Stochastic nonrelativistic approach to gravity as originating from vacuum zero-point field van der Waals forces

Daniel C. Cole
Department of Manufacturing Engineering, Boston University, Boston, Massachusetts 02215

Alfonso Rueda
Department of Electrical Engineering, California State University, Long Beach, California 90840

Konn Danley
Broadcom Corporation, 16215 Alton Parkway, P.O. Box 57013, Irvine, California 92619-7013
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We analyze the proposal that gravity may originate from a van der Waals type of residual force between particles due to the vacuum electromagnetic zero-point field. Starting from the Casimir-Polder integral, we show that the proposed approach can be analyzed directly, without recourse to approximations previously made. We conclude that this approach to Newtonian gravity does not work, at least not with this particular starting point. Only by imposing different or additional physical constraints, or by treating the underlying dynamics differently than what are embodied in the inherently subrelativistic Casimir-Polder integral, can one expect to escape this conclusion.

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The present article analyzes in some detail a specific proposal on the physical origin of gravitation [1]. Most physicists regard gravitation as a very basic phenomenon, on par with the electromagnetic, weak, and strong interactions. However, trying to cast all four of these interactions under one unified theoretical description has proved to be enormously difficult. This difficulty contributed to Sakharov's proposal [2] that the gravitational interaction is not a fundamental interaction at all, but rather that it results from a ''change in the action of quantum fluctuations of the vacuum if space is curved.'' In turn, Sakharov's idea helped to motivate Puthoff's proposal in 1989 that '' . . . gravity is a form of long-range van der Waals force associated with particle *Zitterbewegung* response to the ZP (zero point) fluctuations of the electromagnetic field'' [3].

Several possible starting points were mentioned for the gravity related work in Ref. [1], including (i) Boyer's stochastic electrodynamics (SED) calculation of the van der Waals force between two classical, nonrelativistic, electric dipole harmonic oscillators [4], (ii) Renne's related nonrelativistic quantum electrodynamic (QED) calculation for a quantum harmonic-oscillator model [5], and (iii) fourth-order perturbation theory in QED leading to the (subrelativistic) Casimir-Polder integral [6]. All three of these approaches were discussed and related to each other in Ref. [4]. Since Puthoff explicitly referred to the first term in the Casimir-Polder integral [7], let us begin with this expression [6]:

$$U(R)=-\alpha^2\frac{\hbar c}{\pi}\int_0^\infty du\frac{u^4\omega_0^4}{(c^2u^2+\omega_0^2)^2}\frac{e^{-2uR}}{R^2}\left[1+\frac{2}{uR}+\frac{5}{(uR)^2}+\frac{6}{(uR)^3}+\frac{3}{(uR)^4}\right]. \tag{1}$$

Here, $U(R)$ is the Casimir-Polder potential between two neutral, polarizable particles, R is the distance between the particles, and ω_0 is the resonant frequency associated with the particles when they are treated as harmonic oscillators. The polarizability α is then given by $e^2/(m\omega_0^2)$.

A number of approximations were made to Eq. (1) in Ref. [1]. Only the first term in brackets in Eq. (1) was considered and $\omega_0=0$ was substituted into the integrand, based on the argument of a small effective resonant frequency. The upper limit of ∞ was replaced by an upper cutoff limit $u_c=\omega_c/c$. Some averaging arguments were then made that led to a $1/R$ effective potential between particles. Later, in response [8] to a criticism by Carlip [9] on the calculational procedure of the averaging steps, Puthoff gave some additional arguments and different reasoning to still yield this $1/R$ effective potential, now emphasizing that there should be physical reasons for imposing cutoffs in the integration that enable this $1/R$ form to be obtained.

We wish to make two key points here. First, one cannot simply extract the first term in Eq. (1), as all of the terms contribute on a roughly equal footing in the large distance regime. Second, Eq. (1) can be fully evaluated, as will be done here, and compared with any proposed approximations to the full integral. Unfortunately, as will be seen, the approximations in Refs. [1] and [8] do not hold, at least not without introducing additional assumptions that imply significantly different physical effects not embodied within the inherently subrelativistic full Casimir-Polder integral.

To begin, we make the substitution of $w=uR$ in Eq. (1) to obtain

$$U(R)=-\alpha^2\frac{\hbar c}{\pi}\frac{\omega_0^4}{c^4R^3}I\left(\frac{\omega_0 R}{c}\right), \tag{2}$$

where

$$I(b)\equiv\int_0^\infty dw\,\frac{w^4e^{-2w}}{(w^2+b^2)^2}\left[1+\frac{2}{w}+\frac{5}{w^2}+\frac{6}{w^3}+\frac{3}{w^4}\right]. \quad (3)$$

Thus, $U(R)$ has a functional form of $1/R^3$ times an integral that depends on ω_0R/c. A second argument to this integral could also be included [i.e., $I(b,w_c)$] if we replace the upper integration limit of infinity by a cutoff of $w_c=u_cR=\omega_cR/c$, such as might be imposed if the ZP spectrum was thought to be cutoff at sufficiently large frequencies [10]. Without imposing this cutoff, however, then it is easy to see from the above that if a $1/R$ potential is to emerge for the form of $U(R)$, under whatever limiting conditions one imposes (e.g., large R, small ω_0, etc.), then $I(b)$ must result in a b^{+2} dependence.

However, a full evaluation of Eq. (3) does not reveal any such dependency. As discussed in Ref. [11], each term in Eq. (3) can be analytically evaluated. Indeed, Fig. 1 in Ref. [11] shows a plot of $\ln[I(b)]$ versus $\ln(b)$, revealing that $I(b)$ is bounded from above by two curves that $I(b)$ asymptotically approaches at large and small values of b. For large $b=\omega_0R/c$, the bounding curve is the retarded van der Waals expression of $I_r(b)\equiv 23/4b^{-4}$, yielding an overall $1/R^7$ dependence for $U(R)$ in this regime. At small b, $I(b)$ is bounded by the unretarded van der Waals expression of $I_{ur}(b)\equiv 3\pi/4b^{-3}$, yielding an overall $1/R^6$ dependence for $U(R)$ in this regime. At no point either between these extremes, or at these extremes, is there any behavior that remotely approaches a b^{+2} dependence that would be required to yield a net $1/R$ dependence for $U(R)$.

Reference [11] contains a detailed analysis on how $I_r(b)$ and $I_{ur}(b)$ can be extracted from Eq. (3). Moreover, the question is examined on what happens if an upper cutoff of $w_c=\omega_cR/c$ is imposed in the integration in Eq. (3). As shown there, if $\min(2\omega_0,5c/R)\lesssim\omega_c$, where ω_0 is the resonant frequency of the oscillator system, then the integrations in Eqs. (1) or (3) will be barely affected. Since proposed upper frequency limits for the ZP spectrum are far, far larger than this restriction [10], then we must conclude that imposing a realistic upper frequency cutoff in the integration in Eq. (1) still yields that a Newtonian potential does not arise from the Casimir-Polder integral. An energy based argument discussed in Ref. [11] helps to support this point. It displays the remarkable implausibility of the low frequencies van der Waals force approach to Newtonian gravity formulated in Ref. [8] in response to the objections of Ref. [9]. In conclusion, barring the introduction of additional physical assumptions into the analysis in Refs. [1] and [8], the specific argument presented there involving an average force induced by ZP fields, will not yield a Newtonian gravitational force signature.

