this line constitute the only market of B; but A is able to offer at those points the same prices as B. Were the ratio C_{ab}/C_a greater than one, there would not remain any market for B. These conditions suggest an interpretation for the coefficient C_{ab}. For a given Z_{ab}, its value indicates the minimum carrying cost per ton-mile which will lead the two producers to share the total market. Below that level, i.e., if C_a is lower than C_{ab}, the lowest cost producer would take over the whole market. If $C_{ab}Z_{ab}$ is less than zero, similar cases will arise for the successive negative values of the ratio C_{ab}/C_a. The only difference will be that the hyperbolas will curve around A, which is now the higher cost producer.

The assumption that L_a and L_b are unequal, would not change the analysis very much. Equation (2) would become:

(2a) $$C_a(Z_a - Z_b) = P_b - P_a + L_b - L_a.$$

The right-hand-side of this equation would be defined as equal to $C_{ab}Z_{ab}$, and one would be back to equation (3).

More complex situations would result if the rates C_a and C_b were different and if they were non-linear, either with respect to distance or to quantity.

The transport analog of this spatial competition problem for the case of freight transportation will now be considered.(1)

(1) This extension of the model follows the line of the analog developed by L.N. Moses and H.F. Williamson, Jr., for the case of Urban Transportation, in 'Choice of Mode in Urban Transportation,' Ch. IV, Transportation Center, Northwestern University, 1965. This analog has been mentioned by C.D. and W.P. Hyson in "The Economic Law of Market Areas.", Q.J.E. May 1950, p. 320. J.G. Wardrop uses one of its cases for studying a highway problem in "The Distribution of Traffic on a Road System," in Theory of Traffic Flow, ed. R. Herman, 1961, pp. 57.78.

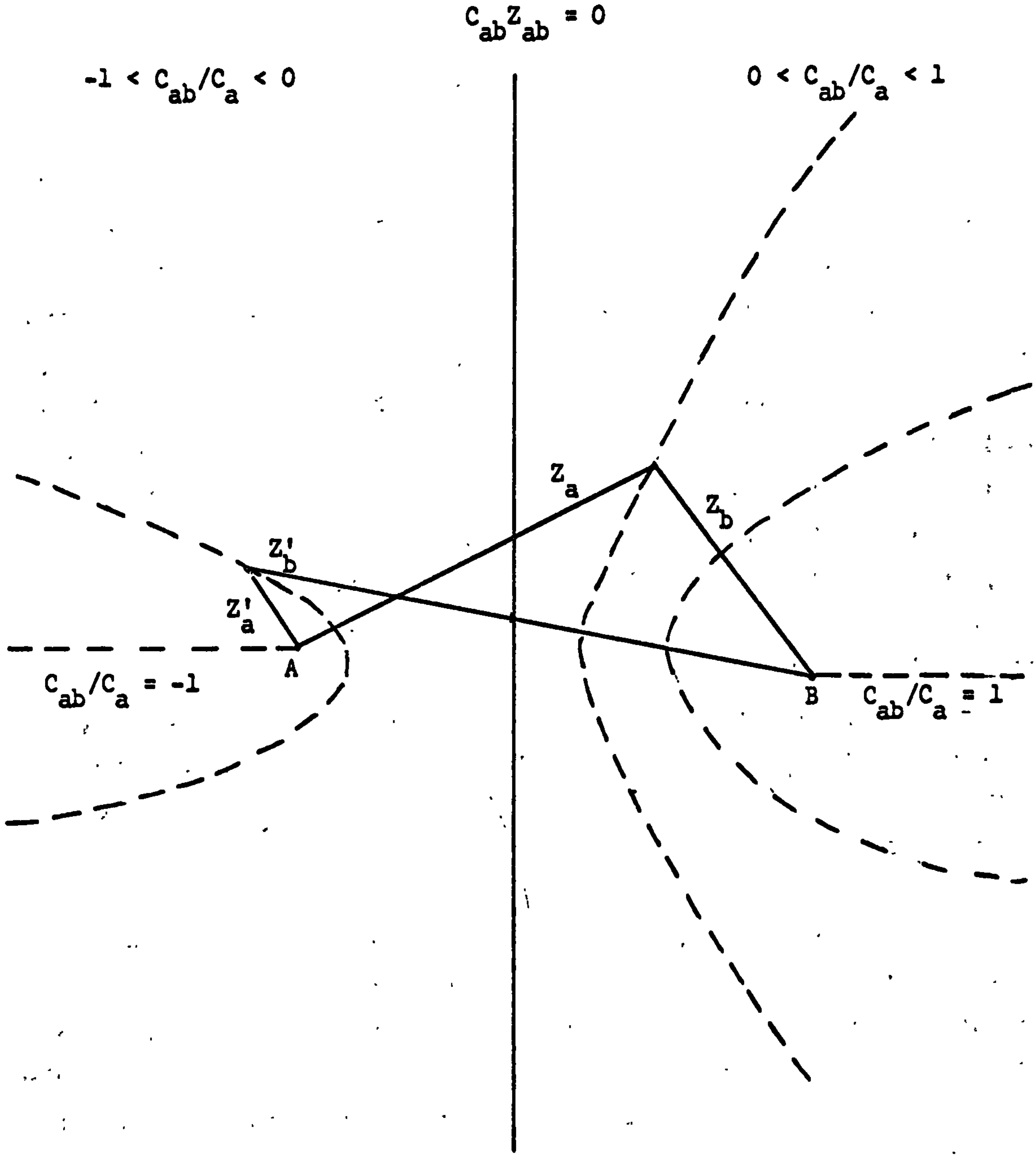


DIAGRAM 1

II. Rail-Water Competition: One Port Problem

Suppose that A, instead of being a production point, is a major market or consumption point, an outlet for the production of grain of the whole area considered. Suppose again that every point of space where the grain is grown is in straight-line connection with every other by road. Make a waterway of the line between A and B, and assume that B is the only port on the waterway shipping grain to A. With this set of assumptions, the grain producer has the choice of sending his grain directly to A by truck or shipping it to B by truck, then having it transferred on barges and carried on water down to A. To simplify the matter at the beginning, it is supposed that there is no difference in the quality of transportation services produced by the truckers and the bargelines, and that the time factor is of no importance in the shipper decision. The problem is to determine which producers will send their grain via B and the waterway, and which producers will only rely on trucks. The solution lies in the derivation of a market boundary or curve along which the producers are indifferent as between the two routes.

At first, the elements of the former problem appear somewhat reversed as the producers are distributed throughout the space and the consumption function centralized at A. However, our present problem is not the one of grain marketing but the one of transport marketing. More precisely, the farmers want to ship their grain to A, and have to choose between two possible routes. One might say that the "good", "to have a ton of grain at A," is produced at two points, A and B. The prices of the "good" at A and B are constant. It is zero at A, so the delivered price equals the cost of transporting one ton of grain from the farm. At B, the price equals the cost of transporting one ton of grain by barge from B to A, while the delivered price is equal to this price at B plus the cost of transporting it from the farm up to B.

Let us define C_t as the constant rate per ton-mile of grain by truck,
C_w as the constant rate per ton-mile of grain by water,
Z_a as the distance in miles between any producing point and A,

Z_b as the distance in miles between any producing point and B,

Z_{ab} as the fixed distance in miles between A and B.

For the time being, let us make an abstraction from all handling costs, including the cost of transferring from trucks to barges. The boundary, or indifference locus for this problem will be given by the following equation:

$$(4) \qquad C_t Z_a = C_t Z_b + C_w Z_{ab},$$

which becomes

$$(5) \qquad Z_a - Z_b = C_w/C_t \cdot Z_{ab}.$$

This equation is similar to equation (3) of the spatial competition model. The shape and the position of the boundary will depend on the same conditions as in the former problem:

If $C_w = C$, $Z_a = Z_b$, and the frontier will be a perpendicular straight line midway between A and B.

If $C_w > 0$ and $C_t > 0$, then

if $C_w/C_t = 1$, $Z_a - Z_b = Z_{ab}$, the boundary degenerates into a simple line from B to the right in the axis of A-b; if $C_w/C_t > 1$, all farmers will ship directly by truck to A; if $0 < C_w/C_t < 1$, the boundary is a hyperbola with vertex to the right of the mid-point and curving around B.

These cases are the most interesting, although one could conceive of cases where either C_w or C_t would be smaller than zero. These would be cases where one of the two modes is subsidized. The conditions for position and shape of the boundary would be similar: if $C_w/C_t < 1$, all grain producers will ship to B and use the waterway;

if $-1 < C_w/C_t < 0$, the boundary will be a hyperbola with vertex to the left of the mid-point between A and B, and curving around A.

These cases are in Diagram 2, which is similar to Diagram 1. Note that, as above, the distance between the mid-point and the vertex is equal to $C_w/2C_t \cdot Z_{ab}$.

Until now, no attention has been paid to the loading and unloading costs. Introducing them in the analysis gives the next equation:

(6) $$C_tZ_a + L_t = C_tZ_b + L_t + C_wZ_{ab} + L_w$$

where L_t and L_w are the costs of loading and unloading a ton of grain, transported by truck and by water respectively. Equation (6) becomes:

(7) $$Z_a - Z_b = C_w/C_t \cdot Z_{ab} + L_w/C_t$$

Thus, if $C_w = 0$, equation (7) becomes:

$$Z_a - Z_b = L_w/C_t,$$

and in place of a straight line through the mid-point, the boundary will be a hyperbola with vertex at the right of the mid-point, since L_w is a constant. In this case, the distance between the mid-point and the vertex is equal to $L_w/2C_t$. When C_w is not equal to zero, this distance $L_w/2C_t$ must be added to $C_w/2C_t \cdot Z_{ab}$, to obtain the total distance from the vertex to the mid-point. This is illustrated in Diagram 3. The major result of introducing the handling cost in the model, is to reduce the number of farmers who use the waterway.

It was assumed that every point in the space was in straight-line connection with every other by means of road and at the same time, that road and water were the only transportation means available. In order to make the model more realistic, we assume now that, while every point in space is still in straight-line connection with B by truck, the farmers can reach A by rail only. This modification hardly brings more realism into the model, but is useful as an expository device.

For the time being this provides us with a new equation:

(8) $$C_rZ_a + L_r = C_tZ_b + L_t + C_tZ_{ab} + L_w$$

where C_r and L_r respectively are the constant rate per ton-mile of grain by rail and the loading-unloading cost by rail. We may define $C_t = kC_r$, and rewrite (8) as

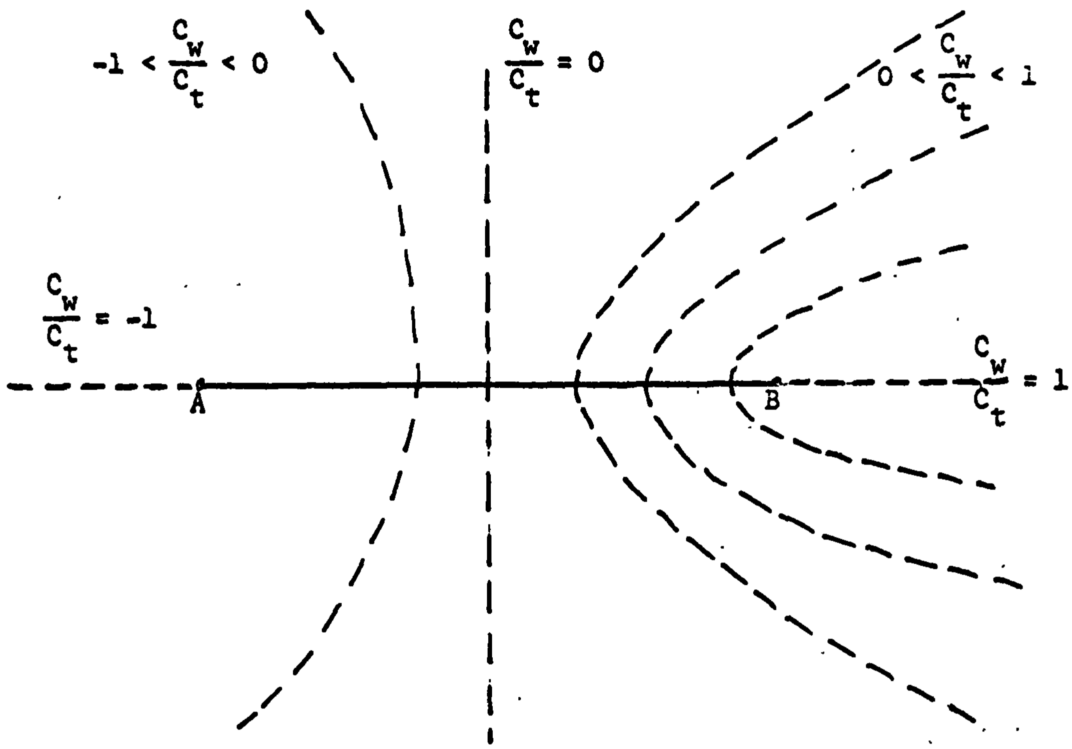


DIAGRAM 2

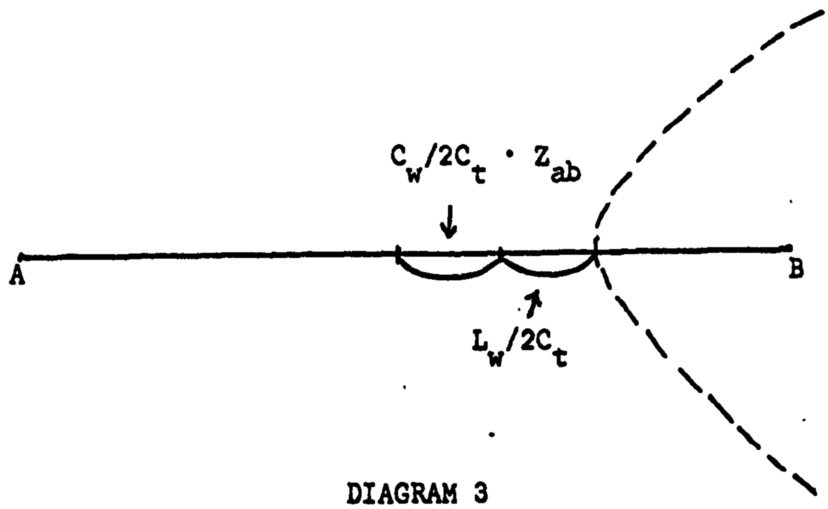


DIAGRAM 3

(9) $(Z_a - kZ_b) = C_w/C_r \cdot Z_{ab} + (L_t + L_w - L_r)/C_r$

The variable Z_b is now weighted by the coefficient k. Equation (9) might still be taken as a hyperbola equation with variable Z_a and kZ_b. However, it is not anymore a hyperbola in the variables Z_a and Z_b. Equation (9) which can be rewritten,

(10) $Z_a - kZ_b = h$

is a different case of the general family of curves, called hypercircles, or Descartes ovals, which are characterized by equation (11):

(11) $Z_a \pm kZ_b = \pm h$

The hyperbola corresponds to the case where

(12) $Z_a - Z_b = \pm h$

Were $h = 0$, and $k \neq 1$, the curve, or indifference locus would be a circle. It would circle around A if $k < 1$, and around B if $k > 1$. Neither A nor B, however, would be the center of such a circle.

The case of interest here is when $h \neq 0$ and $k > 1$. Then, according to the value of k, the curve is as one of the curves in Diagram 4. Similar curves around A would correspond to the case where $k < 1$.

Equation (10) is a complex quartic equation which has the form:

(13) $A x^4 + B x^3 + C x^2 + D x + E y^4 + F y^2 + G x^2y^2 + H x y + K.$

It would be easy, even though tedious, to find out the coefficients of (13). However, there is no point here to go into more details.(1)

(1) See C.D. and H.P. Hyson, 'The Economic Law of Market Areas', Q.J.E., May 1950, pp. 319-327.

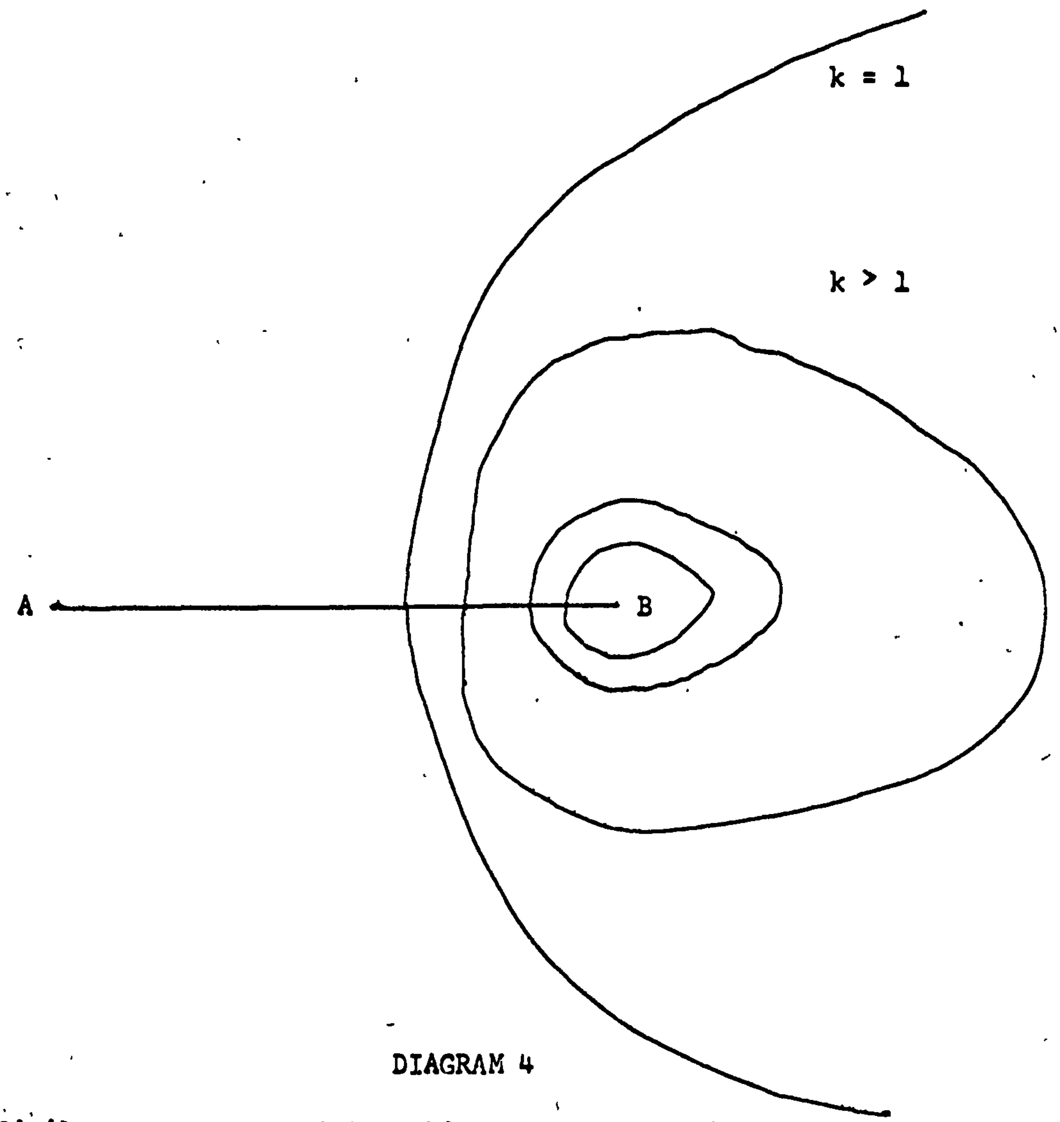


DIAGRAM 4

Similar curves around A would correspond to the case where $k < 1$.

Equation (10) is a complex quartic equation which has the form:

(13) $Ax^4 + Bx^3 + Cx^2 + Dx + Ey^4 + Fy^2 + Gx^2y^2 + Hxy + K.$

It would be easy, even though tedious, to find out the coefficients of (13). However, there is no point here to go into more detail[1].

It was also assumed that the time factor was of no importance for the

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(1) See C.D. and H.P. Hyson, "The Economic Law of Market Areas," Q.J.E. May, 1950, pp. 319-327.

shipper. In the reality of competition between modes of transportation, the difference between the time taken to reach the destination may be of importance. In the case of commuting passengers, the utilities or disutilities of money cost and time are weighed against each other in a complex and unique fashion by each individual. In the case of freight transportation one may suppose that each and every producer is rationally income-maximizing, converting time into money and taking this cost into account in his decision. Let us define:

> V as the value of one ton of grain at the producing place,
>
> I as the interest for one dollar per day,
>
> S_r, S_w and S_t as the speed per day of trains, barges and trucks respectively,
>
> M_r, M_w and M_t as the loading and unloading time in days for a whole train, a complete tow and a truck respectively.

Then,

$$T_r = \frac{V.I}{S_r}$$

is the money cost of time necessary to carry a ton of grain by rail over one mile, and

$$N_r = V.I$$

is the money cost of time necessary to load and unload a train.[1] Similarly, one gets T_w and N_w for water transportation, and T_t and N_t for truck transportation. The boundary equation becomes:

(14) $$(C_r + T_t)\ Z_a + L_r + N_r =$$

$$(C_t + T_t)\ Z_b + L_t + N_t + (C_w + T_w)\ Z_{ab} + L_w + N_w.$$

This equation may be rewritten as:

(15) $$(C_r + T_r)\ Z_a - (C_t + T_t)\ Z_b =$$

$$(C_w + T_w)\ Z_{ab} + L_t + N_t + L_w + N_w - L_r - N_r.$$

The right hand side of equation (15) is constant while the left hand side is made of the difference between two variables, each of which is weighted by a

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(1) See Thomas Thorburn, 'Supply and Demand of Water Transport', F.F.I. Report, The Business Research Institute at the Stockholm School of Economics, 1960.

fixed coefficient. As above, equation (15) can be rewritten in the form of equation (10). However, V, the value of one ton of grain at the producing place, cannot be taken as a constant. V is equal to the market price minus the cost of transportation(1), and varies with the distance from the market. To keep the linearity of the cost functions with respect to distance requires the introduction of an approximation. For instance, one might base the computation of the T_i's and N_i's on the market price minus the average transportation cost of the ith mode, or more simply on the market price only.

Similarly, one could take into account other characteristics of the transportation modes, such as dependability, risk and quality differences. All such factors could be translated in terms of money cost. According to the case, they could be taken either as constant -- as above were the transshipping and loading charges -- or as proportional to the distance -- as the various rates were assumed to be. Following the ideas proposed in Chapters I and II, and using the results of Chapter III, one could introduce the "additional costs" difference in the boundary equation as a constant.

3. The Multiple Ports Problem.

Suppose there is a second port B_2 which, in addition to B_1, ships grain to market A. The producers now have the choice between sending the grain directly to A by truck, shipping it via B_1, or shipping it via B_2. As stated earlier, it is possible to draw a boundary, say b_1, indicating what the choice of the farmers would be according to their position in the space, between the direct route and the one through B_1. Similarly, there is a boundary, say b_2, indicating the preferences between the direct route and the route via B_2. Abstraction being made of the transfer and loading/unloading costs, and considering only truck and water competition as in the first model, we get the following equations:

(16) $C_t Z_a = C_t Z_{b_1} + C_w Z_{ab_1}$ for b_1, and

(17) $C_t Z_a = C_t Z_{b_2} + C_w Z_{ab_2}$, for b_2.

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(1) All costs of transportation other than cost of time.

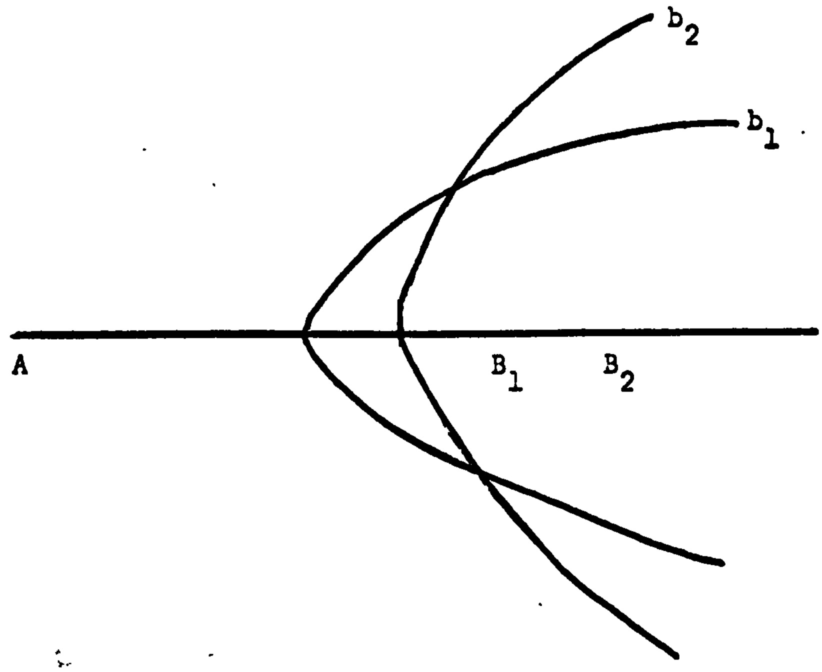


DIAGRAM 5

For the most interesting case where $0 < C_w < C_t$, Diagram 5 shows that these two curves cross. This must be so because, given a particular ratio of rates C_w/C_t, the position and the shape of the hyperbola change with the distance Z_{ab}: the larger will be Z_{ab} the more open will be the hyperbola. In this case Z_{ab_1} is smaller than Z_{ab_2}.

It is easily seen that the producers located in the area completely enclosed by b_1 and b_2 prefer to use the waterway via B_1. Also, the producers located above b_1 but below b_2 in the top part of the diagram, and those located below b_1 but above b_2 in the lower part of the diagram, prefer to use the waterway via B_2. However, it is not yet clear what will be the choice of the farmers to the right of b_2 and inside b_1; some of them could very well prefer to ship via B_1, if there was such a possibility. The solution consists in drawing a new boundary, the locus of the points where the producers would be indifferent to shipping via B_1 or via B_2 where the possibility of direct shipment to A is excluded. The equation of this third boundary is:

(18) $$C_t Z_{b_1} + C_w Z_{ab_1} = C_t Z_{b_2} + C_w Z_{ab_2}$$

which becomes,

(19) $$Z_{b_1} - Z_{b_2} = C_w/C_t \cdot (Z_{ab_2} - Z_{ab_1})$$

Again this is a hyperbola, say b, which is shown in the next diagram. The new hyperbola must cross b_1 and b_2 at the points were these cross each other. At these points, K and H on diagram 5, equations (16) and (17) hold for the same specific value of C_tZ_a; therefore, the right-hand-side of these two equations must be equal. It follows that (18) is verified at points K and H.

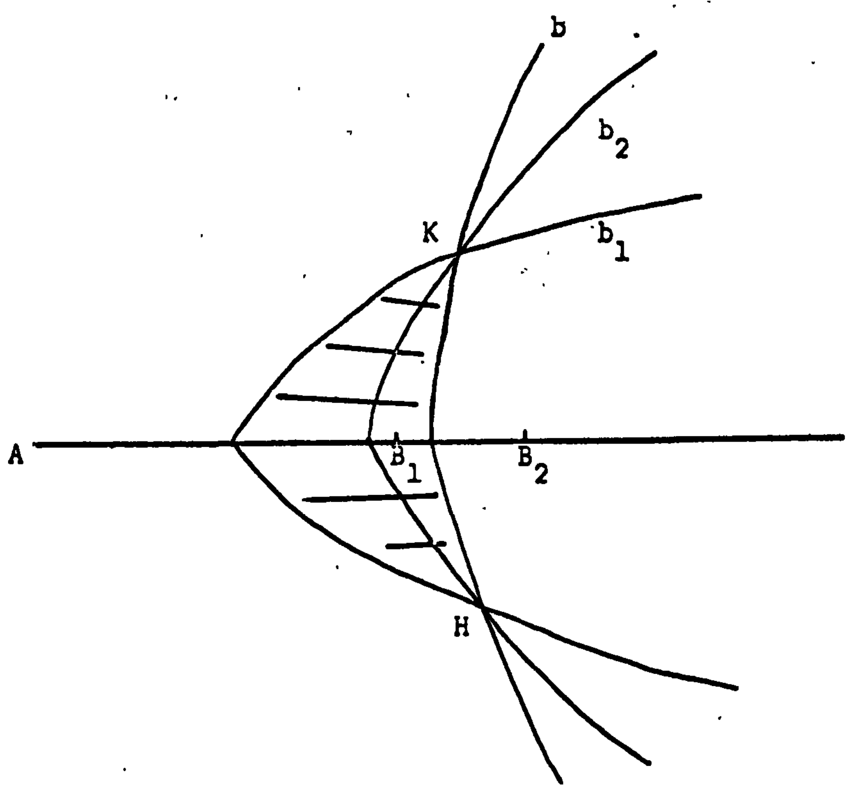


DIAGRAM 6

Now, the preference areas are well defined: producers in the shaded area on the diagram will choose the waterway via B_1; those at the right of b but inside b_2 will prefer B_2; the others will ship directly to A.

It is possible to extend this analysis for cases with more than two ports and find the preference areas for the multiple possible routes. This is illustrated by Diagram 7 for five ports, B_1, B_2, up to B_5. To keep the diagram clear, the additional curves necessary to separate the preference areas for B_1, B_2 have not been drawn. This extension of the model encompasses more of the reality of water transportation; the shipping points on the

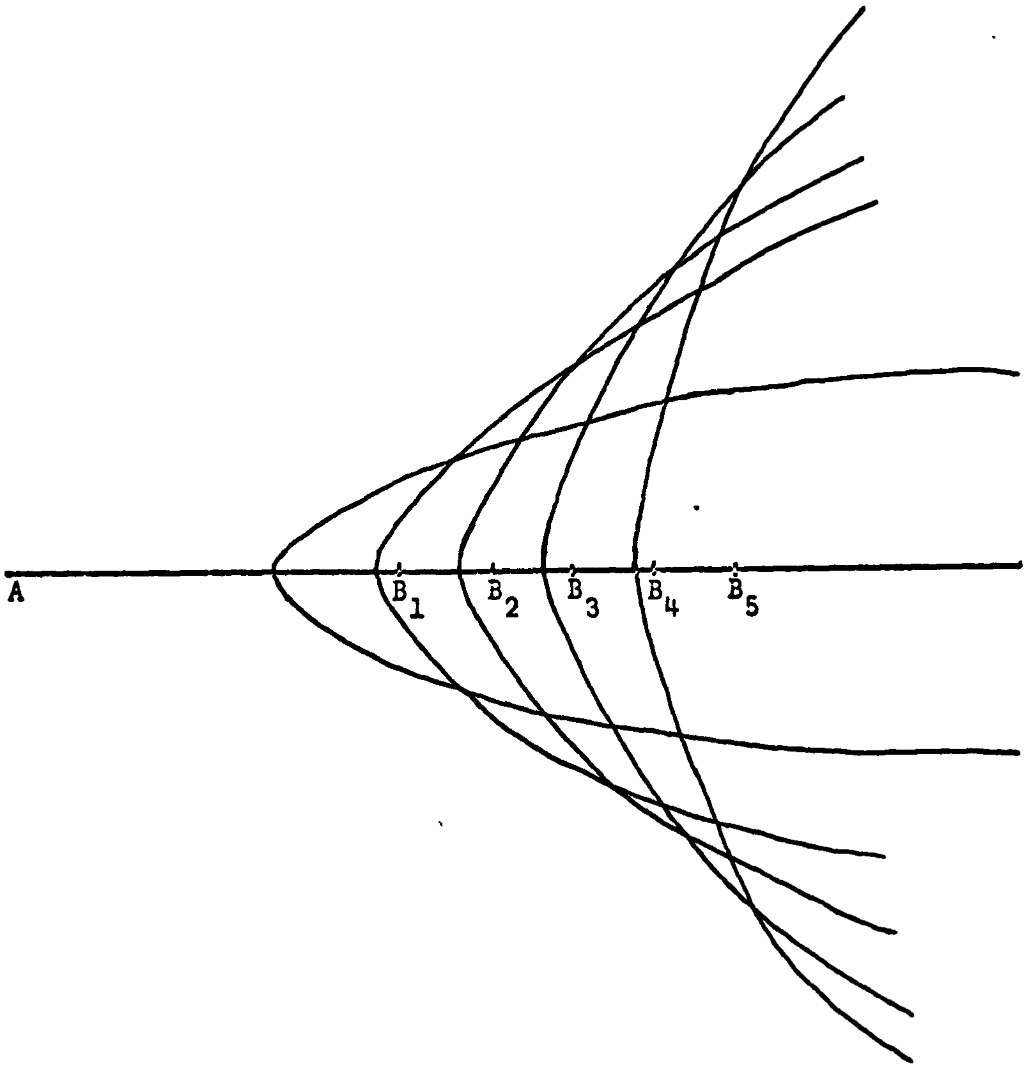


DIAGRAM 7

waterway may be numerous. In fact, in the case of grain transportation, the elevators storing the grain to be loaded on barges may be sometimes located in a quasi-continuous way along the water. Diagram 7 suggests that, when the distance between A and the nearest port decreases, and the number of ports increase, the preference area for the waterway, which is the combination of the successive preference areas for waterway via B_1, B_2, takes the form of a triangle with a corner toward A.

This continuous case may be presented in analytical fashion. For that purpose, we are changing the notation and return to a regular diagram with coordinate axes X and Y. Suppose there are shipping points continuously located along the X axis (the waterway). For some producer located in the space at a point X_0Y_0, the problem is to minimize the cost of transportation to the origin O. In terms of Diagram 8 he may choose between the direct truck route and any indirect route via the shipping points on the waterway leg $X_0 - 0$. More explicitly, his problem is to

(16) $$\underset{X_1}{\text{Min}}\ C = C_t\left[(X_0-X_1)^2 + Y_0^2\right]^{\frac{1}{2}} + C_w X_1, \text{ for } X_1, X_0 \geq 0$$

For the producer the only variable is X_1, which indicates the position of the shipping points on the waterway. The above cost formula includes all the possible routes and will give the cost of the direct one when $X_1 = 0$.

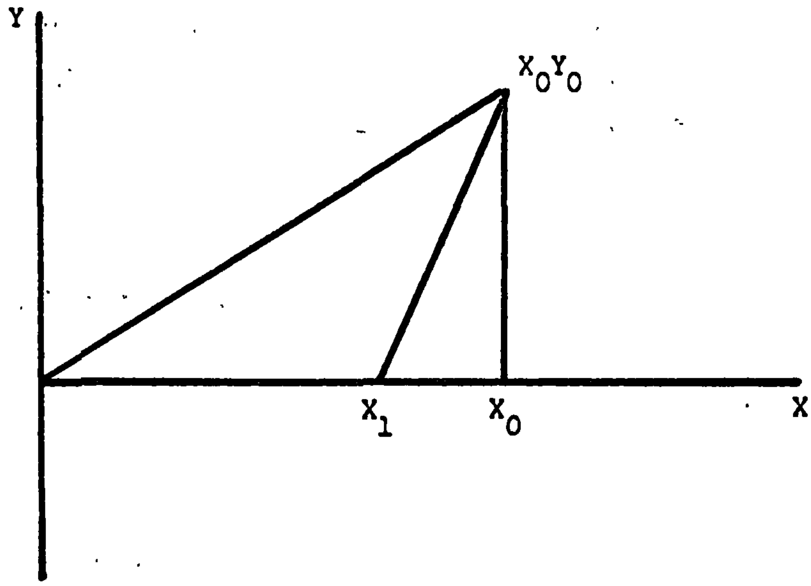


DIAGRAM 8

Taking the first derivative with respect to X_1, gives:

(17) $$\frac{\delta P}{\delta X_1} = - C_t\ ((X_0 - X_1)^2 + Y_0^2)^{-\frac{1}{2}} \cdot (X_0 - X_1) + C_w = 0$$

Multiplying through by $((X_0 - X_1)^2 + Y_0^2)^{\frac{1}{2}}$, and squaring both sides gives:

(18) $$C_w^2\ ((X_0 - X_1)^2 + Y_0^2) = C_t^2\ (X_0 - X_1)^2, \text{ and}$$

(19) $$(X_0 - X_1)^2 = \frac{C_w^2\ Y_0^2}{(C_t^2 - C_w^2)}$$

Taking the square root of both sides one gets

(20) $$X_0 - X_1 = \frac{C_w\ |Y_0|}{\sqrt{C_t^2 - C_w^2}}$$

Note the restriction of Y to its absolute values. This is necessary to ensure the symmetry of the boundary around the river. Negative values of Y would introduce negative costs in the problem, and produce a meaningless boundary. At the points where the direct road is preferred $X_1 = 0$, and

$$X_0 \leq \frac{C_w \mid Y_0 \mid}{\sqrt{C_t^2 - C_w^2}} \tag{21}$$

On the frontier the equality sign holds. Then the indifference locus is a straight line on both sides of the river: the upper boundary has a positive slope, the lower one a negative slope; they meet each other at the origin. The angle they are making with the X axis, can be found through

$$|\tan \theta| = \frac{\sqrt{C_t^2 - C_w^2}}{C_w} \tag{22}$$

It is interesting to note that the angle made by the truck route from the point X_0Y_0 to the waterway is given by

$$|\tan \gamma| = \frac{\sqrt{C_t^2 - C_w^2}}{C_w} \tag{23}$$

which is equal to $|\tan \theta|$. This means that in this simple case, diagram 8 is incorrect and should be drawn as in diagram 9. On this diagram the points C, D, and E represent typical points on the frontier, while F is any point using the combination truck-water as less expensive. Note that all the farmers located on one of the parallel lines joining the X axis use the same shipment point on the river.

One could now introduce rail transportation as the only way to ship grain directly to the market. Assume that $C_r < C_t$. Then, rail transportation will be cheaper for some points in the space. The frontier will still be linear.

Suppose that at point C, the shipper is indifferent between the two routes, because the two routes' costs are identical (see Diagram 10). On the same line originating at O, let us take another point D, such that the length of OD is twice the length of OC. The cost of transportation by rail directly to O is twice the similar cost from C. The triangles OCE and ODF are

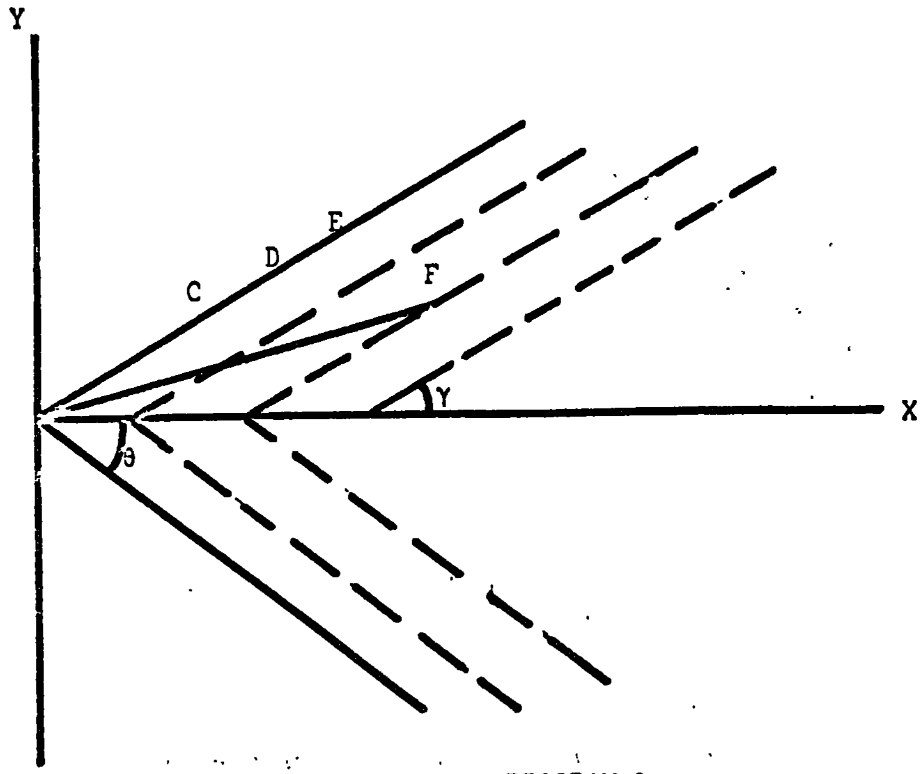


DIAGRAM 9

similar and therefore

$$\frac{OC}{OD} = \frac{OE}{OF} = \frac{CE}{DF}$$

Then, when OD = 2OC, OF = 2OE, and DF = 2CE, so that the costs of the two routes must be equal at D. The frontier is linear as it was constructed, the magnitude of θ depending on the values of C_r, C_t and C_w. Nothing would be changed to the triangular shape of the boundary if additional costs proportional to the distance were added to the model.

It remains then, to introduce costs which are not proportional to the distance and which do not cancel each other as being equal for the two routes. Let us introduce the cost of transshipment and the costs of loading-unloading. As far as the choice of the less expensive truck-barge route is concerned, nothing is changed: the charges or costs L_w and L_t are constant and are applied for all possible indirect routes. Therefore, the angle γ_t is the same as before. However, the frontier does not necessarily reach the origin. In the likely case where $C_w < C_r$, a minimum distance of transportation on the waterway is required in order to make up for the additional cost involved by

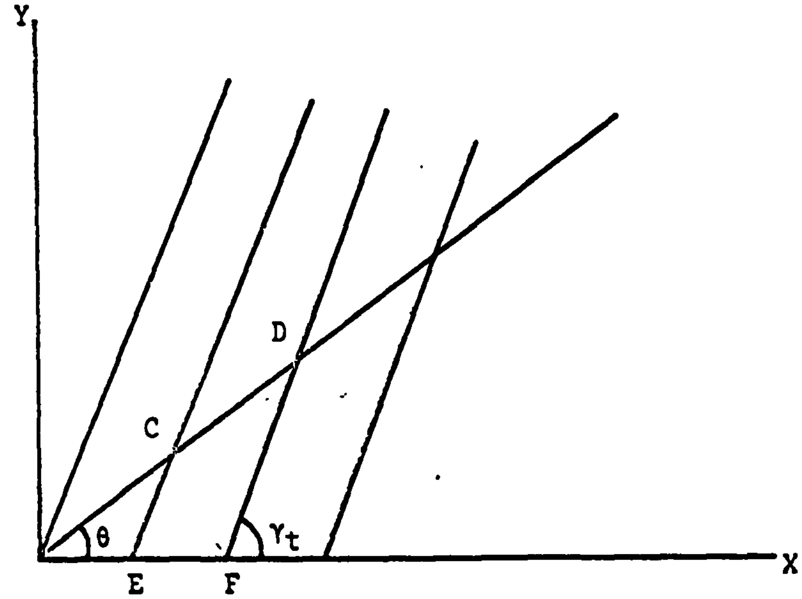


DIAGRAM 10

the road-water route. In terms of diagram 7, the boundary equation is

(24) $$C_t((X_0 - X_1)^2 + Y_0^2)^{\frac{1}{2}} + L_t + C_w X_1 + L_w = C_r(X_0^2 + Y_0^2)^{\frac{1}{2}} + L_r$$

subject to the condition that $X_0 - X_1 = \dfrac{C_w\,|Y_0|}{(C_t^2 - C_w^2)^{\frac{1}{2}}}$ and $X_0, X_1 \geq 0$

(24) may be rewritten more simply:

(25) $$C_t Z_1 + C_w X_1 + L = C_r Z_0$$

where Z_0 represents the square root term on the right-hand-side of the equation, and is equal to the distance between the point X_0Y_0 and the origin;
Z_1 represents the square root term on the left-hand-side and is equal to the distance between the point X_0Y_0 and the point X_1 on the X axis;
$L = L_t + L_w - L_r$

For points on the boundary closer and closer to the horizontal axis, Z_1 approaches zero, and Z_o approaches X_1 so that the boundary tends to the point where

(26) $$X_1 = \frac{L}{C_r - C_w} = \frac{L_t + L_w - L_r}{C_r - C_w}$$

This point is represented by E in diagram 10. However, it must be noted that for the points on the river itself (X axis), $L_t = 0$, so that the boundary has an outgrowth along the X axis to the left. Were $L_w < L_r$, the frontier would reach in that way to the origin. In diagram 11, the frontier actually reaches point F.

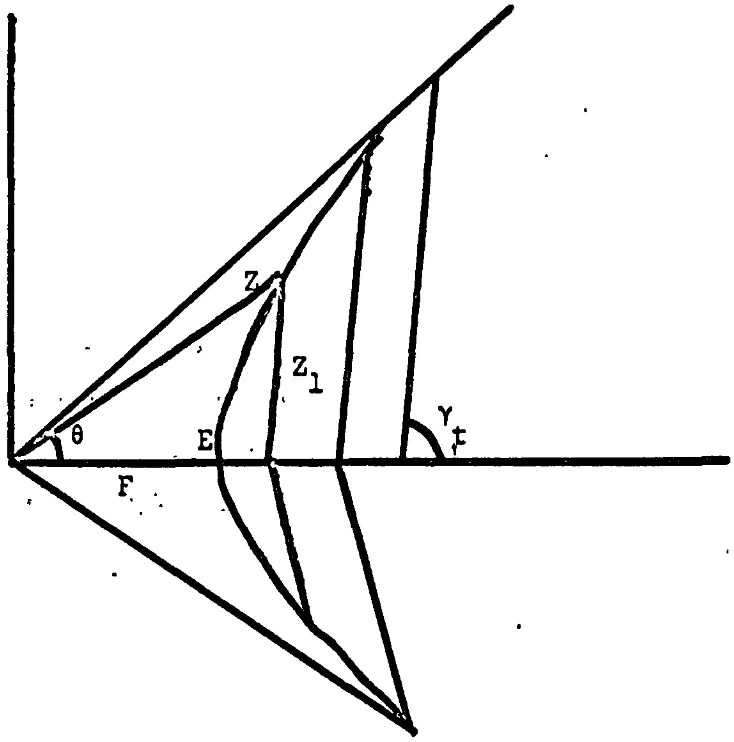


DIAGRAM 11

Furthermore, the boundary is not linear anymore because of the constant term introduced in its equation. The equation of the boundary is equation (24). The minimum cost condition (20), for the truck-water route, must be introduced in (24). It allows us to get rid of X_1:

$$(27)\quad C_t \left[\frac{C_w^2}{C_t^2 - C_w^2} + 1 \right]^{\frac{1}{2}} |Y_0| + L + C_w X_0 - \frac{C_w^2 \, |Y_0|}{(C_t^2 - C_w^2)^{\frac{1}{2}}} = C_r (X_0^2 + Y_0^2)^{\frac{1}{2}}$$

where L is defined as above. Note, once again, the restriction of Y to its absolute values, in order to guarantee the symmetry of the solution on both sides of the X axis. This can be rewritten:

$$(28)\qquad (C_t^2 - C_w^2)^{\frac{1}{2}} \, |Y_0| + L + C_w X_0 = C_r \, (X_0^2 + Y_0^2)^{\frac{1}{2}}$$

Squaring both sides, one obtains an equation corresponding to the general quadratic equation, except for the restriction on the values of Y:

(29) $$AX^2 + BX\,|Y| + CY^2 + DX + E\,|Y| + F = 0.$$

Where
$$A = C_r^2 - C_w^2$$
$$B = 2C_w(C_t^2 - C_w^2)^{\frac{1}{2}}$$
$$C = C_r^2 + C_w^2 - C_t^2$$
$$D = -2\ L\ C_w$$
$$E = -2L\ (C_t^2 - C_w^2)^{\frac{1}{2}}$$
$$F = -\ L^2$$

To find the form of equation (28), it is enough to find the value of its discriminant $B^2 - 4AC$.(1) For doing so, one has to know something about the rates. The interesting case for us is $0 < C_w < C_r < C_t$. Then, the discriminant, which is equal to $4C_r^2C_t^2 - 4C_r^4$, must be positive, and (29) is a hyperbola but for the restriction on Y.

The negative values of Y cannot be used to derive at once the boundary on both sides of the river. They would introduce negative costs in the problem and produce a meaningless boundary in the lower half-space. This is important to point out as the coefficients of (29) are such that the hyperbola could not be symmetric around the X-axis. On the other hand, the cross-product term $BX|Y|$ does not vanish, so that the hyperbola is tilted on the X-axis:(2) on the other hand, the terms D X and E Y do not vanish either, and the hyperbola does not have its center at the origin.

This problem is illustrated by Diagram 12. The lower boundary, symmetric to the upper boundary, can be obtained by rotating the latter 180^o around the X-axis. Bouth boundaries are segments of hyperbolas. But the

(1) As given by G.B. Thomas, '<u>Calculus, And Analytic Geometry</u>', Third edition, Addison-Wesley, 1960, p. 496.

(2) The angle α made by the transverse axis with teh coordinate axes can be found through $\text{Cot } 2\alpha = -\ C_t^2/2\ C_w(C_t^2 - C_w^2)^{\frac{1}{2}}$.

overall locus of indifference cannot be one since the curve is kinked at the point where the two segments meet.

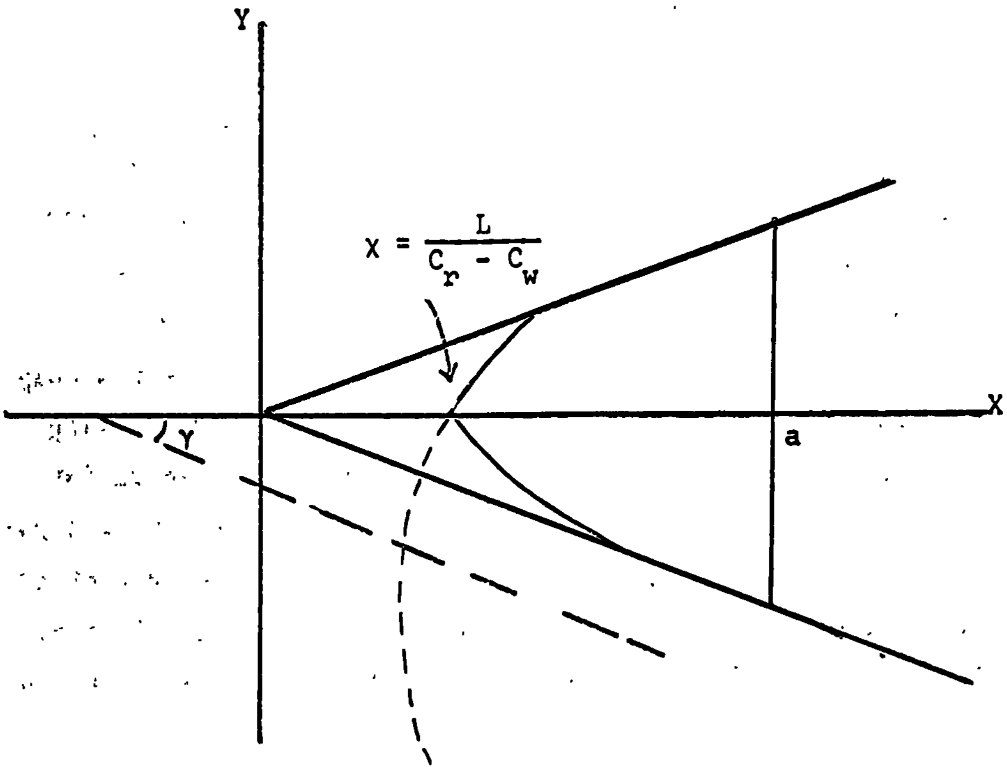


DIAGRAM 12

Using the quadratic formula, it is possible to rewrite (29) as an explicit function

$$(30) \qquad f(X) = \frac{-(BX + E) + \sqrt{(BX + E)^2 - 4C(AX^2 + DX + F)}}{2C}$$

Then, for the case where $C_t > C_r > C_w > 0$, the area of the water transportation market, from its westernmost point up to the vertical line $x = a$, is

$$A = 2 \int_{\frac{L}{C_r - C_w}}^{a} f(x)\, dx$$

Unfortunately, equation (30) is very awkward to handle. Its integration, and for that matter, its differentiation, do not provide interesting expressions. However, there would not be much difficulty in using it in a concrete case.

4. Summary and Conclusion.

In the spatial competition model presented at the beginning of this chapter there were two places where a homogeneous commodity was produced under conditions of perfect competition and constant costs of production. Therefore, the prices of this commodity at the places of production equal the average cost of production and are constant regardless of the quantity produced. It followed that the delivered price of one unit of commodity from either center of production was equal to the relevant cost of production plus the cost of transportation. There was only one mode of transportation in this very simple model. The commodity being homogeneous, the buyers were choosing to buy from the center of production with the lowest delivered price. The boundary of the two production centers' respective markets was given by the locus of the points where the deliveries were equal and the buyers indifferent as to the origin. The amount sold at each point could be determined by the demand curve particular to each buying point confronted with the lowest delivered price as shown in diagram 13. But this determination goes beyond the problem of market boundaries.

In the transportation analog, the homogeneous "good" offered to the farmers was a very special one: 'to have a ton of grain at the market place'. The price of this "good" was zero and constant at the market place itself so that its delivered price anywhere in the space was equal to its transportation cost. Two means of transportation were available: the road (or the rail) using a direct route to the marketplace, and a combination of modes, road-water, through an indirect route. Again, the buyers or farmers were choosing the cheapest way, and boundaries could be derived as the locus of the points where, costs being equal, the farmers were indifferent. At first it was assumed that the transportation services offered by the two routes were identical, and that only their rates, or direct money outlays, were unequal. Then time and service differences were introduced and translated into additional costs for the shippers, under the assumption that they were moneywise maximizing. In some cases, the solution for the boundary gave rise to complex analytical forms, although throughout the analysis it was always assumed that the costs were linearly related to distance.

In the third section, the analysis was extended first to the case of several transshipment points, then to the case of continuous transshipment possibility along the river. In the latter, a simple analytical solution was presented for a rather special case of spatial market competition. In Chapter V, this should be further examined and more complex cases introduced. Chapter V will further generalize the model to the case where more than two modes of transportation are involved.

The model presented in this chapter is essentially a mode choice model. Like the model proposed in the first three chapters, the quantities actually transported do not play any role in the mode choice, and are not estimated by the model. These have to be determined separately given the mode chosen and the cost of production function. This is only one of the similarities between the spatial model and the 'discrimination' model. In fact, they are basically similar. This can be best seen if one considers that the boundaries classify the regions according to the mode they use. In a sense, the boundaries are spatial discriminators. Furthermore, in both approaches, the basic criterion of classification is the same: the relative costs, or, as it will be seen in the next chapter, the relative net incomes. Some differences though are worth pointing out. First, the treatment of time cost is less accurate in the spatial model. Second, it is unable to provide any estimate of 'quality' difference. From that point of view, it has to rely on the estimate generated by the first model, or some additional information. Otherwise, it requires less information: no sample of the origin's choices of mode is needed, but only the rates and other charges of transportation plus, eventually, the market prices.

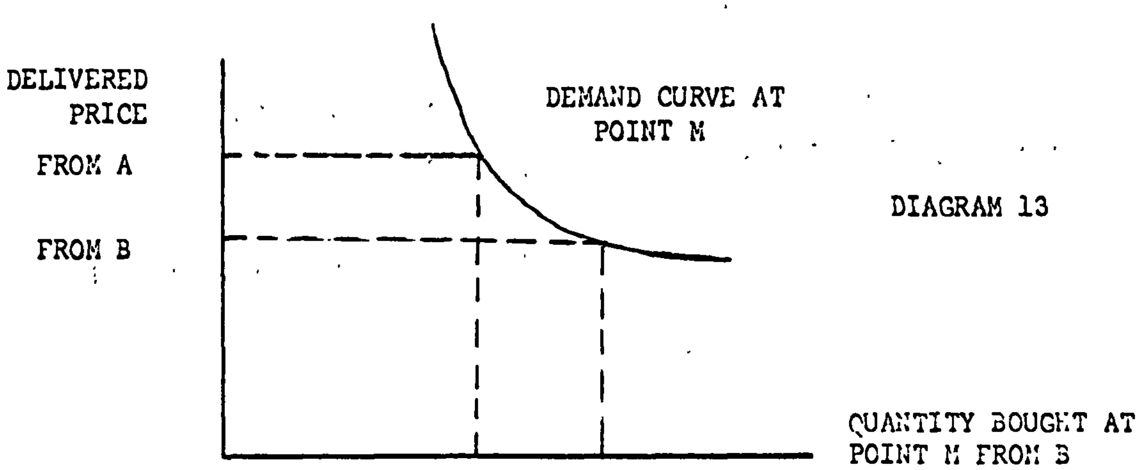


DIAGRAM 13

CHAPTER V.

This chapter attempts to use the basic tools developed in the fourth chapter, to reach more realistic spatial models of modal choice.

1. Three Routes Choice.

Throughout the whole of the fourth chapter, the choice was arbitrarily restricted to two routes: on the one hand the direct route either by road or by rail -- but not both; on the other hand the indirect water route. But what happens when two direct routes, rail and road, are competing against each other as well as against the water route? It is easily possible to decompose the problem in a triple binary choice by analyzing separately the choices between each pair of two routes.

In the fourth chapter, the locus of indifference between the water route and the rail route was defined by equation (24):

$$(1) \qquad C_t\left[(X_0-X_1)^2 + Y_0^2\right]^{\frac{1}{2}} + L_t + C_w X_1 + L_w = C_r(X_0^2 + Y_0^2)^{\frac{1}{2}} + L_r$$

subject to

$$X_0 - X_1 = \frac{C_w Y_0}{(C_t - C_w)^{\frac{1}{2}}}$$

$$X_0, X_1 \geqslant 0$$

When it is assumed that $0 < C_w < C_r < C_t$, this gives rise to a boundary the branches of which, on each side of the river, are segments of hyperbolas. At $Y = 0$,

$$X_0, \text{ say } X_{rw} = \frac{L_t + L_w - L_r}{C_r - C_w}$$

The locus of indifference between the water route and the direct road route can be determined in a similar way:

$$(2) \qquad C_t\left[(X_0 - X_1)^2 + Y_0^2\right]^{\frac{1}{2}} + L_t + L_w + C_w X_1 = C_t(X_0^2 + Y_0^2)^{\frac{1}{2}} + L_t$$

subject to:

$$X_0 - X_1 = \frac{C_w|Y_0|}{\left[C_t^2 - C_w^2\right]^{\frac{1}{2}}}$$

$$X_0, X_1 \geq 0.$$

When manipulated as was equation (24), equation (35) produces a quadratic equation similar to (29). Its coefficients are:

$$A = C_t^2$$

$$B = -2C_w(C_t^2 - C_w^2)^{\frac{1}{2}}$$

$$C = C_w^2$$

$$D = -2L_wC_w$$

$$E = -2L_w(C_t^2 - C_w^2)^{\frac{1}{2}}$$

$$F = -L_w^2$$

It follows that the discriminant equals zero and that the upper and lower boundaries are segments of parabolas. They meet at the point $Y_0 = 0$,

$$X_{tw} = \frac{L_w}{C_t - C_w}$$

The third binary choice is between truck and rail. Here, the indifference locus is determined by

(3) $$C_t(X_0^2 + Y_0^2)^{\frac{1}{2}} + L_t = C_r(X_0^2 + Y_0^2)^{\frac{1}{2}} + L_r$$

or,

(4) $$(C_t - C_r)\left[X_0^2 + Y_0^2\right]^{\frac{1}{2}} = L_r - L_t$$

Again, this produces a quadratic equation. Its coefficients are:

$$A = (C_t - C_r)^2$$

$$B = D = E = 0$$

$$C = (C_t - C_r)^2$$

$$D = -(L_r - L_t)^2$$

The cross-product term vanishes, and $A = C \neq 0$. The boundary is a circle with center at the origin. At $Y_0 = 0$, X, say X_{tr}, is equal to $\frac{L_r - L_t}{C_t - C_r}$.

With the restriction that $0 < C_w < C_r < C_e$, two cases are possible. They are illustrated in Diagram 14, where the total cost by each mode for one unit of weight is given as a function of distance.

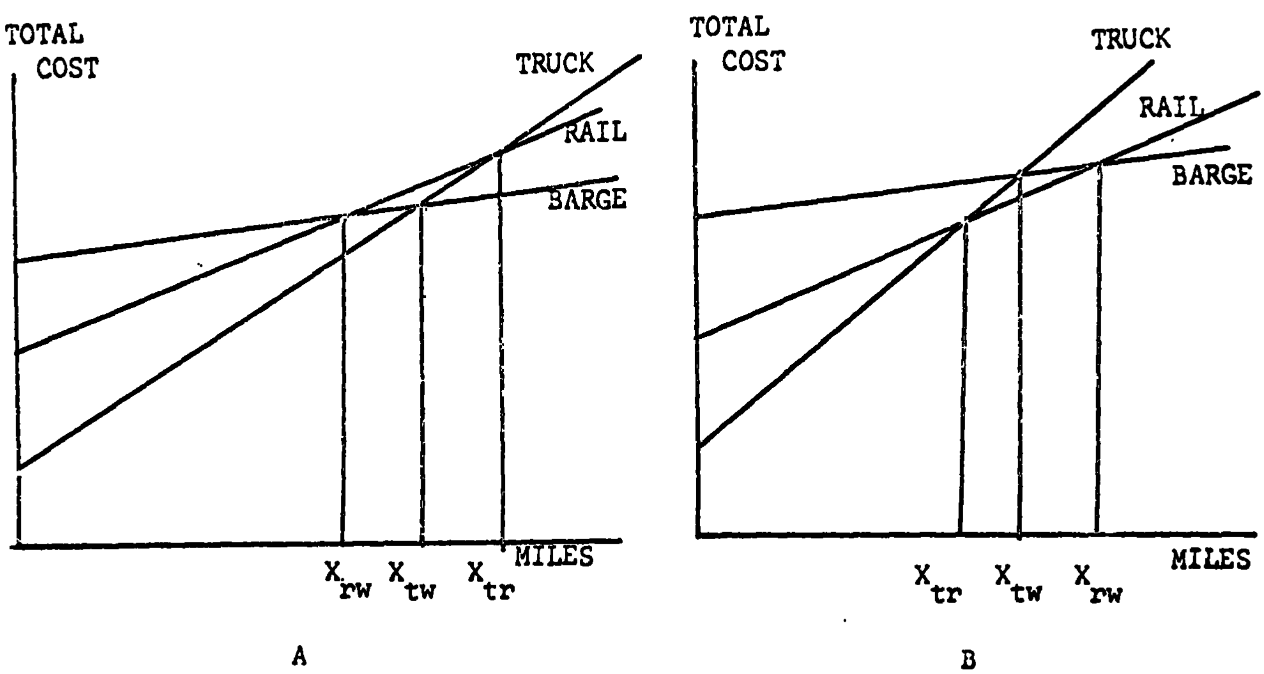


DIAGRAM 14

In case A, the cost curve of truck transportation intersects first the water transportation curve. Then, at a farther point, it intersects the rail transportation cost curve. This case can be summarized as the one where, on the river (or at $Y_0 = 0$), $X_{rw} < X_{tw} < X_{tr}$. Its boundaries and market areas are drawn in Diagram 15. The three boundaries are intersecting at two points, A and B. The respective market areas can be easily deduced, and are indicated in the diagram. Note that the water market boundary is made up of four segments.

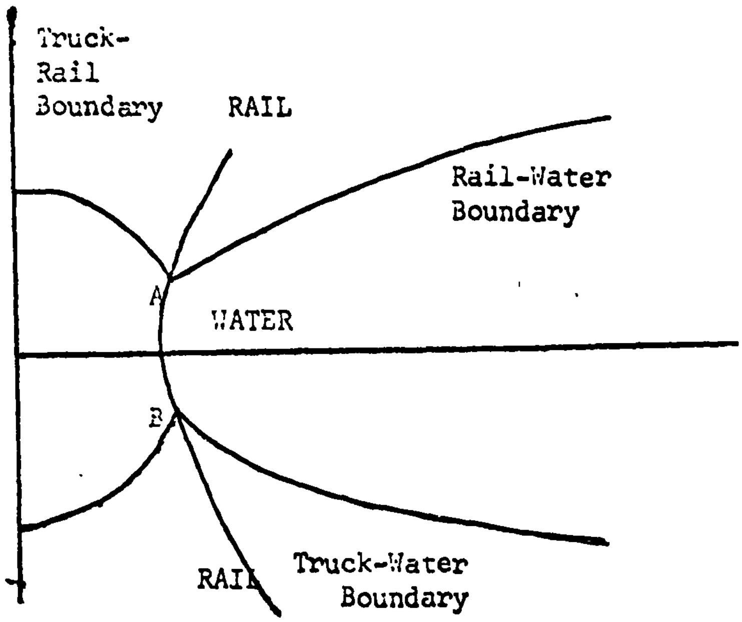


DIAGRAM 15

In case B, the cost curve of truck transportation intersects first the rail curve, then farther, the water curve. Here, $X_{tr} < X_{tw} < X_{rw}$. The boundaries, drawn in Diagram 15, cannot intersect each other,(1) and the water boundary corresponds to the boundary between the rail and the truck-water market.

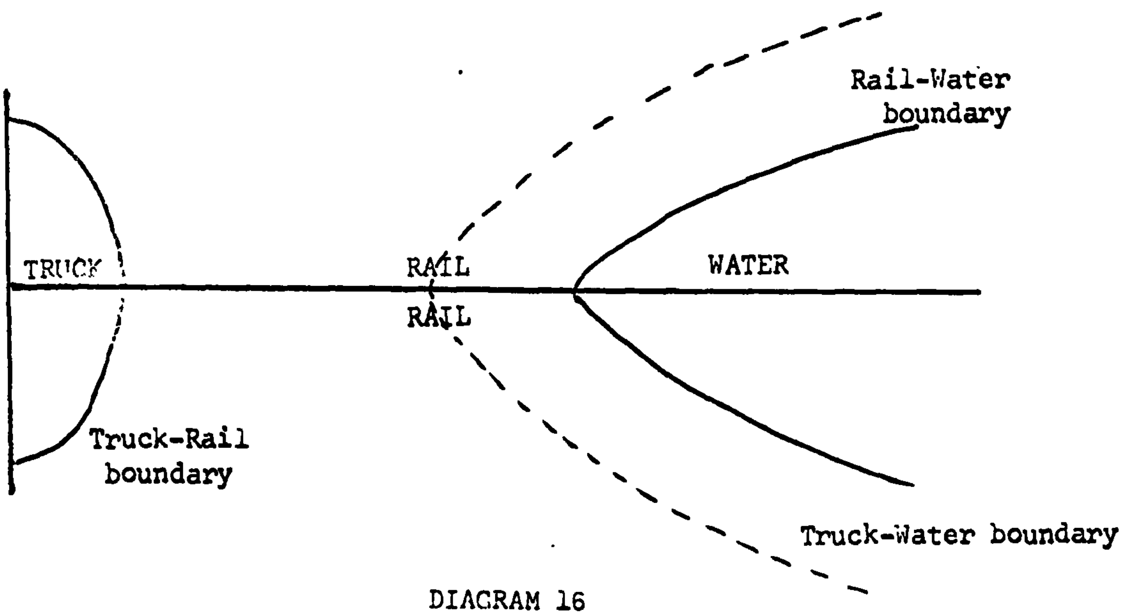


DIAGRAM 16

(1) The case where the three cost curves would intersect at the same point would be similar to this second case: the three boundaries would meet, but not intersect at one point on the river.

2. Two Markets Model.

Suppose that two markets are available at each end of the river, but that the second market can be reached only by water, while the first one can be reached by the three modes. The choice problem is considerably complicated since the number of binary choices, and partial boundaries is augmented by three. However, each of these new boundaries can be derived as easily as were the others.

If the two-market prices are different, they must be taken into account in the shipping decision. For doing so, it is necessary to consider the net income indifference locus rather than the transportation cost indifference locus.

For the competition between market A and market B through truck-water transportation, the boundary is defined by:

$$(5) \qquad P_a - C_t \left[(X_0 - X_1)^2 + Y_0^2\right]^{\frac{1}{2}} - L_t - C_{wa}X_1 - L_{wa} =$$

$$P_b - C_t \left[(X_2 - X_0)^2 + Y_0^2\right]^{\frac{1}{2}} - L_t - C_{wb}(X_3 - X_2) - L_{wb}$$

subject to

$$X_0 - X_1 = \frac{C_{wa}|Y_0|}{(C_t^2 - C_{wa}^2)^{\frac{1}{2}}}$$

and

$$X_2 - X_0 = \frac{C_{wb}|Y_0|}{(C_t^2 - C_{wb}^2)^{\frac{1}{2}}}$$

$$X_0,\ X_1 \geq 0.$$

Where P_a and P_b are the commodity market prices at A and B respectively, X_3 is the distance between A and B, X_2 is the trans-shipment point on the river when the grain is shipped to B, and C_{wa} and C_{wb} are the barge rates to A and B respectively, and L_{wa} and L_{wb} are the fixed costs to A and B respectively. The two restrictions guarantee that optimal trans-shipment points are selected. The second one can be derived as the first one was in Chapter IV.

Introducing these restrictions in (38), and simplifying, gives:

(6) $$P_a - (C_t^2 - C_{wa}^2)^{\frac{1}{2}} Y_0 - C_{wa}X_0 - L_{wa} = P_b - (C_t^2 - C_{wb}^2)^{\frac{1}{2}} Y_0 - C_{wb}X_3 + C_{wb}X_0 - L_{wb}$$

or

(7) $$Y_0 = \frac{P_b - P_a - C_{wb}X_3 + (C_{wb} + C_{wa}) X_0 + L_{wa} - L_{wb}}{\left[(C_t^2 - C_{wb}^2)^{\frac{1}{2}} - (C_t^2 - C_{wa}^2)^{\frac{1}{2}} \right]}$$

The boundary is linear. At $Y_0 = 0$,

$$X_0 = \frac{P_a - P_b + C_{wb}X_3 - L_{wa} + L_{wb}}{C_{wb} + C_{wa}}$$

For the competition between water transportation to B and rail transportation to A, the indifference locus is defined by

(8) $$P_a - C_r(X_0^2 + Y_0^2)^{\frac{1}{2}} - L_r = P_b - C_t\left[(X_2 - X_0)^2 + Y_0^2\right]^{\frac{1}{2}} - L_t - L_{wB} - C_{wB}(X_3 - X_2)$$

subject to $X_2 - X_0 = \dfrac{C_{wb}|Y_0|}{(C_t^2 - C_{wb}^2)^{\frac{1}{2}}}$

and $X_0, X_2 \geqslant 0.$

Simplifying and transforming (4), as was done previously, it can be seen easily that the branches of the boundary are segments of hyperbolas. At $Y_0 = 0$,

$$X_0 = P_a - P_b + C_{wb}X_3 + L_t + L_{wb} - L_r \Big/ C_{wb} + C_r.$$

Similarly, for the competition between water transportation to B and truck transportation to A, the indifference locus is given be

(9) $$P_a - C_t(X_0^2 + Y_0^2)^{\frac{1}{2}} - L_t = P_b - C_t\left[(X_2 - X_0)^2 + Y_0^2\right]^{\frac{1}{2}} - L_t - L_{wb} - C_{wb}(X_3 - X_2)$$

Subject to

$$X_2 - X_0 = \frac{C_{wb}|Y_0|}{(C_t^2 - C_{wb}^2)^{\frac{1}{2}}}$$

$$X_0, X_2 \geq 0.$$

It can readily be seen that it defines a boundary, the branches of which, on each side of the river, are segments of parabolas. At $Y_0 = 0$,

$$X_0 = P_a - P_b + L_{wB} + C_{wb}X_3 \Big/ C_t + C_{wb}$$

At this point, it seems difficult to combine these results in any simple model. One might be interested to assume $C_{wb} < C_{wa}$, and $P_b > P_a$. In these conditions, a remote downstream market would be able to compete for the production of an area close to another market. But these restrictions are not enough to lead to some simple solutions as were obtained above. Too many parameters are involved and the number of particular cases quite large. However, there should not be any particular difficulty in applying these results to a concrete problem.

3. The Circuitous River Case

Until now, it was assumed implicitly or explicitly that the river was as a straight line. One might object that this assumption is farfetched in many cases. However, this assumption is not necessary at all in the simple one port model.

Let us suppose that the river is wiggling as in Diagram 17, and that the grain shipped to A can only be transshipped at the point B. Making abstraction from all handling costs and using the other assumptions made in the one port case above, we get exactly the same boundary equation as (5) in Chapter IV:

(10) $$Z_a - Z_b = C_w/C_r \cdot Z_{ab}$$

This is a hyperbola equation giving a boundary centered around the axis going from A to B. Introducing fixed costs would only change the shape of the hyperbola. Note that here Z_{ab} is the river distance in miles from A to B.

