Applications to p-adic Hodge theory

$K|\mathbb{Q}_p$ Complete discrete with perfect residue field k_K

$K|K_0 = W(k_K)_{\mathbb{Q}}$ $\overline{K}$ alg. closure $C = \widehat{\overline{K}}$

$G_K = \mathrm{Gal}(\overline{K}|K)$

Set $F = R(C) =$ Complete alg. closed field of char. p
($\simeq \widehat{\overline{k((\pi))}}$ by the fields of norms theory)

$$G_K \hookrightarrow \mathrm{Aut}(F)$$

$X =$ Curve associated to F (and $E = \mathbb{Q}_p$)

$$X \circlearrowleft G_K$$

If $t \in P_1$ is a period of $\mu_{p^\infty}/\mathcal{O}_{\overline{K}}$ it defines a point $\infty \in |X|$

[$X \circlearrowleft G_K$ stabilizes ∞ since $\mathbb{Q}_p.t \subset P_1$ is stable under G_K]

<u>Fact</u>: $\infty \in |X|$ is the only closed point whose G_K-orbit is finite.

$$\left.\begin{array}{l} B_{dR}^+ = \widehat{\mathcal{O}_{X,\infty}} \circlearrowleft G_K \\ B_e = \Gamma(X \setminus \{\infty\}, \mathcal{O}_X) \circlearrowleft G_K \end{array}\right\}$$
action induced by the one on X
$\parallel$
usual action

Def: * $\mathrm{Rep}_{B_{dR}^+}(G_K)$ = free of finite rank B_{dR}^+-modules + Continuous semi-linear action of G_K
↳ don't want to enter into the details

* $\mathrm{Rep}_{B_e}(G_K)$ = free of finite rank B_e-modules + semi-linear Continuous action of G_K

* $\mathcal{F}ib_X^{G_K}$ = G_K-equivariant vector bundles (+ continuity condition on the action of G_K)

Thus $\mathcal{F}ib_{X\setminus\{\infty\}}^{G_K} \xrightarrow{\sim} \mathrm{Rep}_{B_e}(G_K)$
$\mathcal{E} \longmapsto H^0(X\setminus\{\infty\}, \mathcal{E})$

Berger's category of B-pairs

and $\mathcal{F}ib_X^{G_K} \xrightarrow{\sim} \{(M, W, u) \,/\, M \in \mathrm{Rep}_{B_e}(G_K), W \in \mathrm{Rep}_{B_{dR}^+}(G_K),$
$u : M \otimes_{B_e} B_{dR} \xrightarrow{\sim} W[\frac{1}{t}]\}$
$\mathcal{E} \longmapsto (\Gamma(X\setminus\{\infty\}, \mathcal{E}), \widehat{\mathcal{E}}_\infty, \mathrm{can})$
iso of B_{dR}-rep. of G_K

Twisting

* If $\underline{\mathcal{E}} \in \mathrm{Bun}_X^{G_K}$ with underlying v. b. $\mathcal{E} \in \mathrm{Bun}_X$ then $\mathrm{Aut}(\mathcal{E}) \circlearrowleft G_K$

$Z^1(G_K, \mathrm{Aut}(\mathcal{E})) \xrightarrow{\sim} \{$set of G_K-equivariant structures on $\mathcal{E}\}$
$c \longmapsto \underline{\mathcal{E}} \overset{G_K}{\wedge} c$

$H^1(G_K, \mathrm{Aut}(\mathcal{E})) \xrightarrow{\sim} \{$iso. classes of $\underline{\mathcal{F}} \in \mathrm{Bun}_X^{G_K}$ s.t. $\mathcal{F} \simeq \mathcal{E}\}$.

Galois action and semi-stability

* <u>Prop:</u> The HN filtration of a G_K-equivariant v.b. is invariant under Galois and is thus a filtration in $\mathrm{Bun}_X^{G_K}$.

$\to$ immediate consequence of the canonicity of the HN filtration
$\left(+ \forall \sigma \in G_K,\ \mu(\sigma^* \mathcal{E}) = \mu(\mathcal{E})\right)$

<u>Ex:</u> $\underline{\mathcal{E}} \in \mathrm{Bun}_X^{G_K}$, $\mathcal{E}$ s.s. $\iff \forall \mathcal{F} \subset \mathcal{E}$ invariant under G_K, $\mu(\mathcal{F}) \leq \mu(\mathcal{E})$.