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EPAPS Supplementary Material for Phys. Rev. A Article, "On a Stochastic Non-relativistic Approach to Gravity as Originating from Vacuum Zero-Point Field van der Waals Forces"

Daniel C. Cole
Dept. of Manufacturing Engineering, Boston University, Boston, MA 02215

Alfonso Rueda
Dept. of Electrical Engineering, California State University Long Beach, Long Beach, CA 90840

Konn Danley
Broadcom Corporation, 16215 Alton Parkway, P.O. Box, Irvine, CA 92619-7013

I. OVERVIEW OF MATERIAL

The material contained here is intended to supplement and support the information reported in our article, "On a Stochastic Non-relativistic Approach to Gravity as Originating from Vacuum Zero-Point Field van der Waals Forces," to be published in Phys. Rev. A. The present material can be obtained from the EPAPS homepage (http://www.aip.org/pubservs/epaps.html) or from ftp.aip.org in the directory /epaps/. The footnotes and reference numbers in this supplementary section refer to references listed at the end of this material, rather than to the reference numbers in the main article.

The information provided here is broken up into three subsequent parts. Section II contains additional background on work related to Sakharov's proposal, on stochastic electrodynamics, and on related work to Puthoff's gravity proposal. Section III then turns to a detailed analysis on how $I_r(b)$ and $I_{ur}(b)$ can be extracted from $I(b)$ in Eq. (3). We are not aware of such calculations being available elsewhere on the connection between the full, retarded, and unretarded van der Waals expressions, so this material should be of general interest for researchers involved with van der Waals and Casimir interaction calculations. Moreover, the question is examined on what happens if an upper cut-off of $w_c = \omega_c R/c$ is imposed in the integration in Eq. (3). Section IV then contains a discussion on the energy aspects associated with variations of the proposal in Ref. [1]. This analysis provides an intuitively compelling argument as to the unlikelihood of the gravitational attraction being a long range van der Waals type force originating in the very low frequencies of the ZP radiation spectrum. Finally, Sec. V provides some concluding comments.

II. BACKGROUND INFORMATION ON RELATED GRAVITATIONAL APPROACHES

Modern views on gravitation are based primarily on Einstein's theory of general relativity. This theory describes the properties of space and time as being altered in the vicinity of massive bodies. Roughly stated, space near a massive body is "bent" in such a way that the shortest distance between two points becomes a curve, rather than a straight line in Euclidean space. One example often given in semipopular writings is the path of a ray of light when passing near a massive body like the Sun. As measured in Euclidean space, the path deviates from that of a straight line. The larger the massive body and the closer the path of the light ray to the body, the greater the deviation.

Most physicists regard gravitation as a very basic phenomenon, on par with the electromagnetic, weak, and strong interactions. Physicists, starting with Einstein, have tried for years to describe the main interactions in nature in terms of a single, coherent framework. However, trying to cast all four of these interactions under one unified theoretical description has proved to be enormously difficult. Indeed, the well-known difficulties in expressing ordinary gravitational theory within a properly quantized framework have occasionally lead to the view that gravitation is merely an effective field or an effective interaction, *i.e.*, essentially phenomenological and thereby susceptible of a more fundamental explanation in terms of other fields. Consequently, several attempts have been launched in this direction [2]- [4]. We refer here in particular to that of Sakharov [5]- [7] and Zel'dovich [8]. Their approach takes a much more radical point of view to gravitation than is conventionally held by most scientists.

As expressed by Sakharov [5], gravitation is not a fundamental interaction at all, but rather a by-product of vacuum quantum fluctuations: "The presence of the (Einstein) action ... leads to a metrical elasticity of space, *i.e.*, to generalized forces which oppose the curving of space ... We consider the hypothesis which *identifies the (Einstein) action ... with the change in the action of quantum fluctuations of the vacuum if space is curved*" (our italics). This came as a natural consequence of trying to express the cosmological constant, Λ, in terms of vacuum fluctuations [8].

This proposal of Sakharov and Zel'dovich elicited considerable interest and received a fair bit of attention, *e.g.*, by Terasawa [9] and collaborators, Hasslacher and Mottola [10], Amati and Veneziano [11], Zee [12], Adler [13], and Yoshimoto [14]. Here we give a rough, qualitative description of the general ideas behind this approach to prepare for the more specific theory that will subsequently be discussed.

A qualitative understanding can be obtained by thinking in terms of bending, by a small amount, a strip of metal with elastic properties. Work must of course be done on this strip to accomplish this bending. This work is stored as potential energy in the strip. When pressure is removed on the strip, the strip will flex back, on its own, and assume its original shape. The elastic potential energy stored in the strip enables this behavior to occur.

The theory by Sakharov and Zel'dovich roughly describes the "bending" of space-time in general relativity in terms of a similar mechanism. When massive bodies are nearby, a sort of "pressure" results that bends space and time; upon removing the bodies, then space-time assumes it's original properties. The more "bending" that occurs, the more energy that is required to be stored in the distorted space-time. Their theory relates gravity to quantum fields by associating the stored energy involved in the bending of space-time to the energy associated with the quantum fields that describe material particles. The presence of a massive body changes the amount of stored energy in these fields, which in turn results in a greater distortion of space-time. Their proposed theory involves an expression that relates G, the gravitational constant, to the speed of light, c, Planck's constant, $\hbar$, and a cutoff length in the integration over zero-point (ZP) field energies. Einstein's equations can be derived with this approach, as well as what are interpreted as higher order corrections to these equations. In this sense, then, gravitation becomes derivable from particle physics. For specific details, Refs. [5]- [8], should be consulted.

The theory we will now turn to and address in detail was motivated in part by this theory by Sakharov and Zel'dovich. This approach [1] uses the techniques of stochastic electrodynamics (SED). SED is classical electrodynamics with the assumption of the existence of a classical random electromagnetic background field that is homogeneous and isotropic and looks the same from all inertial Lorentz frames [15], [16]. The only classical background with such properties is one whose energy density spectrum displays a dependence of the form

$$\rho\left(\omega\right)d\omega = \frac{\hbar\omega^3}{2\pi^2 c^3}d\omega \quad . \tag{A1}$$

This spectrum is exactly the same energy density spectrum found for the vacuum electromagnetic ZP field in quantum theory. Here, however, $\hbar$ enters more as the parameter that fixes the scale of such random electromagnetic background than as a constant in some way connected with quantization, as is traditional in quantum theory.