Diagram 18 corresponds to the two ports case discussed above. The difference is that the two hyperbolas are not necessarily centered around the same axis. This would have been the case only if the second port had been located at D. From these particular boundaries it is easy to deduce the envelope

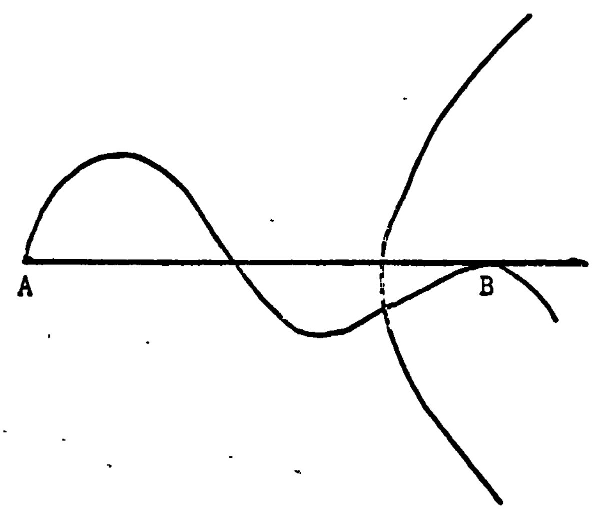


DIAGRAM 17

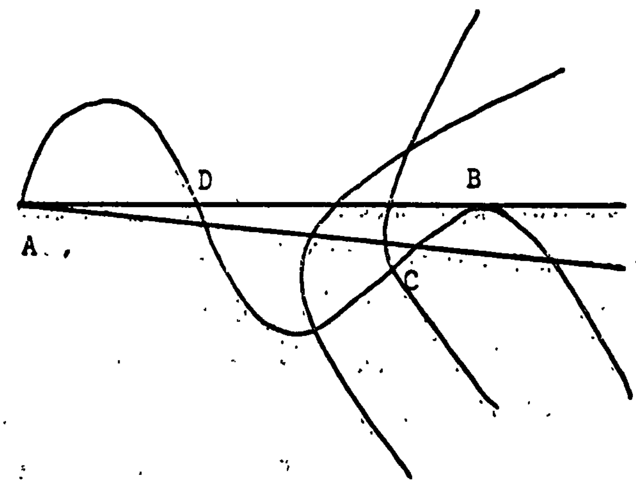
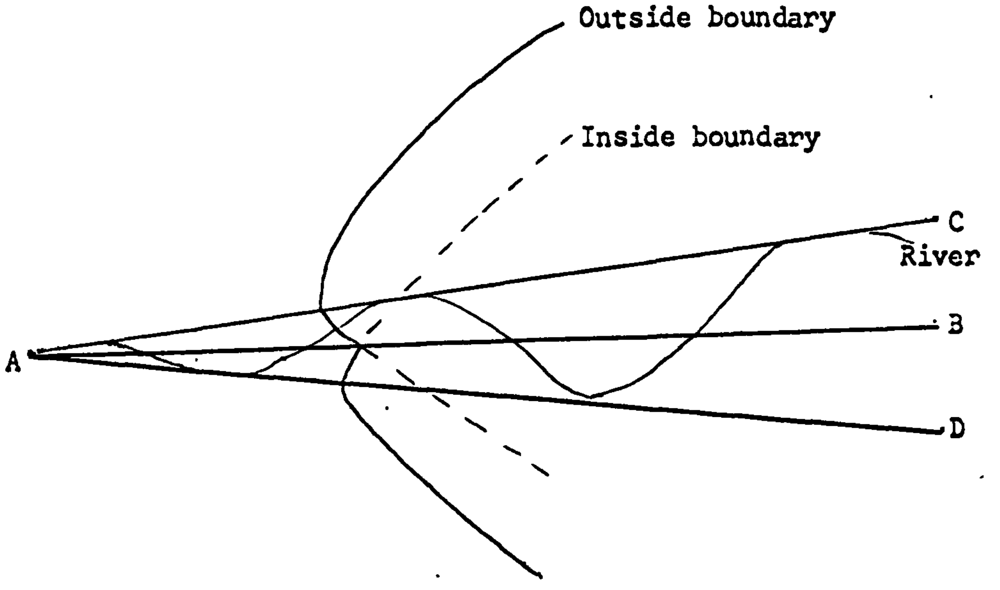


DIAGRAM 18

boundary dividing the space into the respective markets for barge and rail transportation. It is made of the portions of the particular boundaries which are the furthest North and South from the river.

Proceeding in the same way, one could increase the number of ports without any difficulty. However, as the successive hyperbolas are not centered around the same axis, it is not possible to generalize to the continuous case. The envelope boundary would possibly be without kinks -- if the river does not have any --, but its equation would be extremely complex.

DIAGRAM 19



We can only suggest the following. In Diagram 19, two axes, AC and AD, were drawn through the points on the river the furthest north and south. Given some fixed or proportional costs, it is possible to draw the two envelope boundaries centered around those two axes. This gives us an outside boundary defining the largest possible market area for water transportation. It also gives an inside boundary for the smallest possible market for water transportation. In other words, we obtained an upper bound and a lower bound estimate of the market area. An alternative would be to use a centered axis like AB as a linear proxy for the river and a basis for the systems of co-ordinates. The boundary centered around this axis would give some kind of average estimation of the market area for water transportation.

4. Application to Corn Shipments in Illinois

As seen in Chapter III, this particular problem opposes the market of New-Orleans, which can be reached economically only by the combination of road and water transportation, to the market of Chicago where corn is shipped only by road or by rail. The problem is, therefore, to derive the boundaries between three mode-destination market areas. Under some assumptions similar boundaries have already been derived in Sections 2 and 3 of this chapter. These assumptions require a few comments and modifications before these results can be used.

First of all, Figure 1, which reproduces the relevant part of the State of Illinois, shows that here is a case where the river is not a straight line. One of the alternatives proposed in Section 3 was used to solve the problem: an axis fairly in the middle of all the possible axes was drawn from Chicago to Lacon and used as a linear proxy to the river. It is also used as abscissa for a system of co-ordinates with the origin at Chicago.

Furthermore, throughout the presentation of the spatial model the assumption was made that every point in the space was in straight line connection by rail and by road with every other point. It is still possible to proceed as if every point were connected by road and by rail, since practically all actual origins of corn shipments are so connected. As to the other points they do not matter in the least: they eventually grow some corn but do not ship it directly to Chicago or New-Orleans; boundaries can be derived as if they were also connected because it is highly convenient to assign the other points of actual shipments to mode-destination market areas.

However, it is no longer possible to assume straight-line connection between points. This complication has been solved in different ways for the three modes. For road transportation to Chicago, the assumption meant only that every point was in straight-line connection with Chicago. Since it was far from being the case, it was necessary to find a relation between the actual road mileages to Chicago and the corresponding Euclidean distances used

THREE MARKETS BOUNDARIES AND ORIGINS OF THE SAMPLE CORN SHIPMENTS

JANUARY, 1966

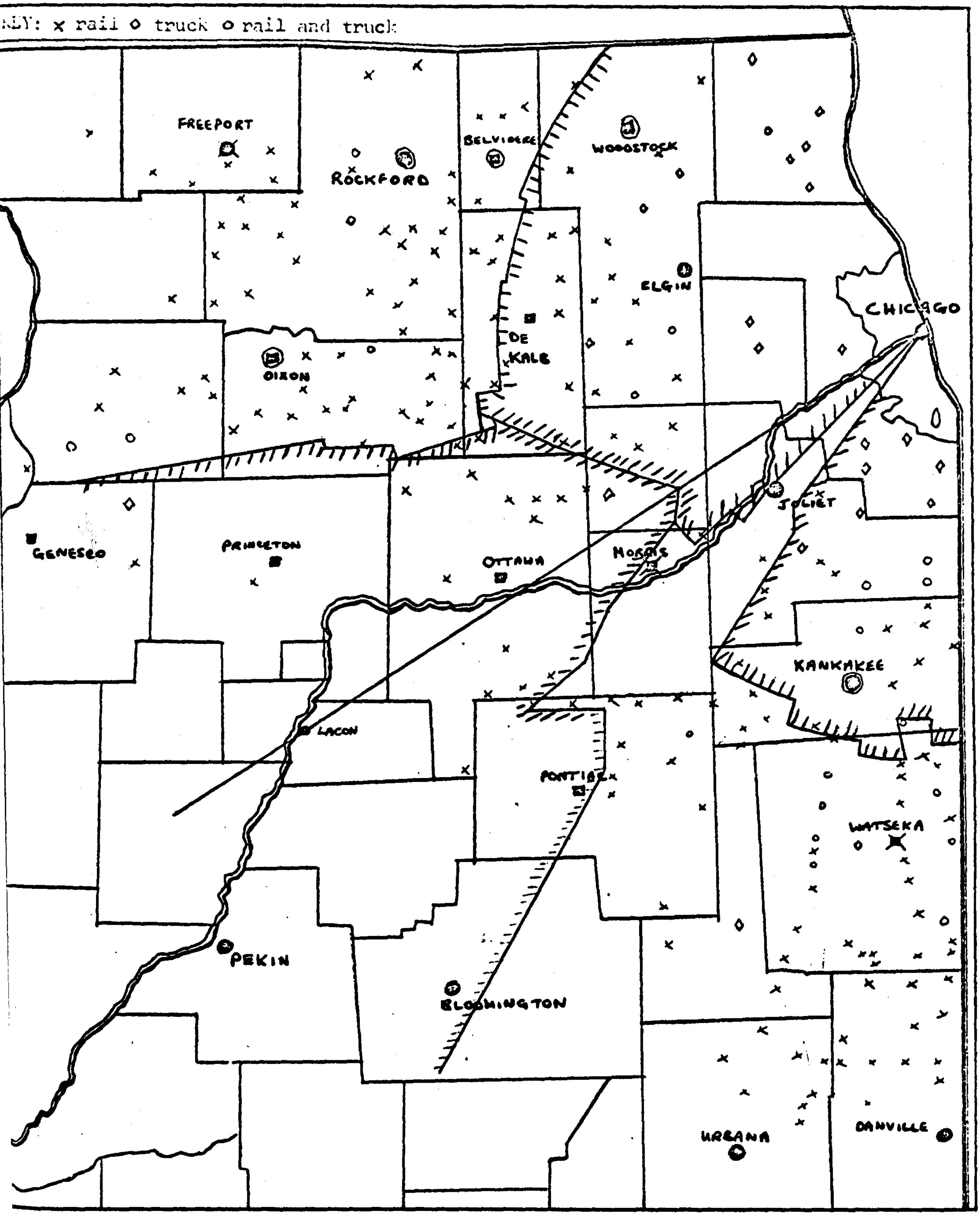


FIGURE I

ORIGINS OF THE SAMPLE CORN SHIPMENTS

FEBRUARY, 1966

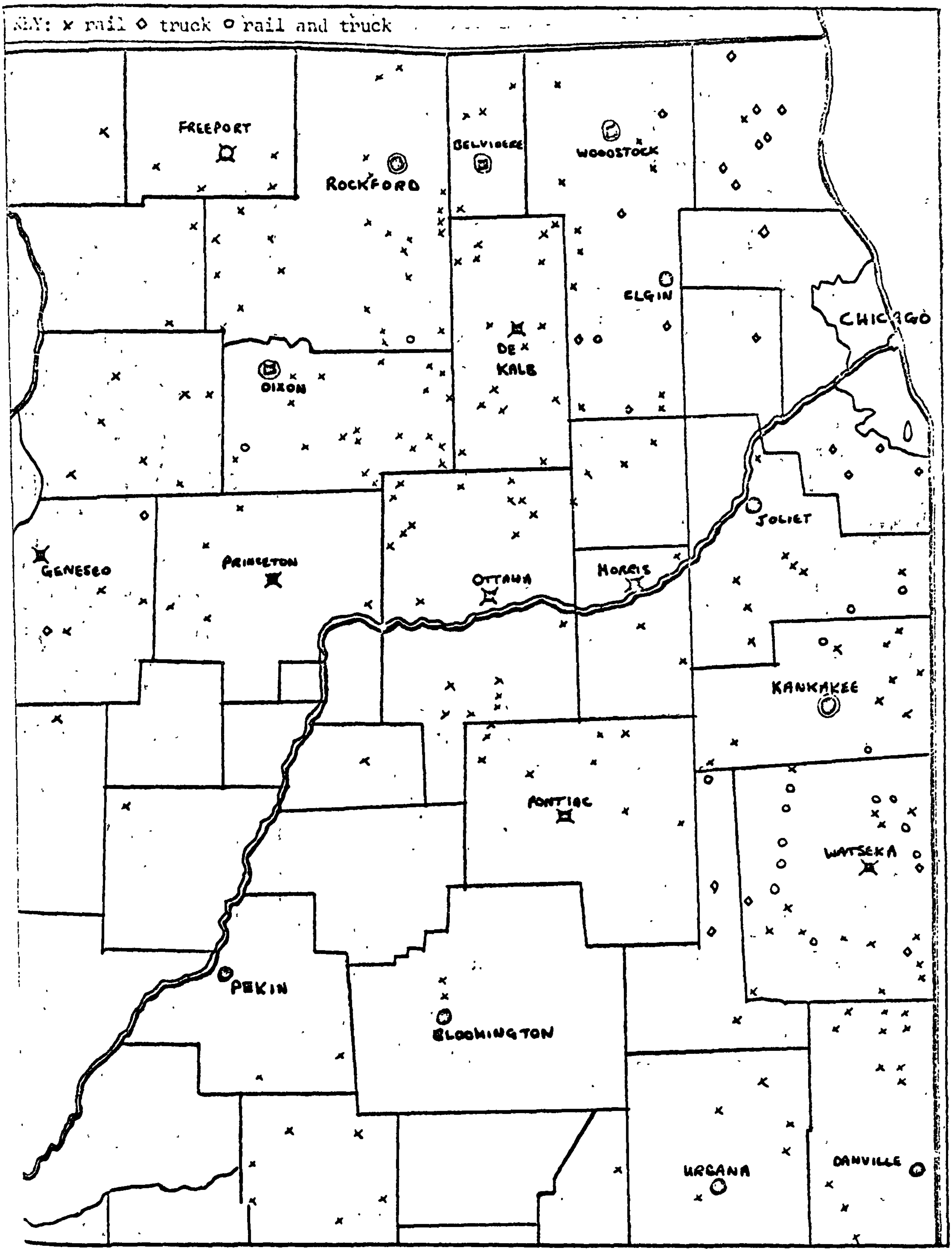


FIGURE II

ORIGINS OF THE SAMPLE CORN SHIPMENTS
JULY, 1966

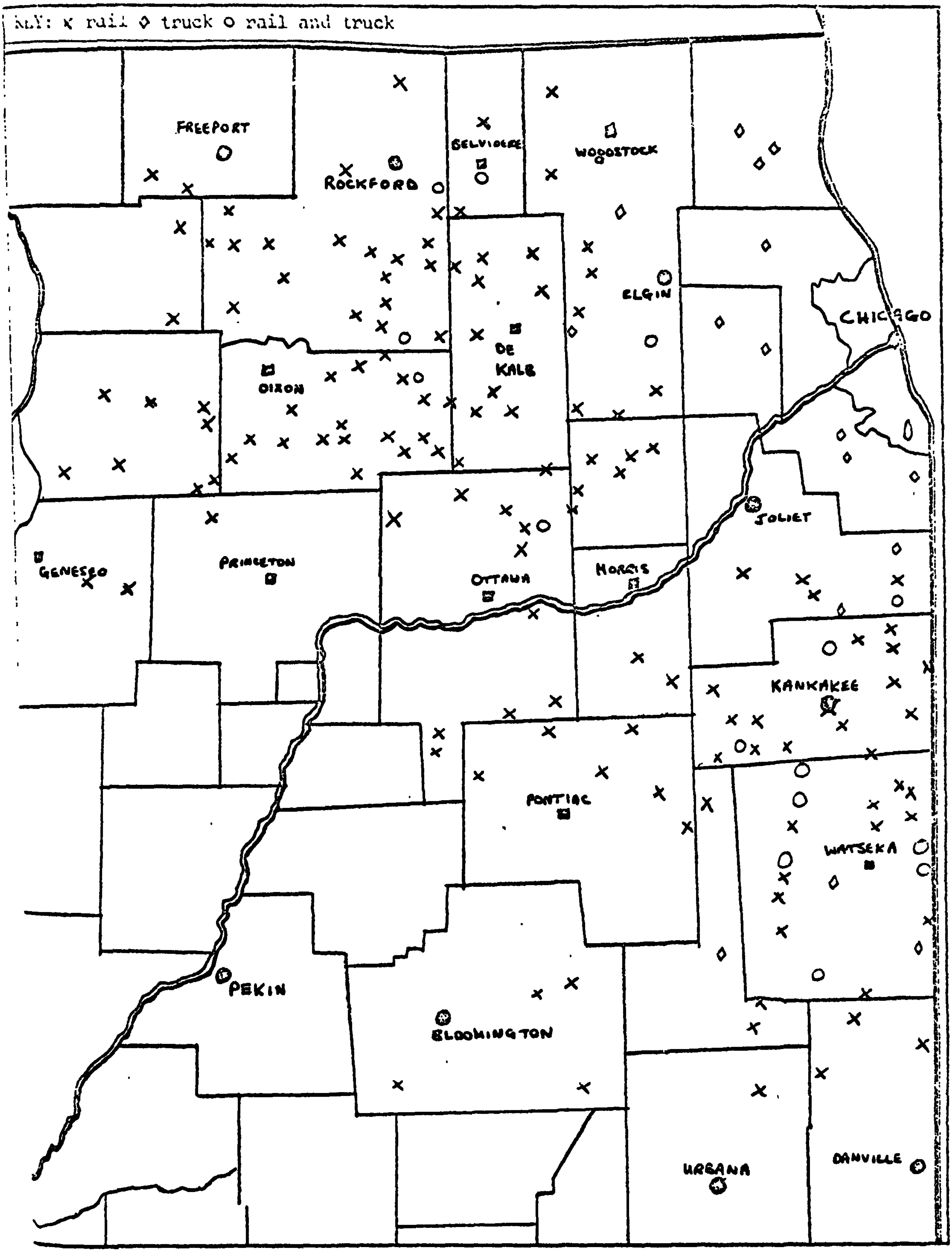


FIGURE III

in the model. The following simple linear regression was run to that effect:

(11) Road miles = 6.842 + .7983 millimeters R^2 = .96
(.02)

where the Euclidean distance is expressed in millimeters according to the scale of the map used for the projection (13 millimeters per 10 miles). The sample was made of 75 origins spread throughout the relevant part of Illinois. Now, if millimeters are substituted as units of measurement of the co-ordinates of X and Y on the map, the truck transportation cost from the point X_oY_o to Chicago can be estimated as

$$L_t + 6.842\ C_t + .7983\ C_t(X_o^2 + Y_o^2)^{\frac{1}{2}},$$

where C_t and L_t have the same meaning as in Section 2, and can be estimated through a linear regression the result of which is given in Table XII. This formula for the truck cost can be easily incorporated in the boundary equations, as will be shown below.

For rail transportation the assumption of straight line connection meant again straight line connection to Chicago. It was found that the linear regression results of the rail rates on the road miles to Chicago were slightly better than those of the regressions of the rail rates on the actual rail distances. Therefore, the above regression (11) could again be used to convert the millimeters of the co-ordinates into road miles. Then the rail transportation cost to Chicago could be estimated as

$$L_r + 6.842\ C_r + .7983\ C_r(X_o^2 + Y_o^2)^{\frac{1}{2}},$$

where L_r and C_r have the same meaning as in Section 2, and are estimated through linear regressions the results of which are given in Table XI. Again this cost formula can be easily included in the boundary equations

Something similar had to be devised for water transportation. Here, the ratio of the actual water mileage from Chicago to Pekin over the distance in millimeters from Chicago to the perpendicular of Pekin along the chosen axis (Chicago-Lacon) was used to convert millimeters in actual water

Table XI. Rail Rates Regressions: Cents (per one hundred pounds) on road miles: $R_r = L_r + C_r \; D_t$. One regression per railway, on the three months sample with one observation per origin.

Railway	Sample Size	L_r	C_r	R^2
A	30	12.145	.015 (.018)	.02
B	50	11.325	.025 (.001)	.85
C	62	10.835	.036 (.004)	.62
D	13	7.465	.068 (.008)	.87
E	50	9.813	.045 (.005)	.64
F	38	10.231	.039 (.006)	.58
G	10	3.715	.112 (.053)	.36
H	14	7.442	.058 (.008)	.81
I	13	9.425	.030 (.008)	.57
J	24	12.500	-	- (1)
K	12	11.450	.032 (.005)	.79

(1) Railway J has constant rates for all observations.

Table XII. Water and Truck Regressions: Cents (per one hundred pounds) on water and road miles respectively.

WATER (to Pekin): Sample Size: 23

R_w	=	L_w	+	$C_w D_w$	R^2
		1.27		.0085	.91
				(.0006)	

TRUCK: Sample Size: 28

R_t	=	L_t	+	$C_t D_t$	R^2
		4.174		.1185	.98
				(.004)	

Sources: 'Guide to Published Barge Rates on Bulk Grain', Schedule No.5, issued by Arrow Transportation Company, April, 12, 1966. The rates used in the regression are discounted by 15 percent (see p. 32-33).

'Agricultural and Materials Tariff', No. 600, of the Illinois Motor Carriers' Bureau, issued by D.S. Mullins, Issuing Officer, May 16, 1966.

mileages. The reason why this ratio was related only to the leg Chicago-Pekin of the Illinois River is simple. Since no country elevators used a transshipment point on the river lower than Pekin, the cost of water transportation from Pekin to New-Orleans could be considered as a fixed cost. Then the water transportation cost to New-Orleans from a point X_2 on the river could be computed as

$$L_{wb} + N_p + C_{wb}\beta_w(X_p - X_2)$$

where N_p is the cost of transportation from Pekin to New-Orleans which is constant,

β_w is the ratio discussed in the above paragraph,

X_p is the distance along the Chicago-Lacon axis between Pekin and Chicago,

X_2 is the point of origin on the same axis,

C_{wb} is the coefficient of the regression of the water rates to Pekin from points between Chicago and Pekin as given in Table XII,

L_{wb} is the usual fixed cost, which is the sum of the intercept of the regression just mentioned and other fixed fees (see Chapter III).

Finally, for the road leg of the combined road-water transportation, only the regression coefficient of (11) was used to convert millimeters into actual road miles. The intercept term was not introduced because of the shortness of the road leg and also because it essentially corresponds to the characteristics of the road network leading to Chicago.

With all these modifications, the basic water-rail boundary equation, as an example, became

$$(12) \quad P_a - C_r\beta_t(X_o^{\,2} + Y_o^{\,2})^{\frac{1}{2}} - L_r - \alpha_r =$$

$$P_b - C_t\beta_t[(X_2 - X_o)^2 + Y_o^{\,2}]^{\frac{1}{2}} - L_t - L_{wb} - N_p - C_{wb}\beta_w(X_p - X_2) + E_{21}$$

$$\text{s.t.} \quad X_2 - X_o = \frac{C_{wb\ w}|Y_o|}{(C_t^{\,2}\beta_t^{\,2} - C_{wb}^{\,2}\beta_w^{\,2})^{\frac{1}{2}}}$$

$$\text{and} \quad X_o, X_2 \geq 0$$

where most of the symbols have the same meaning as before and,

β_t = .7983, the coefficient of regression (11)

$\beta_w = 1$

$\alpha_r = 6.842\ C_r$

$E_{21} = \tilde{X}_{21}$ the money estimate of the quality difference between rail and barge transportation.

This new equation form is identical to the form of equation (8), so that the boundary is made up of segments of hyperbolas. The other boundary equations of Section 2 can be easily modified in the same way. Again, the water-truck boundary is made up of segments of parabolas, and the rail-truck boundary is a circle.

The results of the linear regressions of the rates on distance were very good in the case of river and road transports. As to rail transport results, they were good enough for our purpose, most of the R^2 being above .60, and the coefficients significant. However, railway A's regression was very bad, for the reason that its rates hardly vary with distance, and a few of them are completely out of line(1). A graphic inspection of the rate structures of the railways revealed that they were best summarized by a linear relationship between rate and distance, despite some tariff peculiarities. In these conditions, the linearity assumptions of the model could be used without reservations.

The actual boundary equations can then be computed on the basis of this information plus what has been gathered in Chapter III. These equations are given in Table XIII(2). Note that the time cost was not included in their computation. The little role that time plays in this particular problem did not warrant the additional complication. The next task is to draw all the curves corresponding to these equations on the Illinois map, using the Chicago-Lacon axis as an abscissa. Note that there are one

(1) These rates relate to points of origin which are not located on railway A's network but on another railway feeder line.

(2) In the case of the rail-truck boundaries, only the radius of the circles have been given.

Table XIII. Boundary Equations.

Water - Truck (parabola)

$$-.0001\ Y^2 - .0016\ X|Y| - .0089\ X^2 + 1.0585\ |Y| - .0954\ X + 31.4743 = 0$$

Water - Rail (hyperbolas)

<u>Railway</u>

A	$.0087\ Y^2 - .0016\ X|Y| - .0001\ X^2 - .2243\ |Y| + .0202\ X + 1.4175 = 0$
B	$.0085\ Y^2 - .0016\ X|Y| - .0003\ X^2 - .0828\ |Y| + .0075\ X + .1928 = 0$
C	$.0080\ Y^2 - .0016\ X|Y| - .0008\ X^2 - .0046\ |Y| + .0004\ X + .0006 = 0$
D	$.0059\ Y^2 - .0016\ X|Y| - .0029\ X^2 + .5892\ |Y| + .0532\ X + 9.7763 = 0$
E	$.0076\ Y^2 - .0016\ X|Y| - .0012\ X^2 + .1764\ |Y| - .0159\ X + .8763 = 0$
F	$.0079\ Y^2 - .0016\ X|Y| - .0009\ X^2 + .1054\ |Y| - .0095\ X + .3127 = 0$
G	$.0009\ Y^2 - .0016\ X|Y| - .0079\ X^2 + 1.2391\ |Y| - .1118\ X + 43.2398 = 0$
H	$.0067\ Y^2 - .0016\ X|Y| - .0021\ X^2 + .6064\ |Y| - .0547\ X + 10.3568 = 0$
I	$.0083\ Y^2 - .0016\ X|Y| - .0005\ X^2 + .2688\ |Y| - .0243\ X + 2.0355 = 0$
J	$.0089\ Y^2 - .0016\ X|Y| + .00007 X^2 - .8892 = 0$
K	$.0082\ Y^2 - .0016\ X|Y| - .0006\ X^2 - .1153\ |Y| + .0104\ X + .3744 = 0$

Rail - Truck: Radius of circles in millimeters.

A	82.12
B	80.73
C	85.31
D	61.21
E	79.39
F	79.49
G	-
H	49.20
I	58.99
J	74.39
K	89.87

water-rail equation and one truck-rail radius for each of the railways. But, since each railway network covers only parts of Illinois, only the segments of its curves which correspond to these regions need to be drawn. As for the areas where two railways have a line, only the curve corresponding to the cheaper railway should be kept. In that way it is possible to generate complex rail-water and rail-truck boundaries combining segments of several curves. This is illustrated in Diagram 20 where a typical complex boundary is assembled from segments of two rail-water boundaries. Some links between portions of curves have eventually to be added to obtain a continuous complex boundary. This is the case of the line OP in Diagram 20.

Once the three boundaries corresponding to the three binary choice problems are so derived and drawn, it remains to combine them to obtain the

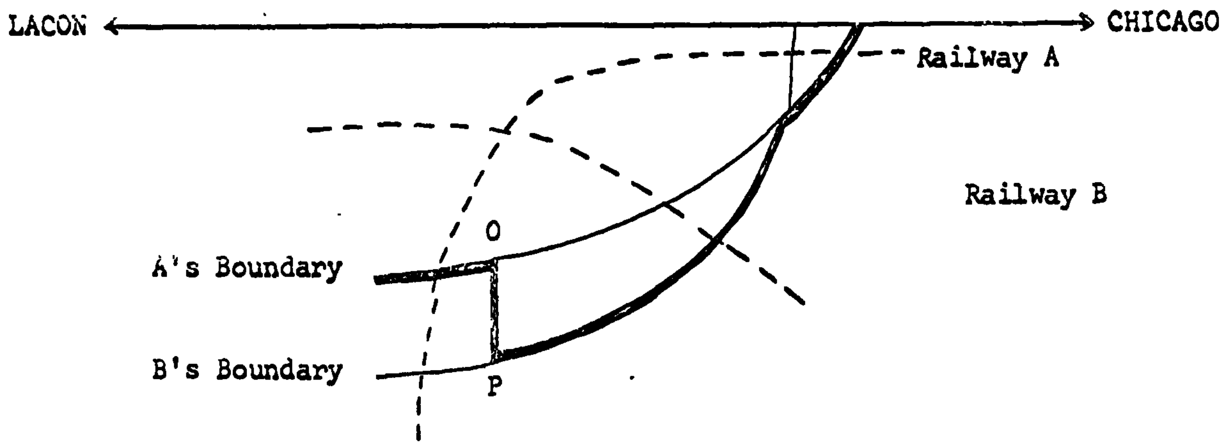


DIAGRAM 20

three mode-destination market areas and their limits. Each mode destination area corresponds to the region where that mode destination is preferred to both others. An example is given in Diagram 21 where the resulting boundaries correspond to the continuous thick lines. Note that it should not be expected that the three boundaries meet at a common point. The money equivalents of the quality differences which were estimated in Chapter III, have been used for their computation and we know that these estimates are somewhat inconsistent. For the same reason as we had an intransitivity area in the classification space, we should have here a region where the preferences conflict.

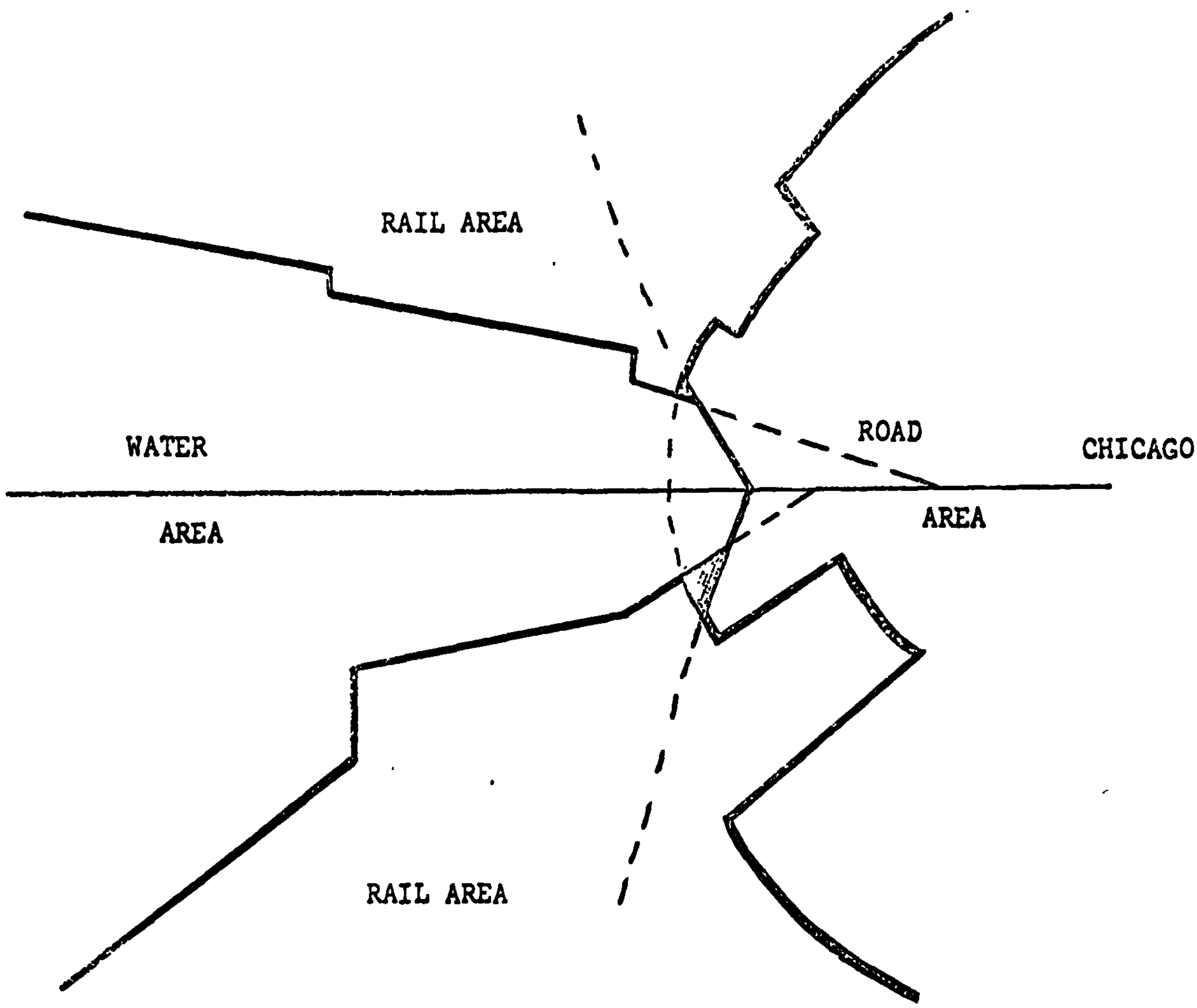


DIAGRAM 21

Figure I shows the actual boundaries and market areas obtained for the month of January: around Chicago is the "truck to Chicago" market area; further away from Chicago along the axis is the "truck-water to New-Orleans" market area; and on both sides of the latter are the "rail to Chicago" market areas.

It is possible to evaluate the boundaries corresponding to each of the binary choice problems as well as the limits of the three market areas (shown in Figure I), by computing the percentages of sample origins which are correctly classified. These percentages are given in Table XIV. They can be compared to the probability of correct classification by the discrimination approach which was given in Tables VI and VII of Chapter III[1]. The

---

(1) Another useful comparison may be made with the probabilities given in Tables VIII and IX.

The percentages are of the same order of magnitude as the probabilities. However, of the rail and truck origins the classification in the binary choice problems is improved, while the classification of the water origin is not as good as before. As to the limits of the three market areas,

TABLE XIV: Percentages of sample origins correctly classified; month of January.

Binary choice boundaries

Problem	Truck-Barge	Rail-Barge	Truck-Rail
MODE			
BARGE	75%	75%	-
RAIL	-	92%	79%
TRUCK	96%	-	58%

Three market boundaries

BARGE	70%
RAIL	73%
TRUCK	55%

they provide a better classification of the rail origins without changing much the two other modes' results. Altogether the results of the spatial model are fairly satisfactory.

Figure I gives enough information(1) to suggest the main defect in the predictions of both the spatial model and the discrimination approach. It appears at once that the boundary which separates the rail and truck areas is at too great a distance from Chicago. A boundary drawn closer to Chicago would certainly improve the classification of rail origins without much affecting that of the truck origins. The reason for this fault should be found in the peculiarities of the sample of truck origins. On one hand, the

(1) The points of origin of truck-water shipments are not given on the maps as this information was of confidential nature.

truck sample is small (48 origins in January). This is due to the fact that truck transportation is particularly favorable for short haul, and little corn is grown within a short distance of Chicago. On the other hand, the few truck origins spread far away from Chicago have a lot of weight (35%) in the sample, although they hardly can be considered as typical origins of truck shipments. In that respect, it is worthwhile to point out that most of these origins are also shipping by rail. They probably use truck shipments occasionally, or have unusual characteristics. Now, the parameters of the population of truck origins have been computed on the basis of the whole sample and have been biased by the peculiar characteristics of these origins. Therefore, the estimates of the money equivalents of quality differences between road transportation and the two other modes have also been biased. It is likely that more realistic estimates would have been obtained on the basis of a truck sample reduced to the "typical" origins. For the boundaries an example shows well how sensitive they are to variations of the money equivalents of quality differences. In the case of Railway C a change of the money equivalent from -3.54 to -2.54 would have reduced the radius of the circumference limit between road and rail areas from 85.31 to about 70 millimeters. Variations of the same order of magnitude have been found for the other railways.

In view of these remarks our confidence in the spatial model is increased. On the basis of correct information it can produce fairly accurate predictions of the choice of modes. As the results take the form of market areas for transportation modes they are particularly convenient to generate estimates of demand for transportation and evaluate investment projects in ways of transportation.

CONCLUSION AND SUMMARY

The theoretical model presented in Chapter I implied that the shipper of a commodity to a particular destination should choose one mode of transportation to the exclusion of all others.

The data concerning corn shipments in Illinois suggested that this was the case for most shippers.

Two forecasting models embodying the idea of exclusive choice were developed and tested on the data. The first uses the statistical approach of discriminant analysis. It predicts the choice of mode made by any shipper on the basis of the relative costs of transportation. Applied systematically to shippers spread over a region of production, it can also predict the regional pattern of use of several modes. Such predictions can be used to generate estimates of demand for a mode of transportation.

The second model is an extension of traditional location theory. Here transportation costs are related to distance and location, and the predictions take the form of boundaries between market areas for the respective modes. In a strict sense, it is a spatial model.

Both models take into account all costs of transportation including time cost and the (relative) costs of 'quality' differences between modes. The latter are rarely available, and it is the advantage of the discrimination model that it estimates the quality differential. The spatial model cannot provide such estimates, and, therefore, requires additional information. Another weakness of the spatial model is that its treatment of time cost rests upon an approximation. In addition, it requires that the costs be linear functions of distance. However, the spatial model has the important advantage that it does not require a sample of actual shipments made by the various modes involved, but only information concerning all costs of transportation. It might be the case that such information can be obtained from trade sources or derived from the technical characteristics of the modes.

In this application of the spatial model use was made of the estimates of 'quality' differences obtained from the discrimination model.

Both models provide very similar and fairly satisfactory results, despite some deficiencies in the sample.

NORTHWESTERN UNIVERSITY

LINEHAUL PROCESS FUNCTIONS FOR RAIL AND INLAND WATERWAY TRANSPORTATION

A DISSERTATION

SUBMITTED TO THE GRADUATE SCHOOL

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JOSEPH S. DE SALVO

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CHAPTER I

INTRODUCTION

Production and Cost Functions in Theory

Purpose. In general, the reasons for developing process functions for transportation modes are much the same as those usually given for developing any production or cost function. With the help of production and cost functions, economists can measure returns to scale. Such information is desirable in determining observed or potential size distribution of firms in the industry. It is possible also, with the aid of production and cost functions, to estimate possible effects of regulation in the industry on costs. One might also wish to know how well observed factor proportions approximate the minimum cost input combinations, given factor prices. Such analyses might also provide information of more direct use to the firm. For example, they could be used to determine the effect of changes in certain parameters on the output of the firm or process.

In particular, the current study was undertaken as part of a larger effort whose goal was to provide an improved methodology for benefit-cost evaluation of proposed public investment projects in inland waterways. Without going into any detail concerning current or proposed benefit-cost analysis, it is clear that benefits will be based to some extent on inland waterway transportation costs before and after the proposed improvements. Improvements may take several forms, e.g., deepening or widening the channel, providing new or larger locks. In order to know how such changes in the operating environment affect the cost of shipping by barge, it is necessary to have a detailed cost-estimating procedure. However, changes in transportation costs on the waterway do not take place in a vacuum; there are repercussions on competing modes, most notably rail. Thus it is desirable to be able to estimate the cost of rail transportation as well as that of inland waterway transportation. It was with these goals in mind that the research in this dissertation was undertaken.

Production and Process Functions.[1] In economic theory a production function is defined as a relationship between inputs and outputs. The production function states the maximum output obtainable from every possible input combination. Inputs are usually defined as any good or service which contributes to the production of an output. The terms good and service may be defined broadly to include other than economic goods and may be either stocks or flows. Inputs are usually thought of as either fixed or variable for a given period of production; however, they may be fixed for one period of time or variable for a longer period. Production functions are defined only for non-negative values of the inputs and outputs.

The most general production function would be a long-run relationship between inputs and output in which all inputs are variable. Short-run production functions could be obtained from the long-run functions by assigning fixed levels to some of the inputs and permitting the others to vary. One might even define various "intermediate" runs depending on which sets of inputs are held constant.

The production function is a very general concept. It may be represented by a single point, a single continuous or discontinuous function, or a system of equations. Moreover, although the production function is usually regarded as describing a firm's activities, this need not be the case. Much less aggregative productive activities may be described by production functions. For example, the firm may be thought of as performing a number of productive activities. Each such activity or process will possess its own production function. In order to avoid confusion with the firm production function, these less aggregative production relations will be called process functions.[2]

A process function may be used in much the same manner as a production function. Productivity relationships for the process may be investigated. Isoquants between inputs to the process may be derived. Returns to scale for the process may be examined. It does not follow, however, that the firm will exhibit the same magnitude or direction in returns to scale as that enjoyed by a process. For example, increasing returns to a process may be offset in the firm by administrative difficulties.[3]

Cost functions. Since inputs are not costless, their use is an economic decision. With this in mind, one may wish to maximize output subject to a cost constraint or to minimize the cost of producing a prescribed level of output. It turns out these two criteria yield the same decision rule, which is to equate the ratio of the marginal productivities of the inputs with the ratio of their prices. Graphically, the optimal input combinations for various cost or output levels consist of the locus of points of tangency between isocost lines and isoquants. This locus is known as the expansion path. If inputs were freely variable, production would take place along the expansion path.

Cost functions may be derived from knowledge of the production function, the expansion path, and input prices. In the short run, however, an input combination on the expansion path cannot always be chosen because of the fixity of one or more inputs. In this case, a "short-run" expansion path may be obtained from the first-order conditions of the usual constrained cost minimization, except that the production function and isocost equation will reflect the condition of input fixity. The system of these equations, consisting of short-run production function, short-run isocost equation, and short-run expansion path, can be reduced to a single equation in which cost is stated as an explicit function of the level of output plus the cost of the fixed inputs, i.e., a short-run total cost curve. However, this is only true where the three functions have the appropriate properties.[4]

In cases where the production function is not amenable to mathematical manipulation, an alternative method may be used to determine cost curves. Of course, both methods should be equivalent.[5] The following steps are involved: (1) choose a level for all inputs; (2) compute cost; (3) compute output; (4) change level of one input; (5) compute cost; (6) compute output; (7) repeat (4), (5), and (6) a number of times to generate a series of cost-output combinations; (8) plot cost against output; (9) repeat process with a new level of the "fixed" input(s) to get another short-run cost curve. By doing this successively for various levels of the fixed inputs, one would generate a family of short-run cost curves. From this family of short-run curves, the envelope or long-run cost curve would be obtained. Although this procedure is less elegant than having a functional form for the long-run cost relationship, it is adequate for purposes of revealing shapes and levels of short-run and long-run cost curves. It is this procedure that will be used below in developing cost curves for the rail linehaul process.

Estimation of Production and Cost Functions

Introduction. There are basically two ways in which production and cost functions may be estimated. These two approaches are referred to as statistical and engineering. The statistical approach to cost and production estimation is clearly the older of the two methods. Cost studies appeared in the late 1930's and early 1940's in the studies of Ehrke,[6] Yntema,[7] and Dean.[8] These early works were based on time-series data and dealt with the firm's short-run cost function. Time-series analysis of production functions appeared with the work of Cobb and Douglas.[9] The use of cross-section data in the estimation of production functions appeared first in the work of Bronfenbrenner and Douglas.[10] Early cross-section cost functions are found in the work of the Temporary National Economic Committee[11] and Dean and James.[12] The statistical approach to cost and production function estimation has received much criticism over the years. This has resulted in improved techniques and probably motivated the development of the engineering approach. The early engineering-type studies were pioneered by Chenery[13] and Ferguson.[14] In what follows, the discussion is limited to the cost and production relations of the firm and the process; more aggregative relationships (such as economy-wide or industry analysis) are ignored. Moreover, the discussion is general in that methods are considered, not specific applications of those methods.

Statistical Approach.[15] Estimation of cost and production functions by statistical methods depends on data generated during the operations of the firm or process. It is unlikely that firms will be in long-run equilibrium at the time they are observed. If they are not, then it is not likely that a long-run cost or production function will be estimated. Therefore, statistically-estimated cost and production functions are merely a means of summarizing actual operating conditions at particular times. These functions do not possess the characteristics of production efficiency and cost minimization displayed by their theoretical counterparts.

Apart from this basic criticism of the statistical approach, there are a host of other problems. In statistical estimation of cost and production functions, choice of an algebraic form of the function is important. This may be based on some knowledge of the physical nature of the productive process or merely on the basis of "goodness of fit" criteria. In simultaneous equation models the problem of identification remains after the form of the model has been chosen, and the problems of estimation vary considerably according to the specification of the model. Since the source of much cost information is accounting data, deficiencies which characterize these data carry over to the estimated functions.

Engineering Approach. The data on which engineering production functions are based consist either of technological information from physical or chemical theory or from empirical analyses of carefully controlled experiments. For example, in the estimation of a production function for electrical transmission, the following relationships were used: the law of energy consumption, the law of heat dissipation in electrical circuits, Ohm's law governing electricity flow in a circuit, and an empirical law concerning the conductive and resistive properties of materials.[16]

There are several advantages in estimating production functions from engineering data and principles. The range of applicability of the function is known in advance; it does not depend, in general, on data limitations. Unlike the information used in cross-section and time-series studies, engineering variables are not typically restricted to the range of actual observation. Moreover, the results of production investigations are not biased by the type of equipment actually installed in a plant. Therefore, production functions based on engineering data and relationships more closely conform to the production function of economic theory. It follows that cost curves derived from such production functions also possess the same advantages and also approximate more closely those of economic theory.

Problems remain, however. If one wishes to estimate a firm's production function, it may be necessary to first estimate process functions for the several processes comprising a firm's total activities. In order to do this, processes must be independent and additive. It has, however, been suggested that this is not an insurmountable problem because engineers also attempt to avoid interaction effects in specifying their processes.[17] Nevertheless, were it possible to estimate independent and additive process functions and to aggregate them in an appropriate manner, a firm production function still might not emerge. This is so because the entrepreneurial inputs are not introduced explicitly into engineering processes (although such inputs may be reflected in the data if these are of the "average experience" type). In addition, it may be difficult or impossible to include nontechnical processes, such as selling activities, in an engineering-type production function. Perhaps, however, a combined engineering-statistical production function could be derived, the statistical approach being used to handle such processes as selling. (This is attempted in Chapter V.)

Summary. Production and cost functions can be estimated in either of two ways. Statistical techniques may be used to synthesize the behavior of a firm or firms in a given industry. Such relationships, whether they be based on time-series or cross-section data, are subject to many reservations and at best summarize what firms have actually been doing. Alternatively, production functions may be derived from technological data and relationships. Production functions so obtained may be used to determine cost functions by associating prices with the inputs of the function. Production and cost functions so derived have several advantages over those statistically obtained. In addition, they more closely conform to their corresponding theoretical counterparts.

Process Functions for Transportation

Transportation Processes. Like most productive activities, transportation may be divided into processes. The discussion will be restricted to freight transportation, but analogous processes could be developed for passenger transportation. Freight transportation consists, in general, of three distinct processes which are connected by a flow of empty and loaded vehicles (e.g., rail freight cars, cargo barges, cargo aircraft, highway vans). The three processes are loading, assembly, and linehaul.[18] The relative importance of these varies as between modes. In rail and barge transportation, the distinction between the three processes is quite clear. There is even a noticeable division of labor. In many cases a large part of the loading process is performed by other than a transportation firm. This is true of barge, rail, and highway, but less true of aircraft. The assembly process is a major activity in the case of rail, where large freight yards are in operation, and consumes a large share of the total shipping time. It is less important in the case of inland waterway transportation, where the typical procedure is for the towboat to pick up loaded barges at a point or points along the river. Assembly is still less important for air and highway transportation. The linehaul process consists of the transfer of assembled vehicles between terminals. In all cases, it consumes a large portion of the time and expense of transportation. It is the linehaul process to which the analysis in this dissertation is directed.

It should be pointed out here that only the point-to-point aspect of the linehaul process is considered. That is, scheduling activities are ignored. Making sure that the right vehicle is at the right place at the right time is an important requisite of any transportation firm. It is a difficult and interesting problem and one on which several studies have dwelt.[19] Moreover, scheduling models can be employed to combine the long-run process functions into a long-run production function for the firm. This will be touched on again in Chapter IV; however, the more fundamental linehaul activity is all that is dealt with in this dissertation.

Linehaul Process. A more detailed discussion of the linehaul process is now undertaken. The linehaul process may be thought of as consisting of several phases: acceleration, cruising, deceleration, delay. The vehicle begins its linehaul activity by accelerating from a stopped position to its cruising speed. The term cruising speed is used to denote the maximum attainable speed of the vehicle given the currently experienced conditions. Cruising speed is not a constant but will vary with many factors. It considerably simplifies the analysis if it is assumed that the vehicle operates at cruising speed when it is not accelerating, decelerating, or stopped. The relevance of this assumption to real-world transportation vehicles is commented upon and tested where appropriate below. Cruising speed, once attained, is maintained until the vehicle is required to or desires to stop. It then decelerates to a stop. The vehicle remains at rest for a period determined by the cause of the stop, e.g., loading or unloading, assembly of cargo vehicles, and delays en route of various sorts. The term delay is used to signify a period during which the vehicle is stopped. The term may also be applied to the extra time required to complete a trip because the vehicle operated at less than cruising speed. The context should denote which of these is meant. The importance of delays, of course, varies with different modes. The primary emphasis in this research is on linehaul delays, i.e., delays en route.

Linehaul Process Functions. The method of obtaining a linehaul process function is presented in Chapter III and applied to inland water and rail transportation in Chapters IV and V. A verbal statement of the approach will be presented in this section.

The output of the linehaul process function is ton-miles per hour. This may be thought of as the product of the cargo tonnage and the average speed of the vehicle. The analysis involves methods for estimating the two multiplicative terms. Cargo tonnage will depend on various characteristics of the transportation vehicle, e.g., size, type, and number of cargo carrying units. It may also vary with commodity type and with institutional restraints. A cargo-tonnage function must be developed for each mode for which a linehaul process function is desired. Average vehicle speed between origin and destination, the second multiplicative term in the process function, is a complicated relationship involving acceleration, deceleration, cruising, and delays. This relationship clearly will depend on the motive power characteristics of the vehicle, the vehicle drag-force characteristics, terrain characteristics, and a variety of delay-causing factors. It is the derivation of the average speed function that consumes most of the pages in what follows. The linehaul process function, therefore, relates the cargo ton-miles per hour produced by a transportation vehicle in the carriage of commodities from origin to destination to a host of variables, the form of the basic relationship being the same for all modes but differing in the actual variables affecting output.

The output of the linehaul process function, ton-miles per unit time, reflects the quantity of cargo carried, the distance it is carried, and the speed of the carrying vehicle. It is not an unambiguous measure of output, but it has been adopted in many studies of transportation. Usually, however, the time unit is longer than the hour. This is usually dictated by the nature of the data which is annually, quarterly, or monthly. Since the linehaul process function is developed from engineering data and relationships, the time unit is not restricted by the data. Moreover, in transportation many factors affect the speed of the vehicle, thereby influencing productivity. A process function that reflects this situation would seem desirable. In addition, implicit in the formulation of the process function is the trip time, route characteristics, and the equipment to be used. Therefore, the function is very detailed and can be made as precise as desirable.

It should be remarked, however, that the prediction of the ton-miles generated by a transportation vehicle over a period of time composed of many point-to-point movements should involve consideration of scheduling problems. It would not be correct to assume that each movement was made optimally since in practice equipment is often not available when and where it should be. If, however, the scheduling conditions were known, then the process function could be used to estimate the output for each of the point-to-point movements involved.

The linehaul process is similar to the job-lot or batch process of production firms. The vehicle may be regarded as producing a lot or batch as it travels from origin to destination or, alternatively, as engaging in a production run. Job lots are usually discussed in the operations research literature;[20] however, batch production functions are reported in Smith.[21] Also, the linehaul process seems to provide an example of the type of production for which <u>volume</u> of output (i.e., the aggregate output produced during one production run) should be distinguished from the <u>rate</u> of output.[22] A given volume linehaul movement (e.g., 100,000 ton-miles) can occur at a variety of rates (e.g., different sized locomotives will pull the same cargo tonnage at different speeds). It would be interesting to investigate the behavior of marginal costs when volume is varied at a given rate and vice versa. That is, what effect does increasing length of haul have on costs given the rate of output? And, in transporting a given amount of cargo a given distance, what effect on costs results from varying the rate of output?[23]

Once the linehaul process function has been obtained, one may investigate the various productivity relationships, isoquants, and returns to scale. It is also possible to obtain cost curves for the linehaul process. However, due to the complexity of the linehaul process function, it may not be possible to obtain the functional form of the expansion path and to derive therefrom functional forms for the cost curves. Cost curves may be obtained from the process function by holding input(s) constant at various levels and varying the other input(s), calculating the total cost of each iteration. (In the cases worked out below, there are ultimately only two inputs: one representing the cargo vehicles and the other representing the motive vehicle. Labor, fuel, etc. vary in known ways with these two inputs.) The family of cost curves obtained in the above manner may be designated short-run costs of the process. A long-run cost curve for the linehaul process may be traced in as the envelope to the family of short-run curves. Economics of scale inherent in the linehaul process will be evident from the shape of the long-run curve.

It should be emphasized again that the terms "short-run," "long-run," "returns to scale," and "economies of scale" all relate to the linehaul process, not to the transportation firm. Specification of functions for the other processes comprising the firm's activities as well as consideration of the scheduling problem are required before firm production functions and cost curves can be derived.

CHAPTER II

REVIEW OF RESEARCH IN COST AND PRODUCTION OF TRANSPORTATION: INLAND WATERWAY AND RAIL

Introduction

There exist a number of studies dealing in part or in whole with cost or production relations for rail and inland water transportation modes. In general, the goals of these studies are similar to the goals of any industry cost or production analysis: examination of extent of returns to scale in order to gain insight into the observed or potential size distribution of firms in the industry; evaluation of efficiency of firms in terms of how well they approximate minimum cost, given factor prices; and, with detailed enough production relations, examination of factor productivities, substitution possibilities, expansion paths, etc. In considering transportation industries, however, additional considerations emerge. For example, transportation regulatory agencies need cost information for their decision-making processes. Also, decisions regarding public investment in waterways improvements or construction require estimation of benefits to the public. These benefit estimates are quite closely connected with the expected cost saving due to the proposed investment and its effect on competing modes.

Approaches toward achieving these goals differ in part because goals have differed: one wishing to answer the question whether economies of scale exist in the transportation firm may use a different method of analysis from someone wishing to determine the cost of a particular trip. Moreover, approaches differ over time because both techniques and theory have changed.

In the review to follow, an attempt is made to bring out both the goal and the method of analysis as well as the results of each piece of research considered. Some attempt is also made to relate the work being discussed to the analysis contained in later chapters of this dissertation. Criticisms are given where they are felt warranted. The review is divided into two parts, each part dealing with the research performed on one mode. In each part, both cost and production studies relating to that mode are examined.

Research in Costs and Production of Inland Waterway Transportation

While the economics literature on cost and production is voluminous, including both theoretical and empirical analyses, there are only a few studies that have dealt with inland waterway transportation. Most important of these are the studies of Charles W. Howe and of Arthur P. Hurter. Although both Howe and Hurter have estimated cost and production functions for waterway operations, their approaches have been quite different, as will be shown below. This review will be restricted to their work and will begin with Howe's analyses.

Howe has developed production and cost functions for both the barge tow, which is the smallest unit of a waterway firm's productive capacity, and the entire bargeline firm. The methods employed by Howe in developing the two sets of functions are different and require separate explanation. The study of a barge tow is taken up first, then the analysis of the waterway firm is discussed.

In his attempt to develop empirical representations of tow performance,[1] Howe began with certain basic engineering relationships. A barge flotilla must be pushed through the water by a towboat. The resistance of the flotilla is a function of the speed at which the flotilla is moving through the water; certain characteristics of the flotilla (length, breadth, and draft); and certain characteristics of the waterway (width and depth). On the other hand, the push generated by the towboat is a function of certain characteristics of the boat (horsepower and speed) and the depth of the waterway. (The width of the waterway was not regarded by Howe to be an important factor in determining a towboat's effective push.) In order for a tow to be operating at a constant speed through still water, flotilla resistance must equal the effective push of the towboat.[2]

Howe then estimated these functional relationships. In order to estimate the resistance function, data were taken from tank tests of barge flotillas, i.e., tests using small scale-model tows in large water-filled test tanks. According to Howe, naval design theory indicates that the resistance function should take a log-linear form. After trying many equations, he decided that the following gave the best fit to his data:

$$R = a_0 e^{a_1/(D-H)} S^{a_2} H^{a_3+a_4/(W-B)} L^{a_5} B^{a_6}, \qquad (2.1)$$

in which R = resistance, in tow-rope horsepower

D = depth of waterway, in feet

H = draft of flotilla, in feet

S = speed, in miles per hour

W = width of waterway, in feet

B = breadth of barge flotilla, in feet

L = length of barge flotilla, in feet.

Equation (2.1) was fitted to five different groups of data, each group corresponding to a particular flotilla configuration (i.e., a given arrangement of barges). A sixth regression was performed using all the data. Although the parameter estimates showed considerable variation between flotilla configurations, Howe chose to confine his analysis to the sixth set of parameter estimates. This was done "on the pragmatic grounds that the resulting function will be used to evaluate a wide variety of tow performances."[3] Moreover, this function "explained" over 92 percent of the observed variance in resistance.

The effective-push function was fitted in a similar manner, but data were for actual towboat operations rather than tank tests and were obtained by Howe from barge operators. The form adopted for fitting was

$$EP = b_1 HP + b_2 HP^2 + b_3 HP \cdot D + b_4 S^2 + b_5 S \cdot HP, \qquad (2.2)$$

in which EP is effective push, HP is horsepower, and the other variables have the same interpretation as in (2.1).

Using the fitted forms of Equations (2.1) and (2.2) and the equilibrium condition that EP = R, Howe solved for S, the equilibrium speed:

$$S = S(HP, L, B, H, D, W). \qquad (2.3)$$

The actual form of the S function is quite complicated and is omitted here. Notice that when L, B, and H are specified, the total net tonnage, T, of the flotilla is determined. Then the output of the tow, measured in ton-miles per hour, TM, is

$$TM = TM(HP, L, B, H, D, W) = S(HP, L, B, H, D, W) \cdot T(L, B, H). \qquad (2.4)$$

Howe referred to (2.4) as a "process function" instead of a production function because it relates to only one of the many production processes of a bargeline firm.

In an attempt to provide an economic analysis of the tow with the aid of the process function, Howe defined productive inputs to be horsepower, which represents the towboat input, and deck area, which represents the barge input. He argued that, for a given tow, labor and fuel are directly related to horsepower and, therefore, may be omitted as inputs. Howe then characterized the process function numerically by tables and graphs. He included the following in his analysis: (1) total product as a function of the barge input for various values of HP, with W and D fixed; (2) marginal product schedules for the barge input and horsepower, with W and D fixed; and (3) isoquants for the inputs, with W and D fixed.

The characteristics of the tow process function may be summarized as (1) marginal productivity of the barge input is always decreasing,

(2) marginal productivity of the boat input is always decreasing, (3) marginal productivities of both inputs eventually become negative. The first two results are not surprising. It is a standard assumption of production theory that beyond some point the marginal productivities of inputs decrease. This behavior of marginal productivity is usually referred to as the law of diminishing marginal productivity.[4] In the case of barge tows, there appears to be no stage of increasing returns for either input. The negativity of the boat-input marginal productivity is probably due to the fact that too large a boat in a given waterway can draw water from under the barge flotilla causing the latter to sink lower in the water thereby increasing resistance substantially. The negativity of the barge-input marginal productivity probably results from the increase in flotilla size relative to the fixed width of the waterway.

In addition to the above results, Howe examined the substitution possibilities of both inputs by means of isoquants. The isoquants exhibited the usual convexity to the origin. Moreover, the spacing of the isoquants indicated decreasing returns to scale for the process.

Howe next derived unit cost curves (average linehaul cost per ton mile) for the tow. These were obtained by attaching factor prices to the boat and barge inputs. With horsepower fixed, varying the barge input permitted the derivation of a "short-run" cost curve. A family of such curves was presented. The short-run curves possessed the typically assumed U-shape. The implied envelope curve was quite flat over a wide range of output rates, "implying a fairly broad range of tow makeups capable of producing output at essentially the same average cost."[5]

Howe investigated the effects of width and depth on output, finding that the former always led to increased output but that the latter's beneficial effect on output was exhausted at widths approximating twice the tow breadth.

It might be commented here that the procedure adopted for measuring the barge input leaves much to be desired. Recall that square feet of deck area was used as a measure of barge input. This was adopted because Howe's process function accommodates only rectangular tow configurations. It is, of course, quite possible to add barges to a tow such that the resulting configuration is not rectangular. This presents a problem in measuring barge input, for adding one barge in different ways will not result in the same affect on output. Therefore, Howe adopted, as a measure of the barge input, the square feet of deck area. Increases in the barge input were achieved by adding square feet in such a manner that a rectangular configuration remained.

Although Howe's process function for a barge tow is a path-breaking analysis of a complicated problem and is likely to be of much importance to anyone interested in linehaul production and costs, some limitations should be pointed out. As was mentioned above, there was considerable variation between coefficients of the resistance functions for different flotilla configurations, yet Howe finally adopted the resistance function which combined all configurations. When the function is used with configurations different from those of the data, it is difficult to know the magnitude of error. Moreover, Howe's function applies only to open-hopper barges (195 feet long and 35 feet wide) organized into rectangular flotillas, and loaded to a uniform draft. Obviously, these conditions are not always met in practice, and again it is difficult to determine the magnitude of error. It was pointed out by Howe that his effective-push function probably should not be extrapolated beyond 5000 horsepower, for that was the highest horsepower in his data. While these limitations appear formidable, they may not seriously impair the usefulness of the process function. This conjecture will receive attention in Chapter IV.

While not a criticism of Howe's analysis, it should be remarked that his process function comprises only a part of the linehaul operations of a barge tow. A complete process function of linehaul tow operations would provide for inclusion of acceleration, deceleration, and delays. These additions are provided in Chapter IV.

As has been indicated Howe's process function will receive some attention at a later point as it forms the basis for the tow linehaul process function to be developed below. It may be pointed out here, however, that some minor alterations and tests of the original functions have been made by Howe in a more recent unpublished paper.[6] These will also be discussed fully in Chapter IV.

Rather than extending the linehaul process function as indicated above, Howe proceeded directly to an analysis of the bargeline firm.[7] In his production analysis of the firm, Howe distinguished between a planning function and a production function. The term planning function was reserved for the relationship between outputs and inputs in which capital stocks are considered inputs. The relationship between output and inputs in which capital services are inputs was called a production function.

The reason given for the distinction between planning and production functions has to do with the way in which capital inputs are used in the bargeline industry. In this industry, capital stock consists of many homogeneous units which, it is contended, if used at all, are used at a uniform rate. This means that the level of output depends on the number of capital input units being employed. That is, at any given moment there is a stock of idle capital units which in no way affects the current rate of output. Granting the above argument, it would seem best to measure capital inputs as flows of services, and this is what is done in the production function analysis of Howe. Nevertheless, since firms must make decisions regarding levels of capital stock, it seemed advisable to study the relationship between the rate of output and the stocks of capital inputs. This relationship was called a planning function.

One might wish to quarrel with the assumption that when capital inputs are used each is necessarily used at a uniform rate. Barges may be loaded to various drafts, so that the quantity of cargo carried by any barge is variable. A barge-hour, even for the same barge, does not always contribute the same amount to production of ton-miles. In addition, tows operate at variable speeds even when pushed by the same towboat at full throttle. This is so because (1) the number of barges in tow varies and affects speed; (2) drafts of barges vary and affect speed; (3) depth, width, and stream velocity all vary and all affect speed. Therefore, a towboat-hour, even for a given towboat, does not contribute a constant amount to the production of ton-miles. This is a problem which occurs in measuring the input services for any production relationship, e.g., a man-hour is not always constant in quality even when defined rather narrowly. However, barge and towboat services seem particularly troublesome in this regard. These problems exist quite apart from the problems associated with using ton-miles as a measure of output. The ton-mile, being the product of two terms, is not an unambiguous unit in which to measure output. Nevertheless, it is a commonly used measure of output in transportation.

Both production and planning functions were estimated by Howe. For the production functions, he used monthly time-series data of three firms. For the planning functions, combined cross-section and annual time-series data for six firms were employed. Howe assumed log-linear production and planning functions. The inputs of the production function were surrogates for barge and towboat services, and the inputs of the planning function were barge and towboat stocks as well as time. Output in both cases was cargo ton-miles. Howe assumed a demand function facing the firm, and with this and the production (planning) function, he maximized a profit function subject to the production (planning) constraint. From the first-order conditions for a maximum, input demand equations were obtained. However, Howe argued that an adjustment lag for the barge and towboat inputs should be assumed for the production function model because "we have omitted stochastic elements from our model and (most important) because the preceding period (month) always leaves a legacy of geographical distribution of equipment...."[8] The introduction of the adjustment lags resulted in lagged values of the barge and towboat inputs appearing in the input demand equations. Finally two equations were added to each model: (1) a fuel input equation, specified as a function of the towboat input, and (2) a labor input equation, also a function of towboat inputs.

Each five-equation model was estimated by single-equation least squares applied to each structural equation in turn. Howe realized that a simultaneous-equation technique was appropriate but justified the single-equation technique as a first approximation. Howe's empirical results for his production and process functions will be summarized below.

The production-function parameters "appear quite reasonable" for two firms, and, with the exception of the towboat input coefficient for one of the two firms, all coefficients were highly significant. Some of the identifying restrictions were not significant however, leaving the identifiability of the production function in doubt. The third firm fared badly in all respects. It would appear that the two firms for which coefficients were significant exhibited constant or slightly increasing returns to scale in terms of the capital-service inputs. The planning function parameters also appeared "quite reasonable" and were highly significant. Decreasing returns to scale were indicated in terms of the stocks of boats and barges for the firms in Howe's sample. Perhaps "the necessity of providing equipment to meet peak loads and providing ready equipment to attract additional volume combines with increasing equipment scheduling difficulties to cause additions to the capital stock to be progressively less effective in increasing output."[9] In addition to information on returns to scale, Howe's analysis indicated a tendency over time toward substitution of barge inputs for towboat horsepower and more efficient use of labor and fuel.

Howe's production and planning models discussed above have been subjected to a more extensive analysis in another paper.[10] He has applied the two-stage least squares estimation procedure to the equations of these models. Nevertheless, the conclusions of the earlier study were not materially affected.

Hurter, following Howe, attempted to summarize the activities of firms in the industry by estimating planning and production functions.[11] Recall that whenever capital stocks are used as inputs, the function is referred to as a planning function; whenever flows of services are used as inputs in a relation, it is called a production function. In Hurter's analysis the sample of bargeline firms was substantially increased over that of Howe. It included all Class A certificated carriers operating on the Mississippi River for three years: 1950, 1957, and 1962. Data were obtained from the Interstate Commerce Commission; they represent annual firm operations and do not relate solely to regulated traffic.

Six least-squares regression equations were fitted for each year; all the equations were in log-linear form. These relationships were (1) annual firm tonnage regressed on the number of boats, the number of barges, annual gallons of fuel, and annual labor hours; (2) the product of annual mileage by towboats and annual tonnage regressed on the same variables as in (1); (3) annual tonnage regressed on average horsepower per towboat, average horsepower age, fuel consumption, and labor hours; (4) annual tonnage regressed on annual mileage, average horsepower per towboat, average horsepower age, fuel-oil consumption, and labor hours; (5) average horsepower per towboat regressed on fuel-oil consumption, towboat miles, and labor hours; and (6) annual tonnage regressed on the product of (a) average horsepower per towboat, (b) number of towboats, and (c) towboat miles and (entering separately) average age per horsepower, fuel consumption, and labor hours.

The results of the regression analysis will be summarized briefly. The regression results for (1) for all three years revealed a relatively stable towboat coefficient ranging from 0.680 to 1.036. For each of these estimates the value of "t" was comparatively large, with the exception of one year (1950). However, the other parameter estimates varied considerably between years. Nevertheless, the labor-hours coefficient showed a downward movement from 1950 to 1962, but the standard errors were quite large. The coefficient of the barge variable, on the other hand, showed upward movement. Little may be said about the parameter estimate of the fuel variable. One percent increases in all inputs in 1950 led to a 1.79 percent increase in tonnage contrasted with a 1 percent increase in 1957 and a 2 percent increase in 1962. Hurter viewed the behavior of tonnage when all inputs are changed proportionately as reflecting the relatively high level of waterways activities in 1957. Finally, the proportion of the observed variance in annual tonnage explained by relation (1) ranged from 64 to 75 percent.

In the next relationship investigated, (2), the towboat coefficients were about the same in 1950 and 1957, but the estimate for 1962 was larger. Although the standard errors were very large, the estimates of the barge coefficient seemed to follow the same pattern. The exponents of the labor-hours term appeared to fall over time. Hurter recognized the possibility of strong collinearity between the fuel and labor variables; he therefore examined the sum of the coefficients of these variables as an indication of the effect of current inputs. This sum was about 0.6 in 1962, 0.93 in 1957, and 1.3 in 1950, clearly, according to Hurter, indicating a diminishing role for the current inputs relative to the capital inputs. Relation (2) explained 75 to 90 percent of the observed variance in ton-miles.

In relationship (3), an upward trend in the exponent of the average horsepower variable seemed to indicate increasing importance of that variable; however, standard errors were large. The estimates of the fuel coefficient were again uninformative, but the estimates of the labor coefficients were quite stable and had reasonably large "t" values. Equation (3) accounted for only 59 to 65 percent of the observed variance in annual tonnage.

Relation (4), which differs from (3) through the introduction of annual firm towboat mileage as an additional explanatory variable, changed considerably from one year to the other. The standard errors of the exponent estimates for the variable miles were very large except for 1962 when the exponent appeared positive and greater than unity. The standard errors of the horsepower-per-towboat term were too large to permit any discussion of their change over time. Other problems appeared in the estimates of this relation; e.g., the estimate of the fuel coefficient was positive in 1957 but negative in 1962; the sum of the fuel and labor coefficients was 0.74 in 1950, 2.5 in 1957, and 0.15 in 1962. Again Hurter noted an indication of a high rate of capacity utilization in 1957 and a general trend toward diminished importance for the current inputs. This relationship accounted for 66 to 72 percent of the variance in annual tonnage.

The last production relationship estimated by Hurter involved an attempt to use a direct measure of the services of the capital inputs as an explanatory variable. The product of average horsepower per towboat, number of towboats, and towboat miles was used as a surrogate for horsepower-hours because horsepower-hours could not be used directly. It was later recognized by Hurter that collinearity between fuel-oil consumption and miles run by towboats would also result in collinearity between the surrogate variable and fuel consumption. The estimates of the coefficient of the surrogate variable were positive and similar in magnitude for 1957 and 1962. The estimates for the labor coefficient fell drastically over the period considered. Relation (6) accounted for 65 to 79 percent of the variance in annual tonnage.

In summarizing his analysis of production, Hurter indicated that increasing returns to scale, of a modest degree, were evident among the firms in the sample investigated. However, it was difficult to conclude anything about the time trend in returns to scale. There did, nevertheless, seem to be an increasing trend in the values of the exponents associated with the capital inputs. Finally, Hurter tested whether the equations were significantly different as between years. He found that indeed (1), (2), (3), (4), and (6) were significantly different in 1957 from their 1950 values and that the same equations in 1950 were significantly different in 1957 from their

1962 counterparts. However, only (1) was significantly different in 1957 from its 1962 version; the others were not significantly different as between 1957 and 1962. These results supported the conclusion that significant changes had occurred since 1950 in the production relationships used by Class A inland waterway carriers operating on the Mississippi River.

The production and planning functions just discussed, both those of Howe and those of Hurter, referred to the activities of an entire firm, as contrasted with the production function for a tow. Theoretically, cost functions for the firm could be developed from a knowledge of the appropriate unit costs of each input and the appropriate planning or production function. This procedure, however, was not used by Hurter in his study of cost relationships for inland waterway operations.[12] Rather, his method was that of statistically relating costs to output or some other measure of firm size. Hurter's empirical cost study made use of annual cross-section data for Class A carriers operating on the Mississippi River System in 1950, 1957, and 1962.

