Notices

OF THE AMERICAN MATHEMATICAL SOCIETY

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Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

Edited by John W. Green and Gordon L. Walker

CONTENTS

MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
624	June 19, 1965	Eugene, Oregon	May 5
625	August 30 - September 3, 1965 (70th Summer Meeting)	Ithaca, New York	July 9
626	October 30, 1965	Cambridge, Massachusetts	Sept. 13
627	November 12-13, 1965	Lexington, Kentucky	Sept. 28
628	November 26-27, 1965	Iowa City, Iowa	Sept. 28
629	December 29, 1965	Berkeley, California	Sept. 28
630	January 24-28, 1966 (72nd Annual Meeting)	Chicago, Illinois	
	August 29 - September 2, 1966 (71st Summer Meeting	New Brunswick, New Jersey	
	January 24-28, 1967 (73rd Annual Meeting)	Houston, Texas	
	August 28 - September 1, 1967 (72nd Summer Meeting)	Toronto, Ontario, Canada	
	August 26-30, 1968 (73rd Summer Meeting)	Madison, Wisconsin	

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for the by title abstracts are April 28, and July 2, 1965.

The *Notices* of the American Mathematical Society is published by the Society in January, February, April, June, August, October and November. Price per annual volume is $7.00. Price per copy $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, 190 Hope Street, Providence, Rhode Island 02906.

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Six Hundred Twenty-First Meeting
University of Chicago
Chicago, Illinois
April 9-10, 1965

PROGRAM

The six hundred twenty-first meeting of the American Mathematical Society will be held at the University of Chicago on April 9 and 10, 1965. Registration and all sessions will be held at the new University of Chicago Conference Center, officially entitled "The Center for Continuing Education" and located at 1307 East 60th Street about a half mile to the southeast of Eckhart Hall. No papers will be presented at Eckhart Hall.

Rooms will be available at the Center at the rate of $10 per single room and $7 per person in a twin-bedded double room. In the event of an overflow, the Center undertakes to place people at nearby hotels, and the hotel in question, will, in that case, confirm the reservation. All meals will be served at the Center and there is a bar which opens daily at 11:30 A.M. The official at the Center in charge of the meeting is Mr. B. Berlin, and inquiries pertaining to the Center may be directed to him.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor C. Ionescu Tulcea of the University of Illinois, Professor J. M. Kister of the University of Michigan, and Professor E. M. Stein of Princeton University will present hour addresses. Professor Ionescu Tulcea will speak on "The lifting property and disintegration of measures," Professor Kister will speak on "Euclidean bundles and topological manifolds," and Professor Stein will speak on "Boundary values of holomorphic and harmonic functions of several variables." Professor Ionescu Tulcea will speak at 2:00 P.M. on Friday, Professor Kister will speak at 11:00 A.M. on Saturday, and Professor Stein will speak at 2:00 P.M. on Saturday. All lectures will be in the Assembly of the Conference Center.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be a special session of 20-minute papers on "Harmonic analysis," organized and chaired by Professor R. P. Boas. The speakers will be Professors A. M. Garsia, Walter Rudin, V. L. Shapiro, M. C. Weiss, and Henry Helson. This session will be given on Saturday starting at 3:15 P.M. in the Assembly.

Sessions for the presentation of contributed papers will be held at 3:15 P.M. on Friday and both at 9:00 A.M. and 3:15 P.M. on Saturday.

There will be a registration fee of $1.50 (no charge for students).

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. To maintain the schedule, the time limit will be strictly enforced.

ALL SESSIONS WILL BE HELD AT "THE CENTER FOR CONTINUING EDUCATION."
1307 East 60th Street

FRIDAY, 2:00 P.M.

Invited address, The Assembly

The lifting property and disintegration of measures
Professor Ionescu Tulcea, University of Illinois

FRIDAY, 3:15 P.M.

Session on Analysis I, The Assembly

3:15 - 3:25
(1) A generalization of dynamical systems
Professor A. J. Heckenbach, Iowa State University (621-63)

3:30 - 3:40
(2) Asymptotic behavior of the solutions of an ordinary nonlinear differential equation
Mr. T. G. Hallam, University of Missouri (621-52)
(Introduced by Professor Seymour Sherman)

3:45 - 3:55
(3) A generalization of a theorem of Bôcher
Professor Morris Marden, The University of Wisconsin-Milwaukee (621-31)

4:00 - 4:10
(4) Cesàro summability for a class of generalized series
Professor P. O. Frederickson, Case Institute of Technology (621-64)

4:15 - 4:25
(5) Uniform convergence of Legendre series. Preliminary report
Professor R. A. Askey, University of Wisconsin (621-62)

4:30 - 4:40
(6) Fourier-Stieltjes-sine-series with finitely many distinct coefficients. Preliminary report
Professor G. W. Goes, Illinois Institute of Technology (621-57)

4:45 - 4:55
(7) Projections onto translation-invariant subspaces of $L^1(G)$
Professor Haskell Rosenthal, University of Minnesota (621-42)

5:00 - 5:10
(8) A characterization of harmonic functions
Professor K. O. Leland, Ohio State University (621-23)

FRIDAY, 3:15 P.M.

Session on Analysis II, Room 2BC

3:15 - 3:25
(9) Fixed points and multiplicative left invariant means. Preliminary report
Dr. Theodore Mitchell, SUNY at Buffalo (621-18)

3:30 - 3:40
(10) Interpolation of operational calculi. Preliminary report
Professor Bertram Walsh, University of California, Los Angeles (621-45)

3:45 - 3:55
(11) Dual generalized bases
Professor W. J. Davis, The Ohio State University (621-14)

4:00 - 4:10
(12) Uniform completeness of sets generated by a single function
Mr. S. W. Young, University of Texas (621-7)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

4:15 - 4:25

(13) On the characterization of compact Hausdorff X for which C(x) is algebraically closed. Preliminary report

Mr. R. S. Countryman, Jr., University of Minnesota (621-5)

4:30 - 4:40

(14) On a class of locally convex spaces

Professor J. E. Simpson, Marquette University (621-4)

4:45 - 4:55

(15) Integration on spaces of real-valued continuous functions whose domain is a compact subset of R_2. Preliminary report

Mr. J. D. Kuelbs, University of Minnesota (621-3)

5:00 - 5:10

(16) Everywhere defined linear transformations affiliated with rings of operators

Dr. E. L. Griffin, University of Pennsylvania (621-2)

FRIDAY, 3:15 P.M.

Session on Algebra and Theory of Numbers, Room 2EF

3:15 - 3:25

(17) Some quartic diophantine equations of genus 3

Professor L. J. Mordell, University of Illinois (621-17)

3:30 - 3:40

(18) A 1965 approach to a 1665 problem

Dr. Maurice Horowitz, The Magnavox Company, Fort Wayne, Indiana (621-67)

3:45 - 3:55

(19) A density inequality for the sum of two sets of lattice points

Mr. A. R. Freedman, Oregon State University (621-53)

4:00 - 4:10

(20) Higher derivations on π-adic fields. Preliminary report

Mr. E. F. Wishart, Florida State University (621-51)

4:15 - 4:25

(21) On the unique factorization problem for finite commutative semigroups. Preliminary report

Mr. Ralph McKenzie, University of Colorado (621-38)

4:30 - 4:40

(22) On d-groups of automorphisms and antiautomorphisms

Professor E. J. Taft, University of Chicago and Rutgers, The State University (621-32)

4:45 - 4:55

(23) Lower radical properties

Professor T. Anderson, University of British Columbia and Professor N. J. Divinsky*, University of Warsaw, Warsaw, Poland (621-28)

5:00 - 5:10

(24) On semi-primary ideals of distributively generated near-rings

Professor J. C. Beidleman, University of Kentucky (621-8)

5:15 - 5:25

(25) Some Wedderburn theorems. Preliminary report

Professor D. J. Rodabaugh, Vanderbilt University (621-1)

SATURDAY, 9:00 A.M.

Session on Analysis, The Assembly

9:00 - 9:10

(26) Estimates for the eigenvalues of a class of non self-adjoint operators

Mr. J. E. Osborn, University of Minnesota (621-12)

9:15 - 9:25

(27) Covering theorems for starlike and convex functions

Professor E. P. Merkes*, University of Cincinnati and Professor W. T. Scott, Arizona State University (621-44)

9:30 - 9:40

(28) A transformation theory for vector measure spaces

Dr. J. K. Brooks, The Ohio State University (621-21)

9:45 - 9:55

(29) An extension of a theorem of Sinkhorn

Professor P. J. Knopp* and Professor Richard Sinkhorn, University of Houston (621-19)

10:00 - 10:10

(30) Some nearness theorems in Banach spaces

Professor J. R. Retherford, Louisiana State University (621-16)

10:15 - 10:25

(31) Concerning local variations in the moment problem

Dr. Gordon Johnson, University of Georgia (621-13)

10:30 - 10:40

(32) Quantitative estimates for nonlinear differential equations by Liapunov functions

Professor J. E. Hall, University of Wisconsin (621-27)

10:45 - 10:55

(33) Singular perturbations on the infinite interval

Mr. F. C. Hoppensteadt, University of Wisconsin (621-24)

SATURDAY, 9:00 A.M.

Session on Geometry and Topology, Room 2BC

9:00 - 9:10

(34) On the non-existence of some (k,n)-arcs in certain projective planes

Professor Adriano Barlotti, University of North Carolina (621-55)
(Introduced by Professor R. C. Bose)

9:15 - 9:25

(35) "Parallel" transport in fibre spaces

Dr. J. D. Stasheff, The Institute for Advanced Study and Notre Dame University (621-37)

9:30 - 9:40

(36) The Haupvermutung and the polyhedral Schoenflies theorem

Professor P. M. Rice, University of Georgia (621-29)

9:45 - 9:55

(37) Čech cohomology of sheaves with values in a category

Professor J. W. Gray, University of Illinois (621-59)

10:00 - 10:10

(38) Transversely cellular mappings of combinatorial manifolds

Mr. M. M. Cohen, University of Michigan (621-34)

10:15 - 10:25

(39) Locally flat k-cells and spheres in E^n are stably flat if $k \leqq 2n/3 - 1$

Professor Prabir Roy, University of Wisconsin (621-66)

10:30 - 10:40

(40) A note on link groups

Professor C. B. Schaufele, Louisiana State University, Baton Rouge (621-20)

10:45 - 10:55

(41) Mappings of circle-like continua onto circle-like continua

Professor Howard Cook, The University of North Carolina (621-15)

SATURDAY, 9:00 A.M.

Session on Applied Mathematics and Probability, Room 2EF

9:00 - 9:10

(42) On Laplace transforms and mixed problems
Professor R. C. MacCamy, Carnegie Institute of Technology (621-35)

9:15 - 9:25

(43) Liapunov functions and global existence
Professor A. S. Strauss, University of Maryland (621-9)

9:30 - 9:40

(44) Boundary value problems for the time-independent Maxwell equations with variable coefficients
Dr. Peter Werner, Mathematics Research Center, The University of Wisconsin (621-54)

9:45 - 9:55

(45) Periodical and almost periodical steady states for physical systems
Dr. J. M. Skowronski* and Dr. Ruey-wen Liu, University of Notre Dame (621-43)

10:00 - 10:10

(46) Integral identities for modified Bessel functions
Professor Thomas Erber, Illinois Institute of Technology (621-39)

10:15 - 10:25

(47) On the Fourier series of a stationary stochastic process
Professor Tatsuo Kawata, Catholic University of America (621-49)

10:30 - 10:40

(48) On the law of the iterated logarithm
Mr. G. R. Andersen, Catholic University of America (621-50)
(Introduced by Professor Tatsuo Kawata)

SATURDAY, 11:00 A.M.

Invited Address, The Assembly

Euclidean bundles and topological manifolds
Professor J. M. Kister, University of Michigan

SATURDAY, 2:00 P.M.

Invited Address, The Assembly

Boundary values of holomorphic and harmonic functions of several variables
Professor E. M. Stein, Princeton University

SATURDAY, 3:15 P.M.

Special Session on Harmonic Analysis, The Assembly

3:15 - 3:35

(49) On the convergence problem for Fourier series
Professor Adriano Garsia, California Institute of Technology (621-48)

3:45 - 4:05

(50) Harmonic analysis on spheres
Professor Walter Rudin, University of Wisconsin (621-30)

4:15 - 4:35

(51) Trigonoetric series and the uniqueness of the heat equation
Professor V. L. Shapiro, University of California, Riverside (621-36)

4:45 - 5:05

(52) An example in the theory of singular integrals
Dr. Mary Weiss, DePaul University (621-68)

5:15 - 5:35

(53) Compact groups with ordered duals

Professor Henry Helson, University of California, Berkeley (621-69)

SATURDAY, 3:15 P.M.

Session on Analysis, Room 2BC

3:15 - 3:25

(54) The Young sigma integral and area

Professor F. M. Wright, Iowa State University (621-58)

3:30 - 3:40

(55) Algebraic periodic solutions of $\ddot{x} + f(x)\dot{x} + x = h(t)$; $h(t + t_0) = h(t)$.

Professor J. C. Wilson, Southern Illinois University (621-56)

3:45 - 3:55

(56) On systems of integrodifferential equations occuring in reactor dynamics

Professor T. A. Bronikowski, Marquette University (621-47)

(Introduced by Professor Seymour Sherman)

4:00 - 4:10

(57) Hölder type inequalities in cones

Professor G. R. Blakley, University of Illinois and Professor D. R. Dixon*, Dayton Campus, Ohio State University (621-46)

4:15 - 4:25

(58) On the size of the Riemann zeta-function at places symmetric with respect to the point 1/2

Professor R. D. Dixon, Dayton Campus, Ohio State University and Mr. Lowell Schoenfeld*, Pennsylvania State University and Mathematics Research Center, the University of Wisconsin (621-41)

4:30 - 4:40

(59) The Ahlfors-Shimizu characteristic function and the area on the Riemann sphere

Mr. Hari Shankar, Ohio University (621-40)

4:45 - 4:55

(60) Extremal problems for sums of powers of complex numbers

Professor J. D. Buckholtz, University of Kentucky (621-33)

5:00 - 5:10

(61) On the boundary behavior of Blaschke products in the unit circle. Preliminary report

Mr. Peter Colwell, University of Minnesota (621-26)

SATURDAY, 3:15 P.M.

Session on Topology, Room 2EF

3:15 - 3:25

(62) A note on countable-dimensional metric spaces

Professor Keio Nagami* and Professor A. H. Roberts, Duke University (621-65)

3:30 - 3:40

(63) On C-embedding

Professor S. G. Mrowka, The Pennsylvania State University (621-61)

3:45 - 3:55

(64) Extended topology: neighborhoods and convergents in isotonic spaces

Mr. G. C. Gastl and Professor P. C. Hammer*, University of Wisconsin (621-60)

4:00 - 4:10

(65) Essential fixed points of almost continuous functions

Professor S. A. Naimpally, Iowa State University (621-25)

4:15 - 4:25

(66) S-Lyapunov stability in dynamics
Dr. J. W. England, University of Virginia (621-22)

4:30 - 4:40

(67) A non-homogeneous inverse limit of homogeneous spaces with covering maps as bonding maps
Professor R. M. Schori, Louisiana State University, Baton Rouge (621-10)

4:45 - 4:55

(68) On open mappings and certain spaces satisfying the first countability axiom
Professor R. W. Heath, Arizona State University (621-6)

5:00 - 5:10

(69) Locally connected 2-cell and 2-sphere-like continua
Professor R. B. Bennett, Knox College (621-11)

Bloomington, Indiana

Seymour Sherman
Associate Secretary

NEWS ITEMS AND ANNOUNCEMENTS

COMPUTER SCIENCE DEPARTMENT
STANFORD UNIVERSITY

As of January 1965 Stanford University has a separate Computer Science Department within the School of Humanities and Sciences. There is a faculty of eight persons, including the following members of the Society: Professors G. E. Forsythe, J. G. Herriot, W. F. Miller, and John McCarthy, and Assistant Professor G. H. Golub. The fields now covered include numerical analysis, programming languages and systems, artificial intelligence, and computer control of external devices. The new department is now authorized to give the M. S. and Ph.D. degrees in Computer Science. Professor Forsythe is Executive Head.

The Computer Science Department and the Stanford Computation Center have both teaching and research assistantships, for which applications are invited. For information, write Executive Head, Computer Science Department, Stanford University, Stanford, California 94305.

PROGRAM IN APPLIED MATHEMATICS
UNIVERSITY OF PENNSYLVANIA

The University of Pennsylvania is now offering a program of study leading to the Ph.D. degree in applied mathematics. Students enrolled in the new program choose a field of application, which may be one of the conventional disciplines that make use of mathematics or an area of applied mathematics itself.

The program is directed by the newly established Graduate Group Committee in Applied Mathematics. The Committee is headed by Professor Herbert Wilf of the mathematics department and is composed of faculty members from the mathematics department and from other disciplines that rely extensively on mathematics, such as astronomy, biology, chemistry, economics, engineering, and physics.

Six Hundred Twenty-Second Meeting
Waldorf-Astoria Hotel
New York, New York
April 12-15, 1965

PROGRAM

The six hundred twenty-second meeting of the American Mathematical Society will be held at the Waldorf-Astoria in New York on April 12-15, 1965.

Contributed papers and invited addresses are scheduled on Monday April 12 and on the morning of Tuesday, April 13. There may be provision for late papers.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor James Eells, Jr. of Cornell University will speak on "A setting for global analysis," at 2:00 P.M. on Monday, April 12 in the Sert Room on the first floor.

By invitation of the same committee, Professor Robert D. M. Accola of Brown University will address the Society on "Some classical theorems on open Riemann surfaces," on Tuesday, April 13 at 11:00 A.M. in the Starlight Ballroom on the eighteenth floor.

SYMPOSIUM IN APPLIED MATHEMATICS

There will be a Symposium on Magneto-fluid and Plasma Dynamics on the afternoon of Tuesday, April 13 and on Wednesday and Thursday, April 14 and 15.

The subject was chosen by the Committee on Applied Mathematics, which consisted of V. Bargmann, G. E. Forsythe, P. R. Garabedian, C. C. Lin, Alfred Schild, Chairman, and David Young.

Financial support comes from the Air Force Office of Scientific Research and the U. S. Army Research Office (Durham).

The Invitations Committee, responsible for the planning of the program and the choice of speakers, consists of

Professor Harold Grad, Chairman, New York University
Dr. Andrew Lenard, Princeton Plasma Physics Laboratory
Professor Marshall N. Rosenbluth, University of California at La Jolla
Professor William R. Sears, Cornell University
Professor Harold Weitzner, New York University

REGISTRATION

On Monday, April 12 and on the morning of Tuesday, April 13 the registration desk will be located in the Silver Corridor on the third floor. Access is by way of the East Elevators. The meetings on Monday and the sessions for contributed papers on Tuesday morning will be readily accessible from the third floor.

On the afternoon of Tuesday, April 13 and on Wednesday, April 14 and Thursday, April 15 the registration desk will be in the foyer leading to the Starlight Ballroom on the eighteenth floor. This is the location of the invited address on Tuesday morning and of the entire symposium on Tuesday, Wednesday, and Thursday. The West Elevators serve this area.

COUNCIL MEETING

The Council will meet on Monday, April 12 at 10:00 A.M. in the Park Avenue Suite South on the fourth floor. It is planned that the meeting will adjourn in time for members to have lunch before the address at 2:00 P.M.

TRAVEL

The Waldorf-Astoria occupies an entire city block on the east side of New York City, from 49th to 50th Street and from Lexington to Park Avenues.

Those arriving by train at Pennsylvania Station may take the Independent Subway System (E or F cars) to the 53rd Street and Lexington Avenue stop, a short walk from the hotel.

From Grand Central Station one may take the I.R.T. Lexington Avenue local subway to the 51st Street stop.

Those arriving by bus may take the Independent Subway System (E or F cars) from the west side bus terminal. There is a shuttle bus service from LaGuardia and Kennedy Airports to the East Side Terminal with a transfer bus to Grand Central Station. (It is suggested that those arriving in a group of three or more may find it as economical to take a taxi directly to the hotel.)

Those arriving at Newark Airport can take a shuttle bus to the west side terminal and use the Independent Subway System (E or F cars) to the 53rd Street stop.

Those arriving by car will find many parking facilities in the neighborhood in addition to those at the hotel. Pickup and delivery service can be arranged through the hotel at a cost of $3.25 for a 24-hour period, plus $1.25 for each pickup-delivery.

RESERVATIONS

Persons intending to stay at the Waldorf-Astoria should make their own reservations with the hotel. A reservation blank and a listing of room rates were on page 259 (the inside back cover) of the February issue of these *Notices*. The deadline beyond which the hotel does not guarantee their reduced rates is March 29, as was noted above the reservation form. That date may precede the date of receipt of this program.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society. The Waldorf-Astoria, 301 Park Avenue, New York, New York 10022.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the ordinary sessions is ten minutes. The papers are scheduled at 15 minute intervals so that listeners can circulate between different sessions. To maintain the schedule, the time limit will be strictly enforced.

MONDAY, 10:00 A.M.

Session on Algebra, Jade Room, Third Floor

10:00 - 10:10

(1) Extensions of completely simple semigroups by completely 0-simple semigroups

Professor R. J. Warne, West Virginia University (622-7)

10:15 - 10:25

(2) Some results on algebraic structures and the digraph topology

Professor T. N. Bhargava and Mrs. S. E. Ohm*, Kent State University (622-54)

10:30 - 10:40

(3) Characterizing ordered groups in terms of real groups

Dr. A. T. Butson * and Dr. J. D. McKnight, University of Miami (622-47)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

10:45 - 10:55

(4) The Frattini subgroup of an E-group

Professor H. F. Bechtell, Bucknell University (622-8)

11:00 - 11:10

(5) Groups of order I. Part II

Professor E. S. Rapaport, Polytechnic Institute of Brooklyn (622-14)

11:15 - 11:25

(6) Distributivity and completeness in implication algebra

Mr. D. L. Pilling* and Professor J. C. Abbott, Unites States Naval Academy (622-36)

11:30 - 11:40

(7) Polynomial symbols and partial algebras

Professor G. A. Gratzer, The Pennsylvania State University (622-37)

11:45 - 11:55

(8) Related pairs of Hasse diagrams. II

Professor A. P. Hillman* and Professor D. G. Mead, University of Santa Clara (622-44)

MONDAY, 10:00 A.M.

Session on Logic and Foundations, Basilidon Room, Third Floor

10:00 - 10:10

(9) The generation of primes by a one-dimensional, real-time array of finite-state machines

Professor P. C. Fischer, Harvard University (622-1)

10:15 - 10:25

(10) Complexity classification of primitive recursive functions by their machine programs

Mr. D. M. Ritchie, Harvard University (622-59)

(Introduced by Professor P. C. Fischer)

10:30 - 10:40

(11) Higher-order indecomposable isols

Mr. A. B. Manaster, Cornell University (622-34)

10:45 - 10:55

(12) Depth of nesting and the Grzegorczyk hierarchy

Mr. A. R. Meyer, Harvard University (622-56)

(Introduced by Professor P. C. Fischer)

11:00 - 11:10

(13) Strong representability of partial functions in arithmetic. Preliminary report

Professor R. W. Ritchie*, University of Washington and Professor P. R. Young, Reed College (622-46)

11:15 - 11:25

(14) The undefinability of the definable

Professor J. W. Addison, University of California, Berkeley (622-71)

11:30 - 11:40

(15) On universal equivalence for ordered groups

Mrs. D. B. Martin and Professor H. B. Ribeiro*, The Pennsylvania State University (622-23)

11:45 - 11:55

(16) Bases in vector spaces and the axiom of choice. Preliminary report

Dr. J. D. Halpern, California Institute of Technology (622-72)

MONDAY, 10:00 A.M.

General Session, Astor Gallery, Third Floor

10:00 - 10:10

(17) A problem related to the approximation of algebraic numbers by rationals
Professor R. T. Bumby, Rutgers, The State University and The University of Michigan (622-39)

10:15 - 10:25

(18) On $y^2 = x^3 + k$
Professor Sarvadaman Chowla, The Pennsylvania State University (622-64)

10:30 - 10:40

(19) On convergence of integrals involving Brownian motion
Dr. Moshe Zakai, Sylvania Applied Research Laboratory, Waltham, Massachusetts, and Professor Eugene Wong*, University of California, Berkeley (622-6)

10:45 - 10:55

(20) A uniformly most powerful test using quantiles
Mr. Isidore Eisenberger, California Institute of Technology (622-45)

11:00 - 11:10

(21) A uniqueness property for bounded observables
Professor S. P. Gudder, Mathematics Research Center, University of Wisconsin (622-3)

11:15 - 11:25

(22) On a general method of constructing triple-systems of all possible orders
Dr. V. Bohun-Chudyniv, Morgan State College (622-61)

11:30 - 11:40

(23) Solution for certain cases of the Cauchy problem for the nonhomogeneous wave equation
Professor E. P. Miles, Jr., Florida State University (622-30)

11:45 - 11:55

(24) The classification of closed-open sets of the Baire space
Mr. R. F. Barnes, Jr., University of California, Berkeley (622-67)

MONDAY, 2:00 P.M.

Invited Address, Sert Room, Lobby Floor

A setting for global analysis
Professor James Eells, Jr., Cornell University

MONDAY, 3:15 P.M.

Session on Analysis, Jade Room, Third Floor

3:15 - 3:25

(25) The abstract time-dependent Cauchy problem for measurable operators affiliated with a discrete ring of operators in a separable Hilbert space
Professor M. M. Hackman, University of Washington (622-65)

3:30 - 3:40

(26) Convexity properties for weak solutions of differential equations in Hilbert spaces
Professor Samuel Zaidman, Unitersité de Montréal (622-66)

3:45 - 3:55

(27) On generalized dynamical systems
Dr. Peter Seibert, Brown University (622-73)

4:00 - 4:10

(28) On the existence of solutions of differential equations in Banach spaces. Preliminary report

Professor M. A. Malik, Université de Montréal (622-62)

(Introduced by Professor Everett Pitcher)

4:15 - 4:25

(29) On the classification of measurable flows and metric automorphisms. Preliminary report

Professor N. F. G. Martin, University of Virginia (622-40)

4:30 - 4:40

(30) A representation theorem for archemedian linear lattices

Professor Leon Brown* and Professor Hidegoro Nakano, Wayne State University (622-24)

4:45 - 4:55

(31) On duality for locally compact groups. Preliminary report

Professor L. T. Gardner, Queens College (622-18)

MONDAY, 3:15 P.M.

Session on Analysis, Astor Gallery, Third Floor

3:15 - 3:25

(32) On weak and strong solutions

Professor Gideon Peyser, Pratt Institute (622-33)

3:30 - 3:40

(33) An ordinate uniqueness theorem for a mildly nonlinear elliptic system

Mr. G. T. McAllister, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland (622-50)

3:45 - 3:55

(34) P and D in $P^{-1}XP = dg(\lambda, \ldots, \lambda) = D$ as matrix functions of X

Professor R. F. Rinehart, Case Institute of Technology (622-26)

4:00 - 4:10

(35) A property of the real nonregular functions C^{∞}

Dr. Hans Hornich, The Catholic University of America (622-38)

4:15 - 4:25

(36) Expansions for Bessel difference systems of zero order

Dr. J. J. Gergen and Dr. F. G. Dressel, Duke University and Dr. G. B. Parrish*, United States Army Research Office, Durham, North Carolina (622-70)

4:30 - 4:40

(37) A note on the Closing Lemma

Professor M. M. Peixoto, Brown University (622-75)

4:45 - 4:55

(38) The Closing Lemma

Mr. C. C. Pugh, University of California, Berkeley (622-19)

MONDAY, 3:15 P.M.

Session on Topology, Basilidon Room, Third Floor

3:15 - 3:25

(39) On the interchangeability of 2-links. Preliminary report

Professor W. C. Whitten, Jr., Drexel Institute of Technology (622-27)

3:30 - 3:40

(40) Reidemeister torsion of 2-spheres in the 4-sphere

Dr. C. H. Giffen, The Institute for Advanced Study (622-49)

3:45 - 3:55

(41) Finding a boundary for an open manifold

Mr. L. C. Siebenmann, Princeton University (622-41)

4:00 - 4:10

(42) Each non-zero-dimensional compactum has a connected one-dimensional subset
Mr. D. W. Henderson, The Institute for Advanced Study (622-69)

4:15 - 4:25

(43) On nearly Lindelöf spaces
Professor S. G. Mrowka, The Pennsylvania State University (622-63)

4:30 - 4:40

(44) Concerning spaces having bases of countable order
Dr. J. M. Worrell, Jr. and Dr. H. H. Wicke*, Sandia Corporation Albuquerque, New Mexico (622-60)

4:45 - 4:55

(45) A theorem on 1-1 mappings
Professor Edwin Duda, University of Miami (622-42)

MONDAY, 3:15 P.M.

Session on Applied Mathematics, Park Avenue Suite, North and Center, Fourth Floor

3:15 - 3:25

(46) Dual formulation for the eigenfunctions corresponding to the bandpass kernel, in the case of degeneracy
Dr. J. A. Morrison, Bell Telephone Laboratories, Murray Hill, New Jersey (622-11)

3:30 - 3:40

(47) Accuracy and dissipation in difference schemes
Professor B. N. Parlett, Stevens Institute of Technology (622-28)

3:45 - 3:55

(48) Dynamic programming existence and uniqueness theorems
Professor E. S. Boylan, Rutgers, The State University (622-25)

4:00 - 4:10

(49) Improvements in the derivation of Runge-Kutta type formulas and computer implementation
Professor D. Sarafyan, Louisiana State University, New Orleans (622-51)

4:15 - 4:25

(50) Simple waves and symmetric hyperbolic systems
Professor R. M. Gundersen, University of Wisconsin-Milwaukee (622-53)

TUESDAY, 9:00 A.M.

Session on Analysis, Jade Room, Third Floor

9:00 - 9:10

(51) An extension to the theory of the F-equation. Preliminary report
Dr. G. O. Peters, General Electric Company, Philadelphia, Pennsylvania (622-74)

9:15 - 9:25

(52) Hermite-Fejér polynomials for functions of several variables
Dr. Oved Shisha* and Dr. Bertram Mond, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio (622-2)

9:30 - 9:40

(53) Proof of global convergence of a class of methods for the solution of polynomial equations
Dr. J. F. Traub, Bell Telephone Laboratories, Murray Hill, New Jersey (622-9)

9:45 - 9:55

(54) On the function theory of occupation number space
Dr. Willard Miller, Jr., New York University (622-20)

10:00 - 10:10

(55) A matrix generalization of Kantorovich's inequality

Dr. Bertram Mond, Wright-Patterson Air Force Base, Ohio (622-4)

10:15 - 10:25

(56) A formal solution of certain dual integral equations

Professor Charles Fox, McGill University (622-13)

10:30 - 10:40

(57) Convergence of an iterated exponential

Professor A. J. Macintyre, University of Cincinnati (622-76)

TUESDAY, 9:00 A.M.

Session on Analysis, Astor Gallery, Third Floor

9:00 - 9:10

(58) Smoothness of Orlicz spaces

Professor M. M. Rao, Carnegie Institute of Technology (622-68)

9:15 - 9:25

(59) Sard's lemma for certain differentiable maps on Hilbert space

Professor Gilbert Stengle, Lehigh University (622-58)

9:30 - 9:40

(60) Some inequalities for the heat operator. Preliminary report

Dr. E. J. Sherry, Sandia Corporation, Albuquerque, New Mexico (622-55)

9:45 - 9:55

(61) On the subspace of L^p invariant under multiplication of transforms by bounded continuous functions

Professor Alessandro Figà-Talamanca, Massachusetts Institute of Technology (622-52)

10:00 - 10:10

(62) A convergent gradient procedure in prehilbert spaces

Professor E. K. Blum, Wesleyan University and United Aircraft Research (622-31)

10:15 - 10:25

(63) Intersections of invariant subspaces

Professor M. J. Sherman, University of California, Los Angeles, (622-15)

10:30 - 10:40

(64) Lower bounds for solutions of differential inequalities in Hilbert space

Professor Hajimu Ogawa, University of California, Riverside (622-5)

TUESDAY, 9:00 A.M.

Session on Algebra, Basilidon Room, Third Floor

9:00 - 9:10

(65) Semi-modular Lie algebras

Professor Bernard Kolman, Drexel Institute of Technology and the University of Pennsylvania (622-16)

9:15 - 9:25

(66) Jordan derivations of symmetric elements. Preliminary report

Professor W. S. Martindale III, University of Massachusetts (622-35)

9:30 - 9:40

(67) The spin model of space-time

Professor W. F. Eberlein, University of Rochester (622-32)

9:45 - 9:55

(68) The differential ideal [uv]

Mrs. K. B. O'Keefe, University of Washington and Dr. E. S. O'Keefe*, The Boeing Company, Seattle, Washington (622-48)

10:00 - 10:10

(69) Matroids and ports

Dr. Alfred Lehman, Walter Reed Army Institute of Research, Washington, D. C. (622-57)

10:15 - 10:25

(70) Applications of M-matrices to non-negative matrices

Professor D. E. Crabtree, University of Massachusetts (622-17)

10:30 - 10:40

(71) Decomposition spectra of rings of continuous functions. Preliminary report

Professor C. W. Kohls, Syracuse University (622-43)

TUESDAY, 9:00 A.M.

Session on Geometry, Park Avenue Suite, North and Center, Fourth Floor

9:00 - 9:10

(72) On the differential geometry of frame bundles

Dr. Tanjiro Okubo, Montana State College (622-21)

9:15 - 9:25

(73) Enumeration of certain types of polyhedra

Professor Hans Rademacher, The Rockefeller Institute, New York, New York (622-22)

9:30 - 9:40

(74) On the curvatures of Riemannian manifolds

Dr. J. A. Thorpe, Massachusetts Institute of Technology (622-29)

9:45 - 9:55

(75) Group theoretical interpretation of incidence theorems

Professor Hans Schwerdtfeger, McGill University (622-10)

10:00 - 10:10

(76) Topology without the union axiom

Mr. Lamar Bentley* and Professor Paul Slepian, Rensselaer Polytechnic Institute (622-12)

TUESDAY, 11:00 A.M.

Invited Address, Starlight Ballroom, Eighteenth Floor

Some classical theorems on open Riemann surfaces

Professor Robert D. M. Accola, Brown University

SYMPOSIUM ON MAGNETO-FLUID AND PLASMA DYNAMICS

TUESDAY, 2:00 P.M.

Session I, Starlight Ballroom, Eighteenth Floor

Chairman: Professor W. R. Sears, Cornell University

Stability of a slightly resistive plasma

Dr. J. M. Greene, Princeton Plasma Physics Laboratory

The aligned magnetic field problem

Professor G. S. S. Ludford, Cornell University

Hydromagnetic instability of the sub-alfven equations

Professor W. V. R. Malkus, University of California at Los Angeles

WEDNESDAY, 9:00 A.M.

Session II, Starlight Ballroom, Eighteenth Floor

Chairman: Professor P. R. Garabedian, Courant Institute of Mathematical Sciences

Singular eigenfunctions and plasma problems
Professor K. M. Case, University of Michigan
A mathematical problem in the quasi-linear theory of plasma waves
Dr. M. Trocheris, Association Euratom, Paris
Some existence theorems and stability properties of the Vlasov equation
Professor H. Weitzner, Courant Institute of Mathematical Sciences

WEDNESDAY, 2:00 P.M.

Session III, Starlight Ballroom, Eighteenth Floor

Chairman: Professor C. S. Morawetz, Courant Institute of Mathematical Sciences

The guiding center plasma
Professor H. Grad, Courant Institute of Mathematical Sciences
Dynamics of a gyroviscous plasma
Dr. W. A. Newcomb, Lawrence Radiation Laboratory, Livermore
Problems in plasma microinstability
Professor M. N. Rosenbluth, University of California at La Jolla

THURSDAY, 9:00 A.M.

Session IV, Starlight Ballroom, Eighteenth Floor

Chairman: Professor J. B. Keller, Courant Institute of Mathematical Sciences

Some remarks on the stability of hydromagnetic shocks
Professor C. K. Chu, Columbia University
Structure of shock waves for magnetodynamics of two fluids
Professor P. Germain, Office Nationale d'Etudes et de Recherche Aéronautique, Paris

THURSDAY, 2:00 P.M.

Session V, Starlight Ballroom, Eighteenth Floor

Chairman: Dr. A. Lenard, Princeton Plasma Physics Laboratory

The present situation in plasma kinetic theory
Professor E. A. Frieman, Princeton Plasma Physics Laboratory
Quasi-particles in a plasma
Professor N. Rostoker, University of California at La Jolla
On Ohm's law by instability
Professor R. Z. Sagdeev, University of Novosibirsk, USSR

Everett Pitcher
Associate Secretary

Bethlehem, Pennsylvania

Six Hundred Twenty-Third Meeting
Stanford, University
Stanford, California
April 24, 1965

PROGRAM

The six hundred twenty-third meeting of the American Mathematical Society will be held on Saturday, April 24, 1965 at Stanford University, Stanford, California.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be addresses by Professor E. G. Straus of the University of California, Los Angeles, and by Professor Harvey Cohn of the University of Arizona. Professor Straus will speak at 11:00 A. M. in Room 100 of the Physics Lecture Hall on the "Arithmetic of analytic functions." The title of Professor Cohn's talk is "Some elementary aspects of modular functions in several variables". This address will be given at 2:00 P.M. in Room 100, Physics Lecture Hall.

There will be sessions for contributed papers at 9:15 A.M. and at 3:30 P.M. in the Alfred P. Sloan Mathematics Center. Abstracts of the papers to be presented at these sessions appear on pages 350-364 of these *Notices*. There are cross references to the abstracts in the program.

Registration for the meeting will take place in the lobby of the Sloan Mathematics Center, beginning at 9:00 A.M.

A tea for persons attending the meeting will be given in the Mathematics Library on the fourth floor of the Sloan Mathematics Center beginning at 4:30 P.M. Luncheon will be available at noon in the Old Union.

Stanford University is about thirty miles south of San Francisco, adjacent to the town of Palo Alto. The Southern Pacific Railroad stops at Palo Alto. Limousine service from the San Francisco International Airport to the Hotel President in Palo Alto is available. It is recommended that taxis be used to get from Palo Alto to the Stanford campus. Persons who drive to the meeting will find ample parking available on the campus.

There are two hotels in Palo Alto and numerous motels on the El Camino Real near the Stanford campus. A complete list of hotels and motels within five miles of the University, giving locations and rates, can be obtained from the Palo Alto Convention and Visitors Bureau, P.O. Box 1321, Palo Alto, California.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals. To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 9:15 A.M.

General Session, Room 380C, Sloan Mathematics Center

9:15 - 9:25

(1) An historical profile of determinants
Professor K. O. May, Carleton College and University of California, Berkeley (623-37)

9:30 - 9:40

(2) Groups graphs and Fermat's last theorem

Professor S. J. Bryant, San Diego State College (623-16)

9:45 - 9:55

(3) On the roots of Euler and Bernoulli polynomials

Mr. John Brillhart, University of San Francisco (623-2)

10:00 - 10:10

(4) New results in lattice integration theory

Mr. S. R. Neal, U. S. Naval Ordnance Test Station, China Lake, California (623-20)

SATURDAY, 9:15 A.M.

Session on Algebra, Room 380F, Sloan Mathematics Center

9:15 - 9:25

(5) Congruence relations on finitely generated free commutative semigroups

Professor T. Tamura and Mr. J. C. Higgins*, University of California, Davis (623-26)

9:30 - 9:40

(6) Attainability and extendability of system of identities on semigroups

Professor T. Tamura, University of California, Davis (623-31)

9:45 - 9:55

(7) On regular semigroups satisfying permutation identities

Professor Miyuki Yamada, Sacramento State College (623-21)

10:00 - 10:10

(8) Direct sums of countable Abelian groups

Professor J. M. Irwin* and Professor Fred Richman, New Mexico State University (623-1)

10:15 - 10:25

(9) Countable direct sums of torsion complete groups

Professor J. M. Irwin, Professor Fred Richman*, and Professor E. A. Walker, New Mexico State University (623-7)

SATURDAY, 9:15 A.M.

Session on Applied Mathematics, Statistics and Probability, Room 380W, Sloan Mathematics Center.

9:15 - 9:25

(10) Mixing as a generalization of independence

Mr. R. M. Fischler, University of Oregon (623-8)

9:30 - 9:40

(11) The time-dependent Poisson queue and moving boundary problems for the heat and wave-equation

Mr. S. F. Neustadter, Sylvania Electronic Systems, Waltham, Massachusetts (623-14)

9:45 - 9:55

(12) On an extension of the Hotelling-Wasow method

Professor W. M. Stone, Oregon State University (621-39)

10:00 - 10:10

(13) Asymptotic nature of the zeros of cross-product Bessel functions

Dr. J. A. Cochran, Bell Telephone Laboratories, Whippany, New Jersey (623-4)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

10:15 - 10:25

(14) Round-off procedures in the numerical treatment of differential equations
Mr. R. A. Hansen, University of Utah (623-32)
(Introduced by Dr. R. E. Barnhill)

SATURDAY, 9:15 A.M.

Session on Analysis, Room 380X, Sloan Mathematics Center

9:15 - 9:25

(15) Hellinger integrals and linear functionals
Professor J. S. MacNerney, University of North Carolina (623-25)

9:30 - 9:40

(16) A Hardy-Bohr theorem
Mr. R. L. Irwin, University of Utah (623-6)
(Introduced by Professor Alexander Peyerimhoff)

9:45 - 9:55

(17) A theorem of the Hardy-Bohr type. Preliminary report
Mr. G. E. Peterson, University of Utah (623-45)
(Introduced by Professor W. J. Coles)

10:00 - 10:10

(18) Criteria for Cesàro convergence
Professor Z. Z. Yeh, University of Hawaii (623-3)

SATURDAY, 11:00 A.M.

Invited Address, Room 100, Physics Lecture Hall

Arithmetic of analytic functions
Professor E. G. Straus, University of California, Los Angeles

SATURDAY, 2:00 P.M.

Invited Address, Room 100, Physics Hall

Some elementary aspects of modular functions in several variables
Professor Harvey Cohn, University of Arizona

SATURDAY, 3:30 P.M.

General Session, Room 380C, Sloan Mathematics Center

3:30 - 3:40

(19) Iterated bounds for error-correcting codes
Dr. R. C. Burton, Brigham Young University (623-44)
(Introduced by Professor R. S. Pierce)

3:45 - 3:55

(20) Multirestricted and rowed partitions
Mr. C. T. Haskell, University of Arizona and California State Polytechnic College (623-12)
(Introduced by Professor M. S. Cheema)

4:00 - 4:10

(21) On the characteristic roots of the product of certain rational integral matrices of order two
Professor L. L. Foster, San Fernando Valley State College (623-17)

4:15 - 4:25

(22) Approximations to quadratic irrationalities
Professor Wolfgang Schmidt, University of Colorado (623-35)

4:30 - 4:40

(23) A characterization of a differential ring of a class of integer-valued entire functions

Professor A. H. Cayford, University of British Columbia (623-43)

4:45 - 4:55

(24) A generalized interpolation by analytic functions

Professor Daihachiro Sato*, University of Saskatchewan, and Professor E. G. Straus, University of California, Los Angeles (623-19)

5:00 - 5:10

(25) The derivatives of some entire functions at algebraic points

Dr. C. F. Osgood, University of Illinois (623-11)

SATURDAY, 3:30 P.M.

Session on Algebra, Room 380F, Sloan Mathematics Center

3:30 - 3:40

(26) On a characterization of semi-simple linear transformations. Preliminary report

Professor D. W. Robinson, Brigham Young University (623-29)

3:45 - 3:55

(27) Primary components in Noetherian rings

Professor W. E. Barnes* and Professor W. M. Cunnea, Washington State University (623-23)

4:00 - 4:10

(28) Separable polynomials over commutative rings. Preliminary report

Mr. G. J. Janusz, University of Oregon (623-28)

4:15 - 4:25

(29) Galois theory in algebras

Mr. F. R. DeMeyer, University of Oregon (623-34)

4:30 - 4:40

(30) A characterization of QF-3 rings

Professor J. P. Jans, University of Washington, Dr. H. Y. Mochizuki, University of California, Berkeley, and Professor L.E.T. Wu*, Western Washington State College (623-36)

4:45 - 4:55

(31) Polynomial rings with a pivotal monomial

Mr. S. K. Jain, University of California, Riverside (623-13)
(Introduced by Professor M. F. Smiley)

5:00 - 5:10

(32) A classification of subdirectly irreducible rings

Professor M. P. Drazin, Purdue University (623-22)

SATURDAY, 3:30 P.M.

Session on Geometry and Topology, Room 380W, Sloan Mathematics Center

3:30 - 3:40

(33) Another property that distinguishes Bing's dogbone space from E^3

Professor L. O. Cannon, University of Utah (623-42)

3:45 - 3:55

(34) Tame subsets of spheres in E

Mr. L. D. Loveland, University of Utah (623-33)

4:00 - 4:10

(35) A product theorem concerning some generalized compactness properties

Mr. L. H. Martin, Harvey Mudd College (623-10)
(Introduced by Professor John Greever)

4:15 - 4:25

(36) Spherical submanifolds of manifolds with positive curvature

Mr. F. J. Flaherty, San Francisco State College (623-9)

4:30 - 4:40

(37) Continuous selections with nonmetrizable range

Professor H. H. Corson and Professor Joram Lindenstrauss*, University of Washington (623-27)

4:45 - 4:55

(38) Minimal subspaces generated by positive bases

Professor J. R. Reay, Western Washington State College (623-40)

5:00 - 5:10

(39) Blaschke sums of bodies of revolution. Preliminary report

Professor W. J. Firey, Oregon State University (623-38)

SATURDAY, 3:30 P.M.

Session on Analysis, Room 380X, Sloan Mathematics Center

3:30 - 3:40

(40) Extensions of the convexity theorem of Study

Professor E. F. Beckenbach and Mr. T. A. Cootz*, University of California, Los Angeles (623-5)

3:45 - 3:55

(41) Images of measurable sets

Professor D. W. Bressler*, University of British Columbia, and Professor A. P. Morse, University of California, Berkeley (623-15)

4:00 - 4:10

(42) Harmonic product and harmonic boundary for bounded complex-valued harmonic functions. Preliminary report

Dr. Linda Lumer-Naim, University of Washington (623-18)

(Introduced by Professor R. S. Pierce)

4:15 - 4:25

(43) An axiomatic treatment of pairs of elliptic differential equations

Dr. P. A. Loeb, University of California, Los Angeles (623-30)

4:30 - 4:40

(44) A linear differential system with general linear boundary conditions. Preliminary report

Mr. R. N. Bryan, University of Utah (623-41)

4:45 - 4:55

(45) An Nth order boundary value problem

Professor W. J. Coles, University of Utah, and Professor T. L. Sherman*, Arizona State University (623-24)

R. S. Pierce
Associate Secretary

Seattle, Washington

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Six Hundred Twenty-Fourth Meeting
University of Oregon
Eugene, Oregon
June 19, 1965

The six hundred twenty-fourth meeting of the American Mathematical Society will be held at the University of Oregon in Eugene, Oregon, in conjunction with a meeting of the Pacific Northwest Section of the Mathematical Association of America. The Society will meet on Saturday, June 19, 1965, and the Association will hold its sessions on Friday, June 18.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, the Society will be addressed at 11:00 A.M. on Saturday by Professor Murray Protter of the University of California at Berkeley. The title of Professor Protter's talk is "The maximum principle". Sessions for contributed papers will be held on Saturday morning and afternoon. All sessions of the meeting will be held in the Science building. The Registration Desk, located in the lobby of the Science building, will be open from 9:00 A.M. to 5:00 P.M. on Friday and Saturday.