Motivated by Sakharov's earlier work [5]- [7] relating ZP fields to gravity, and knowing that Casimir and van der Waals forces are closely tied to electromagnetic ZP fields, in 1989 Puthoff [1] proposed that "... gravity is a form of long-range van der Waals force associated with particle *Zitterbewegung* response to the ZP fluctuations of the electromagnetic field" [17]. According to his analysis, a Newtonian-like $1/R^2$ force arises between distant particles as a residual force due to point-particle dipole interactions with ZP radiation. We show here that this specific attempt to deduce Newton's formula for the gravitational interaction that uses SED in a nonrelativistic approximation, unfortunately, does not work. The approach is interesting and, as noted in Ref. [1], has some connection to Sakharov's proposed idea on the origin of gravitation [5]- [7]. Through the years, several physicists have been attempting other closely related approaches that introduce additional physical considerations to examine whether the general idea and final result might yet hold. Along this vein we would like to mention a recent work by Puthoff [18] that reviews the proposal [19], [20] that gravity may be interpreted as an effect induced by massive bodies on the polarizability of the vacuum medium. At this point, however, this and other speculative developments are still preliminary. To clarify the present situation, we have written the present article to show that unless other physical assumptions are introduced, the original idea in Ref. [1] will not yield a Newtonian gravitational attractive force [21]. Our concerns described here are addressed to the last part of the paper, which is where this inverse square force is derived. We note, however, that other parts of the article we found stimulating. They promote further thought on relating ZP energy, Zitterbewegung behavior, and gravity. Strictly speaking, our argument disproves only the subrelativistic van der Waals forces approach to gravity. A possible relativistic van der Waals forces approach would not be strictly excluded, though its likelihood of success, given the present developments, does not seem promising. Our argument in no way should be construed as an argument against all vacuum or ZP field connections to gravity.

III. ANALYTIC AND NUMERICAL ANALYSIS OF CASIMIR-POLDER POTENTIAL

To facilitate reading, below we first repeat the first three equations in the main part of our article, labeling them as Eqs. (1), (2), and (3), as in the article. Subsequent equations are then continued as Eqs. (A2), (A3), etc.:

$$U\left(R\right)=-\alpha^{2}\frac{\hbar c}{\pi}\int\limits_{0}^{\infty}du\frac{u^{4}\omega_{0}^{4}}{\left(c^{2}u^{2}+\omega_{0}^{2}\right)^{2}}\frac{e^{-2uR}}{R^{2}}\left[1+\frac{2}{uR}+\frac{5}{\left(uR\right)^{2}}+\frac{6}{\left(uR\right)^{3}}+\frac{3}{\left(uR\right)^{4}}\right]\ , \tag{1}$$

$$U\left(R\right)=-\alpha^{2}\frac{\hbar c}{\pi}\frac{\omega_{0}^{4}}{c^{4}R^{3}}I\left(\frac{\omega_{0}R}{c}\right)\ , \tag{2}$$

$$I\left(b\right)\equiv\int\limits_{0}^{\infty}dw\frac{w^{4}e^{-2w}}{\left(w^{2}+b^{2}\right)^{2}}\left[1+\frac{2}{w}+\frac{5}{w^{2}}+\frac{6}{w^{3}}+\frac{3}{w^{4}}\right]\ . \tag{3}$$

As mentioned in the main article, only the first term in brackets in Eq. (1) was considered [22] in Ref. [1]. However, each term in Eq. (3) can be analytically evaluated in terms of the sine and cosine integrals. For example, from Ref. [23], one obtains that

$$\int\limits_{0}^{\infty}\frac{e^{-\mu w}dw}{\left(w^{2}+b^{2}\right)^{2}}=\frac{1}{2\,b^{3}}\left\{\mathrm{ci}(b\mu)\sin(b\mu)-\mathrm{si}(b\mu)\cos(b\mu)-b\mu\left[\mathrm{ci}(b\mu)\cos(b\mu)+\mathrm{si}(b\mu)\sin(b\mu)\right]\right\}\ , \tag{A2}$$

which enables the last term in Eq. (3) (*i.e.*, the $1/w^4$ term) to readily be evaluated ($\mu=2$). The other terms in Eq. (3) can then be found by repeated differentiation of Eq. (A2) with respect to μ.

Figure 1 shows a plot of $\ln\left[I(b)\right]$ versus $\ln(b)$, while Fig. 2 shows a plot of $I(b)$ versus b. As can be seen, $I\left(b\right)$ is bounded from above by two curves that $I(b)$ asymptotically approaches at large and small values of b. For large $b=\omega_0 R/c$, the bounding curve is the retarded van der Waals expression of

$$I_{r}\left(b\right)\equiv\frac{23}{4}b^{-4}\ , \tag{A3}$$

yielding an overall $1/R^7$ dependence for $U(R)$ in this regime. At small b, $I\left(b\right)$ is bounded by the unretarded van der Waals expression of

$$I_{ur}\left(b\right)\equiv\frac{3\pi}{4}b^{-3}\ , \tag{A4}$$

yielding an overall $1/R^6$ dependence for $U(R)$ in this regime. At no point either between these extremes, or at these extremes, is there any behavior that remotely approaches a b^{+2} dependence that would be required to yield a net $1/R$ dependence for $U(R)$.

We now turn to briefly outlining how Eqs. (A3) and (A4) can be extracted from Eq. (3), as this will have bearing on some of the approximations and reasoning described in Puthoff's response [24] to Carlip [25]. Moreover, since we are not aware of such calculations being available elsewhere on the connection between the full, retarded, and unretarded van der Waals expressions, then perhaps this material will be of general interest for researchers involved with van der Waals and Casimir interaction calculations.

Of course, one could simply evaluate Eq. (3) using Ref. [23] and the reasoning described earlier, then take the asymptotic extremes of small b and large b in the resulting expression, but the details of this procedure are long and not very revealing. Hence, we will proceed in a simpler, although more approximate manner, that provides more physical insight. In particular, the following reasoning will provide deeper understanding into the physical argument given by Boyer in Ref. [26] that the major contribution to the long range attractive force between polarizable particles is due to the low frequency contribution of the integral in Eq. (1). Understanding this point more deeply is important as this argument was cited by Puthoff in his response [24] to Carlip [25].

The factor of

$$f\equiv1/\left(w^{2}+b^{2}\right)^{2} \tag{A5}$$

in Eq. (3) is clearly what dictates the formation of the asymptotic regimes for $I(b)$, as the only place b occurs in the integrand in Eq. (3) is within f. The shape of f is roughly described by the following: at $w=0$, f equals its maximum of b^{-4}; f decreases as w increases in value, equaling $1/4$ and $1/25$ of its peak value at $w=b$ and $w=2b$, respectively [see Figs. 3(c) and 3(d)]. For this reason, the largest contribution to the integral in Eq. (3) occurs in the region approximately given by $0\leq w\lesssim 2b$ when $b\lesssim 1$ [see Fig. 3(a)]. For large b, f is no longer the key factor limiting the size of the integrand; instead, the damping due to the factor $\exp(-2w)$ dictates the size of the integrand. Indeed, as can be seen from Figs. 3(b) and 3(d), for $1 \ll b$, f is essentially constant over the region where the integrand is large.

Consequently, for large b,

$$I(b) \approx \frac{1}{b^4} \int_0^\infty dw w^4 e^{-2w} \left[1 + \frac{2}{w} + \frac{5}{w^2} + \frac{6}{w^3} + \frac{3}{w^4}\right]$$
$$= \frac{1}{b^4}\left(\frac{3}{4} + \frac{3}{4} + \frac{5}{4} + \frac{6}{4} + \frac{6}{4}\right) = \frac{23}{4b^4} \ , \tag{A6}$$

yielding $I_r(b)$. A key point here is that each term in the polynomial part of the integrand contributes on a roughly equal basis to the final result, as indicated above. Thus, it is important to note here that for the large R range behavior, one cannot simply restrict attention to the first term in Eq. (3), which corresponds to the same first term in Eq. (1).