The first set of equations estimated by Hurter involved regressing total annual waterline expenses on total annual tonnage. The proportion of the variance explained by these regressions was very low for 1950 but over 60 percent for both 1957 and 1962. Hurter confined his equations to a log-linear form for reasons discussed in his paper. All of the equations indicated economies of scale. Significant differences were found in the extent of the scale economies when

1950 was compared with 1962. Hurter attributed a portion of observed scale economies to scheduling economies associated with large size.

Hurter's next set of regressions estimated the relationship between firm size and profitability. Profitability was measured by (1) the ratio of total freight revenue to total assets, (2) the ratio of total freight revenue minus total waterline expenses to total assets, and (3) the ratio of total annual waterline expenses to total annual freight revenue. The first two measures of profitability were regressed on annual tonnage as a measure of firm size. The third measure of profitability was regressed on annual tonnage and on total assets. No significant relationships among these variables were found.

Hurter was also interested in determining the extent of economies of scale in linehaul operations. He reasoned that if economies of scale could be shown in the linehaul costs, then at least a portion of the overall scale economies was due to advantages of large size in the linehaul operations. Regressions of total annual linehaul costs on total annual tonnage were employed to determine the extent of scale economies. Hurter found that a 1 percent increase in total annual tonnage was accompanied by a 0.65 percent increase in linehaul costs as opposed to about a 0.74 percent increase in total costs for 1957 and 1962.

In order to gain more insight into the sources of scale economies, Hurter investigated the relationship between linehaul costs and the inputs used. Where average horsepower per towboat, annual fuel consumption, and total labor hours were used as explanatory variables in a log-linear regression, a 10 percent increase in all inputs was accompanied by a 13 percent increase in all linehaul costs in 1957 and 1962.

A 10 percent increase in all inputs was also associated with a 28 percent increase in annual tonnage in 1962. These findings seemed to indicate the existence of scale economies in 1957 but not in 1962; the latter year, it will be recalled, showed positive scale economies when tonnage was regressed on linehaul costs. Further support for positive economies of scale was gained from the set of regressions in which stock inputs (boats and barges) and flow inputs (fuel and labor) were regressed on linehaul costs.

Hurter has also provided an analysis of terminal cost, but discussion of that is omitted here since the emphasis of this research is on linehaul operations. Omitted also is a discussion of a barge scheduling model by Hurter and A. Victor Cabot.[13] Reference to the model is made later in the chapter on the tow process function, where it is indicated how one might determine firm costs through use of the tow process function and the Cabot-Hurter scheduling model.

Some general comments on the foregoing research may be appropriate now. The tow process function, as has been pointed out, is incomplete as a description of the barge linehaul process. This is primarily due to the absence of any consideration of linehaul delays but is also due to omission of any consideration of non-constant operating speeds. Whether or not these are important omissions, and if so what to do about them, are questions taken up in Chapter IV of this dissertation. The cost functions derived from the Howe tow process function are, of course, subject to the same reservations as affect the process function.

The work of Howe on the tow process function is in the category of engineering production functions which have several advantages over the statistical approach as indicated in the previous chapter. However, Howe's estimation of the production function of the bargeline firm, as well as Hurter's cost and production function estimations, were statistical. Quite apart from the statistical reliability of the estimates, the reliability of the data, and the randomness of selection of firms to be included in the sample, the resulting functions are not those of economic theory in that they at best display what firms have actually experienced, rather than what they could potentially experience. This issue was discussed in Chapter I.

Research in Cost and Production of Rail Transportation

Railroad costs have long been a subject for study. Initially the major concern was whether or not increasing returns characterized the rail industry. While this question has continued to be of interest, research attention has been directed in more recent years to development of estimates of long-run marginal costs and of costing procedures.

A discussion of the findings of early railway cost studies may be found in an article by Borts.[14] The emphasis, as mentioned above, was on the extent of increasing returns in the railway industry. The degree of increasing returns was felt to be indicated by the ratio of variable cost to constant cost for the rail firm. The smaller was this ratio, the greater was the extent of increasing returns.[15]

In brief, there were two groups of findings. One, due primarily to the work of Ripley,[16] found that rail costs included a very high proportion of constant costs. Typically variable expenses were estimated to constitute less than half the operating expenses and about one-third the total expenses. The other group of findings consisted of those studies which reported that the cost ratio was really much higher than one half. These studies are exemplified by the work of Ford K. Edwards[17] and M. O. Lorenz.[18] It was this body of research, principally the work of Edwards, from which were obtained the ideas and methods employed by the Interstate Commerce Commission in its rail costing procedure.[19] The Interstate Commerce Commission approach to rail cost analysis may serve as the culmination of the earlier cost studies referred to above. A brief exposition and critique of the Interstate Commerce Commission costing procedure is now presented.

A method was developed to estimate the extent of variability of railway expenses with traffic volume. A simple linear regression was used. The dependent variable was operating expense, and the independent variable was gross ton-miles. Both variables were deflated by miles of road in order to eliminate scale effects. The data were total operating expenses, rents, and taxes for a cross section of U.S. rail systems. The intercept value, which may be interpreted as fixed cost, was subtracted from the average value of the dependent variable, which may be interpreted as total cost; the remainder may be viewed as total variable cost. This remainder, expressed as a percentage of total cost, as defined above, was used to measure the percentage of variability among all railway operating expenses. A similar procedure was used to determine the variability of road and equipment capital investment, to which was then applied a cost-of-money factor.

The Interstate Commerce Commission approach to rail costing outlined briefly above has been subjected to some severe criticisms.[20] A summary of these is presented now. It is contended that the very aggregative nature of the cost data hides important details. In addition, the application of an average percentage variable to the cost of a specific railroad may result in error if the railroad has a consistent error of observation in its costs, resulting in a consistent under or overstatement of variable costs. Also, when used separately with component categories of operating expense, the Interstate Commerce Commission method assumes the percentage variable is constant in all these categories. Finally, it is necessary for all subcategories of total operating expense to vary with gross ton-miles, whereas other units may be superior for other categories, e.g., engine miles for measuring the variability of engine maintenance expense.

Meyer, Peck, Stenason, and Zwick (MPSZ) sought to present an approach to rail cost analysis that would answer some of the criticisms leveled at earlier studies and particularly the Interstate Commerce Commission method of rail costing.[21] The approach used by MPSZ in rail cost determination is quite simple in concept.[22] Statistical cost relations of the following form were fitted to the data:

$$E = f(Q, S), \tag{2.5}$$

where E represents an expense or cost account

Q indicates quantity of output or traffic variable

S designates size variable.

In short, cost was estimated as a function of output and plant size. The equations were estimated using linear regression techniques. The output variable was usually gross ton-miles, and the size variable was usually track mileage or number of cars.

The major differences between this and earlier studies of rail costs are that (1) railroad costs were divided into much finer analytical categories in the belief that the behavior of different kinds of railroad costs is likely to be quite dissimilar and (2) unlike most previous statistical analyses of railroad costs, the data used were not in ratio form.

Cross-sectional cost functions were estimated for several different cost accounts for twenty-five Class I U.S. railroads. Separate sets of cost functions were derived for the years 1957-1950 and for the years 1952-1955. The following categories of cost were estimated separately: (1) maintenance of way and structure, (2) maintenance of equipment, (3) administration and legal overhead, (4) selling and marketing, (5) station, (6) yard, and (7) linehaul.

It is clear that no single cost function emerges from this work. Rather, the result is a large number of cost equations for various cost categories. However, according to MPSZ, it is possible to synthesize these individual cost relationships into an estimate of long-run marginal cost. The coefficient on the output variable of each cost equation estimated is interpreted as a component of total long-run marginal cost. The sum of all such components yields an estimate of total long-run marginal cost.

As a general description of the goal of their study, MPSZ state: "...the primary purpose of [the rail cost analysis] is not to obtain cost estimates applicable to specific operations, but to outline a new and, in the belief of the authors, more satisfactory approach to railroad costing problems and to obtain average or typical cost figures that can be used for the broad purpose of regulatory policy."[23]

The Canadian railroads have gone farther with the above approach than did Meyer and his associates.[24] Through the use of statistical methods and engineering analysis, the variability of cost components was investigated in detail. For example, track maintenance expenses were found to vary with the following traffic units: (1) miles of roadway, (2) miles of tunnels (a surrogate variable for terrain conditions), (3) yard locomotive miles, and (4) gross ton-miles. A cross-section multiple linear regression of expenses on relevant traffic units was used to determine the degree of variability. The data consisted of observations from each of the divisions of the railroad (e.g., the Canadian National Railways has sixteen divisions). The coefficients of such regressions were considered to be marginal costs per unit of the traffic unit. Not all cost variability was estimated by regression analysis. Some was determined by engineering studies, and others were standard cost-accounting determinations.

The outcome of this process, which was considerably more extensive than the brief discussion above would indicate, was a cost coefficient for each of the various traffic units. In order to determine the cost of a particular rail movement, the relevant traffic units were computed (e.g., the gross ton miles to be generated, the trip time, the number of cars, etc.). Each is then multiplied by its respective cost co-efficient. The products obtained this way are summed to obtain the cost of a train movement. The resulting figure is regarded as the long-run marginal cost of the movement and is used in pricing decisions.

The MPSZ and Canadian approaches to rail costing seem to be considerable improvements over the Interstate Commerce Commission methods currently in use: disaggregation of costs, more explanatory variables, more careful use of statistical methods. However, since any statistical analysis is only as good as the data it uses, the Canadian results are probably more reliable than those of MPSZ for costing purposes, quite aside from the particular years MPSZ used in their study, because the Canadian railroads used their own internally generated data while MPSZ had to use Interstate Commerce Commission published data. The latter are likely to be less homogeneous in quality than the Canadian data and are probably less accurate, although this is just a conjecture.

The extent to which domestic railroads use costing procedures similar to those of the Canadian roads is not known. It is likely that they recognize the variability of costs with time, distance, and gross tonnage, but it is not known how they determine the relationships among these factors and costs.

It has been recognized that there are many problems associated with the estimation of rail cost relationships from cross-section data. Several of these were discussed in the Meyer volume referred to above, and a more intensive discussion was presented in a paper by Meyer and

Kraft.[25] In addition, Borts has also been concerned with the problem raised by the attempts to estimate rail costs from cross-section data and has proposed a method of statistical analysis which he regards as an improvement over current methods.[26] Borts argued that biases have crept into statistical cost estimation primarily because the size of the firm has been treated incorrectly in previous studies. He proposed as a solution stratification of firms by size.

To illustrate his proposal Borts presented a stratified cost function estimated from a cross-section sample of firms. He also stratified by region. For each size class, estimates were made of average cost, marginal cost, and the elasticity of cost. (Cost elasticity is defined as the percentage variation in cost accompanying a 1 percent change in output.) Further, the marginal cost and elasticity of cost between cost classes have been computed as a check on the possible error of estimating a function which is not the true long-run cost function.

The basic cost-output relation (estimated by least-squares) was of the following form:

$$C = a + bX + c(Z/X), \qquad (2.6)$$

where C = freight operating expenditures

X = total loaded and empty freight car miles

Z = total freight carloads.

The variable (Z/X) may be viewed as a measure of carload density; its reciprocal can be interpreted as a measure of the average length of haul. The variable (Z/X) was used instead of Z alone in order to avoid high correlation between the independent variables and to permit non-linearity into the function.

With this formulation of the cost function, Borts was able to examine (1) the marginal cost of carloads for a given average length of haul, (2) partial cost elasticities with respect to X and Z, (3) the marginal cost of car miles for a given level of carloads, and (4) the partial cost elasticity with respect to length of haul.

Three size classes were determined based on cost of reproduction and track mileage. Regions used were East, South, and West. Data used were Interstate Commerce Commission rail cost data for the year 1948. Allocations of freight costs were made by Borts. "Elements of cost were eliminated where they did not appear related to the production of freight service. Whenever possible, items of cost were excluded when they were shared jointly between freight and passenger service."[27]

It was found that long-run increasing cost prevailed only in the Eastern region, while long-run decreasing and constant costs occurred in the South and West. Cost elasticities computed between size classes confirmed these findings.

It is doubtful whether any real conclusions can be drawn from the rail cost analysis of Borts, if only because the data were fourteen years old when his article was published and because they were annual costs cross-sectionally for one year only. Additionally, the cost allocations were done by the author, and the degree of arbitrariness is unclear. It is unfair to be too critical of Borts regarding the independent variables in his estimation equations, for he was attempting to determine the effects of size and region on cost. Nevertheless, using only car miles and carloads (the latter deflated by the former) seems a tremendous oversimplification of cost-determining factors. Even Borts himself, in a different article (to be discussed below), used a much more disaggregative approach and many more explanatory variables. Borts' real contribution would seem to lie in his use of covariance analysis in the study of costs. This is a statistical technique that permits investigation of the effects on regression equations of various categorizations of the data. It is particularly useful when comparisons between categories is desired but the number of sample observations is small.[28]

The only estimation of rail production functions to be found in the economics literature is the work of Borts.[29] His study was designed to determine whether increasing returns existed in the railway industry. A model of production was specified for two processes in railway technology, and a production function was estimated from cross-section data for each process.

It was hypothesized that railway technology consisted of the simultaneous operation of three distinct processes which were physically connected. The three processes were loading of cargo cars, switching of cargo cars, and the transfer of assembled trains between terminals (linehaul). Because a large proportion of loading activity was carried on by other than transport firms, models of production relations were constructed for the switching and linehaul processes only.

In the switching process--which consists of picking up, assembling, and sorting cargo cars into trains--the following input services were employed: labor, fuel, equipment, and fixed plant. Output was measured by number of transferred cargo cars. In addition to the inputs given above, the following were used: (1) switch engine miles, to account for the distance over which the switch engine must travel, which will vary from yard to yard; and (2) switch-engine hours, to account for circuity in routing of switch locomotives required by the scatter of cargo cars.

In the linehaul process--which consists of the transfer of loaded and empty cargo cars between terminals--the following inputs were used: labor, fuel, equipment, and fixed plant. The outputs considered were loaded cargo miles, loaded transported cars, and empty cargo car miles.

The form of the production relations for both processes was the same:

$$Y_i = f_i(Z_1, Z_2, \ldots, Z_n), \qquad i = 1, \ldots, m \tag{2.7}$$

where the Y's represent the minimum values of the variable inputs, and the Z's represent the outputs and the fixed inputs. In both models four variable inputs were specified. For the linehaul process the variable inputs were (1) labor services; (2) fuel consumption; (3) flow of equipment services, measured by expenditures on maintenance of freight equipment, exclusive of depreciation; and (4) flow of track and structure services, measured by expenditures on maintenance of track and structure for freight service purposes, exclusive of depreciation. The outputs were represented by (1) loaded freight car miles, (2) carloads of freight, and (3) empty freight car miles. The fixed inputs were represented by (1) total tractive capacity of freight locomotives and (2) miles of mainline track. In an alternative specification of the model, a variable was employed to measure all of the physical capital in use by the railway firm.

In the switching process the following variables were used to measure the flow of input and output services. The variable inputs were the same as those used for the linehaul model, except, of course, they were specified to yard operations. The outputs were represented by (1) yard switching locomotive miles, (2) yard switching locomotive hours, and (3) carloads of freight. The fixed inputs were (1) miles of yard switching track, (2) total tractive capacity of yard locomotives, and (3) average number of freight cars standing on line.

Two statistical models were formulated and their coefficients estimated. In the first model, each variable input was treated as a linear function of the fixed inputs and of the outputs. The second model was derived from the first by normalizing the variables by a measure of capacity.

The data consisted of observations on seventy-six Class I railways in the United States for the year 1948. All observations were from Interstate Commerce Commission data. The coefficients of the equations for each model were estimated by least-squares methods. From the estimated regression coefficients (of which slightly fewer than one-third were not significantly different from zero) elasticities of factor use were derived. The elasticity of factor use is defined to be the percentage change in factor use accompanying a 1 percent change in all of the output variables, holding constant the stocks of equipment. Decreasing returns to scale are implied by a value of the elasticity coefficient which is greater than unity; increasing returns are implied by a value of the coefficient which is less than unity. The elasticity of variable cost was also estimated. This is defined as the percentage change in variable cost accompanying a 1 percent change in the output variables, holding constant the stock of equipment. This particular elasticity was calculated by taking a weighted average of the factor-use elasticities, where the weights consisted of the proportion of total variable cost expended on the particular inputs. These proportions were derived from the aggregate expense accounts of all the railway firms.

The models of the switching process indicated a cost elasticity not significantly different from unity, implying constant returns to scale. For the linehaul process, there was a considerable difference between the cost elasticity coefficients indicated by the two models: the first model exhibited a coefficient close to unity, but the second showed a coefficient of one-half, implying increasing returns to scale.

As in the cost study discussed previously, Borts used annual Interstate Commerce Commission cross-section data for the year 1948. In addition, the variable cost proportions were based on figures summed across firms, and it is possible that the proportions used have little relevance to the actual proportions of individual firms. Finally, it should be mentioned again that slightly fewer than one-third of the estimated coefficients were not significantly different from zero.

Moreover, the form of the production relations estimated by Borts permits little flexibility. Input elasticities may be calculated, but a process function which relates inputs to outputs does not emerge. Factor substitutability cannot be examined, nor can marginal or total productivities be calculated. Returns to scale are investigated indirectly through factor elasticities rather than directly by means of a well-articulated production or process function.

In summary, it seems that past rail cost analyses have been designed to answer one of two questions: (1) What is the extent of increasing returns in the railway firm? and (2) What is the long-run marginal cost of the railway firm? Attempts to answer the first question are exemplified by the work of the early railway economists, as noted above, and also by later economists such as Borts. The second question has been pondered by the Interstate Commerce Commission, John Meyer and his associates, and the railroads themselves, as exemplified by the Canadian approach to rail costing. Because of the difference in approach between the two groups of researchers, the results of the one have not aided in answering the question posed by the other. Were it possible to derive the long-run average cost curve of the railroad firm, then both questions could be answered by the same study. This has not yet been accomplished. However, the rail process function of Chapter V will be used to derive a long-run cost curve for the linehaul activities of the rail firm, and it will be shown how to integrate this with statistical cost analysis to determine the firm's short and long-run average costs. The only other production analysis of rail is the work of Borts which was discussed above.[30] As noted, his approach was to divide the rail firm's activities into non-overlapping processes and to analyze each separately via statistical methods. No attempt was made to determine cost curves; but, as noted, cost elasticities were calculated.

CHAPTER III

METHOD OF ANALYSIS

Introduction

The purpose of the research reported in this dissertation is to develop process functions for the linehaul, or point-to-point, operations of freight transport vehicles. Loading and unloading activities are excluded from the production process although these terminal operations could easily be incorporated into the model as well be indicated later. Another quasi-terminal activity--assembly and disassembly of the transport vehicle or train of vehicles--is analyzed and included as a linehaul function for the case of barge tows.

The "output" of transportation is the physical movement of a commodity from one place to another in a given time. Such an output will differ as commodity, time required for shipment, and distance between origin and destination differ. Therefore, the customary output measure for freight transport--the ton-mile per unit time--is inadequate in several respects. While the ton-mile per hour will be used as the output measure in the following analysis, implicit in the formulation is the trip time, the route characteristics, and the equipment utilized. The inputs to the productive process may of course be any factor that even remotely affects the output. However, it is desired to restrict the number of inputs to manageable proportions while including those over which someone has control. In the case of transportation, the inputs controlled by the productive unit may be such factors as the horsepower of the motive unit, the size and type of carrying capacity, labor, fuel, etc. Some factors not controllable by the productive unit are delays en route due to weather, breakdown, etc. Some inputs may be under the control of one mode and not controllable by another. "Roadbed" is an example of this kind of input. Barging firms, for example, have no direct control over the depth, width, and stream velocity of the waterways on which they operate. Railroads, on the other hand, may alter the roadbeds over which they travel. The inputs of the productive processes will be discussed more fully later. In the next sections the method of articulating a linehaul transportation process function will be developed. In later chapters this method will be applied to water and rail transport.

Form of Linehaul Process Function

The ton-miles per hour generated by a transportation vehicle is the product of the cargo tonnage and the average speed at which the vehicle travels between origin and destination. Cargo tonnage and average speed are functions of the size and type of cargo-carrying units comprising the transport vehicle. Important inputs for these functions will be specified below; this discussion is focused on the form of the production relationship.

The process function has the following form:

$$Q = C \cdot V_e, \qquad (3.1)$$

in which Q = ton-miles per hour

C = cargo tonnage

V_e = average (effective) speed.

Since a transport vehicle must be in one of four states (at rest, at constant velocity, accelerating, or decelerating) at any moment of time, the following travel time expression may be defined:[1]

$$T_t = T_r + T_s, \tag{3.2}$$

in which T_t = travel time of vehicle

T_r = running time of vehicle, i.e., time during which the vehicle is moving

T_s = delay time of vehicle, i.e., time during which the vehicle is stopped for one reason or another.

This travel-time expression may be further disaggregated as follows:

$$T_r = T_f + T_a N_a + T_d N_d, \tag{3.3}$$

in which T_f = time during which the vehicle operates at "full" speed, i.e., the equilibrium speed of the vehicle for a given power input

T_a = time required for vehicle to accelerate from a stop to full speed

N_a = number of accelerations occurring during a trip

T_d = time required for vehicle to decelerate from full speed to a complete stop

N_d = number of decelerations occurring during a trip.

It will be convenient to define T_f as follows:

$$T_f = D_f/V_f = (D - D_aN_a - D_dN_d)/V_f, \tag{3.4}$$

in which D_f = distance traveled at full speed during a trip

D = distance to be traveled during a trip

D_a = distance traveled during one acceleration

D_d = distance traveled during one deceleration

V_f = full speed of vehicle, i.e., the equilibrium speed of the vehicle for a given power input.

Finally, the travel-time expression may be written

$$T_t = [(D - D_aN_a - D_dN_d)/V_f] + T_aN_a + T_dN_d + T_s. \tag{3.5}$$

The total trip time, T_t, required to travel distance D is the sum of four terms: (1) the time during which the vehicle travels at maximum speed, (2) the additional time required because the vehicle accelerates, (3) the additional time required because the vehicle decelerates, and (4) the amount of time during which the vehicle is stopped.

Effective speed may now be defined as

$$V_e = D/T_t = D\left\{[(D - D_aN_a - D_dN_d)/V_f] + T_aN_a + T_dN_d + T_s\right\}^{-1}. \tag{3.6}$$

If it is desired to divide D into I segments of length D_i each, then

$$V_e = D\left[\sum_{i=1}^{I}\left\{[(D_i - D_{a_i}N_{a_i} - D_{d_i}N_{d_i})/V_{f_i}] + T_{a_i}N_{a_i} + T_{d_i}N_{d_i} + T_{s_i}\right\}\right]^{-1}. \tag{3.7}$$

Equation (3.7) allows for the possibility that factors affecting trip time may not be constant over the entire distance D but are constant over the length D_i.

The process function, then, takes the following form

$$C \cdot D \left[\sum_{i=1}^{I} \left\{ [(D_i - D_{a_i} N_{a_i} - D_{d_i} N_{d_i}) / V_{f_i}] + T_{a_i} N_{a_i} + T_{d_i} N_{d_i} + T_{s_i} \right\} \right]^{-1} \qquad (3.8)$$

Of course, Equation (3.8) may be simplified considerably. If, for example, it were decided that acceleration and deceleration were unimportant but that route segmentation were important, the linehaul process function would become

$$Q = C \cdot D \left[\sum_{i=1}^{I} (D_i / V_{f_i}) + T_{s_i} \right]^{-1} . \qquad (3.9)$$

In addition, if it were decided to ignore the effects of variability in route characteristics, the process function would take the following form:

$$Q = C \cdot D \left[(D/V_f) + T_s \right]^{-1} . \qquad (3.10)$$

Analysis of Transportation Vehicle in Motion[2]

Introduction. This subsection discusses the determination of the constant running speed, V_f, the acceleration time and distance, T_a and D_a, and the deceleration time and distance, T_d and D_d. For the purpose of analyzing the motion of a transport mode, the vehicle, or train of vehicles, is considered to be a point moving in a straight line. This assumption is standard in physics and engineering, and should not materially affect the result. Constant velocity, acceleration, and deceleration will be discussed in that order.

Constant Velocity State. The cruising speed of a vehicle depends on the resistance characteristics or drag forces acting on the vehicle and the tractive-effort or motive properties of the propulsion system. These forces are each functions of velocity as well as vehicle characteristics, "roadbed," etc. The equation of motion for the vehicle is[3]

$$F = F_t(v) - F_d(v) = m(dv/dt), \qquad (3.11)$$

in which F = resultant force

$F_t(v)$ = tractive force

$F_d(v)$ = drag force

dv/dt = acceleration.

Equation (3.11) is a form of Newton's second law of motion: force is the product of mass and acceleration. An alternative statement is

$$dv/dt = [F_t(v) - F_d(v)]/m. \qquad (3.12)$$

If the vehicle is operating at limiting velocity V_f^*, i.e., the maximum attainable velocity for a given power input, then $dv/dt = 0$, and it may be possible to solve Equation (3.12) for $v = V_f^*$. For water transportation vehicles V_f^* must be adjusted for stream velocity in order to obtain V_f, which is speed relative to the land. For land transportation vehicles, $V_f^* \equiv V_f$.

In the case of barge tows, tractive-effort and drag-force functions exist which are quadratic in velocity, i.e.,

$$F_d(v) = A_0 + A_1v + A_2v^2 \qquad (3.13)$$

$$F_t(v) = B_0 + B_1v + B_2v^2, \qquad (3.14)$$

where the A's and B's are parameters which express towboat, barge, or waterway characteristics.[4] Utilizing (3.13) and (3.14), one may express Equation (3.11) as

$$dv/dt = k_0 + k_1v + k_2v^2, \qquad (3.15)$$

where

$$k_0 = (B_0 - A_0)/m \qquad (3.16)$$

$$k_1 = (B_1 - A_1)/m \qquad (3.17)$$

$$k_2 = (B_2 - A_2)/m. \qquad (3.18)$$

Then, setting $dv/dt = 0$, Equation (3.15) may be solved for $v = V_f^*$ by using the familiar quadratic formula. Of the two roots, which will be real and unequal, one will correspond to the maximum velocity of the vehicle; the remaining root has no physical significance and exists because of the quadratic functions used to describe the system.[5]

In the case of rail trains, the drag-force function is quadratic, but the tractive-effort function is a rectangular hyperbola, i.e.,

$$F_d(v) = A_0 + A_1v + A_2v^2 \qquad (3.19)$$

$$F_t(v) = B/v.^6 \qquad (3.20)$$

Therefore, the equation of motion for the system is

$$dv/dt = [(B/v) - (A_0 + A_1 v + A_2 v^2)]/m. \qquad (3.21)$$

When dv/dt = 0, Equation (3.21) will have the following form

$$v^3 + k_2 v^2 + k_1 v + k_0 = 0, \qquad (3.22)$$

where $k_0 = -B/A_2$, $k_1 = A_0/A_2$, and $k_2 = A_1/A_2$, which may be solved for $v = V_f$. Of the resulting roots, one will be real and positive and will be the equilibrium speed of the train.[7]

Acceleration State. It is desired to determine estimates of a vehicle's acceleration time and the distance traveled during an acceleration from v = 0 to $v = V_f^*$. The formulas derived below could be generalized to include acceleration from any one speed to any other. However, these situations are not emphasized here, for they are ignored in the linehaul process functions. If the differential equation constituting the vehicle equation of motion, Equation (3.12), could be solved for velocity as a function of time (yielding an "acceleration-speed-time" function),[8] then the above time and distance estimates could be obtained. Specifically, if v = v(t) were the acceleration curve, then, in order to determine acceleration time, one would obtain the inverse function t = t(v). The time required to accelerate from

$v = 0$ to $v = V_f^*$ would be given by $T_a = t(V_f^*)$. The distance traveled during an acceleration from $v = 0$ to $v = V_f^*$ would be given by

$$D_a = \int_0^{V_f^*} v(t)dt. \qquad (3.23)$$

For a quadratic equation of motion, Equation (3.15), these formulas may be explicitly derived. The solution of the differential equation, Equation (3.15), obtained by Haase and Holden,[9] is

$$v = \frac{R_1 - SR_2e^{Kt}}{2k_2(Se^{Kt}-1)} , \qquad (3.24)$$

in which $K = (k_1^2 - 4k_0k_2)^{\frac{1}{2}}$

$R_1 = k_1 - K$

$R_2 = k_1 + K$

$S = \dfrac{2k_2V_0 + R_1}{2k_2V_0 + R_2}$

V_0 = velocity at time $t = 0$.

Equation (3.24) holds under the following conditions: $k_0 \neq 0$, $k_1 \neq 0$, $k_2 \neq 0$, and $k_1^2 > 4k_0k_2$. For the special case in which initial velocity is zero, i.e., $V_0 = 0$, Equation (3.24) becomes

$$v = \frac{R_1(1 - e^{Kt})}{2k_2[(R_1/R_2)e^{Kt} - 1]} .[10] \qquad (3.25)$$

The graphical expression of Equation (3.25) is known as an acceleration speed-time curve and is given schematically in Figure 3.1.[11]

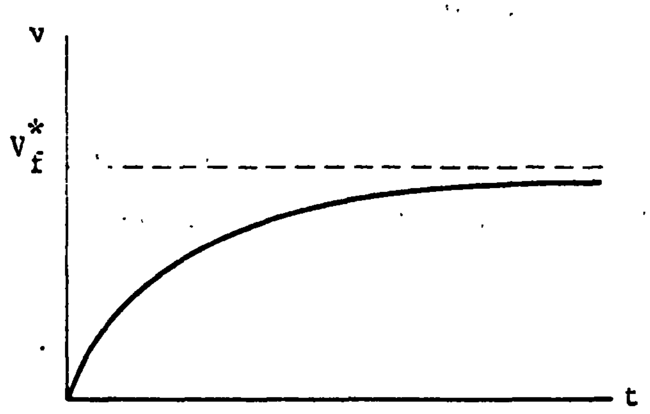

Figure 3.1. Acceleration Speed-Time Curve

It would be desirable to let $v = V_f^*$ in Equation (3.25) and solve for t, which would give an expression for the time required to accelerate from zero to V_f^*. However, because of the quadratic nature of Equation (3.15), this is not possible, i.e., an attempt to solve for acceleration time results in an indeterminate form because the acceleration speed-time relationship approaches V_f^* asymptotically. Therefore, a velocity ρV_f^* is defined, where $0 < \rho < 1$. Setting $v = \rho V_f^*$ and solving Equation (3.25) for $t = T_a$ (i.e., the time required to acceleration from zero to 100 percent of limiting speed), one obtains

$$T_a = \frac{1}{K} \ln \left[\frac{R_2(2k_2\rho V_f^* + R_1)}{R_1(2k_2\rho V_f^* + R_2)} \right] . \qquad (3.26)$$

To determine the distance traveled during an acceleration from $v = 0$ to $v = \rho V_f^*$, the definite integral of Equation (3.25) from $t = 0$ to $t = T_a$ is obtained, i.e., where D_a is acceleration distance,

$$D_a = \int_0^{T_a} v\,dt = \frac{1}{k_2}\left[\ln\left|\frac{R_1 - R_2}{R_1 e^{KT_a} - R_2}\right|\right] - \frac{R_1 T_a}{2k_2}. \tag{3.27}$$

No general solution, such as Equation (3.24), exists in the case of the equation of motion applicable to rail trains, Equation (3.21). There are, however, various ways to estimate the acceleration time and distance for vehicles possessing such an equation of motion. These will be presented in Chapter V in the discussion of the rail process function.

Deceleration State. Deceleration is a good deal more complicated than is acceleration because (1) it may consist of coasting and braking stages and (2) additional resistance forces, e.g., engine drag, may occur. That is, if the motive force ceases, the vehicle will coast to a stop. One can hasten the process by adding drag forces such as braking or reversing engines. One must make assumptions about how much drag forces are increased; and, apparently, engineering practice is to assume coasting and braking at constant rates.[12] Then the time, T_c, required to coast from one constant velocity, V, to another, V_c, at a constant rate c is

$$T_c = (V - V_c)/c.^{13} \tag{3.28}$$

And the distance traveled, D_c, during a coasting phase is

$$D_c = \int_0^{T_c} (V - ct)dt = VT_c - \tfrac{1}{2}cT_c^2.^{14} \qquad (3.29)$$

The braking phase of the deceleration may be handled in the same manner as above. Suppose braking occurs at the rate b at the end of the coasting phase until the vehicle comes to a stop. Then the time, T_b, required to brake from a velocity V_c to a velocity zero is

$$T_b = V_c/b.^{15} \qquad (3.30)$$

The distance traveled, D_b, during the braking phase is given by

$$D_b = \int_0^{T_b} bT_b dt = \tfrac{1}{2}bT_b^2.^{16} \qquad (3.31)$$

Of course the method by which a vehicle decelerates depends on the type of vehicle and the accepted practice of its operators. Deceleration characteristics would therefore have to be determined for each vehicle-type studied. If, in addition, the constant rates of coasting and braking did not turn out to be good approximations, then other formulations, such as exponential or quadratic, could be used.

Delays and Cargo Capacity

In the preceding sections of this chapter, expressions concerning maximum speed, acceleration time and distance, deceleration time and distance were presented. It was shown above how these relationships enter the linehaul process function. In the next two chapters, specific forms for each of these relationships will be derived for rail and inland water transportation modes. In order, however, to specify a complete linehaul process function for each mode, two further relationships are required as indicated above.

A transport vehicle may be required to stop en route for a variety of reasons. Therefore, delays must be analyzed. There does not appear, however, to be a general theory of delay that would be applicable to all transportation modes for all types of delays. Different transportation modes need not encounter the same types of delays, nor will the same factor affect different modes identically. Therefore, it seems necessary to analyze individual delays separately for each mode. This will be done in the next two chapters.

The second relationship required to complete the process function is the cargo-tonnage function. The cargo carrying capability of a transportation vehicle will vary with the size, type, and number of units comprising the whole vehicle. In addition, cargo tonnage may vary with commodity characteristics such as weight per unit volume and with institutional constraints such as load limits. It seems impossible to develop a general cargo tonnage relationship that would be applicable to all modes. Therefore, the derivation of cargo functions is postponed to the next two chapters.

CHAPTER IV

LINEHAUL PROCESS FUNCTION FOR BARGE TOWS

Introduction

A barge tow consists of a flotilla of barges rigidly lashed together and pushed by a towboat.[1] The flotilla may consist of any number of barges from one to more than forty. Barges are constructed in a variety of models, e.g., open and covered hopper barges, tank barges, deck barges, etc. The shape is similar for most barges: a rectangular steel box with two raked ends. So-called integrated tows, however, consist of square-end barges inserted between a front barge with a raked bow and square stern and a rear barge with a raked stern and square bow. Semi-integrated tows consist of partly integrated sections and partly non-integrated sections. Integrated tows offer less resistance to the water and therefore travel faster than comparable non-integrated tows for a given horsepower towboat. Integrated and semi-integrated tows are generally used only for trips that are to be made repeatedly, for square-end barges when not part of an integrated unit create much resistance and slow the tow. The barge type in most common use is the Jumbo (195 feet by 35 feet) open or closed hopper with bow and stern rakes. The analysis to follow deals exclusively with this type of barge, but it will be shown how other types and sizes may be handled.

A trip by barge tow may be thought of as consisting of several phases. Before the linehaul movement can begin, the barges intended to comprise the flotilla must be gathered and assembled into a unit; this process is known as "making tow." Once the tow is made, it begins the trip by accelerating to its cruising speed. The time required to accelerate and the distance covered during acceleration depend on the power of the towboat, the number and draft of barges (determined by size, tare weight, and cargo weight) and waterway characteristics. If there were no delays en route the rest of the trip would take place at the maximum cruising speed of the tow. Tows typically operate at maximum attainable speed, although they will occasionally reduce speed in certain situations such as light fog. It should be pointed out that the maximum attainable speed of the tow need not be a constant for a trip. Rather, the speed will be greatly influenced by waterway characteristics such as depth, width, and stream current, all of which may vary over the length of a trip. For this reason, it has become common in the barging industry to divide waterways into segments having fairly uniform characteristics. This procedure is adopted in the following analysis to account for the variability of waterway characteristics. A typical trip is interrupted by many delays such as locking, bad weather, ice, and running aground, so that most trips do not proceed from origin to destination at maximum attainable speed. In addition, each stop has an acceleration and a deceleration associated with it. When the tow reaches its destination it disassembles the flotilla ("break tow"). While it is common practice for a towboat to assemble and disassemble its own tow, the larger ports offer assistance in the form of a harbor boat. The towboat will not usually wait while barges are loaded or unloaded.

Form of Process Function

The process function for barge tows has the same form as Equations (3.8) - (3.10) above, except that some more specification of the variables is required. The total delay (time stopped), T_s, may be written as

$$T_s = T_L + T_m + T_b + T_o, \tag{4.1}$$

the sum of four delay factors relating to tow operations:

T_L = locking time

T_m = make-tow time

T_b = break-tow time

T_o = miscellaneous delay time.

Methods of estimating these delays are developed below. Also, the number of accelerations, N_a, and decelerations, N_d, for the case in which the waterway is assumed to be uniform throughout the length of the trip, are defined as

$$N_a = N_d = N_L + N_o D + 1, \tag{4.2}$$

in which

N_L = number of locks to be traversed during the trip

N_o = number of miscellaneous delays per mile.

In Equation (4.2) the number of accelerations required during a trip of length D is the sum of the number of locks (since each locking delay requires an acceleration), the number of miscellaneous delays, and an initial acceleration. Similarly, the number of decelerations is the number of locks plus the number of miscellaneous delays plus a final deceleration. For the case in which the waterway is segmented, N_a and N_d must be defined differently,

$$N_{a_i} = \begin{cases} N_{L_i} + N_{o_i} D_i + 1 & i=1 \\ N_{L_i} + N_{o_i} D_i & i = 2, \ldots, I \end{cases} \quad (4.3)$$

$$N_{d_i} = \begin{cases} N_{L_i} + N_{o_i} D_i & i = 1, \ldots, I-1 \\ N_{L_i} + N_{o_i} D_i + 1 & i = I \end{cases} \quad (4.4)$$

in which the new variables are

N_{L_i} = number of locks in ith segment

N_{o_i} = number of miscellaneous stops per mile of the ith segment.

For any given trip, the number of locks is known in total and for each segment. Discussion of the frequency of miscellaneous delays is left for a later section.

Given these additional specifications--Equations (4.1) and (4.2) or (4.3) and (4.4)--the tow process function has the form given in (3.8) - (3.10). The problem now becomes that of specifying the methods of estimating the values of the variable entering into the process function. For several of the variables, of course, no estimation is required, i.e., their values can be expected to be known for any given trip. Into this category fall (1) the length of the trip, D; (2) the number of locks en route, N_L; and (3), in some cases, the cargo tonnage of the tow, C. However, even given the cargo tonnage for a particular trip, that tonnage may be achieved in a variety of ways, e.g., fewer barges with deeper drafts, more barges with smaller drafts, and different arrangements of a given number of barges. It is desirable, therefore, to have a formulation of C that will account for these possible alternatives. This is discussed in the next section. There remain a large number of components of the tow process function for which methods of estimation must be developed. Functional forms for V_f, D_a, D_d, T_a, and T_d are developed below based upon the analysis of Chapter III and specific tractive-effort and drag-force functions applicable to barge tows. This chapter also develops procedures for estimating T_L, T_m, T_b, T_o, and N_o. Locking time is estimated by a relatively simple queuing model. Make-tow and break-tow time are estimated from towboat log data by least-squares procedures. Miscellaneous delay time is estimated from aggregate delay data provided by a barging firm and, in addition, utilizes the analysis of a tow in motion which was developed in an earlier section of this chapter. Finally, an estimate of N_o is obtained from towboat log data. Moreover, these formulations of the inputs to the process function will be amenable to changes in variables under control of the firm (e.g., horsepower of towboat, draft of barges, number of barges comprising a flotilla, etc.) as well as variables under control of the Army Corps of Engineers (e.g., waterway depth, waterway width, stream velocity, and size of locks).

Cargo Capacity of Tow

In the following analysis only one barge size will be used, that of the Jumbo (195 feet by 35 feet) double-raked hopper barge. As was mentioned earlier, this particular barge is probably the most common model in use today. Nevertheless, the general applicability of the analysis to follow is restricted somewhat by the above assumption, however, perhaps less than might be thought. Most double-raked barges, no matter what type and size, have essentially the same hull shape. The most important differences among barge types are tare weight. These differences introduce little difficulty into the following analysis. It will be seen that the operative variables with respect to the flotilla of barges are (1) length of flotilla, L; (2) breadth of flotilla, B; and (3) draft of flotilla, H. Given L and B and the barge type comprising the flotilla, one can easily determine the number of barges in the flotilla, whether the barge type is Jumbo hopper or some other. Moreover, there is a regular relationship between the cargo weight and the draft of a barge, which can be estimated quite accurately by a second degree polynomial in H. For the Jumbo hopper barge, the relationship is

$$W_c = -605{,}120 + \underset{(125)}{380{,}000\,H} + \underset{(100)}{2{,}210\,H^2} \qquad (4.5)$$

in which W_c = cargo weight of barge, in pounds; H = draft of flotilla, in feet; coefficient of determination = 0.99; standard errors are given in parentheses; and sample size was 109. Equation (4.5) was estimated from a capacity table for the Jumbo hopper barge.[2] From capacity tables for other barge types and sizes, similar relationships could be developed. So, knowing the length, breadth, and draft of a flotilla of barges of given size and type, one can determine the cargo capacity of the flotilla; or, in symbols,

$$C = C(L, B, H) = N_b(L, B) \cdot W_c(H) = N_b(-605{,}120 + 380{,}000\,H + 2{,}210\,H^2) \qquad (4.6)$$

in which N_b = the number of barges in the flotilla, as determined by the length and breadth of the flotilla

W_c = cargo weight of a barge as determined from Equation (4.5) or similar relations for other barge types and sizes.

Formulating the cargo capacity of the flotilla in this manner brings out three implicit assumptions which are somewhat restrictive and intractable. They are: (1) the barges comprising the flotilla are of the same size and type; (2) the configuration of barges in the flotilla is rectangular; and (3) all barges in the flotilla are loaded to the same draft. These assumptions are not always fulfilled in practice. Nevertheless, it will be argued below that these assumptions do not adversely affect the tow speed prediction equation.

Analysis of Tow in Motion

Introduction. In order to apply the relationships developed in Chapter III, tractive-effort and drag-force functions must be obtained for barge tows. Several drag-force functions (generally called resistance functions for water vehicles) have been developed. These will be discussed and compared with the resistance function of Charles W. Howe[3] which is the relationship adopted in this dissertation. In contrast to resistance functions for barge tows, of which there are several, there appears to be only one tractive-effort function that has been developed. This also is the work of Howe.[4]

Tow Resistance Formulations. Howe has estimated a resistance function for barge tows, using data taken from tank tests of model barge flotillas.[5] According to Howe, naval design theory indicates that the resistance function should be of log-linear form. The functional form adopted by Howe was the following:

$$F_d(v) = a_0 e^{a_1/(D-H)} v^{a_2} H^{a_3 + a_4/(W-B)} L^{a_5} B^{a_6}, \qquad (4.7)$$

in which $F_d(v)$ = resistance, in pounds

D = depth of waterway, in feet

H = uniform draft of flotilla, in feet

v = speed of tow, in miles per hour

W = width of waterway, in feet

B = breadth of flotilla, in feet

L = length of flotilla, in feet.

Equation (4.7) reduces to linear form under a logarithmic transformation and exhibits the following plausible limiting properties:

$$\lim_{H \to D} F_d = \lim_{B \to W} F_d = \infty \qquad (4.8)$$

The function was originally fit by least squares to five groups of data, each group corresponding to a different flotilla configuration, e.g., one group of data was for flotillas arranged four barges long and two barges wide. A sixth regression was performed using all the data. Although the parameter estimates showed considerable variation among flotilla configurations, Howe chose to confine his analysis to the function fitted to all the data. This was done "on the pragmatic grounds that the resulting function will be used to evaluate a wide variety of tow performances."[6] Table 4.1 presents information about the regression.

Table 4.1

STATISTICAL RESULTS OF TOW RESISTANCE ESTIMATION

Parameter	a_0	a_1	a_2	a_3	a_4	a_5	a_6
Estimate	0.07125	1.46	2.0	0.60	16.83	0.38	1.19
Standard Error		0.0416	0.0253	0.0173	0.4675	0.0277	0.0196

Coefficient of Determination 0.92

Sample Size 1,632

SOURCE: Howe, "Methods ...," p.28.

Although the estimated function fits the results of tank tests quite well, it seems desirable to determine whether or not other resistance formulas which have been developed might either perform better or perform equally well but be simpler in form. Howe has recently reported such an investigation.[7]

Two tow resistance functions were compared by Howe with his function. They are:

$$F_d'(v) = 1.722H^{-(4/3)}v^2 \qquad (4.9)$$

and

$$F_d''(v) = 100.838\ (\Delta/L)v^2, \qquad (4.10)$$

in which Δ is gross displacement of flotilla, in short tons; the other variables are as defined before. The F' function is due to Langbein,[8] and the F" function is the work of Rouse.[9]

Howe tested all three of the functions against tank test data, using the reported model resistance as the norm. In all cases involving deep-water tests, Howe's function was markedly superior to the other two. Specifically, the following statistics were all lower for the Howe function than for the other two: (1) the arithmetic average difference of predicted from actual, (2) the average absolute difference of predicted from actual, and (3) the maximum absolute difference of predicted from actual. For example, the average absolute deviations ranged from 11 to 25 percent of the data-set averages for Howe's function and over 100 percent of the data-set averages for the others. Shallow-water comparisons could only be made against Langbein's functions since Rouse's function applies solely to deep-water operations. Here, again, Howe's resistance function appears decidedly superior; in all cases the three statistics were markedly lower for the Howe-function predictions than for the Langbein-function predictions.

In the process of performing the above tests, Howe discovered that his resistance function consistently produced large errors for those cases in which flotilla width approached channel width, in which flotilla draft approached channel depth, or in which speeds were greater than 11 miles per hour. Using the data subsets for which $v \leq 11$ miles per hour and W = maximum (28, 225 feet), Howe modified his resistance function as follows:

$$F_d(v) = 0.07289^{1.46/(D-H)}v^{2.0}H^{0.6+50/(W-B)}L^{0.38}B^{1.19}. \quad (4.11)$$

The modified function performed well on the test data subsets for which the above conditions were met. The F_d function as given by Equation (4.11) is used in the remainder of this chapter.

Howe has further refined his resistance function to account for a factor known as slope-drag.[10] When a tow is proceeding upstream there is a loss of speed, in addition to that due to the stream current, because the unit is actually going uphill. The reverse is the case for downstream movements. Howe feels that the slope-drag adjustment could become important for tow operations on waterways with a steep gradient. This may well be true, and, if so, the slope-drag adjustment could easily be incorporated into the process function; however, it is not used here.

<u>Tractive-Effort Function</u>. Howe has estimated a tractive-effort, or effective-push, function for barge tows. It has the following form

$$F_t(v) = b_1HP + b_2HP^2 + b_3HP \cdot D + b_4v^2 + b_5v \cdot HP. \quad (4.12)$$

in which $F_t(v)$ = tractive-effort, or effective-push, in pounds

HP = rated brake horsepower of towboat

D = depth of waterway, in feet

v = speed of tow, in miles per hour.

The HP^2 term allows for increasing or decreasing effectiveness of horsepower in determining push. The cross-product term, HP·D, reflects the fact that gains from greater depth are greater for larger boats and vice versa. The v·HP term reflects the fact that $\partial F/\partial v$, evaluated at a given speed, is a decreasing function of horsepower, a fact that was determined from plots of the data. Waterway width was not found to be an important variable in determining effective push.

Equation (4.12) was estimated by least squares; Table 4.2 contains information about the estimates.

Table 4.2

STATISTICAL RESULTS OF TOW EFFECTIVE-PUSH ESTIMATION

Parameter	b_1	b_2	b_3	b_4	b_5
Estimate	31.82	0.0039	0.379	-172.05	-1.14
Standard Error	0.96	0.0003	0.058	11.50	0.07
Coefficient of Determination			0.98		
Sample Size	145				

SOURCE: Howe, "Methods ...," p.29.

The data used in the estimation of Equation (4.12) were obtained from barging firms and contained observations on eleven towboats ranging in horsepower from 500 to 3500. Observations included a range of waterway depths of from 15 to 26 feet and a range of towboat speeds up to 14 miles per hour. While the estimation is quite good, it is impossible to compare the function with others as was done for the resistance function, for there are no others. Rather, the predictive ability of the speed function determined from the resistance and effective-push functions will be examined below. The speed function is derived in the next section, and some tests are presented in a following section.

Constant Velocity State. Given resistance and tractive-effort functions, the results obtained in Chapter III may be applied to barge tows.

Equation (3.13), the quadratic drag-force function, becomes Howe's resistance function when

$$A_0 = 0$$

$$A_1 = 0$$

$$A_2 = 0.07289e^{1.46/(D-H)}H^{0.6+50/(W-B)}L^{0.38}B^{1.19}. \qquad (4.13)$$

Equation (3.14), the quadratic tractive-effort function, becomes Howe's effective-push relation when

$$B_0 = 31.82HP - 0.0039HP^2 + 0.38HP \cdot D$$

$$B_1 = 1.14HP$$

$$B_2 = 172.05. \qquad (4.14)$$

Defining

$$k_0 = \frac{1}{m}j_0 = \frac{1}{m}(31.82HP - 0.0039HP^2 + 0.38HP \cdot D)$$

$$k_1 = \frac{1}{m}j_1 = \frac{1}{m}(-1.14HP)$$

$$k_2 = \frac{1}{m}j_2 = \frac{1}{m}[-172.05 - 0.07289e^{1.46/(D-H)}H^{0.6+50/(W-B)}L^{.38}B^{1.19}], \qquad (4.15)$$

then from the earlier discussion the limiting velocity of a tow is given by

$$V_f^* = -\frac{(k_1 + K)}{2k_2} = -\frac{(j_1 + J)}{2j_2} \qquad (4.16)$$

where

$K = (k_1^2 - 4k_0k_2)^{\frac{1}{2}}$ and $J = (j_1^2 - 4j_0j_2)^{\frac{1}{2}}$, which is the familiar quadratic formula. Note that the m's cancel out.

Equation (4.16) gives the speed of a tow relative to the water. In order to obtain the speed relative to the land, V_f, an adjustment for stream current is required:

$$V_f = V_f^* - \delta S_w \qquad (4.17)$$

in which

S_w = stream velocity, in miles per hour

$$\delta = \begin{cases} 1 & \text{upstream} \\ -1 & \text{downstream.} \end{cases}$$

Tests of Tow Speed Function. Although the effective-push and drag-force functions estimated by Howe seem to be quite good statistically and although the resistance function seems to be better than any other available, it remains to be shown that the speed function is capable of approximating speeds of tows under actual working conditions. In addition, several simplifying assumptions were made: uniform waterway depth, width, and stream velocity; uniform draft of barges comprising the flotilla; and rectangular configuration of flotilla. These assumptions will not always be realized in practice, and one would like to know whether the formula can be applied without resulting in large errors.

Howe's speed function has been applied to two sets of data. The first is composed of tests performed by the Army Corps of Engineers with a 1500-horsepower towboat pushing tows of 4 and 8 fully integrated and semi-integrated barges.[11] These tests were performed in 1947 on reaches of the lower Illinois and Upper Mississippi. All variables necessary to the application of Howe's formula were carefully recorded by Corps personnel, except for stream velocity. Stream velocity was assumed to be 2 feet per second or 1.36 miles per hour. This assumption was based on discussions with two Corps hydrology experts who independently advanced this figure as a good "approximate average." A better estimate of stream velocity would require obtaining the discharge of the waterway (in cubic feet per second), obtaining an approximation of the cross section of the waterway (in square feet), and dividing the latter into the former. The resulting velocity in feet per second could then be transformed into a mile-per-hour equivalent. While such a calculation is possible, it would add little, for the tests all occurred within a few days of one another over the same stretches of the two waterways involved. These data contained 24 observations, half upstream and half downstream. The ranges of the variables are shown below:

(1) Horsepower: 1500
(2) Length of flotilla: 390 feet and 780 feet
(3) Breadth of flotilla: 70 feet
(4) Draft of flotilla: 7.5 feet
(5) Depth of water: 9.5 feet-30.0 feet
(6) Width of channel: 700 feet-1500 feet
(7) Stream velocity: 1.36 miles per hour (assumed).

As a test of the relationship between actual speeds attained by the tows, V_f^a, and their predicted speeds, V_f^p, a straight line was fit to the data points by the method of least squares. As far as determining the extent of linear correlation between the two sets of speeds, it is immaterial which variable is used as the "dependent" and which the "independent" variable. However, if hypothesis testing and prediction are desired, then there are good statistical reasons for using V_f^a as the independent variable and V_f^p as the dependent variable.[12]

If Equation (4.17) provided a perfect prediction of actual speed and if actual speed were measured without error, the least-squares line would possess a slope coefficient of unity and an intercept coefficient of zero. This statement is strictly true only if all tows used to generate the actual speeds were non-integrated (i.e., all barges comprising the flotillas were double-raked), for this was the condition under which Howe's resistance function was estimated. Since the prototype data are for fully and semi-integrated tows, one would expect a priori a negative intercept, i.e., the prediction equation will consistently underestimate the actually attained speeds.

The estimated relationship is as follows:

$$V_f^p = \underset{(0.87)}{-0.35} + \underset{(0.11)}{1.01}V_f^a, \qquad (4.18)$$

with (1) coefficient of determination = 0.79
(2) standard error of estimate = 0.69
(3) sample size = 24
(4) standard errors shown in parentheses under the estimated coefficients.

Upon testing the hypotheses indicated above, it was found that the slope coefficient was not significantly different from unity (t = 0.09) and that the intercept coefficient was not significantly different from zero (t = 0.40). Both of these t values are so small that the null hypotheses could not be rejected at any conventional level of significance. As was expected on a priori grounds, the prediction equation consistently underestimated the actual values by about 0.35 miles per hour. As an illustration of how the tow-speed prediction equation could be corrected for different tow types, the following relation could be used to estimate the speed of fully and semi-integrated tows:

$$\hat{V}_f = V_f + .35. \tag{4.19}$$

If such a correction were going to be used in practice, it would be desirable to have many more observations over a wider range of variables than the data provided. In addition, one would undoubtedly wish separate adjustments for fully integrated tows as against semi-integrated tows. Nevertheless, this serves as an example of the applicability of the previous analysis to different tow types.

To provide tests of his function, Howe obtained a set of 224 movements gathered from towboat log books of Ohio River operations.[13] The log data gave actual tow characteristics--length, breadth, and draft--which are required for the use of Equation (4.16). Average tow speeds were calculated from the log data because the speed of the tow was not recorded directly in the logs. Port delays and waiting time at locks were excluded from the trip times, but unfortunately, time actually spent in locks could not be excluded.

The range of variables was as follows:

(1) Horsepower of towboat: 1400-3200
(2) Length of flotilla: 350 feet-1180 feet
(3) Breadth of flotilla: 35 feet-135 feet
(4) Draft of flotilla: 1 foot-9.3 feet
(5) Depth of water: 13.2 feet, 14.5 feet, and 18.3 feet (assumed)
(6) Width of channel: 500 feet and 550 feet (assumed)
(7) Stream velocity: 0.61 miles per hour-4.0 miles per hour (assumed).

The values of the last three variables for each of the 224 observations had to be assumed, for there were no data in the logs pertaining to depth, width, and stream velocity of the waterway.[14]

As before, predicted speed was regressed on actual speed with the following results:

$$V_f^p = \underset{(0.27)}{2.12} + \underset{(0.06)}{0.94}V_f^a \qquad (4.20)$$

in which (1) coefficient of determination = 0.49
(2) standard error of estimation = 2.19
(3) sample size = 224
(4) standard errors shown in parentheses under the estimated coefficients.

In this case, as in the other, the slope coefficient is not significantly different from unity (t = 1) for any conventional level of significance. Of course, the intercept is significantly greater than zero (t = 7.85) which was to be expected since locking time was included in the average speed observations.

These two sets of data fail to provide adequate tests of the hypothesis that the intercept term in the regression equations is zero, for in the first case tows were semi- and fully integrated and in the second locking time was included. An ideal test would involve obtaining carefully collected observations (like those of the prototype data above) on operations of non-integrated tows composed of Jumbo hopper barges. Nevertheless, the results of the tests are in the right direction: the intercept being negative for the case in which it was expected the predictions would understate actual speeds and positive for the case in which overstatement was expected. The two sets of data do confirm the unit slope of the fitted line. This confirmation indicates that the tow speed prediction equation, Equation (4.17), forecasts actual speeds without bias.

In summary, the tests performed and reported here seem to provide convincing evidence in favor of the predictive quality of Equation (4.17). Conclusive evidence has by no means been presented, however. Better data would be needed for that. In addition, a method has been demonstrated for adapting the model to fit situations other than those for which it was designated, as illustrated by Equation (4.19).

On the basis of the very good statistical estimation equations derived by Howe for the resistance and tractive-effort functions of barge tows, on the basis of the favorable comparative tests of Howe's resistance function with others, and on the basis of the predictive quality of the speed function as indicated by the previous analysis, there seems sufficient evidence favorable to the continued use of Howe's resistance and tractive-effort functions in the analysis of tow movements.

Acceleration and Deceleration. The acceleration time and distance formulas were presented in Chapter III above and are given in Equations (3.26) and (3.27). Using the definitions of k_0, k_1, k_2, and K given in (4.15) and an appropriate value of ζ, these formulas may be applied to barge tows. Since tows typically operate at full throttle,[15] accelerations from velocities other than zero to velocities other than V_f were not considered.

The method of obtaining the factor ζ was as follows: Equation (3.25), as applied to barge tows, was plotted for three sets of tow-waterway characteristics. These graphs are shown in Figures 4.1, 4.2, and 4.3. A velocity of $0.99V_f$ seemed to include most of the time required for acceleration while not including a large amount of time on the asymptotic portion of the acceleration curve.

Examples of acceleration times and distances as predicted by Equations (3.26) and (3.27) are given in Table 4.3 for five different flotilla combinations. In all five cases the waterway characteristics are assumed to be the same, viz., width equal to 225 feet and depth equal to 12 feet. These values could easily be changed; the particular values presumably typify the Illinois Waterway.[16] Horsepower of the towboat was assumed to be 3000 and flotilla draft was assumed to be 8.5 feet. These also could be varied. Five flotilla sizes were assumed: (1) a four-barge flotilla, arranged two barges long and two barges wide (2 x 2); (2) a nine-barge flotilla, arranged three barges long and three barges wide (3 x 3); (3) a twelve-barge flotilla, arranged four barges long and three barges wide (4 x 3); (4) a fifteen-barge flotilla, arranged five barges long and three

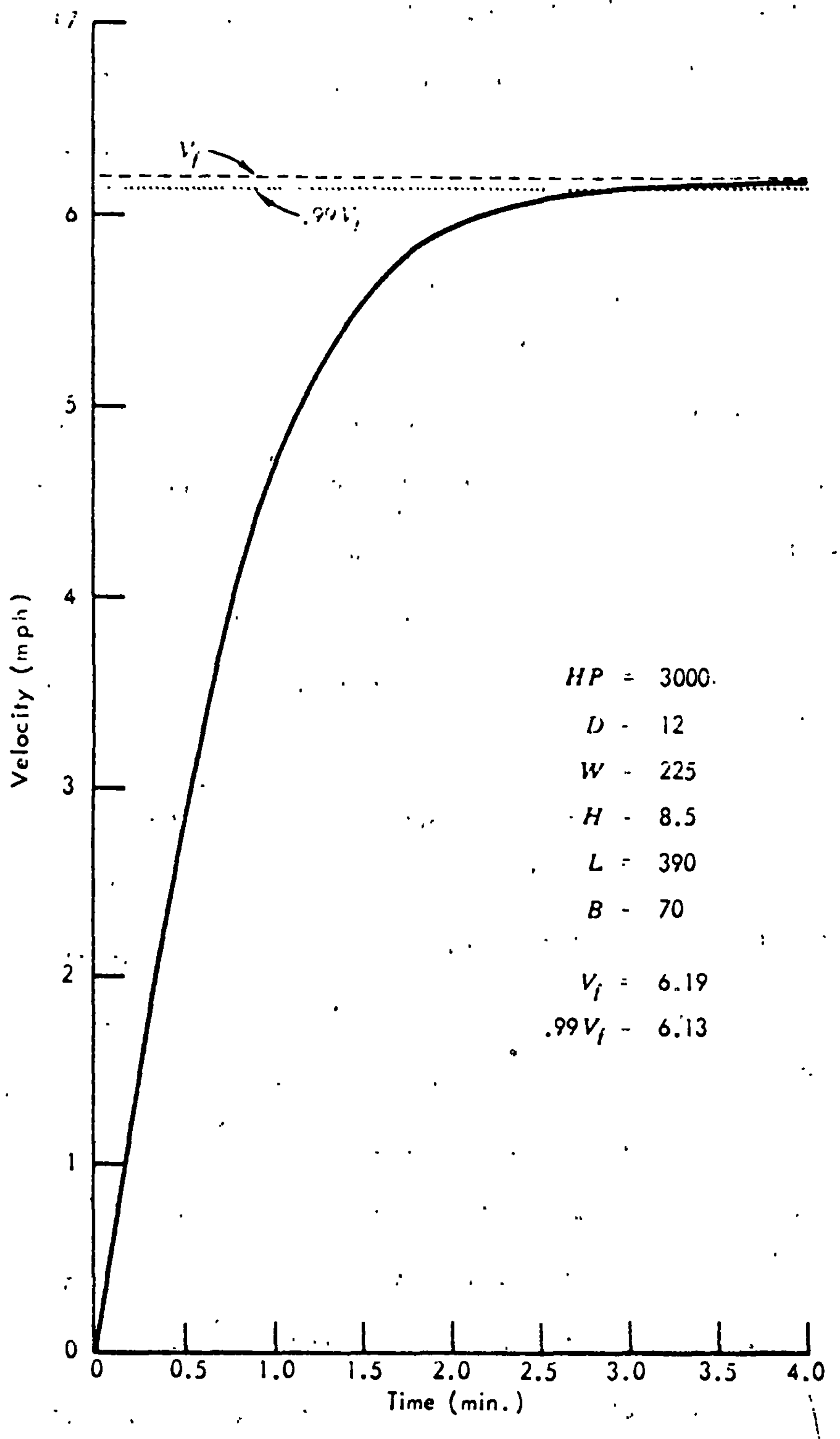

Fig. 4.1 -- Acceleration-Speed-Time Curve for Barge Tow (3000-Horsepower Towboat, 4 Jumbo Open-Hopper Barges)

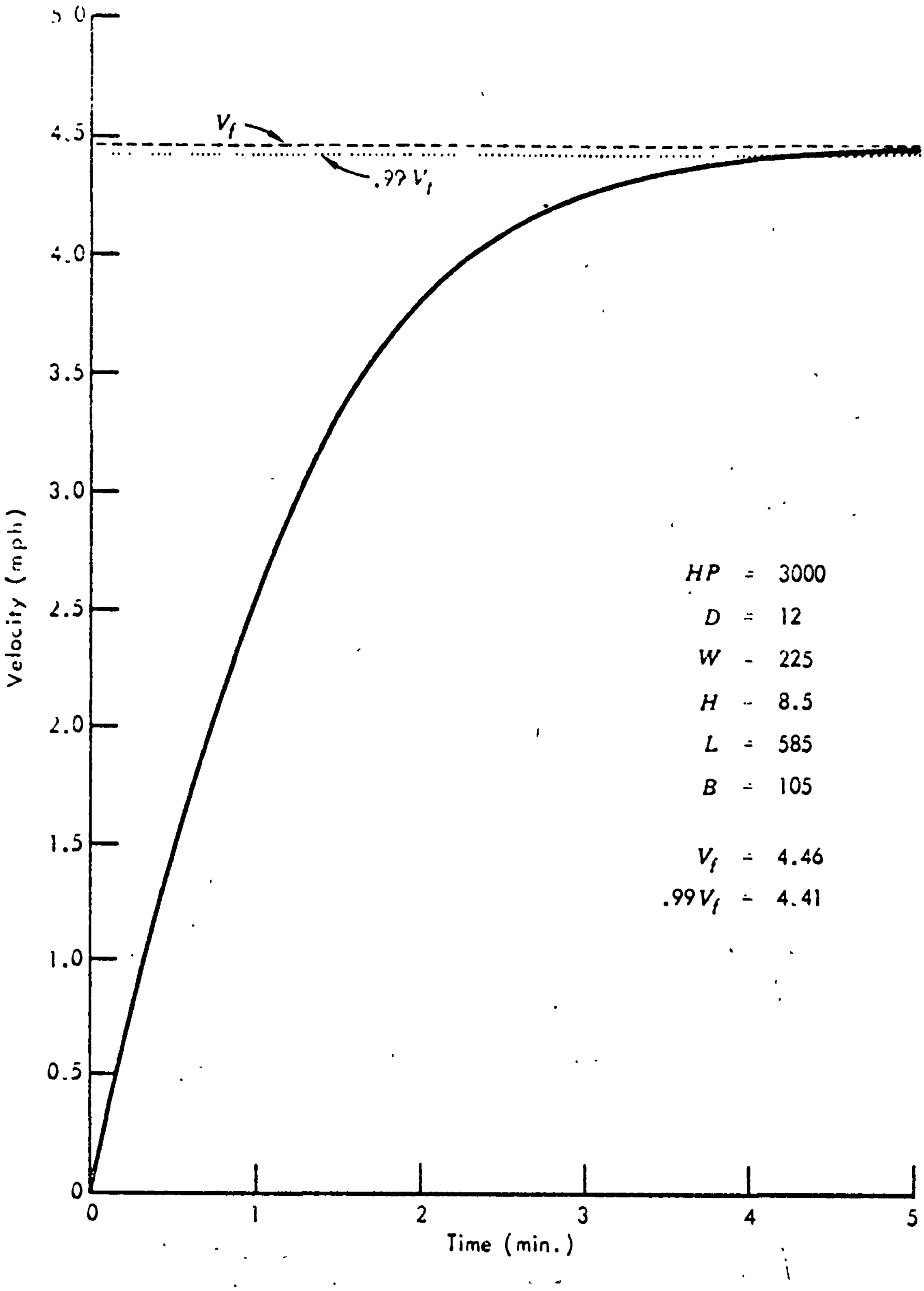

Fig. 4.2 -- Acceleration-Speed-Time Curve for Barge Tow (3000-Horsepower Towboat, 9 Jumbo Open-Hopper Barges)

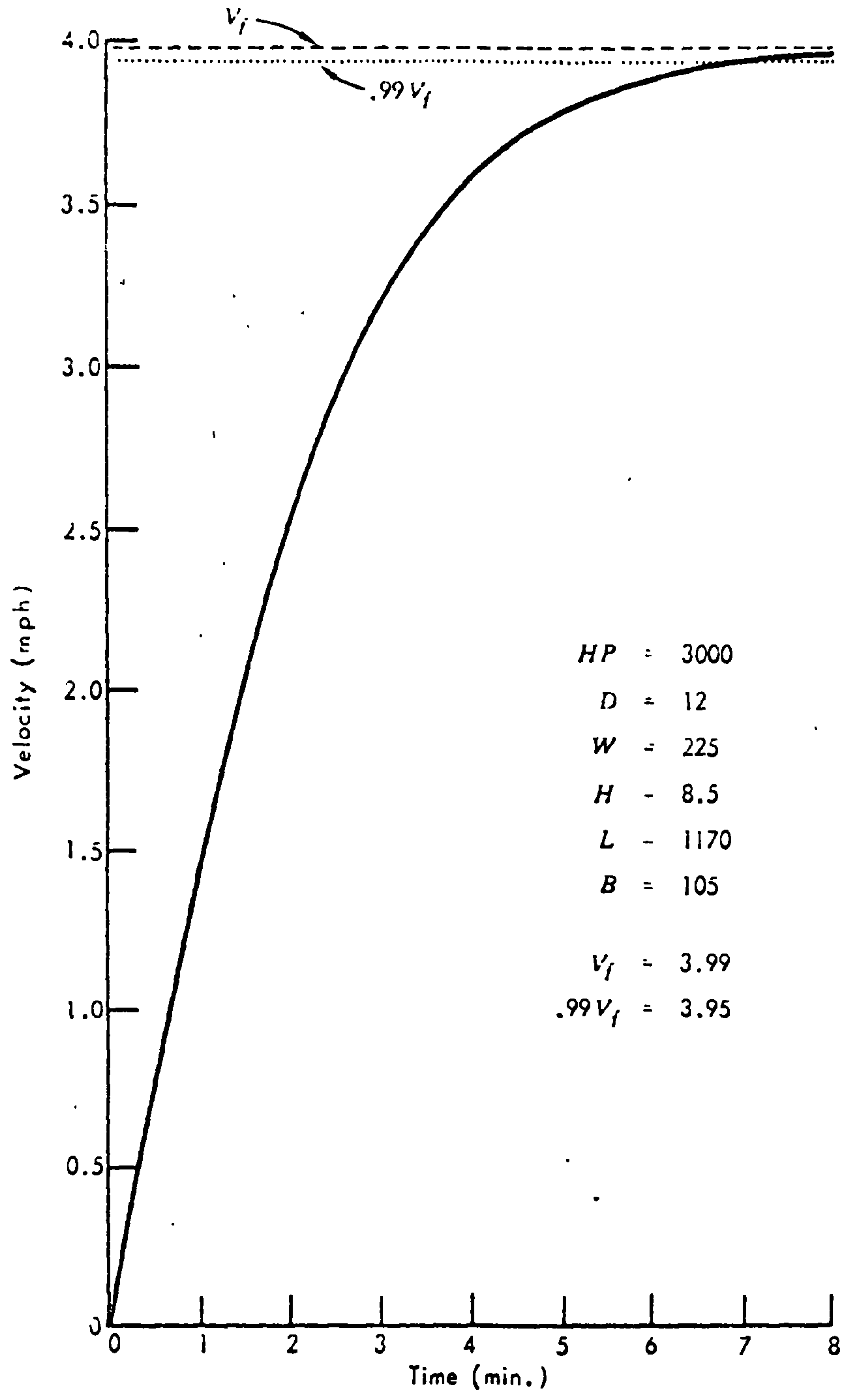

Fig. 4.3 -- Acceleration-Speed-Time Curve for Barge Tow (3000-Horsepower Towboat, 18 Jumbo Hopper Barges)

barges wide (5 x 3); and (5) an eighteen-barge flotilla, arranged six barges long and three barges wide (6 x 3). The configurations correspond to typical tows operating on the Illinois Waterway.

Table 4.3

ACCELERATION TIMES AND DISTANCES FOR BARGE TOWS

Flotilla Configuration (no. barges long by no. barges wide)	Acceleration Time (min.)	Acceleration Distance (ft.)
2 x 2	2.75	1,127
3 x 3	4.15	1,218
4 x 3	5.23	1,464
5 x 3	6.26	1,690
6 x 3	7.25	1,901

The figures in Table 4.3 seem to correspond well with the industry rule of thumb that a tow can accelerate from a stop to full speed in 1½-2 times the length of the tow.[17] Moreover, and most importantly, the figures indicate the relative insignificance of acceleration time and distance per acceleration. Since the frequency of stops en route is not likely to be large, as will be shown below, the error introduced by ignoring acceleration altogether is slight. Moreover, since tows typically decelerate by reversing towboat engines at full throttle, deceleration time and distance will be even less than acceleration time and distance; the rule of thumb in this case is that it requires 1-1½ times a tow length to bring the tow to a stop from full speed.[18] The insignificance of acceleration time and distance, as discussed above, applies _a fortiori_ to the case of deceleration of barge tows.

A final note concerning the effect of stream velocity on predicted acceleration and deceleration times and distances is in order. If a constant velocity of water flow, S_w, is assumed, a tow beginning to accelerate upstream will at time t = 0 be operating at a velocity equal to the negative of the stream current, $-S_w$. If the tow's full speed in still water is V_f^*, then its attainable speed for the upstream movement is $V_f^* - S_w$. If it is assumed that no changes occur in the resistance and push functions and that stream current only affects the tow's speed additively, then the tow will require the same time to accelerate from $v = -S_w$ to $v = V_f^* - S_w$ as it will to accelerate from $v = 0$ to $v = V_f^*$. The same reasoning applies _mutatis mutandis_ for downstream acceleration, and similar arguments for both situations may be made for the case of deceleration. Therefore, stream current may be ignored in considering time required and distance traveled in acceleration and deceleration of barge tows.

