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Methodology for the Calibration of and Data Acquisition with a Six-Degree-of-Freedom Acceleration Measurement Device

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16. Abstract

This report describes a methodology for calibrating and gathering data with a six-degree-of-freedom acceleration measurement device that is intended to measure head acceleration of anthropomorphic dummies and human volunteers in automotive crash testing and head impact trauma studies. Error models (system equations) were developed for systems using six accelerometers in a coplanar (3-2-1) configuration, nine accelerometers in a coplanar (3-3-3) configuration and nine accelerometers in a non-coplanar (3-2-2-2) configuration and the accuracy and stability of these systems were compared. In particular, an array of linear accelerometers in the non-coplanar (3-2-2-2) configuration was found to be most stable (computationally) and is treated in great detail. A methodology for determining the system coefficients in the laboratory is described. A correction algorithm is also described that is used to correct data taken in the field using the coefficients obtained in the laboratory. The model was verified under various input and computational conditions. Results of parametric sensitivity analyses which included parameters such as system geometry, coordinate system location, data sample rate and accelerometer cross axis sensitivities are presented. Recommendations to optimize data collection and reduction are given. Complete source listings of all of the software developed are presented. Portions of this report have appeared previously as internal documentation at the Transportation Systems Center.

17. Key Words
Head Impact, Crash Tests, Biomechanics, Anthropomorphic Dummy, Trauma, Acceleration, Angular Acceleration, Cross Axis Sensitivity, Error Model, Sample Rate, Accelerometer

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PREFACE

The work described herein was sponsored by the Office of Research and Development at the National Highway Traffic Safety Administration (NHTSA) under Project Plan Agreement HS-76. Mr. Mark Haffner, the NHTSA project manager, provided crucial guidance and support throughout this activity.

The report describes the development of a methodology for the calibration, test and use of a six-degree-of-freedom acceleration measurement system intended to measure the linear and angular head accelerations of anthropomorphic dummies, human volunteers, cadavers and primates during crash test conditions. Three devices, using arrays of linear accelerometers, were examined in detail. They were; (1) a six-accelerometer coplanar array, (2) a nine-accelerometer coplanar array, and (3) a nine-accelerometer non-coplanar array. In support of the development of the calibration procedure: (1) error models were developed, (2) computer simulations were performed to examine the stability of each of the above arrays, (3) a methodology was developed for the determination of the system error coefficients, (4) an algorithm for field data correction was defined, (5) parametric sensitivity analyses were performed to determine the variables that contribute most to system error, and (6) recommendations for optimizing data collection and reduction are made. This report documents the findings of these activities to date. Laboratory verifications of these techniques will be undertaken during 1988-1989.

Appendix A was prepared by H. Weinstock, M. Coltman and H. Lee at the Transportation Systems Center and contains the development of the six-degree-of-freedom accelerometer system equations and comparisons of three different arrays. Appendix B was prepared by A. Boghani, K. Carlson, M. Cohen and R. Spencer at Arthur D. Little, Inc. and contains the development of system equation software and a methodology for laboratory calibration of a six-degree-of-freedom device.

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METRIC / ENGLISH CONVERSION FACTORS

ENGLISH TO METRIC

LENGTH (APPROXIMATE)

1 inch (in) = 2.5 centimeters (cm)
1 foot (ft) = 30 centimeters (cm)
1 yard (yd) = 0.9 meter (m)
1 mile (mi) = 1.6 kilometers (km)

AREA (APPROXIMATE)

1 square inch (sq in, in^2) = 6.5 square centimeters (cm^2)
1 square foot (sq ft, ft^2) = 0.09 square meter (m^2)
1 square yard (sq yd, yd^2) = 0.8 square meter (m^2)
1 square mile (sq mi, mi^2) = 2.6 square kilometers (km^2)
1 acre = 0.4 hectares (he) = 4,000 square meters (m^2)

MASS - WEIGHT (APPROXIMATE)

1 ounce (oz) = 28 grams (gr)
1 pound (lb) = .45 kilogram (kg)
1 short ton = 2,000 pounds (lb) = 0.9 tonne (t)

VOLUME (APPROXIMATE)

1 teaspoon (tsp) = 5 milliliters (ml)
1 tablespoon (tbsp) = 15 milliliters (ml)
1 fluid ounce (fl oz) = 30 milliliters (ml)
1 cup (c) = 0.24 liter (l)
1 pint (pt) = 0.47 liter (l)
1 quart (qt) = 0.96 liter (l)
1 gallon (gal) = 3.8 liters (l)
1 cubic foot (cu ft, ft^3) = 0.03 cubic meter (m^3)
1 cubic yard (cu yd, yd^3) = 0.76 cubic meter (m^3)

TEMPERATURE (EXACT)

[(x - 32)(5/9)] °F = y °C

METRIC TO ENGLISH

LENGTH (APPROXIMATE)

1 millimeter (mm) = 0.04 inch (in)
1 centimeter (cm) = 0.4 inch (in)
1 meter (m) = 3.3 feet (ft)
1 meter (m) = 1.1 yards (yd)
1 kilometer (km) = 0.6 mile (mi)

AREA (APPROXIMATE)

1 square centimeter (cm^2) = 0.16 square inch (sq in, in^2)
1 square meter (m^2) = 1.2 square yards (sq yd, yd^2)
1 square kilometer (km^2) = 0.4 square mile (sq mi, mi^2)
1 hectare (he) = 10,000 square meters (m^2) = 2.5 acres

MASS - WEIGHT (APPROXIMATE)

1 gram (gr) = 0.036 ounce (oz)
1 kilogram (kg) = 2.2 pounds (lb)
1 tonne (t) = 1,000 kilograms (kg) = 1.1 short tons

VOLUME (APPROXIMATE)

1 milliliter (ml) = 0.03 fluid ounce (fl oz)
1 liter (l) = 2.1 pints (pt)
1 liter (l) = 1.06 quarts (qt)
1 liter (l) = 0.26 gallon (gal)
1 cubic meter (m^3) = 36 cubic feet (cu ft, ft^3)
1 cubic meter (m^3) = 1.3 cubic yards (cu yd, yd^3)

TEMPERATURE (EXACT)

[(9/5) y + 32] °C = x °F

QUICK INCH-CENTIMETER LENGTH CONVERSION

INCHES	0	1	2	3	4	5	6	7	8	9	10

CENTIMETERS: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 | 25.40

QUICK FAHRENHEIT-CELCIUS TEMPERATURE CONVERSION

°F	-40°	-22°	-4°	14°	32°	50°	68°	86°	104°	122°	140°	158°	176°	194°	212°
°C	-40°	-30°	-20°	-10°	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°

For more exact and/or other conversion factors, see NBS Miscellaneous Publication 286, Units of Weights and Measures. Price $2.50. SD Catalog No. C13 10 286.

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

LIST OF FIGURES

LIST OF FIGURES (Cont'd)

LIST OF FIGURES (Cont'd)

LIST OF FIGURES (Cont'd)

LIST OF FIGURES (Cont'd)

FIGURE PAGE

List of Tables

1. INTRODUCTION

The accurate measurement of head acceleration is required by the NHTSA in order to establish reasonable head injury criteria for use in motor vehicle safety assessments. If the linear and angular head accelerations which cause cerebral concussion were known, it would be of great benefit in the development of improved protective headgear for motorcyclists, design of advanced crash test dummies, and safer automotive interiors. These data would greatly enhance the biomechanics data base, and be of great value to policy makers and the engineering community and would contribute significantly to the Departmental goal of reducing the frequency and severity of head injuries in motor vehicle accidents. In order to gather this data, accurate and reliable instrumentation is required for the measurement of the desired accelerations. In addition, a reliable means of calibrating this instrumentation must be established. The effort reported here describes the development of a methodology for measuring the linear and angular head acceleration of anthropomorphic dummies and the calibration and test of the instrumentation used for these measurements.

1.1 BACKGROUND

In recent years, considerable evidence [1 2 3] has been gathered that angular, as well as linear acceleration is a significant factor in head injury sustained in automotive crashes. Efforts to measure these parameters have utilized several different methodologies with varying success.

A commercially available angular accelerometer (ENDEVCO Model 7302A) was initially attractive because it measured angular acceleration directly, eliminating the algebraic manipulation and errors associated with other methods employing arrays of linear accelerometers. Subsequent tests[4] however, identified problems with reliability when the device was subjected to a high translational acceleration field. The model was ultimately discontinued in favor of a larger model (7302B) which had more application in the manufacturing community.

Another device developed for use in studies of human response to impact events used technology derived from fiber optic gyroscopes.[5] The device consisted of a fiber optic coil with countercirculating light beams created by a laser. When the coil was rotated, there was a phase shift between the two beams causing a measurable intensity change in the interference pattern on a detector surface. Acoustic and thermal noise problems proved too great to produce a reliable device.

The most reliable and widely used methodology has been several configurations of linear accelerometer arrays used to measure the head acceleration of primates, cadavers, human volunteers and anthropomorphic dummies. The three principal configurations that have been used to date are, (1) the nine-accelerometer coplanar[6] (3-3-3 configuration), (2) the nine-accelerometer non-coplanar[7] (3-2-2-2 configuration) and (3) the six-accelerometer non-coplanar [8 9] (3-2-1 configuration). It has been shown in other studies[7] and confirmed here (see Appendix A) that the 3-3-3 configuration and the 3-2-1 configuration are mathematically unstable under certain circumstances and their use should be limited.

1.2 SUMMARY

The purpose of the work reported here was to develop a methodology for calibration of, and data acquisition with, a six-degree-of-freedom acceleration measurement device. To accomplish this, an examination of the most commonly used linear accelerometer arrays was conducted. Details of this portion of the work can be found in Appendix A, "Comparison of Translational Accelerometer Configurations for Measuring Angular Accelerations of a Rigid Body". A nine-accelerometer array in the 3-2-2-2 configuration was determined to be the most reliable array. Sources of error were identified and equations of motion were developed to describe system behavior. The equations of motion consist of six equations for three measured angular accelerations and three measured linear accelerations when the actual angular and linear inputs are known.

Software was developed to implement the system equations in the work described above. The first phase of software development is discussed in

Appendix B, "Simulation and Calibration of Nine-Accelerometer Package (NAP) for Anthropomorphic Dummies". This software was refined to be more user friendly and listings for the revised program (designated NAPLABG) are given in Appendix C. In addition, software for the correction of data taken in the field was developed and this is discussed in Section 4 and listed in Appendix D. Model verification and an examination of system parametric sensitivities were conducted using this software. Throughout most of the work presented here, the acceleration pulses used as input to the model were made intentionally unrealistic (large magnitude with long duration) to present a worst case situation for the various algorithms used. This allowed an easier assessment of parameter sensitivities and model stability. However, estimates of residual error associated with input pulses of more reasonable magnitude and duration can easily be made (see Figures 45, 46, 60, and 61 and Sections 6.11 and 6.12). Parametric sensitivity studies assuming perfect transducers in one case and perfect geometry in another case were conducted. It was determined that the software algorithms for the system equations and data correction were valid representations of the 3-2-2-2 configuration of the NAP. It was also found that data sample rate was one of the most important parameters with regard to system accuracy. The choice of location of the coordinate system was also important in the reduction of residual errors (errors remaining after data correction). Listings for all the software discussed are contained in the appendices.

A methodology for the laboratory calibration of a six-degree-of-freedom acceleration measurement device was developed and the results are presented in Appendix B. The methodology consists of vibrating the accelerometer system in modes that will allow for the determination of the coefficients of the system equations. These coefficients are then used to create the necessary coefficients of the data correction algorithms (NAPFLDEUL or NAPFLDRK). The data correction algorithms are used to correct the field data gathered with the accelerometer system in question.

2. MEASUREMENT SYSTEM DEFINITION

2.1 SYSTEM GEOMETRY

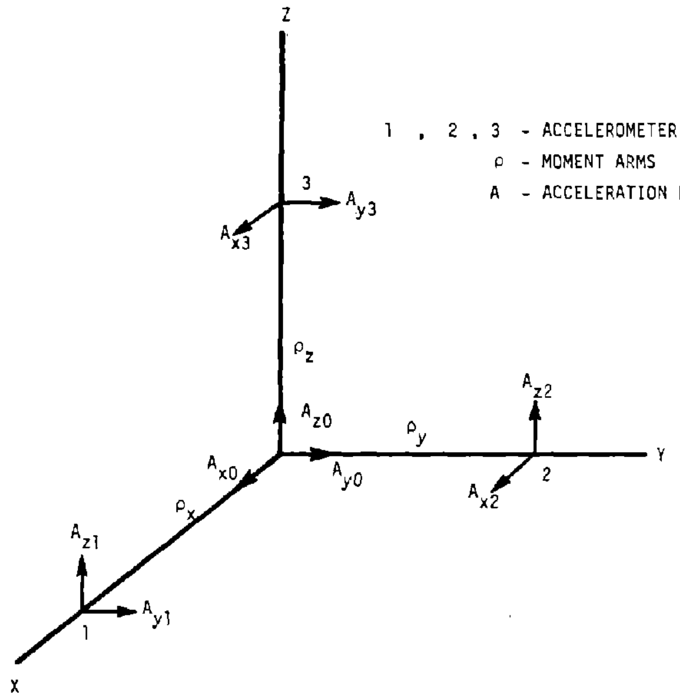
A comparison of three different configurations of linear accelerometers used for measuring linear and angular head acceleration is given in Appendix A. The three configurations considered were the nine-accelerometer coplanar (3-3-3 configuration), the nine-accelerometer non-coplanar (3-2-2-2 configuration), and the six-accelerometer non-coplanar (3-2-1 configuration). The results of this comparison show that the most stable and reliable of the configurations tested was the 3-2-2-2 configuration.

2.2 3-2-2-2 CONFIGURATION

The geometry of the 3-2-2-2 configuration is shown in Figure 1. This configuration, developed mainly at Wayne State University[7] has the advantage of providing signals directly proportional to the angular accelerations with no need to estimate angular velocities. The geometry of this configuration allows the algebraic elimination of velocity squared terms and velocity cross product terms from the equations of motion. This, in turn, eliminates the requirement for numerical integration of the data which may lead to instability in certain cases. The equations for the output of this system are also shown in Figure 1. These equations presume perfect transducers (no manufacturing defects, etc.) and perfect transducer alignment in the array.

The ideal geometry for the 3-2-2-2 configuration would allow mounting of the three centrally located accelerometers such that their seismic centers are at the same point in space (the origin). In theory, this can be achieved with a special transducer design[10] incorporating forked cantilever beams as illustrated in Figure 2 but this configuration is not available as a reliable, off-the-shelf item. The ideal geometry is useful, however, when simulated with a computer model to examine the effects of transducer generated errors without introducing geometric errors.

Deviations from the ideal case of perfect geometry and perfect transducers result in errors that must be accounted for. System equations (hereafter



$$\dot{\omega}_x = \frac{(A_{z2} - A_{z0})}{2\rho_y} - \frac{(A_{y3} - A_{y0})}{2\rho_z}$$

$$\dot{\omega}_y = \frac{(A_{x3} - A_{x0})}{2\rho_z} - \frac{(A_{z1} - A_{z0})}{2\rho_x}$$

$$\dot{\omega}_z = \frac{(A_{y1} - A_{y0})}{2\rho_x} - \frac{(A_{x2} - A_{x0})}{2\rho_y}$$

FIGURE 1. 3-2-2-2 CONFIGURATION (IDEAL CASE)

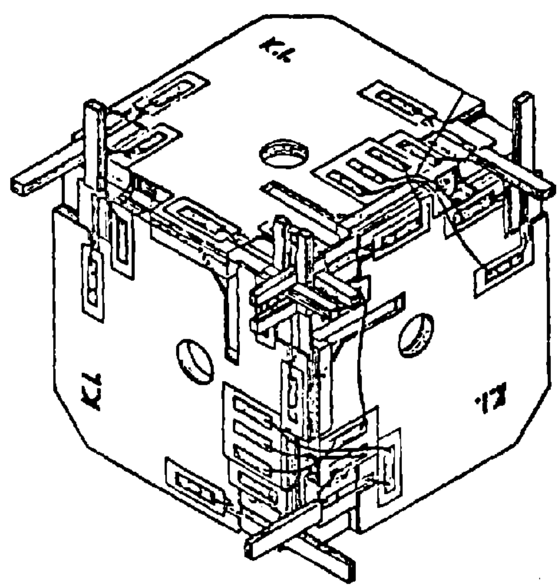


FIGURE 2. EXPERIMENTAL 3-2-2-2 SYSTEM

referred to as the "laboratory system equations") were developed to account for these errors (see Appendix A and Section 3.1). The model represented by these equations is used to characterize the system and allow the calculation of system coefficients using known inputs. Software has been developed for this purpose. In addition to the model, a test methodology for the purpose of determining the values of the system coefficients represented in the laboratory system equations has been developed. Both the software model and test methodology are contained in Appendix B. In addition to the laboratory system equations, a set of inverse equations, hereafter referred to as the "field system equations" were developed (Section 4). These equations allow for the correction of data taken in the field. After the system coefficients are determined in the laboratory, their values may be used in parametric studies with the laboratory system or converted to the coefficients associated with the field data correction algorithm of the field system equations.

2.3 SOURCES OF ERROR

The sources of the system errors are reflected in the coefficients of the equations of motion. The actual source of error represented by any particular coefficient is dependent entirely upon the methodology used to gather the acceleration information (i.e., an array of linear accelerometers, differentiated velocity transducers, etc.). The report reproduced in Appendix A derives the system coefficient values for a 3-2-2-2 configuration as well as a 3-3-3 configuration. That is, each coefficient is defined in terms of system geometry errors, individual transducers errors, etc. When the system is calibrated in the laboratory, under various vibrational modes, the actual source of the error is of little consequence as the calibration determines the values of the coefficients regardless of the actual source of the errors. As long as the errors are represented by the system coefficients, they are compensatable errors. That is, as long as the coefficients remain constant across the test conditions, then the errors encountered will be compensatable. If the coefficients are not constant, if they vary with frequency for example, then the issue is much more complex and error compensation becomes very difficult. The extent to which this type of problem exists will become apparent as experience grows with the calibration methodology.

3. THE LABORATORY MODEL

3.1 LABORATORY SYSTEM EQUATIONS

The laboratory model consists of six equations for three measured angular accelerations and three measured linear accelerations when the actual angular and linear accelerations are known. These equations are shown in Figure 3. The resulting measured values are in error due to accelerometer misalignments, sensitivity mismatches and mispositioning as reflected in the system coefficients. The original algorithm developed for these equations was refined to be more user friendly with regard to choice of input files, double precision option and numerous other variables. The designation for the revised program is NAPLABG and a listing of the program is found in Appendix C.

3.2 COMPUTER MODEL

The computer model of the laboratory system equations is a useful tool for examining parametric sensitivities. The first thing the computer program for the laboratory model does is prompt the user for the name of the error term file to be used (see Appendix E). The error term file is a matrix of error terms describing accelerometer characteristics such as bias, cross-axis sensitivity, scale factor and accelerometer location errors. The program then gives the user the option of entering analytical (half-sine pulse) or experimental input data. If the choice is analytical, it then prompts the user for the magnitude and duration of the half-sine pulse as well as the integration time step and the desired name of the output file. System coefficients are then calculated based upon the error term matrix chosen. Angular velocity is calculated with a fourth order Runge-Kutta numerical integration subroutine. If the integration time step is chosen such that the intervals are not multiples of the sampling rate, an extrapolation subroutine is used to estimate values between sample points. The resulting output is an array of data containing the actual linear and angular accelerations, estimated angular velocities, and estimated linear and angular accelerations. The estimated linear and angular accelerations contain errors due to accelerometer and system geometry errors and must be processed with the field data correction algorithm to remove these errors.

$$
\begin{aligned}
\dot{\Upsilon}_x = \; & \dot{\omega}_x + AA0 + AA1\,\dot{\omega}_x + AA2\,\dot{\omega}_y + AA3\,\dot{\omega}_z \\
& + AA4\,\omega_x^2 + AA5\,\omega_y^2 + AA6\,\omega_z^2 \\
& + AA7\,\omega_x\omega_y + AA8\,\omega_x\omega_z + AA9\,\omega_y\omega_z \\
& + AA10\,\ddot{x} + AA11\,\ddot{y} + AA12\,\ddot{z}
\end{aligned}
$$

$$
\begin{aligned}
\dot{\Upsilon}_y = \; & \dot{\omega}_y + BA0 + BA1\,\dot{\omega}_x + BA2\,\dot{\omega}_y + BA3\,\dot{\omega}_z \\
& + BA4\,\omega_x^2 + BA5\,\omega_y^2 + BA6\,\omega_z^2 \\
& + BA7\,\omega_x\omega_y + BA8\,\omega_x\omega_z + BA9\,\omega_y\omega_z \\
& + BA10\,\ddot{x} + BA11\,\ddot{y} + BA12\,\ddot{z}
\end{aligned}
$$

$$
\begin{aligned}
\dot{\Upsilon}_z = \; & \dot{\omega}_z + CA0 + CA1\,\dot{\omega}_x + CA2\,\dot{\omega}_y + CA3\,\dot{\omega}_z \\
& + CA4\,\omega_x^2 + CA5\,\omega_y^2 + CA6\,\omega_z^2 \\
& + CA7\,\omega_x\omega_y + CA8\,\omega_x\omega_z + CA9\,\omega_y\omega_z \\
& + CA10\,\ddot{x} + CA11\,\ddot{y} + CA12\,\ddot{z}
\end{aligned}
$$

$$
\begin{aligned}
\ddot{X} = \; & \ddot{x} + AL0 + AL1\,\dot{\omega}_x + AL2\,\dot{\omega}_y + AL3\,\dot{\omega}_z \\
& + AL4\,\omega_x^2 + AL5\,\omega_y^2 + AL6\,\omega_z^2 \\
& + AL7\,\omega_x\omega_y + AL8\,\omega_x\omega_z + AL9\,\omega_y\omega_z \\
& + AL10\,\ddot{x} + AL11\,\ddot{y} + AL12\,\ddot{z}
\end{aligned}
$$

$$
\begin{aligned}
\ddot{Y} = \; & \ddot{y} + BL0 + BL1\,\dot{\omega}_x + BL2\,\dot{\omega}_y + BL3\,\dot{\omega}_z \\
& + BL4\,\omega_x^2 + BL5\,\omega_y^2 + BL6\,\omega_z^2 \\
& + BL7\,\omega_x\omega_y + BL8\,\omega_x\omega_z + BL9\,\omega_y\omega_z \\
& + BL10\,\ddot{x} + BL11\,\ddot{y} + BL12\,\ddot{z}
\end{aligned}
$$

$$
\begin{aligned}
\ddot{Z} = \; & \ddot{z} + CL0 + CL1\,\dot{\omega}_x + CL2\,\dot{\omega}_y + CL3\,\dot{\omega}_z \\
& + CL4\,\omega_x^2 + CL5\,\omega_y^2 + CL6\,\omega_z^2 \\
& + CL7\,\omega_x\omega_y + CL8\,\omega_x\omega_z + CL9\,\omega_y\omega_z \\
& + CL10\,\ddot{x} + CL11\,\ddot{y} + CL12\,\ddot{z}
\end{aligned}
$$

FIGURE 3. LABORATORY SYSTEM EQUATIONS

3.3 DETERMINATION OF SYSTEM COEFFICIENTS (SYSTEM CALIBRATION)

The values of the system coefficients can be determined in the laboratory by vibrating the system in the modes prescribed in Appendix B. This calibration approach is different from previous work[11] performed at the Central Inertial Guidance Test Facility at Holloman Air Force Base in that it calibrates the package as a system and does not calibrate individual accelerometers. In fact, the method with which the system under test derives the three angular and three linear accelerations is not critical in the calibration methodology. In the case of a nine-accelerometer array, the system coefficients can be shown to be derived from individual accelerometer error terms as well as accelerometer location errors. However, if a system utilizes another methodology to measure the six accelerations in question, the system equations still contain the same coefficients but in this case the coefficients are generated by a different set of characteristics. The calibration methodology, however, is valid in either case, as it evaluates the system coefficients without regard to the system characteristics that generate these coefficients.

4. THE FIELD MODEL

4.1 FIELD SYSTEM EQUATIONS

Data gathered in the field for linear and angular head acceleration contains errors due to accelerometer misalignment, cross-axis sensitivities and bias. These errors may be removed from the data using the findings of the laboratory system calibration. A data correction algorithm was developed for use with data gathered with a 3-2-2-2 NAP that has been calibrated in the manner suggested in Appendix B to evaluate the system coefficients. This is referred to as the field model and the software to implement the equations representing this model corrects the actual data taken in the field. The system coefficients for the field model are derived from the system coefficients determined for the laboratory model. The field system equations are essentially the inverse of the laboratory system equations and are illustrated in matrix form with the laboratory equations in Figure 4. The field equations are shown in Figure 5. A matrix inversion subroutine used to develop the field system equations is given in Appendix F.

4.2 COMPUTER MODEL

The simultaneous solutions of the field equations is accomplished using simple Euler integration techniques, or by linking to a Runge-Kutta subroutine in the IMSL* library. The two programs developed for this purpose are designated NAPFLDEUL and NAPFLDRK and listings can be found in Appendix D.

Using the coefficients derived in the laboratory, the software for the field model performs the required matrix products and invertions to solve the field equations. The user is first prompted for a system coefficient file name.

---

*IMSL, Inc., Houston, Texas - Supplier of scientific and mathematical FORTRAN subroutines.

$$\begin{bmatrix} \dot{\gamma}_x \\ \dot{\gamma}_y \\ \dot{\gamma}_z \\ \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} = \begin{bmatrix} AA0 \\ BA0 \\ CA0 \\ AL0 \\ BL0 \\ CL0 \end{bmatrix} + [A+I] \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} + [B] \begin{bmatrix} \omega_x^2 \\ \omega_y^2 \\ \omega_z^2 \\ \omega_x\omega_y \\ \omega_x\omega_z \\ \omega_y\omega_z \end{bmatrix}$$

Laboratory Equations

$$\begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = [A+I]^{-1} \begin{bmatrix} \dot{\gamma}_x \\ \dot{\gamma}_y \\ \dot{\gamma}_z \\ \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} - [A+I]^{-1}[B] \begin{bmatrix} \omega_x^2 \\ \omega_y^2 \\ \omega_z^2 \\ \omega_x\omega_y \\ \omega_x\omega_z \\ \omega_y\omega_z \end{bmatrix} - [A+I]^{-1} \begin{bmatrix} AA0 \\ BA0 \\ CA0 \\ AL0 \\ BL0 \\ CL0 \end{bmatrix}$$

Field Equations

FIGURE 4. LABORATORY AND FIELD EQUATIONS (MATRIX FORM)

$$\dot{\omega}_x = \dot{\gamma}_x + DA0 + DA1\,\dot{\gamma}_x + DA2\,\dot{\gamma}_y + DA3\,\dot{\gamma}_z$$
$$+ DA4\,\gamma_x^{\,2} + DA5\,\gamma_y^{\,2} + DA6\,\gamma_z^{\,2}$$
$$+ DA7\,\gamma_x\gamma_y + DA8\,\gamma_x\gamma_z + DA9\,\gamma_y\gamma_z$$
$$+ DA10\,\ddot{X} + DA11\,\ddot{Y} + DA12\,\ddot{Z}$$

$$\dot{\omega}_y = \dot{\gamma}_y + EA0 + EA1\,\dot{\gamma}_x + EA2\,\dot{\gamma}_y + EA3\,\dot{\gamma}_z$$
$$+ EA4\,\gamma_x^{\,2} + EA5\,\gamma_y^{\,2} + EA6\,\gamma_z^{\,2}$$
$$+ EA7\,\gamma_x\gamma_y + EA8\,\gamma_x\gamma_z + EA9\,\gamma_y\gamma_z$$
$$+ EA10\,\ddot{X} + EA11\,\ddot{Y} + EA12\,\ddot{Z}$$

$$\dot{\omega}_z = \dot{\gamma}_z + FA0 + FA1\,\dot{\gamma}_x + FA2\,\dot{\gamma}_y + FA3\,\dot{\gamma}_z$$
$$+ FA4\,\gamma_x^{\,2} + FA5\,\gamma_y^{\,2} + FA6\,\gamma_z^{\,2}$$
$$+ FA7\,\gamma_x\gamma_y + FA8\,\gamma_x\gamma_z + FA9\,\gamma_y\gamma_z$$
$$+ FA10\,\ddot{X} + FA11\,\ddot{Y} + FA12\,\ddot{Z}$$

$$\ddot{x} = \ddot{X} + DL0 + DL1\,\dot{\gamma}_x + DL2\,\dot{\gamma}_y + DL3\,\dot{\gamma}_z$$
$$+ DL4\,\gamma_x^{\,2} + DL5\,\gamma_y^{\,2} + DL6\,\gamma_z^{\,2}$$
$$+ DL7\,\gamma_x\gamma_y + DL8\,\gamma_x\gamma_z + DL9\,\gamma_y\gamma_z$$
$$+ DL10\,\ddot{X} + DL11\,\ddot{Y} + DL12\,\ddot{Z}$$

$$\ddot{y} = \ddot{Y} + EL0 + EL1\,\dot{\gamma}_x + EL2\,\dot{\gamma}_y + EL3\,\dot{\gamma}_z$$
$$+ EL4\,\gamma_x^{\,2} + EL5\,\gamma_y^{\,2} + EL6\,\gamma_z^{\,2}$$
$$+ EL7\,\gamma_x\gamma_y + EL8\,\gamma_x\gamma_z + EL9\,\gamma_y\gamma_z$$
$$+ EL10\,\ddot{X} + EL11\,\ddot{Y} + EL12\,\ddot{Z}$$

$$\ddot{z} + \ddot{Z} + FL0 + FL1\,\dot{\gamma}_x + FL2\,\dot{\gamma}_y + FL3\,\dot{\gamma}_z$$
$$+ FL4\,\gamma_x^{\,2} + FL5\,\gamma_y^{\,2} + FL6\,\gamma_z^{\,2}$$
$$+ FL7\,\gamma_x\gamma_y + FL8\,\gamma_x\gamma_z + FL9\,\gamma_y\gamma_z$$
$$+ FL10\,\ddot{X} + FL11\,\ddot{Y} + FL12\,\ddot{Z}$$

FIGURE 5. FIELD SYSTEM EQUATIONS

A system coefficient file may be generated from an error term matrix for the specific situation under consideration (See Appendix G, Computer Program for Creating Laboratory System Matrices from Error Term Matrices) or may be determined from a laboratory calibration. The user is then prompted for the type of input file (analytic, experimental data file, or output from the laboratory model); the name of the data input file, the number of samples in the data, the integration time step desired, and the name desired for the resulting output file. The output file produced will then contain data on all pertinent parameters, such as actual and measured accelerations and velocities and must be further processed to examine any specific parameter graphically. The program CONV contained in Appendix H converts output data from either the laboratory system software or the field system software into a file for the specific parameter desired. Sample time and parameter values are written to the new file. The new file may then be used as input to appropriate graphics software in order to produce acceleration and velocity time histories.

5. MODEL VERIFICATION

Model verification was performed by putting known input signals into the laboratory model which would produce erroneous data at the output in accordance with the system coefficients. This erroneous signal was then used as input to the field model which corrects the errors and returns the original signals.

5.1 ERROR TERM MATRICES

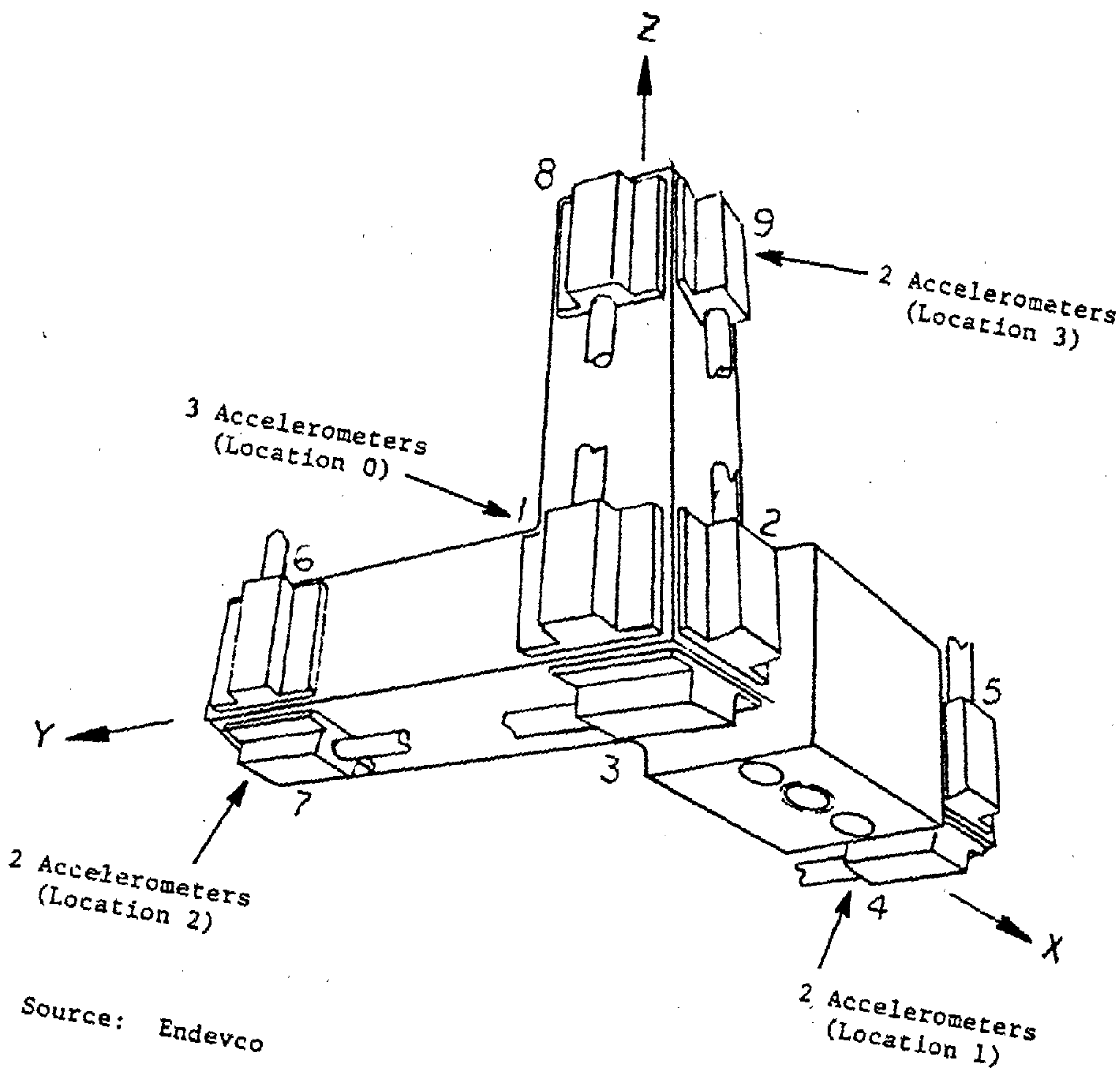
Error term matrices (see Section 3.2 Computer Model) were calculated from a standard ENDEVCO mount illustrated in Figure 6. This mount was designed to be used with model 2264 accelerometers but these accelerometers have been replaced by a newer model 7264 and the mount is no longer being manufactured. It is representative, however, of a 3-2-2-2 mounting configuration using typical miniature piezoresistive accelerometers and is used as a baseline model for the parametric sensitivity studies. Variations in the calculation of error matrices are due to the choice of location of the coordinate system and the uncertainty (tolerance) associated with the location of the transducer seismic center. The error matrices currently on file for use are illustrated in Appendix E. The error term matrices are used to generate laboratory system coefficient matrices for use in parametric sensitivity analysis or algorithm validation. The laboratory system matrices are generated with the program CREATMAT shown in Appendix G.

5.2 VERIFICATION PROCEDURE

A typical model verification procedure is conducted as follows:

1. Run NAPLABG

    a. Choose an appropriate error term file.
    b. Choose an appropriate input file.
    c. Choose an integration time step.



Source: Endevco

FIGURE 6. STANDARD ENDEVCO MOUNT

NAPLABG allows you to choose a half-sine or experimental pulse as the input file. The output file created represents measured values and contains errors due to the particular system geometry chosen and errors in transducer characteristics.

2. Run NAPFLDEUL or NAPFLDRK

a. Choose an appropriate system coefficient file (created with CREATMAT from the error file used in step 1).

b. Choose an appropriate input file (from NAPLABG output).

c. Choose an integration time step.

The output created should be identical to the original input to NAPLABG. Discrepancies are a measure of the fidelity of the algorithm.

Files designated as UDS files at the NHTSA may be used as inputs, by first processing them with the programs UDS2ASCII (Appendix I) and then EXPERFILE (Appendix J). This will convert a UDS file to ASCII format and then to a file that can be read by NAPLABG, NAPFLDEUL or NAPFLDRK.

5.3 HALF SINE INPUTS

The initial inputs used to verify the model were half sine inputs. The acceleration pulse experienced by an occupant in a crash may often be approximated by a half sine function. Representative input pulses are shown in Figure 7. In this figure and all that follow, the nomenclature used above each graph to indicate the graphed quantity corresponds to the definitions shown in Table 1. These pulses in Figure 7 (a half-sine pulse about each axis) were used with the verification procedure detailed in the previous section and the error matrix ERTERM009.DAT which presumes that the siesmic centers of the accelerometers lie in the three principal planes of the system (see Appendix E). When the only inputs to the system are pure angular accelerations with no linear acceleration components, there exists an apparent measurement of linear accelerations due to the geometry offsets (from the ideal) of the accelerometers in the particular array in question. The apparent linear accelerations before correction with the field system algorithm are shown in Figure 8. The residual errors after correction with the field system algorithm are shown in Figure 9. As can be seen the apparent linear accelerations in Figure 8 are quite large ($>$ 2000g) but after correction become insignificant ($<$ 10g).

WX =	ESTIMATED ANGULAR VELOCITY ABOUT THE X-AXIS	(R/S)
WY =	ESTIMATED ANGULAR VELOCITY ABOUT THE Y-AXIS	(R/S)
WZ =	ESTIMATED ANGULAR VELOCITY ABOUT THE Z-AXIS	(R/S)
RX =	ACTUAL LINEAR ACCELERATION ALONG THE X-AXIS	(G'S)
RY =	ACTUAL LINEAR ACCELERATION ALONG THE Y-AXIS	(G'S)
RZ =	ACTUAL LINEAR ACCELERATION ALONG THE Z-AXIS	(G'S)
QDOTX =	ESTIMATED LINEAR ACCELERATION ALONG THE X-AXIS	(G'S)
QDOTY =	ESTIMATED LINEAR ACCELERATION ALONG THE Y-AXIS	(G'S)
QDOTZ =	ESTIMATED LINEAR ACCELERATION ALONG THE Z-AXIS	(G'S)
WDOTX =	ACTUAL ANGULAR ACCELERATION ABOUT THE X-AXIS	$(R/(S^2)$
WDOTY =	ACTUAL ANGULAR ACCELERATION ABOUT THE Y-AXIS	$(R/(S^2)$
WDOTZ =	ACTUAL ANGULAR ACCELERATION ABOUT THE Z-AXIS	$(R/(S^2)$
GDOTX =	ESTIMATED ANGULAR ACCELERATION ABOUT THE X-AXIS	$(R/(S^2)$
GDOTY =	ESTIMATED ANGULAR ACCELERATION ABOUT THE Y-AXIS	$(R/(S^2)$
GDOTZ =	ESTIMATED ANGULAR ACCELERATION ABOUT THE Z-AXIS	$(R/(S^2)$

Table 1. Nomenclature Used for Graphic Time Histories.

5.4 NOISY SIGNAL INPUTS

To verify that the system's good performance was not due to the fact that the half sine pulses were so "well behaved", a noisy signal was also used as input to the laboratory system. Validation tests were run with noisy signals derived from UDS files. The presumption was that cumulative errors associated with integration and roundoff would be more pronounced when using a noisy (ill behaved) signal. As there were few files of angular head acceleration available, files of linear head acceleration were used as though they were angular acceleration. Head acceleration data from the following UDS files at the NHTSA were input to NAPLAB.

$\ddot{x}$ - V0200AA00.001

$\ddot{y}$ - V0200AA00.002

$\ddot{z}$ - V0200AA00.003

$\ddot{\theta}_x$ - V0201AA00.001

$\ddot{\theta}_y$ - V0201AA00.002

$\ddot{\theta}_x$ - V0201AA00.003

The signals used for angular acceleration were multiplied by 1000 to study the effects of very large angular inputs. This scale factor was chosen because signals of these magnitudes (but not necessarily of the durations used here) have been reported in laboratory tests with cadavers. The signals were used as input to the laboratory system of equations to investigate the error introduced by very large angular acceleration and the geometry of a standard ENDEVCO mount. These input files are illustrated in Figure 10.

Tests were run with the UDS files listed above using every point in the data set (1,993 points). As with the half-sine inputs, the error matrix used was ERTERM009.DAT. The magnitude of the signals used for angular acceleration coupled with the duration of the signals drawn from the UDS files, result in acceleration data that is physically unrealizable for a dummy (i.e., the head would rotate more than 360 degrees). Nonetheless, the signals were used as input to the system for the purpose of exercising the system equations in the presence of very large angular accelerations.

The original input signals for the x, y and z linear accelerations, the values after processing with NAPLABD and the final corrected values after processing with NAPFLDRK are shown in Figure 11. It is apparent that large angular accelerations can cause significant errors in the measurement of linear acceleration. On a larger scale, the original input signals for the x, y and z linear accelerations and the final corrected values after processing with NAPFLDRK are shown in Figure 12. It can be seen that the original unputs and corrected outputs are virtual overlays. These same variables processed in the same manner but using 250 data points are shown in Figure 13. It can be seen that the error after correction is significant in this case. Further tests to examine the effect of sample rate were performed and the results are given in Section 6.1.