To obtain the unretarded regime described by $b \ll 1$, one simply needs to note that now the integrand in Eq. (3) is essentially determined only by the factor of f [see Figs. 3(a) and 3(c)]. Since this factor is then only large when $0 \leq w \ll 1$, the rest of the integrand can be approximated by its value at $w = 0$; *i.e.*, $\left(w^4 + 2w^3 + 5w^2 + 6w + 3\right) e^{-2w} \approx 3$. Hence, for $b \ll 1$:

$$I(b) \approx 3 \int_0^\infty dw \frac{1}{\left(w^2 + b^2\right)^2} = \frac{3\pi}{4b^3} \ , \tag{A7}$$

yielding $I_{ur}(b)$.

Now let us return to the question of what happens if an upper cut-off of $w_c \equiv \omega_c R/c$ is imposed in the integration in Eq. (3). As can be seen from Figs. 3(a) and 3(b), if w_c is approximately equal to or larger than about $2b = 2\omega_0 R/c$ for $b \lesssim 1$, or, indeed, if w_c is larger than about 5 for any value of b [Figs. 3(a) and 3(b)], then the integration over w in Eq. (3) will contain the largest contribution from the integrand and the character of the plot in Fig. 1 will be hardly affected at all. We checked this assertion via numerical calculations and found that if w_c is made equal to $2b$ instead of ∞, then the calculation of the integral in Eq. (3) is decreased by a maximum of about 4% at small values of b and monotonically decreases to 0% at large values of b. The curve $\ln I(b)$ in Fig. 1 would then be visibly changed only at small values of b. Thus, if $\min\left(2\omega_0, \frac{5c}{R}\right) \lesssim \omega_c$, where ω_0 is the resonant frequency of the oscillator system, then the integration in Eq. (3) will be barely affected.

IV. ZP RADIATION AND THE $1/R$ GRAVITATIONAL POTENTIAL - PLAUSIBILITY CONSIDERATIONS

In Ref. [24], Puthoff added the further argument that to obtain a net $1/R$ potential dependence, an upper effective frequency limit of ω_i needed to be imposed in the integration, where $\omega_i < \omega_0$, followed by the condition that the limit of $\omega_0 \to 0$ was to be taken. In his justification for this reasoning, he used an argument by Boyer in Ref. [26] that only the low frequency contribution of the interaction between polarizable particles should be effective in yielding the long range attractive force due to correlated motion. Figure 3(b) shows that at sufficiently large distances, the significant frequency contributions in the Casimir-Polder integration lie in the range of $0 \leq \omega \lesssim 5c/R$ (*i.e.*, for large distances, $5c/R$ is much smaller than $2\omega_0$). Thus, for large distances, the significant frequency contributions to the integration are indeed the very low frequency regime. However, applying this reasoning leads to the retarded van der Waals expression in Eq. (A3) with its $1/R^7$ dependence, rather than to a $1/R$ dependence.

Moreover, there is an additional point of concern here. For macroscopic distances and in particular for astronomical distances where Newton's law has been shown to work quite well, the energies available in the relevant part of the ZP field, *i.e.*, in the low frequency limit for $0 \leq \omega \leq 5c/R$, would be incredibly small. Integrating the spectrum $\left(\hbar\omega^3\right)/\left(2\pi^2 c^3\right)$ over that range and applying the result to the Earth-Moon distance ($R \cong 3.844 \times 10^{10}$ cm) we obtain that in a cubic volume V_0 of side $2R$ that is able to enclose the Earth-Moon orbit inside, the amount of energy in that whole volume ($V_0 \cong 4.544 \times 10^{32}\,\text{cm}^3$) would only be on the order of 0.3×10^{-13} eV! However, according to Ref. [24], only this tiny part of the ZP field would be responsible for the enormous Newtonian gravity force required to attract the Moon to the Earth (and vice-versa), which seems unreasonable. This is of course due to the very small amounts of energy available in the ZP field at long wavelengths. As a sanity check on the above estimate, when exactly the same estimate is made for $R \approx 4\,\mu$m, the effective cut-off frequency of $5c/R$ changes, yielding an energy in this much smaller cubic box to be on the order of 3 eV. This order of magnitude estimate seems reasonable for the energy amounts that should be relevant in a van der Waals attraction between molecules.

In the previous two examples of the last paragraph, the maximal frequency ω_c corresponds to a minimal wavelength, $\lambda_{\rm min} = 2\pi c/\omega_c$, which is on the order of the separation distance between the attracting bodies. The spectral energy density for ZP radiation wavelengths longer than $\lambda_{\rm min}$, $\lambda \geq \lambda_{\rm min}$, is $\rho_{\rm E} = \hbar\omega_c^4/\left(8\pi^2 c^3\right)$. In both examples we have considered as effective in the purported van der Waals attractive interaction only the ZP radiation in that range, $0 \leq \omega \leq \omega_c$, enclosed in a volume V_0 that we estimated to be on the order of $\lambda_{\rm min}^3$, $V_0 \cong \lambda_{\rm min}^3$. Now, if the associated motions are strictly quasistatic, it might be argued that such a volume is perhaps too small and that in the hypothetical extreme case of a strictly static situation, the volume should become indefinitely large; *i.e.*, rather one should use V_0 on the order of the "volume of the Universe" (whatever that really means). Of course, ordinary gravitating astrophysical bodies do not exist in static situations, but undergo motions (*e.g.*, orbits) with respect to each other. In spite of this, we introduce an additional counter-example where the volume V_0 is on the order of the "volume of the Universe." Newton's gravitational theory has been thoroughly tested in the Solar System and even beyond and found to amply hold. Let us consider a $\lambda_{\rm min}$ on the order of 10^{-3} light years, that roughly corresponds to a distance on the order of the orbital radius of Pluto around the Sun. In such a case, $\omega_c = 2\pi c/\lambda_{\rm min} \cong 2 \times 10^{-4}$ sec^{-1} and the corresponding energy density is $\rho_{\rm E} \cong 8 \times 10^{-76}$ erg cm^{-3}. Subsequently, let us consider as the "volume of the Universe" a volume $V_{\rm T}$ equal to that of a cube of side $L = 2 \times 10^{10}$ light years, *i.e.*, $V_{\rm T} \cong 7 \times 10^{84}$ cm^3. This gives a total energy available for the purported relevant part of the ZP field that is active for the corresponding Pluto to Sun gravitational attraction of $E \cong \rho_{\rm E} V_{\rm T} \cong 6 \times 10^9$ erg = 600 Joules (!). Clearly the energies and changes of energy (kinetic and/or potential) associated with motions of planets like Pluto around the Sun are enormously larger than this tiny energy, confirming thereby the fact adduced to above that, in the low frequencies of the ZP radiation, energies are almost negligible and that consequently gravitation cannot plausibly be conjectured to be a long range van der Waals force induced by the extremely low frequency ZP field components. As Newton's gravitational law has been successfully applied to the trajectories of comets, and comets are known to go up to the Oort Cloud at a distance from the Sun of at least a few light days (and even more), an additional and even more compelling argument may be easily constructed for that case with $\lambda_{\rm min} \approxeq 10^{-2}$ light years, $\rho_{\rm E} \cong 8 \times 10^{-80}$ erg/cm^3, and if the same $V_{\rm T} \cong 7 \times 10^{84}$ cm^3 volume is assumed, the energy comes out to be $E = \rho_{\rm E} V_{\rm T} = 6 \times 10^{-2}$ Joules! Of course the changes in kinetic and in potential energy associated with the motion of a comet from perihelion to aphelion and back are in absolute value much larger than this extremely small energy [27].