* $\lambda \in \mathbb{Q}$, $\mathcal{O}_X(\lambda)$ has a canonical G_K-equivariant structure $\underline{\mathcal{O}}_X(\lambda)$

One has $\mathrm{End}(\mathcal{O}_X(\lambda)) = D_\lambda^{\mathrm{opp}}$ where D_λ = division alg. w.t. invariant λ

G_K acts via $G_K \longrightarrow \mathrm{Gal}(\overline{\mathbb{F}}_p | \mathbb{F}_p) \longrightarrow \mathrm{Aut}(D_\lambda)$

$\mathrm{Frob} \longmapsto [x \mapsto \pi x \pi^{-1}]$

if $\lambda = \frac{d}{h}$, $(d,h) = 1$, π = unif. element of D_λ.

Set ${}^{\lambda}\mathrm{Bun}_X^{G_K} = \{ \underline{\mathcal{E}} \in \mathrm{Bun}_X^{G_K} \mid \mathcal{E} \text{ is s.s. of slope } \lambda \}$.

<u>Prop:</u> ~~[illegible]~~ $\mathrm{Rep}_{D_\lambda^{\mathrm{opp}}}(G_K) \xrightarrow{\sim} {}^{\lambda}\mathrm{Bun}_X^{G_K}$

$V \longmapsto V \otimes_{D_\lambda} \underline{\mathcal{O}}_X(\lambda)$

$\uparrow$ V = G_K-eq. v.b. on $\mathrm{Spec}(\mathbb{Q}_p)$, pull it back to X.

$$\boxed{\text{Ex: } \mathrm{Rep}_{\mathbb{Q}_p}(G_K) \simeq {}^{0}\mathrm{Bun}_X^{G_K}}$$

$V \longmapsto V \otimes \underline{\mathcal{O}}_X$

$H^0(X, \underline{\mathcal{E}}) \longleftarrow \underline{\mathcal{E}}$

$\to$ Consequence of the classification theorem + twisting procedure.

G_K- equivariant vector bundles on $X \setminus \{\infty\}$

<u>Th:</u> $\mathrm{Bun}^{G_K}_{X\setminus\{\infty\}} = \mathrm{Rep}_{B_e}(G_K)$ is an abelian (Tannakian) category.

<u>Proof</u>: If $\mathcal{F}$ is a G_K- equivariant coherent sheaf on $X \setminus \{\infty\}$ then the finite set $\mathrm{Supp}(\mathcal{F}_{\mathrm{tor}}) \subset |X|$ is stable under G_K. But $\infty \in |X|$ is the only point of $|X|$ whose G_K-orbit is finite.

$\Rightarrow \mathrm{Coh}^{G_K}_{X\setminus\{\infty\}} = \mathrm{Bun}^{G_K}_{X\setminus\{\infty\}}$ $\square$

$\rightarrow$ We are going to see this ~~[illegible]~~ Tannakian category generalizes the Tannakian category of isocrystals.

<u>Recall</u>:
$$\varphi\text{-}\mathrm{Mod}_{K_0} \longrightarrow \mathrm{Bun}^{G_K}_X$$
$$(D,\varphi) \longmapsto \mathcal{E}(D,\varphi) = \Big(\bigoplus_{d\geq 0} (D\otimes B)^{\varphi = p^d}\Big)^{\sim}$$
$$\circlearrowleft G_K$$

<u>Prop</u>:
$$\varphi\text{-}\mathrm{Mod}_{K_0} \underset{D_{\mathrm{cris}}\ \leftarrow\ \text{left}}{\overset{\text{right}\ \swarrow\ V_{\mathrm{cris}}}{\rightleftarrows}} \mathrm{Rep}_{B_e}(G_K)$$

defined by $V_{\mathrm{cris}}(D,\varphi) = (D \otimes_{K_0} B[\tfrac{1}{t}])^{\varphi = \mathrm{Id}}$

$D_{\mathrm{cris}}(M) = \Big((M \otimes_{B_e} B[\tfrac{1}{t}])^{G_K}, \mathrm{Id}\otimes\varphi\Big)$

are adjoint functors. Moreover V_{cris} is fully faithfull:

$$\mathrm{Id} \xrightarrow{\sim} D_{\mathrm{cris}} \circ V_{\mathrm{cris}}$$

and $\dim_{K_0} D_{\mathrm{cris}}(M) \leq \mathrm{rg}_{B_e} M$ with equality iff M is in the essential image of V_{cris}.

