

Medical Imaging

MVA 2023-2024

<http://www-sop.inria.fr/teams/asclepios/cours/MVA/>

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Analysis in the space of Covariance Matrices



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Medical Image Processing – MVA 2023-2024

Tuesday afternoon

Course notes : <http://www-sop.inria.fr/teams/asclepios/cours/MVA/>

- Tue Oct 3, ENSPS 2E30, Introduction to Medical Image Acquisition and Image Filtering, [HD]
- Tue Oct 10, ENSPS 3E34, Medical Image Registration [XP]
- Tue Oct 17, ENSPS 2E30, Riemannian Geometry & Statistics [XP]
- Tue Oct 24, ENSPS 1B18, Basis of Image Segmentation [HD]
- Tue Nov 7, ENSPS 2E30, Image Segmentation based on Clustering and Markov Random Fields [HD]
- Tue Nov 14, ENSPS 3E34, Shape constrained image segmentation and Biophysical Modeling [HD]
- Tue Nov 21, ENSPS 1N82, Analysis in the space of Covariance Matrices [XP]
- Tue Nov 28, ENSPS 2E30, Diffeomorphic Registration and computational anatomy [XP]
- Tu Dec 5, VISI Exam [HD, XP]

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Course overview

Computing with tensor images

- Diffusion tensor imaging
- Statistical computing on manifolds
- An affine invariant Riemannian metric
- Interpolation, filtering, diffusion PDEs on Tensors
- Other metrics

Applications of manifold valued image processing

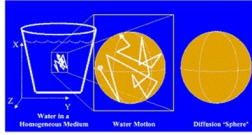
- Diffusion tensor imaging
- Morphometry of sulcal lines on the brain

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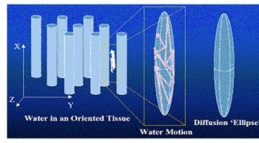
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Introduction to diffusion tensor imaging (DTI)

Isotropic/Anisotropic Diffusion



Anisotropic diffusion
($\lambda_1 > \lambda_2 > \lambda_3$)



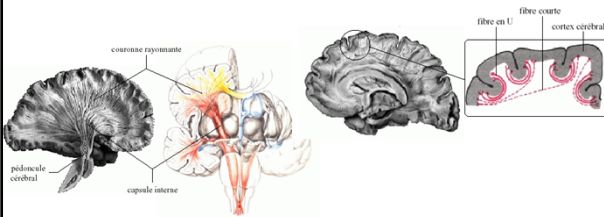
Introduction to diffusion tensor imaging (DTI)

Classical MRI:

- white matter, grey matter, CSF

Diffusion MRI:

- MR technique born in the mid-80ies (Basser, LeBihan).
- in vivo imaging of the white matter architecture
- set of nervous fibers (axons): information highways of the brain!



The Tenseur model

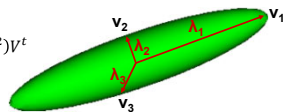
Mathematical definition:

- A multilinear application (generalization of matrices in linear algebra).

In medical imaging: real positive definite symmetric matrix

- All eigenvalues λ_i are real and positive (>0)
- Quadratic form = ellipsoid ($x^t \Sigma x = 1$)

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} = V \text{Diag}(\lambda^2) V^t$$



Stejskal & Tanner equation

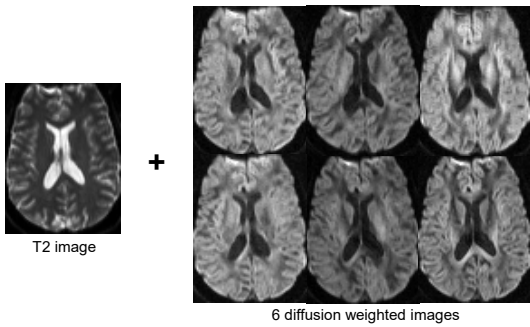
Signal attenuation related to tensor using:

$$S_i = S_0 \exp(-b \cdot \vec{g}_i^t \cdot D \cdot \vec{g}_i)$$

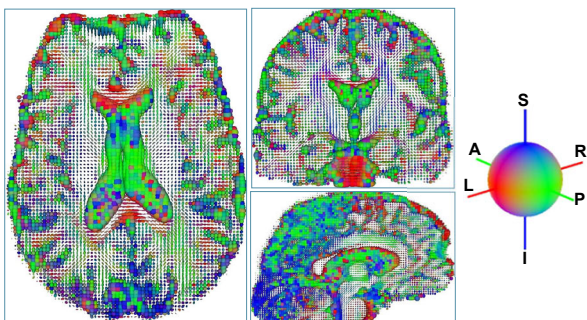
- S_i : diffusion weighted images;
- S_0 : T2 image;
- \vec{g}_i : spatial direction of the diffusion gradient;
- b : b-value (related to physical parameters of the acquisition, including field strength and diffusion time);
- D : diffusion tensor;

https://www.youtube.com/watch?v=J_aamnpRJE8

Anatomy of a diffusion MRI

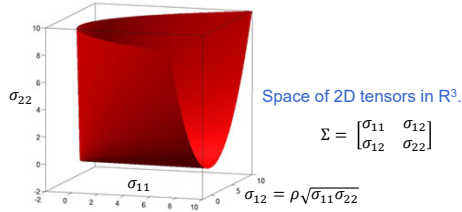


Visualization using ellipsoids



The Tensor Space is not a Vector Space

Tensors = Cone of positive definite matrices



Matrices with null eigenvalues are reachable in a finite time

- Null and negatives eigenvalues are not physical

Intrinsic computing on Manifold-valued images?