V. ADDITIONAL REMARKS

If the *additional* constraints are imposed of (i) an upper limit of ω_i in the integration, with $\omega_i < \omega_0$, as well as that (ii) the limit of $\omega_0 \to 0$ be taken, then this procedure is equivalent to saying that additional physical effects need to be imposed that are not present in the full Casimir-Polder expression. Indeed, after reading Refs. [1] and [24], one might have the impression that the Casimir-Polder expression reduces to a $1/R$ potential if one could only calculate it appropriately under the correct conditions. Instead, the Casimir-Polder expression clearly contains the retarded van der Waals expression with its $1/R^7$ dependence as a limiting case (Figs. 1 and 2). The physical reasoning of the largest contribution to this van der Waals force result being due to the small frequency regime is indeed correct and is not an additional requirement that needs to be imposed when evaluating the integral in Eq. (1). Instead, the requirements of Puthoff in Ref. [24] involving ω_i, $\omega_i < \omega_0$, and $\omega_0 \to 0$, constitute additional physical impositions (as they are not derivable from proper mathematical arguments), that are not contained within the Casimir-Polder equation. Consequently, without any further justification available, we conclude that Puthoff's proposal that the Newtonian gravitational attraction arises from a van der Waals force-like interaction [1], [24], is presently unsupported. However, it does seem reasonable to expect there are ties yet to be discovered between ZP fields and gravitation, as emphasized by the cosmological constant problem [30].

Finally, we should note that two of us (KD and AR) carried out integration analysis for the first term of Eq. (1) above in work reported in Ref. [31]. This was the same term used by Puthoff [1], [24], as the objective was to check in detail the technique of Refs. [1], [24]. Even here, for realistic upper frequency limits, a $1/R$ potential did not arise. One may still speculate that a fully relativistic analysis involving the ZP high frequencies and very fast oscillatory motion might still reveal a main term with a $1/R$ potential dependence, but this remains to be proven (or disproved).

In Ref. [1], Puthoff assumed that *Zitterbewegung* (very fast) oscillations occur for charged particles or subparticles (partons) due to the influence of electromagnetic ZP field fluctuations. As in quantum theory it is reasonable to expect that those motions are ultrarelativistic. Hence one can argue that a fully relativistic analysis should, for consistency, have been implemented in Ref. [1]. On these grounds, it can be argued that the subrelativistic procedure of Ref. [1] is suspect and the Newtonian gravitational inverse square force should not be expected to result. Moreover, one can argue that since the various ZP fields, namely, the ground states of the various interactions, clearly make a contribution to the observed mass of particles (*i.e.*, ZP fields are a widely accepted component of the mass renormalization process that thereby contribute to inertial mass [32]- [35]), then via the equivalence principle, we should expect some ZP field contributions to gravity. Nevertheless, we do not expect that a relativistic reanalysis of the relativistic van der Waals forces analysis of Puthoff [1] will yet recover a $1/R$ potential, but rather that the mass contribution of ZP fields and its ultimate connection with gravity, will show up in other ways, perhaps more along the lines of Sakharov's and others' [5]- [14] or Dicke's [20], Wilson's [19], and Puthoff's [18] proposals.

Thus, we have (a) explicitly evaluated the Casimir-Polder integral, (b) shown the connection to the unretarded and retarded expressions, (c) concluded that, barring the introduction of additional physical assumptions into the analysis in Refs. [1] and [24], the specific argument presented there involving an average force induced by ZP fields, will not yield a Newtonian gravitational force signature, and finally (d) we have provided an energy-based argument why the gravitational attraction is not likely to be a long range van der Waals type force originating in the very low frequencies of the ZP radiation spectrum, $0 \leq \omega \leq \omega_c$, $\omega_c = 2\pi c/\lambda_{\min}$ ($\lambda_{\min}$ is on the order of the separation distance between the attracting bodies).

Figures

FIG. 1: Plot containing $\ln[I(b)]$, $\ln[I_r(b)]$, and $\ln[I_{ur}(b)]$ vs. $\ln(b)$. Here, $\ln[I_r(b)]$ and $\ln[I_{ur}(b)]$ appear as straight lines with slopes of -4 and -3, respectively.

FIG. 2: Plot containing $I(b)$, $I_r(b)$, and $I_{ur}(b)$ vs. b.

FIG. 3. (a) Plot of the integrand in Eq. (3), scaled by b^4, vs. w, for a range of smaller values of b. Notice that the largest contribution for each curve falls in the region approximately given by $0 \leq w \leq 2b$. (b) Same as (a), but for larger values of b. The $b = 50, 100$ curves cannot be distinguished here. (c) Plot of $f \cdot b^4 = b^4/\left(w^2+b^2\right)^2$ vs. w for corresponding values of b as in (a). (d) Plot of $f \cdot b^4$ vs. w for corresponding values of b as in (b).