On Friday night, June 17, there will be a no-host banquet. Persons who plan to attend are asked to make reservations in advance. The desired number of tickets should be requested from Glenn T. Beelman, Department of Mathematics, University of Oregon, Eugene, Oregon. On Saturday afternoon, a tea will be held.

Dormitory space will be available on campus for the nights of June 17, 18, and 19. The rates are $3.50 per person for adults and $1.00 for each child under 10. Reservations for dormitory accommodations also should be sent to Professor Beelman at the address above. Requests should include the number and names of the adults and children, and the dates and times of arrival and departure.

The following is a list of Eugene motels which are within easy walking distance of the meetings.

	Single	Double	Twin
Flagstone Motel, 1601 Franklin Boulevard	$7.50	$9.00	$10.00
Hyatt Chalet Motel, 1857 Franklin Boulevard	$7.00	$10.00	$12.00
New Oregon Motel, 1655 Franklin Boulevard	$8.00	$10.00	$13.00
Travel-Inn Motel, 2121 Franklin Boulevard	$8.00	$10.00	$12.00

Anyone who wishes to stay in a motel should write directly and as soon as possible to the motel for reservations.

Meals will be available in Erb Memorial Union.

Eugene is served by the Southern Pacific Railway, United Airlines, West Coast Airlines, and the Greyhound Bus Company. Persons who drive to the meetings will find ample free parking on campus.

R. S. Pierce
Associate Secretary

Seattle, Washington

Seventieth Summer Meeting
and
Forty-Third Colloquium

Cornell University
Ithaca, New York
August 31—September 3, 1965

The American Mathematical Society will hold its seventieth summer meeting at Ithaca, New York from Tuesday through Friday, August 31-September 3, 1965.

All sessions will be held in lecture rooms and classrooms on the campus of Cornell University.

Professor A. P. Calderón of the University of Chicago will present the Forty-Third Colloquium in a set of four lectures with the title "Singular integrals." The first lecture will be given in the Alice Statler Auditorium of Statler Hall on Tuesday, August 31 at 2:00 P.M. The second will be in the same auditorium on Wednesday, September 1 at 9:00 A.M. The third and fourth will be on Thursday, September 2 at 2:00 P.M. and on Friday, September 3 at 9:00 A.M., respectively, but in Room B17 of Upson Hall.

By invitation of the Committee to Select Hour Speakers for Summer and Annual Meetings, Professor George Lorentz of Syracuse University will address the Society on Friday, September 3 at 2:00 P.M. in the Alice Statler Auditorium. His topic will be in the field of approximation of functions.

There will be sessions for contributed papers on Tuesday, August 31 in the afternoon; Wednesday, September 1 in the morning; Thursday, September 2 in the afternoon; and Friday, September 3 both morning and afternoon.

Abstracts of contributed papers should be sent to the American Mathematical Society, Providence, Rhode Island 02904 so as to arrive prior to the deadline of July 9. According to authorization of the Council, the number of contributed papers will be limited to 170. Contributed papers which meet the standard set in Article X, Section 5 of the By-Laws will be accepted in order of their receipt until 170 have been accepted or July 9 has arrived. There will be no provision for late papers.

Several organizations will cooperate in holding meetings or council meetings on the same campus as the Society and at approximately the same time. These include Pi Mu Epsilon, Mu Alpha Theta, and the Society for Industrial and Applied Mathematics. In particular SIAM will present the John von Neuman Lecture in the Alice Statler Auditorium on Thursday, September 2 at 8:00 P.M. The invited speaker is Professor Freeman J. Dyson of the University of California at San Diego, whose topic is "Applications of group theory in particle physics."

The Mathematical Association of America will hold their forty-sixth summer meeting from Monday, August 30 through Thursday, September 2. This meeting marks the fiftieth anniversary of the Association, an event which will be heralded in the special character of their program. In deference to the Association, the Society has yielded a half day of time normally devoted to Society sessions in order to permit the Association to present an expanded anniversary program.

COUNCIL AND BUSINESS MEETINGS

The Council of the Society will meet at 5:00 P.M. on Tuesday, August 31 in Room 217 of Ives Hall. There will be an intermission for dinner unless the agenda promises to be brief.

The Business Meeting of the Society will be held in Room B17 of Upson Hall on Thursday, September 2 at 4:45 P.M.

ADVANCE REGISTRATION

The advance registration procedure will be used. On the inside back cover of this issue of the Notices is a registration form. The same form will appear in the June *Notices* but it will not appear in the August issue. The form provides for registration together with badge, registration fee, and information packet for all persons attending the meeting. It also provides for a parking permit and for dormitory room reservations, barbecue tickets, SIAM beer party tickets, and the non-mathematicians excursion to Corning Glass, together with relevant advance payments.

A copy of the registration form may be obtained by request to the American Mathematical Society, Providence, Rhode Island 02904.

REGISTRATION

The Registration Desk will be in the Memorial Lobby of Willard Straight Hall. This is at the north end of the main floor. It will be open on Sunday, August 29 from 2:00 to 8:00 P.M.; on Monday, August 30 from 8:00 A.M. to 5:00 P.M.; on Tuesday through Thursday, August 31 through September 2, from 9:00 A.M. till 5:00 P.M.; and on Friday, September 3 from 9:00 A.M. till 3:30 P.M.

The registration fees will be as follows:

Member	$2.00
Member's family	.50

for the first such registration and no charge for additional registrations,

Student	No fee
Others	$5.00

The preferred procedure is to register in advance, as described in the section titled ADVANCE REGISTRATION, and to complete the process by picking up the badge and information packet at the registration desk. It is desirable to have one's local address already established when completing registration as this information will be recorded at the registration desk for the visual index. In particular, persons with dormitory reservations should go first to University Hall I before completing registration. See the section on DORMITORY HOUSING.

It is possible to register at the desk without advance registration. However, one may find that the facilities for which attendance is estimated and guaranteed will be sold out.

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be in Rooms 112, 114 and 116 of Ives Hall. It will be open Tuesday through Thursday, August 31 through September 2, from 9:00 A.M. to 5:00 P.M. on each of the three days. Attention is invited to the announcement of the Employment Register on page 296, in particular to the deadline dates for application to the register and to the necessity for prompt registration at the Employment Register Desk by both applicants and employers.

EXHIBITS

Various publishers will have exhibits of books on Tuesday through Thursday in the Memorial Room of Willard Straight Hall. This is in the northwest corner of the main floor, to the west of the lobby which contains the registration desk.

BOOK SALE

Books published by the Society will be sold for cash prices somewhat below the usual prices when these same books are sold by mail on invoice.

DORMITORY HOUSING

Dormitory rooms will be available in University Halls and Baker Dormitories. The rates are $5.50 per day for a single room and $3.50 per day per person for a double room. Soap, towels, and maid service are provided.

Rooms will be available from Saturday, August 28 to Saturday, September 4. Rooms must be vacated by 9:00 A.M. on September 4.

Dormitory room reservations are to be made on the advance registration form already described. The reservation is to be accompanied with payment for one night, as noted on the registration form.

Reservations will be confirmed. A local map will accompany the confirmation.

When people with dormitory reservations arrive, they should go directly to University Hall I, without going first to the meeting registration desk. The office in University Hall I will be open 24 hours a day to give room assignments and keys and to collect the remainder of the room rent.

Persons arriving without dormitory reservations may be able to obtain dormitory accommodations through a dormitory representative at the registration desk during the hours that desk is open and at University Hall I during other hours. This is not the recommended procedure.

During the day, bellhops will be available. They will accept tips. It is not necessary to use their services.

Dormitory rooms are not air-conditioned.

There are no special provisions for small children. For an older child (but not for an adult) a cot 30 inches by 6 feet can be put in a double room at a cost of $2.00 per night.

United Rent-Alls, 363 Elmira Road, Ithaca, New York (Phone Area 607, AR3-1807) has standard cribs at $3.50 per week and portacribs at $2.50 per week. Their local supply is limited, so that it is advisable to begin negotiations with them early.

There will be a short list of baby sitters at the registration desk.

MOTELS AND HOTELS

There are a number of motels in the Ithaca area, including the following:

Collegetown Motor Lodge - 312 College Avenue - 41 rooms - single $9, double $12 - 10 minute walk from campus - air-conditioned.

Howard Johnson Motor Lodge - North Triphammer Road and route 13 - 72 rooms - swimming pool - single $10.50 to $16.50, doubles $14.50 to $18.50 and family units - 15-20 minute drive from campus - air-conditioned.

Meadow Court - 529 South Meadow Street - 50 rooms - singles $8 and up, doubles $12 and $13, family units $20 - 15 minute drive from campus - air-conditioned.

Plaza Motel - corner Meadow Street and Elmira Road - 84 rooms - swimming pool - singles $8, doubles $12 to $13, for three $15 to $17, for four $17 - 15 minute drive from campus - air-conditioned.

Wonderland Motel - 654 Elmira Road - 27 rooms - swimming pool - singles $8 to $13, doubles $10 to $14, family units $19 to $21 - 25 minute drive from campus - air-conditioned.

Hillside Inn - 518 Stewart Avenue - 41 rooms - singles $6, doubles $8, family units - 35 rooms have private baths and are air-conditioned - 10 minute walk from campus.

Ithaca Hotel - 219 East State Street - 73 rooms, of which about three-quarters are air-conditioned - singles $7.50 to $8.50, doubles $12 to $14 - on bus line which goes to campus.

Persons wishing to reserve motel or hotel accommodations should write before July 1 directly to Mr. B. Anderson, Meadow Court, 529 South Meadow Street, Ithaca, New York 14850, listing three choices and mentioning the meeting of mathematicians. Beyond the date of July 1 it is less reasonable to suppose that accommodations will be available.

CAMPING

There are three state parks in the area with tent and trailer sites for camping. The location and the mail address for information and reservations follows.

Buttermilk Falls State Park - 2 miles south of Ithaca on route 13. R. D. 5, Ithaca, New York 14850.

Robert H. Treman State Park - 5 miles south of Ithaca on route 13. R.D. 5, Ithaca, New York 14850.

Taughannock Falls State Park - 8 miles north of Ithaca on route 89. R.D. 3, Trumansburg, New York 14850.

MEALS

The Willard Straight cafeteria will be open all day for meals, beginning at 7:00 A.M., and through the evening for light refreshment. The Sage Hall cafeteria will be open all day for meals. Both of these operate on a cash basis.

The Statler Hall dining room will be open for breakfast, lunch, and dinner. One should realize that it is more expensive than the cafeterias and that reservations may be necessary.

A list of local restaurants will be at the registration desk.

ENTERTAINMENT

The Society for Industrial and Applied Mathematics will sponsor their traditional Beer Party at Noyes Lodge on the evening of Monday, August 30 at 8:00 P.M. Tickets will be sold through the advance registration form at a price of $1.25 each. It is possible that some additional tickets will be sold at the registration desk.

There will be a chicken barbecue on Wednesday evening at a precise time and place to be announced. Tickets will be sold through the advance registration form at $2.75 per person and for children up to the age of 12 at $1.50 for a half portion. It is possible that some additional tickets will be sold at the registration desk.

For non-mathematicians, there will be an excursion by bus on Thursday, September 2 to the Corning Glass factory. The round trip by bus will cost $1.75 per person, payable in advance through the advance registration procedure. Departure time will be immediately after lunch.

Persons registered at the meeting will be able to use the Cornell Golf Course by showing their badges and paying a greens fee of $2.00 on weekdays and $3.00 on weekends. Golf clubs can be rented.

There is swimming and picnicking at the state parks and at other places. There are tennis courts adjacent to the dormitories and bowling alleys near the campus. Information about these diversions may be obtained at the registration desk.

TRAVEL

Ithaca is centrally located in the Finger Lakes region of New York State. Mohawk Airlines serves the region with connecting flights from the principal surrounding large cities — New York, Newark, Boston, Pittsburgh, Cleveland, Buffalo, Detroit. The Tompkins County Airport is two miles from the University with taxi, limousine, and car rental service available. American Airlines serves this area through Syracuse. Empire State Airlines connects Binghamton, Ithaca, Syracuse, Elmira, and New York, thus connecting Ithaca with major air carriers. Also, Commuter Airlines provide service between Binghamton and Washington, D.C. The Airline Guide should be consulted for flight times.

There is no direct railroad service to Ithaca, but the New York Central goes to Syracuse and the Erie Lackawanna to Binghamton and Owego. Buses connect Syracuse and Binghamton with Ithaca, but there is no public transportation between Owego and Ithaca. Owego is approximately 30 miles from Ithaca.

The Greyhound Bus Company runs several buses daily that connect with New York, Buffalo, Rochester, Syracuse, Binghamton, and Scranton.

Ithaca can be reached by private car by using an excellent system of connecting highways. Coming from the west one uses the New York Thruway to Waterloo and then connects with New York Route 89 to Ithaca; coming from the New England area one uses the New York Thruway to Syracuse where U. S. Route 11 and New York Route 13 lead to Ithaca; coming from New York City one uses New York Route 17 to Owego where New York Route 96 connects directly to Ithaca. From a southerly direction, one can take the northerly extension of the Pennsylvania Turnpike from Philadelphia to Scranton and then Interstate Highway 81 to Binghamton, and then proceed as if from New York City.

The City of Ithaca operates a bus line which provides infrequent local service within the city. Persons employed by certain universities and government agencies are entitled to car rental discounts. Participants should determine if they qualify for this discount. Car rental agencies exist both in Ithaca and in the surrounding cities of Syracuse, Binghamton, and Elmira.

PARKING

The Safety Division (campus police) will issue cards which will allow parking on the campus. Permits will be made available to those who indicate on the advance registration form that they will drive to the meeting.

WEATHER

The mean temperature during the week of August is 67°. During this period the average maximum is 79°, and the average minimum is 55°. However, maxima as high as 90° are not impossible. Minima as low as 40° have also been known to occur. The average rainfall is 69/100 inches for the week. The humidity is between 50 and 60 percent in the afternoon and rises to an average of 90 percent at night.

MEDICAL SERVICES

The staff of the Gannett Clinic will provide people attending the meeting with daytime out-patient medical care for acute illness and injury. Such medical care will be charged, at a reasonable fee, to the individual seeking medical care.

In general, those requiring hospitalization and night-time emergency care will rely on private medical facilities of the community. The Gannett Clinic physician who is on emergency call will assist in arranging such care if his help is needed, and can be contacted by calling Sage Hospital, AR 2-6962, or Gannett Clinic, Ext. 3493.

ADDRESS FOR MAIL AND TELEGRAMS

The address for mail and telegrams is in care of Mathematics Meetings, Willard Straight Hall, Cornell University, Ithaca, New York 14850. Individuals should check for mail from time to time in the vicinity of the registration desk.

COMMITTEE

The committee on arrangements consist of

H. L. Alder — A. Rosenberg, Chairman
S. U. Chase — G. Sacks
R. Greenblatt — G. L. Walker
May Kinsolving — R. J. Walker
E. Pitcher — S. Wainger
G. S. Rinehart — H. Widom

Everett Pitcher
Associate Secretary

Bethlehem, Pennsylvania

ACTIVITIES OF OTHER ASSOCIATIONS

CANADIAN MATHEMATICAL CONGRESS
1965 SEMINAR AND CONGRESS

Since the announcement that appeared in the February issue of the *Notices*, new dates have been scheduled for the tenth biennial seminar and the seventh congress of the Canadian Mathematical Congress, to take place at Laval University, Quebec. The seminar will be held from August 16 to September 3, with lectures beginning on August 16; the congress will be held from September 4 to September 7, with registration on September 3.

AMS Summer Seminar on Relativity Theory and Astrophysics

The Fourth Summer Seminar, sponsored by the American Mathematical Society, will take place at Cornell University in Ithaca, New York, from July 25 to August 20, 1965. Financial support of the Seminar will be provided by the Air Force Office of Scientific Research, Atomic Energy Commission, National Aeronautics and Space Administration, National Science Foundation, and the Office of Naval Research.

Since the Seminar program is largely instructional in purpose, graduate students and recent Ph.D.'s interested in the fields of relativity and astrophysics are encouraged to apply for admission. It is hoped that the Seminar will help to increase the number of workers in the field.

Graduate students, postdoctoral scientists, and staff members from active centers of research in relativity and astrophysics, such as Princeton University, Syracuse University, the University of Chicago, and the University of Texas, are expected to attend. Theoretical progress in general relativity will be presented, including the general relativistic treatment of static and dynamic behavior of large masses and their gravitational fields. Past, recent, and proposed experimental tests of the general theory of relativity will be discussed.

Ample opportunity will be provided for the participants to engage in free discussion in small informal groups among themselves and with the distinguished speakers on the program. This opportunity should prove to be one of the lasting benefits to be gained from the Seminar.

The members of the Joint Invitations and Organizing Committee are A. H. Taub (Chairman), Director, Computer Center, University of California at Berkeley; S. Chandrasekhar, Yerkes Observatory; C. C. Lin, Massachusetts Institute of Technology; A. Schild, University of Texas; and C. W. Misner, University of Maryland. Members of the Committee will give lectures on their recent work.

A preliminary version of the formal program of lecturers and the topics upon which they will speak is given below.

Week of July 26-30

A. Schild	General Relativity
L. Woltjer	Galactic Dynamics and Galactic Structure
L. I. Schiff	Experimental Tests of General Relativity (Gyroscope)
E. M. Burbidge	Stellar-like Radio Sources
W. B. Bonnor	Jeans' Criterion for Stability

Week of August 2-6

A. Schild	General Relativity
L. Woltjer	Galactic Dynamics and Galactic Structure
E. E. Salpeter	Stellar Structure Leading up to White Dwarfs and Neutron Stars
E. L. Schucking	Theoretical Cosmology
D. Lynden-Bell	Cooperative Phenomena in Stellar Dynamics

Week of August 9-13

R. K. Sachs	Gravitational Radiation
D. Lynden-Bell	Cooperative Phenomena in Stellar Dynamics

E. E. Salpeter	Stellar Structure Leading up to White Dwarfs and Neutron Stars
B. B. Rossi	Cosmic Rays
J. Linsley	Cosmic Rays
I. Robinson	Gravitational Radiation
I. R. King	Stellar Clusters
R. P. Kerr	Solutions of the Einstein Equations
K. H. Prendergast	Barred Spirals
S. Chandrasekhar	Stellar Stability
J. Weber	Measurements in Gravitational Radiation
R. Penrose	Solutions of the Einstein Equations
C. C. Lin	Normal Spirals
G. C. McVittie	Gravitational Collapse

Week of August 16-20

F. J. Dyson	Experimental Tests of General Relativity (Radar)
C. C. Lin	Normal Spirals
S. Chandrasekhar	Stellar Stability Instability Problems
A. H. Taub	Relativistic Hydrodynamics
C. Hunter	Fragmentation
M. May	Numerical Calculations in Relativistic Hydrodynamics
D. R. Layzer	The Energy Equation and the Virial Theorem for Cosmic Distributions, and The Formation of Stars and Galaxies: Unified Hypotheses or: The Nature and Origin of Nonthermal Radio Sources
C. W. Misner	Gravitational Collapse

Admission and Financial Assistance

Application blanks for admission and for financial assistance can be obtained from Dr. Gordon L. Walker, Executive Director, American Mathematical Society, 190 Hope Street, Providence, Rhode Island 02906.

Completed application blanks should be sent to Professor A. H. Taub, Director, Computer Center, University of California, Berkeley, California. In view of the limited accommodations, the Committee requests that applications reach Professor Taub as soon as possible and no later than April 12, 1965.

An applicant should state his scientific background and interests, and a graduate student should ask his faculty advisor to write to the Committee concerning his ability and promise. Anyone who wishes a grant-in-aid should indicate so on his application. A limited amount of financial help is available.

Tuition will be charged at the rate of $100 per week for participants from industry. Participants from academic institutions and government agencies may apply for waiver of tuition.

The dormitories and other housing and dining facilities of Cornell University will be available to participants and their families. Participants will receive a detailed announcement which will include a complete program of the Seminar and information about registration, rooms and meals, entertainment and recreation, and transportation.

NEWS ITEMS AND ANNOUNCEMENTS

FIFTH SYMPOSIUM ON MATHEMATICAL STATISTICS AND PROBABILITY

The Symposium will be held at the University of California, Berkeley, from June 21 to July 18, 1965. It is being organized by the Statistical Laboratory of the University with the financial support of NSF, ARO(D), AFOSR, and NIH (National Institutes of Health).

The American Mathematical Society will be represented on the Advisory Committee of the Symposium by Professor J. L. Doob. The other members of this Committee are Professors S. Karlin and H. E. Robbins from the Institute of Mathematical Statistics, and Professor D. Burkholder, editor of the Annals of Mathematical Statistics. The Committee on Local Arrangements consists of Professors E. W. Barankin, E. Fix, L. LeCam, J. Neyman, and E. L. Scott.

The preliminary program lists over 100 lectures. As a rule, four lectures will be scheduled for each day of the Symposium. The program will also include discussion sessions. It is expected that there will be approximately fifty foreign participants.

Lectures on theory will take place for the duration of the Symposium. Starting July 1, however, there will be several sessions devoted to applications, arranged by agreement with Professor L. Moses, Chairman of the Statistics Department, Stanford University. One day of lectures will be held at Stanford.

Additional Sessions will be scheduled in connection with the AAAS meeting to be held at Berkeley in December, 1965.

SUMMER CONFERENCE IN RELATIVITY

A relativity conference will be given at Arlington State College from June 14 to July 2 (3 weeks). It is sponsored by the National Science Foundation and some stipends will be available. The dates for this conference will not conflict with the American Mathematical Society summer seminar in "Relativity Theory and Astrophysics," which will be held July 25 - August 20, 1965 at Cornell University. (See page 292, these *Notices*).

Most lectures will be restricted both in material and in presentation to that which could reasonably be given to undergraduates, although some lectures will go beyond this to give additional depth.

The geometry of spacetime will be emphasized in establishing the fundamentals of relativity.

In addition to special relativity there will be some cosmology and non-mathematical general relativity. The program will consist of lectures, discussions, problems, study projects, and some experimental work. Printed notes and the periodical, SPACETIME, will be free to participants.

The visiting lecturer-leaders will be professors Vaclav Hlavaty of The Graduate Institute of Mathematics and Mechanics at Indiana University, J. B. Crabtree of Stevens Institute of Technology, Richard Schlegel of Michigan State University, Isidore Hauser of Illinois Institute of Technology, Richard A. Mould of Stony Brook (State University of New York). The Arlington State College professors will be Ulrich Herrmann, Nolan Massey, and Jason Ellis.

Arlington State College is located in Northern Texas about halfway between the twin cities of Dallas and Fort Worth. All buildings involved are fully air conditioned. There are lakes and recreation areas nearby.

Letters of application and inquiry should be submitted as soon as possible to the conference director, Dr. Jason Ellis, Department of Physics, Arlington State College, Arlington, Texas.

LECTURE NOTES FROM THE LEHIGH SUMMER INSTITUTE

The lecture notes from the 1964 Lehigh Summer Institute for Advanced Graduate Students in Analysis, sponsored by the American Mathematical Society, are now for sale. The notes consist of the following:

Topics in the Theory of Functions of One Complex Variable
W. H. J. Fuchs in collaboration with A. Schumitzky
190 pages Price $3.00

Introduction to Functional Analysis
D. A. Edwards; Lecture notes by A. J. Ellis
267 pages Price $3.75

Orders should be addressed to:
Professor Everett Pitcher
Department of Mathematics and Astronomy
Lehigh University
Bethlehem, Pennsylvania

A check payable to Lehigh University should accompany the order. Notes will be mailed postage prepaid.

ALL BACK VOLUMES OF MATHEMATICS OF COMPUTATION NOW AVAILABLE

Back Volumes. The reprinting of all the out-of-print back issues of MATHEMATICS OF COMPUTATION is a publishing event that has been long-awaited by everyone concerned with the rapidly growing field of uses of mathematics for computational purposes. It is now possible for private, industrial, and academic libraries to have a complete collection of this standard journal of tables and other aspects of numerical mathematics.

Any of the back volumes, from Volume 1, 1943-1945, through Volume 18, 1964, may be purchased from the American Mathematical Society at $20.00 list price, $17.00 agents' price*, and $15.00 Society members' price. Orders are being filled now.

Also available are some single issues from Volumes 2, 7 through 12, and 14 through 18. More specific information will be given upon request. The prices for single issues are $5.00 list, $4.25 agents' price*, and $3.75 Society members' price.

Subscriptions. Beginning with Volume 19, 1965, the prices for a yearly subscription of four issues of MATHEMATICS OF COMPUTATION are $16.00 list price, $13.60 to agents*, and $8.00 to Society members.

*The agents' price is allowed only for subscriptions and back volumes mailed directly from the Society to foreign addresses.

SYMPOSIUM ON INEQUALITIES

A Symposium on Inequalities will be held at Wright-Patterson Air Force Base during August 19-27, 1965, under the sponsorship of Aerospace Research Laboratories, a component of the Office of Aerospace Research.

The symposium is planned to include, in addition to formal lectures, informal workshop-like meetings of small groups engaging in exchange of ideas and in actual research, and also short talks.

A partial list of participants follows. Professors E. F. Beckenbach, K. Fan, J. B. Diaz, M. Marcus, H. Minc, T. S. Motzkin, I. Olkin (tentatively), G. Pólya, I. J. Shoenberg, G. Szegö, Dr. O. Taussky Todd, and J. Todd.

The proceedings of the symposium will be published by some well known publisher.

For further information please write to Dr. Oved Shisha, ARL(ARM), Building 450, Wright-Patterson Air Force Base, Ohio. Please indicate whether or not you would like to contribute actively, and in what manner.

MEMORANDA TO MEMBERS

MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The latest compilations of available positions and of applicants for positions in the mathematical sciences may be purchased from the Mathematical Sciences Employment Register, to be mailed on May 15, 1965. The List of Applicants is available for $7.50; the List of Positions costs $3.00.

At the 1965 Summer Meeting in Ithaca, New York, the Employment Register will again schedule interviews and distribute a listing of applicants and positions. The Register will be open from 9:00 A.M. to 5:00 P.M. on Tuesday, August 31 through Thursday, September 2 in Rooms 112, 114 and 116 of Ives Hall.

There is no charge for registration, either to job applicants or to employers, except when the late registration fee of $5.00 for employers is applicable. Provision will be made for anonymity of applicants upon request and upon payment of $5.00 to defray the cost involved in handling anonymous listings.

Job applicants and employers who wish to be listed will please write to the Employment Register, Providence, Rhode Island 02904 for application forms or for position description forms. These forms must be completed and returned to Providence not later than July 1, 1965, in order to be included in the listings at the Summer Meeting in Ithaca. Position Description Forms which arrive after this closing date, but before August 1 will be included in the register at the meeting for a late registration fee of $5.00. The printed listings will be available for distribution, both during and after the meeting.

It is essential that applicants and employers register at the Employment Register Desk promptly upon arrival at the meeting to facilitate the arrangement of appointments.

The Mathematical Sciences Employment Register is sponsored jointly by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics.

LETTERS TO THE EDITOR

Editor, the *Notices*

As pointed out in a recent letter from Richard Askey et al., revision of the Library of Congress classification is desirable. However, it should be pointed out that no classification system can achieve contiguous shelving of related books or avoid multiple classification of each book. Indeed the key factor is not call number assignment but an adequate system of cross referencing and multiple indexing. Each of the books listed in the Askey letter should be indexed under several subjects, and if this is done the shelf assignment is a rather minor matter of convenience when picking up several related books.

Kenneth O. May

PERSONAL ITEMS

Professor C. R. ADAMS has been named Professor Emeritus of Mathematics at Brown University and has accepted an appointment as Executive Secretary of the Rhode Island Commission for Higher Education Facilities.

Associate Professor P. M. ANSELONE of the Mathematical Research Center of the University of Wisconsin has been appointed to a professorship at Oregon State University.

Assistant Professor P. J. ARPAIA of Clarkson College of Technology has been appointed to an assistant professorship at the C. W. Post College of Long Island University.

Mrs. J. M. BAKER of Regis College has been appointed to an assistant professorship at the George Mason College of the University of Virginia.

Dr. A. H. BRADY of the National Bureau of Standards has been appointed an Assistant Professor of Computing Science at the University of Notre Dame.

Professor G. F. CARRIER of the Mechanical Engineering Department at Harvard University will be on sabbatical leave for the academic year 1964-1965. He was awarded a Fulbright grant and is spending the year doing research at the University of Western Australia, Nedland, Australia.

Dr. C. J. CLARK of Sylvania Electronic Defense Laboratory has accepted a position as Staff Scientist, Mathematician with the Texas Instruments Incorporated, Dallas, Texas.

Dr. J. P. CLAY of Univac Division of Sperry Rand Corporation has been appointed to an assistant professorship at Drexel Institute of Technology.

Dr. H. S. M. COXETER of the University of Toronto is on leave to serve as Distinguished Visiting Professor of Mathematics at Florida Atlantic University.

Associate Professor D. G. DE FIGUEIREDO of The Universidade de Brasilia, Brasilia, Brazil has been appointed a Research Member at the Mathematics Research Center of the University of Wisconsin.

Mr. H. H. DIEKHANS of the University of Illinois has been appointed to an associate professorship at Indiana State College.

Mr. G. E. DIMITROFF of the University of Oregon has been appointed to an assistant professorship at Knox College.

Assistant Professor S. A. FOOTE of Rutgers, The State University has received a National Science Foundation Science Faculty Fellowship and will be on leave for the academic year 1964-1965 at the University of Oregon.

Dr. G. E. FORSYTHE of Stanford University has been appointed Executive Director of the newly formed Computer Science Department at Stanford University.

Mr. P. O. FREDERICKSON of the University of Nebraska has been appointed to an assistant professorship at Case Institute of Technology.

Dr. R. M. FREYRE of the College of Advanced Science has been appointed to an assistant professorship at Lowell Technical Institute.

Mr. J. C. GRIMBERG of Aerospace Corporation has accepted a position as Operations Analysis Section Manager with the Bunker-Ramo Corporation, Canoga Park, California.

Mr. S. P. HASTINGS of the Massachusetts Institute of Technology has been appointed to an assistant professorship at Case Institute of Technology.

Professor CHARLES HATFIELD of the University of Missouri at Rolla has been named Chairman of the Department of Mathematics.

Professor EDWIN HEWITT of the University of Washington delivered the third annual DeLong Lectures at the University of Colorado during March 17-22, 1965.

Mr. J. N. ISSOS of Auburn University has been appointed to an assistant professorship at Michigan Technological University.

Assistant Professor SHMUEL KAN-

IEL of the University of Chicago has been appointed to a visiting assistant professorship at Stanford University.

Mr. ABRAHAM KAREN of Reeves Instrument Corporation has accepted a position as Assistant Director of the Office of Scientific Research at New York University.

Mr. W. E. KIRWAN II of Rutgers, The State University, has been appointed to an assistant professorship at the University of Maryland.

Mr. S. K. KNAPOWSKI of the University of Marburg, Germany has been appointed to an associate professorship at the University of Florida.

Dr. MANFRED KOCHEN of the Thomas J. Watson Research Center of the International Business Machines Corporation has been appointed an Associate Professor of Mathematical Biology at the University of Michigan.

Associate Professor V. V. KOTA of the State University College of New York at Fredonia has been appointed to a professorship at Atlanta University.

Mr. J. P. LABUTE of the University of Windsor, Canada has been appointed a Research Assistant at Harvard University.

Mr. RICHARD LEE of the University of British Columbia has been appointed to an assistant professorship at the University of New Brunswick, Fredericton, New Brunswick, Canada.

Assistant Professor T. L. MCCOY of Illinois Institute of Technology has been appointed to an assistant professorship at Michigan State University.

Dr. R. E. MESSICK of California Institute of Technology has been appointed to an assistant professorship at Case Institute of Technology.

Mr. JONAH MANN of Yeshiva University has been appointed to an assistant professorship at City College of the City University of New York.

Mr. J. P. MILLER of Space Technology Laboratories has accepted a position as a Member of the Technical Staff with the Aerospace Corporation, San Bernardino, California.

Mr. D. A. MORAN of the University of Chicago has been appointed to an assistant professorship at Michigan State University.

Professor RUFUS OLDENBURGER, Director of the Automatic Control Center at Purdue University has received the American Society of Mechanical Engineers 1964 Machine Design Award.

Mr. WILLIAM PARRY of the University of Birmingham, England has been appointed a Lecturer at the University of Sussex, England.

Associate Professor L. E. PAYNE of the University of Maryland has been appointed to a professorship at Cornell University.

Mr. J. M. ROBERTSON of the University of Utah has been appointed to an assistant professorship at Washington State University.

Dr. M. L. ROCKOFF of the National Bureau of Standards has been appointed a Research Associate at the Institute for Fluid Dynamics and Applied Mathematics, University of Maryland.

Dr. H. L. ROLF of Vanderbilt University has been appointed to a professorship at Baylor University.

Mr. P. L. SADAGURSKY of the University of Illinois has been appointed to an assistant professorship at the George Washington University.

Mr. NOBUO SHIMADA of Nagoya University, Japan has been appointed to a professorship at the Research Institute for Mathematical Science, Koyoto University, Koyoto, Japan.

Mr. ANTHONY SPINGOLA of General Precision Incorporated has accepted a position as Programmer Analyst with the Grumman Aircraft Corporation, Bethpage, New York.

Dr. DANIEL TEICHROEW of Stanford University has been appointed to a professorship and Head of the Division of Organizational Sciences at Case Institute of Technology.

Dr. PETER WERNER of the Technical University of Karlsruhe, Germany has been appointed a Visiting Research Mathematician at the Mathematics Research Center, U. S. Army, at the University of Wisconsin.

Mrs. M. K. WINTER of Northwestern University has accepted a position as Assistant Mathematician in the Applied Mathematics Division of the Argonne National Laboratory, Argonne, Illinois.

The following promotions are announced:

R. K. BHATTACHARYA, University of Arizona, to an assistant professorship.

FLORA DINKINES, University of Illinois, to a professorship.

M. P. DOLCIANI, Hunter College, to a professorship.

R. E. DOWDS, Butler University, to a professorship.

N. E. FOLAND, Kansas State University, to an associate professorship.

EISHI HONGO, Kyushu Institute of Technology, Tobata, Japan, to a professorship.

F. G. MAPA, University of the Philippines, to an associate professorship.

W. E. STUMPF, International Business Machines Corporation, Poughkeepsie to a Senior Associate Programmer.

The following appointments to Instructorships are announced:

Massachusetts Institute of Technology: E. G. K. LOPEZ-ESCOBAR; University of Massachusetts: R. G. BAUER; College of New Rochelle: EUGENE SANTORO; Princeton University: P. C. GREINER; Purdue University: D. R. BEUERMAN.

Deaths:

Professor B. A. BERNSTEIN of the University of California, Berkeley died September 25, 1964 at the age of 83. He was a member of the Society for 50 years.

Professor PHILIP FRANKLIN of the Massachusetts Institute of Technology died on January 27, 1965 at the age of 66. He was a member of the Society for 45 years.

NEW AMS PUBLICATIONS

MEMOIRS

Number 53

JORDAN ALGEBRAS OF SELF-ADJOINT OPERATORS

By David M. Topping

48 pages; List Price $1.50; Member Price $1.13.

This Memoir presents a real non-associative counterpart to the theory of von Neumann algebras. The algebras studied are weakly closed real Jordan algebras of bounded self-adjoint operators, for which an intrinsic classification into the classical five types is shown to exist. A theory of relative dimensionality based on symmetries, culminating in a comparison theorem for projections, is developed and is exploited to analyze the structure of these algebras. The work of von Neumann and Kaplansky on continuous geometries occupies a central role in the dimension theory, and a final refinement is achieved through application of recent results of A. B. Ramsay on dimension structure in orthomodular lattices. A new kind of factor, not present in the von Neumann theory, is exhibited. This factor is "discrete" and of "finite class", yet is infinite dimensional as a real linear space.

SUPPLEMENTARY PROGRAM—Number 31

During the interval from January 15, 1965 through February 17, 1965 the papers listed below were accepted by the American Mathematical Society for presentation by title. After each title on this program there is an identifying number. The abstracts of the papers will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these *Notices*.

One abstract presented by title may be accepted per person per issue of the *Notices*. Joint authors are treated as a separate category; thus in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

(1) Summability of real-valued set functions. Preliminary report
Professor W. D. L. Appling, North Texas State University (65T-160)

(2) Normal extensions of formally normal ordinary differential operators. II
Mr. Richard Balsam, University of California, Los Angeles (65T-197)

(3) The Frattini subgroup of a p-group. Preliminary report
Professor H. F. Bechtell, Bucknell University (65T-174)

(4) Transfinite automata recursions
Professor J. R. Büchi, The Ohio State University (65T-177)

(5) Groups and graphs. II. Preliminary report
Professor C. Y. Chao, University of Pittsburgh (65T-196)

(6) The decidability of the derivability problem for one-normal systems
Mr. S. A. Cook, Harvard University (65T-205)

(7) A symmetric integral of order two
Professor G. E. Cross, University of Waterloo (65T-187)

(8) Semigroups having left or right zeroid elements
Professor D. F. Dawson, North Texas State University (65T-180)

(9) A comparison of inversive and conformal differential geometries
Professor John DeCicco and Mr. Stavros Busenberg, Illinois Institute of Technology (65T-183)

(10) Commutators of singular integrals
Mr. E. B. Fabes and Mr. N. M. Riviere, University of Chicago (65 T-193)

(11) Results concerning models of Peano's arithmetic
Dr. Haim Gaifman, The Hebrew University (65T-195)

(12) A partial binary relation algebra
Mr. D. S. Geiger, University of Illinois (65T-171)

(13) Cross sections of 2-spheres in the 4-sphere
Dr. C. H. Giffen, The Institute for Advanced Study (65T-200)

(14) Singularities of analytic functions having integral representations with a remark about the elastic unitarity integral
Professor R. P. Gilbert, Professor H. C. Howard and Mr. S. O. Aks, University of Maryland (65T-188)

(15) Deterministic context free languages. II. Preliminary report
Dr. Seymour Ginsburg, System Development corporation, Santa Monica, California and Dr. Sheila Greibach, Harvard University (65T-164)

(16) Sectional curvatures and Euler-Poincaré characteristic of homogeneous spaces
Professor Werner Greub, University of Toronto and Dr. P. M. Tondeur, Harvard University (65T-190)

(17) Compactness and the projection mapping from a product space. Preliminary report
Mr. A. W. Hager and Professor S. G. Mrowka, Pennsylvania State University (65T-167)

(18) L^2 series expansions for functions with the Huygens property
Professor D. T. Haimo, Southern Illinois University and Harvard Uni-

versity (65T-163)

(19) Post normal systems: The unrestricted halting problem
Mr. P. K. Hooper, Harvard University (65T-178)
(Introduced by Professor Hao Wang)

(20) Pure systems of binary relations
Mr. Michel Jean, University of California, Berkeley (65T-189)

(21) Constructible sets and weakly compact cardinals. Preliminary report
Professor H. J. Keisler, University of Wisconsin and Dr. Frederick Rowbottom, University of California, Berkeley (65T-186)

(22) On a property of infinitely divisible distribution in a Hilbert space
Professor R. G. Laha, The Catholic University of America (65T-170)

(23) Necessary density conditions for local harmonic analysis and interpolation
Mr. H. J. Landau, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (65T-202)

(24) On the zeros of solutions of a second-order linear differential equation
Professor Walter Leighton, Western Reserve University (65T-172)

(25) Dehn's algorithm
Professor R. C. Lyndon, Queen Mary College, London, England and University of Michigan (65T-175)

(26) Splitting and decomposition by regressive sets
Dr. T. G. McLaughlin, University of Illinois (65T-191)

(27) Semigroups of connected functions
Professor K. D. Magill, Jr., SUNY at Buffalo (65T-181)

(28) Cardinal sums and Beth's theorem in infinitary languages
Mr. J. I. Malitz, University of California, Berkeley (65T-203)

(29) Analogues of highly composite and related numbers. Preliminary report
Mr. A. A. Mullin, University of California, Livermore (65T-179)

(30) A generalization of the Kuratowski embedding theorem
Professor S. A. Naimpally, Iowa State University (65T-162)

(31) Norm-differentiability of evolution operators
Professor E. T. Poulsen, Heidelberg Universität, Heidelberg, Germany and Aarhus Universität, Aarhus, Denmark (65T-176)

(32) A hierarchy for objects of type 2
Professor J. R. Shoenfield, Stanford University (65T-173)

(33) Weakly prime alternative rings
Mr. M. B. Slater, University of Chicago (65T-166)

(34) Extensions of semigroups to groups
Professor A. H. Smith, California State College at Long Beach (65T-185)

(35) On unitary perfect numbers
Professor M. V. Subbarao, University of Alberta (65T-169)

(36) On rational characters of a finite group
Professor Shuichi Takahashi, University of Montreal (65T-168)

(37) On uniqueness of generalized direct products with amalgamated subgroups. Preliminary report
Professor C. Y. Tang, Illinois Institute of Technology (65T-194)

(38) A homogeneous line is completely homogeneous
Mr. S. S. Wagner, 340 Pendleton Road, Clemson, South Carolina (65T-158)
(Introduced by Professor W. S. Mahavier)

(39) Connectivity versus diameter in graphs
Professor M. E. Watkins, University of North Carolina (65T-182)

(40) Sentences preserved under unions. Preliminary report
Mr. J. M. Weinstein, University of Wisconsin (65T-159)

(41) Characterizations and representations of semi-normed algebras. II. Preliminary report
Professor Chien Wenjen, California State College at Long Beach (65T-161)

(42) Completeness and certain implicit upper semicontinuous decompositions
Dr. J. M. Worrell, Jr., Sandia Corporation, Albuquerque, New Mexico (65T-199)

(43) On the existence of a weighted

Stieltjes mean sigma integral. II
Professor F. M. Wright, Iowa State University (65T-165)

(44) Sieve-generated sequences of natural numbers. II
Professor Marvin Wunderlich, SUNY at Buffalo (65T-192)

NEWS ITEMS AND ANNOUNCEMENTS

SEMINAR ON PARTIAL DIFFERENTIAL EQUATIONS
Université de Montréal, June 28 to August 6, 1965

Under the sponsorship of the North Atlantic Treaty Organization (NATO) and the Canadian Mathematical Congress, the fourth session of the University of Montreal international "SEMINAIRE DE MATHEMATIQUES SUPERIEURES" will be held next summer, from June 28 to August 6.

This year, the Seminar will be on Partial Differential Equations. The program will consist of six main courses given by the following lecturers:

Shmuel AGMON, Université de Jérusalem. "Unicité et propriétés de convexité dans les problèmes differentiels"

Marcel BRELOT, Université de Paris. "Théorie axiomatique du potentiel s'appliquant aux équations aux dérivées partielles du second ordre"

Felix BROWDER, University of Chicago. "Problèmes non-linéaires"

Guido STAMPACCHIA, Université de Pise. "Equations elliptiques du second ordre à coefficients discontinus"

José BARROS-NETO, Université de Montréal. "Problèmes aux limites non-homogènes"

Samuel ZAIDMAN, Université de Montréal. "Equations différentielles abstraites"

Apart from these courses, the program will include a certain number of lectures given by guest speakers. Registrants may make application for financial assistance to cover travelling and living expenses. To obtain further information and registration forms, please write to:

Département de Mathématiques,
Université de Montréal
Case postale 6128
Montreal 3, Quebec (CANADA)

ABSTRACTS OF CONTRIBUTED PAPERS

The April Meeting in Chicago, Illinois
April 9-10, 1965

621-1. D. J. RODABAUGH, Box 1631, Vanderbilt University, Nashville, Tennessee 37203. Some Wedderburn theorems. Preliminary report.

The Wedderburn Principal Theorem is proved for a class of algebras which are known to be alternative when they are semisimple. Let A be a power associative algebra (finite dimensional) such that A - N (N is the maximal nil ideal) is without simple nodal subalgebras and let F be algebraically closed. Let A satisfy one of the following sets of identities: (1) the (γ,δ) identifies with $\delta \neq 0,1$ and char. $\neq 2,3,5$; (2) $(x,x,x) = 0$ and $a(y,x,x) - (a+1)(x,y,x) + (x,x,y) = 0$ for $a \neq 1, -1/2, -2, -1, 0$; (3) $(x,x,x) = 0$, $(x,x,y) = (y,x,x)$ with $A_{ii}^2(e) \subseteq A_{ii}(e)$ for all idempotents. Under these assumptions, there exists a semisimple subalgebra B in A with $A = B + N$. (Received October 30, 1964.)

621-2. E. L. GRIFFIN, University of Pennsylvania, Philadelphia, Pennsylvania 19104. Everywhere defined linear transformations affiliated with rings of operators.

Let T be a linear transformation defined everywhere in a complex Hilbert space H. If T commutes with every member of a ring of operators M, then T is bounded if and only if it is bounded on each minimal projection in M. Consequently, if M has no type I part, T must be bounded. Also, if the coupling operator of the pair M', M is essentially bounded, then T is bounded. This extends an unpublished result of J. R. Ringrose on transformations commuting with maximal abelian rings. (Received November 2, 1964.)

621-3. J. D. KUELBS, 400 Ford Hall, University of Minnesota, Minneapolis, Minnesota. Integration on spaces of real-valued continuous functions whose domain is a compact subset of R_2. Preliminary report.

Suppose X is a compact subset of R_2 and C(X) is the space of real-valued continuous functions on X. Let $\underline{a} = \min(s:(s,t) \in X)$ and $\underline{c} = \min(t:(s,t) \in X)$. Theorem 1. There exists a family N of regular probability measures on C(X) such that N is homeomorphic to $((s,t): s \leqq \underline{a}, t \leqq \underline{c})$. These measures include as a special case the measure given by James Yeh (Trans. Amer. Math. Soc. vol. 95, 433-450). Theorem 2. If $X = [\underline{a},\underline{b}] \otimes [\underline{c},\underline{d}]$ where $\underline{a} \leq \underline{b}$, $\underline{c} \leq \underline{d}$, and m_1 and m_2 are distinct elements in the interior of N then there exists a subset E of C(X) such that $m_1(E) = 1$ and $m_2(E) = 0$. That is, m_1 is not absolutely continuous with respect to m_2. (Received November 30, 1964.)

621-4. J. E. SIMPSON, Marquette University, 1131 West Wisconsin Avenue, Milwaukee, Wisconsin. On a class of locally convex spaces.

Some implications of the following condition on a locally convex topological vector space, E, are discussed. Definition: The dual, E', of such a space is said to be countably total if there is a sequence $\{B_n\}$ of equicontinuous subsets of E', whose union is total (in the usual sense that $\langle x, x' \rangle = 0$ for all x' in all B_n implies x = 0). Sample Theorem: Let $\mathfrak{B}$ be a bounded σ-complete Boolean algebra of projections on E, E' countably total. Then the strong closure of $\mathfrak{B}$ is complete. This theorem is useful in applying a multiplicity theory for Boolean algebras of projections to the resolution of the identity of a scalar operator on E. The author would welcome conditions equivalent to the above defined "countably total". (Received December 2, 1964.)

621-5. R. S. COUNTRYMAN, JR., 811 28th Avenue South, Minneapolis, Minnesota 55406. On the characterization of compact Hausdorff X for which C(x) is algebraically closed. Preliminary report.