Diffusion Tensor Imaging

Covariance of the Brownian motion of water
-> Architecture of axonal fibers

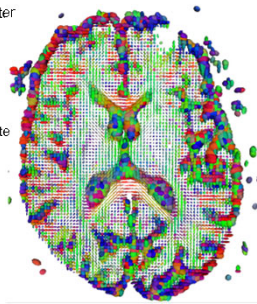
Symmetric positive definite matrices

- Cone: Convex operations are stable
 - mean, interpolation
 - Null eigenvalues are reachable in a finite time : not physical!
- More complex operations are not
 - PDEs, gradient descent...

SPD-valued image processing

- Clinical images: Very noisy data
- Robust estimation
- Filtering, regularization
- Interpolation / extrapolation

Intrinsic computing on Manifold-valued images?



Diffusion Tensor Field
(slice of a 3D volume)

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- Diffusion tensor imaging
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Bases of Algorithms in Riemannian Manifolds

Riemannian metric :

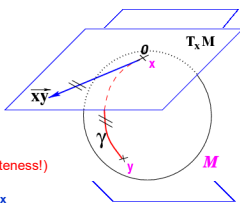
- Dot product on tangent space
- Speed, length of a curve
- Shortest path: Riemannian Distance
- Geodesics characterized by 2nd order diff eqs: **locally unique for initial point and speed**

Exponential map (Normal coord. syst.) :

- Geodesic shooting: $Exp_x(v) = \gamma_{(x,v)}(1)$
- Log: find vector to shoot right (**geodesic completeness!**)

Reformulate algorithms with exp_x and log_x

- Vector -> Bipoint (no more equivalent class)



Operator	Euclidean space	Riemannian manifold
Subtraction	$\bar{xy} = y - x$	$\bar{xy} = Log_x(y)$
Addition	$y = x + \bar{xy}$	$y = Exp_x(\bar{xy})$
Distance	$dist(x,y) = \ y - x\ $	$dist(x,y) = \ \bar{xy}\ _x$
Gradient descent	$x_{t+\epsilon} = x_t - \epsilon \nabla C(x_t)$	$x_{t+\epsilon} = Exp_{x_t}(-\epsilon \nabla C(x_t))$

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Fréchet expectation (1944)

Minimizing the variance $E[x] = \operatorname{argmin}_{y \in M} (E[dist(y, x)^2])$

Existence

- Finite variance at one point

Characterization as an exponential barycenter (P(C)=0)

$$\operatorname{grad}(\sigma_x^2(y)) = 0 \Rightarrow E[\bar{xx}] = \int_M \bar{xx} \cdot p_x(z) \cdot dM(z) = 0$$

Uniqueness Karcher 77 / Kendall 90 / Afsari 10 / Le 10

- Unique Karcher mean (thus Fréchet) if distribution has support in a regular geodesic ball with radius $r < r^* = \frac{1}{2} \min(inj(M), \pi/\sqrt{K})$ (K upper bound on sectional curvatures on M)

Other central primitives $E^\alpha[x] = \operatorname{argmin}_{y \in M} (E[dist(y, x)^\alpha])^\alpha$

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First Statistical Tools: Moments

Gauss-Newton Geodesic marching

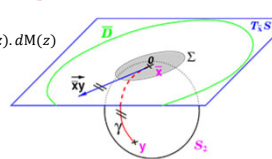
$$\bar{x}_{t+1} = \exp_{\bar{x}_t}(\tau v) \quad \text{with} \quad v = E[\bar{y}\bar{x}] = \frac{1}{n} \sum_{i=1}^n \operatorname{Log}_{\bar{x}_t}(x_i)$$

Covariance (PCA) [higher moments]

$$\Sigma_{xx} = E[(\bar{xx}) \cdot (\bar{xx})^T] = \int_M (\bar{zz}) \cdot (\bar{zz})^T \cdot p_x(z) \cdot dM(z)$$

Principal component analysis

- Tangent-PCA: principal modes of the covariance
- Principal Geodesic Analysis (PGA) [Fletcher 2004, Sommer 2014]



[Oller & Corcuera 95, Battacharya & Patrangenaru 2002, Pennec, JMIV06, NSIP'99]

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Computing on manifolds: a summary

The Riemannian metric easily gives

- Intrinsic measure and probability density functions
- Expectation of a function from M into R (variance, entropy)

