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Performance Evaluation and Complexity analysis of Re-jagged AR4JA code over AWGN channel

Abhishek Kumar,Madhusmita Mishra,Sarat Kumar Patra

Abstract— A flurry of low-density parity check codes from small template graphs(protographs)LDPC code that perform at rates extremely close to the Shannon capacity has been discussed here.Finally a new method for designing protograph codes similar to AR4JA codes of different rates,satisfying the linear minimum distance property is being proposed.The advantage of using this code is that LDPC codes of arbitrarily larger size can be constructed by expanding individual protographs. The method of encoding used in the literatures has encoding complexity prohibitively complex as we move to long codes of length of the order of 10^5 or 10^6. Here using structured parity check matrices we have reduced this encoding complexity. Simulations has been done with Hard-decision decoding to compare the performance of this code with the rates 1/2 ,2/3,3/4 and 4/5 over BIAWGN channel.Finally through analysis of BER performance it is proved that this code has equivalent performance with the long Irregular code taken in the literature with less encoding complexity

Index Terms Accumulate repeat by 4 jagged accumulate(AR4JA), BIAWGN , encoding, irregular codes, hard decision decoding, Low-density parity check code(LDPC), Protograph codes,

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1. INTRODUCTION

Low-density parity-check (LDPC) codes were proposed by Gallager [8] in 1962.After 1993 there have been many contributions to the design and analysis of LDPC codes.Recently, Circulant and block-circulant LDPC codes have been found that provide both excellent error correction performance and well structured decoder architectures [3], [4].The methods for constructing LDPC codes can be divided into random and algebraic construction methods. The performance of irregular LDPC codes with long code lengths constructed by random construction [1][2] has been observed to be close to the Shannon limit. However, the encoding of these LDPC codes is quite complex because of the lack of code structure such as cyclic or quasi-cyclic structure.Recent advances in LDPC code construction have resulted in the development of new codes with improved performance over Gallager codes. One class of these codes, irregular LDPC codes, demonstrates improved performance in the waterfall region. Disadvantages of irregular codes, however, include an increase, in general, in the number of iterations required for decoding convergence and an unequal error protection between code bits resulting from the irregular structure. Therefore, we concentrate here on algebraic constructions of LDPC codes that belong to a class of quasi-cyclic (QC) codes. The circulant generator matrix is a square binary matrix where each row is constructed from the previous row by a single right cyclic shift; we do not require that each row has Hamming weight 1.The row and column weights of a circulant are the same, say w. A circulant is completely characterized by its first row (or first column) which is called the generator of the circulant.For a $b \times b$ circulant A over GF(2), if its rank is b, then all its rows (or columns) are linearly independent. If its rank r is less than b, then any consecutive r rows (or columns) of A may be regarded as linearly independent and the other $b - r$ rows (or columns) are linearly dependent. This is due to the cyclic structure of A.For simplicity, we always take the first (or the last) r rows (or columns) of A as the independent rows (or columns).The density of each circulant matrix is indicated by the corresponding value in an $r \times n$ base matrix H_{base} . The Tanner graph corresponding to this matrix is called a protograph [3]. Entries greater than 1 in the base matrix correspond to multiple edges in the protograph. Base matrices can be expanded into block-circulant LDPC codes by replacing each entry in H_{base} with a circulant containing rows of the specified Hamming weight; the resulting codes are quasi cyclic.Quasi-cyclic (QC)low-density parity-check (LDPC) codes form an important subclass of LDPC codes. QC LDPC code is equivallent to circulant LDPC code added with irregularity of degree distributions and random permutations.These codes have encoding advantage over the other types of LDPC codes. A QC-LDPC code is given by the null space of a parity-check matrix that is an array of sparse circulants of the same size [1], [6]. For systematic encoding, it is necessary to find the generator matrix of the code in systematic form.Based on this systematic-circulant form of a generator matrix, several types of encoding circuits using simple shift-registers are devised. It is shown that the encoding complexity of a QC-LDPC code is linearly proportional to the number of parity-check bits of the code for serial encoding and to the length of the code for high speed parallel encoding. This is a substantial improvement over the quadratic complexity involved in general type of LDPC codes [7].Recently another