Note on Calculation of Mass. For the computation of acceleration time, the mass of the tow is required. Mass is defined as weight divided by gravity: $m = w/g$. The quantity g is usually approximated as 32 feet/second^2. Since the tow speed is in units of miles per hour (mph), the units of g must be altered in order to achieve the requisite dimensionality of acceleration time. Specifically, if these times are to be in minutes (min.), then g = 1309 mph/min.; in hours (hr.), g = 78,540 mph/hr.

The other component of mass is weight. The gross weight of the entire tow is required. The cargo weight of a barge is a function of its draft, and capacity tables exist from which one can determine the cargo weight for given drafts (as was noted above) as well as the tare weight. Using the capacity table for a 195-foot by 35-foot open-hopper barge, it was found that the following quadratic function of draft gave an excellent representation (coefficient of determination was 0.99) of the gross weight of the barge:

$$W_b = -15{,}830 + \underset{(125)}{380{,}060H} + \underset{(100)}{2{,}210H^2}, \qquad (4.21)$$

in which H is draft, in feet, and W_b is gross barge weight, in pounds. The figures in parentheses are standard errors. Sample size was 109. The weight of a flotilla consisting of N_b uniformly loaded barges, then, is N_bW_b.

Howe has developed a similar relationship for the gross weight of a towboat.[19] It is

$$W_t = 24{,}300 + 350HP - 0.021HP^2, \qquad (4.22)$$

in which W_t is the gross towboat weight, in pounds, and HP is towboat rated brake horsepower. While the coefficient of determination and standard errors were not reported for this estimation, it probably gives close enough towboat weights for the computation of the mass.

Delay Analysis of Tows

Introduction. Many delays occur in the movement of commodities by barge. In addition there is some increase in travel time each time the tow stops because of the attendant acceleration and deceleration. It is desired to provide analyses of tow delays in order better to predict the time required for travel by barge between origin and destination.

In subsequent usage delay will mean any period of time during which the tow is not moving. Such delays result from a variety of factors. One classification of delays and the relative importance of them is given in Table 4.4

Table 4.4

PERCENTAGE OF ACTIVE TIME ACCOUNTED FOR BY VARIOUS DELAYS FOR NINETEEN TOWBOATS, 1960-64

Delay Category	Percentage of Active Time
1. Locking: waiting for and being serviced by a lock	7.29
2. Weather: unnavigable weather conditions such as fog, ice, and snow	2.35
3. Repair: Minor repairs (major overhauls not included)	3.94
4. Supplies: taking on supplies	0.65
5. Waiting barges: Frequently barges are transferred from one towboat to another. When one of the boats arrives late, the other is delayed	1.87
6. Load and unload: towboat waiting while barges are loaded and unloaded	0.75
7. Bridges: waiting for lift bridges to operate	0.12
8. Running aground	0.24
9. Make and break tow: assembly of tow at beginning of trip and disassembly at end	3.16
10. Other: includes waiting for orders, waiting for arrival of crew, and waiting for clear channel	3.91

SOURCE: Common carrier barge firm which requested not to be identified.

The data for Table 4.4 were obtained from a common carrier barge firm which operated on the Lower Mississippi, the Illinois, and the Ohio. Active time, which is used for the base in the percentages, includes all delays and running time but excludes any time during which the towboat was undergoing major repairs or for which it was tied up for long periods. Although loading and unloading time is quite small relative to other delays for these data, this delay factor could be large for contract carriers whose boats are often required to wait during loading and unloading. In any case, the subsequent analysis will not consider this terminal delay. If an analysis of loading and unloading were available, however, it could be included in the tow process function as an additional T in Equation (4.1). Below are analyzed locking delay, T_L, making and breaking tow, T_m and T_b, and miscellaneous delays, T_o. In addition, the frequency of miscellaneous delays will be examined.

<u>Locking</u>. Instrumental in analyzing the benefits and costs of a waterway investment is the determination of the waterway capacity and the delay involved in locking. In addition, locking time is an important component of the tow process function. Therefore, a method is desired for predicting the waiting time in queue and in lock for a tow operating on a waterway with several locks. The model, to be discussed in this subsection, should be sensitive to changes in total tonnage carried on the waterway (i.e., demand for transport), changes in lock capacity, and changes in flotilla characteristics. Waterway capacity considerations will be taken up in the next subsection.

Given the tonnage, O, to be carried on the waterway and the tonnage, $\overline{C}$, of an optimal or average tow, one can determine the number of trips required to achieve O, viz.,

$$\lambda = O/\overline{C}, \tag{4.23}$$

where λ is the number of trips per time period. A trip is defined as any point-to-point movement. While λ is the number of trips required to produce a waterway tonnage of O, there may not be a one-to-one relationship between λ and the number of trips by tows operating on the waterway because of the possibility of trips involving empty tows. This consideration may be accounted for by adjusting λ by the factor $(1 + p)$, where p represents the percentage of unproductive trips (i.e., with unloaded barges) which requires an empty barge backhaul. Let λ^* be the adjusted trip rate:

$$\lambda^* = (1 + p)\lambda, \tag{4.24}$$

where $0 \leq p \leq 1$.

It is assumed that the possibility of a tow's being at any given point on the waterway is independent of the time since the last tow was there. This implies a Poisson distribution for the arrival of tows; that is, the probability that n arrivals occur within an interval of time of duration t is given by

$$P(t) = \frac{(\lambda^* t)^n e^{-\lambda^* t}}{n!} \tag{4.25}$$

This probability distribution corresponds to completely random arrivals. The mean of the distribution is λ^* and is identified with the λ^* discussed above. In queuing theory Equation (4.25) is called an arrival distribution and is an important input to a model capable of predicting waiting time.[20]

Suppose a lock exists through which all tows must pass in order to complete a trip. The number of tows that are handled by the lock in a given period of time is called the service rate of the lock. Several operations are included in the "service" of a tow through a lock: (1) preparation of the lock, (2) rearrangement of the tow for locking, (3) entrance of tow, (4) locking, (5) rearrangement of tows for departure, and (6) departure of tow. Since each of these operations is likely to be stochastic in its duration, it is plausible to treat the service rate of the lock as a random variable. Specifically, it is assumed that the service rate has a Poisson distribution whose mean, the average service rate, is denoted by μ. Such a distribution implies that the probability of prolongation of service is independent of how long ago the service began. Many service operations exhibit a Poisson distribution, e.g., telephone conversations, grocery checkout facilities, various repair operations, etc.[21] In addition, use of a Poisson service rate coupled with a Poisson arrival rate permits the resulting queuing model to be relatively simple. Furthermore, it will be possible to judge the quality of the resulting model to see if undue error is caused by these assumptions.

It seems likely that μ, the average service rate, can be closely approximated for a proposed lock by experience with locks already in existence. For example, the locks on the Illinois River have been intensively studied, and their service rates are likely to apply to similar newly-built locks on other waterways. A complicating factor here concerns single versus multiple locking procedures. It is typical, for example, on the Illinois River for tows to be composed of 16 barges. Since the locks are 600 feet by 110 feet, the tow is locked in two passes, the first consisting of 9 barges, the second of 7 barges and the towboat. It would be desirable therefore, to know the service rate of the lock for each type of locking procedure, i.e., single, double, or triple (though the latter is not common practice at this time). One could then evaluate different locking procedures. This sort of information should not be very difficult to obtain. In any case, μ is taken as given and no analytical procedure as to how it might be estimated is presented here.

Given λ^* and μ and assuming Poisson distributions for the arrival rate and service rate, queuing theory may be used to derive an estimate of the expected waiting time in queue and in service for a tow at a lock. In particular, the relevant formulas are [22]

$$T_L = \frac{1}{\mu - \lambda^*} \qquad (4.26)$$

and

$$T_{Lq} = \frac{\lambda^*}{\mu(\mu - \lambda^*)}, \qquad (4.27)$$

where T_L is the expected total locking time and T_{Lq} is the expected waiting time in queue. The expected service time is given by the reciprocal of μ. So far the analysis is applicable only to waterways with a single lock. The next step is to extend the model to permit estimation of T_L and T_{Lq} for a tow traversing a multiple-lock waterway.

It has been shown[23] that, for the arrival and service distributions assumed above, efflux from a service facility has the same distribution and mean as the arrival rate. It must also be assumed that the mean service rate of the facility is greater than the mean arrival rate. This last proviso is not very restrictive since a service facility must possess a mean service rate greater than the mean arrival rate or such long queues will develop that the facility will not be able to handle its customers. Therefore all locks on the waterway will have the same arrival rate, λ^*.

Locks may be considered independently if one is willing to assume the possibility of infinite queues between locks. This, however, is not the drastic assumption it may appear. The case in which only finite queues are permitted between locks approaches the case of infinite queues rather rapidly. It has been shown, for example, that when a maximum queue length of 19 units is permitted between the first and second stage of a two-stage sequential system, the situation is little different from one which allows infinite queues between this pair of facilities.[24] However, as the number of sequential stages in the system increases, the maximum number of units permitted between stages must increase also in order for the infinite-queue assumption to be justified. Since the distances between locks are large relative to the size of the tows--the smallest distance between adjacent locks on the Illinois is 5 miles, the average being 35 miles--the assumption of infinite queues between locks seems reasonable.

If all locks on the waterway are treated as independent of one another, then the average waiting and service time for a tow would be given by

$$T_L = \sum_{i=1}^{r} \frac{1}{\mu_i - \lambda^*} , \qquad (4.28)$$

where μ_i represents the mean service rate of the ith lock and there are r locks. For the case in which all locks have the same mean service rate, the formula becomes

$$T_L = \frac{r}{\mu - \lambda^*} . \qquad (4.29)$$

To provide examples of this model, the conditions obtaining on the Illinois waterway in the years 1949 and 1950 are used. (More recent data were, unfortunately, not available when this analysis was performed.) Locking operations were studied intensively for these years as part of an analysis of the economic feasibility of constructing larger locks. Five of the seven locks on the waterway were studied in enough detail to yield information needed for the following examples.[25]

For 1949 the average loaded tow size seems to have been about five barges each carrying 1,000 tons of cargo. These figures give a $\bar{C}$ of 5,000. The average tonnage through the locks for 1949 was 8,164,869. Therefore, 0 is approximated as 8,000,000. Then $\lambda = 0/\bar{C}$ is 1,600 trips per year. This is an estimate of the number of loaded tows that passed through the waterway in 1949. On the basis of the ratio of empty barges to loaded barges, p is estimated to be 0.90, i.e., 90 percent of all loaded tows make a return trip empty. This statement is not strictly true, for undoubtedly many tows contained both loaded and empty barges. This does not change the analysis, however; and it is easier to speak of empty and full tows rather than tows some of whose barges may be full and others empty. Therefore, λ^* is approximately 3,000 tows per year or 0.34 tows per hour.

The average service times per tow for each of the five locks are available from the Corps study. Thus the waiting times, T_L and T_{Lq} may be calculated. These quantities are presented in Table 4.5 along with the actual values of these variables for 1949.

Table 4.5

AVERAGE LOCKING TIME PER TOW FOR FIVE LOCKS ON THE ILLINOIS WATERWAY, 1949

Lock	Predicted Total Waiting Time (min.)	Reported Total Waiting Time (min.)	Predicted Time in Queue (min.)	Reported Time in Queue (min.)
Lockport	75.9	68.4	22.8	15.3
Brandon Road	89.5	75.8	30.2	16.5
Dresden Is.	57.7	55.5	14.3	12.1
Marseilles	70.6	64.2	20.1	13.7
Starved Rock	47.6	50.1	10.1	12.6

SOURCE: Interim Survey Report: Duplicate Locks, Illinois Waterway.

For 1950 the average tow size seemed about the same as for 1949, but the average tonnage per loaded barge was larger at about 1,200. Therefore, $\bar{C}$ is 5,000 for 1950. Total tonnage on the waterway was also larger in 1950, averaging 10,457,127 through the five locks. Therefore, O is approximated as 10,500,000 tons, and λ becomes 1,750 trips per year. It appears that empty hauls were 85 percent of loaded ones in 1950; therefore p is approximated by 0.85. Adjusting for p yields λ^* = 3,278, which is approximately 0.38 tows per hour. The results of the model are shown in Table 4.6 compared with the actual figures for 1950.

Table 4.6

AVERAGE LOCKING TIME PER TOW FOR FIVE LOCKS ON THE ILLINOIS WATERWAY, 1950

Lock	Predicted Total Waiting Time (min.)	Reported Total Waiting Time (min.)	Predicted Time in Queue (min.)	Reported Time in Queue (min.)
Lockport	80.0	78.4	26.9	25.3
Brandon Road	95.2	82.9	35.9	23.6
Dresden Is.	60.0	59.9	16.6	16.5
Marseilles	74.1	68.2	23.6	17.7
Starved Rock	49.2	54.2	11.7	16.7

SOURCE: Interim Survey Report: Duplicate Locks, Illinois Waterway.

These two examples should be viewed as no more than a first application of the techniques. It was not intended to develop a model of locking on the Illinois Waterway in 1949 and 1950. If that was the goal, one should have data on the exact number of lockages that occurred each year in order to derive the arrival rate. Then total waterway tonnages and average tow sizes would not have been needed. The examples are intended to show the workability of the model and that even approximations of tonnage and tow size would give results fairly close to those actually reported.

This model provides good predictions of the locking time of tows. Furthermore, it can be used to predict the consequences of changes in decision variables. In particular, changes in total tonnage carried on the waterway will be reflected in the arrival rate of tows and thereby in the locking time involved. Changes in tow size will be reflected in the arrival rate also. Finally, changes in lock sizes will be reflected in the service rate, μ.

Note on Capacity of a Waterway and Optimal Tolls. The model of locking employed above might be used to determine the capacity of a waterway and to measure the congestion which ensues from having an additional tow on the waterway.

As noted above, locking consists of the operations of (1) waiting for permission to approach the lock, (2) approaching the lock and maneuvering into the chamber, (3) filling or emptying the lock, (4) multiple locking if necessary, and (5) leaving the lock and maneuvering out into the channel. Operations (1) and (3) will be independent of the number of barges in the tow, but the other operations will be directly related to the number of barges. One might show this relationship, $T_L(N_b)$, as in Figure 4.4 where T_L is the total service time (assuming a constant amount of time is spent in queue) and N_b is the number of barges in the tow. Note that there are sharp discontinuities when double or triple locking is necessary, for part of the tow must be tied off, the lock must go through one cycle empty, the rest of the tow must be pulled into or out of the lock from the shore, and the tow must be reassembled.

One measure of the capacity of a waterway would be the maximum feasible number of barges that could be locked through in a given period of time. This idea was explored earlier by Bottoms.[26] Since

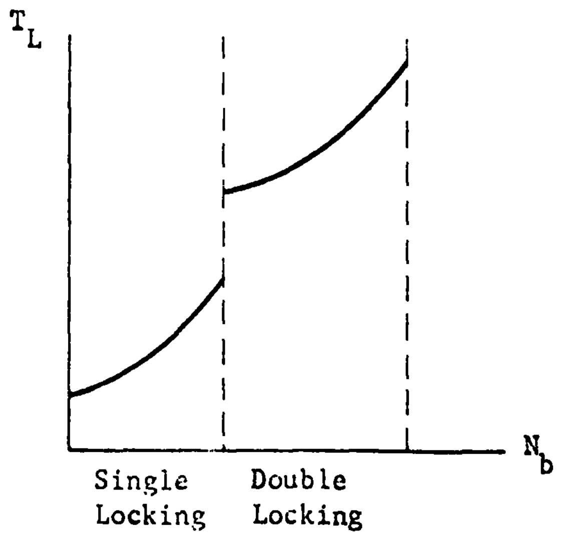

Fig. 4.4 -- Locking time

$T_L(N_b)$ is the time, in hours, it takes to service a tow of N_b barges, the number of tows that could be locked through in a year, N_T, is

$$N_T = \frac{8760}{T_L(N_b)} \, , \tag{4.30}$$

where 8,760 is the number of hours in a year, and it is assumed all tows have N_b barges. The number of barges that could be serviced in a year, N_B, under the same assumptions, is given in Equation (4.31)

$$N_B = N_T N_b \; . \tag{4.31}$$

The most efficient tow size, from the standpoint of the lock, is probably that involving no more than a single locking.

While the above N_B is one measure of the capacity of a waterway, it is a capacity that would never be attained in practice. Tows are not uniform; therefore, capacity would depend on the distribution of tow sizes experienced in practice. Secondly, and more important, the above model assumes that there is always a tow ready to be serviced when the previous lockage is completed. In practice, the capacity of a waterway is determined more directly by the time which tows must wait to be serviced than by the physical capacity of the lock.

Suppose that a total tonnage of 0 per year must be moved over a particular waterway. Suppose further that the tonnage, $\overline{C}$, of an average tow is given. Then $\lambda = 0/\overline{C}$ full tows must traverse the waterway (and each lock) per year. Let p be a factor measuring the amount of empty backhaul; p is defined by the relation $(1 + p)\lambda = \lambda^*$, where λ^* is the total number of tows traversing the waterway per year. This rate may be put on a per-hour basis by $\lambda^*/8,760$. Table 4.7 shows the relationship between λ^*, the percentage utilization of the lock (assuming a service time of one hour), and total locking time T_L. (The remaining entries in the table will be explained below.) Note that T_L does not begin to rise rapidly until the lock is utilized more than 50 percent of the time and it rises very rapidly for a utilization greater than 80 percent.

For any given λ^*, adding another barge tow to the waterway would result in higher utilization of the lock and greater total waiting time. The amount of added queuing time is spread across all existing tows, so the increase in T_L is quite small. Nonetheless, the addition of one extra tow can give rise to a substantial increase in queuing time in total.

Table 4.7

INDIVIDUAL AND SYSTEM EFFECTS ON TOWS DUE TO INCREASED ARRIVAL RATES AT LOCK

λ^*	u	T_L	τ	$M\tau(\lambda^*)$	$M\tau'(\lambda^*)$	$-M\tau(\mu)$
1	.01	1.00	1	1.00	0.00	1
100	1.14	1.01	101	1.02	0.01	102
1000	11.4	1.13	1129	1.28	0.15	1280
2000	22.9	1.30	2592	1.69	0.39	3380
2190	25.0	1.33	2920	1.77	0.44	3876
3000	34.3	1.52	4563	2.31	0.79	6930
4000	46.7	1.84	7361	3.39	1.55	13560
5000	57.1	2.33	11648	5.43	3.10	27150
6000	68.5	3.18	19043	10.11	6.93	60660
7000	80.0	4.98	34841	24.80	19.82	173600
8000	91.4	10.52	92210	132.76	121.19	1061680
8750	99.9	876.00	766500	767376.00	766500.00	6714548000

μ = average number of tows serviced per hour by the lock, assumed equal to unity

λ^* = number of tows locked per year

u = utilization rate of lock: $u = \lambda^*/8760\mu$

T_L = average locking time (including waiting time), in hours
$T_L = 8760/(8760\mu - \lambda^*)$

τ = total locking time per year for all tows, in hours
$\tau = \lambda^* T_L = 8760\lambda^*/(8760\mu - \lambda^*)$

$M\tau(\lambda^*)$ = marginal locking time due to the addition of a tow
$M\tau(\lambda^*) = [8760/(8760\mu - \lambda^*)]^2\mu = T_L{}^2\mu$

$M\tau(\mu)$ = marginal time lost due to an increase in the service rate
$M\tau(\mu) = -[8760/(8760\mu - \lambda^*)]^2\lambda^* = T_L{}^2\lambda^*$

$M\tau' = M\tau(\lambda^*) - T_L$.

Also given in Table 4.7 is $\tau = \lambda^* T_L$, the total waiting time for all tows during the year. The increase in total waiting time due to an extra tow is, therefore,

$$d\tau / d\lambda^* = M\tau(\lambda^*) = \left(\frac{8760}{8760\mu - \lambda^*} \right)^2 \cdot a. \qquad (4.32)$$

This figure is shown in the table as $M\tau(\lambda^*)$. Note that $M\tau(\lambda^*)$ rises very rapidly as the percentage of utilization rises.

From the above discussion, it is clear that an additional tow imposes a cost (in terms of additional waiting time) of $M\tau(\lambda^*)$ on all existing operations. Even one tow increases congestion. In practice, an additional tow does experience some part of the cost of this congestion, for T_L increases. However, the cost to all tows is considerably greater than the cost borne by the individual tow. The social or system cost of an additional tow is $M\tau(\lambda^*)$; the private cost (that borne by the extra tow) is T_L. The difference between social and private cost is

$$M\tau(\lambda^*) - T_L = M\tau'(\lambda^*). \qquad (4.33)$$

On a waterway at any given time, congestion would be decreased, and the savings to the whole system would be $M\tau'(\lambda^*)$, if a single tow were persuaded to exit. If it costs C_T dollars per hour to operate a tow, then the system would be willing to pay $C_T M\tau'(\lambda^*)$ to any tow if it would leave the system. Alternatively, each tow should be charged a like amount if it insists on staying. Thus, each tow on the waterway is costing all other users $M\tau'(\lambda^*)$ hours, and rational allocation of resources requires that a tow not be operated unless it is generating $C_T M\tau'(\lambda^*)$ in profit.

Note that the above formulation gives T_L and τ as functions of λ^*, p, and lock capacity. For a given lock capacity, one could determine how much equipment would be required to move a given number of bargeloads per year. Note further that this analysis provides a way of optimizing lock capacity. $M\tau(\lambda^*)$ is the increment to total waiting time from the last tow. One can define $M\tau(\mu) = d\tau/d\mu$ which will measure the change in total waiting time as the service time is changed. This is

$$M\tau(\mu) = -\left(\frac{8760}{8760\mu - \lambda^*}\right)^2 \lambda^*. \qquad (4.34)$$

One would then expand capacity until the cost to tows of waiting an additional hour equals the cost of expanding the lock to save that hour.

An example will demonstrate how to determine when a lock should be expanded on the basis of the queuing model and its extensions developed above. Assume it costs $17,000,000 to build new locks twice as large as the old locks. (The figure $17,000,000 is the average estimated cost of building second locks of twice the capacity of the existing seven locks on the Illinois Waterway.)[27] It is assumed that doubling the size of the lock doubles the service rate. If double locking is the practice on the waterway before the change, the increased service rate may in fact be achieved by doubling the size of the locks. If some other situation prevails, it may not be possible to increase the service rate so dramatically. The principle developed below will be the same, however. Next assume that the annual cost of the lock is \$656,000. (Again, this is the average cost of the seven proposed Illinois Waterway locks.)[28]

If there is a single lock on the waterway and the cost per hour of tows is \$100,[29] then the lock should be improved if at least 6,500 tow-hours can be saved per year. Such a saving can be obtained if $\lambda^* = 5{,}000$. For this number of tows operating on the waterway, τ, the total locking time, will be 11,650 at the old service rate $\mu = 1$ per hour. At the new service rate $\mu = 2$, τ will be 3,450, a saving of 8,200 tow-hours which exceeds the amount required to justify the lock. Note that this example implies that a lock should be expanded, given the above assumptions, when that lock's utilization is approximately 57 percent.

Another extension of the above analysis is the determination of the optimal toll for the use of the lock. In general, the optimal toll per tow is the additional cost imposed on all other tows, i.e., $M\tau'(\lambda^*) \cdot C_T$, where C_T is the per-hour cost of a tow and $M\tau'(\lambda^*)$ is defined above. For the previous example $M\tau'(\lambda^*) = 0.27$, after lock expansion, $C_T = \$100$, so the toll per tow should be \$27. Before improvement of the lock $M\tau'(\lambda^*) = 3.10$, so the optimal toll should have been \$310 per tow.

The government should incur the cost of lock expansion (according to the above criterion) only if barge traffic is behaving in accordance with the toll scheme outlined above. Given the number of tows on the waterway (all of which are paying $M\tau'(\lambda^*)\cdot C_T$ or could pay it), the waterway ought to be expanded according to the rule developed above, i.e., expand lock capacity until the cost to tows of waiting an additional hour equals the cost of expanding the lock to save that hour.

The discussion has been in terms of a single lock only. However, as was shown above, under reasonable assumptions, all locks on a waterway may be treated independently. Therefore, if tows arrive completely independently of each other, and if locks are sufficiently far enough apart, at least several times the length of a tow, then the above analysis may be applied individually to each lock on a multiple-lock waterway.

A caveat should be issued at this point. In the preceding discussion, the term optimal toll was used to designate a toll that, if imposed, would equate private and social costs. The idea of using a toll (or, for that matter any kind of tax or subsidy) for this purpose is, of course, well known in the literature of welfare economics.[30] In the above case, the external diseconomy imposed on all other tows by a single tow's operations is "internalized" by means of the toll. However, the adjective optimal may be misleading. If it were true that all the marginal conditions for achieving a Pareto optimum would be satisfied, except for the existence of an external diseconomy such as the one described above, then the proposed toll scheme would indeed be optimal.[31] If, on the other hand, it were not true or one were not willing to assume that the marginal conditions would be satisfied, then the toll scheme above would not achieve a Pareto optimum and, in that sense, would not be optimal.

Making and Breaking Tow. At the beginning and end of each trip, the flotilla of barges must be assembled and disassembled. As Table 4.4 shows, these processes--known as making and breaking tow--comprised 3.19 percent of the active time over a five-year period for all boats of the barge firm which supplied the data. This is the fourth largest delay-causing factor in Table 4.4, and it is the second largest of non-miscellaneous delays. It is desired, therefore, to present some analysis of this major delay category so that one might be able to predict the time required to assemble and disassemble tows.

Data on making and breaking tows were collected from two towboat logs which were supplied by a barging firm. A towboat log reports in detail on a year's operations of a towboat; in particular, delay times and causes are given. It was possible to collect a large number of making and breaking times with the attendant number of barges involved.

In the case of making tow, 242 observations were obtained. The average time required to make tow was 1.7 hours; the number of barges included in the sample tows ranged from 1 to 24, and the average tow size was 3.4 barges. In the cases of breaking tow, 247 observations were obtained. The average time to break tow was about 1 hour; the number of barges involved ranged from 1 to 16, and the average tow size was 3.1 barges. The small average tow size is due to the fact that the observations contain many additions to or subtractions from tows of only one or a few barges at a time. These "drops" and "adds" are counted because they occur frequently and the time required should not differ importantly from that required in making or breaking tows of only one or a few barges. An examination of the data showed: (1) on the average, making tow required considerably more time than breaking tow and (2) both making and breaking tow were affected by the number of barges involved in the process. Both of these points seemed plausible a priori. The possible reason for (1) is that in assembling the barges the towboat captain attempts to achieve a flotilla configuration that reduces resistance of the tow. On the other hand, when the destination is reached, the flotilla may be disassembled in any order. Point (2) seems even more obvious. Since each barge must be collected and tied securely either to the towboat or to other barges, the time required to do this is clearly a function of the number of the barges comprising the tow. In the case of breaking tow, the barges may be let off individually or in two's at the terminal. Again, the time required for this procedure would seem to be a function of the number of barges.

With these ideas in mind, two linear regressions were performed: (1) making-tow time regressed on number of barges added to the tow and (2) breaking-tow time regressed on number of barges dropped from the tow. The results are,

where T_m = time required to make tow

T_b = time required to break tow

N_b = number of barges added or removed

$$T_m = \underset{(0.061)}{0.21} + \underset{(0.015)}{0.44}N_b, \qquad (4.35)$$

with (1) coefficient of determination = 0.78

(2) standard error of estimate = 0.97

(3) F statistic = 848;

$$T_b = \underset{(0.029)}{0.34} + \underset{(0.010)}{0.20}N_b, \qquad (4.36)$$

with (1) coefficient of determination = 0.63

(2) standard error of estimate = 0.53

(3) F statistic = 411.

Both regressions are highly significant, which means that the estimated time to assemble or disassemble a flotilla consisting of a given number of barges obtained from Equations (4.35) and (4.36) is a better prediction than using the average sample values. In addition, the linearity of the estimating equations was corroborated by log-linear regressions not reported here.

<u>Miscellaneous Delays</u>. Estimation procedures for two of the most important delay causes have been developed above. It is conceivable that similar approaches could be used in analyzing each of the remaining delay categories given in Table 4.4. However, this procedure would be impossible given the data availability. A simpler approach and one amenable to the available data is, therefore, desired.

To simplify the analysis all delay-causing factors other than locking, load-unload, and make-break tow are aggregated into a single category to be called "miscellaneous delays." On the basis of the delay categorization of Table 4.4, miscellaneous delays include all delays due to weather, repair, supplies, waiting barges, waiting bridges, running aground, and other. These delays were approximately 13 percent of active time for the total of 59 boat-year observations summarized in Table 4.4. Note that for any given trip, the expected locking time and the expected assembly and disassembly time for the tow may be predicted using the analyses reported above. Now, it is desired to know how much additional time might be expected because of all other delays en route.

Two types of data are available for estimating miscellaneous delay time:

1. Point-to-point data: These data are derived from logs of two towboats for the year 1964. The logs give information on the number of stops made each day, the duration of each stop, where each stop occurred, and, in most cases, the cause of the stop.
2. Aggregate delay data: These data consist of summary sheets which show the total delay time for each of the ten delay categories discussed above. In addition, the total active time for each boat per year is given. Observations are available for 19 towboats across 5 years, giving 59 towboat-year combinations in all (not all towboats operated each year). Table 4.4

is based on these data. They were obtained from towboat logs just as the other data were.

Two types of analyses of miscellaneous delays are possible. On the one hand, one might take a very detailed approach using the logs. These data have the advantage of indicating the location of a towboat at the time the delay occurred. This information could be used to investigate the relationship between location and frequency and duration of delay. Such analysis could provide a detailed relationship between the characteristics of the waterway (as specified by the location of the towboat) and the total miscellaneous delay time. Analysis conducted in this detail would require many logs representing the various waterways.

On the other hand, a more aggregate analysis might be used. Here locational differences could not be investigated. Such an analysis would prove to be more tractable and could be done with the second type of data discussed above. A sample observation of the aggregate data contains the same information (but without the details) as is available from the logs. For example, if one wishes to know how long a given towboat was delayed because of bad weather in a certain year, this time could be found by reading it directly from the summary sheet for the particular boat and year, or it could be determined by looking up the time lost for each weather delay for an entire year.

The aggregate delay data are easier to use and available in much greater quantity than are the log data. On the other hand, only the log data can be used to infer the relationship between waterway characteristics and delays. Therefore, in order to justify using the aggregate data in the following analysis, it is necessary to presume that the waterway characteristics have little effect on delay time. On an a priori basis, it seems that some of the miscellaneous delays would depend on waterway characteristics while others would not.

Table 4.8

WATERWAY-DEPENDENT AND WATERWAY-INDEPENDENT MISCELLANEOUS DELAYS

Dependent on Waterway		Independent of Waterway	
Delay	Percentage of Active Time	Delay	Percentage of Active Time
Weather	2.35	Repair	3.94
Bridge	.12	Supply	.65
Running aground	.24	Waiting barge	1.87
One-third of "other"	1.30	Two-thirds of "other"	2.61
Total	4.01	Total	9.07

Source: Table 4.4 above.

In order to get an upper bound on the error introduced by neglecting waterway characteristics, the following percentages were calculated: (1) total waterway-dependent miscellaneous delays expressed as a percentage of total active time and (2) total waterway-independent miscellaneous delays expressed as a percentage of total active time. Table 4.8 shows the breakdown into dependent and independent delays and the percentages of active time involved. The "other" category consists of the following three delay factors: (1) waiting orders, (2) waiting for crew, (3) waiting for a clear channel. Since the only one of these that is likely to be affected by waterway characteristics is (3) and since the relative magnitudes of the three are unknown, one-third of the "other" category is allocated to waterway-dependent delays and two-thirds to waterway-independent delays. When this is done, it is seen that total waterway-dependent delays cannot amount to more than 4.01 percent of active time, while total waterway-independent delays amount to 9.07 percent of active time. Or, in other words, a maximum of one-third of miscellaneous delays may be waterway dependent. This seems to be an upper bound and that in all likelihood the proportion is much lower than one-third, for not every weather delay, running-aground delay, and waiting-clear-channel delay is dependent on waterway characteristics. Moreover, these delays are not ignored in the following analysis, but, rather, they are assumed to be equally spread among waterways regardless of waterway characteristics.

Next, a framework is established in which the aggregate delay data may be used to estimate the miscellaneous delay time for individual trips. The following relationship among the aggregate variables is defined:

$$t_t = t_a + t_n = t_r + \sum_{i=1}^{10} t_i + t_n, \qquad (4.37)$$

in which t_t = total time in a year, in hours, (observable from data)

t_a = total active time in a year, in hours (observable from data)

t_n = total inactive time in a year, in hours (observable from data)

t_r = total running time in a year, in hours (observable from data)

t_i = total delay time in a year for the ith delay, in hours, i = 1, ..., 10 (observable from data).

Equation (4.37) divides the total time in a year into the sum of the time during which a towboat is actually making trips and the time during which it is inactive because of major repairs or lack of business. From Equation (4.37) running time is the difference between active time and total delay time.

$$t_r = t_a - \sum_{i=1}^{10} t_i . \tag{4.38}$$

The aggregate running time may be related to individual trip time by

$$t_r = \sum_{j=1}^{J} T_{rj} , \tag{4.39}$$

in which T_{rj} is the time required to make the jth trip, and there are J trips in a year. Note that the previous analysis of the tow in motion permits the estimation of trip running time. Repeating Equation (3.3) above.

$$T_r = T_f + T_a N_a + T_d N_d , \tag{4.40}$$

i.e., for any given trip, the running time is the sum of the time during which the tow operates at full speed, T_f, and the time during which it is accelerating or decelerating, $T_a N_a + T_d N_d$.

Define total miscellaneous delay time in a year as

$$t_o = \sum_{\substack{i=1 \\ i \neq 1,\ 6,\ 9}}^{10} t_i . \tag{4.41}$$

The ratio, α, of total miscellaneous delays per year to total yearly running time is defined:

$$\alpha = t_o/t_r. \tag{4.42}$$

Note also that $t_o = \alpha t_r$. Since the previous analysis permits the estimation of T_r, which is the point-to-point counterpart of t_r, an estimate of miscellaneous delay time expected to be encountered on any given trip, T_o, is

$$T_o = \alpha T_r. \tag{4.43}$$

The factor of proportionality, α, must be estimated from the aggregate delay data. Note that t_o and t_r are available from the aggregate data for each of the 59 boat-year observations. A sample of α's, defined as in Equation (4.42), are therefore available.

The central limit theorem leads one to believe that the distribution of α should be approximately normal, for miscellaneous delays are the sum of seven independent random variables. The data were tested for normality with a standard chi-square test employing seven cells. With four degrees of freedom, the calculated statistic is 0.12; the statistic would have to exceed 9.49 for significance of conventional levels. Therefore, it is concluded that the distribution of miscellaneous delays is not significantly different from normal.

Given a normally distributed random variable, probably the best estimate of the mean for most purposes is the sample arithmetic mean.

More confidence may be attached to this estimate if the distribution has a small variance. The aggregate delay data give $\bar{\alpha}$ = 14 percent with a standard deviation of 5 percent. The standard deviation is large relative to the mean, however, but this seems the best that can be done given the data.

In summary, the miscellaneous delay time for any given tow trip may be estimated by $T_o = \bar{\alpha}T_r$, where $\bar{\alpha} = 0.14$ and T_r is estimated from the relationships developed above. Note that T_r contains acceleration and deceleration time as well as time during which the tow travels at full throttle. It was noted above that, for long trips and/or those in which few locks are encountered, acceleration and deceleration time is likely to be insignificant relative to the time during which the tow is operating at full speed. If this is the case, the miscellaneous delay time for a trip may be estimated by

$$T_o = \bar{\alpha}T_f, \qquad (4.44)$$

in which $\bar{\alpha} = 0.14$ as before, but T_f is the time required to make the trip at full speed. In either formulation, T_o enters the tow process function as shown earlier. See Equation (4.1).

Note on Frequency of Miscellaneous Delays. Recall from earlier sections that in order to adjust for acceleration and deceleration time lost and distance traveled incidental to each stop, the number of stops en route must be known. The number of locking stops should be known for any given trip. While there must be an acceleration at the beginning of the trip and a deceleration at the end, it is not known how many stops are due to miscellaneous delays.

In order to get an idea of the frequency of miscellaneous delays, the two towboat logs were examined. For each day, the number of stops due to miscellaneous factors, still using the term "miscellaneous" in the manner described above, was counted and the miles traveled that day were recorded. It turned out that the average number of miscellaneous stops per mile was 0.01, with a standard deviation of 0.02. In other words, there is on average one stop due to miscellaneous delay factors for every 100 miles of travel, and in most cases the number of miscellaneous stops will be between zero and 3 for 100 miles of travel. As a point estimate of N_o, the number of stops due to miscellaneous delay factors, the following relationship could be used:

$$N_o = 0.01D, \qquad (4.45)$$

where D is, as before, the length of the trip, in miles. In most cases N_o would be a rather small number. Moreover, since it has been determined that accelerations and decelerations take little time and account for little distance traveled for most trips, it is probably best to ignore the slight adjustment factor of Equation (4.45).

Recapitulation of Tow Linehaul Process Function

The output of a barge tow is the cargo ton-miles per hour generated in the process of hauling commodities from one point to another. Ton-miles per hour may be expressed as the product of (1) the cargo tonnage of the tow and (2) average effective speed of the tow between origin and destination. It is the latter of these two that has required considerable analysis in the preceding sections of this chapter.

Cargo tonnage of a tow was expressed as a simple relationship between number of barges and their draft. Average speed, on the other hand, was seen to be dependent in complicated ways on a wide variety of variables. In the development of a method of determining the average speed of a tow, it was convenient to think of any tow trip as consisting of various phases and to develop models for predicting the time spent in any phase. Given the time spent by the tow in each phase of the trip and the length of the trip, then the average effective speed is determined. This, as was pointed out above, when multiplied by the cargo tonnage of the tow, provided the ton-miles per hour generated by the transport vehicle. Analyses were required concerning such things as acceleration, deceleration, cruising speed, locking time, assembly and disassembly time, and other delay times.

The analyses of acceleration, deceleration, and cruising speed were based on estimated tractive-effort and drag-force functions for barge tows. It was found that the time spent in acceleration was quite small relative to the time during which the tow would typically operate at its cruising speed. Also, it was found that the distance traveled by a tow during acceleration was small relative to typical trip distances. Likewise, deceleration was determined to be insignificant. The model predicting the cruising speed of the tow, then, is the major input to the process function as far as tow movement is concerned. That would be the end of things were it not for the fact that tows are delayed en route. Delays occur because of, locking, assembly and disassembly of the tow, and other miscellaneous reasons. Locking delay was handled by a simple queuing model A delay relationship for making and breaking tow was estimated from towboat log data. Finally, all other linehaul delays were grouped together and estimated from delay data collected by a common carrier barge firm.

Given the above relationships, the ton-miles per hour generated by any given tow for any given trip may be estimated. Moreover, the predicted ton-miles per hour figure is amenable to changes in variables under the control of barging firms (e.g., horsepower of towboats) as well as variables under the control of the Corps of Engineers (e.g., depth, width, and stream velocity of waterway; lock size). In addition, as will be shown in the next section, cost per ton-mile may be determined with the aid of the process function developed above.

Productivity relationships and isoquants will not be investigated for the tow process function. As was noted in Chapter II, Howe has investigated these relationships for his tow process function. The difference between the tow linehaul process function presented in this chapter and Howe's function is that the former includes delay. To the extent, therefore, that the variables affecting delay time also affect the running time of the tow, productivity relationships and isoquants based on the process function of this chapter will differ from those derived from Howe's process function.

The last few paragraphs have given a verbal recapitulation of the results of this chapter. These results are now gathered together and presented in their algebraic form. The most practical form of process function appears to be that form in which the effects of acceleration and deceleration are ignored, viz.,

$$Q = C \cdot D(T_f + T_L + T_m + T_b + T_o)^{-1}, \qquad (4.46)$$

which when dealing with a segmented waterway becomes

$$Q = C \cdot D\left(\sum_{i=1}^{I} T_{f_i} + T_L + T_m + T_b + T_o\right)^{-1}. \qquad (4.47)$$

The estimating equations for the variables entering Equation (4.46) are as follows, with appropriate changes required for Equation (4.47). The equation numbers in which these were first stated are given in the right-hand margin.

$$C = N_b W_c = N_b(-605{,}120 + 380{,}000H + 2{,}210H^2) \qquad (4.6)$$

$$T_f = (D/V_f) \qquad (3.4)$$

$$V_f = V_f^* - \delta S_w \qquad (4.17)$$

$$V_f^* = - \frac{(j_1 + J)}{2j_2} \qquad (4.[illegible])$$

$$T_L = \sum_{i=1}^{r} \frac{1}{\mu_i - \lambda^*} \qquad (4.28)$$

$$\lambda^* = (1 + p)\lambda \qquad (4.24)$$

$$\lambda = Q/\overline{C} \qquad (4.23)$$

$$T_m = 0.21 + 0.44N_b \qquad (4.35)$$

$$T_b = 0.34 + 0.20N_b \qquad (4.36)$$

$$T_o = 0.14T_f \qquad (4.44)$$

The variables are defined as follows:

Q = ton-miles per hour

C = cargo tonnage of tow, in short tons

D = distance of trip, in miles

T_f = time required to travel D at full speed, in hours

T_L = locking time, in hours

T_m = make-tow time, in hours

T_b = break-tow time, in hours

T_o = miscellaneous delay time, in hours

N_b = number of barges in tow

W_c = cargo tonnage of a barge, in short tons

H = draft of barges, in feet

V_f = full speed of tow, in miles per hour, adjusted for stream current

V_f^* = full speed of tow, in miles per hour, in still water

$$\delta \begin{cases} = 1 \text{ upstream} \\ = -1 \text{ downstream} \end{cases}$$

S_w = speed of water, in miles per hour

HP = rated brake horsepower of towboat

D = depth of channel, in feet

W = width of channel, in feet

B = breadth of flotilla of barges, in feet

L = length of flotilla of barges, in feet

r = number of locks to be traversed during trip

μ_i = service rate of ith lock, in tows per hour

λ^* = arrival rate at locks, in tows per hour

O = total waterway tonnage required to pass through locks in a given period of time

$\bar{C}$ = cargo tonnage, in short tons, of an average tow

p = percentage of trips requiring empty backhaul.

Obtaining Costs from Process Function for Barge Tows

The tow process function provides an estimate of the ton-miles per hour generated by a tow, Q. In order to convert this into the cost per ton-mile of operating the tow, two more inputs are needed: (1) the cost per hour of operating towboats of various horsepower, C_t, and (2) the cost per hour of a Jumbo hopper barge, C_b. Both of these items are available from the Corps of Engineers, which collects cost data from barging firms and calculates the per-hour costs, C_t and C_b. In addition, the Corps has per-hour costs of various other barge types as well. And, as was indicated above, the process function may be adapted with little difficulty to other barge types.

Given Q, C_t, and C_b and knowing the number of barges in the tow, N_b, then the cost per ton-mile of a tow trip, C_{tm}, may be determined from

$$C_{tm} = \frac{C_t + N_b C_b}{Q} . \tag{4.48}$$

Note that the cost per ton-mile given by Equation (4.48) refers only to a point-to-point movement. Such a cost, therefore, does not take into consideration any adjustments in the barging firm's other transportation activities. If the firm, for example, had to divert barges from some other traffic to make a given trip, then the real cost per ton-mile might not be reflected in the cost per ton-mile for the point-to-point movement. Moreover, a change in the depth of a waterway, for example, would affect the cost per ton-mile of the point-to-point barge movement over that waterway, but might also affect the operations on other waterways as well. The effect of these other adjustments would not be reflected in the point-to-point cost per ton-mile over the improved waterway.

In order to determine the cost per ton-mile of the firm's operations, one would need to know the entire distribution of tow trips undertaken by the firm. In addition, to know the change in the firm's costs due to a waterway improvement, one would again need information on the firm's entire activities. In order, therefore, to know firm costs as opposed to individual trip costs, the tow process function must be incorporated into a scheduling model of the barging firm. Simple scheduling problems, such as contract carriage, can be handled by the above analysis, but for more complex problems such as those of a common carrier, a scheduling model for the barge firm has been developed by Hurter and Cabot.[32] Without going into detail, the model provides optimal scheduling of barges and towboats in order to satisfy the transport demands on the firm and to maximize the firm's profits. An important input into the Cabot-Hurter model is the time required for any tow to travel between ports designated in the model. This information could be supplied by the point-to-point analysis detailed above. Tow cost analysis will not be pursued further here.

CHAPTER V

RAIL LINEHAUL PROCESS FUNCTION

Introduction

In Chapter III a method of formulating a linehaul process function for a transportation mode was presented. The method was applied to barge tows in Chapter IV. In this chapter a linehaul process function for rail freight transportation is presented. The next section develops the form of the rail linehaul process function. Functions are developed for the cargo weight and light weight of a train in the following sections. A subsequent section analyzes the train in motion, including constant velocity, acceleration, and deceleration. There is no discussion of tests of the speed prediction function as there was in the chapter on barge tows, for the tractive-effort and drag-force functions employed in the train process function are both well-known relationships that have received wide approval and use. A section on rail linehaul delays provides hardly anything that is new. This is in marked contrast to the comparable sections in the last chapter. This should not be taken to imply that linehaul delays are unimportant for rail or that the definitive delay analysis has been done. Quite the contrary, rail linehaul delays are important, as will be shown; and almost no one would agree that no more analysis is needed on them. Nevertheless, that analysis is not provided in this dissertation. Concluding sections contain economic analysis of the rail linehaul process function, including various productivity relationships, isoquants, and cost curves. The final section includes a derivation of the railroad firm's cost curves using both engineering and statistical methods. This last should be viewed as merely an example of what can be done and not as an attempt to estimate present day rail costs since the data on which the statistical results are based are a decade old.

Form of Rail Linehaul Process Function

The process function for rail trains has the same form as Equations (3.8)-(3.10) above. Recall that Equation (3.8) allows for the effects of acceleration and deceleration on the output produced by the vehicle, whereas Equations (3.9) and (3.10) are the process functions when these factors are ignored. Also, note that the process function allows for the trip mileage to be segmented. This was done to account for variability of factors en route that may affect the vehicle's performance. In the case of barge tows, such variables as depth, width, and stream velocity can vary over a route between two points. In order to account for variability, the route may be divided into many segments, each of which is fairly uniform with respect to depth, width, and stream velocity.

In the case of railroads, the primary factors that are likely to vary over the length of a trip are the grade of terrain and the degree of track curvature. The operating characteristics of a freight train will vary considerably with these two factors. The method for dealing with grade and curvature is to divide the trip into segments with respect to these variables. For example, suppose a train is engaged in a trip of length ten miles of which the first two-mile stretch has an upgrade of 2 percent, the next two miles a downgrade of 1 percent, and the remaining six miles are on level ground. The trip would consist of three segments over which the speed and, therefore, the ton-miles per hour generated by the train would be different. The trip may be further segmented with respect to curvature since the length and degree of each curve encountered can be determined. Of course, the two factors of curvature and grade will be combined on several segments, e.g., a half-mile curve of two degrees may occur on a 1 percent grade.

Another factor that must be considered in the rail linehaul process function is the speed limit. Any existing speed limit must be added as a constraint to the function. For example, the analysis may indicate a feasible speed of 100 miles per hour, whereas the speed limit is 75 miles per hour. Again, the route may be segmented into sections over which the speed limit constraint is operative and over which it is not.

In the process function, time spent delayed en route is represented by T_s. It was convenient to disaggregate delay time into several components for the tow analysis. However, in the case of rail all delays en route could perhaps be treated as one. Delay time due to acceleration and deceleration will also be examined below. If these delays are found to be large relative to total trip time, then they may be integrated into the rail process function by using forms of the function which include acceleration and deceleration, Equation (3.8). Moreover, some analysis of the number of stops en route would be required in order to include the effects of acceleration and deceleration on a train's output.

Cargo Capacity and Light Weight of Rail Train

Recall that the linehaul process function is the product of two terms: (1) the cargo tonnage of the vehicle and (2) the average speed of the vehicle between origin and destination. The second of these terms will be developed in the next and following sections of this chapter for the case of rail. The cargo tonnage of a train will be discussed in this section beginning first with a discussion of the cargo tonnage of a rail car and then generalizing this discussion to the cargo tonnage of a rail train. The remarks made here concerning the cargo tonnage of rail trains are applicable to all different types of rail cars and to trains composed of a variety of types of cars. Note that this was not the case in the tow process function. In the analysis of tow cargo capacity, a particular barge type was selected and used exclusively in the analysis, although methods were suggested for generalizing this procedure. The use of one type of barge was necessitated by the fact that the tow resistance formulation had been estimated for only one barge type. In the case of rail, on the other hand, the resistance function is not based solely on one rail car type, nor is it dependent on a particular configuration of rail cars, i.e., the order of the rail cars comprising a train does not affect the total resistance.

There are several types of freight cars and many sizes for each type. For each size and type of car, one can determine the cargo capacity in cubic feet, the light weight, and the load limit.[1] Load limits, set by the Association of American Railroads (AAR), may not be exceeded and are generally much less than the actual cubic capacity would permit. For example, a 50-foot general box car may have a capacity of 5000 cubic feet but a load limit of 70 tons. A typical medium density commodity, such as corn, may weigh 45 pounds per cubic foot. This means that if the above box car were loaded to capacity, the cargo weight would be 112.5 tons which far exceeds the load limit. However, a low density commodity like oats, which weighs about 26 pounds per cubic foot, would be space-limited rather than weight-limited, for the freight car when filled to capacity would weigh 65 tons, less than the load limit of 70 tons.

To put this argument into symbols, the following definitions are needed:

K_{ij} = capacity of ith car for jth commodity, in tons

L_i = load limit of ith freight car, in tons

x_i = cargo volume of ith car, in cubic feet

g_j = weight of jth commodity per unit volume, in tons per cubic foot

k_i = proportion of capacity of ith car utilized, $0 \leq k_i \leq 1$

c_i = cargo weight of ith car, in tons.

Capacity is defined as follows:

$$K_{ij} = \min (L_i, x_i g_j). \qquad (5.1)$$

Then the cargo weight of the ith car for the jth commodity is

$$c_i = k_i K_{ij}, \ 0 \le k_i \le 1. \qquad (5.2)$$

An example will illustrate the above relationships. Suppose for a given freight car and a given commodity, the following hold:

L_i = 70 tons

g_j = 0.01 tons/cu. ft.

x_i = 5000 cubic feet.

Then K_{ij} = min (70 tons, 50 tons) = 50 tons; i.e., the car is space limited for the assumed commodity, and the cargo weight of the car is given by Equation (5.2), where k_i refers to the fraction of the ith car's capacity utilized. If another, denser commodity is considered, say the rth commodity, where $g_r = 0.025$, then K_{ir} = min (70, 125) = 70, and the car is weight limited. The cargo weight is again given by Equation (5.2).

The cargo weight of a train of freight cars will be the sum of the cargo weights of the individual freight cars comprising the train, or, in symbols,

$$C = \sum_{i=1}^{N_c} c_i = \sum_{i=1}^{N_c} k_i K_{ij}, \qquad (5.3)$$

where N_c is the number of cars in the train and j represents the commodity with which the car is loaded. This formulation allows for N_c different cars, N_c different commodities (but only one to a car) and N_c different utilization rates. If all cars were identical and hauled the same amount of the same commodity, and if the cars were weight-limited for that commodity, then the cargo weight of a train would be given by

$$C = N_c kL. \qquad (5.4)$$

In addition to the cargo weight of a train, it will be necessary in the subsequent analysis to know the tare or light weight of the cars and the gross weight of the locomotive. Again, for any given freight car, one can easily determine the tare weight of the car, w_{t_i}.[2] Therefore, for any given train of cars, one can determine the light weight of the train of cars

$$W_t = \sum_{i=1}^{N_c} w_{t_i}, \qquad (5.5)$$

where N_c is the number of cars in the train. The gross weight of one freight car is

$$w_{c_i} = c_i + w_{t_i}, \qquad (5.6)$$

and the gross weight of the train of cars is

$$W_c = C + W_t. \qquad (5.7)$$

The gross weight of a locomotive may be similarly obtained.[3] However, the gross weight of a locomotive seems to be related to its horsepower and number of axles, i.e., for a given number of axles

(4, 6, or 8), the weight of the locomotive is an increasing function of horsepower. And for any given horsepower, weight is an increasing function of number of axles. In the subsequent analysis, it will prove convenient to have a functional relationship among horsepower, weight, and axles. The following relationship will be used:

$$w_L = -3.62 + \underset{(0.00194)}{0.00834HP_L} + \underset{(1.724)}{27.140a_L} \qquad (5.8)$$

in which w_L = weight of locomotive unit in tons

HP_L = horsepower of locomotive unit

a_L = number of axles of locomotive unit.

Both coefficients are significantly different from zero. The t-values for horsepower and axles are, respectively, 4.3 and 15.7. Standard errors are given in parentheses below the estimates to which they relate. The coefficient of determination is 0.98. This relationship was obtained by regression analysis applied to 24 sample observations of diesel-electric locomotives contained in Car & Locomotive Cyclopedia. It should prove applicable to present day, United States and Canadian diesel-electric locomotives.

Horsepower may be added to a train in one of two ways: (1) using a larger horsepower locomotive and (2) using additional locomotives. For example, a motive force of 6,000 horsepower may be obtained from one 6,000-horsepower locomotive or from four 1,500-horsepower locomotives. It would be interesting to be able to investigate these alternative methods of obtaining horsepower. This can be accomplished by defining locomotive gross weight W_L, as

$$W_L = \sum_{i=1}^{N_L} w_{L_i}, \tag{5.9}$$

where N_L is the number of locomotive units and w_{L_i} is the weight of the ith locomotive unit. If $N_L = 1$, then $W_L = w_L$. This formulation will permit investigation of the effect of changing horsepower on the ton-miles per hour generated by the train, where horsepower can be changed either by adding locomotives or by using a locomotive of greater horsepower.

The gross weight of the train is expressed as

$$W = W_L + W_c = \sum_{i=1}^{N_L} w_{L_i} + \sum_{i=1}^{N_c} c_i + \sum_{i=1}^{N_c} w_{t_i}. \tag{5.10}$$

The preceding discussion has developed a formulation of the cargo and gross weight of a train. Both of these are components of the train process function, for the cargo tonnage transported by the train certainly affects the ton-miles per hour produced by the vehicle, and the gross weight of the train affects the speed at which it may travel. In the next section the analysis of a train in motion is begun. This analysis is necessary in determining the average effective speed of the train between origin and destination. Another factor affecting average speed--delays en route--will be taken up in a later section.

Analysis of Train in Motion

Introduction. In order to apply the relationships developed in Chapter III, tractive-effort and drag-force functions must be obtained for rail trains. This is a much easier task than it was for the case of barge tows, for there already exist in the railroad engineering literature well-known and accepted functions.

Tractive-effort Function. A general tractive-effort function for diesel-electric locomotives is derivable from the definition of horsepower. Moreover, it takes account of mechanical and electrical losses from auxiliary units of the locomotive. This function is[4]

$$F_t(v) = 375(HP - HP_a)(e/v), \qquad (5.11)$$

in which $F_t(v)$ = tractive effort, in pounds

HP = total rated horsepower of locomotive units

HP_a = horsepower used by auxiliaries

e = an efficiency factor, mechanical-electrical, taken as 82.2 percent

v = velocity, in miles per hour.

When these factors are given numerical values from conventional design and operation, the tractive-effort equation reduces to

$$F_t(v) = (308HP)/v. \qquad (5.12)$$

Drag-force Function. The most commonly used drag-force function in railroad engineering is the Davis formulation.[5] This relationship is usually presented as follows, applicable both to rail cars and locomotives:

$$F_d(v) = (1.3 + 29/w + bv + SXv^2/wa)wa, \qquad (5.13)$$

in which $F_d(v)$ = drag-force, or resistance of car or locomotive, in pounds

w = weight per axle of car or locomotive, in tons

a = number of axles

b = coefficient of flange friction = 0.045 for freight cars; 0.03 for locomotives

S = drag coefficient of air = 0.0025 for locomotives; 0.0005 for freight cars

X = cross-sectional area = 105-120 square feet for locomotives; 85-90 square feet for freight cars.

Frontal area could be determined for each particular locomotive and car, but since the subsequent analysis is to be applied to a large variety of cars and locomotives and since the range of variation of X is small, it will be assumed to be equal to 112.5 for locomotives and 87.5 for freight cars (the arithmetic averages of the end points of the ranges of variation).

Equation (5.13) may be put into a more useful form if the implied multiplication of the bracketed expression by wa is carried out. Note that wa is the gross weight of the vehicle. Substituting the values of b, S, and X, two drag-force functions are obtained, one for a locomotive and one for a freight car, viz.,

$$F_d^L(v) = 1.3w_L + 29a_L + 0.03w_Lv + 0.28125v^2 \qquad (5.14)$$

and

$$F_d^c(v) = 1.3w_c + 29a_c + 0.045w_c v + 0.04375v^2, \qquad (5.15)$$

in which $F_d^L(v)$ = drag-force of locomotive, in pounds

$F_d^c(v)$ = drag-force of freight car, in pounds

w_L = gross weight of locomotive, in tons

w_c = gross weight of freight car, in tons

a_L = number of axles of locomotive

a_c = number of axles of freight car

v = velocity of locomotive or car, in miles per hour.

The above formulas give resistance of only one locomotive and one freight car. Total train resistance is the sum of the locomotive resistance and the resistances of all the freight cars, or

$$F_d(v) = \sum_{i=1}^{N_L} F_{d_i}^L(v) + \sum_{i=1}^{N_c} F_{d_i}^c(v), \qquad (5\ 16)$$

where $F_d(v)$ = total train resistance

$F_{d_i}^L(v)$ = locomotive resistance of ith unit

N_L = number of locomotive units in train

$F_{d_i}^c(v)$ = freight car resistance of ith car

N_c = number of freight cars in train.

Upon substituting Equations (5.14) and (5.15) into Equations (5.16), one obtains the drag-force function for a train:

$$F_d(v) = 1.3W + 29A + 0.03W_L + 0.045W_c v + (0.28125N_L + 0.04375N_c)v^2, \quad (5.17)$$

in which $W = W_L + W_c$

$$W_L = \sum_{i=1}^{N_L} w_{L_i} = \text{gross weight of locomotive units}$$

$$W_c = \sum_{i=1}^{N_c} w_{c_i} = \text{gross weight of freight cars}$$

$$A = A_L + A_c$$

$$A_L = \sum_{i=1}^{N_L} a_{L_i} = \text{total number of axles of locomotive units}$$

$$A_c = \sum_{i=1}^{N_c} a_{c_i} = \text{total number of axles of freight cars.}$$

If there is only one locomotive unit and all freight cars are identical, then the drag-force function reduces to

$$F_d(v) = 1.3(w_L + N_c w_c) + 29(a_L + N_c a_c) + (0.03w_L + 0.045N_c w_c)v + (0.28125 + 0.04375N_c)v^2. \quad (5.18)$$

Adjustment for Grade and Curvature. In the case of barge tows, there were several factors concerning the "roadbed" that affected the speed, e.g., depth, width, and stream velocity of the waterway. The only factors comparable to these for the case of the rail are the gradient of the terrain and the degree of track curvature.

It is not difficult to adjust the preceding drag-force function to account for the effect of grade. Let the extra resistance on the train due to grade be denoted by F_s, which may be negative or positive depending upon whether the train is going downhill or uphill, respectively. F_s is equal to 20 pounds per ton per percentage of grade.[6] For example, a train climbing a 2 percent grade encounters additional resistance equal to 40 pounds per ton of train weight. The relationship among grade resistance, train weight, and percentage grade may be expressed as

$$F_s = 20Ws, \qquad (5.19)$$

in which F_s = grade resistance, in pounds

W = total weight of train (engine and cars) in tons

s = percentage gradient of terrain.

In order to obtain total train drag force including grade resistance, one need merely add the grade resistance to the train resistance, i.e.,

$$F_{ds}(v) = F_d(v) + F_s,$$

or

$$F_{ds}(v) = (1.3 + 20s)W + 29A + (0.03W_L + 0.045W_c)v$$

$$+ (0.28125N_L + 0.04357N_c)v^2, \qquad (5.20)$$

where $F_{ds}(v)$ means drag-force adjusted for gradient of terrain, and the other variables have the same meanings as before. Of course, should the train be operating on level ground, s = 0, and Equation (5.20) reduces to Equation (5.17).

Track curvature is another important factor affecting the performance of freight trains. When a train rounds a curve, resistance is encountered in addition to train resistance and grade resistance. This additional resistance due to track curvature is known as curve resistance. There are several notions as to why curve resistance occurs which are unnecessary to discuss here.[7] In any case, it is easy to adjust the forgoing resistance relationship for the additional resistance of curvature.

On the basis of railroad tests, the American Railway Engineering Association (AREA) has adopted a recommended value for curve resistance of 0.8 pounds per ton per degree of curvature.[8] The degree of curvature is a measure of the sharpness of the curve and is a standard railroad engineering concept. Total curve resistance, F_c, for a train which has a gross ton weight of W rounding a curve of c degrees is given by

$$F_c = 0.8Wc. \qquad (5.21)$$

Curve resistance, like grade resistance, enters the resistance function additively. The resistance function adjusted for curve and grade is, therefore,

$$F_{dsc}(v) = F_d(v) + F_s + F_c = (1.3 + 20s + 0.8c)W + 29A + (0.03W_L + 0.045W_c)v + (0.28125N_L + 0.04375N_c)\ v^2. \tag{5.22}$$

Constant Velocity State. Given the tractive-effort function of Equation (5.12) and the drag-force function of Equation (5.22), the analysis developed in Chapter III may now be applied to rail trains. The equation of motion for a rail train operating at cruising speed is

$$F_t(v) - F_{dsc}(v) = 0. \tag{5.23}$$

Substituting the functional forms for F_t and F_{dsc}, one obtains the following cubic equation:

$$308HP - [(1.3 + 20s + 0.8c)W + 29A]v - (0.03W_L + 0.045W_c)v^2 - (0.28125N_L + 0.04375N_c)v^3 = 0. \tag{5.24}$$

Solving this cubic equation for velocity will yield the cruising speed, V_f, of a rail train as a function of HP, s, c, W, A, W_L, N_L and N_c. Fortunately, a solution to a cubic equation of this form exists, and a standard computer program is available for obtaining it. Of the three possible sets of roots that may be obtained from cubic equations (see Chapter III), the set resulting from the solution of Equation (5.24) will contain one real, positive root. That this should be the case may be seen from examination of the tractive-effort and drag-force functions. The tractive-effort function is a rectangular hyperbola, see Equation (5.12). The drag-force function, Equation (5.22), is quadratic in velocity. The latter reaches a minimum value at a negative velocity and proceeds into the first quadrant positively sloped.

This can be demonstrated as follows. Write the drag-force function in the general form of Equation (3.13)

$$F_d = A_0 + A_1 v + A_2 v^2, \tag{5.25}$$

where $A_0 = (1.3 + 20s + 0.8c)W + 29A > 0$

$A_1 = 0.03W_L + 0.045W_c > 0$

$A_2 = 0.29125N_L + 0.04375N_c > 0.$

Then $dF/dv = A_1 + 2A_2 v$. Setting this equal to zero and solving gives a first-order condition for an extremum as $v = -A_1/2A_2$ which is negative. Second-order condition is $d^2F/dv^2 = 2A_2$, which is positive ensuring a minimum and ensuring that $F_d(v)$ is positively sloped to the right of its minimum. Graphically, these results may be shown as follows

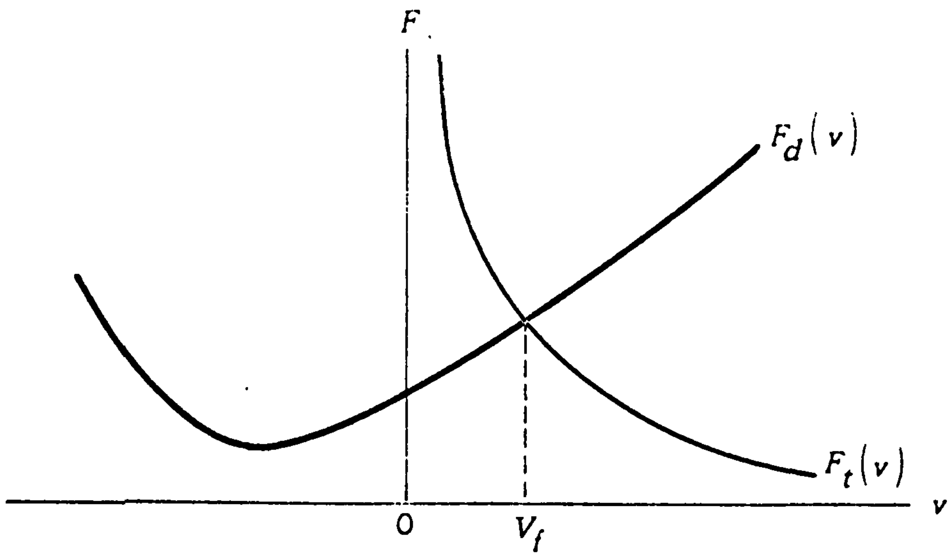

Fig. 5.1 -- Determination of cruising speed of train

These characteristics of the two functions ensure an intersection in the first quadrant (therefore a real, positive root). The other two roots may be conjugate and imaginary (if $4A_0A_2 > A_1^2$); real, negative, and unequal (if $4A_0A_2 < A_1^2$); and real, negative, and equal (if $4A_0A_2 = A_1^2$).

<u>Acceleration</u>. The following acceleration equations are commonly found in railroad engineering literature:[9]

$$T_a = (95.6/F_a')(V_2 - V_1) \tag{5.26}$$

and

$$D_a = (70/F_a')(V_2^2 - V_1^2), \tag{5.27}$$

where T_a = acceleration time, in seconds

D_a = acceleration distance, in feet

V_1 = initial velocity, in miles per hour

V_2 = final velocity, in miles per hour

F'_a = force available for acceleration, in pounds per ton.

F'_a is obtained as follows: (1) calculate the tractive effort, in pounds, of the motive units minus the resistance, in pounds, of the motive units; (2) calculate the resistance, in pounds, of the train of cars; (3) divide the resulting figure by the weight, in tons, of the entire train. The figure obtained by the operation described in (1) is commonly referred to as "drawbar pull"; it is the amount of pulling force left after the resistance of the locomotive units is accounted for. All of these computations are made for a given speed, usually the mean obtained over the entire interval considered. If one were interested in the time and distance required to accelerate from a speed of V_1 to V_2, then (1) and (2) could be calculated using $(V_2 - V_1)/2$. This would represent a linear approximation to the acceleration curve and would result in an underestimate of acceleration time and distance. Better estimates could be obtained by using smaller speed intervals and then adding up the resulting magnitudes. For example, if one were interested in the time required to accelerate from 0-20 miles per hour, the time required to accelerate from 0-2, 2-4, 4-6, ..., 18-20 could be calculated and the resulting time figures added together to get the required result.

Such a procedure is followed by Hay in determining the acceleration characteristics of a train of fifty 70-ton cars pulled by four 2,000-horsepower diesel-electric units.[10] Using the equations for acceleration time and distance given above, Equations (5.25) and (5.26), and 2-miles-per-hour speed intervals, Hay calculates it would require 49,295 feet and 694.29 seconds to achieve the cruising speed of 60 miles per hour. While these appear to be large figures, it is interesting to calculate the error resulting from ignoring acceleration. Suppose it is determined the train operates at a cruising speed of 60 miles per hour. At this speed the time required to cover 49,295 feet is 560 seconds. Therefore, the error in ignoring acceleration time is an underestimate by the amount 134 seconds or slightly over 2 minutes. Distance traveled would be overestimated by 10,777 feet or less than 2 miles. It would seem that unless one were concerned with short trips and/or those involving frequent stops, acceleration time and distance can be ignored with only small resulting error.

Deceleration. There exist in the railroad engineering literature formulas for obtaining the time required and the distance traveled in the deceleration of trains. For example, Hay derives quite complicated deceleration relationships by equating the kinetic energy of the train with the work done in stopping the vehicle.[11] In this derivation, consideration is made of such things as the time required for brakes to act, weight of train, brake cylinder pressure, etc.

An alternative, and much simpler, procedure for obtaining estimates of deceleration time and distance was outlined in Chapter III. Assume a constant rate of deceleration d; e.g., 0.75 miles per hour per second, suggested in Car and Locomotive Cyclopedia. Then the time required to stop a train from full speed is

$$T_d = V_f/d = (4/3)V_f, \qquad (5.28)$$

where T_d is in seconds. The distance traveled in such a deceleration is given by

$$D_d = (1/2)dT_d^2 = V_f^2/2d = (2/3)V_f^2, \qquad (5.29)$$

where D_d is in feet.

The deceleration figures produced by Equations (5.27) and (5.28) are likely to be overestimates of actual deceleration time and distance. This observation is based on actual braking tests reported in Hay.[12] The tests were performed on a train consisting of fifty 70-ton freight cars pulled by four 2,000-horsepower locomotives. The distance traveled in stopping from 60 miles per hour on various grades ranged from about 900 feet to 1350 feet; the time required to stop varied from 20 seconds to about 28 seconds. Unless one were concerned with short trips involving many stops or slow-downs, it seems safe to omit calculations of deceleration times and distances in determining the time required to make a trip.

Rail Linehaul Delays

Introduction. In the general form of the process function of a transport mode, there is a term to represent the time during which the unit is stopped. See Equations (3.8) - (3.10). In the case of barge tows, it was found convenient to disaggregate total delay time per trip into several components, e.g., make tow, break tow, locking, etc. But some tow delays were best treated by lumping them together into a single category which was called "miscellaneous delays." These miscellaneous delays were those due to weather, running aground, waiting for bridges, etc. In the case of rail, for a given level of traffic, the delays encountered en route are similar to those in the miscellaneous category for barge tows, i.e., there are a large number of random and independent delay-causing factors which may be combined for treatment. However, train delays are also related to congestion en route.

In the next subsection, a brief discussion of delay-estimating procedures is presented. Some data obtained from a railway are also discussed to give an idea of the magnitude of linehaul delays. Apart from delays en route, trains are detained at terminals for inspection and assembly. Although this category of delay will not be discussed below, several studies of yard operations are available,[13] the results of which could be incorporated into the rail linehaul process function in the same manner in which make and break tow delays were incorporated into the tow linehaul process function.

Delays en Route. The time in hours, T_f, required to travel the distance D miles at cruising speed of V_f miles per hour was given in Equation (3.4) as

$$T_f = D/V_f. \qquad (5.30)$$

Preceding sections have examined the relationships required to obtain V_f, i.e., the tractive-effort and drag-force functions. Since D is assumed known for any given trip, the trip time would be determined from Equation (5.29) were it not for delays.

In reality, for a given level of traffic, there is a dispersion of trip times about an average, and the average itself increases as traffic congestion rises. Since the variations in conditions affecting trip time are largely random--being primarily speed variations due to weather, individual engine efficiency, and operator procedures--the plot of the relative frequencies of vehicles against trip time has been approximated by a normal distribution.[14]

While, for a given level of traffic, the delay time is distributed about a mean delay, the average delay itself is a function of the level of traffic. In railroad parlance, "interference time" increases as more trains operate on the same route. This interference time is primarily due to trains pulling into sidings to allow on-coming trains to pass, but it may also result from trains being retarded behind more slowly moving units.

These ideas have been incorporated into an analysis of line-haul delays.[15] The following trip-time expression is postulated:

$$T_t = T_f + \beta N_t, \qquad (5.31)$$

where T_t = road time, in hours

T_f = minimum road time, in hours

β = delay factor, in hours per train

N_t = number of trains on a given route in a 24-hour period. The factors governing T_f have already been given considerable attention; T_f is the time that would result if there were no delays en route. In practice, an "average" train is considered for which T_f is calculated. The average train is usually selected as that train whose cargo weight is the average of all those trains traversing the route in a given period of time. Next, a sample of actual road times is collected along with the corresponding number of trains operating on that route in a 24-hour period, i.e., for each observation on road time, T_t, there is an associated measure of traffic density, N_t. These observations are collected either from test results or dispatchers' records. From these data are calculated the average road time, $\bar{T}_t$, and the average number of trains per day, $\bar{N}_t$. Then the delay factor is obtained as

$$\beta = (\bar{T}_t - T_f)/\bar{N}_t. \qquad (5.32)$$

An alternative procedure for obtaining an estimate of delays would be to perform a simple regression of road time, T_t, on traffic density, N_t. An estimate of trip time would be obtained in the same manner as before: calculate T_f and add to it $\beta^* N_t$, where β^* is the slope coefficient in the regression. Although this is a feasible alternative to the former approach, no instance of its use has been found in the railroad engineering literature. On the other hand, the former procedure for estimating trip time seems to have become the accepted approach by railroad engineers. Another estimate of delay time would be simply the arithmetic average delay time over a sample of train trips. This, of course, ignores congestion and would relate to a specific route only.