Integral or sum in M: minimize an intrinsic functional

- Fréchet / Karcher mean: minimize the variance
- Filtering, convolution: weighted means
- Gaussian distribution: maximize the conditional entropy

The exponential chart corrects for the curvature at the reference point

- Gradient descent: geodesic walking
- Covariance and higher order moments
- Laplace Beltrami for free

[Pennec, NSIP'99, JMIV 2006, Pennec et al, IJCV 66(1) 2006, Arsigny, PhD 2006]

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GL(n)-Invariant Metrics on SPD matrices

Action of the Linear Group GL_n

$$A * \Sigma = A \Sigma A^T$$

Invariant metric $\langle W_1 | W_2 \rangle_{\Sigma} = \langle A W_1 A^T | A W_2 A^T \rangle_{A \Sigma A^T} \stackrel{def}{=} \langle \Sigma^{-1/2} * W_1, \Sigma^{-1/2} * W_2 \rangle_{Id}$

- Isotropy group at the identity: Rotations
- All rotationally invariant scalar products:

$$\langle W_1 | W_2 \rangle_{Id} \stackrel{def}{=} \text{Tr}(W_1^T W_2) + \beta \text{Tr}(W_1) \cdot \text{Tr}(W_2) \quad (\beta > -1/n)$$

- Geodesics at Id $\Gamma_{Id,W}(t) = \exp(tW)$
- Exponential map $Exp_{\Sigma}(\overline{\Sigma\Psi}) = \Sigma^{1/2} \exp(\Sigma^{-1/2} \overline{\Sigma\Psi} \Sigma^{-1/2}) \Sigma^{1/2}$
- Log map $\overline{\Sigma\Psi} = Log_{\Sigma}(\Psi) = \Sigma^{1/2} \log(\Sigma^{-1/2} \Psi \Sigma^{-1/2}) \Sigma^{1/2}$

Distance $dist(\Sigma, \Psi)^2 = \langle \overline{\Sigma\Psi} | \overline{\Sigma\Psi} \rangle_{\Sigma} = \left\| \log(\Sigma^{-1/2} \Psi \Sigma^{-1/2}) \right\|_{Id}^2$

GL(n)-Invariant Metrics on Tensors

$$\|W\|_{\Sigma}^2 = \text{Tr}(W \Sigma^{-1} W \Sigma^{-1}) + \beta \text{Tr}(W \Sigma^{-1})^2 \quad (\beta > -1/n)$$

Space of Gaussian distributions (Information geometry) ($\beta=0$)

- o Fisher information metric [Burbea & Rao J. Multivar Anal 12 1982, Skovgaard Scand J. Stat 11 1984, Calvo & Oller Stat & Dec. 9 1991]
- o Tensor segmentation [Lenglet RR04 & JMIV 25(3) 2006]

Affine-invariant metrics for DTI processing ($\beta=0$)

- o [Pennec, Fillard, Ayache, IJCV 66(1), Jan 2006 / INRIA RR-5255, 2004]
- o PGA on tensors [Fletcher & Joshi CVMIA04, SigPro 87(2) 2007]

Geometric means ($\beta=0$)

- o Covariance matrices in computer vision [Forstner TechReport 1999]
- o Math. properties [Moakher SIAM J. Matrix Anal App 26(3) 2004]
- o Geodesic Anisotropy [Batchelor MRM 53 2005]

Homogeneous Embedding ($\beta=-1/(n+1)$)

- o [Lovric & Min-Oo, J. Multivar Anal 74(1), 2000]

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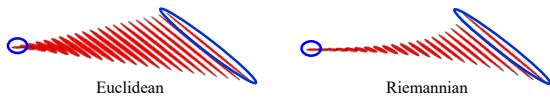
Applications of manifold valued image processing

- o Diffusion tensor imaging
- o Morphometry of sulcal lines on the brain

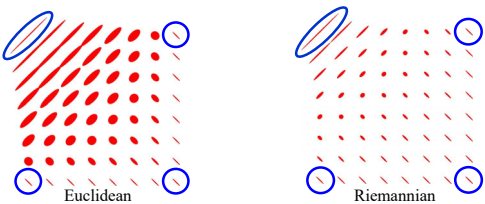
Tensor interpolation

Geodesic walking in 1D

$$\Sigma(t) = \exp_{\Sigma_1}(t \vec{\Sigma_1 \Sigma_2})$$

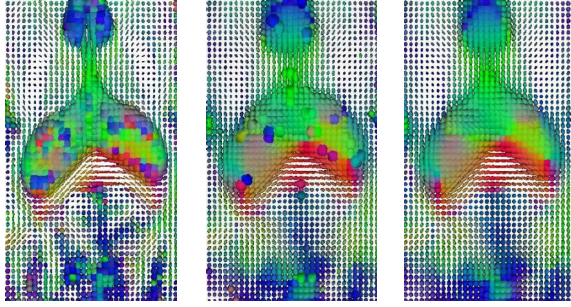


Bivariate weighted linear interpolation $\sum w_i(x) \text{dist}(\Sigma, \Sigma_i)^2$