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- *Abhishek kumar is currently pursuing masters degree program in electronics engineering in National Institute of TechnologyRourkela, INDIA, PH-08763835225. E-mail: aaryan.aabhi@gmail.com*
- *Madhusmita Mishra is currently pursuing doctoral program in electronic engineering in National Institute of TechnologyRourkela, INDIA, PH-09778582363. E-mail: madhusmita.nit@gmail.com*
- *Sharat k. Patra is professor in electronics engineering in National Institute of TechnologyRourkela, INDIA. E-mail: skpatra@nitrkl.ac.in,PH-0661-2462457*

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class of LDPC codes has been developed using algebraic construction based on finite geometries providing very low error floors and very fast iterative convergence.These belong to the AR4JA (Accumulate, Repeat-by-4, and Jagged Accumulate) family [5]. This paper addresses the issue of efficient encoding of cyclic,QC LDPC codes,AR4JA and the modified AR4JA.

2. DESIGN CONCEPTS

2.1. Theoretical Overview of AR4JA and the Re-jagged AR4JA LDPC codes

When designing a communications link, selection of the error correcting code requires a trade-off of several parameters. Dominant parameters typically include power efficiency, code rate (a high code rate may be required to meet a bandwidth constraint with the available modulations), and the block length (shorter blocks reduce latency on low data-rate links, and reduce encoder and decoder complexity) [5].LDPC codes serve well when bandwidth is constrained, typically for higher data-rate links. The AR4JA LDPC code combines the structure Quasi cyclic added with permutation basing on the basic protograph structure.The various code rates are generated by expanding, copying and permuting the protograph structure.The parity check matrix of this code is similar in shape to that of Quasi cyclic code with the difference that here the sub blocks are related through permutation to make a systematic structure.The advantage of this code over the Quasi cyclic code is that the BER convergence is faster with lesser number of decoder iterations.

The second type is the Re-jagged AR4JA code,where each of the non-empty submatrices of AR4JA matrix are replaced by the same submatrix structure of one fourth size.The main difference between this Re-jagged AR4JA matrix and the AR4JA matrix is that here the sub matrix is quasi cyclic in nature while that in AR4JA matrix is circulant in nature. The advantage of this structure over AR4JA is that here the BER performance is better than AR4JA in the low SNR region.The parity check matrix for rate 1/2 is shown in fig.1 below.The hardware encoder diagram for this modified encoding process is given in the next sub-section.

2.2. Design Steps for Hardware Encoer

A direct implementation of this modified AR4JA encoder is shown in figure.2 [3]–[5]. For n and k being the codeword and information lengths respectively and M being the block length,the set of $(n-k)*16$ cyclic shift registers at the top of the figure, each of length $T(M/16)$,are loaded with the circulant patterns for the first row of G. For each message bit these registers are cycled once and, if message bit = 1, exclusive-ORed with the $(n-k)*16$ symbol output register. When each row of circulants is completed, sequences for the next row of circulants in G are loaded into the shift registers.The output register is cyclicly shifted in a hardware encoder rather than in the circulant registers. In this way, the circulant patterns need not be stored in registers at all, but can be generated as simple combinatorial functions of a symbol counter.With the switches set as drawn, the k message bits are fed through the encoder one at a time, and the registers are updated and shifted once per bit. Then the switches are changed and the contents of the registers are sequentially read out as the parity portion of the codeword. This encoder has been implemented in hardware. It requires $(n-k)*16$ D latches, $(n-k)*16$ exclusive-OR gates, and a modest amount of additional combinatorial logic. The size (k = 1024, n = 2048) LDPC code fits comfortably in a VERTEX-II Field Programmable Gate Array (FPGA), and runs at 100Msymbols/second. Speed is determined by the maximum clock rate of the FPGA. The maximum supported code size is determined primarily by the number of D-latches required to accumulate the parity, and so scales linearly with $(n-k)$.

3. EVALUATION OF SIMULATION RESULTS AND COMPLEXITY ANALYSIS

LDPC code of almost any rate and block length can be designed only from the specification of target parity check matrix. This system is simulated at the transmitter side by encoding the serial stream of data by the LDPC encoder and then converting it into a BPSK modulated wave and passing it into the AWGN channel.