In order to get an idea of the magnitude of linehaul delays, a sample of 100 unit-train movements was obtained. These data all relate to the same origin-destination combination: a one-way distance of 227 miles. Information on the composition of each train was obtained, specifically: (1) number of cars, (2) tare weight of cars, (3) loaded weight of cars, (4) number of axles on each car, (5) number of locomotive units, (6) horsepower of each locomotive unit, and (7) number of axles on each locomotive unit. Detailed terrain analysis of the route was made by the railway; it was decided that there was no appreciable gradient or curvature along the route. Given this information--train characteristics and route characteristics--it was possible to determine the speed of each sample train from the solution to Equation (5.24) above. Applying Equation (5.24) to each of the 100 data points in turn yielded 100 predicted speeds V_f and, by Equation (5.29), 100 predicted trip times, T_f. These were then subtracted from the observed trip times supplied by the railway. The result in each case was an estimate of delay time en route. It should be stressed that no terminal time was included in these observations; therefore, assuming the predicted times are correct, deviations from predicted trip times are due to delays en route involving slow-downs as well as stops. The results are shown in Table 5.1.

Table 5.1

STATISTICAL ANALYSIS OF TRIP TIMES AND DELAY TIMES FOR 100 UNIT TRAIN MOVEMENTS

	Predicted Trip Time (hours)	Actual Trip Time (hours)	Delay Time (hours)
Range	7.17 - 9.41	8.42 - 22.58	0.61 - 14.32
Mean	8.15	12.13	4.14
Standard Deviation	0.47	3.26	3.92

SOURCE: Midwestern Railway which requested not to be identified.

An estimate of the average delay time per train is 4.14 hours for the 100 sample movements. The standard deviation is large relative to the mean, however, indicating, as does the range of actual trip times, a great dispersion from the mean. Unfortunately, the data did not disclose the nature of the delays, and it was impossible to deduce delay-causes except for a few cases. Based on an examination by railroad personnel of some of the very large trip times, it can be concluded that some atypical delays occurred, e.g., a derailment. Removing these observations from the sample reduces the mean delay time to 3.85 hours and the standard deviation to 3.09 hours. Even with these observations removed, however, the mean delay is large relative to the predicted mean trip time of 8.15 hours and to the delay standard deviation. This signifies the importance of linehaul delays, even for unit-train movements which are supposed to be unimpeded en route. However, while recognizing the importance of linehaul delays, no additional analysis of these will be provided here. Rather, economic analysis of the linehaul process function, exclusive of delays, is begun in the next section. To the extent that linehaul delays are related to variables which also enter the train speed function and train cargo function, the productivity relationships and input tradeoffs derived below will be biased. However, until a reliable linehaul delay analysis is available, little can be done about this problem.

Rail Productivity Relationships

It is interesting at this point to investigate the rail linehaul process function, ignoring route segmentation, acceleration, deceleration, and linehaul delays. Route segmentation, by its very nature, is specific to a given trip. Ignoring it in the following analysis does not preclude examination of the effects of route characteristics on the output of a train. It was argued above that, in most cases, acceleration and deceleration may be safely ignored. Delays en route are important, as was indicated above, but will be omitted nevertheless. The form of the linehaul process function which omits acceleration, deceleration, route segmentation, Equation (3.10) above, reduces to the following when delays are ignored:

$$Q = V_f C, \tag{5.33}$$

where Q = output of the train, in ton-miles per hour

V_f = cruising speed of the train, in miles per hour

C = cargo weight of train, in tons.

Each of the terms V_f and C represents a function. In general, they may be specified as follows:

$$V_f = f(HP_{L_i}, N_L, a_{L_i}, s, c, N_c, a_{c_j}, k_j, x_j, g_j) \qquad (5.34)$$

$$C = g(N_c, k_j, L_j, x_j, g_j)$$
$$i = 1, \ldots, N_L$$
$$j = 1, \ldots, N_c, \qquad (5.35)$$

in which HP_{L_i} = horsepower of ith locomotive unit

N_L = number of locomotive units

a_{L_i} = number of axles per ith locomotive unit

s = gradient of terrain, in percent

c = degree of curvature, in degrees

N_c = number of cars

a_{c_j} = number of axles per jth car

k_j = proportion of the jth car loaded, $0 \leq k_j \leq 1$

L_j = load limit of jth car, in tons

x_j = cargo volume of jth car, in cubic feet

g_j = weight of jth commodity per unit volume, in tons per cubic foot.

Note that the above formulation is quite general, allowing for almost any conceivable combination of the inputs. To simplify matters somewhat, the following assumptions are made: (1) all freight cars in the train are identical; (2) all locomotive units on a given train are identical; (3) each freight car contains the same amount of cargo weight as each of the others in a given train; (4) freight cars are weight-limited rather than space-limited; another way of stating this is that the commodities being transported are so dense that the load limit of a freight car is reached before its cubic capacity is exhausted.[16] These assumptions permit the following simplification of the speed and cargo functions:

$$V_f = h(HP_L, N_L, a_L, s, c, N_c, a_c, k, L) \qquad (5.36)$$

$$C = N_c kL. \qquad (5.37)$$

With the aid of a production function, one may examine the effect of the individual inputs on the productive unit's output, given constant levels of the other inputs. In the economist's parlance, productivity relationships may be investigated. One is concerned with the total, average, or marginal productivity of the productive unit with respect to a particular input. In addition, the substitution possibilities between inputs may be investigated through the production function. That is, isoquants may be determined. From these and the related input costs, expansion paths and cost functions may be obtained.

These properties of a production function may be determined analytically if the production function relation is sufficiently simple to permit this. Functional forms may be obtained for such things as productivity curves, isoquants, and expansion paths.

However, in the case of the rail linehaul process function developed above, such analytical treatment is precluded by the complexity of the relationship. (Recall that the function V_f results from the solution of a cubic equation.) The alternative method of obtaining the desired results is to characterize the function numerically using tables and graphs. This is the procedure followed below.

In the case of rail, the most important effects to be examined seem to be (1) the effect of the locomotive input on train performance, and (2) the effect of the car input on train performance. The most important input substitution to consider is that between locomotive power and cars. In addition to these relationships it is also of interest to investigate the effect of terrain, as represented by gradient and curvature, on train performance. Finally, one might like to know the effect upon train output of less-than-carload shipments. All of these are discussed below.

First, the relationship between the locomotive input and the train's output is investigated. The locomotive input in the process function is represented by three variables: (1) horsepower of a locomotive unit, (2) number of axles of a locomotive unit, and (3) number of locomotive units. It is total horsepower of the motive unit(s) that determines the tractive effort of the vehicle. (See Equation (5.12).) Under the assumptions stated above, horsepower may be obtained in either of two ways. A given horsepower may be achieved by using a single locomotive unit or several locomotive units of the same horsepower. (Recall that unequally-powered locomotive units for a given train are excluded by assumption.)

Figure 5.2 displays total productivity of cars for various levels of horsepower. The values of the variables entering the process function are given in the box accompanying the figure. They represent one locomotive pulling standard, fully loaded 70-ton box cars over straight, level track. These cars have a light weight of 30 tons each. The following observations are made. (1) The total productivity curves for fixed horsepower are strictly concave, i.e., the marginal productivity of cars is monotone decreasing, or $(\partial^2 Q/\partial N_c^2) < 0$. (2) The marginal productivity of horsepower diminishes. This is seen from Figure 5.2 by the curves getting closer together as one moves upward from a given number of cars, i.e., $(\partial^2 Q/\partial HP^2) < 0$. (3) The marginal products of both inputs remain positive within the ranges considered, i.e., $(\partial Q/\partial HP) > 0$ and $(\partial Q/\partial N_c) > 0$ for $500 \leq HP \leq 6000$ and $10 \leq N_c \leq 100$. It is likely that the marginal product of cars would become negative at some point beyond a train length of 100 cars, at least for the low horsepower trains. The positivity of the marginal products means that as the train gets larger and larger, the resulting decrease in speed is more than offset by the resulting increase in additional carrying capacity.

The productivity curves of Figure 5.2 also permit the examination of substitution possibilities between the car and locomotive inputs. By selecting a particular rate of output and drawing at that level of output a horizontal line across the family of total product curves, the combinations of the two inputs yielding the given level of output are obtained. The curve drawn through such a collection of points is called an isoproduct curve or isoquant.

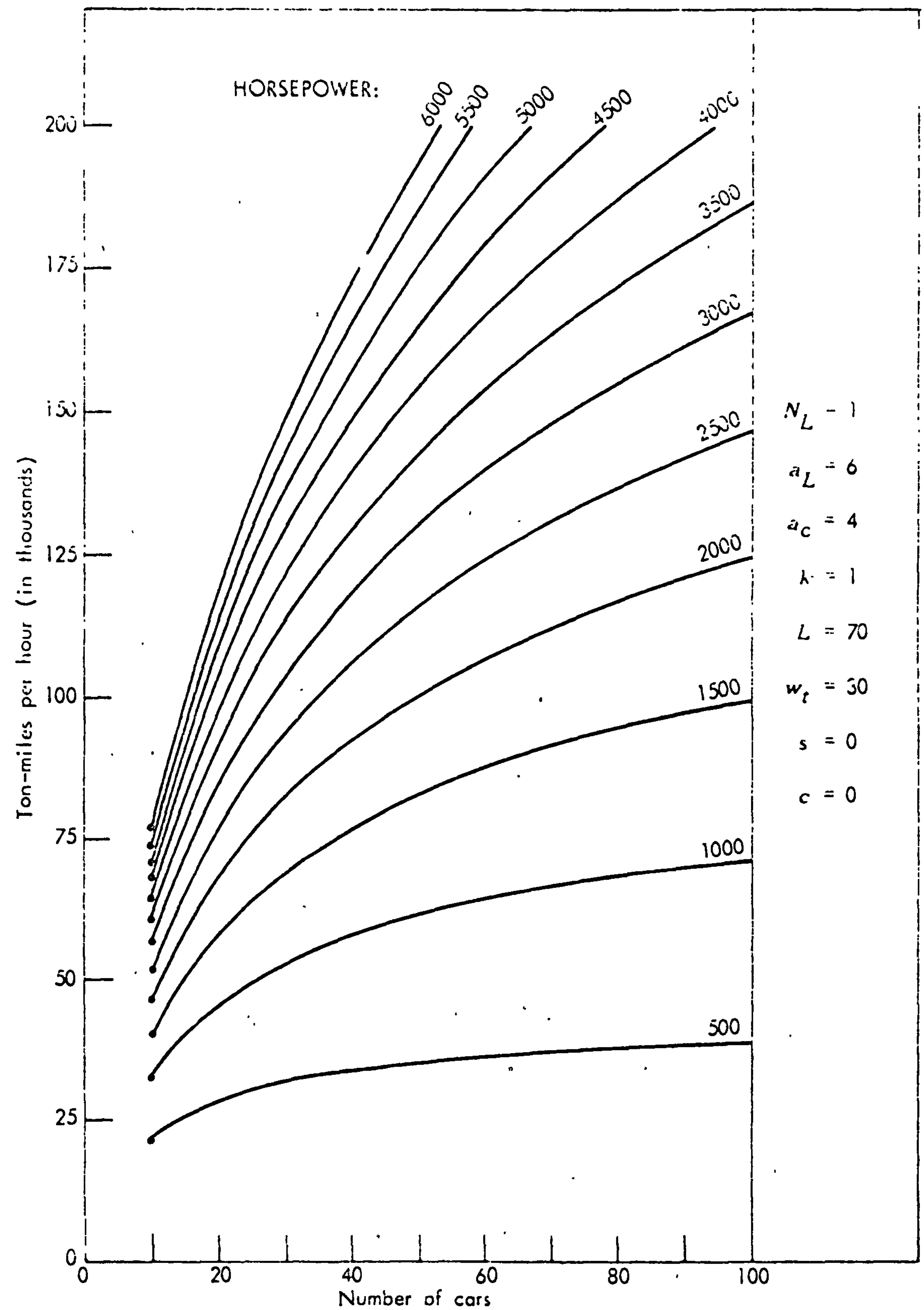

Fig. 5.2 -- Productivity Curves for Rail Linehaul Process Function

By examining the isoquants shown in Figure 5.3, a couple of interesting properties of rail transportation may be determined. The isoquants are negatively sloped throughout, indicating that the two inputs are technical substitutes for one another, i.e., if fewer cars are used, more horsepower must be used in order for total output to remain unchanged. Moreover, the isoquants are convex to the origin, which means that, while the two inputs are technical substitutes, they are not perfect substitutes. More cars will just compensate for smaller and smaller amounts of horsepower and vice versa.

Finally, isoquants can be used to investigate the returns to scale exhibited by the productive process. The term returns to scale describes the output response to a proportionate increase in all inputs. If output increases in the same proportion, returns to scale are constant. They are increasing if output increases in greater proportion and decreasing if it increases in smaller proportion than the increase in inputs.[17] There are two equivalent ways one may determine which of these three categories prevails when an isoquant graph is available. In both methods straight lines must be drawn from the origin of the horsepower-car axes into the plane. Three of these are shown in Figure 5.3. Upon measuring the distance along a line from the origin between points on successive isoquants which increase by a constant amount, one will find that these distances become smaller. This shows that a given increment in output may be obtained with a proportionately smaller increase in both inputs, a condition of increasing returns to scale. This condition prevails along all of the lines drawn in Figure 5.3. Alternatively, one could

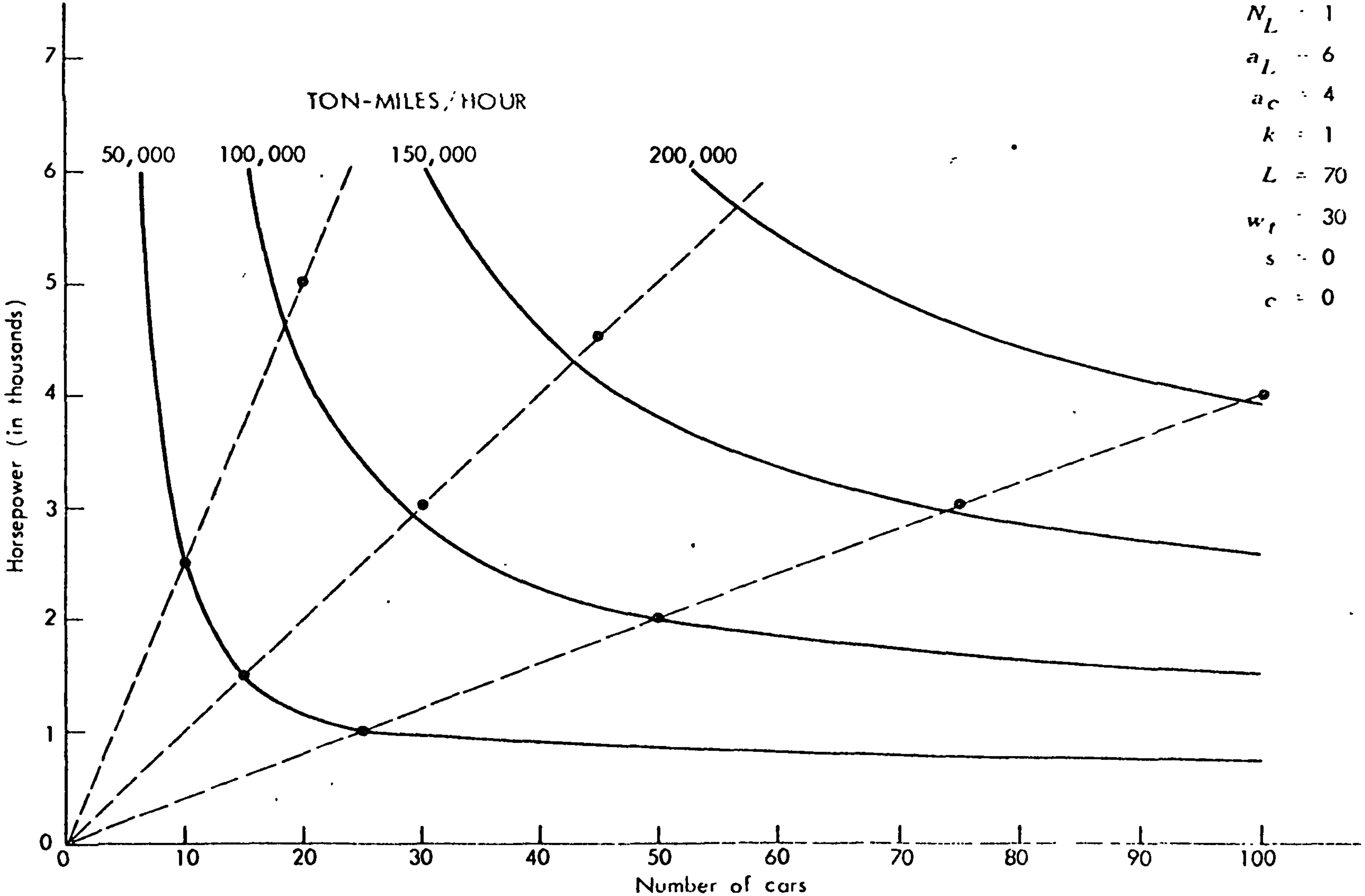

Fig. 5.3 -- Isoquants of Rail Linehaul Process Function

note the points along a line from the origin which represent given proportionate increases in inputs. These are shown in Figure 5.3 by dots along the three straight lines. The first set of dots were selected to lie on the isoquant representing 50,000 ton-miles per hour. Successive sets of dots represent, respectively, increases of 100 percent, 50 percent, and 33-1/3 percent. It can be observed that the isoquants representing increases in output of 100, 50, and 33-1/3 percent respectively, lie below the corresponding points on the line from the origin. Moreover, the isoquants lie increasing distances below these points the greater is the rate of output. Again, this situation indicates increasing returns to scale because proportionate increases in all inputs are accompanied by greater than proportionate increases in output. It is interesting to note that Howe found decreasing returns to scale for tow production.[18] This property of the tow process function was attributed to the increased resistance encountered as the tow widened relative to the width of the waterway. There is no factor comparable to the width of the waterway in the train process function, and the absence of such a resisting factor may account for the apparent increasing returns observed.

A qualification of the above results should be voiced at this point. Recall that it was assumed in the above calculations that only one locomotive unit was employed. Thus motive power was added to a train by using a more powerful locomotive. The rail process function allows for the possibility of multiple locomotive units, and the assumption of a single unit was made for simplification only. Multiple units would typically be used when total motive power exceeded 6,000 horsepower, for at present the largest locomotive units are of that size. However, it is also conceivable that multiple locomotive units would be employed even where the total horsepower could be obtained from one unit. This could occur for two reasons. First, scheduling problems might dictate the use of multiple units where a single unit would be more efficient. Second, in certain circumstances, multiple units might be more efficient than a comparable single unit. Such a situation occurs whenever the total weight of the multiple units is less than that of a comparable single unit and when the total number of axles of the multiple unit is less than or equal to that of a single unit. This is so because both weight and number of axles enter the train resistance function positively (see Equation (5.22)); and, thus, increases in these factors, certeris paribus, decrease output. In most instances, however, for total horsepower less than 6,000, single locomotive units will be more efficient than multiple units. In any case, the process function may be used to rule out inefficient alternatives. Table 5.2 illustrates how certain multiple units compare with their corresponding single unit locomotives in pulling a given train.

Table 5.2

EFFECT OF HORSEPOWER ON TRAIN PERFORMANCE FOR SINGLE AND MULTIPLE LOCOMOTIVE UNITS FOR 50-CAR TRAIN

Total Horsepower	Ton-miles per Hour: Multiples of 2000-HP (4-axle locomotives)	Ton-miles per Hour: Single Unit
2000	101,374	101,374 (4 axles)
4000	152,765	155,108 (6 axles)
6000	187,600	193,909 (8 axles)

In the examination of barge tows, it was found that the tow's output was materially affected by characteristics of the waterway, namely, width, depth, and stream velocity. The comparable variables in the case of rail transportation are the gradient and curvature of the track. The effect of gradient of terrain on train performance is illustrated in Table 5.3. The train whose output is examined in that table consists of fifty fully-loaded, 70-ton freight cars pulled by one 2,000-horsepower locomotive. The table ends with a grade of 3 percent because few grades exceed that amount.[19]

Table 5.3

EFFECT OF GRADIENT ON TRAIN PERFORMANCE FOR FIFTY 70-TON BOX CARS PULLED BY 2000-HORSEPOWER LOCOMOTIVE

Gradient (percent)	Speed (miles/hour)	Output (ton-miles/hour)
0	28.96	101,374
.5	9.31	32,600
1.0	5.30	18,540
1.5	3.69	12,903
2.0	2.82	9,885
2.5	2.29	8,009
3.0	1.92	6,731

Curvature, the other terrain factor to be considered, also impedes train performance, although not so dramatically as does gradient. The following table illustrates the effect of curvature on train performance for the same train considered above. Hay reports that railroad curves for high speed traffic--70 to 100 miles per hour--are held to 1 to 2 degrees and 2 to 3 degrees for moderate speeds of 45 to 69 miles per hour. Curvature may go as high as 30 to 40 degrees, but curvature above 20 degrees is not recommended.[20]

Curvatures of greater than 5 degrees are not considered in the table.

Table 5.4

EFFECT OF CURVATURE ON TRAIN PERFORMANCE FOR FIFTY 70-TON BOX CARS PULLED BY 2000-HORSEPOWER LOCOMOTIVE

Curvature (degrees)	Speed (miles/hour)	Output (ton-miles/hour)
0	28.96	101,374
.5	27.18	95,137
1.0	25.54	89,384
1.5	24.03	84,092
2.0	22.64	79,235
2.5	21.37	74,734
3.0	20.20	70,706
3.5	19.13	66,970
4.0	18.16	63,546
4.5	17.26	60,404
5.0	16.43	57,517

The final factor to be investigated is the effect of loading on train performance. It is common knowledge that less-than-full carloadings are discriminated against in rail transport pricing. While this is surely a question of relative costs of transporting carloads versus less-than-full carloads, it is of interest to look at this question from the production side in order possibly to obtain some evidence on it. The process function permits investigation of this point, for it has built into it a variable representing the loading characteristics of a train. Recall that the simplifying assumptions given earlier require that all cars be loaded to the same extent. The process function, however, permits investigation of any conceivable loading condition, if desired.

Table 4 shows, for the same train considered above, what happens to speed and output as loading varies from empty to full.

Table 5.5

EFFECT OF CARLOADING ON TRAIN PERFORMANCE FOR FIFTY 70-TON BOX CARS PULLED BY 2000-HORSEPOWER LOCOMOTIVE

Proportion Loaded (percent)	Speed (miles/hour)	Output (ton-miles/hour)
0	40.90	0
10	39.25	13,738
20	37.74	26,416
30	36.34	38,156
40	35.05	49,063
50	33.85	59,229
60	32.73	68,734
70	31.67	77,644
80	30.72	86,021
90	29.81	93,916
100	28.96	101,374

Although output increases considerably as the cars become fuller, there is diminishing marginal productivity associated with the factor "proportion loaded." If hourly operating costs of the train were constant no matter what the load, then unit cost (i.e., cost per ton-mile) would be decreasing throughout as would marginal cost. This would constitute an argument in favor of pricing so as to achieve full carloading.

Rail Costs

Introduction. In the preceding sections of this chapter a rail linehaul process function has been developed. Some properties of the process function were investigated in the last section. Now it is desired to use the process function as an aid in determining the behavior of rail costs. First, the costs of the linehaul process are investigated by deriving a family of short-run average cost curves. These are analogous to the plant unit cost curves of the firm. However, here the "plant" is a railroad freight train, and plant "scale" varies as the size of the locomotive changes. Information on the extent of economies of scale is provided by the envelope to the short-run unit cost curves. Secondly, the railway firm's costs are investigated by means of the linehaul process function and some statistical cost analysis. In this case the rail linehaul process function again provides the basis for the analysis of linehaul costs, but the costs comprising the railway firm's activities other than linehaul are provided by the statistical cost analysis. The rail firm's short-run average cost curves are obtained, and again the implied long-run envelope curve is used to draw conclusions about economies of scale.

This last exercise, that of grafting the linehaul costs obtained via the process function onto the other costs obtained from a statistical analysis, should be viewed as merely an example of how one might obtain cost curves for the firm when process functions do not exist for all the processes comprising the firm's activities. A pure process function approach to obtaining the rail firm's costs would be better, in this writer's view, than the "mongrel" approach illustrated below. The reasons for this belief are the same as those given in Chapter I for preferring an engineering approach to a statical one in the examination of production and cost functions. The technological approach is inherently more flexible than the statistical, and in addition, does not require the assumption of long-run equilibrium necessary to statistical estimations of long-run costs. Production functions based on technological data more closely approach the long-run production functions of economic theory than do statistical estimations based solely on observed behavior. It is unfortunate, therefore, that similar technological information has not been successfully employed for the other cost components of train operations.

Costs of Linehaul Process. Aside from depreciation and interest, the primary components of linehaul costs are crew wages and fuel expenses. Others such as lubricants and water are so small as to be safely ignored.[21] In order to determine the train's cost curves, methods of estimating the individual cost components must be developed.

A typical freight train crew consists of an engineer, a conductor, and two brakemen.[22] The rate at which crew members are paid is based upon the distance traveled. Time is converted into distance by the assumption of an average freight-train speed of 12.5 miles per hour. Therefore, an 8-hour period is equivalent to a 100-mile trip. For purposes of determining the crew cost of a trip, the first 100 miles is charged at one rate and the remaining miles at another, lower rate. These mileage rates are effectively independent of the actual trip time involved. If, however, the train actually attains an average speed for the trip of less than 12.5 miles per hour, then "overtime" is paid on the basis of the miles that could have been traveled at a speed of 12.5 miles per hour. Thus, if a 125-mile trip which "should" have taken 10 hours, actually takes 12 hours, then it is equivalent to a 150-mile trip for crew-wage purposes. The additional 25 miles being charged at the higher rate. These conditions may be stated algebraically as follows:

$$C_W = \begin{cases} r_1 D & \text{if } D \leq 100 \text{ and } T \leq D/12.5 \\ 100r_1 + r_2(D - 100) & \text{if } D > 100 \text{ and } T \geq D/12.5 \\ 12.5 r_1 T & \text{if } D \leq 100 \text{ and } T > D/12.5 \\ 100r_1 + r_2(12.5T - 100) & \text{if } D > 100 \text{ and } T > D/12.5 \end{cases} \quad (5.38)$$

where C_W = crew member's cost to the trip

r_1 = crew member's wage rate per mile for first 100 miles

r_2 = crew member's wage rate per mile after first 100 miles

D = actual trip distance in miles

T = actual trip time in hours.

All crew members are paid on the basis of the same formula set out above. There are, of course, differences in the size of the rates. In addition, the rates vary with other factors. An engineer receives a higher wage rate for operating two, rather than one, engine units, and a higher one still if there are three engine units on the train. The wage rates of the conductor and brakemen rise with the number of cars in the train. The relationships are summarized in the following table. These figures are those used by a Midwestern railroad as of July, 1967.

The contribution of one crew member to the trip cost is given in general by Equation (5.38). Table 5.6 presents rates currently used by a Midwest railroad. If it is assumed that the trip is

Table 5.6

FREIGHT TRAIN CREW WAGE RATES, 1967

Crew Member	Rates/Mile for First 100 Miles	Rates/Mile after First 100 Miles
Engineer		
1 engine unit	$ .2564	$ .2389
2 engine units	.2657	.2482
3 engine units	.2775	.2575
4 engine units	.2840	.2665
Conductor		
1-81 cars	.2216	.2041
82-105 cars	.2251	.2076
106-125 cars	.2291	.2116
126-145 cars	.2316	.2141
146-165 cars	.2326	.2151
over 165 cars	a	a
Brakeman		
1-81 cars	.2104	.1860
82-105 cars	.2139	.1895
106-125 cars	.2179	.1935
126-145 cars	.2204	.1960
146-165 cars	.2214	.1970
over 165 cars	a	a

[a] $.20 for each additional block of 20 cars or portion thereof.

SOURCE: Midwestern railroad which requested not to be identified.

100 miles in length and that the crew consists of an engineer, a conductor and two brakemen, then the following total crew costs for the trip are defined:

1. If train speed is less than 12.5 miles per hour, then crew cost is given by

   a. 1 - 81 cars: $11.235T
   b. 82 - 105 cars: 11.366T
   c. 106 - 125 cars: 11.516T
   d. 126 - 145 cars: 11.610T
   e. 146 - 165 cars: 11.648T
   f. 166 - 185 cars: 11.723T
   g. 186 - 205 cars: 11.798T

2. If train speed is greater than or equal to 12.5 miles per hour, then the crew cost is given by

   a. 1 - 81 cars: $89.88
   b. 82 - 105 cars: 90.93
   c. 106 - 125 cars: 92.13
   d. 126 - 145 cars: 92.88
   e. 146 - 165 cars: 93.18
   f. 166 - 185 cars: 93.78
   g. 186 - 205 cars: 94.38.

Fuel cost is the second major component of train expenses. The amount of fuel consumed on a trip is primarily a function of the horsepower, thermal efficiency of the engine units, and the energy content of the fuel. A formula is developed for predicting train fuel consumption per hour. Knowing this and the price per gallon of fuel, the fuel cost per hour may be calculated.

The AREA Manual states that the thermal efficiency, defined as the ratio of work performed to energy consumed of rail diesel engines, is 25 to 30 percent.[23] This may be stated in symbols as $W/E = 0.25$, where W is work performed and E is energy consumed. In order that W and E be in the same units, work (which is defined as the product of horsepower and time) is converted to Btu. by the following relation: 2545 Btu. = 1 horsepower-hour. Since there are approximately 139,000 Btu. in a gallon of diesel fuel,[24] the energy content of N gallons is $E = 139{,}000N$. Thus, the number of gallons consumed per hour, n, by a diesel engine of HP horsepower is obtained from the following expression

$$0.25 = \frac{2545HP}{(139{,}000)n}, \tag{5.39}$$

which when solved for n yields

$$n = 0.073HP. \qquad (5.40)$$

Equation (5.40) gives the number of gallons per hour that will be consumed by engine units generating HP horsepower.

Given the price p per gallon of fuel oil, fuel cost per hour (C_f) is obtained as

$$C_f = p \cdot n = 0.073pHP. \qquad (5.41)$$

In general, fuel cost per hour is given by Equation (5.41). Fuel cost per trip may be found by multiplying trip time in hours by the fuel cost per hour. If it is assumed that p = \$0.10, i.e., that diesel fuel is \$0.10 per gallon,[25] then fuel cost per trip is given by

$$F = \$0.0073HP \cdot T. \qquad (5.42)$$

This expression is true regardless of the rated horsepower of the engine units. However, since the horsepower actually generated by the locomotive units over the entire trip cannot be known, it will be assumed to be equal to the rated horsepower of the engine units. What this means is that the train is assumed to operate at full power during those periods when it is moving at all. This is clearly an oversimplification and will result in an over-estimate of fuel consumption and cost when actual utilized horsepower departs from the maximum attainable horsepower of the engine units.[26] Since trains utilize full throttle whenever possible,[27] this assumption should not do too much violence to the facts, however.

The next cost to be associated with linehaul operations is depreciation of equipment. Cars and locomotives depreciate partly through use and partly through the passage of time. No distinction is made between these two causes of depreciation, however. Moreover, for convenience, straight-line depreciation is assumed. While this assumption is justified only under certain circumstances,[28] it is quite common accounting practice and is used here merely for convenience.

Table 5.7 shows freight car prices by car types. These figures are averages of actual purchase prices reported to the Interstate Commerce Commission by railroads. Prices will vary for cars of a given type as size and other factors vary, but these figures should serve where more exact ones are not available.

The following equation represents diesel locomotive prices as a function of horsepower. The equation is a least-squares regression based on 22 sample observations from Interstate Commerce Commision data.[29] Locomotive cost (C_L) is in thousands of dollars.

$$C_L = -4.313 + \underset{(0.007)}{0.094} HP_L \qquad (5.43)$$

Again, better information could be obtained for specific locomotive types, but estimates based on the above equation should be fairly accurate. The F value of 188 indicates a significant relationship

Table 5.7

FREIGHT CAR PURCHASE PRICES BY TYPE, 1964

Type of Car	Cost
Box (general service)	$ 13,083
Box (special service)	17,534
Flat	15,462
Stock	11,546
Gondola	12,504
Hopper (open)	10,380
Hopper (covered)	14,073
Rack	12,000
Refrigerator	21,914
Tank	19,339
Caboose	17,759

SOURCE: Interstate Commerce Commission, Seventy-Eighth Annual Report on Transport Statistics in the United Stated for the Year Ended December 31, 1964 (Washington, D.C.: U.S. Government Printing Office, 1965).

between horsepower and cost. The standard error of the slope coefficient is shown in parentheses below the estimated value. The coefficient of determination is 0.90.

Following the AAR, equipment life of 15 years and salvage value of 10 percent of purchase price are assumed.[30] Straight-line depreciation based on the purchase price less salvage value is used as the estimate of current depreciation expense per annum. This figure is converted to a per-hour basis by dividing the annual figure by the number of hours per year (8760).

Depreciation expense per trip for a given train would be calculated as follows: (1) determine number and horsepower of engine units, (2) determine number and type of cars, (3) estimate purchase price for each locomotive unit from Equation (5.43) and for each car from Table 5.7, (4) subtract 10 percent of this amount to obtain depreciable portion of the investment, (5) divide the resulting figure by (15 x 8760) to obtain the depreciation charge per hour, (6) determine time required for the trip from the analysis of train operations presented above, and (7) multiply this figure by the per-hour depreciation charge to get total depreciation charge allocable to the trip.

In particular, if it is assumed that the cars comprising the train are each identical fully loaded general service box cars, then the car depreciation charge allocable to the trip is given by

$$D_c = \frac{[\$13,083 - 0.10(\$13,083)]T \cdot N_c}{(15)(8760)} = \$0.0896T \cdot N_c, \qquad (5.44)$$

where D_c = total car depreciation per trip

T = actual trip time, in hours

N_c = number of cars.

A similar calculation produces the locomotive depreciation allocable to the trip. The purchase price of a locomotive is estimated via Equation (5.43). Given this quantity, one may determine the depreciation charges per locomotive per trip, D_L, as

$$D_L = \frac{[(\$94HP - \$4313) - 0.10(\$94HP - \$4313)]T}{(15)(8760)}$$

$$= (\$0.0007HP - \$0.0295)T. \qquad (5.45)$$

The final cost to be allocated to the linehaul process is interest on investment in road equipment. Here, as with depreciation, the total interest charge over the life of the equipment will be reduced to a per-hour amount and allocated to the linehaul process on the basis of the time required to complete the trip.

The total interest charge incurred over the life of the equipment will be estimated by the following relation.

$$I = \tfrac{1}{2}WLr, \tag{5.46}$$

where I = total interest charge

W = purchase price of unit of capital equipment

L = length of life of unit of capital equipment

r = annual interest rate.

Equation (5.46) gives the total interest paid (or foregone) on the investment of W dollars for L years at an annual interest rate r. It assumes that the amount W is paid off at the rate W/L continuously over the life of the equipment and is paid off completely at time L. This assumption may not be justified in practice, but alternatives to it would be equally suspect. In any case, Equation (5.46) is quite commonly used for costing purposes[31] and is very easy to apply.

Again, it is desired to express I on a per-hour basis and relate it to specific units of equipment. Assume r = 0.065; no justification is given for this rate. It may, however, be regarded as the rate paid to bondholders by the firm. Then assuming L years equipment life and using the locomotive purchase price equation, Equation (5.43), the per-hour interest cost allocable to a locomotive unit,

I_L, is given by

$$I_L = \frac{(0.5)(\$94HP_L - \$4313)(L)(r)}{8760L}$$

$$= \$0.0003511HP_L - \$0.016003. \quad (5.47)$$

Similarly, using the purchase price in Table 5.7, the hourly interest cost of a standard box car, I_c, is given by

$$I_c = \frac{(0.5)(\$13,083)(L)(r)}{8760L} = \$0.048367. \quad (5.48)$$

Thus, the total interest cost of equipment to be allocated to a trip of T hours is

$$I_T = (I_L N_L + I_c N_c)T, \quad (5.49)$$

where I_T = total interest cost of equipment for the trip

N_L = number of locomotive units

N_c = number of cars.

In summary, per-hour estimation equations for crew expenses, fuel expenses, depreciation, and interest have been derived above. Therefore, to determine the cost of a particular train trip, one needs to know the trip time in hours as well as the hourly cost components. The product of trip time and hourly cost gives total trip cost. Note that trip cost is a function of (1) trip time, (2) number of cars, (3) number of locomotives, and (4) horsepower of each locomotive. See Equations (5.38), (5.42), (5.44), (5.45), and

(5.49). In addition, recall that trip time is given by Equation (5.30) and is itself a function of (1) trip distance, (2) locomotive horsepower, (3) number of locomotives, (4) number of axles on each locomotive, (5) gradient of terrain, (6) degree of curvature, (7) number of cars, (8) number of axles on each car, (9) loading percentage of each car, and (10) load limit of each car. See Equation (5.36) above. Given these relationships, the cost behavior of trains under a variety of circumstances may be investigated.

Cost Curves for Linehaul Process. Given the cost estimation equations of the preceding subsection and the train linehaul process function, it is possible now to investigate the cost behavior of a train in the process of hauling freight between two points. Specifically, it is desired to develop short-run and long-run unit cost curves for the freight train. Unit cost will be measured in the units of dollars per ton-mile and will be composed of all major linehaul expenses including interest and depreciation on equipment (no distribution of administrative expenses is included, however). The output measure is the ton-mile per hour. A "plant" consists of a locomotive of given horsepower; output varies in the short run as the number of cars comprising the train varies. In the long run the locomotive horsepower as well as the number of cars can vary. This is expressed by a family of short-run unit cost curves, each representing a given horsepower. The implied envelope curve to this family of short-run cost curves represents the long-run unit cost curve of the linehaul process.

Figure 5.4 displays a set of short-run average cost curves for the linehaul operation of a freight train. The assumptions under which these curves were obtained are stated here: (1) trip distance of 100 miles; (2) zero gradient of terrain; (3) zero track curvature; (4) no delays on route; (5) one 6-axle engine unit; (6) identical, fully loaded 4-axle box cars; (7) weight-limited commodity; and (8) costs given by Equation (5.38) and Table 5.6, Equation (5.42), Equation (5.45), Equation (5.47), Equation (5.48), and Equation (5.49). The assumed physical characteristics of the trip are summarized in Figure 5.4.

Figure 5.4 presents the cost per ton mile of the assumed trip as horsepower and number of cars change. These were obtained by expressing trip cost on a per-hour basis and dividing the result by the appropriate ton-mile per hour figure. Each curve represents a given horsepower locomotive pulling cars of from 10 to 200 in number. Each curve may be thought of as a "plant" unit cost curve, where horsepower represents the fixed input and number of cars represents the variable input. The unit cost curves for 500 to 3000 horsepower display the U-shape usually assumed to characterize plant unit cost curves. However, beyond 3000 horsepower the curves are continually downward sloping. This implies that trains substantially longer than 200 cars would be required to make the plant unit cost curves rise. An implied envelope curve (not drawn), representing the long-run average cost of the train trip, clearly indicates economies of scale throughout the range of output from zero to 196,500 ton-miles per hour (a horsepower range up to 3000).

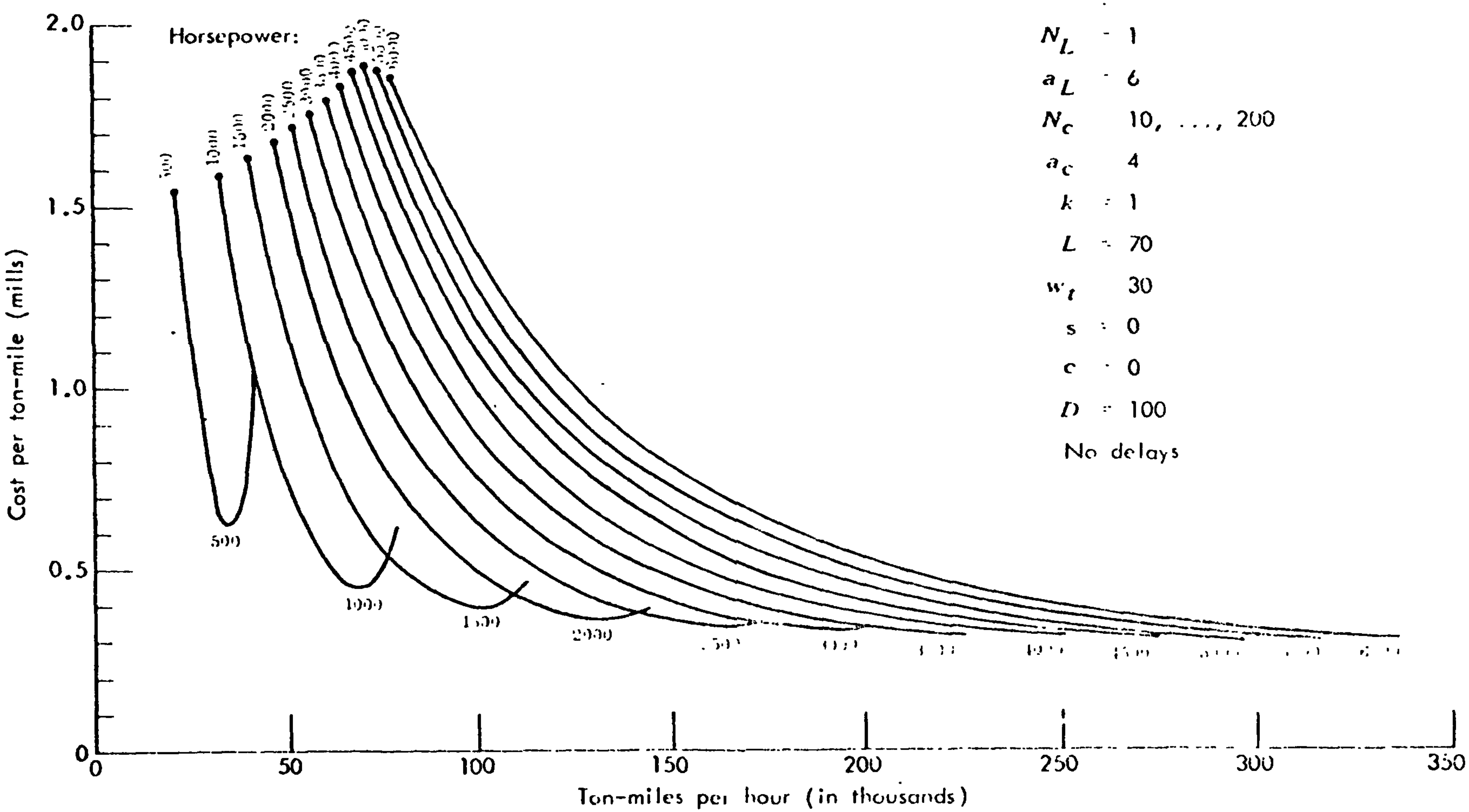

Fig. 5.4 -- Rail Linehaul Cost Curves

Economies of scale, in fact, seem to persist throughout the range of output shown on the graph although at a much diminished rate beyond an output of 196,500 ton-miles per hour.

Curves such as those of Figure 5.4 can be derived for any proposed trip by specifying the necessary values of the appropriate variables. The results could be used in the selection of equipment for various train trips. In addition the linehaul costs generated via the process function could be used with other cost determination methods for the remaining rail processes to estimate the rail firm's costs of a train trip. One way of achieving this last suggestion is illustrated in the next section.

Cost Curves for Railway Firm. Meyer, Peck, Stenason, and Zwick (MPSZ) have estimated the long-run marginal cost of freight train transportation.[32] Their general approach was to divide total rail operating costs into several components. Each component account was then subjected to individual analyses. These analyses consisted in cross-section regressions of cost on output and size variables. The output variables were usually gross ton-miles of freight and of passenger traffic, although other output variables were used where they seemed appropriate or gave better results. The size variable was, usually, miles of track, although, again, others were used.

The components of total operating cost were: train (linehaul) expenses, station expenses, yard expenses, traffic (selling and marketing) expenses, general (administrative and legal overhead) expenses, variable portion of maintenance expenses, variable portion of depreciation expenses, and variable portion of capital costs.

The analyses of these cost categories were synthesized to yield over-all relationships between cost and output. MPSZ did this by employing certain assumptions about the relationships between ton-miles and certain other operating measures such as yard hours and train miles. Based on these assumptions and the statistical analyses, the long-run marginal costs shown in Table 5.8 are obtained. Using MPSZ's results, if one wanted to estimate the long-run marginal cost of a freight train trip, he would determine the amount of gross ton-miles to be generated and multiply this by 4.28 mills, which is the price-corrected version of the MPSZ figure. Such a calculation would require knowledge of the distance to be traversed by the train, the cargo tonnage of the train, and the tare weight of the train.

MPSZ say, in reference to the above sort of calculation, "It should be noted that these estimates are based on a sort of central tendency and are typical figures that will apply to freight movements only of a very average or ordinary kind."[33] The reason for this is in part due to the nature of the estimation procedure and in part due to the assumptions required to use some of the results. The variability of costs with output was investigated, as mentioned above, by means of linear regression analysis. This by its very nature makes the estimates "sort of central tendencies." In addition, certain assumptions were employed to achieve the figure of long-run marginal cost on a gross ton-mile basis. For example, it was assumed that it required "approximately 24 minutes to originate, classify, and terminate the typical merchandise car, that this car goes approximately 400 miles, and that it has a load of 25 tons."[34] These assumptions

Table 5.8

LONG-RUN MARGINAL COST OF RAIL FREIGHT TRANSPORTATION

Cost Category	Cost Per Gross Ton-Mile of Freight Traffic: 1947-50 Mills	Cost Per Gross Ton-Mile of Freight Traffic: 1965 Mills[b]	Percentage of Total
Train[a]	0.411000	0.809670	18.90
Station	0.001799	0.003544	0.08
Yard	0.268441	0.528829	12.34
Traffic	0.070860	0.139594	3.26
General	0.126000	0.236400	5.52
Maint. of Way & Struct.	0.284000	0.559480	13.06
Freight Car Maint.	0.162000	0.319140	7.45
Yard Engine Maint.	0.004000	0.007880	0.18
Road Engine Maint.	0.151000	0.297470	6.94
Joint Equip. Repair	0.088900	0.175133	4.09
Equip. Depreciation[a]	0.165360	0.325759	7.60
Road Struct. Depreciation	0.046640	0.091881	2.14
Capital	0.401100	0.790167	18.44
Totals	2.175100	4.284947	100.00

[a]To be estimated from an engineering production function.

[b]The price indices used for correcting the 1947-50 figures were obtained from Association of American Railroads, Railway Statistics Manual (Washington, 1964), p. 10.14, "Indexes of Average Charge-Out Prices of Railway Material and Supplies and Straight Time Hourly Wage Rate Class I Railroads," for the period to 1963, and from Association of American Railroads, "Indexes of Railroad Material Prices and Wage Rates (Railroads of Class I)," Series Q-MPW-51, May 17, 1966, for the period 1963-65.

SOURCE: John R. Meyer *et al.*, *The Economics of Competition in the Transportation Industries* (Cambridge, Mass., 1960), pp. 46-47, 51, 56, 60, 62, Appendix B.

were used to reduce yard expenses to a gross ton-mile basis. Later, it was assumed "that any shipment under analysis is normal in the sense that it approximates the 1947 to 1950 averages of having 3,000 gross ton-miles of freight moved for every road mile of diesel engine operations and requiring one diesel engine yard hour for every 69,000 gross ton-miles of freight traffic."[35] These assumptions were used to translate the yard engine and road engine maintenance figures into gross ton-mile equivalents.

It is possible even within the context of the MPSZ work to avoid making such assumptions. For example, MPSZ provide equations from which can be estimated yard time.[36] If one does not necessarily wish to relate all costs to a gross ton-mile basis, the original estimating equations can be used rather than the aggregate figures of Table 5.8. In most cases, they will be no different because the gross ton-mile is the predominant output variable used, but in some cases, for example, road engine maintenance, other variables are included in the estimation, necessitating either assumptions such as those used by MPSZ or actual values for a particular situation.

While it is possible to use MPSZ results without some of their assumptions, it is not possible to use them and avoid the central-tendency aspect mentioned above. One way to avoid this problem is to provide an alternative cost estimation procedure for some or all of the components of total operating expenses. Clearly one of the most important of the cost categories is train expenses. It is proposed, therefore, to estimate train expenses by use of the rail process function and direct costs for crew and fuel. The train expenses obtained this way may then be added on to the other operating expenses as estimated by MPSZ, yielding a combined engineering-statistical cost estimate.

It is proposed to substitute direct estimates of certain costs for those estimates of MPSZ. Specifically, the following costs will be estimated directly: (1) train expenses, i.e., crew costs and fuel costs, and (2) locomotive and car depreciation expenses. Another cost that could be estimated directly, locomotive and car capital expenses, will not be. The reason for this is that the MPSZ estimate of capital cost includes not only locomotives and cars but investment in road as well. It would be difficult to disentangle the various capital items in the MPSZ analysis, and therefore, it is more convenient to use their capital cost estimate exclusively. Those other costs for which the MPSZ figures will be used are: station, yard, traffic, general, maintenance of way and structure, freight car maintenance, joint equipment repair, and road structure depreciation. Directly estimated costs (train and equipment depreciation) comprise about 25.5 percent of the total, based on Table 5.8.

For any given trip, crew wages may be determined from Equation (5.38) and Table 5.6, with the following additional information: trip distance, number of engine units, number of cars, and trip time. Fuel cost for a trip may be calculated from Equation (5.42) if one knows trip time, engine horsepower, and price per gallon of diesel fuel. Equipment depreciation expenses allocable to the trip may be determined from Equation (5.45) for locomotives and Equation (5.44) for boxcars. (If one were interested in a different type of freight car, an equation similar to Equation (5.44) could be developed with the use of Table 5.7, freight car prices.)

For those costs not estimated as described above, the price-corrected figures of MPSZ are used.[37] Referring to Table 5.8, it is seen that MPSZ have estimated long-run marginal cost per gross ton-mile of freight to be 4.28 mills. Train and equipment depreciation expenses were estimated by MPSZ to be 1.13 mills per gross ton-miles of freight. In order, then, to find the contribution to the firm's total costs of an additional trip (excluding crew, fuel, and equipment depreciation costs of a trip), one must determine the gross ton-miles to be generated during the trip and multiply this figure by 3.15 mills. Note that this assumes marginal cost equal to average cost. It is not in general true that marginal cost times output yields total cost. However, since MPSZ claim they have estimated long-run marginal cost, the identification of marginal with average is justified in this case.

Putting the preceding discussion into symbols, one may express the cost (C) to the firm of a train trip as follows:

$$C = W + F + D_c + D_L + A, \tag{5.50}$$

where W = crew wages

F = fuel cost

D_c = depreciation of cars

D_L = depreciation of locomotives

A = all other costs.

The estimating methods for each of the first four cost components were presented above. Utilizing the MPSZ estimate, corrected for price changes, all other costs of the train trip may be estimated by

$$A = \$0.00315 \text{ (gross ton-miles)}. \qquad (5.51)$$

The rail process function estimates train speed and the ton-miles per hour produced by the train. The inputs required for the use of the process function are (1) number of cars, (2) tare weight of each car, (3) load limit of each car (if commodity to be transported is not very dense, then cubic capacity of each car and weight per cubic foot of the commodities must be known), (4) proportion of each car filled, (5) number of axles on each car, (6) horsepower of locomotive(s), (7) number of locomotives, (8) number of axles on each locomotive, (9) gradient of terrain, and (10) degree of curvature.

Given the speed of the train from the process function, the trip time is easily calculated, assuming no delays. If an estimate of delay time is available, this may be used in conjunction with train speed to determine trip time. For any given route and any given train, the trip cost may be determined from the cost estimation procedures discussed above, using the train process function to estimate trip time and to calculate the rate of output (in ton-miles per hour) produced by the train. Moreover, the behavior of cost as horsepower and number of cars vary may be observed.

In addition to the assumptions made above, it is further assumed for purposes of the example presented below: (1) there are no delays en route, (2) there is no grade, (3) there is no track curvature, (4) the commodities are dense enough that the cars are weight limited rather than space limited, (5) all cars are fully loaded, (6) trip length is 100 miles, (7) cars are standard box cars with tare weight of 30 tons each and load limit of 70 tons each, and (8) only one locomotive is used.

Figure 5.5 presents the cost per ton-mile of the assumed trip as horsepower and the number of cars change. These were obtained by expressing trip cost on a per-hour basis and dividing the result by the appropriate ton-mile per hour figure. Each curve represents a given horsepower locomotive pulling cars of from 10 to 200 in number. Each curve may be thought of as a "plant" unit cost curve, where horsepower represents the fixed input and number of cars represents the variable input. The unit cost curves for 500 to 2500 horsepower display the U-shape usually assumed to characterize plant cost curves. However, beyond 2500 horsepower the curves are continually downward sloping. This implies that trains substantially longer than 200 cars would be required to make the unit cost curves rise. An envelope curve could be drawn to the individual unit cost curves. This would represent the long-run average cost of the train trip to the firm. Although such a curve is not drawn in Figure 5.5, it is clear that economies of scale exist in the range of output from zero to 180,000 ton-miles per hour (a horsepower range of from 500 to 2500). Economies of scale, in fact, seem to persist throughout the range of output shown on the graph although at a much diminished rate beyond an output of 180,000 ton-miles per hour.

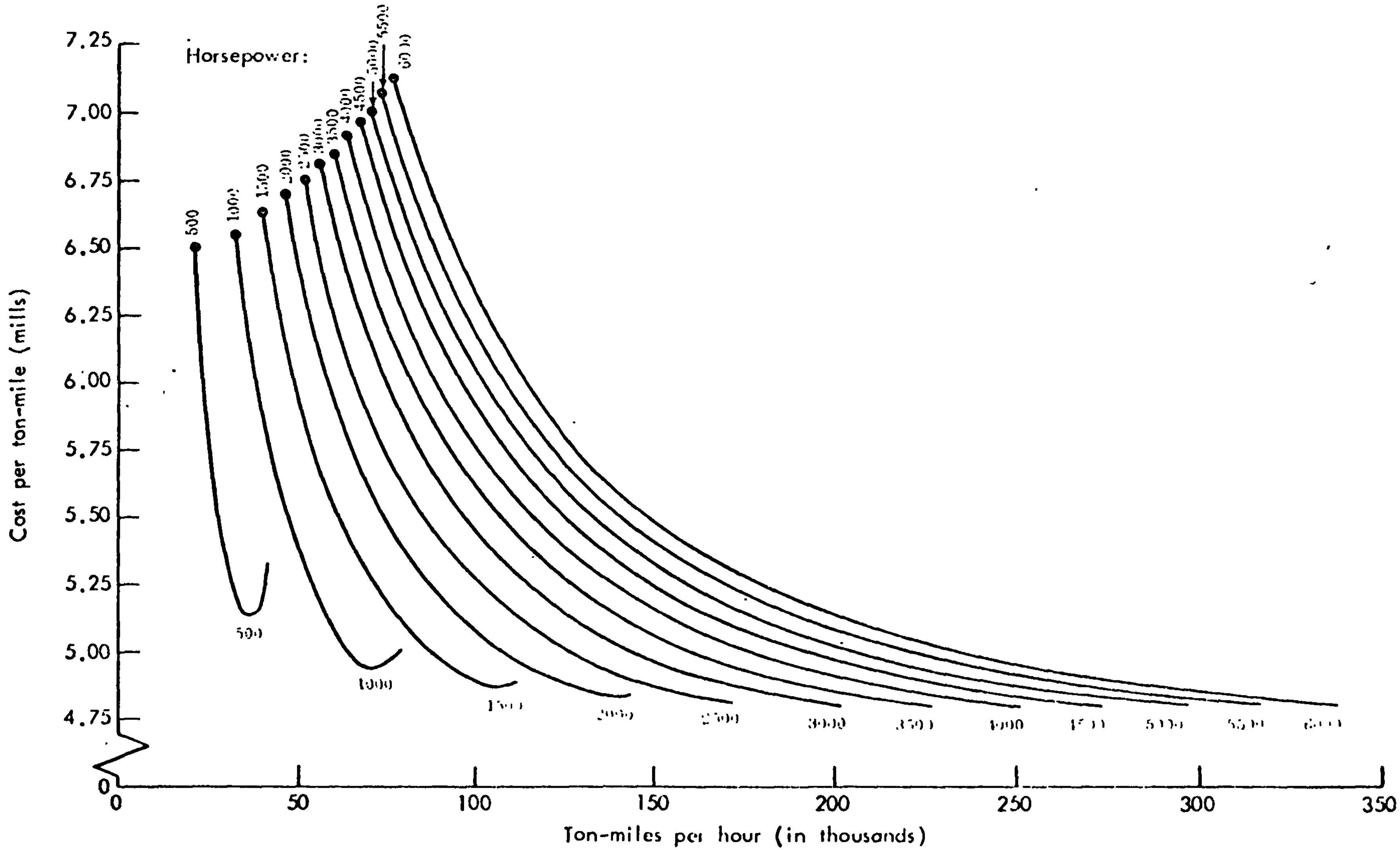

Fig. 5.5 -- Railway Firm Cost Curves

Summary. In this section, an attempt has been made to show how the rail linehaul process function may be used to generate linehaul costs. Linehaul plant unit cost curves were derived and some implications of them discussed. An example was also provided to show how an engineering-type process function could be used in conjunction with statistical analysis to provide estimates of the rail firm's costs. Unit cost curves were again derived and their implications discussed.

CHAPTER VI

SUMMARY AND CONCLUSIONS

The purpose of this dissertation is to provide a method for developing process functions for the linehaul operations of transport modes and to apply that method to two modes. An introductory chapter distinguishes between a production function and a process function, the latter being used throughout the dissertation to describe a production function for one process out of several that comprise the production activities of a transport mode. The derivation of cost functions from production functions is discussed in the introduction with an eye on the future use to be made of this discussion in Chapter V. A distinction is made between the statistical approach to estimating production and cost functions and the engineering approach. It is argued the latter possesses several advantages over the former. The engineering approach is followed in the dissertation.

Chapter II contains a rather detailed and complete survey of the previous research in cost and production relationships for inland waterway and rail transportation. Emphasis is on method but results are thoroughly discussed. Criticisms are made where they seem appropriate, but the review is a survey rather than a critique.

The method of analysis to be used throughout the remainder of the study is contained in Chapter III. This chapter details the form of the transportation linehaul process function which in its most general form makes provision for acceleration, deceleration, and delays as well as cruising speed of the vehicle. The general equation of motion of a vehicle is presented, and specific forms of it to be used later are also discussed. General acceleration equations are obtained for the case of a quadratic equation of motion, to be applied later to the case of barge tows. A brief discussion of delays and cargo capacity concludes the chapter. Since the latter two items are specific to a transport mode, no general delay or cargo analyses are provided.

Chapter IV contains the application of methods developed in Chapter III to the case of barge tows. Much of the analysis in this chapter relies on the work of Charles W. Howe. Howe's analysis is deficient in that no consideration of delays or of acceleration and deceleration is presented. This chapter rectifies those deficiencies by providing analyses of locking, making and breaking tow, miscellaneous delays (e.g., weather, running aground, etc.) as well as acceleration and deceleration.

It was found that the industry rule of thumb concerning acceleration is probably correct, i.e., it requires a distance of 1½ to 2 times the length of the tow to reach cruising speed from a stopped position. This ordinarily involves distances of between 1,000 and 2,000 feet. In the examples given in the text, acceleration time ranged from about 2½ minutes to 7½ minutes. It was concluded that, unless trips were very short or involved many stops and starts, acceleration could be ignored in the linehaul process function. A comparable deceleration analysis was not provided, but the conclusions reached for acceleration, which takes longer than deceleration, should apply _a fortiori_.

Of the myriad delays encountered by tows, locking and tow assembly and disassembly account for the major portion of delay time. A simple queuing model is developed for predicting locking delay. The locking model estimates the amount of time a tow can expect to spend in the process of locking on any given trip. The model is sensitive to the number of tows operating on the waterway (which is estimated from total tonnage to be transported on the waterway, average tow size, and an empty backhaul factor) and to the characteristics of the locks themselves. The model is checked against actual average locking times and found to predict well.

The time involved in assembly and disassembly of tows is found to be accurately estimated by linear relationships in which the number of barges comprising the tow is the independent variable. That is, making and breaking tow each requires a constant amount of time irrespective of the number of barges in the tow and an amount of time proportionate to the number of barges in the tow.

All other delay time (e.g., that due to weather, running aground, etc.) is classified as miscellaneous. The time consumed by miscellaneous delays is found to be proportionate to tow running time, the factor of proportionality being estimated as 14 percent. That is, a trip which would require 10 hours with no delays could be expected to encounter miscellaneous delays amounting to 1.4 hours.

Given these delay analyses, the tow linehaul process function is complete and may be used to estimate productivity and cost relationships in the same manner in which these were performed by Howe for the incomplete tow process function. Howe found decreasing returns to scale for his tow process function, i.e., equiproportionate increases in the barge and towboat inputs resulted in less than proportionate increases in output. While similar computations are not made above, there is no reason to believe Howe's result is changed. In fact, it seems likely that a greater degree of decreasing returns to scale would be exhibited by the linehaul process function inclusive of delays, for delay time is in all cases positively related to the number of barges in the tow. Making and breaking tow require more time the greater the size of the tow, and a larger towboat would not likely help very much in reducing that time (although this conjecture is not investigated in the preceding analysis). Likewise, locking time will increase as tow size increases; this is especially true where double and triple locking would be required from increases in tow size. Finally, even miscellaneous delays are likely to be greater as tow size increases. It was found that miscellaneous delay time is proportionate to running time, but running time is likely to be greater for large tows than for smaller ones since the former will travel more slowly. This may be thought to be offset to some extent by large towboats but, since Howe found decreasing returns to scale and diminishing returns to the towboat input, this offset is not likely to exist. All in all, it seems reasonable to say that the tow process function (including delays) will exhibit decreasing returns to scale. It is also likely, based on the above discussion, that Howe's conclusion of a flat long-run unit cost curve over a wide range of output would have to be altered. This would probably result in a U-shaped long-run average cost curve, which if flat at all would be flat over a narrower range of output possibilities. Of course, the level of the curve would be higher than that calculated by Howe, even if the same input costs were used, simply because of the inclusion of delay time.

Though not a component of the tow process function, an analysis of waterway capacity and tolls is presented in Chapter IV. This is an outgrowth of the locking model and may be viewed as an application of welfare economics to tow and waterway operations. The analysis provides a method of measuring the capacity of a waterway in terms of the utilization of its locks. A method of calculating the optimal toll to be charged for use of the waterway is also developed. The toll is optimal in the sense that if imposed it would equate private and social costs. Methods are, of course, provided for calculating these costs. A recapitulation of results ends the chapter.

Chapter V is an application of the method of Chapter III to the case of rail freight trains. For the purpose of the rail analysis, there is no previously existing process function as there was for the case of barge tows. However, there are much more widely accepted and well known engineering relationships available for rail operations than there are for barge operations. The chapter proceeds to exploit these relationships in developing the rail linehaul process function.

The two most important functions in determining the speed of a rail train are, as discussed in Chapter III, the tractive-effort function and the drag-force function. Both of these have been estimated, and their forms are well known in the railroad engineering literature. The tractive-effort function is taken over from the literature without modification. The drag-force function, however, is modified to include variables such as number of cars, number of locomotives, gradient of terrain, and degree of track curvature.

Given these two functions, one can determine the cruising speed of a train as a function of many interesting variables. The cruising speed of a train multiplied by the cargo tonnage of the train gives the ton-miles per hour produced by that vehicle, ignoring acceleration, deceleration, and delays and assuming that the train operates at full throttle. The implications and importance of these assumptions are discussed in Chapter V. No analysis of delays is provided in this chapter. A discussion of acceleration and deceleration, along with some estimating equations, is presented but no use is made of them in the remainder of the chapter. It may be noted that the rail linehaul process function, exclusive of delays and acceleration-deceleration, is similar to the original tow process function of Howe. That is, it is incomplete in several respects but may be used to provide insights into rail freight operations.

The chapter proceeds to investigate various productivity relationships of the rail freight train. Under certain simplifying assumptions, it is found that the marginal productivity of the car input is monotone decreasing. It is further shown that the marginal productivity of the locomotive input diminishes. The marginal products of both inputs remain positive within the ranges considered.

Isoquants between car and locomotive inputs are drawn and returns to scale investigated. It is found that the inputs are technical substitutes, but not perfect substitutes, and that the process function exhibits slightly increasing returns to scale. In addition to these findings, effects of gradient of terrain and track curvature on train performance are investigated. Increases in both of these factors result in decreases in train output, most notably for gradient where very small increments result in very large reductions in output. Finally the effect of carloading on train performance is investigated. A rationale is provided for the less-than-carload discriminatory pricing practiced by railroads.

Finally, the rail linehaul process function is used to estimate rail costs. In order to do this, however, the cost components of linehaul operations must be investigated. This leads to development of methods for estimating crew costs, fuel costs, interest cost on equipment, and equipment depreciation costs. All of these costs relate to the linehaul movement of commodities; there are no station expenses, administrative costs, etc. Given these cost estimation relationships and the rail linehaul process function, it is then possible to produce unit cost curves for the train trip. A short run average cost curve is developed by assuming a locomotive of given size but varying numbers of cars. A family of such curves is obtained by varying the size of locomotives. The typically assumed U-shaped average cost curve is obtained for small horsepower locomotives. But as locomotive horsepower increases (i.e., "firm" size increases) the short-run average cost curves become flatter and flatter. An envelope curve may be drawn to this family of short-run curves. Such an envelope represents the long-run unit cost curve of the train. The implied envelope curve indicates great economies of scale at low levels of output but much smaller economies at higher levels of output.

It is also shown in Chapter V how one might use both statistically derived costs and the rail process function to estimate the rail _firm's_ costs of a train trip. In order to illustrate this the statistical cost analysis of Meyer, Peck, Stenason, and Zwick is employed. They had divided total rail operating costs into several categories, e.g., train (linehaul), station, yard, etc. Each category was analyzed individually by means of regression analysis, with the goal of synthesizing the resulting relationships into an estimate of the rail firm's costs.

Since the analysis of this dissertation provides a method for estimating the train expense portion of the firm's costs as described above, that analysis was substituted for the statistical estimation of Meyer, Peck, Stenason, and Zwick. Thus, it was possible to generate cost curves for the rail firm in a manner similar to that described above. Were process functions available for other cost categories, such as yard operations, these could be incorporated into the statistical analysis as train expenses were. Thus it is seen that a combination of statistical and engineering costing may be used to generate the firm's cost curves.

The reader should be reminded that in the rail linehaul process function delays were ignored. Therefore, the productivities reported in the dissertation will in fact be lower if delays occur. In like manner cost curves will be higher than those shown above. Moreover, to the extent that delays are themselves functions of some of the same variables in the process function, then productivities and cost curves may assume different forms than those shown in Chapter V. It is difficult to say anything more on a priori grounds about the effect of delays on the shape of the productivity and cost curves. It is left to someone else to provide the required delay analysis for rail.

In conclusion, it might be said that the results of this dissertation should be of interest to two groups of people: to those who are interested in seeing examples of the "engineering approach" to the development of production functions and to those who are interested in the costing of rail and water freight transportation. It is hoped that the former group will be satisfied. Indeed, it is further hoped that they will see the possibilities of applying the method developed here to other transportation modes as well. Unfortunately for the latter group, the last word has by no means been said. It is hoped, however, that they recognize an approach to costing that could be fruitful and which is comparable across modes.

FOOTNOTES

Chapter I

1. Unless otherwise noted, the discussion in this and in the next subsection may be found in any intermediate price-theory textbook, e.g., James M. Henderson and Richard E. Quandt, Microeconomic Theory: A Mathematical Approach (New York, 1958), pp. 42-75.

2. The distinction between process and production functions made in the text may be found in A. A. Walters, "Production and Cost Functions: An Econometric Survey," Econometrica, XXXI (Jan.-Apr. 1963), pp. 11-14. An alternative use of the term is found in Alan S. Manne and Harry M. Markowitz (eds.), Studies in Process Analysis: Economy-Wide Production Capabilities (New York, 1963). To Manne and Markowitz "'process analysis' always refers to the construction and use of industry-wide, multi-industry, and economy-wide models which attempt to predict production relationships on the basis of technological structure." (p.4)

3. See Walters, p.13.

4. See Henderson and Quandt, p. 55 and the example on pp. 66-67.

5. An example of this approach may be found in Charles W. Howe, "Process and Production Functions for Inland Waterway Transportation" (Institute for Quantitative Research in Economics and Management, Paper No. 65, Purdue University, 1964), pp. 20-31.

6. Kurt Ehrke, Die Uebererzeugung in der Zementindustrie von 1858-1913 (Jena, 1933).

7. Theodore O. Yntema, United States Steel Corporation T.N.E.C. Papers, Comprising the Pamphlets and Charts Submitted by United States Steel Corporation to the Temporary National Economic Committee (Washington, 1940), I, 223-301.

8. Joel Dean, Statistical Determination of Costs with Special Reference to Marginal Costs (Chicago, 1963).

9. C. W. Cobb and P. H. Douglas, "A Theory of Production," American Economic Review, XVIII (March 1928), 139-65.

10. M. Bronfenbrenner and P. H. Douglas, "Cross Section Studies in the Cobb-Douglas Function," Journal of Political Economy, XLVII (Dec. 1939), 761-85.

11. Temporary National Economic Committee, "The Relative Efficiency of Large, Medium-sized, and Small Business," Monograph 13 (Washington, 1941).

12. J. Dean and R. W. James, "The Long-run Behavior of Costs in a Chain of Shoe Stores," Studies in Business Administration, XV (Chicago, 1942).

13. H. B. Chenery, "Engineering Production Functions," Quarterly Journal of Economics, LXIII (Nov. 1949), 507-31.

14. Allan R. Ferguson, "Commercial Air Transportation in the United States," Studies in the Structure of the American Economy, ed. Wassily Leontief (New York, 1953), pp. 412-47.

15. Statistical estimation of cost and production functions has been subjected to a great deal of scrutiny. One of the best surveys on this subject, in addition to the Walters article cited above, is John R. Meyer and Gerald Kraft, "The Evaluation of Statistical Costing Techniques as Applied to the Transportation Industry," American Economic Review, LI (May 1961), 313-34.

16. See Vernon L. Smith, Investment and Production: A Study in the Theory of Capital-Using Enterprise (Cambridge, Mass., 1966) pp. 24-30.

17. Walters, p. 12.

18. This classification has been used in statistical estimation of a rail production function. See George H. Borts, "Production Relations in the Railway Industry," Econometrica, XX (Jan. 1952), 71-79.

19. See, for example, Nancy Lou Schwartz, "Economic Transportation Fleet Composition and Scheduling, with Special Reference to Inland Waterway Transport" (Institute for Research in the Behavioral, Economic, and Management Sciences, Paper No. 91, Purdue University, 1964); M. L. Burstein, et al., The Cost of Trucking: Econometric Analysis (Dubuque, 1965).

20. See, for example, Charles R. Carr and Charles W. Howe, Quantitative Decision Procedures in Management and Economics: Deterministic Theory and Applications (New York, 1964), pp. 10-20.

21. Smith, pp. 45-55.

22. See, for an elaboration of this distinction, Jack Hirshleifer, "The Firm's Cost Function: A Successful Reconstruction?" Journal of Business, XXXV (July 1962), 235-55.

23. The latter of these is examined in Chapter V.

Chapter II

1. Howe, "Process and Production Functions ...." The portion of this paper relating to the process function has been published as "Methods for Equipment Selection and Benefit Evaluation in Inland Waterway Transportation," Water Resources, I (First Quarter 1965), 25-39. Citations will be made to the first of these papers.

2. A more general treatment of this assertion is presented in Chapter III.

3. Howe, p. 10.

4. See, for example, Henderson and Quandt, p. 46.

5. Howe, p. 29.

6. Charles W. Howe, "The Performance of Barge Tows: A Mathematical Representation" (unpublished paper, Resources for the Future, Inc., 1966).