Moreover $\big[V_{\mathrm{cris}} : \{\text{sub-isocristals of } (D,\varphi)\} \xrightarrow{\sim} \{\text{sub-representations of } V_{\mathrm{cris}}(D,\varphi)\}\big]$.

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$\to$ Consequence of $\boxed{B\left[\frac{1}{t}\right]^{G_K} = K_0}$

crystalline B_e-rep. of G_K

Def: $\mathrm{Rep}^{\mathrm{cris}}_{B_e}(G_K) \subset \mathrm{Rep}_{B_e}(G_K)$ is the subabelian category that is the essential image of V_{cris}

$= \{ M \in \mathrm{Rep}_{B_e}(G_K) \mid \dim_{K_0} D_{\mathrm{cris}}(M) = \mathrm{rk}_{B_e} M \}$

Thus: $\boxed{V_{\mathrm{cris}} : \varphi\text{-}\mathrm{Mod}_{K_0} \xrightarrow{\sim} \mathrm{Rep}^{\mathrm{cris}}_{B_e}(G_K)}$

Thus $\exists$ Tannakian category that generalizes isocrystals: $\mathrm{Rep}_{B_e}(G_K)$.

Crystalline vector bundles

Def: $\underline{\mathcal{E}} \in \mathrm{Bun}_X^{G_K}$ is crystalline $\iff$ $\underline{\mathcal{E}}_{|X \setminus \{\infty\}}$ is.

Ex: The composite

$$\mathrm{Rep}_{\mathbb{Q}_p} G_K \xrightarrow{\sim} {}^{0}\mathrm{Bun}_X^{G_K} \xrightarrow{\text{restriction}} \mathrm{Bun}_{X \setminus \{\infty\}}^{G_K} \xrightarrow{D_{\mathrm{cris}}} \varphi\text{-}\mathrm{Mod}_{K_0}$$

is Fontaine's functor V_{cris}. Thus for $V \in \mathrm{Rep}_{\mathbb{Q}_p} G_K$,

$V \otimes \mathcal{O}_X$ is crystalline $\iff$ V is crystalline.

Classification:

If $(D,\varphi) \in \varphi\text{-Mod}_{K_0}$ then $\widehat{\mathcal{E}(D,\varphi)_\infty} = D \otimes_{K_0} B^+_{dR} \in \mathrm{Rep}_{B^+_{dR}}(G_K)$.

Prop: Let W be a finite dimensional K-vector space. Then the application $\mathrm{Fil}^\bullet W \longmapsto \mathrm{Fil}^0(W \otimes_K B_{dR})$ induces a bijection between finite decreasing filtrations of W and B^+_{dR}-lattices in $W \otimes_K B_{dR}$ stable under G_K.

Def: $(D,\varphi,\mathrm{Fil}^\bullet D_K) \in \varphi\text{-ModFil}_{K/K_0}$. Set $\mathcal{E}(D,\varphi,\mathrm{Fil}^\bullet D_K) \in \mathrm{Bun}_X^{G_K}$ to be the G_K-equivariant modification of $\mathcal{E}(D,\varphi)$ at ∞ s.t. $\widehat{\mathcal{E}(D,\varphi,\mathrm{Fil}^\bullet D_K)_\infty} = \mathrm{Fil}^0(D \otimes_{K_0} B_{dR})$

Preceding proposition

$$\Rightarrow \boxed{\mathcal{E}(-) : \varphi\text{-ModFil}_{K/K_0} \xrightarrow{\sim} \mathrm{Bun}_X^{G_K,\mathrm{cris}}}$$

Moreover $\left[H^0(X, \mathcal{E}(D,\varphi,\mathrm{Fil}^\bullet D_K)) = V_{\mathrm{cris}}(D,\varphi,\mathrm{Fil}^\bullet D_K)\right]$

usual Fontaine's V_{cris}

$$* \quad \deg(\mathcal{E}(D,\varphi,\mathrm{Fil}^\bullet D_K)) = \underbrace{\deg \mathcal{E}(D,\varphi)}_{-t_N(D,\varphi)} + \underbrace{\left[\mathrm{Fil}^0 D\otimes B_{dR} : D\otimes B^+_{dR}\right]}_{t_H(\mathrm{Fil}^\bullet D_K)}$$

$$= \deg(D,\varphi,\mathrm{Fil}^\bullet D_K)$$

<u>Prop</u>: Let $A \in \varphi\text{-ModFil}_{K/K_0}$ and $0 = A_0 \subsetneq \dots \subsetneq A_r = A$ be its H.N. filtration. Then $0 = \underline{\mathcal{E}}(A_0) \subsetneq \dots \subsetneq \underline{\mathcal{E}}(A_r) = \underline{\mathcal{E}}(A)$ is the H.N. filtration of $\underline{\mathcal{E}}(A)$.