Gaussian filtering: Gaussian weighted mean

$$\Sigma(x) = \min \sum_{i=1}^n G_{\sigma}(x - x_i) \text{ dist}(\Sigma, \Sigma_i)^2$$



Raw

Coefficients $\sigma=2$

Riemann $\sigma=2$

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PDE for filtering and diffusion

Harmonic regularization

$$C(\Sigma) = \int_{\Omega} \|\nabla \Sigma(x)\|_{\Sigma(x)}^2 dx$$

- Gradient = manifold Laplacian

$$\Delta \Sigma(x) = \sum_{i=x,y,z} \partial_i^2 \Sigma - \sum_{i=x,y,z} (\partial_i \Sigma) \Sigma^{(-1)} (\partial_i \Sigma) + \sum_u \frac{\Sigma(x)\Sigma(x+u)}{\|u\|^2} + O(\|u\|^2)$$

- Trivial intrinsic numerical schemes with exponential maps!
- Geodesic marching $\Sigma_{t+1}(x) = \exp_{\Sigma_t(x)}(-\epsilon \nabla C(\Sigma)(x))$

Anisotropic diffusion

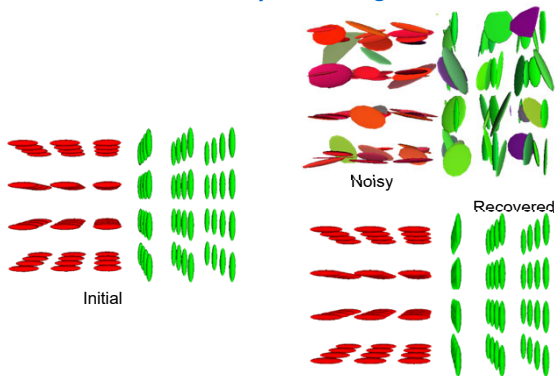
- Perona-Malik 90 / Gerig 92 $\Delta_w \Sigma(x) = \sum_u w(\|\partial_u \Sigma(x)\|_{\Sigma(x)}) \Delta_u \Sigma(x)$
- Robust functions $\text{Reg}(\Sigma) = \int \Phi(\|\nabla \Sigma(x)\|_{\Sigma(x)}^2) dx$

[Pennec, Fillard, Arsigny, IJCV 66(1), 2005, ISBI 2006]

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Anisotropic filtering



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Anisotropic filtering

$$\bar{\Delta}\Sigma(x) = \sum_u w(\|\partial_u \Sigma(x)\|_{\Sigma(x)}) \Delta_u \Sigma(x) \quad \text{with} \quad w(t) = \exp(-t^2/\kappa^2)$$

$$\Delta_u \Sigma(x) = \partial_u^2 \Sigma - (\partial_u \Sigma) \Sigma^{-1} (\partial_u \Sigma) \approx \overrightarrow{\Sigma(x)\Sigma(x+u)} / \|u\|^2$$

Raw
Riemann Gaussian
Riemann anisotropic

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Extrapolation by Diffusion

$$C(\Sigma) = \frac{1}{2} \int_{\Omega} \sum_{i=1}^n G_{\sigma}(x-x_i) \text{dist}(\Sigma(x), \Sigma_i)^2 dx + \frac{\lambda}{2} \int_{\Omega} \|\nabla \Sigma(x)\|_{\Sigma(x)}^2$$

$$\nabla C(\Sigma)(x) = - \sum_{i=1}^n G_{\sigma}(x-x_i) \overrightarrow{\Sigma(x)\Sigma_i} - \lambda(\Delta \Sigma)(x)$$

Original tensors
Diffusion $\lambda=0.01$
Diffusion $\lambda=\infty$

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Log Euclidean Metric on Tensors

Exp/Log: global diffeomorphism Tensors/sym. matrices

- Carry vector space structure from tg space to manifold
 - Log. product $\Sigma_1 \otimes \Sigma_2 \equiv \exp(\log(\Sigma_1) + \log(\Sigma_2))$
 - Log scalar product $\alpha \bullet \Sigma \equiv \exp(\alpha \log(\Sigma)) = \Sigma^\alpha$
 - Bi-invariant metric

$$\text{dist}(\Sigma_1, \Sigma_2)^2 \equiv \|\log(\Sigma_1) - \log(\Sigma_2)\|_F^2$$

Properties

- Invariance by the action of rotations + scale only
- Very simple algorithmic framework

[Arsigny, MICCAI 2005 & MRM 56(2), 2006]

Riemannian Frameworks on tensors

Affine-invariant Metric (Curved space – Hadamard)

- Dot product $\langle V | W \rangle_\Sigma = \langle AVA^T | AW A^T \rangle_{\text{vec}, \Sigma} = \langle \Sigma^{-1/2} V \Sigma^{-1/2} | \Sigma^{-1/2} W \Sigma^{-1/2} \rangle_{\text{vec}}$
- Geodesics $\text{Exp}_\Sigma(t, \Sigma \Psi) = \Sigma^{1/2} \exp(t \Sigma^{-1/2} \Sigma \Psi \Sigma^{-1/2}) \Sigma^{1/2}$
- Distance $\text{dist}(\Sigma, \Psi)^2 = \langle \Sigma \Psi | \Sigma \Psi \rangle_\Sigma = \|\log(\Sigma^{-1/2} \Psi \Sigma^{-1/2})\|_{\text{vec}}^2$