Let X be a compact Hausdorff space, C(X) the algebra of complex-valued continuous functions on X. X is said to be a C-space in case X does not contain: (1) two connected closed sets M and N such that $M \cap N$ is separated, or (2) a sequence $M_1, M_2, \ldots$ of disjoint connected closed sets having a non-degenerate sequential limit set. Theorem. If X is a compact metric space, then a necessary and sufficient condition that the algebra C(X) be algebraically closed is that X be a C-space. The proof of this theorem is essentially a generalization of that used by Don Deckard and Carl Pearcy to prove Theorem 1 in their paper On algebraic closure in function algebras (Proc. Amer. Math. Soc. 15 (1964), 259-263). The condition that X be a C-space is necessary even without the hypothesis of metric. (Received November 27, 1965.)

621-6. R. W. HEATH, Arizona State University, Tempe, Arizona 85281. On open mappings and certain spaces satisfying the first countability axiom.

A. H. Stone [Proc. Amer. Math. Soc. 7 (1956), 690-700] showed that a regular space E is metrizable and locally separable if E is the image of a locally separable metric space under an open mapping f such that, for each $p \in E$, $f^{-1}(p)$ is separable. S. Hanai [Proc. Jap. Acad. 37 (1961), 233-238] showed that a T_1-space satisfies the first countability axiom if and only if it is the open continuous image of a metric space. Similar or related theorems are due to Ponomarev [Bul. Pol. Acad. 8 (1960), 127-134], Arhangel'ski [Sov. Math. Dokl. 3 (1962), 953-956] and Nagami [Proc. Jap. Acad. 37 (1961), 356-357]. In this paper necessary and sufficient conditions are given for the open continuous image of a metric space to be (1) a semi-metric space, (2) a developable space, (3) a Nagata space or (4) metrizable. For example: Theorem. A T_1-space Y is developable if and only if there is an open mapping f from some metric space X onto Y such that, for everv $p \in Y$ and everv open set R, $p \in R$, there is an $\epsilon > 0$ such that $f(S[f^{-1}(p), \epsilon]) \subset R$. ($S[k,\epsilon] = \{x: \mathrm{dist}(x,k) < \epsilon\}$). Also given is a characterization of Nagata spaces similar to that for semi-metric, developable and metric given by the author in [Pacific J. Math. 12 (1962), 1301-1319]. (Received November 23, 1964.)

621-7. S. W. YOUNG, University of Texas, Austin, Texas 7812. Uniform completeness of sets generated by a single function.

Let $C_0[0,1]$ denote the real function space of continuous functions g on the interval $[0,1]$ such that $g(0) = 0$ and $\|g\|$ is the uniform norm. Theorem 1 gives necessary and sufficient conditions on a function f continuous on $[0,\infty)$ and $f(0) = 0$ in order that there exist a sequence of positive numbers, $\{k_n\}$ such that $\{f(k_n x)\}$ is uniformly complete in $C_0[0,1]$. Theorem 2 deals with collections each member of which is a function f continuous on $[0,\infty)$ and $f(0) = 0$. The following is a Corollary: If $\{C_n\}$ is a sequence of positive numbers, then $\{X^{c_n}\}$ is uniformly complete in $C_0[0,1]$ if and only if the function q can be uniformly approximated by finite linear combinations of $\{X^{c_n}\}$ where $q(x) = 2x$ for $0 \leqq x \leqq 1/2$ and $q(x) = 1$ for $1/2 \leqq x \leqq 1$. Theorem 3 gives a necessary and sufficient condition for a function $f(x) = \sum_{n=1}^{\infty} a_n x^n$ $(0 \leqq x \leqq b > 0)$ in order that there exist a sequence of numbers $\{k_n\}$ in $[0,b]$ such that $\{f(k_n x)\}$ is uniformly complete in $C_0[0,1]$. Theorem 4. There exists a function f continuous on $[0,\infty)$ and $f(0) = 0$ and a sequence of ordered number pairs $\{(a_n,b_n)\}$ such that $\{a_n f(b_n x)\}$ is dense in $C_0[0,1]$. (Received November 27, 1964.)

621-8. J. C. BEIDLEMAN, University of Kentucky, Lexington, Kentucky. On semi-primary ideals of distributively generated near-rings.

A distributively generated near-ring R with identity is called semi-primary if every non-zero right ideal is a direct summand. Let R be a distributively generated near-ring with identity. An ideal B of R is called semi-primary if R/B is a semi-primary near-ring. The intersection of all semi-primary ideals of R is called the semi-primary radical of R. It is understood that if R contains no semi-primary ideals, then the semi-primary radical of R is R itself. An element $a \in R$ is called semi-idempotent if there exists a semi-primary ideal B such that $a^2 - a \in B$ and $a \notin B$. An ideal A of R is called semi-nilpotent if it contains no semi-idempotent elements. The following theorems are proved: Theorem 1. A distributively generated near-ring R is semi-primary iff it can be expressed as a finite direct sum of minimal right ideal. Theorem 2. A semi-primary ideal of R contains every nil right ideal of R. Theorem 3. A semi-primary ideal is a semi-prime ideal that contains the radical of R (See Abstract 64T-161). Theorem 4. The semi-primary radical is the largest semi-nilpotent ideal of R. Theorem 5. If the radical J(R) of R is semi-primary, then it coincides with the semi-primary radical and every semi-prime ideal that contains J(R) is semi-primary. (Received December 18, 1964.)

621-9. A. S. STRAUSS, University of Maryland, College Park, Maryland. Liapunov functions and global existence.

Definition: V(t,x) is a Liapunov function for the ordinary differential equation (E) $x' = f(t,x)$, where f is continuous and locally Lipschitzian on $D = E^1_+ \times E^n$ and $f(t,0), = 0$, if V(t,x) is non-negative, continuous, locally Lipschitzian on D, $V(t,0) = 0$ and $V'(t,x) \leqq 0$ (see Antosiewicz, A survey of Lyapunov's second method, Annals of Math. Studies 41). Definition 2: V(t,x) is mildly unbounded if for every $T > 0$, $V(t,x) \to \infty$ as $|x| \to \infty$ uniformly in t, $0 \leqq t \leqq T$. Theorem. The solution $F(t,t_0,x_0)$ of (E) can be continued to $[t_0,\infty)$ for every (t_0,x_0) in D if and only if there exists on D a mildly unbounded Liapunov function V(t,x) for (E). Furthermore, this function is positive definite if and only if the zero solution of (E) is stable. The asymptotic-stability-in-the-large theorems can now be generalized by replacing the statement "$V(t,x) \to \infty$ as $|x| \to \infty$ uniformly in t, $0 \leq t < \infty$" by "$V(t,x)$ is mildly unbounded." (Received December 18, 1964).

621-10. R. M. SCHORI, Louisiana State University, Baton Rouge, Louisiana 70803. A nonhomogeneous inverse limit of homogeneous spaces with covering maps as bonding maps.

In this paper we construct an inverse sequence of compact 2-manifolds where the bonding maps are covering maps and prove that the limit space is not homogeneous. These coordinate spaces are clearly homogeneous. This construction establishes that a theorem stated by Jack Segal in Abstract 551-15, these *Notices* 5 (1958), 687, is not valid in its full generality. However, a weaker version of this theorem has been verified by M. C. McCord, Abstract 63T-267, these *Notices* 10 (1962), 499. The crucial lemma in proving the nonhomogeneity of the example is: Let (M_i, f^i_{i-1}) be an inverse sequence of compact manifolds such that each f^i_{i-1} is a (at least 2-fold) covering map. Let M be the limit of (M_i, f^i_{i-1}) and let f_1 be the projection of M onto M_1. If M is homogeneous, $\epsilon > 0$, and $r_1 \in M_1$, then there exist distinct points r and s of $f_1^{-1}(r_1)$ and a homeomorphism $h: M \to M$ such that $\|h - id\| < \epsilon$ and $h(r) = s$. (Received November 27, 1964.)

621-11. R. B. BENNETT, Knox College, Galesburg, Illinois 61401. Locally connected 2-cell and 2-sphere-like continua.

A 2-cell-like continuum is one that can be ϵ-mapped onto a 2-cell for every positive number ϵ. Theorem 1. Every locally connected 2-cell-like continuum can be embedded in the plane. This theorem can be shown using Whyburn's cyclic element theory, a characterization of planar peanian continua due to W. W. S. Claytor and the fact that every 2-cell-like continuum is unicoherent. Corollary 2. Every locally connected 2-cell-like continuum has the fixed point property. By somewhat the same methods as used for Theorem 1, one can show Theorem 3. A locally connected continuum is 2-sphere-like if and only if a 2-sphere or a nondegenerate dendrite. Theorem 3 is a generalization of a theorem of Mardesic and Segal [Trans. Amer. Math. Soc. 109 (1963), 146-164]. (Received January 4, 1965.)

621-12. J. E. OSBORN, 114 Main Engineering, University of Minnesota, Minneapolis, Minnesota. Estimates for the eigenvalues of a class of non self-adjoint operators.

Let L be the inverse of a positive definite, self-adjoint, Hilbert-Schmidt operator defined on a Hilbert space. Let A be a bounded linear operator defined on the domain of L; A is not assumed to be self-adjoint. Let the eigenvalues of L be numbered as follows: $0 < \lambda_1 \leq \lambda_2 \leq \ldots \to \infty$. Then if $\|A\| < \lambda_1$ and the circles $C_j = \{z \mid |z - \lambda_j| \leq \|A\|\}$, $j = 1,2,\ldots$, are all disjoint, the operator $\tilde{L} = L + A$ has a countable set of eigenvalues, one in each C_j. The Rayleigh-Ritz method is applied to $\tilde{L}$, using the eigenvectors of L as a basis, to obtain approximations to the eigenvalues of $\tilde{L}$ and estimates are derived for the errors which arise. More precisely we have the following situation. Let μ_p be the eigenvalue of $\tilde{L}$ in C_p. For $n \geq p$ let $\eta_1^n, \eta_2^n, \ldots, \eta_n^n$ be the eigenvalues of $\tilde{L}_n = (P_n \tilde{L})|_{V_n}$, where V_n is the subspace spanned by the first n eigenvectors of L and P_n is the projection onto that subspace. Then if $|||\widetilde{L}_n^* - \widetilde{L}_n|||\max\sum_{\ell\neq k}|\eta_k^n - \eta_\ell^n|^{-1} < 1$, where $|||\cdot|||$ denotes the Hilbert-Schmidt norm, we obtain a bound for $|\mu_p - \eta_p^n|$ which under certain other conditions converges to zero in n. This bound is readily computable and examples are presented. (Received January 11, 1965.)

621-13. GORDON JOHNSON, University of Georgia, Athens, Georgia. Concerning local variations in the moment problem.

Let $\{c_i\}_{i=0}^{\infty}$ be a real number sequence such that the function sequence $\{\phi_i\}_{i=1}^{\infty}$ is uniformly bounded on $[0,1]$ where for each positive integer n, $\phi_n(x) = 0$ if $x \leqq 0$, $\phi_n(x) = c_0$ if $x \geqq 1$ and $\phi_n(x) = \sum_{t=0}^{k} C(n,t)\sum_{i=0}^{n-t} C(n-t,i)(-1)^i c_{i+t}$ if $x \in (0,1) \cap [k/n,(k+1)/n)$. Then (1) if $0 \leqq a < b \leqq 1$ and $\epsilon > 0$ there is a number $N > 0$, so that if n is an integer greater than N there is a number $M > 0$ so that if m is an integer greater than M then $V_a^b\phi_n < \epsilon + V_{a-\epsilon}^{b+\epsilon}\phi_m$, (2) there is a number $A > 0$ so that if n is a positive integer $V_0^1\phi_n \leqq A\sqrt{n}$ and (3) if $x \in [0,1]$ the point set $C_x = \{y | y$ is a sequential limit point of a subsequence of $\{\phi_i(x)\}_{i=1}^{\infty}\}$ is a continuum. (Received January 13, 1965.)

621-14. W. J. DAVIS, Ohio State University, 231 W. 18th Avenue, Columbus, Ohio 43210. Dual generalized bases.

A generalized basis is a maximal biorthogonal system, $\{x_a;\phi_a\}$, such that $\{\phi_a\}$ is total in the linear topological space, X [Arsove and Edwards, Generalized bases, 1960]. The dual situation is a maximal system such that $[x_a] = X$. A pair of systems $\{x_a;\phi_a\} \subset (X,X^*)$ and $\{y_a;\psi_a\} \subset (Y,Y^*)$ are similar if $\{\phi_a(X)\} = \{\psi_a(Y)\}$, and are *-similar if $\{\hat{x}_a(X^*)\} = \{\hat{y}_a(Y^*)\}$. [$\hat{x}_a$ and $\hat{y}_a$ are the natural images of x_a and y_a in X^{**} and Y^{**}, respectively.] Arsove and Edwards show that complete metric linear spaces containing similar generalized bases are isomorphic. The following theorems are proved: Theorem 1. If (X,X^*) and (Y,Y^*) (with suitable topologies) contain *-similar dual generalized bases, the spaces X and Y are isomorphic. Theorem 2: If (X,X^*) and (Y,Y^*) contain similar maximal biorthogonal systems, the quotient spaces X/N_1 and Y/N_2 are isomorphic. [$N_1 = \bigcap\mathcal{N}(\phi_a)$, the null spaces, and $N_2 = \bigcap\mathcal{N}(\psi_a)$]. (Received January 18, 1065.)

621-15. HOWARD COOK, University of North Carolina, Chapel Hill, North Carolina 27515. Mappings of circle-like continua onto circle-like continua.

Suppose that P is a sequence $p_1, p_2, p_3, \ldots$ of non-negatigeve integers, M is a circle-like continuum, and $C_1, C_2, C_3, \ldots$ is a sequence of circular chains covering M such that mesh C_i approaches 0 as i increases without bound and C_{i+1} circles C_i only p_i times (R. H. Bing, Embedding circle-like continua in the plane, Canad. J. Math. 14 (1962), 113-128). Then M will be called a P-adic circle-like continuum. Theorem. If the circle-like continuum N is a continuous image of the circle-like continuum M then there exist sequences $P = p_1, p_2, p_3, \ldots$ and $Q = q_1, q_2, q_3, \ldots$ of non-negative integers such that (1) M is a P-adic circle-like continuum, (2) N is a Q-adic circle-like continuum, and (3) for each i, either $q_i = 0$ or p_i/q_i is an integer. Theorem. If each of two solenoids is a continuous image of the other, they are topologically equivalent. (Received January 21, 1965.)

621-16. J. R. RETHERFORD, Louisiana State University, Baton Rouge, Louisiana. Some nearness theorems in Banach spaces.

If $\{E_i\}$ is a sequence of continuous, orthogonal linear projections ($E_i^2 = E_i$, $E_iE_j = 0$, $i \neq j$) from a Banach space X into itself and if M_i = Range of E_i then $\{M_i\}$ is a Schauder basis of subspaces for X if and only if $x = \sum_1^\infty E_i(x)$ for each $x \in X$. We write: $\{M_i, E_i\}$ is a Sbos for X. A Banach space analogue of a Theorem of Markus [Amer. Math. Soc. Transl. 1 (1960), 600, Thm. 1] is proved: If $\{M_i, E_i\}$ is a Sbos for X and if $\{D_i\}$ is a sequence of non-trivial, continuous, orthogonal, linear projections from X into X satisfying $\sum_1^\infty \|E_i - D_i\| = K < +\infty$ then $\{N_i, D_i\}$, N_i = Range D_i, is a Sbos for the linear closure of $\bigcup_{i=1}^\infty N_i$. In particular, if $K < 1$ then $\{N_i, D_i\}$ is a Sbos for X. It is observed that Hilding's Theorem [Ann. of Math. (2) 49 (1948), 953-955] is valid in Banach spaces and this yields several nearness theorems of the Paley-Wiener type previously known only for Hilbert space. (Received January 25, 1965.)

621-17. L. J. MORDELL, University of Illinois, Urbana, Illinois. Some quartic diophantine equations of genus 3.

About 45 years ago, the writer put forward the conjecture that a curve of genus > 1 has only a finite number of rational points. Though it has since become widely known, no progress has been made with this conjecture. It may therefore be of interest to prove it in some special cases, and in particular that given by the Theorem. The quartic equation of genus 3, $((br - cq)/a)^3x^4 + ((cp - ar)/b)^3y^4 + ((aq - bp)/c)^3z^4 = 0$, has at most eight integer solutions (and these can be given explicitly if (1) $a > 0$, $b > 0$, $c > 0$, $a \equiv b \equiv c \equiv -1 \pmod 8$; $(b,c) = (c,a) = (a,b) = 1$. (2) $p \equiv 0 \pmod 8$, $q \equiv r \equiv -1 \pmod 8$. (3) $(br - cq)/a \equiv (cp - ar)/b \equiv (aq - bp)/c \equiv 0 \pmod 1$, and the positive odd factors of the three terms are $\equiv 1 \pmod 8$. (Received January 25, 1965.)

621-18. THEODORE MITCHELL, State University of New York at Buffalo, Michael Hall, Buffalo, New York 14214. Fixed points and multiplicative left invariant means. Preliminary report.

Let S be a semigroup. Definition. A subset $S' \subseteq S$ is called left thick in S if for every finite subset $S'' \subseteq S$, there exists $s'' \in S$ such that $S''s'' \subseteq S'$. Theorem. The following are equivalent: (a) S has a multiplicative left invariant mean. (b) For each compact Hausdorff space X, and for each homomorphic representation $\mathscr{S}$ of S as a semigroup (under functional composition) of continuous maps of X into itself, there is in X a common fixed point of the family $\mathscr{S}$. (c) For each finite collection of subsets $S_i \subseteq S$ such that $S = \bigcup_{i=1}^n S_i$, there exists an S_i in the collection which is left thick in S. Remark. If S is any semigroup such that for every $s_1, s_2 \in S$, there exists $s_3 \in S$ such that $s_1s_3 = s_2s_3$, then S has properties (a), (b) and (c) above. (Received February 1, 1965.)

621-19. P. J. KNOPP and RICHARD SINKHORN, University of Houston, Houston, Texas. An extension of a theorem of Sinkhorn.

The following theorem is proved. Theorem. Let M and N be compact topological spaces and let μ and ν be regular Borel measures on M and N, respectively, such that $\mu(M) > 0$ and $\nu(N) > 0$. Let $h(x,y)$, $F(x)$, and $G(y)$ be positive and continuous on $M \times N$, M, and N, resp., such that $\int_M F d\mu =$

$\int_N G d\nu$. Then there exist functions f(x) and g(y) positive and continuous on M and N, resp., such that $F(x) = \int_N f(x)h(x,y)g(y)d\nu(y)$ and $G(y) = \int_M f(x)h(x,y)g(y)d\mu(x)$. The form f(x)h(x,y)g(y) can be obtained as a limit to the iteration of alternately scaling the function h to have the correct integrals over M and then N. If, in addition, each nonvoid open set in M and N has positive measure then f(x)h(x,y)g(y) is unique and the functions f and g are unique up to a positive scalar multiple. (Received February 8, 1965.)

621-20. C. B. SCHAUFELE, Louisiana State University, Baton Rouge, Louisiana. A note on link groups.

Every tame link L in S^3 possesses a reduced surface. By a reduced surface S for L, we mean a surface of type (p,μ,r), where μ is the number of components of L and p is the genus of L, and where the inclusions $i: S^3 - S \rightarrow S^3 - L$ and $i_k: \overset{\circ}{S}_k \rightarrow S^3 - L$ induce monomorphisms on the fundamental groups (S_k, k = 1,...,r, are the components of S). (See these *Notices*, January 1964). Theorem. If L has a reduced surface of type (p,μ,r), then $\pi_1(S^3 - L)$ can be mapped epimorphically to a free group of rank r. Corollary 1. If $p \geqq 1$ or if p = 0 and $\mu \neq 1$ is odd, then $\pi_1(S^3 - L)$ contains a free group of rank n for any $n \leqq \infty$. Corollary 2. If $p \geqq 1$ or if p = 0 and $\mu > 2$, then $\pi_1(S^3 - L)$ is not solvable. (Received February 8, 1965.)

621-21. J. K. BROOKS, 1126 Grandview Avenue, Columbus 12, Ohio. A transformation theory for vector measure spaces.

Let T be a transformation whose domain is a measure space $(S,\underline{M},\mu)$ and whose range is a measure space $(S',\underline{M}',\mu')$. Reichelderfer has developed a transformation theory for non-negative measure spaces (see Abstract 61T-267, these *Notices* 8 (1961), 518). Hypotheses are given to extend this theory to X-vector measure spaces, where X denotes a Banach space over the complex numbers with a separable conjugate space X*. A weight function W' is a non-negative real-valued function defined on $S' \times \underline{D}$, where $\underline{D}$ is a certain subfamily of $\underline{M}$. In addition to certain "continuity" conditions relative to $\underline{D}$, W' satisfies the following conditions: (1) $W'(\cdot,D) = 0$ a.e. μ' on S' - TD, $D \in \underline{D}$; (2) $W'(\cdot,D)$ is measurable μ' for each $D \in \underline{D}$. T is ACW' if $W'(\cdot,D)$ is integrable μ' for each $D \in \underline{D}$ and if there exists a complex-valued integrable μ function f defined on S such that $\int_D f d\mu = \int_{S'} W'(\cdot,D)d\mu'$, $D \in \underline{D}$. Theorem. Fix D in $\underline{D}$ and assume T is ACW'. Let H' be a complex-valued measurable μ' function defined on S' such that $H'W'(\cdot,D)$ is integrable μ' and $\int H' \circ T f d(x^*\mu)$ is uniformly absolutely continuous with respect to $\{x^*\mu: x^* \in X^*, |x^*| \leqq 1\}$ on D. Then $\int_D H' \circ T f d\mu = \int_{S'} H'W'(\cdot,D)d\mu'$. (Received February 8, 1965.)

621-22. J. W. ENGLAND, University of Virginia, Charlottesville, Virginia. S-Lyapunov stability in dynamics.

Let (X,T,π) be a transformation group, X a uniform space with uniformity U and S a subset of T. A point x in X is said to be S-Lyapunov stable with respect to a set B in X provided that for each α in U there exists a β in U such that if $y \in B \cap x\beta$ then $yt \in (xt)\alpha$ for all t in S. This type of stability is studied in relation to almost periodicity and uniform almost periodicity. This then gives a theorem of the type of Nemickiĭ (Qual. Theory of Diff. Equations, Princeton Math. Series, No. 22). Theorem: If X is a compact uniform space then a necessary and sufficient condition that it be the closure of an almost periodic point is that it be the space of a (necessarily compact) topological group. (Received February 8, 1965.)

621-23. K. O. LELAND, Ohio State University, 231 W. 18th Avenue, Columbus, Ohio 43210. A characterization of harmonic functions.

Let F be a family of continuous functions on open subsets of a Euclidean space E into the reals, closed under the operations of addition, multiplication by a scalar, and linear translation and rotation, and satisfying the Maximum Modulus Theorem. Then the elements of F are harmonic functions. For each function f on the boundary M of the closed unit sphere V of E, such that f is the restriction to M of an element g of F containing V in its domain, define $L_1(f) = g(0)$, $L_2(f) = \int_M f d\mu$, and $L_3(f) = \int_V f dm$. Mapping the space of continuous functions on M into the space of continuous functions on the group G of rotations of E, it is shown in view of the uniqueness of Haar measure on G, that $L_1 = L_2 = L_3$. This characterization is used in the resolution of removable singularity problems by the methods of Topological Analysis (cf. these *Notices* 11 (1964), 586). A byproduct is an intrinsic characterization of surface area μ on M. (Received February 9, 1965.)

621-24. F. C. HOPPENSTEADT, 515 Van Vleck Hall, University of Wisconsin, Madison, Wisconsin. Singular perturbations on the infinite interval.

A theorem of A. N. Tihonov (Mat. Sb. (N.S.) 31 (73) (1952), 575-586) indicates the behavior for small positive ϵ of the solution of the (k + j)-dimensional initial-value problem (1) $x' = f(t,x,y,\epsilon)$, $\epsilon y' = g(t,x,y,\epsilon)$, $x(t_0) = x_0$, $y(t_0) = y_0$, when attention is restricted to compact intervals. It can be assumed without loss of generality that $f(t,0,0,0) = 0$ and $g(t,x,0,0) = 0$ for $t_0 \leqq t < \infty$ and $|x| \leqq R < \infty$. Use is made of two associated systems—(2) $x' = f(t,x,0,0)$, $x(t_0) = x_0$, and (3) $dy/ds = g(\alpha,\beta,y,0)$, $y(0) = y_0$, where $0 \leqq s < \infty$ is a new independent variable and α and β are treated as parameters. Let $x = \phi(t,\epsilon)$, $y = \psi(t,\epsilon)$ be the solution of the initial-value problem (1), and let $x = \Phi(t)$ be the solution of the initial-value problem (2). It is shown that under suitable stability conditions on the zero solutions of systems (2) and (3) and suitable smoothness conditions on the functions f and g, $\phi(t,\epsilon) \longrightarrow \Phi(t)$ and $\psi(t,\epsilon) \longrightarrow 0$ as $\epsilon \longrightarrow 0^+$ uniformly on closed subsets of $t_0 < t < \infty$. The hypotheses reduce to those needed by Tihonov when compact t-intervals are considered; moreover, a series of examples shows that no substantial weakening of the required hypotheses is possible. (Received February 12, 1965.)

621-25. S. A. NAIMPALLY, Iowa State University, Ames, Iowa 50010. Essential fixed points of almost continuous functions.

Essential fixed points were first introduced by Fort (Amer. J. Math. 72 (1950), 315-322) and almost continuous functions by Stallings (Fund. Math. 47 (1959), 249-263). Let X be a topological space with the fixed point property, C the set of all continuous selfmappings of X, and A the set of all almost continuous selfmappings of X. When X is a compact metric space, A is made a pseudometric space by the use of the Hausdorff metric on the hyperspace of all nonempty closed subsets of $X \times X$. When X is compact Hausdorff a new function space topology is introduced in A (which is equivalent to u.c. topology in C). In both the cases Fort's results are generalised, the following being the principal result. Theorem. Each $f \in A$ can be approximated arbitrarily closely by a $g \in C$ such that all fixed points of g are essential. (Received February 15, 1965.)

621-26. PETER COLWELL, 600 W. Franklin Avenue, Minneapolis, Minnesota 55405.
On the boundary behavior of Blaschke products in the unit circle. Preliminary report.

Let $B(z;A)$ be a Blaschke product analytic in $\{|z| < 1\}$ with $B(z;A) = 0$ for z in A. Let A' denote the derived set of A on $\{|z| = 1\}$ and $B(e^{i\theta})$ denote the radial limit of $B(z;A)$ at $e^{i\theta}$, if it exists. Theorem 1. Let E be a set on $\{|z| = 1\}$. There exists $B(z;A)$ with $B(e^{i\theta})$ defined and of modulus one for all θ such that $A' = E$ if, and only if, E is closed and nowhere dense in $\{|z| = 1\}$. Theorem 2. Let $B(z;A)$ be given with $B(e^{i\theta})$ defined and of modulus one for all θ. As a function of θ, $B(e^{i\theta})$ is discontinuous at $\theta = \theta_0$ if, and only if, $e^{i\theta_0}$ is in A'. Theorem 3. Let $B(z;A)$ be given with $B(e^{i\theta})$ defined and of modulus one for all θ. If A' is countable, $B(e^{i\theta})$ assumes no value more than countably many times. The radial variation of $B(z;A)$ at $e^{i\theta}$ is defined as $V(B;\theta) = \int_0^1 |B'(re^{i\theta};A)|dr$. Cargo [J. London Math. Soc. 36 (1961), 424-430] proves: The radial variations of $B(z;A)$ and all its subproducts are uniformly bounded if, and only if, $\sum_A(1 - |a|/|e^{i\theta} - a|) < \infty$. Theorem 4. Let $B(z;A)$ be given such that $\sum_A(1 - |a|/|e^{i\theta} - a|) < \infty$ for every θ. As a function of θ, $V(B;\theta)$ is discontinuous at $\theta = \theta_0$ if, and only if, $e^{i\theta_0}$ is in A'. Similar theorems hold for functions of the form $P(z; \{\theta_n, b_n\}) = \exp[\sum_{n=1}^{\infty} b_n(e^{i\theta_n} + z/e^{i\theta_n} - z)]$, for $0 < b < 1$, and $\sum_{n=1}^{\infty} b_n < \infty$. (Received February 15, 1965.)

621-27. J. E. HALL, Windsor, Wisconsin 53598. Quantitative estimates for nonlinear differential equations by Liapunov functions.

Consider the nonlinear system of ordinary differential equations (1) $\dot{x} = \sum_{n=1}^{N} c_n m_n(t,x,z)g_n(z_n)$, $\dot{z}_n = - h_n(t,x,z)g_n(z_n) - m_n(t,x,z)\sigma(x)$, $n = 1,\dots,N$, where x is scalar, $z = (z_1,\dots,z_N)$; $g_1,\dots,g_N$, σ are nonlinear springs; $h_1,\dots,h_N$ are given non-negative functions; and c_n is a positive constant, $n = 1,\dots,N$. By considering suitable Liapunov functions, sufficient conditions are given for global existence, boundedness, decay, and exponential decay of solutions of (1). Various generalizations are considered. For a special case of (1), the rate of decay is independent of N, and this result is applied to obtain global existence, boundedness, and exponential decay of the solution of the integro-differential problem $\dot{u}(t) = - \int_0^{\pi} a(x)T(x,t)dx$, $aT_t(x,t) = bT_{xx}(x,t) + \eta(x)\sigma(u(t))$ on the region $0 < x < \pi$, $0 < t < \infty$, subject to the initial and boundary conditions $u(0) = u_0$; $T(0,t) = T(\pi,t) = 0$, $0 < t < \infty$; and $\lim_{t\to 0^+} T(x,t) = f(x)$. (Received February 15, 1965.)

621-28. T. ANDERSON and N. J. DIVINSKY, University of British Columbia, Vancouver, British Columbia, Canada and A. SULINSKI, University of Warsaw, Warsaw, Poland. Lower radical properties

A radical theory for groups has been recently developed by Kurosh and independently by P. Hall. Shuken has shown that the construction of lower radical properties for groups terminates at ω_0, the first infinite ordinal. His argument uses group theoretic properties and thus cannot be used for rings. In this paper we prove that the construction of lower radical properties for associative rings also terminates at ω_0. For alternative rings the construction terminates at ω_0^2. (Received February 15, 1965.)

621-29. P. M. RICE, University of Georgia, Athens, Georgia. The Hauptvermutung and the polyhedral Schoenflies Theorem.

M. L. Curtis and E. C. Zeeman (On the polyhedral Schoenflies Theorem, Proc. Amer. Math. Soc. 11 (1960), 888-889) conjectured that the double suspension of a Poincaré manifold is the 5-sphere. Theorem. There is a noncombinatorial triangulation of some manifold if and only if there is a combinatorial manifold K which is topologically not a sphere and an integer n such that the n-fold suspension of K is a topological sphere. Corollary. Modulo the Poincaré conjecture, the following statements are equivalent: (i) If a (compact) combinatorial n-cell B is embedded as a subcomplex of a triangulated n-sphere S, then $S\backslash B$ is simply connected. (ii) Every triangulation of every manifold is combinatorial. (Received February 1, 1965.)

621-30. WALTER RUDIN, University of Wisconsin, Madison, Wisconsin. Harmonic analysis on spheres.

Various aspects of harmonic analysis on spheres will be contrasted with their analogues on abelian groups. The following topics will be touched on: Operators which commute with rotations. Gap series. Expansions of measures in series of spherical harmonics. A convolution measure-algebra. Most of the results have been obtained by two of the speaker's students, D. G. Rider and C. F. Dunkl. (Received February 16, 1965.)

621-31. MORRIS MARDEN, The University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53211. A generalization of a theorem of Bocher.

The principal result of this paper is the following. Let E be a vector space over an algebraically closed field K. Let $H_j(x,y)$, $j = 1,2$, be two Hermitian symmetric forms defined on E, with values in K, such that there are subspaces $E_j = \{x: x \in E, H_j(x,x) > 0, x \neq 0\}$, $j = 1,2$, with the properties $x \notin E_1 \Rightarrow x \in E_2$ and $x \notin E_2 \Rightarrow x \in E_1$. Let $P_j(x)$, $j = 1,2,\ldots,q$, defined for $x \in E$, with values in K, be homogeneous polynomials of degree n_j and let $P_j(x,x_1)$ be the first polar of $P_j(x)$ with respect to x_1, $x_1 \in E$. With $\{m_j\}$ a set of real numbers such that $\sum_1 q_{m_j} = 0$, form $\Phi(x,x_1) = \sum_{j=1}^{q} m_j P_1(x) \ldots P_{j-1}(x) P_j(x,x_1) P_{j+1}(x) \ldots P_q(x)$. If $m_j > 0$ and $P_j(x) \neq 0$ for $x \in E_1$, when $j = 1,2,\ldots,p < q$, and $m_j < 0$ and $P_j(x) \neq 0$ for $x \in E_2$, when $j = p+1, p+2,\ldots,q$, then $\Phi(x,x_1) \neq 0$ when $x \in E_1 \cap E_2$. This result generalizes to abstract spaces a well-known theorem of M. Bôcher [Proc. Amer. Acad. Sci. 40 (1904), 469-484] concerning the Jacobian of two binary forms. It is analogous to the generalization of Laguerre's theorem due to L. Hörmander [Math. Scand. 2 (1954), 55-64]. (Received January 29, 1965.)

621-32. E. J. TAFT, University of Chicago, Chicago, Illinois 60637. On d-groups of automorphisms and antiautomorphisms.

Let A be a finite-dimensional associative algebra over a field F. Let R be the radical of A, and assume A/R is a separable algebra. Let G be a completely reducible group of automorphisms and antiautomorphisms of A. Then G will leave invariant a maximal separable subalgebra of A (i. e., A has a G-invariant Wedderburn decomposition into its radical and a separable subalgebra) if any of the following four conditions is satisfied: (1) F has characteristic zero. (2) G is finite, of order not divisible by the characteristic of F, (3) G is a d-group (i. e., every element is semisimple) and F is algebraically closed, (4) G is locally nilpotent and F is perfect. Conditions (1) and (2) are not new [cf. G. D. Mostow, Amer. J. Math. 78 (1956), 200-221, and E. J. Taft, Illinois J. Math. 1 (1957), 565-573]. Condition (a) is a generalization of a result in Abstract 614-10, these *Notices* 11 (1964), 529, where G is assumed to be abelian. The proof involves extending G to its algebraic hull $\underline{G}$ in the Zariski topology, and considering a certain representation of $\underline{G}$. A similar result holds if A is a Jordan or alternative algebra, where characteristic F is not two, except that, in the Jordan case, it must also be assumed that A/R has no special simple ideal whose degree is divisible by the characteristic of F. (Received February 17, 1965.)

621-33. J. D. BUCKHOLTZ, University of Kentucky, Lexington, Kentucky. Extremal problems for sums of powers of complex numbers.

Given n complex numbers $z_1, z_2, \ldots, z_n$ with $z_1 = 1$, let $s_k = \sum_{j=1}^{n} z_j^k$, $k = 1,2,\ldots,n$. For fixed $a > 0$ and $b > -1$, lower bounds are obtained for (*) $\max k^a |s_k|$, $1 \leq k \leq n$, and (**) $\sum_{k=1}^{n} k^b |s_k|$. If $n > (3 + 3a)^3$, then (*) is greater than $(1/2)(\sqrt{(1 + a)} - 1)^2 \log n$. If $n > (6 + 3b)^3$, then (**) is greater than $(1/2)(1 + b) \log n$. Examples are constructed to show that in the second case the coefficient of log n is best possible. This implies that, in the first case, the coefficient of log n cannot be replaced by a number greater than $(1 + a)/2$. For $b = -1$ it is shown that (**) is greater than 0.278, and that this result is, essentially, best possible. The problem for $a = 0$ has been considered previously. F. V. Atkinson [Acta Math. Acad. Sci. Hungar. 12 (1961), 185-188] has shown that in this case (*) is greater than 1/6. (Received February 18, 1965.)

621-34. M. M. Cohen, University Michigan, Ann Arbor, Michigan. Transversely cellular mappings of combinatorial manifolds.

Let M^n be a close combinatorial n-manifold. A subpolyhedron X of M^n is called transversely cellular in M^n if, given any subpolyhedron M^i of M^n, containing X, such that $M^i = M^n$ or M^i is a combinatorial i-sphere ($i < n$), the regular neighborhood of X in M^i is a combinatorial i-ball. For example, X is transversely cellular if (1) X is collapsible or (2) X is cellular and $n \leqq 3$. Theorem: If T is a simplicial complex and if there exists a simplicial mapping f of M^n onto T such that $f^{-1}(x)$ is transversely cellular for each x in T, then T is combinatorially equivalent to M^n. There are examples to show that, if the word "cellular" is substituted for "transversely cellular", this theorem becomes false in dimensions $n \geqq 5$. (Received February 18, 1965.)

621-35. R. C. MacCamy, Carnegie Institute of Technology, Schenley Park, Pittsburgh, Pennsylvania 15213. On Laplace transforms and mixed problems.

The application of Laplace transform techniques to mixed initial boundary value problems is studied. Parabolic problems of the form $Lu = u_t$, L a second-order elliptic operator with coefficients depending only on x, are studied in a cylinder $R \times [0,\infty)$, where R is a bounded domain in E^n. This is a special case of the problems studied by Friedman [see, for example, J. Math. Mech. 8 (1959), 387-392]. Existence theorems and large t behavior can be obtained by studying the solution of the transformed equation as a function of transform variable. It is observed that the same methods can be used to study exterior problems for the wave equation and that results of the type of Morawetz, Phillips and Lax [Comm. Pure Appl. Math. 16 (1963), 477-486] can be obtained. (Received February 18, 1965.)

621-36. V. L. SHAPIRO, University of California, Riverside, California. Trigonometric series and the uniqueness of the heat equation.

A function u(x,t) is said to be a solution of the heat equation in the strip $0 < t < c$ if u(x,t) is in class $C^{(2)}$ in this strip and if $u_{xx}(x,t) = u_t(x,t)$ for every point of the strip. For $t > 0$, $\|u(x,t)\|_\infty$ will designate $\sup_{-\infty < x < \infty} |u(x,t)|$. Motivated by previous results in the uniqueness of trigonometric series, a theorem is obtained which contains the following theorem as a corollary: Theorem. Let u(x,t) be a solution of the heat equation in the strip $0 < t < c$ and be bounded in every substrip of the form $0 < t_0 \leqq t < c$. Suppose that (i) $\|u(x,t)\|_\infty = o(t^{-1})$ as $t \to 0$, and, (ii) for every x, $u(x,t) \to 0$ as $t \to 0$. Then u(x,t) is identically zero in the strip $0 < t < c$. A consideration of the functions k(x,t) and $k_x(x,t)$, where $k(x,t) = t^{-1/2} e^{-|x|^2/4t}$ shows that, from a certain point of view, this result is best possible in two different senses, i. e., (i) cannot be weakened to read "O(t)" and (ii) cannot be weakened to read "for every x but one". (Received February 18, 1965.)

621-37. J. D. STASHEFF, The Institute for Advaced Study, Princeton, New Jersey 08540. "Parallel" transport in fibre spaces.

The covering homotopy property of a fibre space $p: E \to B$, with fibre F, is used to define a "transport" $\theta_1: \Omega B \times F \to F$, where ΩB is the space of loops on B. For each $\lambda \in \Omega B$, $\theta(\lambda, \ ): F \to F$ is a homotopy equivalence. θ need not be transitive but, if not, auxiliary homotopies $\theta_i: I^{i-1} \times (\Omega B)^i \times F \to F$ can be constructed which serve to classify the fibre space up to fibre homotopy equivalence. In particular, given such maps θ_i, there is a method for constructing the corresponding fibre space. The maps θ_i can also be used to construct a map of B into $B_{H(F)}$, the universal base space for H(F), the monoid of homotopy equivalences of F into itself. The homotopy class of this map also classifies the fibre space so the usual classification of bundles is recovered [cf. Stasheff, Topology 2 (1963), 239-246]. (Received February 19, 1965.)

621-38. RALPH McKENZIE, Room 207, Temporary Building 1, University of Colorado, Boulder, Colorado. On the unique factorization problem for finite commutative semigroups. Preliminary report.

For definitions, see Jónsson and Tarski, Direct decompositions of finite algebraic systems, Notre Dame University, Notre Dame, Indiana, 1947. Let O_n be the semigroup with n elements such that $|O_n^2| = 1$. Let $\mathfrak{A} = (A, \circ)$ be any finite commutative semigroup, and let $\theta^{\mathfrak{A}}$ be the equivalence relation with field A defined by $(x,y) \in \theta^{\mathfrak{A}}$ if and only if $x, y \in A$ and, for all $u \in A$, $xu = yu$. Theorem 1. If $\mathfrak{A} = (A, \circ)$ is a finite commutative semigroup such that $A^2 = A$, and $1 \leqq n < \omega$, then $\mathfrak{A} \times O_n$ has the unique factorization property. Theorem 2. If $\mathfrak{A} = (A, \circ)$ is a finite commutative semigroup and $n, m > 1$ are positive integers satisfying the following conditions, then $\mathfrak{A}$ does not have the unique factorization property: (i) $(n,m) = 1$; (ii) for all $u \in A$, $nm \mid |u/\theta^{\mathfrak{A}}|$; (iii) for all $u \in A$, $|u/\theta^{\mathfrak{A}} \cap A^2| \leqq \min\{|u/\theta^{\mathfrak{A}}|/n, |u/\theta^{\mathfrak{A}}|/m\}$; (iv) there exists $u \in A$ such that $|u/\theta^{\mathfrak{A}} \cap A^2| > |u/\theta^{\mathfrak{A}}|/nm$. Theorem 2 permits the construction of a commutative semigroup with 12 elements which does not have the unique factorization property, thus answering a question posed by Chang-Jónsson and Tarski [Refinement properties for relational structures, Fund. Math. (to appear)]. (Received February 19, 1965.)

621-39. THOMAS ERBER, Illinois Institute of Technology, Chicago, Illinois 60616. Integral identities for modified Bessel functions.

Carleman's sufficient conditions for the Stieltjes' moment problem have been used to prove the following integral identities for modified Bessel functions: $K_\nu(z) = \sqrt{(2z/\pi)} \int_0^\infty d\tau (\cosh\tau)^{\nu+1/2} \cdot K_{\nu-1/2}(z\cosh\tau)$, $z > 0$, $I_m\{\nu\} = 0$; $\int_y^\infty dx K_{5/3}(x) = (\sqrt{3}/\pi) y \int_0^\infty dx \{\cosh^5 x K_{2/3}^2((y/z)\cosh^3 x) + \cosh^3 x \sinh^2 x K_{1/3}^2((y/z)\cosh^3 x)\}$, $y > 0$. (Received February 19, 1965.)

621-40. HARI SHANKAR, Ohio University, Athens, Ohio 45701. The Ahlfors-Shimizu characteristic function and the area on the Riemann sphere.

For the notations and terminology cf. W. K. Hayman, Meromorphic functions, Oxford, 1964. Let f(z) be meromorphic and nonconstant in the open plane and of finite nonzero order ρ. T, t, respectively denote, as $r \to \infty$, the lim sup and lim inf of $T_0(r,f)/r^\rho$; A, a denote the lim sup and lim inf of $A(r,f)/r^\rho$, where $T_0(r,f)$ is the Ahlfors-Shimizu characteristic of f(z) and A(r,f) is the area of the image of $|z| < r$ on the Riemann sphere by f(z). The object of this paper is to reexamine and sharpen the inequalities involving the numbers T, t, A and a, proved earlier by the author [Shankar, Tohoku Math. J. 9 (1957), 243-246]. Among other results the basic one is Theorem 1. $A\exp(\rho t/A - 1) \leqq \rho T$; $a\exp(\rho t/a - 1) \leqq \rho T$. Theorem 2. $A/\rho t \leqq g(T/t)$; $a/\rho t \geqq h(T/t)$, where g is a unique continuous function increasing from 1 to ∞ with T/t, and h is also unique and continuous but decreases from ∞ to 1 as $T/t \to 1$. Theorem 3. (i) $a \leqq A\exp(a/A - 1) \leqq A\exp(\rho t/A - 1) \leqq \rho T \leqq A \leqq e\rho T$, $t < \infty$, $A \neq 0$. (ii) $\rho t h(A/\rho t) \leqq \rho t h(T/t) \leqq a \leqq \rho t \leqq a(1 + \log(\rho T/a)) \leqq a(1 + \log(A/a)) \leqq A \leqq \rho t g(T/t) \leqq \rho t g(A/\rho t)$, $t < \infty$, $a \neq 0$. (Received February 19, 1965.)

621-41. R. D. DIXON and LOWELL SCHOENFELD, Mathematics Research Center, University of Wisconsin, Madison, Wisconsin. On the size of the Riemann zeta-function at places symmetric with respect to the point 1/2.

As usual, let $s = \sigma + it$. In a paper to appear in the Duke Math. J. R. Spira proves that $|\zeta(1 - s)| > |\zeta(s)|$ provided $t \geqq 10$, $1/2 < \sigma < 1$ and s is not a zero of ζ. Here we give a much simpler proof that the above inequality holds if the following weaker conditions are satisfied: $|t| \geqq 6.8$ and $\sigma > 1/2$. Aside from the functional equation and Stirling's formula, the proof uses only the elementary relation $(\partial/\partial\sigma) \log |f(\sigma + it)| = \mathrm{Re}(d/ds)\log f(s)$. (Received February 19, 1965.)

621-42. HASKELL ROSENTHAL, University of Minnesota, Minneapolis, Minnesota. Projections onto translation-invariant subspaces of $L^1(G)$.

The notation is as used in Rudin, Fourier analysis on groups, Interscience, New York, 1962. Let G be a LCA group endowed with a fixed Haar measure; Γ the dual group of G; let A be a non-zero closed translation-invariant subspace of $L^1(G)$, and define the hull of A as $\{\tau \in \Gamma: \hat{f}(\tau) = 0$ for all $f \in A\}$. Theorem 1. If there is a bounded linear projection for $L^1(G)$ onto A, the hull of A must belong to the coset-ring of Γ_d. Theorem 2. Let G be the group of real numbers; then if there is a bounded linear projection onto A, the hull of A must be discrete; in fact, the hull of A must differ by a finite set from a finite union of stretched cosets of the integers. Corollary 3. Let G be the group of integers; then there is a bounded linear projection onto A if and only if the hull of A is finite. Corollary 4. Let G be an arbitrary LCA group; then $L^1(G)/A$ is not linearly isomorphic to L^1 of any measure space unless the hull of A belongs to the coset-ring of Γ_d. (Received February 10, 1965.)

621-43. J. M. SKOWRONSKI and RUEY-WEN LIU, University of Notre Dame, Notre Dame, Indiana. Periodical and almost periodical steady states for physical systems.