<u>Proof</u>: $\mu(\underline{\mathcal{E}}(B)) = \mu(B)$ for $B \in \varphi\text{-ModFil}_{K/K_0}$.

Moreover, the HN filtration of $\underline{\mathcal{E}}(A)$ is G_K-invariant. But:

locally direct factor

$$
\begin{array}{c}
\text{Sub } G_K\text{-invariant v.b. of } \underline{\mathcal{E}}(A) \\
\wr\!\wr \\
\text{Sub } G_K\text{-invariant v.b. of } \underline{\mathcal{E}}(A)_{|X \setminus \{\infty\}} \\
\uparrow\!\wr \; \mathcal{V}_{cris} \\
\text{Sub filtered } \varphi\text{-modules of } A
\end{array}
$$

$\Longrightarrow \square$

Thus $\forall \lambda \in \mathbb{Q}$

s.s. of slope λ

$$
{}^{\lambda}\left(\varphi\text{-ModFil}_{K/K_0}\right) \xrightarrow{\sim} {}^{\lambda}\mathrm{Bun}_X^{G_K, cris}
$$

In particular for $\lambda = 0$, weakly admissible $\Leftrightarrow$ s.s. of slope 0

$$
\boxed{
\begin{array}{ccccc}
\varphi\text{-ModFil}_{K/K_0}^{wa} & \xrightarrow{\sim} & {}^{0}\mathrm{Bun}_X^{G_K, cris} & = & \mathrm{Rep}_{\mathbb{Q}_p}^{cris}(G_K) \\
 & & V \otimes \mathcal{O}_X & \longleftarrow\!\shortmid & V
\end{array}
}
$$

Weakly admissible $\Leftrightarrow$ admissible:

If $\mathcal{E} \in \mathrm{Bun}_X$ one checks using the classification theorem:

$$\dim_{\mathbb{Q}_p} H^0(X, \mathcal{E}) = \begin{cases} +\infty \\ \leq \mathrm{rg}\, \mathcal{E} \end{cases}$$

and $\dim_{\mathbb{Q}_p} H^0(X, \mathcal{E}) = \mathrm{rg}\, \mathcal{E} \Leftrightarrow \mathcal{E}$ is trivial that is to say s.s. of slope 0.

Thus, if $A \in \varphi\text{-}\mathrm{ModFil}_{K/K_0}$,

$$\begin{array}{c} A \text{ admissible} \\ \Updownarrow \text{def.} \\ \dim_{\mathbb{Q}_p} V_{\mathrm{cris}}(A) = \mathrm{rk}\, A \\ \Updownarrow \\ \dim_{\mathbb{Q}_p} H^0(X, \mathcal{E}(A)) = \mathrm{rk}\, \mathcal{E}(A) \\ \Updownarrow \\ \mathcal{E}(A) \text{ s.s. of slope } 0 \\ \Updownarrow \\ A \text{ weakly admissible.} \end{array}$$

Rem: In fact one has the stronger statement: let $A \in \varphi\text{-}\mathrm{ModFil}_{K/K_0}$ then:

* If $t_H(A) > t_N(A)$ $\quad \dim_{\mathbb{Q}_p} V_{\mathrm{cris}}(A) = +\infty$
* If $t_H(A) < t_N(A)$ $\quad \dim_{\mathbb{Q}_p} V_{\mathrm{cris}}(A) = \begin{cases} +\infty \\ < \mathrm{rk}(A) \end{cases}$
* If $t_H(A) = t_N(A)$ $\quad \dim_{\mathbb{Q}_p} V_{\mathrm{cris}}(A) = \begin{cases} +\infty \\ \mathrm{rk}(A) \end{cases}$

[Thus A admissible $\Leftrightarrow$ $t_H(A) = t_N(A)$ and $\dim_{\mathbb{Q}_p} V_{\mathrm{cris}}(A) < +\infty$.]

($\Uparrow$ def Admissible)

$\rightarrow$ use the classification theorem.

Other results and perspectives

In the article: * a proof of de Rham $\Rightarrow$ potentially s.s.