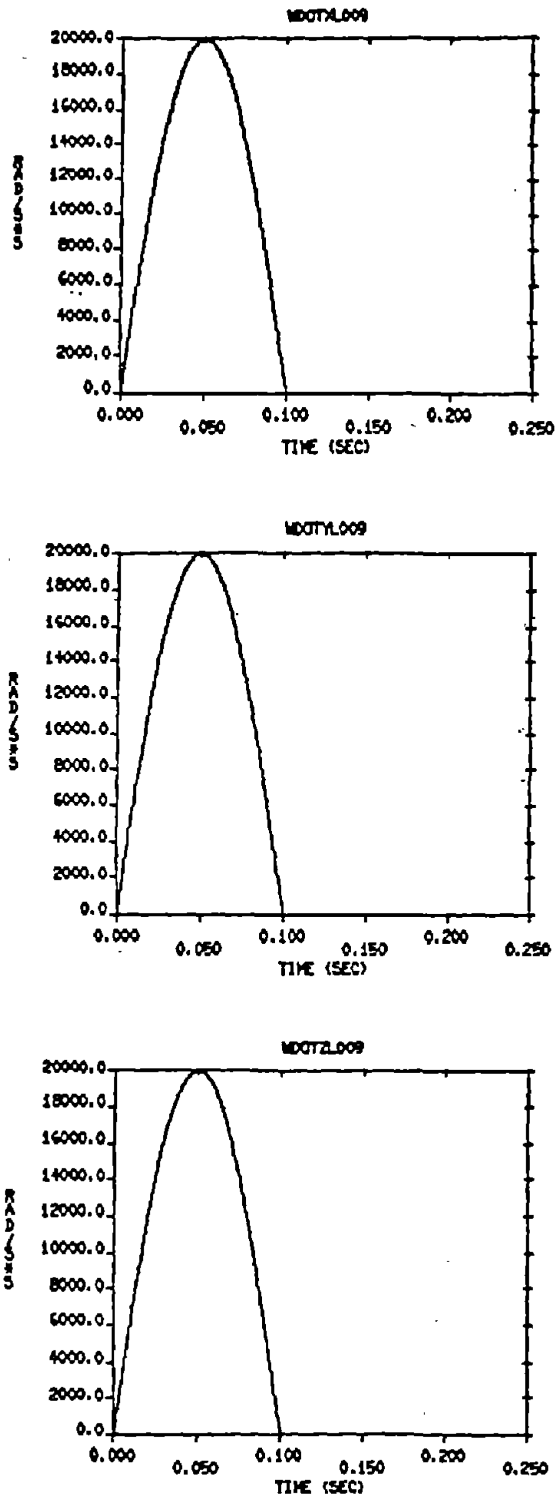


FIGURE 7. REPRESENTATIVE HALF-SINE INPUT PULSE

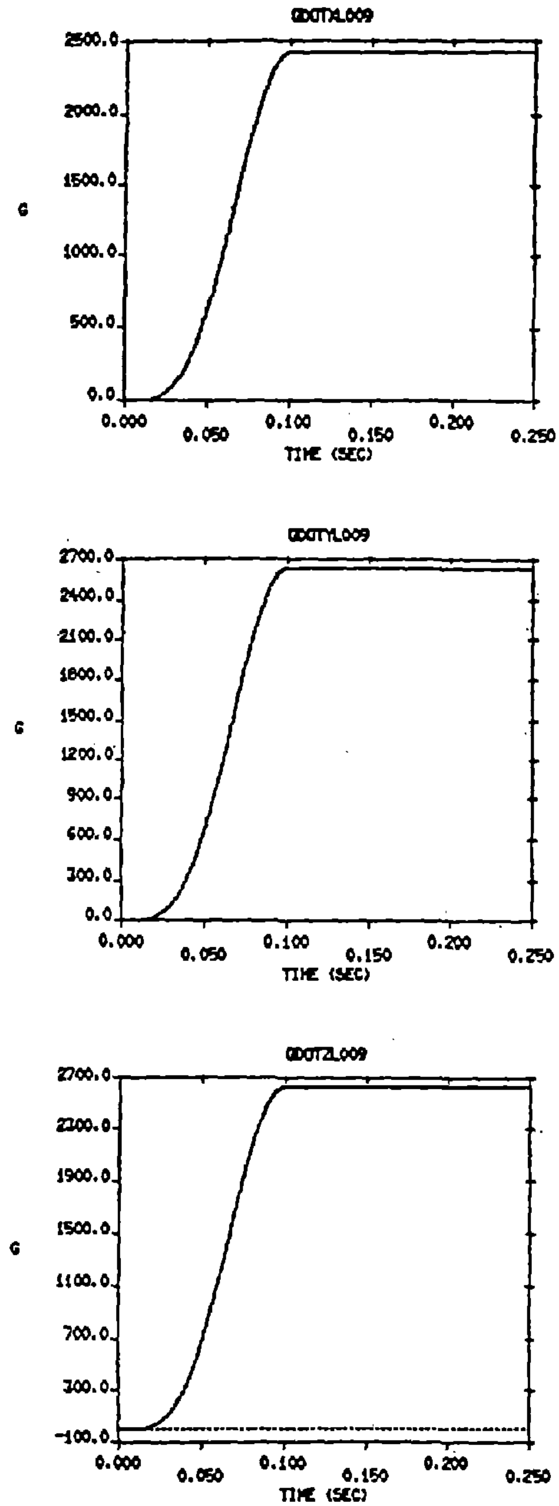


FIGURE 8. APPARENT LINEAR ACCELERATIONS (HALF-SINE PULSES)

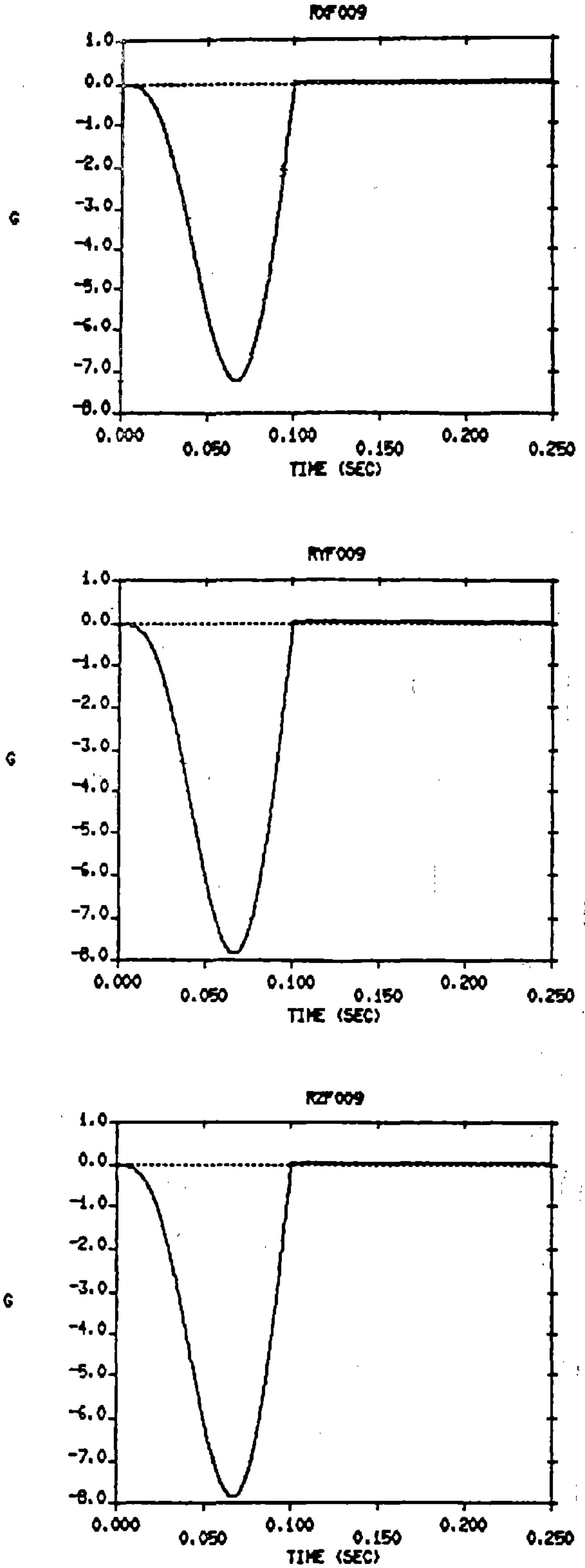


FIGURE 9. RESIDUAL ERRORS AFTER CORRECTION (HALF-SINE PULSES)

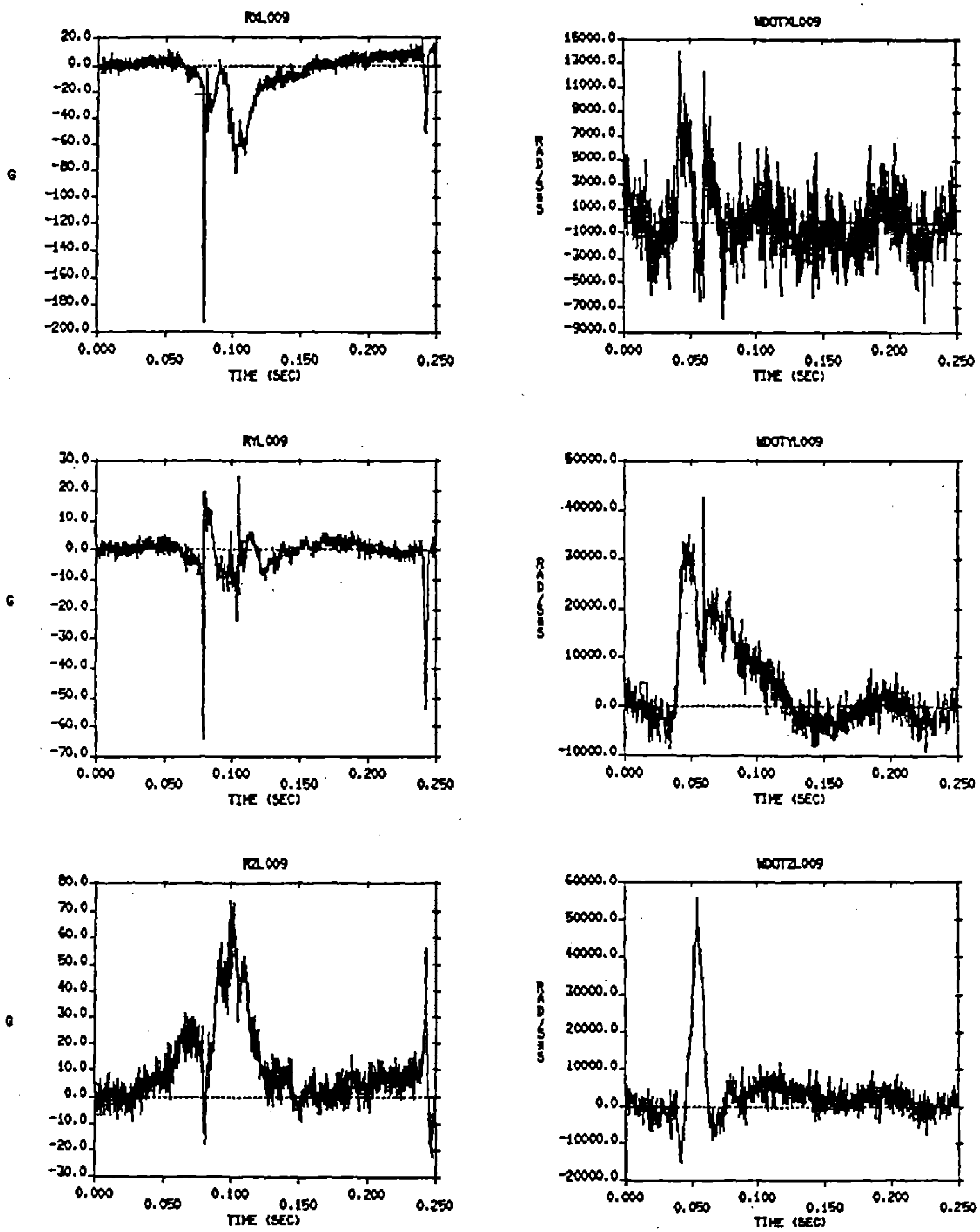


FIGURE 10. INPUTS TO NAPLABD (NOISY SIGNALS, 1,993 POINTS)

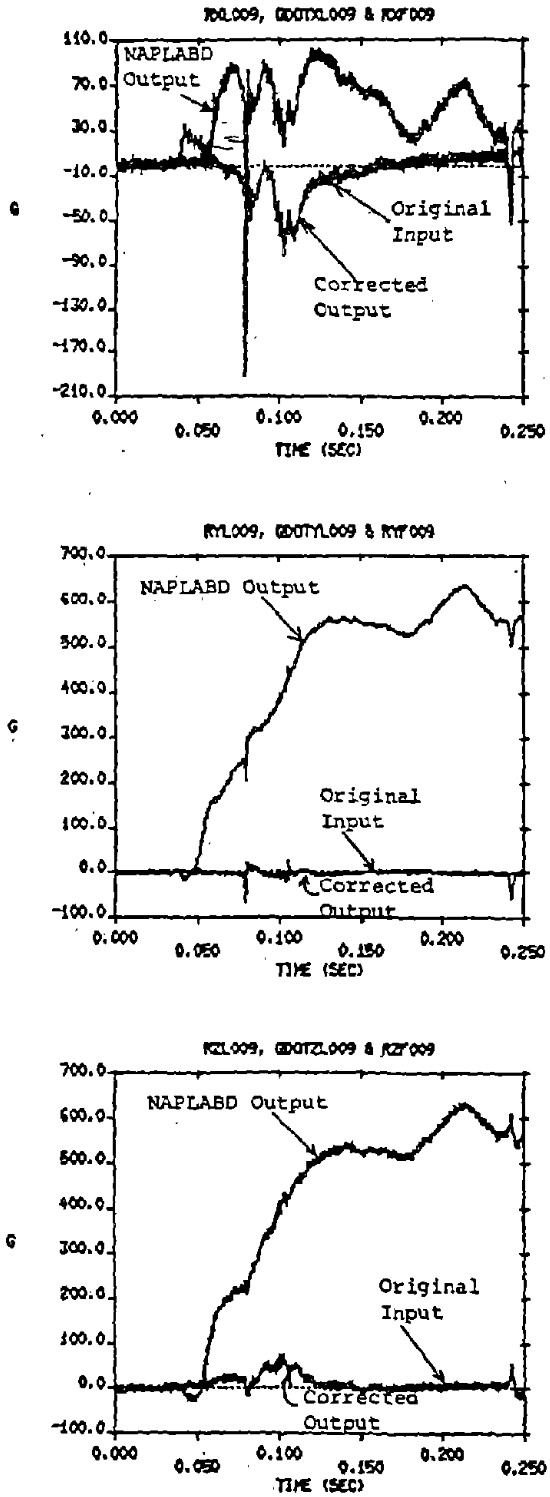


FIGURE 11. ORIGINAL INPUT, NAPLABD OUTPUT, AND CORRECTED OUTPUT (NOISY SIGNALS 1,993 POINTS)

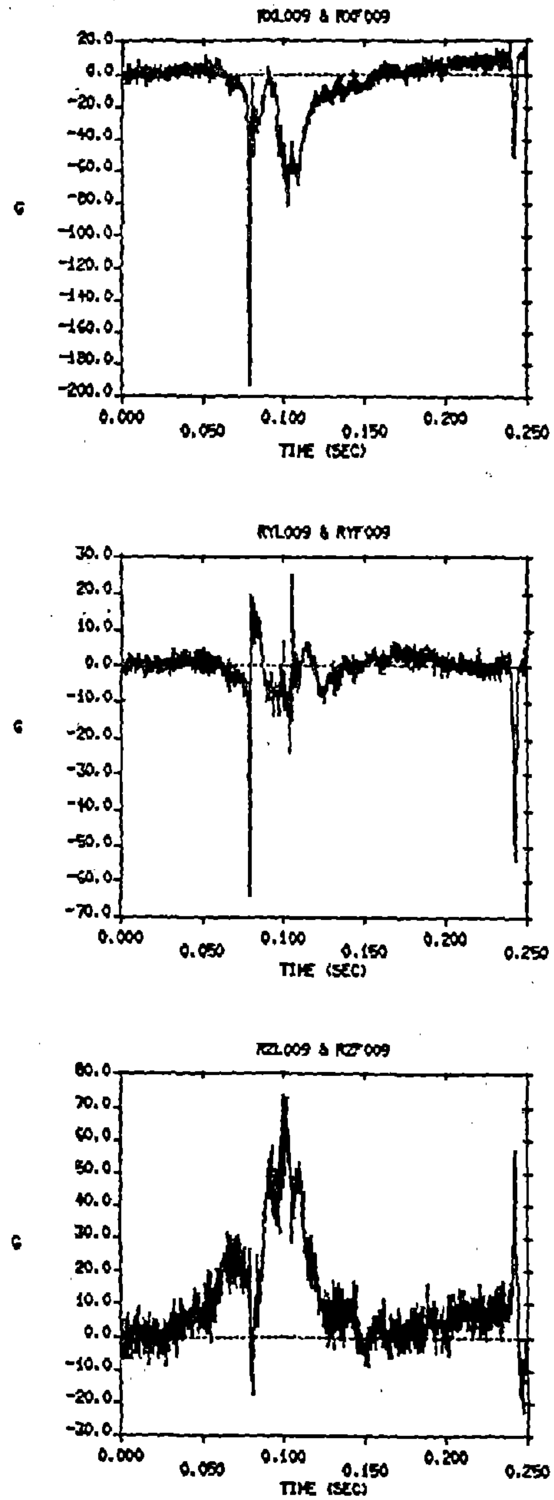


FIGURE 12. ORIGINAL INPUTS AND CORRECTED OUTPUTS (NOISY SIGNALS 1,993 POINTS)

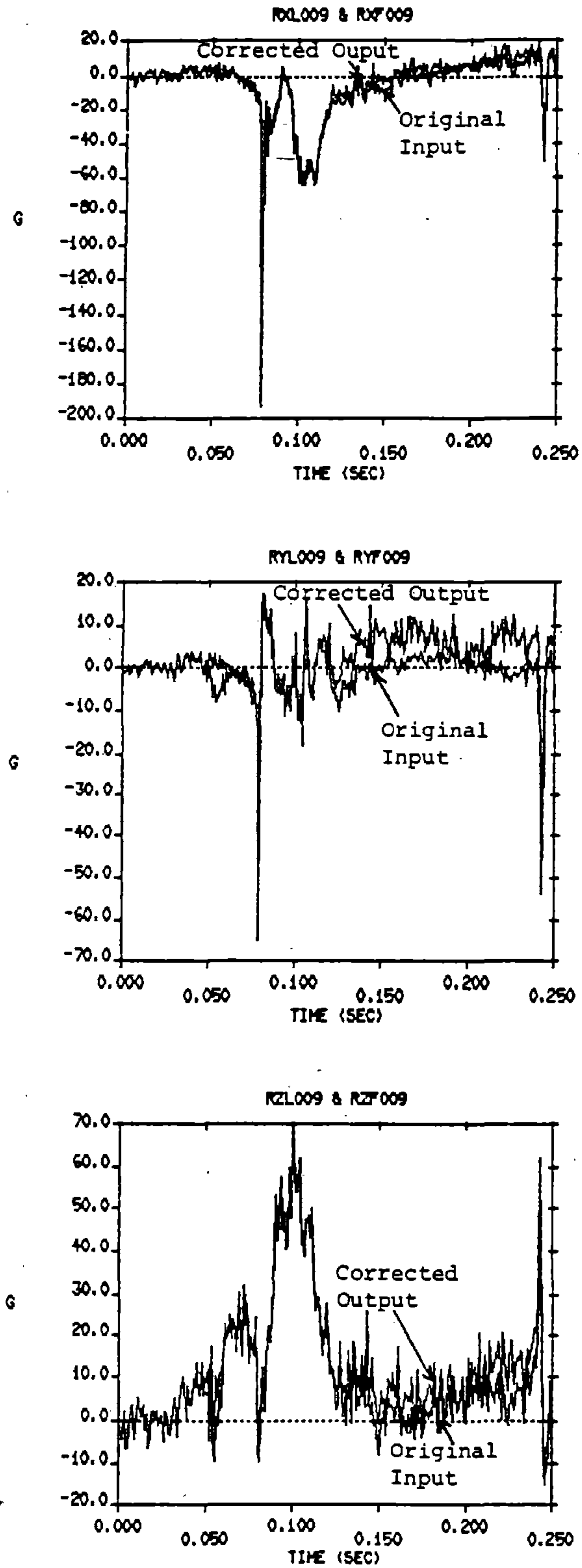


FIGURE 13. ORIGINAL INPUTS AND CORRECTED OUTPUTS (NOISY SIGNALS 250 POINTS)

5.5 DOUBLE PRECISION

The algorithm designated NAPLAB was modified to include the option of double precision output; the modified version being designated NAPLABD. It was thought that this output, used as input to NAPFLDRK would result in lower residual error in the corrected data. It was found, however, that the use of double precision resulted in no reduction in the residual error observed after correction by NAPFLDRK.

5.6 INTEGRATION TECHNIQUES

As mentioned in Section 4.2, two integration techniques were used in the development of the field model data correction algorithm (Appendix D). The first was Euler's integration formula given as:

$$y_{n+1} = y_n + \Delta x \left(\frac{dy}{dx}\right)_n$$

This formulation was used in NAPFLDEUL. (Appendix D-1). The second technique is Runge-Kutta integration provided as a subroutine in the IMSL library. This technique appears in NAPFlDRK (Appendix D-2). Tests were run to determine if one or the other of these techniques is more accurate. Inputs for these tests are shown in Figure 14. The error term matrix used was ERTERM004.DAT and 250 points were sampled. A low sample rate was used to make integration errors more pronounced. The results of processing with NAPLAB and then correcting with NAPFLDEUL for the X, Y and Z axes are shown in Figures 15, 16 and 17, respectively. The results of processing with NAPLAB and then correcting with NAPFLDRK are shown in Figures 18, 19 and 20, respectively. On close inspection, it appears that NAPFLDRK corrected the data slightly better than NAPFLDEUL. The differences were slight, however, and as is shown in Section 6.2, the biggest contributor to the error is low sample rate and not the integration scheme.

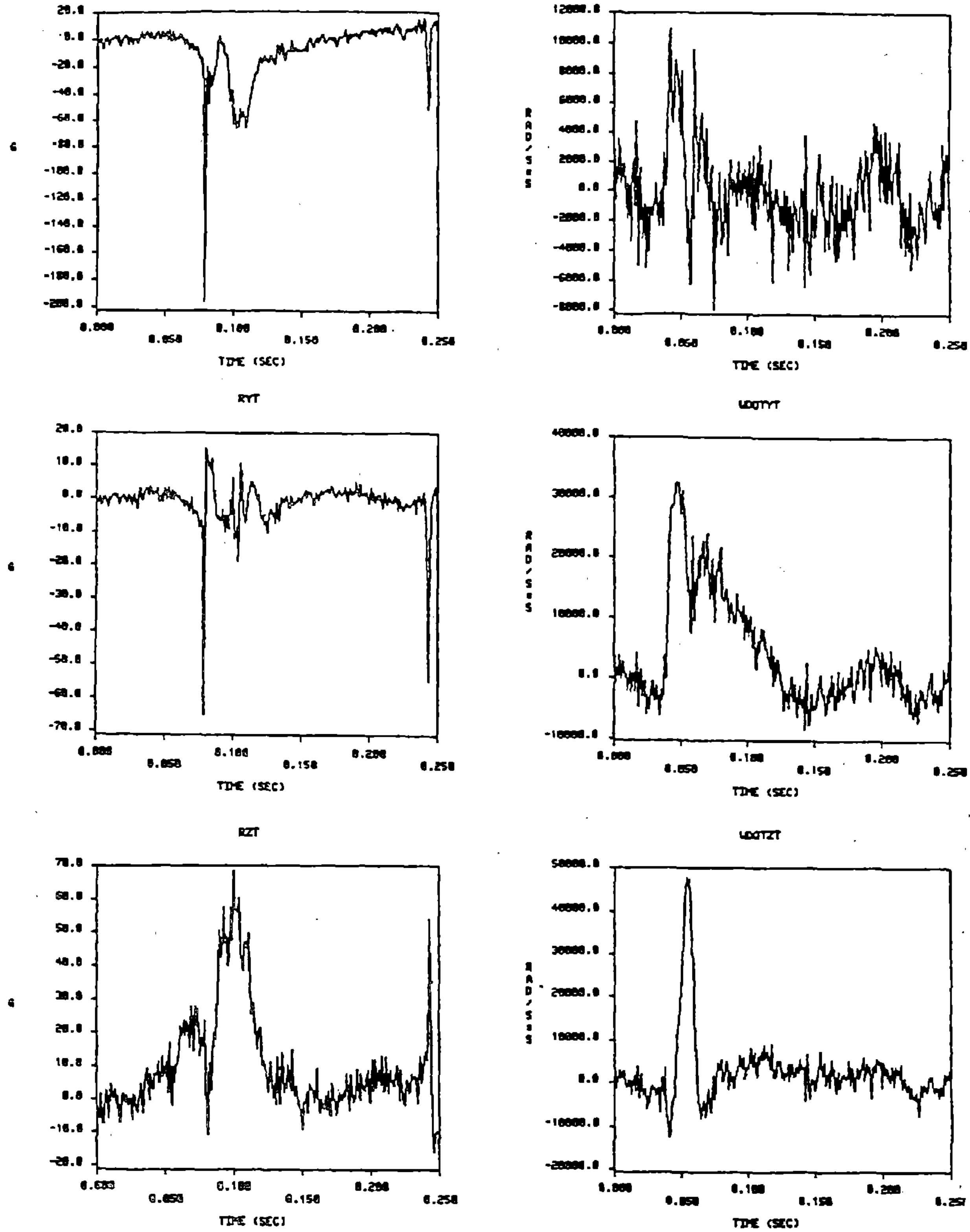


FIGURE 14. ORIGINAL INPUTS FOR INTEGRATION TESTS

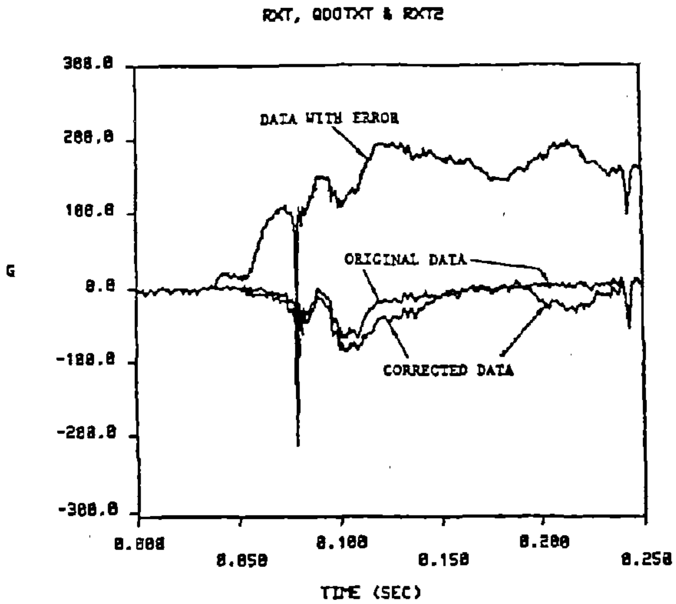


FIGURE 15. COMPARISON OF ORIGINAL AND CORRECTED DATA FOR X LINEAR ACCELERATION USING NAPFLDEUL

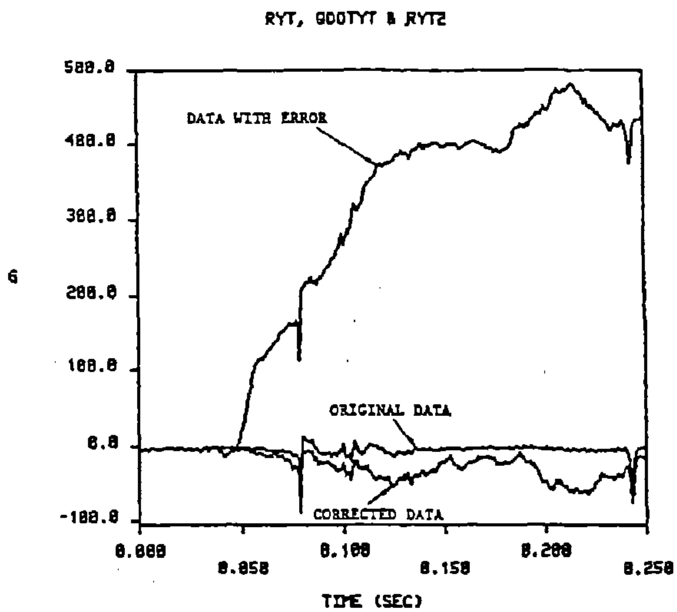


FIGURE 16. COMPARISON OF ORIGINAL AND CORRECTED DATA FOR Y LINEAR ACCELERATION USING NAPFLDEUL

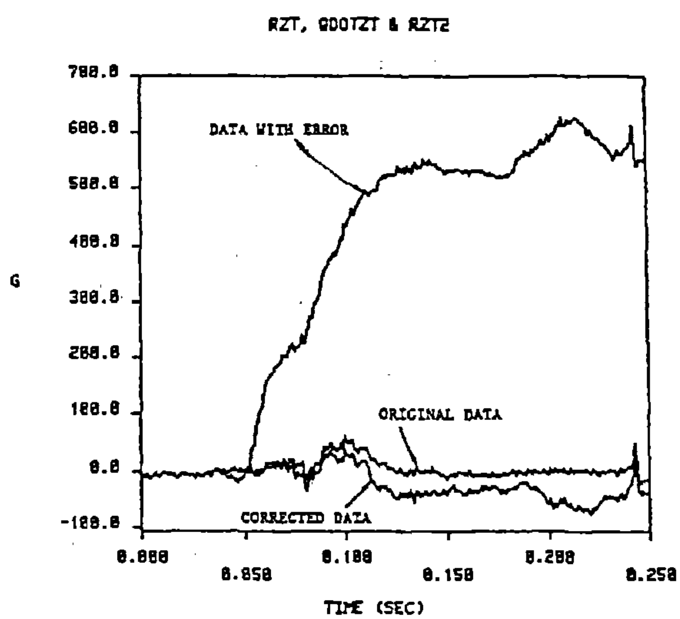


FIGURE 17. COMPARISON OF ORIGINAL AND CORRECTED DATA FOR Z LINEAR ACCELERATION USING NAPFLDEUL

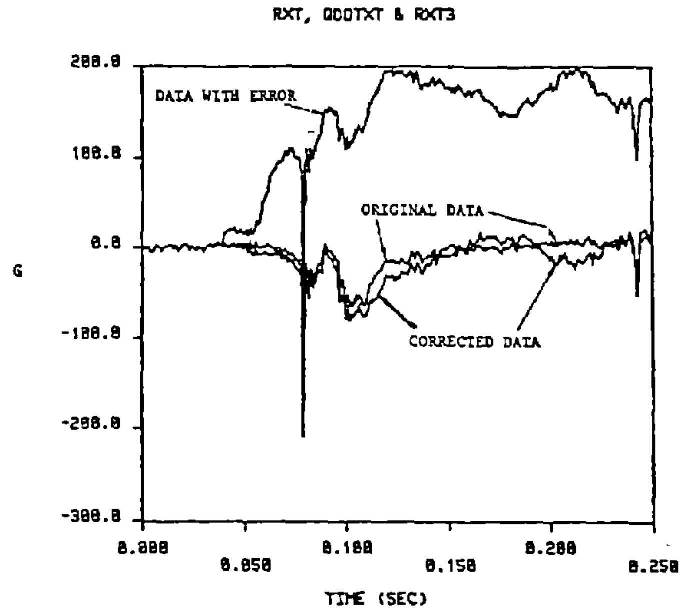


FIGURE 18. COMPARISON OF ORIGINAL AND CORRECTED DATA FOR X LINEAR ACCELERATION USING NAPFLDRK

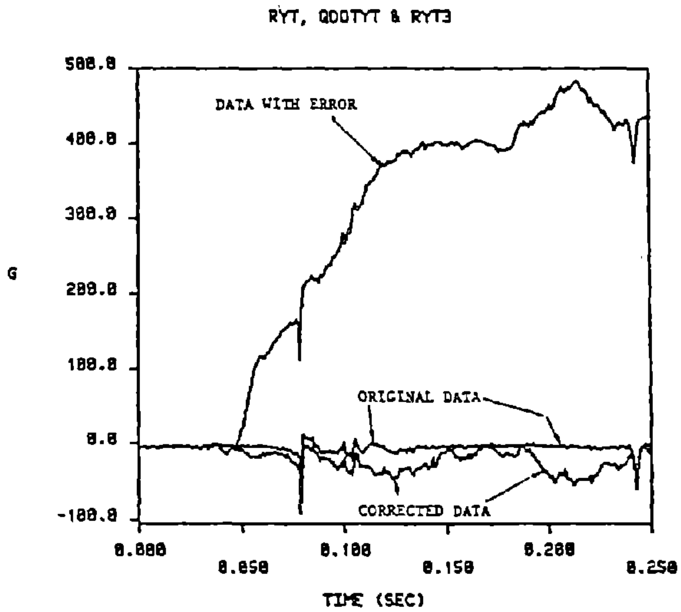


FIGURE 19. COMPARISON OF ORIGINAL AND CORRECTED DATA FOR Y LINEAR ACCELERATION USING NAPFLDRK

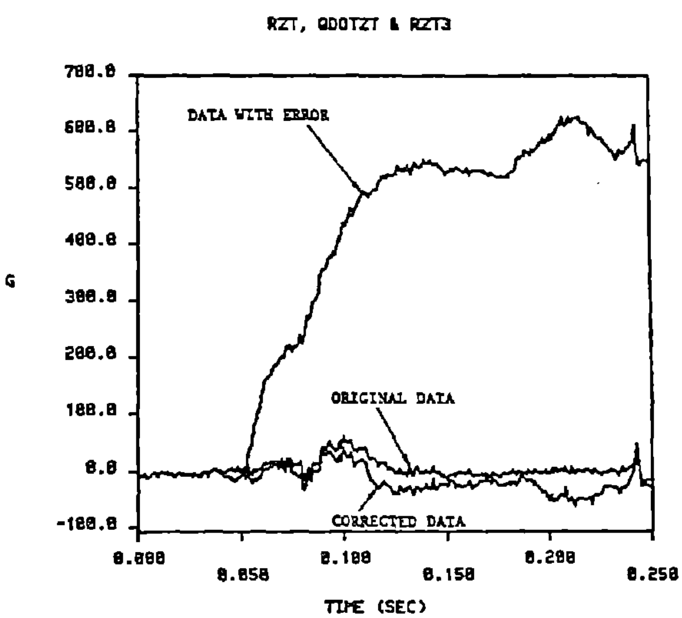


FIGURE 20. COMPARISON OF ORIGINAL AND CORRECTED DATA FOR Z LINEAR ACCELERATION USING NAPFLDRK

6. PARAMETRIC SENSITIVITY INVESTIGATION (PERFECT TRANSDUCERS)

6.1 SYSTEM GEOMETRY, COORDINATE SYSTEM SELECTION AND SAMPLE RATE

6.1.1 ENDEVCO Configuration

If perfect accelerometers are presumed and an ENDEVCO mount as shown in Figure 6 is used, there are three coordinate configurations that are immediately suggested. The first would have its principal axes along the edges of the mount with the origin at the apex of the mount; the second would place the origin at the intersection of the sensitive axes of the three central accelerometers with the three principal axes parallel to the mount edges; and the third would have principal axes such that the seismic centers of the acclerometers are located in the three planes formed by the coordinate system and the sensitive axes are normal to these planes . The designation of any one of these coordinate systems is arbitrary and will result in different system error matrices.

To investigate the effect of the choice of coordinate system location on measurement errors, tests were run using the three error term matrices ERTERM001 (axes along the edges of the mount), ERTERM002 (axes along the sensitive axes of the three centrally located transducers) and ERTERM009 (seismic centers located in the three planes formed by the coordinate system) Two simple input situations were used; (1) angular acceleration about the y-axis only and (2) equal angular acceleration about all three axes. The input linear accelerations were set to zero. Large half-sine inputs (20,000 rad/sec^2 peak) were used. The results of these exercises (Tests 1 through 7) are shown in Figures 21 through 41.

After examining the tests that used angular acceleration about the y-axis only (Tests 1, 3 and 5), larger errors were encountered when using the error term ERTERM002 smaller errors when using the error term ERTERM001 and even smaller errors when using error term ERTERM009. This is because choosing the axes such that the seismic centers are located in the three planes of the coordinate system causes the coefficients of the velocity squared term (about the y-axis) to go to zero. This velocity squared term is the major contributor to the errors seen with the other two axes selections. This may be seen from the following equations.

For input about the y-axis only:

QDOTX = AL0 + AL2*WDOTY + AL5*WY*WY
QDOTZ = CL0 + CL2*WDOTY + CL5*WY*WY

QDOTX and QDOTZ are the errors in the measured linear accelerations in the x-and z-directions. These errors are equal to zero only if AL0 = AL2 = AL5 = 0 and CL0 = CL2 = CL5 = 0. They are small if AL5 = 0 and CL5 = 0 which is the case when the axes are chosen such that seismic centers are located in the three planes formed by the coordinate system. In addition, the residual error after correction with NAPFLDRK appears to be proportional to WY*WY so if AL5 and CL5 are non-zero the errors have the potential for becoming relatively large. When AL5 = 0 and CL5 = 0, the correction algorithm reduced the residual errors to essentially zero.

The advantage to choosing one coordinate system over another is not as apparent when equal angular accelerations are input about all three axes (Tests 2, 4 and 6). This is probably due to the fact that there is now additional error buildup due to angular velocity cross product terms in the system equations. In all of the cases with inputs of 20,000 rad/sec^2 about all three axes, the residual errors in the measurement of linear acceleration were from 29g to 35g.

It must be remembered that the test situation was made intentionally unrealistic to present a worst case situation to the correction algorithm. Such large angular accelerations (20,000 rad/sec^2) with linear accelerations equal to zero would not be expected in a crash. The use of a worst case input allows us to more easily compare the options with regard to coordinate selection. In this regard, for the given test scenarios, the best choice appears to be that in which the seismic centers of the accelerometers are located in the three planes formed by the coordinate system (e.g. error term matrix ERTERM009).

For the given test scenario, if the angular acceleration is reduced by an order of magnitude (to 2,000 rad/sec^2), the residual errors are reduced by two orders of magnitude (to less than 1g) as shown in Figures 39-41. This is to be expected, given the relationship of angular acceleration to the velocity squared and cross product terms.

Further tests were run with half-sine input pulses to determine the effects of input pulse magnitude, input pulse duration and data sampling frequency on the residual errors measured. Figure 42 illustrates typical input signals used. The linear acceleration inputs were set to zero in these tests. Figure 43 illustrates the apparent linear acceleration measured and Figure 44 shows typical residual errors after correction with NAPFLDRK. A summary of the results of these tests is given in Figures 45 and 46. Improved data correction as sample rate increases is apparent. It is also seen that the larger the pulse (greater magnitude or longer duration), the greater the residual error. Figures 45 and 46 may be used by the investigator to estimate the maximum error to be expected as a result of system geometry and computational characteristics.

Tests were also run with field data (Tests 16 through 19) to examine the effects of sample rate. Data was input about only the y-axis in Figures 47 and 50. The NAPLABD outputs in Figures 48 and 51 are seen to be proportional to the input as expected, since the omega squared and cross product terms are zero. Residual errors (Figures 49 and 52) are small and equal for both tests. Tests 18 and 19 had inputs about all three axes (Figures 53 and 56). The NAPLABD outputs are seen in Figures 54 and 57 and the residual errors are seen in Figures 55 and 58. It can be seen that the residual errors in the test that used 1,993 points are significantly smaller than in test that used 250 points. It is apparent then that significant error buildup may occur in the output values for linear acceleration when there are angular inputs about more than one axis and the data sample rate is too low. The above condition is due to cumulative error that occurs in the integration routines of the corrective algorithm. It is also apparent that the residual error is inversely proportional to the integration sample rate and in general, the errors appear to be governed by the following relationship:

$$E = K \frac{M^2 T}{R}$$

where:

E = Residual error (g)

M = Peak value of half-sine pulse (rad/sec^2)

T = Duration of half-sine pulse (sec)

R = Data sample rate (samples/sec)

K = constant

For the ENDEVCO configuration and error matrix ERTERM009:

$$K = 7.2 \times 10^{-4}$$

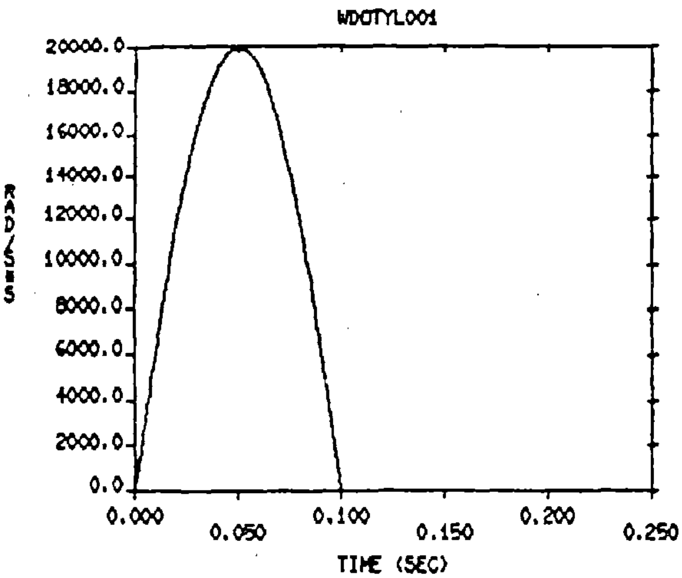
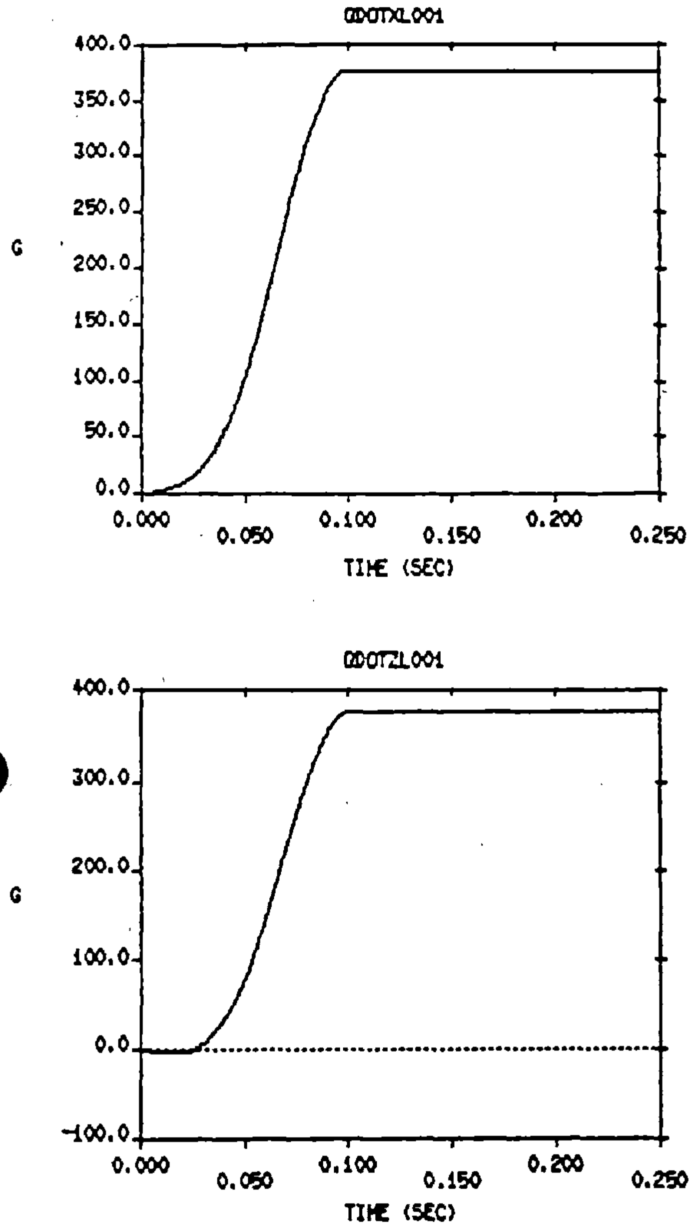


FIGURE 21. TEST #1 - INPUT PULSE
ENDEVCO MOUNT ERROR
TERM MATRIX = ERTERM009
Input = 20,000 rad/sec^2 0.1 sec, duration



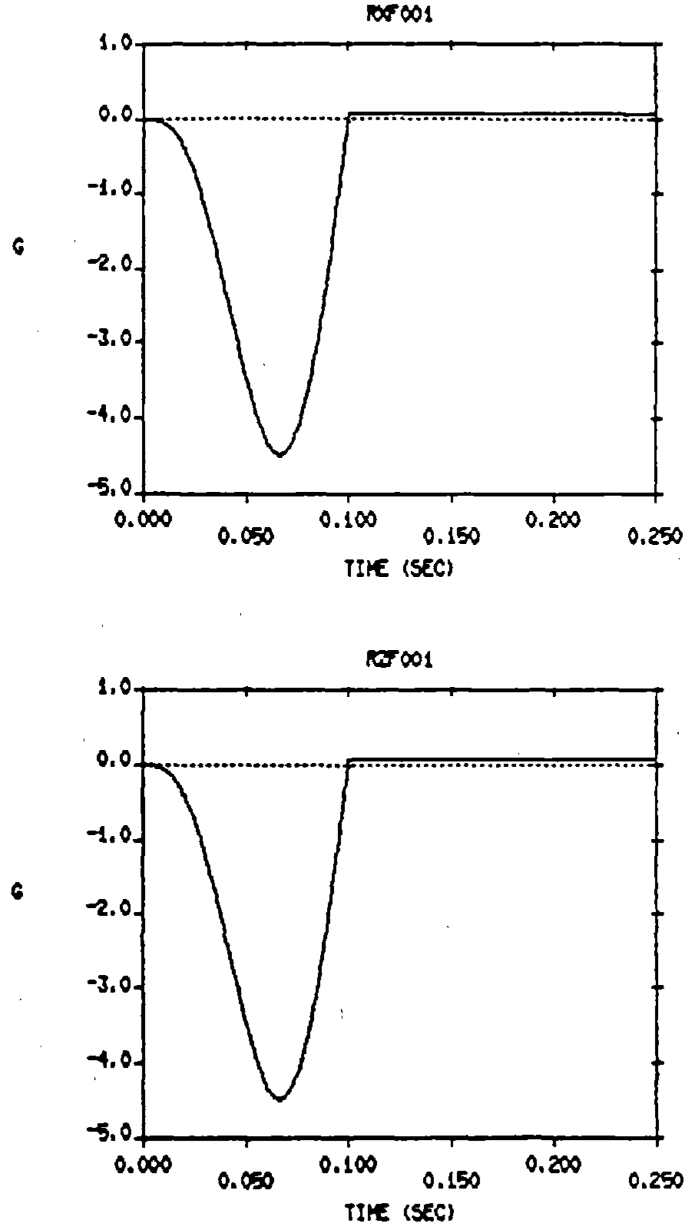


FIGURE 22. TEST #1 - APPARENT LINEAR ACCELERATIONS MEASURED BY NAPLABD SYSTEM

FIGURE 23. TEST #1 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK

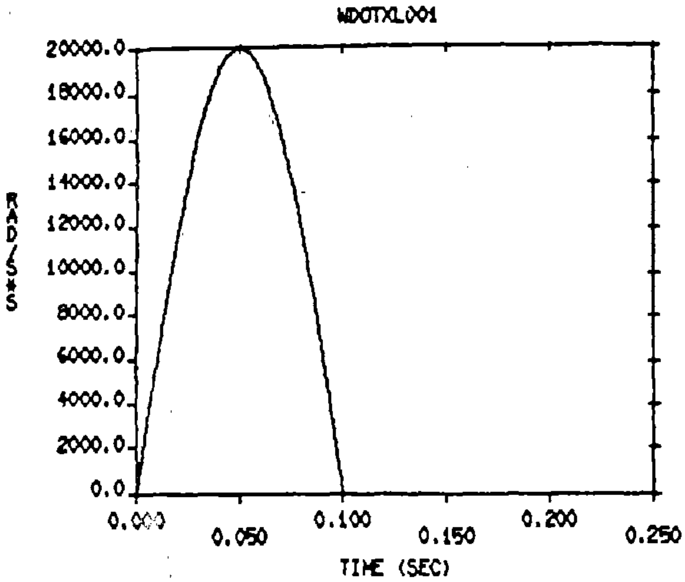
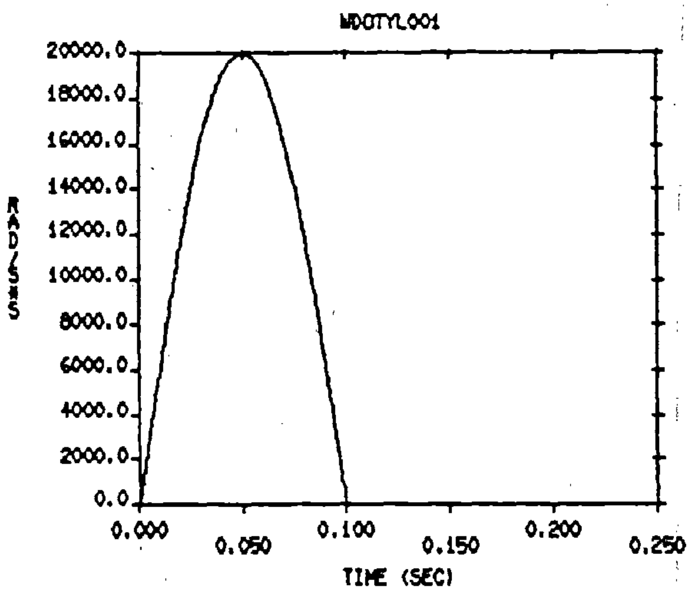
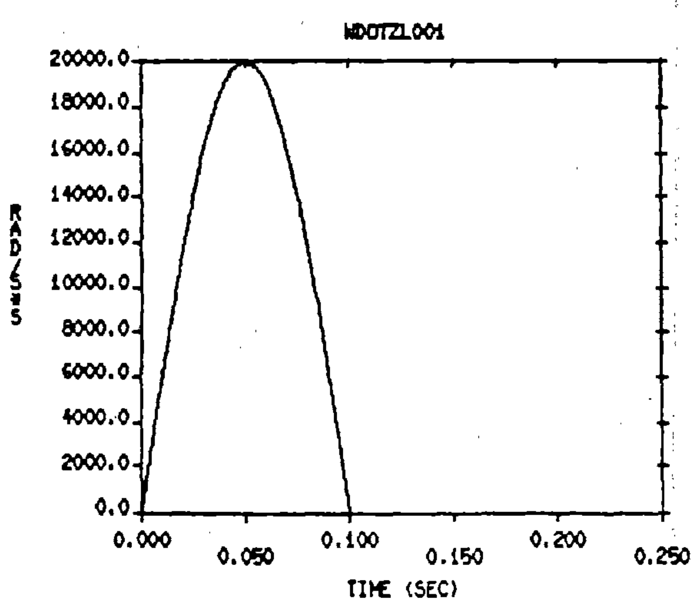