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Figure 1

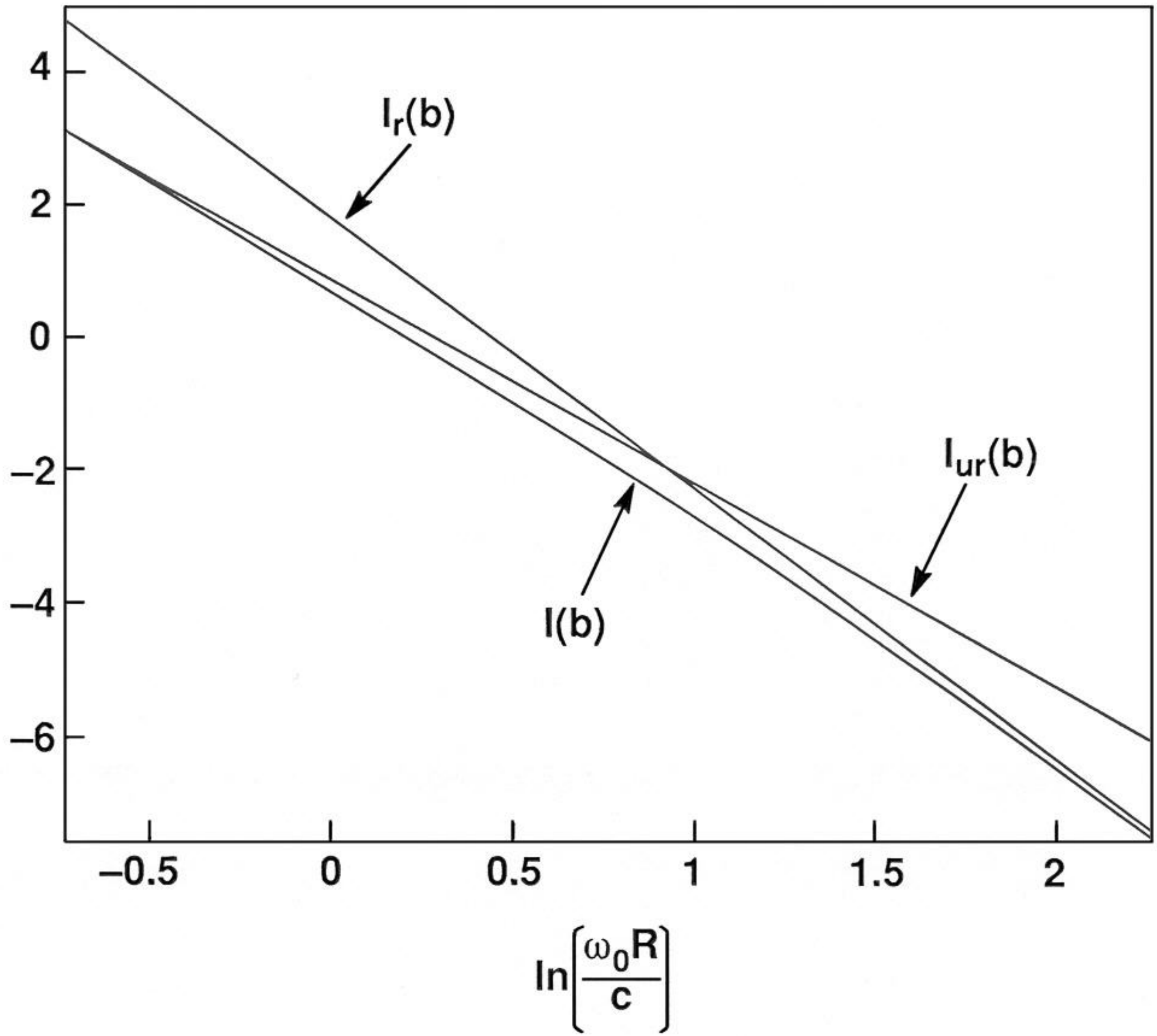
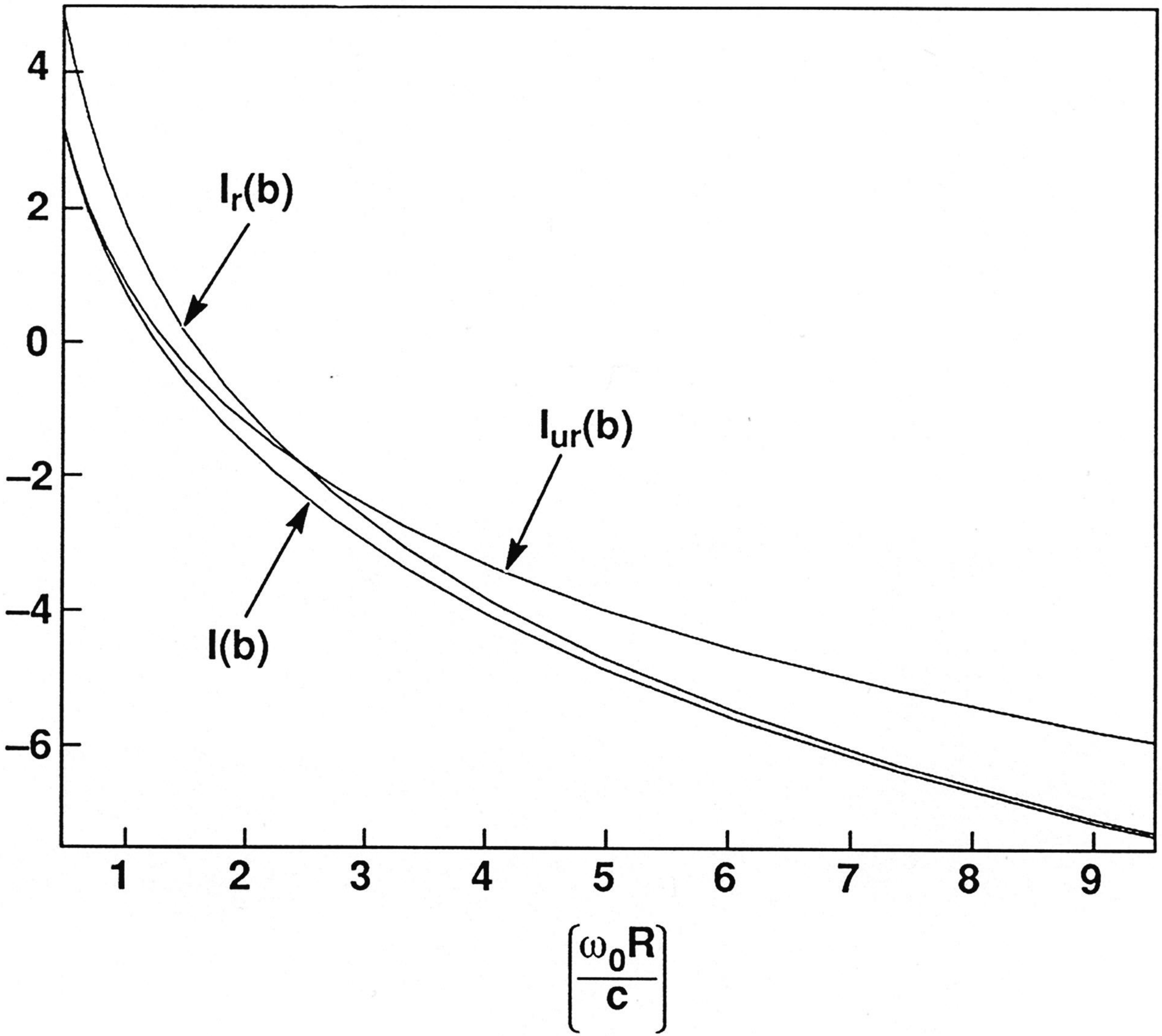