7. Howe, "Process and Production Functions ...," pp. 35-61.

8. Ibid., p. 48.

9. Ibid., p. 60.

10. Charles W. Howe, "Models of a Barge Line: An Analysis of Returns to Scale in Inland Waterway Transportation" (Institute for Research in the Behavioral, Economic, and Management Sciences, Paper No. 77, Purdue University, 1964).

11. Arthur P. Hurter, Jr., "Production Relationships for Inland Waterway Operations on the Mississippi River: 1950, 1957, 1962" (unpublished paper, The Transportation Center, Northwestern University, 1965).

12. Arthur P. Hurter, Jr., "Cost Relationships for Inland Waterways Operations on the Mississippi River: 1950, 1957, 1962" (unpublished paper, The Transportation Center, Northwestern University, 1965).

13. A. Victor Cabot and Arthur P. Hurter, Jr., "Equipment Scheduling in River System Transportation" (unpublished paper, The Transportation Center, Northwestern University, 1965).

14. George H. Borts, "Increasing Returns in the Railway Industry," Journal of Political Economy, LXII (Aug. 1954), 316-33, esp. pp. 318-21.

15. As Borts points out, much of the work done on rail costs was done before the long-run envelope curve of economic theory had been developed. There is, therefore, little distinction made between long-run and short-run costs. Borts feels that the dichotomy between "variable" costs and "constant" costs is identical to the current economics usage only when the short-run cost function is linear. Otherwise, the early railway economists were including some variable cost in their constant cost category. The extent of increasing returns, as this term was used by the early railway economists, referred to the relationship between fixed and variable costs and is not the "economies of scale" resulting from firm size. See Borts, pp. 318-21.

16. W. Z. Ripley, Railroads, Rates, and Regulation (New York, 1927).

17. Rail Freight Service Costs in the Various Rate Territories of the United States (Washington, 1943).

18. "Cost and Value of Service in Railroad Rate Making," Quarterly Journal of Economics, XXX (Feb. 1916), 109-12.

19. The I.C.C. cost analysis is explained in I.C.C. Bureau of Accounts, Explanation of Rail Cost Finding Procedures and Principles Relating to the Use of Costs, Statement No. 7-63 (Washington, 1963), Chs. 1 and 2. References to the work of early railway economists are found in these chapters also.

20. See, for example, John R. Meyer, et al., The Economics of Competition in the Transportation Industries (Cambridge, Mass., 1960), Appendix A, pp. 274-76. Also see, W. J. Stenason and R. A. Bandeen, "Transportation Costs and Their Implications: An Empirical Study of Railway Costs in Canada," Transportation Economics (New York, 1965), pp. 121-23.

21. Meyer, et al., pp. 33-63, 177-320

22. Only a brief description of Meyer's study will be given here, for a much more detailed discussion is presented in Chapter V in illustrating the integration of statistical cost analysis with the engineering process function for rail.

23. Meyer, et al., p. 43.

24. The discussion of the Canadian rail costing methods is based on Stenason and Bandeen, pp. 123-38, and on Royal Commission on Transportation, III (Ottawa, 1962), pp. 193-365. Discussions with Mssrs. George Hanks and Victor Alalouf of the Canadian National Railways were very helpful also.

25. Meyer and Kraft, pp. 313-40.

26. George H. Borts, "The Estimation of Rail Cost Functions," Econometrica, XXVIII (Jan. 1960), 108-31.

27. Ibid., p. 118, n. 13.

28. Ibid., p. 120, See also G. W. Snedecor, Statistical Methods (Ames, Iowa, 1946), pp. 318-74.

29. George H. Borts, "Production Relations in the Railway Industry," pp. 71-79.

30. Soberman has provided a rail linehaul simulation model as part of a larger study being performed at Harvard University. While the Soberman model provides neither a production function nor a cost function for rail transport, it does use some of the basic engineering relationships to be employed below. Richard Soberman, "A Railway Performance Model" (unpublished paper, Harvard Transportation and Economics Seminar, Paper No. 45, Harvard University, 1966).

Chapter III

1. Travel-time expressions similar to those in the text may be found in R. H. Haase and W. H. T. Holden, Performance of Land Transportation Vehicles, RAND Memorandum RM-3966-RC (Santa Monica, 1964), pp. 11-13 and Appendix D. These expressions are not related by Haase and Holden to any sort of transportation production analysis.

2. Some of the notation of Haase and Holden has been adopted in this section. Where specific analysis of theirs is used, this will be indicated by individual citations.

3. Haase and Holden, p. 4.

4. The actual functions will be given in Chapter IV.

5. Haase and Holden, p. 97.

6. The actual functions will be given in Chapter V.

7. There are in fact three different possible sets of roots: (1) one real, two conjugate imaginary; (2) three real of which at least two are equal; and (3) three real and unequal roots. See Robert C. Weast et al. (eds.), Handbook of Mathematical Tables (Cleveland, 1964), p. 464. The fact that a unique real, positive root results from the solution of the rail equation of motion is discussed in Chapter V.

8. Haase and Holden, p. 107ff.

9. Ibid., p. 96.

10. Ibid.

11. See Haase and Holden, p. 107ff; also see Figures 4.1 to 4.3 below.

12. Haase and Holden, p. 121.

13. Ibid.

14. Ibid.

15. Ibid., p. 123.

16. Ibid.

Chapter IV

1. Much of the descriptive material in this subsection is based on talks with personnel of barging firms and on the volume Big Load Afloat (Washington, 1956) prepared by the American Water-Ways Operators, Inc.

2. Such a capacity table is reproduced in Howe, "Methods ...," p. 39.

3. Ibid., pp. 25-39.

4. Ibid.

5. U.S. Army Engineer Division, Ohio River, Corps of Engineers, Cincinnati, Ohio, Resistance of Barge Tows: Model and Prototype Investigation, August, 1960.

6. Howe, "Methods ...," p. 27.

7. Howe, "The Performance of Barge Tows ...," pp. 3-11.

8. W. B. Langbein, Hydraulics of River Channels as Related to Navigability, Geological Survey Water-Supply Paper 1539-W (Washington, 1962).

9. Hunter Rouse, Elementary Fluid Mechanics, (New York, 1946), p. 234ff.

10. Howe, "The Performance of Barge Tows ...," p. 14.

11. U.S. Army Division, Upper Mississippi Valley Division, Memorandum Report on Prototype Barge Resistance Tests, September 21, 1949.

12. See Robert H. Strotz and Robert M. Coen, "Estimating Explanatory Variables in Single and Simultaneous Equation Models" (unpublished paper, Northwestern University, n.d.), pp. 1-6.

13. Howe, "The Performance of Barge Tows ...," p. 22, Appendix C.

14. Ibid., pp. 14-17

15. Based on discussions with personnel of barging firms and with personnel of towboat and barge construction firms.

16. The Illinois Waterway project width and depth are given respectively as 225 ft. and 9 ft. in Big Load Afloat, p. 73. However, personnel of the Army Corps of Engineers suggested a depth of 12 ft. would be a better approximation.

17. Confirmed in a letter to the author from A. M. Martinson, Jr., Chief Marine Engineer, Dravo Corporation, Feb. 24, 1967.

18. Dravo Corporation, Push Towing, Bulletin No. 250 (Pittsburgh, n.d.), p. 5. Also letter from Martinson cited above.

19. Howe, "The Performance of Barge Tows ...," p. 13.

20. See, for example, Philip M. Morse, Queues, Inventories, and Maintenance (New York, 1958), pp. 12-13.

21. Ibid., pp. 7-12.

22. Ibid., p. 22

23. Gordon C. Hunt, "Sequential Arrays of Waiting Lines," Journal of the Operations Research Society of America, IV (Dec. 1956), p. 676.

24. Ibid. p. 682.

25. All data are from U.S. Army Corps of Engineers, Chicago District, Interim Survey Report: Duplicate Locks, Illinois Waterway, Jan. 25, 1967.

26. Eric Bottoms, "Practical Tonnage Capacity of Canalized Waterways," Journal of the Waterways and Harbors Division, Proceedings of the American Society of Civil Engineers, XCII (Feb. 1966), 33-46.

27. U.S. Army Engineers Division, Chicago District, Interim Survey Report ..., pp. A-8 to A-21.

28. Ibid.

29. Hourly tow operating costs may range from about $25.00, for a low-horsepower boat pushing one small barge, to over $500, for a large boat pushing a number of large expensive barges (e.g., liquid sulfur barges). These cost figures are based on Corps of Engineers data which are priviledged. Therefore, more specific information on tow operating costs cannot be given here. The figure of $100 used in the text is, of course, only for illustrative purposes.

30. See, for example, A. C. Pigou, Economics of Welfare 4th ed. (London, 1932), Part II, Chs. IX-XI.

31. For a discussion of the marginal conditions required to achieve a Pareto optimum, see M. W. Reder, Studies in the Theory of Welfare Economics (New York, 1947), pp. 21-38. A discussion of externalities may be found in ibid., pp. 62-67.

32. See Chapter II.

Chapter V

1. See C. L. Combes (ed.), 1966 Car and Locomotive Cyclopedia of American Practice (Chicago, 1966).

2. Ibid.

3. Ibid.

4. See W. W. Hay, An Introduction to Transportation Engineering (New York, 1961), pp. 200-201.

5. W. V. Davis, "Tractive Resistance of Electric Locomotives and Cars," General Electric Review, XXIX (Oct. 1926), 685-708. See also Hay, pp. 174-75.

6. For a derivation of the grade-adjustment factor, see W. W. Hay, Railroad Engineering (New York, 1953), I, 82-83.

7. But see ibid., pp. 52-55.

8. American Railway Engineering Association, Manual (Chicago, 1961), pp. 16-3-35.

9. For a derivation of these equations, see Hay, Railroad Engineering, pp. 134-39.

10. Ibid., p. 139.

11. Ibid., pp. 141-46.

12. Ibid., pp. 146-47.

13. Examples are: (1) M. Beckmann, C. B. McGuire, and C. B. Winsten, Studies in the Economics of Transportation (New Haven, 1956), Ch. VIII; (2) David Nippert, "Simulation of Terminal Operations," Simulation of Railroad Operations (Chicago, 1966), pp. 169-79.

14. Such an approximation is found in several places. See, for example, Hay, An Introduction to Transportation Engineering, p. 267 or E. E. Kimball, "Track Capacity and Train Performance," American Railway Engineering Association Bulletin, XLVIII (Nov. 1946), 133. It has been questioned by Mostafa K. K. Mostafa, "Actual Track Capacity of a Railroad Division" (unpublished Ph.D. Thesis, Department of Civil Engineering, University of Illinois, 1951), Ch. III. Mostafa suggests a Pearson's Type III Frequency Curve to fit the data.

15. This approach to estimating train linehaul delays is based primarily on the work of Kimball, pp. 125-44. The article referred to was a report of Committee 16, Economics of Railway Location and Operation, to the American Railway Engineering Association. The same approach was adopted, except for the minor criticism noted above, by Mostafa. Some additional applications of the approach are given in E. E. Kimball, "Report on Assignment 1: Revision of Manual," American Railway Engineering Association Proceedings, XLIX (March 1948), 2-14. The approach has been enshrined in a textbook by Hay, An Introduction to Transportation Engineering, pp. 266-71.

16. An additional assumption, of less importance, is that the number of axles is six. As was shown in Eq. (5.8) the number of axles is related to the weight of the locomotive. The above assumption tends to overestimate the weight of locomotives whose horsepowers range from 500 to 2,000, since these usually have four axles, and to underestimate the weight of locomotives whose horsepowers range from 4,500 to 6,000, since these usually have eight axles.

17. See Henderson and Quandt, p. 62.

18. Howe, "Methods ...," p. 32.

19. Hay, An Introduction to Transportation Engineering, p. 221.

20. Ibid., p. 445.

21. In 1965 the following expense categories relating solely to freight linehaul service for all rail districts amounted to $1,005,631,294: enginemen ($278,777,773), trainmen ($451,420,683), fuel ($258,117,538), water ($500,377), and lubricants ($16,824,923). The last two items constituted 1.7 percent of the total of these five cost categories. See Interstate Commerce Commission, Seventy-Ninth Annual Report of Transport Statistics in the United States for the Year Ended December 31, 1965 (Washington, 1966), Part 1, p. 93.

22. The description of train crew make-up, method of payment, and wage rates contained in this subsection were obtained from a Midwestern railroad.

23. American Railway Engineering Association, p. 16-3-35.

24. The figure used in the text for the Btu. content of a gallon of diesel fuel was obtained from the Marketing Department of the American Oil Company's Chicago Office. The range of variation was given as 135,680 to 141,700 Btu.'s per gallon.

25. This figure was used in the A.A.R. See Association of American Railroads, Incremental Railroad Costs in the Tennessee-Tombigbee (unpublished report, March 9, 1966), p. 22.

26. This, of course, assumes that thermal efficiency remains constant, an assumption which is implicitly made in engineering textbooks. See, for example, Robert G. Hennes and Martin I. Eske, Fundamentals of Transportation Engineering (New York, 1955), pp. 339-40.

27. Railroad engineers assume full power operation of trains in their calculations of trip time and average speed. See the discussion of plotting a "velocity profile" in any transportation engineering textbook, e.g., Hennes and Eske, p. 336.

28. See Vernon L. Smith, Investment and Production (Cambridge, Mass., 1961), p. 109.

29. Interstate Commerce Commission, Seventy-Eighth Annual Report on Transport Statistics in the United States for the Year Ended December 31, 1964 (Washington, 1965).

30. See Association of American Railroads, p. 22.

31. See Association of American Railroads, pp. 22-23. The Army Corps of Engineers also uses this method for calculating the capital cost of barges and towboats. See, for example, "Estimated Costs of Operating Towboats on Mississippi River System, January 1966,"

(unpublished report, Jan. 1966). This report and a similar one for barges are prepared annually by the Transportation Economics section.

32. A full reference citation to this work is contained above in note 20 to Ch. II.

33. Meyer, et al., p. 63.

34. Ibid., p. 48.

35. Ibid., p. 52.

36. Ibid., pp. 308-15.

37. The price-adjusted cost figures in the text do not reflect technological and institutional improvements that occurred in the intervening years. Since such improvements did occur, it is likely that the price-adjusted figures are too high. Ideally, one would want to perform the statistical analysis using more recent data, but this is precluded here by the magnitude of the task. Moreover, the purpose of this subsection is primarily to illustrate a method of combining statistical and engineering cost analysis. No claim is made that current rail costs are estimated.

BIBLIOGRAPHY

American Railway Engineering Association. Manual. Chicago, 1961.

American Waterways Operators, Inc. Big Load Afloat: U.S. Inland Water Transportation Resources. Washington, 1965.

Association of American Railroads. Incremental Railroad Costs in the Tennessee-Tombigbee Area. Unpublished report, 1966.

_______. Railway Statistical Manual. Washington, 1964.

Baier, L. A. Supplement to Resistance of Barge Tows: Model and Prototype Investigations. Cincinnati, 1963.

Beckmann, Martin, C. B. McGuire, and C. B. Winsten. Studies in the Economics of Transportation. New Haven, 1956.

Borts, George H. "Increasing Returns in the Railway Industry," Journal of Political Economy, LXII (Aug. 1954), 316-33

_______. "Production Relations in the Railway Industry," Econometrica, XX (Jan. 1952), 71-79.

_______. "The Estimation of Rail Cost Functions," Econometrica, XXVIII (Jan. 1960), 108-31.

Bottoms, Eric. "Practical Tonnage Capacity of Canalized Waterways," Journal of the Waterways & Harbors Division, Proceedings of the American Society of Civil Engineers, XCII (Feb. 1966), 33-46.

Bronfenbrenner, M. and P. H. Douglas. "Cross Section Studies in the Cobb-Douglas Function," Journal of Political Economy, XLVII (Dec. 1939), 761-85.

Bureau of Railway Economics, Association of American Railroads. A Guide to Railroad Cost Analysis. Washington, 1964.

Burstein, M. L., A. Victor Cabot, John W. Egan, Arthur P. Hurter, Jr., and Stanley L. Warner. The Cost of Trucking: Econometric Analysis. Dubuque, Iowa, 1965.

Cabot, A. Victor and Arthur P. Hurter, Jr. "Equipment Scheduling in River System Transportation." The Transportation Center. Northwestern University, 1965.

Carr, Charles R. and Charles W. Howe. Quantitative Decision Procedures in Management and Economics: Deterministic Theory and Applications. New York, 1964.

Cobb, C. W. and P. H. Douglas. "A Theory of Production," American Economic Review, XVIII (March 1928), 138-65.

Combes, C. L. (ed.). 1966 Car and Locomotive Cyclopedia of American Practice. Chicago, 1966.

Davis, W. V. "Tractive Resistance of Electric Locomotives and Cars," General Electric Review, XXIX (Oct. 1926), 685-708.

Dean, Joel and R. W. James. "The Long-run Behavior of Costs in a Chain of Shoe Stores," Studies in Business Administration. Chicago, 1942.

_______. Statistical Determination of Costs with Special Reference to Marginal Costs. Chicago, 1963.

Dravo Corporation. Personal letter from A. M. Martinson, Jr., Chief Marine Engineer. Feb. 24, 1967.

_______. Push Towing. Bulletin No. 260. Pittsburgh, n.d.

Edwards, Ford K. Rail Freight Service Costs in the Various Rate Territories of the United States. Washington, 1943.

Ehrke, Kurt. Die Ueberzeugung in der Zementindustrie von 1858-1913. Jena, 1933.

Ferguson, Allan R. "Commercial Air Transportation in the United States," Studies in the Structure of the American Economy. Ed. by Wassily Leontief. New York, 1953.

Haase, R. H. and W. H. T. Holden. Performance of Land Transportation Vehicles. RAND Memorandum RM-3966-RC. Santa Monica, 1964.

Hay, William W. An Introduction to Transportation Engineering. New York, 1961.

_______. Railroad Engineering. Vol. I. New York, 1953.

Henderson, James M. and Richard E. Quandt. Microeconomic Theory: A Mathematical Approach. New York, 1958.

Hennes, Robert G. and Martin I. Eske. Fundamentals of Transportation Engineering. New York, 1955.

Hirshleifer, Jack. "The Firm's Cost Function: A Successful Reconstruction?" Journal of Business, XXXV (July, 1962), 235-55.

Howe, Charles W. "Methods for Equipment Selection and Benefit Evaluation in Inland Waterway Transportation," Water Resources Research, I (First Quarter 1965), 25-39.

________. "Models of a Bargeline: An Analysis of Returns to Scale in Inland Waterway Transportation." Institute for Research in the Behavioral, Economic, and Management Sciences, Paper No. 77. Purdue University, 1964.

________. "Process and Production Functions for Inland Waterway Transportation." Institute for Quantitative Research in Economics and Management, Paper No. 65. Purdue University, 1964.

________. "The Performance of Barge Tows: A Mathematical Representation." Unpublished paper. Resources for the Future, Inc., 1966.

Hunt, Gordon C. "Sequential Arrays of Waiting Lines," Operations Research, IV (Dec. 1956), 674-83.

Hurter, Arthur P., Jr. "Cost Relationships for Inland Waterways Operations on the Mississippi River:, 1950, 1957, 1962." The Transportation Center. Northwestern University, 1965.

________. "Production Relationships for Inland Waterway Operations on the Mississippi River: 1950, 1957, 1962." The Transportation Center. Northwestern University, 1965.

Interstate Commerce Commission, Bureau of Accounts. Explanation of Rail Cost Finding Procedures and Principles Relating to the Use of Costs. Statement No. 7-63. Washington, 1963.

________. Seventy-Eighth Annual Report of Transport Statistics in in the United States for the Year Ended December 31, 1964. Washington, 1965.

________. Seventy-Ninth Annual Report of Transport Statistics in the United States for the Year Ended December 31, 1965. Washington, 1966.

Johnston, J. Statistical Cost Analysis. New York, 1960.

Kimball, E. E. "Report on Assignment 1: Revision of Manual," American Railway Engineering Association Proceedings, XLIX (March, 1948), 2-14.

________. "Track Capacity and Train Performance," American Railway Engineering Association Bulletin, XLVIII (Nov. 1946), 125-44.

Langbein, W. B. Hydraulics of River Channels as Related to Navigability: Geological Survey Water-Supply Paper 1539-W. Washington, 1962.

Leontief, Wassily (ed.). Studies in the Structure of the American Economy. New York, 1953.

Lorenz, M. O. "Cost and Value of Service in Railroad Rate Making," Quarterly Journal of Economics, XXX (Feb. 1916), 109-12.

Manne, Alan S. and Harry M. Markowitz (eds.). Studies in Process Analysis: Economy-Wide Production Capabilities. New York, 1963.

Meyer, John R. and Gerald Kraft. "The Evaluation of Statistical Costing Techniques as Applied in the Transportation Industry," The American Economic Review, LI (May 1961), 313-40.

Meyer, John R., Merton J. Peck, John Stenason, and Charles Zwick. The Economics of Competition in the Transportation Industries. Cambridge, 1960.

Morse, Philip M. Queues, Inventories and Maintenance: The Analysis of Operational Systems with Variable Demand and Supply. New York, 1958.

Mostafa, Mostafa K. K. "Actual Track Capacity of a Railroad Division." Unpublished Ph. D. Thesis. Department of Civil Engineering, University of Illinois, 1951.

National Bureau of Economic Research. Transportation Economics. New York, 1965.

Nippert, D. "Simulation of Terminal Operations," Simulation of Railroad Operations. Chicago, 1966.

Pigou, A. C. Economics of Welfare. 4th ed. London, 1932.

Railway Systems and Management Association. Simulation of Railroad Operations. Chicago, 1966.

Reder, M. W. Studies in the Theory of Welfare Economics. New York, 1947.

Ripley, W. Z. Railroads, Rates, and Regulation. New York, 1927.

Rouse, Hunter. Elementary Fluid Mechanics. New York, 1946.

Royal Commission on Transportation. 3 Vols. Ottawa, 1961.

Schwartz, Nancy Lou. "Economic Transportation Fleet Composition and Scheduling, with Special Reference to Inland Waterway Transport." Institute for Research in the Behavioral, Economic, and Management Sciences, Paper No. 91. Purdue University, 1964.

Smith, Vernon L. Investment and Production. Cambridge, 1961.

Soberman, Richard. "A Railway Performance Model." Unpublished paper, Harvard Transportation and Economics Seminar, Paper No. 45. Harvard University, 1966.

Stenason, W. J. and R. A. Bandeen. "Transportation Costs and Their Implications: An Empirical Study of Railway Costs in Canada," Transportation Economics. New York, 1965.

Strotz, Robert H. and Robert M. Coen. "Estimating Explanatory Variables in Single and Simultaneous Equation Models." Unpublished paper. Northwestern University, n.d.

Temporary National Economic Committee. "The Relative Efficiency of Large, Medium-sized, and Small Business." Monograph 13. Washington, 1941.

U.S. Army Corps of Engineers, Chicago District. Interim Survey Report: Duplicate Locks, Illinois Waterway. Chicago, 1957.

U.S. Army Corps of Engineers. Resistance of Barge Tows: Model and Prototype Investigations. Cincinnati, 1960.

U.S. Army Corps of Engineers, Upper Mississippi Valley Division. Memorandum Report on Prototype Barge Resistance Tests. Unpublished report. U.S. Army Corps of Engineers, 1949.

Walters, A. A. "Production and Cost Functions: An Econometric Survey," Econometrica, XXXI ((Jan.-April 1963), 1-66.

Weast, Robert C., Samuel M. Selby, and Charles D. Hodgman (eds.). Handbook of Mathematical Tables. 2nd ed. Cleveland, 1964.

Yntema, Theodore O. United States Steel Corporation T.N.E.C. Papers, Comprising the Pamphlets and Charts Submitted by United States Steel Corporation to the Temporary National Economic Committee. Washington, 1940.

VITA

Joseph S. DeSalvo was born on August 6, 1938, in Jacksonville, Florida. He attended primary and secondary schools there. After graduation from high school in 1956, he attended the University of Florida. Following the receipt of a B.A. in Economics in 1960, Mr. DeSalvo entered graduate school in the field of Economics at the University of Florida, where he was awarded a master's degree in 1961. Mr. DeSalvo accepted an Instructorship in Economics at The Virginia Military Institute and taught there for two years. While at V.M.I., he was awarded an Economics-in-Action Fellowship by Case Institute of Technology and spent the summer of 1962 at that institution. Mr. DeSalvo left V.M.I. to enter the Northwestern graduate program in 1963, from which he received a Ph.D. in Economics in June, 1968. Mr. DeSalvo is at present a member of the professional staff of The RAND Corporation. He is married and has two children.

NORTHWESTERN UNIVERSITY

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TABLE OF CONTENTS

CHAPTER I

INTRODUCTION AND REVIEW OF THE LITERATURE

What is the cost of moving a ton of freight from some origin A to some destination B? Answers to this question are necessarily complex since they depend upon the mode of transport used, the length of haul, the commodity shipped, and whether a backhaul is required. This study examines this question for one mode of transport: Barge transportation on inland waterways.

Essentially, two different methods can be employed to arrive at estimates of transport cost. One method is based on the use of engineering production functions. The engineering relationship shows the technological substitution possibilities between the various inputs (which should be stated in stocks <u>and</u> flows) required to perform a given task (for barge transportation, the point-to-point movement of commodities and equipment). If the prices of the inputs are known, long-run and short-run cost relationships can be derived. In the case of barge transportation, one needs the costs associated with various kinds of equipment (towboats and barges), costs of a crew, the characteristics of the waterway, and a measure of delay time. With this information the cost of a particular point-to-point movement can be determined. An engineering relationship developed in this way could be termed a "process function" since it describes the alternative ways of performing a particular task. It does not describe the production relationship when several tasks are performed either simultaneously, or sequentially (under some conditions it can be aggregated), nor does it describe the cost-output relationship.

The second method is the use of statistical estimation procedures. This is a more direct and traditional approach to obtaining cost estimates. "Basically, statistical methods are a substitute for experimental controls in attempting to establish and measure causal relationships, statistical techniques being used when controls are either unavailable or too expensive."[1] The statistical approach is based on observed rates of output and expenditure collected by firms. Cross-section or time-series data are used in the framework of an appropriate statistical model to determine the parameter values. The selection of the model may be based on theoretical or technological knowledge, or simply on goodness of fit.

The two methods are not mutually exclusive. Statistical methods are frequently employed in deriving the technological parameter estimates of engineering relationships; knowledge of technological constraints can be used to select the appropriate model for statistical production or cost estimates. The two approaches do, however, possess different interpretations. The engineering relationships, as they embody the available technology, describe the set of alternative ways of performing a particular task independent of the relative prices of the inputs. The engineering relationships thus describe the technologically most efficient ways of accomplishing the task. Given the prices of the various inputs, the economically efficient production method can be selected, that is, the method which produces a given output at the least cost. Cost relationships obtained in this way, then, represent the cost curves of economic theory.

*All footnotes appear following text, beginning on p. 152.

On the other hand, the statistical cost-output relationships, since they are based on actual rates of output and expenditure, are a summary of the average operating conditions for the period observed. Since they do not carry the interpretation of technological efficiency or cost minimization at each output level, they do not represent the cost relationships of economic theory.

Nevertheless, statistical relationships, if properly interpreted, can yield extremely useful information about the operations of a firm. One could assume, as is frequently done, that the observed data do represent minimum cost points, that is, that the firm is at, or near, an optimum. The estimated results can then be interpreted as the long-run costs of economic theory. This, however, is not likely to be true. The observations will most certainly reflect the operation of a firm where some short run adjustments are still taking place. Another interpretation seems more reasonable: Assume that the observations represent a certain average level of efficiency which has been achieved in the past and is likely to prevail for some time. The cost relationships could then be used to draw conclusions, for example, about the expected effects of increasing firm size.

Since statistical relationships are derived from the operations of firms, one can be sure that these functions represent actual operations. No such guarantee can be given for engineering functions. We know that some inputs (such as management) are omitted. Even the engineering process functions are simplifications rather than the exact physical relations. Thus, there is no guarantee that these relations can be realized in actual operations.

Both the engineering and statistical methods have been employed in studies of rail and barge transport cost. A summary of the more significant studies in this area is presented below.

A. Review of Cost and Production Literature in Rail and Barge Transportation

Rail Cost Literature

Considerable attention has been devoted in the past to studies of rail transport costs. Although the primary concern of this study is the cost of barge transportation, investigators of rail cost have dealt with problems common to both modes. For this reason a survey of the more recent studies of rail cost have been included in this review.

Meyer, Peck, Stenason, and Zwick (MPSZ) have estimated long-run cost relationships for freight and passenger rail traffic.[2] Their study is distinguished from those done in the past in two ways. The first is that they stratify railroad costs into much finer categories than in the past. This is a recognition that railroad transportation is a many faceted operation. "Specifically, total railroad operating expense includes all the labor, fuel, and miscellaneous variable costs associated with the operation of trains, yards, and stations . . ." (p. 34). In addition there are the costs of marketing, supervision, and maintenance. Obviously the behavior of cost in relation to output will be quite different for different parts of the operation. Also a single output measure will not adequately describe the activities of all aspects of rail transport.

Five categories of operating cost are included by MPSZ: general (administrative and legal overhead), traffic (selling and marketing), station, train (line-haul), and yard. Maintenance cost, depreciation, and capital outlays are also treated as separate categories. The data used were cross-section observations on the twenty-seven largest United States class I rail systems (excepting the New York Central and the Pennsylvania Railroad). The sample was taken from data collected by the Interstate Commerce Commission from 1947 to 1955.

The second distinguishing feature of the MPSZ study is that the data are not used in ratio form. A difficulty with cross-section studies is that almost all the differences between observations in a sample seem to be due to the influence of firm size. A common method employed to eliminate this influence has been to deflate the cost and output observations by the size of the firm. An important consequence of this procedure for linear regression equations is to overstate the constant term and hence overstate the extent of scale economies. According to MPSZ, ". . . these scale economies probably are best attributed to nonoptimal factor proportions at lower levels of output . . ." (p. 38).

As noted above, the MPSZ study is based on cross-section data. As a result, the authors consider their estimates to represent long-run cost relationships. The possibility of transient short-run influences in cross-section data are recognized; however, these influences are discounted by the authors because, ". . . interfirm variance over the cross section is almost surely many times the intrafirm variance attributable to these short-run influences . . ." (p. 41). This is particularly true in the MPSZ case, since the authors use two and four year averages in estimating their cost functions.

The authors estimate relationships for each cost category. A description of the procedures followed for each category of cost would necessarily be extremely detailed and has been omitted. The procedure followed was to regress each cost category on its associated output measure. Linear regression equations were employed.

The authors simplify their analysis of different cost components into "typical" over-all relationships between costs and gross ton-miles. This is done by making "normative" assumptions about the relationships between gross ton-miles and the other output measures (such as yard hours or train-miles) employed in the individual category regressions. These normative assumptions are apparently industry rules-of-thumb obtained from discussions with railroad operating officials.

The results of this summarization are presented in Table 1. To obtain the total cost of a particular trip, one calculates the number of gross ton-miles to be generated and then multiplies this by the relevant total long-run marginal cost coefficient. The coefficients can be adjusted by price indices when considering subsequent periods. In this way, the total cost (or cost of particular operations) can be determined for freight and passenger rail traffic; however, as the authors point out, reasonable caution should be employed in making such estimates. "It should be noted that these estimates are based on a sort of central tendency and are typical figures that will apply to freight movements only of a very average or ordinary kind." (p. 63). This is due primarily to the nature of the normative assumptions that were made. In reducing yard expenses to a gross ton-mile basis, for example, the authors assume that twenty-four minutes are required ". . . to originate, classify, and terminate the typical merchandise car, [and] that this car goes approximately 400 miles, and that it has a load of 25 tons . . ." (p. 48). Nevertheless, the MPSZ results are potentially quite useful in determining the cost of typical railway traffic and should be useful in making regulatory deci-

TABLE 1

SUMMARY ESTIMATES OF RAILROAD OPERATING COSTS THAT VARY IN THE LONG RUN*

	1947-1950 results (1947-1950 mills)		1952-1955 results (1952-1955 mills)	
	Per G.T.M. of freight traffic	Per G.T.M. of passenger traffic	Per G.T.M. of freight traffic	Per G.T.M. of passenger traffic
Long-run marginal train, station, yard, traffic, and general expenses...	0.8721	4.2330	1.700	4.50
User cost portion of maintenance costs...	.6900	1.4100	0.910	1.53
Variable portion of depreciation expenses...	.2120	0.9760	.216	1.00
Total long-run marginal operating costs...	1.7741	6.6190	2.826	7.03
Variable portion of capital costs at 6 1/2 percent interest rate...	0.4011	1.4079	0.445	1.48
Total long-run marginal operating and capital costs...	2.1752	8.0269	3.271	8.51

*Source: John R. Meyer et al., The Economics of Competition in the Transportation Industries (Cambridge, Mass.: Harvard University Press, 1964), p. 62.

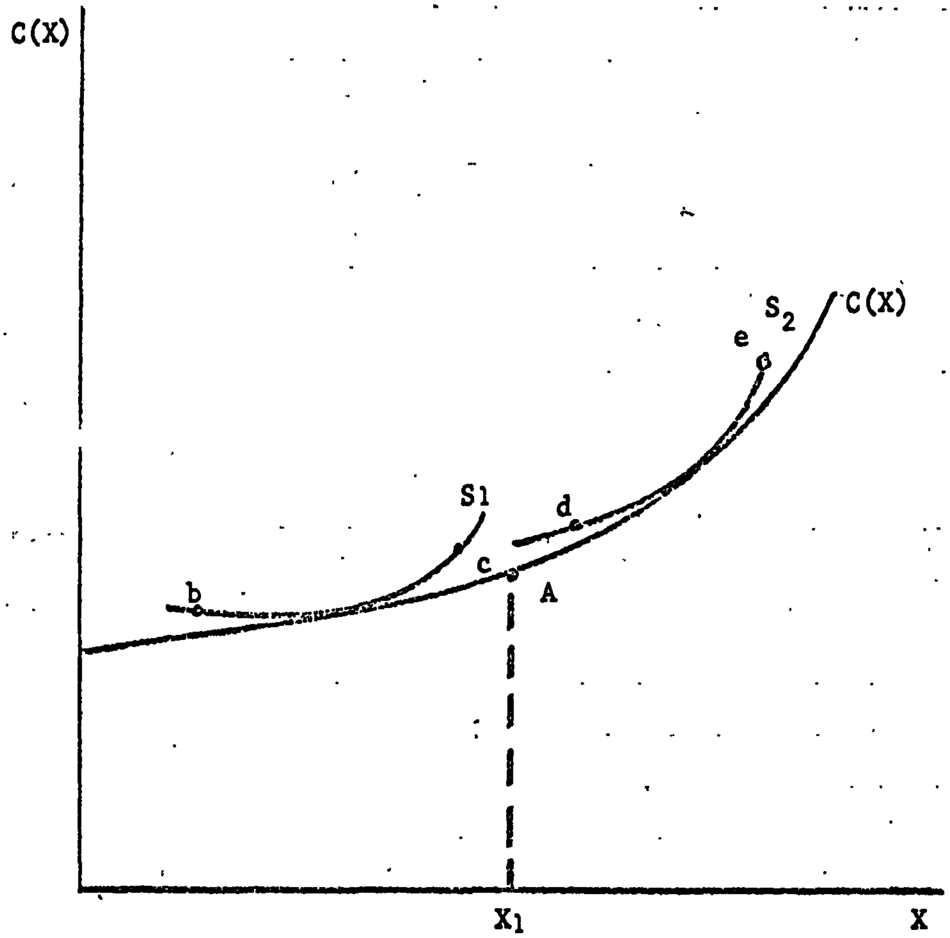
sions particularly with respect to rate making. Finally as the authors point out, even if their results are not perfect they may represent the best alternative approach to determining rail costs, and ". . . cost determination is almost always not a matter of obtaining an exact answer but of getting the best available answer." (p.44).[3]

Borts[4] argues that the use of cross-section data in statistical cost estimation can produce biases in the measurement of returns to scale. After discussing the source of these biases, Borts outlines a method for avoiding them and applies this method to cross-section data for United States railroads.

His argument is illustrated in Figure 1. (Borts, p. 110). C(X) is the long-run total cost function for each firm in the industry. S_1 and S_2 are short-run total cost curves for two plants, one large and one small. X_1 is the output where the firm begins to experience increasing returns to scale. If all of the observations of a cross-section data set lie on C(X), and if all of the observations are for plant sizes below X_1, then the intercept term of a simple linear regression equation will provide a reasonable estimate of return to scale in the industry.

However, it is more likely that the data will represent points such as b, c, d, and e in Figure 1. In this case, the estimated regression equation will lie above C(X) throughout its range yielding an over-estimate of the intercept term. According to Borts, the most frequently employed method of correcting for this problem is to explicitly introduce some measure of plant size in the regression equation. The size coefficient can then be interpreted as meaning ". . . the extent to which the firm is operating above its long-run cost function as well as the position of the plant curve relative to the envelope." (p. 109).

FIGURE 1



Results obtained in this way, however, may be misleading due to the influence of the regression fallacy. Firms are subject to random fluctuations in the demand for their product. As a result, the output produced by each firm is a random variable and the variation of output about the mean value is not controlled by the firm. The firm then tries to optimize its plant size for the production of some distribution of outputs. When firms are observed at a point in time, as in cross-section data, firms with the largest outputs are most likely to be producing at unusually high output levels (such as points c and e in Figure 1). The opposite is likely to be true for firms with the lowest output. Thus, classification of the firms by actual output, may lead to an understatement or overstatement of the firm's long-run (planned) output rate. "Under these conditions, the interpretation of the estimated size coefficient is ambiguous." (p. 110). For example, a positive sign for the coefficient of firm size may indicate that abnormally large expenditures were incurred which are attributable to a divergence of the planned and actual output rates. It may not indicate, according to the usual interpretation, that the firm could reduce its cost by selecting a smaller size plant.

Borts proposes an alternative method of cost estimation, that of stratifying the observations by firm size. Besides avoiding the bias due to the regression fallacy, this approach ". . . permits a test of the presence of the regression fallacy through a comparison of between size- and within size-class estimates of marginal and average costs . . ." (p. 114).

This method is applied by Borts to a cross-section of sixty-one United States railroads. Linear regression techniques are employed.

The effects of two types of classification are examined: the size of firm and the region in which the railroad operates. The regional classification is used to account for spatial price differences, and regional differences in physical operating conditions. Due to the small sample size, shift parameters are used to take account of regional and size differences. Covariance analysis is then used to examine the influence of these variables. The hypotheses tested are that the cost-output relationships are significantly affected by both the size of firm and the region of operation.

Two types of cost behavior are observed in Borts results; each is associated with different regions in Borts classification. "It was found that long-run increasing costs prevailed only in the Eastern region, while long-run decreasing or constant costs occurred in the South and West." (p. 116). According to Borts, this evidence on regional differences suggests that different regulatory policies should be designed to meet the varying conditions among regions. In the East, ". . . the question of basing charges on identifiable cost can be reopened, once it is possible to ignore the problem of allocating indirect expenses . . . [In the West and South] the evidence indicates a continuation of the historical problems raised by overbuilding, because of the possibility of continued increasing returns." (p. 128).

In a study of railroad cost in Canada, Stenason and Bandeen provide a critique of current cost finding techniques of the Interstate Commerce Commission.[5] The authors present an alternative costing system used by the Canadian National Railroad and the Canadian Pacific Railroad Company.

According to the authors,

> . . . the ICC procedures are a compromise between the need of the accountant to record all the costs in a specific set of accounts, often on a somewhat arbitrary basis, and the desire of the economist to trace expenses to the output that occasions them, in order to determine the costs associated with specific changes in service or output. (p. 122).

Current ICC cost finding techniques are based on eighty year old cost classifications which succeed in assuring uniform reporting of expenses by the carriers, but are not adaptable to cost analysis.

In the ICC procedures total cost in each category is allocated to five service areas (linehaul, switching, station, special services, and overhead). The allocation is quite arbitrary. For example, an analysis of time spent in different stations by trains is used to allocate station labor costs among different service categories. To determine the variable portion of total expense, simple linear regression techniques are employed using cross-section data. The procedure is to regress total operating expense per mile of road on gross ton-miles per mile of road (density). The intercept coefficient of the regression equation is then subtracted from the average value of expense for United States railroads. The figure derived from this calculation is expressed as a percent of the average value of expense per mile of road. This percentage is then interpreted as the percent of total expense which varies with output.

The authors find this procedure unacceptable for several reasons. One is that the arbitrary allocation of costs to separate service categories does not allow ". . . an estimate of the extent to which total expenses will change as a result of change in traffic volume or in its mix." (p. 123). A second criticism is that the use of ratios in the regression equations will produce biased results unless the numerator and denominator of the ratio are homogeneously distributed. In addition, the authors point out that even in the absence of statistical bias, the regression procedure is invalid as it ignores the length of haul, traffic volume, traffic density, and the commodity carried. Finally, the authors note that the association of variable costs with a single output measure, gross ton-miles, ignores the multi-product character of railroad output.

The authors are led to reject the ICC procedures for use on Canadian railroads. They describe research conducted in Canada which has resulted in a more sophisticated cost finding procedure. This procedure has been adopted by Canadian regulatory agencies for their cost analysis. The procedure is based on actual recorded costs in the Canadian system of railroad accounts and a knowledge of technical engineering and operating features of railroad systems.

The procedure is to first define and quantify measures of output corresponding to different aspects of the transport process and which are applicable to particular categories of traffic. For example, a determination was made of the number of minutes required to switch freight cars of different types and having particular origins and destinations. The principle output measures (traffic units) analyzed are: switching minutes per car, gross ton-miles, train miles, diesel unit miles (reflecting the number of diesel units used on a train for each train run), car-days, and car-miles.

The next step was to develop a set of cost factors or coefficients which reflect expenses that are variable with respect to the different output measures. "This involved estimating many hundreds of regression models and selecting those which best fitted the many tests made of them." (p. 127).

Once the service or work requirements of particular shipments are known and knowing the marginal cost associated with each type of service, the marginal cost of any particular point-to-point movement can be calculated. The various factors and procedures have been formalized so that they can easily be applied in particular circumstances. Quite apparently, such a cost finding technique is superior to the one employed by the ICC. It avoids arbitrary allocations and recognizes the complexity of the rail transportation process.

Barge Transport Literature

The work of Charles W. Howe and Arthur P. Hurter represents the entire body of literature concerned with production and cost relationships for barge transportation.

Howe[6] has developed a production function for the point-to-point movement of a barge tow based upon engineering relationships taken from naval design theory. His approach is to solve a system of equations for the equilibrium speed of a tow. The model, in functional notation, is:

(1) $R = f(L, B, H, S, D, W)$

(2) $EP = g(HP, S, D)$

(3) $EP = R$

where: R = resistance of flotilla in tow-rope horsepower

EP = effective push of towboat

D = depth of waterway in feet

H = draft of barge in feet

S = speed in miles per hour

W = width of waterway in feet

B = breadth of flotilla in feet

L = length of flotilla in feet

HP = horsepower of the towboat

The resistance of a flotilla as it is pushed through the waterway depends on its velocity, the characteristics of the flotilla (length, breadth, and draft), and the characteristics of the waterway (depth, and width). The push generated by a towboat depends upon the horsepower and speed of the boat and the depth of the waterway. The equilibrium condition, given in (3), must hold for a tow operating at a constant speed.

Since naval design theory does not provide the exact form of either the resistance function or the effective push function, it was necessary for Howe to estimate the parameters of the two relationships statistically. The data used to estimate the functions were derived from tank and prototype tests conducted by the U. S. Army Corp of Engineers and barge equipment manufacturers, and speed tests conducted with actual equipment by barge operators.

According to Howe, "Extensive use of regression analysis showed that the following functional form fitted the resistance test data quite well:" (p. 27).

$$(4) \quad R = \alpha_0 \, e^{\alpha_1 \, (1/D-H)} \, S^{\alpha_2} \, H^{\alpha_3 + \alpha_4 \, (1/W-B)} \, L^{\alpha_5} \, B^{\alpha_6}$$

This, Howe used as a modified log-linear resistance function which is ". . . consistent with what is known about resistance phenomena." (p. 27).

The resistance data were stratified into five groups according to flotilla configuration. Equation (4) was then estimated for each configuration separately and again for a weighted average of the five groups. His results show significant parameter difference among the stratified regressions. Nevertheless, Howe adopts the combined regression to use in his analysis. Howe recognizes that this is a questionable procedure since structural differences have been observed, however, he proceeds ". . . on the pragmatic grounds that the resulting function will be used to evaluate a wide variety of tow characteristics." (p. 27).

The effective push relationship was examined in a similar fashion. The adopted form was:

$$(5) \quad EP = B_1HP + B_2HP^2 + B_3HP \cdot D + B_4S^2 + B_5S \cdot HP$$

According to Howe, "The HP^2 term allows for increasing or decreasing effectiveness of HP in determining push. [The coefficient of HP^2 was negative, indicating diminishing productivity of HP in higher HP ranges.] The cross-product term, HP·D, reflects the fact that gains from greater depth are greater for larger boats and vice versa. [The coefficient of HP·D was positive, an indication that this prediction of navel design theory is correct.] Finally, the S·HP term reflects the fact that $\partial EP/\partial S$ is a decreasing function of HP . . ." (p. 29). The coefficient of S·HP was negative, indicating that the push generated by a towboat of given horsepower declines as the velocity of the tow increases.

Using the estimated relationships (1) and (2) and the equilibrium condition for constant speed (3), Howe was able to solve the system of equations for the equilibrium speed, S. In functional notation, we have:

(6) $S = h(HP, L, B, H, D, W)$

The actual form of the equilibrium speed relationship is very complex and has been omitted. However, it should be noted that when L, B, and H are specified, the cargo tonnage of the tow is also specified. Thus by rewriting the function (6), we can derive the output of the tow in cargo ton-miles per hour:

(7) $TM(HP, L, B, H, D, W,) \equiv S(HP, L, B, H, D, W) \cdot T(L, B, H)$

where TM represents the output of the tow measured in net cargo (short) ton-miles per hour. Such a function is of considerable interest since it relates the tow's output to (1) waterway characteristics under the control of the Army Corp of Engineers, and (2) to the tow inputs (horsepower and flotilla configuration) under the control of the individual barge firm.

To derive the unit cost functions from his analysis, Howe defined his productive inputs to be horsepower of the towboat (assuming that labor and fuel costs are proportional to horsepower) and the deck area of the barge input. Howe then obtained from barge firms average daily operating costs of towboats of various horsepowers and of open hopper jumbo barges.

Using (7) and the related cost information, Howe was able to generate (1) total product curves as a function of flotilla area for various horsepowers, holding waterway characteristics constant, (2) iso-quants for horsepower and deck area, holding waterway characteristics constant, (3) a family of average cost curves for particular flotillas, allowing horsepower to vary, and (4) the effects on tow performance of changes in waterway characteristics, W and D.

Since the function (7) is discontinuous in the barge inputs, it was necessary to evaluate the function over a set of specified values of all inputs. According to Howe, "these values cover farily well the range of operating conditions found on the Ohio, Upper Mississippi, Missouri, Illinois Waterway, and the Gulf Intracoastal Canal." (p. 30).

The set of derived relationships exhibits reasonable properties. In particular it was discovered that, (1) both the tow boat and barge inputs showed diminishing productivity (2) the tow shows "decreasing returns to scale" which can be attributed to the waterway inputs, and (3) "U-shaped" average cost curves, costs rising beyond some point ". . . because speed reductions more than offset the effects of additional carrying capacity." (p. 36).

Howe recognized that his process function was not adequate for describing the activities of a barge firm in organizing its traffic into tows and in scheduling the movement of those tows across the waterway. As was demonstrated by the process function, the point-to-point operation of a tow was subject to decreasing returns to scale due to the limitations of the waterway. However, the firm " . . . may experience constant returns because of its ability to duplicate individual operating units as total output grows."[7]

Howe then turns to developing production functions for the barge firm. Of particular interest is his discussion of how the capital inputs should enter the production function. Howe argues that the entire capital stock in barge transportation is not always used at any one time. Observed current output is thus related to the portion of the capital stock actually used. It is not related to the capital stock which remains idle. Under these circumstances, "It would not be adequate to measure capital utilization simply by counting the number of units which were utilized . . ." (p. 40). The appropriate measure for the capital input is thus the flow of capital services. Howe called a function estimated using capital services a production function.

The idle capital stock, however, is important for meeting the firm's overall objectives. This can easily be seen when it is realized that barge firms (and other transport firms as well) are subject to random fluctuations in the demand for their services. Firms must make decisions about the optimal stock of equipment to have on hand as well as what equipment to use in meeting current demands. Thus, the relationship between capital stock and output is also of interest. However, Howe recognizes that such a relationship extends beyond the technological constraints embodied in a production function, involving ". . . all of the variables which must be considered in dynamic long-run profit maximization." (p. 40). Howe refers to such a relationship as a "planning function."

The two relationships are viewed as providing complimentary information on the firm's operation. The two functions can be compared to gain insights into returns to scale. That is, suppose the firm has increasing returns to the production process (because a large cargo volume allows for a high proportion of larger more efficient tows), but experiences decreasing returns reflected through the planning function. This might indicate increasing complexities in scheduling so that the stock of capital must be greatly increased as the rate of output increases.

Howe estimates both kinds of functions. His production function uses monthly time-series data for three firms representative of different operating conditions on the Mississippi River System. The production function estimated was:

$$(8) \quad TM = \alpha_0 B^{\alpha_1} H^{\alpha_2} \mu$$

where: TM = cargo ton-miles

B = equivalent barge days underway, full and empty

H = operating horsepower hours

The log-linear form of the function was estimated. Note that the inputs are measured as flows. The function was estimated separately for each firm.

Howe's planning function uses combined cross-section, time-series data for six firms. The estimated function was:

$$(9) \quad TM = \alpha_0 B^{\alpha_1} HP^{\alpha_2} T^{\alpha_3} \mu$$

where: TM = annual cargo ton-miles

B = number of equivalent jumbo barges owned or leased by the firm

T = time (1950 = 01, 1962 = 13)

Once again the log-linear form of the function was estimated. The inputs are measured as stocks.

The results for the production function exhibited reasonable properties for two of the firms in the sample. The coefficients of H and B were significant for these firms and showed constant or slightly increasing returns to scale for the capital input. The third firm had neither significant coefficients nor reasonable parameter estimates (the sign of the barge input was negative).

The planning function also exhibited reasonable properties and had significant parameter estimates. The results indicate decreasing returns to scale when the firm's stock of equipment is considered. The time trend coefficient was positive, an indication of technological improvements in the industry.

A possible conclusion is that scheduling difficulties increase as the volume of output grows, making additions to the capital stock less productive overall. Although Howe's process function showed decreasing returns, the results for the production function indicate that the firm is able to schedule tows out of a fixed stock of equipment subject to increasing returns, as would be expected. However, the planning function indicated that as the firm adds to its capital stock to meet the requirements of growing demand (i.e., as the total number of tows to be scheduled increases) it does so subject to decreasing returns.

In the work of Hurter, statistical cost and production relationships were estimated for three years (1950, 1957, 1962) using annual Interstate Commerce Commission data on Class A certificated waterway carriers operating on the Mississippi River.

Hurter's production study[8] followed closely the methodology developed by Howe. Hurter estimated six production relationships, three of which he characterizes as planning functions in the sense in which Howe employed the term. The stated objective of Hurter's study is to describe the production and planning functions applicable to the firms in the sample and to make "year-wise" comparisons in order to determine ". . . the change in input combinations that have taken place in response to technological and environmental change."[9] The six relationships are:

(10) $Z = f_1\ (B, b, G, T)$

(11) $Z = f_2\ (B, b, G, T)$

(12) $Z = f_3\ (P, Q, G, T)$

(13) $Z = f_4\ (M, P, G, T, Q)$

(14) $Z = f_5\ (P \cdot B \cdot M, Q, G, T)$

(15) $P = f_6\ (G, M, T)$

where: G = gallon's of fuel oil consumed

M = miles run by towboats

T = total labor hours by all employees

B = number of towboats owned by each firm

b = number of barges owned by each firm

Q = average age per horsepower for each firm: horsepower (age)/total horsepower

P = average horsepower per towboat per fleet

Z = total freight tonnage carried by each firm

(10), (11), and (12) are characterized by Hurter as planning functions. All six are estimated in log-linear form.

Due to the varied nature of the results and uncertain interpretation of the variables, the results are difficult to summarize. The aggregate nature of the data used prevented Hurter from obtaining measures of input stocks and services suited to his purpose. All the planning functions (10, 11, 12) show increasing returns to scale in all three years (with the exception of equation 12 in 1962, which shows nearly constant returns). Note that these results are the opposite of Howe's, who found decreasing returns to the stock input relationships. No regularities over time can be noted with respect to returns to scale in Hurter's results.

Of the production functions, (13, 14, 15), equation (15) produced no statistically meaningful results. (13) exhibited increasing returns to scale each year. On the other hand, (14) showed decreasing returns for 1957 and 1962.

Finally, Hurter tests the hypotheses that there are no significant differences between the three years in the sample for each estimated relationship. Applying the standard tests developed by Chow,[10] Hurter found that significant differences did indeed exist between 1950-57 and 1950-62 for all relationships. Only (10) showed a significant difference for 1957-62. Hurter thus concluded that significant adjustment in the productive process had taken place up to 1957.

Hurter then turned his attention to estimating cost relationships.[11] Once again, the data used were annual cross-section observations for Class A carriers operating on the Mississippi system in 1950, 1957, and 1962. His work can be summarized in three parts: The first part relates annual water line expense to each of two measures of firm size (total annual cargo tonnage, and asset size of firm). Both linear and log-linear results were obtained although Hurter preferred the log-linear results since the firm size coefficient will be biased for the linear regressions. All four regressions indicated the existence of economies of scale in each year. Hurter attributes these scale economies to the barge firm's scheduling operations. However, only statistically significant firm size coefficients were obtained for 1957 and 1962 for all four relationships. The regressions using total output in ton-miles (linear and log-linear) and the linear regressions of cost on asset size did not produce significant results and had extremely low coefficients of determination (.12 to .24).

The second part of Hurter's cost study was an attempt to relate firm size to "profitability." Three measures of profitability were used: (1) the ratio of total freight revenue to total assets; (2) the ratio of net revenue to assets; and, (3) the ratio of total waterline costs to total freight revenue (the so-called "operating ratio"). No significant relationships were found.

For the third part of the cost study, Hurter separated out annual line-haul costs from total waterline expense. Linear and log-linear estimates were made of line-haul expense on annual tonnage. Significantly higher scale economies were indicated by the results for the line-haul operation relative to the terminal and administrative operations. Once again, scheduling economies are a possible source of these scale economies.

Hurter also related line-haul costs to inputs used in the line-haul operations (average horsepower per towboat, annual fuel consumption, and total labor hours). These variables represent the input flow variables of Hurter's production functions. A log linear relationship was estimated. It was found that a 10 percent increase in all inputs was accompanied by a 13 percent increase in all line-haul costs in 1957 and 1962. At the same time, a 10 percent increase in all inputs was found to be associated with a 28 percent increase in total output for 1957; a 10 percent increase in total output in 1962. This indicates strong economies of scale in 1957, but does not support scale economies for 1962. Additional support for positive scale economies was obtained by relating the stock input variables to line-haul costs.

One of the difficulties that have beset investigators of the barge industry has been the unavailability of important (and relevant) sources of data. Despite the use of sophisticated statistical techniques, Hurter's cost and production studies produced few important results. This stems partly from the data he used; partly from a seeming lack of theoretical justification for many of his estimated relationships. The latter criticism, no doubt, stems from the first. Hurter was simply not able to obtain direct measures of the variables of interest and had to make do with less than satisfactory surrogates.

The data used by Hurter, and to a lesser extent by Howe, are taken from the annual KA reports submitted by common carriers to the ICC. The information is highly aggregate and it is not possible to separate certain cost and output quantities into relevant categories. For example, total labor expenses may include terminal labor, linehaul labor, maintenance labor (of all sorts, from barge and towboat maintenance to repairs of office structures), janitorial services, administrative clerks, and so on. Fuel costs include fuel for harbor tugs as well as linehaul boats. The reports do not consider measures of towboat or barge services which can be related to different activities or services offered by the firm (for example, ton-miles carried of each commodity on each waterway).

The reports are designed to relate to the common carriage portion of a firm's business. By the nature of the record keeping process, however, it is difficult for the firms to assign a portion of total cost to the common carriage part of their operation. The reports, at best, represent industry guesswork as to the correct cost allocation.

The barge operators themselves have little respect for the information reported. Many firms do, however, collect quite detailed data on their operations. Unfortunately, this data has not been made generally available to investigators of barge operations in the past. Access to the records of the barge firms would allow a detailed analysis of several aspects of their operations. Common costs, for example, could be allocated to different types of service offered by the firm. Use of heretofore unavailable data in examining these questions is one of the major contributions of this study.

II. Plan of Study

As noted above, this study presents the results of statistical cost and production functions estimated using actual operating and cost data from several inland waterway operators. Chapter II considers the problem of selecting an output measure for barge transportation. The rest of the paper outlines the statistical methods and results. Chapter III presents a statistical production function which draws upon engineering knowledge, but which differs from engineering production functions such as Howe's in that it describes actual operations. Chapter IV is an analysis of direct operating costs for towboats and barges. One would expect that the costs of operating each type of equipment will bear a different relationship to the work performed. Thus, a separate cost analysis has been made for both types of equipment. Chapter V is a detailed analysis of the costs of a single firm using time-series data. It was possible to obtain extremely detailed data for this one firm including ton-miles of each commodity transported by river district. Thus, it has been analyzed separately. Chapter VI reports on a combined cross-section, time-series analysis of five barge firms. Chapter VII summarizes and expands on the major conclusions of the study.

The reader should note several themes running throughout the entire study. One is the importance of the operating environment in which the firm operates, the waterway. This is shown to have a significant effect on cost and output. Second, is the multi-product character of barge transport. This is especially evident in Chapter V, where the detailed data on cost and output make it possible to examine the problem of common costs. Finally, a third theme, is the lack of adequate output measures to describe all of the many services performed by barge firms (and transport firms in general).

CHAPTER II

TRANSPORT OUTPUT AND THE ALLOCATION OF COMMON COSTS

For the firm producing a single, homogeneous product under conditions of perfect competition, short-run and long-run cost relationships are easily defined. At the other extreme is the multiproduct firm. The costs associated with the production of each of many products and services are complex, interdependent, and difficult to measure.

A real difficulty arises in the case of joint costs: when a given expenditure is associated with the production of multiple products in fixed proportions. In this case, it is impossible to completely separate the costs of producing each product. For example, if a barge firm produces services by moving commodities in one direction, it necessarily must produce the service of a return haul. Allocation of cost between the trips is necessarily arbitrary. As a result, an arbitrary element enters the rate structure; no particular pricing policy can be justified on the basis of marginal cost. Price is then determined by the conditions of demand, or "what the traffic will bear."

A system of rates, such as the actual railroad rate structure in the United States, constitutes an elaborate system of price discrimination. This price discrimination, however, is not easy to disentangle from variations in rates based on variations in the cost of different services provided.

More frequently encountered is the situation where multiple products or services can be produced in varying proportions by a single operating unit. Certain expenditures may be shared by (or common to) all products, as in the joint cost case. However, since the output of the individual products are capable of independent variation, it is possible, at least conceptually, to determine the marginal costs associated with each by varying the output of one product, and fixing the outputs of all other products. Note that the marginal cost schedules obtained for each product may depend upon the level of output of all other products.

A major difficulty in examining the cost of services provided by transport firms is that the services cannot be measured along a single dimension. The service performed in handling, for example, a ton-mile of coal is not the same as the service performed in handling a ton-mile of steel wire. This is true even though the same equipment may be used and the same route traveled in both cases. The character of the output (and the cost-output relationship) may vary because of time requirements to be met, special handling, or equipment necessary for certain commodities, route traveled, and amount of empty backhaul.

As a result, many of the costs that are incurred cannot directly be attributed to any one particular type of service provided by the firm. This problem of common costs occurs at several levels of analysis, depending upon how one disaggregates the firm's output. Looking at the problem of common costs at the highest level of output aggregation (taking, for example, ton-miles as a single measure of total output) some costs can be directly identified as "due" to particular shipments that are made. Such costs are referred to in the barge industry as "direct" or "operating" costs. These are the costs associated with actually moving and handling the freight: fuel, maintenance, labor, etc. Such costs vary directly with the total number of ton-miles produced.

Additional costs, however, are incurred which cannot be identified with a general output measure such as ton-miles. These are the costs associated with the sale of services, record keeping, and general office and administrative expense. Such costs are generally termed "indirect" costs. At this level of output aggregation these indirect costs cannot be allocated to particular shipments, although certain shipments may have been more costly than others.

Most barge firms analyze their costs exactly in this way. That is, individual expense items are aggregated into the two categories, direct and indirect expense.[1] However, analyzing costs in this way obscures many important relationships.

This can be seen by disaggregating total output (ton-miles) in various ways. As has already been argued, a ton-mile of one commodity is not the same as a ton-mile of a different commodity. A ton-mile on one route, or over one river district, may have a different relationship with cost than a ton-mile on other river districts. When output is disaggregated, the direct costs also take on the attribute of commonness. A profit maximizing firm would need information on the marginal costs associated with these different outputs. That is, if it were possible to allocate direct costs to particular types of service the firm would want to set the marginal cost of each type of service equal to the marginal revenue derived from the sale of that service.

The problem of allocating common costs then, may be approached by segregating costs that are directly associated with particular shipments. It may be possible to allocate other costs, if not to individual shipments, at least to classes of shipments. One might think of a hierarchy of costs ranked by the degree of output aggregation. Some costs are allocable to individual shipments, others to classes of shipments somehow defined, while still others are not easily allocable to any services provided.

The crucial question is, what is the marginal cost of the service? What would be saved if the service was not provided? For example, the extra wear on a barge from carrying certain commodities is allocable as the cost of that shipment.

In examining the question of output measures for transportation, Wilson[2] provides an example of such an allocation for the case of motor freight transport. Wilson argues that transport firms provide a wide variety of services. This heterogeneity of transport output is, however, frequently collapsed into a single output measure, the ton-mile. But the ton-mile is itself a heterogenous output measure. ". . . it is evident that the costs for 100 ton-miles must differ, depending on the proportion of tons and miles involved." (p. 272).

Examining ICC cost data for various combinations of weight and distance, Wilson is able to derive total and average cost schedules for weight and distance. He finds that average cost declines as either tons or miles increases, holding one variable constant. He also finds cost interdependence, that is ". . . the percent variability of weight (or distance) is greater the greater the constant distance (or weight) which one uses." (p. 274).

Several measures of current transport output are available: tons, ton-miles (cargo or gross), number of trips or barge loads, and equivalent barge miles (EBM). None of the output measures are completely satisfactory in describing waterway transport output. This is because the character of the output measured, for example, in ton-miles, varies with respect to the commodity handled, equipment used, length of haul, size of shipment, route taken, special handling required, and time spent in transit. In general, these dimensions cannot be collapsed into a single variable such as ton-miles. An "ideal" single output measure would be an index, calculated by weighting in some fashion the different aspects of the services performed. Since no such comprehensive weighting scheme exists it is necessary to use one of the "standard" output measures.

One measure has recently received considerable attention: the EBM. The EBM is a measure of output designed to determine the relative amount of towboat effort (and thus towboat cost) required to move barges of various sizes and loads over waterways with different characteristics. This concept is of particular interest since it has been adopted by the ICC as part of its cost-finding procedure.[3] An EBM is defined by the ICC as the movement of a fully loaded jumbo barge one mile. To this definition should be added the characteristics of the waterway in which the movement occurs.

The EBM is a recognition of the fact, well established in engineering relationships and confirmed by operator experience, that towboat output, or effort, is not proportional to the number of tons or ton-miles being pushed. In particular, a towboat does not expend twice the "effort" in pushing a barge loaded with 1200 tons of cargo as it does with 600 tons of cargo. Yet, for each mile towed, twice as many ton-miles are produced by towing the 1200 ton barge. This tends to make costs rise more than in proportion to increases in output. This is because the relationship between the draft of the barge (as determined by the weight of the cargo in the barge) and the resistance of the flotilla is not proportional. As draft increases, the amount of turbulence in the water also increases, thus requiring increasing marginal effort by the towboat to achieve any given speed. There is an additional interaction with waterway depth: In general, the deeper the water the less severe the turbulence for any draft.

In addition to this engineering phenomena, two additional factors alter the draft, or cargo load, and output relationship: waterway congestion and locking time. If, for example, a large portion of the total time taken on any trip is spent navigating locks, an empty or partially loaded barge becomes just as difficult to push as does a fully loaded barge.

All of these factors are closely related to the waterway conditions which prevail for any movement (thus the insistence above on including waterway characteristics in the EBM definition). The number of EBM's per hour produced by a given boat will vary with the physical conditions of the waterway (depth, width, stream flow), average congestion experienced, and with the time required to pass through locks. It should be noted that, viewed in this way, the EBM provides a way of dividing the river systems into uniform river districts.

A number of reasons for preferring the EBM as an output measure can be cited. It is only slightly more difficult to calculate than a measure such as cargo ton-miles (or gross ton-miles). More importantly, it represents an attempt to account for the effort expended in pushing empty, or partly loaded barges. Over a uniform stretch of waterway, a towboat should generate the same number of EBM's per hour, no matter how many barges are being pushed, or what the amount of cargo is. If fewer barges are being pushed, or the barges contain less cargo, the speed of the flotilla increases. The EBM concept can be thought of as describing the relationship between number of barges, cargo weight, and the speed of the flotilla (waterway characteristics, congestion, and towboat characteristics held constant). Needless to say, these relationships are more complicated than those embodied in the ICC formulation; the latter must be viewed as an approximation over a reasonable range.

The EBM, then, is itself an output index, although it is still imperfect as an output measure. However, it does attempt to collapse certain features of waterway transport output into a single measure by recognizing characteristics of waterway transportation that have an important influence on the relationship between costs and output.

Because of these reasons, and because considerable interest has been generated in the barge industry regarding its use, the EBM has been selected as the measure of current output for this study.

In attempting to allocate the costs of the barge firm to each shipment it carries, one would have to specify, at least, what commodity was being carried, the length of the haul, the size and configuration of the flotilla, the characteristics of the equipment used, the route traveled, and the current level of total output. Allocations at this level of disaggregation are extremely difficult to obtain, particularly when a single output index is used. In addition, results would be extremely detailed, relating only to the specific traffic being considered.

However, it is possible to examine interesting questions relating to the expenses of the firm by taking an intermediate position in the cost-output hierarchy. The approach taken in this study is to look at the total output of the firm (measured in EBM's) and attempt to analyze the effect on costs of providing certain classes of transport service (assuming an average length of haul, and an average collection of equipment).

For example, we would certainly expect that the type of traffic the firm chooses to carry will influence its cost-output relationship. Certain liquid bulk material, for example, must be carried in relatively expensive, specialized barges often built specifically for a single commodity. If the firm chooses to haul some combination of bulk commodities, such as coal and grain, extra cost will be incurred in cleaning and preparing the barges for the grain movement. In addition, the kind of traffic will influence labor costs to the extent special handling is involved in either the terminal or the "on-line" operation.

We know from the engineering production function that an important input into the barge transport process is the physical operating environment for the tow. In examining the cost-output relationship it is of interest to determine the effect of changes in operating environment on cost. Once again the aggregative point of view can be taken. Specifying an average or typical barging operation, one looks at the operation under a variety of physical conditions. It is possible then, to relate systematically the observed physical conditions to the estimated differences in cost.

CHAPTER III

A STATISTICAL PRODUCTION FUNCTION FOR INLAND WATERWAY TOWBOATS

A firm producing transportation on inland waterways must make optimizing decisions similar to those made by any transportation firm. Freight is offered at various points along the waterway for transportation to other points. The firm must determine what equipment is to be used in a movement and the way in which the service is to be performed. In particular, the firm must allocate barges to haul the freight, it must decide how deeply to load the barges, how many barges to include in a tow, what size towboat to use in pushing the tow and when to schedule the movement.

Two types of exogenous factors influence the firm's decisions. One type is under no one's control (for example, weather conditions). The second type is under the control of the U. S. Army Corps of Engineers (such as, waterway characteristics, including depth and width, distance between locks and amount of traffic on the waterway). In order to make waterway transportation efficient, the Corps of Engineers must adjust public investment in waterways in accordance with the demand for waterway transportation, the equipment and operating decisions of carriers and exogenous factors.

The resistance of a flotilla of barges increases as the draft and breadth of the flotilla increase and decreases as the depth and width of the waterway increase. Other factors influencing towboat performance include the speed of the current, the amount of congestion and the distance between locks. To produce water transportation cheaply, the firm must optimize the factors under its control: towboat horsepower, towboat features (including the age and special features), crew size and various operating rules. These decisions must be tailored to the nature of demand and the characteristics of the waterway.

Here, these optimization decisions are approached by estimation of a statistical production function for a towboat. This function explains the productivity of a towboat in terms of towboat characteristics (such as horsepower), waterway characteristics, and seasonal variation, as shown in Tables 2-7. By estimating this function with data from a number of firms, it may be possible to gain insight into the decisions necessary to optimize waterway transportation.

A. Engineering Process Functions

In order to optimize the barge firm's operation, one must have a function which holds constant all other factors while focusing on the effect of varying a single parameter. Two techniques have been used to generate data of this nature. One technique involves performing speed tests with actual equipment under carefully controlled conditions. A flotilla is run over a course (generally pooled water) with speed, waterway and flotilla characteristics carefully noted. These prototype data are exceedingly useful, although quite expensive to obtain. Furthermore, it is difficult to get a waterway which is uniform and displays characteristics of interest.

A second way of generating data involves the construction of laboratory test facilities. Test tanks and scale model towboats and barges are constructed. The models are propelled through the tank with measurements taken of the resistance of the flotilla. It is comparatively inexpensive to try many new designs and ideas. The basic limitation of these tests is that they are only simulations. One is never sure whether a result is an abberation due to the test procedure or a true indication of the performance that might be expected in a prototype test.

Between laboratory and prototype tests, it has been possible to estimate towboat process functions.[1] These engineering functions relate the speed of a flotilla to the characteristics of flotilla and waterway. Such a function is extremely useful in optimizing the depth to which barges should be loaded, and the number of barges that should be placed in a flotilla. To a lesser extent, this function is also useful in deciding what size towboat is optimal for a firm's operation. Unfortunately, a whole host of additional considerations intrude in the latter decision. Towboats must operate on many different waterways, under a vast number of conditions. The function could determine which boat is best for any particular conditions, but it is not easily adapted to determining the best boat for a range of operations.

Equipment selection must take into account the entire matrix of the firm's shipments. The demands for transport services presented to the firm vary as to commodity hauled, length of haul, frequency of shipment, and shipper requirements. The major role of the firm is the coordination of many such point-to-point movements. Time, for example, is an important element in equipment selection. If the firm's demands are presented infrequently, or if shippers do not require immediate delivery, a smaller towboat may be optimal for the firm even though a larger boat would yield greater speed for a given waterway.

In addition, the firm must operate the same equipment over waterways which have widely varying characteristics. Thus, what may be an optimal piece of equipment for a particular point-to-point movement may not be satisfactory when viewed from the standpoint of the firm's scheduling requirements.

Another difficulty with the engineering approach is the nature of the approximation it makes. The technological constants in the function are based upon a tow moving through still water and over a waterway of uniform depth and width. Actual waterway profiles are quite irregular. A "9 foot" waterway refers to the minimum controlling depth. The actual depth varies between nine feet and several hundred feet. Thus, when one evaluates a tow's performance with the engineering function based, for example, upon a nine foot "bathtub" one is introducing a downward bias of unknown size.

The engineering process function describes the speed of a flotilla under carefully specified conditions. There is a vast set of conditions which are encountered during normal operations. One might apply the engineering function to each of these and then aggregate them in some fashion to get a measure of "normal operation." Such a task would be tedious and it is not evident what weights should be used for aggregation. Finally, there is some question about the accuracy with which the process function would predict the actual speed that would be observed.

The last difficulty can be handled by observing that the engineering process function is likely to be accurate for small changes, in the variables, even if it is not accurate for predicting absolute speeds. One way to use the engineering process function would be to observe the speed of a ton under a particular set of conditions and then use the function to predict the change in speed that would occur by changing one or more conditions. This technique could be used both to optimize the operating characteristics of a flotilla (including the derivation of a specific measure of marginal cost associated with changing characteristics) and to determine the marginal benefit to be derived by improving various waterway characteristics.