FIGURE 24. TEST #2 - INPUT PULSES ENDEVCO MOUNT ERROR TERM MATRIX = ERTERM001. Input = 20,000 rad/sec^2, About All Three Areas, 0.1 sec duration





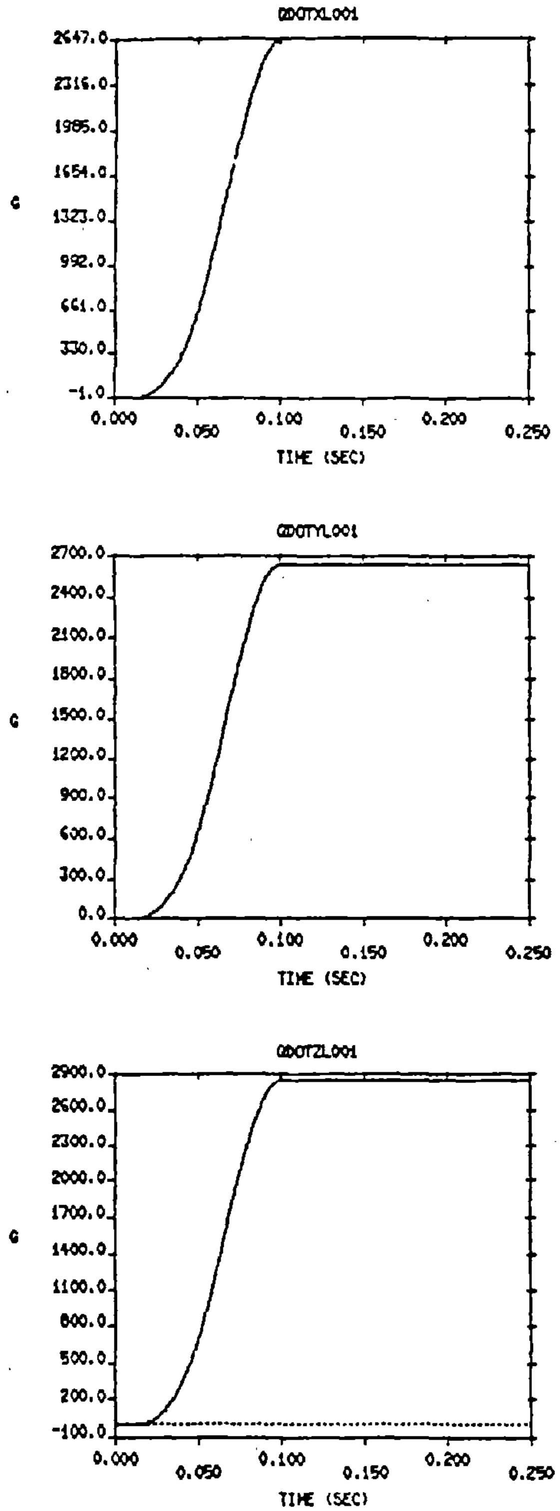


FIGURE 25. TEST #2 - APPARENT LINEAR ACCELERATIONS MEASURED BY THE NAPLABD SYSTEM

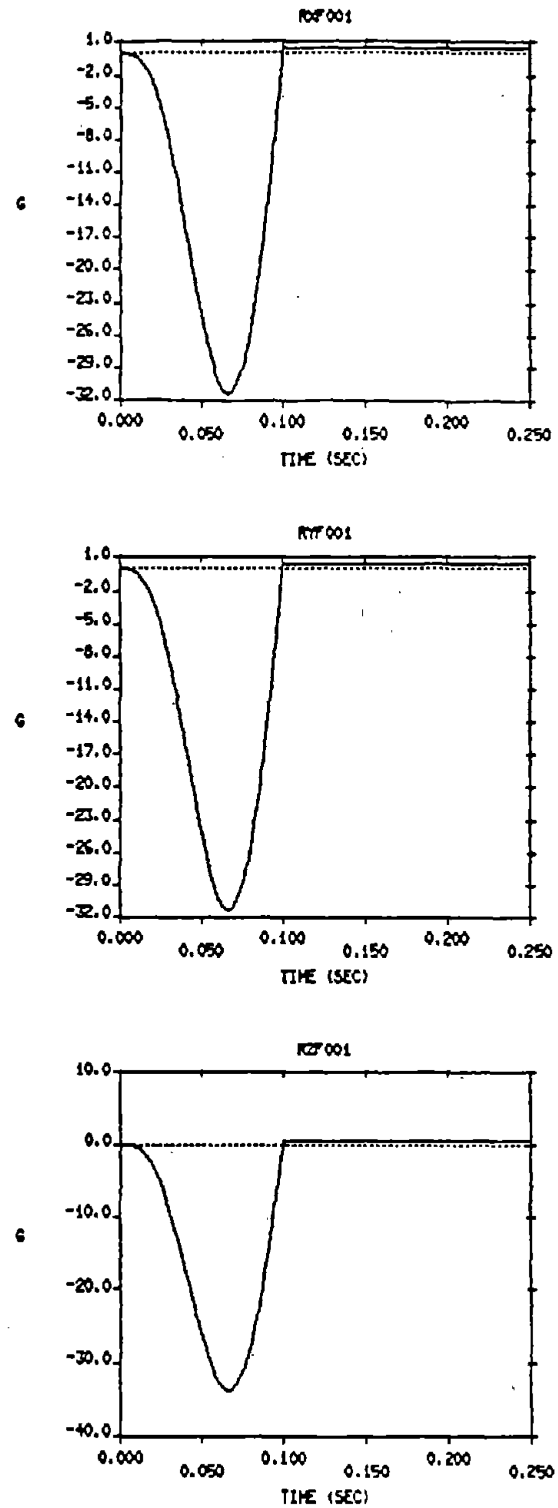


FIGURE 26. TEST #2 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK

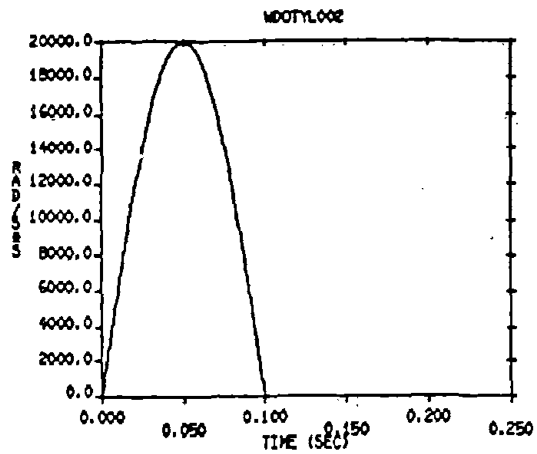


FIGURE 27. TEST #3 - INPUT PULSE ENDEVCO MOUNT ERROR TERM MATRIX = ERTERM002 Input = 20,000 rad/sec^2, 0.1 sec duration

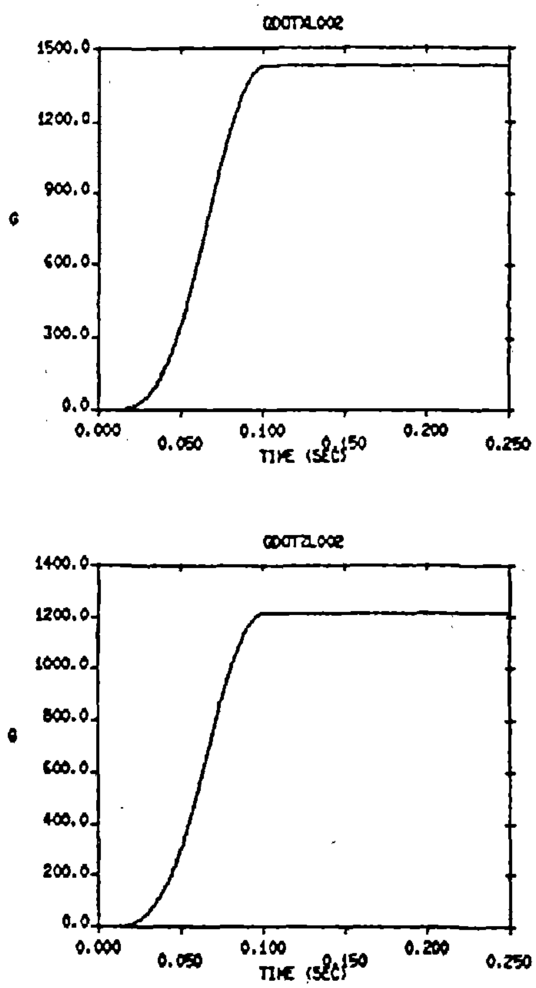


FIGURE 28. TEST #3 - APPARENT LINEAR ACCELERATIONS MEASURED BY NAPLABD SYSTEM

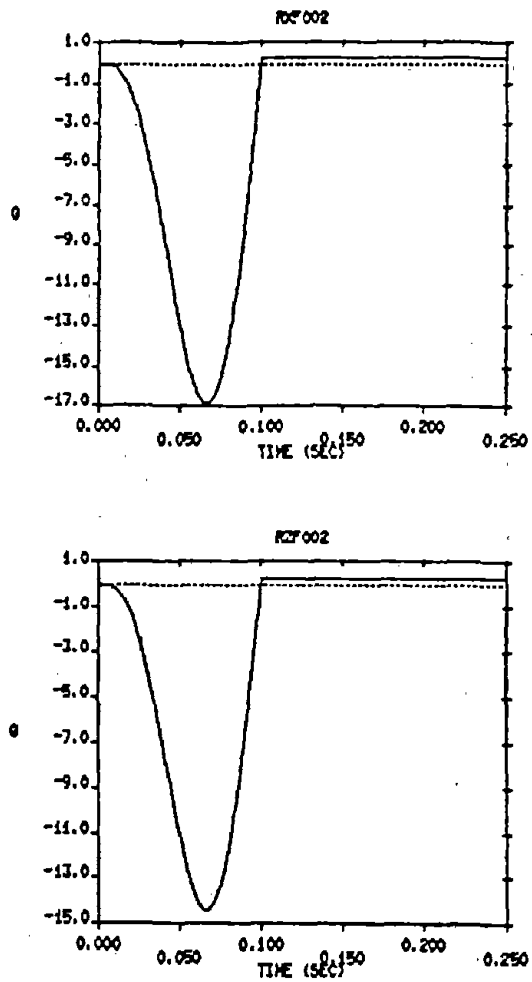


FIGURE 29. TEST #3 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK

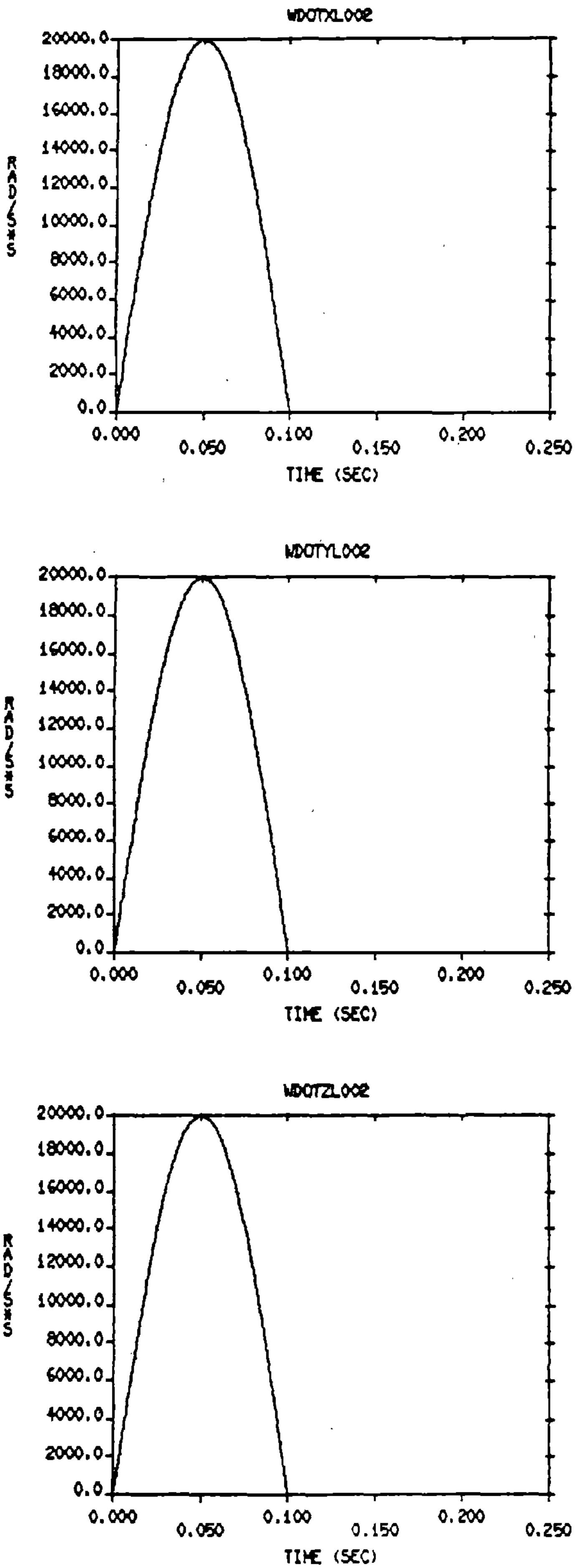


FIGURE 30. TEST #4 - INPUT PULSES ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM002 Input = 20,000 rad/sec^2. About All Three Axes, 0.1 sec duration

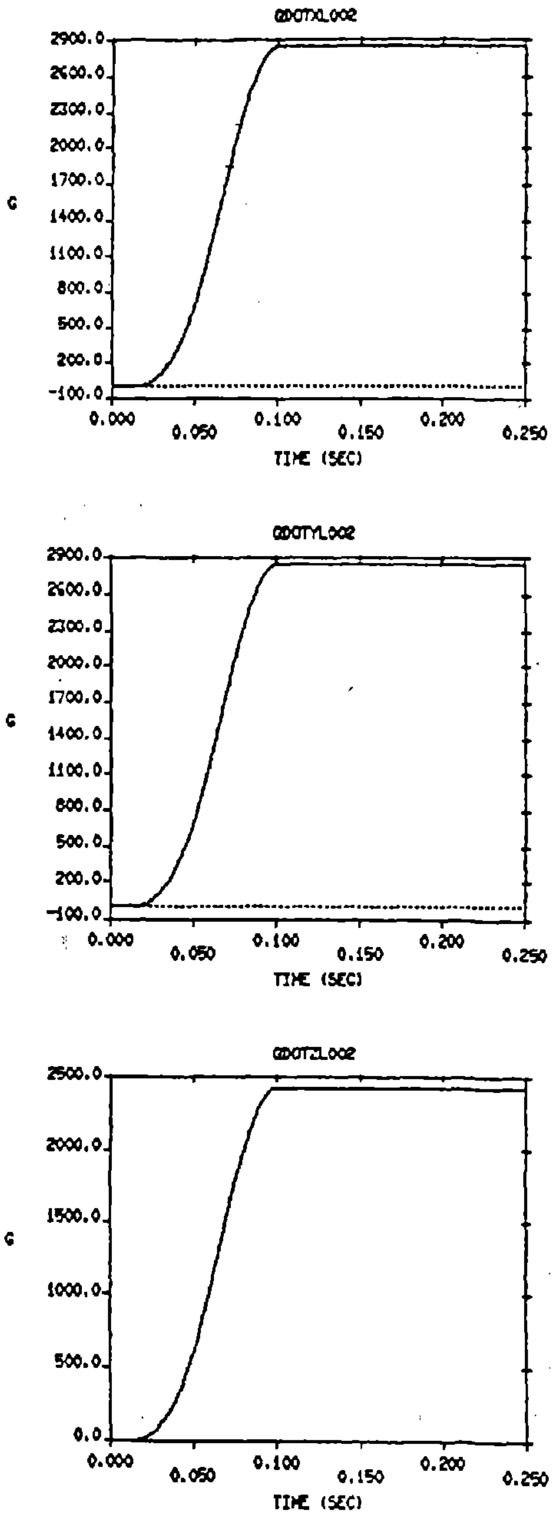


FIGURE 31. TEST #4 - APPARENT LINEAR ACCELERATIONS MEASURED BY NAPLABD SYSTEM

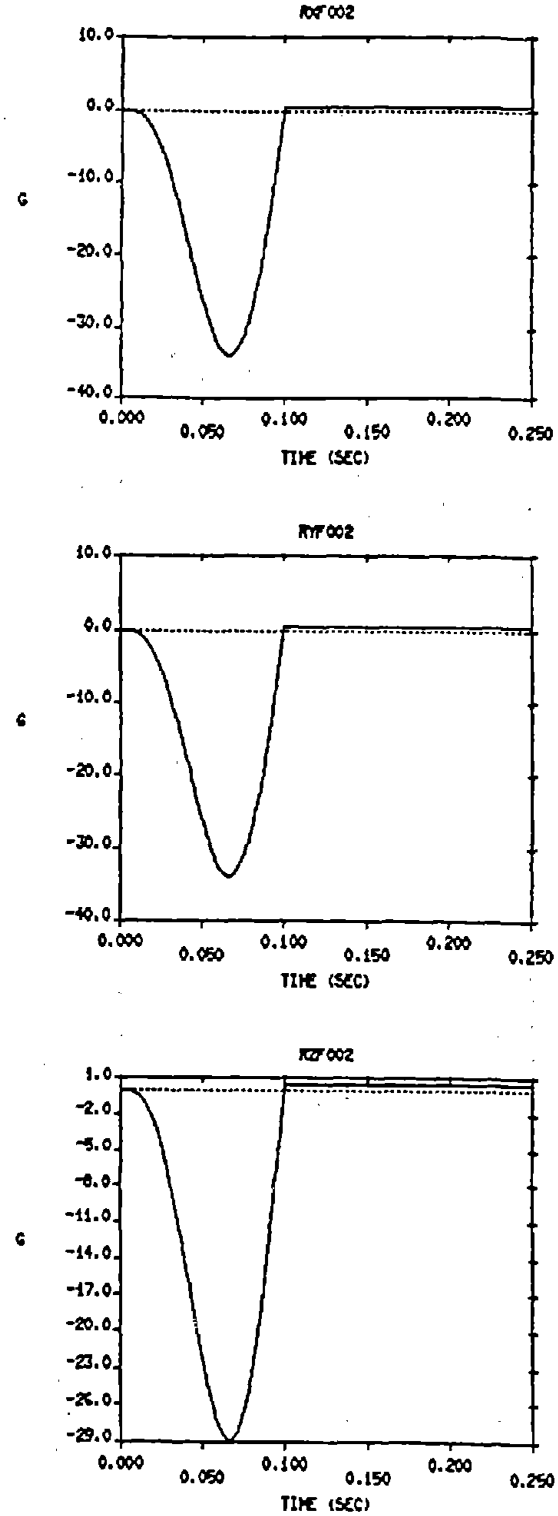


FIGURE 32. TEST #4 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK

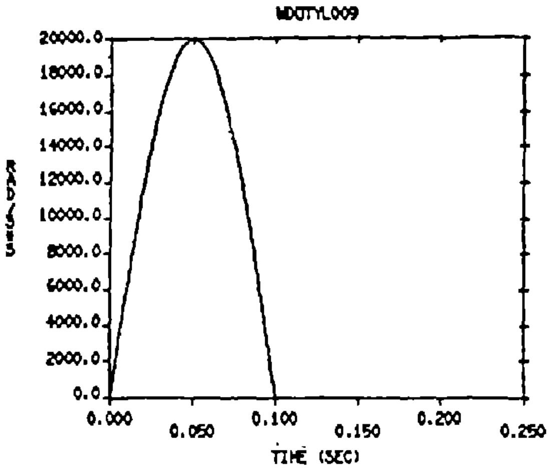


FIGURE 33. TEST #5 - INPUT PULSE ENDEVCO MOUNT ERROR TERM MATRIX = ERTERM009 Input = 20,000 rad/sec^2, 0.1 sec duration

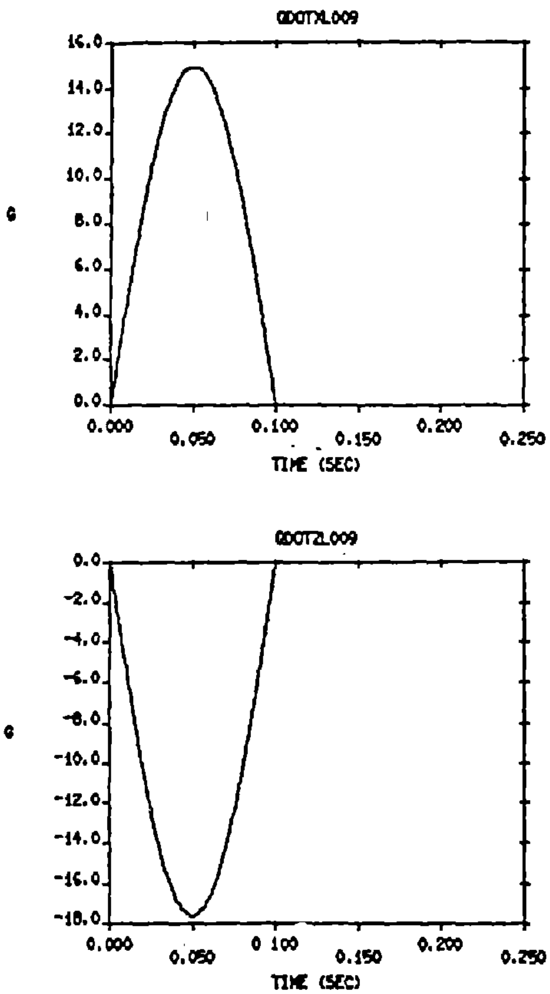


FIGURE 34. TEST #5 - APPARENT LINEAR ACCELERATIONS MEASURED BY NAPLABD SYSTEM

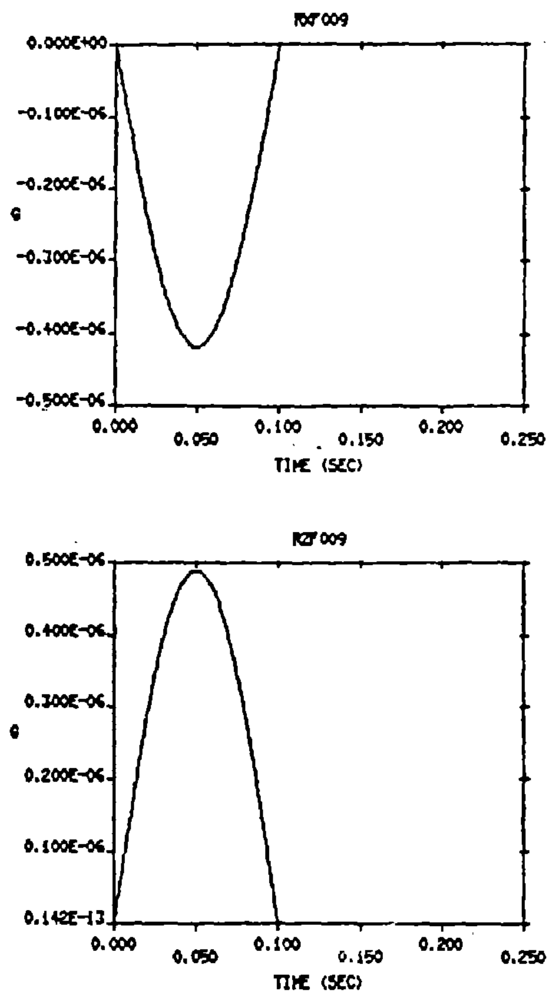


FIGURE 35. TEST #5 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK

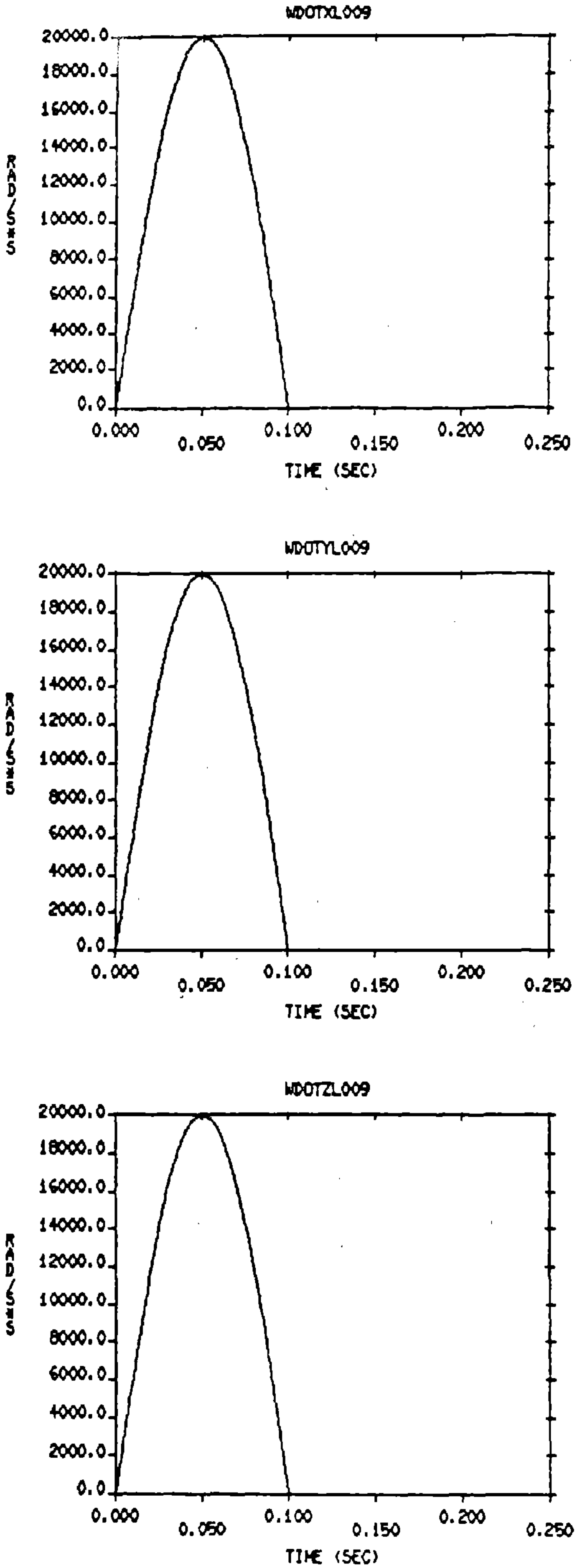


FIGURE 36. TEST #6 - INPUT PULSES ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM009 Input = 20,000 rad/sec^2. About All Three Axes, 0.1 sec duration

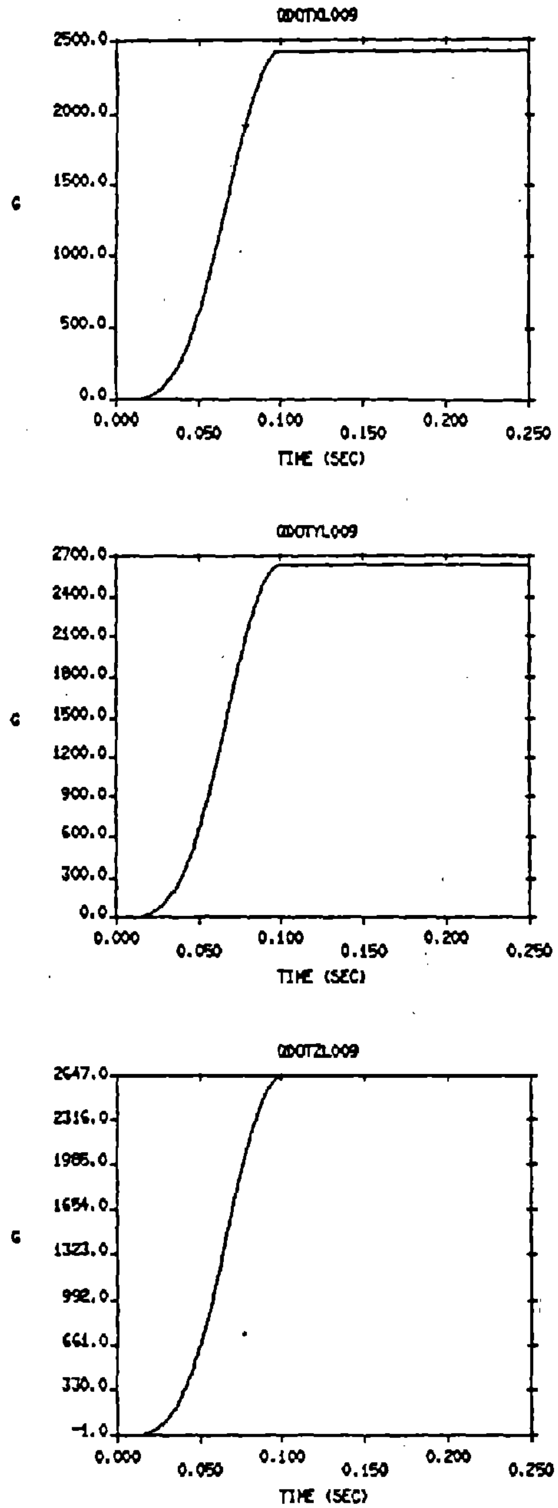


FIGURE 37. TEST #6 - APPARENT LINEAR ACCELERATIONS MEASURED BY NAPLABD SYSTEM

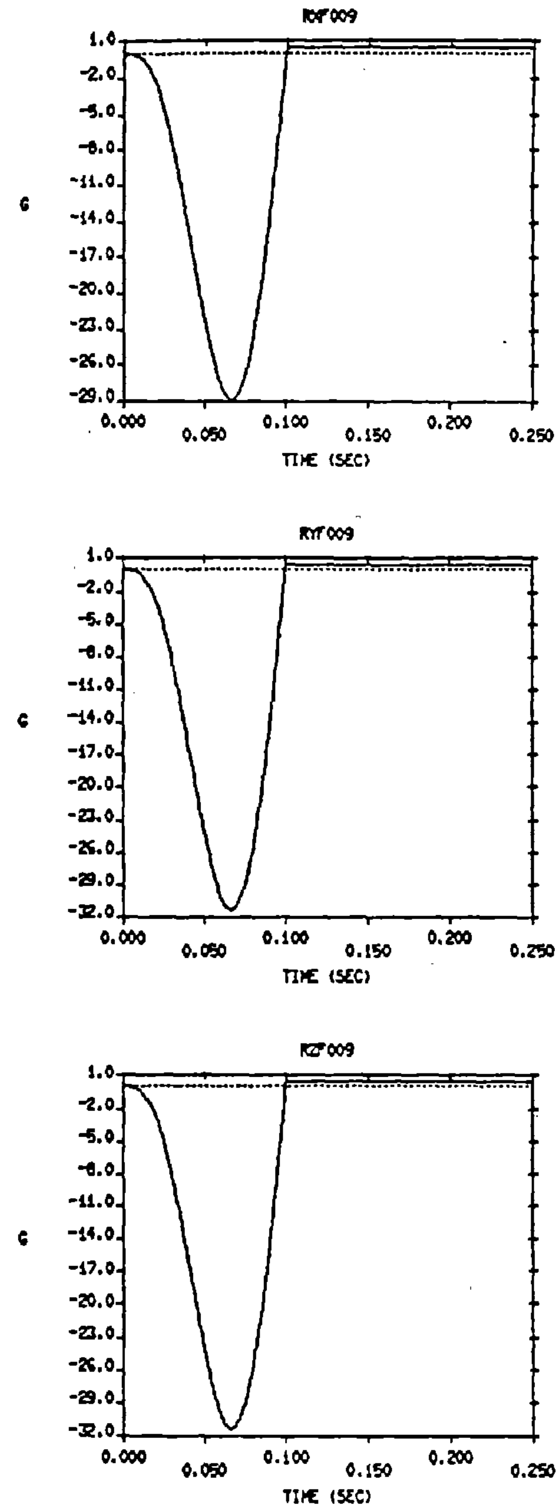


FIGURE 38. TEST #6 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK

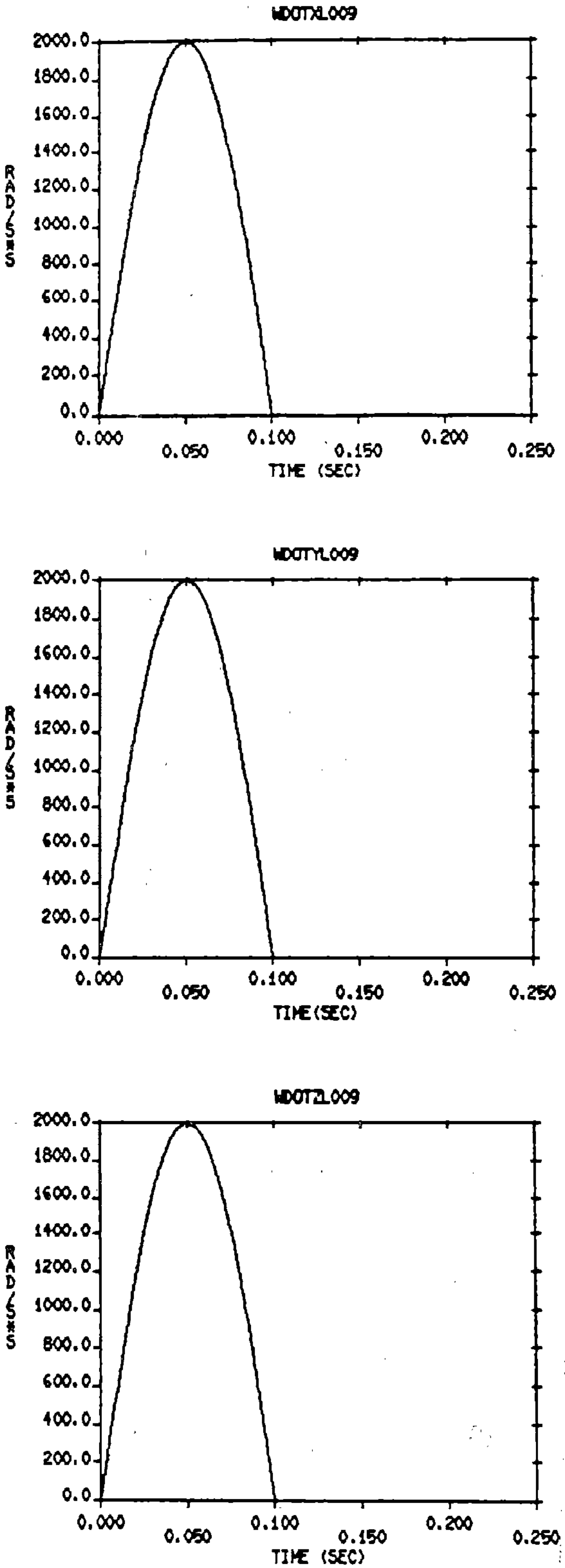


FIGURE 39. TEST #7 - INPUT PULSES ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM009 Input = 2,000 rad/sec^2. About All Three Axes, 0.1 sec duration

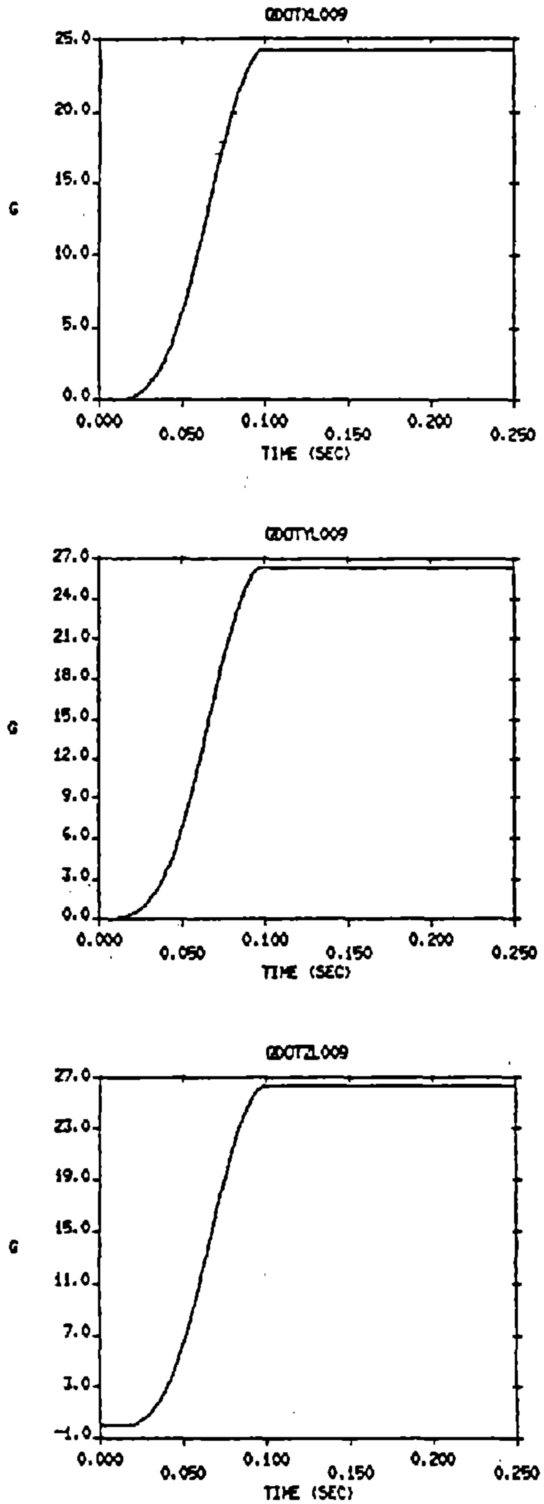


FIGURE 40. TEST #7 - APPARENT LINEAR ACCELERATIONS MEASURED BY NAPLABD SYSTEM

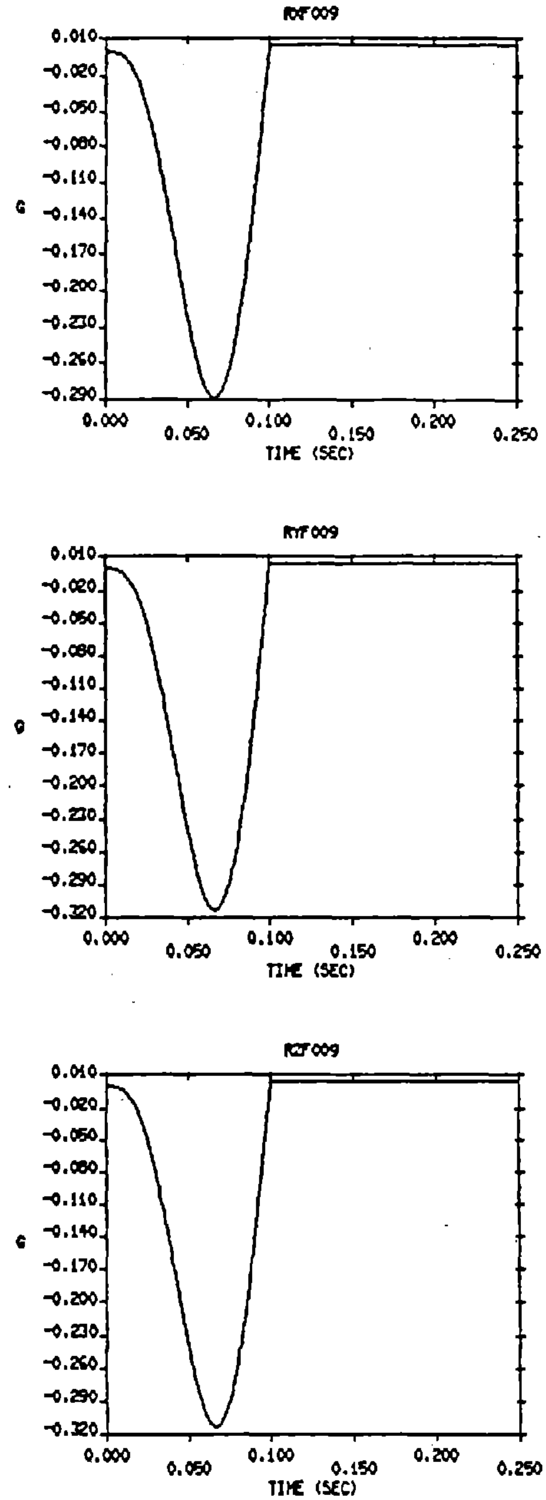
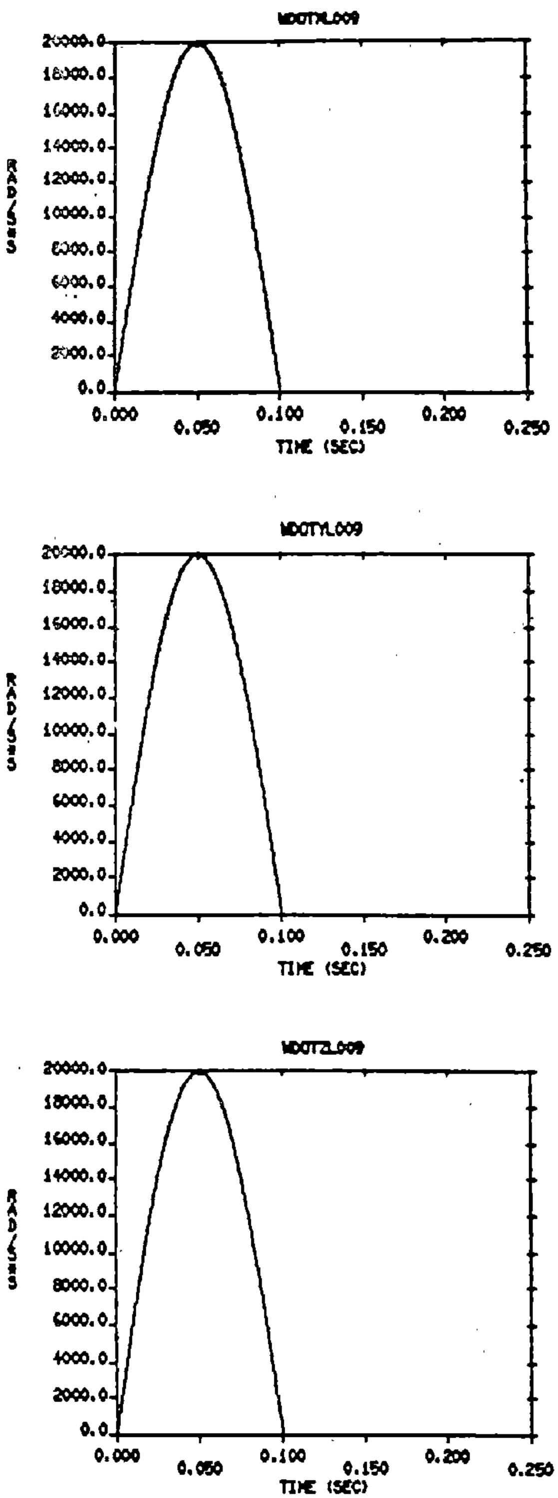


FIGURE 41. TEST #7 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK



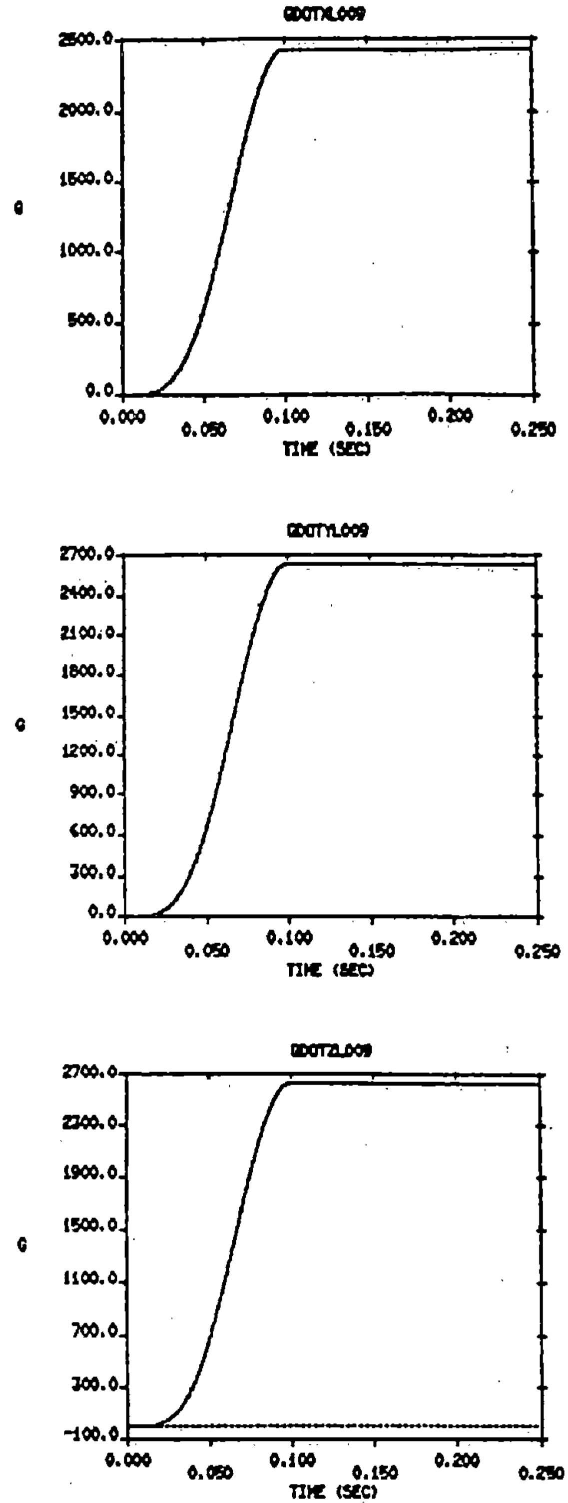


FIGURE 42. TYPICAL INPUT PULSES (SAMPLE RATE TESTING)

FIGURE 43. TYPICAL APPARENT LINEAR ACCELERATIONS (SAMPLE RATE TESTING)