Figure 2



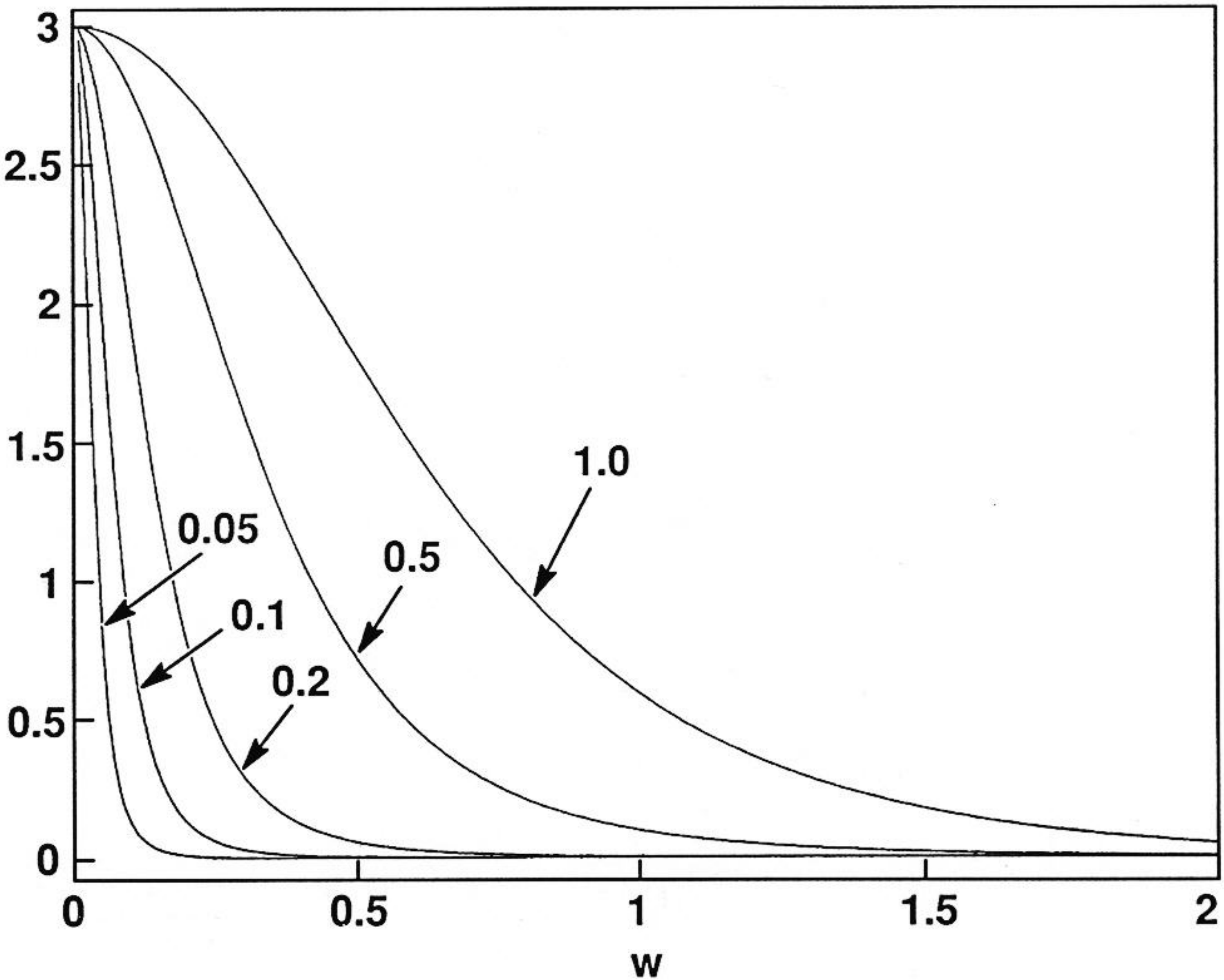


Figure 3a

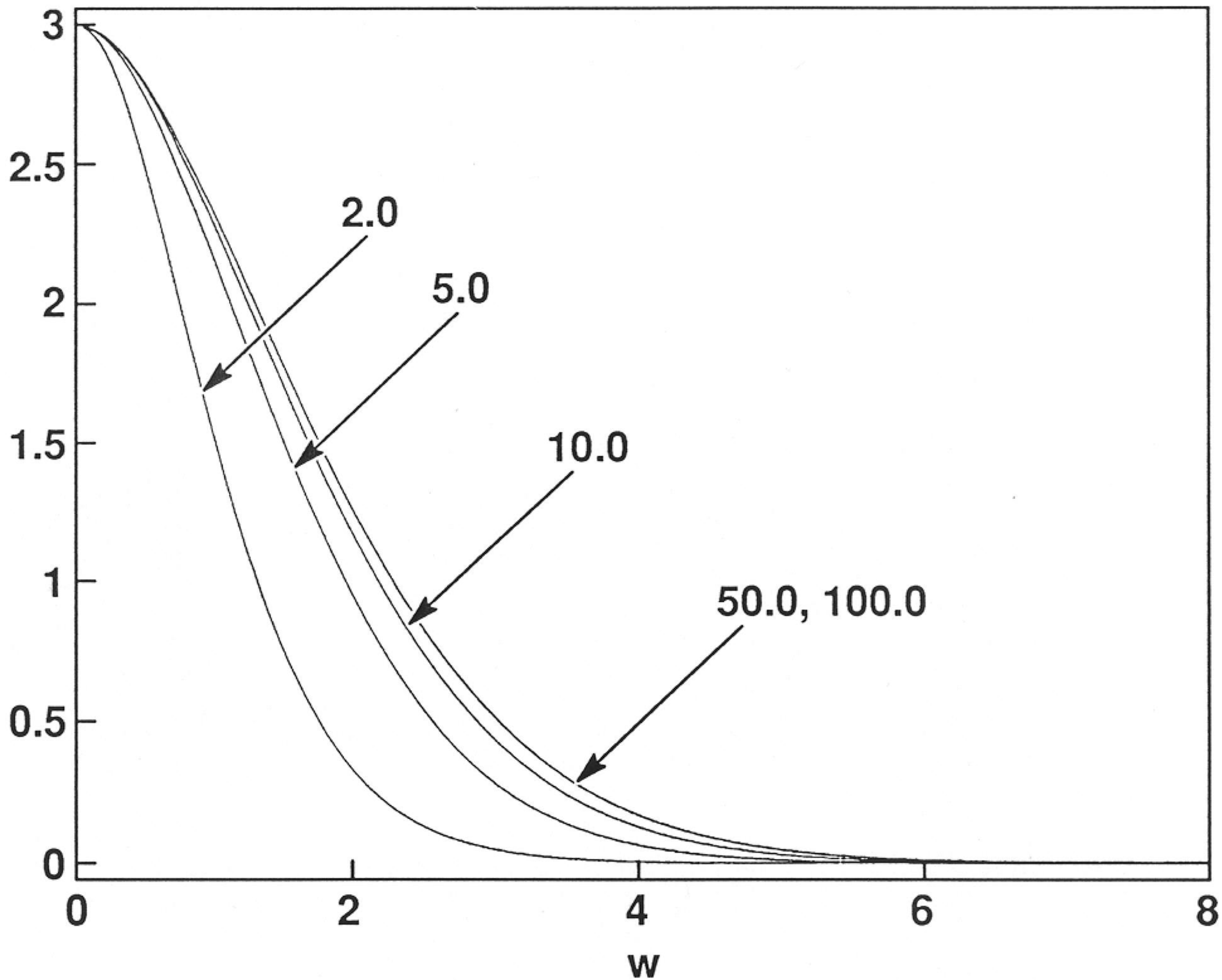