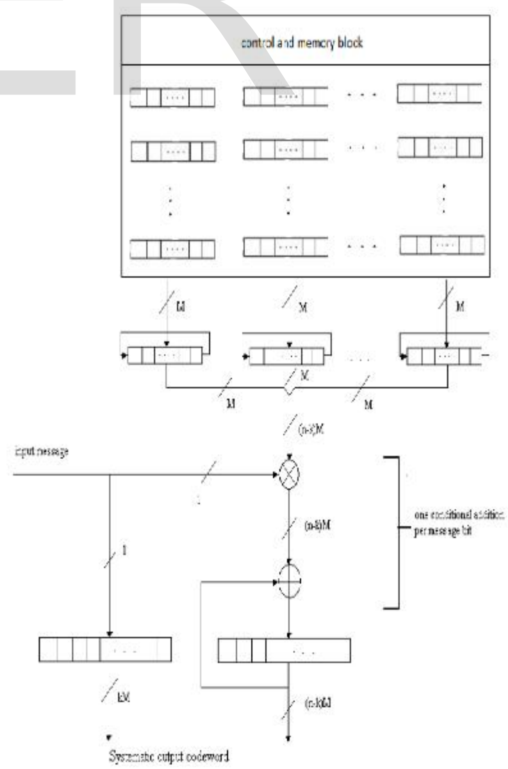

Fig. 2. Modified AR4JA LDPC encoder

At the receiver side, the demodulator calculates the llr (log likelihood ratios) followed by the LDPC decoder (Hard-decision) [2].The system was simulated over the AWGN channel with various code rates as 1/2, 2/3, 3/4 and 4/5 respectively. The LDPC decoder is of Hard- decision SPA type [2]. The information is binary in nature.The encoder is the quasi-cyclic type [4].The output block sizes varies with different rates.The encoder implementations has a complexity linearly proportional to the number of parity- check bits of the code for serial encoding.This is illustrated in Table.1 and Table.2 below for AR4JA and Re-jagged AR4JA code respectively.It shows that the output Blocksize(n)and the encoding time complexity is linearly varying with different rates with respect to Information length (k = 1024) and Block length(M = 512). The most interesting point here is for different rates of AR4JA and the Re-jagged AR4JA LDPC code the encoding times are less in case of Re-jagged AR4JA code.While the encoding time in the literatures [1], [2] with long irregular LDPC code is large compared to the Quasi encoder.This encoding time for Re-jagged AR4JA code is shown in Table.3. The BER plots for all rates in case of AR4JA code and Re-jagged AR4JA code are shown below in the figures 3 and 4 respectively.From the figures and tables it is visible that performance of Re-jagged AR4JA code is very close to that of AR4JA code in the low SNR region and in the High SNR region the performance is slightly better.