[Pennec, Fillard, Ayache, IJCV 66(1), 2006, Lenglet JMIV'06, etc]

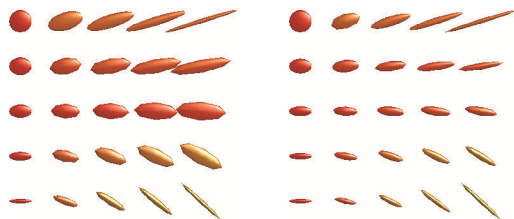
Log-Euclidean similarity invariant metric (vector space)

- Transport Euclidean structure through matrix exponential
- Dot product $\langle V | W \rangle_\Sigma = \langle \partial_V \log(\Sigma) | \partial_W \log(\Sigma) \rangle_{\text{vec}}$
- Geodesics $\text{Exp}_\Sigma(t, \Sigma \Psi) = \exp(\log(\Sigma) + t \partial_{\Sigma \Psi} \log(\Sigma))$
- Distance $\text{dist}(\Sigma_1, \Sigma_2)^2 \equiv \|\log(\Sigma_1) - \log(\Sigma_2)\|_F^2$

[Arsigny, Pennec, Fillard, Ayache, SIAM'06, MRM'06]

Log Euclidean vs Affine invariant

- Means are geometric (vs arithmetic for Euclidean)
- Log Euclidean slightly more anisotropic
- Speedup ratio: 7 (aniso. filtering) to >50 (interp.)



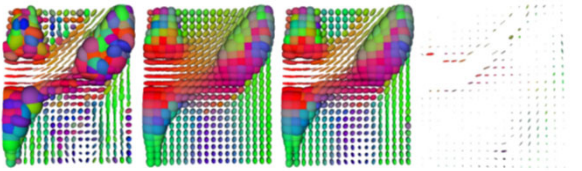
Euclidean

Affine-invariant

Log Euclidean vs Affine invariant

Real DTI images: anisotropic filtering

- Difference is not significant
- Speedup of a factor 7 for Log-Euclidean



Original Euclidean Log-Euclidean Diff. LE/affine (x100)

Some other metrics on tensors

Riemannian metrics

- Cholesky [Wang Vemuri et al, IPMI'03, TMI 23(8) 2004]
- Size and shape space [Dryden, Koloydenko & Zhou, 2008]
- Power Euclidean [Dryden & Pennec, arXiv 2011]
- Mixed-Power-Euclidean [Thanwerdas 2019]
- Alpha-Procrustes [HaQuang 2019]
- Polar-Affine [Su12], Bogoliubov-Kubo-Mori [Michor 2000]

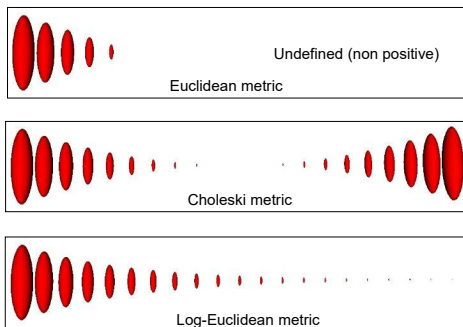
Non Riemannian distances

- J-Divergence [Wang & Vemuri, TMI 24(10), 2005]
- Geodesic Loxodromes [Kindlmann et al. MICCAI 2007]

4th order tensors

- [Gosh, Descoteau & Deriche MICCAI'08]

Geodesic shooting in tensors spaces



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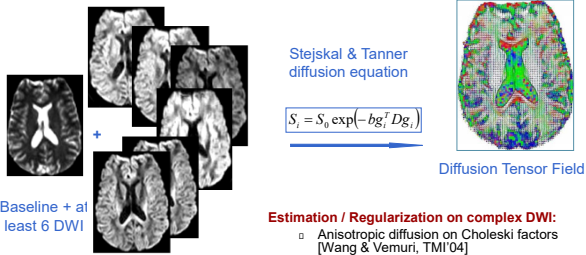
Applications of manifold valued image processing

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DTI Estimation from DWI



Stejskal & Tanner diffusion equation

$$S_i = S_0 \exp(-bg_i^T Dg_i)$$

Diffusion Tensor Field

Noise is Gaussian in complex DW signal

- Rician = amplitude of complex Gaussian
- LSQ on Rician noise = bias for low SNRs [Sijbers, TMI 1998]

Estimation with a Rician noise

- Smoothing DWI before estimation [Basu & Fletcher, MICCAI 2006]
- ML (MMSE) [Aja-Fernández et al, TMI 2008]
- MAP with log-Euclidean prior [Fillard et al., ISBI 2006, TMI 2007]