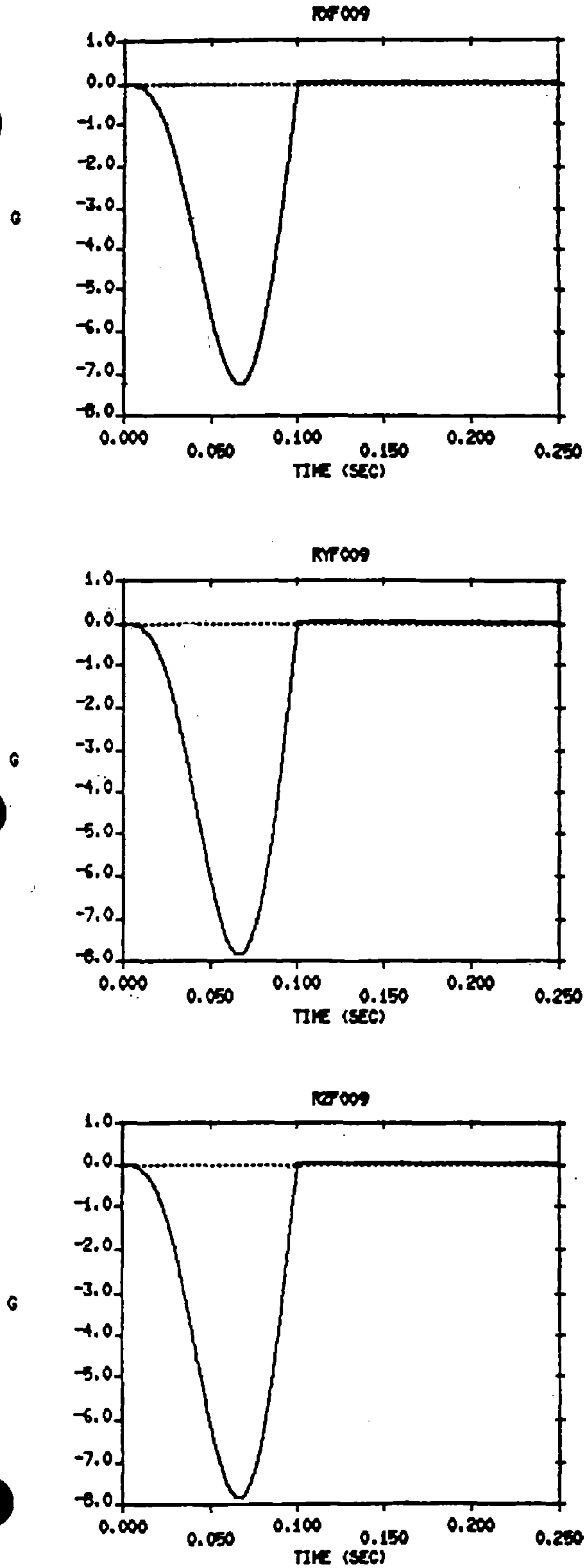


FIGURE 44. TYPICAL RESIDUAL ERRORS (SAMPLE RATE TESTING)

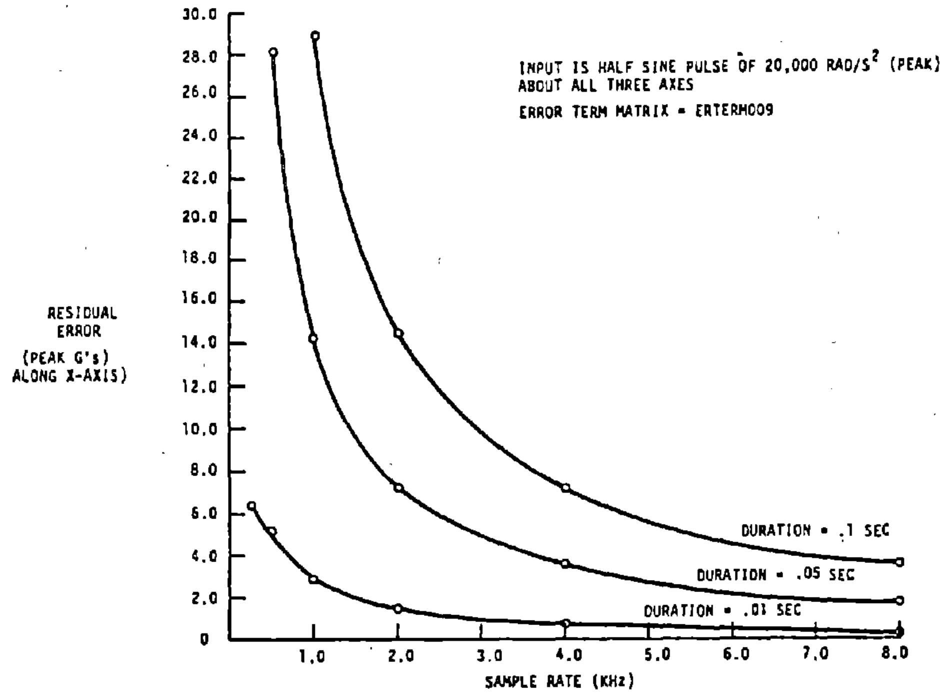


FIGURE 45. RESIDUAL ERRORS VS. SAMPLE RATE ENDEVCO CONFIGURATION, INPUT = 20,000 rad/sec^2

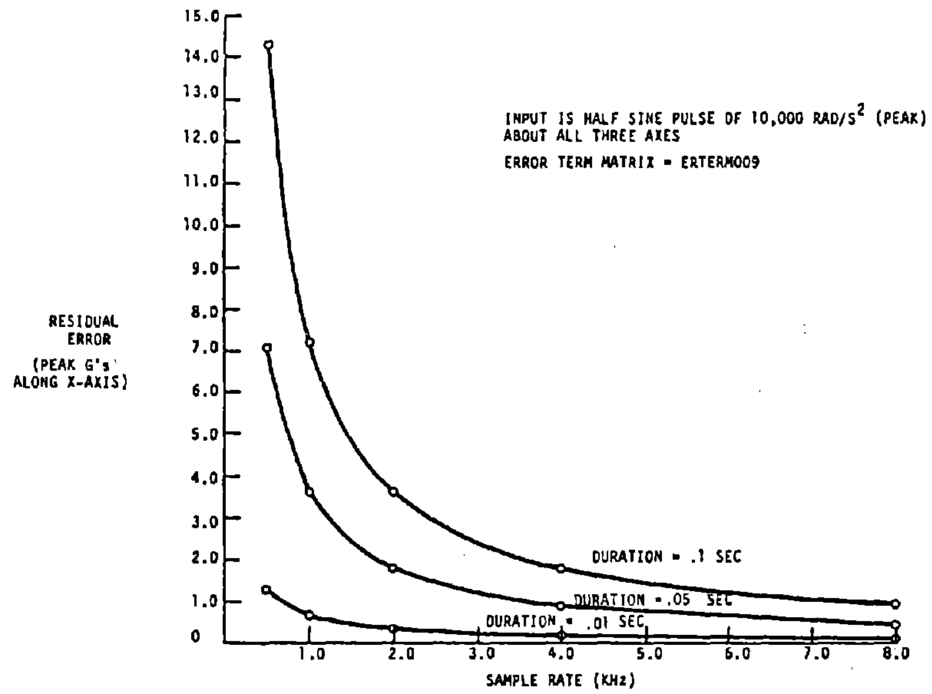


FIGURE 46. RESIDUAL ERRORS VS. SAMPLE RATE ENDEVCO CONFIGURATION, INPUT = 10,000 rad/sec^2

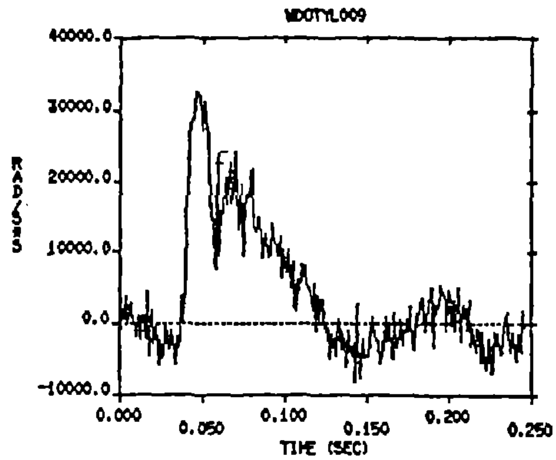


FIGURE 47. TEST #16 - FIELD DATA INPUT PULSE (250 POINTS) ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM009

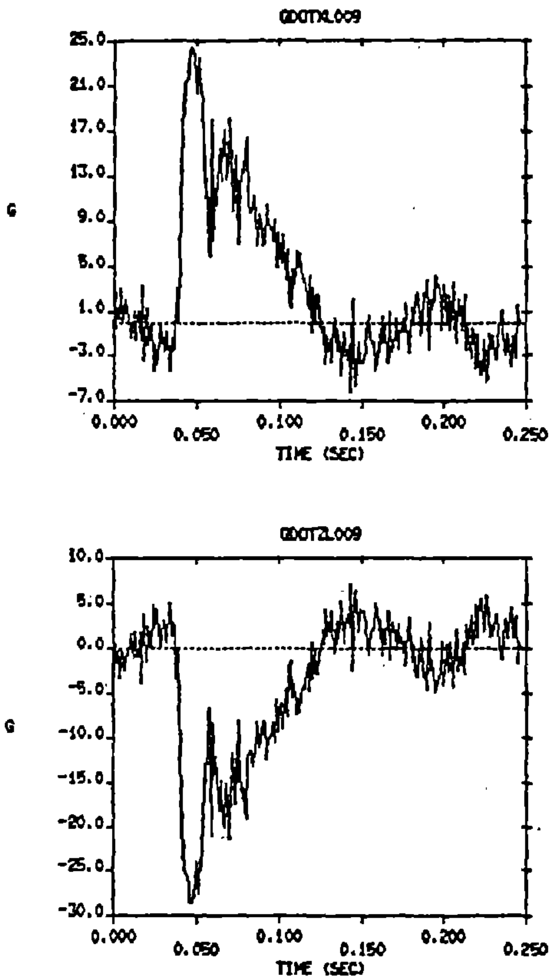


FIGURE 48. TEST #16 - APPARENT LINEAR ACCELERATIONS MEASURED BY NAPLABD SYSTEM

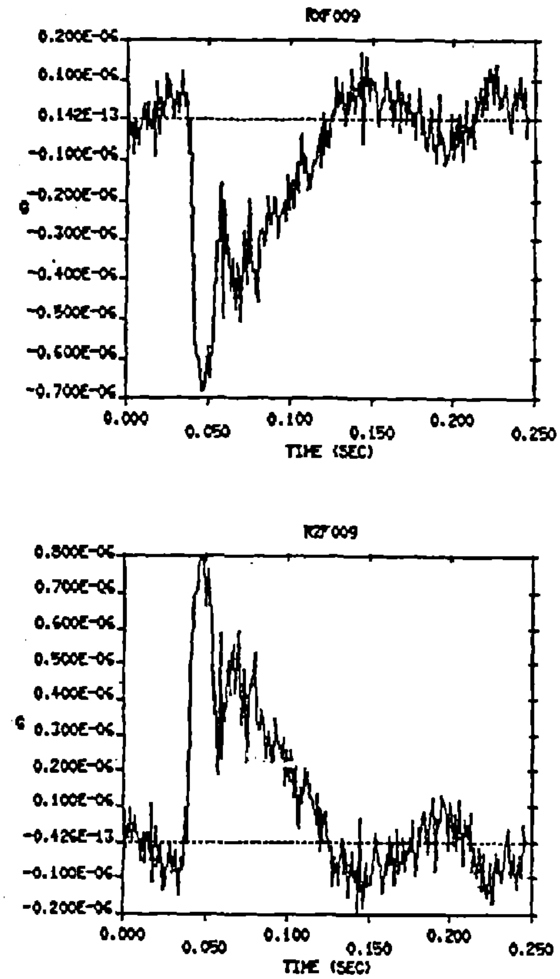


FIGURE 49. TEST #16 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK

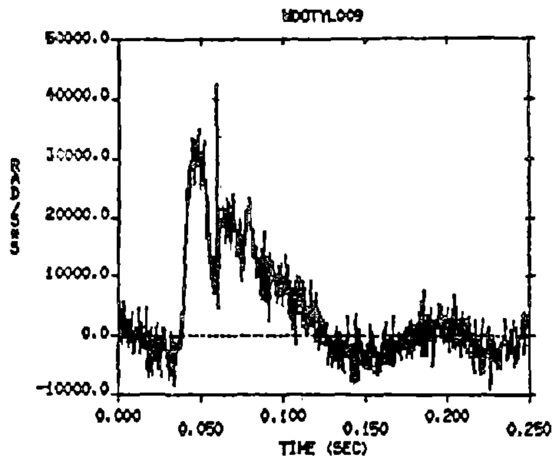
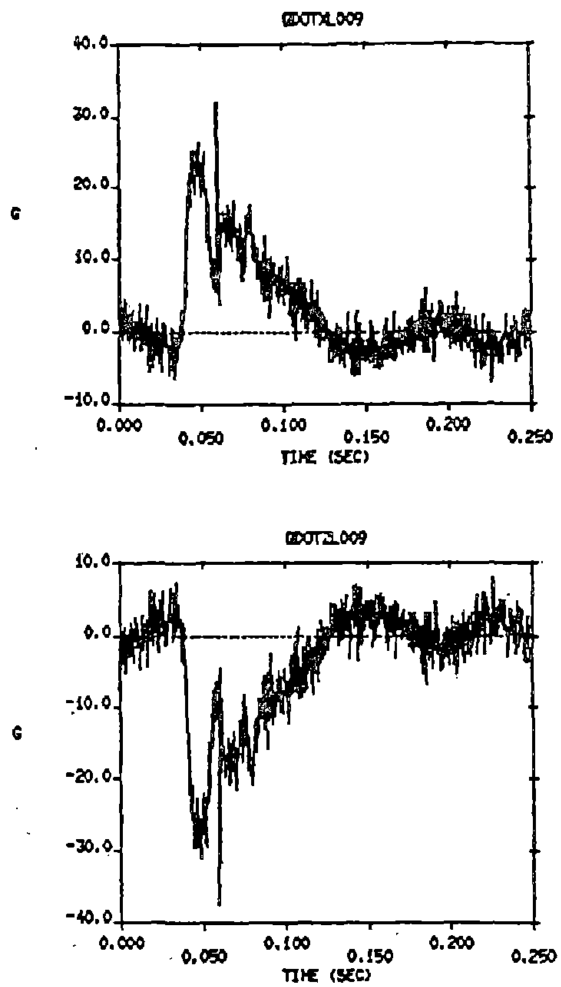


FIGURE 50. TEST #17 - FIELD DATA INPUT PULSE (1,993 POINTS) ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM009



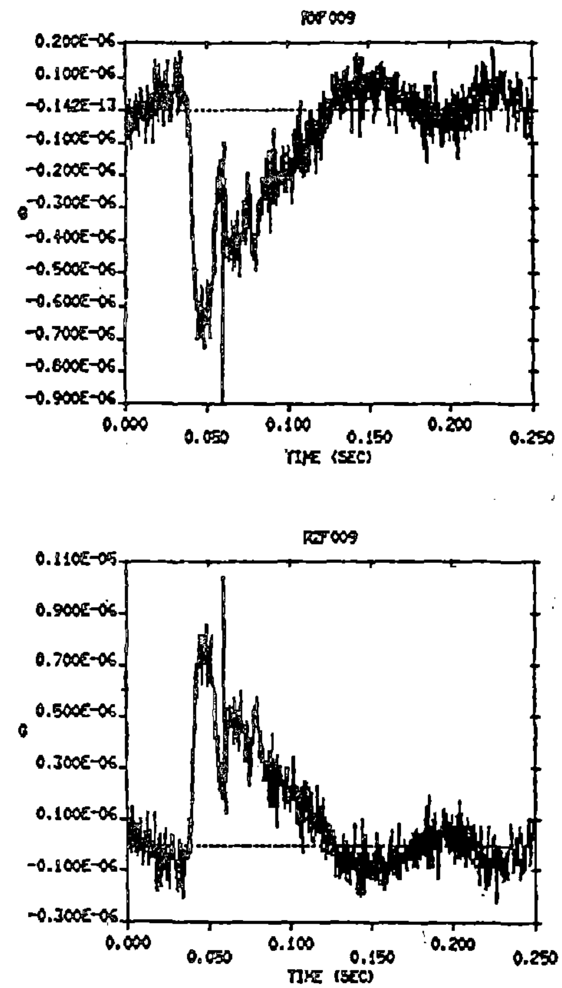


FIGURE 51. TEST #17 - APPARENT LINEAR ACCELERATIONS MEASURED BY NAPLABD SYSTEM

FIGURE 52. TEST #17 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK

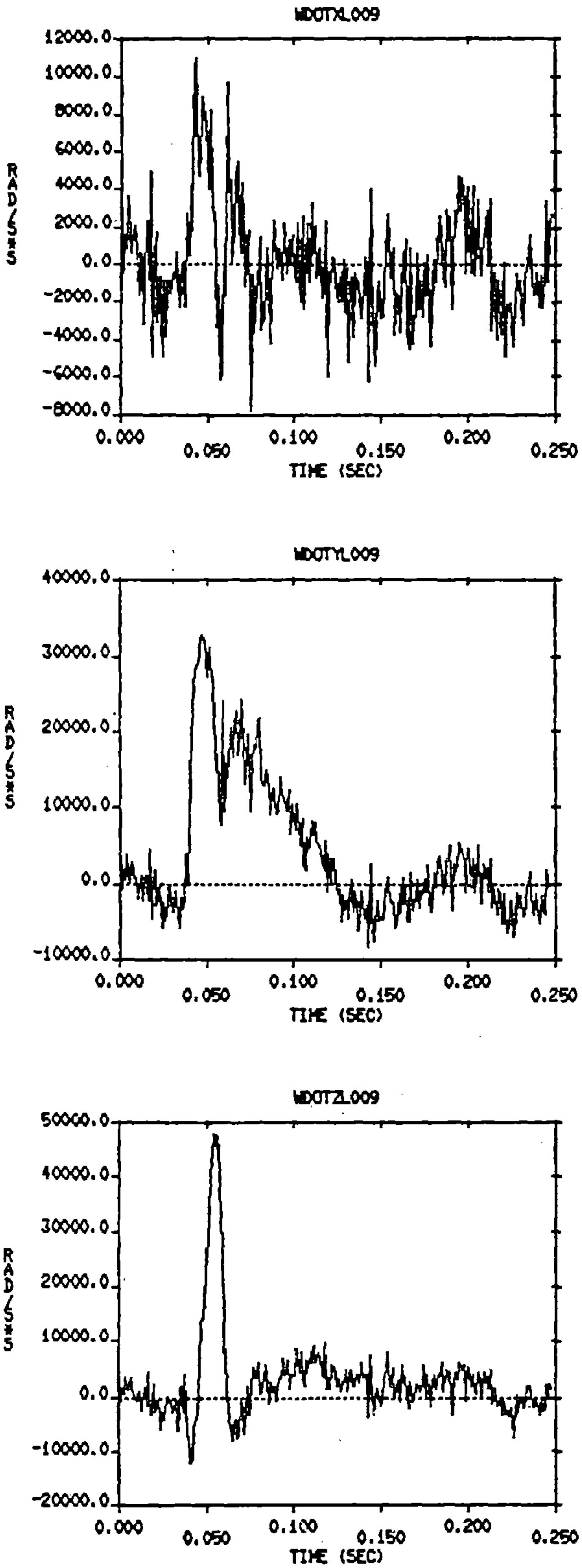


FIGURE 53. TEST #18 - FIELD DATA INPUT PULSE (250 POINTS) ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM009

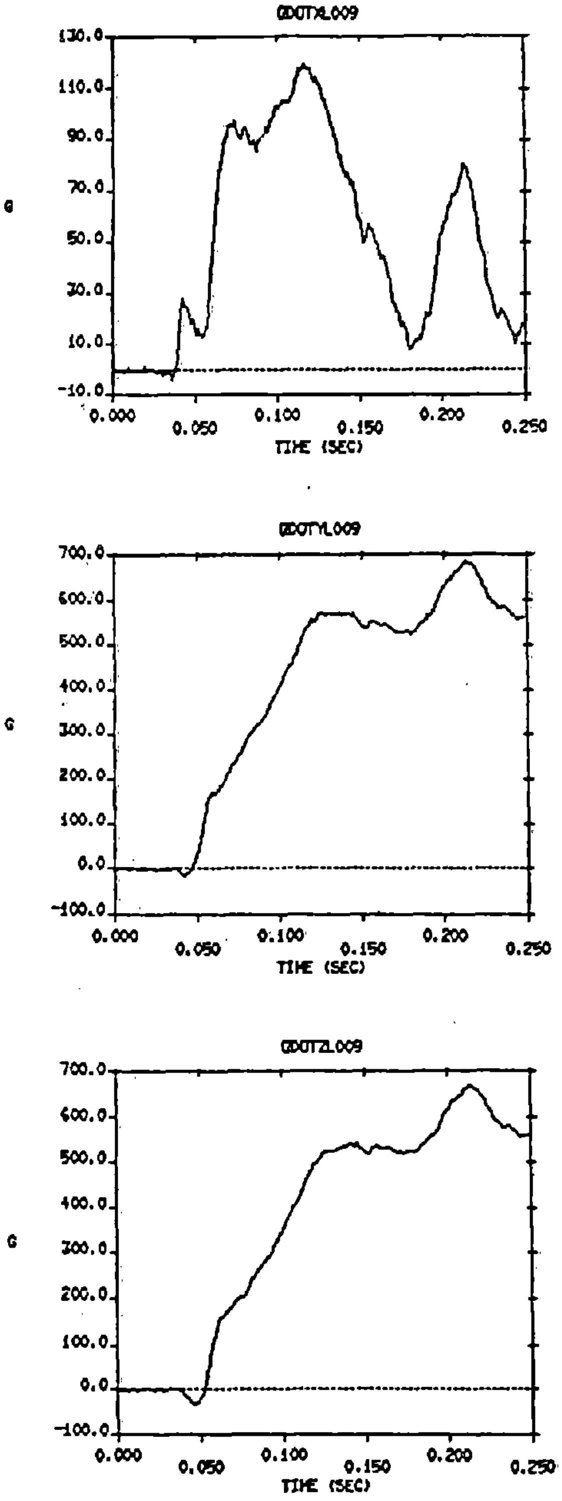


FIGURE 54. TEST #18 - APPARENT LINEAR ACCELERATIONS MEASURED BY NAPLABD SYSTEM

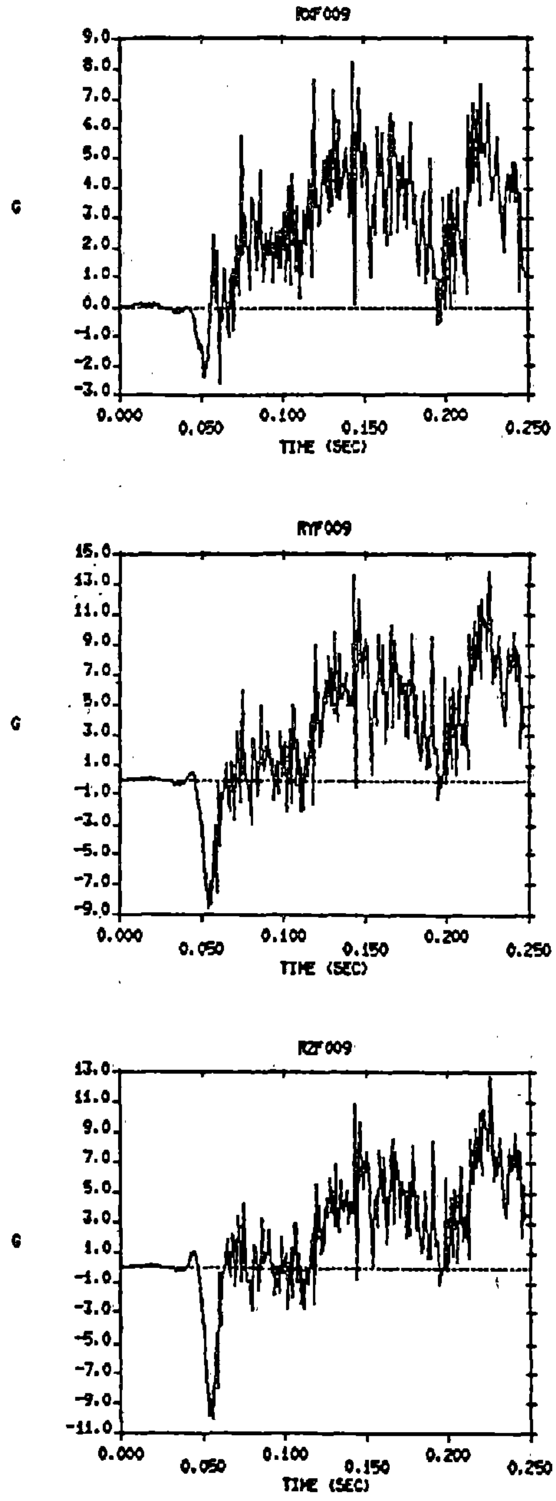
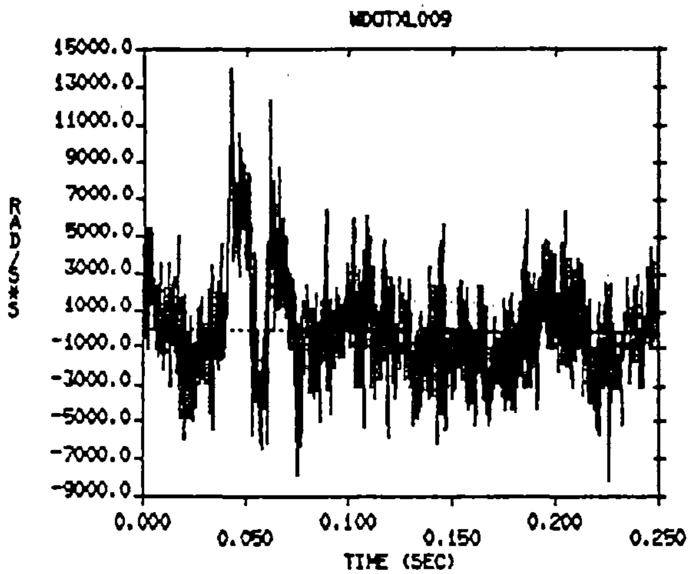
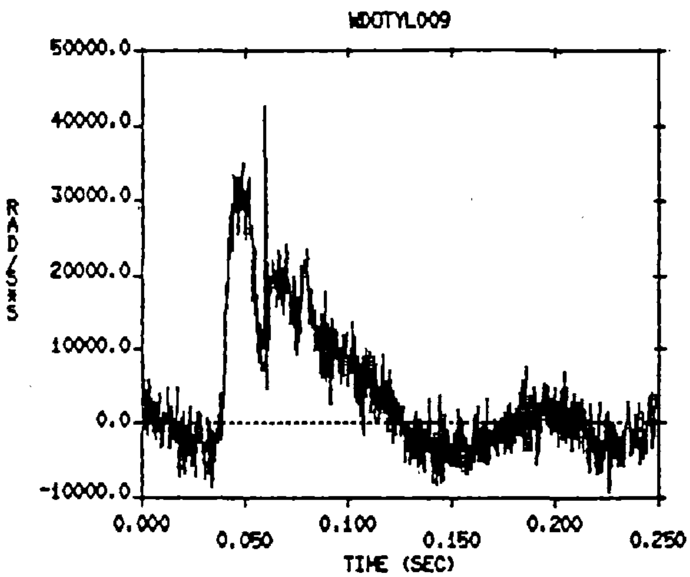


FIGURE 55. TEST #18 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK





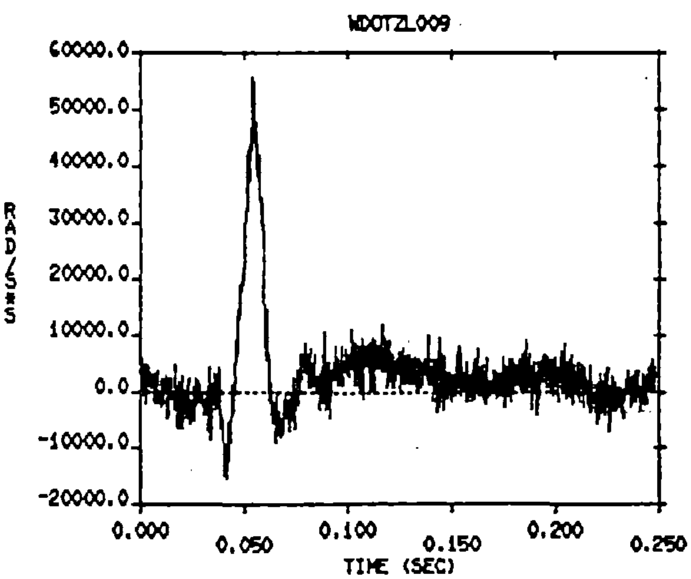


FIGURE 56. TEST #19 - FIELD DATA INPUT PULSES (1,993 POINTS) ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM009

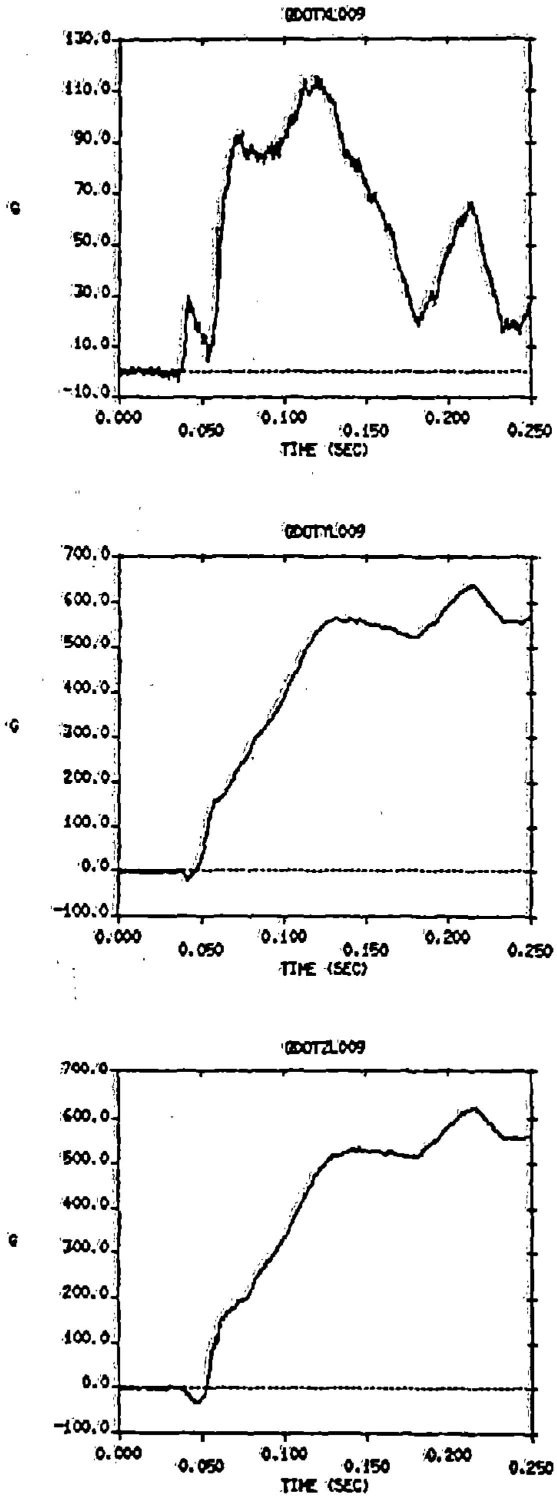


FIGURE 57. TEST #19 - APPARENT LINEAR ACCELERATIONS MEASURED BY NAPLABD SYSTEM

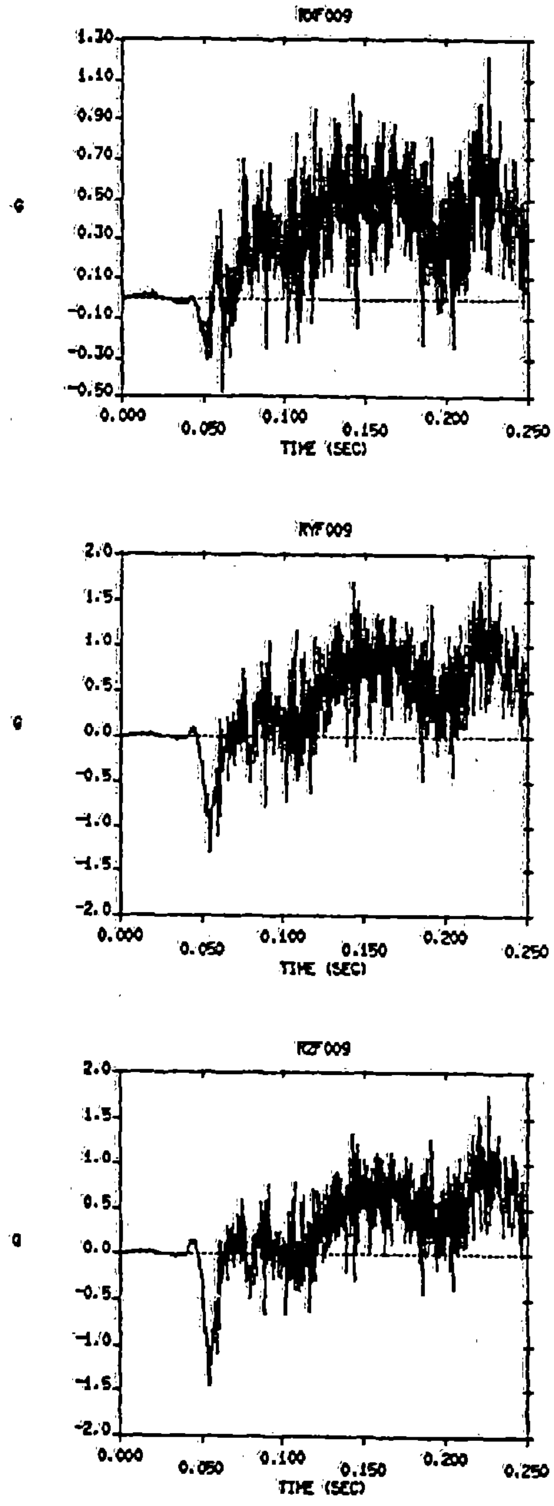


FIGURE 58. TEST #19 - RESIDUAL ERRORS AFTER CORRECTION WITH NAPFLDRK

6.1.2 VRTC Head Configuration

The Vehicle Research and Test Center (VRTC) in East Liberty, Ohio has developed, with Robert Denton, Inc., an anthropomorphic head with an integral nine-accelerometer system in the 3-2-2-2 configuration. The geometry of this head is illustrated in Figure 59. One of the principal differences between this configuration and the standard ENDEVCO mount is that the VRTC unit has unequal moment arms. Presumably, the VRTC unit is more rigid than the ENDEVCO mount. The particular configuration is of little consequence when calibrating the system, as the calibration procedure essentially treats the system as a black box without regard to the details of the geometry.

Additional computer simulations were run using the Robert Denton NAP configuration with 1.5 inch moment arms. 1.5 inch moment arms will allow direct comparison with the data reported for the ENDEVCO standard mount. Equal half-sine input pulses about all three axes were used to determine the effects of input pulse magnitude, input pulse duration and data sampling frequency on the residual errors measured. The error term matrix used was ERTEMR010 which presumes coordinate axes along the sensitive axes of the three centrally located accelerometers. The linear acceleration inputs were set to zero in these tests. The results of these tests are shown in Figures 60 and 61.

As was illustrated in the previous section, the residual errors appear to be governed by the relationship:

$$E = K \frac{M^2 T}{R}$$

In the case of the Robert Denton configuration with a 1.5" moment arm and the error term matrix chosen above, the value of the constant is:

$$K = 5.08 \times 10^{-4}$$

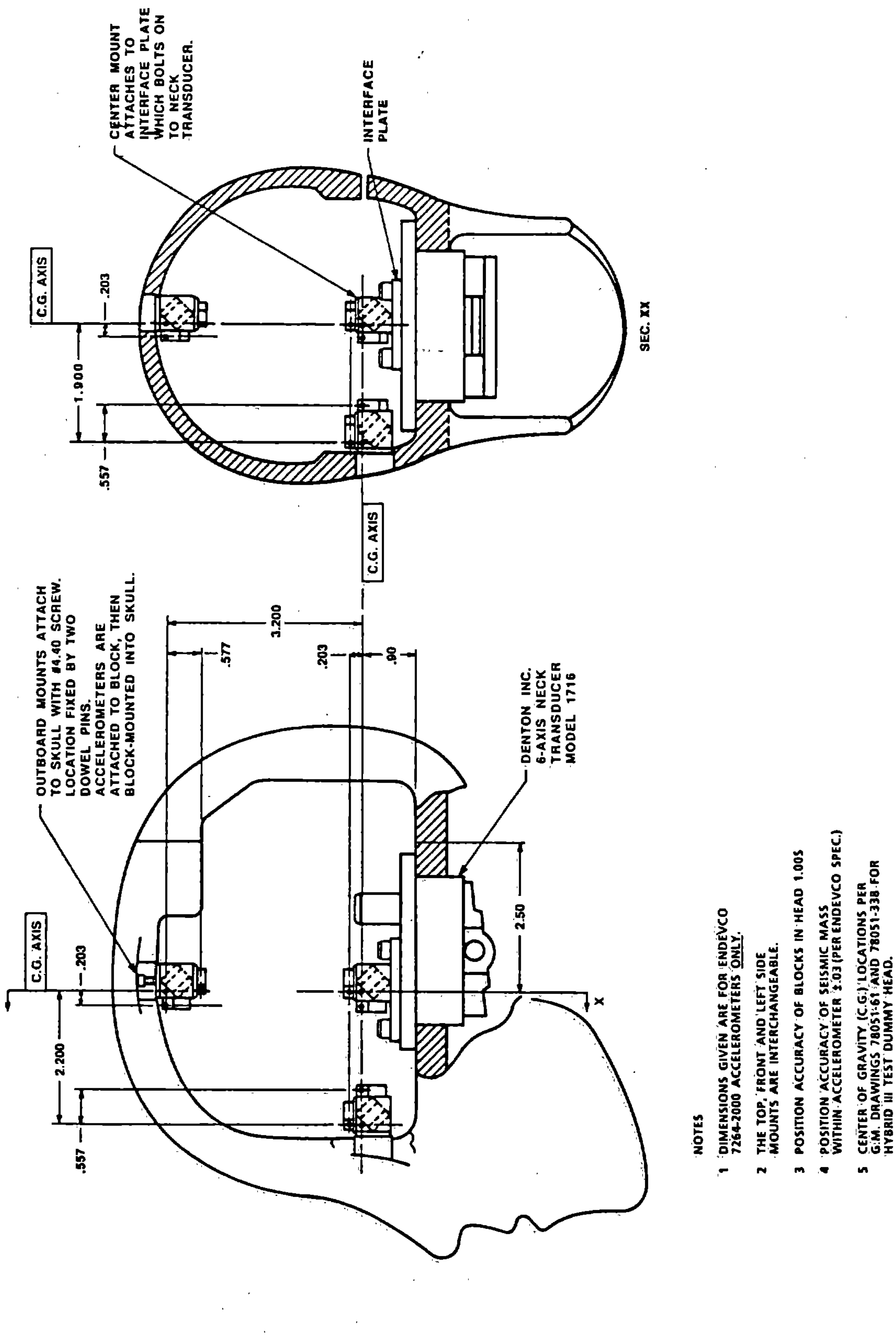


FIGURE 59. VRTC HEAD CONFIGURATION

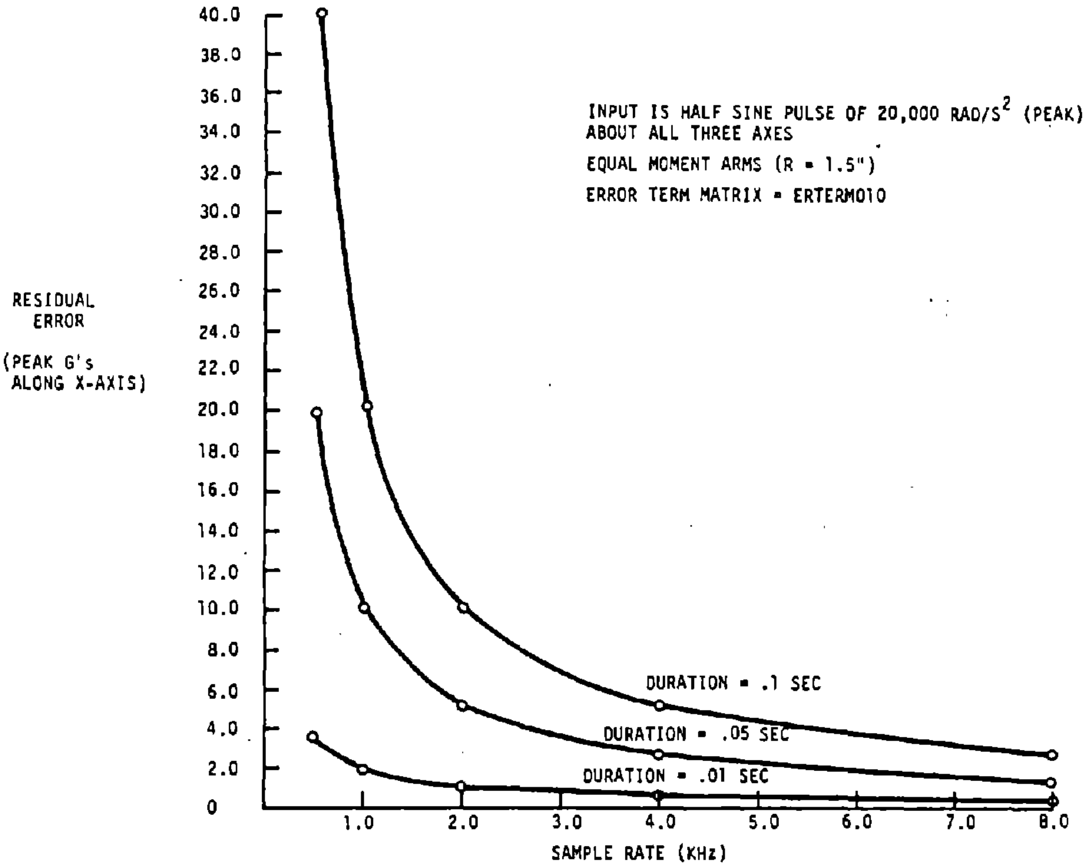


FIGURE 60. RESIDUAL ERRORS VS. SAMPLE RATE VRTC CONFIGURATION, INPUT = 20,000 rad/sec^2

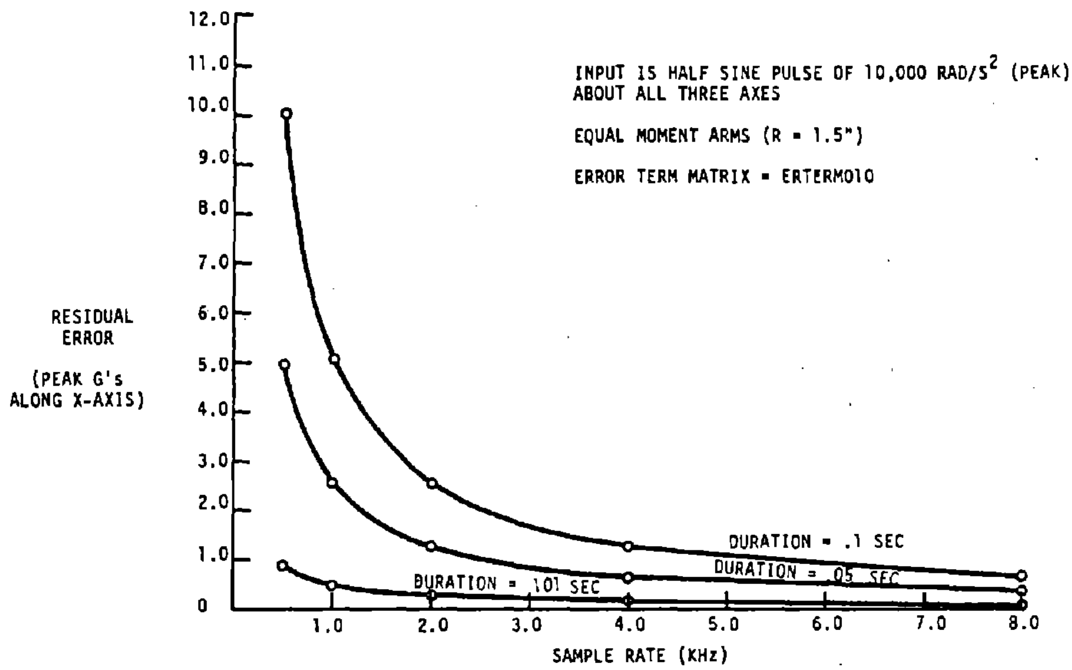


FIGURE 61. RESIDUAL ERRORS VS. SAMPLE RATE VRTC CONFIGURATION, INPUT = 10,000 rad/sec^2

6.1.2 VRTC Head Configuration

The Vehicle Research and Test Center (VRTC) in East Liberty, Ohio has developed, with Robert Denton, Inc., an anthropomorphic head with an integral nine-accelerometer system in the 3-2-2-2 configuration. The geometry of this head is illustrated in Figure 59. One of the principal differences between this configuration and the standard ENDEVCO mount is that the VRTC unit has unequal moment arms. Presumably, the VRTC unit is more rigid than the ENDEVCO mount. The particular configuration is of little consequence when calibrating the system, as the calibration procedure essentially treats the system as a black box without regard to the details of the geometry.

Additional computer simulations were run using the Robert Denton NAP configuration with 1.5 inch moment arms. 1.5 inch moment arms will allow direct comparison with the data reported for the ENDEVCO standard mount. Equal half-sine input pulses about all three axes were used to determine the effects of input pulse magnitude, input pulse duration and data sampling frequency on the residual errors measured. The error term matrix used was ERTEMR010 which presumes coordinate axes along the sensitive axes of the three centrally located accelerometers. The linear acceleration inputs were set to zero in these tests. The results of these tests are shown in Figures 60 and 61.

As was illustrated in the previous section, the residual errors appear to be governed by the relationship:

$$E = K \frac{M T^2}{R}$$

In the case of the Robert Denton configuration with a 1.5" moment arm and the error term matrix chosen above, the value of the constant is:

$$K = 5.08 \times 10^{-4}$$

7. PARAMETRIC SENSITIVITY INVESTIGATION (PERFECT SYSTEM GEOMETRY)

7.1 NON-IDEAL ACCELEROMETER CHARACTERISTICS

One of the geometric uncertainties in a commercial, single-degree-of-freedom beam accelerometer is the uncertainty in the location of the seismic center. This uncertainty is equivalent to an error in the mounting location of the accelerometer and is very small compared to, for example, the geometric offsets required when mounting the three accelerometers at the origin of the selected coordinate system in a nine accelerometer array. The uncertainty in the location of the seismic center becomes part of the mounting location uncertainty and is accounted for in the laboratory calibration.