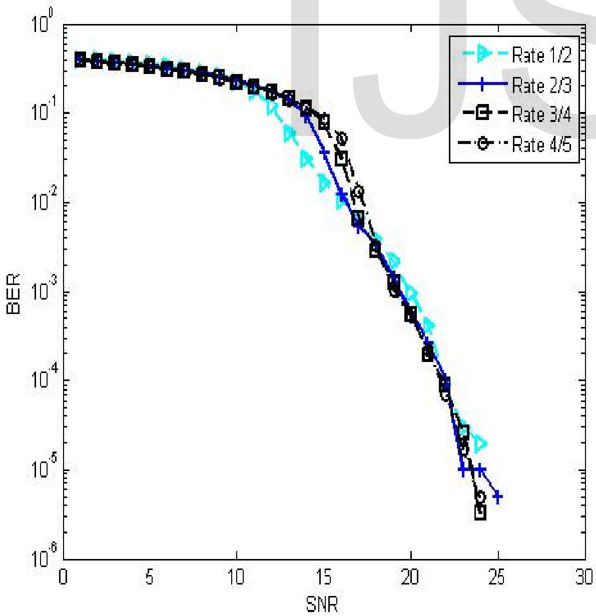

Fig. 3. BER PLOT FOR AR4JA LDPC CODED BPSK WAVE OVER AWGN CHANNEL

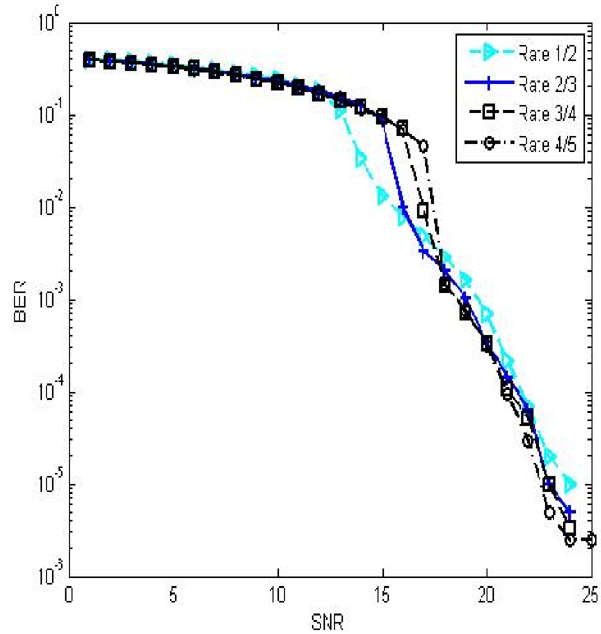

Fig. 4. BER PLOT FOR Re-jagged AR4JA LDPC CODED BPSK WAVE OVER AWGN CHANNEL

TABLE I

OUTPUT BLOCKSIZE AND QUASI ENCODING TIME FOR DIFFERENT RATES OF AR4JA LDPC CODE

Code Rates	N	AR4JA Encoding time
1/2	2560	0.132261
2/3	3584	0.157087
3/4	4608	0.175156
4/5	5632	0.206130

TABLE II

OUTPUT BLOCKSIZE AND QUASI ENCODING TIME FOR DIFFERENT RATES RE-JAGGED AR4JA LDPC CODE

Code Rates	N	Re-jagged AR4JA Encoding time
1/2	2560	0.122104
2/3	3584	0.150088
3/4	4608	0.162018
4/5	5632	0.185178

4. CONCLUSION

The proposed Re-jagged AR4JA LDPC codes according to the present disclosure can simultaneously achieve better error floor

T ABLE III

Out put block size and encoding time for different rates of RE-JAGGED ARJJA LDPC code with forward substitution encoding

Code Rates	N	Re-jagged AR4JA Encoding time
1/2	2560	0.25427
2/3	3584	0.37889
3/4	4608	0.52464
4/5	5632	0.69777

performance while providing various code rates.The interesting point is one encoder and one decoder can sup- port different code rates.Compared to AR4JA LDPC code its performance is better after SNR of 10.Its error floor for rate 1/2 is at 10^{-2} for SNR of 15, while that of AR4JA LDPC code is less than 10^{-2}.Similarly for all other rates its performance is better than that of AR4JA after SNR of 10.The quasi encoder implemented here is providing less encoding time as compared to the forward substitution encoding method provided in the literature.The most important factor of this proposed Re-jagged AR4JA LDPC code is that it is giving same error floors with lesser number of block lengths as that of the long Irregular LDPC code discussed in literature with hard- decision SPA decoder.Hence it can be used in any Practical Communication environment.

REFERENCES

[1] Madhusmita Mishra,S.K.Patra,A K Turuk " Long Irregular LDPC coded OFDM with Soft Decision", International Journal of Com- puter Applica- tions(0975 8887),Volume 56 No.4, October 2012.

[2] Madhusmita Mishra,S.K.Patra,A K Turuk," Performance anal- ysis of ensemble of long irregular ldpc code over various chan- nels with cutoff rate",International Journal of Applied Information Systems (IJAIS)ISSN : 2249-0868,Foundation of Computer Sci- ence FCS, New York,USA,Volume 4

[3] Kenneth Andrews, Sam Dolinar,and Jeremy Thorpe, "Encoders for Block- Circulant LDPC Codes",Jet Propulsion Laborato- ry,California Institute of Technology,Pasadena, CA, USA. No.8, December ,2012

[4] Zongwang Li,Lei Chen, Lingqi Zeng and Shu Lin,Wai Fong, "Effi- cient Encoding of Quasi-Cyclic Low-Density Parity-Check Codes",IEEE GLOBECOM 2005 proceedings.

[5] Research and Development for Space Data System Stand- ards,"LOW DENSITY PARITY CHECK CODES FOR USE IN NEAR-EARTH AND DEEP SPACE APPLICTIONS " , EXPERIMENTAL SPECIFICATION, CCSDS 131.1-O-2,Orange Book, Issue 2,September 2007.

[6] David J.C.MacKay," Good Error-Correcting Codes based on very Sparse Matrices",IEEE Transaction on Inforation Theory, January, 1999.

[7] R.Gallager, Low-density parity-check codes, IEEE Trans.Iform. Theory, vol. 8, no. 1, pp. 2128, Jan. 1962.

[8] Vo Tam Van,Hajime Matsui,Seiichi Mita,"Low complexity en- coder for generalized quasi-cyclic codes coming from finite ge- ometries",