- ### Estimation / Regularization on complex DWI:
- Anisotropic diffusion on Choleski factors [Wang & Vemuri, TMI'04]

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DTI Estimation from DWI

Maximum Likelihood Estimation

$$S_i = S_0 \exp(-b \mathbf{g}_i^T \Sigma \mathbf{g}_i) + \text{Noise}$$

- ML with Log-Gaussian noise:

- linear system on the log-Images $\arg \min_D \sum_i (\log(S_i / S_0) + b \mathbf{g}_i^T D \mathbf{g}_i)^2$

- ML with Gaussian noise on the MRIs:

- non-linear optimization $\arg \min_D \sum_i (S_i - S_0 \exp(-b \mathbf{g}_i^T D \mathbf{g}_i))^2$

Actual MRI Noise

- Gaussian on the complex signal
- Rician on the amplitude

$$\hat{S}_i = \sqrt{(S_i + N_1(0, \sigma))^2 + N_2(0, \sigma)^2}$$

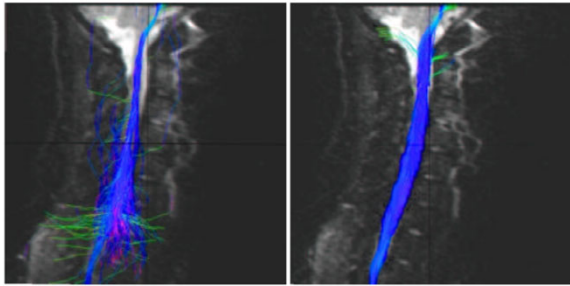
- This leads to a bias for low SNRs [Sijbers, TMI 1998]

$$E[\hat{S}_i] \approx E[S_i] + \frac{\sigma^2}{2S_i}$$

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Clinical DTI of the spinal cord: fiber tracking



Standard

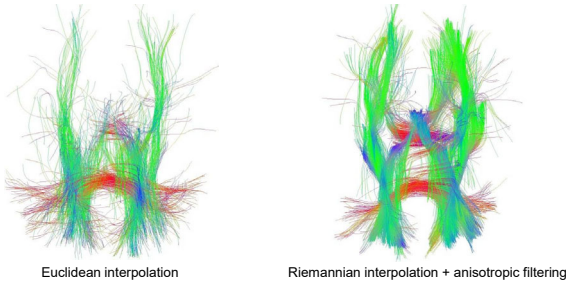
MAP Rician

[Fillard, Arsigny, Pennec, Ayache ISBI'06, TMI 26(11) 2007]

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Impact on fibers tracking



Euclidean interpolation

Riemannian interpolation + anisotropic filtering

[Fillard, Toussaint et al, MedINRIA: DTI Processing and Visualization Software, 2006]

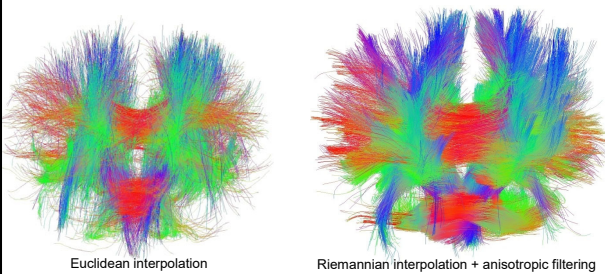
From images to anatomy

- Classify fibers into tracts (anatomo-functional architecture)?
- Compare fiber tracts between subjects?

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Impact on fibers tracking



Euclidean interpolation

Riemannian interpolation + anisotropic filtering

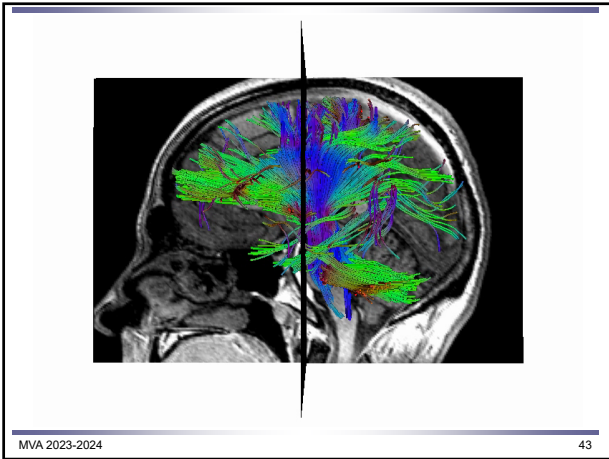
[Fillard, Toussaint et al, MedINRIA: DTI Processing and Visualization Software, 2006]