Another deviation from the ideal experienced with typical piezoresistive accelerometers is in the frequency response and phase shift characteristics of the transducer. A cantilever beam piezoresistive accelerometer behaves as a single-degree-of-freedom spring mass system. The sensitivity of such a transducer is essentially constant at frequencies much lower than the natural frequency of the system and it has essentially zero phase shift at those frequencies. These are desireable characteristics for accelerometers used in a nine-accelerometer array, as selected pairs of these accelerometers are used to measure very small acceleration differences in the calculation of angular acceleration and therefore must be very well matched with regard to gain and phase shift. It is recommended by most manufacturers that the natural frequency of a piezoresistive beam accelerometer be at least five (5) times the anticipated information bandwidth[12]. This will insure operation in a region with virtually constant gain and minimal phase shift.

Amplitude linearity (deviation from constant sensitivity vs. input amplitude) of piezoresistive accelerometers is excellent (generally less than typical errors in the procedures employed to calibrate these devices), which may not be the case with piezoelectric accelerometers.

7.2 CROSS-AXIS SENSITIVITIES

Angular misalignment of the attached beam in the manufacturing process is the primary cause of cross-axis sensitivities. This error may also be introduced when the accelerometer is mounted in its fixture. Tests were run to determine sensitivity with regard to cross-axis errors in individual accelerometers. The geometry of the system with regard to the placement of the transducers was assumed to be ideal (no positional offsets). The input excitation was a simple case of angular acceleration in a single plane and linear acceleration along a single axis in the same plane. An error matrix (ERTERM012 - see Appendix F) was constructed to represent a 5% cross-axis error in the transducers. The signs of the terms were chosen to give maximum error for the selected test condition. The inputs and affected transducers are shown in Figure 62. The moment arms are presumed to be 1.5". The following half-sine pulses were used as inputs:

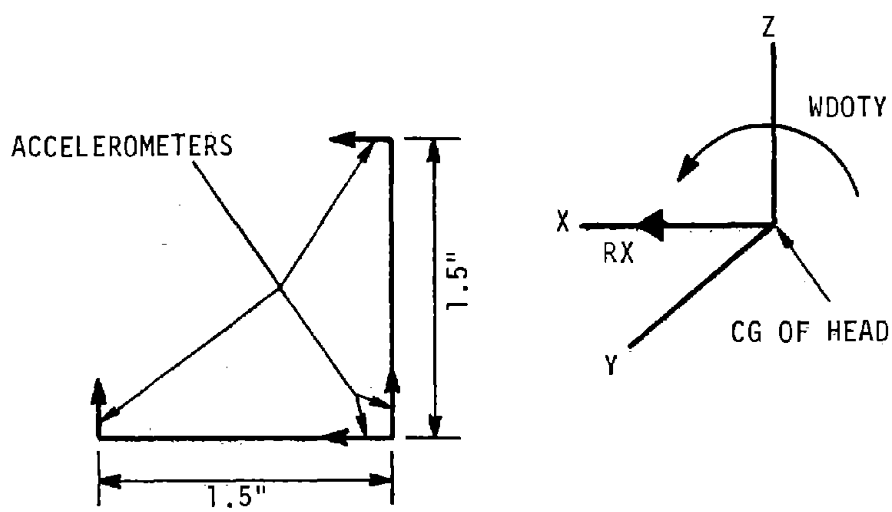


Figure 62 - Conditions for Angular Misalignment Tests

The input pulses are shown in Figures 63 and 64. The same input pulses were used in Tests 20 through 24. The results of these tests are shown in Figures 65 through 67. Additional error matrices ERTERM013 through ERTERM016 were constructed to represent 4%, 3%, 2% and 1% cross-axis errors in the transducers and similar tests were run. The results of these tests are shown in Figures 68 through 76.

The significant outputs from NAPLABD using ERTERM012 are shown in Figures 65 through 66. It is obvious that the uncorrected estimate for the angular acceleration (Figure 66) contains considerable error (+46.9% - peak value) due to the angular misalignment of the transducers. The estimates for the linear acceleration for this particular test condition (Figure 65) contain no errors. In fact, in this particular case, had there been no linear input, the results for the angular measurements would have been unchanged. Figure 67 shows the output for angular acceleration after correction with NAPFLDRK. As can be seen, the corrected output looks exactly like the original input of Figure 63. The actual peak value of this corrected pulse was 9,989 rad/sec^2 compared to 10,000 rad/sec^2 for the original pulse, representing a residual error of .1%. In a similar manner, ERTERM013 through ERTERM016 were used to represent 4% through 1% cross-axis error. The results are shown in Figures 68 through 76. It can be seen (Figures 66, 68, 70, 72 and 74) that the error in the estimate of the angular acceleration before correction is reduced when the cross-axis sensitivity of the individual accelerometers is reduced. The peak values of the estimated angular acceleration before correction with NAPFLDRK were noted for each level of cross-axis sensitivity and the errors in these estimates are summarized in Figure 76.

While it appears from the data that virtually complete correction is achieved with the correction algorithm (NAPFLDRK), it is probably prudent (since the errors before data correction are so high) to choose accelerometers that have low cross-axis sensitivities. Manufacturers usually allow the buyer to do this at a slightly higher cost for the accelerometers.

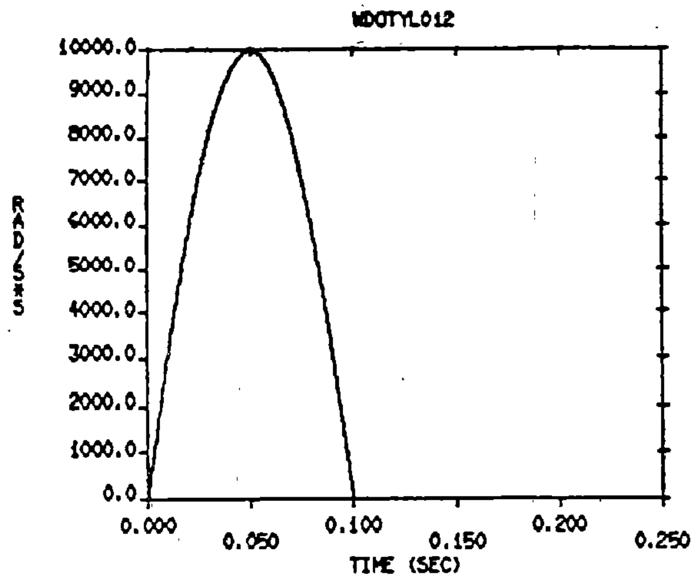


FIGURE 63. TEST #20 - ANGULAR INPUT ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM012. 5% Cross-Axis Sensitivity Input = 10,000 rad/sec^2, 0.1 sec duration

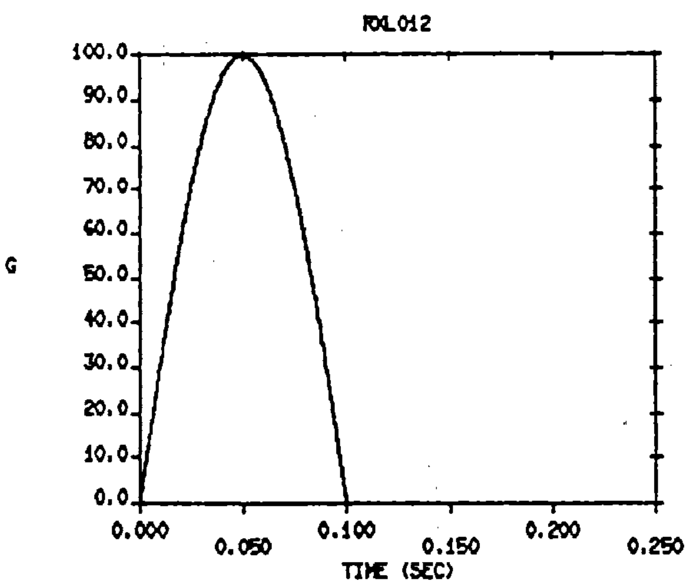


FIGURE 64. TEST #20 - LINEAR INPUT

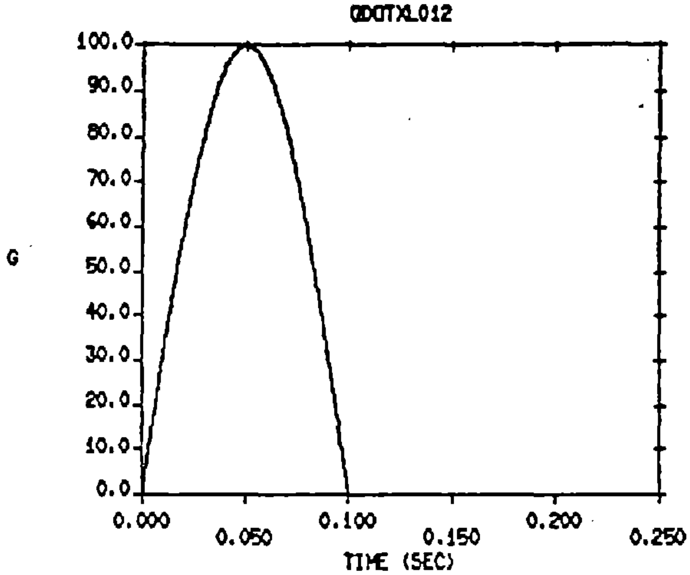


FIGURE 65. TEST #20 - ESTIMATED LINEAR OUTPUT

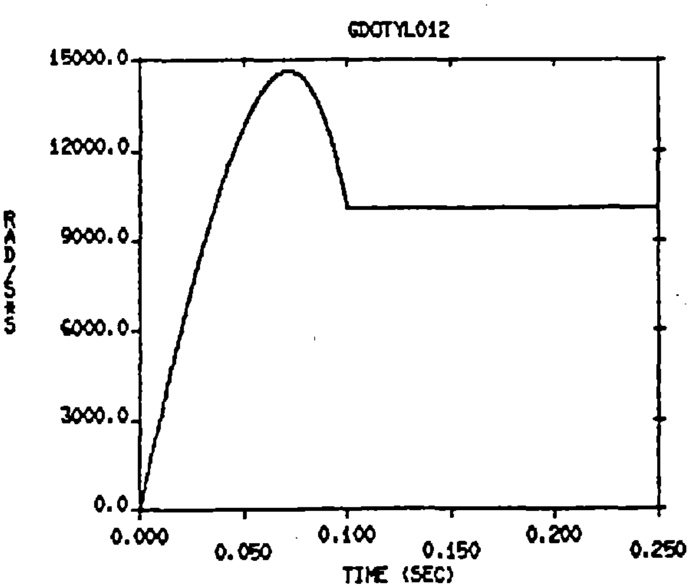


FIGURE 66. TEST #20 - ESTIMATED ANGULAR OUTPUT. Peak Acceleration = 14,688 rad/sec^2, @ 0.0715 sec

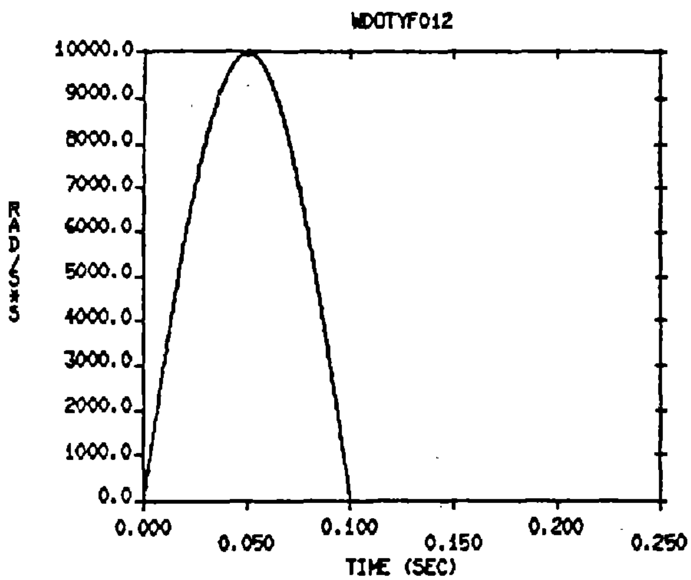


FIGURE 67. TEST #20 - CORRECTED ANGULAR OUTPUT
Peak Acceleration = 9,989 rad/sec^2

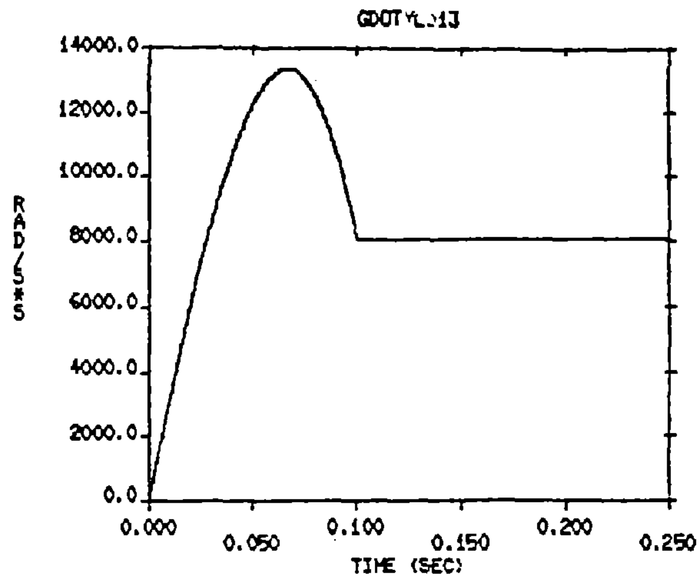


FIGURE 68. TEST #21 - ESTIMATED ANGULAR OUTPUT ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM013
4% Cross-Axis Sensitivity
Peak Acceleration = 13,393 rad/sec^2 @ 0.0672 sec

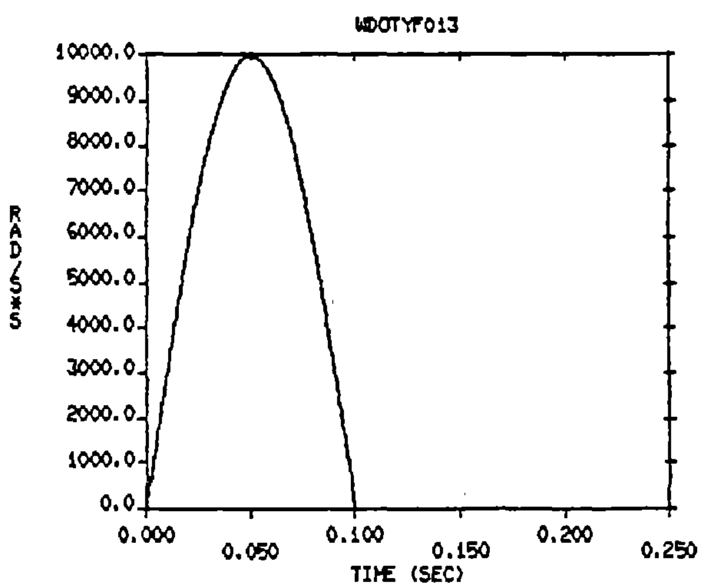


FIGURE 69. TEST #21 - CORRECTED ANGULAR OUTPUT

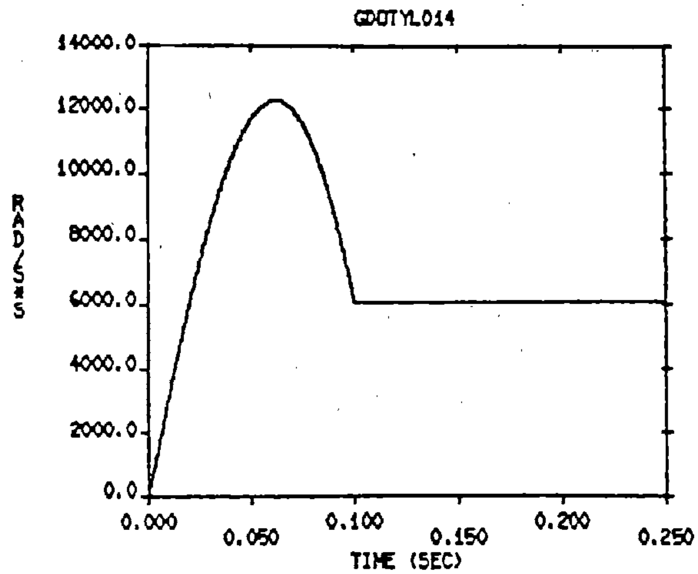


FIGURE 70. TEST #22 - ESTIMATED ANGULAR OUTPUT ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM014
3% Cross-Axis Sensitivity
Peak Acceleration = 12,282 rad/sec^2 @ 0.0625 sec

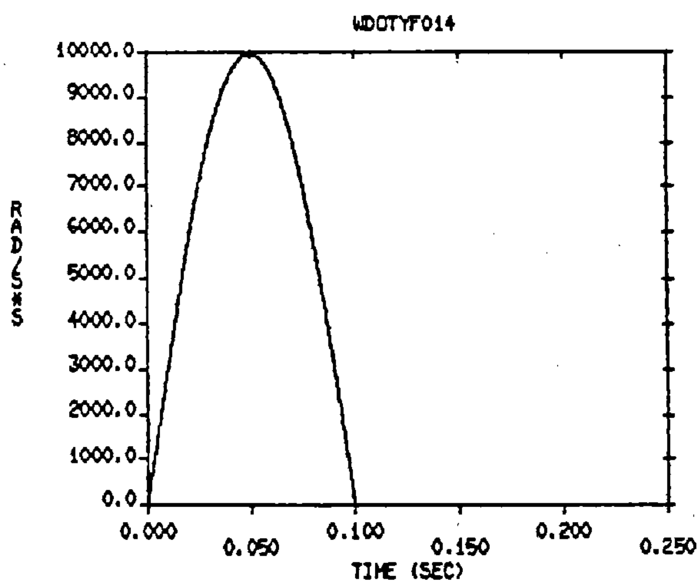


FIGURE 71. TEST #22 - CORRECTED ANGULAR OUTPUT

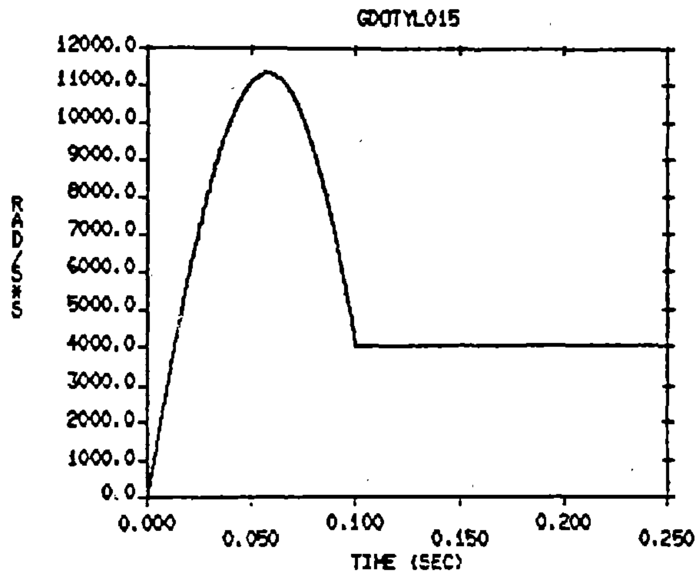


FIGURE 72. TEST #23 - ESTIMATED ANGULAR OUTPUT ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM015
2% Cross-Axis Sensitivity
Peak Acceleration = 11,362 rad/sec^2 @ 0.0578 sec

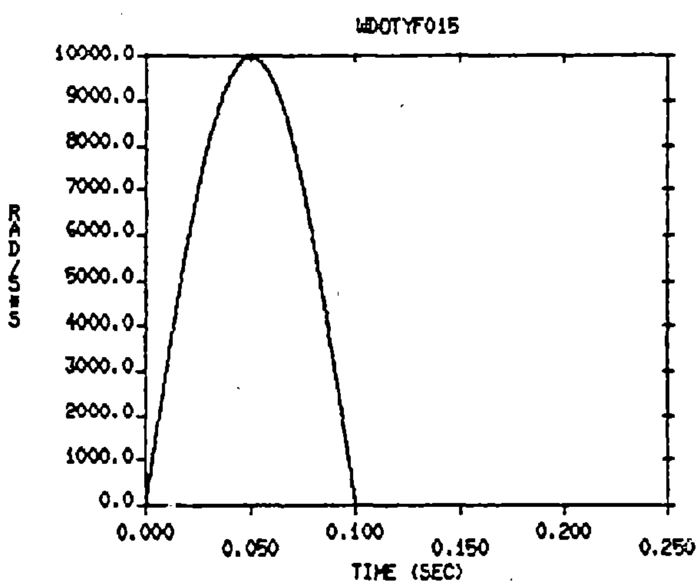


FIGURE 73. TEST #23 - CORRECTED ANGULAR OUTPUT

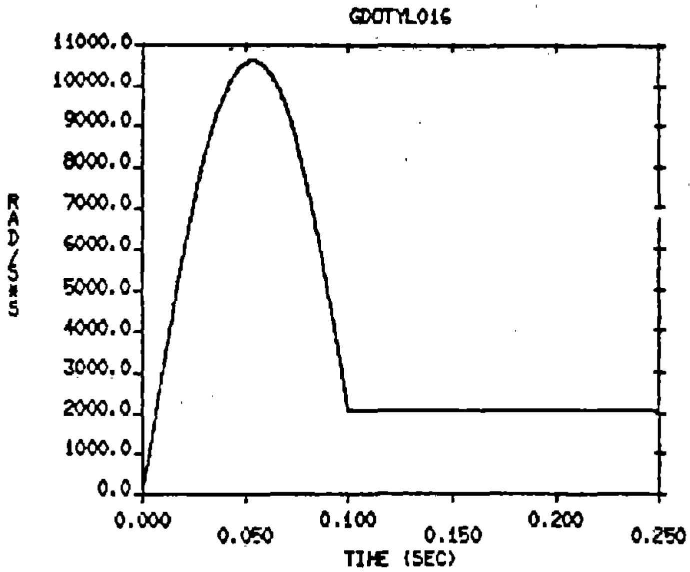


FIGURE 74. TEST #24 - ESTIMATED ANGULAR OUTPUT ENDEVCO MOUNT. ERROR TERM MATRIX = ERTERM016
1% Cross-Axis Sensitivity
Peak Acceleration = 10,613 rad/sec^2 @ 0.0535 sec

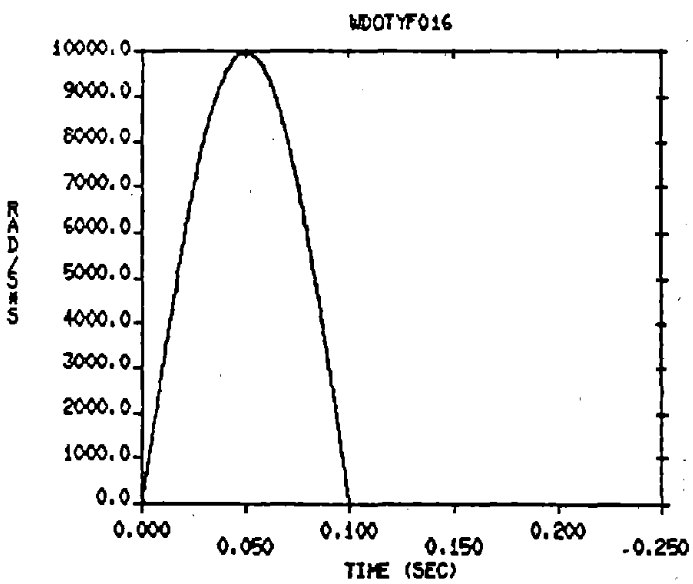


FIGURE 75. TEST #24 - CORRECTED ANGULAR OUTPUT

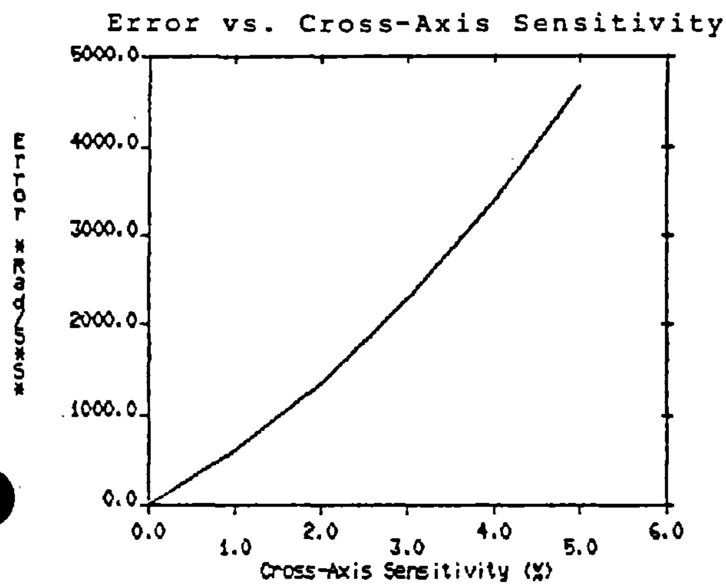


FIGURE 76. ERROR VERSUS CROSS-AXIS SENSITIVITY

8. CONCLUSIONS AND RECOMMENDATIONS

8.1 CONCLUSIONS

1. The laboratory and field models are valid representations of a nine-accelerometer package (NAP) in the 3-2-2-2 configuration.

2. Essentially complete correction of noisy field data is possible with either of the correction algorithms NAPFLDEUL or NAPFLDRK provided the data sample rate is sufficiently high (i.e., 8,000 samples/sec.).

3. The use of double precision did not result in a reduction in the residual error observed after correcting field data with NAPFLDRK.

4. Runga - Kutta integration techniques were only slightly more accurate than simple Euler integration.

5. Coordinate system location has a significant effect on measurement accuracy and the reduction of residual errors.

6. Significant error buildup may occur in the output values for linear acceleration when there are angular inputs about more than one axis and the data sample rate is too low.

7. The above condition is due to cumulative error that occurs in the integration routines of the corrective algorithm.

8. Residual errors (errors after correction) appear to be governed by the relationship:

$$E = K \frac{M^2 T}{R}$$

where:

E = Residual error (g)

M = Peak value of half-sine pulse (rad/sec^2)

T = Duration of half-sine pulse (sec)

R = Data Sample Rate (samples/sec)

K = Constant

9. Cross-axis sensitivities on the order of 5% may result in significant error before applying the corrective algorithm.

8.2 RECOMMENDATIONS

1. Use the correction algorithm NAPFLDRK instead of NAPFLDEUL if the IMSL Runga - Kutta integration routine is available. This will result in more accurate integration within the program and slightly better correction of the data.

2. Use a data sample rate of 8,000 samples/sec. or greater.

3. For the ENDEVCO configuration, choose the coordinate system where the seismic centers of the centrally located transducers are located in the three planes formed by the coordinate system (ERTERM009).

4. If other than an ENDEVCO configuration, create error term matrices for the configuration and run NAPLABG to determine the best locations for the coordinate system.

5. If possible, choose accelerometers with low cross-axis sensitivity (3.0%).

6. Use longest moment arms possible to maximize the signal to noise ratio.

APPENDIX A

COMPARISON OF TRANSLATIONAL ACCELEROMETER CONFIGURATIONS FOR MEASURING ANGULAR ACCELERATIONS OF A RIGID BODY

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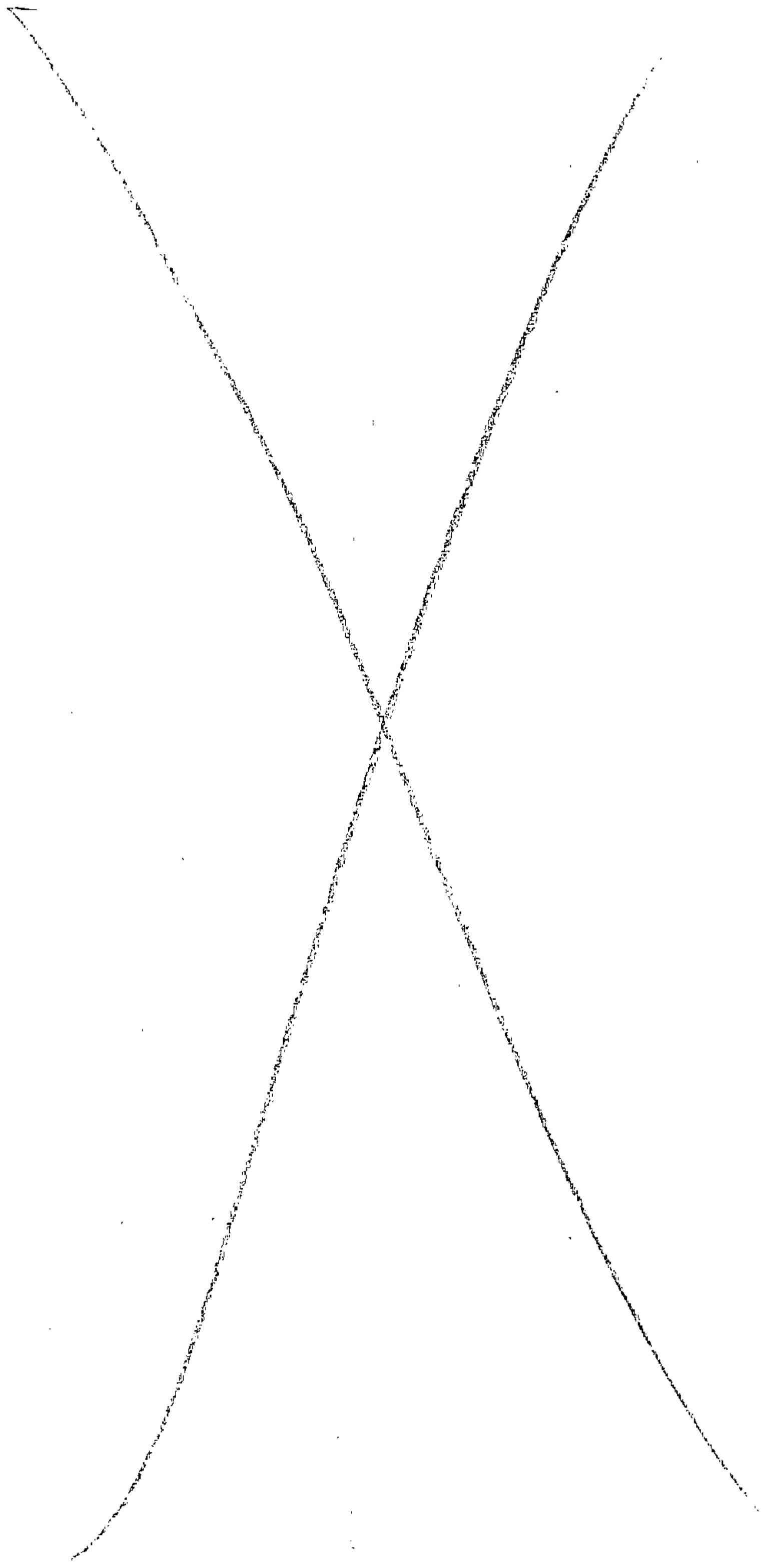
PREFACE

The Transportation Systems Center (TSC) is currently engaged in a number of activities in support of the Crashworthiness Division of the National Highway Traffic Safety Administration (NHTSA). In particular, instrumentation support is provided to the NHTSA through the Biomechanics Technology Project at TSC. As part of this project, requirements are to be established for the fabrication, calibration and test of a standard nine-accelerometer array for use in measuring the linear and angular head accelerations of anthropomorphic dummies. Comparisons are made among the three arrays that have been used for these measurements; a six-accelerometer coplanar, a nine-accelerometer coplanar (3-3-3 configuration), and a nine-accelerometer non-coplanar (3-2-2-2 configuration) array. An error model is developed and computer simulations are performed to examine the stability of each system.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES



1. SUMMARY

References 1 , 2 and 3 contain descriptions of three alternative proposed methods of measuring the translational and rotational accelerations using translational accelerometers mounted at various locations. The problem of measurement of the angular acceleration of a rigid body by translational accelerometers in a general translational and rotational motion is complicated by the presence of angular velocity squared (centripetal) and angular velocity cross product acceleration components. If the motions of the body are small (compared to say, 15^{o} of rotation) these components will be small compared to the angular acceleration terms. However, if the motions are large, these components will be of the same order or larger than those due to the angular acceleration. In general, the acceleration at an arbitrary point in a rigid body at an instant of time will be a function of nine independent variables; the three translational accelerations at the center of the coordinate system, R_x, R_y, R_z, the three rotational accelerations $\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$ and the three angular velocity components ω_x, ω_y, ω_z. If the angular velocities are not known, a minimum of nine independent acceleration measurement locations are required to determine the linear and rotational accelerations. The nine accelerometer system described in Reference 1 defines a set of measurement locations where signals can be scaled and linearly combined to eliminate the angular velocity induced accelerations and provide signals directly proportional to the translational and rotational accelerations (assuming perfect positioning and transducers).

Other configurations containing the required nine independent measurement locations and orientations can be used to develop the translational and rotational acceleration components (Reference 2), however, more complex algebraic operations may be required to isolate the acceleration components. However, any system where all measurements are located in the same plane cannot provide the nine independent accelerations required as discussed in the body of this memo.

If estimates of the angular velocities of the rigid body are available, six independent measurement locations can be used to obtain the angular accelerations. However, if the estimate of angular velocity is derived from a knowledge of the initial state of the body and a numerical integration of the estimated angular acceleration, the errors due to location errors and accelerometer imperfections will grow rapidly with time. As an example, for accelerometer bias errors equivalent to an error of 100 rad/sec^2 at the initialization of the integration, the angular acceleration error will be of the order of 1700 rad/sec^2 for an integration step of one millionth of a second carrying the computation for a one second period of time. An apriori knowledge of the mechanics of the motion could be used to reduce the error as well as a reduction of the period over which the integration is carried. In some cases, however, more accurate estimates of the angular accelerations will be obtained by assuming zero angular velocity rather than attempting the numerical integration.

The following paragraphs contain a more detailed discussion and description of:

1 - Configurations of transducer locations and directions to measure angular acceleration.

2 - The influence of transducer location errors and imperfections on signal measurement accuracy.

3 - Rate of growth of angular acceleration measurement errors due to numerical integration.

4 - Initialization and calibration of measurement system.

2. MEASUREMENT CONFIGURATIONS

For a rigid body in which we fix a coordinate system x, y, z centered at a point in the body, the acceleration at any point is given by:

$$A_{xi} = R_x + r_{zi}(\dot{\omega}_y - \omega_x\dot{\omega}_z) - r_{yi}(\dot{\omega}_z + \omega_x\omega_y) + r_{xi}(\omega_y^2 + \omega_z^2) \qquad \text{A-1}$$

$$A_{yi} = R_y + r_{xi}(\dot{\omega}_z - \omega_x\omega_y) - r_{zi}(\dot{\omega}_x + \omega_y\omega_z) + r_{yi}(\omega_x^2 + \omega_y^2) \qquad \text{A-2}$$

$$A_{zi} = R_z - r_{xi}(\dot{\omega}_y - \omega_x\omega_z) + r_{yi}(\dot{\omega}_x - \omega_y\omega_z) + r_{zi}(\dot{\omega}_x^2 + \omega_z^2) \qquad \text{A-3}$$

where A_{xi}, A_{yi}, A_{zi} are the x, y, z components of acceleration at location i; R_x, R_y, R_z are the components of the rigid body acceleration at point 0; $\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$ are the angular accelerations; ω_x, ω_y, ω_z are the angular velocity components; and r_{xi}, r_{yi}, r_{zi} are the coordinates of location i in the coordinate system measured from point o.

2.1 NINE ACCELEROMETERS (3-2-2-2 CONFIGURATIONS)

If three accelerometers measuring in the x direction, three in the y direction and three in the z direction are mounted at locations that permit the accelerations to be independent, we have nine equations in nine unknowns (R_x, R_y, R_z, $\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$, ω_x, ω_y, ω_z) that can be solved by algebraic manipulation.

If the locations at the accelerometers are arbitrary, the algebraic manipulations can be quite cumbersome. Some simplification exists if we insist that the three measurements in the x direction be located in the yz plane ($r_x = 0$), the measurements in the y direction in the xz plane ($r_y = 0$), and measurements in the z direction be located in the xy plane ($r_z = 0$). This reduces equations A-1 to A-3 to:

$$A_{xi} = R_x + r_{zi}(\dot{\omega}_y - \omega_x\omega_z) - r_{yi}(\dot{\omega}_z + \omega_x\omega_y) \qquad \text{A-4}$$

$$A_{yj} = R_y + r_{xj}(\dot{\omega}_z - \omega_x\omega_y) - r_{zj}(\dot{\omega}_x + \omega_y\omega_z) \qquad \text{A-5}$$

$$A_{zk} = R_z - r_{xk}(\dot{\omega}_y + \omega_x\omega_z) + r_{yk}(\dot{\omega}_x - \omega_y\omega_z) \qquad \text{A-6}$$

With 3 independent locations for each measurement direction, this set of equations can be solved for the nine unknowns by linear operations. If we locate A_{x1}, A_{y1} and A_{z1} at 0,0,0, we have:

$$A_{x1} = R_x, \quad A_{y1} = R_y, \quad A_{z1} = R_z \qquad \text{A-7}$$

Locating A_{x2} at 0,0,r, provides:

$$A_{x2} = R_x + r(\dot{\omega}_y - \omega_y\omega_z) \qquad \text{A-8}$$

and A_{x3} at 0,r,0, yields:

$$A_{x3} = R_x - r(\dot{\omega}_z + \omega_x\omega_y) \qquad \text{A-9}$$

which provides

$$(\dot{\omega}_y - \omega_x\omega_z) = \frac{A_{x2} - A_{x1}}{r} = Q_{y1} \qquad \text{A-10}$$

$$(\dot{\omega}_z + \omega_x\omega_y) = -\left[\frac{A_{x3} - A_{x1}}{r}\right] = -Q_{z2} \qquad \text{A-11}$$

Similarily if we locate A_{y2} at r,0 ,0 , we have:

$$(\dot{\omega}_z - \omega_x\omega_y) = \frac{A_{y2} - A_{y1}}{r} = Q_{z1} \qquad \text{A-12}$$

and A_{y3} at 0 ,0 ,r, we have:

$$(\dot{\omega}_x + \omega_y\omega_z) = -\left[\frac{A_{y3} - A_{y1}}{r}\right] = -Q_{x2} \qquad \text{A-13}$$

completing the nine measurements we locate A_{z2} at r,0 ,0 , and A_{z3} at 0,0 ,r to obtain:

$$-(\dot{\omega}_y + \omega_x\omega_z) = \frac{A_{z2} - A_{z1}}{r} = Q_{y2} \qquad \text{A-14}$$

and

$$(\dot{\omega}_x - \omega_y\omega_z) = \frac{A_{z3} - A_{z1}}{r} = Q_{x1} \qquad \text{A-15}$$

permitting solutions for $\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$ as:

$$\dot{\omega}_x = \frac{Q_{x1} - Q_{x2}}{2} \tag{A-16}$$

$$\dot{\omega}_y = \frac{Q_{y1} - Q_{y2}}{2} \tag{A-17}$$

$$\dot{\omega}_z = \frac{Q_{z1} - Q_{z2}}{2} \tag{A-18}$$

This would provide direct signals proportional to the angular acceleration with no need for estimating the angular velocities. The locations given above represent the 3-2-2-2 configuration for the nine accelerometer system. As noted above, other configurations can be used to produce the required results at the expense of additional algebraic manipulation of the signals.

The 3-3-3 transducer configuration [2], however, does not provide the nine independent measurements required to eliminate the angular velocity terms. If three, three-axis accelerometer clusters are used, the locations of the three clusters lie in a single plane. If we define the z axis of a coordinate system normal to this plane, equations A-1, A-2 and A-3 become:

$$A_{xi} = R_x + r_{xi}(\omega_y^2 + \omega_z^2) - r_{yi}(\dot{\omega}_z + \omega_x\omega_y) \tag{A-19}$$

$$A_{yi} = R_y + r_{xi}(\dot{\omega}_z - \omega_x\omega_y) + r_{yi}(\omega_x^2 + \omega_z^2) \tag{A-20}$$

$$A_{zi} = R_z - r_{xi}(\dot{\omega}_y + \omega_x\omega_z) + r_{yi}(\dot{\omega}_x - \omega_y\omega_z) \quad \text{A-21}$$

Appropriate placement of 3 accelerometers in the x direction and 3 accelerometers in the y direction permits the use of equations A-19 and A-20 to determine R_x, R_y and $_z$ without apriori knowledge of the angular velocities. However, it is not possible with a configuration with all measurement locations coplanar to separate $_x$ and $_y$ $_z$ by simple algebraic operations. (Although it is possible to use equations A-19 and A-20 to develop quartic solutions for x, y, and z the solutions are multi-valued). By the use of scaling and linear combinations, with nine accelerometers located in the same plane with 3 in each direction we can obtain the signals:

$$Q_z = \dot{\omega}_z \quad \text{A-22}$$

$$Q_y = \dot{\omega}_y + \omega_x\omega_z \quad \text{A-23}$$

$$Q_x = \dot{\omega}_x - \omega_y\omega_z \quad \text{A-24}$$

In theory, these equations could be solved by numerical integration to determine the angular velocities.

The simplest integration scheme would be:

$$\dot{\Omega}z_i = Q_z \qquad \text{A-25}$$

$$\dot{\Omega}y_i = Q_y - \Omega x_{i-1}\Omega z_{i-1} \qquad \text{A-26}$$

$$\dot{\Omega}x_i = Q_x - \Omega y_{i-1}\Omega z_{i-i} \qquad \text{A-27}$$

$$\Omega x_i = \Omega x_{i-1} + \dot{\Omega}x_{i-1}\Delta t \qquad \text{A-28}$$

$$\Omega y_i = \Omega y_{i-1} + \dot{\Omega}y_{i-1}\Delta t \qquad \text{A-29}$$

$$\dot{\Omega}z_i = \Omega z_{i-1} + \dot{\Omega}z_{i-1}\Delta t \qquad \text{A-30}$$

where $\dot{\Omega}_I$ is the estimated angular acceleration at the I^{th} time step on t. This method of developing the angular acceleration results in an increase in the error in the estimate of angular acceleration as a function of the duration of the integration process and is discussed further in section 4.

3. TRANSDUCER SYSTEM ERRORS

Errors are introduced to the measurement system by imperfections in the accelerometers and by inaccuracies in transducer location and direction. Typical accelerometers have output signals that are given by:

$$Q = R_u + \epsilon_1 R_u + \epsilon_2 R_v + \epsilon_3 R_w + \epsilon_0 + \epsilon_4 R_u^2 + \epsilon_5 R_{uv} + \epsilon_6 R_u R_w \qquad \text{A-31}$$

where R_u is the acceleration along the transducer axis and R_v and R_w are the accelerations in the directions perpendicular to the sensitive axis, ε_o is the combination of the accelerometer bias term and noise signals that the transducer system may generate, ε_1 represents the uncertainty in the accelerometer scale factor and ε_2, ε_3 are accelerometer cross axis sensitivities due to accelerometer imperfections and mounting misalignments. The terms ε_4, ε_5 and ε_6 represent the sensitivities of some accelerometers to acceleration squared effects. Neglecting the acceleration squared terms and assuming location inaccuracies in the position of the accelerometers of δ_{xi}, δ_{yi} and δ_{zi} the estimated angular accelerations of the 3-2-2-2 transducer configuration are derived in Appendix A1 as:

$$\dot{\Omega}_x = \dot{\omega}_x + A_0 + A_1\dot{\omega}_x + A_2\dot{\omega}_y + A_3\dot{\omega}_z + A_4\omega_x^2 + A_5\omega_y^2 + A_6\omega_z^2 + A_7\omega_x\omega_y + A_8\omega_x\omega_z + A_9\omega_y\omega_z + A_{10}R_x + A_{11}R_y + A_{12}R_z \qquad \text{A-32}$$

$$\dot{\Omega}_y = \dot{\omega}_y + B_0 + B_1\dot{\omega}_y + B_2\dot{\omega}_x + B_3\dot{\omega}_z$$
$$+ B_4\omega_x^2 + B_5\omega_y^2 + B_6\omega_z^2 + B_7\omega_x\omega_y \qquad \text{A-33}$$
$$+ B_8\omega_x\omega_z + B_9\omega_y\omega_z + B_{10}R_x + B_{11}R_y + B_{12}R_z$$

$$\dot{\Omega}_z = \dot{\omega}_z + C_0 + C_1\dot{\omega}_z + C_2\dot{\omega}_y + C_3\dot{\omega}_x$$
$$+ C_4\omega_x^2 + C_5\omega_y^2 + C_6\omega_z^2 + C_7\omega_x\omega_y \qquad \text{A-34}$$
$$+ C_8\omega_x\omega_z + C_9\omega_y\omega_z + C_{10}R_x + C_{11}R_y + C_{12}R_z$$

where for the estimated angular acceleration $\dot{\Omega}_x$ we have, neglecting terms of 2nd order (i.e. $\varepsilon\delta/r$)

$$A_0 = \frac{\epsilon_{2z0} - \epsilon_{0z0} - \epsilon_{3y0} + \epsilon_{0y0}}{2r} \qquad \text{A-35}$$

$$A_1 = \frac{\epsilon_{2z3} + \epsilon_{3yz}}{2} + \frac{\delta_{2zy} - \delta_{0zy} + \delta_{3yz} - \delta_{0yz}}{2r} \qquad \text{A-36}$$

$$A_2 = \frac{\epsilon_{3y1}}{2} + \frac{\delta_{2zx} - \delta_{0zx}}{2r} \qquad \text{A-37}$$

$$A_3 = \frac{-\epsilon_{2z1}}{2} - \frac{\delta_{3yx} - \delta_{0yx}}{2r} \qquad \text{A-38}$$

$$A_4 = \frac{-(\epsilon_{2z2} - \epsilon_{3z3})}{2} + \frac{(\delta_{3yy} - \delta_{0yy} + \delta_{0zz} - \delta_{2zz})}{2r} \qquad \text{A-39}$$

$$A_5 = \frac{\epsilon_{3y3}}{2} + \frac{\delta_{0zz} - \delta_{2zz}}{2r} \qquad \text{A-40}$$

$$A_6 = \frac{-\epsilon_{3y3}}{2} + \frac{\delta_{3yy} - \delta_{0yy}}{2r} \qquad \text{A-41}$$

$$A_7 = \frac{\epsilon_{2z1}}{2} + \frac{\delta_{0yx} - \delta_{3yx}}{2r} \qquad \text{A-42}$$

$$A_8 = \frac{-\epsilon_{3y1}}{2} + \frac{\delta_{2zx} - \delta_{0zx}}{2r} \qquad \text{A-43}$$

$$A_9 = \frac{\epsilon_{2z3} - \epsilon_{3yz}}{2} + \frac{(\delta_{2zy} - \delta_{0zy} - \delta_{3yz} + \delta_{0yz})}{2r} \qquad \text{A-44}$$

$$A_{10} = \frac{\epsilon_{2y1} - \epsilon_{3y1}}{2r} \qquad \text{A-45}$$

$$A_{11} = \frac{\epsilon_{2z2} - \epsilon_{0z2} - \epsilon_{3y2} - \epsilon_{0y2}}{2r} \qquad \text{A-46}$$

$$A_{12} = \frac{\epsilon_{2z3} - \epsilon_{0z3} - \epsilon_{3y3} + \epsilon_{0y3}}{2r} \qquad \text{A-47}$$

where the notation ε_{2z0} refers to the bias error (as indicated by the subscript 0) in the accelerometer measuring in the z direction at location 2 defined in Figure A-1c. The notation δ_{2zy} refers to the error in position of the accelerometer at location 2, for an accelerometer measuring in the z direction, with the error in the y direction.