The response $x(x^0,t_0,t)$ of a general type of physical (mechanical and electrical) system is considered locally in $\Omega_H \subset \Omega \subset E^{n+1}(x_1,\dots,x_n,t)$: $\dot{x} = f(x,t)$, with assigned assumptions on f. $\Omega = \mathscr{T} \times \Delta, \mathscr{T}$: $t \in [t_0,\infty)$, Δ-open set in E^n; $(x^0,t_0) \in \Omega_a$, Ω_a: $(|x^0| < a, t_0 \in [0,\infty))$; f, a such that $\Omega_a \subset \Omega_H$, where Ω_H is defined by some property of H(x) — total energy of the system. Assuming uniform, ultimate boundedness of x in $\Omega_a \subset \Omega_H$, the behavior of x is analysed in terms of assigned H, $\dot{H}$ along $\mathscr{T}$, $H = H(H^0,t_0,t)$. Let u,x be two arbitrary particular responses: $u = x(u^0,t_0,t)$; given $t \in \mathscr{T}$: $\delta x = u - x$, $\delta H = H(u) - H(x)$, $\delta\dot{H} = \dot{H}(u) - \dot{H}(x)$. Given characteristics of the system (assumptions on f), the conditions for the existence of a limit steady state set: $|\delta H| \to 0$ for $t \to \infty$, are studied. If so, f periodical or almost periodical in t imply the periodical or almost periodical steady states. Pliss and Zubov convergence is discussed. (Received February 22, 1965.)

621-44. E. P. MERKES, University of Cincinnati, Cincinnati, Ohio 45221 and W. T. SCOTT, Arizona State University, Tuscon, Arizona. Covering theorems for starlike and convex functions.

Let U_n denote the class of univalent functions $f(z) = z + a_2z^2 + \dots$, $a_2 \geqq 0$, in $|z| < 1$, and let S_n and C_n, respectively, denote the subclasses of starlike functions and of convex functions. For f in U_n, $\rho(\phi,f)$ represents the distance along a fixed ray arg $w = \phi$ from $w = 0$ to the nearest boundary point of the image of $|z| < 1$ by $w = f(z)$. Let $u(\phi) = \inf \rho(\phi,f)$, for f in U_n. Define $s(\phi)$ for S_n and $c(\phi)$ for C_n analogously. In conjunction with a previous result of Scott [Amer. Math. Monthly 64 (1957), 90-94] it is shown that $1/2 = u(\phi) = s(\phi) < c(\phi) \leqq \pi/4$ for $0 \leqq |\phi| \leqq \pi/2$, $1/4 = u(\pi) = s(\pi) < c(\pi) = 1/2$, and $u(\phi) < s(\phi) < c(\phi)$ for $\pi/2 < |\phi| < \pi$. Estimates for $u(\phi)$, $s(\phi)$, $c(\phi)$ are given in the latter interval. (Received February 22, 1965.)

621-45. BERTRAM WALSH, University of California, 405 Hilgard Avenue, Los Angeles, California 90024. Interpolation of operational calculi. Preliminary report.

Let E_1, E_2 be Banach spaces contained in some linear space, with $E_1 \cap E_2$ dense in either E_i; let F be an interpolation space for E_1 and E_2, e. g., $E_1 = L^p$, $E_2 = L^q$, $F = L^r$, $p \leqq r \leqq q$. Let T be an operator defined on $E_1 \cap E_2$ which has a continuous extension to both E_i, thus to F; suppose both these extensions to E_i admit operational calculi for C^∞ functions, i. e., are "generalized scalar operators" in the sense of Foias [Bull. Sci. Math. 84(1960), 147-158]. If the spectra of these extensions are real, or under a mild additional hypothesis otherwise, the extension of T to F is also generalized scalar. Further, the spectra of all three extensions coincide, and some fine structure properties of the spectra are preserved. (Received February 22, 1965.)

621-46. G. R. BLAKLEY, University of Illinois, Urbana, Illinois and D. R. DIXON, Dayton Campus, Ohio State University, Dayton, Ohio. Holder type inequalities in cones.

A simple proof is given of the inequality $(u,u)^k(u,S^{j+k}u)^j \geqq (u,S^ju)^{j+k}$, where k is a positive integer, $j = 1$, u is a nonnegative vector, and S is a nonnegative symmetric matrix. This was first proved by Blakley and Roy. A conjecture of theirs that the above inequality holds for j positive and odd is verified in the presence of an additional condition on (u,Su). (Received February 22, 1965.)

621-47. T. A. BRONIKOWSKI, Marquette University, Milwaukee, Wisconsin 53233. On systems of integrodifferential equations occurring in reactor dynamics.

Consider the real linear system (i) $u'(t) = -\int_0^c \alpha(x)T(x,t)\,dx$, $aT_t(x,t) = bT_{xx} + \eta(x)u(t)$, in the region $0 < x < c$, $0 < t < \infty$. The functions $\alpha, \eta \in L_2(0,c)$ and the positive constants a,b are given. The initial and boundary conditions are $u(0) = u_0$, $T(x,0) = f(x)$ $(0 < x < c)$, $T(0,t) = T(c,t) = 0$ $(0 < t < \infty)$. Modifying techniques employed by Levin and Nobel [J. Math. Mech. 9 (1960), 347-368; Arch. Rational Mech. Anal 11 (1962), 210-243], It is shown that the preceding is a properly posed problem. Moreover, under suitable additional conditions, it is shown that the solution u(t), T(x,t) tends exponentially to zero as $t \to \infty$. Also, the behavior of the solutions as $b \to 0$ and as $c \to \infty$ is investigated. Several generalizations of (I) are considered. (Received February 22, 1965.)

621-48. ADRIANO GARSIA, California Institute of Technology, Pasadena, California. On the convergence problem for Fourier series.

Salem [Nederl. Akad. Wettensch. Proc. Ser. A. 57 (1954), 550-555] proved that if a function f(x) satisfies the condition (*) $(1/\Delta)\int_0^\Delta(f(x + t) - f(x - t))\,dt = o(1/\log(1/\Delta))$ uniformly at the points of an interval, then, in every compact subinterval, the Fourier series of f(x) converges uniformly. The results of some calculations involving the Dirichlet kernel will be presented which seem to throw some light on the role of the condition (*). A new proof of Salem's result is thus obtained along with some new global integral conditions for convergence. (Received February 22, 1965.)

621-49. TATSUO KAWATA, Catholic University of America, Washington, D. C. 20017.
On the Fourier series of a stationary stochastic process.

Let X(t) be a weakly stationary process with mean 0. Many applications have been made of the Fourier series of the process, particularly in engineering mathematics. However, few papers have been devoted to a mathematical analysis of these applications. Let us take a sample of the process X(t) $(0 < t < T)$. It is known that the Fourier series of X(t) converges in quadratic mean for every $0 < t < T$, T being fixed. It is shown that if $\int_{-\infty}^{+\infty}\log^{+}|x|dF(x) < \infty$, where F(x) is the spectral distribution of X(t), then the Fourier series converges almost everywhere with probability 1 and if $\int_{-\infty}^{\infty}|x|^{\alpha}\, dF(x) < \infty$, $\alpha > 1$, the Fourier series converges absolutely with probability 1. The behavior of the Fourier coefficients when $T \to \infty$ is also considered. It is shown that if X(t) has a spectral density and so is of the type $\int_{-\infty}^{\infty}C(t - s)dy(s,\omega)$, $C(t) \in L_2(-\infty,\infty)$ and if $y(s,\omega)$ has independent increments with $E|dy(s)|^2 = ds$ and $C(t) \in L_1(-\infty,\infty)$, then the joint distribution of the Fourier coefficients $\sqrt{T}\{A_0\pi, A_1,\dots,A_n, B_1,\dots,B_n\}$ converges to the $(2n + 1)$-fold convolution of the distribution $N(0, 4\pi(\int_{-\infty}^{\infty}C(s)ds)^2)$. (Received February 22, 1965.)

621-50. G. R. ANDERSON, Catholic University of America, Washington, D. C. 20017.
On the law of the iterated logarithm.

Let $\{X_k\}$ be a sequence of positive independent random variables. If $\phi_k(\cdot,\lambda)$, $\lambda > 0$, is the Poisson integral of the characteristic function, f_k, of X_k $(k = 1,2,\dots)$ then, for each $\lambda > 0$ and k, $k = 1,2,\dots$, $\phi_k(\cdot,\lambda)\phi_k^{-1}(0,\lambda)$ is the Fourier transform of a distribution function $F_k(\cdot,\lambda)$. Let $\{X_k(\lambda)\}$ denote a sequence of independent random variables defined by the sequence $\{F_k(\cdot,\lambda)\}$. $\{X_k(\lambda)\}$ is called the Poisson-shifted sequence of random variables generated by $\{X_k\}$ and λ. A characterization of the a.e. convergence of $\sum X_k$ is provided in terms of $\{X_k(\lambda_n)\}$, $n = 1,2,\dots$, $\lambda_n > 0$. Assume, further, that X_k $(k = 1,2,\dots)$ has finite variance, $s_n^2 = \mathrm{Var}(X_1 + \dots + X_n)$, $s_n \to +\infty$, $\max_{k\leq n}EX_k = o(s_n(\mathrm{lglg}\, s_n^2)^{-1/2})$. Then if $L_n(s_n(\mathrm{lglg}\, s_n^2)^{-1/2}) = o(1)$, where L_n denotes the Lindeberg quotient, it is shown that there exists a numerical sequence $\lambda_n \searrow 0$ such that $\lim\sup(2s_n^2\mathrm{lglg}\, s_n^2)^{-1/2}S_n(\lambda_n) = 1$, a.e., where $S_n(\lambda_n) = X_1'(\lambda_n) + \dots + X_n'(\lambda_n)$, $X_k'(\lambda_n)$ being the centered Poisson-shifted random variable generated by X_k and λ_n, and $s_n^2 \sim B_n^2$, the variance of $S_n(\lambda_n)$. (Received February 22, 1965.)

621-51. E. F. WISHART, Florida State University, Tallahassee, Florida. Higher derivations on π-adic fields. Preliminary report.

Let K_e be an Eisenstein extension of a p-adic field K by a root, π, of $f(x) = x^e + p\sum_{i=1}^{e-1}a_ix^i$. Results of Heerema and Neggers are extended as follows: (1) If $(e,p) = 1$, then every infinite higher derivation on the residue field k is induced via the place by an infinite higher derivation on K_e. π-adic fields with this property are said to have property H^{∞}. (2) If $p = e$, necessary and sufficient conditions on the polynomial f(x) are derived for K_p to have property H^{∞}. Also the interconnections among the following are investigated: (1) property H^{∞}, (2) the representation of inertial automorphisms using higher derivations and (3) the factor groups of the ramification groups of K_e. (Received February 22, 1965.)

621-52. T. G. HALLAM, University of Missouri, Columbia, Missouri. Asymptotic behavior of the solutions of an ordinary nonlinear differential equation.

The nth order differential equation (*) $u^{(n)} + f(t,u) = h(t)$, where f(t,u) and h(t) are real functions of a real variable, is considered, subject to the following hypotheses: (1) $|f(t,u)| \leqq g(t)|u|^r$, where the inequality need hold only for large values of t, $r > 0$, and $\lim_{t\to\infty} t^p g(t) = c \neq 0$ for some p; (2) $h(t) = h_m(t)t^m + R(t)$, where $m > -1$, $\lim_{t\to\infty} h_m(t) = a_m \neq 0$, and $R(t) = o(t^m)$ as $t \to \infty$. If the existence of solutions and the integrability of h(t) and g(t) are assumed, then the following result is obtained. Theorem. If $p > rm + rn - m$, then all solutions u(t) of (*), valid for large t, have the asymptotic behavior $u(t)/t^{m+n} \sim k \neq 0$. A partial converse for the above theorem shows the result is almost the best possible. Other types of asymptotic behavior are considered. If (*) is homogeneous and $f(t,u) = g(t)u^{2n+1}$, where n is a nonnegative integer, a necessary and sufficient condition for the asymptotic behavior $u(t)/t \sim k \neq 0$ is given. (Received February 22, 1965.)

621-53. A. R. FREEDMAN, Oregon State University, Corvallis, Oregon. A density inequality for the sum of two sets of lattice points.

Let I^n denote the set of all n-tuples $(x_1,\ldots,x_n)$, where each x_i is a non-negative integer and $\sum x_i > 0$. For X, $D \subseteq I^n$, let X(D) denote the cardinality of the set $X \cap D$. The density of a set $A \subseteq I^n$ is defined to be the greatest lower bound of the fractions $A(D)/I^n(D)$, where D ranges over all sets of the form $\{\bar{y} | \bar{y} = (y_1,\ldots,y_n) \in I^n, 0 \leqq y_i \leqq x_i, i = 1,2,\ldots,n\}$ for some $(x_1,\ldots,x_n) \in I^n$. Let A, $B \subseteq I^n$ and let A + B denote the set $A \cup B \cup \{\bar{a} + \bar{b} | \bar{a} \in A, \bar{b} \in B\}$. Let α, β, γ be the densities of A, B, A + B, respectively. It is shown that $\gamma \geqq \alpha + (1 - (1 - \alpha)^{1/(n-1)})(\beta/n)(1 - \alpha)$. This improves a conjecture of F. Kasch [J. Reine Angew. Math. 197 (1957), 208-215]. The proof is a refinement of a method used by Kasch in proving the above inequality for the special case n = 2. Use is made of a result of Loomis and Whitney [Bull. Amer. Math. Soc. 55 (1949), 961-962]. (Received February 22, 1965.)

621-54. PETER WERNER, The University of Wisconsin, Madison, Wisconsin 53706. Boundary value problems for the time-independent Maxwell equations with variable coefficients.

The determination of the stationary electromagnetic field with frequency ω which is produced by the reflection of a given incoming field at a perfect conductor with the surface S leads to the following boundary value problem: Find two vector fields, E(x) and H(x), continuously differentiable in the exterior D of S and continuous in D + S, such that (a) E and H satisfy in D the time-independent Maxwell equations $\nabla \times E - i\omega\mu H = 0, \nabla \times H + i\omega\epsilon E = 0$, (b) E satisfies the boundary condition $n \times E = c$ on S where c is a prescribed tangential field, (c) each component of E and H satisfies the Sommerfeld radiation conditions $(\partial/\partial r - ik)U = o(r^{-1})$ as $r \to \infty$ with $k^2 = \omega^2\epsilon\mu$, Im $k \geqq 0$. This boundary value problem has been discussed by several authors in the special case in which ϵ and μ

are constant. In this paper these methods are extended to variable coefficients ϵ and μ. By using the theory of dipole fields contained in a previous paper [Arch. Rational Mech. Anal. 16 (1964), 1-33], the boundary value problem is reduced to a uniquely solvable Fredholm integral equation. As a consequence, several existence and dependence theorems are obtained. A discussion of the behavior of the solution as $\omega \rightarrow 0$ is included. (Received February 19, 1965.)

621-55. ADRIANO BARLOTTI, University of North Carolina, Chapel Hill, North Carolina. On the non-existence of some (k,n)-arcs in certain projective planes.

A (k,n)-arc of a finite projective plane of order q, $\pi(q)$, is a set of k points such that no n + 1 of them are collinear. A line of the plane which contains r $(1 \leqq r \leqq n - 1)$ points of a given (k,n)-arc is called a tangent of order r to the (k,n)-arc. The following theorem holds: The necessary condition for the existence in $\pi(q)$ (with q equal to the power of a prime) of an $\{(n-1)q+1,n\}$-arc having the property that through each of its points there passes exactly one tangent of order one is that either $q \equiv 0 \pmod n$ or $q \equiv 1 \pmod n$. (Received February 23, 1965.)

621-56. J. C. WILSON, Southern Illinois University, Carbondale, Illinois. Algebraic periodic solutions of $\ddot{x} + f(x)\dot{x} + x = h(t)$; $h(t + t_0) = h(t)$.

We give examples of algebraic curves whose closed components in the phase plane represent periodic solutions of $\ddot{x} + f(x)\dot{x} + x = h(t)$, where h has period t_0. Also, we obtain an algebraic approximation to the periodic solution of a certain perturbed Van der Pol equation. (Received February 23, 1965.)

621-57. G. W. GOES, Illinois Institute of Technology, Chicago 16, Illinois. Fourier-Stieltjes-sine-series with finitely many distinct coefficients. Preliminary report.

Let $\sum_{j=1}^{\infty} b_j \sin jt$ be a Fourier-Stieltjes-series with only finitely many distinct coefficients. Then $\{b_j\}$ is ultimately periodic as shown by Helson [Proc. Amer. Math. Soc. 6 (1955), 235-242]. Additionally it can be shown: If $\{b_j\}$ $(r \leqq j < \infty)$ is the periodic part with period p, then there exists to every $b_j \geqq 0$ $(r \leqq j \leqq r + p - 1)$ a $b_{j'}$ $(r \leqq j' \leqq r + p - 1)$ with $b_{j'} = - b_j$. In particular: (i) If all $b_j \geqq 0$ for $j \geqq r$, then $b_j = 0$ for all $j \geqq r$. (ii) If p is odd, then at least one b_j is zero in each period. For cosine series the situation is (of course) different. (Received February 23, 1965.)

621-58. F. M. WRIGHT, Iowa State University, Ames, Iowa. The Young sigma integral and area.

The author has described a rather standard method for associating with a quasi-monotone non-decreasing real-valued function g on E^2 an outer area function A_g^*, as well as what is believed to be a novel method for associating with g an inner area function $A_{\underline{*g}}$ (Abstract 620-22, these *Notices* 12 (1965), 216). In this paper an upper Young sigma integral $Y\sigma \iint_I f(x,y)dg(x,y)$ and a lower Young sigma integral $Y\sigma \underline{\iint}_I f(x,y)dg(x,y)$ are considered for I a nondegenerate finite closed interval of E^2, and for f a bounded real-valued function on I. It is shown that if S is a bounded subset of E^2, then, for I a nondegenerate finite closed interval of E^2 which contains S, and for θ the real-valued function on I such that $\theta(x,y) = 1$ or 0 according as (x,y) is in S or not, it follows that $A^*_g(S) = Y\sigma \overline{\iint}_I \theta(x,y)dg(x,y)$ and also that $A_{*g}(S) = Y\sigma \underline{\iint}_I \theta(x,y)dg(x,y)$. A result is established relative to the Young sigma integral $Y\sigma\iint_I f(x,y)dg(x,y)$ for f a bounded real-valued function on a nondegenerate finite closed interval I of E^2, defined in terms of the above upper Young sigma integral and lower Young sigma integral, and the Stieltjes sigma integral of f with respect to g over I involving certain one-sided Young sigma integrals over parts of the boundary of I. (Received February 23, 1965.)

621-59. J. W. GRAY, University of Illinois, Urbana, Illinois. Cech cohomology of sheaves with values in a category.

A projective object P in a category is called small if Hom(P,-) preserves direct limits. Assume that $\underline{A}$ is an abelian category with a generating family of small projectives and with exact direct limits. Let X denote a paracompact, Hausdorff space, let $\underline{S}$ denote the category of sheaves on X with values in $\underline{A}$, and let $\check{H}(X,-)$ denote Cech cohomology. Theorem 1. $\check{H}(X,-)$ is a cohomological functor on $\underline{S}$. Proof. One can show in this context that, à la Serre, for a presheaf F whose associated sheaf is zero one has $\check{C}(X,F) = 0$. Theorem 2. $\check{H}(X,-)$ agrees with the cohomology given by injective resolutions. Proof. One shows, à la Godement, that a short exact sequence of sheaves whose first term is flasque is exact as presheaves. The interesting thing is that these theorems depend not only on the base space X, but also on the value category $\underline{A}$. (Received February 23, 1965.)

621-60. G. C. GASTL and P. C. HAMMER, University of Wisconsin, 435 N. Park Street, Madison, Wisconsin. Extended topology: Neighborhoods and convergents in isotonic spaces.

Let M be our space and $\mathcal{M}$ the class of its subsets. Suppose u: $\mathcal{M} \to \mathcal{M}$ is an isotonic function and v = cuc its dual isotonic function, where c is the complement function. Then we define neighborhoods and convergents as follows: $X \subseteq M$ is a convergent of p if $p \in uX$, and $Y \subseteq M$ is a neighborhood of p if $p \in vY$. We show that a set is a neighborhood of p if and only if it samples every convergent of p, and a set is a convergent of p if and only if it samples every neighborhood of p. The duality of convergent and neighborhood is clear. We see how certain conditions, e.g., additivity and idempotence, placed on u and v affect the class of convergents and the class of neighborhoods of a point. We also consider bases for these classes. (Received February 23, 1965.)

621-61. S. G. MROWKA, The Pennsylvania State University, 227 MacAllister Hall, University Park, Pennsylvania. On C-embedding.

Y denotes a completely regular superspace of X. Theorem. If X is locally compact Lindelöf, then X is C-embedded in Y iff there is a continuous function g: $Y \to R$ (R = the reals) such that g is unbounded on every $A \subset X$ with $cl_X(A)$ noncompact. This yields the following property of locally compact Lindelöf spaces X: (P) there is one continuous function f: $X \to R$ such that X is C-embedded in Y iff this function f can be continuously extended over Y. Spaces with property (P) are examined. (Received February 23, 1965.)

621-62. R. A. ASKEY, University of Wisconsin, Madison, Wisconsin. Uniform convergence of Legendre series. Preliminary report.

Suetin [Dokl. Akad. Nauk SSSR 158 (1964), 1275-1277] has shown that if $f \in \mathrm{Lip}\,\alpha$, $\alpha > 1/2$, then its Legendre series converges uniformly. Here Lip α is defined by $|f(x) - f(y)| \leqq A|x - y|^{\alpha}$. The same theorem is shown to hold if Lip α is defined by $|f(x;h) - f(x)| \leqq A|1 - h|^{\alpha/2}$, where $f(x;h) \sim \sum a_n P_n(x) P_n(h)$ is the generalized translation of f in the sense of Levitan. The proof uses elementary facts about fractional integrals. The advantage of this proof over the one given by Suetin is that this proof probably goes over to the sphere and so will answer the question of uniform convergence of Laplace series. (Received February 23, 1965.)

621-63. A. J. HECKENBACH, Iowa State University, Ames, Iowa. A generalization of dynamical systems.

Let (X,d) be a metric space, R the reals and f a map of $X \times R^2$ to X. (X,R,f) is called an N-system provided: (i) for fixed p,q, f(x,p,q) is a homeomorphism of X onto X; (ii) f(x,p,q) is continuous in the pair (x,q); (iii) f(x,p,0) = x for all x,p; (iv) f(f(x,p,q),p + q,r) = f(x,p,q + r) for all x,p,q,r. A dynamical system (X,π) is an N-system (X,R,f), where $\pi(x,t) = f(x,p,t)$. For fixed x,p, the set $O_p(x) = \{y: y = f(x,p,q),\ q \in R\}$ is called the p-orbit of x and the function f(x,p,q) is called the p-motion of x. If the system of differential equations x' = F(t,x) has unique solutions which are continuable for all t, there is an N-system whose p-orbits are the solution curves of this equation. Let the p-motion f(x,p,q) have a compact positive limit set and let there be a real sequence $t_n \longrightarrow \infty$ such that $f(x,p + t_n,q)$ converges to f(x,p,q) for all x,p,q. Sufficient conditions, depending on the sets on which this convergence is uniform, are given for the existence of bounded, pointwise recurrent and recurrent p-motions. (Received February 24, 1965.)

621-64. P. O. FREDERICKSON, Case Institute of Technology, University Circle, Cleveland, Ohio 44106. Cesàro summability for a class of generalized series.

The set M of all multi-indices may be characterized as a countable weak direct product in which each factor is the set of non-negative integers. The generalized sequences and series under consideration are those which have M as index set; they are readily seen to have many of the properties of ordinary sequences and series. Linear summability methods, for example, may be defined for these generalized sequences. In particular, Cesàro summability is defined in terms of the generalized binomial coefficients $C_{m,n} = \prod C_{m_s,n_s}$. The fact that little is lost in this generalization is evident from the following Theorem. If $\sum a_m$ is (C,p) summable to A and $\sum b_m$ is (C,q) summable to B, then their Cauchy product $\sum c_n$ is (C,p + q + 1) summable to AB. The above is useful in obtaining a series solution to certain differential equations. (Received February 24, 1965.)

621-65. KEIO NAGAMI and J. H. ROBERTS, Duke University, Durham, North Carolina. A note on countable-dimensional metric spaces.

A space is called countable-dimensional (respectively strongly countable-dimensional) if it is the countable sum of finite-dimensional (respectively, finite-dimensional closed) subsets.

Theorem 1. A metric space X is countable-dimensional if and only if for every sequence of pairs of disjoint closed sets C_1, C'_1, C_2, C'_2,... there exist separating closed sets B_i between C_i and C'_i, i = 1,2,..., such that $\{B_i\}$ is point-finite. Theorem 2. A metric space X is strongly countable-dimensional if and only if there exists a sequence U_1, U_2,... of open coverings of X such that (i) $\{St(x,U_i^{\Delta}), i = 1,2,...\}$ is a local base of $x \in X$, (ii) sup ord $(x,U_i) < \infty$ for $x \in X$. Theorem 3. K_ω has no metric-completion which is even countable-dimension. Here K_ω is the universal strongly countable-dimensional space due to Nagata which is strongly countable-dimensional. These are supplementary results to Nagata's characterization of countable-dimensional spaces. (Received February 24, 1965.)

621-66. PRABIR ROY, 213 Van Vleck Hall, University of Wisconsin, Madison, Wisconsin.
Locally flat k-cells and spheres in E^n are stably flat if $k \leqq 2n/3 - 1$.

A k-cell D in E^n is locally flat means that for each point p in D there is a neighborhood C and a homeomorphism h of E^n with h(C) the unit ball in E^n and $h(C \cap D)$ the unit ball in a k-hyperplane in E^n; it is flat if there is a homeomorphism h of E^n with h(D) the unit ball in a k-hyperplane; it is stably flat if there is a flattening homeomorphism (as above) which in addition is stable, i.e., composite of finite number of homeomorphisms each equal to identity on some open set. It is known that locally flat k-cells in E^n are flat. The proof is by induction on k and engulfing. (Received February 24, 1965.)

621-67. MAURICE HOROWITZ, The Magnavox Company, 2131 Bueter Road, Fort Wayne, Indiana 46803. A 1965 approach to a 1665 problem. Preliminary report.

A progress report on attempting to understand Fermat's Last Theorem, Euler's Conjecture, and other similar conjectures is made. A general problem is stated thus: Find constraints on N so that $F = F_K(X_S) = F(X_1,X_2,...,X_S,...,X_{K+1},N) = 0$ has a solution with S = 1,2,...,K + 1 and when X_S,N,F, and K are positive integers. By information theory, perhaps, $H(N) + \sum_{S=1}^{K+1} H(X_S) \geqq H(F)$, where H(P) is the nonredundant bit length of the integer P, and H(F) is the length in bits needed to distinguish F. This information conjecture simply says if K + 1 variables exist which contain H(K) nonredundant bits, then no function which is a positive integer of the K + 1 variables exists which contains more than H nonredundant bits; i.e., information is not "created" except via irrational numbers. The applications to Fermat's Last Theorem and Euler's Conjecture immediately follow, taking advantage of an inequality relating H(P) and $H(P^N)$, and of the fact that if X_S is a solution, so is MX_S, where M is a positive integer, however large. The validity of attempting to employ information theory in this fashion is discussed. (Received February 12, 1965.)

621-68. MARY WEISS, DePaul University, Chicago, Illinois. An example in the theory of singular integrals.

It is known that if $f(x) \in L\ (E\ )$, $p > 1$, and $K(x) = |x|^{-n}\Omega(x')$ where x' is the projection of x onto the unit sphere Σ, then the convolution integral $(K * f)(x)$ exists a.e. provided $|\Omega|\log^*|\Omega|$ is integrable over Σ and $\int_\Sigma \Omega = 0$. The integrability of $|\Omega|\log^*|\Omega|$ is also a necessary condition for the existence of the convolution integral and we give an example to show that this is so. (Received February 24, 1965.)

621-69. HENRY HELSON, University of California, Berkeley, California 94720. Compact groups with ordered duals.

Let K be a compact abelian group with dual Γ, $\Gamma \subset R_d$ (the group of reals in discrete topology), but assume K is not a circle. For each real t let e_t be the element of K such that $e_t(\lambda) = \exp(i\lambda t)$, $\lambda \in \Gamma$. These points form a dense one-parameter subgroup of K, dual in a sense to the order relation in Γ. Say a function f in $L^2(K)$ is analytic if its Fourier coefficients $a(\lambda)$ vanish for all $\lambda < 0$. Theorem. f in $L^2(K)$ is analytic iff $f(x + e_t)$ is analytic as a function of t for almost all x, in the sense that the Fourier transform of $f(x + e_t)/(1 - it)$ vanishes a.e. on the negative half-line. A subspace $\mathcal{M}$ of $L^2(K)$ is called simply invariant if multiplication with a character λ carries functions of $\mathcal{M}$ into $\mathcal{M}$ for each $\lambda > 0$, and if $\lambda \cdot \mathcal{M} \subset \mathcal{M}$, $\lambda \cdot \mathcal{M} \neq \mathcal{M}$, for $\lambda > 0$. A function called a cocycle, previously introduced in a paper with Lowdenslager, is attached to each such $\mathcal{M}$: $A = A(t,x)$ is a measurable function of modulus 1 a.e. defined on the product of the line with K, satisfying $A(t + u,x) = A(t,x)A(u, x + e_t)$. If $\mathcal{M}$ is suitably normalized, a function f in $L^2(K)$ is in $\mathcal{M}$ iff $A(t,x)f(x + e_t)$ is analytic in t for almost all x. Inclusion of subspaces is related to factoring of cocycles, and an operation on subspaces is determined corresponding to multiplying their cocycles. (Received March 1, 1965.)

The February Meeting in New York
February 27, 1965

620-23. G. G. WEILL, 601 West 183rd Street, New York, New York 10033. On Bergman's kernel function for some uniformly elliptic partial differential equations.

A generalization of the theory of Bergman's kernel function is presented for uniformly elliptic partial differential equations of the second order, of divergence type $D_k(a_{ik}(x)D_i u) = 0$ where $x \in \Omega$, Ω a regular open set in R^n. Let $E \subset \Omega$ be compact, and consider the family of regular solutions of $D_k(a_{ik}(x)D_i u) = 0$ vanishing at $x_0 \in \Omega$. The existence of the kernel follows from an estimate $|u(x)|^2 \leqq C(E) \int_\Omega a_{ik}(x)D_i u D_k u dx$ for $x \in E$ where $c(E)$ is a constant whose value depends only on E. (Received February 15, 1965.)

The April Meeting in New York
April 12-15, 1965

622-1. P. C. FISCHER, Pierce Hall, Harvard University, Cambridge 38, Massachusetts. The generation of primes by a one-dimensional, real-time array of finite-state machines.

A sequential device is said to generate an infinite sequence a_1, a_2, a_3, ... in real time if a_t is generated by time t. A one-dimensional iterative array is an infinite set of identical finite-state machines M_1, M_2, M_3, ... in which the state of machine M_i at time t + 1 depends only on the states of M_{i-1}, M_i, and M_{i+1} at time t. All machines except M_1 are started in the same state at time 0, and designated outputs of M_1 are the output of the array. Theorem. Let $a_t = 1$ if t is prime and $a_t = 0$ if t is not prime. Then there exists a one-dimensional iterative array which generates $a_1, a_2, a_3, \ldots$ in real time. (Received November 23, 1964.)

622-2. OVED SHISHA and BERTRAM MOND, Aerospace Research Laboratories, Wright-Patterson AFB, Ohio. Hermite-Fejér polynomials for functions of several variables.

The Hermite-Fejér polynomials $H_n(f,x)$ are important approximating polynomials to a given real function f(x) defined on [- 1,1]. For every positive integer n, $H_n(f,x)$ is of degree $\leqq 2n - 1$ and satisfies $H_n(f,x_k^{(n)}) = f(x_k^{(n)})$, $H_n'(f,x_k^{(n)}) = 0$ where $x_k^{(n)} = \cos[(2k - 1)\pi/2n]$ $(k = 1,2,\ldots,n)$. A classical result of Fejér [Göttinger Nachrichten (1916), 66-91] states that if f is continuous on [- 1,1], then $H_n(f,x)$ converges uniformly to f on [- 1,1]. We define an analogue of $H_n(f,x)$ for functions $f(x_1,x_2,\ldots,x_p)$ of several variables; this analogue is a polynomial in these variables. Results are obtained on the rapidity of convergence of these polynomials to $f(x_1,x_2,\ldots,x_p)$. (Received November 30, 1964.)

622-3. S. P. GUDDER, University of Wisconsin, Madison, Wisconsin. A uniqueness property for bounded observables.

Let L be an orthocomplemented lattice. (For definitions and notation, cf. Varadarajan, Comm. Pure Appl. Math. 15 (1962), 189-217.) We assume that L has the following property. If m(b) = 1 whenever m(a) = 1 for a,b in L and states m, then $a \leqq b$. If x is a bounded observable and m a state, the expectation of x under m is defined as $m(x) = \int \lambda m(x(d\lambda))$. Theorem 1. If the bounded observables x,y satisfy m(x) = m(y) for every state m on L, then $x(\lambda) = y(\lambda)$ for every real number λ. Theorem 2. If x,y are bounded observables, x has countable spectrum, and m(x) = m(y) for every state m on L, then x = y. (Received January 11, 1965.)

622-4. BERTRAM MOND, ARL (ARM) Building 450, Wright-Patterson AFB, Ohio. A matrix generalization of Kantorovich's inequality.

Let A be a positive definite hermitian matrix with eigenvalues $\lambda_1 \geqq \lambda_2 \geqq \ldots \geqq \lambda_n$, $\lambda_1 > \lambda_n$.

There exists a unitary matrix U such that $A = U^*[\lambda_1,\ldots,\lambda_n]U$. Powers of A are defined by $A^q = U^*[\lambda_1^q,\ldots,\lambda_n^q]U$. Set $\gamma = \lambda_1/\lambda_n$ and let r and s be real numbers, $r < s$, $rs \neq 0$. Theorem. For any n-dimensional vector x of unit norm, $1 \leqq (A^s x,x)^{1/s}/(A^r x,x)^{1/r} \leqq \{[r(\gamma^s - \gamma^r)]/[(s - r)(\gamma^r - 1)]\}^{1/s} \times \{[s(\gamma^r - \gamma^s)]/[(r - s)(\gamma^s - 1)]\}^{-1/r}$. Let $y = Ux$, U a unitary matrix such that $A^s = U^*[\lambda_1^s,\ldots,\lambda_n^s]U$ and $A^r = U^*[\lambda_1^r,\ldots,\lambda_n^r]U$. Let S be the set of all k (k = 1,...,n) such that $y_k \neq 0$. Equality on the right holds if and only if there exists a p, $1 \leqq p < n$, such that $\sum_{k=1}^{p} |y_k|^2 = [r/(\gamma^r - 1) - s/(\gamma^s - 1)]/(s - r)$, $\lambda_k = \lambda_1$ (k = 1,...,p) and $\lambda_k = \lambda_n$ for every k satisfying $p < k$, $k \in S$. Equality on the left holds if and only if all λ_k ($k \in S$) are equal. Corresponding results are obtained for r or s equal to zero where $(A^0x,x)^{1/0}$ means $\lim_{q\to 0}(A^q x,x)^{1/q}$. Letting s = 1 and r = - 1, the theorem yields $1 \leqq (Ax,x)(A^{-1}x,x) \leqq (\lambda_1 + \lambda_n)^2/4\lambda_1\lambda_n$, the well-known Kantorovich inequality. [Cf. Cargo and Shisha, Bounds on the ratio of means, J. Res. NBS 66B (1962), 169-170.] (Received January 18, 1965.)

622-5. HAJIMU OGAWA, University of California at Riverside, Riverside, California 92502. Lower bounds for solutions of differential inequalities in Hilbert space.

Let A be an operator on a Hilbert space and let u(t) be in the domain of A for each $t \geqq 0$. Assume u is strongly differentiable, Au strongly continuous and du/dt strongly piecewise continuous, all with respect to t, and define $Lu = du/dt - Au$. For A self-adjoint, Agmon and Nirenberg [Properties of solutions of ordinary differential equations in Banach space, Comm. Pure Appl. Math. 16 (1963), 121-239] proved: Theorem. Let $|Lu(t)| \geqq \phi(t)|u(t)|$. (i) If $\phi \in L_p(0,\infty)$ for some p with $2 \leqq p \leqq \infty$, then $|u(t)| \geqq |u(0)|\exp[\lambda t - \mu(t + 1)^{2-2/p}]$. (ii) If $\phi(t) \leqq K(t + 1)^{\alpha}$, $\alpha > 0$, then $|u(t)| \geqq |u(0)|\exp[\lambda t - \mu(t + 1)^{2\alpha+2}]$. In each case, λ is a constant depending on u, while μ is a constant depending only on ϕ. In this paper the theorem is proved for A symmetric. (Received January 22, 1965.)

622-6. EUGENE WONG, University of California, Berkeley, California, and MOSHE ZAKAI, Sylvania Applied Research Laboratory, Waltham, Massachusetts. On convergence of integrals involving Brownian Motion.

Let $y(t,\omega)$ be a Brownian Motion process $(E\{|y(t) - y(s)|^2\} = |t - s|)$. Let $\{y_n(t,\omega)\}$ be a sequence of approximations to $y(t,\omega)$ such that $y_n(t,\omega)$ has a piecewise continuous derivative and $\lim_{n\to\infty} y_n(t) \xrightarrow{a.s.} y(t)$. In this paper we study differential equations of the form $dx_n(t) = m(x_n(t),t)dt + \sigma(x_n(t),t)dy_n(t)$. It is shown that under quite general conditions $\lim_{n\to\infty} x_n(t) \overset{a.s.}{=} x(t)$, where x(t) is the unique solution of the stochastic differential equation $dx(t) = m(x(t),t)dt + (1/2)(\partial\sigma(x(t),t)/\partial x(t))\sigma(x(t),t)dt + \sigma(x(t),t)dy(t)$. (Received January 25, 1965.)

622-7. R. J. WARNE, West Virginia University, Morgantown, West Virginia. Extensions of completely simple semigroups by completely 0-simple semigroups.

Let S and T be disjoint semigroups, T having a zero element. A semigroup $(V,\circ)$ is called an extension of S by T if it contains S as an ideal and if the Rees factor semigroup V/S is isomorphic to T. We shall say that V is determined by a partial homomorphism if there exists a partial homomorphism $A \to \bar{A}$ of $T\backslash 0$ into S such that $A \circ B = AB$ if $AB \neq 0$, $A \circ B = \bar{A}\bar{B}$ if $AB = 0$, $A \circ s = \bar{A}s$, $s \circ A = s\bar{A}$, and $s \circ t = st$ where s,t in S and the operations in S and T are denoted by juxtaposition. Theorem. An extension V of a completely simple semigroup S by a completely 0-simple semigroup T is given by a partial homomorphism if and only if there exists in V at most one idempotent of S under each idempotent of $T\backslash 0$. (Received January 26, 1965.)

622-8. H. F. BECHTELL, Bucknell University, Lewisburg, Pennsylvania 17837. The Frattini subgroup of an E-group.

An E-group G has the property that $\Phi(N) \leqq \Phi(G)$ for each subgroup $N \leqq G$, $\Phi(N)$ the Frattini subgroup of a finite group N. (All finite nilpotent groups are E-groups.) Denote the automorphism group of a group N by $\mathscr{A}$ and its inner automorphism group by $\mathscr{I}(N)$. It is shown that a necessary condition that a nilpotent group N be $\Phi(G)$ for some E-group G is that $\mathscr{A}$ contains a subgroup $\mathscr{A}'$ such that $\Phi(\mathscr{A}') = \mathscr{I}(N)$. Furthermore if M is a subgroup of N invariant under $\mathscr{A}'$ and induced automorphisms $\mathscr{K}$ by $\mathscr{A}'$, $\Phi(\mathscr{K}) = \mathscr{I}(M)$, then a further necessary condition on N is that $N = MB$, $[M,B] = 1$, $M \cap B \neq 1$ unless $M = 1$, B invariant under $\mathscr{A}'$. If $\mathscr{G}$ denotes the induced automorphisms of B by $\mathscr{A}'$, $\Phi(\mathscr{G}) = \mathscr{I}(B)$. This improves the general result by Gaschütz that $\mathscr{I}(N) \leqq \Phi(\mathscr{A}')$ for an E-group. (Received January 26, 1965.)

622-9. J. F. TRAUB, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey 07971. Proof of global convergence of a class of methods for the solution of polynomial equations.

We give an algorithm, easily realizable in practice, for generating pth order iteration functions $\phi_p(\lambda,t)$ for λ an arbitrary nonnegative integer. These functions are ratios of polynomials of the same degree and hence defined at ∞. Theorem. Let the zeros ρ_i of the polynomial P be distinct and let $|\rho_1| > |\rho_i|$, $i = 2,3,\ldots,n$. Let t_0 be an arbitrary point in the extended complex plane such that $t_0 \neq \rho_2,\rho_3,\ldots,\rho_n$, and let $t_{i+1} = \phi_p(\lambda,t_i)$. Then for λ sufficiently large but fixed, the sequence t_i is defined for all i and $t_i \longrightarrow \rho_1$. Alternative formulation of Theorem. Let t_0 be an arbitrary point in the extended complex plane and let $t_{i+1} = \phi_p(\lambda,t_i)$. Then for λ sufficiently large but fixed, the sequence t_i is defined for all i and $t_i \longrightarrow \rho_j$ for some j. The theory can be extended to multiple zeros, complex dominant zeros, and subdominant zeros. (Received January 29, 1965.)

622-10. HANS SCHWERDTFEGER, McGill University, Montreal, Province of Quebec, Canada. Group theoretical interpretation of incidence theorems.

In an earlier report (these *Notices* 11 (1964), 216, or Canad. J. Math. 16 (1964), 683-700) it has been shown that the affine group $x \longrightarrow ax + \alpha$, $a \neq 0$, $a,\alpha \in F$, where F is an arbitrary field, constitutes a modified projective plane where the group elements $A = (a,\alpha)$ are the points, the normalizers $\mathfrak{N}_A$ and the greatest normal subgroup $\mathfrak{H}$ $(H = (1,\eta))$ and their cosets are the straight lines This plane is obtained from the projective plane P over F by eliminating two different straight lines of P. Every T_1-group $\mathfrak{G}$ (cf. *Notices* 9 (1962), 33, or Arch. Math. 13 (1962), 283-289, or Math. Reviews 6 #3770) appears as modified projective plane. Depending on the situation of the configurations with respect to the two exceptional lines, the geometrical incidence theorems admit a variety of different interpretations in the group. If the vertices of the two perspective triangles are situated on the exceptional lines, then Desargues' theorem is equivalent with the fact that all elements of order 2 form a class of conjugate elements. If the vertices of the Pappus hexagon are situated on the exceptional lines, then Pappus' theorem expresses the fact that all the normalizers and the normal subgroup of $\mathfrak{G}$ are commutative. (Received February 1, 1965.)

622-11. J. A. MORRISON, Room 3D-278, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey 07971. Dual formulation for the eigenfunctions corresponding to the bandpass kernel, in the case of degeneracy.

Previously [Quart. Appl. Math. 21 (1963), 13-19] the author considered the finite integral equation with bandpass difference kernel. It was shown that, in the case of degeneracy, which occurs for a specific relationship between the two parameters in the kernel, one eigenfunction is the continuous solution of a certain fourth-order linear differential equation, containing two parameters which must be determined from prescribed conditions. The second eigenfunction is the derivative of the first one. Here the investigation of these degenerate eigenfunctions is continued by considering the dual formulation of the problem, in which the integral equation corresponds to the one-band, two-interval problem. This approach leads to solutions of a second-order linear differential equation which are subject to two integral conditions. A consequence of transcribing some of the results of the previous paper is closed form expressions for prolate spheroidal wave functions of odd order, for specific values of the parameter, explicit results being given for order one and order three. Those for order one are already known. The asymptotic behavior of the curves of degeneracy is investigated, together with that of the corresponding eigenfunctions. (Received February 2, 1965.)

622-12. LAMAR BENTLEY and PAUL SLEPIAN, Rensselaer Polytechnic Institute, Troy, New York. Topology without the union axiom.

The present paper investigates the more general structure which results from the omission of the union axiom from the definition of a topology. Definition: G is a colander if and only if $a \in G$ and $b \in G$ implies for some $v \in G$, $v \subset a \cap b$. The name colander is due to the similarity to filters. Theorems are proved about certain maps between colanders and about product and quotient colanders. Definition: T is a basis if and only if $x \in a \in T$ and $x \in b \in T$ implies for some $v \in T$, $x \in v \subset a \cap b$. Note that T is a basis if and only if T is a basis for some topology in the usual sense. Let T be a basis and let S be a basis. Let F and K be the topologies generated by T and S respectively. Continuity between two bases is defined so that f is (T,S)-continuous if and only if f is (F,K)-continuous in the usual sense. The relation between colanders and bases is pointed out by the following Theorem: T is a basis if and only if $x \in \cup T$ implies $\{a \mid x \in a \in T\}$ is a colander. (Received February 4, 1965.)

622-13. CHARLES FOX, McGill University, Montreal, Quebec, Canada. A formal solution of certain dual integral equations.

Dual integral equations occur in many problems of mathematical physics. An example is $\int_0^\infty u^a J_\mu(ux)f(u)du = g(x)$, when $0 < x < 1$ and $\int_0^\infty u^\beta J_\nu(ux)f(u)du = h(x)$ when $x > 1$. Here g(x) and h(x) are both given, $J_\lambda(x)$ is the usual Bessel function and f(x) is to be found. I consider dual integral equations of a very general nature in which the Bessel functions are replaced by H functions. The Mellin transform of an H function, considered as a function of s, consists of n factors, the ith being $\Gamma(\alpha_i + a_i s)/\Gamma(\beta_i - a_i s)$ where $a_i > 0$. When n = 1 the H functions reduce to Bessel functions multiplied by powers of x. For such general dual integral equations with H kernels we show that it is possible to write down by inspection, quite simply, a formal solution of the dual system by means of two operations of fractional integration. (Received February 5, 1965.)

622-14. E. S. RAPAPORT, Polytechnic Institute of Brooklyn, 333 Jay Street, Box 175, Brooklyn 1, New York. Groups of order 1. II.

Let R_1, R_2 be elements of the free group F_2 generated by x_1 and x_2, and N the normal closure of the subgroup R_1 and R_2 generate in F_2. Suppose that F_2/N has order 1. Let also R_1^* and R_2^* have the property that F_2/N^* has order 1, and let (M) be the set of all mappings $M(R_i) = R_i^*$ between all possible pairs of such elements. Finally, if w_i designate arbitrary elements of F_2, K_i an arbitrary consequence of R_i in F_2, i: 1,2, then let $Q_1(R_1) = w_1 R_1^{\pm 1} w_1^{-1} K_2$ and $Q_1(R_2) = R_2$, while Q_2 is similarly defined. Then the normal closure of the subgroup generated by $Q_1(R_1)$ and $Q_1(R_2)$ is again F_2, and mappings of type Q may again be applied to these words. Let (Q) be the set of maps generated by such repeated applications of the Q_i. I define a special set (P) of mappings in the set (M) above. Theorem. The set ((Q),(P)) generates (M) and (Q) $\neq$ ((Q),(P)). Corollary. The subgroup generated by R_1 and R_2 does not contain a simple word (generator) of F_2, in general. The theorem has a bearing on the Poincaré conjecture (and answers a question of M. L. Curtis), the corollary on a problem of Magnus. (Received February 8, 1965.)

622-15. M. J. SHERMAN, University of California, Los Angeles, California 90024.
Intersections of invariant subspaces.