There is no easy way of adapting the engineering process function to describe actual operations. The engineering relation is an ideal one, much like the production and cost functions of economic theory. Actual operations consist of equipment which is outmoded or not operating at peak efficiency; it consists of bad weather and channels which are more shallow than they are supposed to be. Some adjustments can be made, such as incorporating a delay analysis; but, basically, it is evident that the engineering relation does not describe actual operations. The question of how closely it approximates actual operations is an empirical one.

B. A Statistical Production Function

Economists have estimated statistical production functions for many industries.[2] After obtaining observations on each of many production units, multivariate techniques can be used to isolate the effect of each input factor on output. The estimated function is especially valuable in that it estimates the production relation presently in existence, that is, the one which applies for firms currently going about their business. This relation is not the optimal production relation of the economics textbook.

Along with its advantages, the statistical production function also has a number of drawbacks. Firms tend to cluster around one set of equipment and operating rules (that representing "current practices"). Insofar as this is true, there will be no observations on other techniques of production. Even where some differences can be observed, firms are unlikely to impose a Latin-square experimental design on their operations. Thus, the observations on most variables will show limited range and many variables will be collinear. While a vast number of techniques have been developed for estimation, it remains true that there is basically no answer to the problems of collinearity and limited range of observations.

There are two ways to estimate production functions. One involves building in all a priori information in order to get the best possible estimates. A second way is to make use of none of the a priori information and simply let the estimates come out as they will. The former technique generates better estimates. However, there is always a question about what is known a priori, and what is only convention. A priori information often consists of ranges of reasonable parameter values rather than single values. One reason for using the second approach is that any a priori information can be used to check the estimated parameters. For example, one can check the estimated production function against the accumulated experience of men who have worked in the industry. In the case of Barge Transportation, some of this experience has been quantified in ICC cost finding procedures. Finally, there are investigations of the physical-engineering relations between changes in such characteristics as towboat horsepower and tow size and the resulting change in tow speed.

Three major inland waterway firms provided data for the period 1964-66. An observation consists of the operations of a particular towboat for a one-month period. Roughly 100 towboats were observed operating on sixteen river districts over this period. For each towboat month, data were available on: (1) the amount of time it operated in each river district (broken down by the time going downstream and time going upstream), (2) the horsepower of the towboat, (3) whether it was equipped with Kort nozzles, (4) the size of the crew, (5) the time it had received its last major overhaul, and (6) the number of EBM's per hour produced on each river district during operation. (A towboat operates 24 hours a day unless it is in drydock for repairs or is deactivated). A number of variables were tried with both linear and log-linear specifications.

The horsepower of the towboat has been chosen as a measure of the capital input. Towboat sizes are most frequently stated in terms of horsepower. In addition, one would expect that, ceteris paribus, the greater the horsepower, the larger the output of the towboat.

Crew size represents the labor input; it generally varies directly with the number of barges being towed and the horsepower of the towboat. It varies directly with the age of the towboat. Any particular towboat, however, tends to have the same crew size over time.

A dummy variable which increases linearly with the age of the towboat has also been included to pick up the effect of two reinforcing phenomena. The first is the expectation that newer boats have better engineering characteristics and more efficient engines. The second is the physical deterioration of the boat. The age of the towboat is taken from the data of its last major overhaul. This implicity assumes that an overhaul restores a boat to "new" efficiency.

Another dummy variable indicates the presence or absence of Kort nozzles on the towboat. Kort nozzles are tunnels which surround the towboat's propellers and are designed to reduce turbulence around the propellers and hence to improve performance.

The operations of an inland waterway company are quite sensitive to seasonal variations in waterway conditions and to the demand for transport services. A set of dummy variables has been included to estimate this element of seasonal variation. Stream flow and weather conditions are determinants of the maximum draft and number of barges that can be included in a tow. In addition, during peak seasons we should observe a more intensive use of capital stock as reflected in increased tow sizes and average load of barges. To the extent that this occurs on particular river districts, the coefficients for those districts will generally be larger. Likewise, if the increase in output is distributed more generally over the entire firm's operations, the coefficients for these peak seasons should be larger than the coefficients for less active periods.

Two Specification Problems

The river districts differ markedly in physical characteristics affecting towboat operations. One way to measure this effect is to use the output produced on each waterway (corrected for differences in tow and towboat characteristics) as the dependent variable in a regression. Differences in output would be attributed to changes in such characteristics as the depth, width, stream flow, number of locks and amount of traffic on each river district. In such a regression, the coefficients of each characteristic variable would measure the change in output resulting, for example, from changing depth or stream flow. Such a model would provide a direct way of estimating the effect of changes in waterway characteristics on barge operations and costs. This approach is empirically direct and simple.

There are also a number of disadvantages. This analysis is essentially a reworking of the investigation of the physical engineering relationship. While the direct approach might be desirable under some conditions, it is much simpler than the engineering relationship and could hardly be expected to be as good.

A second method, the one used in this analysis, is to include a set of dummy variables for each river district on which the towboat operated. The coefficient of the dummy variable associated with a particular river district would show how the production surface should be shifted when a towboat of given characteristics is operated on that river district.[3] The coefficients would represent a relative evaluation of the "difficulty" or the "penalty" to be attached to operating on each waterway. This "penalty" is due both to physical constrictions in the waterway (such as shallow stretches) and to congestion resulting from other traffic.

A second problem is involved in specifying production function changes when the tow is headed upstream rather than downstream. While there is no particular reason to suspect that the <u>form</u> of the production function changes as the waterway input changes, the same may not be true of the direction the towboat is traveling, that is, upstream or downstream. The standard approach is to correct for direction by subtracting the speed of the stream. However, this ignores the effects on performance of additional turbulence due to stream flow and the downward slope of the waterway in the direction of stream flow.

To determine the effect of direction on performance, a dummy variable has been included. Also, separate runs were made for the upstream and downstream data. In this way, it is possible to test the hypothesis that the performance of a towboat, upstream and downstream, differs only by the magnitude of stream flow.

The Results

The first problem investigated was the similarity of the production functions for upstream and downstream movements. The null hypothesis is that the production functions are identical, except for a dummy variable indicating direction. (The effect of the dummy variable is to shift the entire production surface depending on whether the movement was upstream or downstream). This hypothesis was tested with an analysis of covariance.[4] After taking account of the degrees of freedom, an F-test showed that the variance of the residual in the regression using the pooled data was significantly greater than the sum of the variances of the residuals in the two separate regressions. Therefore, the null hypothesis of identical production functions is rejected. (The value of F was always significant well beyond the .01 level.)

As indicated earlier, a predetermined concept of the specification of the function was not formed. Thus, both linear and log-linear specifications were estimated for each company separately and for all companies together. Since the production function for upstream movement differed from that for downstream movement separate estimations were made for each. (In the log-linear regression the dummy variables were left untransformed.)

The next hypothesis was that the production functions for all companies were identical, except for a dummy variable. The null hypothesis is that the production functions for all three companies are identical. Again, an analysis of covariance was performed; again the null hypothesis was rejected at a level of significance beyond .01 for all regressions (upstream and downstream; linear and log-linear specifications).

The results of this test are somewhat surprising. Firms performing similar services on similar waterways should arrive at similar operating practices. There seems to be two possible explanations for these differences.

The first is measurement error. Firms may compile their data in different ways, leaving errors of unknown size. Certainly, some errors of this sort do occur. The basic data are the towboat logs that each captain keeps. It is possible that some captains keep less accurate logs--or have divergent interpretations of the reporting method. But, there is no reason to suspect that such errors are systematically related to different companies.

In gathering the data, one cannot help but be impressed by the uniformity of the methods employed by the different companies in their individual data collection. (Thus minimizing the importance of the measurement error.) The division of the Mississippi River System into river districts was, with one exception, the same for all companies. All companies gathered towboat performance data monthly, by towboat and direction. All companies used the same definition of operating hours and all reported cargo ton-miles, tare ton-miles and empty ton-miles.

A more plausible hypothesis about the firm differences is that there are, in fact, basic differences in each firm's operation that affect overall performance. Different rules-of-thumb may be applied to determine how full the barges are to be loaded, how many barges are put in a flotilla and in what configuration. There are differences in the type of traffic handled, the amount of empty backhaul, the average length of haul and the distribution of the firm's total traffic among the various river districts. One would expect that average performance would improve the greater the length of haul. A firm which operates many unit-tows will experience a different level of performance than a carrier which deals primarily in common carriage. Thus, it is argued that the difference between companies is really a difference in the primary type of service performed.

The final specification problem involves choosing between linear and log-linear forms of the equations. So far as the coefficient of determination is concerned, there is little to differentiate the two forms; both are quite large. The two specifications gave statistically comparable results, with one exception: the river district coefficients for the linear specification accorded more closely with a priori expectations (see discussion below). The results of both the linear and the log-linear specifications are reported in Tables 2-7. With these specification problems out of the way, the individual parameter estimates can be discussed.

The most striking variable is horsepower. In all 16 regressions, the coefficient is highly significant. Furthermore, the coefficients

TABLE 2

LINEAR DOWNSTREAM REGRESSION COEFFICIENTS

Variable Name	Company One Reg. Coeff.	Company One t-Value	Company Two Reg. Coeff.	Company Two t-Value	Company Three Reg. Coeff.	Company Three t-Value	Combined Reg. Coeff.	Combined t-Value
River Districts								
Port of Chicago	-0.09812	-2.29394	-0.18535	-7.18904	-0.02662	-0.80761	-0.03057	-1.49458
Tennessee	0.52442	6.32513					0.39062	3.92938
Gulf of Mexico	0.39234	9.07865					0.34176	8.08419
Cairo to St. Louis	0.53470	14.26803	0.47962	6.90990	0.04370	0.93167	0.34182	10.65100
St. Louis to Grafton	-0.01631	-0.29853					0.01563	0.23218
Cairo to Mouth of Miss.	0.63719	21.37967	1.57041	38.04025	0.93620	22.83751	0.84336	37.58672
Cairo to Cincinnati	0.34959	2.72070	0.20413	6.42601	0.09778	3.20360	-0.01325	-0.61252
Cincinnati to Dam 23	0.18459	1.43658	0.16198	4.30791	0.17216	6.32202	0.06657	3.25946
Dam 23 to Pittsburgh			-0.15207	-4.73193			-0.32619	-10.69112
St. Louis to Minneapolis			0.07008	2.43436	-0.27039	-1.95822	0.00731	0.26373
Monongahela					-0.23925	-3.65824	-0.29608	-4.00632
Port of Pittsburgh					-0.21169	-5.09393	-0.27822	-6.47615
Kanawha to Monica					0.06840	2.60817	0.01144	0.55382
Kanawha					-0.00402	-0.08556	0.09315	1.98303
Towboat Characteristics								
Horsepower (times 10^{-4})	0.98328	5.99971	0.65912	4.46520	0.66773	5.38189	1.10163	14.88296
Kort nozzles	0.14868	4.63161	0.03869	1.72628	0.15056	6.90008	0.12898	9.45143
Crew Size (times 10^{-2})	12.42605	12.49200	-1.43760	-0.79942	7.46569	5.23677	10.95867	15.24768
Linear Age (times 10^{-1})	0.21422	6.47709	0.01099	0.55851	-0.04165	-1.91714	-0.01785	-1.66029

TABLE 2--Continued

Variable Name	Company One Reg. Coeff.	Company One t-Value	Company Two Reg. Coeff.	Company Two t-Value	Company Three Reg. Coeff.	Company Three t-Value	Combined Reg. Coeff.	Combined t-Value
Month								
January	-0.09467	-1.99514	-0.00216	-0.06290	0.05099	1.65056	-0.01450	-0.58075
February	-0.03891	-0.81306	0.04613	1.34226	-0.04813	-1.34091	-0.04669	-1.77957
March	0.01414	0.28470	0.15455	4.67984	0.11426	3.99957	0.10489	4.34597
April	0.13571	2.76299	0.16941	5.05616	0.04423	1.55179	0.09581	3.96492
May	0.09132	1.84063	0.12123	3.63094	0.01709	0.58761	0.05076	2.07837
June	0.03406	0.67575	0.02241	0.67366	-0.00812	-0.26075	-0.01904	-0.75635
July	0.06176	1.21282	0.06165	1.87044	-0.05611	-1.85428	-0.01385	-0.55796
August	-0.01479	-0.29935	-0.00720	-0.22459	-0.06414	-2.18826	-0.05428	-2.25129
September	-0.04621	-0.92748	-0.00126	-0.03900	-0.06429	-2.25589	-0.05254	-2.19421
October	0.09766	2.02642	0.02592	0.77830	-0.04337	-1.54912	-0.01959	-0.82146
November	0.06293	1.29246	0.04308	1.30676	-0.05799	-2.03624	-0.02041	-0.84936
Year								
1964	-0.21201	-6.29756	-0.03236	-1.98050			-0.08948	-5.90145
1965	-0.05493	-1.98258	-0.00679	-0.40592	0.00368	0.27902	-0.00789	-0.69192
Dependent Variable: EBM/HR (times 10^{-2})								
r^2	.9083		.9173		.6660		.7934	
Standard error	0.17423		0.18346		0.19253		0.22815	
Intercept	-1.5936		0.3317		-0.6925		-1.2064	
Sample size	349		743		951		2043	

TABLE 3

LINEAR UPSTREAM REGRESSION COEFFICIENTS

Variable Name	Company One Reg. Coeff.	Company One t-Value	Company Two Reg. Coeff.	Company Two t-Value	Company Three Reg. Coeff.	Company Three t-Value	Combined Reg. Coeff.	Combined t-Value
River Districts								
Port of Chicago	-0.08867	-3.6550	-0.07878	-5.2396	-0.05663	-2.6645	-0.07245	-6.5343
Tennessee	0.11258	2.6597					0.05999	1.2294
Gulf of Mexico	0.13774	5.6821					0.09404	4.1014
Cairo to St. Louis	0.14082	8.6864	0.07237	3.5261	-0.07615	-3.0041	0.06334	5.2752
St. Louis to Grafton	-0.09143	-3.3125					-0.08522	-2.5995
Cairo to Mouth of Miss.	0.10975	6.9720	0.28593	12.5021	0.13086	5.0100	0.14583	12.3962
Cairo to Cincinnati	0.31634	4.3052	0.02303	1.2805	0.03784	1.9238	0.01014	0.9031
Cincinnati to Dam 23	0.31634	4.3052	0.02947	1.5548	0.11593	6.5340	0.08181	7.8515
Dam 23 to Pittsburgh	0.05134	0.6987	-0.19278	-10.4603			-0.20124	-13.0902
St. Louis to Minneapolis			-0.00435	-0.2380	-0.21721	-2.3636	-0.01582	-1.0042
Monongahela					-0.12843	-4.2121	-0.13576	-4.9823
Port of Pittsburgh					-0.11342	-5.2281	-0.12128	-7.1618
Kanawha to Monica					0.01615	0.9291	0.00561	0.5006
Kanawha					0.03503	1.1173	0.03188	1.2013
Towboat Characteristics								
Horsepower (times 10^{-4})	0.95214	10.5881	1.00327	12.3443	0.69701	8.7461	0.79823	21.4626
Kort nozzles	0.08541	5.2319	0.04936	3.7269	0.10134	7.4298	0.07455	10.5093
Crew Size (times 10^{-2})	3.67580	6.8629	-1.87375	-1.7595	5.05772	5.5754	3.52121	9.2280
Linear Age (times 10^{-1})	0.05939	3.4153	-0.00235	-0.1998	0.01091	0.7926	0.02511	4.5262

TABLE 3--Continued

Variable Name	Company One Reg. Coeff.	Company One t-Value	Company Two Reg. Coeff.	Company Two t-Value	Company Three Reg. Coeff.	Company Three t-Value	Combined Reg. Coeff.	Combined t-Value
Month								
January	-0.01703	-0.6872	-0.01462	-0.7264	-0.02282	-1.2061	-0.01512	-1.1747
February	-0.04504	-1.7736	-0.00454	-0.2270	-0.09184	-3.9952	-0.04276	-3.1131
March	0.00514	0.2017	-0.01039	-0.5373	-0.02547	-1.4172	-0.01046	-0.8379
April	-0.01068	-0.4332	0.01791	0.9462	-0.04548	-2.5293	-0.01473	-1.1943
May	0.01615	0.6351	0.00782	0.4027	-0.02255	-1.2296	-0.00490	-0.3876
June	0.02768	1.0870	0.01964	1.0096	0.01908	0.9847	0.02200	1.6984
July	0.02827	1.1063	0.03895	1.9939	-0.00849	-0.4411	0.01664	1.2853
August	0.04232	1.5862	0.04426	2.2711	0.04697	2.5187	0.04622	3.5914
September	-0.04462	-1.7839	0.00813	0.4116	0.00815	0.4374	-0.00100	-0.0784
October	0.07407	2.9879	0.03777	1.9032	0.00626	0.3519	0.02833	2.2602
November	0.05243	2.0500	0.00502	0.2579	-0.00592	-0.3247	0.00711	0.5610
Year								
1964	-0.00832	-0.4660	-0.00638	-0.6727			-0.00054	-0.0691
1965	-0.00633	-0.4394	-0.02572	-2.6893	-0.00829	-1.0014	-0.01010	-1.7122
Dependent Variable: EBM/HR (times 10^{-2})								
r^2	.8508		.8127		.6195		.7178	
Standard error	0.10010		0.11741		0.12826		0.12775	
Intercept	-0.4653		0.2756		-0.4969		-0.3434	
Sample Size	424		914		1065		2403	

TABLE 4

LOG-LINEAR DOWNSTREAM REGRESSION COEFFICIENTS

Variable Name	Company One Reg. Coeff.	Company One t-Value	Company Two Reg. Coeff.	Company Two t-Value	Company Three Reg. Coeff.	Company Three t-Value	Combined Reg. Coeff.	Combined t-Value
River Districts								
Port of Chicago	-0.54509	-17.6538	-0.35729	-18.0708	-0.05272	-1.9048	-0.27553	-17.7132
Tennessee	0.18771	3.1880					0.02529	0.3761
Gulf of Mexico	0.20224	6.5469					0.09006	3.1687
Cairo to St. Louis	0.32578	12.2768	0.26647	6.7716	0.00590	0.1564	0.21324	9.8909
St. Louis to Grafton	-0.05671	-1.4566					-0.05121	-1.1348
Cairo to Mouth of Miss.	0.35008	16.6685	0.53185	24.1166	0.42878	13.1628	0.37325	25.6628
Cairo to Cincinnati	0.30644	3.3684	0.10746	5.9932	0.03020	1.2384	0.03392	2.3656
Cincinnati to Dam 23	0.20203	2.2207	0.09929	4.6668	0.09368	4.2514	0.09012	6.5840
Dam 23 to Pittsburgh			-0.18636	-10.1112			-0.22849	-11.2003
St. Louis to Minneapolis			0.04656	2.7743	-0.43020	-3.8911	0.03269	1.7431
Monongahela					-0.31412	-5.9855	-0.30059	-6.0635
Port of Pittsburgh					-0.28760	-8.5705	-0.26997	-9.3431
Kanawha to Monica					0.00941	0.4405	0.02541	1.8294
Kanawha					-0.01803	-0.4844	0.03710	1.1757
Towboat Characteristics								
$\log_{10}$ Horsepower (times 10^{-4})	0.47509	7.8211	0.51673	10.2347	0.58533	9.9264	0.44742	14.9251
Kort Knozzles	0.11426	4.9445	0.05847	4.4951	0.14983	8.6605	0.10644	11.6179
$\log_{10}$ Crew Size (times 10^{-2})	2.44631	15.8897	-0.18005	-0.7556	1.36704	4.1702	1.81511	16.4901
Linear Age (times 10^{-1})	0.06357	2.9427	0.00263	0.2479	-0.09113	-5.6954	-0.00186	-0.2620

TABLE 4--Continued

Variable Name	Company One Reg. Coeff.	Company One t-Value	Company Two Reg. Coeff.	Company Two t-Value	Company Three Reg. Coeff.	Company Three t-Value	Combined Reg. Coeff.	Combined t-Value
Month								
January	-0.09778	-2.9133	0.00288	0.1482	0.04360	1.7629	-0.01333	-0.7969
February	-0.06742	-1.9902	0.02301	1.1826	-0.05399	-1.8803	-0.04817	-2.7403
March	0.00743	0.2115	0.11359	6.0780	0.09227	4.0355	0.07778	4.8108
April	0.08627	2.4826	0.11665	6.1511	0.04867	2.1333	0.06984	4.3139
May	0.06298	1.7936	0.07952	4.2048	0.02773	1.1911	0.03810	2.3279
June	0.01257	0.3523	0.03446	1.8292	0.01706	0.6845	0.00566	0.3352
July	0.02273	-0.6308	0.04836	2.5914	-0.02348	-0.9695	-0.00873	-0.5248
August	0.00325	0.0930	0.01429	0.7867	-0.02131	-0.9085	-0.01672	-1.0347
September	-0.03989	-1.1318	0.01859	1.0154	-0.02536	-1.1117	-0.02004	-1.2495
October	0.04807	1.4101	0.03650	1.9357	-0.00851	-0.3797	0.00832	0.5209
November	0.04178	1.2121	0.03947	2.1148	-0.02390	-1.0487	0.00244	0.1513
Year								
1964	-0.15876	-6.9232	-0.03111	-3.3634			-0.07235	-7.1595
1965	-0.06234	-3.1934	-0.02607	-2.7509	-0.01096	-1.0389	-0.02464	-3.2279
Dependent Variable: $\log_{10}$ EBM/HR (times 10^{-2})								
r^2	.9287		.9242		.6843		.8049	
Standard error	0.12329		0.10387		0.15411		0.15287	
Intercept	2.0643		-0.3256		1.2376		1.4970	
Sample Size	349		743		951		2043	

TABLE 5

LOG-LINEAR UPSTREAM REGRESSION COEFFICIENTS

Variable Name	Company One Reg. Coeff.	Company One t-Value	Company Two Reg. Coeff.	Company Two t-Value	Company Three Reg. Coeff.	Company Three t-Value	Combined Reg. Coeff.	Combined t-Value
River Districts								
Port of Chicago	-0.42727	-13.3649	-0.09178	-4.0727	-0.11542	-4.9902	-0.18450	-13.0303
Tennessee	0.11110	2.0324					-0.05969	-1.0557
Gulf of Mexico	0.22575	7.1560					0.16238	6.1756
Cairo to St. Louis	0.11203	5.3706	-0.06242	-2.5115	-0.15601	-5.8465	0.00069	0.0499
St. Louis to Grafton	-0.09911	-2.7763					-0.10076	-2.6783
Cairo to Mouth	0.09343	4.6062	0.05734	2.1229	0.02333	0.8545	0.04967	3.7365
Cairo to Cincinnati	0.28732	3.0469	-0.05512	-2.5053	-0.04085	-1.9777	-0.02104	-1.6319
Cincinnati to Dam 23	0.28732	3.0469	-0.04059	-1.7374	0.02934	1.5521	0.04581	3.7884
Dam 23 to Pittsburgh	0.07700	0.8166	-0.28628	-12.5773			-0.24588	-13.8858
St. Louis to Minneapolis			-0.09529	-4.1851	-0.45808	-4.7778	-0.07227	-3.9586
Monongahela					-0.21593	-6.7262	-0.17823	-5.6873
Port of Pittsburgh					-0.19747	-8.5437	-0.15829	-8.0822
Kanawha to Monica					-0.03998	-2.1537	-0.00428	-0.3311
Kanawha					0.07427	2.2909	0.12115	3.9784
Towboat Characteristics								
$\log_{10}$ Horsepower (times 10^{-4})	0.59697	9.6466	1.05504	18.9280	0.79985	16.0680	0.71113	26.8758
Kort nozzles	0.09395	4.4501	0.10544	6.4395	0.12754	9.1000	0.10010	12.2901
$\log_{10}$ Crew Size (times 10^{-2})	1.78892	11.8469	-0.80135	-2.7953	0.97165	3.5299	1.07602	10.7736
Linear Age (times 10^{-1})	0.06463	3.1898	-0.00186	-0.1355	-0.03394	-2.5871	0.02029	3.2285

TABLE 5--Continued

Variable Name	Company One Reg. Coeff.	Company One t-Value	Company Two Reg. Coeff.	Company Two t-Value	Company Three Reg. Coeff.	Company Three t-Value	Combined Reg. Coeff.	Combined t-Value
Month								
January	-0.04946	-1.5551	0.01825	0.7449	-0.02933	-1.4867	-0.01525	-1.0335
February	-0.05526	-1.6951	0.02067	0.8497	-0.11125	-4.6406	-0.04469	-2.8385
March	-0.00380	-0.1163	0.04536	1.9272	-0.02490	-1.3287	0.00251	0.1755
April	-0.01348	-0.4262	0.05984	2.5969	-0.04534	-2.4184	-0.00366	-0.2587
May	0.02883	0.8836	0.05406	2.2845	-0.03193	-1.6693	0.00714	0.4924
June	0.03370	1.0318	0.06833	2.8837	0.01918	0.9495	0.03681	2.4795
July	0.02975	0.9073	0.08416	3.5381	-0.01600	-0.7973	0.02841	1.9140
August	0.04118	1.2033	0.09245	3.8938	0.04114	2.1155	0.05784	3.9210
September	-0.06365	-1.9824	0.06866	2.8546	0.01111	0.5721	0.01544	1.0541
October	0.04913	1.5448	0.08196	3.3925	0.00808	0.4357	0.04220	2.9378
November	0.04404	1.3421	0.05906	2.4916	-0.01337	-0.7032	0.02117	1.4579
Year								
1964	-0.04627	-2.1036	-0.00479	-0.4151			-0.00162	-0.1822
1965	-0.02900	-1.5777	-0.03710	-3.1854	-0.00135	-0.1566	-0.01254	-1.8569
Dependent Variable: $\log_{10}$ EBM/Hour (times 10^{-2})								
r^2	.8661		.7972		.7538		.7630	
Standard Error	0.12846		0.14295		0.13376		0.14643	
Intercept	1.4166		-0.6848		0.9056		0.8564	
Sample Size	424		914		1065		2403	

TABLE 6

VARIABLE MEANS FOR DOWNSTREAM REGRESSIONS

A. Linear Regression (Arithmetic Means)

Variable Name	Company One	Company Two	Company Three	Combined
Port of Chicago	0.06877	0.14266	0.08307	0.10230
Tennessee	0.01719			0.00294
Gulf of Mexico	0.11461			0.01958
Cairo to St. Louis	0.09169	0.01077	0.02208	0.02986
St. Louis to Grafton	0.03438			0.00587
Cairo to Mouth of Miss.	0.37249	0.11709	0.04416	0.12677
Cairo to Cincinnati	0.00573	0.14266	0.15352	0.12433
Cincinnati to Dam 23	0.00573	0.06460	0.22608	0.12971
Dam 23 to Pittsburgh		0.10767		0.03916
St. Louis to Minneapolis		0.11306	0.00210	0.04209
Monongahela			0.01052	0.00489
Port of Pittsburgh			0.03365	0.01566
Kanawha to Monica			0.24395	0.11356
Kanawha			0.02839	0.01322
Horsepower (times 10^{-4})	0.26346	0.33161	0.30103	0.30573
Kort nozzles	0.47564	0.64949	0.80336	0.69506
Crew Size (times 10^{-2})	0.11284	0.12340	0.11831	0.11923
Linear Age (times 10^{-1})	1.26304	0.66231	0.32828	0.07493
January	0.10315	0.07402	0.07045	0.07734
February	0.10315	0.07402	0.04627	0.06608
March	0.08023	0.08614	0.09148	0.08762
April	0.08596	0.08075	0.09253	0.08713
May	0.08023	0.08075	0.08623	0.08321
June	0.07450	0.08210	0.06730	0.07391
July	0.07163	0.08479	0.07571	0.07832
August	0.08023	0.09556	0.08412	0.08762

TABLE 6--Continued

Variable Name	Company One	Company Two	Company Three	Combined
September	0.07736	0.09287	0.09253	0.09006
October	0.09169	0.08075	0.09989	0.09153
November	0.08596	0.08479	0.09253	0.08860
1964	0.39255	0.34186		0.19139
1965	0.33238	0.30283	0.45110	0.37690
Dependent Variable: EBM/HR (times 10^{-2})	0.66636	0.63836	0.55095	0.60245

B. Log - Linear Regression (Geometric Means of Transformed Variables only)

Variable Name	Company One	Company Two	Company Three	Combined
$\log_{10}$ Horsepower (times 10^{-4})	-0.63112	-0.54140	-0.57349	-0.57166
$\log_{10}$ Crew Size (times 10^{-2})	-0.95432	-0.91001	-0.92767	-0.92580
$\log_{10}$ EBM/HR (times 10^{-2})	-0.36790	-0.35478	-0.33535	-0.34798

TABLE 7

VARIABLE MEANS FOR UPSTREAM REGRESSIONS

A. Linear Regression (Arithmetic Means)

Variable Name	Company One	Company Two	Company Three	Combined
Port of Chicago	0.05896	0.11816	0.07418	0.08822
Tennessee	0.01887			0.00333
Guld of Mexico	0.09906			0.01748
Cairo to St. Louis	0.19575	0.08425	0.03286	0.08115
St. Louis to Grafton	0.03774			0.00666
Cairo to Mouth of Miss.	0.33255	0.09847	0.04319	0.11527
Cairo to Cincinnati	0.00472	0.14004	0.15211	0.12151
Cincinnati to Dam 23	0.00472	0.09519	0.22535	0.13691
Dam 23 to Pittsburgh	0.00472	0.10722		0.04161
St. Louis to Minneapolis		0.08643	0.00188	0.03371
Monongahela			0.02254	0.00999
Port of Pittsburgh			0.06667	0.02955
Kanawha to Monica			0.21878	0.09696
Kanawha			0.02441	0.01082
Horsepower (times 10^{-4})	0.27719	0.34560	0.30168	0.31406
Kort nozzles	0.52123	0.70022	0.80845	0.71660
Crew Size (times 10^{-2})	0.11406	0.12430	0.11840	0.11988
Linear Age (times 10^{-1})	1.28891	0.68359	1.30854	1.06737
January	0.09670	0.07440	0.07887	0.08032
February	0.08962	0.07659	0.04413	0.06450
March	0.08255	0.08972	0.09671	0.01955
April	0.09670	0.09737	0.09484	0.09613
May	0.08255	0.08643	0.08826	0.08656
June	0.08019	0.08534	0.07230	0.07865
July	0.08019	0.08425	0.07418	0.07907
August	0.06604	0.08534	0.08263	0.08073

TABLE 7--Continued

Variable Name	Company One	Company Two	Company Three	Combined
September	0.08726	0.08096	0.08263	0.08281
October	0.09198	0.07877	0.09859	0.08989
November	0.07783	0.08534	0.08920	0.08573
1964	0.38443	0.33260		0.19434
1965	0.33019	0.32057	0.45164	0.38036
Dependent Variable: EBM/HR (times 10^{-2})	0.41738	0.43478	0.41986	0.45510

B. Log - Linear Regression (Geometric Means of Transormed Variables only)

Variable Name	Company One	Company Two	Company Three	Combined
$\log_{10}$ Horsepower (times 10^{-4})	-0.60632	-0.52300	-0.56918	-0.55816
$\log_{10}$ Crew Size (times 10^{-2})	-0.94929	-0.90689	-0.92728	-0.92340
$\log_{10}$ EBM/HR (times 10^{-2})	-0.48584	-0.45640	-0.44333	-0.45580

are roughly comparable across all companies (see Table 2) and for both upstream and downstream. Apparently, horsepower is just as useful for a tow heading downstream as it is for a tow heading upstream. At first, this result may seem paradoxical. After all, in heading downstream the current is doing much of the work. However, horsepower is required to control the tow (by steering and power backing). Carriers find that using a towboat on a downstream run can achieve high speeds, but it also has a high probability of having an accident.

With the exception of company two, the coefficient of crew size is always positive and significant. The negative sign for company two is implausible since it indicates that additional crew members decrease the productivity of a towboat. In attempting to explain this sign, one should note that company two operates larger crews than either of the other companies. Thus, it is quite possible that they are at the point of negative marginal productivity. Alternatively, one might note that company two operates larger boats and pushes larger tows than do the other two companies. This is most evident in the newest large boats where automated equipment has replaced several crew functions. Since company two operates larger boats, it is quite possible that larger outputs are associated with smaller crew sizes (that is, a negative coefficient for crew size).

The coefficient for labor is much larger for the downstream regressions than for the upstream regressions. According to the estimates, labor is four times as productive on downstream runs. In part, this additional productivity comes from higher average speed on the downstream run; there is more work for the crew per hour of running time.

A towboat is an expensive piece of capital equipment (costing between 1/2 million and 1 million dollars when new) and can be assumed to embody the best technology available when it was constructed (or had a major overhaul). Insofar as this technology has improved over time, newer boats should be more productive. Thus, a linear measure of the age of a towboat should have a negative coefficient. In looking at the results, only company one has a significant coefficient. Two of six coefficients are negative, and coefficients of companies two and three change signs between the upstream and downstream regressions. Apparently, there have not been significant improvements in design over this period.

This lack of significance in design inprovements was checked with naval architects. They expressed the opinion that there has been only one major improvement: the Kort nozzle. These impressions are confirmed by the regressions. In all cases, the coefficient of Kort nozzles is positive and significant. Again, they seem to add more to productivity in downstream runs than in upstream runs.

Over the last two or three decades, the price of waterway transportation either remained constant or declined slightly. Over this same period, the price of fuel has risen sharply, as has the cost of equipment and crew. Apparently, there has been a significant increase in productivity over this period. The increased productivity could be due to: (1) changes in equipment design, (2) improvements in labor productivity or (3) changes in operating rules, such as the number of barges in a tow. If the increased productivity took the form of improved equipment, it could be thought of as "capital embodied technological change."[5] If the productivity were due to labor, it would be termed "labor embodied technological change." If it can be ascribed to neither capital nor labor, it is "neutral technological change."

The age of the towboat gives a general indication of capital embodied technological change. As pointed out above, there is little evidence that this change is important. A particular measure of capital embodied technological change is the presence of Kort nozzles. These seem to be effective in improving the productivity of towboats. Over the last three decades, crew size has fallen (holding towboat size constant). However, this change took place before 1964, when the data series began. A measure of labor embodied technological change has not been pursued. However, the experience of operators is that this effect has not been significant.

Insofar as capital embodied technological change has been accounted for, any remaining increase in productivity must be classified as labor embodied or neutral. In fact, a more direct measure of neutral technological change is available. Controlling for capital and labor variables, how did towboat productivity change between 1964 and 1966? This question is answered by looking at the year dummy variables. Since 1966 is omitted, it is defined as having a coefficient of zero (to serve as a base of comparison). In the downstream regression for company one, the coefficient of 1964 is -.20 and for 1965 -.05. Thus, 1966 experienced a neutral technological improvement of .21 over 1964 and .05 over 1965. These coefficients indicate an impressive amount of neutral technological change.[6]

For the river district dummy variables, the omitted district was the Lower Illinois River (which thus has a coefficient of 0). The estimated coefficients carry the interpretation of being corrections to the productivity experienced on the Lower Illinois River. For example, the Port of Chicago has a lower productivity (equal to about .09 EBM's per hour) after accounting for all other factors. In contrast, the Lower Mississippi River (downstream) gives about .6 more EBM's per hour. Any river district which is more "difficult" to operate on than the Lower Illinois River should have a negative coefficient; positive coefficients indicate rivers which are "easier" to operate on.

The estimated coefficients indicate the penalty to be assigned to operating on each river district. Thus, if a company were offered freight in the Port of Chicago, which is to be delivered in the Port of Chicago, the price per ton-mile should be relatively higher (other things held equal) because it is more difficult to operate there. (These coefficients give quantitative information as to the size of the penalty to be attached to each river district.)

One way of checking these river district coefficients is to rank them and compare this ranking with a similar one embodied in Barge Form C. Table 8 presents a ranking of the coefficients for both upstream and downstream movements of the company one linear regression. The ranking varies between the two as expected. Some river districts are completely enclosed by locks. Current is minimal, and it makes little difference whether a tow is headed upstream or downstream. Other river districts have completely open water (such as the lower Mississippi). For example, in the upstream run, the lower Mississippi probably has the most rapid current, while the Ohio River is completely dammed up (except during spring flood). For operations on the Ohio River it should prove as easy to go one way as the other.

TABLE 8

COMPANY ONE LINEAR REGRESSION RIVER DISTRICT COEFFICIENT COMPARISON

Districts	Coefficient	Empty Barge Factor
Downstream:		
Cairo to Mouth of Miss.	0.637*	.6
Cairo to St. Louis	0.535*	.6
Tennessee	0.524*	.7
Gulf of Mexico	0.392*	.7
Cairo to Cincinnati	0.349*	.7
Cincinnati to Dam 23	0.184	.7
Lower Illinois	0.000	.7
St. Louis to Grafton	-0.016	.7
Port of Chicago	-0.098*	1.0
Upstream:		
Cincinnati to Cairo	0.316*	.7
Cincinnati to Dam 23	0.316*	.7
Cairo to St. Louis	0.141*	.6
Gulf of Mexico	0.138*	.7
Tennessee	0.113*	.7
Cairo to Mouth of Miss.	0.110*	.6
Dam 23 to Pittsburgh	0.051	.75
Lower Illinois	0.000	.7
Port of Chicago	-0.089*	1.0
St. Louis to Grafton	-0.091*	.7

*Indicates that coefficient was significant (t-value greater than ±2.0)

Barge Form C contains a table of the relative weight given to towing an empty barge on each river district. The lowest weight indicates that (on the Lower Mississippi) an empty barge requires about 60 per cent of the effort required to push a full barge. The highest weight indicates that (for the Port of Chicago) it takes the same effort to push an empty barge as a full one. In the port of Chicago, average speed is governed by congestion and a towboat can never use all of its power to gain speed.

In general, the higher the weight assigned to an empty barge, the more congested the river district will be. These weights also tend to reflect shallow water, swift current and locking delays as well as congestion. These factors are approximately the same as would be expected to produce penalties in our estimation scheme. For downstream movements, river districts are ranked identically by both the river district coefficients and the Barge Form C weights. On upstream movements the ranking is less perfect. While this rank correlation tends to strengthen one's faith in the estimated conditions, it should be pointed out that the Barge Form C weights are somewhat arbitrary and do not represent any systematic collection of data.

These estimated coefficients are of direct interest to waterway carriers. They provide a measure of the relative difficulty of operating in each waterway. These coefficients might be used to modify the rate structure to bring price more closely in line with marginal cost. *Ceteris paribus*, the price (per ton-mile) of carrying freight on each river district should reflect these operating penalties.

The final set of variables accounts for seasonality in operations. The month of December is defined to have a coefficient of zero; each of the other monthly coefficients indicates the relative difficulty of operating in that month. The changes between the months appear to establish a regular pattern. The winter months have negative coefficients, indicating difficulties of icing and bad weather. The spring and early summer months have large positive coefficients, indicating they are the easiest months in which to operate. Finally, the early fall carries a negative coefficient due to low water (in general there is little rain in summer and so river depth falls). Again, these monthly variables could be used to redesign the rate structure to bring prices in line with costs.

Summary

A statistical production function for towboats on inland waterways was estimated from data from three companies over a three-year period; approximately 4500 observations on 100 towboats operating on 16 river districts were used. Both linear and log-linear relationship were estimated.

The reduction function for downstream operations was found to differ significantly from that for upstream movements. It was also found that the production functions differed between companies.

The productivity of a towboat (measured in terms of the number of EBM's per hour produced) was regressed on the horsepower and also age of the towboat, its crew size and whether it had Kort nozzles. Additional variables were the river in which it operated and the month and year of the observation. Horsepower is a measure of the size of the towboat and had a consistently positive, significant coefficient. With the exception of one company, crew size had positive, significant coefficients. The age of the towboat added little to the regression, indicating few improvements in towboat design in recent years. The one significant improvement was the Kort nozzle; thus there is evidence of some capital embodied technological change (in the Kort nozzles). There was also evidence of neutral technological change. The variables for river district and month showed effects consistent with the experience of waterway operators.

CHAPTER IV

AN ANALYSIS OF DIRECT TOWBOAT AND BARGE COSTS

Inland waterway firms are the operating managers of large, varied, and costly assortments of floating equipment. This study is an attempt to determine the influence of the operating environment of inland waterway barge firms upon the direct operating costs associated with their towboat and barge fleets.

A. An Analysis of Direct Towboat Costs

The cost of operating a towboat should be directly related to the amount of "work" (measured in EBM's or ton-miles) the boat performs. However, towboats possess a range of capabilities (as measured by their horsepower and other physical characteristics). In addition, the environment in which the towboat operates (the waterway) directly affects the output of the towboat. Thus, one cannot look simply at the average cost of an EBM. The approach used here is to specify how average cost varies with output (EBM's) as the characteristics of the equipment and of the waterway are changed.

The Data

Data on towboat costs have been gathered from six inland waterway operators. The services performed by these six firms are widely distributed over the entire Mississippi and Gulf River systems. The six firms are varied as to size and type of traffic carried. Together, the six firms operate a towboat fleet which possess a wide range of physical characteristics.

The observations range over a period of five years, from 1962 to 1966. Annual costs were obtained for each towboat the firms actually operated during each year. In addition, information was gathered on the number of hours that each towboat operated on each river district in the Mississippi and Gulf River systems during each year in the sample.[1]

From the standpoint of determining costs, there are two dominant features of towboat operations. The first feature is the characteristics of the equipment itself. The most general indicator of towboat size (and thus of its productive capacity) is its horsepower. As larger horsepower towboats are used to produce a given output, one would expect total cost to rise (due to increased fuel consumption, greater maintenance expense, etc). However, since certain components of total cost, such as fuel consumption, wages, and maintenance, would not be expected to increase in proportion to horsepower (and since some components of total cost are not even related to horsepower, e.g., radio repair) one would expect that increases in total cost would be less than proportional to increases in horsepower. Therefore, we would expect average cost to fall, ceterus paribus as horsepower increases.

It is to be expected that the age of a towboat will directly affect the cost of operating it. An old boat is a less efficient resource than a new boat. One would expect costs to be higher for older boats. A linear measure of towboat age was used in the statistical production function without much success. Here, the effect of age has been included with the use of dummy variables. Towboat ages were grouped into five year intervals. A dummy variable was assigned to each group. In the regression the dummy variable for the newest boats was excluded. If our age hypothesis is correct, the coefficients of the remaining dummy variables should all be positive.

Other characteristics of the towboat most certainly will affect cost in some way. Some features, such as the extent of automation in the engine operation will affect cost directly by reducing the amount of the labor input and hence wage cost (there will of course be an increase in cost for the capital input). Other features, such as electronic guidance equipment, will serve to improve the average performance of the boat and lower cost for any given output. Most of these factors are extremely hard to quantify and have not been included in the statistical model. Information on one feature, the Kort nozzle, is available and has been included in the analysis. The Kort nozzle is a tunnel which surrounds the towboat's propellers and creates a channel through which the water is propelled. Its purpose is to reduce power loss due to turbulence and thus improve performance. Its effect on cost is of interest since some controversy among naval architects surrounds its use. The presence of Kort nozzles on a boat in our sample is denoted by a dummy variable. If the Kort nozzle does perform as expected we would expect the dummy variable associated with its presence to have a negative sign.

The second dominant feature of towboat operations is the operating environment provided by the waterway. The physical characteristics of the waterway (depth, width, stream flow) affect the speed at which a given towboat can travel and thus help determine the maximum output the towboat can produce per unit-time. Other features of the waterway, such as the number of locks and conjestion, will affect the average level of performance of the boat over many trips.

Towboats generally do not operate on one river district alone. Rather, their operations may be spread over three or four districts. From time-to-time the assignments of the towboats will change and the boats will be assigned to a different set of river districts. One would expect the cost of operation for any towboat to vary with its assignment. That is, one would expect that average costs will be lower for operations on a less restrictive waterway than for operations on a river district with a narrow channel and congested traffic. Thus, in estimating a cost relationship for towboats, an allowance must be made for where the towboat operated.

One way to include the effect of river districts would be to look at the distribution of total output among river districts, measured in EBM's, for each towboat in each year. However, this approach is not satisfactory because the period over which the output is produced is not known. It could be argued that the number of EBM's produced on a river district and the time spent operating on a river district for any one boat will be highly correlated. This will in general be true. But, in comparing towboats cross-sectionally, an implicit assumption would be introduced: that each boat spends the same number of hours in operation during the year. This is, however, a poor approximation to actual operations. Towboats will typically spend time "tied up" either for repairs or in waiting for business. This time may vary from a few hours due to severe ice on the waterway, to several weeks for a major overhaul.[2]

A more direct method, and the one employed in this study, is to weight a towboat's operation on the various river districts by the number of operating hours spent on each river district. This is done in the statistical model by calculating the percent of total hours of operation spent on each river district. A set of variables has been included in the statistical model which represent the relative operating time spent on each river district for each boat. The interpretation of these variables would be similar to that of dummy variables. One of the percentages must be excluded. Then, the question could be posed: How would costs change if the towboat operated 1% more of the time on river district X_1 and 1% less on the excluded river district? The answer would, of course, depend upon the magnitude and sign of the coefficient of X_1 (note that a 1% increase, in this case, is not the same as a one percentage point increase. For example, if a boat spent 50% of its time on the Lower Illinois River, an increase of 1% of time spent on this river district would be a .5 percentage point increase). Thus, the river district coefficients should show the "costliness" of operating on each river district relative to the excluded river district.

In summary, an observation for the towboat cost function consists of the operating cost of a towboat for each year in the sample, the characteristics of the towboat (its horsepower, age, and whether it has Kort nozzles), the total output of the towboat for the year (measured in EBM's), and the distribution of the time spent in actual operation for twenty-one river districts of the Gulf and Mississippi River systems.

The Statistical Model

A statistical cost function has been estimated with the above described data. The form of the function is:

$$\log_{10}\text{Cost/EBM} = \log_{10}\alpha_0 + \alpha_1\log_{10}\text{HP} + \alpha_2\log_{10}\text{EBM} + \alpha_3\text{KN} + \sum_{i=1}^{20} B_iR_i + \sum_{j=1}^{3} \gamma_jA_j + \sum_{k=1}^{5} \delta_kC_k + \sum_{\ell=1}^{4} \lambda_\ell Y_\ell$$

where: Cost = annual total direct towboat cost for each boat

EBM = Equivalent Barge Miles produced by each boat in each year

HP = towboat horsepower

KN = Kort nozzle dummy variable (1 = presence of Kort nozzles)

R_i = percent time towboat operated on ith river district (Lower Illinois excluded)

A_j = dummy variables for towboat age (A_1 = 1 indicates the towboat was built prior to 1950. 1960-1965 excluded)

C_k = dummy variables for each company

Y_ℓ = dummy variables for year (Y_1 = 1 is 1962, etc., 1966 excluded)

In many regression models in economics, economic theory does not indicate the correct specification of the function to be estimated. Frequently, however, statistical theory indicates a choice. One model may be preferred in particular circumstances because it avoids statistical biases which may occur due to the nature of the data. One example of this is the choice between the total and average functional forms for cost estimation.

In using, for example, cross-section cost data, it is frequently the case that the variance of total cost increases as output increases. This phenomena is known as heteroscedasticity. This would be depicted by a "fanning-out" of the scatter of points in the cost-output plane as output increases. The presence of heteroscedasticity will produce biases in the estimated function. One way of correcting for this bias is to deflate cost by output, that is, to specify the average cost function.

Plots of the towboat cost data indicate that the data tend to disperse at higher outputs. Thus, the average form of the function was specified.

A second specification problem involves the choice between the linear and log-linear forms. The log-linear form is more likely to approximate true average costs than the linear form if the average cost curve is "U-shaped." Both linear and log-linear forms were estimated. The results bear out the expectation of U-shaped average costs. For example, the r^2 for the linear regression was .16; the standard error of the estimate was .14, which can be compared to the mean value of the dependent variable of .19. Thus, the log-linear form on the average cost function was specified.

The Results

The results of the log-linear estimation are presented in Table 9. Note that the dummy variables for year have been omitted. An F-test was performed to determine the significance of each variable or set of variables. The results of these tests are reported in Table 10. As can be seen from Table 10, all variables, or sets of variables,

TABLE 9

TOWBOAT COST REGRESSION RESULTS

r^2 = .85
Sample Size = 266
Standard Error of Estimate = 0.14883
Intercept = -0.36759

Variable Name	Mean	Regression Coefficient	t-Value
Kort Nozzles	0.60150	-0.07181	-2.96398
Towboat Age			
1956 - 1960	0.27820	-0.03936	-0.92493
1951 - 1955	0.21053	0.01693	0.44163
Before 1950	0.33835	0.06774	1.98602
River Districts (percent times 10^{-2})			
Chicago Port	0.08023	0.01841	0.25051
Tennessee	0.00598	0.18884	0.46628
West Intercostal Canal	0.02857	-0.16024	-1.63870
East Intercostal Canal	0.00241	-1.21041	-1.45835
Gulf of Mexico	0.01857	-0.07897	-0.83839
Cairo to St. Louis	0.06481	-0.18085	-1.44655
St. Louis to Grafton	0.00402	0.82517	0.87359
Cairo to Mouth of Mississippi	0.18763	0.03639	0.57067
Cincinnati to Cairo	0.04395	0.06897	0.50527
Dam 23 to Cincinnati	0.03207	0.17160	0.81126
Pittsburgh to Dam 23	0.04797	0.00230	0.02480
Port of Pittsburgh	0.00526	1.37323	0.69112
St. Louis to Minnesota	0.07530	0.01127	0.15281
Dam 23 to Cairo	0.08252	-0.07361	-0.87499
Kanawha River	0.00752	0.04017	0.25541
Monongahela	0.00421	0.35108	0.48631
Missouri	0.00361	0.64024	1.09926
Unit Tow Movement	0.06177	0.23119	3.84230
Kanawha to Monaca	0.04241	-0.20394	-0.67019
Missouri to Ohic to Tennessee	0.01410	0.23829	0.60023
Company Dummy Variables			
Company 1	0.11654	-0.03766	-0.62727
Company 2	0.19173	0.15438	3.80294
Company 3	0.02632	0.19398	2.91415
Company 4	0.18421	-0.00853	-0.19389
Company 5	0.01880	-0.31262	-0.81462
$\log_{10}$ Horsepower (times 10^{-4})	-0.61664	-0.26136	-2.23274
$\log_{10}$ EBM (times 10^{-6})	-0.73099	-0.53895	-19.21339

Dependent Variable: $\log_{10}$ Cost/EBM (times 10^{0}) 0.19820

TABLE 10

F-TESTS FOR TOWBOAT COST REGRESSIONS

Excluded Variables	Degrees Of Freedom	Mean Square	F-Value (Significance Level)
Yearly Dummy Variables	4	.02132	0.9621
None	230	.02216	(Not Significant)
Towboat Dummy Age Variables	3	.09292	4.1931
None	230	.02216	(.99)
River District Percentages	20	.05551	2.5051
None	230	.02216	(.99)
Firm Dummy Variables	5	.11424	5.1550
None	230	.02216	(.99)

are significant at the .99 level, except for year (which has an F-value of less than one). Thus, the function was re-estimated excluding year.

Looking first at the results for towboat characteristics, we see that the coefficient of $\log_{10}$ horsepower is negative and significant. One would expect the horsepower of a towboat to be one of the most significant elements in determing towboat cost. The negative sign for horsepower indicates that the average cost of an EBM declines as larger boats are employed. It is possible to think of horsepower as a measure of "plant" size. The curve of the function traced out by increasing horsepower (allowing EBM's to adjust optimally) could be viewed as a long-run average cost curve. This curve would be downward sloping throughout the range of actual operations. Viewed in this way, towboat operations exhibit long-run decreasing costs.

Care should be exercised, however, in making this observation. We know, from the production function, that for operations on a given river district, horsepower exhibits diminishing productivity. Thus, it must be true that the average cost curve will turn up after some point. That is, it is certainly not possible to increase horsepower indefinitely and expect long-run average costs to fall. There is, nevertheless, reason to believe that average towboat costs are falling throughout a wide range of current operations.

The results for Kort nozzles show that the presence of this feature on a towboat has a significant influence on average cost. The coefficient for Kort nozzles is negative and significant. Use of the Kort nozzle implies an overall reduction of 15% in average cost. For example, multiplying mean average cost and mean EBM, the total cost of mean EBM (almost

200,000 EBM per year) comes to about \$300,000 per year. A 15% cost reduction for Kort nozzles would be \$45,000 per year.

The results for towboat age are generally consistent with expectations. Age of the boat was taken from the date it was built, or the date of its last major overhaul. Newer boats represent an investment in a newer technology. To the extent that there has been any technological improvement in towboats, costs for newer boats should be lower. An implicit assumption is that when a boat is overhauled the technology current at the time of overhaul is incorporated into the towboat. The newest boats (dated from 1961 to 1966) represent the omitted dummy variable.

The results show that the next newest boats (dated from 1955 to 1960) have lower average costs than the newest boats, although the coefficient for this variable is not significant. The coefficients for boats built from 1951 to 1955 is positive (.01693). The coefficient for the oldest boats (built prior to 1951) is positive and significant, and is larger algebraically (.06774), than the coefficients for any of the newer boats. The oldest boats are 17% more costly to operate than the newest boats.

The river district variables are ranked by the algebraic magnitude of their coefficients in Table 11. Also shown, in Table 11 for each river district, are the empty barge factors from ICC Barge Form C. The factors are a rough indication of the "difficulty" or "penalty" to be attached to that river district depending upon the restrictiveness of the channel and delays necessary to pass through any locks. The rankings are generally consistent. However, it is apparent that the Barge

TABLE 11

RANKING OF RIVER DISTRICT COEFFICIENTS FOR TOWBOAT COST REGRESSION

River District	Regression Coefficients	Barge Form C Empty Factors
Port of Pittsburgh	1.373	1.0
St. Louis to Grafton	0.825	.7
Missouri	0.640	.9
Monongahela	0.351	.9
Missouri to Ohio to Tennessee	0.238	
Unit Tow Movement	0.231	
Tennessee	0.189	.7
Dam 23 to Cincinnati	0.172	.7
Cairo to Mouth of Mississippi	0.036	.6
Port of Chicago	0.018	1.0
St. Louis to Minnesota	0.011	.7
Pittsburgh to Dam 23	0.002	.75
Lower Illinois	0.000	.7
Dam 23 to Cairo	-0.074	.7
Gulf of Mexico	-0.079	.7
West Intercostal Canal	-0.160	.7
Cairo to St. Louis	-0.181	.6
Kanawha to Monaca	-0.204	.7
East Intercostal Canal	-1.210	.7

Form C weights have limited accuracy. One would expect, for example, that operating conditions are not the same for the Tennessee River and the Gulf of Mexico, yet both have a weight of .7. The factors are included merely as a rough check on the tendencies shown by the river district coefficients.

In looking at the river district coefficients, there is only one that is quite surprising. That is the coefficient shown for the Lower Mississippi. Since the Lower Mississippi is probably the least restrictive of all river districts and contains no locks, one would expect it to have a relatively large, negative coefficient. Other than the coefficient for the Lower Mississippi, the relative magnitudes of the river district coefficients appear reasonable.

It is worth noting that, although individually some of the coefficients are not significant, nevertheless they are significant as a group (see Table 10). In addition, from an economic standpoint, most of the river district coefficients are significant relative to average cost. For example, by increasing operations on the Central Ohio (Cincinnati to Dam 23) 10 percentage points (reducing operations by a like amount on the Lower Illinois) average cost will fall by 4% at all levels of output. Taking, once again, $300,000 as the representative annual total cost, this change would increase total cost by $12,000.

That the results show significant differences existing between firms is no surprise. One would expect the cost to vary due to regional differences in wage contracts and other prices; differences in typical operating conditions (river districts where the firm is licensed to operate, rules concerning how deeply to load barges, flotilla make-up, and frequency of equipment maintenance); differences in the type of commodity carried; and differences in equipment used. Due to the varied circumstances it is difficult to draw definite conclusions from the firm dummy variable coefficients. For example, company three, with the largest positive (and significant) coefficient is a specialized company dealing with common carriage almost exclusively, and operates to a great extent on some of the more restrictive waterways. Company four, on the other hand, is one of the largest firms in the industry, carries almost exclusively bulk commodities and has several unit tow operations. One would expect its cost to be generally lower than the other firms.

Finally, the coefficient of $\log_{10}$ EBM is negative, indicating an inverse relationship between total output and average cost. Viewing the towboat as a firm once again, the relationship between EBM's and Cost per EBM can be characterized as short-run cost. Holding horsepower (firm size) constant and varying EBM's a family of short run curves can be derived.

At the point of means the average cost of an EBM is $1.578. A more intuitive feel for this figure can be gained by recalling the definition of an EBM: An EBM is the movement of a fully loaded barge for one mile under given physical operating conditions. A fully loaded barge will weigh about 1400 tons. Thus, an EBM, ignoring its special physical characteristics is approximately equal to 1400 ton-miles. Meyer reports the total average cost of a ton-mile in the barge industry to be about two to two and one-half mills.[3] This is a cost per 1400 ton-miles of $2.80 to $3.50. No indication is given in the Meyer study as to the level of output to which his estimate corresponds. However, our figure of $1.578 does appear quite reasonable. One would have to add an estimate of direct barge expense and indirect expense to our estimate to make it directly comparable to Meyer's figure.

In summary, the results for the towboat cost function are quite generally consistent with a priori expectations and result in reasonable overall predictions of average cost. The evidence presented indicates that there is strong dependence of cost upon towboat characteristics and the physical characteristics of the various waterways. Both long-run and short-run average costs are declining, indicating that if output was to be expanded, a major category of total average cost would be declining, at least for a wide range of output.

B. An Analysis of Direct Barge Costs

In the analysis of towboat costs reported in the previous section, cost was related to equipment and waterway characteristics, as well as the general output level. It was found, in particular, that significant cost differences existed for shipments on different waterways. A further attempt is made in this section to allocate a general cost category (direct barge costs) to classes of service barge firms perform. In addition to equipment and waterway characteristics, the effect of the actual commodity shipped is examined.

Direct barge costs are primarily maintenance costs. Maintenance is required for two reasons: (1) physical deterioration due to the passage of time, and (2) wear occasioned by actual use. Maintenance cost due to time should be related to the age of the barge. One would expect that wear due to use would be systematically related to the commodity carried and the waterway over which the movement occurs. In addition, cost is related to the size and type of barge as well as the total output of each barge.

The Data

Data on individual barge costs have been obtained from one inland waterway operator for a period of three years. Although cross-section data would be preferable it was not possible to gather equivalent data from other firms. However, due to the homogeniety of the equipment, there is no reason to suspect that barge costs should vary systematically with different companies. In addition, a strong argument can be made that the data obtained are representative of the entire industry.

The firm in the sample operates a large and varied selection of barges, including both standard and jumbo open and covered barges, various types of tank barges, and several deck barges. The size of the fleet in each year is about 330 barges, an average size for the major freight carriers.

Although it is generally thought to be primarily a general cargo carrier, the firm in the sample derived an average of 49.2% of its revenue from bulk commodities for the period of the study (1964-1966). The distribution of its output (in cargo ton-miles) between all bulk commodities and general cargo was 61.3% and 38.7% respectively. In addition, the firm's operations cover the major portions of the Gulf and Mississippi River Systems.

The observations range over a period of three years from 1964 to 1966. Annual costs were obtained for each barge the firm actually operated during the year (including long-term charters). In addition, the total number of cargo ton-miles carried by each barge during a year was obtained as a measure of work performed (Empty barge movements by river district for each barge were not available. As a result the EBM measure could not be calculated).

Several types of barges are in use on the inland waterways. Prior to about 1950 the most common type of barge in use was the "standard" size barge. Standard barges are generally 175 feet long and 26 feet wide. The size of the barge was a reflection of the lock sizes then in use on most waterways. Locks were usually 56 feet or 80 feet wide and thus would allow two or three standard size barges to pass abreast through the lock. Several hundred of these standard barges are still in use.

The most common type of barge in use today is the "jumbo" barge. Jumbo barges are 195 feet in length and 35 feet wide. Several reasons might account for the use of these larger barges. A "tow" consists of several barges lashed together. The fewer the number of barges required to make up any given tow size, the more stable or rigid the tow will tend to be. This is an advantage, not only in tow make-up time, but the stability also serves as an aid to navigation. In addition, most modern locks, constructed in the past decade, have length and width conforming to multiples of the 195 by 35 foot barge. Also, large shipment sizes are almost certainly cheaper to handle in one barge than in two. Cost per shipment would be lower because of reductions in loading and unloading time and the shifting of fewer barges. If shippers have also tended to increase shipment size, further savings will have accrued to the scheduling operation.

Other barges in use, primarily tank barges, are specialized in use to one or two particular commodities. Tank barges are generally custom built to meet special shipper requirements. Certain acid barges, for example, have special corrosion resistant tank linings. Other barges may contain heating or refrigeration equipment for special cargos.

One would expect that these differences between barge types would have an effect on the cost-output relationship. This is especially true since the major part of barge cost is expenditure on maintenance of the hull and ancillary equipment. For example, both standard and jumbo barges come with or without covers. The covers are expensive metal lids which are easily damaged. One would expect that barges with covers would tend to be slightly higher cost barges than those without the covers.

As a result each of the barges in the sample has been classified into nine types (see Table 14). In the statistical model each type is identified by a dummy variable. Thus, it will be possible to determine the relative cost of operating barges with different physical characteristics.

Two additional aspects of barge freight operations should be significant elements in barge cost determination. The two aspects can be seen as different ways of partitioning total output into types of special services produced by the firm.

The first way of partitioning total output is by commodity shipped. Barge cost should be quite sensitive to the commodity being shipped for several reasons. One reason is found in the way commodities are loaded and unloaded. Coal, for example, is generally unloaded by bucket clamshells which are dropped into the barge. Even when reasonable care is exercised the buckets frequently damage the sides and bottom of the barge. Eventually, the hull damage will require that the barge spend time in dry-dock to be patched. Occasionally, the buckets will drop through the bottom or side of the barge requiring substantial structural repair.

Grain, on the other hand, is loaded and unloaded by more "gentle" equipment. However, grain shipments require special handling in that barges used for shipping grain must be cleaned of other bulk materials and also must be covered to prevent mildew of the grain.

Bulk commodities, including coal, generally tend to be more abrasive than, for example, grain. In addition, some bulk commodities come in large size pieces, such as coal and some ores, whereas others, such as sand or grain, come in small particles. As bulk commodities are loaded into a barge, generally by dumping, the large size commodities can inflict additional damage on the hull.

As mentioned earlier, liquid commodities are shipped in tank barges. Some tanks are used to ship more than a single commodity. To prevent contamination when this occurs, tank barges must be cleaned. In addition, some liquid commodities are corrosive, thus tanks may have to be occasionally relined.

To capture the effect of commodity shipped on cost, information on the number of ton-miles of each commodity shipped by each barge has been obtained. Commodities are classified in the barge industry according to one of several standard commodity classifications. The classification used by the firm in this sample is the ICC "Standard Commodity Classification for Transportation." This commodity code is an extremely disaggregate taxonomy of bulk, agricultural, and manufactured products. The sample firm carried an average of 60 commodities, as listed in the commodity code, in each of the three years in the sample.

To reduce the problem to manageable proportions the sixty detailed commodities were combined into six commodity groups. A two-fold classification is frequently used in the barge industry: general cargo and bulk commodities. However, there are important commodity differences which this classification would obscure. Bulk commodities were classified into five types, general cargo becoming the sixth type, as follows:

1. grain
2. coal

3. other bulk
4. petroleum products
5. other liquid bulk
6. general cargo

This classification is a reasonable reflection of differences among commodities as to commodity characteristics and special handling required.

After aggregating the commodities into the six groups for each barge in each year, on the basis of cargo-ton-miles, the group percentages for each barge were calculated. This yields a measure of the relative output produced by each barge of the six commodity groups.

When the percentages are used in this way in the statistical model, the coefficients of each type of service will yield a measure of the relative "costliness" of producing each class of service. The interpretation of the coefficients for these variables is the same as dummy variables. One variable must be excluded to prevent singularity of the variance -- covariance matrix. The question can then be posed: How will costs be affected if the firm ships 1% more grain (and 1% less of the excluded commodity)? The answer will, of course, depend upon the sign and magnitude of the grain coefficient.

An additional variable, relating to commodities, has been included: the number of commodities shipped by each barge during the year. As mentioned earlier, barges must frequently be cleaned before different commodities can be shipped. It would be possible, of course, for the firm to avoid such expenses by always using the barge in the same service. This would be done, however, at the expense of increased scheduling costs. In addition, the firm would most likely have to maintain a larger barge fleet. Scheduling costs cannot be determined here, however, one would expect direct barge costs to be higher the greater number of commodities carried by each barge.

The second point of view that can be taken in disaggregating total output is by the geographic area of operation. The inland waterways do not represent a homogeneous "roadbed" for freight operations. This is especially important in considering towboat costs where the physical characteristics of the waterway have been shown to have a significant effect on output and cost (see the previous section).

These effects should be evident for the barge fleet as well, particularly when maintenance cost is considered. Barge equipment faces a continual rust problem. To prevent rust, each barge is treated with a special rust-resistant paint. These treatments can be quite expensive, often running in excess of $5,000 for any one barge. One would expect that waterways with relatively higher salt content (or other corrosive material) such as the entire Gulf River System would also have higher barge costs associated with movements on them.

Barges also receive additional wear due to physical contact with other barges or structures such as locks and piers. Such effects are quite likely to be associated with the river district over which a barge operates. For example, a river district which has a large number of locks that must be traversed and difficult navigation conditions will tend to have a higher incident of collision and scraping. Barge damage and thus barge maintenance cost will be higher for such river districts. A similar argument could be made for river districts with conjested traffic.

In addition, river districts with shallow stream depths present a persistent problem for barge navigation. Barges are frequently loaded to 8 1/2 foot or 9 foot drafts. On a nine-foot waterway, such as the Illinois River, a barge may frequently scrape along the river bottom or may clear the bottom only by a few inches. In addition to wear on the barge from scraping, any obstruction encountered under these conditions can inflict considerable damage on the barge.

The total output of each barge (in cargo ton-miles) was obtained for each of twelve river districts over which it operated during each year in the sample. The twelve districts cover the major portion of the Mississippi and Gulf River Systems. The percent cargo-ton-miles by river district was calculated for each barge. The interpretation of these percent variables is the same as for the percent commodity variables described above.

Additional information has been obtained on the characteristics of each barge, in particular, barge age and cubic capacity. Barge age would attempt to capture two reinforcing phenomena. First, if there has been embodied technological change in barge construction one would expect newer barges to have generally lower costs. Secondly, newer barges may require less maintenance to the extent that barge wear is cumulative with time. Thus, newer barges should be associated with lower costs.

The cubic capacity of two barges of the same type is not necessarily the same. Variations occur due to the number of rakes, hull construction, and height of combing. The capacity variable could be viewed as a measure of "plant" size. The relationship described by increasing cubic capacity (allowing cargo ton-miles to adjust optimally) could be viewed as long run cost.

The barge type dummy variables really serve two purposes. Its major purpose is as an indicator of design or physical differences between barges. The second use of barge type could be as a discrete measure of barge capacity. However, there are variations in capacity within any one barge type, with some overlapping of sizes. In addition, the discrete nature of barge type does not lend itself to analysis in a long-run cost framework. Thus, in order to obtain a more meaningful cost-capacity relationship, the continuous cubic capacity measure has been included in the model.

In summary, an observation for the barge cost study consists of a barge (identified by type, age, and cubic capacity); the total number of cargo ton-miles carried during each year by each barge; the percent cargo ton-miles carried by each barge of six commodity groups and the total number of commodities carried; the percent cargo ton-miles carried by each barge for twelve river districts; the annual expense for each barge; and the year in which the observation was made.

The Statistical Model

A statistical cost function for barges has been estimated using the data described above. The form of the function is:

$$\log_{10}\text{Cost/CTM} = \log_{10}\alpha_0 + \alpha_1\log_{10}\text{CTM} + \alpha_2\log_{10}S + \alpha_3 N + \sum_{i=1}^{11} B_i R_i + \sum_{j=1}^{5} \theta_j C_j + \sum_{k=1}^{8} \delta_k T_k + \sum_{\ell=1}^{2} \lambda_\ell Y_\ell$$

where: Cost = annual total direct barge operating cost for each barge

CTM = cargo ton-miles carried by each barge in each year

S = barge size, measured in cubic feet of cargo space

N = number of different commodities carried by each barge during one year

R_i = percent CTM carried by each barge on the ith river district (Lower Mississippi excluded)

C_j = percent CTM carried by each barge of six commodity groups (general cargo excluded)

T_k = dummy variables for barge type (jumbo covered excluded)

Y_ℓ = dummy variables for year (1966 excluded)

Once again, the average form of the function has been specified. The reasons for this specification are the same as given in the preceding section on towboat costs. The log-linear form is used to take into account the expected curvature of the average cost curve.

Note that in the above specification the variables for barge age have been omitted. Two ways of including barge age were tried. The first was a linear index, which increased with barge age. The coefficient of the linear age measure was not significant (t-value of .97) and was excluded.

The second way of including age was to partition barge ages into five year groups by using dummy variables. An F-test performed on this set of dummy variables led to the failure to reject the null hypothesis (F-value of less than one). Thus, the function was re-estimated excluding the age variables.

Several factors could explain the absence of significant efforts due to barge age. First, if no embodied technological change has occurred in barge construction over the range of ages in our sample the coefficients of the dummy age variables would show no significant differences between age groups. A jumbo hopper barge is a fairly simple piece of equipment. According to naval architects, few design changes in barge construction have taken place since World War II.

Second, if barges receive preventative maintenance to arrest corrosion and hull damage, rather than being allowed to deteriorate until maintenance is absolutely necessary, the effects of age on maintenance cost would disappear. Casual inspection of the firms' records indicates that this may be the case, many barges receiving, for example, painting in successive years.

Table 13 presents the results of F-tests performed on each variable finally included. With the exception of cubic capacity, all variables are significant at the .999 level of probability. Although the F-value of the cubic capacity variable was not significant, a one-tailed t-test performed on the capacity coefficient yields a significant value at the .90 level of probability. Because of interest in the capacity variable as a quasi-plant size measure it has been included in the results below.

The Results

The results of the log-linear estimation are presented in Table 12. Looking first at the results for total current output, we see that the coefficient of cargo ton-miles is negative and significant. The negative sign for cargo ton-miles is an indication that the average cost of a cargo ton-mile declines throughout the range of current output actually experienced. If one takes the view that each barge is a "plant" for the production of cargo ton-miles of freight, the average

TABLE 12

BARGE COST REGRESSION RESULTS

$r^2 = .63$
Sample Size = 991
Standard Error of Estimate = 0.31022
Intercept = -0.72980

Variable Name	Mean	Regression Coefficient	t-Value
1964	0.31382	0.12851	4.90390
1965	0.34511	0.05558	2.26840
Barge Type			
Standard Open	0.08073	-0.14828	-1.75352
Standard Cover	0.16448	-0.13739	-2.04134
Jumbo Open	0.03734	0.09554	1.76755
Gulf	0.04743	0.17957	1.19173
Petroleum	0.03027	0.66234	4.53281
Acid	0.01514	-1.58922	-10.08618
Chemical	0.04036	0.29873	2.13220
Deck	0.00303	0.53075	2.26636
River Districts (percent times 10^{-2})			
Illinois River	0.20800	0.32822	5.69710
Port of Chicago	0.03466	0.36686	2.66019
Upper Mississippi	0.15879	0.13617	1.89436
Ohio River	0.03268	-0.06366	-0.29551
Tennessee River	0.00700	-0.01193	-0.02137
Cumberland River	0.00010	0.12623	0.02746
West Intercostal Canal	0.03322	0.40043	2.12541
East Intercostal Canal	0.00359	2.88486	1.59912
Gulf of Mexico	0.01365	0.20679	0.35863
Lake Michigan	0.00103	-0.41600	-0.21558
Atchtafalaya River	0.00944	0.52900	0.74934

TABLE 12--Continued

BARGE COST REGRESSION RESULTS

Variable Name	Mean	Regression Coefficient	t-Value
Commodity Shipped (percent times 10^{-2})			
Grain	0.34030	0.05175	1.22945
Coal	0.01290	0.21012	1.45622
Other Bulk	0.09413	0.03337	0.56460
Petroleum	0.02249	-0.90644	-3.73632
Liquid Bulk	0.06181	-0.09444	-0.67989
$\log_{10}$ Cubic Feet of Capacity (times 10^{-5})	-0.27828	-0.30018	-1.30661
$\log_{10}$ Cargo Ton-Mile (times 10^{-7})	-0.46448	-0.45160	-12.39779
Number of Commodities Shipped (times 10^{-1})	0.39364	0.24000	3.46528

Dependent Variable: $\log_{10}$ Cost/Cargo Ton-Mile (times 10°) -0.16661

TABLE 13

F-TESTS FOR BARGE COST REGRESSIONS

Excluded Variables	Degrees Of Freedom	Mean Square	F-Value (Significance Level)
Yearly Dummy Variables	2	1.1588	12.0403
None	961	0.0962	(.999)
Barge Type Dummy Variables	8	5.3195	55.2734
None	961	0.0962	(.999)
River District Percentages	11	0.4669	4.8516
None	961	0.0962	(.999)
Commodity Group Percentages	5	0.4646	4.8271
None	961	0.0962	(.999)
Cubic Capacity	1	0.1643	1.7071
None	961	0.0962	(Not significant)

cost-cargo ton-mile relationship would have the interpretation of short-run average costs. A family of short-run relationships exists for each plant size (cubic capacity). Average cost at the point of means is .68 mills/CTM.