If we assume that the errors are statistically independent (at least to the degree that the knowledge that one is positive does not carry implications on the sign of the other terms), the standard deviations of the error coefficients, A_0 - A12, are given by:

$$\sigma(A_0) = \frac{\sigma(\epsilon_0)}{r} \qquad \text{A-48}$$

$$\sigma(A_1) = \sqrt{\frac{\sigma^2(\epsilon_1)}{2} + \frac{\sigma^2(\delta)}{r^2}} \qquad \text{A-49}$$

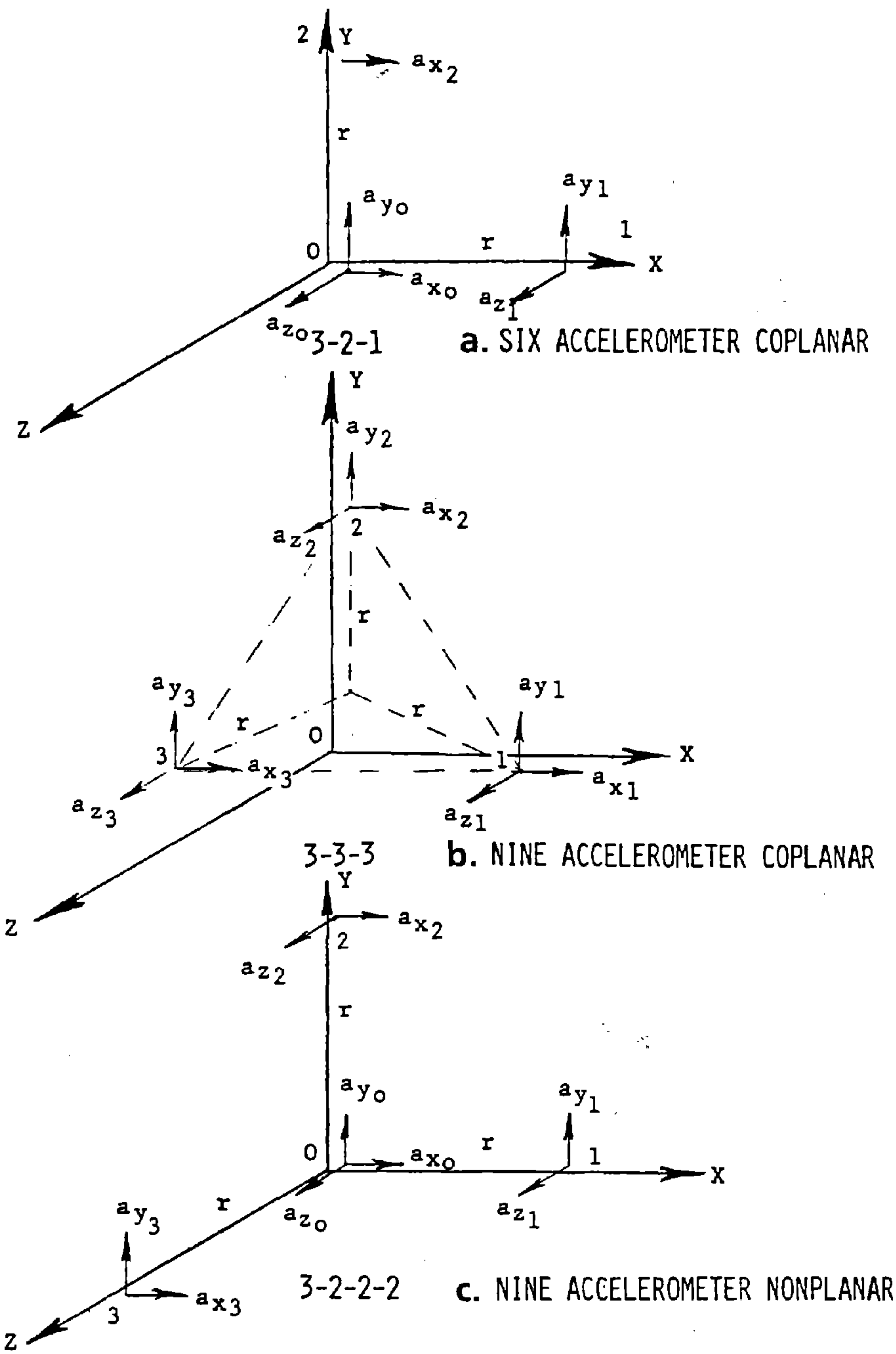


FIGURE A-1. THE THREE CONFIGURATIONS OF TRANSLATIONAL ACCELEROMETERS USED TO MEASURE ANGULAR ACCELERATIONS OF A RIGID BODY

$$\sigma(A_2) = \sqrt{\frac{\sigma^2(\epsilon_1)}{2^2} + \frac{\sigma^2(\delta)}{(\sqrt{2}\ r)^2}} \quad \text{A-50}$$

$$\sigma(A_3) = \sigma(A_2) \quad \text{A-51}$$

$$\sigma(A_4) = \sqrt{\frac{\sigma^2(\epsilon_2)}{(\sqrt{2})^2} + \frac{\sigma^2(\delta)}{r^2}} \quad \text{A-52}$$

$$\sigma(A_5) = \sqrt{\frac{\sigma^2(\epsilon_3)}{2^2} + \frac{\sigma^2(\delta)}{(\sqrt{2}\ r)^2}} \quad \text{A-53}$$

$$\sigma(A_6) = \sigma(A_5) \quad \text{A-54}$$

$$\sigma(A_7) = \sigma(A_3) \quad \text{A-55}$$

$$\sigma(A_8) = \sigma(A_5) \quad \text{A-56}$$

$$\sigma(A_9) = \sigma(A_4) \quad \text{A-57}$$

$$\sigma(A_{10}) = \frac{\sigma(\epsilon_1)}{\sqrt{2}\ r} \quad \text{A-58}$$

$$\sigma(A_{11}) = \frac{\sigma(\epsilon_2)}{r} \qquad \text{A-59}$$

$$\sigma(A_{12}) = \frac{\sigma(\epsilon_3)}{r} \qquad \text{A-60}$$

The influence of both the accelerometer bias error and the accelerometer location errors are seen to decrease with increasing r, the distances between transducers. Increasing the distance between transducers also decreases the errors due to the presence of translational acceleration components (A_{10}, A_{11}, A_{12}).

For an accelerometer capable of measuring an acceleration of 200g it would be reasonable to assume error coefficients standard deviations of:

$$\sigma(\epsilon_0) = 1\text{g} \qquad \text{A-61}$$

$$\sigma(\epsilon_1) = 5\% \text{ of signal} \qquad \text{A-62}$$

$$\sigma(\epsilon_2) = \sigma(\epsilon_3) = 3\% \text{ of signal} \qquad \text{A-63}$$

and assuming r = 4" and a standard deviation of the location error of 0.01", the standard deviation of the error coefficients in estimating the angular accelerations are:

$$\sigma_{A_0} = 96.5 \text{ rad/sec}^2 \qquad \text{A-64}$$

$$\sigma_{A_1} = 3.54\% \text{ of signal} \qquad \text{A-65}$$

$$\sigma_{A_2} = 1.51\% \text{ of signal} \qquad \text{A-66}$$

$$\sigma_{A_3} = 1.51\% \qquad \text{A-67}$$

$$\sigma_{A_4} = 2.14\% \qquad \text{A-68}$$

$$\sigma_{A_5} = 1.51\% \qquad \text{A-69}$$

$$\sigma_{A_6} = 1.51\% \qquad \text{A-70}$$

$$\sigma_{A_7} = 1.51\% \quad \text{A-71}$$

$$\sigma_{A_8} = 1.51\% \quad \text{A-72}$$

$$\sigma_{A_9} = 3.54\% \quad \text{A-73}$$

$$\sigma_{A_{10}} = 0.53\% \quad \text{A-74}$$

$$\sigma_{A_{11}} = 1.03\% \quad \text{A-75}$$

$$\sigma_{A_{12}} = 1.03\% \quad \text{A-76}$$

For the six accelerometer system, defined in figure A-1a, the combined signals will provide:

$$Q_x = \dot{\omega}_x + \omega_y\omega_z + E_{x1} \tag{A-77}$$

$$Q_y = \dot{\omega}_y - \omega_x\omega_z + E_{y1} \tag{A-78}$$

$$Q_z = \dot{\omega}_z + \omega_x\omega_y + E_{z1} \tag{A-79}$$

To estimate the angular acceleration we set

$$Q_x = \dot{\Omega}_x + \Omega_y\Omega_z \tag{A-80}$$

$$Q_y = \dot{\Omega}_y - \Omega_x\Omega_z \tag{A-81}$$

$$Q_z = \dot{\Omega}_z + \Omega_x\Omega_y \tag{A-82}$$

so that

$$\dot{\Omega}_x = \dot{\omega}_x + \omega_y\omega_z - \Omega_y\Omega_z + E_{x1} \tag{A-83}$$

$$\dot{\Omega}_y = \dot{\omega}_y - \omega_x\omega_z + \Omega_x\Omega_z + E_{y1} \tag{A-84}$$

$$\dot{\Omega}_z = \dot{\omega}_z + \omega_x\omega_y - \Omega_x\Omega_y + E_{z1} \tag{A-85}$$

While for the coplanar location of nine transducers, defined in Figure A-1b, we have:

$$\dot{\Omega}_x = \dot{\omega}_x + \omega_y\omega_z - \Omega_y\Omega_z + E_{x3} \qquad \text{A-86}$$

$$\dot{\Omega}_y = \dot{\omega}_y - \omega_x\omega_z + \Omega_x\Omega_z + E_{y2} \qquad \text{A-87}$$

$$\dot{\Omega}_z = \dot{\omega}_z + E_{z2} \qquad \text{A-88}$$

As shown in appendix A2, E_x, E_y and E_z will have the same form as the error terms for the nine accelerometer package of reference 1.

4. ERROR GROWTH DUE TO NUMERICAL INTEGRATION

In this section the three accelerometer configurations are compared to evaluate the importance of numerical integration and other factors. The three configurations are illustrated in Figure A-1.

As previously discussed, a transducer configuration which does not provide nine independent translational acceleration measurements i.e., the six-accelerometer system and the nine-accelerometer coplanar system, requires an estimate of the angular velocities in order to obtain the angular acceleration for even perfect transducers and transducer locations. For the six accelerometer system, the estimated angular accelerations obtained by combining the accelerometer signals through linear algebraic operations are:

$$\dot{\Omega}_x = \dot{\omega}_x + \omega_y\omega_z - \Omega_y\Omega_z + E_x \qquad \text{A-89}$$

$$\dot{\Omega}_y = \dot{\omega}_y - \omega_x\omega_z + \Omega_x\Omega_z + E_y \qquad \text{A-90}$$

$$\dot{\Omega}_z = \dot{\omega}_z + \omega_x\omega_y - \Omega_x\Omega_y + E_z \qquad \text{A-91}$$

where E_x, E_y, and E_z are errors due to transducer and transducer location imperfections. If the angular velocities are estimated from integration of the estimated angular accelerations by the algorithm:

$$\Omega x_I = \Omega x_{I-1} + \dot{\Omega} x_{I-1}(\Delta t) \qquad \text{A-92}$$

$$\Omega y_I = \Omega y_{I-1} + \dot{\Omega} y_{I-1}(\Delta t) \qquad \text{A-93}$$

$$\Omega z_I = \Omega z_{I-1} + \dot{\Omega} z_{I-1}(\Delta t) \qquad \text{A-94}$$

The errors in the estimated angular acceleration may grow with time. The errors that develop are a function of the transducer errors (E_x, E_y, and E_z), the integration time step (Δt) and the period of time for which the integration is carried.

In a similar manner, as shown in appendix A2 for the nine-accelerometer, coplanar system the estimated angular acceleration components obtained from a least squares fit which weights each accelerometer output signal equally are given by:

$$\dot{\Omega}_z = \dot{\omega}_z + E_z \qquad \text{A-95}$$

$$\dot{\Omega}_y = \dot{\omega}_y + \omega_x\omega_z - \Omega_x\Omega_z + E_y \qquad \text{A-96}$$

$$\dot{\Omega}_x = \dot{\omega}_x - \omega_y\omega_z - \Omega_y\Omega_z + E_x \qquad \text{A-97}$$

The error terms due to transducer and location errors are derived in Appendix A2 as:

$$\begin{aligned} E_x = C_0 &+ C_1\dot{\omega}_x + C_2\dot{\omega}_z + C_3\dot{\omega}_y + C_4\omega_x^2 + C_5\omega_y^2 + C_6\omega_z^2 \\ &+ C_7\omega_x\omega_y + C_8\omega_x\omega_z + C_9\omega_y\omega_z + C_{10}R_x + C_{11}R_y \\ &+ C_{12}R_z \end{aligned} \qquad \text{A-98}$$

$$\begin{aligned} E_y = B_0 &+ B_1\dot{\omega}_y + B_2\dot{\omega}_x + B_3\dot{\omega}_z + B_4\omega_x^2 + B_5\omega_y^2 + B_6\omega_z^2 \\ &+ B_7\omega_x\omega_y + B_8\omega_x\omega_z + B_9\omega_y\omega_z + B_{10}R_x + B_{11}R_y \\ &+ B_{12}R_z \end{aligned} \qquad \text{A-99}$$

$$\begin{aligned} E_z = A_0 &+ A_1\dot{\omega}_z + A_2\dot{\omega}_x + A_3\dot{\omega}_y + A_4\omega_x^2 + A_5\omega_y^2 + A_6\omega_z^2 \\ &+ A_7\omega_x\omega_y + A_8\omega_x\omega_z + A_9\omega_y\omega_z + A_{10}R_x + A_{11}R_y \\ &+ A_{12}R_z \end{aligned} \qquad \text{A-100}$$

The relationship between the error coefficients and the transducer imperfections and mislocations are given in Appendix A2. The following paragraphs describe the errors that would be obtained for selected rigid body acceleration time histories and transducer errors.

4.1 EFFECTS OF NEGLECTING ANGULAR VELOCITY TERMS

During the integration of the angular accelerations, errors may accumulate in the angular velocity crossproduct terms. These terms may be neglected for certain special conditions of rigid body acceleration. The simplest case is when the angular displacements are small, making the angular velocity terms small compared to the angular accelerations. Another case is when all the angular motion is about a single, principal axis of the accelerometer configuration. For the six-accelerometer, coplanar configuration, if two of the angular velocity terms are zero, then the crossproduct terms will be zero.

For this reason the coplanar accelerometer configurations should be placed, whenever possible, so that all the angular motion is about one of the principal axes of the configuration. As discussed below, error terms resulting in erroneous signals on the other axes may still lead to instablities. For the nine-accelerometer, coplanar configuration, the motion should be restricted to be in the same plane as the plane formed by the location of the three triads making one of the angular velocity terms zero. If the expected rotations are about axes in the x-y plane, then the configuration should be positioned in the x-y plane.

For cases that include arbitrary angular accelerations and velocities about each of the three principal axes, the importance of estimating the angular velocities depends on the amount of angular displacement taking place. As an example, consider the case of the constant angular acceleration, a, about an axis, q which has equal components in the x, y, and z directions. The angular accelerations and velocities about each axis may be written as:

$$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 1/\sqrt{3}\ a \qquad \text{A-101}$$

$$\omega_x = \omega_y = \omega_z = 1/\sqrt{3}\ a\ t \qquad \text{A-102}$$

The velocity crossproduct terms are then:

$$\omega_x{*}\omega_y = \omega_x{*}\omega_z = \omega_y{*}\omega_z = 1/3\ a^2\ t^2 \qquad \text{A-103}$$

Writing the acceleration, velocity and angular displacement about the q axis produces:

$$\dot{\omega}_q = a \qquad \text{A-104}$$

$$\omega_q = a\ t \qquad \text{A-105}$$

$$\theta_q = 1/2\ a\ t^2 \qquad \text{A-106}$$

Substituting for the angular displacement, $_q$, into the angular velocity crossproduct terms expression produces:

$$\omega_x{*}\omega_y = \omega_x{*}\omega_z = \omega_y{*}\omega_z = 2/3\ a\ \theta_q \qquad \text{A-107}$$

If the angular displacement is less then 15 degrees, the error that will result from assuming the angular velocities to be zero will be less than 20% of the angular acceleration. However, if the displacement is 85^{o}, then the error will be equal to the acceleration being measured. Large displacements require that the angular velocity terms be included.

4.2 ZERO ACCELERATION WITH BIAS ERRORS

In this case we consider the situation where the rigid body is stationary at the time the data acquisition system and processing is started and remains stationary for one second. Further, if we assume that bias errors are the only errors in the accelerometer configuration, for a 1 g. bias error and a distance of 4" between transducers, erroneous constant angular acceleration signals, $E_x = E_y = E_z = 100$ rad/sec^2 would be produced. The non-coplanar, 3-2-2-2 configuration[1] would report a constant angular acceleration error, while the nine coplanar 3-3-3 configuration[2] would report angular accelerations which grow, in an oscillatory manner, with time. Figure A-2 shows the acceleration estimate $\dot{\Omega}_x$ that would be reported as a function of time for integration time steps of one millionth, one ten thousandth and one thousandth of second. Figure A-3 shows the error size after a one second integration versus the integration time step size.

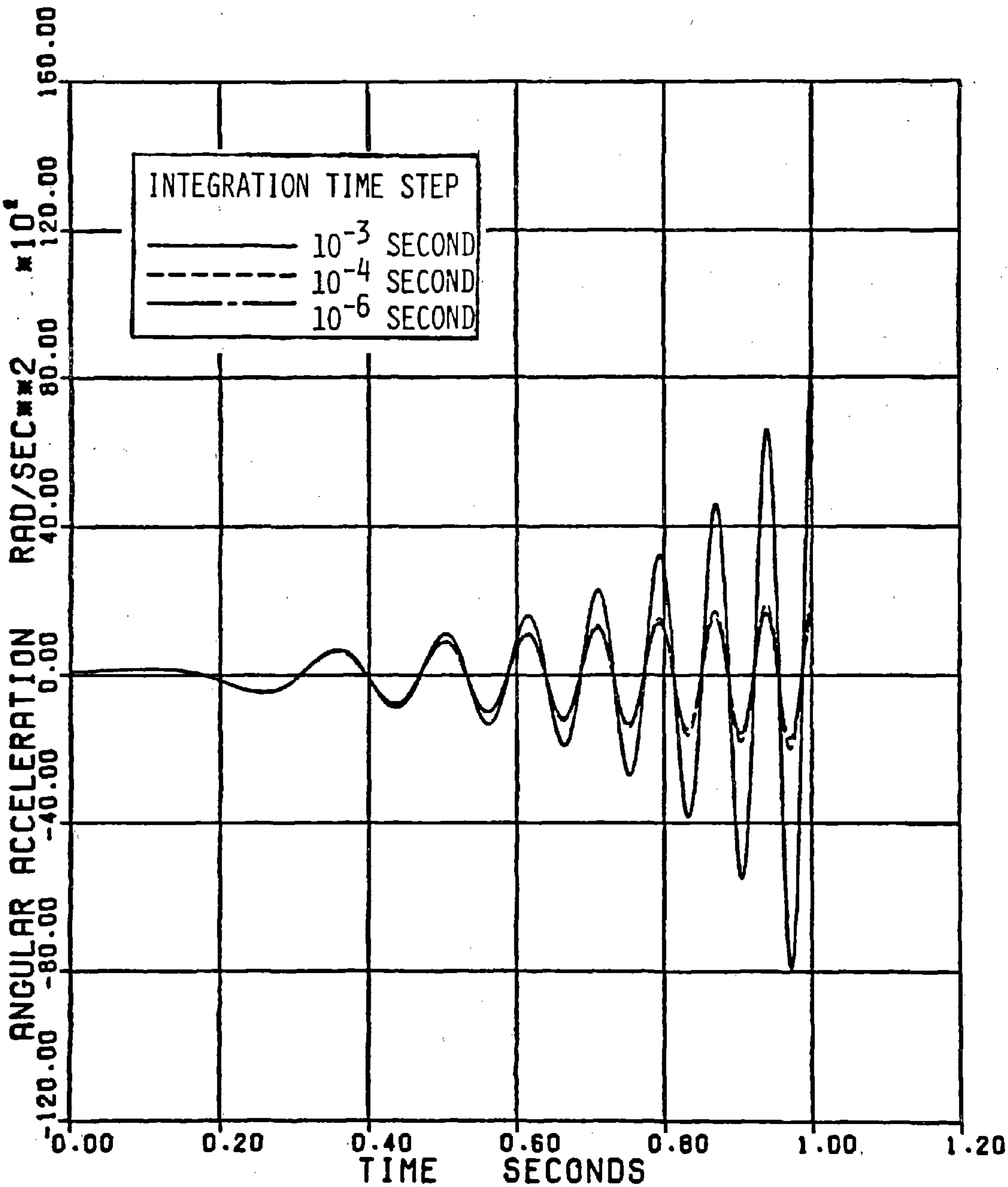


FIGURE A-2: ERRONEOUS ANGULAR ACCELERATION SIGNALS DUE TO CONSTANT BIAS ERROR OF 100 rad/sec^2. THE EFFECT OF INTEGRATION TIME STEP SIZE ON ERROR GROWTH IN $\dot{\Omega}_x$ FOR 9 ACCELEROMETER IN-PLANE CONFIGURATION

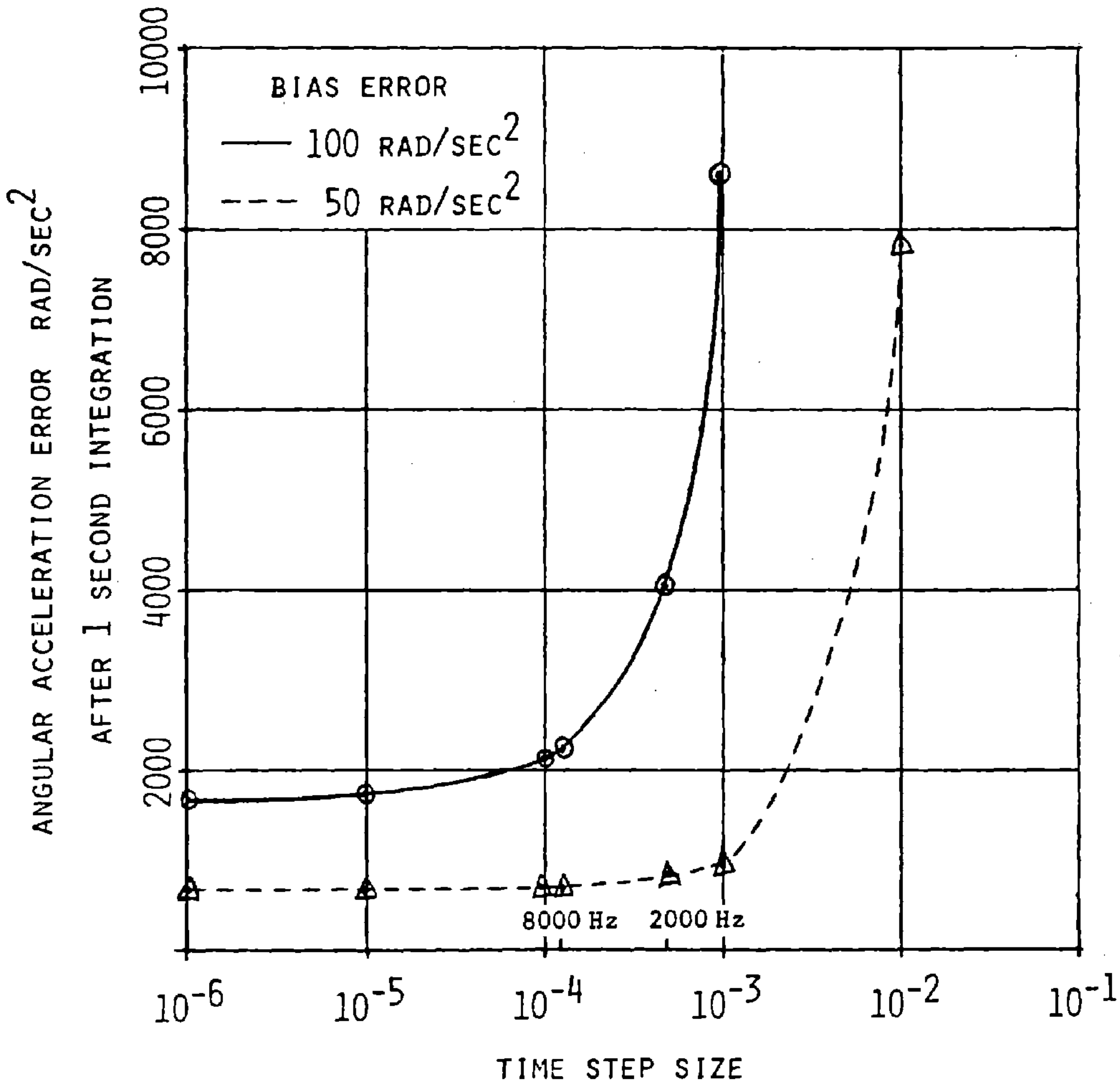


FIGURE A-3. ERROR AFTER 1 SECOND INTEGRATION VERSUS INTEGRATION STEP SIZE FOR 9 ACCELEROMETER IN-PLANE CONFIGURATION

Interestingly, for this case, the six accelerometer coplanar configuration (1) does not lead to large error estimates. After one second the erroneous signal is compensated for by computed constant angular velocities.

4.3 SINUSOIDAL ANGULAR ACCELERATION ABOUT A SINGLE AXIS WITH SENSITIVITY, CROSSTALK, AND LOCATION ERRORS

Reference 1 discusses the stability of the six-accelerometer configuration with respect to the nine, non-coplanar accelerometer configuration. Using a hypothetical signal, shown in Figure A-4, representing a sinusoidal angular acceleration about the y axis with an amplitude of 11,580 rad/sec^2 and a period of 100 msec and with a 2% error due to primary axis sensitivity, alignment and crossaxis sensitivity errors, the six accelerometer configuration is shown to produce unstable estimates of the angular accelerations about the x and z axes after less than 0.15 seconds for certain conditions. The nine-accelerometer, non-coplanar configuration is stable. The estimate of the angular accelerations about the x axis from the six-accelerometer configuration is shown in Figure A-5. The nine-accelerometer, coplanar configuration is also, for these conditions, stable over the one second integration period with a one thousandth of a second time step size as seen in Figure A-6. If, however, the angular acceleration is about the z axis rather than the y axis, the nine accelerometer, coplanar configuration is unstable even for a time step of one ten thousandth of second due to error build up in the angular velocity crossproduct terms. In Figure A-7, the estimated angular acceleration about the x axis for the nine accelerometer, coplanar configuration with the actual angular acceleration about the z axis is shown for two integration time step sizes.

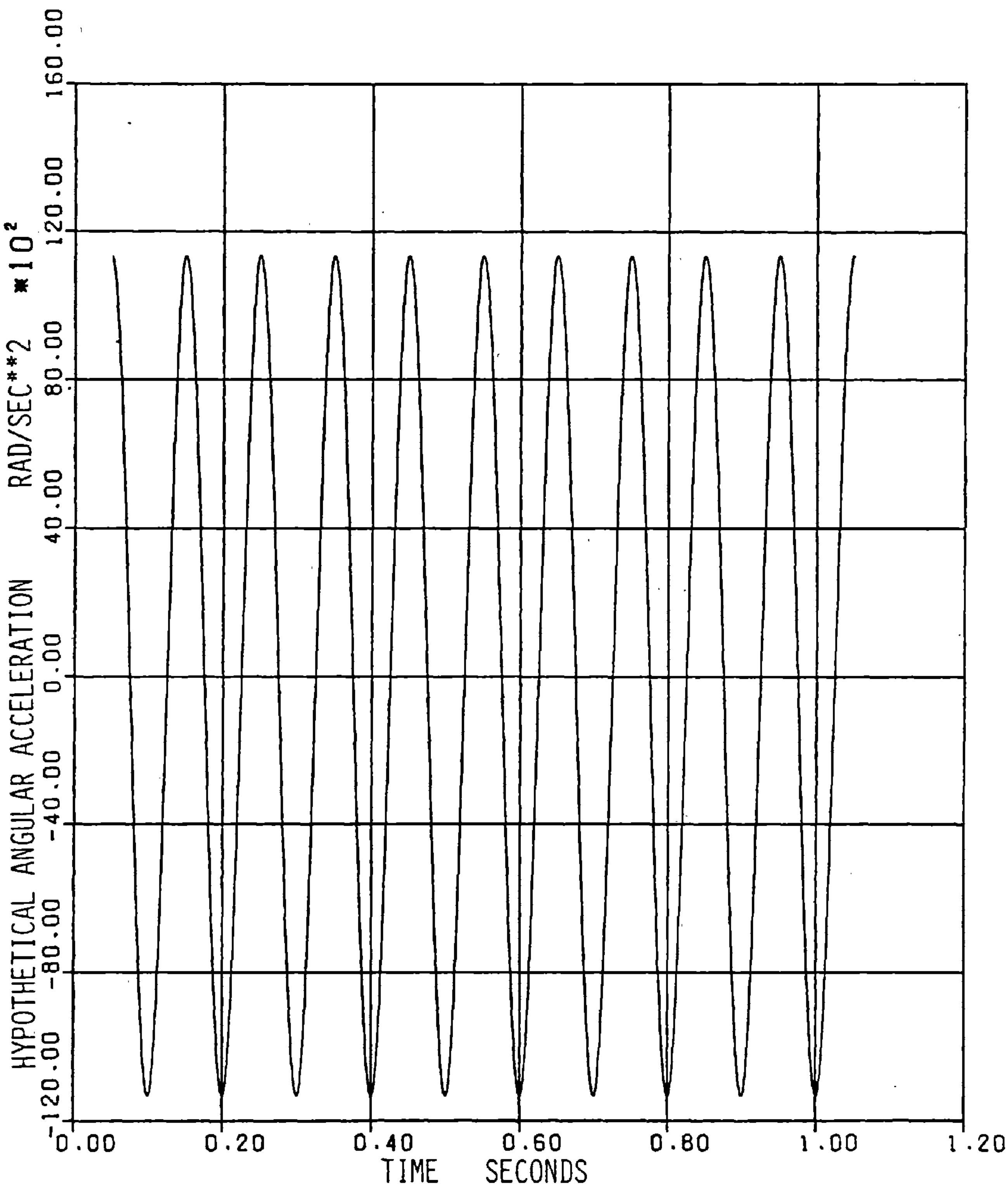


FIGURE A-4. HYPOTHETICAL SINUSOIDAL ANGULAR ACCELERATION ABOUT Y AXIS STARTING AT T+50 MSEC. AMPLITUDE = 11580 rad/sec^2

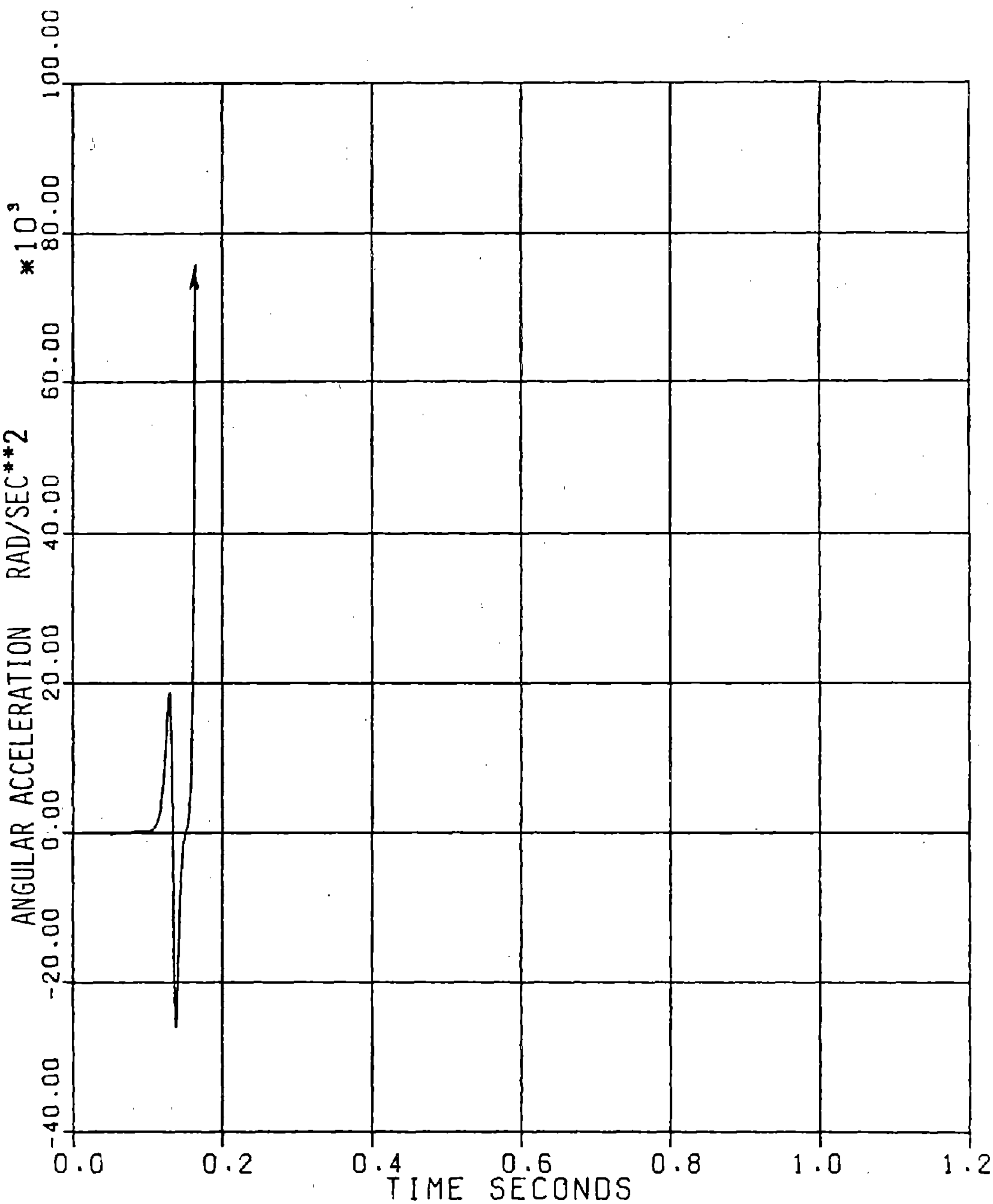


FIGURE A-5. ESTIMATE OF ANGULAR ACCELERATION ABOUT X AXIS FOR SIX ACCELEROMETER CONFIGURATION WITH SINUSOIDAL ANGULAR ACCELERATION ABOUT Y AXIS

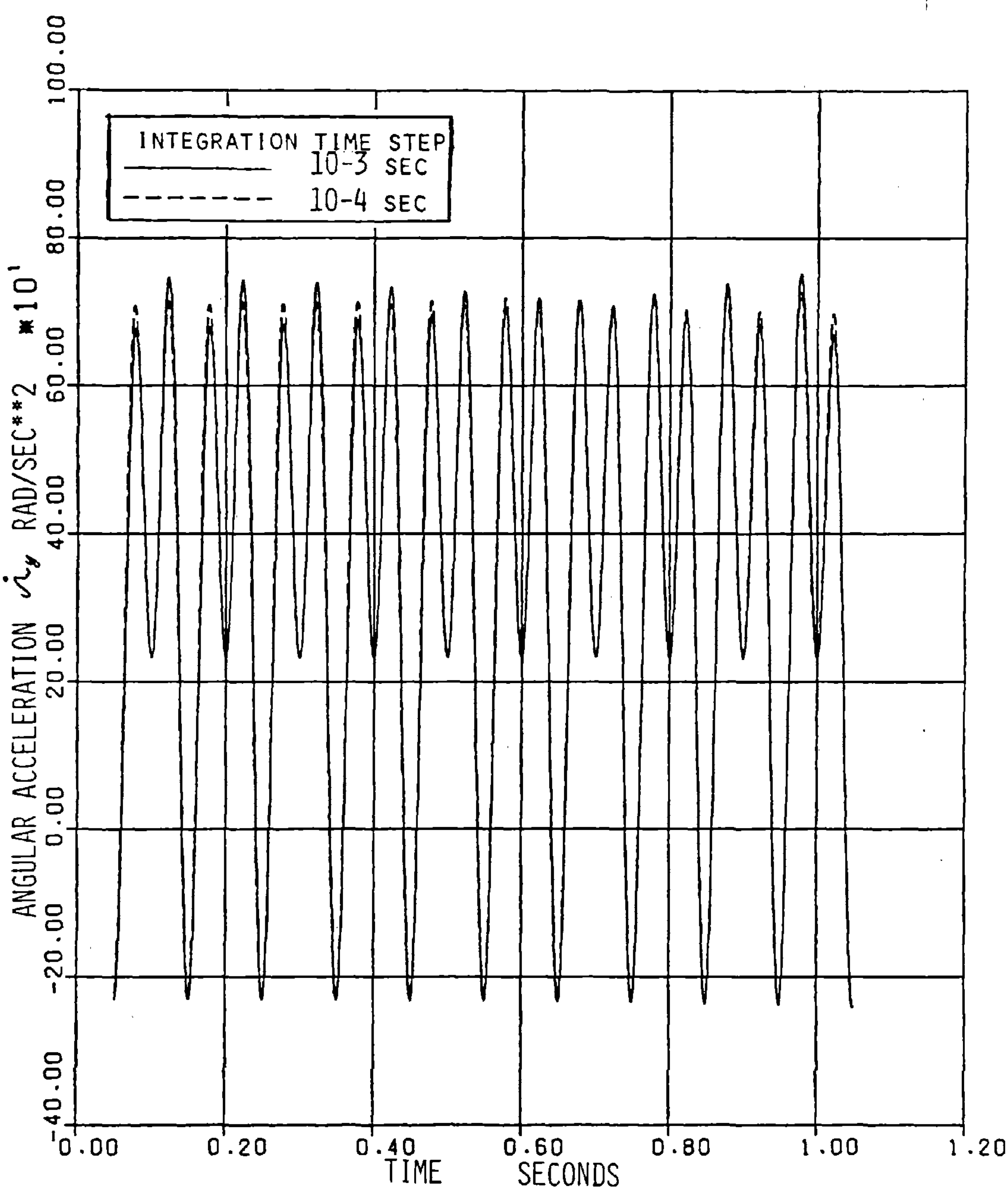


FIGURE A-6. ESTIMATED $\dot{\Omega}_x$ FROM 9 ACCELEROMETER COPLANER CONFIGURATION DUE TO SENSITIVITY ERRORS WITH SINUSOIDAL ANGULAR ACCELERATION ABOUT Y AXIS

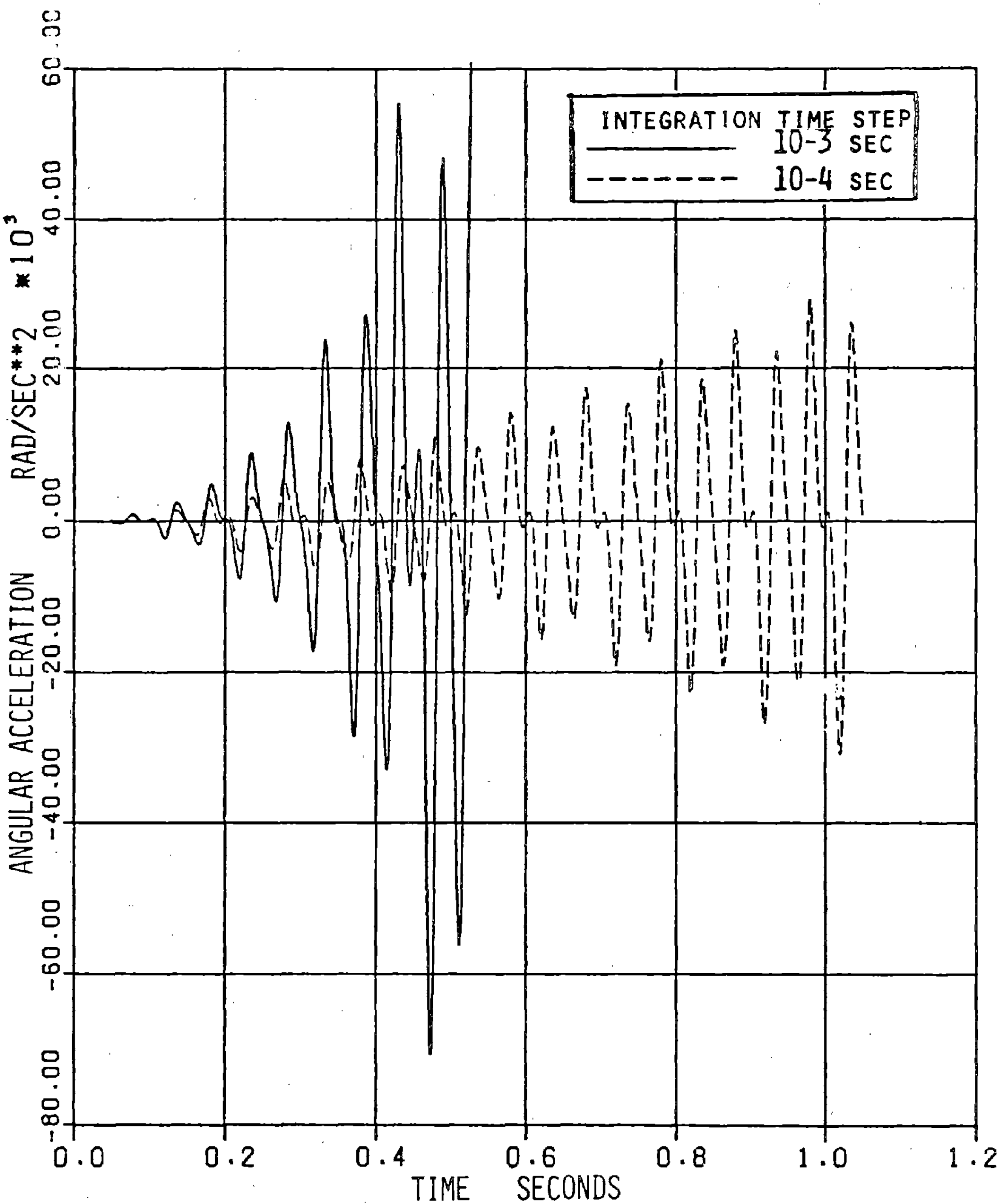


FIGURE A-7. ESTIMATED $\dot{\Omega}_x$ FROM NINE ACCELEROMETER COPLANER CONFIGURATION DUE TO SENSITIVITY ERRORS WITH SINUSOIDAL ANGULAR ACCELERATION ABOUT Z AXIS

As is seen in Figure A-7, integration time step size can affect the stability of the numerical integration. Roundoff errors due to computer word length can have a similar effect especially when algebraically combining the transducer signals to form the angular acceleration estimates. The results shown here were produced using a word size with about 35 significant figures which is probably larger than would be used in a typical data aquisition system.

4.4 SINUSOIDAL NOISE ERRORS

A common error source is 60 cycle noise appearing as a bias error. In Figure A-8, the angular acceleration time history resulting from a 80 rad/sec^2 sinusoidal bias error is shown. No perceptible build up of error occurs.