Let $H^2_{\mathscr{A}}$ denote the space of all weakly measurable functions defined on the unit circle of the complex plane taking values in the separable Hilbert space $\mathscr{A}$, whose pointwise norms are square-integrable with respect to Lebesgue measure and which have analytic extensions to the disk. It is known (H. Helson, Lectures on invariant subspaces, pp. 61-64) that every closed subspace of $H^2_{\mathscr{A}}$ invariant under the shift operator $(SF)(e^{ix}) = e^{ix}F(e^{ix})$ is of the form $\mathscr{U}H^2_{\mathscr{A}}$, where $\mathscr{U}(e^{ix})$ is a.e. a partial isometry of $\mathscr{A}$ acting pointwise on elements of $H^2_{\mathscr{A}}$. If $\mathscr{U}$ is unitary a.e. and $\mathscr{U}H^2_{\mathscr{A}} \subseteq H^2_{\mathscr{A}}$, we say that $\mathscr{U}$ is inner. Theorem 1. If $\mathscr{A}$ is finite-dimensional, the set of all functions of the form $\mathscr{U}^*\mathscr{V}$, with $\mathscr{U},\mathscr{V}$ inner, is the same as the set of all $\mathscr{U}\mathscr{V}^*$. This is proved false when $\mathscr{A}$ is infinite-dimensional, the result being a consequence of the following. Theorem 2. If $\mathscr{A}$ is infinite-dimensional, there exist inner functions $\mathscr{U}$, $\mathscr{V}$ such that $\mathscr{U}H^2_{\mathscr{A}} \cap \mathscr{V}H^2_{\mathscr{A}} = (0)$. The inner functions used in the proof of Theorem 2 arise from bounded operators on $\mathscr{A}$, using a construction due to Rota and Lowdenslager (Helson, pp. 103-104). (Received February 8, 1965.)

622-16. BERNARD KOLMAN, 32nd and Chestnut Streets, Philadelphia, Pennsylvania 19104.
Semi-modular Lie algebras.

A finite-dimensional Lie algebra is called distributive, modular, upper semi-modular or lower semi-modular if its lattice of all subalgebras has the corresponding property. An $(n + 1)$-dimensional, $n \geqq 1$, Lie algebra over a field of any characteristic is called almost abelian if it has a basis $e_0, e_1, \ldots, e_n$ such that $[e_i, e_0] = e_i$, and $[e_i, e_j] = 0$ for $i, j \geqq 1$. Also, a simple nonsplit three-dimensional Lie algebra over a field of characteristic zero will be called special simple. Theorem 1. Let L be a Lie algebra over a field of characteristic zero. Then L is upper semi-modular if and only if L is abelian, almost abelian, or special simple. Moreover, the theorem holds if "upper semi-modular" is replaced by "modular". Theorem 2. Let L be a Lie algebra over an algebraically closed field of characteristic zero. Then L is lower semi-modular if and only if L is solvable, or L is a direct sum of its radical S and a simple ideal isomorphic to A_1. In this paper other properties of upper semi-modular, lower semi-modular, modular Lie algebras are also studied. (Received February 8, 1965.)

622-17. D. E. CRABTREE, University of Massachusetts, Amherst, Massachusetts 01003. Applications of M-matrices to non-negative matrices.

A square matrix A is called an M-matrix if it has the form $kI - B$, where B is a non-negative matrix, I is the identity matrix, and $k > p(B)$, the Perron maximal characteristic root of B. In this paper we generalize a result of Ky Fan on M-matrices [Quart. J. Math. (2) 11 (1960), 43-49] in order to obtain new bounds for p(B). In particular, the fact that $p(B) \geqq p(C)$ for each principal submatrix C of B is improved. Using other known properties of M-matrices, we also improve the fact that p(B) lies between the smallest and the largest row sums of B. (Received February 9, 1965.)

622-18. L. T. GARDNER, 610 W. 113th Street, New York, New York 10025. On duality for locally compact groups. Preliminary report.

A universal algebra for a topological group G is defined to be a W*-algebra A with a "σ-map" (a σ-weakly continuous representation) Φ of G into the unitary group of A, satisfying (i) A is generated by $\Phi(G)$, and (ii) if ϕ is a σ-map of G into (the unitary group of) W*-algebra B, there exists a normal *-homomorphism $\tilde{\phi}$ of A into B such that $\phi = \tilde{\phi} \circ \Phi$. Theorem. If G is locally compact, a universal algebra for G exists, and is essentially unique. A is constructed as the strong closure of the image of the universal representation of C*(G). Uniqueness then shows that for separable G, A is Ernest's "big group algebra". (See J. Ernest, A new group algebra for locally compact groups, Amer. J. Math. 86 (1964), 467-492.) Thus A is isometrically isomorphic to the second dual of C*(G), an apparent sharpening of Ernest's Theorem 3.1. Theorem. The complex linear span V of the positive-definite functions on locally compact G is a commutative Banach *-algebra (pointwise operations, conjugation as involution), when normed as the predual of A. When G is abelian, V is simply the measure algebra of the dual group $\hat{G}$. For compact G, the ring V is again familiar. The subspace of A consisting of homomorphisms (resp. *-homomorphisms) of V onto the complex numbers is discussed. (Received February 10, 1965.)

622-19. C. C. PUGH, University of California, Berkeley, California. The closing lemma.

Let M be a compact C^∞ n-manifold and let $\mathscr{X} = \mathscr{X}(M)$ be the space of C^∞ tangent vector fields on M under the C^1 topology. Then the following has been conjectured by R. Thom and is here proved: Closing Lemma. If $X \in \mathscr{X}$ and p is a nonwandering point of the X-flow, $X_p \neq 0$, then arbitrarily near X in $\mathscr{X}$ there lies X' such that X' has a closed orbit through p. From the Closing Lemma follow: (1) Seifert's Conjecture is true for an open dense subset of the nonvanishing fields in $\mathscr{X}(S^3)$; (2) Thom's First Integral Theorem — "the set of X in $\mathscr{X}$ which fails to have a global first integral is residual in $\mathscr{X}$" — holds (residual = the complement of a countable union of nowhere dense sets); (3) For any $X \in \mathscr{X}$ let $\overline{\Gamma}(X)$ denote the closure of all periodic points of the X-flow (that is, the closure of all the closed orbits and zeros of X). Then $\{X \in \mathscr{X}$: (a) the X-flow has generic periodic points, (b) has its stable and unstable manifolds in general position, (c) has all its nonwandering points contained in $\overline{\Gamma}(X)\}$ is residual in $\mathscr{X}$. (3) has been proved by Smale and Kupka if (c) is omitted. (3) implies that (a), (b), (c) are necessary for X to be structurally stable. (c) is Axiom 3a of Smale, Proc. Internat. Congress of Mathematicians, Stockholm, 1962. (Received February 10, 1965.)

622-20. WILLARD MILLER, JR., Courant Institute of Mathematical Sciences, New York University, 25 Waverly Place, New York, New York. On the function theory of occupation number space.

The occupation number representation for bosons is shown to be closely related to the representation theory of a certain four-dimensional real Lie group H_4. The unitary representations of H_4 are exactly the projective representations of the Euclidean group in the plane. A family of such representations is studied and the Clebsch-Gordan coefficients for the tensor product of any two of these representations are computed. With respect to a suitable basis the matrix elements of these representations are shown to be functions of the paraboloid of revolution. This connection between Lie groups and special functions leads to a number of important results in the theory of the above-named functions as well as for the associated Laguerre polynomials. (Received February 10, 1965.)

622-21. TANJIRO OKUBO, Montana State College, Bozeman, Montana 59715. On the differential geometry of frame bundles.

This work is the continuation and completion of the previous one introduced in Abstract 65T-49, these *Notices* 12 (1965), 140. Let X_n be a smooth connected differentiable manifold and its frame bundle be denoted by $F(X_n)$. Then, regardless whether the dimension is even or not, $F(X_n)$ admits a tensor field H of type (1.1) and of rank n^2 satisfying $H^3 = H$. If we put $\Psi = -(3/2)H^2 + (\sqrt{3}/2)H - E$, Ψ satisfies $\Psi^3 = -E$, where E is the unit tensor of rank $n + n^2$. H decomposes this tensor into two complementary tensor fields, P of rank n and Q of rank n^2; on the former H acts as an annihilator and on the latter H acts as an almost product structure. P and Q yield n and n^2 vectors spanning the horizontal and the vertical distributions. The horizontal subspace is holonomic if and only if X_n is flat. Given a vector field in X_n, we introduce three kinds of lift, horizontal, vertical and complete, and with respect to which one gets the theorem similar to that derived in $F(X_n)$, $n = 2m$. When X_n is Riemannian, one can introduce the metric G in $F(X_n)$ such that it leads to the theorem that the group of tangent bundle of $F(X_n)$ is reduced to $O(n^2) \times O(n)$. (Received February 11, 1965.)

622-22. HANS RADEMACHER, The Rockefeller Institute, New York, New York 10021. Enumeration of certain types of polyhedra.

A polyhedron (in 3-space) is a complex of vertices, edges, and faces. A class of polyhedra isomorphic to each other is called a (morphological) type. The number of types of trihedral polyhedra of f faces is known only for small f. For certain trihedral polyhedra singled out by Th. P. Kirkman (1858), namely those which have among their f faces one which is a polygon of (f - 1) sides, the number of types is found here as an explicit function of f. Types are considered in the strict sense, in which isomorphism also requires the preservation of orientation, and in the wider sense in which this requirement is dropped. For both cases explicit formulae are given. (Received February 15, 1965.)

622-23. DIANA BRIGNOLE MARTIN and H. B. RIBEIRO, Pennsylvania State University, McAllister Building, University Park, Pennsylvania. On universal equivalence for ordered groups.

Gurevich and Kokorin have recently shown that any (first-order) universal sentence valid in one linearly ordered abelian group is valid in all. For such classes without finite systems, this property of "universal equivalence" coincides with that of universal completeness which has been discussed by one of the present authors. Using results of this discussion, and after a simpler proof of the above, it is established that the (only) class of infinite relational systems of one binary operation and one binary relation which contains all linearly ordered abelian groups and is maximal with the property of universal equivalence is the class of those linearly ordered abelian cancellative semigroups in which the ordered group of the integers is (isomorphically) imbeddable. (Received February 15, 1965.)

622-24. LEON BROWN and HIDEGORO NAKANO, Wayne State University, Detroit, Michigan 48202. A representation theorem for archemedian linear lattices.

We present a characterization of archemedian linear lattices as quotient spaces of function spaces. If S is an archemedian linear lattice, then there exist a function lattice F and a σ-closed semi-normal manifold N of F such that S is lattice-isomorphic to F/N. Conversely, if F is a function lattice and N is a σ-closed semi-normal manifold of F, then F/N is an archemedian linear lattice. (Received February 15, 1965.)

622-25. E. S. BOYLAN, Rutgers, The State University, New Brunswick, New Jersey. Dynamic programming existence and uniqueness theorems.

Let g(x), a nonnegative function defined for all $x \geqq 0$, h(x) a monotonically nondecreasing function defined for all $x \geqq 0$, $h(0) = 0$, F(x) a distribution function on $[0,\infty)$ and a positive constant a, $0 < a < 1$, be given. The following dynamic programming equation arises in the study of inventory control: (1) $f(x) = \inf_{y \geqq x}(g(y) + h(y - x) + a\int f(y - z)dF(z))$ for all $x \geqq 0$, where $f(y - z) = f(0)$ for $y > z$. The existence and uniqueness of a function f which satisfies (1) has been shown only for the case where g(x) is bounded for all x. (See, for example, Chapter 4 of Bellman's book Dynamic programming.) The main result of this paper is the following Theorem. Let g(x) be uniformly continuous

for all $x \geqq 0$ or let $g(x)$ be continuous for all $x \geqq 0$ and $h(x)$ be continuous for all $x > 0$. If $\lim_{x \to \infty} g(x) = \infty$, then there exists a unique nonnegative continuous function f which satisfies (1).
(Received February 15, 1965.)

622-26. R. F. RINEHART, Case Institute of Technology, 10900 Euclid Avenue, Cleveland, Ohio 44106. P and D in $P^{-1}XP = dg(\lambda_1,\dots,\lambda_n) = D$ as matrix functions of X.

For X belonging to the open set K_n of $n \times n$ complex matrices with distinct characteristic roots, $P^{-1}XP = dg(\lambda_1, \lambda_2,\dots, \lambda_n) = D$ defines D as a Hausdorff-differentiable matrix function of X throughout K_n, and for a suitably "stabilized" choice of the eigenvectors of X, the matrix P is similarly a single-valued and Hausdorff-differentiable function of the matrix X in a neighborhood N_0 of any $X_0 \in K_n$. The Hausdorff derivatives of the functions D and P with respect to X have the respective values 1 and 0, throughout K_n and N_0, respectively. The first of these results implies as a corollary that each characteristic root λ_i of $X = \|x_{rs}\|$ satisfies the partial differential equation $\sum_{j=1}^{n} \partial \lambda_i / \partial x_{jj} = 1$ throughout K_n. (Received February 15, 1965.)

622-27. W. C. WHITTEN, JR., 246 South 44th Street, Philadelphia, Pennsylvania 19104. On the interchangeability of 2-links. Preliminary report.

Let $L = K_1 + K_2$ be an oriented 2-link tamely imbedded in S^3. L is interchangeable if there is an autohomeomorphism of S^3 taking K_1 onto K_2 and K_2 onto K_1. Problem (R. H. Fox): Find conditions on L which must necessarily hold if L is interchangeable. If L is nonsplittable, it possesses exactly one prime factor 2-link which shall be called the hub of L, and which is uniquely determined up to orientation-preserving autohomeomorphisms of S^3. Theorem 1. Let $L = K_1 + K_2$ be nonsplittable, and let $L^* = k_1^* + k_2^*$ be the hub of L. Then L is interchangeable if and only if the following three conditions hold: (1) L^* is interchangeable; (2) k_1 and k_2 are of the same knot type, where $k_i \# k_i^* = K_i$ $(i = 1,2)$; (3) there exist autohomeomorphisms f_1, f_2 of S^3 such that f_1 interchanges L^*, $f_2(k_2) = k_1$, with f_1 and f_2 both orientation-preserving or both orientation-reversing. Theorem 2. If L is splittable and K_1 and K_2 are of the same knot type, then L is interchangeable. (Received February 16, 1965.)

622-28. B. N. PARLETT, Stevens Institute of Technology, Castle Point Station, Hoboken, New Jersey 07030. Accuracy and dissipation in difference schemes.

We consider the stability of difference schemes for solving the initial value problem for first-order hyperbolic systems of differential equations with variable coefficients. Theorem. Let the coefficient matrices in the differential and difference operators be hermitian, uniformly bounded, and uniformly Lipschitz continuous. If, for some positive integer r, the difference operator is (i) accurate of degree 2r, (ii) dissipative of order 2r - 2, and (iii) simple, then it is stable in the sense of Lax and Richtmyer. The essence of the theorem is the construction of a new norm. Kreiss proved the existence of such a norm under slightly more restrictive conditions. The direct construction simplifies the arguments and suggests the extension. (Received February 16, 1965.)

622-29. J. A. THORPE, Room 2-278, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. On the curvatures of Riemannian manifolds.

Let X be a Riemannian manifold of dimension n. For p an even integer with $2 \leqq p \leqq n$, let γ_p denote the pth sectional curvature of X. (For definition of γ_p, see Ann. of Math. (2) 80 (1964), 429-443.) γ_p is a smooth function on the bundle of p-planes over X. Theorem. Suppose γ_p is constant for some p. Let P be a (p + q)-plane on X (q even). Then $\gamma_{p+q}(P)$ is equal to the constant value of γ_p multiplied by the average value of γ_q over all q-planes contained in P. Corollary 1. Let X be compact, orientable, and of even dimension n. Suppose γ_p is constant and that γ_{n-p} keeps constant sign for some p. Then the Euler-Poincaré characteristic χ of X has the same sign as $\gamma_p \gamma_{n-p}$. Corollary 2. Let X be as in Corollary 1. Assume $\gamma_{n-2} = K$ is constant. Then $\chi = (2K/n(n-1)c_n) \int_X \rho dV$, where c_n is the volume of the unit n-sphere and ρ is the scalar curvature of X.
(Received February 19, 1965.)

622-30. E. P. MILES, JR., Florida State University, Tallahassee, Florida. Solutions for certain cases of the Cauchy problem for the nonhomogeneous wave equation.

Let $P(x_1,x_2,\dots,x_k)$ and $Q(x_1,x_2,\dots,x_k)$ be polyharmonic of order p and q respectively. Let $G(x_1,x_2,\dots,x_k,t)$ be of the type $\sum_{n=0}^{N} R_n(x_1,x_2,\dots,x_k)t^n$, where R_n is polyharmonic of order r_n. Let Δ denote the Laplacian operator in k variables. The Cauchy problem $\Delta u - u_{tt} = G$, $u|_{t=0} = P$, $u_t|_{t=0} = Q$, is shown to have solution $u = P + \sum_{j=1}^{p-1} \Delta^j(P)\cdot t^{2j}/(2j)! + Q\cdot t + \sum_{j=1}^{q-1} \Delta^j(Q)\cdot t^{2j+1}/(2j+1)! + \sum_{n=0}^{N} [R_n\cdot t^{n+2}/(n+1)(n+2) + \sum_{j=1}^{r_n - 1} \Delta^j(R_n)\cdot t^{n+2+2j}/(n+1)(n+2)\dots(n+2+2j)]$. This extends to the nonhomogeneous case results obtained earlier for the homogeneous wave equation by the author and E. Williams (The Cauchy problem for linear partial differential equations with restricted boundary conditions, Canad. J. Math. 8 (1956), 426-431).
(Received February 19, 1965.)

622-31. E. K. BLUM, Wesleyan University, Middletown, Connecticut 06457. A convergent gradient procedure in prehilbert spaces.

Let H be a real prehilbert space with inner product $\langle u,v \rangle$. For $u \in H$ write $\bar{u} = u/\|u\|$. Let J, ψ_i, $1 \leqq i \leqq p$, be real functionals on H. Let u* be a relative minimum of J satisfying $\psi_i(u^*) = 0$, $1 \leqq i \leqq p$. Definition. A neighborhood, N, of u* is regular if, for $u \in N$, (1) the gradients $\nabla\psi_i(u)$ are continuous and not zero; (2) $\nabla J(u)$ is continuous and its projection, $\nabla J_G(u)$, on the subspace spanned by $\{\psi_i(u)\}$ is nonzero; (3) $\{\nabla\psi_i(u)\}$ are linearly independent; (4) for $\theta(u) = \arcsin(\|\nabla J(u) - \nabla J_G(u)\|/\|\nabla J(u)\|)$, $\nabla\theta(u)$ exists, is nonzero for $u \neq u^*$, and the weak differential, $d\theta(u^*,h)$, exists and $\langle \nabla\theta(u), h\rangle \longrightarrow d\theta(u^*,h)$ as $u \longrightarrow u^*$; and (5) for $u = u^* + \Delta u$, $\Delta u \neq 0$, and $a_i = \arccos\langle \overline{\nabla\psi_i(u)}, \overline{\Delta u}\rangle$, $a_0 = \arccos\langle \overline{\nabla J(u) - \nabla J_G(u)}, \overline{\Delta u}\rangle$, $\beta = \arccos\langle \overline{\nabla\theta(u)}, \overline{\Delta u}\rangle$, $\gamma = \arccos\langle \overline{\nabla\theta(u)}, \overline{\nabla J(u) - \nabla J_G(u)}\rangle$, there are constants $a_1 > 0$, $a_2 > 0$ such that $\sum_1^p \cos^2 a_i + \cos a_0 \cos\beta/\cos\gamma > a_1$ and $|\cos\gamma| > a_2$. Theorem. For $u = u^* + \Delta u \in N$, $\Delta u \neq 0$, let $h_G = \sum_1^p(\psi_i(u)/\|\nabla\psi_i(u)\|)\nabla\psi_i(u)$ and $h_T = -(\|\nabla J_G(u)\|\cdot|\langle\nabla\theta(u), \nabla J - \nabla J_G\rangle|)^{-1}(\nabla J - \nabla J_G)$. There exist positive constants K,d,r with $K < 1$ such that for $\|\Delta u\| < r$ and $d/2 < s < d$, $\|u + sh_T + sh_G - u^*\| < K\|\Delta u\|$.
(Received February 22, 1965.)

622-32. W. F. EBERLEIN, University of Rochester, Rochester, New York 14627. The spin model of space-time.

The spin model $\underline{E}_3$ of Euclidean 3-space is the vector space of self-adjoint linear transformations of trace 0 in a 2-dimensional unitary space H_2 plus the inner product $(A,B) = (AB + BA)/2$. The obvious extension of $\underline{E}_3$ is the model M_4 of space-time consisting of all self-adjoint linear transformations in H_2, but the inner product loses the Jordan form above. We construct a spin model $\underline{E}_4$ of space-time — consisting of linear transformations in a Euclidean 4-space —that contains $\underline{E}_3$ as a subspace and preserves the Jordan form of the inner product. There is a natural isomorphism of M_4 onto $\underline{E}_4$ leaving $\underline{E}_3$ pointwise fixed. The connection between 2- and 4-component spinors becomes very transparent, while the Dirac equation and its relavitistic "invariance" properties undergo a drastic simplification and clarification. (Received February 17, 1965.)

622-33. GIDEON PEYSER, Pratt Institute, Brooklyn 5, New York. On weak and strong solutions.

Consider the first-order linear system of partial differential equations: (*) $Lu \equiv \sum_{i=1}^{m} A_i D_i u + Bu = f$. On the boundary S, the boundary conditions are: (**) $u \in N(x)$, $x \in S$, where N is a smoothly varying linear vector space. Define the boundary matrix $\Delta = \sum A_i n_i$, where n_i are the direction cosines of the normal to S. If S is smooth and Δ is of constant rank, and possibly singular, on and near S, then it is shown that a weak solution of (*) satisfying (**) weakly is also a strong solution of (*) satisfying (**) strongly. For regular Δ this includes the result of Lax-Phillips. For the case of a corner, a weak solution is shown to be also a strong solution under additional restrictions of the equation and the boundary conditions. The principal tools of the proofs are mollifiers whose support is shifted differently for different elements of the vector functions. (Received February 17, 1965.)

622-34. A. B. MANASTER, Cornell University, Ithaca, New York 14850. Higher-order indecomposable isols.

For definitions of isols and ideals of isols see Dekker and Myhill, Recursive equivalence types, Univ. California Publ. Math. (N.S.) 3, No. 3 (1960), 67-214. Let Δ be a class of isols. An isol X is called indecomposable over Δ if whenever $X = Y + Z$, then $Y \in \Delta$ or $Z \in \Delta$. X is called highly decomposable over Δ if whenever $Y \leqq X$ and $Y \notin \Delta$, there exist U and V not in Δ such that $Y = U + V$. Define $I(\alpha)$, $P(\alpha)$, and $S(\alpha)$ by induction on the ordinals. Let $I(0)$ be the class of finite isols. Let $P(\alpha)$ be the isols indecomposable over $\bigcup_{\alpha'<\alpha} I(\alpha')$. Let $S(\alpha)$ be the isols highly decomposable over $\bigcup_{\alpha'<\alpha} I(\alpha')$. Let $I(\alpha)$ be the ideal generated by $P(\alpha) \cup S(\alpha)$. Theorem 1. If α is greater than zero and countable, both $P(\alpha) - \bigcup_{\alpha'<\alpha} I(\alpha')$ and $S(\alpha) - \bigcup_{\alpha'<\alpha} I(\alpha')$ have cardinality c. Theorem 2. Every isol belongs to $I(\alpha)$ for some countable α. (Received February 18, 1965.)

622-35. W. S. MARTINDALE III. University of Massachusetts, Amherst, Massachusetts. Jordan derivations of symmetric elements. Preliminary report.

If S is a Jordan subring of a ring R, then a Jordan derivation of S into R is an additive mapping J of S into R such that $J(st + ts) = J(s)t + tJ(s) + sJ(t) + J(t)s$, $s,t \in S$. Theorem. Let R be a simple ring (char. $\neq 2$) with identity and with an involution. If R contains a nontrivial symmetric idempotent, then every Jordan derivation of the symmetric elements S into R can be extended uniquely to an associative derivation of R. (Received February 19, 1965.)

622-36. D. L. PILLING and J. C. ABBOTT, U. S. Naval Academy, Annapolis, Maryland 21402.
Distributivity and completeness in implication algebra.

An implication algebra is a set I closed under ab satisfying (P1) (ab)a = a, (P2) (ab)b = (ba)a, and (P3) a(bc) = b(ac). It is characterized as a union semi-lattice in which every principal ideal is boolean. A boolean algebra is an implication algebra satisfying (P4) $\exists$ o such that oa = aa. An ideal H satisfies: $a \in H$, $x \in I$ imply $xa \in H$, and $a,b \in H$, $\exists\, a \cap b$ imply $a \cap b \in H$. The implication product of two ideals is given by $HK = \{x \in K \mid hx = x \ \forall\, h \in H\}$. HK satisfies (P1), (P3) and (P4). In this paper we investigate (P2). The operation $H \to (HK)K$, K fixed, $H \subseteq K$, is a closure operation. HK is a relative pseudo-complement of H in K satisfying $HK = \mathrm{Max}\{X \subseteq K \mid H \cap X = \emptyset\}$. The distribution laws $H(\bigcup H_i) = \bigcup HH_i$ and $H \cap H_i = \bigcap HH_i$ and the de Morgan law $(H_1 \cup H_2)K = H_1K \cap H_2K$ hold, but $(H_1 \cap H_2)K = [(H_1K \cup H_2K)K]K$. The set of regular ideals defined by $H \cup HK = K$ is a boolean algebra and the set of normal ideals defined by $(HK)K = H$ is a complete boolean algebra. If $H\downarrow\uparrow$ is the set of upper bounds for the set of lower bounds of H, then $H\downarrow\uparrow = (HI)I$, and the set of cuts (closed, bounded ideals) is a complete implication algebra. (Received February 19, 1965.)

622-37. G. A. GRÄTZER, The Pennsylvania State University, University Park, Pennsylvania.
Polynomial symbols and partial algebras.

Let $\tau = \langle n_0,\dots,n_\gamma,\dots\rangle$, $\gamma < O(\tau)$, be a type of algebras. The set $P^{(\alpha)}(\tau)$ of α-ary polynomial symbols is defined recursively by the rules: (1) $\underline{e}_\gamma^\alpha \in P^{(\alpha)}(\tau)$, for $\gamma < \alpha$; (2) $\underline{p}_1,\dots,\underline{p}_{n_\gamma} \in P^{(\alpha)}(\tau) \to \underline{f}_\gamma(\underline{p}_1,\dots,\underline{p}_{n_\gamma}) \in P^{(\alpha)}(\tau)$. Operations are defined on $P^{(\alpha)}(\tau)$ by: $f_\gamma(\underline{p}_1,\dots,\underline{p}_{n_\gamma}) = \underline{f}_\gamma(\underline{p}_1,\dots,\underline{p}_{n_\gamma})$. The α-ary polynomial algebra is $\langle P^{(\alpha)}(\tau);F\rangle$. If $\langle A;F\rangle$ is an algebra, $\bar{a} \in A^\alpha$ then $p(\bar{a})$ for $\underline{p} \in P^{(\alpha)}(\tau)$ can be defined recursively using $e_\gamma^\alpha(\bar{a}) = a_\gamma$. If $\langle A;F\rangle$ is a partial algebra, then $p(\bar{a})$ can be defined only for some $\underline{p} \in P^{(\alpha)}(\tau)$, including all $\underline{e}_\gamma^\alpha$. Theorem. Define $\theta_{\bar{a}}$ by: $\underline{p} \equiv \underline{q}(\theta_{\bar{a}})$ iff $\underline{p} = r(\underline{p}_1,\dots,\underline{p}_n)$, $\underline{q} = r(\underline{q}_1,\dots,\underline{q}_n)$ ($r \in P^{(n)}(\tau)$) and $p_i(\bar{a})$, $q_i(\bar{a})$ are defined, $p_i(\bar{a}) = q_i(\bar{a})$. Then $\theta_{\bar{a}}$ is a congruence relation of $\langle P^{(\alpha)}(\tau);F\rangle$ and the subalgebra of $\langle A;F\rangle$ generated by $\bar{a}$ can be embedded in $\langle P^{(\alpha)}(\tau)|\theta_{\bar{a}};F\rangle$. The freeness of the latter can also be shown. (Received February 22, 1965.)

622-38. HANS HORNICH, The Catholic University of America, Washington, D. C. 20017.
A property of the real nonregular functions C^∞.

For the solution of a linear differential equation $y^{(n)} + \sum_{i=0}^{n-1} \sigma_i y^{(i)} = \phi$ on $[0, \mathfrak{B}]$ with $y(0) = \dots = y^{(n-1)}(0) = 0$, there exists an inequality from which it follows that the solution becomes arbitrarily small for sufficiently large order n and bounded integrable coefficients $\sigma_i \phi$. Therefore, a nonregular function in C^∞ cannot be a solution of a linear differential equation with bounded coefficients for sufficiently large order n. (Received February 22, 1965.)

622-39. R. T. BUMBY, University of Michigan, Ann Arbor, Michigan. <u>A problem related to the approximation of algebraic numbers by rationals.</u>

The problem of finding good rational approximants to a real algebraic number α (of degree n) can be formulated as follows. Let $|\ |_i$ stand for the archimedean valuations on $Q(\alpha)$, with $|\ |_1$ reserved for the valuation associated with the real completion in which we first met α. Define $H(\xi) = \max |\xi|_i$ for $\xi \in Q(\alpha)$. We can find Q-linear functions L_j, $j = 1,\ldots,n-2$, such that $L_j(\xi) = 0$ for all j if and only if $\xi = x + y\alpha$ with x,y rational. A well-known unsolved problem then becomes: Can one find algebraic integers $\xi \in Q(\alpha)$ which simultaneously satisfy $H(\xi)\cdot|\xi|_1 < \epsilon$ and $|L_j(\xi)| < 1$, $j = 1,\ldots,n-2$. This formulation suggests other problems. In particular, $H(\xi)|\xi|_1 \prod_{j=1}^{n-2} |L_j(\xi)| < \epsilon$ can be shown to be always solvable for algebraic integers $\xi \neq 0$ if $n > 2$. (Received February 22, 1965.)

622-40. N. F. G. MARTIN, University of Virginia, Charlottesville, Virginia. <u>On the classification of measurable flows and metric automorphisms.</u> Preliminary report.

Let $(X, \mathscr{M}, \mu)$ be a totally finite, normalized measure space and let G denote a compact topological group. Let ν be the normalized Haar measure on G. A measurable flow $\{T_g : g \in G\}$ is defined on $(G \times X, \mathscr{B} \times \mathscr{M}, \nu \times \mu)$ such that for any measurable flow $\{t_g : g \in G\}$ on $(X, \mathscr{M}, \mu)$ there exists a T_g-invariant σ-subalgebra $\mathscr{A}$ of $\mathscr{B} \times \mathscr{M}$ such that t_g is conjugate to T_g restricted to $\mathscr{A}$. This is an extension of Rota's result [Proc. Amer. Math. Soc. 13 (1962), 659-662], where G was taken to be the circle group. Furthermore a characterization of those T_g-invariant σ-subalgebras of $\mathscr{B} \times \mathscr{M}$ which arise from some measurable action of G on $(X, \mathscr{M}, \mu)$ is given. Let Z denote the integers and let $\mathscr{P}$ denote the σ-algebra of subsets of Z. A finitely additive measure λ is defined on $\mathscr{P} \times \mathscr{M}$ and a metric automorphism T is defined on $(Z \times X, \mathscr{P} \times \mathscr{M}, \lambda)$ such that if t is any metric automorphism of $(X, \mathscr{M}, \mu)$ there is a T-invariant σ-subalgebra $\mathscr{A}$ of $\mathscr{P} \times \mathscr{M}$ such that $(Z \times X, \mathscr{A}, \lambda)$ is measure-theoretically isomorphic to $(X, \mathscr{M}, \mu)$ and t is conjugate to T restricted to $\mathscr{A}$. A characterization of the T-invariant subalgebras of $\mathscr{P} \times \mathscr{M}$ which arise from some metric automorphism of $(X, \mathscr{M}, \mu)$ is given. (Received February 22, 1965.)

622-41. L. C. SIEBENMANN, 29 Wiggins Street, Princeton, New Jersey. <u>Finding a boundary for an open manifold.</u>

Necessary and sufficient conditions are given that an open manifold W of dimension $\geqq 6$ be the interior of a compact manifold. (The smooth category or else the piecewise linear category is understood throughout.) Necessarily W has finitely many ends $\epsilon_1,\ldots,\epsilon_s$. An end ϵ_i is called tame if the fundamental group satisfies a certain stability condition at the end ϵ_i, and W is suitably dominated by a finite complex at ϵ_i. For each tame ϵ_i an obstruction $\sigma(\epsilon_i)$ is defined in the projective class group $\widetilde{K}_0[Z\pi_1(\epsilon_i)]$, where $\pi_1(\epsilon_i)$ is the inverse limit of the fundamental groups of the connected neighborhoods of ϵ_i. <u>Theorem</u> I: W is the interior of a compact manifold if and only if its ends ϵ_i are tame and $\sigma(\epsilon_i) = 0$, $i = 1,\ldots,s$. <u>Remark.</u> If the ends of W are tame, $W \times M$ is the interior of a compact manifold whenever M is closed and $\chi(M) = 0$. But examples exist where $\sigma(\epsilon_i) \neq 0$. An application of Theorem I (relativized) is <u>Theorem</u> II: With W as above, let N be a properly imbedded submanifold (without boundary) of codimension $\geqq 3$. If W and N separately admit a boundary, there exists a compact manifold pair $(\overline{W},\overline{N})$ such that $\dot{W} = \text{Int}\,\overline{W}$, $N = \text{Int}\,\overline{N}$. This is false for codimensions 1 and 2. (Received February 22, 1965.)

622-42. EDWIN DUDA, University of Miami, Coral Gables, Florida. A theorem on one-to-one mappings.

Theorem. Let X be a locally connected generalized continuum with the property that the complement of each compact set has only one nonconditionally compact component. If $f(X) = E^2$ is a 1-1 continuous function, where E^2 is Euclidean 2-space, then f is a homeomorphism. The proof consists of proving a series of five statements about the structure of X if f is not a homeomorphism. Then with the aid of a theorem of G. T. Whyburn (Theorem 7, Proc. Nat. Acad. Sci. U.S.A. 52, No. 6) a contradiction is obtained. The statements are: (i) X contains simple closed curves, (ii) Every simple closed curve J in X separates X and is the boundary of an open 2-cell which is an open subset of X, (iii) Each compact nondegenerate cyclic element of X is topologically a closed 2-cell, (iv) There is only one noncompact cyclic element in X, (v) Let M be the noncompact cyclic element and B the set of points of M not in an open 2-cell. The set B is a topological line. (Received February 22, 1965.)

622-43. C. W. KOHLS, Syracuse University, Syracuse, New York 13210. Decomposition spectra of rings of continuous functions. Preliminary report.

Let C(X) be the ring of all continuous real-valued functions on a completely regular space X. A cozero-set in X is the set of all points at which some function in C(X) is nonzero. The concept of decomposition spectrum has been considered for topological spaces by Flachsmeyer [Math. Ann. 144 (1961), 253-274] and for ordered sets by Rinow [Z. Math. Logik Grundlagen Math. 10 (1964), 331-360]. A decomposition spectrum of a ring A is defined to be an inverse system of residue class rings of A. Theorem. Let U be a cozero-set in a completely regular space X. Then C(U) is isomorphic to the inverse limit of a decomposition spectrum of C(X). Combined with a result of Fine, Gillman and Lambek, this yields the Corollary. The classical ring of quotients of C(X) has a representation as a direct limit of inverse limits of residue class rings of C(X). (Received February 22, 1965.)

622-44. A. P. HILLMAN and D. G. MEAD, University of Santa Clara, Santa Clara, California. Related pairs of Hasse diagrams. II.

Let $\{x_1,\ldots,x_s\}$ be partially ordered by a Hasse diagram D satisfying $x_1 < x_i$ for $i > 2$, $x_3 < x_j$ for $j > 3$, $x_1 \nless x_2$, $x_2 \nless x_1$, $x_2 \nless x_3$, and $x_3 \nless x_2$. Let D' result from D when $x_1 \nless x_2$ is replaced by $x_1 < x_2$. Let f(n) and f'(n) be the numbers of realizations of D and D' by families of s subsets of a fixed set of n elements. Using extensions of methods in [Hillman, Proc. Amer. Math. Soc. 6 (1955), 542-548], it is shown that $f(n) = T^{-1}(T^2 - 1)f'(n)$, where T is the operator of Hillman's paper, defined by $Tg(n) = \sum_{i=0}^{n} C_{n,i}\,g(i)$. Also see [Stroot and Grassl, Abstract 64T-426, these *Notices* 11 (1964), 672]. (Received February 22, 1965.)

622-45. ISIDORE EISENBERGER, Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California. A uniformly most powerful test using quantiles.

Given a set of n sample values, $x_1, x_2, \ldots, x_n$, taken from a normally distributed population with density function g(x), the following test is considered: H_0: $g(x) = g_1(x) = N(\mu_1, \sigma)$, H_1: $g(x) = g_2(x) = N(\mu_2, \sigma)$, where σ is known, $\mu_2 > \mu_1$ ($\mu_2 < \mu_1$) and n is large. The test statistics are based on one, two and four sample quantiles. The optimum power function in each case is derived and the efficiency relative to the best test using all the sample values is determined. The minimum efficiency is about .61 for one quantile, about .80 for two quantiles and about .91 for four quantiles. The tests are also performed when the assumption that σ is known is dropped, with a surprisingly small loss in efficiency. (Received February 22, 1965.)

622-46. R. W. RITCHIE, University of Washington, Seattle, Washington 98105 and P. R. YOUNG, Reed College, Portland 2, Oregon. Strong representability of partial functions in arithmetic. Preliminary report.

Notation and terminology are from Introduction to mathematical logic by Mendelson (Van Nostrand, 1964). Let K be a first-order theory of arithmetic such that m equals n whenever $\vdash_K \bar{m} = \bar{n}$. Definition. A partial function $\phi(x_1, \ldots, x_n)$ is represented in K by the wf $A(x_1, \ldots, x_n, x_{n+1})$ with the free variables $x_1, \ldots, x_n, x_{n+1}$ if (1) and (2) below hold; it is strongly represented if (1) and (2') hold. (1) The relation $\phi(k_1, \ldots, k_n) = k_{n+1}$ holds iff $\vdash_K A(\bar{k}_1, \ldots, \bar{k}_n, \bar{k}_{n+1})$. (2) If $\phi(k_1, \ldots, k_n)$ is defined, then $\vdash_K (E_1 x_{n+1}) A(\bar{k}_1, \ldots, \bar{k}_n, x_{n+1})$. (2') $\vdash_K (E_1 x_{n+1}) A(x_1, \ldots, x_n, x_{n+1})$. Theorem 1. If a partial function ϕ is represented in K then some extension of ϕ is strongly represented in K. (This answers a generalization of the problem in [Mendelson, p. 135].) Let R be R. M. Robinson's very weak subsystem of arithmetic [Tarski, Mostowski and Robinson, Undecidable theories, North Holland, 1953]. Theorem 2. If K is a consistent extension of R, then every partial recursive function has an extension which is strongly represented in K. (Received February 22, 1965.)

622-47. A. T. BUTSON and J. D. McKNIGHT, University of Miami, Coral Gables, Florida 33124. Characterizing ordered groups in terms of real groups.

It is shown that a fully ordered group G with three Archimedean classes is order-isomorphic to a group H constructed as follows: An element of H has the form (c,b) where $c \in C$, $b \in B$ and C and B are real groups (subgroups of the additive group of the reals with the usual order); the elements of H are ordered lexicographically; and the operation on H is given by $(c_1, b_1)(c_2, b_2) = (c_1 + c_2, \mu(c_1, c_2) + \lambda(c_2)(b_1 + b_2))$, where μ: $C \times C \rightarrow B$ such that $\mu(c,0) = \mu(0,c) = \mu(c,-c) = 0$, and λ is a homomorphism of C into the group of order automorphisms of B (these automorphisms constituting a subgroup of the multiplicative group of the positive reals). The functions μ are determined explicitly by choosing a Hamel basis for C. This procedure can be extended to characterize ordered groups with a finite number of Archimedean classes. (Received February 22, 1965.)

622-48. K. B. O'KEEFE and E. S. O'KEEFE, 2428 Boyer Avenue East, Seattle, Washington 98102. The differential ideal [uv].

Let R$\{u,v\}$ be a Ritt algebra in the indeterminates u,v, $\Omega = [uv]$ be the differential ideal generated by the form $X = uv$, and $P = u_{i_1} \dots u_{i_k} v_{j_1} \dots v_{j_\ell}$ be a power product of signature $\langle k, \ell \rangle$ in u,v and their derivatives. The weight of P is $w(P) = \sum_{s=1}^{k} i_s + \sum_{t=1}^{\ell} j_t$. It is known that for q sufficiently large, $(u_i v_j)^q \equiv 0\,[uv]$. An unsolved problem of J. F. Ritt is to find the smallest such exponent. The solution is given in Theorem 1. The smallest q such that $(u_i v_j)^q \equiv 0\,[uv]$ is $q = i + j + 1$. The methods used to prove Theorem 1 are applied to more general power products P, yielding the converse of H. Levi's theorem: Theorem 2. If $P \equiv 0\,[uv]$, but no proper factor of P is in [uv], then $w(P) < k \cdot \ell$. (Received February 22, 1965.)

622-49. C. H. GIFFEN, The Institute for Advanced Study, Princeton, New Jersey 08540. Reidemeister torsion of 2-spheres in the 4-sphere.

A tent is a regular polyhedral (4,2)-ball pair (S_*, K_*) such that there is a general position semilinear map $f_*: S_* \longrightarrow [0,1]$ satisfying: (0) $f_*^{-1}1$ is a point; (1) $S_t = f_*^{-1}t \cong S^3$ for all $t \in [0,1)$; (2) $K_* \subset f_*^{-1}[0,1)$; (3) $(f_*|K_*)^{-1}(0,1/2) \supset$ all hyperbolic points; (4) $(f_*|K_*)^{-1}(1/2,1) \supset$ all elliptic points; (5) $K_0 = K_* \cap S_0$ is a simple closed curve. Theorem. The spine of the complementary space $Q_* = S_* - K_*$ of a tent (S_*, K_*) is at most 2-dimensional. Milnor (Ann. Math. 76 (1962), 137-147) defines the torsion of homology circles and shows that it is $A(t)/(t-1)$ for 2-complexes, where A(t) is the Alexander polynomial of the fundamental group of the complex. Corollary. The torsion of Q_* is $A_*(t)/(t-1)$, where $A_*(t)$ is the Alexander polynomial of the tent (S_*, K_*). In Abstract 65T-200, Cross sections of 2-spheres in the 4-sphere, we show that any polyhedral 2-sphere in S^4 can be decomposed as the sum of two tents with their boundaries glued together. Using the duality theorem for torsion, we get the following. Theorem. If (S,K) is a locally flat polyhedral (4,2)-sphere pair, the torsion is $A(t)/A(1/t)(t-1)$ where A(t) is the Alexander polynomial of (S,K). (Received February 22, 1965.)

622-50. G. T. McALLISTER, Ballistic Research Laboratories, Computing Laboratory, Aberdeen Proving Ground, Maryland. An ordinate uniqueness theorem for a mildly nonlinear elliptic system.

Let (u,v) be a pair of non-negative Hölder continuous functions such that $\Delta u = v^2$, $\Delta v = u^2$ and $u = \phi_1 \geqq 0$, $v = \phi_2 \geqq 0$ on the boundary of a plane bounded region Ω. Let (w,z) be any other such pair. If there exists a $P \in \Omega$ such that $u(P) = w(P)$ or $v(P) = z(P)$, then $u(Q) = w(Q)$ and $v(Q) = z(Q)$ for every $Q \in \Omega$. (Received February 22, 1965.)

622-51. D. SARAFYAN, Louisiana State University, New Orleans, Louisiana 70122. Improvements in the derivation of Runge-Kutta type formulas and computer implementation.

The derivation of Runge-Kutta type formulas, for the numerical solution of differential equations $y' = f(x,y)$, is done in two stages. First, nonlinear algebraic equations should be determined, then these should be solved. Both of these problems are quite tedious and time consuming. Various methods, more or less similar, have been known and used for the purpose of making the derivation of these equations less laborious. A method will be established, totally different from these familiar ones, which will further simplify considerably the derivation of the algebraic equations. This method, which makes repeated use of a formula involving some simple operators, does not require the analyticity of f(x,y). Furthermore, this fundamental formula enables one to derive the Runge-Kutta algebraic equations in about a minute (for up to the tenth order) through the electronic computers. This introduces new possibilities both for computers and the numerical solution of differential equations by higher-order approximation methods. (Received February 22, 1965.)

622-52. ALESSANDRO FIGÀ TALAMANCA, Massachusetts Institute of Technology, Cambridge 39, Massachusetts. On the subspace of L^p invariant under multiplication of transforms by bounded continuous functions.

Theorem. Let $f \in L^p(-\infty, +\infty)$, for $1 < p < 2$ and let $\hat{f}$ be its Fourier transform. Suppose that, for each bounded continuous function ϕ, there exists $g \in L^p$ such that $\phi\hat{f} = \hat{g}$; then $f = 0$ a.e. A paper containing this theorem and other related results will be published in Rend. Sem. Mat. Univ. Padova. (Received February 22, 1965.)

622-53. R. M. GUNDERSEN, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53211. Simple waves and symmetric hyperbolic systems.

Consider the partial differential system $U_t + AU_x = 0$, where U is a column vector with n-components and A is an $n \times n$ real symmetric matrix with all diagonal elements equal. With some minor modifications, the equations governing one-dimensional magnetohydrodynamic flow form such a system. By looking directly for simple wave solutions, sufficient conditions are derived for the existence of $n - 2$ independent integrals so that the system is reduced to only two equations; these may be integrated and generalized Riemann invariants defined. In particular, for $n = 3$, this situation obtains if any nondiagonal element vanishes. (Received February 22, 1965.)

622-54. T. N. BHARGAVA and S. E. OHM, Kent State University, Kent, Ohio 44240. Some results on algebraic structures and the digraph topology.

Let us define an ideal for a halfgroupoid (for notations and definitions see Doyle and Warne [Amer. Math. Monthly 70 (1963), 1051-1057] and Bruck [A survey of binary systems, Springer, Berlin, 1958]) to be a subset $I \subseteq H$ such that $I \cdot (H - I) \subseteq I$ and $(H - I) \cdot I \subseteq I$; then we can easily establish a topology on a halfgroupoid by letting an ideal be an open set. If we now consider the digraph topology as described by Bhargava [Abstract 64T-138, these *Notices* Amer. Math. Soc. 11 (1964), 230] and Bhargava and Ahlborn [Abstract 611-80, these *Notices* Amer. Math. Soc. 11 (1964), 341], we find that various relationships can be established among the three mathematical systems of half groupoids, the digraph topology, and directed graphs. For example, we prove Theorem 1. A digraph is weakly connected $\Leftrightarrow$ the digraph topology is connected $\Leftrightarrow$ every nonempty $I \subset H$ for which $H - I$ is a subgroupoid such that either $I \cdot (H - I)$ or $(H - I) \cdot I$ is nonempty, where $I \cdot (H - I) = \{(a,b);\ a \in I,\ b \in (H - I),\ a \cdot b \text{ is defined in } H\}$; Theorem 2. A digraph is a null graph $\Leftrightarrow$ the digraph topology is a T_2 space $\Leftrightarrow$ for every $a \in H$, $a\cdot(H - a) = (H - a)\cdot a = \emptyset$ and $a\cdot a = a$, or is undefined. We further note that a digraph may be obtained directly from a halfgroupoid by allowing a directed edge from a to c if $a\cdot b = c$ for some $b \in H$. From this, several interesting results follow, e.g., a cyclic groupoid yields a Hamiltonian line whose length is the order of the groupoid. (Received February 22, 1965.)