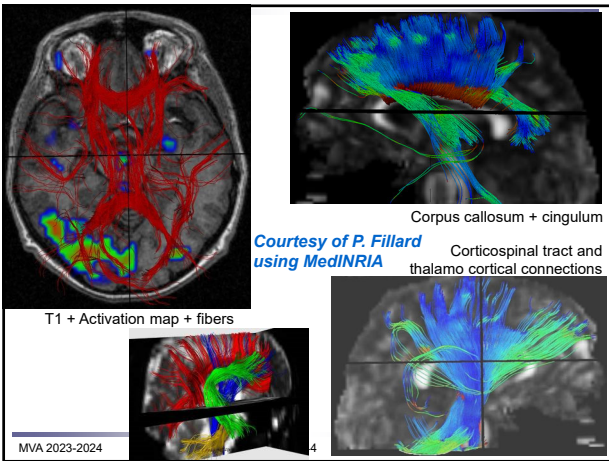
From images to anatomy

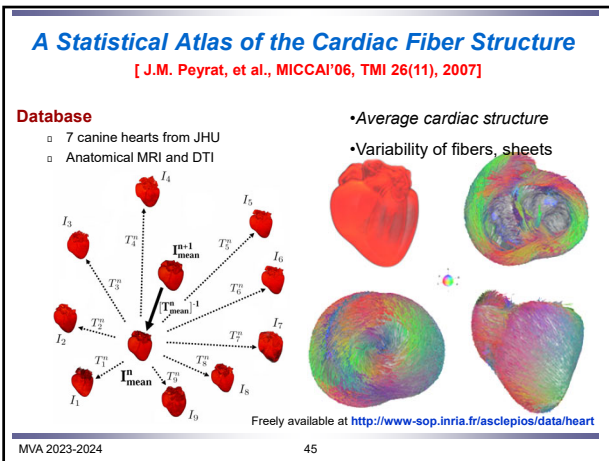
- Classify fibers into tracts (anatomo-functional architecture)?
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A Statistical Atlas of the Cardiac Fiber Structure

[J.M. Peyrat, et al., MICCAI'06, TMI 26(11), 2007]

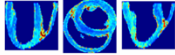
Database

- 7 canine hearts from JHU
- Anatomical MRI and DTI

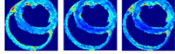
Method

- Normalization based on aMRIs
- Log-Euclidean statistics of Tensors

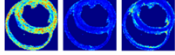
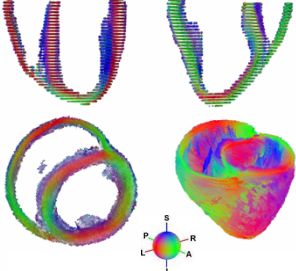
Norm covariance



Eigenvalues covariance (1st, 2nd, 3rd)



Eigenvectors orientation covariance (around 1st, 2nd, 3rd)

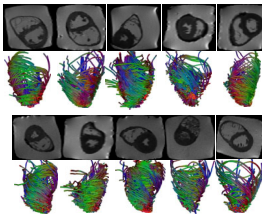



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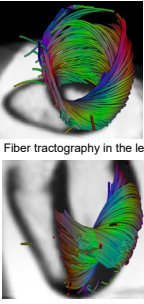
Diffusion model of the human heart

10 human ex vivo hearts (CREATIS-LRMN, Lyon, France)

- Classified as healthy (controlling weight, septal thickness, pathology examination)
- Acquired on 1.5T MR Avento Siemens
 - bipolar echo planar imaging, 4 repetitions, 12 gradients
- Volume size: 128x128x52, 2 mm resolution



Fiber tractography in the left ventricle



Helix angle highly correlated to the transmural distance

[H. Lombaert Statistical Analysis of the Human Cardiac Fiber Architecture from DT-MRI, ISMRM 2011, FIMH 2011]

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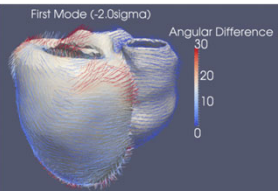
A Statistical Atlas of the Cardiac Fiber Structure

10 human ex vivo hearts (CREATIS-LRMN, France)

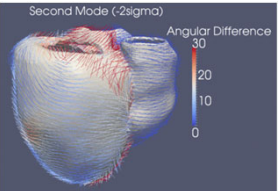
- Classified as healthy (controlling weight, septal thickness, pathology examination)
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 - bipolar echo planar imaging, 4 repetitions, 12 gradients
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[R. Mollero, M.M Rohé, et al, FIMH 2015]

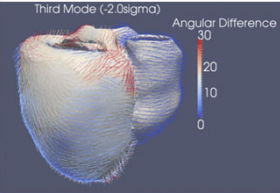
First Mode (-2.0sigma) Angular Difference



Second Mode (-2.0sigma) Angular Difference



Third Mode (-2.0sigma) Angular Difference



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Course overview

Computing with tensor images

- Diffusion tensor imaging
- Statistical computing on manifolds
- An affine invariant Riemannian metric
- Interpolation, filtering, diffusion PDEs on Tensors
- Other metrics

Applications of manifold valued image processing

- Diffusion tensor imaging
- Morphometry of sulcal lines on the brain

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Morphometry of the Cortex from Sulcal Lines

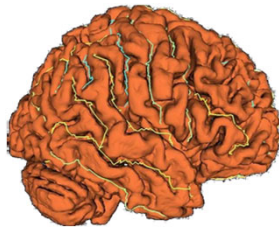
Associated team Brain-Atlas (2001-2006)