More generally an equivalence of category

$$(\varphi, N)\text{-ModFil}_{K/K_0} \xrightarrow{\sim} \mathrm{Bun}_X^{G_K, dR} = \left\{ \underline{\mathcal{E}} \in \mathrm{Bun}_X^{G_K} \,/\, \underbrace{\widehat{\mathcal{E}}_\infty\left[\tfrac{1}{t}\right]}_{} \in \mathrm{Rep}_{B_{dR}}(G_K) \right\}$$

equivariant v.b. on the formal punctured disk $\mathrm{Spec}(B_{dR})$ is flat that is to say generated by its G_K-invariants

* For $\underline{\mathcal{E}} \in \mathrm{Bun}_X^{G_K}$ a definition of $H^i_{G_K}(X, \underline{\mathcal{E}}) = G_K$-equivariant cohomology of $\underline{\mathcal{E}}$ + spectral sequence

$$H^i(G_K, \underbrace{H^j(X, \underline{\mathcal{E}})}_{\substack{\text{Banach space + continuous} \\ \text{action of } G_K}}) \xRightarrow{\text{degenerates}} H^{i+j}_{G_K}(X, \underline{\mathcal{E}})$$

generalizes galois cohomology: for $V \in \mathrm{Rep}_{\mathbb{Q}_p}(G_K)$, $H^i_{G_K}(X, V \otimes \mathcal{O}_X) = H^i(G_K, V)$

+ generalized Bloch-Kato exponential...

Hope for a general Tate duality if $[K : \mathbb{Q}_p] < +\infty$:

$H^i_{G_K}(X, \underline{\xi}) = 0$ if $i \notin \{0,1,2\}$

$\dim_{\mathbb{Q}_\ell} H^i_{G_K}(X, \underline{\xi}) < +\infty$

$$H^i_{G_K}(X, \underline{\xi}) \times H^{2-i}(X, \underline{\xi}^\vee \otimes \mathbb{Q}_\ell(1)) \xrightarrow{\cup} H^2(G_K, \mathbb{Q}_\ell(1)) = \mathbb{Q}_\ell$$

perfect

$$+ \sum_i (-1)^i \dim H^i_{G_K}(X, \underline{\xi}) = [K:\mathbb{Q}_p] \cdot \mathrm{rg}\, \xi.$$

itten but not in the article : F perfect non-necessarily alg. closed

$\overline{F}|F$ algebraic closure

$X_F = \mathrm{Proj}\left(\bigoplus_{d \geq 0} B_F^{\varphi = p^d}\right)$ is a curve but if $t \in B_F^{\varphi = p} \setminus \{0\}$

$B_e = B_F\left[\frac{1}{t}\right]^{\varphi = \mathrm{Id}}$ is not a PID anymore in general, only Dedekind

Moreover if $x \in |X_F|$, $k(x)|\mathbb{Q}_p$ is a complete perfectoid extension

$F \hookrightarrow R(k(x))$ finite extension

and if $\deg(x) = [R(k(x)) : F] < +\infty$ then

$\forall f \in \mathbb{Q}_p(X)^\times, \deg(\mathrm{div} f) = 0$

$$\begin{array}{c} X_{\hat{\overline{F}}} \\ \pi \big\downarrow \circlearrowleft \mathrm{Gal}(\overline{F}|F) \\ X_F \end{array}$$

$\forall x \in |X_F|$, $\deg(x) = \# \pi^{-1}(x)$

Moreover if $y \in |X_{\hat{\overline{F}}}|$ either:

* $\mathrm{Gal}(\overline{F}|F).y$ is infinite $\Rightarrow \pi(y)$ is the generic point of X_F
* $\mathrm{Gal}(\overline{F}|F).y$ is finite $\Rightarrow \pi(y) \in |X_F|$ and $\deg \pi(y) = \#(\mathrm{Gal}(\overline{F}|F).y)$.

Descent of vector bundles: $\pi^*: \boxed{\mathrm{Bun}_{X_F} \xrightarrow{\sim} \mathrm{Bun}_{X_{\hat{\overline{F}}}}^{\mathrm{Gal}(\overline{F}|F)}}$

Ex: $\mathrm{Pic}^0(X_F) = \mathrm{Hom}(\mathrm{Gal}(\overline{F}|F), \mathbb{Z}_p^\times)$
$= \mathcal{C}\ell(B_e)$

(⚠: On X_F the H.N. filtration is not split: $H^1(X_F, \mathcal{O}_{X_F}) = H^1(\mathrm{Gal}(\overline{F}|F), \mathbb{Q}_p) = \mathrm{Hom}(\mathrm{Gal}(\overline{F}|F), \mathbb{Q}_p))$