622-55. E. J. SHERRY, Systems Analysis Division II - 5253, Sandia Corporation, Box 5800, Albuquerque, New Mexico 87115. Some inequalities for the heat operator. Preliminary report.

This paper extends to the L^p norms for $p \geqq 2$ and the heat operator $P(D) = \partial/\partial t - \sum_{i=1}^n \partial^2/\partial x_i^2$ the method used to derive global energy inequalities of the kind found in Trevés [Relations de dominations entre opérateurs différentiels, Acta Math. 101 (1959), 1-139]. Theorem. Let $u(x,t) \in C_0^\infty(R_x^n \times R_t)$, $\lambda > 0$, and $p \geqq 2$; then for $P(D) = \partial/\partial t - \sum_{i=1}^n \partial^2/\partial x_i^2$, $\|e^{-\lambda t}u\|_{L^p} \leqq (1/\lambda)\|e^{-\lambda t}P(D)u\|_{L^p}$, and $\|e^{-\lambda t}\partial u/\partial x_i\|_{L^p} \leqq (p/4\sqrt{\lambda})\|e^{-\lambda t}P(D)u\|_{L^p}$ $(i = 1,\dots,n)$. (Received February 22, 1965.)

622-56. A. R. MEYER, 298 Pierce Hall, Harvard University, Cambridge, Massachusetts 02138. Depth of nesting and the Grzegorczyk hierarchy.

A primitive recursive function is in K_n if its definition requires nesting primitive recursions to a depth of at most n. (See Axt, Abstract 597-182, these *Notices* 10 (1963), 113.) Lemma 1. (Axt) $K_n \subset \mathscr{E}^{n+1}$, for $n \geqq 0$, where $\mathscr{E}^m$ is the mth Grzegorczyk class. Lemma 2. For $n \geqq 2$, $f(\bar{x}) \in \mathscr{E}^{n+1} \Rightarrow \exists g(\bar{x}) \in K_n$ s.t. $f(\bar{x}) \leqq g(\bar{x})$. Lemma 3. $O(e,\bar{x},y) \in K_{n_0}$ where $O(e,\bar{x},y)$ = output of the eth Turing Machine (T.M.) with input $\bar{x}$ if it halts in $\leqq$ y steps, otherwise it = 0. Theorem 1 (Announced by A. Cobham). For $n \geqq 3$, $f(\bar{x}) \in \mathscr{E}^n \Leftrightarrow$ a T.M. computing $f(\bar{x})$ halts in $\leqq \sigma_f(\bar{x})$ steps and $\sigma_f(\bar{x}) \in \mathscr{E}^n$. Theorem 2. $\mathscr{E}^{n+1} \subset K_n$, for $n \geqq n_0$ (if $f(\bar{x}) \in \mathscr{E}^{n+1}$, $f(\bar{x}) = O(e,\bar{x},\sigma_f(\bar{x})) = O(e,\bar{x},g(\bar{x})) \in K_n$ for some fixed e with $\sigma_f \in \mathscr{E}^{n+1}$ and $g \in K_n$). Corollary (Announced by P. Axt in a private communication). For $n \geqq n_0$, $K_n = \mathscr{E}^{n+1}$. D. M. Ritchie (see Abstract 622-59 these *Notices* 12 (1965), 343) has shown $n_0 \leqq 4$. Theorem 3. There is no effective procedure to find the minimal K or $\mathscr{E}$ class containing a given function. (Received February 22, 1965.)

622-57. ALFRED LEHMAN, Walter Reed Army Institute of Research, Washington, D. C. 20012. Matroids and ports.

A port $\mathscr{P}$ is a family of finite subsets of a set S which has property (Δ) and in which no subset properly contains any other subset. (Δ) For $P_1, P_2, P_3 \in \mathscr{P}$ and $a \in (P_1 \cap P_2) - P_3$, there exists $P_4 \in \mathscr{P}$ such that $P_4 \subset (P_1 \cup ((P_2 \cup P_3) - (\bigcap_{P \subset P_2 \cup P_3} P))) - \{a\}$. This property ($\Delta$) is weaker than the tree (Steinitz exchange) and circuit properties of a matroid [(B_2) and (C_2) of H. Whitney, Amer. J. Math. 57 (1935), 509-533]. For example, the collection of all maximal trees of a matroid forms a port; so also does the collection of all circuits which intersect a fixed set. Nevertheless, the theory of ports can be subsumed under the theory of matroids via the following result. Theorem. $\mathscr{P}$,S is a port if and only if M, $S \cup \{e\}$, where $e \notin S$ and $M = \{P \cup \{e\} \mid P \in \mathscr{P}\} \cup$(minimal non-null members of $\{(P_1 \cup P_2) - (\bigcap_{P \subset P_1 \cup P_2} P) \mid P_1, P_2 \in \mathscr{P}\}$), is a matroid. Extending matroid duality to ports, the dual of a port $\mathscr{P}$ (where S is finite) is the family $\mathscr{K}$ of minimal sets K which intersect every P in $\mathscr{P}$. Thus $\mathscr{K}$, which is the Boolean dual of $\mathscr{P}$, is a port. The theorem also answers the question posed implicitly in [A. Lehman, J. Soc. Indust. Appl. Math. 12 (1964), p. 718, lines 28-30]. (Received February 22, 1965.)

622-58. GILBERT STENGLE, Lehigh University, Bethlehem, Pennsylvania. Sard's lemma for certain differentiable maps on Hilbert space.

Let f be a differentiable map of separable Hilbert space H into itself. Define a critical value of f to be a y such that the adjoint of f'(y) is not one-to-one. Let f be of the form f(x) = x + Rh(Sx), where h is differentiable and R and S are linear operators of trace class. Let K be the set of critical values of f. Then there is a symmetric operator $\sigma_0(f)$ of trace class such that, if P is any Gaussian measure on H, with a covariance operator dividing σ_0, then $P\{f(K)\} = 0$. (Received February 22, 1965.)

622-59. D. M. RITCHIE, 298 Pierce Hall, Harvard University, Cambridge, Massachusetts 02138. Complexity classification of primitive recursive functions by their machine programs.

A type of computer is presented which computes exactly the primitive recursive functions. The computer has available a number of memory cells, each of which may hold any non-negative integer, and a repertoire of 3 instructions: MAD, RPT, and END. MAD causes the contents of one memory cell to be transferred to another and a constant added; RPT causes the instructions up to a balancing END to be repeated a number of times equal to the contents of some memory cell at the time the RPT is encountered. The programs for the computer, and, by extension, the functions computed by them, may be assigned to classes M_{ij}, where i measures the depth of nesting of RPT/END sequences, and j measures the number of outer RPT/END's. Theorem. The classes of functions M_{ij} ordered by the lexicographical ordering of i and j form a strictly increasing hierarchy of order type ω^2. Theorem. For $i \geqq n_0$, $\bigcup_j M_{ij} = \mathscr{E}^{i+1} = K_i$, where $\mathscr{E}$ and K are the Grzegorczyk and Axt hierarchies, respectively, and $n_0 = 4$. This strengthens the independent results of Axt and A. R. Meyer (see Abstract 622-56, these *Notices* 12 (1965), 342) who had $n_0 = 6$ and $n_0 = 9$, respectively. Theorem. For each $i > 4$, M_{ij} has the same closure properties under composition and limited recursion as the classes F_j of R. Ritchie. (Received February 22, 1965.)

622-60. J. M. WORRELL, JR., and H. H. WICKE, Sandia Corporation, Sandia Base, Division 5251, Albuquerque, New Mexico. Concerning spaces having bases of countable order.

Characterizations are given of essentially T_1 spaces having bases of countable order and of regular T_1 spaces satisfying Aronszajn's axiom [Fund. Math. 15, 231-232.] These characterizations have, as one of two conditions, the following base property. There exists a sequence $H_1, H_2, \ldots$ of well-ordered collections of open sets such that, if P is a point of an open set D, there exist integers k and n such that n elements h of H_k contain P and the nth such h is a subset of D. The following theorems are obtained as corollaries. Theorem 1. A paracompact Hausdorff space is metrizable if and only if (1) the above base condition is satisfied locally and (2) closed sets are locally inner limiting sets. Theorem 2. A paracompact Hausdorff space is metrically topologically complete if and only if (1) the above base condition is satisfied locally and (2) the space is complete in the sense of Čech locally. (Received February 23, 1965.)

622-61. V. BOHUN-CHUDYNIV, Morgan State College, Baltimore, Maryland. On a general method of constructing triple-systems of all possible orders.

Examples of triple-systems of order $n = 6q + r$ ($r = 1$ or 3), (a), were constructed by Kirkman (1847), Steiner (1953), De Vries (1889, 1894), Cayley (1850), Netto (1893), White (1913), and others, all using different particular methods which even together do not yield all possible triple-systems satisfying (a). In 1893 E. Moore established the existence of at least two nonequivalent types of triple-systems of order greater than 13. The present author, in a paper presented to MAA (1964) proved the existence of at least 8 triple-systems of order $n > 13$. E. Netto (M.A. 42 (1893), 143-152) derived 5 particular methods for constructing triple-systems for all $n < 100$ satisfying (a) with the exception of $n = 25$ and $n = 85$. The present author, in a paper presented to the 14th Scandinavian Mathematical Congress (1964) established a method of constructing, from any given triple-system satisfying (a), a triple-system of order $2n + 1$, simpler than that of E. Netto. E. Netto raised the question of whether triple-systems exist for every positive integer n satisfying (a), which was answered in the affirmative by the present author (Int. Cong. of Math., Stockholm, 1962). The present paper presents a general method for constructing triple-systems of all orders n satisfying (a) without exception. (Received February 23, 1965.)

622-62. M. A. MALIK, Université de Montréal, Montréal, Canada. On the existence of solutions of differential equations in Banach spaces. Preliminary report.

Let B be a Banach space, A a closed linear operator with dense domain D_A in B, $\mathscr{L}(B,D_A)$ the set of linear continuous operators from B to D_A (D_A with the graph topology). Let also $\mathscr{D}(R^n)$ be the space of infinitely differentiable scalar functions with compact support equipped with Schwartz topology and $\mathscr{D}'(R^n,X)$ the space of vector X-valued distributions, that is $\mathscr{D}'(R^n,X) = \mathscr{L}(\mathscr{D}(R^n),X)$. A distribution $E \in \mathscr{D}'(R^1, \mathscr{L}(B,D_A))$ is called an elementary solution for the operator $(-\ id/dt) - A = L$, if $LE = \delta \otimes I$, δ being the Dirac distribution and I the identity operator. Theorem 1. If B is reflexive and, in any interval $(a \leqq t \leqq b) \subset R^1$, the relations $Lu = 0$, $u(t) \in C^1(a,b;B)$ and $u(t) \in D_A$ imply that $u(t) = 0$ on $a \leqq t \leqq b$, then there exists at least one $f(t) \in \mathscr{D}(R^1,B)$ such that the equation $Lu = f$ has no solution $u \in \mathscr{D}'(R,D_A)$. In the proof we use the following Lemma. Under the hypothesis of Theorem 1 (B need not be reflexive) the operator L has no elementary solution. Theorem 2. If (1) $R(\lambda;A) = (\lambda - A)^{-1} \in \mathscr{L}(B,D_A)$ in $\Sigma \equiv \{|\mathrm{Im}\,\lambda| \leqq (1/\epsilon)\log|\mathrm{Re}\,\lambda|, |\lambda| \geqq N_0 > 0, \epsilon > 0\}$ and in Σ, $\|R(\lambda;A)\| \leqq \mathrm{const}|\lambda|^s e^{\Delta|\mathrm{Im}\,\lambda|}$, $s \geqq -1$, $\Delta > 0$. (2) The complement of the spectrum $\sigma(A)$ is arcwise connected; then the operator L has an elementary solution belonging to $C^k(|t| > (s + k + 1)\epsilon + \Delta; \mathscr{L}(B,D_A))$. These results are connected with a recent paper by S. Agmon and L. Nirenberg. (Received February 23, 1965.)

622-63. S. G. MROWKA, Pennsylvania State University, 227 MacAllister Hall, University Park, Pennsylvania. On nearly-Lindelöf spaces.

A completely regular space X is nearly-Lindelöf provided that every transfinite decreasing sequence $F_0, F_1, \ldots, F_\xi, \ldots$, $\xi < \alpha$, of nonempty closed subsets of X with $cf(\alpha) > \omega$ has the nonempty intersection. This notion is a result of some theorems which the author was trying to prove for Lindelöf spaces. Example: If X is Lindelöf, then (a) for every compact Y, the projections onto Y of closed subsets of $X \times Y$ are real compact; however, if (a) holds, then the author is able to prove only that X is nearly-Lindelöf. The author does not know whether every nearly-Lindelöf space is Lindelöf. Yu. M. Smirnov (Izv. Akad. Nauk SSSR 14 (1950), 155-178) has a theorem which yields a positive answer (Theorem 1, p. 158); his proof, however, seems to have a gap. (Received February 23, 1965.)

622-64. SARVADAMAN CHOWLA, 413 E. Mitchell Avenue, State College, Pennsylvania 16801. On $y^2 = x^3 + k$.

Improvement of Chang's results (Quart. J. Math., Oxford 19 (1948), 181-188). (Received February 23, 1965.)

622-65. M. M. HACKMAN, University of Washington, Seattle, Washington 98105. The abstract time-dependent Cauchy problem for measurable operators affiliated with a discrete ring of operators in a separable Hilbert space.

Theorem. Let X be a separable Hilbert space, let $\mathscr{A}$ be a discrete ring of operators in X, let $\mathscr{M}(\mathscr{A})$ be the algebra of operators affiliated with $\mathscr{A}$ measurable in the sense of Segal, let tr be an essential trace on $\mathscr{A}$, and let $X_0 = \{x \in X: (Tx,x) \leqq k(x) \operatorname{tr} T \ \forall T \in \mathscr{A}^+\}$ (X_0 is dense in X). $\forall t \in [a,b]$, let $A_0(t) = A_1(t) + A_2(t)$, where $A_1(\cdot) \in L^1(B(X),[a,b])$ (B(X) = bounded operators in X), and where $A_2(\cdot)$ is a measurable function from $[a,b]$ to $\mathscr{M}(\mathscr{A})$ having $\operatorname{tr} A_2(\cdot)^* A_2(\cdot) \in L^1([a,b])$. Then for $a \leqq s \leqq t \leqq b$, $\exists B(t,s) \in \mathscr{M}(\mathscr{A})$ such that (1) $B(t,t) = I \ \forall t \in [a,b]$, (2) $\exists$ central $A \in B(X)$, 1-1 with dense range in the domain of every B(t,s), having $B(\cdot,s)A \in L^\infty(B(X),[s,b]) \ \forall s \in [a,b]$ (having, indeed, B(t,s)A uniformly bounded over all $a \leqq s \leqq t \leqq b$), and (3) if $x \in X_0$, $B(\cdot,s)Ax$ is absolutely continuous on $[s,b]$ and a.e. strongly differentiable on $[s,b]$ with derivative $A_0(\cdot)B(\cdot,s)Ax$. These properties uniquely characterize B(t,s). Further, (4) for $x \in X_0$, $B(t,\cdot)Ax$ is absolutely continuous on $[a,t]$ and a.e. strongly differentiable on $[a,t]$ with derivative $-B(t,\cdot)AA_0(\cdot)x$, (5) if $\bar{B}(t,s)$ is related to $-A_0(\cdot)^*$ as B(t,s) is related to $A_0(\cdot)$, $\bar{B}(t,s)$ has all the previous properties for the same A, and $\bar{B}(t,s) = B(t,s)^{-1}$, (6) $\forall s_0 \in [s,t]$, $B(t,s_0)B(s_0,s) = B(t,s)$, and (7) B(t,s)A is jointly strongly continuous in t and s. (Received February 24, 1965.)

622-66. SAMUEL ZAIDMAN, Université de Montréal, Montréal, Canada. Convexity properties for weak solutions of differential equations in Hilbert spaces.

Let H be a Hilbert space, $(\cdot)$ and $|\ |$ being the scalar product and the norm. In H consider a family B(t), $0 \leqq t \leqq T$, of closed linear operators with dense domain D(B(t)), and denote by $B^*(t)$ the adjoint of B(t). Let $L^2(0,T;H)$ be the space of Bôchner square-integrable H-valued functions. Theorem. Consider a function u(t) from $0 \leqq t \leqq T$ to H, such that: (i) $u(t) \in L^2(0,T;H)$, $u(t) \in L^2(0,T;H)$, $u(t) \in D(B(t)) \cap D(B^*(t))$ a.e., $|u(t)| > 0$, $u'(t) - B(t)u(t) = 0$; (ii) The derivative $(d/dt)\operatorname{Re}(B(t)u(t),u(t)) \in L^1(\alpha \leqq t \leqq \beta)$ if $0 < \alpha < \beta < T$; (iii) For a constant $k \geqq 0$ and an increasing twice continuously differentiable function $\omega(t)$, $0 \leqq t \leqq T$, the inequality $(d/dt)\operatorname{Re}(B(t)u(t), u(t)) \geqq (1/2)|(B(t) + B^*(t))u(t)|^2 + (\omega''/\omega') \operatorname{Re}((B(t) - k)u,u)$ holds a.e. in $0 \leqq t \leqq T$. Then, $\log e^{-kt}|u(t)|$ is a convex function of $s = \omega(t)$.

The result is a generalization of a recent one by Agmon-Nirenberg. It permits also a partial extension of the theorem on backward unicity for parabolic equations by Lions and Malgrange.
(Received February 24, 1965.).

622-67. R. F. BARNES, JR., 1127D 9th Street, Albany, California 94710. The classification of closed-open sets of the Baire space.

Let N be the set of natural numbers and let $\mathfrak{N}$ be the Baire (metric) space $\langle N^N, d\rangle$, where, for any distinct α, β in N^N, $d(\alpha,\beta) = (1 + \min\{n: \alpha(n) \neq \beta(n)\})^{-1}$. For any m,n in N and any α in N^N let $m\alpha(0) = m$, $m\alpha(n + 1) = \alpha(n)$; for any set X of $\mathfrak{N}$ and any n in N let $X_{(n)} = \{\alpha: n\alpha \in X\}$. Let $Ka_0 = \{\phi, N^N\}$ and for any positive ordinal ν let $Ka_\nu = \{X: \forall n\ X_{(n)} \in \bigcup\{Ka_\mu: \mu < \nu\}\}$. Then $Ka|\Omega$ is a hierarchy invariant under isometry and $Ka_\Omega = \bigcup\{Ka_\nu: \nu < \Omega\}$. Strong separation principle. For any disjoint closed sets X, Y of $\mathfrak{N}$: X,Y are Ka_Ω-separable (i.e., separable by a set in Ka_Ω). Corollary (Construction principle). For any set X of $\mathfrak{N}$: X is closed-open $\Leftrightarrow X \in Ka_\Omega$. Let $\mathscr{S}$ be $\bigcup\{N^n: n \in N\}$; for any s in $\mathscr{S}$ let $[s] = \{\alpha: s \subseteq \alpha\}$. For any set X of $\mathfrak{N}$ let $\partial X = \{s: |s| \not\subseteq X \wedge [s] \not\subseteq N^N \sim X\}$. For any subset S of $\mathscr{S}$ let $D_0 S = S$ and for any positive ordinal ν let $D_\nu S = \{s: \exists n\ sn \in \bigcap \{D_\mu S: \mu < \nu\}\}$. Separability test. For any disjoint sets X,Y of $\mathfrak{N}$ and any ordinal ν: X,Y are Ka_ν-separable $\Leftrightarrow$ $D_\nu(\partial X \cap \partial Y) = \emptyset$. Corollary (Classification test). For any set X of $\mathfrak{N}$ and any ordinal ν: $X \in Ka_\nu$ $\Leftrightarrow D_\nu \partial X = \emptyset$. These results consolidate and extend hitherto unconnected work of Luzin (Lecons sur les ensembles analytiques, p. 85), Kalmar (Colloq. Math. 5 (1957), 1-5), and Kleene (Colloq. Math. 6 (1958), 67-78). (Received February 24, 1965.)

622-68. M. M. RAO, Carnegie Institute of Technology, Pittsburgh, Pennsylvania 15213. Smoothness of Orlicz spaces.

Let L^Φ, L^Ψ be complementary Orlicz spaces on (Ω,Σ,μ), μ with finite subset property, and $\Phi(1) + \Psi(1) = 1$. $N_\Phi(f) = \inf\{k > 0, \int_\Omega \Phi(f/k)d\mu \leq \Phi(1)\}$ is the norm in L^Φ and similarly $N_\Psi(\cdot)$ for L^Ψ. [For geometric terminology below, see M. M. Day's Normed linear spaces.] Φ', Ψ' are derivatives of Φ, Ψ. Theorem 1. (a) Sufficient conditions for the smoothness and uniform rotundity of L^Φ are: (i) for each $\alpha > 1$, $\exists k_\alpha > 1 \ni \Phi'(\alpha u) \geqq k_\alpha \Phi'(u)$, $u \geqq 0$; (ii) $\exists C < \infty \ni \Phi(2u) \leqq C\Phi(u)$, $u > 0$. (b) If L^Φ is reflexive, Φ', Ψ' are continuous, then L^Φ is both rotund and smooth. Theorem 2. Let L^Φ be reflexive, $\mathfrak{X}$ a reflexive B-space. Let the norm of $\mathfrak{X}$ $[\mathfrak{X}^*]$ be weakly [strongly] differentiable. If $L^\Phi(\mathfrak{X})$ is the Orlicz space of $\mathfrak{X}$-valued functions, then for each ℓ in $[L^\Phi(\mathfrak{X})]^*$ there is a unique unit vector g_ℓ in $L^\Phi(\mathfrak{X})$ such that, for all $f \in L^\Phi(\mathfrak{X})$, $\ell(f) = \|\ell\| \int_\Omega \Phi'(|g_\ell|)(d/dt)[|g_\ell + tf|]_{t=0} d\mu$, ($|\cdot|$ is the norm in $\mathfrak{X}$). Theorem 3. Let L^Φ be reflexive and Φ', Ψ' be continuous, and $\{X_n\}$ be a sequence of random variables in the unit ball of L^Φ, and $Y \in L^\Psi$. If $\int_\Omega X_n Y d\mu \to N_\Psi(Y) > 0$, then there is a unique unit vector X in L^Φ, $\ni X_n \to X$ strongly, and $X = \Psi'(|Y|/N_\Psi(Y))\ \mathrm{sgn}(Y)$, a.e., as $n \to \infty$. (Received February 24, 1965.)

622-69. D. W. HENDERSON, The Institute for Advanced Study, Princeton, New Jersey 08540. Each non-zero-dimensional compactum has a connected one-dimensional subset.

This result is proved using the fact that each infinite-dimensional compactum contains, for each n, hereditarily indecomposable continua of dimension greater than n. (See R. H. Bing, Higher-dimensional hereditarily indecomposable continua, Trans. Amer. Math. Soc. 71 (1951), 267-273.) It is an immediate consequence of the inductive definition of dimension that all n-dimensional ($n \neq \infty$) compacta have compact k-dimensional subsets for each $k \leqq n$. Van Heemert (Indag. Math. 8 (1946), 564-569) purported to show that each infinite-dimensional compactum has compact subsets of dimensions one and two, but his arguments are incorrect. No other results along these lines are known (for compacta). (Received February 24, 1965.)

622-70. J. J. GERGEN, F. G. DRESSEL, Duke University, Durham, North Carolina and G. B. PARRISH, U. S. Army Research Office, Durham, North Carolina. Expansions for Bessel difference systems of zero order.

The authors consider the difference system (1): $4(u_1 - u_0)/h^2 + \lambda^2 u_0 = 0$, $(u_{j+1} + u_{j-1} - 2u_j)/h^2 + (u_{j+1} - u_{j-1})/(2jh^2) + \lambda^2 u_j = 0$, $u_n = 0$, $j = 1,2,\ldots,n - 1$, where n is a positive integer and $h = 1/n$. This system corresponds to the Bessel differential system $y'' + y'/x + \Lambda^2 y = 0$, $x \in (0,1]$, $y \in C''[0,1]$, $y(1) = 0$. It has been shown by Boyer that the system (1) has n positive real eigenvalues: $\lambda_1^2 < \ldots < \lambda_n^2$, $0 < \lambda_j$. The authors [Abstract 604-6, these *Notices* 10 (1963), 569] have shown that, if $\Lambda_1 < \Lambda_2 < \ldots$ are the positive real zeros of $J_0(x)$, then $0 < \Lambda_k - \lambda_k < C_1\Lambda_k^3 h^2$, C_1 an absolute const. In this paper the authors consider the eigenvectors $\{u_0(k), u_1(k), \ldots, u_n(k)\}$, $u_0(k) = 1$, $k = 1,2,\ldots,n$, corresponding to the eigenvalues $\lambda_1^2, \ldots, \lambda_n^2$, and Fourier expansions in terms of these eigenvectors. It is shown that $|u_j(k) - J_0(jh\Lambda_k)| < C_2(\Lambda_k h)^{3/2} \cdot j^{-1/2}$, C_2 const, $1 \leqq j \leqq n$, $1 \leqq k \leqq n/2$; and that, if $f(x) \in C''[0,1]$, $f'(0) = f(1) = 0$, then $\sum_{k=1}^n m_k^2(a_k - A_k)^2 < C_3 h^4$, C_3 const, where $a_k = m_k^{-2}h^2\sum_{j=0}^{n-1} e_j u_j(k)f(jh)$, $m_k^2 = h^2\sum_{j=0}^{n-1} e_j u_j^2(k)$, $A_k = M_k^{-2}\int_0^1 xJ_0(x\Lambda_k)f(x)dx$, $M_k^2 = \int_0^1 xJ_0^2(x\Lambda_k)dx$ and $e_0 = 1/8$, $e_j = j$, $j \neq 0$. Values are given for C_2, C_3. These results are applied to obtain the truncation error for a boundary value problem for the equation $u_{xx} + u_x/x + u_{yy} = 0$. (Received February 24, 1965.)

622-71. J. W. ADDISON, University of California, Berkeley, California 94720. The undefinability of the definable.

Is the class of sets (explicitly) definable in a language itself definable in that language? asked Tarski (J. Symbolic Logic 13 (1948), 107-111), who considered, in particular, applied predicate languages L for number theory. For third- and higher-order L he gave negative answers, and negative answers for certain infinitary first-order L and (assuming the axiom of constructibility) for second-order L were given at the 1957 Cornell Institute (Summaries, pp. 355-362); but the finitary first-order case has remained open. In trying to generalize the easy proof that the class of recursive subsets of N is not recursive, one is led naturally to the concepts of forcing and generic set (first discovered by Cohen in his independence proofs (Berkeley Symposium, 1963) and developed by Feferman (loc. cit.) for first-order L), and thereby to a solution of Tarski's problem for first-order L: (I) The class of arithmetical subsets of N is not arithmetical. The analysis suggests, moreover, a general method for extending results about recursiveness to higher levels of definability; e.g. the method can be combined with Shoenfield's category technique (Proc. Amer. Math. Soc. 9 (1958), 630-692) and Alexandrov's theorem to refine (I) to: (II) For any n in N and any Π_{n+2} subset A of N^N if $A \supseteq \Sigma_n \cap N^N$ then A has $2^{\aleph_0}$ elements in every interval. (Received February 24, 1965.)

622-72. J. D. HALPERN, California Institute of Technology, Pasadena, California. Bases in vector spaces and the axiom of choice. Preliminary report.

We are concerned with the relationship between the axiom of choice (AC) and the following two theorems about vector spaces: (I). Every vector space has a basis and (II) Any two bases of a vector space have the same cardinal number. Both of these theorems are consequences of AC and the problem arises as to whether or not they individually or in conjunction imply the axiom of choice. It can be shown that (II) is a consequence of the Boolean prime ideal theorem and hence by itself does not imply AC. (See the author's paper, Independence of A.C. from the Boolean prime ideal theorem, Fund. Math 55 (1964).) Whether or not (I) implies AC is still unknown. However it can be shown that a natural strengthening of (I) is already equivalent to AC: Theorem. If every vector space V has the property that every spanning subset of V includes a basis, then AC. It seems doubtful that the conjunction of (I) and (II) imply AC and in particular we see no way to use the theorem to obtain this result. For other results see M. Bleicher, Some theorems on vector spaces and AC, Fund Math. 54 (1964). (Received February 24, 1965.)

622-73. PETER SEIBERT, Division of Applied Mathematics, Brown University, Providence, Rhode Island 02912. On generalized dynamical systems.

A generalized dynamical system is defined as a collection of "orbits", i.e., continuous maps ϕ from $R^+ = [0,\infty)$ to a locally compact metric space X. Notations: $\Phi_x = \{\phi \in \Phi | \phi(0) = x\}$, $F(x,t) = \{\phi(t) | \phi \in \Phi_x\}$. Axioms: (I) $F(x,t) \neq \emptyset$; (II) and (III) concern "piecing together" and "truncating" of orbits and imply the semi-group-property of F; (IV) F is upper-semi-continuous; (V) F(x,t) is compact. The following kinds of limit sets are introduced: $\Lambda(\phi) = \bigcap \{\overline{\phi([t,\infty))} | t \in R^+\}$, $\Lambda(x) = \bigcup \{\Lambda(\phi) | \phi \in \Phi_x\}$, $\Lambda^*(x) = \bigcap \{\overline{F(x,[t,\infty))} | t \in R^+\}$. Let $M \subset X$ be compact, A a compact neighborhood of M, and $F(A,R^+) \subset A$. M is called an attractor [a strong attractor] relative to A if $x \in A \Rightarrow \Lambda(x) \subset M$ $[\Lambda^*(x) \subset M]$. Stability of M (in the sense of Liapunov) is defined as usual. Theorem: If M is an attractor and stable, it is a strong attractor. Theorem: M is a stable attractor relative to A iff there exists a continuous map V from A to R^+ with the following properties: (a) $V^{-1}(0) = M$; (b) $x \in A \setminus M$ and $x \neq y \in F(x,R^+)$, imply $V(y) < V(x)$. (Received February 24, 1965.)

622-74. G. O. PETERS, General Electric Company, Re-entry Systems Department, Philadelphia, Pennsylvania. An extension to the theory of the F-equation. Preliminary report.

Clifford A. Truesdell called attention to the unification in the theory of special functions achieved through systematic exploitation of properties of the solutions of the F-equation: $\partial F(z,\alpha)/\partial z = F(z,\alpha + 1)$. Letting $\beta = -\alpha$, this can be written: $D_z F_2(z,\beta) = F_2(z,\beta - 1)$. The author suggests other interesting results might be obtained by variations of the F-equation, such as by replacing the differentiation operation by the difference, Δ; mean, $\nabla = 1 + 1/2\Delta$; or (Dalet) $7 = 1 + 1/2D$ operation: and also by considering the equation: $D_z F_3(z, a, \mu) = F_3(z, a+1, \mu+1) = F_4(z, \beta - 1, \eta - 1)$, where: $\beta = -a$ and $\eta = -\mu$. The Appell polynomials, $B_\nu^{(n)}(z), E_\nu^{(n)}(z), \zeta_\nu^{(n)}(z), b_\nu^n(z), e_\nu^n(z), Z_\nu^n(z)$ (Bernoulli, Euler, and Boole polynomials of higher order, of the first and second kind) each satisfy two such equations. Two other solutions of such equations involve the Beta function, $B(z,a)$: $\Delta_z(-1)^a B(z,a) = (-1)^{a+1} B(z, a+1); \Delta z[(z+a)B(z, a+1)]^{-1} = [(z+a)B(z+1, a)]^{-1}$. The author shows some relations of such functions. (Received February 24, 1965.)

622-75. M. M. PEIXOTO, Brown University, Providence, Rhode Island. A note on the Closing Lemma.

The problem of the Closing Lemma is the following. Let M^n be a compact differentiable manifold, $n \geqq 2$, and B^r, $r \geqq 1$, the space of all vector fields on M^n endowed with the C^r topology. Let $\gamma = \gamma(p)$, $p \in M^n$, be a positive (negative) semi-trajectory of a vector field $X \in B^r$ which is recurrent, i.e. such that p belongs to the ω-limit (α-limit) set of γ. Given any neighborhood U of X on B^r can one always find a vector field $Y \in U$ such that its trajectory through p is a closed orbit? The problem was formulated by the author (Topology 1 (1962), 101-120), and Pugh (Bull. Amer. Math. Soc. 70 (1964), 584-587) announced its solution in the affirmative for $n = 2$, $r = 1$. In a remarkable forthcoming paper Pugh extends his proof for the case of an arbitrary dimension and draws important consequences for the theory of structural stability. But his proof covers only the case $r = 1$ and even the case $n = 2$, $r > 1$ remains open. The aim of this note is to fill this gap, i.e., to prove the Closing Lemma for $n = 2$, $r > 1$. The proof, of a very global nature, is too complicated to be sketched here. (Received February 24, 1965.)

622-76. A. J. MACINTYRE, University of Cincinnati, Cincinnati 21, Ohio. Convergence of an iterated exponential.

Convergence of $i^{i^{i\cdots}}$ is established by observing that $w - \exp(\pi i z/2)$ maps a portion of the plane containing i into its interior. (Received February 21, 1965.)

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623-1. J. M. IRWIN and FRED RICHMAN, Box 396, New Mexico State University, University Park, New Mexico. Direct sums of countable Abelian groups.

Let G be an Abelian group. $G^1 = \bigcap_n nG$. Theorem: If G^1 is a direct sum of countable groups and G/G^1 a direct sum of cyclic groups, then G is a direct sum of countable groups. Corollary: If G is a direct sum of countable torsion groups and $nG \subset H \subset G$, for some n, then H is a direct sum of countable groups. (Received November 24, 1964.)

623-2. JOHN BRILLHART, University of San Francisco, San Francisco, California. On the roots of Euler and Bernoulli polynomials.

The question whether an Euler polynomial other than $E_5(x) = (x - 1/2)(x^2 - x - 1)^2$ can have multiple roots has been raised by Leonard Carlitz. This question is answered in the negative in this paper by simple arguments mod 2 which show that $x^2 - x - 1$ is the only possible multiple factor, and that this factor only occurs as a multiple factor in $E_5(x)$. It is also proven in a similar way that the Bernoulli polynomials of odd suffix do not have multiple roots, and that the only rational roots an Euler polynomial $E_n(x)$ can have are $x = 1/2$ for n odd, and $x = 0$ and 1 for n even and > 0. (Received December 1, 1964.)

623-3. Z. Z. YEH, University of Hawaii, Honolulu 14, Hawaii. Criteria for Cesàro convergence.

Given a sequence $X = \{x_i\}$, each of its subsequences is uniquely determined by a strictly increasing function ϕ of the set of natural numbers into itself. The subsequence of X determined by ϕ is denoted by $X_\phi = \{x_{\phi(i)}\}$. X_ϕ is said to be a steady subsequence of X if $K(\phi) = \sup\{\phi(i+1) - \phi(i)\}$ is finite. The Cesàro derived sequemce X' of X is defined by $X' = \{x_i'\}$ with $x_i' = (x_1+x_2+\ldots.+x)/i$ Theorem 1. A sequence Cesàro derived from a bounded sequence is convergent if and only if it has a convergent steady subsequence. Example. Every periodic sequence is Cesàro convergent to the mean of its period. A much more general theorem is the following. Theorem 2. Let $X = \{x_i\}$ be such that $\lim_{i\to\infty}(x_{i+1} - x_i)/i) = 0$. Then X' is convergent if and only if X'_ϕ is convergent for some ϕ with a finite $K(\phi)$. (Received January 7, 1965.)

623-4. J. A. COCHRAN, Bell Telephone Laboratories, Inc., Whippany, New Jersey. Asymptotic nature of the zeros of cross-product Bessel functions.

The two cross-product combinations of Bessel functions $f_\nu(k,z) = J_\nu(z)Y_\nu(kz) - Y_\nu(z)J_\nu(kz)$ and $g_\nu(k,z) = J_\nu'(z)Y_\nu'(kz) - Y_\nu'(z)J_\nu'(kz)$ appear regularly in physical problems having circular or cylindrical geometry. Asymptotic expansions for the larger z-zeros of these expressions with given real or complex ν and k, $|\arg k| < \pi$, are known (McMahon, Ann. of Math. 9 (1894), 23-30). Similar results for the larger ν-zeros occurring under various limiting regimes in k and z are of distinct practical importance. In this paper expansions for these ν-zeros are established for (i) $z \to \infty$, k = constant; (ii) $k \to \infty$, z = constant; (iii) $k \to 1$, $(k - 1)z$ = constant. Our unified approach utilizes Olver's asymptotic expressions for $J_\nu(\nu z)$ and $Y_\nu(\nu z)$ (Phil. Trans. A 247 (1954), 328-368) and differs significantly from Buchholz's earlier analysis of case (iii) (Z. Angew. Math. Mech. 29 (1949), 356-367). (Received January 7, 1965.)

623-5. E. F. BECKENBACH and T. A. COOTZ, University of California, 405 Hilgard Avenue, Los Angeles, California 90024. Extensions of the convexity theorem of Study.

The conformal-mapping theorem of Study has been extensively generalized (see, for example, E. F. Beckenbach and E. W. Graham, Nat. Bur. Standards Appl. Math. Ser. No. 18 (1952), 247-254). Some analogous results are known for the Green's function of regions in E_3, and recently Stoddard (Michigan Math. J. 11 (1964), 225-229) obtained corresponding results in E_3 for harmonic functions, with the pole of the Green's functions replaced by a continuum on which the functions are constant. The interplay of the two sets of results, in E_2 and E_3, is explored in the present paper. It is shown, for example, that if $f(z)$ is analytic for $r_1 < |z| < r_2$ and continuous for $r_1 \leqq |z| \leqq r_2$, and if the maps of $|z| = r_1$ and $|z| = r_2$ are both star-shaped with respect to a point 0, then the map of each $|z| = r$, $r_1 < r < r_2$, also is star-shaped with respect to 0. (Received January 15, 1965.)

623-6. R. I. IRWIN, University of Utah, Salt Lake City, Utah 84112. A Hardy-Bohr theorem.

A summability method B is said to have the absolute Hardy-Bohr property if $\sum_{v=0}^{\infty} a_v \epsilon_v \in |B|$ whenever $\sum_{v=0}^{\infty} a_v \in |B|$ where ϵ_v is given by $\epsilon_v = \sum_{\mu=v}^{\infty} \hat{b}_{\mu v} c_\mu$ $(c_\mu = O(1))$. The notation $|A| \dot{\supseteq} |B|$ will mean if $s_n \in |B|$ then $s_{n-1} \in |A|$. Theorem: If B has the absolute Hardy-Bohr property, P a weighted arithmetic mean, $A = BP$, $|A| \supseteq |B|$, $|A| \dot{\supseteq} |B|$ then A has the absolute Hardy-Bohr property. The theorem contains all Cesàro means C_α: $0 \leqq \alpha \leqq 2$. (Received January 20, 1965.)

623-7. J. M. IRWIN, FRED RICHMAN and E. A. WALKER, New Mexico State University, University Park, New Mexico. Countable direct sums of torsion complete groups.

Theorem. Let G be a direct sum of countably many torsion complete groups. Then any two direct sum decompositions of G have isomorphic refinements. This theorem generalizes and extends results of Kolettis (Proc. Amer. Math. Soc. 11 (1960), 200-205) on semi-complete groups. Corollary 1. If G is a direct sum of countably many torsion complete groups so is any summand of G. Corollary 2. If G is semi-complete so is any summand of G. (Received January 25, 1965.)

623-8. R. M. FISCHLER, University of Oregon, Eugene, Oregon. Mixing as a generalization of independence.

Mixing as introduced by Renyi (Acta. Math. Acad. Sci. Hungar. 9 (1958), 215-228) is a form of asymptotic independence — particular independent sequences are mixing — and the question arises how far one can extend various results of probability theory with the hypothesis of independence weakened to mixing. The following topics are considered: various aspects of the strong law; the central limit theorem; Borel-Cantelli type theorems; and a characterization of mixing sequences in terms of subsequences whose indicators obey the strong law. For the latter and other results, use is made of theorems of Sucheston (J. Math. Anal. Appl. 6 (1963), 447-456). (Received January 29, 1965.)

623-9. F. J. FLAHERTY, San Francisco State College, San Francisco, California 94132. Spherical submanifolds of manifolds with positive curvature.

Recently some progress has been made in discovering the topological structure of riemannian manifolds with sectional curvature positive and bounded away from 0, see for example the work of Berger, Klingenberg, Rauch, and Topogonov. A related question is: To what extent does the imbedding of a submanifold and the curvature of the surrounding riemannian manifold determine the structure of the submanifold? Some work has been done in the case of submanifolds of euclidean spaces. A partial answer to our question is the following theorem: Let M be a simply connected riemannian manifold whose sectional curvature lies in the interval $]1/4,]$ and let N be a closed, connected riemannian submanifold of dimension at least 2, all of whose eigenvalues of second fundamental forms lie in the interval $[b\sqrt{k},b]$, where k is the minimum sectional curvature of M and $b \geqq 0$, then N is a homotopy sphere and thus is homeomorphic to a sphere for dimension $\neq 3,4$. (Received February 1, 1965.)

623-10. L. H. MARTIN, Harvey Mudd College, Claremont, California. A product theorem concerning some generalized compactness properties.

Let P(X) be the collection of all subsets of the set X. Let Q be a function that associates with each topological space X a collection of subsets of P(X). Then $Q(X) = \{q_\lambda(X): q_\lambda(X) \subset P(X), \lambda \in \text{some index } \Lambda\}$. Definition. Q is said to be slattable over X if and only if, whenever Y is a topological space and $Q(X \times Y) = \{q_\gamma(X \times Y): q_\gamma(X \times Y) \subset P(X \times Y), \gamma \in \text{index } \Gamma\}$, then for each $\lambda \in \Lambda$ there exists a $\gamma(\lambda) \in \Gamma$ such that $G \subset L \times Y$ for some $L \in q_\lambda(X)$ whenever $G \in q_{\gamma(\lambda)}(X \times Y)$. Suppose that Q is slattable over all topological spaces and that n is an infinite cardinal. Definition. Q_n (Q_n^n) is the class of all topological spaces X such that if $\mathscr{C}$ is an open cover of X ($\mathscr{C}$ is an open cover of X with card $\mathscr{C} \leqq n$), then there exists an open refinement $\mathscr{R}$ of $\mathscr{C}$ such that for some $q_\lambda(X) \in Q(X)$, each element of $q_\lambda(X)$ intersects fewer than n sets of $\mathscr{R}$. Theorem. Let X be a compact space and $Y \in Q_n$ or Q_n^n. Then $(X \times Y) \in Q_n$ or Q_n^n, respectively. This theorem has as immediate corollaries well-known theorems of Tychonoff, J. Dieudonné, C. H. Dowker, and Y. Hayashi. Corollary. Let X be a compact space and Y be a space with property P, then $X \times Y$ has property P, where P is any one of the following properties: compactness, countable compactness, Lindelöf, paracompactness, countable paracompactness, metacompactness, or countable metacompactness. (Received February 1, 1965.)

623-11. C. F. OSGOOD, University of Illinois, 273 Altgeld Hall, Urbana, Illinois. The derivatives of some entire functions at algebraic points.

Let f(z) be an entire transcendental function of finite order $\rho \geqq 0$. Let K be a finite extension of the rationals of dimension $\ell \geqq 1$ over the rationals. (1) Theorem: Suppose f(z) satisfies (a) $g(z)f^{(n)}(z) = p(z, f(z), \ldots, f^{(n-1)}(z))$ where both g and p are polynomials with coefficients in K, $g(z) \not\equiv 0$, and $n \geqq 1$; then, f(z) can have all of its derivatives in K at a maximum of $|\ell \cdot \rho|$ regular points of (a) in K. (2) Let $w_1, \ldots, w_t$ be distinct regular points of (a) in K. When (a) is a linear differential equation and $t > |\ell \cdot \rho|$ then a result is established about the simultaneous approximation of the $f^{(i)}(w_j)$ ($i = 0, 1, \ldots, n - 1$; $j = 1, \ldots, t$) by numbers in K. (Received February 8, 1965.)

623-12. C. T. HASKELL, California State Polytechnic, San Luis Obispo, California. Multirestricted and rowed partitions.

q(n,r) denotes number of partitions of n into at most r parts. p(n,r;m), p'(n,r;m), P(n,r;m), and P'(n,r;m) respectively denote those into r parts none less than m which are unrestricted, distinct, odd, and odd distinct. The latter four equal q(N,r), N = N(n,r;m) being known for each case. Several algebraical identities emerge, which are generalizations of those found in Hardy and Wright. Of interest in evaluating and finding the asymptotic behavior of t(n;k), the number of k-rowed partitions of n, is p(n; - k), the coefficients generated by the kth power of the generating function of p(n), the number of unrestricted partitions of n. Methods like those of Hardy-Ramanujan-Rademacher give $2^{-k-1/2}\pi^{-k-1}d^{k+1}/d_n^{k+1}\,(\sinh(\pi(((4k + 2)/3) \cdot (n - (2k + 1)/24))^{1/2})/(n - (2k + 1)/24)^{1/2})$ as an approximation for p(n; - 2k - 1) (which, at k = 0, is the main term in Rademacher's series for p(n)); and, as an approximation for p(n; - 2k), $(k/3)^{1/2}2^{-k}\pi^{-k+1}i^{-1} \times d^k/d_n^k(J_1(2\pi i((k/3)(n - k/12))^{1/2})/(n - k/12)^{1/2}$. For k = 1, and for those small values of n which have been tested, the above give good results. Recursive formulae have been found for t(n;k), in terms of p(n) and the sigma function and its variants; and, for special k, in terms of t(n;r), where r is less than or equal to k. (Received February 8, 1965)

623-13. S. K. JAIN, University of California, Riverside, California 92502. Polynomial rings with a pivotal monomial.

Amitsur [Proc. Amer. Math. Soc. 11 (1960), 28-31] has shown that a division ring D is finite-dimensional over its center if and only if D[x] has a J-pivotal monomial. This paper extends Amitsur's result. For example, we prove Theorem 1. Let R be a ring having a nilpotent Jacobson radical. Then R[x] has a J-pivotal monomial if and only if R[x] satisfies a polynomial identity. Another result we obtain is Theorem 2. Let R be a primitive algebra over its centroid C. Then R is finite-dimensional over C if and only if (1) R has at most a finite number of orthogonal idempotents and (2) R has a nonzero one-sided ideal I such that I[x] has a J-pivotal monomial. Related, but subsidiary, results are also discussed. (Received February 11, 1965.)

623-14. S. F. NEUSTADTER, Applied Research Laboratory, Sylvania Electronic Systems, 40 Sylvan Road, Waltham, Massachusetts 02154. The time-dependent Poisson queue and moving boundary problems for the heat and wave equation.