4.5 TRANSLATIONAL IMPULSE WITH SENSITIVITY ERRORS

Often the event of interest occurs during a very short time. For this case consider a translational acceleration pulse producing erroneous angular acceleration signals due to sensitivity and crosstalk errors. Figure A-9 shows the estimated angular acceleration time history due to a 10 msec, half sine translational pulse with a 100 g peak. If there were no error build up in the angular velocity crossproduct terms the peak angular acceleration would be 240 rad/sec^2 based on a distance of 4" between transducers and a 1% sensitivity error. For this case the signal is so short that no substantial error can build up in the angular velocity terms and after one second the angular acceleration estimates have all nearly returned to zero. In Figures A-10a and A-10b the estimated angular acceleration time histories for similar

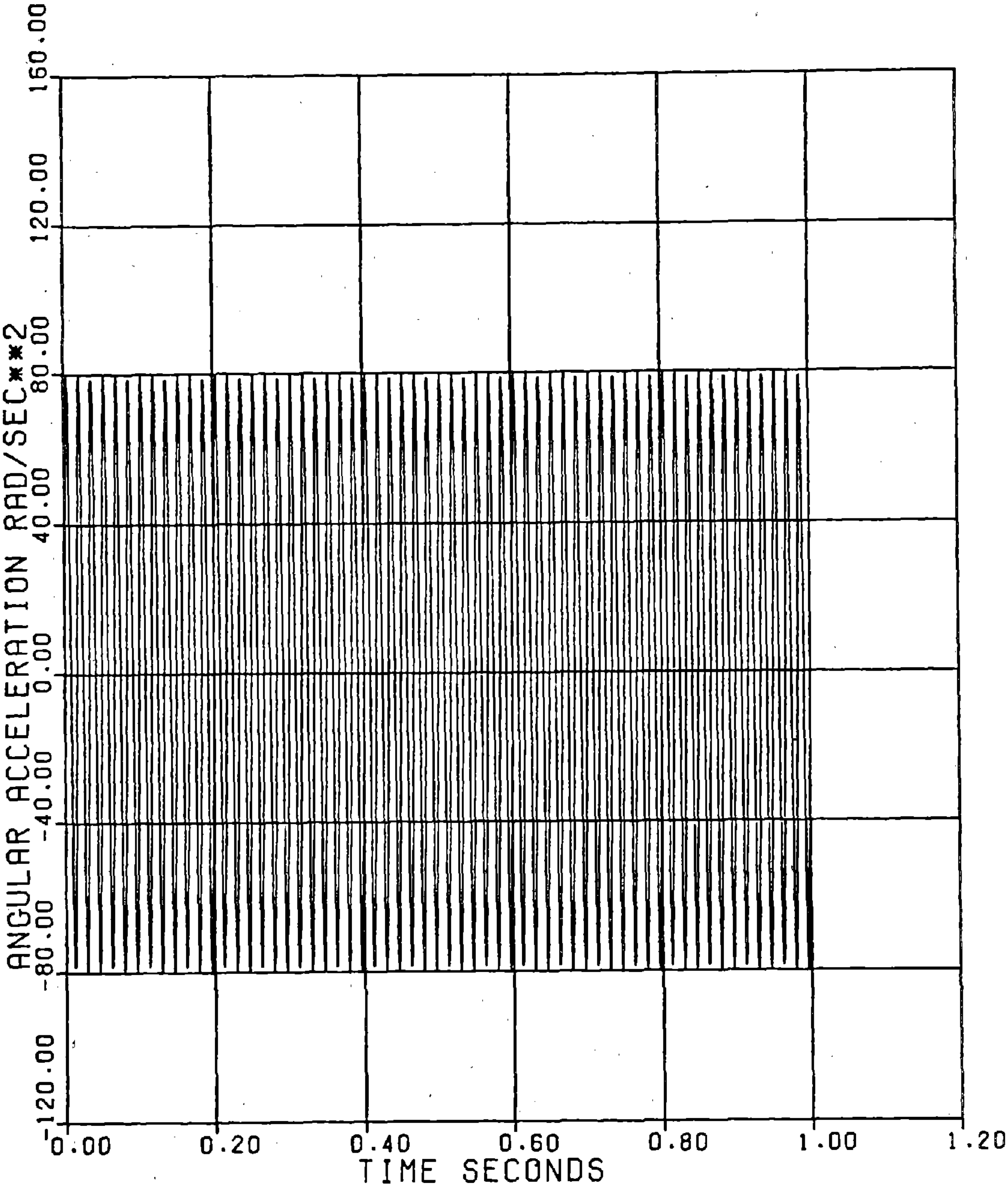


FIGURE A-8. $\dot{\Omega}_x$, RESULTING FROM 60 HERTZ NOISE BIAS ERROR OF 80 rad/sec^2. INTEGRATION TIME STEP 10^{-4} SECONDS

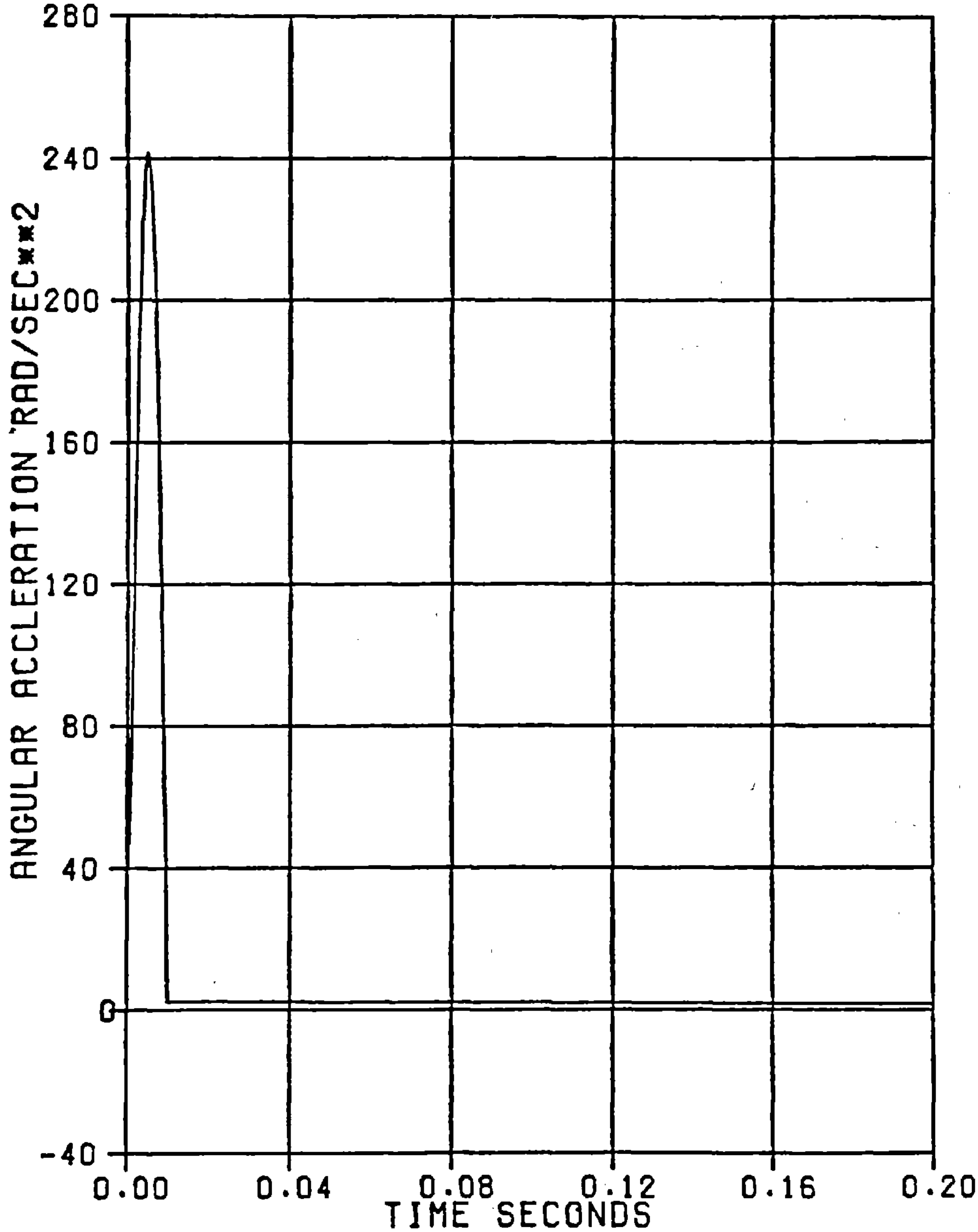


FIGURE A-9. ANGULAR ACCELERATION TIME HISTORY FOR $\dot{\Omega}_x$ FROM NINE ACCELEROMETER, CO-PLANAR CONFIGURATION WITH 100 g, 10 MSEC TRANSLATIONAL PULSE AND SENSITIVITY ERRORS

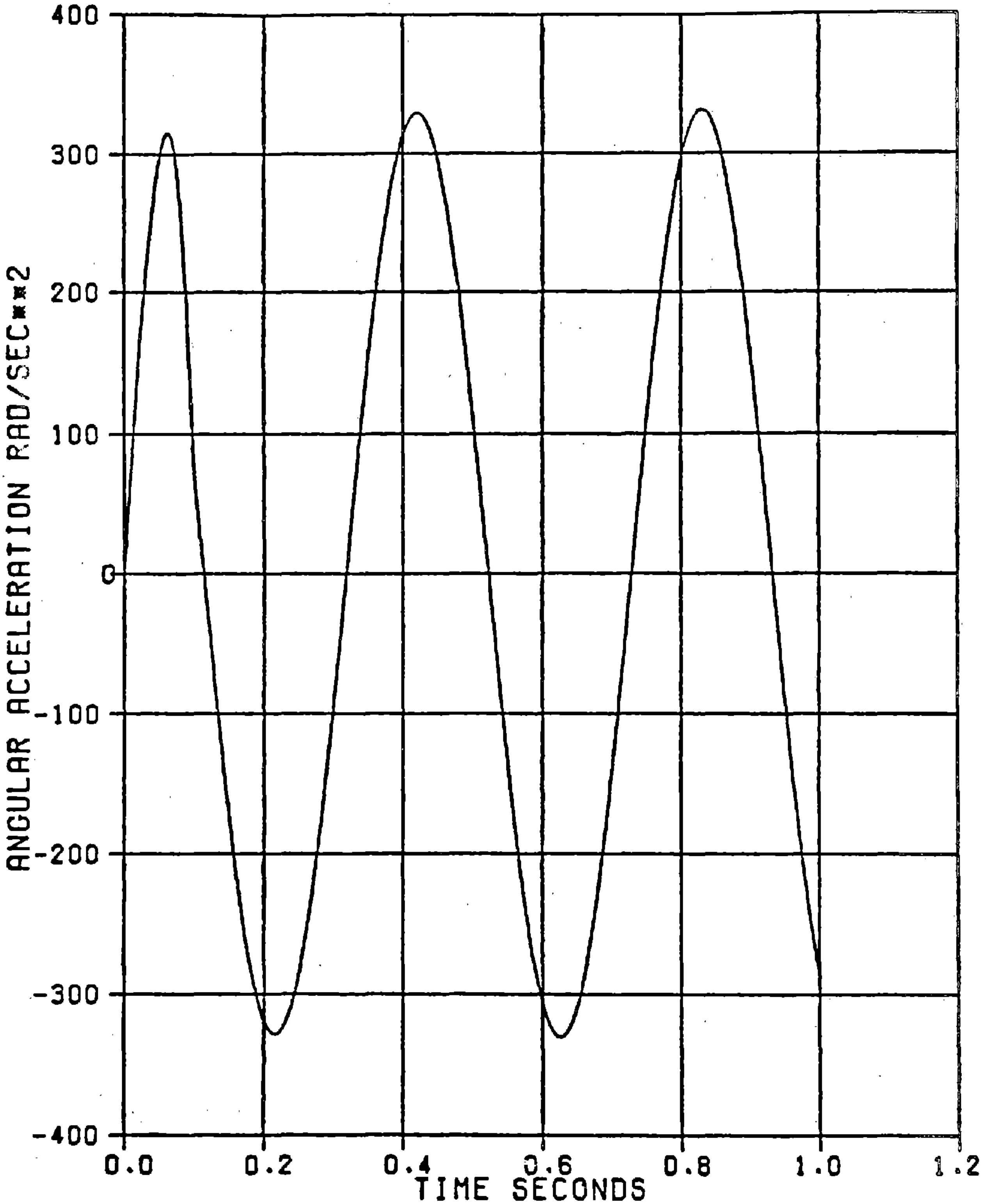


FIGURE A-10a. $\dot{\Omega}_x$ TIME HISTORY RESULTING FROM 100 g TRANSLATIONAL PULSE LASTING 100 MSEC, WITH TRANSDUCER SENSITIVITY ERRORS FOR NINE ACCELEROMETER, COPLANAR CONFIGURATION

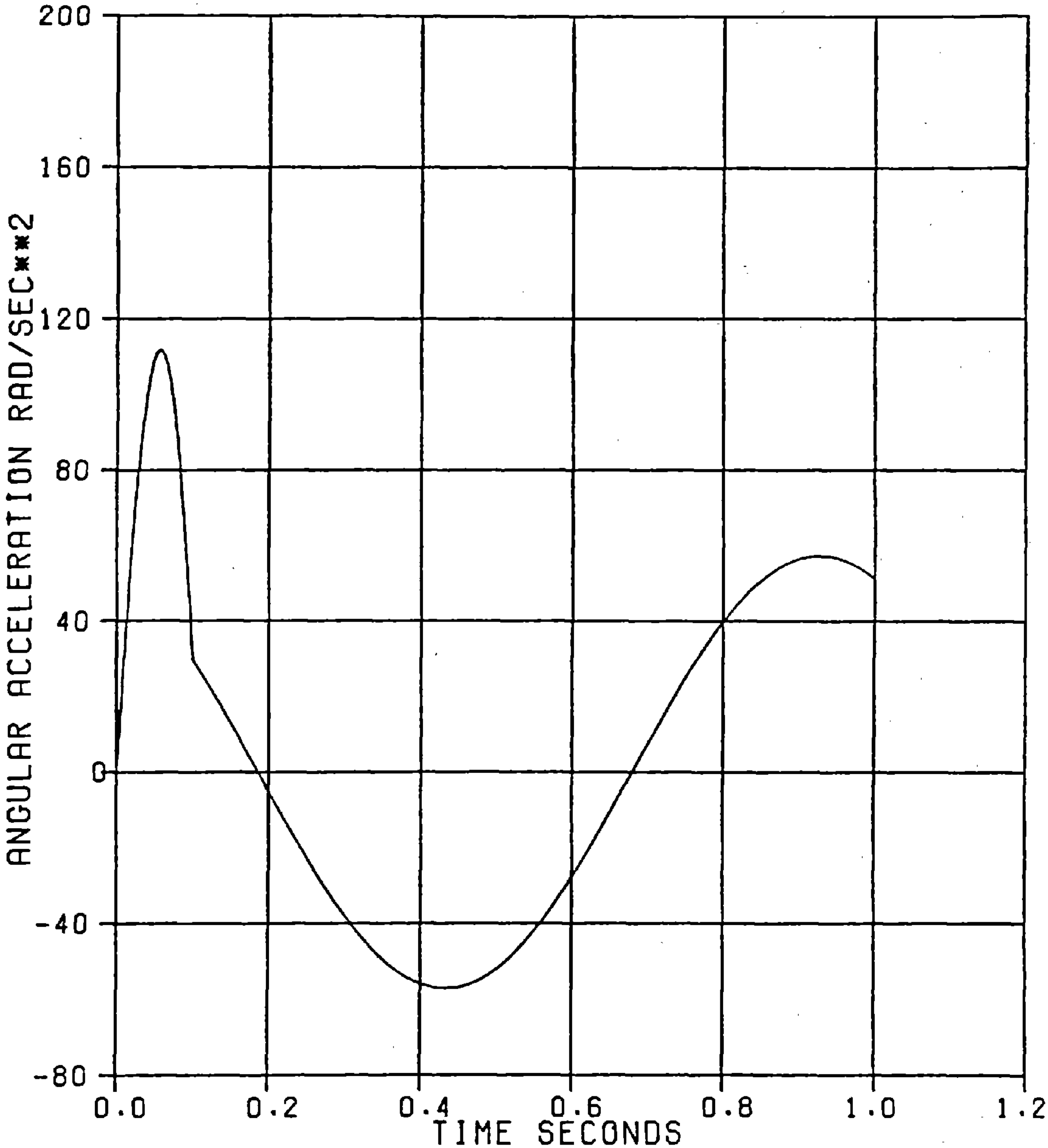


FIGURE A-10b. $\dot{\Omega}_x$ TIME HISTORY RESULTING FROM 100 rad/sec^2 HALF SINE PULSE LASTING 100 MSEC, WITH TRANSDUCER SENSITIVITY ERRORS, NINE ACCELEROMETER, COPLANAR CONFIGURATION translational pulses, in this case lasting 100 msec., are shown. The first case corresponds to the 100 g. half sine pulse discussed above, while the other corresponds to a signal which would produce a 100 rad/sec^2 estimate if there were no error build up. In both these cases the angular velocity terms are sufficiently large after the 100 msec. pulse has ended to produce significant erroneous angular acceleration estimates. The angular acceleration estimates oscillate after the signal has ended and in each case the peak is slightly larger after each oscillation.

5.0 - Initialization and Calibration of Measurement System

Once the transducer package has been assembled, the signals from each of the transducers are combined algebraically either through electronics contained in the package or through computer operations to provide six output variables VR_x, VR_y, VR_z, $V\dot{\omega}_x$, $V\dot{\omega}_y$, and $V\dot{\omega}_z$, where the quantities, VR_x, VR_y, and VR_z are estimates of the translational components of acceleration R_x, R_y, and R_z and the quantities $V\dot{\omega}_x$, $V\dot{\omega}_y$, and $V\dot{\omega}_z$ are from the nine independent accelerometer package estimates of the angular accelerations $\dot{\omega}_x$, $\dot{\omega}_y$, and $\dot{\omega}_z$. For the six accelerometer package the $V\dot{\omega}_x$, $V\dot{\omega}_y$, and $V\dot{\omega}_z$ are estimates of the quantities $(\dot{\omega}_x + \omega_y*\omega_z)$, $(\dot{\omega}_y - \omega_x*\omega_z)$, and $(\omega_z + \dot{\omega}_x*\dot{\omega}_y)$. For the nine accelerometer, coplanar configuration the $V\dot{\omega}_x$, $V\dot{\omega}_y$, and $V\dot{\omega}_z$ are estimates of $(\dot{\omega}_x - \omega_y*\omega_z)$, $(\dot{\omega}_y + \omega_x*\omega_z)$, and $(\dot{\omega}_z)$. In all cases the errors in the estimates are of the form:

$$E = C_0 + C_1\omega_x + C_2\omega_y + C_3\omega_z + C_4\omega_x^2 + C_5\omega_y^2 + C_6\omega_z^2 + C_7\omega_x*\omega_y + C_8\omega_x*\omega_z + C_9\omega_y*\omega_z + C_{10}R_x + C_{11}R_y + C_{12}R_z \qquad \text{A-108}$$

The relationship between these coefficients and the individual transducer and transducer location errors are given in section 3 and Appendices A and B.

To the degree that these coefficients are constant and invariant with time, the values of these coefficients can be determined by laboratory calibration using vibration test equipment to produce the translational accelerations and turntables, to produce angular velocities. Once the coefficients are known several options are available for improving the estimate. One is to examine the relationship between the transducer errors, location errors and the coefficients to correct accelerometer sensitivities and distances. A second is to use the estimates of the angular velocities and accelerations to correct the output computation. A third would be to use the coefficients measured in the laboratory to algebraically solve for improved estimates in terms of combinations of VR_x, VR_y, VR_z, $V\dot{\omega}_x$, $V\dot{\omega}_y$, and $V\dot{\omega}_z$. The relative benefits of these approaches depends on the size of the error coefficients and the particular mechanizations used to combine the signals.

Since the accelerometer errors may drift with time it would be desirable to calibrate the instrumentation package as near to the test as possible. Most tests do provide some level of redundant information, through photographic coverage or the use of redundant accelerometers. Improved estimates of the error coefficients and the accelerations can be achieved with the use of the independent knowledge of the states, such as the position, velocity and acceleration immediately before and after the test. Through more sophisticated techniques, use of average accelerations estimated from velocity changes combined with an understanding of the mechanics of the impact could be applied to further improve the estimates.

If the angular velocities must be obtained by integration of the estimated accelerations, significant error reductions could be achieved by delaying the start of the integration until the start of the impact event, possibly using an acceleration threshold to indicate the start of the integration.

Prior to exploring the above techniques in more detail it would be desirable to have a more complete definition of the scenarios for application of the measurements and the calibration test data for typical transducer packages.

APPENDIX A1

DERIVATION OF EQUATIONS FOR ESTIMATED ANGULAR ACCELERATIONS FOR NINE ACCELEROMETER, NON-COPLANAR CONFIGURATION

In this Appendix, the estimated angular accelerations are derived in terms of accelerometer location error δ and accelerometer output error ε. Figure A1-1 shows the fixed coordinate system Ixyz with unit vectors $\bar{i}\bar{j}\bar{k}$, and the moving coordinate system $O\hat{x}\hat{y}\hat{z}$ with unit vectors $\bar{u}_1\bar{u}_2\bar{u}_3$. Point P is an arbitrary point representing the transducers. The components of the position vector $\bar{R}$ of the moving coordinates $O\hat{x}\hat{y}\hat{z}$ are,

$$\bar{R} = X\bar{i} + Y\bar{j} + Z\bar{k} \tag{A1-1}$$

$$= X_o\bar{u}_1 + Y_o\bar{u}_2 + Z_o\bar{u}_3 \tag{A1-2}$$

resolved in the direction of the fixed and moving coordinate axis respectively. Similarly, the components of the position vector $\bar{r}$ of point P relative to the fixed coordinates are,

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k} \tag{A1-3}$$

$$= x_o\bar{u}_1 + y_o\bar{u}_2 + z_o\bar{u}_3 \tag{A1-4}$$

and the components of the position vector $\bar{\rho}$ of point P with respect to the moving coordinates are,

$$\bar{\rho} = \hat{x}\bar{u}_1 + \hat{y}\bar{u}_2 + \hat{z}\bar{u}_3 \tag{A1-5}$$

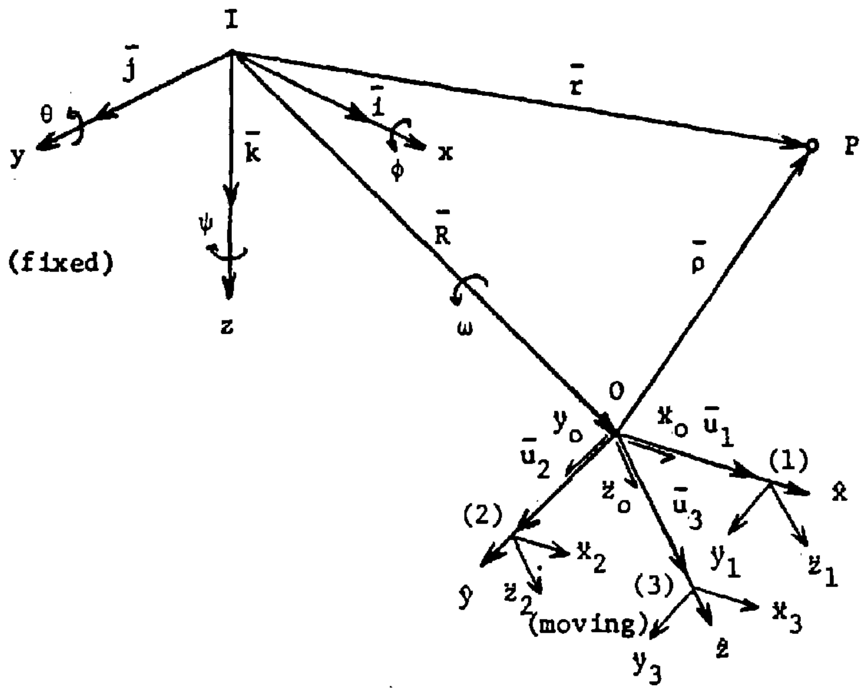


FIGURE A1-1. COORDINATE SYSTEM FOR NON-COPLANAR ACCELEROMETER CONFIGURATION

The angular velocity vector $\bar{\omega}$ is expressed in velocity components $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$ as follows:

$$\bar{\omega} = \dot{\phi}\bar{i} + \dot{\theta}\bar{j} + \dot{\psi}\bar{k} \qquad \text{(A1-6)}$$

$$\bar{\omega} = \dot{\phi}_o\bar{u}_1 + \dot{\theta}_o\bar{u}_2 + \dot{\psi}_o\bar{u}_3 \qquad \text{(A1-7)}$$

To express the three angular accelerations of a rigid body in terms of the linear accelerations without the angular velocity product terms appearing in the equation, it is necessary to have nine independent linear acceleration measurements. For a nine-accelerometer technique, a set of three accelerometers is located at the origin 0 which measures the three orthogonal directions $(\ddot{x}_o, \ddot{y}_o, \ddot{z}_o)$, two accelerometers are located on the $\hat{x}$-axis at position 1 which measures in the y and z directions $(\ddot{y}_1, \ddot{z}_1)$, two on the $\hat{y}$-axis at position 2 $(\ddot{x}_2, \ddot{z}_2)$, and two on the $\hat{z}$-axis at position 3 $(\ddot{x}_3, \ddot{y}_3)$.

The angular accelerations can be obtained from the general kinematic equation of relative motion for the absolute acceleration of point P,

$$\ddot{\bar{r}} = \ddot{\bar{R}} + \ddot{\bar{\rho}} + (\dot{\bar{\omega}} \times \bar{\rho}) + \bar{\omega} \times (\bar{\omega} \times \bar{\rho}) + (2\bar{\omega} \times \dot{\bar{\rho}}) \qquad \text{(A1-8)}$$

The components of the acceleration vector $\ddot{\bar{r}}$ and relative position vector $\bar{\rho}$ at each of the accelerometer locations are:

$$\ddot{\bar{r}}_o = \ddot{x}_o\bar{u}_1 + \ddot{y}_o\bar{u}_2 + \ddot{z}_o\bar{u}_3 \qquad \bar{\rho}_o = 0 \qquad \text{(A1-9)}$$

$$\ddot{\bar{r}}_1 = \ddot{y}_1\bar{u}_2 + \ddot{z}_1\bar{u}_3 \qquad \bar{\rho}_1 = \rho_1 u_1 \qquad \text{(A1-10)}$$

$$\ddot{\bar{r}}_2 = \ddot{x}_2\bar{u}_1 + \ddot{z}_2\bar{u}_3 \qquad \bar{\rho}_2 = \rho_2 u_2 \qquad \text{(A1-11)}$$

$$\ddot{\bar{r}}_3 = \ddot{x}_3\bar{u}_1 + \ddot{y}_3\bar{u}_2 \qquad \bar{\rho}_3 = \rho_3 u_3 \qquad \text{(A1-12)}$$

(Note that $x_o = X_o, y_o = Y_o, z_o = Z_o$)

Since there are no relative motions between the accelerometers and the moving coordinate, $\ddot{\rho} = 0$, $\dot{\rho} = 0$, and Equation A1-8 reduces to,

$$\ddot{\bar{r}} = \ddot{\bar{R}} + (\dot{\bar{\omega}} \, X \, \bar{\rho}) + \bar{\omega} \, X \, (\bar{\omega} \, X \, \bar{\rho}) \qquad \text{(A1-13)}$$

By substituting the vectors in terms of their components into the kinematic equation (Equation A1-13) and solving for the angular velocities yields the following:

$$\ddot{\phi}_o = (\ddot{z}_2 - \ddot{Z}_o)/2\rho_2 - (\ddot{y}_3 - \ddot{Y}_o)/2\rho_3 \qquad \text{(A1-14)}$$

$$\ddot{\theta}_o = (\ddot{x}_3 - \ddot{X}_o)/2\rho_3 - (\ddot{z}_1 - \ddot{Z}_o)/2\rho_1 \qquad \text{(A1-15)}$$

$$\ddot{\Psi}_o = (\ddot{y}_1 - \ddot{Y}_o)/2\rho_1 - (\ddot{x}_2 - \ddot{X}_o)/2\rho_2 \qquad \text{(A1-16)}$$

If perfect transducers were located at precisely the specified locations, the angular acceleration estimates would be exactly obtained from:

$$\dot{\Omega}\phi = (Q_{2z} - Q_{0z})/2\rho_2 - (Q_{2y} - Q_{0y})/2\rho_3 \qquad \text{(A1-17)}$$

$$\dot{\Omega}\theta = (Q_{3x} - Q_{0x})/2\rho_3 - (Q_{1z} - Q_{0z})/2\rho_1 \qquad \text{(A1-18)}$$

$$\dot{\Omega}\Psi = (Q_{1y} - Q_{0y})/2\rho_1 - (Q_{2x} - Q_{0x})/2\rho_2 \qquad \text{(A1-19)}$$

where Q_{ix}, and Q_{iz} are the output signals of the accelerometers. However, errors in the indicated accelerations come about as a result of errors in locating the accelerometers and inability to locate multiple accelerometers at the same point in space, as well as instrument imperfections which result in bias errors, cross axis sensitivities, and in some cases acceleration squared sensitivities. The following paragraphs relate the output signals and angular acceleration estimates to the actual translational and rotational motions of the rigid body to provide an estimate of errors that can be expected in the measurement of angular acceleration.

To provide an estimate of the angular acceleration from measured output signals, functions are derived that relate the output signal in terms of the actual rotational variables and error parameters. Two sources of errors can be identified in a measurement: accelerometer location error δ_{ijk}, and accelerometer output error ε_{ijk}, where,

i: accelerometer locations (0,1,2,3)

j: accelerometer measurement directions (x,y,z)

k: error directions (1,2,3 corresponding to x,y,z)

ε_{ijo} - bias error

(δ_{1x2} is the location error at position 1, for an accelerometer measuring the acceleration in the x-direction, whose location error is displaced in the y-direction)

Due to uncertainties in accelerometer location, each accelerometer can deviate from its nominal location ρ by a location error δ. The position vectors $\bar{q}_{ij}$ of each accelerometer at the i<u>th</u> position in the j-direction are:

$$\bar{q}_{ix} = \rho_i \bar{u}_i + \delta_{ix1}\bar{u}_1 + \delta_{ix2}\bar{u}_2 + \delta_{ix3}\bar{u}_3 \quad (i=0,2,3) \qquad \text{(A1-20)}$$

$$\bar{q}_{iy} = \rho_i \bar{u}_i + \delta_{iy1}\bar{u}_1 + \delta_{iy2}\bar{u}_2 + \delta_{iy3}\bar{u}_3 \quad (i=0,1,3) \qquad \text{(A1-21)}$$

$$\bar{q}_{iz} = \rho_i \bar{u}_i + \delta_{iz1}\bar{u}_1 + \delta_{iz2}\bar{u}_2 + \delta_{iz3}\bar{u}_3 \quad (i=0,1,2) \qquad \text{(A1-22)}$$

(at $i=0$, $\rho_o=0$)

In an actual accelerometer, the output signal Q is sensitive to the acceleration in directions other than the direction it's intended to measure. Therefore, each accelerometer contains an acceleration component in all three directions (x,y,z) that influences the output signal. The output signal Q_{ij} at the i<u>th</u> position in the j-direction are:

$$Q_{ix} = \ddot{x}_{ix} + \varepsilon_{ix0} + \varepsilon_{ix1}\ddot{x}_{ix} + \varepsilon_{ix2}\ddot{y}_{ix} + \varepsilon_{ix3}\ddot{z}_{ix} \quad (i=0,2,3) \qquad \text{(A1-23)}$$

$$Q_{iy} = \ddot{y}_{iy} + \varepsilon_{iy0} + \varepsilon_{iy1}\ddot{x}_{iy} + \varepsilon_{iy2}\ddot{y}_{iy} + \varepsilon_{iy3}\ddot{z}_{iy} \quad (i=0,1,3) \qquad \text{(A1-24)}$$

$$Q_{iz} = \ddot{z}_{iz} + \varepsilon_{iz0} + \varepsilon_{iz1}\ddot{x}_{iz} + \varepsilon_{iz2}\ddot{y}_{iz} + \varepsilon_{iz3}\ddot{z}_{iz} \quad (i=0,2,3) \qquad \text{(A1-25)}$$

Since it's the angular acceleration that's of interest, the output signal Q in Equations A1-23, A1-24, A1-25 must be expressed in terms of rotational variables. From Equation A1-13, the equation of relative motion is,

$$\ddot{r} = \ddot{\bar{R}} + (\dot{\bar{\omega}} \times \bar{q}) + \bar{\omega} \times (\bar{\omega} \times \bar{q}) \qquad \text{(A1-26)}$$

The components of acceleration in the x, y, and z directions for the acceleration vectors $\ddot{\bar{r}}_{ij}$ of the ith position in the j-direction are:

$$\ddot{r}_{ix} = \ddot{x}_{ix}\bar{u}_1 + \ddot{y}_{ix}\bar{u}_2 + \ddot{z}_{ix}\bar{u}_3 \quad (i=0,2,3) \tag{A1-27}$$

$$\ddot{r}_{iy} = \ddot{x}_{iy}\bar{u}_1 + \ddot{y}_{iy}\bar{u}_2 + \ddot{z}_{iy}\bar{u}_3 \quad (i=0,1,3) \tag{A1-28}$$

$$\ddot{r}_{iz} = \ddot{x}_{iz}\bar{u}_1 + \ddot{y}_{iz}\bar{u}_2 + \ddot{z}_{iz}\bar{u}_3 \quad (i=0,1,2) \tag{A1-29}$$

Substituting Equations A1-20, A1-21, A1-22 and A1-27, A1-28, A1-29 into A1-26, the linear acceleration components $\ddot{x}$, $\ddot{y}$, $\ddot{z}$, for all the accelerometers can be expressed in terms of the angular variables ϕ, θ, and Ψ, and the location error δ:

$$\ddot{x}_{ij} = f(\ddot{X}_o, \ddot{\theta}_o, \ddot{\Psi}_o, \dot{\theta}_o, \dot{\Psi}_o, \delta_{ijk}) \tag{A1-30}$$

$$\ddot{y}_{ij} = f(\ddot{Y}_o, \ddot{\phi}_o, \ddot{\Psi}_o, \dot{\phi}_o, \dot{\Psi}_o, \delta_{ijk}) \tag{A1-31}$$

$$\ddot{z}_{ij} = f(\ddot{Z}_o, \ddot{\phi}_o, \ddot{\Psi}_o, \dot{\phi}_o, \dot{\theta}_o, \delta_{ijk}) \tag{A1-32}$$

By substituting Equations A1-30, A1-31, A1-32 into Equations A1-23, A1-24, A1-25, the output signal Q can be expressed in terms of the rotational variables and error parameters. The estimated angular accelerations $\dot{\Omega}_\phi$, $\dot{\Omega}_\theta$, and $\dot{\Omega}_\psi$ can then be evaluated from Equations A1-17, A1-18, A1-19 as follows:

$$\dot{\Omega}_\phi = (Q_{2z} - Q_{0z})/2\rho_2 - (Q_{2y} - Q_{0y})/2\rho_3 \tag{A1-33}$$

$$\dot{\Omega}_\theta = (Q_{3x} - Q_{0x})/2\rho_3 - (Q_{1z} - Q_{0z})/2\rho_1 \tag{A1-34}$$

$$\dot{\Omega}_\psi = (Q_{1y} - Q_{0y})/2\rho_1 - (Q_{2x} - Q_{0x})/2\rho_2 \tag{A1-35}$$

Table A1-1
Angular Acceleration Estimates
3-2-2-2 Configuration

$$\dot{\Omega}_x = \dot{\omega}_x + A_0 + A_1\dot{\omega}_x + A_2\dot{\omega}_y + A_3\dot{\omega}_z + A_4\omega_x^2 + A_5\omega_y^2 + A_6\omega_z^2 + A_7\omega_x\omega_y + A_8\omega_x\omega_z + A_9\omega_y\omega_z + A_{10}\ddot{X} + A_{11}\ddot{Y} + A_{12}\ddot{Z}$$

$$\dot{\Omega}_y = \dot{\omega}_y + B_0 + B_1\dot{\omega}_x + B_2\dot{\omega}_y + B_3\dot{\omega}_z + B_4\omega_x^2 + B_5\omega_y^2 + B_6\omega_z^2 + B_7\omega_x\omega_y + B_8\omega_x\omega_z + B_9\omega_y\omega_z + B_{10}\ddot{X} + B_{11}\ddot{Y} + B_{12}\ddot{Z}$$

$$\dot{\Omega}_z = \dot{\omega}_z + C_0 + C_1\dot{\omega}_x + C_2\dot{\omega}_y + C_3\dot{\omega}_z + C_4\omega_x^2 + C_5\omega_y^2 + C_6\omega_z^2 + C_7\omega_x\omega_y + C_8\omega_x\omega_z + C_9\omega_y\omega_z + C_{10}\ddot{X} + C_{11}\ddot{Y} + C_{12}\ddot{Z}$$

Table A1-2
Angular Error Coefficients
3-2-2-2 Configuration

$$A_0 = \frac{\varepsilon_{2z0} - \varepsilon_{0z0}}{2\rho_2} - \frac{\varepsilon_{3y0} - \varepsilon_{0y0}}{2\rho_3}$$

$$A_1 = \frac{1}{2}(\varepsilon_{2z3} + \varepsilon_{3y2}) + \frac{\delta_{2z2} - \delta_{0z2}}{2\rho_2} + \frac{\delta_{3y3} - \delta_{0y3}}{2\rho_3}$$
$$- \frac{\delta_{2z3}\varepsilon_{2z2} - \delta_{0z3}\varepsilon_{0z2}}{2\rho_2} + \frac{\delta_{2z2}\varepsilon_{2z3} - \delta_{0z2}\varepsilon_{0z3}}{2\rho_2}$$
$$+ \frac{\delta_{3y3}\varepsilon_{3y2} - \delta_{0y3}\varepsilon_{0y2}}{2\rho_3} - \frac{\delta_{3y2}\varepsilon_{3y3} - \delta_{0y2}\varepsilon_{0y3}}{2\rho_3}$$

$$A_2 = -\frac{1}{2}\varepsilon_{3y1} + \frac{\delta_{2z1} - \delta_{0z1}}{2\rho_2} + \frac{\delta_{2z3}\varepsilon_{2z1} - \delta_{0z3}\varepsilon_{0z1}}{2\rho_2}$$
$$- \frac{\delta_{2z1}\varepsilon_{2z3} - \delta_{0z1}\varepsilon_{0z3}}{2\rho_2} - \frac{\delta_{3y3}\varepsilon_{3y1} - \delta_{0y3}\varepsilon_{0y1}}{2\rho_3}$$
$$+ \frac{\delta_{3y1}\varepsilon_{3y3} - \delta_{0y1}\varepsilon_{0y3}}{2\rho_3}$$

$$A_3 = -\frac{1}{2}\varepsilon_{2z1} - \frac{\delta_{3y1} - \delta_{0y1}}{2\rho_3} - \frac{\delta_{2z2}\varepsilon_{2z1} - \delta_{0z2}\varepsilon_{0z1}}{2\rho_2}$$
$$+ \frac{\delta_{2z1}\varepsilon_{2z2} - \delta_{0z1}\varepsilon_{0z2}}{2\rho_2} + \frac{\delta_{3y2}\varepsilon_{3y1} - \delta_{0y2}\varepsilon_{0y1}}{2\rho_3}$$
$$- \frac{\delta_{3y1}\varepsilon_{3y2} - \delta_{0y1}\varepsilon_{0y2}}{2\rho_3}$$

$$A_4 = -\frac{1}{2}(\varepsilon_{2z2} - \varepsilon_{3y3}) - \frac{\delta_{2z3} - \delta_{0z3}}{2\rho_2} + \frac{\delta_{3y2} - \delta_{0y2}}{2\rho_3}$$
$$- \frac{\delta_{2z2}\varepsilon_{2z2} - \delta_{0z2}\varepsilon_{0z2}}{2\rho_2} - \frac{\delta_{2z3}\varepsilon_{2z3} - \delta_{0z3}\varepsilon_{0z3}}{2\rho_2}$$
$$+ \frac{\delta_{3y2}\varepsilon_{3y2} - \delta_{0y2}\varepsilon_{0y2}}{2\rho_3} + \frac{\delta_{3y3}\varepsilon_{3y3} - \delta_{0y3}\varepsilon_{0y3}}{2\rho_3}$$

$$A_5 = \frac{1}{2}\,\varepsilon_{3y3} - \frac{\delta_{2z3} - \delta_{0z3}}{2\rho_2} - \frac{\delta_{2z1}\varepsilon_{2z1} - \delta_{0z1}\varepsilon_{0z1}}{2\rho_2}$$

$$- \frac{\delta_{2z3}\varepsilon_{2z3} - \delta_{0z3}\varepsilon_{0z3}}{2\rho_2} + \frac{\delta_{3y1}\varepsilon_{3y1} - \delta_{0y1}\varepsilon_{0y1}}{2\rho_3}$$

$$+ \frac{\delta_{3y3}\varepsilon_{3y3} - \delta_{0y3}\varepsilon_{0y3}}{2\rho_3}$$

$$A_6 = -\frac{1}{2}\,\varepsilon_{2z2} + \frac{\delta_{3y2} - \delta_{0y2}}{2\rho_3} - \frac{\delta_{2z1}\varepsilon_{2z1} - \delta_{0z1}\varepsilon_{0z1}}{2\rho_2}$$

$$- \frac{\delta_{2z2}\varepsilon_{2z2} - \delta_{0z2}\varepsilon_{0z2}}{2\rho_2} + \frac{\delta_{3y1}\varepsilon_{3y1} - \delta_{0y1}\varepsilon_{0y1}}{2\rho_3}$$

$$+ \frac{\delta_{3y2}\varepsilon_{3y2} - \delta_{0y2}\varepsilon_{0y2}}{2\rho_3}$$

$$A_7 = \frac{1}{2}\,\varepsilon_{2z1} - \frac{\delta_{3y1} - \delta_{0y1}}{2\rho_3} + \frac{\delta_{2z2}\varepsilon_{2z1} - \delta_{0z2}\varepsilon_{oz1}}{2\rho_2}$$

$$+ \frac{\delta_{2z1}\varepsilon_{2z2} - \delta_{0z1}\varepsilon_{0z2}}{2\rho_2} - \frac{\delta_{3y2}\varepsilon_{3y1} - \delta_{0y2}\varepsilon_{0y1}}{2\rho_3}$$

$$- \frac{\delta_{3y1}\varepsilon_{3y2} - \delta_{0y1}\varepsilon_{0y2}}{2\rho_3}$$

$$A_8 = -\frac{1}{2}\,\varepsilon_{3y1} + \frac{\delta_{2z1} - \delta_{0z1}}{2\rho_2} + \frac{\delta_{2z3}\varepsilon_{2z1} - \delta_{0z3}\varepsilon_{0z1}}{2\rho_2}$$

$$+ \frac{\delta_{2z1}\varepsilon_{2z3} - \delta_{0z1}\varepsilon_{0z3}}{2\rho_2} - \frac{\delta_{3y3}\varepsilon_{3y1} - \delta_{0y3}\varepsilon_{0y1}}{2\rho_3}$$

$$- \frac{\delta_{3y1}\varepsilon_{3y3} - \delta_{0y1}\varepsilon_{0y3}}{2\rho_3}$$

$$A_9 = \frac{1}{2}\,(\varepsilon_{2z3} - \varepsilon_{3y2}) + \frac{\delta_{2z2} - \delta_{0z2}}{2\rho_2} - \frac{\delta_{3y3} - \delta_{0y3}}{2\rho_3}$$

$$+ \frac{\delta_{2z3}\varepsilon_{2z2} - \delta_{0z3}\varepsilon_{0z2}}{2\rho_2} + \frac{\delta_{2z2}\varepsilon_{2z3} - \delta_{0z2}\varepsilon_{0z3}}{2\rho_2}$$

$$- \frac{\delta_{3y3}\varepsilon_{3y2} - \delta_{0y3}\varepsilon_{0y2}}{2\rho_3} - \frac{\delta_{3y2}\varepsilon_{3y3} - \delta_{0y2}\varepsilon_{0y3}}{2\rho_3}$$

$$A_{10} = \frac{\varepsilon_{2z1} - \varepsilon_{0z1}}{2\rho_2} - \frac{\varepsilon_{3y1} - \varepsilon_{0y1}}{2\rho_3}$$

$$A_{11} = \frac{\varepsilon_{2z2} - \varepsilon_{0z2}}{2\rho_2} - \frac{\varepsilon_{3y2} - \varepsilon_{0y2}}{2\rho_3}$$

$$A_{12} = \frac{\varepsilon_{2z3} - \varepsilon_{0z3}}{2\rho_2} - \frac{\varepsilon_{3y3} - \varepsilon_{0y3}}{2\rho_3}$$

$$B_0 = \frac{\varepsilon_{3x0} - \varepsilon_{0x0}}{2\rho_3} - \frac{\varepsilon_{1z0} - \varepsilon_{0z0}}{2\rho_1}$$

$$B_1 = \frac{1}{2}\,\varepsilon_{3x2} - \frac{\delta_{1z2} - \delta_{0z2}}{2\rho_1} - \frac{\delta_{3x3}\varepsilon_{3x2} - \delta_{0x3}\varepsilon_{0x2}}{2\rho_3}$$

$$+ \frac{\delta_{3x2}\varepsilon_{3x2} - \delta_{0x2}\varepsilon_{0x2}}{2\rho_3} + \frac{\delta_{1z3}\varepsilon_{1z2} - \delta_{0z3}\varepsilon_{0z2}}{2\rho_1}$$

$$- \frac{\delta_{1z2}\varepsilon_{1z3} - \delta_{0z2}\varepsilon_{0z3}}{2\rho_1}$$

$$B_2 = \frac{1}{2}\,(\varepsilon_{3x1} + \varepsilon_{1z3}) + \frac{\delta_{3x3} - \delta_{0x3}}{2\rho_3} + \frac{\delta_{1z1} - \delta_{0z1}}{2\rho_1}$$

$$+ \frac{\delta_{3x3}\varepsilon_{3x1} - \delta_{0x3}\varepsilon_{0x1}}{2\rho_3} - \frac{\delta_{3x1}\varepsilon_{3x3} - \delta_{0x1}\varepsilon_{0x3}}{2\rho_3}$$

$$- \frac{\delta_{1z3}\varepsilon_{1z1} - \delta_{0z3}\varepsilon_{0z1}}{2\rho_1} + \frac{\delta_{1z1}\varepsilon_{1z3} - \delta_{0z1}\varepsilon_{0z3}}{2\rho_1}$$

$$B_3 = -\frac{1}{2}\,\varepsilon_{1z2} - \frac{\delta_{3x2} - \delta_{0x2}}{2\rho_3} - \frac{\delta_{3x2}\varepsilon_{3x1} - \delta_{0x2}\varepsilon_{0x1}}{2\rho_3}$$

$$+ \frac{\delta_{3x1}\varepsilon_{3x2} - \delta_{0x1}\varepsilon_{0x2}}{2\rho_3} + \frac{\delta_{1z2}\varepsilon_{1z1} - \delta_{0z2}\varepsilon_{0z1}}{2\rho_1}$$

$$- \frac{\delta_{1z1}\varepsilon_{1z2} - \delta_{0z1}\varepsilon_{0z2}}{2\rho_1}$$

$$B_4 = -\frac{1}{2}\,\varepsilon_{3x3} + \frac{\delta_{1z3} - \delta_{0z3}}{2\rho_1} - \frac{\delta_{3x2}\varepsilon_{3x2} - \delta_{0x2}\varepsilon_{0x2}}{2\rho_3}$$

$$- \frac{\delta_{3x3}\varepsilon_{3x3} - \delta_{0x3}\varepsilon_{0x3}}{2\rho_3} + \frac{\delta_{1z2}\varepsilon_{1z2} - \delta_{0z2}\varepsilon_{0z2}}{2\rho_1}$$

$$+ \frac{\delta_{1z3}\varepsilon_{1z3} - \delta_{0z3}\varepsilon_{0z3}}{2\rho_1}$$