Ex: $K|\mathbb{Q}_p$, $\overline{K}|L|K$ arithmetically profinite, $F = \mathcal{R}(\hat{L})$ = Completion of the perfection of the $\overbrace{\text{field of norms}}$ of $L|K \simeq k((\pi))^{\mathrm{perf}}$

Suppose $L|K$ is Galois, $\Gamma = \mathrm{Gal}(L|K)$. Then

$$\boxed{\mathrm{Bun}_{X_{\hat{\overline{F}}}}^{G_K} \simeq \mathrm{Bun}_{X_F}^{\Gamma}}$$

<u>Perspectives:</u> <u>Generalization of Kisin's theory:</u>

* $\mathcal{C}$ = Category of couples (M, φ)

free of finite rank $W(\mathcal{O}_F)[\frac{1}{\pi}]$-module

Semi-linear endomorphism s.t. Coker φ is killed by a finite degree element of $W(\mathcal{O}_F)$

$\sum_{n\geq 0} [x_n]\pi^n$ s.t. $\exists n, x_n \in \mathcal{O}_F^\times$

$\Updownarrow$

Coker φ is killed by $m_1 \dots m_r$ for some $m_i \in |Y|$, $i=1,\dots,r$.

Then Coker φ is a finite lenght $W(\mathcal{O}_F)[\frac{1}{\pi}]$-module

Set $\mathrm{div}(M/\varphi(M)) = \sum_{i=1}^{r} [m_i] \in \mathrm{Div}^+(|Y|)$ if $JH(M/\varphi(M)) = (m_i)_{i=1,\dots,r}$

with multiplicities.

* $\mathcal{M}$ = Category of $\mathcal{E}_0 \hookrightarrow \mathcal{E}_1$. Set $\mathrm{div}(\mathcal{E}_1/\mathcal{E}_0) = \sum_{i=1}^{r} [x_i] \in \mathrm{Div}^+(X)$

modification i.e. $\mathcal{E}_1/\mathcal{E}_0$ is torsion.

trivial vector bundle

vector bundle

if $JH(\mathcal{E}_1/\mathcal{E}_0) = (i_{x_i *} k(x_i))_{1\leq i\leq r}$

<u>Conjecture:</u> * There exists an essentially surjective functor

$$\mathcal{C} \longrightarrow \mathcal{M}$$

such that if $(M, \varphi) \longmapsto (\mathcal{E}_0 \hookrightarrow \mathcal{E}_1)$ then

$$\sum_{n\in\mathbb{Z}} \varphi^n(\mathrm{div}(M/\varphi(M))) = \mathrm{div}(\mathcal{E}_1/\mathcal{E}_0) \in \mathrm{Div}^+(X) = \mathrm{Div}^+(\cancel{Y}/\varphi^{\mathbb{Z}})$$

* For $(M,\varphi) \in \mathcal{C}$ set $M^{(\sigma)} = M \otimes_{W(O_F)[\frac{1}{p}],\varphi} W(O_F)[\frac{1}{p}]$

$\circlearrowright$

$\varphi = \varphi \otimes \varphi \qquad (M^{(\sigma)},\varphi) \in \mathcal{C}.$

There is a morphism $(M^{(\sigma)},\varphi) \longrightarrow (M,\varphi)$

$m \otimes 1 \longmapsto \varphi(m)$

<u>Conjecture:</u> The preceding functor induces an equivalence

$$\mathcal{I}^{-1}\mathcal{C} \xrightarrow{\sim} \mathcal{M}$$

where $\mathcal{I}$ = set of maps $(M^{(\sigma)},\varphi) \longrightarrow (M,\varphi)$.

<u>Conjecture:</u> $C = W(O_F)[\frac{1}{p}]/m$, $m \in |Y|$.

semi-linear endo. s.t. $mM \subset \varphi(M) \subset M$

p-divisible groups$/O_C \simeq \{(M,\varphi)\}$

maybe do trivial mod p.

free of finite rank $W(O_F)$-module

and if $(M,\varphi) \longleftrightarrow$ p-div. group H/O_C then $(\mathcal{E}_0 \hookrightarrow \mathcal{E}_1) = (V_p(H) \otimes O_X \hookrightarrow \mathcal{E}(\mathbb{D}(H)))$

$\updownarrow$

$(\mathcal{E}_0 \hookrightarrow \mathcal{E}_1)$

$$0 \to V_p(H) \otimes O_X \to \mathcal{E}(\mathbb{D}(H)) \to i_{\infty *}\mathrm{Lie}\, H \to 0$$