An integral transform is obtained for the transient state-probabilities of the time-dependent Poisson single channel Erlang queue. The inversion of the transform is accomplished by the method of Ribaric (Arch. Rational Mech. Anal. 3 (1959), 45-50). Some moving boundary problems for partial differential equations can also be solved by this technique: We show how to find $u(x,t)$ satisfying the heat conduction equation for $0 \leqq g(t) < x < \infty$, $0 < t < \infty$, with $u(x,0) = u_0(x)$ for $0 < x < \infty$ and $u(g(t),t) = f(t)$ for $0 < t < \infty$. The corresponding problem for the one-dimensional wave equation is also treated. (Received February 11, 1965.)

623-15. D. W. BRESSLER, University of British Columbia, Vancouver 8, British Columbia, Canada and A. P. MORSE, University of California, Berkeley 4, California. Images of measurable sets.

For a finitely additive and countably multiplicative family H, Measurable H is the family of all sets which are measurable by every Carathéodory outer measure by which the members of H are measurable and complements of members of H are approximable from within. A relation contained in a topological product space is subvalent, if for some countable ordinal α, each horizontal section of the relation has an empty derived set of order α. A topological space is Borelcompact if it and the difference of any two of its closed compact subsets are countable unions of closed compact sets. It is shown that if X and Y are Borelcompact, Hausdorff spaces with countable bases and R is an analytic and subvalent subset of the cartesian product of X with Y, then the direct R-image of A is Measurable$\mathscr{F}(Y)$ whenever A is Measurable$\mathscr{F}(X)$. ($\mathscr{F}(X)$ is the family of closed subsets of X.) If X and Y are complete, separable, metric spaces and R is an analytic and subvalent subset of $X \times Y$, the same conclusion can be drawn. (Received February 15, 1965.)

623-16. S. J. BRYANT, San Diego State College, San Diego, California. Groups, graphs and Fermat's last theorem.

If the square of every element of a finite group is given, then a graph can be obtained by drawing a directed line from x to x^2 for each x in the group. Any Abelian group with fewer than $(1093)^2$ elements is determined up to isomorphism by its graph, i. e., two such groups with isomorphic graphs are isomorphic. A stronger statement can be made: If the square of every element is known, then the group is known up to isomorphism. This says that if the diagonal of the group table is given, then the isomorphism type of the group can be computed. The prime 1093 is the first prime which satisfies $2^{p-1} \equiv 1 \bmod p^2$. Such primes are related to Fermat's Theorem, i. e., if no one of x, y, z are divisible by p and $2^{p-1} \not\equiv 1 \bmod p^2$ then $x^p + y^p \neq z^p$. (Received February 17, 1965.)

623-17. L. L. FOSTER, San Fernando Valley State College, Northridge, California. On the characteristic roots of the product of certain rational integral matrices of order two.

This paper deals with a special case of the following problem: Let A, B be matrices of order n over the rational integers. Compare the algebraic number field generated by the characteristic roots of AB with those generated by A, B. Specifically, we let M(r,s) denote the companion matrix of $x^2 + rx + s$, for rational integers r and s, and let $N(r,s) = M(r,s)(M(r,s))'$. We let F(M(r,s)) and F(N(r,s)) denote the fields generated by the characteristic roots of M(r,s) and N(r,s) over the rational field. This paper is concerned with F(N(r,s)), especially in relation to F(M(r,s)). F(N(r,s)) is characterized. Sequences (r_n,s_n) are obtained such that $F(M(r_n,s_n))$ and $F(N(r_n,s_n))$ are related in a certain sense. Results are given which limit the pairs (r,s) for which F(M(r,s)) and F(N(r,s)) can coincide. Methods employed are those of elementary number theory, especially diophantine analysis. Certain results from algebraic number theory are also employed. (Received February 17, 1965.)

623-18. LINDA LUMER-NIAM University of Washington, Seattle, Washington 98105. Harmonic product and harmonic boundary for bounded complex-valued harmonic functions. Preliminary report.

Let $\mathscr{H}^\infty$ be the set of all bounded complex-valued harmonic functions in a Green space Ω. An abstract harmonic product is defined in $\mathscr{H}^\infty$, which makes it a commutative Banach algebra with identity, under the sup norm; the harmonic product of u, v $\in \mathscr{H}^\infty$, initially defined without using any boundary for Ω, amounts to be in fact the solution of the Dirichlet problem for the Martin boundary Δ of Ω, and boundary values equal to the product of the fine boundary values of u and v respectively (which are known to exist); the algebra $\mathscr{H}^\infty$ is isomorphic to $L^\infty(\Delta)$ (with respect to harmonic measure). The harmonic boundary of $\mathscr{H}^\infty$ is defined as the maximal ideal space X of this algebra; it is compact, may be continuously mapped into the Martin boundary of Ω, and each $u \in \mathscr{H}^\infty$ is represented by a continuous function $\hat{u}$ on X, which is "approximately" the fine boundary function of $\hat{u}$ on Δ; every continuous u on X represents a unique $u \in \mathscr{H}^\infty$. The theory extends to bounded h-harmonic functions (quotients of the harmonic functions by a fixed >0 harmonic function h), and also to the general set-up of Brelot's or Bauer's axiomatic theory. (Received February 18, 1965.)

623-19. DAIHACHIRO SATO, 12 Falcon Bay, Regina, Saskatchewan, Canada and E. G. STRAUS, University of California, Los Angeles, California. A generalized interpolation by analytic functions.

Theorem: Let S be a set of complex numbers such that $\mathrm{glb}_{s \in S}|z - s| \leq |z|^{1-\epsilon}$ for some $\epsilon > 0$ and all sufficiently large $|z|$. Let $\{z_h\}$ be a sequence of complex numbers without finite limit points, then there exist entire functions F(z) with $F^{(m)}(z_h) \in S$ for $m = 0,1,2,\ldots$; $h = 1,2,3,\ldots$. The set of such functions has the power of the continuum, even when a finite number of the values of $F^{(m)}(z_h)$ are prescribed arbitrarily in S. Corollary: There exist entire functions which together with all their derivatives assume prime numbers (Gaussian primes) at all prime numbers (Gaussian prime numbers). These properties can be generalized to an interpolation by functions analytic on some domain or on a Riemann surface. The generalization and the theorems concerning the impossibilities of the interpolation of this type will be touched upon. (Received February 18, 1965.)

623-20. S. R. NEAL, U. S. Naval Ordnance Test Station, China Lake, California. New results in lattice integration theory.

Dilworth [Ann. of Math. (2) 51 (1950), 348-359] has characterized the lattice congruence relation $\theta(a,b)$, $a \geqq b$. A somewhat different characterization is used to establish Theorem 1: If L is a nonmodular lattice, then there is a least congruence relation θ on L such that (i) L/θ is modular and (ii) there is a one-to-one, linear and isotone map $\mu \to \mu_\theta$ of $\Phi(L)$ (the family of all valuations on L) onto $\Phi(L/\theta)$ satisfying $\mu_\theta(\bar{x}) = \mu(x)$, for all x in L. A modification of Theorem 1 is valid for the class L' of all projectivity-invariant and additive interval functions on L studied by Alfsen [Math. Ann. 149 (1963), 419-461]. This version of Theorem 1 will allow most of part two of Alfsen's work to be extended to the nonmodular case. These results include a Lebesgue decomposition for L^* (the largest directed subspace of L') and an "optimal Lebesgue-Radon-Nikodym inequality" for L^*.
(Received February 19, 1965.)

623-21. MIYUKI YAMADA, Sacramento State College, 6000 J Street, Sacramento, California. On regular semigroups satisfying permutation identities.

Let p: $\begin{pmatrix} 1 & 2 & 3 \dots & n \\ p(1) & p(2) & p(3) \dots & p(n) \end{pmatrix}$ be a nontrivial permutation. Then, the identity $x_1 x_2 x_3 \dots x_n = x_{p(1)} x_{p(2)} x_{p(3)} \dots x_{p(n)}$ is called a permutation identity. Commutativity xy = yx and normality xyzw = xzyw are clearly permutation identities. Special kinds of regular semigroups satisfying permutation identities have been studied in many papers. For example, the structure of commutative regular semigroups was determined by A. H. Clifford (Ann. of Math. 42 (1941), 1037-1049), and the structure of bands satisfying normality (i. e. normal bands) was also completely determined by N. Kimura and the author (Proc, Japan Acad. 34 (1958), 110-112). The purpose of this paper is to present a structure theorem for regular semigroups satisfying permutation identities, and some relevant matters. The main results are as follows: Theorem 1. For regular semigroups, any permutation identity implies normality. Theorem 2. A regular semigroup is isomorphic to the spined product of a commutative regular semigroup and a normal band if and only if it satisfies a permutation identity. (For the definition of spined products, see the author, Inversive semigroups. I, Proc. Japan Acad. 39 (1963), 100-103.) (Received February 19, 1965.)

623-22. M. P. DRAZIN, Purdue University, Lafayette, Indiana 47907. A classification of subdirectly irreducible rings.

The following strikingly general result is established: Let R be any nonzero associative ring, let Z denote its center and let S denote the intersection of all the nonzero ideals of R. Then exactly one of the following three possibilities must hold: (1) $S^2 = 0$, (2) $S^2 = S \neq 0$ and $S \cap Z = 0$, (3) R is simple with unit. The proof is short and strictly elementary. Besides potential applications of the full force of this result as stated (using the representability of arbitrary associative rings as subdirect sums of subdirectly irreducible R's), several more or less known facts about subdirectly irreducible rings follow as immediate corollaries. (Received February 19, 1965.)

623-23. W. E. BARNES and W. M. CUNNEA, Washington State University, Pullman, Washington 99163. Primary components in Noetherian rings.

Let R be a commutative Noetherian ring, A an ideal and P an associated prime ideal of A. Then there exists n such that $(A:P^m) \cap (A + P^m) = (A:P^n) \cap (A + P^n)$ for all $m \geq n$. For such an n, $(A + P^n)_P$ is a P-primary component of A. Moreover, if $P_1, \ldots, P_k$ are the associated primes of A with $n_1, \ldots, n_k$ as above, then $A = \bigcap_{i=1}^k (A + P_i^{n_i})_{P_i}$ is a normal primary representation of A. Considering A_P, for P an associated prime of A, we obtain that $(A_P:P^m) \cap (A_P + P^m) = (A_P:P^n) \cap (A_P + P^n)$ for all $m \geq n$ iff $(A_P + P^n)_P = (A + P^n)_P$ is a P-primary component of A_P. (Received February 22, 1965.)

623-24. W. J. COLES, University of Utah, Salt Lake City, Utah and T. L. SHERMAN, Arizona State University, Tempe, Arizona 85281. An nth order boundary value problem.

Let $F(x, u_1, \ldots, u_{n-1})$ be continuous and satisfy $|F(x, y, \ldots, y^{(n-1)}) - F(x, u, \ldots, u^{(n-1)})| \leq p(x)|y - u|$, where p(x) is continuous on |a,b|. It is proved that the problem $y^{(n)} + F(x, y, \ldots, y^{(n-1)}) = 0$, $y(a_i) = y'(a_i) = \ldots = y^{(k_i - 1)}(a_i) = 0$, $k_i \geq 0$, $1 \leq i \leq r$, $a = a_1 < a_2 < \ldots < a_r = b$, $\sum_{i=1}^r k_i = r$ has one and only one solution provided b is less than the first conjugate point of a with respect to the equation $u^{(n)} + p(x)u = 0$, and also less than the first conjugate point of a with respect to the equation $u^{(n)} - p(x)u = 0$. (Received February 22, 1965.)

623-25. J. S. MAC NERNEY, University of North Carolina, Chapel Hill, North Carolina 27515. Hellinger integrals and linear functionals.

Suppose $\{S,Q\}$ is a complete inner product space and ϕ is a nondecreasing function from [0,1] to a set of Hermitian transformations of $\{S,Q\}$. Theorem. If E is a linear family of functions f from [0,1] to S such that the Stieltjes integrals $F(u) = \int_0^u d\phi \cdot f$ exist for u in [0,1] and $\int_0^1 Q(f, dF) = \int_0^1 Q(f, d\phi \cdot f)$, and L is a linear function from E to the plane, these are equivalent: (1) There is a number b such that $|L(f)|^2 \leq b \int_0^1 Q(f, d\phi \cdot f)$ for each f in E; (2) There is a function λ from [0,1] to S such that the Hellinger integral $\int_0^1 Q(|d\phi|^{-1/2} d\lambda, |d\phi|^{-1/2} d\lambda)$ exists and $L(f) = \int_0^1 Q(f, d\lambda)$ for each f in E. Concerning the indicated Hellinger integral see, for instance, the author's Hermitian moment sequences [Trans. Amer. Math. Soc. 103 (1962), 45-81, esp. pp. 76-77]. Remark. If S is finite-dimensional, E can be realized as the set of all continuous functions from [0,1] to S; otherwise, there exists a projection-valued ϕ such that, for some continuous f from [0,1] to S, the Stieltjes integral $\int_0^1 d\phi \cdot f$ fails to exist. (Received February 22, 1965.)

623-26. T. TAMURA and J. C. HIGGINS, University of California, Davis, California. Congruence relations on finitely generated free commutative semigroups.

Let S be the direct power of the semigroup P of all positive integers with addition: $S = P \oplus \ldots \oplus P = \{(a_1, \ldots, a_n) \mid a_i \in P\}$ where $(a_1, \ldots, a_n) + (b_1, \ldots, b_n) = (a_1 + b_1, \ldots, a_n + b_n)$. Let G be the smallest group containing S. Any congruence on S is determined in terms of a system $\mathscr{I}$ of ideals of S and a system $\mathscr{G}$ of subgroups of G under certain restriction. Further the authors refer to the relationship between $\{\mathscr{I}, \mathscr{G}\}$ and the lattice of all congruences on S. (Received February 22, 1965.)

623-27. H. H. CORSON and JORAM LINDENSTRAUSS, University on Washington, Seattle, Washington. Continuous selections with nonmetrizable range.

E. Michael (Ann. of Math. (2) 63 (1956), 361-382) has proved the following selection theorem: Let X be a paracompact space and let Y be a subset of a Fréchet space. Let ϕ be a lower semi-continuous map from X into the set of nonempty subsets of Y such that $\phi(x)$ is closed and convex for every $x \in X$. Then there is a continuous function f from X into Y such that $f(x) \in \phi(x)$ for every x. In the present paper this selection theorem is shown to hold for some nonmetrizable Y, provided suitable conditions are imposed on X. It is shown for example that the theorem above holds if X is a complete separable metric space and Y the unit cell of a (not necessarily separable) $L_p(\mu)$ space, for some measure μ and $1 < p < \infty$, in the w topology. Examples are given which show that even for compact metric X the selection theorem fails unless Y is a w compact subset of a Banach space. Some applications are given. The proofs are based on theorems asserting that certain function spaces are Lindelöf spaces. The paper contains a fairly complete answer to the question: For which metric spaces X and locally convex spaces L is the space of all continuous functions from X to L a Lindelöf space in the topology of pointwise convergence or in the compact open topology. (Received February 22, 1965.)

623-28. G. J. JANUSZ, University of Oregon, Eugene, Oregon. Separable polynomials over commutative rings. Preliminary report.

Let R be a commutative ring with no idempotents except 0 and 1. A commutative, separable, R-algebra S is a Galois extension of R with group G in case G is a finite group of R-automorphisms of S which leave only R element-wise fixed. In Mem. Amer. Math. Soc. (1965), No. 52, Chase, Harrison, and Rosenberg have proved the fundamental theorem of Galois theory for Galois extensions of R. In this paper we consider the elementwise aspects of Galois theory. We call a monic polynomial $f(x) \in R[x]$ separable in case $R[x]/(f(x))$ is a separable R-algebra. We give necessary and sufficient conditions that f(x) be separable in terms of Galois extensions of R. As an application we have Theorem: If R is a semi-local ring then any Galois extension S of R with no proper idempotents is of the form $S = R[a]$ where $a \in S$ is a root of a separable polynomial. We can define a "Galois group" for a separable polynomial and prove the Galois theorem on the solvability of the equation $f(x) = 0$ by radicals assuming R semi-local and certain integer multiples of the identity of R are units in R. (Received February 22, 1965.)

623-29. D. W. ROBINSON, 269 ESC, Brigham Young University, Provo, Utah. On a characterization of semi-simple linear transformations. Preliminary report.

Let L be the algebra of linear transformations on a finite-dimensional vector space over a field F. For A in L let Δ_A be the mapping on L given by $X\Delta_A = XA - AX$. Let A in L be called semi-simple if the minimum polynomial of A is relatively prime to its derivative. It is known that if A is semi-simple, then $X\Delta_A^2 = 0$ implies $X\Delta_A = 0$. The converse is also valid, and is a consequence of the demonstrated fact that if A is cyclic with minimum polynomial M(x), then the difference between the nullities of Δ_A^2 and Δ_A is given by the nullity of M'(A). Several corollaries of this characterization of semi-simple are given. For example, if F is of characteristic prime p and A is in L, then there is some positive power of A which is semi-simple. Also, it is noted that if F is of characteristic zero, A is in L with semi-simple part S (see N. Jacobson, Lie algebras, Interscience Tracts in Pure and Applied Mathematics 10, p. 98) and m is an integer ≧ 2, then $B\Delta_X^m = 0$ whenever $X\Delta_A^m = 0$ if and only if B is a polynomial in S with coefficients in F. (Compare Abstract 64T-143, these *Notices* 11 (1964), 231.) (Received February 22, 1965.)

623-30. P. A. LOEB, University of California, Los Angeles, California 90024. An axiomatic treatment of pairs of elliptic differential equations.

Let W be a locally compact Hausdorff space which is connected and locally connected but not compact. Let $\mathscr{A}$ and $\mathscr{K}$ be harmonic classes of functions on W in the sense of Brelot (Lectures on potential theory, Tata Institute of Fundamental Research, Bombay, India). Assume that the nonnegative functions in $\mathscr{A}$ are superharmonic (in the sense of Brelot) with respect to $\mathscr{K}$. For example, let W be a region in Euclidean n-space and let L be the elliptic differential operator defined on W by $L(u) = \sum a_{ij}u_{x_i x_j} + \sum b_i u_{x_i}$, where $\sum a_{ij}x_i x_j$ is a positive definite quadratic form and the coefficients of L satisfy a local Lipschitz condition. Let $\mathscr{A}$ and $\mathscr{K}$ be the solution classes of $L(u) = Qu$ and $L(u) = Pu$ respectively, where P and Q satisfy a local Lipschitz condition, $P \geqq Q$ and $P \geqq 0$. It is shown that for a relatively compact region in W, the Dirichlet problem can be solved for $\mathscr{A}$ iff it can be solved for $\mathscr{K}$. If V is a strictly positive, superharmonic function with respect to $\mathscr{A}$, then $\mathscr{B}(V)\mathscr{A} = \{h \in \mathscr{A} \mid \exists \text{ constant } M \text{ with } |h| \leqq MV\}$ is a Banach space with norm $\|h\| = \sup_{x \in W}|V^{-1}h(x)|$. For the same V, $\mathscr{B}(V)\mathscr{K}$ is also a Banach space. It is shown that there exists an isometric isomorphism from $\mathscr{B}(V)\mathscr{K}$ into $\mathscr{B}(V)\mathscr{A}$. (Received February 22, 1965.)

623-31. T. TAMURA, University of California, Davis, California. Attainability and extendability of system of identities on semigroups.

Let $\mathscr{I}$ be a system of identities on semigroups, ρ a congruence on a semigroup S such that every identity in $\mathscr{I}$ identically holds in S/ρ. The congruence ρ is called a $\mathscr{I}$-congruence on S. Let ξ_S denote the smallest $\mathscr{I}$-congruence on S. $\mathscr{I}$ is called attainable on all semigroups if, for every semigroup S, the following condition is satisfied: If a congruence class U of S modulo ξ_S is a subsemigroup of S, then U is $\mathscr{I}$-indecomposable, namely $|U/\xi U| = 1$. The systems $\{x = x\}$ and $\{x = y\}$ are attainable, but we call them trivial. Theorem 1. The system $\{x^2 = x, xy = yx\}$ is the only nontrivial attainable system of identities on all semigroups. Next we define extendability of system of identities on all semigroups. Let S be any semigroup, S_α a congruence class modulo a $\mathscr{I}$-congruence ρ on S. $\mathscr{I}$ is called extendable on all semigroups if the following condition is satisfied: If S_α is a subsemigroup and if ρ_α is a $\mathscr{I}$-congruence on S_α, then there is a $\mathscr{I}$-congruence ρ' on S such that ρ_α is the restriction of ρ' to S_α and $\rho' \subset \rho$. Theorem 2. The system $\{x^2 = x, xy = yx\}$ is the only nontrivial extendable system of identities on all semigroups. (Received February 22, 1965.)

623-32. R. A. HANSEN, Math 201, University of Utah, Salt Lake City, Utah. Round-off procedures in the numerical treatment of differential equations.

A study of round-off errors in the numerical treatment of the initial value problem using one-step methods is presented. The problems of inherent and induced round-off errors, as defined by

Henrici in Discrete variable methods in ordinary differential equations, are treated in detail. New nonstatistical round-off error bounds and new rounding-off procedures are developed. The new round-off procedures, in effect, minimize the effect of round-off errors on the numerical approximate solution. (Received February 22, 1965.)

623-33. L. D. LOVELAND, University of Utah, Salt Lake City, Utah. Tame subsets of spheres in E^3.

Let F be a closed subset of a 2-sphere S in E^3 such that the diameters of the components of F have a positive lower bound, and let V be a component of $E^3 - S$. We say that F can be locally spanned from V if for each point p in F and for each positive number ϵ there are disks R and D such that $p \in \text{Int}\, R \subset S$, $\text{Bd}\, R = \text{Bd}\, D$, $\text{Int}\, D \subset V$, and $\text{diam}(R + D) < \epsilon$. Burgess proved that S is tame from V if S can be locally spanned from V [cf. Theorem 8 of Characterizations of tame surfaces in E^3, Trans. Amer. Math. Soc. (to appear)]. Theorem 1. If F can be locally spanned from each component of $E^3 - S$, then F lies on a tame 2-sphere. Property (*,F,S) is defined in [Abstract 619-178, these *Notices* 12 (1965), 110]. Theorem 2. If for each point p in F and for each $\epsilon > 0$ there is a 2-sphere S' such that $p \in \text{Int}\, S'$, $\text{diam}\, S' < \epsilon$, and $S \cap S'$ is a continuum satisfying Property $(*, S \cap S' S)$, then F lies on a tame 2-sphere. Theorem 3. If $F_1, F_2, \ldots, F_n$ is a finite collection of closed subsets of S such that Property $(*, F_i, S)$ holds for each i and S is locally tame modulo $\sum_{i=1}^{n} F_i$, then S is tame. (Received February 22, 1965.)

623-34. F. R. DeMEYER, University of Oregon, Eugene, Oregon. Galois theory in algebras.

Let S be a ring with 1 and with no other nontrivial central idempotents. Let K be a subring (with 1) of the center C of S. Let G be a finite group of automorphisms of S so that $S^G = \{x \in S \mid \sigma(x) = x \,\forall \sigma \in G\} = K$. Call S a Galois extension of K if there exists $x_1, \ldots, x_n$; $y_1, \ldots, y_n \in S$ such that $\sum_{i=1}^{n} x_i \sigma(y_i) = \delta_{\sigma,1}$. Theorem 1. If S is a Galois extension of K with the group G then there is a normal subgroup H of G so that $C = S^H$. S is a Galois extension of K with group G/H and S is a Galois extension of C with group H. Theorem 2. Let K = C, then $S = KG_a$ $(a \in H^2(G, U(K)))$ is a twisted group algebra over K with group G. Employing Theorem 2, a decomposition theorem for central Galois extensions with an abelian Galois group and a determination of the G-isomorphism classes of central Galois extensions of a commutative ring with odd order abelian Galois group G are obtained. Corollary: If G is cyclic and S is a Galois extension of K with group G, then S is commutative. (Received February 18, 1965.)

623-35. WOLFGANG SCHMIDT, University of Colorado, Boulder, Colorado. Approximations to quadratic irrationalities.

Let $N(\alpha;h)$ be the number of integers q, $1 \leq q \leq h$, such that the distance from αq to the nearest integer does not exceed $\psi(q)$, where $\psi(q) > 0$ is a given function. It is shown that $N(\alpha;h)$ is asymptotically equal to $2\sum_{q=1}^{h}\psi(q)$ if α is a quadratic irrationality and if both $\psi(q)$ and $(q\psi(q))^{-1}$ decrease to zero. This generalizes a result of S. Lang. (Received February 23, 1965.)

623-36. J. P. JANS, University of Washington, Seattle, Washington 98105, H. Y. MOCHIZUKI, University of California, Berkeley, California and L. E. T. WU, Western Washington State College, Bellingham, Washington. A characterization of QF-3 rings.

A left QF-ring R is one which $_RR$, the ring considered as a left module over itself, can be embedded in a projective injective left R-module. Let $\hat{L}$ denote the class of left torsionless R-modules and $\hat{T}$ the class of left R-modules M for which Hom(M,R) = 0. Theorem. If R is a ring with minimum condition on left ideals, then R is left QF-3 if and only if (1) $\hat{L}$ is closed under extension and (2) $\hat{T}$ is closed under taking submodules. (Received February 23, 1965.)

623-37. K. O. MAY, University of California, Berkeley, California. An historical profile of determinants.

Some results of a survey of about 2500 publications through 1964. Some 1700 items from 1800 through 1920 were assigned to the following (overlapping) categories in the percentages indicated. RES (nontrivial new theorems or proofs) — 10%. TRV (trivia, e. g. easily obtainable from previous results) — 43%. DUP (duplications, exclusive of nearly simultaneous publications) — 21%. APP (new applications) — 12%. SYE (systematizations and expositions addressed to professionals) — 10%. TXT (texts and expositions addressed to students) — 13%. PDH (pedagogy and history) — 4%. AND (anticipations and new directions) —4%. With small and large fluctuations, the annual output varies from less than one in the first decade to a maximum of fifty in 1878 and declines thereafter. RES papers are randomly distributed with little relation to total output whose variations are explained mainly by changes in TRV and DUP papers triggered apparently by 'major' RES-SYE-AND publications. Case histories and historical conjectures. (Received February 23, 1965.)

623-38. W. J. FIREY, Oregon State University, Corvallis, Oregon 97331. Blaschke sums of bodies of revolution. Preliminary report.

For each Borel set ω on the unit spherical surface Ω in E_3, the value $S(K,\omega)$ of the area function of convex body K is the area of $\bigcup_{u \in \omega}(\Pi(u) \cap K)$, where $\Pi(u)$ is the support plane of K with outer normal u. In virtue of an existence theorem of Minkowski, Fenchel and Jessen, the set function $(1 - \vartheta)S(K_0,\omega) + \vartheta S(K_1,\omega)$ for $0 \leq \vartheta \leq 1$, is the area function of a convex body K_ϑ to be called the weighted Blaschke sum of K_0, K_1. If V(K), S(K), M(K) denote the volume, surface area and total mean curvature of K, then $S(K_\vartheta)$ is linear in ϑ and it has been shown by H. Kneser and W. Suss that $V^{2/3}(K_\vartheta)$ is concave. If K_i, i = 0,1, are coaxial convex bodies of revolution, then $M^2(K_\vartheta)$ is convex in ϑ, but for arbitrary K_i this need not be true. Again, for coaxial bodies of revolution K_i, $a(K_\vartheta)M(K_\vartheta)$ is convex, $V(K_\vartheta)/a(K_\vartheta)$ is concave, $a^2(K_\vartheta)$ linear, where a(K) denotes the equatorial radius of K. A body of revolution can be arbitrarily well approximated by weighted Blaschke sums of coaxial double cones (i. e. bodies swept out by revolving triangles about their longest sides). This, together with the preceding, allows the deduction of certain inequalities, first proved by Hadwiger, connecting V, S, M, a for bodies of revolution. (Received February 23, 1965.)

623-39. W. M. STONE, Oregon State University, Corvallis, Oregon 97331. On an extension of the Hotelling-Wasow method.

The Hotelling-Wasow method of relating a more general distribution to the error function, $\int_{-\infty}^{x}\phi(y)\,dy = \int_{-\infty}^{t} f_n(\delta,y)dy$, involves the solution of a sequence of linear differential equations to arrive at an asymptotic relationship between x and t (Proc. Sixth Symp. Appl. Math., Amer. Math. Soc., Providence, R. I., 1956). In similar fashion, the noncentral chi-square distribution may be related to the chi-square distribution; the asymptotic relationship takes the form $x = e^{-\delta/2n}(t + \sum_{1}^{\infty} P_k(t,\delta)/n^k)$, where n is the number of degrees of freedom, δ is the noncentrality parameter. The first two polynomials are identically zero and the sequence of polynomials does not involve the solution of a sequence of differential equations. For the interesting applications discussed by Fix, Hodges and Lehmann (Probability and statistics, Wiley, New York, 1959) the present procedure serves to extend the region covered by their table. (Received February 23, 1965).

623-40. J. R. REAY, Western Washington State College, Bellingham, Washington. Minimal subspaces generated by positive bases.

The set B positively spans the linear space L if each point of L may be represented as a linear combination of the points of B using only positive coefficients. If B is a minimal such set, we call B a positive basis for L. M is a spanned linear subspace if $M = \text{pos}(M \cap B)$. If card $(M \cap B) = 1 + \dim M$ as well, then $(M \cap B)$ is called a minimal positive basis for M, and $M = \text{pos}(M \cap B)$ is called a minimal subspace of L. L is always the linear sum of its minimal subspaces. Theorem: For a given positive basis of a linear space, (1) Distinct minimal bases for minimal subspaces are pairwise disjoint iff (2) Distinct minimal subspaces have only 0 in common iff (3) L is the direct linear sum of its minimal subspaces. The equivalence of (2) and (3) is well-known, and the necessity of (1) is clear. The sufficiency is interesting in view of known examples of positive bases which may be partitioned into distinct minimal bases which are pairwise disjoint, and yet (2) and (3) fail. It is proved that such examples occur only in spaces of dimension 5 or greater. (Received February 24, 1965.)

623-41. R. N. BRYAN, University of Utah, Salt Lake City, Utah 84112. A linear differential system with general linear boundary conditions. Preliminary report.

Suppose U is a bounded linear transformation from C, the space of $n \times n$ continuous matrices on [a,b], to the space of matrix constants. There exists a $B \in C$ such that the system $Y' = BY$; $U(Y) = 0$ has no nontrivial solution iff there is a nonsingular differentiable $\Phi \in C$ such that $U(\Phi)$ is nonsingular, in which case U is said to be nonsingular. If U is nonsingular, then the system $Y' = AY + R$; $U(Y) = K$ is equivalent to a Fredholm integral equation, the operator of which is compact on C and L^2. Thus, the Fredholm alternative applies. Expansion theorems follow in the L^2 case. A differential system adjoint to the given differential system is obtained. It has the same order of compatibility and the same eigenvalues as the original system. The results obtained include some of those of R. H. Cole in General boundary conditions for an ordinary linear differential system, Trans. Amer. Math. Soc. 111 (1964), 521-550. (Received February 24, 1965.)

623-42. L. O. CANNON, University of Utah, Salt Lake City, Utah 84112. Another property that distinguishes Bing's dogbone space from E^3.

In A decomposition of E^3 into points and tame arcs such that the decomposition space is topologically different from E^3 (Ann. of Math. (2) 65 (1957), 484-500), Bing defined an upper semi-continuous decomposition G of E^3 such that the decomposition space E^3/G (the dogbone space) is not homeomorphic to E^3. He established that if g is a nondegenerate element of G then there is no neighborhood of g in E^3/G which is homeomorphic to E^3. Using preliminary theorems from the above paper and methods suggested by Bing (Pointlike decompositions of E^3, Fund. Math. 50 (1962), 431-453) we prove the following Theorem: If g is a nondegenerate element of G, there is an open set in E^3/G containing g and contained in V, then Bd V is not a 2-sphere. Armentrout has announced (Abstract 612-61, these *Notices* 11 (1964), 369) that, in the above notation, there is an open set U in E^3/G containing g which contains no simply connected open set V containing g. (Received February 24, 1965.)

623-43. A. H. CAYFORD, University of British Columbia, Vancouver 8, British Columbia, Canada. A characterization of a differential ring of a class of integer-valued entire functions.

Let $\mathscr{D}$ be a differential ring of entire functions satisfying the condition: If $f \in \mathscr{D}$ then the values of the derivatives $f^{(n)}(z_r)$ ($r = 1,\ldots,s$, $n = 0, 1,\ldots$) at the points z_r are Gaussian integers and if ρ is the order of f(z) then $\rho < s/2$. Then there exists a function h(z) such that every $f(z) \in \mathscr{D}$ can be written $f(z) = \sum_{n=-m}^{m} c_n(h(z))^n$, where c_n is constant. For a given ring $\mathscr{D}$, h(z) is one of the functions exp(z) or $\exp(\sqrt{D}\, z)$, where D is a fixed rational integer and the c_n are respectively either of the form a_n or $a_n + b_n\sqrt{D}$ with a_n and b_n rational. In the proof the form $f(z) = \sum_{i=1}^{N} p_i(x) \exp(\nu_i z)$ where $p_i(z)$ is a polynomial with Gaussian integer coefficients is obtained. It is shown that $p_i(z)$ is a constant c_i and that there exists ν such that $\nu_i = r_i \nu$ with r_i an integer for each i. Finally either c_i is rational and $r_i \nu$ an integer or $c_i = a_i + b_i\sqrt{D}$ and $r_i\nu = n\sqrt{D}$ for some integer n and rational a_i and b_i. (Received February 24, 1965.)

623-44. R. C. BURTON, Brigham Young University, Provo, Utah. Iterated bounds for error-correcting codes.

A linear (n,k,d) code is a k-dimensional subspace of the vector space of all n-tuples over a finite field with the property that every non-null vector of the subspace has d or more non-zero coordinates. A matrix whose rows are a basis for the subspace is called a generator matrix of the code. Jump-down Lemma: If there exists an (n,k,d) code such that C columns of a generator matrix of the code depend on c columns, where $c < k$ and $c \leq C$, then there exists an $(n - C, k - c, d)$ code. In order to use the Lemma, suppose we have any bound (e. g. the bound of Hamming or Johnson) which tells us that for a given n and r ($r = n - k$), it must be that $d \leq D$. Then any set of n r-tuples contains some subset of $C = D$ r-tuples which depend on $c = D - 1$ r-tuples. Example: Assume there exists a (16,6,7) code over the field with two elements. Taking $n = 16$, $r = 6$, the Hamming bound gives $D = 4$. Therefore a (12,3,7) code must exist. Lemma. If $\underline{x}_1, \underline{x}_2,\ldots,\underline{x}_n$ are k-tuples over a field with q elements and if $n > [(q^k - 1) Q + q^k - q^{k-v}]/(q - 1)$ where Q and v are non-negative integers and $v \leq k$, then $C = (Q + 1)(q^{v+1} - 1)/(q - 1)$ of the k-tuples depend on $c = v + 1$ k-tuples. The above example may be continued letting $n = 12$, $k = 3$, giving $C = 6$ and $c = 2$. Contradiction. (Received February 24, 1965.)

623-45. G. E. PETERSON, University of Utah, Salt Lake City, Utah. A theorem of the Hardy-Bohr type. Preliminary report.

Let $C = (c_{n\nu})$ be any triangular summability matrix; then if the sequence $s_n = \sum_{\nu=0}^{n} a_\nu$ is such that $\sum_{\nu=0}^{n} c_{n\nu} s_\nu = \sum_{\nu=0}^{n} \bar{c}_{n\nu} a_\nu$ converges, we will say that $s_n \in C$. Suppose $B = (b_{n\nu})$ is a regular, triangular and normal matrix method with a mean-value theorem, and that $P^{(i)} = (p_\nu^{(i)}/P_n^{(i)})$ is a weighted arithmetic mean for each i. Define $A = B\prod_{i=1} P^{(i)}$ and suppose that $b_{n+1,\,n+1} = O(b_{nn})$ and also that $p_{n+1}^{(i)} = O(p_n^{(i)})$ for $1 \leqq i \leqq k$, where k is any positive integer. Then necessary and sufficient conditions that $\sum a_\nu \epsilon_\nu$ converges whenever $\sum a_\nu \in A$ are (i) $\epsilon_n = \sum_{\nu=n}^{\infty} a_{\nu n} \alpha_\nu$, where $\sum |\alpha_\nu| < \infty$ and (ii) $\epsilon_n = O(a_{nn})$. (Received February 24, 1965.)

ABSTRACTS PRESENTED BY TITLE

65T-158. S. S. WAGNER, 340 Pendleton Road, Clemson, South Carolina 29631. A homogeneous line is completely homogeneous.

L denotes a simply ordered set, with topology determined by the order relation. Two subsets of L are said to be congruent (in the algebraic sense) if there is an automorphism (an order-preserving transformation from L onto L) throwing one onto the other. Assuming that L is complete (in the Dedekind sense), homogeneous (any two points congruent), and non-degenerate, but not isomorphic to the integers, 3 lemmas are proved: 1. Some segment is autocongruent—that is, there is an automorphism leaving two points fixed but moving every point which is between them. 2. An autocongruent segment is congruent to each of its subsegments. 3. An autocongruent segment is congruent to each segment which includes it. These give the Theorem: If L is connected and homogeneous, it is completely homogeneous (any two segments are congruent). The topological analogue is also proved. (Received December 4, 1964.)

65T-159. J. M. WEINSTEIN, University of Wisconsin, Madison, Wisconsin 53706. Sentences preserved under unions. Preliminary report.

Let L be a first-order language with equality, B an L-structure, C a set of L structures. Definition. B is the union of C if the universe of B is the union of the universes of the structures of C and for each predicate P of L the relation denoted by P in B is the union of the relations denoted by P in the structures of C. Theorem. A sentence is preserved under unions iff it is equivalent to a finite conjunction of sentences of the forms $\forall v_0 S$, $\forall v_0 \ldots v_r (P(v_0, \ldots, v_r) \rightarrow S)$, where P is a predicate other than equality and S is a formula built up—using $\exists, \wedge, \vee$ — from atomic formulas, equations, and negated equations. The above definition is due to Tarski; Keisler (Proc. Amer. Math. Soc. 15 (1964), 540-545) has characterised sentences preserved under another kind of union. The present theorem is proved using special models in combination with generalised atomic formulas (cf. cited paper and its reference 4); the same method also yields characterisations of sentences preserved under various other union-like constructions. (Received December 2, 1964.)

65T-160. W. D. L. APPLING, North Texas State University, Denton, Texas. Summability of real-valued set functions. Preliminary report.

Suppose U is the set of all real nonnegative-valued functions on a field S of subsets of a set R, and U* is the set of all finitely additive elements of U. All integrals discussed are Hellinger type limits of the appropriate sums. Suppose m is in U*. Suppose W is the set of all Q in U such that for each $K \geqq 0$, $\int_R \min\{K, Q(I)\} m(I)$ exists, and l.u.b. $\{z \mid z = \int_R \min\{K, Q(I)\} m(I),\ 0 \leqq K\} < \infty$. If Q is in W, we let s(Q) denote the function on S such that for each V in S, s(Q)(V) = l.u.b. $\{z \mid z = \int_V \min\{K, Q(I)\} m(I),\ 0 \leqq K\}$; and we see that s(Q) is in U*. We let W_D denote the set of all functions on S of the form H - L for H and L in W. If each of H and L is in W, then we let s*(H - L) =

$s(H) - s(L)$. Suppose each of H and L is in W_D. Theorem 1. Each of $H + L$, $\max\{H,L\}$ and $\min\{H,L\}$ is in W_D, and for each V in S, $s^*(H + L) = s^*(H) + s^*(L)$, $s^*(\max\{H,L\})(V) = \int_V \max\{s^*(H)(I), s^*(L)(I)\}$ and $s^*(\min\{H,L\})(V) = \int_V \min\{s^*(H)(I), s^*(L)(I)\}$. Theorem 2. If L is bounded, then LH is in W_D and $\int_R |s^*(LH)(V) - \int_V L(I)s^*(H)(I)| = 0$. Theorem 3. If $|H| - c$ is in U for some $c > 0$, then $\int_R H(I)^{-1}m(I)$ exists. (Received December 7, 1965.)

65T-161. CHIEN WENJEN, California State College at Long Beach, California. Characterizations and representations of semi-normed algebras. II. Preliminary report.

Theorem 3. A satisfying the condition (I) is equivalent to $C(T,K)$, with compact-open topology, where T is a locally compact paracompact space T if and only if there exists a cardinal number ξ such that every net of power $\geqq \xi$ of the homomorphisms from A to K has a cluster point in the weak topology. (See Arens, ibid.) Theorem 4. Every representation of A satisfying the condition (I) is given by the formula $R_x = \int_{\mathfrak{M}} x(M)dP(M)$, where $\mathfrak{M}$ is the space of all closed maximal ideals of A and $P(\Delta)$ is a spectral measure on $\mathfrak{M}$. Theorem 4 gives rise to a new proof of the spectral theorem for unbounded self-adjoint operators in Hilbert spaces which relates unbounded operators to unbounded continuous functions. (Received December 1, 1964.)

65T-162. S. A. NAIMPALLY, Iowa State University, Ames, Iowa 50010. A generalization of the Kuratowski embedding theorem.

By using the notion of a generalized metric due to Kalisch (Bull. Amer. Math. Soc. 52 (1946), 936-939) the following result is obtained: Every Hausdorff uniform space can be embedded isometrically (in the sense of the generalized metrics) as a closed subset of a convex subset of a complete real linear topological space. In this connection see Kuratowski (Fund. Math. 25 (1935), 534-545); Arens and Eells (Pacific J. Math. 6 (1956), 397-403). (Received December 21, 1964.)

65T-163. D. T. HAIMO, 77 Snake Hill Road, Belmont, Massachusetts. L^2 series expansions for functions with the Huygens property.

A function $u(x,t) \in H^*$ iff it is a C^2 solution of the equation $\Delta_x u(x,t) = (\partial/\partial t)u(x,t)$, where $\Delta_x f(x) = f''(x) + (2\nu/x)f'(x)$, ν a fixed positive number, and is such that $u(x,t) = \int_0^\infty G(x,y;t - t')u(y,t')d\mu(y)$, $d\mu(x) = c_\nu^{-1}x^{2\nu}dx$, the integral converging absolutely for every t,t', $a < t' < t < b$, where $G(x,y;t) = (1/2t)^{\nu+1/2}\exp[-(x^2 + y^2)/4t]\mathscr{I}(xy/2t)$, $\mathscr{I}(z) = c_\nu z^{1/2-\nu}I_{\nu-1/2}(z)$, and $c_\nu = 2^{\nu-1/2}\Gamma(\nu + 1/2)$. Let $P_{n,\nu}(x,t) = \sum_{k=0}^{n} 2^{2k}\binom{n}{k}[\Gamma(\nu + 1/2 + n)/\Gamma(\nu + 1/2 + n - k)]x^{2n-2k}t^k$ and $W_{n,\nu}(x,t) = G(x;t)P_{n,\nu}(x/t, -1/t)$, where $G(x,0;t) = G(x;t)$. Theorem: Let $u(x,t) \in H^*$, $-\sigma \leqq t < 0$, and $u(x,t)[G(x,-t)]^{1/2} \in L^2$, $-\sigma \leqq t < 0$, $0 \leqq x < a$. Then, for $-\sigma \leqq t < 0$, $\lim_{N\to\infty}\int_0^\infty G(x,-t)|u(x,t) - \sum_{n=0}^{\infty} a_n P_{n,\nu}(x,-t)|^2 d\mu(x) = 0$ and $\int_0^\infty G(x,-t)|u(x,t)|^2 d\mu(x) = \sum_{n=0}^\infty |a_n|^2 b_n t^{2n}$, where $b_n = 2^{4n}n!\,\Gamma(\nu + 1/2 + n)/\Gamma(\nu + 1/2$ and $a_n = 1/b_n \int_0^\infty u(y,t)W_{n,\nu}(y,-t)d\mu(y)$. Theorem: If $u(x,t) \in H^*$, $0 < t \leqq \sigma$, and if $u(ix,t)[G(x,t)]^{1/2} \in L^2$, $0 < t \leqq \sigma$, $0 \leqq x < \infty$, then, for $0 < t \leqq \sigma$, $\lim_{N\to\infty}\int_0^\infty G(x,t)|u(ix,t) - \sum_{n=0}^{N} a_n P_{n,\nu}(x,-t)|^2 d\mu(x) = 0$, and $\int_0^\infty G(x,t)|u(ix,t)|^2 d\mu(x) = \sum_{n=0}^\infty |a_n|^2 b_n t^{2n}$, where $a_n = 1/b_n \int_0^\infty u(ix,t)W_{n,\nu}(x,t)d\mu(x)$. Theorem: If $u(x,t) \in H^*$, $0 < \sigma \leqq t$, and if $u(x,t)[G(ix,t)]^{1/2} \in L^2$, $0 < \sigma \leqq t$, $0 \leqq x < \infty$, then, for $0 < \sigma \leqq t$,

$\lim_{N\to\infty}\int_0^\infty G(ix,t)|u(x,t) - \sum_{n=0}^N a_n W_{n,\nu}(x,t)|^2 d\mu(x) = 0$, and $\int_0^\infty G(ix,t)|u(x,t)|^2 d\mu(x) = \sum_{n=0}^\infty t^{-2n} b_n (2t)^{-2\nu-1} |a_n|^2$, where $a_n = 1/b_n \int_0^\infty u(x,t)P_{n,\nu}(x, -t)d\mu(x)$. (Received January 14, 1965.)

65T-164. SEYMOUR GINSBURG, System Development Corporation, 2500 Colorado, Santa Monica, California and SHEILA GREIBACH, Harvard University, Cambridge, Massachusetts. Deterministic context free languages. II. Preliminary report.

Deterministic is to mean deterministic language (see Part I). $\{a^n b^n | n \geq 0\} \cup \{a^n b^{2n} | n \geqq 0\}$ is an example of a context free language which is not deterministic. If a deterministic language contains a sequence, then it contains an ultimately periodic sequence. As a corollary, it is solvable whether a deterministic language contains a sequence. It is unsolvable whether a context free language is deterministic. If L_1 and L_2 are deterministic, then it is unsolvable (a) whether $L_1 \subseteq L_2$, (b) whether $L_1 \cup L_2$ is deterministic. Finally, a number of operations are shown to preserve deterministic languages. In particular, if R is regular, S is a generalized sequential machine, and L is deterministic; then LR and $S^{-1}(L)$ are deterministic. Another operation: if L is deterministic then $\max(L) = \{w \text{ in } L | w\Sigma^* \cap L = \{w\}\}$ is deterministic. Also, $L/R = \{w | \exists v \in R, wv \in L\}$ is deterministic. (Received January 14, 1965.)

65T-165. F. M. WRIGHT, Iowa State University, Ames, Iowa. On the existence of a weighted Stieltjes mean sigma integral. II.

In Part I a weighted Stieltjes mean sigma integral was introduced. In this part it is first shown that if g is a continuous real-valued function of bounded variation on the closed interval [a,b] of the real axis, and if f is a bounded real-valued function on [a,b] such that $[F,(w_1,w_2,\ldots,w_p)]\int_a^b f(x)dg(x)$ exists, then the ordinary Riemann-Stieltjes norm integral $RS\int_a^b f(x)dg(x)$ exists. This result constitutes an extension of Corollary 4 given by Porcelli (Illinois J. Math. 2 (1958), 124-128) for the Stieltjes mean sigma integral. The author's proof of the above result for the weighted Stieltjes mean sigma integral is believed to be relatively straightforward and fairly simple, and this proof does not involve use of the quantity $C_{g*}M(f,k)$ featured by Porcelli. This result and the last result presented in Part I yield a theorem which provides necessary and sufficient conditions for the existence of $[F,(w_1,w_2,\ldots,w_p)]\int_a^b f(x)dg(x)$ in case g is a real-valued function of bounded variation on [a,b], and f is a bounded real-valued function on [a,b]. It is interesting to compare this theorem with Theorem 14 on page 278 of Graves (The Theory of Functions of a Real Variable, 1946) dealing with both the Riemann-Stieltjes norm integral and the Riemann-Stieltjes sigma integral. (Received November 30, 1964.)