Continuing with the plant analogy, we note also that the coefficient of cubic capacity is negative. Viewed in this way, barging operations exhibit long-run decreasing costs.

If these short and long-run relationships hold true for the entire range of output two predictions can be made for barge operations, one in the short-run and one in the long-run. The long-run tendency would be toward employing larger sized barges. In the previous discussion of barge size the limiting factor on the use of large barge sizes seemed to be the physical waterway characteristics, especially the size of locks. However, as lock size has been increased and as waterways have been deepened there has in fact been a tendency to use larger barges.

Unfortunately, the extent of this tendency cannot be easily determined. Nevertheless, barge size could not be expected to increase indefinitely even with accomodating changes in waterway characteristics. A major limitation on the optimal size barge is shipment size and scheduling costs. One could conceive of an extremely large barge sufficient in size to carry the entire cargo load of a flotilla of 16 jumbo barges. Typical shipment sizes presented to the firm, however, would not fill such a flotilla. A large flotilla, at any one time, will be carrying a variety of commodities with different origins and destinations. Barges are added and dropped from the flotilla as it moves along the waterway. Unless shipment size increases accordingly, scheduling costs for such a barge would rise considerably, and would tend to rise with increases in barge size. The firm in co-ordinating its shipments and selecting its equipment has to trade off the scale economies and the additional scheduling costs due to increases in barge size.

The short-run expectation is due to the indication of falling short-run average costs. If, for any given barge size, the average cost of a CTM falls, one would expect a tendency for barges to be loaded to capacity (as determined either by the cubic capacity of the barge or the depth of the waterway). This in fact seems to be the industry practice. Various operating rules-of-thumb have been developed in the industry as a result of experience. One rule-of-thumb expressed by all operators is that barges are loaded to within about 6 inches of the river bottom. Use of such a rule usually results in some barges scraping the river bottom in shallow stretches. That is, the rule says load barges as deeply as you can and still be able to navigate the river. During periods of high water the average load per barge tends to increase. One operator indicated that if the waterway channel was deepened he would immediately begin loading his barges to deeper drafts.

Once again, it is necessary to point out that this relationship holds only over a reasonable range of output. We know from the engineering production function for towboats that tow speed is a decreasing function, inter alia, of the draft of the flotilla. As draft increases speed falls and costs will rise. However, it is reasonable to conclude that falling short-run barge costs are consistent with expectations and appear to be confirmed by actual industry practice.

Table 14 ranks the coefficients for each of the barge types. Jumbo covered barges, the omitted type, has a "coefficient" of zero. The rank-

TABLE 14

BARGE TYPE REGRESSION COEFFICIENTS

Type of Barge	Regression Coefficients
Petroleum	0.66234
Deck	0.53075
Chemical	0.29873
Gulf	0.17957
Jumbo Open	0.09554
Jumbo Covered	0.00000
Standard Covered	-0.13739
Standard Open	-0.14828
Acid	-1.58922

ing is quite consistent with a priori expectations. Standard barges tend to be lower cost than the larger jumbos with covered standards being slightly more expensive. Jumbo open hopper barges are very slightly more expensive than jumbo covered barges. There might be a tendency for this to occur since covered barges carry mostly grain and general cargo which probably contribute less to barge maintenance problems than coal, for example, which often moves in the open hopper barges.

Gulf barges, a jumbo size barge with higher sides to allow deep water operations in the Gulf of Mexico,are higher cost barges. This is to be expected since gulf barges operate almost entirely in salt water areas where the corrosive environment requires frequent preventative maintenance.

Tank barges, with the exception of the acid barges, are high cost barges. This is certainly a reflection of the additional cost required to maintain the more sophisticated design of such barges.

Acid barges present a problem, in that their coefficient is the lowest, by an order of magnitude, than the coefficient for the other types and has a t-value significant at the .9995 level of probability. In checking the records for these barges it is apparent that little maintenance is performed on them. However, all of the acid barges are operated under a long term lease. A possible explanation is that any maintenance is performed by the leasing agency.

Tables 15 and 16 present rankings for the river district and commodity variables respectively. These two sets of variables, it should be recalled, represent alternative ways of dis-aggregating total output.

TABLE 15

RIVER DISTRICT REGRESSION COEFFICIENTS

River District	Regression Coefficients
East Intercostal Canal	2.88486
Atchtafalaya River	0.52900
West Intercostal Canal	0.40043
Port of Chicago	0.36686
Illinois River	0.32822
Gulf of Mexico	0.20679
Upper Mississippi	0.13617
Cumberland River	0.12623
Lower Mississippi	0.00000
Tennessee River	-0.01193
Ohio River	-0.06366
Lake Michigan	-0.41600

TABLE 16

PERCENT COMMODITY REGRESSION COEFFICIENTS

Commodity Group	Regression Coefficients
Coal	0.21012
Grain	0.05175
Other Bulk	0.03337
General Cargo	0.00000
Liquid Bulk	-0.09444
Petroleum	-0.90644

The coefficients of each set represent the relative cost of maintaining the barge fleet in the different types of service.

The relative order of the coefficients in both sets are consistent with reasonable expectations. For example, the Gulf, the East Intercoastal Canal, the West Intercoastal Canal, waterways with high salt content, are all high cost service areas. In addition both the East and West Canals are constricted waterways containing several locks.

An example perhaps best illustrates the use of the service variables. Looking at the commodity coefficients, suppose the firm has an opportunity to expand its coal operation. In the short-run this expansion will mean a reduction of the amount of some other service it offers. What will be the effect on barge costs of an increase in coal service?

Increasing coal service by 10 percentage points (decreasing by the same amount the output of general cargo) would increase average costs per cargo ton-mile by 5% at all levels of output. If the coal service was increased by reducing the amount of "other bulk" materials shipped (instead of general cargo) the net effect would be to increase the average cost of the mean output per barge by .029 mills/CTM, an increase of 4.3%. This represents an average increase in total barge operating expense of $98 per barge, or an increase in total barge cost of $32,500 per year for the entire fleet. Adding to this figure changes in other direct and indirect costs would yield an average measure of marginal cost which could be compared with the marginal revenue to be derived from the new business.

Finally, the coefficient for the number of commodities shipped per barge is positive and significant. The results show that if one barge was to ship one more commodity, average costs for that barge would increase by about 5% at all output levels. This represents an average increase of about $116 for each barge which carries an additional commodity.

For both towboat and barge operating costs it appears that many of the costs incurred by the firm in providing freight transport services are sensitive to the exact nature of the service performed. In particular, it has been found that for both towboat and barge costs, changes in equipment and waterway characteristics produce significant differences in marginal cost. In addition, for the barge analysis, the effect of the commodity carried on barge maintenance costs indicates that competitive rate differentials between commodities can be justified on the basis of differences in marginal cost.

CHAPTER V

A TIME SERIES ANALYSIS OF BARGE FIRM COSTS

In the preceding parts of this study, the costs associated with the linehaul equipment have been analyzed. As has been pointed out, however, this equipment must be co-ordinated by the barge firm in order to satisfy, at the least cost, the demands presented to it. In this chapter, the analysis is extended to cost relationships for the barge firm. As in the two preceding chapters, the approach is to examine how variations in waterway characteristics, commodity shipped, and seasonality are related to changes in the cost of transporting freight.

A. The Data

Detailed monthly cost and production data have been gathered from a single firm for a period of three years. The selected firm is among the major inland waterway operators. In terms of total output the firm operates at the mid-range for the industry. The firm operates a wide selection of floating equipment, varied as to age and size. Its operations cover a major portion of the Gulf and Mississippi River systems. Approximately 60% of its output (in cargo ton-miles) is bulk freight. Its revenue is derived in about equal proportions from bulk commodities and general cargo.

The total expenses of the firm have been broken down into three expense categories: overhead expense, direct barge expense, and direct terminal expense. Direct barge costs are all those costs related to the actual movement of freight. Included here are the costs of operating and maintaining their floating equipment and fees or expenses incurred in moving barges (mooring charges, port expenses, etc.). Overhead expenses are all those expenses which cannot directly be assigned to the freight operation (management and office salaries, building rents, sales expenses, etc.). Terminal costs are the direct operating expenses, primarily labor and maintenance, associated with the operations of terminals owned by the firm. Terminal expenses do not include the cost of loading and unloading all of the companies' cargo. Many terminal services are performed for the firm by other companies or by independent operators. The cost of these services are borne by the shipper and do not enter the analysis (nor are they part of direct barge costs). These expense categories are standard for the industry. The sample firm adhered strictly to the "Uniform System of Accounts for Carriers by Inland and Coastal Waterways" in accordance with I.C.C. regulations. These accounts represent a detailed reporting method which most firms in the industry use.

The number of EBM's produced on each of eleven river districts where the firm operates have been calculated for each month. The question to be asked of the river district variables is: How will costs be affected if the firm produces more services on one river district and less on another river district? To accomplish this task a measure of the relative output the firm produced on each river district is necessary. This is done by calculating the percent of total EBM's produced on each river district. These percent variables are similar in nature to dummy variables. Using the coefficient for one river district it is possible to determine the change in cost occasioned by increasing the proportion of total output produced on that river district. This method provides a link between all river districts for determining the effect on cost of relative shifts in the firms operation from one part of the waterway to another.

Detailed records of the firm's shipments were obtained for the three years in the sample. These records provide information on the number of tons and ton-miles of each commodity carried by the firm. The sample firm carried over sixty different commodities each year as detailed by the I.C.C. "Standard Commodity Classification for Transportation." Due to the small sample size it was necessary to aggregate the commodities in some fashion.

Freight can be loosely classified into bulk freight and general cargo. Bulk freight is perhaps best characterized by commodities which can be dumped or poured into a barge. Bulk commodities represent the "bulk" of the traffic handled by the barge industry. General cargo is freight which has undergone some processing and is usually "packaged" in some way.

This two-fold classification, however, obscures some important differences. Bulk commodities, for example, come in both liquid and solid form. Coal, usually carried under long term contracts which specify frequency of delivery, is often carried in unit tows (i.e., with equipment "dedicated" to that particular traffic). Equipment used in unit tows can be scheduled separately from the rest of the firm's traffic. Special cleaning and preparation of barges is necessary for grain shipments. The basic differences in handling and in equipment used for the commodities carried by the firm suggest that the following classification would be a reasonable approximation to capturing the special features of transporting different commodities:

1. grain products
2. coal
3. other solid bulk (primarily ores)
4. petroleum and petroleum products
5. other liquid bulk
6. general cargo

The commodities carried by the firm were aggregated, on the basis of cargo ton-miles, into one of the six groups above for each month in the sample. Percentages for each category were calculated. The interpretation of these percent variables is the same as for the river district percentages described above.

Casual observation would lead one to expect that costs for general cargo will be higher than for most bulk products. General cargo requires special handling which would raise terminal costs. The loading and unloading of general cargo frequently damages barges. Also, since general cargo is specialized as to specific commodity, shipper, and origin and destination, one would expect that sales expenses and scheduling costs will be generally higher than for the bulk commodities. Of the bulk commodities, one would expect that the costs of shipping coal would be the lowest of all the commodity groups considered. This is due to the ease of scheduling and absence of special handling or preparation of barges.

Finally, a measure of firm size has been included. At the conceptual level, several measures can be suggested. Among these are total horsepower, number of barges, barge carrying capacity in cubic feet or cargo tons, horsepower or number of barges weighted by age. With the exception of the age weighted variable all of the proposed measures are essentially identical, as measured by their simple correlation coefficients. The age weighted variables proved to be unrelated to the dependent variables and were excluded on statistical grounds. The measure selected was cargo tons of barge capacity. One might argue that this is a better measure of firm size than, for example, cubic feet of cargo space since commodities typically carried in barges have high weight to volume ratios. This assumes then, that the relevant constraint in loading barges is cargo weight, rather than the cargo's volume.

In summary, an observation for the single firm study consists of the monthly percent distribution of cargo ton-miles among six commodity groups; the monthly distribution of EBM's among eleven river districts; total EBM's per month, a measure of firm size, and a set of dummy variables denoting the month. For the terminal costs, total cargo tons was substituted for EBM's. EBM's are calculated to correct for conditions peculiar to the linehaul operation and do not relate to the loading and unloading of cargo.

B. The Statistical Model

Separate statistical cost functions have been estimated for the three cost categories. The general form of the functions estimated was the same for all three categories. The general form used was:

$$\log_{10} \text{Cost/EBM}_i = \log_{10}\alpha_0 + \alpha_1\log_{10}\text{EBM}_i + \alpha_2\log_{10}S_i + \sum_{i=1}^{11}\beta_i M_i + \sum_{j=1}^{10}\gamma_j R_{ij} + \sum_{k=1}^{5}\delta_k C_{ik}$$

where: EBM_i = the number of equivalent barge miles produced each month

S_i = firm size measured in total net tons of barge carrying capacity

M_i = monthly dummy variables (December excluded)

R_{ij} = percent of total EBM's produced on river district j in the ith month (Lower Mississippi excluded)

C_{ik} = percent of total ton-miles carried of kth commodity group in the ith month (general cargo excluded)

The reasons for the specification of the average cost function in a log-linear form are given in Chapter IV.

As noted above, the measure of output used in the terminal analysis was cargo tonnage. In the estimation of overhead costs use of the monthly dummy variables resulted in an F-value for the regression which was not significant at the .90 probability level. Thus, the monthly dummy variables are excluded for the overhead results.

C. The Results

Results for each cost category are presented in Tables 17-19. Before examining the results, a general comment on the regressions is in order. The sample size is quite small (36 observations). With the relatively large number of variables used the number of degrees of freedom is quite small. On the positive side it is worth noting that the regression results are significant. F-values for the analysis of variance of the three regressions are presented in Table 20. Although care must be exercised in using the regressions for predictive purposes, the results are quite consistent with a priori expectations and can be used to estimate the direction and magnitude of changes in cost due to specified changes in the variables.

Direct Barge Costs

The results for the direct barge cost estimation are presented in Table 17. Direct barge costs include all those costs we would expect to change if one additional ton of cargo was transported. As expected, short-run average costs are falling as current output increases. The range of output produced by the firm is between 200,000 and 400,000 EBM's. Average cost for this firm thus varies from $2.86 per EBM to $1.54 per EBM. Assuming an EBM to be 1400 cargo ton-miles, average cost per ton-mile ranges from 2.1 mills to 1.1 mill. Recall that the costs reported by Meyer ranged from 2 to 2.5 mills per ton-mile.[1] These estimates reported here are to be preferred to Meyer's, first, since they are more current and secondly, because the relationship between average cost and output is explicit.

The relationship of the estimates presented here and past estimates, such as Meyer's, that are available, have been influenced in three important ways. First, the barge industry has experienced

TABLE 17

DIRECT BARGE EXPENSE

$r^2 = .94$
Sample size = 36
Standard Error of Estimate = 0.03498
Intercept = -0.23800

Variable Name	Mean	Regression Coefficient	t-Value
Commodity Shipped (percent times 10^{-2})			
Grain	0.30111	-0.22344	-1.06479
Coal	0.00806	-1.05039	-0.69376
Other Bulk	0.12583	-0.15056	-0.55241
Petroleum products	0.08917	-0.48547	-0.85037
Other liquid bulk	0.08861	-0.40087	-0.98271
River Districts (percent times 10^{-2})			
Lower Illinois	0.14694	0.21486	0.27631
Chicago Port	0.02417	1.77304	1.04040
Tennessee	0.04111	-0.49696	-0.30993
W I C C	0.04250	-1.16258	-0.88053
E I C C	0.00222	-2.14679	-0.72300
Gulf	0.06833	-0.64112	-1.12297
Cairo - St. Louis	0.10556	-0.47782	-0.35987
St. Louis - Grafton	0.02278	-4.28935	-1.75228
Grafton - Minneapolis	0.00278	-1.75541	-0.56699
Cairo - Cincinnati	0.03833	2.04464	1.65515
Monthly Dummy Variables			
January	0.08333	-0.11094	-1.46574
February	0.08333	-0.09831	-1.94714
March	0.08333	-0.07816	-1.76284
April	0.08333	-0.03305	-0.59840
May	0.08333	-0.07039	-1.25853
June	0.08333	-0.12112	-1.90933
July	0.08333	-0.08488	-1.54296
August	0.08333	-0.02014	-0.40840
September	0.08333	-0.03360	-0.43063
October	0.08333	-0.04345	-0.91529
November	0.08333	-0.04931	-1.28813
$\log_{10}$ EBM (times 10^{-6})	-0.56578	-0.90051	-3.49150
$\log_{10}$ Net Tons of Cargo Capacity (times 10^{-6})	0.56021	-0.74976	-0.22836
Dependent Variable: $\log_{10}$ Cost/EBM (times 10°)	0.33743		

TABLE 18

INDIRECT EXPENSE

$r^2 = .77$
Sample size = 36
Standard Error of Estimate = 0.07017
Intercept = -2.88572

Variable Name	Mean	Regression Coefficient	t-Value
Commodity Shipped (percent times 10^{-2})			
Grain	0.30111	0.39329	1.66889
Coal	0.00806	-0.27541	-0.16023
Other Bulk	0.12583	0.23339	0.73634
Petroleum products	0.08917	-0.40578	-0.59953
Other liquid bulk	0.08861	0.58189	1.00984
River Districts (percent times 10^{-2})			
Lower Illinois	0.14694	0.29158	0.23695
Chicago Port	0.02417	0.24184	0.13171
Tennessee	0.04111	1.58667	1.17173
W I C C	0.04250	-2.09464	-1.07996
E I C C	0.00222	11.74213	2.57471
Gulf	0.06833	0.82765	0.89386
Cairo - St. Louis	0.10556	3.34299	2.93826
St. Louis - Grafton	0.02278	-0.14480	-0.03895
Grafton - Minneapolis	0.00278	-2.21722	-0.65286
Cairo - Cincinnati	0.03833	0.88130	0.50978
$\log_{10}$ EBM (times 10^{-6})	-0.56578	-0.63702	-1.70245
$\log_{10}$ Net Tons of Cargo Capacity (times 10^{-6})	-0.56021	-1.75342	-0.43830
Dependent Variable: $\log_{10}$ Cost/EBM (times 10°)	-0.27948		

TABLE 19

TERMINAL EXPENSE

r^2 = .91
Sample Size = 36
Standard Error of Estimate = 0.09458
Intercept = -1.92645

Variable Name	Mean	Regression Coefficient	t-Value
Commodity Shipped (percent times 10^{-2})			
Grain	0.30111	-0.93996	-1.56756
Coal	0.00806	6.80497	1.68074
Other bulk	0.12583	-1.77043	-2.41563
Petroleum products	0.08917	-0.62837	-0.40276
Other liquid bulk	0.08861	-0.50185	-0.45147
River Districts (percent times 10^{-2})			
Lower Illinois	0.14694	3.51629	1.97465
Chicago Port	0.02417	-0.41507	-0.08936
Tennessee	0.04111	5.89744	1.37863
West Intercoastal Canal	0.04250	-4.42486	-1.21512
East Intercoastal Canal	0.00222	2.38680	0.31098
Gulf	0.06833	-4.78775	-3.05355
Cairo - St. Louis	0.10556	4.11378	1.10961
St. Louis - Grafton	0.02278	1.07093	0.15063
Grafton - Minneapolis	0.00278	0.72734	0.08133
Cairo - Cincinnati	0.03833	-5.15640	-1.49995
Monthly Dummy Variables			
January	0.08333	-0.17086	-0.83604
February	0.08333	-0.00568	-0.04666
March	0.08333	0.02756	0.23059
April	0.08333	0.05754	0.39828
May	0.08333	-0.11066	-0.72972
June	0.08333	-0.15505	-0.90428
July	0.08333	-0.17728	-1.09764
August	0.08333	-0.00378	-0.02837
September	0.08333	-0.55436	-2.50447
October	0.08333	-0.30962	-2.53681
November	0.08333	-0.15355	-1.56261
$\log_{10}$ Cargo Ton-miles (times 10^{-9})	-0.72592	-0.27221	-0.42996
$\log_{10}$ Net Tons of Cargo Capacity (times 10^{-})	-0.56021	-3.09194	-0.32019
Dependent Variable: $\log_{10}$ Cost/EBM (times 10	-0.17567		

TABLE 20

F-TESTS FOR TIME-SERIES COST REGRESSIONS

Source Of Variation	Degrees Of Freedom	Mean Square	F-Value (Significance Level)
1. Direct Cost Regression			
Due to Regression	28	.00482	3.9412
Deviation About Regression	7	.00122	(.95)
2. Overhead Cost Regression			
Due to Regression	18	.01615	3.2806
Deviation About Regression	17	.00492	(.99)
3. Terminal Cost Regression			
Due to Regression	28	.02444	2.7328
Deviation About Regression	7	.00894	(.90)

considerable growth in the past twenty years. This is reflected by both a growth in total output of the industry and a growth in the size of individual firms. In terms of the model presented here, this means there has been both a shift (to the right) in the short run cost curve, and a movement along any one short-run curve to higher output levels.

Secondly, the barge industry has experienced considerable technological progress due to the kinds and characteristics of equipment used, changes in operating methods (for example, unit tows), and in the scheduling of barge movements. Thus, ceterus paribus, one would expect to find the short-run cost curves estimated from actual data to shift downward over time. To the extent that the prices of inputs used in barge transportation have increased, however, the cost curves would have shifted upward. Since our cost estimates are lower, in general, than the figures given by Meyer, one might tentatively conclude that the first two factors have offset the effect of rising input prices.

Controlling for price changes, the above arguments would indicate that barge transportation experiences long-run decreasing costs. Although the period of time over which our sample is taken is too short to allow definite conclusions to be drawn, the hypothesis of decreasing costs is given tentative support by the fact that the coefficient estimated for firm size has a negative sign.

The coefficients for the commodity variables conform quite closely to expectations. The results indicate that each of the commodity groups can be shipped for lower average cost than general cargo, the excluded category. The average cost of shipping coal, in particular is considerably lower than the cost of general cargo. For example, increasing coal shipments by one percentage point, costs per EBM would fall, at all levels of output, by 2.4%. At typical output levels for this firm, a 2.4% average cost reduction would reduce total cost by over $15,000 per month.

The coefficients for the river district percentages show a wide range of values. Unfortunately, the coefficients do not show significant statistical differences from the omitted river district (the Lower Mississippi). The coefficients for St. Louis to Grafton, and Cairo to Cincinnati however, are significant for a one-tailed t-test at the .95 level of probability. Economically, however, the coefficients indicate that large cost differentials may exist between operations on different river districts. A one percentage point increase, for example, of traffic in the Port of Chicago would increase average cost by more than 4% at all output levels.

Indirect Costs

Indirect costs represent the portion of total cost which cannot directly be assigned to particular outputs. These are the costs primarily associated with the scheduling of equipment and the sale of services. The estimates can serve two purposes. First, it may be possible to associate variations in indirect cost with variations in particular services offered. Secondly, the estimates could be used to supplement the other statistical estimates of direct cost (both direct barge and direct terminal) to arrive at a measure of total firm cost. The estimates could also be used in conjunction with direct cost estimates derived from an engineering production function.

Substantial difficulties are inherent in attempting to estimate the indirect cost associated with a movement of cargo. Cargo must be found and coordinated. In general, the balance of traffic and coordination are much less than perfect and a high percentage of barge miles are taken up with empty barges. The scheduling problem is the major difficulty in estimating indirect costs. One might view the firm as an organization designed to coordinate various point-to-point movements. Various demands are presented to the firm and the scheduling problem is to manage to satisfy the demands with its current equipment at the least cost. In practice, this generally involves carrying as many shipments as possible given the stock of equipment.

Estimates of direct cost can come from two sources. The first source is the direct operating expense reported by a firm. The results presented in the preceding section are of this type. These direct barge costs are related to the commodity shipped and the location of services provided. The advantage of these estimates is that they reflect the average actual performance of the firm for the broad characteristics of its operation.

A second source of direct cost is the engineering production function. The engineering function relates the output of a tow directly to the physical characteristics of the waterway (depth, width, stream flow) and the characteristics of the flotilla (length, breadth, draft). The advantage of this approach is its disaggregative nature. Having determined the output of a tow, one can go on to calculate the direct cost associated with a particular movement. There is a cost associated with each piece of equipment (which can be expressed on an hourly basis); knowing output (expressed in ton-miles per hour), one can calculate the cost per ton-mile. This cost is only part of the marginal cost of the movement; no provision is made for scheduling or other overhead costs or for empty backhaul.

Changes in output, and hence changes in cost, can be calculated, for example, by adding a barge, or by loading it to a deeper draft. Thus, the engineering function can be used to provide estimates of direct cost for any particular point-to-point movement that might be of interest. Using estimates of indirect cost obtained from the statistical model together with the direct cost of a point-to-point movement would provide an estimate of the total cost associated with that movement.

The EBM might be integrated into the engineering production function. The function can then be used to generate an estimate of the number of EBM's per hour produced by any towboat on any river district. An analysis of delay times could be included to modify this estimate to take account of locks and congestion.

Using this estimate of EBM's per hour produced by a towboat provides considerable simplification in planning the operations of the towboat. Since EBM's per hour are constant, one can easily calculate the effect of adding additional barges or cargo. It should be noted, however, that estimates obtained in this way would be most relevant for determining the cost of a marginal movement (adding one more barge to an already scheduled tow). Two kinds of scheduling costs can be identified: Indirect supervisory costs, and direct operating costs due to the necessity of shipping extra empty barges. Since the engineering estimate would not include the latter kind of cost its most appropriate use would be for the marginal movement where the marginal direct scheduling cost is zero. The results of the indirect cost estimation are presented in Table 18. Looking at the coefficient for firm size we find that it has a negative sign. Firm size should be an important determinant of scheduling costs. In general, we should expect economies of scale in the scheduling of barges.

The coefficient of EBM's is also negative. This is not surprising since average costs will continually fall until total cost begins to rise at an increasing rate. This is certainly not what we would expect total indirect costs to do. In fact, we should expect total indirect cost to remain fairly constant over wide ranges of output. We can infer the shape of the total cost function from our results. The coefficient of $\log_{10}$EBM is interpreted as the elasticity of the average cost curve. If the coefficient were equal to -1.0 the total cost curve would be constant over all output values. The coefficient is equal to -.63702 which would indicate that total indirect cost increases at a decreasing rate with respect to output, although it is by no means constant.

Note also that it is possible to use these estimates to allocate part of indirect cost to particular services the firm renders. In the statistical model these effects are derived by looking at the coefficients for the commodity, and river district percentages and the monthly dummy variables. Several examples have already been given of the use of these variables.

Direct Terminal Costs

Very little is known about the cost associated with terminal operations in inland waterway transportation. This is primarily due to the high degree of specialization of terminal facilities. Specialization occurs at two levels. First, different commodities require their own particular terminal services. Secondly, different concepts in the provision of terminal services can be applied depending upon the rate of output required, the nature of the interface (whether the terminal connects with truck, rail, or both) and the size and topology of the terminal location. Different types of facilities are also required for loading and unloading. As a result of the specilization it is extremely difficult to make any generalizations about terminal cost.

The firm in our sample, however, operates two terminals which are devoted entirely to general cargo. Terminal facilities for handling general cargo are less specialized than terminals for bulk commodities. In addition, the same facilities can be used at both ends of the handling operation. Input requirements are fairly standard: lift trucks, one or two cranes (an extra crane is generally kept on a standby basis since only one crane can be used at a time on any one barge), and stevadores. The process of general cargo handling is labor intensive relative to the handling of bulk commodities.

The terminal results are quite tentative, particularly since an adequate output measure is not available. Cargo ton-miles, however, should be highly correlated with cargo tons. Since the coefficient of the surrogate measure is negative, short run terminal costs are falling over the range of the sample. (See Table 19.)

We would expect to observe falling short-run average costs for terminal operations due to the necessity for considerable excess capacity. The demand for terminal services is random. As a result, terminals must be of sufficient size to handle peak loads or else lengthy (and costly) queues will form. For a given size facility, then, average costs will be lower the greater the volume of cargo handled.

CHAPTER VI

A CROSS SECTION ANALYSIS OF FIRM COSTS

This chapter presents the results of a statistical investigation of the relationship between output and costs for five major inland waterway operators. The data used are actually combination cross-section and time-series. Quarterly observations over a period of five years have been employed.

Chapter V reported the results of a detailed examination of cost-output relationships for a single firm. The approach taken in Chapter V was to relate categories of firm cost to special types of service offered by inland waterway carriers. In this study it has not been possible to disaggregate output in the same way. Instead, a more traditional approach has been employed; the costs of the firms in the sample have been related to their total current output and the size of the firm, adjusting the observations for the passage of time.

A. The Data

As mentioned above, the data consist of quarterly observations of cost, output, and firm size for a period of five years for five inland waterway operators. The cost data have been separated into direct and indirect components. Separate cost relationships have been estimated for each cost category and for total cost. Once again, the measure of current output used in the statistical model was the EBM.

The five firms in the sample represent major inland waterway carriers. Taken as a group, their barging operations are highly diversified. They operate on all of the navigable tributaries of the Mississippi and Gulf Intercostal River Systems, including the Missouri River, the Gulf of Mexico, and Lake Michigan. In addition, the five firms offer essentially a complete range of water transport services, both with respect to commodities handled, and equipment available for freight transportation.

Although the firms in the sample are among the largest firms in the industry (measured either in current output and revenue, or by size of equipment stocks), nevertheless, there are considerable differences in their absolute size. For example, the largest firm, as measured by total available horsepower, was on the average 3.3 times larger than the smallest firm in the sample.

Several surrogate measures could be used to correct for firm size in the statistical model. Since the firm size-cost relationship carries the interpretation of long-run costs, the measure selected should be a reflection of the productive capacity of the firm at any one time. That is, any paired cost-size observation should represent the least cost way of operating a firm of particular size. It should be noted, however, that when historical data are used in statistical cost estimation these theoretical efficiency assumptions are not necessarily met. The data, in general, do not describe the least cost solution for a particular firm; rather they represent the actual cost of operating particular firm sizes included in the sample.

One could assume that the observations do represent minimum cost points and interpret the estimated results as the long-run costs of economic theory. This, however, is certainly not likely to be true. That is, the observations will reflect the operation of a firm where some short-run adjustments are still taking place. Another interpretation is possible: Assume that the observations made represent a certain average level of efficiency which has been achieved in the past and is likely to prevail for some time. The long-run relationship could then be used to draw conclusions, for example, about the expected effects of increasing firm size.

Four measures of firm size are readily available. These measures are also reflections of the firm's productive capacity: towboat horsepower, number of towboats, cargo tons of barge capacity, and the number of barges. Since no a priori judgement can be made as to which of the measures is the "correct one," all four have been gathered for each firm in the sample. Each could then be tried in the statistical model and the results compared. If the four measures yield similar results we would gain some confidence in the use of the variables as measures of firm size.

Since the observations range over a period of five years, it is of interest to ask how costs have changed over time. This period is reputed to be one in which the industry made substantial efforts to improve its efficiency. As a result considerable attention has been paid to the equipment used, to scheduling, and other operating rules-of-thumb. One reflection of this movement is the increasing use of sophisticated cost accounting techniques (for example, much of the data used in this study was simply not available before 1962, at least in a form which would be useful as a guide to production control). To attempt to measure the extent of this activity, a dummy variable which increases linearly with time has been included in the statistical model. This variable should indicate the extent of neutral technological change which has taken place during the sample period. If the hypothesis of positive neutral technological change is correct, this variable should have a negative coefficient. According to representatives in the industry, the period reported in our sample did not experience significant amounts of inflation for inputs important to barge transport (a major labor contract was renegotiated in 1967, after our sample period). As a result, the data have not been deflated.

An additional time measure has been included: the quarter the observation was made. This is done by assigning a dummy variable to each quarter. In the statistical model, one dummy variable must be excluded. The coefficients of the remaining variables will reflect the shift in the cost relationship to be attributed to operations at that time of year relative to the omitted quarter. This, then, will give an indication of seasonal variation in water transport costs. The fourth calendar quarter was the omitted dummy variable.

These variations may occur for two basic reasons: One is that the volume of cargo offered to the firm varies seasonally (for example, the grain products). Secondly, operating conditions vary during the year due to weather (icing, fog, etc.), steamflow, and water level. Seasonality of the firm's demand will affect cost primarily by increasing scheduling difficulties at peak periods and by requiring the employment of less efficient factors. Operating conditions affect cost primarily by affecting towboat performance. (See Chapter III.)

Thus, an observation for the cross-section study consists of a measure of current output (EBM's); a measure of firm size (total towboat horsepower, number of towboats, net tons of barge cargo capacity, and the number of barges); a linear annual time trend; a set of dummy variables to identify the quarter of the year; and quarterly costs; total costs and two subdivisions of total cost; direct operating (or linehaul costs) and indirect operating expense.

B. The Statistical Model

Separate statistical cost functions have been estimated for three categories of cost: indirect, direct, and total cost, each for the four measures of firm size. The general form of all twelve functions estimated is the same:

$$\log_{10} \frac{\text{Cost}}{\text{EBM}_{ijk}} = \log_{10}\alpha_0 + \alpha_1 \log_{10}\text{EBM}_{ijk} + \alpha_2 \log_{10} S_{ijk} + \alpha_3 T + \sum_{1=j}^{3} B_j Q_j$$

where: EBM_{ijk} = current output, measured in EBM's, for the ith firm, during the jth year and kth quarter

S_{ijk} = a measure of firm size, either total towboat horsepower, number of towboats, net tons of barge cargo capacity, or number of barges for the ith firm, during the jth year and kth quarter

T = annual time trend variable (1962 = 1, . . . 1966 = 5)

Q_k = quarterly dummy variables (fourth quarter omitted)

The estimating equations are linear in logarithms (the time trend variables, however, are left untransformed). The average cost and log-linear form of the functions have been specified. The arguments for this specification have already been given in Chapter IV.

C. The Results

Results for all cost categories are presented in Tables 21-23. Four regressions are presented for each category of cost, one for each alternative measure of firm size, to allow comparison. With the exception of indirect cost there is little to distinguish between the four regressions for each cost category. Since the results using the number of barges as the firm size variable appear to be generally better, these results have been used in the discussion below.

Indirect Costs

Indirect costs represent expenses incurred by the firm which cannot be allocated to particular levels of output. These are the costs associated with general office and management expense, record keeping, sale of services, and the scheduling and coordination of shipments and equipment.

Table 21 presents the indirect cost results for each of the firm size measures. In comparing the results across firm size measures, an apparent contradiction is evident in the firm size coefficients. The two towboat measures show a strong positive relationship with average indirect cost, whereas the two barge measures show a strong negative relationship with average indirect cost.

It would be expected, ceterus paribus, that if the "correct" firm size measure was used, average indirect costs would fall as firm size increased. For example, we would expect to find economies

TABLE 21

CROSS SECTION INDIRECT COST

A. Total Horsepower

$r^2 = .57$
Sample Size = 84
Standard Error of Estimate = 0.11626
Intercept = -1.13876

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	0.04107	0.43167
First Calendar Quarter	0.25000	0.09268	2.34259
Second Calendar Quarter	0.25000	0.05015	1.39720
Third Calendar Quarter	0.25000	0.04941	1.37036
$\log_{10}$ Total Horsepower (times 10^{-5})	-0.45576	0.09230	0.49784
$\log_{10}$ EBM (times 10^{-7})	-0.19924	-0.70037	-3.35663
Dependent Variable: $\log_{10}$ Indirect Cost/EBM (times 10°)	-0.44298		

B. Number of Towboats

$r^2 = .61$
Sample Size = 84
Standard Error of Estimate = 0.11089
Intercept = -1.22234

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	0.04862	0.53887
First Calendar Quarter	0.25000	0.06884	1.89323
Second Calendar Quarter	0.25000	0.05169	1.50993
Third Calendar Quarter	0.25000	0.05424	1.58103
$\log_{10}$ Number of Towboats (times 10^{-2})	-1.02363	0.23946	2.81235
$\log_{10}$ EBM (times 10^{-7})	-0.96562	-0.99916	-0.59324
Dependent Variable: $\log_{10}$ Indirect Cost/EBM (times 10°)	-0.44298		

CROSS SECTION INDIRECT COST

C. Net Tons of Barge Capacity

r^2 = .65
Sample Size = 84
Standard Error of Estimate = 0.10520
Intercept = -0.91246

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	0.01911	0.22338
First Calendar Quarter	0.25000	0.13510	3.99459
Second Calendar Quarter	0.25000	0.04656	1.43373
Third Calendar Quarter	0.25000	0.04145	1.27516
$\log_{10}$ Net Tons of Barge Capacity (times 10^{-6})	-0.37330	-0.66452	-4.16424
$\log_{10}$ EBM (times 10^{-7})	-0.96562	-0.16503	-1.36981
Dependent Variable: $\log_{10}$ Indirect Cost/EBM (times 10°)	-0.44298		

D. Number of Barges

r^2 = .64
Sample Size = 84
Standard Error of Estimate = 0.10626
Intercept = 0.10626

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	-0.02011	-0.22993
First Calendar Quarter	0.25000	0.13822	4.00942
Second Calendar Quarter	0.25000	0.04630	1.41121
Third Calendar Quarter	0.25000	0.04095	1.24661
$\log_{10}$ Number of Barges (times 10^{-3})	-0.45985	-0.64527	-3.93314
$\log_{10}$ EBM (times 10^{-7})	-0.96562	-0.12424	-0.91772
Dependent Variable: $\log_{10}$ Indirect Cost/EBM (times 10°)	-0.44298		

of scale in both the scheduling of equipment and the sale of services. We would also expect to find little association of indirect cost and current output (EBM's). This is in fact what we find for the barge-firm size regressions; both coefficients of EBM being small and insignificant, both barge-firm size measures are negative and significant. For the towboat-firm size regressions, however, the EBM coefficients are large and significant, the towboat-firm size measure coefficients are positive.

A clue to the explanation of this apparent contradiction can be found in the fact that the contradiction occurs across the two sets of firm size measures, each representing two different types of productive capacity. No one of the firm size measures can be said to be better than any of the others. These measures represent inputs to the productive process that are quite different and enter the production function in quite different ways.

Suppose that the barge measure was the "correct" one, that is, every firm in the industry uses barges in their production function in exactly the same way. Suppose also, that the same is not true for towboats. In particular, suppose that the barge fleet is used to capacity; excess barges being chartered to other firms. On the other hand, towboats are not used to capacity; if no work is available for a boat it is allowed to sit in port. Under these assumptions, one would expect larger increases in output from increasing the barge fleet by 10% than from increasing the towboat input a proportional amount. Controlling for output (EBM's) we would then expect to find lower average costs per EBM to be associated with increases in the barge fleet; higher costs per EBM to be associated with larger towboat fleets.

Indirect cost is the same for towboats and barges whether they are in or out of service. If towboats or barges are out of service, they produce no EBM's. The greater the non-service time of towboats or barges, the higher the indirect costs per EBM will be. If towboats are not used to capacity, increasing the number of towboats will increase total idle time and increase indirect cost per EBM. If barges, on the other hand, are fully utilized, increasing the number of barges will increase the number of EBM's produced. With relatively constant indirect cost, indirect cost per EBM will fall as the size of the barge fleet increases.

This argument is consistent with the operation of the lease markets for towboat and barge equipment. Barges are extremely portable between firms. They are frequently chartered, even for short periods. This is less true for towboats (except indirectly through outside towing). The results may then reflect the fact that there is a more active market for barges than for towboats.

If the above arguments about the firm size measures are valid, the best (although not perfect) measure of firm size would be one of the barge size measures. For either one of the barge size measures, indirect cost is relatively constant at all current output levels. (The coefficient for both barge measures are also not significantly different from zero.) Average indirect cost at the point of means is $.35 per EBM (about .2 mills per cargo ton-mile).

These indirect cost estimates provide a supplement to other estimates of direct cost that can be made. In particular, it is possible to obtain from the engineering production function the direct costs associated with any particular trip. Direct cost estimates obtained in this way represent the cost for the most efficient way of providing that individual trip. The firm, however, must sell and coordinate this shipment with the many others that it produces. Thus, a measure of per unit indirect costs can be added to the engineering direct cost estimate to yield a measure of the marginal cost of an additional trip. The indirect cost estimate would have to be adjusted for the level of output at which the additional trip occurred, as well as for the size of firm making the shipment.

Total and Direct Costs

Both the direct cost and total cost results are quite similar. This is not surprising since direct cost is about 75% of total cost. Only the average direct cost results will be discussed, however, the discussion will also be applicable to total average costs.

As expected, the average direct cost of an EBM falls as output increases. The coefficient of EBM's is negative and significant for all regressions. Evidently, upward short run adjustments in output can be accomplished with lower per unit costs over a wide range of output actually experienced. Average direct cost per EBM at the point of means is $2.08 (about 1.5 mills per cargo ton-mile).

A puzzling result is once again obtained for the firm size measures. The coefficients for all measures are positive. Larger firms appear to be associated with higher costs. This is certainly strange in view of results obtained in other parts of this study which indicate substantial amounts of technological change occurring in the industry, and that economies of scale are to be expected with respect

TABLE 22

CROSS SECTION DIRECT COST

A. Total Horsepower

$r^2 = 0.68$
Sample Size = 84
Standard Error of Estimate = 0.04050
Intercept = 0.01241

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	-0.12615	-3.80639
First Calendar Quarter	0.25000	0.02181	1.58225
Second Calendar Quarter	0.25000	0.01044	0.83503
Third Calendar Quarter	0.25000	0.00472	0.37566
$\log_{10}$ Total Horsepower (times 10^{-5})	-0.45576	0.19924	3.09328
$\log_{10}$ EBM (times 10^{-7})	-0.96562	-0.44389	-6.10697
Dependent Variable: $\log_{10}$ Direct Cost/EBM (times 10°)	0.31778		

B. Number of Towboats

$r^2 = .69$
Sample Size = 84
Standard Error of Estimate = 0.04009
Intercept = 0.06900

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	-0.13265	-4.06703
First Calendar Quarter	0.25000	0.02517	1.91486
Second Calendar Quarter	0.25000	0.01035	0.83632
Third Calendar Quarter	0.25000	0.00383	0.30897
$\log_{10}$ Number of Towboats (times 10^{-2})	-1.02363	0.10369	3.36840
$\log_{10}$ EBM (times 10^{-7})	-0.96562	-0.40251	-7.20547
Dependent Variable: $\log_{10}$ Direct Cost/EBM (times 10°)	0.31778		

TABLE 22--Continued

CROSS SECTION DIRECT COST

C. Net Tons of Barge Capacity

r^2 = .72
Sample Size = 84
Standard Error of Estimate = 0.03827
Intercept = 0.06104

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	-0.13201	-4.24030
First Calendar Quarter	0.25000	0.02546	2.06905
Second Calendar Quarter	0.25000	0.01072	0.90738
Third Calendar Quarter	0.25000	0.00342	0.28936
$\log_{10}$ Net Tons of Barge Capacity (times 10^{-6})	-0.37330	0.25913	4.46305
$\log_{10}$ EBM (times 10^{-7})	-0.95562	-0.40072	-9.14191
Dependent Variable : $\log_{10}$ Direct Cost/EBM (times 10°)	0.31778		

D. Number of Barges

r^2 = .73
Sample Size = 84
Standard Error of Estimate = 0.03701
Intercept = 0.04806

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	-0.11298	-3.70748
First Calendar Quarter	0.25000	0.02172	1.80837
Second Calendar Quarter	0.25000	0.01105	0.96735
Third Calendar Quarter	0.25000	0.00407	0.35596
$\log_{10}$ Number of Barges (times 10^{-3})	-0.45985	0.29494	5.16078
$\log_{10}$ EBM (times 10^{-7})	-0.96562	-0.44869	-9.51454
Dependent Variable: $\log_{10}$ Direct Cost/EBM (times 10°)	0.31778		

TABLE 23

CROSS SECTION DIRECT COST

A. Total Horsepower

$r^2 = .69$
Sample Size = 84
Standard Error of Estimate = 0.04428
Intercept = 0.05131

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	-0.10722	-2.95923
First Calendar Quarter	0.25000	0.03143	2.08579
Second Calendar Quarter	0.25000	0.01370	1.00215
Third Calendar Quarter	0.25000	0.00928	0.67572
$\log_{10}$ Total Horsepower (times 10^{-5})	-0.45576	0.17761	2.51543
$\log_{10}$ EBM (times 10^{-7})	-0.96562	-0.46311	-5.82781
Dependent Variable: $\log_{10}$ Direct Cost/EBM (times 10°)	0.39592		

B. Number of Towboats

$r^2 = .74$
Sample Size = 84
Standard Error of Estimate = 0.04091
Intercept = 0.07273

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	-0.11022	-3.31178
First Calendar Quarter	0.25000	0.02773	2.06743
Second Calendar Quarter	0.25000	0.01403	1.11135
Third Calendar Quarter	0.25000	0.00987	0.77999
$\log_{10}$ Number of Towboats (times 10^{-2})	-1.02363	0.014262	4.54054
$\log_{10}$ EBM (times 10^{-7})	-0.96562	-0.51004	-8.94762
Dependent Variable: $\log_{10}$ Direct Cost/EBM (times 10°)	0.39592		

TABLE 23--Continued

CROSS SECTION DIRECT COST

C. Net Tons of Barge Capacity

r^2 = .70
Sample Size = 84
Standard Error of Estimate = 0.04416
Intercept = 0.10912

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	-0.11380	-3.16856
First Calendar Quarter	0.25000	0.03759	2.64764
Second Calendar Quarter	0.25000	0.01368	1.00371
Third Calendar Quarter	0.25000	0.00760	0.55689
$\log_{10}$ Net Tons of Barge Capacity (times 10^{-6})	-0.37330	0.17431	2.50220
$\log_{10}$ EBM (times 10^{-7})	-0.96562	-0.38789	-7.67006
Dependent Variable: $\log_{10}$ Direct Cost/EBM (times 10°)	0.39592		

D. Number of Barges

r^2 = .71
Sample Size = 84
Standard Error of Estimate = 0.04343
Intercept = 0.09785

Variable Name	Mean	Regression Coefficient	t-Value
Time Index (times 10^{-1})	0.32857	-0.10020	-2.80227
First Calendar Quarter	0.25000	0.03453	2.45018
Second Calendar Quarter	0.25000	0.01396	1.04077
Third Calendar Quarter	0.25000	0.00814	0.60595
$\log_{10}$ Number of Barges (times 10^{-3})	-0.45985	0.20771	3.09741
$\log_{10}$ EBM (times 10^{-7})	-0.96562	-0.42705	-7.71734
Dependent Variable: $\log_{10}$ Direct Cost/EBM (times 10°)	0.39592		

to scheduling. In addition, we observe a recent trend toward individual firm expansion along with considerable merger activity. Unless gross irrationality exists in the industry, another explanation of the results must be found.

A possible explanation may be found in the hypothesis that large firms offer a wider range of special services than smaller firms. The problem may then be with the output measure. It is not an adequate measure of the work done by the firm. The EBM (as well as the ton-mile) is a measure of work done by the carrier's equipment in the linehaul operation, not of services performed by the carrier for customers. A large firm may be more willing to allow commodities to be stored in a barge at port for long periods; it may provide special handling of particular commodities; it may incur a higher percentage of empty barge movements, that is, it may be quite willing to accomodate certain customers with equipment on demand. It will, of course, perform these services at additional expense, but it also will receive additional revenue.

Smaller firms may attempt to only haul traffic with a short turn around time and with little extra service. Larger firms may find it more profitable to offer these special services. It will do so at the expense of some efficiency in the strictly line-haul part of their operation, that is, for a higher cost per EBM. It can be argued, therefore, that the association of higher average costs with larger firms comes as a result of the failure of the output measure used in the statistical models to incorporate the special aspects of water transport services.

Looking at the time variables, it is worth noting that average costs have fallen significantly over the five year period in the sample. An average decrease of about 2 1/2 percent per year is indicated.

The quarterly dummy variables show average cost declining steadily throughout the year. This accords reasonably well with a priori expectations. The first quarter (the only dummy variable which proves to be statistically different from the omitted variable) has significantly higher costs than other quarters. This is to be expected since the winter months represent generally adverse operating conditions for tows.

In summary, it appears that the barge industry is capable of expanding its output over a wide range at lower unit costs in the short run. In addition there is evidence that the industry has experienced considerable cost saving reductions over the past few years. The results for the cost-size relationship are mixed, yet they point to at least two interesting issues. One is the indication that the different types of operating equipment (towboats and barges) are not used with equal efficiency. This may in part be due to imperfections in the market for chartered towboats.

The second issue is the difficulty of obtaining satisfactory measures of the output and size of transport firms. The EBM does represent an improvement as it corrects for differences in equipment and operating conditions. It does not correct, however, for special features of waterway transport service which apparently have significant effects upon the operating costs of the industry.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The major results obtained in this study and their implications are summarized below in three sections: (A) returns of scale, (B) technological change, and (C) costs and the rate structure. Following this summary, two methods of obtaining estimates of cost using these results are discussed.

A. Returns to Scale

Two indications of returns to scale can be obtained from the statistical results. One relates to the major operating unit of waterway transportation, the towboat. The second indication of returns to scale relates to the firm.

With respect to the towboat, we find in the towboat production function a general indication of increasing returns to scale, as measured by the capital and labor inputs. The sum of the capital and labor coefficients of the log-linear regressions are greater than one for companies one and three and the combined regressions, both upstream and downstream.

Company two is a notable exception; for both the upstream and downstream regressions, firm two shows decreasing returns for the towboat. This seeming contradiction, however, may be an important indication of the extent of scale economies in towboat size. One would predict, from the horsepower coefficients alone, that there is some upper limit to increases in horsepower. Horsepower consistently shows diminishing productivity (with the single exception of the company two upstream coefficient, 1.05). It is evident that the labor input alone is sufficient to assure constant or increasing returns for companies one and three and the combined production functions.

However, it must be remembered that the results are derived from the limited range of actual practice. Although, proportional increases in the labor and horsepower inputs for companies one and three indicate increasing returns we may not be able to extrapolate this result beyond the ranges of labor and horsepower used by the two companies. A priori, we would expect eventual decreasing returns for very high horsepower and labor input combinations.

A possible explanation for the company two results is that company two in fact operates in the range where decreasing returns have set in. Note that company two has both the highest average crew size and towboat horsepower. In addition, firm two, which has the smallest towboat fleet of all companies, has the largest number (four) of very large towboats (6000 horsepower and above). Firms one and three each have only one towboat as large as 5500 horsepower.

If the above arguments are true, decreasing returns to towboat size must become important for towboats in the 4000-6000 horsepower range and for crew sizes of 12 men and above.

Supporting evidence on returns to scale for towboats is found in the estimation of towboat cost. Towboat cost declines throughout the range of actual horsepowers, as would be expected. Note however, that the coefficients of the dummy variables for two companies (two and three) are positive and significant, indicating that these firms have the highest towboat costs of all firms in the sample. Company two is the same as company two in the production function. Company three is similar company two in that it also operates a small fleet (eight boats) of very large horsepower towboats (average towboat size is 4300 horsepower, even higher than that for company two). While these coefficients do not measure returns to scale, it does appear that decreasing returns have already been attained for some firms in their current operations.

Measures of returns to scale for the firm are given in both the single firm and the cross-section regressions. In the single firm study the coefficient of firm size (cargo tons of barge capacity) is negative for both the direct and indirect cost regressions. Moderate increasing returns are indicated; however, the coefficients are not significantly different from zero.

In the cross-section results, four different measures of firm size were employed in estimating direct, indirect, and total cost per EBM. They were (1) total available horsepower, (2) number of towboats, (3) total available barge capacity, measured in net cargo tons, and (4) the number of barges. The results were quite surprising. For the direct and total cost regressions, all four measures of firm size have positive and significant coefficients. In addition both of the towboat firms size measures were positive for the indirect cost regressions.

The most plausible interpretation of these results seems to be that large firms offer a diversity of high cost services. The measure of output used, however, is an index of work done in actually moving commodities. This may represent only part of the work performed for the customer, excluding such services as storage, special handling, and faster delivery. If the numbers of these services are expanded as the firm grows, its cost (but also its revenue) per EBM will increase. Thus, neither the single firm nor the cross section results can be used to draw unambiguous conclusions about returns to scale for barge firms.

B. Technological Change

The several parts of this study contain various measures of capital embodied and neutral technological change (labor embodied technological change was not examined). The results are remarkable for their consistency.

Two measures of capital embodied technological change for towboats were examined in the towboat production and cost functions: towboat age and Kort nozzles. The results for towboat age indicate that there have been no significant changes in towboat design for the range of ages in the sample (about 25 years). This indication has been confirmed by naval architects.

However, it is possible that minor changes have occurred that would not show up in our data. For example, newer towboats are generally equiped with sophisticated navigation and communication equipment as they are built. Such equipment should tend to raise the productivity of the towboat. The improvements, however, are easily installed on older boats. In fact, almost all the boats in the sample were so equiped by the beginning of our sample period. Thus, our analysis is not capable of picking up these effects.

The coefficients of the dummy variables for towboat age in the towboat cost regression do indicate that the very oldest boats are less efficient than the newest boats. Boats built prior to 1950 are approximately 17% more costly to operate than the newest boats. Boats built between 1950 and 1955 are 4% more costly to operate, although the coefficient of the 1950-55 dummy variable is not significant. However, for boats built between 1956 and 1960, the results indicate a <u>reduction</u> in operating cost of 9% over the newest boats. The latter coefficient is also not significant.

A dummy variable for the Kort nozzle was included in both the towboat cost and towboat production functions. For the combined log regressions of the towboat production function, Kort nozzles improved average productivity by more than 26% for both upstream and downstream movements. Both coefficients are significant. The Kort nozzle coefficients are positive for all six of the individual company log regressions. Four of the six coefficients are significant. The range of improvement for the individual companies is from 14% to 41%. The linear regressions indicate that Kort nozzles may be slightly more important for downstream movements.

Supporting evidence of Kort nozzles is obtained from the towboat cost function. Here, the Kort nozzle coefficient is negative and significant. A 15% reduction in average towboat cost is indicated for use of the Kort nozzle.

As reported in Chapter IV, an attempt was made to measure capital embodied technological change for the barge equipment. Both a linear age index and dummy variables for age groups were tried. F-tests performed on these variables indicated that no significant cost differences were present with respect to barge age. Thus, we may conclude that there have not been significant technological improvements in barge design for the period of our sample (about 30 years). This, once again, is confirmed by the opinion of naval architects.

Evidence on neutral technological change can be obtained from the towboat production function, the barge cost function, and the cross-section firm cost analysis. The results are quite consistent, showing considerable neutral technological improvement for the period of our sample (five years in the cross-section study).

In the cross-section study, the effect of time was embodied in a linear index which increased with time (ranging from one to five). For all of the average direct operating cost and total cost regressions, the coefficients of the index were negative and significant. (The results for indirect cost are inconclusive; one coefficient was negative, but three were positive; none were significant). An average annual cost reduction of 2.3% to 3% is indicated.

In the barge cost results dummy variables were used to denote the passage of time. 1966 was the omitted dummy variable, and thus has a "coefficient" of zero. The coefficients of the 1964 and 1965 dummy variables are positive and significant. The coefficients become less positive over time, thus cost falls during this period. An average annual reduction in direct barge operating cost of 12% is indicated.

It should be pointed out that the indicated cost savings may be understating the real reductions. This is because the cost data are not deflated for price changes, since inflation of barge factor input prices was not found to be significant for the period of our sample. To the extent any inflation did occur, the reductions in cost would be greater.

Supporting results for neutral technological change are given in the towboat production function. Dummy variables are used to denote the year. Once again, 1966 was the omitted year, thus to show productivity increases, all the dummy variable coefficients should have negative signs. In looking at the results, the dummy variables for 1964 and 1965 are negative in all 16 regressions, with one exception. In general, the coefficients become less negative over time. Average annual increases in productivity appear to be in the range of 5% to 9% for the combined downstream regressions.

The barge industry appears to have experienced considerable technological improvement over the past few years. A possible source of these improvements is the scheduling of equipment. Barge firms typically have difficulty in scheduling their equipment and are constantly seeking improvements in their methods. To the extent they have been successful we would expect the productivity of the towboat, measured in EBM's per hour, to rise (as unnecessary empty movements are eliminated) and the cost per EBM to fall.

C. Costs and the Rate Structure

A major objective of this study was to examine the problem of common costs. In particular, one would expect cost to vary with the commodity carried, the waterway on which the shipments are made, the time of year, and the length of haul. All of these represent aspects of the actual transport process which can be identified with each shipment made. To the extent costs are different for each of these variables, we would expect different rates to be charged (with competitive markets) which reflect these cost differences.

It was not possible to pursue the question of the length of haul in this study. Implicitly, then, an average trip length has been assumed. However, the other three effects have been examined. The towboat production function, for example, employs dummy variables to determine productivity shifts between sixteen river districts. Dummy variables are also used for each month to capture seasonal output variations. The direct barge cost and single firm cost studies include a set of variables for the commodity shipped, as well as the seasonal and river district variables.

When taken together, the results are extremely detailed. That is, it is possible to determine the marginal cost, at any output level, for shipments of particular commodities, river districts, and seasons (16 river districts, times 12 months, times 6 commodity groups, yields 1152 possible cost estimates). In addition, these costs can be adjusted for the equipment (towboats and barges) used.

It is impossible to summarize all the results here. The individual results are discussed in each chapter. In general, it was found that these variables had significant effects on cost. F-tests were performed on each group of dummy variables to see if they could be excluded from the individual regressions. The hypothesis of no significant differences in the residual regression variance was consistantly rejected. The coefficients were found to generally accord with a priori expectations as to their signs and magnitudes.

If the barge industry was characterized by perfect competition we would expect the rate structure to reflect these differences in marginal cost. Barge rates, however, are not always competitive ones. Many commodities are, of course, regulated by the ICC and various state transport agencies. A considerable amount of traffic, however, is carried exempt from regulation or under contract, where presumably competitive rates are established. The cost estimates that can be derived from this study could be used as a guide to these rate-making decisions.

As just one example, consider the effect on cost of shipping additional grain on the Lower Illinois relative to its cost on the Lower Mississippi (using the single firm results). Suppose we allow traffic on the Lower Illinois River to increase by 1%. (Recall that the river district variables are in terms of percentages. Thus, a 1% increase in traffic on the Illinois, at its mean value, represents a .147 percentage point increase).

A 1% increase in Illinois River traffic will result in a .23% increase in cost over the Lower Mississippi for any level of output. Marginal cost of shipments on the Illinois, then, are .23% higher than for the Lower Mississippi. For roughly comparable distances, however, the price per ton-mile of grain on the Illinois is 11% higher than on the Lower Mississippi, according to one industry representative. Illinois grain rates are regulated by the state. In the absence of such regulation one would predict that the rate for grain would fall.

D. Cost Finding Techniques

It should be noted that the cost finding procedure implicit in this study can be combined with other methods of cost estimation to obtain particular results. One possibility is to use estimates of direct cost derived from the engineering production function.

For example, consider a marginal shipment from Cairo to New Orleans (one extra barge). The engineering function is capable of providing direct cost estimates of this sort, taking into account waterway characteristics, and the change in tow configuration. To this estimate of direct cost could be added the indirect cost of the additional movement based on the statistical indirect cost function. One difficulty with the estimate made is that part of indirect cost is due to scheduling. There is no scheduling problem for this one barge, thus a slight over estimate would be made. In this case one would have an accurate direct cost estimate, and a less accurate indirect cost estimate.

Alternatively, the statistical towboat and barge cost functions could be used to provide a direct cost estimate for this movement. Again, the statistical indirect cost estimate could be added to the direct cost figure to determine the marginal cost of the trip. In this case, however, the direct cost estimate is less satisfactory, since the statistical direct cost functions do not take into account changes in tow configuration.

For the "average" movement, that is, viewing the problem from the standpoint of the firm that has demands it must satisfy at a certain time, it would be possible to use the engineering function to determine cost. However, direct scheduling costs would be left out of account. The best cost finding procedure in this case would be the statistical one. The estimates obtained would be representative of the costs the firm would actually experience for the conditions which prevail.

The results presented in this study permit a wide variety of cost estimates to be made. These estimates are sensitive to changes in the commodity shipped, the type of equipment used, the waterway characteristics and seasonal changes. The results should be of interest to inland waterway operators, since they can be employed in rate making and in examining sources of extra profit. In addition, the results should be of assistance to the Army Corp of Engineers in calculating the costs and benefits of additional investment in waterway projects.

FOOTNOTES

CHAPTER I

[1]John R. Meyer and Gerald Kraft, "The Evaluation of Statistical Costing Techniques as Applied in the Transportation Industry," A. E. R., LI (May, 1961), 313.

[2]John R. Meyer, Merton J. Peck, W. John Stenason, and Charles Zwick, The Economics of Competition in the Transportation Industries (Cambridge, Mass.: Harvard University Press, 1964). Subsequent references to this edition will appear in the text.

[3]It should be noted that if superior estimates of long-run marginal cost could be found for any particular cost category, these estimates could be substituted for the values estimated by MPSZ. A recent study (Joseph S. DeSalvo and Lester B. Lave, "A Statistical-Engineering Approach To Estimating Railway Cost Function," P-3781, Rand Corporation, March, 1968.) suggests the possibility of replacing two of MPSZ's estimates (train expense, and locomotive and car depreciation expense) by cost estimates obtained via an engineering production function for the point-to-point movement of a train. This approach would yield an estimate of cost for the two categories which could be associated with particular train movements (specifying the characteristics of the locomotive, the type and number of cars, and the roadbed over which the train travels). Thus, a more specific estimate of long-run incremental costs could be obtained.

[4]George H. Borts, "The Estimation of Rail Cost Function," Econometrica, XXVIII, No. 1 (Jan., 1960), 108-131. Subsequent references to this article will appear in the text.

[5]W. John Stenason and R. A. Bandeen, "Transportation Costs and Their Implications: An Empirical Study of Railway Costs in Canada," Transportation Economics, National Bureau of Economic Research, Special Conference 17 (New York: Columbia University Press, 1965), pp. 121-138. Subsequent references to this edition will appear in the text.

[6]Charles W. Howe, "Methods for Equipment Selection and Benefit Evaluation in Inland Waterway Transportation," Water Resources Research, I, No. 1 (1965), 25-39; and Charles W. Howe, "Process and Production Functions for Inland Waterway Transportation," Institute for Quantitative Research in Economics and Management (Herman C. Krannert Graduate School of Industrial Administration, Purdue University, Institute Paper No. 65, January, 1964). (Mimeographed.) Subsequent references to Howe, 1965, will appear in the text.

Footnotes--Continued

[7]Howe, Jan., 1964, p. 36. Subsequent references to this paper will appear in the text.

[8]Arthur P. Hurter, Jr., "Production Relationships for Inland Waterway Operations on the Mississippi River 1950, 1957, 1962 for the project The Economics of Inland Waterway Transportation" (Transportation Center, Northwestern University, September, 1965). (Mimeographed.)

[9]Ibid., p. 8.

[10]Gregory C. Chow, "Tests of Equality Between Sets of Coefficients in Two Linear Regressions," Econometrica, XXVIII, No. 3 (July, 1960), 591-605. A brief explanation of the Chow test also appears in J. Johnston, Econometric Methods (New York: McGraw-Hill, 1963), pp. 136-138.

[11]Arthur P. Hurter, Jr., "Cost Relationships for Inland Waterways Operations on the Mississippi River 1950, 1957, 1962 for the project The Economics of Inland Waterway Transportation" (Transportation Center, Northwestern University, September, 1965). (Mimeographed.)

CHAPTER II

[1]Direct expenses are usually further subdivided into those costs associated with towboats and those associated with barges. In the case of barges, the expense is primarily for maintenance. Separate analyses of towboat and barge direct costs are made in Chapter iv.

[2]George W. Wilson, "On the Output Unit in Transportation," Land Economics, XXXV (Aug., 1959), 266-276. Subsequent references to this article will appear in the text.

[3]Interstate Commerce Commission Bureau of Accounts, "Formula for Determining the Cost of Transporting Freight by Barge Lines," Prepared by the Cost Finding Section (Barge Form C), Washington D.C., September, 1964.

An EBM is defined as the movement of a jumbo barge for one mile when loaded to a draft of 8.5 feet (with 1,350 net tons of cargo). Such a barge movement would have an EBM weight of 1.00. Conceptually, different weight can be attached to each barge movement depending upon:

1. The characteristics of the barge (its size and capacity)
2. The size of the load in the barge, in tons
3. The river district, that is, the characteristics of the waterway over which the movement occurs (depth, stream flow, width, number of locks, congestion, etc.)

Barge Form C considers only (1) and (2) above, listing weighting schemes for:

1. Each type of barge, fully loaded with an average density

commodity.

2. The amount of cargo in a barge. There is an empty barge weight for each river district. A simple formula is given for interpolating the weights for loads between empty and completely full. The formula makes the EBM weight of any specific load a linear function of the draft of the vessel.

No detailed account is given of the derivation of the weights. They were apparently taken from tank tests made at the University of Michigan by Professor L. A. Baier and from individual carrier experience.

CHAPTER III

[1]Howe, Water Resources Research, I, No. 1 (1965), 25-39, and Howe, "Process and Production Functions . . . . ," January, 1964.

[2]A. A. Walters, "Production and Cost Functions: An Econometric Survey," Econometrica, XXI (Jan.-April, 1963), 1-66.

[3]It is assumed that other parameters of the production function remain the same across all River Districts. Thus, the river district dummy variables represent a shift in the whole production surface.

[4]See: J. Johnston, Econometric Methods (New York: McGraw Hill, 1963), pp. 136-138.

[5]For a discussion of technological change see: Lester B. Lave, Technological Change: Its Conception and Measurements (Englewood Cliffs: Prentice Hall, 1966).

[6]Ibid., Chap. 4. This can be compared to an overall rate of technological change in manufacturing of about 2 1/2% per year.

CHAPTER IV

[1]A river district is a stretch of the waterway system which has roughly the same characteristics (depth, width, and stream flow) over its length. All of the firms in the sample collect information on their operations by river district (ton-miles produced, towboat hours of operation, etc.) This information is taken from the towboat logs that a captain keeps for each boat. Fortunately, the definition of the various river districts (about 27 in all) is, with one exception, the same for all the firms. The same definitions are found also in Interstate Commerce Commission literature about the barge industry (See I.C.C. Barge Form C).

Footnotes--Continued

[2]This is also why the total EBMs produced by each boat have been included as a separate independent variable. That is, it might be argued that since towboat and waterway characteristics have been specified, the number of EBMs/Hour produced is determined, and thus total EBMs need not be included separately. In general, this is true. However, since towboats do not operate the same number of hours in a year the number of EBMs per year is not determined. Thus, total EBMs have been included as a separate explanatory variable.

[3]Meyer et al., The Economics of Competition . . . , p. 121.

CHAPTER V

[1]Ibid.

BIBLIOGRAPHY

Rail Cost Literature

Borts, George H. "The Estimation of Rail Cost Function," Econometrica, XXVIII, No 1 (Jan., 1960), 108-131.

DeSalvo, Joseph S., and Lave, Lester B. "A Statistical-Engineering Approach To Estimating Railway Cost Function," P-3781, Rand Corporation, March 1968.

Meyer, John R., et al. The Economics of Competition in the Transportation Industries. Cambridge, Mass.: Harvard University Press, 1964.

Stenason, W. John, and Bandeen, R. A. "Transportation Costs and Their Implications: An Empirical Study of Railway Costs in Canada," Transportation Economics. National Bureau of Economic Research, Special Conference 17. New York: Columbia University Press, 1965.

Barge Literature

American Waterways Operators. Big Load Afloat: U.S. Inland Water Transportation Resources. Washington, 1965.

Howe, Charles W. "Methods for Equipment Selection and Benefit Evaluation in Inland Waterway Transportation," Water Resources Research, I, No. 1 (1965), 25-39.

________. "Process and Production Functions for Inland Waterway Transportation." Institute for Quantitative Research in Economics and Management. Herman C. Krannert Graduate School of Industrial Administration, Purdue University, Institute Paper No. 65, January, 1964. (Mimeographed.)

Hurter, Arthur P., Jr. "Cost Relationships for Inland Waterways Operations on the Mississippi River 1950, 1957, 1962 for the project The Economics of Inland Waterway Transportation." Transportation Center, Northwestern University, September, 1965. (Mimeographed.)

________. "Production Relationships for Inland Waterway Operations on the Mississippi River 1950, 1957, 1962 for the project The Economics of Inland Waterway Transportation." Transportation Center, Northwestern University, September, 1965. (Mimeographed.)

Interstate Commerce Commission Bureau of Accounts. "Formula for Determining the Cost of Transporting Freight by Barge Lines." Prepared by the Cost Finding Section (Barge Form C), Washington, D.C., September, 1964.

Statistical Methods

Anderson, R. L., and Bancroft, T. A. Statistical Theory in Research. New York: McGraw-Hill, 1952.

Chow, Gregory C. "Tests of Equality Between Sets of Coefficients in Two Linear Regressions," Econometrica, XXVIII, No. 3 (July, 1960), 591-605.

Johnston, J. Econometric Methods. New York: McGraw-Hill, 1963.

Meyer, John R., and Kraft, Gerald. "The Evaluation of Statistical Costing Techniques as applied in the Transportation Industry," A. E. R., LI (May, 1961), 313-340.

Suits, Daniel B. "Use of Dummy Variables in Regression Equations," Journal of the American Statistical Association, LII (December, 1957), 548-551.

Walters, A. A. "Production and Cost Functions: An Econometric Survey," Econometrica, XXI (Jan.-April, 1963), 1-66.

General Literature

Lave, Lester B. Technological Change: Its Conception and Measurements. Englewood Cliffs: Prentice Hall, 1966.

Transportation Economics. A Conference of the Universities-National Bureau Committee for Economic Research. Special Conference 17. New York: Columbia University Press, 1965.

Wilson, George W. "On the Output Unit in Transportation," Land Economics, XXXV (Aug., 1959), 266-276.

VITA

Name: Leland Stewart Case

Birth Place: Salt Lake City, Utah

Date of Birth: August 28, 1942

Education:

University of Utah, September 1959 to June 1963.

B. S. Degree in Economics received June, 1963.

Northwestern University, September 1963 to June 1967.

M. A. Degree in Economics received June, 1967.

Ph. D. Degree in Economics received August, 1968.

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Leon N. Moses and Lester B. Lave, James P. Stucker, William B. Allen, Michael V. Beuthe, Joseph S. DeSalvo, Leland S. Case, Richard B. Hefelbower, Andrew J. Gold

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13. ABSTRACT

This research is directed towards improvement in procedures for the estimation of that portion of inland waterway transportation benefits which contribute to national income. The report is in three volumes (1) Summary Report, (2) Demand and Cost Analysis and (3) Regulatory Policy and Intermode Competition. The first volume develops a conceptual framework which reflects interaction of supply and demand as they relate to the evaluation of transportation alternatives. A model for demand for transportation is developed, the application of modal split is introduced and engineering and statistical cost functions are developed. In the second volume, Stucker, Allen and Beuthe deal with the development of demand and modal split analysis, whereas DeSalvo and Case deal with the estimation of cost functions for rail and waterway transportation. Hefelbower and Gold develop, in the third volume, an analysis of regulatory policy, its influence upon intermode competition and a model of regulatory behavior based upon modern utility theory.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Transportation Demand						
Modal Split						
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Regression Analysis						
Benefits						
Transportation Benefits						
National Income Benefits						
Process Cost Functions						
Line Haul Cost Functions						
Joint Costs						
Production Function of Transportation Modes						
Engineering Cost Functions						
Statistical Cost Functions						
Intermodal Competition						
Private Costs						
Public Costs						
Externalities						
Regulatory Behavior						