- LONI (UCLA) : P. Thompson et al.
- ASCLEPIOS (INRIA): V. Arsigny, N. Ayache, P. Fillard, X. Pennec



Neuroanatomical reference:

- 72 sulcal lines manually extracted and labeled
- 700 subjects



Alternative

- Automatic extraction
- JF. Mangin, D. Rivière, 2003, SHFJ-CEA

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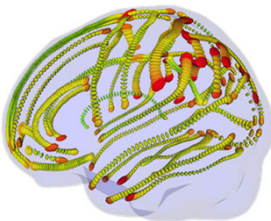
50

Morphometry of the Cortex from Sulcal Lines

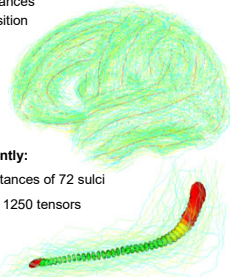
Computation of the mean sulci: Alternate minimization of global variance

- Dynamic programming to match the mean to instances
- Gradient descent to compute the mean curve position

Extraction of the covariance tensors



Currently:
80 instances of 72 sulci
About 1250 tensors



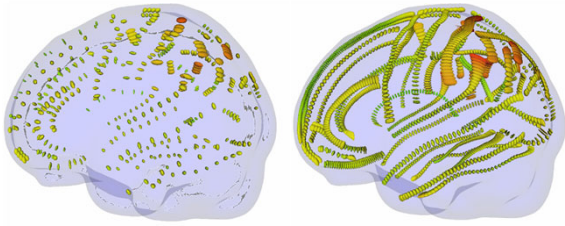
Covariance Tensors along Sylvius Fissure

Collaborative work between Asclepios (INRIA) and LONI (UCLA) P. Thompson
[Fillard et al IPMI05, LNCS 3565:27-38, NeuroImage 34(2):639-650, January 2007]

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Compressed Tensor Representation



Representative Tensors (250)

Reconstructed Tensors (1250)
(Riemannian Interpolation)

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Extrapolation by Diffusion

Minimize

$$C(\Sigma) = \frac{1}{2} \int_{\Omega} \sum_{i=1}^n G_{\sigma}(x - x_i) \text{dist}(\Sigma(x), \Sigma_i)^2 dx + \frac{\lambda}{2} \int_{\Omega} \|\nabla \Sigma(x)\|_{\Sigma(x)}^2 dx$$

Evolution Equation : $\Sigma_{t+1}(x) = \exp_{\Sigma_t(x)}(-\varepsilon \nabla C(\Sigma)(x))$

$$\nabla C(\Sigma)(x) = - \sum_{i=1}^n G_{\sigma}(x - x_i) \overrightarrow{\Sigma(x)\Sigma_i} - \lambda(\Delta \Sigma)(x)$$

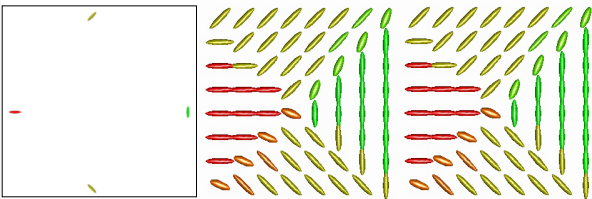
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Extrapolation by Diffusion

$$C(\Sigma) = \frac{1}{2} \int_{\Omega} \sum_{i=1}^n G_{\sigma}(x - x_i) \text{dist}(\Sigma(x), \Sigma_i)^2 dx + \frac{\lambda}{2} \int_{\Omega} \|\nabla \Sigma(x)\|_{\Sigma(x)}^2 dx$$

$$\nabla C(\Sigma)(x) = - \sum_{i=1}^n G_{\sigma}(x - x_i) \overrightarrow{\Sigma(x)\Sigma_i} - \lambda(\Delta \Sigma)(x)$$



Original tensors

Diffusion $\lambda=0.01$

Diffusion $\lambda=\infty$

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Full Brain extrapolation of the variability

$$C(\Sigma) = \frac{1}{2} \int_{\Omega} \sum_{i=1}^d G_{\sigma}(x-x_i) \text{dist}(\Sigma(x), \Sigma_i)^2 dx + \frac{\lambda}{2} \int_{\Omega} \|\nabla \Sigma\|_{\Sigma(x)}^2 dx$$

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Comparison with cortical surface variability

P. Thompson et al, HMIP, 2000
Average of 15 normal controls by non-linear registration of surfaces

P. Fillard et al, IPMI 05
Extrapolation of our model estimated from 98 subjects with 72 sulci.

Consistent low variability in phylogenetical older areas

- (a) superior frontal gyrus

Consistent high variability in highly specialized and lateralized areas

- (b) temporo-parietal cortex

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Quantitative Evaluation: Leave One Sulcus Out

- Remove data from one sulcus
- Reconstruct from extrapolation of others

Sylvian Fissure
Superior Temporal
Inferior Temporal

Original tensors **Leave one out reconstructions**

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