65T-166. M. B. SLATER, University of Chicago, Chicago, Illinois 60637. Weakly prime alternative rings.

Let R be a ring with nucleus $N \neq R$ and center Z. We say R is weakly prime provided (i) Z (as a subring) has no zero-divisors; (ii) If M is an ideal of R and $M \subseteq N$, then M = (0); (iii) If V is an ideal of R and $V^2 = (0)$, then $V = (0)$. It may easily be verified that any prime (non-associative) ring is weakly prime. Now suppose R is alternative. If $3R = R$, then $Z \neq (0)$, and we may form $R' = R \otimes_Z K$, where K is the quotient field of Z. We have the Theorem: If $R = 3R$ is weakly prime alternative, then the natural map of R into R' is an imbedding, and R' is a Cayley-Dickson algebra over K. The proof uses the results of Kleinfeld, Alternative nil rings, Ann. of Math. 66, p. 395. The restriction on the characteristic is required only to ensure the validity of Kleinfeld's Lemma 1. (Received February 15, 1965.)

65T-167. A. W. HAGER and S. G. MROWKA, McAllister Building, University Park, Pennsylvania 16802. Compactness and the projection mapping from a product space. Preliminary report.

Let π_y denote the projection from $X \times Y$ onto Y. (All spaces are Hausdorff completely regular.) "π_y is closed (Q-closed, sequentially-closed)" means "$\pi_y[F]$ is closed (Q-closed, seq-closed) for each closed $F \subset X \times Y$." "$\pi_y|\mathcal{Z}$ is closed (Q-closed, seq-closed)" means "$\pi_y[Z]$ is closed (Q-closed, seq-closed) for each zero-set $Z \subset X \times Y$." Theorems. 1. X is compact iff $\pi_y|\mathcal{Z}$ is closed, for every space Y. 2. X is Lindelof iff π_y is Q-closed, for every Y. Corollary. If X is Lindelöf and Y is realcompact, then $\pi_y[F]$ is realcompact for each closed $F \subset X \times Y$. 3. If X has a dense Lindelöf subspace, then $\pi_y|\mathcal{Z}$ is Q-closed, for every Y. 4. (a) X is countably compact iff (b) π_y is seq-closed, for every Y iff (c) π_y is seq-closed for some non-seq-discrete Y. 5. (a) X is pseudocompact if (b) $\pi_y|\mathcal{Z}$ is seq-closed, for every Y iff (c) $\pi_y|\mathcal{Z}$ is seq-closed for some non-seq-discrete Y. 4 and 5 are similar to results of H. Tamano [J. Math. Kyoto U., 1,2, 1962]. The generalizations of 2,4,5 to the appropriate statements for "higher cardinality" are valid. (Received January 15, 1965.)

65T-168. SHUICHI TAKAHASHI, University of Montreal, Montreal, Canada. On rational characters of a finite group.

Let G be a finite group. By a rational character I mean the trace of a matrix representation of G with entries in Q. By a theorem of Artin, any rational character χ is a linear rational combination of those characters each of which is induced from the identity character of a subgroup of G. So there is the smallest positive integer $d = d(\chi)$ such that $d \cdot \chi$ is a linear integral rational combination of those induced characters. Now Theorem A: If G is a nilpotent group of odd order then for any rational character χ, $d(\chi) = 1$. But Theorem B: For any integer $d \geqq 0$, there is a metacyclic group G and a rational character χ of G such that $d(\chi) \geqq d$. Proof of the theorem A is group theoretical. On the other hand my proof of theorem B is rather number theoretical and the main point of the proof is to compute certain norm residu symbols. (Received January 15, 1965.)

65T-169. M. V. SUBBARAO, University of Alberta, Edmonton, Alberta, Canada. On unitary perfect numbers.

Define an integer N to be unitary perfect if the sum of its unitary divisors (namely, divisors d prime to N/d), including N and unity, is 2N. It is shown here, among other things, that every such N should be even and, excluding the first three of these numbers (6, 60 and 90), N should be $\equiv 0$ or 128 (mod 384) and should have at least eight distinct odd prime divisors and be $> 10^{12}$. Conjecture: There is no unitary perfect number $N > 90$. (Received January 19, 1965.)

65T-170. R. G. LAHA, Statistical Laboratory, The Catholic University of America, Washington, D. C. 20017. On a property of infinitely divisible distribution in a Hilbert space.

The following theorem is proven: Theorem. Let $\{\xi_n\}$ be a sequence of independently and identically distributed random variables taking value in a real separable Hilbert space $\mathcal{H}$. Let $\{A_n\}$ be a sequence of bounded self-adjoint operators in $\mathcal{H}$ satisfying the conditions (i) $\sup_n \|A_n\| < 1$; (ii) $\sum_{n=1}^{\infty} \|A_n\|^2 < \infty$. Suppose that the sum $\sum_{n=1}^{\infty} A_n \xi_n$ is identically distributed as ξ_1. Then the common distribution of each ξ_n is infinitely divisible. (Received November 23, 1964.)

65T-171. D. S. GEIGER, P. O. Box 33, Station A, Champaign, Illinois. A partial binary relation algebra.

We define a P.B.R.A. as a set of binary relations closed under relative multiplication, intersection $\cap$, and inversion $^{-1}$. Let M and M_{ij} (i,j = 1,2,3,...) be relations satisfying $M = M^{-1} \geqq M_{ij}$, $M_{ij}^{-1} = M_{ji}$, $M_{ij}M_{jk} = M_{ik}$ and $(M_{11} \cap M_{22} \cdots \cap M_{nn})M_{n+1\ n+1} = M$ for n = 1,2,3... . Then either $M_{ij} = M$ for all i,j or the M_{ij} generate a P.B.R.A. $\mathcal{M}$. Let $\mathcal{N}$ be a P.B.R.A. with maximum element N and Ers. (equivalence relations) J,K,L and L_i. We write $J \sim K$ iff there exists an $f \in \mathcal{N}$ such that $ff^{-1} = J$, $f^{-1}f = K$ and $ff^{-1}f = f$; $J > K$ iff $J \geqq K$ and JK = J; $J \supset K$ iff $J \geqq K$ and JK = K; $(J \otimes K) \sim L$ iff there exist Ers. L_1, L_2, L_3 such that $L_3NL_3 = L_3$, $J \sim L_1$, $K \sim L_2$, $L_1, L_2 < L_3$, $L_1L_2 = L_3$ and $L_1 \cap L_2 = L$. Theorem. $\mathcal{N}$ is isomorphic to $\mathcal{M}$ iff: (1) $ff^{-1}f = f$ for all $f \in \mathcal{N}$. (2) There exists an Er. $K \in \mathcal{N}$ such that for any Er. J we have $J \sim N$ or $J \sim K$ or $J \sim (K \otimes K)$ or $J \sim (K \otimes (K \otimes K))$... . (3) For every Er. J there exists an Er. L such that JL = J and $L < N$. (4) If J,K and L are Ers. satisfying NJ = NK = NL then $(J \cap K)L = JL \cap KL$. (5) Ascending chains of Ers. connected by $<$ and descending chains connected by $\subset$ are finite. (6) $\mathcal{N}$ has a countable number of relatons. (Received January 20, 1965.)

65T-172. WALTER LEIGHTON, Western Reserve University, Cleveland, Ohio 44106. On the zeros of solutions of a second-order linear differential equation.

In this paper various necessary conditions and sufficient conditions are given for the conjugacy of a solution y(x) of the differential equation $y'' + p(x)y = 0$ on an interval I: [a,b]. A typical theorem is: Let p(x) be positive, convex, and of class C' with $p'(x) > 0$ on I, and let $x_1 (a < x_1 < b)$ be a solution of the equation $p(x) = xp'(x)$. If $(b - a)^3 \geqq 9\pi^2/4p'(x_1)$, y(x) has a zero on the interval $a < x \leqq b$. It follows, for example, that a solution of the system $y'' + (1 + x^2)y = 0$, $y(0) = 0$, has a zero on the interval $0 < x < (3\pi)^{2/3}/2$. (Received January 21, 1965.)

65T-173. J. R. SHOENFIELD, Stanford University, Stanford, California. A hierarchy for objects of Type 2.

Let j be a jump in the sense of Enderton (Trans. Amer. Math. Soc., 111, 457-471). Define 0, $<_0$ and H_a for a in 0 as in Kleene's definition of the hyperarithmetical hierarchy with the following two changes: $H_{a*} = j(H_a)$; and in place of (03), we put $3^b 5^e$ in 0 if b is in 0; $\{e\}^{H_b}(n)$ is defined and in 0 for all n; $b <_0 \{e\}^{H_b}(0) <_0 \{e\}^{H_b}(1) <_0 \ldots$. Then a predicate is recursive in H_a for some a in 0 iff it is recursive in j (in the sense of Kleene, considering j as a higher type object).

The proof requires a uniqueness theorem analogous to Spector's uniqueness theorem for the hyperarithmetical hierarchy. (Received January 26, 1965.)

65T-174. H. F. BECHTELL, Bucknell University, Lewisburg, Pennsylvania 17837. The Frattini subgroup of a p-group. Preliminary report.

Restricting attention to finite p-groups G and denoting the Frattini subgroup of G by Φ, it is shown that no non-Abelian subgroup $M \leqq \Phi$ and normal in G can have a cyclic center. This generalizes the known result by W. Burnside that no non-Abelian p-group having cyclic center can be the derived group of a p-group and the analogous theorem for Frattini subgroups by C. Hobby. The approach used leads to the classification of the non-Abelian groups of order p^4 that are Frattini subgroups of some p-group. (Received January 26, 1965.)

65T-175. R. C. LYNDON, Queen Mary College, London E.1, England. Dehn's algorithm.

A unified topological proof is given of results of Dehn (Math. Ann., 1912), Britton (Proc. Glasgow Math. Assoc., 1957), Greendlinger (Comm. Pure Appl. Math., 1960; Doklady, 1964), and of new results. Let F be a free group; for w in F, $|w|$ = length of w. Let $R \subseteq F$, and r in R imply that r is cyclically reduced and every cyclically reduced conjugate of $r^{\pm 1}$ is in R. Condition $C(\lambda)$: ab, ac in R, $ab \neq ac$ implies $|a| < \lambda |ab|$. Condition T: a, b, c in R implies one of ab, bc, ca reduced. Let $w \neq 1$ be in the normal closure of R. Theorems. $C(\lambda)$, $\lambda \leqq 1/5$, implies w contains more than $(1 - 3\lambda)|r|$ of some r in R. $C(\lambda)$, $\lambda \leqq 1/3$, and T, imply w contains more than $(1 - 2\lambda)|r|$ of some r in R. Analogous theorems for free products. Proofs reduce to two easy Lemmas. Let a finite graph H decompose the closed disc D into simply connected regions. Let each vertex not on the boundary B of D have degree $\geqq 3$ $[\geqq 4]$, and each region not on B have $\geqq 6$ $[\geqq 4]$ edges on its boundary. Then H has two regions, each with $\leqq 3$ $[\leqq 2]$ edges on its boundary that do not lie on B. (Received January 28, 1965.)

65T-176. E. T. POULSEN, Heidelberg Universität, Institut fur angewandte Mathematik, Heidelberg, Germany. Norm-differentiability of evolution operators.

Consider the evolution equation (E): $U_t(t,s) = A(t)U(t,s)$ with initial condition $U(s,s) = I$, where the $U(t,s)$ are bounded linear operators in a B-space X, while the $A(t)$, $t \geqq 0$, are closed, densely defined linear operators in X. We assume that (1) all operators A(t) have the same domain D, and (2) there exist real numbers $\theta > 0$ and $M < \infty$ such that $\|(\lambda - A(t))^{-1}\| \leqq M(1 + |\lambda|)^{-1}$ for $|\mathrm{Arg}\,\lambda| \leqq \pi/2 + \theta$ and all t. Tanabe [Osaka Math. J. 13 (1960), 363-376] proved that if (3) $B(t) = A(t)A(0)^{-1}$ is Hölder-continuous in Norm, then (E) has a unique solution U(t,s), which is strongly continuous in (t,s) for $t \geqq s$, while, for $t > s$, $U(t,s)X \subset D$, the derivative $U_t(t,s)$ exists as strong derivative and is strongly continuous in (t,s). Komatsu [J. Fac. Sci. Univ. Tokyo 9 (1961), 1-11] proved that if (3a) B(t) is analytic, then U(t,s) is analytic in (t,s) for $t > s$. We prove that if (3) holds, then "strong" can be replaced by "norm-" in Tanabe's result for $t > s$. (Received January 19, 1965.)

65T-177. J. R. BUCHI, Ohio State University, 231 W. 18th Avenue, Columbus, Ohio 43210. Transfinite automata recursions.

ω_1 = the first noncountable ordinal. Let r denote an ω_1-sequence of members of the finite set S (states of r), i.e., $r: \omega_1 \longrightarrow S$. For a limit $x < \omega_1$, $\sup_{t<x}(rt)$ denotes the set of states occurring cofinal to x. Recursions $r = \zeta i$ of the form $r0 = A$, $rx' = H[rx, ix]$, $rx = U[\sup_{t<x}(rt)]$ for x a limit, constitute a natural extension of finite automata. The behavior, beh(ζ,W), is the set of all z-sequences $i[0,z)$, $z < \omega_1$, such that $W[rz]$ holds for $r = \zeta i$. All basic facts about regular events carry over naturally to ω_1-behaviors. In particular, the projection of a behavior is again a behavior, i.e., Theorem: If B(i,j) is the behavior of an automaton $r = \zeta i,j$ with output W, then $(\exists j)B(i,j)$ is again behavior of an automaton $s = \eta i$ with output Q. $[\eta, Q]$ can be effectively constructed from $[\zeta, W]$.

(Received January 21, 1965.)

65T-178. P. K. HOOPER, Computation Laboratory, Harvard University, Cambridge 38, Massachusetts. Post Normal Systems: The unrestricted halting problem.

A natural and rather interesting problem arises when one considers, for a given Post Normal System, whether or not the system halts (eventually) for every word over its alphabet. If one permits the system to be multigenic, the problem can be shown recursively unsolvable by reducing to it the problem of solvability for an arbitrary domino set, proved unsolvable by Berger (these *Notices* 11 (1964), 537). If we represent each domino of the set $D = \{D_i\}$ by a quintuple (X,a,b,c,d), meaning that domino D_X has its edges marked with a,b,c, and d, clockwise from the top, our normal system, N(D), will be over alphabet $\{d_i\} \cup \{*\}$ with productions: $d_x d_y P \longrightarrow P d_z d_z$ for every triple of dominos of the form (X,a,b,c,d), (Y,e,f,g,h), (Z,i,j,e,b). $d_x * d_y P \longrightarrow P d_u d_u * d_v d_v$ for every set of pairs of the form (X,a,b,c,d), (U,j,k,l,b); (Y,e,f,g,h), (V,m,n,e,p). It is quite easy to show that D has a (periodic) solution if and only if there is a word (with no occurrences of "*") on which N(D) does not halt, establishing our result. However, the unrestricted halting problem for monogenic normal systems is still open. (Received February 1, 1965.)

65T-179. A. A. MULLIN, University of California, Lawrence Radiation Laboratory, Box 808, Livermore, California 94551. Analogues of highly composite and related numbers. Preliminary report.

S. Ramanujan [Collected papers, Cambridge, 1927, p. 86] defines a highly composite natural number n to be one satisfying $d(m) < d(n)$ for every natural number $m < n$. Clearly, there are infinitely many highly composite numbers, which even satisfy a "Bertrand's Postulate" that the prime numbers also satisfy. Since, in many respects, submosaics operate analogously to divisors, one can define a highly decomposable natural number n to be one satisfying $N(m) < N(n)$ for every natural number $m < n$, where N(n) is the number of distinct submosaics in the mosaic for n. (Erratum: $N(n) \leqq 1 + d(n)$ for almost all n.) Lemma 1. There exist infinitely many highly decomposable numbers; in fact, if j(n) is the least prime number not in the mosaic of n, then for every natural number $n \geqq 3$ there exists a highly decomposable M satisfying $n < M \leqq n \cdot j(n) < n^2$. Define an involuted natural number n to be one satisfying $\sigma^*(m) < \sigma^*(n)$ for every natural number $m < n$, where σ^*, an analogue of classical σ, is defined in the author's Abstract 65T-64 (these *Notices* 12

(1965), 217). Lemma 2. There exist infinitely many involuted numbers; in fact, as before, for every $n \geqq 3$ there exists an involuted M satisfying $n < M \leqq n \cdot j(n) < n^2$. (Received February 1, 1965.)

65T-180. D. F. DAWSON, North Texas State University, Denton, Texas. Semigroups having left or right zeroid elements.

It is well-known that if a semigroup S contains a zeroid element, then S contains a unique idempotent e such that eS = Se is a group, namely the group of zeroid elements of S (Clifford and Miller, Amer. J. Math. 70 (1948), 117-125). Theorem 1. If S is a semigroup with left zeroid μ and $L = \{x \in S | x\mu = \mu\}$, then each of the following conditions on the semigroup L is sufficient for S to have an idempotent e such that eS is a group and Se is regular: (1) L has left zeroid idempotent, (2) L is degenerate, (3) L has a right zeroid, (4) L is regular, (5) L is simple and contains an idempotent. T. Tamura (Kōdai Math. Sem. Rep. 6 (1954), 93-95) showed that if a semigroup S contains exactly one idempotent e, then e is a left zeroid of S if and only if e is a right zeroid of S. Theorem 2. If S is a semigroup which contains among its idempotents one and only one left (right) zeroid e, then e is a zeroid of S. Theorem 3. If a semigroup S contains a unique least idempotent e (K. Iséki, Proc. Japan Acad. 32 (1956), 225-227) and e is a left or right zeroid of S, then e is a zeroid of S. Theorem 4. If S is a semigroup with left zeroid μ, then S has a left zeroid idempotent if and only if the equation $\mu = (\mu\mu)x$ has a solution $x \in S$. (Received February 1, 1965.)

65T-181. K. D. MAGILL, JR., State University of New York at Buffalo, Buffalo, New York 14214. Semigroups of connected functions.

Let T(X) denote the semigroup of all connected functions mapping the topological space X into itself under the composition operation. A class of spaces is said to be T-admissible if for any two spaces X and Y of the class, any isomorphism ϕ from T(X) onto T(Y) is of the form $\phi(f) = h \circ f \circ h^{\leftarrow}$ for some homeomorphism h from X onto Y. Definition. A space X is a T-space if it is connected and for any connected subset K of X, there is a function $f \in T(X)$ such that $f[X] = K$. Theorem. Let X and Y be T-spaces. A mapping ϕ from T(X) onto T(Y) is an isomorphism if and only if there exists a biconnected mapping h from X onto Y such that $\phi(f) = h \circ f \circ h^{\leftarrow}$ for each $f \in T(X)$. Examples are given to show that the class of T-spaces is not T-admissible. However, using a result of Pervin and Levine on biconnected mappings, we obtain the following Theorem. The class of locally connected, compact Hausdorff, T-spaces it T-admissible. This class includes Peano spaces Finally, homomorphisms from T(X) into T(Y) are investigated and it is shown that if X is a completely regular, connected, Hausdorff space with cardinality $\leqq c$, and Y is any space with more than one point, then any epimorphism from T(X) onto T(Y) is an isomorphism. (Received February 2, 1965.)

65T-182. M. E. WATKINS, University of North Carolina, Chapel Hill, North Carolina 27515. Connectivity versus diameter in graphs.

Let G denote a finite undirected graph with no loops or multiple edges. Let V denote its vertex set and let $|V|$ be the cardinality of V. Let δ and λ denote the diameter and the vertex-connectivity of G, respectively. Theorem 1: In a graph G, if $\lambda \geqq 1$ and $\delta \geqq 1$, then $|V| \geqq F(\delta, \lambda) \equiv \lambda(\delta - 1) + 2$. The function F is the best possible result, as indicated by Theorem 2: Given positive integers δ and λ, there exists a graph G with vertex set V such that $|V| = F(\delta, \lambda)$. Both proofs depend upon H. Whitney's characterization of vertex-connectivity (Amer. J. Math. 54 (1932)). (Received February 4, 1965.)

65T-183. JOHN DE CICCO and STAVROS BUSENBERG, Illinois Institute of Technology, Technology Center, Chicago, Illinois 60616. A comparison of inversive and conformal differential geometries.

By use of the Schwarzian derivative $\{z,t\}$, the differential and integral invariants of a regular arc C are derived. If M_{12} denotes Kasner's conformal measure of a horn angle of order two of which the sides are C_1 and C_2 and if C_1 is the osculating circle of $C_2 = C$ at the vertex, then M_{12} is equal to $-1/(8\epsilon\Gamma)$, where ϵ is + 1 or - 1 according as $d\kappa/ds$ of the curve C is positive or negative at its initial point, and Γ is the inversive curvature of the curve C at its vertex. It is established that two regular curves C of the inversive Moebius plane, are inversely congruent if and only if the corresponding inversive curvatures Γ are given by the same function of the inversive arc length σ. Characterizations of the conformal and Moebius groups are obtained by use of the Schwarzian derivative. See Kasner: Conformal geometry, Proceedings of the Fifth International Congress of Mathematics, Cambridge, Vol. 2, 1912, pp. 81-90. (Received February 5, 1965.)

65T-184. WITHDRAWN.

65T-185. A. H. SMITH, California State College at Long Beach, Long Beach, California. Extensions of semigroups to groups.

The object is to associate with each semigroup M a group G(M), uniquely determined by M, such that if M can be embedded in a group then M can be embedded in G(M). G(M) is constructed as follows: Form all possible pairs (a, ϵ) where a is an element of M and ϵ is an integer (positive, negative, or zero). Consider all finite words $(a_1, \epsilon_1)(a_2, \epsilon_2) \ldots (a_n, \epsilon_n)$. Define an equivalence relation between words by the rule: $(a_1, \epsilon_1) \ldots (a_n, \epsilon_n) \sim (b_1, \eta_1) \ldots (b_m, \eta_m)$ if and only if for every group F and homomorphism $h: M \to F$, $[h(a_1)]^{\epsilon_1} \ldots [h(a_n)]^{\epsilon_n} = [h(b_1)]^{\eta_1} \ldots [h(b_m)]^{\eta_m}$. The equivalence classes $[(a_1, \epsilon_1) \ldots (a_n, \epsilon_n)]$ of words $(a_1, \epsilon_1) \ldots (a_n, \epsilon_n)$ are the elements of G(M) and multiplication is defined by $[(a_1, \epsilon_1) \ldots (a_n, \epsilon_n)] \cdot [(b_1, \eta_1) \ldots (b_m, \eta_m)] = [(a_1, \epsilon_1) \ldots (a_n, \epsilon_n)(b_1, \eta_1) \ldots (b_m, \eta_m)]$. Define a homomorphism $i: M \to G(M)$ by $i(a) = [(a,1)]$. The homomorphism i is also an injection (one-to-one into mapping) if and only if for each pair of distinct elements a,b in M there exists a group F and homomorphism $h: M \to F$ such that $h(a) \neq h(b)$ (different h and F may be cited for each pair of elements in M). This property characterizes those semigroups which can be embedded in a group. (Received February 5, 1965.)

65T-186. H. J. KEISLER, University of Wisconsin, Madison, Wisconsin 53706, and FREDERICK ROWBOTTOM, University of California, Berkeley, California. Constructible sets and weakly compact cardinals. Preliminary report.

We use the notation of Gödel, Consistency of the continuum hypothesis, Princeton, 1940. Lemma. Suppose m is a set of ordinals which is closed under the operations C, K_1, K_2, J_0, ..., J_8. Assume that $\overline{\overline{m}} - m$ is nonempty and its least element is γ. Then: (1) m is not constructible; (2)

γ has a nonconstructible subset; (3) γ is an uncountable weakly compact cardinal in the model Δ. B. Jónsson has asked: Is there an algebra of power α with no proper subalgebra of power α? The lemma shows that if V = L, then the answer is "yes" for all cardinals α. Theorem 1. Let $\alpha \leqq \beta$ be cardinals such that $\langle F''\beta, \epsilon \rangle$ has an elementary substructure of power α which does not include $F''\alpha$. Then there exists $\gamma < \alpha$ such that (2) and (3) hold. Theorem 2. Let $\alpha \leqq \beta$ be cardinals such that for all $\delta < \alpha$, $\langle F''\beta, \epsilon \rangle$ has an elementary substructure of power α which includes $F''\delta$ but not $F''\alpha$. Then α is a limit of ordinals γ which are weakly compact cardinals in the model Δ. This improves a result of Rowbottom, Doctoral dissertation, University of Wisconsin, Madison, 1964. The proofs use results in Keisler's abstract, Extending models of set theory, to appear in J. Symbolic Logic. Weakly compact cardinals were defined in Tarski's article on pp. 125-135 in Logic, methodology, and philosophy of science, Stanford, 1962. (Received February 8, 1965.)

65T-187. G. E. CROSS, University of Waterloo, Waterloo, Ontario, Canada. A symmetric integral of order two.

A symmetric derivative of order two (denoted by $SCD^2F(x)$) is defined for Lebesgue integrable F(x) after the manner in which the symmetric derivative SCDF(x) was defined by J. C. Burkill for Perron integrable F(x). [J. C. Burkill, Integrals and trigonometric series, Proc. London Math. Soc. 1 (1951), 46-57.] If f(x) is finite-valued everywhere and there exists $F(x) \in L$ such that (a) $SCD^2F(x) = f(x)$ in $[a - 2\pi, a + 2\pi]$, (b) $F(a + 2\pi) = F(a - 2\pi)$, (c) $F(a) = 0$, (d) F(x) is c-continuous at a, $a + 2\pi$, $a - 2\pi$, (e) $\lim_{h \to 0}(1/h)\int_x^{x+h}F(t)dt$ exists everywhere, then f(x) is SCP^2-integrable over $[a, a + 2\pi]$ and $F(a + 2\pi) = SCP^2\int_a^{a+2\pi}f(t)dt$. It is shown that (i) if the series $\sum_0^\infty c_n e^{inx}$ is summable (C,1) everywhere to f(x) then $c_n = (1/2\pi^2)\, SCP^2\int_a^{a+2\pi}f(t)e^{-int}dt$ for suitably chosen a; (ii) if the series $a_0/2 + \sum_1^\infty a_n \cos nx + b_n \sin nx$ is summable (C,2) everywhere to f(x) and a_n, $b_n = O(n^\alpha)$, $0 \leqq \alpha < 1$, then $a_n = (1/\pi^2)SCP^2\int_0^{2\pi}f(t)\cos nt\, dt$, $n \geqq 0$, and a similar formula for b_n, $n > 0$; (iii) SCP^2-integrability implies J_4-integrability [P. S. Bullen, Primitives of generalized derivatives, Canad. J. Math. 13 (1961), 48-58] and when f(x) is periodic a relation between the two definite integrals is demonstrated. (Received February 9, 1965.)

65T-188. R. P. GILBERT, H. C. HOWARD and S. O. AKS, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland. Singularities of analytic functions having integral representations with a remark about the elastic unitarity integral.

In this paper a survey is given of some results which have been obtained recently concerning the singularities of holomorphic functions having integral representations. These results are all essentially extensions or modifications of those developed by Hadamard (for the proof of his multiplication of singularities theorem) to the case of several complex variables. As a concluding remark we consider the connection between the original Hadamard idea and the elastic unitarity integral of the quantum theory of fields. (To appear in the Journal of Mathematical Physics.) (Received February 9, 1965.)

65T-189. MICHEL JEAN, University of California, Berkeley 4, California. Pure systems of binary relations.

Let A be of finite cardinality n, $R_1 \cup \ldots \cup R_t \subseteq A^2$, and $\mathfrak{A} = \langle A, R_1, \ldots, R_t \rangle$. For $k < n$, $\mathfrak{A}$ is k-pure iff any two subsystems of $\mathfrak{A}$ of cardinality k are isomorphic. $\mathfrak{A}$ is pure iff $\mathfrak{A}$ is k-pure for each $k < n$. $\mathfrak{A}$ is purely orderable iff there is a linear ordering < of A such that $(\mathfrak{A}, <)$ is pure (cf. Fraissé, Alger-Math. 1 (1954), 93). Theorem. If $3 \leqq k \leqq n - 3$ and if $\mathfrak{A}$ is k-pure, then $\mathfrak{A}$ is purely orderable. Also if $4 \leqq n$ and if $\mathfrak{A}$ is (n - 2)-pure, then $\mathfrak{A}$ is 2-pure. R. M. Robinson has found, for every prime n congruent to 3 modulo 4, an (n - 2)-pure system of cardinality n which is not pure. (Received February 11, 1965.)

65T-190. WERNER GREUB, University of Toronto, Toronto, Ontario, Canada and P. M. TONDEUR, Harvard University, Cambridge, Massachusetts 02138. Sectional curvatures and Euler-Poincaré characteristic of homogeneous spaces.

Let G/H be a homogeneous space of a compact Lie group G. A bi-invariant metric on G defines an orthogonal decomposition $\mathfrak{G} = \mathfrak{H} \oplus \mathfrak{M}$. Assume G/H to be locally symmetric with respect to this decomposition and consider G/H with its natural induced G-invariant metric. Theorem. For any even integer p with $0 < p \leqq n = \dim G/H$, the pth sectional curvature γ_p is non-negative. For the definition of γ_p, see J. A. Thorpe [Ann. of Math. (2) 80 (1964), 429-443]. The proof consists in an explicit computation of γ_p. Corollary (Hopf-Samelson). Let G/H be as before. Then the Euler-Poincaré characteristic of G/H is non-negative. The proof follows from the theorem by applying the generalized Gauss-Bonnet theorem. This answers, for the class of spaces considered, the question raised by Samelson [Michigan Math. J. 5 (1958), p. 13, 1.5]. (Received February 11, 1965.)

65T-191. T. G. McLAUGHLIN, University of Illinois, Urbana, Illinois. Splitting and decomposition by regressive sets.

Let α, β, γ be infinite sets of natural numbers. α is weakly decomposed by β and $\gamma \leftrightarrow_{df} \alpha \subseteq \beta \cup \gamma$ and each of β, γ splits α. (If β, γ are r.e., this gives the usual notion of α being decomposable.) It follows immediately from a theorem announced earlier in Abstract 64T-441 (these *Notices* 11 (1964), 676) that there is a set α which can be split by an r.e. set but cannot be weakly decomposed by any pair β, γ of regressive sets. We state two further results. Theorem. There is a cohesive set α which can be split by a retraceable set but cannot be weakly decomposed by any pair of regressive sets. Theorem. There is a set α which can be decomposed by a pair of r.e. sets, but which cannot be weakly decomposed by any pair of regressive sets β, γ such that neither β nor γ is nonrecursive r.e. (and hence, in particular, cannot be weakly decomposed by any pair of retraceable sets). (Received February 11, 1965.)

65T-192. MARVIN WUNDERLICH, State University of New York at Buffalo, Buffalo, New York 14214. Sieve-generated sequences of natural numbers. II.

The following is a generalization of the set of results announced in Abstract 619-154 (these *Notices* 12 (1965), 102) and presented by the author to the winter meeting in Denver. We will first define a class S of sequences of natural numbers. Let $A = \{2,3,4,\ldots\}$. Suppose inductively that $A_k = \{a_{k,1}, a_{k,2}, \ldots\}$ has been defined. We now define A_{k+1} as follows: Let $a_{k,k} = a$ and for each integer $m \geqq 0$, choose an arbitrary element from the set $\{a_{k,ma+k+1}, a_{k,ma+k+2}, \ldots, a_{k,ma+k+a}\}$. Delete these elements from sequence A_k to form A_{k+1}. The sequence $A = \{a_k\}$ is then defined to be the sequence $\{a_{k,k}\}$ which is the set of natural numbers $\geqq 2$ which survive each execution of the sieve process. The class of sequences S consists of all the sequences A which can be generated by this sieve process. The results given below are for an arbitrary sequence $\{a_k\} \in S$. Definition: Let $f_k(n)$ be the number of integers $m < a_n$ which are eliminated at the kth execution of the sieve process. Definition: $\ell(n)$ is the number of k such that $f_k(n) = 1$. Definition: $d(n) = n/n + \ell(n)$. Theorem 1. $a_n \sim n \log n$ iff $\sum_{k=2}^{n} d(k)/k \sim d(n) \log n$. Theorem 2. $1/2 - \epsilon < a_n/n \log n < 2 + \epsilon$. Theorem 3. $\liminf a_n/n \log n \leqq 1$ and $\limsup a_n/n \log n \geqq 1$. (Received February 12, 1965.)

65T-193. E. B. FABES and N. M. RIVIERE, University of Chicago, Chicago, Illinois 60637. Commutators of singular integrals.

Let x,y denote points in E^n. For $f \in C_0^\infty(E^n)$ and $\epsilon > 0$, set: $(S_{j,\epsilon}^{\alpha} f)(x) = C_{\alpha,j} \int_{|y|>\epsilon} (y_j/|y|^{n+1+\alpha}) f(x-y)\,dy$ $(0 < \alpha < 1)$. The limit is taken in the $L^p(E^n)$ sense $(1 < p < \infty)$; $C_{\alpha,j}$ is a constant depending only on α and j, and chosen so that $\widehat{(S_{j,\epsilon} f)}(x) \to (x_j/|x|^{1+\alpha})\hat{f}(x)$ (in the sense of $L^2(E^n)$). The notation used below is taken from A. P. Calderón and A. Zygmund, Singular integral operators and differential equations (Amer. J. Math. 79 (1957), 901-921). Define $\Lambda^\alpha = \sum_{j=1}^{n} R_j S_j^\alpha$, where R_j is the jth Riesz transform. For $f \in C_0^\infty(E^n)$, $\widehat{\Lambda^\alpha f}(x) = c|x|^\alpha \hat{f}(x)$, c an absolute constant. Theorem. Let K_1, K_2 be two operators of class C_β^∞ $(0 < \beta \leqq 1)$. If $0 < \alpha < \beta$, then (i) $K_1\Lambda^\alpha - \Lambda^\alpha K_1$; (ii) $(K_1 \circ K_2 - K_1K_2)\Lambda^\alpha$; (iii) $(K_1^* - K_1^\#)\Lambda^\alpha$ are bounded operators from $L^p(E^n)$ into $L^p(E^n)$ $(1 < p < \infty)$. (Received February 12, 1965.)

65T-194. C. Y. TANG, Illinois Institute of Technology, Chicago, Illinois 60616. On uniqueness of generalized direct products with amalgamated subgroups. Preliminary report.

We shall define an amalgam A to be a set of groups G_i, $i \in I$, together with a set of subgroups H_{ij}, $i \neq j$, $H_{ij} \subset G_i$, and a set of isomorphisms θ_{ij} mapping H_{ij} onto H_{ji}. Definition: Two amalgams A and A' are said to be isomorphic if there exists an 1-1 correspondence between the groups of A and A' such that if G_i corresponds to G_i' then there exists an isomorphism ϕ_i mapping G_i onto G_i' satisfying the condition $\phi_j \theta_{ij} = \theta_{ij}' \phi_i$. In general the generalized direct products of isomorphic amalgams may not be isomorphic. Theorem 1. The generalized direct products of isomorphic amalgams of two groups are isomorphic. Theorem 2. The generalized direct products of isomorphic reduced amalgams of three groups are isomorphic. Here a reduced amalgam is an amalgam in which G_i is generated by the subgroups H_{ij}, $i \neq j$, $j \in I$, for each $i \in I$. Examples of nonisomorphic generalized direct products of isomorphic amalgams consisting of more than two groups and non-isomorphic generalized direct products of isomorphic reduced amalgams consisting of more than three groups have been constructed. (Received February 15, 1965.)

65T-195. HAIM GAIFMAN, The Hebrew University, Jerusalem, Israel. Results concerning models of Peano's arithmetic.

T is any extension of Peano's arithmetic. M_i are models of T. Terminology: A minimal model is one having no elementary proper submodels. A minimal extension of M is an elementary proper extension of M, in which every elementary submodel, which properly includes M, is equal to the whole model. M_2 is an end extension of M_1 if it is an elementary proper extension of M_1 and every element of $M_2 - M_1$ follows every element of M_1, in the ordering of M_2. A minimal end extension is one which is both minimal and an end extension. Two models are isomorphic over M if M is a submodel of both and there is an isomorphism of one onto the other leaving every element of M fixed. Theorem 1. Every model of T has two minimal end extensions which are not isomorphic over it. Using this, one can construct models of T, M_λ, where λ varies over all ordinals, such that M_0 is minimal (the existence of which is not difficult to show), $M_{\lambda+1}$ is a minimal end extension of M_λ, for all λ, and $M_\nu = \bigcup_{\lambda<\nu} M_\lambda$ if ν is a limit ordinal > 0. A model is called rigid if it has no nontrivial automorphisms. Theorem 2. Every M_λ is rigid, and if M'_λ are likewise constructed and τ is an isomorphism of M_λ onto M'_ν then $\lambda = \nu$ and $\tau(M_\alpha) = M'_\alpha$, for all $\alpha \leqq \lambda$. Corollary. For every infinite cardinal δ, there are 2^δ rigid nonisomorphic models of T. (Received February 15, 1965.)

65T-196. C. Y. CHAO, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Groups and graphs. II. Preliminary report.

An algorithm is presented for constructing all graphs (directed or undirected, with or without loops) of n vertices, each of whose group of automorphisms $\supseteq$ a given permutation group on n letters. (This is a generalization of a result of the author's article Groups and graphs, to appear in Trans. Amer. Math. Soc.) Theorem. For any order k of some element g of a finite group G of order n there exists a graph X_k with n vertices consisting of t directed k-cycles such that G is isomorphic to a subgroup of the group of automorphisms of X_k where t is the index of the cyclic subgroup generated by g in G. Another version of the theorem is: Let G be a finite group of order n. Then for any order k of some element of G, G can be imbedded in the group of centralizers $C(\sigma)$ of a permutation σ in the symmetric group of n letters, where σ consists of $t = (n/k)$ disjoint k-cycles, i.e., $\sigma = (1,2,\ldots,k)(k+1,k+2,\ldots,2k)\ldots((t-1)k+1,\ldots,tk)$. When $k = 1$, this is the well-known Cayley's theorem. The order of $C(\sigma)$ is $t!k^t$. (Received February 15, 1965.)

65T-197. RICHARD BALSAM, University of California, Los Angeles, California 90024. Normal extensions of formally normal ordinary differential operators. II.

Let L be a first-, second-, or third-order ordinary linear differential operator, with coefficients infinitely differentiable on a real interval (a,b). Assume $L^+L = LL^+$ and let N be the minimal formally normal operator in $\mathscr{L}^2(a,b)$, induced by L. Theorem. N has a normal extension N_1 in $\mathscr{L}^2(a,b)$ if and only if: (1) L is a polynomial in some formally self-adjoint differentiable operator L_s, such that S, the minimal symmetric operator in $\mathscr{L}^2(a,b)$ induced by L_s, has equal deficiency indices, and (2) N_1 is the same polynomial in some self-adjoint extension of S. It is also shown that for a class of third-order L, suitable (a,b) can be chosen so that N has no normal extension in $\mathcal{L}^2(a,b)$ or in any larger Hilbert space containing $\mathcal{L}^2(a,b)$. (Received February 15, 1965.)

65T-198. WITHDRAWN.

65T-199. J. M. WORRELL, JR., Division 5251, Sandia Corporation, Sandia Base, Albuquerque, New Mexico 87115. Completeness and certain implicit upper semicontinuous decompositions.

To say here that a space S is complete in Sense A means that there exists a base B for S such that the closures of the elements of every monotonic subcollection of B have a common part. Let S denote a subspace of a regular T_1 space R that is complete in Sense A and has a base of countable order. Let G denote an upper semicontinuous decomposition of S into bicompact point sets. Theorem. S is dense in some subspace R' of R such that (1) R' is complete in Sense A and (2) G is a subcollection of some upper semicontinuous decomposition of R' all elements of which are bicompact. Corollary (Vaĭnšteĭn). If U is an upper semicontinuous decomposition of a metrizable space M into compact point sets, M is dense in some metrically topologically complete space having an upper semicontinuous decomposition F such that U is a subcollection of F and all elements of F are compact. (Received February 16, 1965.)

65T-200. C. H. GIFFEN, The Institute for Advanced Study, Princeton, New Jersey 08540. Cross sections of 2-spheres in the 4-sphere.

Let $I = [-1,1]$ and $I/2 = [-1/2,1/2]$. Theorem. If K is a polyhedral 2-sphere in the 4-sphere S, there is a general position semilinear map $f: S \longrightarrow I$ such that: (0) $S_t = f^{-1}(t) \cong S^3$ for all $t \in \mathring{I}$; (1) $f^{-1}(\dot{I}) \cong S^0$; (2) $f^{-1}(\mathring{I}) \supset K$; (3) $(f|K)^{-1}(\mathring{I}/2) \supset$ all saddle (hyperbolic) points; (4) $(f|K)^{-1}(I - I/2)$ $\supset$ all elliptic points; (5) $K_0 = K \cap S_0$ is a simple closed curve. The 4-disk $S_+ = f^{-1}[0,1]$ admits the map $f_+ = f|S_+$: $S_+ \longrightarrow I_+ = [0,1]$ such that S_+ and $K_+ = K \cap S_+$ satisfy appropriate restrictions of (0)-(5); such a (4,2)-disk pair (S_+,K_+) is called a tent. Hence, any polyhedral (4,2)-sphere pair can be decomposed as the sum of two tents with their boundaries identified. Theorem. The boundary (3,1)-sphere pair of a locally flat tent is a ribbon knot in a natural way. Hence, if (S,K) is a locally flat (4,2)-sphere pair, the "equatorial" cross section (S_0,K_0) is a ribbon knot. Therefore 4-dimensional knot theory by cross sections need consider only ribbon knots (rather than slice knots) and the tents they bound. These theorems have many applications. (Received February 16, 1965.)

65T-201. WITHDRAWN.

65T-202. H. J. LANDAU, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey 07971. Necessary density conditions for local harmonic analysis and interpolation.

For a compact set S in R^N, let B(S) denote the subspace of $L^2(R^N)$ consisting of those functions whose Fourier transforms are supported on S. Call a subset Λ of R^N uniformly discrete if the distance between any two points of Λ exceeds some positive quantity. If Λ is a uniformly discrete set such that, for some K, $\|f\|^2 \leqq K \sum_\Lambda |f(\lambda)|^2$ for every $f \in B(S)$, call Λ a set of harmonic analysis for S. If Λ is a set such that, given an arbitrary square-summable collection $\{a_\lambda\}$ of complex numbers, there exists $f \in B(S)$ with $f(\lambda) = a_\lambda$ for $\lambda \in \Lambda$, call Λ a set of interpolation for B(S). The following connection is established between the density of such sets and the measure of S. Theorem 1.

With I_0 a set in R^N and $r > 0$, let I_r be any translate of the set of points rx with $x \in I_0$. If Λ is a set of harmonic analysis for S, then for any $\epsilon \, 0$, $n(I_r) \geqq (1 - \epsilon)m(S)m(I_0)(r/2\pi)^N + o(r^N)$, and if Λ is a set of interpolation for B(S), then for any $\epsilon > 0$, $n(I_r) \leqq (1 + \epsilon)m(S)m(I_0)(r/2\pi)^N + o(r^N)$, where $n(I_r)$ denotes the number of points of Λ contained in I_r, and m is Lebesgue measure in R^N. In R^1 this can be made more precise. Theorem 2. If S is the union of k intervals and I_r any interval of length r, then if Λ is a set of harmonic analysis for S, $n(I_r) \geqq m(S)r/2 \; - a \log^+ r - b$, and if Λ is a set of interpolation for B(S), $n(I_r) \leqq m(S)r/2\pi + a \log^+ r + b$, where the constants a and b depend on S and Λ but not on r or I_r. (Received February 10, 1965.)

65T-203. J. I. MALITZ, University of California, Berkeley, California 94720. Cardinal sums and Beth's theorem in infinitary languages.

Let α, β be infinite cardinals. We denote by $L(\alpha,\beta)$, the first-order language with equality that allows conjunction involving less than α conjuncts and quantification over less than β variables. Two structures $\mathfrak{A}$ and $\mathfrak{A}'$ are $L(\alpha,\beta)$ equivalent (written $\mathfrak{A} \equiv_{\alpha,\beta} \mathfrak{A}'$) if the $L(\alpha,\beta)$ sentences that are true in $\mathfrak{A}$ are true in $\mathfrak{A}'$. The cardinal sum and direct product of $\mathfrak{A}$ and $\mathfrak{B}$ are denoted by $\mathfrak{A} \oplus \mathfrak{B}$ and $\mathfrak{A} \odot \mathfrak{B}$ respectively. We say that $L(\alpha,\beta)$ equivalence is preserved under cardinal sum (direct product) if whenever $\mathfrak{A} \equiv_{\alpha,\beta} \mathfrak{A}'$ and $\mathfrak{B} \equiv_{\alpha,\beta} \mathfrak{B}'$ then $\mathfrak{A} \oplus \mathfrak{B} \equiv_{\alpha,\beta} \mathfrak{A}' \oplus \mathfrak{B}'$ ($\mathfrak{A} \odot \mathfrak{B} \equiv_{\alpha\beta} \mathfrak{A}' \odot \mathfrak{B}'$). Theorem. $L(\alpha,\beta)$ equivalence is preserved under cardinal sum (direct product) if and only if α is strongly inaccessible. For $\alpha = \omega_1$, $\beta = \omega$ this solves a problem posed by E.G.K. Lopez-Escobar in Infinitely long formulas with countable quantifier degrees, Doctoral Dissertation, Berkeley, California, 1964, p. 131. This theorem is also used in proving that Beth's theorem (and hence Craig's theorem) fails for $L(\alpha,\beta)$ whenever $\beta > \omega$. This contradicts the interpolation theorem stated by S. Maehara and G. Takeuti in A formal system of first-order predicate calculus with infinitely long expressions, J. Math. Soc. Japan 13 (1961), p. 365. (Received February 17, 1965.)

65T-204. WITHDRAWN.

65T-205. S. A. COOK, Conant 25, Harvard University, Cambridge, Massachusetts 02138. The decidability of the derivability problem for one-normal systems.

A one-normal system is a Post production system on a finite alphabet $\{s_1, s_2, \ldots, s_n\}$ with productions $s_i P \rightarrow PE_{ij}$, where i ranges over a subset of $\{1,2,\ldots,n\}$ and, for fixed i, j takes on the values $1,2,\ldots,n_i$. The following derivability problem is shown to be decidable for each such system: Given two words P and Q, can Q be derived from P by successive applications of the production rules? Hao Wang has proved decidability for the case in which the system is monogenic (i.e. each $n_i = 1$) (Math. Ann. 152 (1963), 65-74) and suggested the more general problem to the author. (Received February 17, 1965.)

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Advance Registration, Dormitory Housing Reservation and Entertainment Ticket Form

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Please fill out the form below for Advance Registration, Dormitory Housing Reservation and tickets for the three recreational events listed in the Preliminary Announcement. Return the form with your payment **no later than August 1, 1965** to:

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