Figure 3b

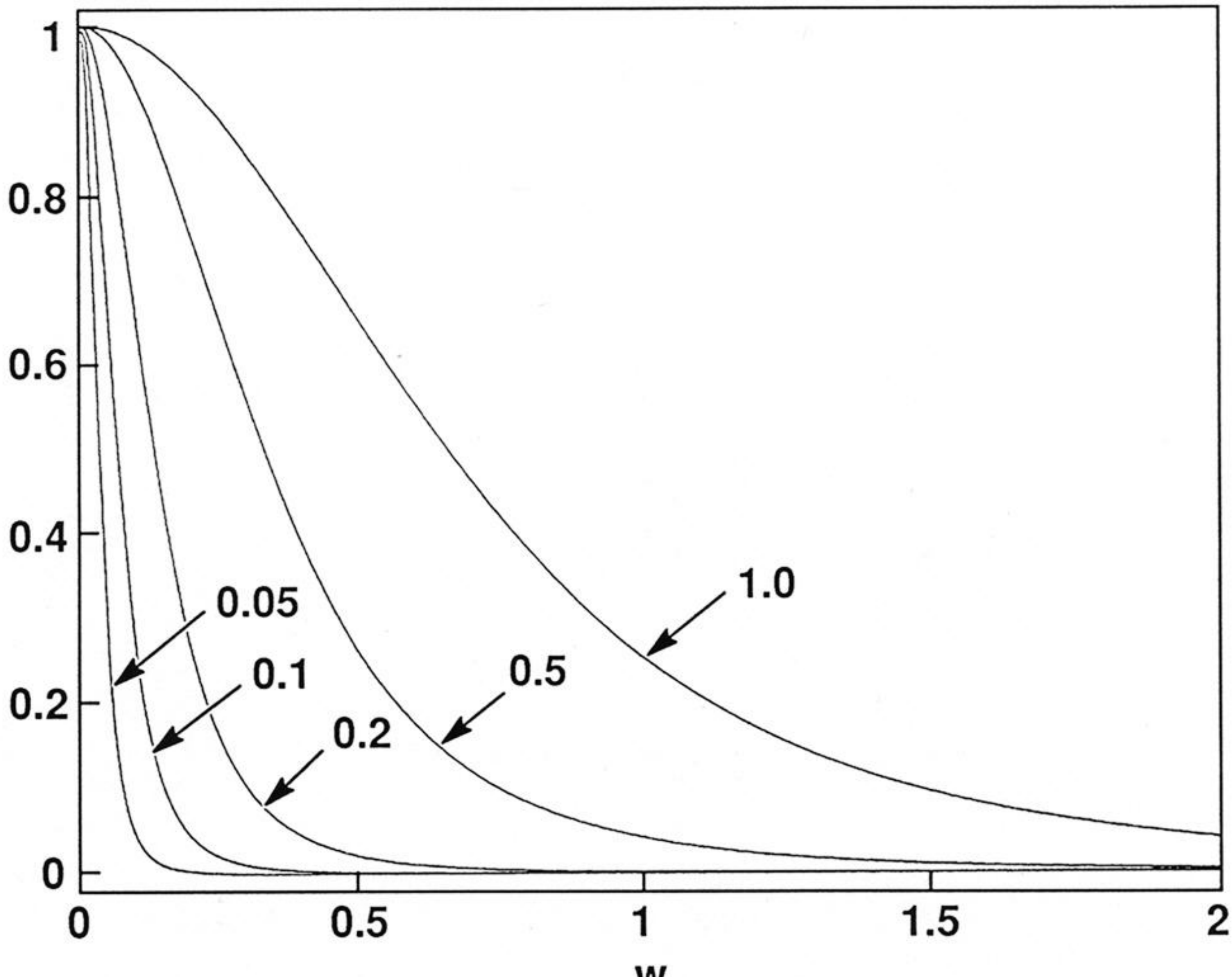


Figure 3c

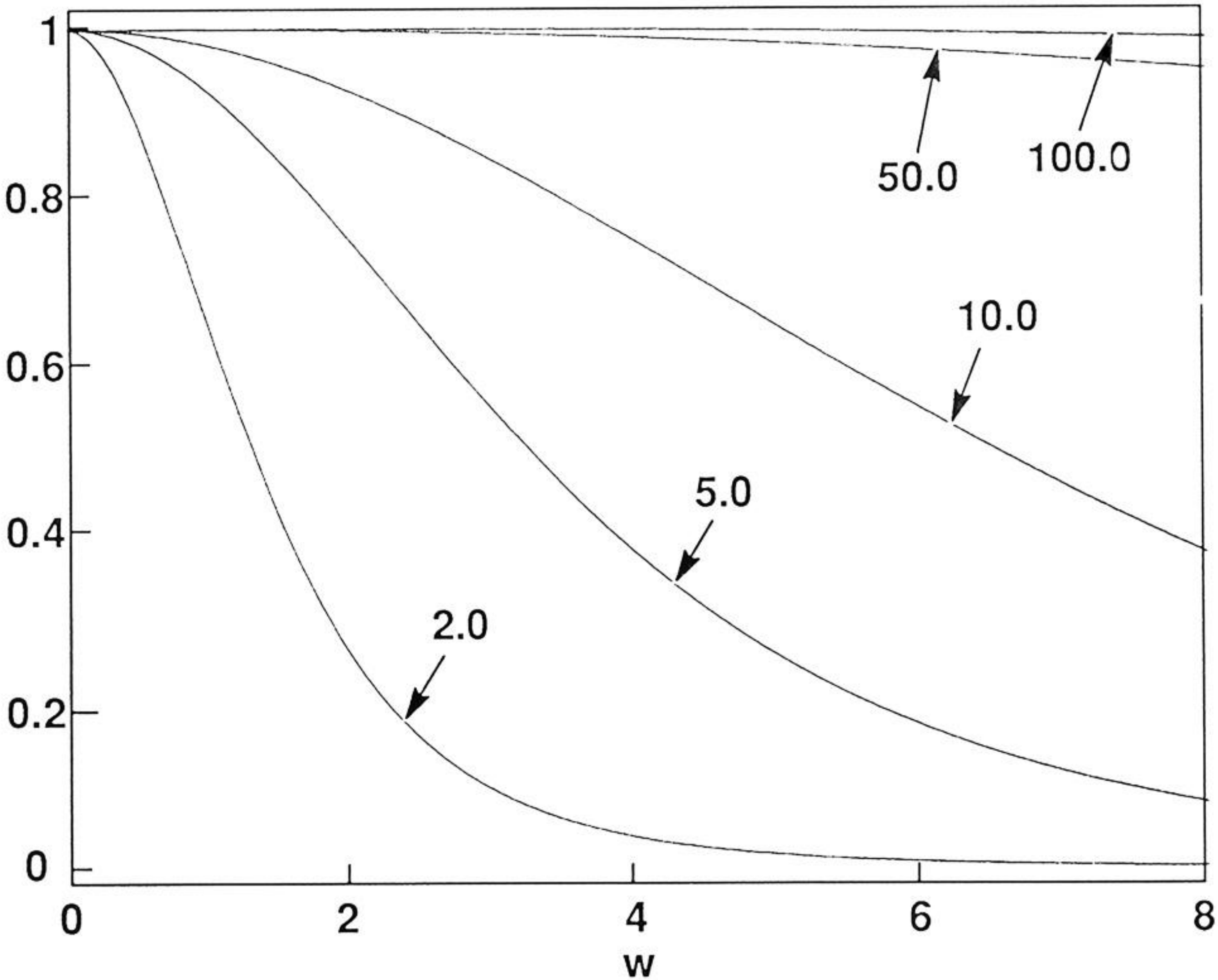


Figure 3d