$$B_5 = -\frac{1}{2}\,(\varepsilon_{3x3} - \varepsilon_{1z1}) - \frac{\delta_{3x1} - \delta_{0x1}}{2\rho_3} + \frac{\delta_{1z3} - \delta_{0z3}}{2\rho_1}$$

$$- \frac{\delta_{3x1}\varepsilon_{3x1} - \delta_{0x1}\varepsilon_{0x1}}{2\rho_3} - \frac{\delta_{3x3}\varepsilon_{3x3} - \delta_{0x3}\varepsilon_{0x3}}{2\rho_3}$$

$$+ \frac{\delta_{1z1}\varepsilon_{1z1} - \delta_{0z1}\varepsilon_{0z1}}{2\rho_1} + \frac{\delta_{1z3}\varepsilon_{1z3} - \delta_{0z3}\varepsilon_{0z3}}{2\rho_1}$$

$$B_6 = \frac{1}{2}\,\varepsilon_{1z1} - \frac{\delta_{3x1} - \delta_{0x1}}{2\rho_3} - \frac{\delta_{3x1}\varepsilon_{3x1} - \delta_{0x1}\varepsilon_{0x1}}{2\rho_3}$$

$$- \frac{\delta_{3x2}\varepsilon_{3x2} - \delta_{0x2}\varepsilon_{0x2}}{2\rho_3} + \frac{\delta_{1z1}\varepsilon_{1z1} - \delta_{0z1}\varepsilon_{0z1}}{2\rho_1}$$

$$- \frac{\delta_{1z2}\varepsilon_{1z2} - \delta_{0z2}\varepsilon_{0z2}}{2\rho_1}$$

$$B_7 = -\frac{1}{2}\,\varepsilon_{1z2} + \frac{\delta_{3x2} - \delta_{0x2}}{2\rho_3} + \frac{\delta_{3x2}\varepsilon_{3x1} - \delta_{0x2}\varepsilon_{0x1}}{2\rho_3}$$

$$+ \frac{\delta_{3x1}\varepsilon_{3x2} - \delta_{0x1}\varepsilon_{0x2}}{2\rho_3} - \frac{\delta_{1z2}\varepsilon_{1z1} - \delta_{0z2}\varepsilon_{0z1}}{2\rho_1}$$

$$- \frac{\delta_{1z1}\varepsilon_{1z2} - \delta_{0z1}\varepsilon_{0z2}}{2\rho_1}$$

$$B_8 = \frac{1}{2}\,(\varepsilon_{3x1} - \varepsilon_{1z3}) + \frac{\delta_{3x3} - \delta_{0x3}}{2\rho_3} - \frac{\delta_{1z1} - \delta_{0z1}}{2\rho_1}$$

$$+ \frac{\delta_{3x3}\varepsilon_{3x1} - \delta_{0x3}\varepsilon_{0x1}}{2\rho_3} + \frac{\delta_{3x1}\varepsilon_{3x3} - \delta_{0x1}\varepsilon_{0x3}}{2\rho_3}$$

$$- \frac{\delta_{1z3}\varepsilon_{1z1} - \delta_{0z3}\varepsilon_{0z1}}{2\rho_1} - \frac{\delta_{1z1}\varepsilon_{1z3} - \delta_{0z1}\varepsilon_{0z3}}{2\rho_1}$$

$$B_9 = \frac{1}{2}\,\varepsilon_{3x2} - \frac{\delta_{1z2} - \delta_{0z2}}{2\rho_1} + \frac{\delta_{3x3}\varepsilon_{3x2} - \delta_{0x3}\varepsilon_{0x2}}{2\rho_3}$$

$$+ \frac{\delta_{3x2}\varepsilon_{3x3} - \delta_{0x2}\varepsilon_{0x3}}{2\rho_3} - \frac{\delta_{1z3}\varepsilon_{1z2} - \delta_{0z3}\varepsilon_{0z2}}{2\rho_1}$$

$$- \frac{\delta_{1z2}\varepsilon_{1z3} - \delta_{0z2}\varepsilon_{0z3}}{2\rho_1}$$

$$B_{10} = \frac{\varepsilon_{3x1} - \varepsilon_{0x1}}{2\rho_3} - \frac{\varepsilon_{1z1} - \varepsilon_{0z2}}{2\rho_1}$$

$$B_{11} = \frac{\varepsilon_{3x2} - \varepsilon_{0x2}}{2\rho_3} - \frac{\varepsilon_{1z2} - \varepsilon_{0z2}}{2\rho_1}$$

$$B_{12} = \frac{\varepsilon_{3x3} - \varepsilon_{0x3}}{2\rho_3} - \frac{\varepsilon_{1z3} - \varepsilon_{0z3}}{2\rho_1}$$

$$C_0 = \frac{\varepsilon_{1y0} - \varepsilon_{0y0}}{2\rho_1} - \frac{\varepsilon_{2x0} - \varepsilon_{0x0}}{2\rho_2}$$

$$C_1 = -\frac{1}{2}\varepsilon_{2x3} - \frac{\delta_{1y3} - \delta_{0y3}}{2\rho_1} - \frac{\delta_{1y3}\varepsilon_{1y2} - \delta_{0y3}\varepsilon_{0y2}}{2\rho_1}$$

$$+ \frac{\delta_{1y2}\varepsilon_{1y3} - \delta_{0y2}\varepsilon_{0y3}}{2\rho_1} + \frac{\delta_{2x3}\varepsilon_{2x2} - \delta_{0x3}\varepsilon_{0x2}}{2\rho_2}$$

$$- \frac{\delta_{2x2}\varepsilon_{2x3} - \delta_{0x2}\varepsilon_{0x3}}{2\rho_2}$$

$$C_2 = -\frac{1}{2}\varepsilon_{1y3} - \frac{\delta_{2x3} - \delta_{0x3}}{2\rho_3} + \frac{\delta_{1y3}\varepsilon_{1y1} - \delta_{0y3}\varepsilon_{0y1}}{2\rho_1}$$

$$- \frac{\delta_{1y1}\varepsilon_{1y3} - \delta_{0y1}\varepsilon_{0y3}}{2\rho_1} - \frac{\delta_{2x3}\varepsilon_{2x1} - \delta_{0x3}\varepsilon_{0x1}}{2\rho_2}$$

$$+ \frac{\delta_{2x1}\varepsilon_{2x3} - \delta_{0x1}\varepsilon_{0x3}}{2\rho_2}$$

$$C_3 = \frac{1}{2}(\varepsilon_{1y2} + \varepsilon_{2x1}) + \frac{\delta_{1y1} - \delta_{0y1}}{2\rho_1} + \frac{\delta_{2x2} - \delta_{0x2}}{2\rho_2}$$

$$- \frac{\delta_{1y2}\varepsilon_{1y1} - \delta_{0y2}\varepsilon_{0y1}}{2\rho_1} + \frac{\delta_{1y1}\varepsilon_{1y2} - \delta_{0y1}\varepsilon_{0y2}}{2\rho_1}$$

$$+ \frac{\delta_{2x2}\varepsilon_{2x1} - \delta_{0x2}\varepsilon_{0x1}}{2\rho_2} - \frac{\delta_{2x1}\varepsilon_{2x2} - \delta_{0x1}\varepsilon_{0x2}}{2\rho_2}$$

$$C_4 = \frac{1}{2}\varepsilon_{2x2} - \frac{\delta_{1y2} - \delta_{0y2}}{2\rho_1} - \frac{\delta_{1y2}\varepsilon_{1y2} - \delta_{0y2}\varepsilon_{0y2}}{2\rho_1}$$

$$- \frac{\delta_{1y3}\varepsilon_{1y3} - \delta_{0y3}\varepsilon_{0y3}}{2\rho_1} + \frac{\delta_{2x2}\varepsilon_{2x2} - \delta_{0x2}\varepsilon_{0x2}}{2\rho_2}$$

$$+ \frac{\delta_{2x3}\varepsilon_{2x3} - \delta_{0x3}\varepsilon_{0x3}}{2\rho_2}$$

$$C_5 = -\frac{1}{2}\varepsilon_{1y1} + \frac{\delta_{2x1} - \delta_{0x1}}{2\rho_2} - \frac{\delta_{1y1}\varepsilon_{1y1} - \delta_{0y1}\varepsilon_{0y1}}{2\rho_1}$$

$$- \frac{\delta_{1y3}\varepsilon_{1y3} - \delta_{0y3}\varepsilon_{0y3}}{2\rho_1} + \frac{\delta_{2x1}\varepsilon_{2x1} - \delta_{0x1}\varepsilon_{0x1}}{2\rho_2}$$

$$+ \frac{\delta_{2x3}\varepsilon_{2x3} - \delta_{0x3}\varepsilon_{0x3}}{2\rho_2}$$

$$C_6 = -\frac{1}{2}(\varepsilon_{1y1} - \varepsilon_{2x2}) - \frac{\delta_{1y2} - \delta_{0y2}}{2\rho_1} + \frac{\delta_{2x1} - \delta_{0x1}}{2\rho_2}$$

$$- \frac{\delta_{1y1}\varepsilon_{1y1} - \delta_{0y1}\varepsilon_{0y1}}{2\rho_1} - \frac{\delta_{1y2}\varepsilon_{1y2} - \delta_{0y2}\varepsilon_{0y2}}{2\rho_1}$$

$$+ \frac{\delta_{2x1}\varepsilon_{2x1} - \delta_{0x1}\varepsilon_{0x1}}{2\rho_2} + \frac{\delta_{2x2}\varepsilon_{2x2} - \delta_{0x2}\varepsilon_{0x2}}{2\rho_2}$$

$$C_7 = \frac{1}{2}(\varepsilon_{1y2} - \varepsilon_{2x1}) + \frac{\delta_{1y1} - \delta_{0y1}}{2\rho_1} - \frac{\delta_{2y2} - \delta_{0y2}}{2\rho_2}$$

$$+ \frac{\delta_{1y2}\varepsilon_{1y1} - \delta_{0y2}\varepsilon_{0y1}}{2\rho_1} + \frac{\delta_{1y1}\varepsilon_{1y2} - \delta_{0y1}\varepsilon_{0y2}}{2\rho_1}$$

$$- \frac{\delta_{2x2}\varepsilon_{2x1} - \delta_{0x2}\varepsilon_{0x1}}{2\rho_2} - \frac{\delta_{2x1}\varepsilon_{2x2} - \delta_{0x1}\varepsilon_{0x2}}{2\rho_2}$$

$$C_8 = \frac{1}{2}\varepsilon_{1y3} - \frac{\delta_{2x3} - \delta_{0x3}}{2\rho_2} + \frac{\delta_{1y3}\varepsilon_{1y1} - \delta_{0y3}\varepsilon_{0y1}}{2\rho_1}$$

$$- \frac{\delta_{1y1}\varepsilon_{1y3} - \delta_{0y1}\varepsilon_{0y3}}{2\rho_1} - \frac{\delta_{2x3}\varepsilon_{2x1} - \delta_{0x3}\varepsilon_{0x1}}{2\rho_2}$$

$$- \frac{\delta_{2x1}\varepsilon_{2x3} - \delta_{0x1}\varepsilon_{0x3}}{2\rho_2}$$

$$C_9 = -\frac{1}{2}\varepsilon_{2x3} + \frac{\delta_{1y3} - \delta_{0y3}}{2\rho_1} + \frac{\delta_{1y3}\varepsilon_{1y2} - \delta_{0y3}\varepsilon_{0y2}}{2\rho_1}$$

$$+ \frac{\delta_{1y2}\varepsilon_{1y3} - \delta_{0y2}\varepsilon_{0y3}}{2\rho_1} - \frac{\delta_{2x3}\varepsilon_{2x2} - \delta_{0x3}\varepsilon_{0x2}}{2\rho_2}$$

$$- \frac{\delta_{2x2}\varepsilon_{2x3} - \delta_{0x2}\varepsilon_{0x3}}{2\rho_2}$$

$$C_{10} = \frac{\varepsilon_{1y1} - \varepsilon_{0y1}}{2\rho_1} - \frac{\varepsilon_{2x1} - \varepsilon_{0x1}}{2\rho_2}$$

$$C_{11} = \frac{\varepsilon_{1y2} - \varepsilon_{0y2}}{2\rho_1} - \frac{\varepsilon_{2x2} - \varepsilon_{0x2}}{2\rho_2}$$

$$C_{12} = \frac{\varepsilon_{1y3} - \varepsilon_{0y3}}{2\rho_1} - \frac{\varepsilon_{2x3} - \varepsilon_{0x3}}{2\rho_2}$$

APPENDIX A2

DERIVATION OF EQUATIONS FOR ESTIMATED ANGULAR ACCELERATIONS FOR NINE ACCELEROMETER, COPLANAR CONFIGURATION

In this appendix, the estimated angular accelerations for the nine, coplanar configuration are derived in terms of the accelerometer location and output errors. The development closely follows the discussion in Appendix A1.

The configuration for the nine accelerometer configuration analyzed is shown in Figure A2-1.

The accelerometer signals can be expressed in terms of the rigid body motions, without error terms, as:

A2-1 $$A_{x1} = R_x - \frac{\sqrt{3}}{2} r \left(\omega_y^2 + \omega_z^2\right) + \frac{1}{2} r \left(\dot{\omega}_z - \omega_x \omega_y\right)$$

A2-2 $$A_{x2} = R_x - r \left(\dot{\omega}_z - \omega_x \omega_y\right)$$

A2-3 $$A_{x3} = R_x + \frac{\sqrt{3}}{2} r \left(\omega_y^2 + \omega_z^2\right) + \frac{1}{2} r \left(\dot{\omega}_z - \omega_x \omega_y\right)$$

A2-4 $$A_{y1} = R_y + \frac{\sqrt{3}}{2} r \left(\dot{\omega}_z + \omega_x \omega_y\right) + \frac{1}{2} r \left(\omega_x^2 + \omega_z^2\right)$$

A2-5 $$A_{y2} = R_y + r \left(\omega_x^2 + \omega_z^2\right)$$

A2-6 $$A_{y3} = R_y - \frac{\sqrt{3}}{2} r \left(\dot{\omega}_z + \omega_x \omega_y\right) + \frac{1}{2} r \left(\omega_x^2 + \omega_z^2\right)$$

A2-7 $$A_{z1} = R_z - \frac{\sqrt{3}}{2} r \left(\dot{\omega}_y - \omega_x \omega_z\right) - \frac{1}{2} r \left(\dot{\omega}_x + \omega_y \omega_z\right)$$

A2-8 $$A_{z2} = R_z + r \left(\dot{\omega}_x + \omega_y \omega_z\right)$$

A2-9 $$A_{z3} = R_z + \frac{\sqrt{3}}{2} r \left(\dot{\omega}_y - \omega_x \omega_z\right) - \frac{1}{2} r \left(\dot{\omega}_x + \omega_y \omega_z\right)$$

Eliminating the angular velocity squared terms through algebraic manipulation or solving for the angular accelerations in a least squared sense produces:

A2-10 $$\dot{\Omega}_z = \frac{A_{x1} + A_{x3} - 2A_{x2}}{6r} + \frac{A_{y1} - A_{y3}}{2\sqrt{3}\, r}$$

A2-11 $$\dot{\Omega}_y + \Omega_x \Omega_z = \frac{A_{z3} - A_{z1}}{\sqrt{3}\, r}$$

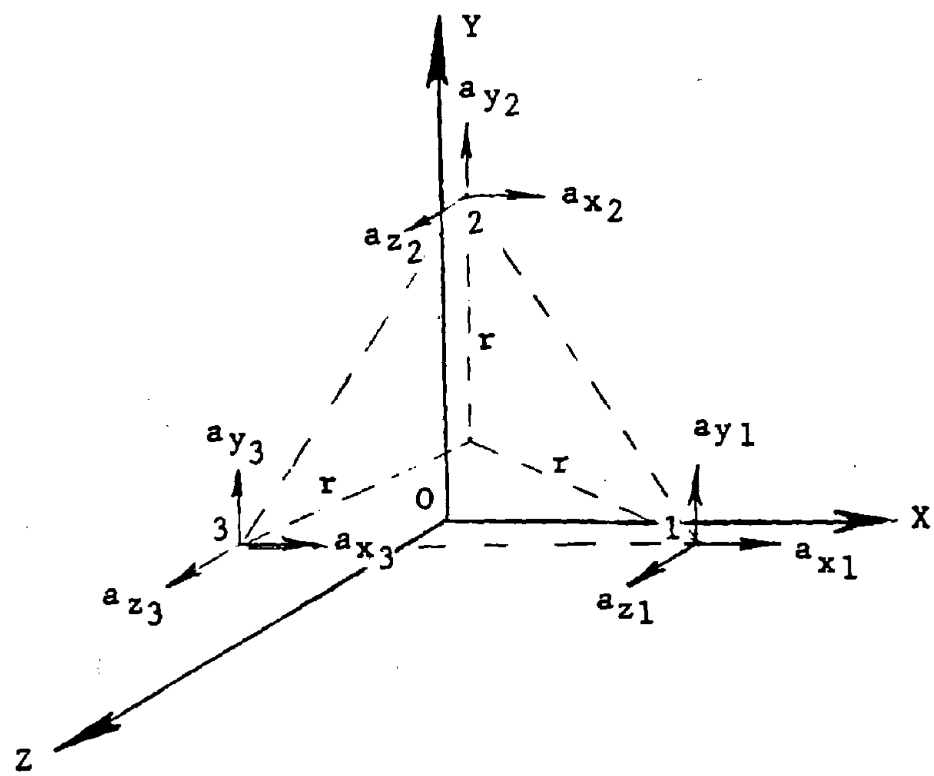


FIGURE A2-1. NINE ACCELEROMETER, COPLANAR CONFIGURATION

A2-12
$$\dot{\Omega}_x - \Omega_y \Omega_z = \frac{2A_{z2} - A_{z1} - A_{z3}}{r}$$

where A_{x1} refers to the accelerometer at location 1 measuring in the x direction.

Using these equations with the actual error terms, the angular acceleration estimates may be expressed in terms of the rigid body motions and transducer location errors. Neglecting terms of second order, the equations can be written as shown in Tables A2-1 and A2-2.

Assuming that the errors are statistically independent, the standard deviations of the error coefficients are shown in Table A2-3.

As an example, if we assume r = 4 inches, location errors = 0.01 inches, sensitivity errors are 5%, and cross axis sensitivity errors are 3%, the standard deviation of error coefficients in estimating the angular accelerations are shown in Table A2-4.

Table A2-1
Angular Acceleration Estimates
3-3-3 Configuration

$$\dot{\Omega}_z = \dot{\omega}_z + A_0 + A_1\dot{\omega}_2 + A_2\dot{\omega}_x + A_3\dot{\omega}_y + A_4\omega_x^2 + A_5\omega_y^2$$

$$+ A_6\omega_z^2 + A_7\omega_x\omega_y + A_8\omega_x\omega_z + A_9\omega_y\omega_z + A_{10}R_x + A_{11}R_y + A_{12}R_z$$

$$\dot{\Omega}_y + \Omega_x\Omega_z = \dot{\omega}_y + \omega_x\omega_y + B_0 + B_1\dot{\omega}_y + B_2\dot{\omega}_x + B_3\dot{\omega}_z$$

$$+ B_4\omega_x^2 + B_5\omega_y^2 + B_6\omega_z^2 + B_7\omega_x\omega_y + B_8\omega_x\omega_z$$

$$+ B_9\omega_y\omega_z + B_{10}R_x + {}_{11}R_y + {}_{12}R_z$$

$$\Omega_x - \Omega_y\Omega_z = \dot{\omega}_x - \omega_y\omega_z + C_0 + C_1\omega_x + C_2\omega_y + C_3\omega_z$$

$$+ C_4\omega_x^2 + C_5\omega_y^2 + C_6\omega_z^2 + C_7\omega_x\omega_y + C_8\omega_x\omega_z$$

$$+ C_9\omega_y\omega_z + C_{10}R_x + C_{11}R_y + C_{12}R_z$$

Table A2-2
Angular Error Coefficients
3-3-3 Configuration

$$A_0 = \frac{1}{6r} (\varepsilon_{x10} + \varepsilon_{x30} - 2\varepsilon_{x20} + \sqrt{3}\ \varepsilon_{y10} - \sqrt{3}\ \varepsilon_{y30})$$

$$A_1 = \frac{\sqrt{3}}{12} (\varepsilon_{x1y} - \varepsilon_{x3y} + \varepsilon_{y1x} - \varepsilon_{y3x}) + \frac{1}{12} (3\varepsilon_{y2y} + \varepsilon_{x1x} + \varepsilon_{x3x} + 4\varepsilon_{x2x})$$

$$+ \frac{1}{2\sqrt{3}r} (\delta_{y1x} - \delta_{y3x}) + \frac{1}{6r} (\delta_{x3y} - \delta_{x1y} - 2\delta_{x2y})$$

$$A_2 = \frac{1}{12} (\sqrt{3}\ \varepsilon_{y3z} - \sqrt{3}\ \varepsilon_{y1z} - 4\varepsilon_{x2z} - \varepsilon_{x3z} - \varepsilon_{x1z}) + \frac{1}{2\sqrt{3}r} (\delta_{y3z} - \delta_{y1z})$$

$$A_3 = \frac{\sqrt{3}}{12} (\varepsilon_{x3z} - \varepsilon_{x1z}) - \frac{1}{4}(\varepsilon_{y1z} + \varepsilon_{y3z}) + \frac{1}{6r} (\delta_{x1z} + \delta_{x3z} - 2\delta_{x2z})$$

$$A_4 = \frac{1}{12} (\sqrt{3}\ \varepsilon_{y3y} - \sqrt{3}\ \varepsilon_{y1y} - 4\varepsilon_{x2y} - \varepsilon_{x3y} - \varepsilon_{x1y}) + \frac{1}{2\sqrt{3}r} (\delta_{y1y} - \delta_{y3y})$$

$$A_5 = \frac{\sqrt{3}}{12} (\varepsilon_{x1x} - \varepsilon_{x3x}) + \frac{1}{4} (\varepsilon_{y1x} + \varepsilon_{y3x}) + \frac{1}{6r} (\delta_{x1x} + \delta_{x3x} - 2\delta_{x2x})$$

$$A_6 = \frac{\sqrt{3}}{12} (\varepsilon_{x3x} + \varepsilon_{x1x}) + \frac{1}{4} (\varepsilon_{y1x} + \varepsilon_{y3x}) + \frac{\sqrt{3}}{2} (\varepsilon_{y3y} - \varepsilon_{y1y}) + \frac{1}{6r} (\delta_{x1x} + \delta_{x3x}$$

$$- 2\delta_{x2x}) + \frac{1}{2\sqrt{3}r} (\delta_{y1y} - \delta_{y3y})$$

$$A_7 = \frac{1}{12} (\varepsilon_{x1x} + \varepsilon_{x3x} + 4\varepsilon_{x2x} + \sqrt{3}\ \varepsilon_{y1x} - \sqrt{3}\ \varepsilon_{y3x} - \sqrt{3}\ \varepsilon_{x1y} + \frac{\sqrt{3}}{1} \varepsilon_{x3y} - 3\varepsilon_{y3y}$$

$$- \frac{1}{6r} (\delta_{x3y} + \delta_{x1y} - 2\delta_{x2y}) + \frac{1}{2\sqrt{3}r} (\delta_{y3x} - \delta_{y1x})$$

$$A_8 = \frac{\sqrt{3}}{12} (\varepsilon_{x3z} - \varepsilon_{x1z}) - \frac{1}{4} (\varepsilon_{y1z} + \varepsilon_{y3z}) + \frac{1}{6r} (2\delta_{x2z} - \delta_{x1z} - \delta_{x3z})$$

$$A_9 = \frac{1}{12} (\varepsilon_{x1z} + \varepsilon_{x3z} + 4\varepsilon_{y2z} + \sqrt{3}\ \varepsilon_{y1z} - \sqrt{3}\ \varepsilon_{y3z}) + \frac{1}{2\sqrt{3}r} (\delta_{y3z} - \delta_{y1z})$$

$$A_{10} = \frac{1}{6r} (\varepsilon_{x1x} + \varepsilon_{x3x} - 2\varepsilon_{x2x}) + \frac{1}{2\sqrt{3}r} (\varepsilon_{y1x} - \varepsilon_{y3x})$$

$$A_{11} = \frac{1}{6r} (\varepsilon_{x1y} + \varepsilon_{x3y} - 2\varepsilon_{x2y}) + \frac{1}{2\sqrt{3}r} (\varepsilon_{y1y} - \varepsilon_{y3y})$$

$$A_{12} = \frac{1}{6r} (\varepsilon_{x1z} + \varepsilon_{x3z} - 2\varepsilon_{x2z}) + \frac{1}{2\sqrt{3}r} (\varepsilon_{y1z} - \varepsilon_{y3z})$$

$$B_0 = \frac{1}{\sqrt{3}r} (\varepsilon_{z30} - \varepsilon_{z10})$$

$$B_1 = \frac{1}{2}(\varepsilon_{z1z} + \varepsilon_{z3z}) - \frac{1}{\sqrt{3}r} (\delta_{z1x} + \delta_{z3x})$$

$$B_2 = \frac{\sqrt{3}}{6} (\varepsilon_{z1z} - \varepsilon_{z3z}) + \frac{1}{\sqrt{3}r} (\delta_{z3y} - \delta_{z1y})$$

$$B_3 = \frac{\sqrt{3}}{6} (\varepsilon_{z3x} - \varepsilon_{z1x}) + \frac{1}{\sqrt{3}r} (\delta_{z1y} - \delta_{z3y})$$

$$B_4 = \frac{\sqrt{3}}{6} (\varepsilon_{z1y} - \varepsilon_{z3y}) + \frac{1}{\sqrt{3}r} (\delta_{z3z} - \delta_{z1z})$$

$$B_5 = -\frac{1}{2} (\varepsilon_{z1x} + \varepsilon_{z3x})$$

$$B_6 = -\frac{1}{2} (\varepsilon_{z1x} + \varepsilon_{z3x}) + \frac{1}{\sqrt{3}r} (\delta_{z3y} - \delta_{z1y})$$

$$B_7 = \frac{\sqrt{3}}{6} (\varepsilon_{z3x} - \varepsilon_{z1x}) + \frac{1}{2} (\varepsilon_{z1y} + \varepsilon_{z3y})$$

$$B_8 = \frac{1}{2} (\varepsilon_{z1z} + \varepsilon_{z3z}) - \frac{1}{\sqrt{3}r} (\delta_{z1x} + \delta_{z3x})$$

$$B_9 = \frac{\sqrt{3}}{6} (\varepsilon_{z3z} - \varepsilon_{z1z}) + \frac{1}{\sqrt{3}r} (\delta_{z1y} - \delta_{z3y})$$

$$B_{10} = \frac{1}{\sqrt{3}r} (\varepsilon_{z3x} - \varepsilon_{z1x})$$

$$B_{11} = \frac{1}{\sqrt{3}r} (\varepsilon_{z3y} - \varepsilon_{z1y})$$

$$B_{12} = \frac{1}{\sqrt{3}r} (\varepsilon_{z3z} - \varepsilon_{z1z})$$

$$C_0 = \frac{1}{3r}(2\varepsilon_{z20} - \varepsilon_{z10} - \varepsilon_{z30})$$

$$C_1 = \frac{1}{3}(2\varepsilon_{z2z} + \frac{1}{2}\varepsilon_{z1z} - \frac{1}{2}\varepsilon_{z3z}) + \frac{1}{3r}(2\delta_{z2y} - \delta_{z1y} - \delta_{z3y})$$

$$C_2 = \frac{\sqrt{3}}{6}(\varepsilon_{z1z} - \varepsilon_{z3z}) + \frac{1}{3r}(\delta_{z1x} + \delta_{x3x} - 2\delta_{x2x})$$

$$C_3 = \frac{1}{3}(-2\varepsilon_{z2x} - \frac{1}{2}\varepsilon_{z1x} - \frac{1}{2}\varepsilon_{z3x} + \frac{\sqrt{3}}{2}\varepsilon_{z1y} + \frac{\sqrt{3}}{2}\varepsilon_{z3y})$$

$$C_4 = \frac{1}{3}(2\varepsilon_{z2y} + \frac{1}{2}\varepsilon_{z1y} + \frac{1}{2}\varepsilon_{z3y}) + \frac{1}{3r}(2\delta_{z2z} - \delta_{z1z} - \delta_{z3z})$$

$$C_5 = \frac{1}{3}(-\frac{\sqrt{3}}{2}\varepsilon_{z1x} + \frac{\sqrt{3}}{2}\varepsilon_{z3x})$$

$$C_6 = \frac{1}{6}(-\frac{\sqrt{3}}{2}\varepsilon_{z1x} + \frac{\sqrt{3}}{2}\varepsilon_{z3x} + 4\varepsilon_{z2y} + \varepsilon_{z1y} + \varepsilon_{z3y}) + \frac{1}{3r}(2\delta_{z2z} - \delta_{z1z} - \delta_{z3z})$$

$$C_7 = \frac{1}{3}(-2\varepsilon_{z2x} - \frac{1}{2}\varepsilon_{z1x} - \frac{1}{2}\varepsilon_{z3x} + \frac{\sqrt{3}}{2}\varepsilon_{z1y} - \frac{\sqrt{3}}{2}\varepsilon_{z3y})$$

$$C_8 = \frac{1}{3}(\frac{\sqrt{3}}{2}\varepsilon_{z1z} - \frac{\sqrt{3}}{2}\varepsilon_{z3z}) + \frac{1}{3r}(-2\delta_{z2x} + \delta_{x1z} - \delta_{z3x})$$

$$C_9 = \frac{1}{3}(2\varepsilon_{z2z} + \frac{1}{2}\varepsilon_{z1z} - \frac{1}{2}\varepsilon_{z3z}) + \frac{1}{3r}(2\delta_{z2y} - \delta_{z1y} - \delta_{z3y})$$

$$C_{10} = \frac{1}{3r}(2\varepsilon_{z2x} - \varepsilon_{z1x} - \varepsilon_{z3x})$$

$$C_{11} = \frac{1}{3r}(2\varepsilon_{z2y} - \varepsilon_{z1y} - \varepsilon_{z3y})$$

$$C_{12} = \frac{1}{3r}(2\varepsilon_{z2z} - \varepsilon_{z1z} - \varepsilon_{z3z})$$

Table A2-3
Standard Deviations
of Error Coefficients

$$\sigma_{A0} = \frac{\sqrt{3}}{3r} \sigma(\varepsilon_0)$$

$$\sigma_{A1} = \sqrt{\frac{\sigma^2(\varepsilon_2)}{12} + \frac{27\,\sigma^2(\varepsilon_1)}{144} + \frac{\sigma^2(\delta)}{18r^2}}$$

$$\sigma_{A2} = \sqrt{\frac{\sigma^2(\varepsilon_2)}{6} + \frac{\sigma^2(\delta)}{6r^2}}$$

$$\sigma_{A3} = \sqrt{\frac{\sigma^2(\varepsilon_2)}{6} + \frac{\sigma^2(\delta)}{6r^2}}$$

$$\sigma_{A4} = \sqrt{\frac{\sigma^2(\varepsilon_1)}{24} + \frac{\sigma^2(\varepsilon_2)}{8} + \frac{\sigma^2(\delta)}{6r^2}}$$

$$\sigma_{A5} = \sqrt{\frac{\sigma^2(\varepsilon_1)}{24} + \frac{\sigma^2(\varepsilon_2)}{8} + \frac{\sigma^2(\delta)}{6r^2}}$$

$$\sigma_{A6} = \sqrt{\frac{\sigma^2(\varepsilon_1)}{12} + \frac{\sigma^2(\varepsilon_2)}{8} + \frac{\sigma^2(\delta)}{3r^2}}$$

$$\sigma_{A7} = \sqrt{\frac{27\,\sigma^2(\varepsilon_1)}{144} + \frac{\sigma^2(\varepsilon_2)}{12} + \frac{\sigma^2(\delta)}{3r^2}}$$

$$\sigma_{A8} = \sqrt{\frac{\sigma^2(\varepsilon_2)}{6} + \frac{\sigma^2(\delta)}{6r^2}}$$

$$\sigma_{A9} = \sqrt{\frac{\sigma^2(\varepsilon_2)}{6} + \frac{\sigma^2(\delta)}{6r^2}}$$

$$\sigma_{A_{10}} = \sqrt{\frac{\sigma(\varepsilon_1)}{6r^2} + \frac{\sigma^2\ (\varepsilon_2)}{6r^2}}$$

$$\sigma_{A_{11}} = \sqrt{\frac{\sigma\ (\varepsilon_1)}{6r^2} + \frac{\sigma\ (\varepsilon_2)}{6r^2}}$$

$$\sigma_{A_{12}} = \sqrt{\frac{\sigma\ (\varepsilon_1)}{6r^2} + \frac{\sigma\ (\varepsilon_2)}{6r^2}}$$

$$\sigma_{B_0} = \frac{\sqrt{6}}{3r} \sigma (\varepsilon_0)$$

$$\sigma_{B_1} = \sqrt{\frac{\sigma^2 (\varepsilon_1)}{2} + \frac{2 \sigma^2 (\delta)}{3r^2}}$$

$$\sigma_{B_2} = \sqrt{\frac{\sigma^2 (\varepsilon_1)}{6} + \frac{2 \sigma^2 (\delta)}{3r^2}}$$

$$\sigma_{B_3} = \frac{\sqrt{6}}{3} \sigma (\varepsilon_2)$$

$$\sigma_{B_4} = \sqrt{\frac{\sigma^2 (\varepsilon_2)}{6} + \frac{2 \sigma^2 (\delta)}{3r^2}}$$

$$\sigma_{B_5} = \frac{1}{\sqrt{2}} \sigma (\varepsilon_2)$$

$$\sigma_{B_6} = \sqrt{\frac{\sigma^2 (\varepsilon_2)}{2} + \frac{2 \sigma^2 (\delta)}{3r}}$$

$$\sigma_{B_7} = \frac{\sqrt{6}}{3} \sigma (\varepsilon_2)$$

$$\sigma_{B_8} = \sqrt{\frac{\sigma^2 (\varepsilon_1)}{6} + \frac{2 \sigma^2 (\delta)}{3r^2}}$$

$$\sigma_{B_9} = \sqrt{\frac{\sigma^2(\varepsilon_1)}{6} + \frac{2\sigma^2(\delta)}{3r^2}}$$

$$\sigma_{B_{10}} = \frac{\sqrt{6}}{3r}\sigma(\varepsilon_2)$$

$$\sigma_{B_{11}} = \frac{\sqrt{6}}{3r}\sigma(\varepsilon_2)$$

$$\sigma_{B_{12}} = \frac{\sqrt{6}}{3r}\sigma(\varepsilon_1)$$

Table A2-4. EXAMPLE CALCULATION OF ERROR COEFFICIENTS

$R = 4$ inches $\delta = 0.01$ inches

$\varepsilon_0 = 1g$ $\varepsilon_1 = 5\%$ of signal $\varepsilon_2 = 3\%$ of signal

$A_0 = 55.7$ rad/sec^2	$B_0 = 78.8$	$C_0 = 78.8$
$A_1 = 2.34\%\ \dot{\omega}_z$	$B_1 = 3.54$	$C_1 = 3.54$
$A_2 = 1.23\%$	$B_2 = 2.05\%$	$C_2 = 2.05\%$
$A_3 = 1.23\%$	$B_3 = 2.45\%$	$C_3 = 2.45\%$
$A_4 = 1.48\%$	$B_4 = 1.24\%$	$C_4 = 2.13\%$
$A_5 = 1.48\%$	$B_5 = 2.12\%$	$C_5 = 1.22\%$
$A_6 = 1.80\%$	$B_6 = 2.13\%$	$C_6 = 2.46\%$
$A_7 = 2.17\%$	$B_7 = 2.45\%$	$C_7 = 2.45\%$
$A_8 = 1.23\%$	$B_8 = 3.54\%$	$C_8 = 2.05\%$
$A_9 = 1.23\%$	$B_9 = 2.05\%$	$C_9 = 3.54\%$
$A_{10} = 0.60\%$	$B_{10} = 0.61\%$	$C_{10} = 0.61\%$
$A_{11} = 0.60\%$	$B_{11} = 0.61\%$	$C_{11} = 0.61\%$
$A_{12} = 0.60\%$	$B_{12} = 1.02\%$	$C_{12} = 1.02\%$

APPENDIX A3

DERIVATION OF EQUATIONS FOR ESTIMATED TRANSLATIONAL ACCELERATIONS FOR NINE ACCELEROMETER NON-COPLANAR CONFIGURATION

In this appendix the estimated translational accelerations for the nine accelerometer, non-coplanar configuration are derived in terms of the accelerometer locations and output errors. The development closely follows the discussions of the previous appendices.

The translational accelerations, without errors terms, are simply the accelerometer signals at the origin or:

$$R_x = A_{x1}$$
$$R_y = A_{y1}$$
$$R_x = A_{z1}$$

Using these equations with the actual error terms the translational acceleration estimates may be expressed in terms of the rigid body motions and transducer location errors. Neglecting terms of second order, the equations can be written as shown in Table A3-1 and the error coefficients are given in Table A3-2.

Table A3-1
Translational Acceleration Estimates
3-2-2-2 Configuration

$$Q_{0x} = \ddot{X}_0 + A_0 + A_1\dot{\omega}_x + A_2\dot{\omega}_y + A_3\dot{\omega}_z + A_4\omega_x^2$$
$$+ A_5\omega_y^2 + A_6\omega_z^2 + A_7\omega_x\omega_y + A_8\omega_x\omega_z$$
$$+ A_9\omega_y\omega_z + A_{10}\ddot{X}_0 + A_{11}\ddot{Y}_0 + A_{12}\ddot{Z}_0$$

$$Q_{0y} = \ddot{Y}_0 + B_0 + B_1\dot{\omega}_x + B_2\dot{\omega}_y + B_3\dot{\omega}_z + B_4\omega_x^2$$
$$+ B_5\omega_y^2 + B_6\omega_z^2 + B_7\omega_x\omega_y + B_8\omega_x\omega_z$$
$$+ B_9\omega_y\omega_z + B_{10}\ddot{X}_0 + B_{11}\ddot{Y}_0 + B_{12}\ddot{Z}_0$$

$$Q_{0z} = \ddot{Z}_0 + C_0 + C_1\dot{\omega}_x + C_2\dot{\omega}_y + C_3\dot{\omega}_z + C_4\omega_x^2$$
$$+ C_5\omega_y^2 + C_6\omega_z^2 + C_7\omega_x\omega_y + C_8\omega_x\omega_z$$
$$+ C_9\omega_y\omega_z + C_{10}\ddot{X}_0 + C_{11}\ddot{Y}_0 + C_{12}\ddot{Z}_0$$

Table A3-2
Translational Error Coefficients
3-2-2-2 Configuration

$$A_0 = \varepsilon_{0x0}$$

$$A_1 = -\delta_{0x3}\varepsilon_{0x2} + \delta_{0x2}\varepsilon_{0x3}$$

$$A_2 = \delta_{0x3} + \delta_{0x3}\varepsilon_{0x1} - \delta_{0x1}\varepsilon_{0x3}$$

$$A_3 = -\delta_{0x2} - \delta_{0x2}\varepsilon_{0x1} + \delta_{0x1}\varepsilon_{0x2}$$

$$A_4 = -\delta_{0x2}\varepsilon_{0x2} - \delta_{0x3}\varepsilon_{0x3}$$

$$A_5 = -\delta_{0x1} - \delta_{0x1}\varepsilon_{0x1} - \delta_{0x3}\varepsilon_{0x3}$$

$$A_6 = -\delta_{0x1} - \delta_{0x1}\varepsilon_{0x1} - \delta_{0x2}\varepsilon_{0x2}$$

$$A_7 = \delta_{0x2} + \delta_{0x2}\varepsilon_{0x1} + \delta_{0x1}\varepsilon_{0x2}$$

$$A_8 = \delta_{0x3} + \delta_{0x3}\varepsilon_{0x1} + \delta_{0x1}\varepsilon_{0x3}$$

$$A_9 = \delta_{0x3}\varepsilon_{0x2} + \delta_{0x2}\varepsilon_{0x3}$$

$$A_{10} = \varepsilon_{0x1}$$

$$A_{11} = \varepsilon_{0x2}$$

$$A_{12} = \varepsilon_{0x3}$$

$$B_0 = \varepsilon_{0y0}$$

$$B_1 = -\delta_{0y3} - \delta_{0y3}\varepsilon_{0y2} + \delta_{0y2}\varepsilon_{0y3}$$

$$B_2 = \delta_{0y3}\varepsilon_{0y1} - \delta_{0y1}\varepsilon_{0y3}$$

$$B_3 = -\delta_{0y2}\varepsilon_{0y1} + \delta_{0y1} + \delta_{0y1}\varepsilon_{0y2}$$

$$B_4 = -\delta_{0y2} - \delta_{0y2}\varepsilon_{0y2} - \delta_{0y3}\varepsilon_{0y3}$$

$$B_5 = -\delta_{0y1}\varepsilon_{0y1} - \delta_{0y3}\varepsilon_{0y3}$$

$$B_6 = -\delta_{0y1}\varepsilon_{0y1} - \delta_{0y2} - \delta_{0y2}\varepsilon_{0y2}$$

$$B_7 = \delta_{0y2}\varepsilon_{0y1} + \delta_{0y1} + \delta_{0y1}\varepsilon_{0y2}$$

$$B_8 = \delta_{0y3}\varepsilon_{0y1} + \delta_{0y1}\varepsilon_{0y3}$$

$$B_9 = \delta_{0y3} + \delta_{0y3}\varepsilon_{0y2} + \delta_{0y2}\varepsilon_{0y3}$$

$$B_{10} = \varepsilon_{0y1}$$

$$B_{11} = \varepsilon_{0y2}$$

$$B_{12} = \varepsilon_{0y3}$$

$$C_0 = \varepsilon_{0z0}$$

$$C_1 = -\delta_{0z3}\varepsilon_{0z2} + \delta_{0z2} + \delta_{0z2}\varepsilon_{0z3}$$

$$C_2 = \delta_{0z3}\varepsilon_{0z1} - \delta_{0z1} - \delta_{0z1}\varepsilon_{0z3}$$

$$C_3 = -\delta_{0z2}\varepsilon_{0z1} + \delta_{0z1}\varepsilon_{0z2}$$

$$C_4 = -\delta_{0z2}\varepsilon_{0z2} - \delta_{0z3} - \delta_{0z3}\varepsilon_{0z3}$$

$$C_5 = -\delta_{0z1}\varepsilon_{0z1} - \delta_{0z3} - \delta_{0z3}\varepsilon_{0z3}$$

$$C_6 = -\delta_{0z1}\varepsilon_{0z1} - \delta_{0z2}\varepsilon_{0z2}$$

$$C_7 = \delta_{0z2}\varepsilon_{0z1} + \delta_{0z1}\varepsilon_{0z2}$$

$$C_8 = \delta_{0z3}\varepsilon_{0z1} + \delta_{0z1} + \delta_{0z1}\varepsilon_{0z3}$$

$$C_9 = \delta_{0z3}\varepsilon_{0z2} + \delta_{0z2} + \delta_{0z2}\varepsilon_{0z3}$$

$$C_{10} = \varepsilon_{0z1}$$

$$C_{11} = \varepsilon_{0z2}$$

$$C_{12} = \varepsilon_{0z3}$$

APPENDIX A4

DERIVATION OF EQUATIONS FOR ESTIMATED TRANSLATIONAL ACCELERATIONS FOR NINE ACCELEROMETER COPLANAR CONFIGURATION

In this appendix, the estimated translational accelerations for the nine accelerometer, coplanar configuration are derived in terms of the accelerometer location and output errors. The development closely follows the discussions of appendices A1 and A2. Algebraically manipulating the accelerometer signal as presented in appendix A2, the translational accelerations can be written, without errors terms,

as: $$R_x = \frac{A_{x1} + A_{x2} + A_{x3}}{3}$$

$$R_y = \frac{A_{y1} + A_{y2} + A_{y3}}{3}$$

$$R_z = \frac{A_{z1} + A_{z2} + A_{z3}}{3}$$

Using these equations with the actual error terms the translational acceleration estimates may be expressed in terms of the rigid body motions and transducer location errors. Neglecting terms of second order the equation can be written as shown in Table A4-1 and the error coefficients are given in Table A4-2.

Table A4-1

Translational Acceleration Estimates

3-3-3 Configuration

$$Q_{0x} = \ddot{X}_0 + A_0 + A_1\dot{\omega}_x + A_2\dot{\omega}_y + A_3\dot{\omega}_z + A_4\omega_x^2$$
$$+ A_5\omega_y^2 + A_6\omega_z^2 + A_7\omega_x\omega_y + A_8\omega_x\omega_z$$
$$+ A_9\omega_y\omega_z + A_{10}\ddot{X}_0 + A_{11}\ddot{Y}_0 + A_{12}\ddot{Z}_0$$

$$Q_{0y} = \ddot{Y}_0 + B_0 + B_1\dot{\omega}_x + B_2\dot{\omega}_y + B_3\dot{\omega}_z + B_4\omega_x^2$$
$$+ B_5\omega_y^2 + B_6\omega_z^2 + B_7\omega_x\omega_y + B_8\omega_x\omega_z$$
$$+ B_9\omega_y\omega_z + B_{10}\ddot{X}_0 + B_{11}\ddot{Y}_0 + B_{12}\ddot{Z}_0$$

$$Q_{0z} = \ddot{Z}_0 + C_0 + C_1\dot{\omega}_x + C_2\dot{\omega}_y + C_3\dot{\omega}_z + C_4\omega_x^2$$
$$+ C_5\omega_y^2 + C_6\omega_z^2 + C_7\omega_x\omega_y + C_8\omega_x\omega_z$$
$$+ C_9\omega_y\omega_z + C_{10}\ddot{X}_0 + C_{11}\ddot{Y}_0 + C_{12}\ddot{Z}_o$$

Table A4-2

Translational Error Coefficients

3-3-3 Configuration

$$A_0 = \frac{1}{3}(\varepsilon_{1x0} + \varepsilon_{2x0} + \varepsilon_{3x0})$$

$$A_1 = -\frac{1}{3}(\delta_{1x3}\varepsilon_{1x2} + \delta_{2x3}\varepsilon_{2x2} + \delta_{3x3}\varepsilon_{3x2})$$

$$+ \frac{1}{3}\,a(-\frac{1}{2}\varepsilon_{1x3} + \varepsilon_{2x3} - \frac{1}{2}\,\varepsilon_{3x3})$$

$$+ \frac{1}{3}(\delta_{1x2}\varepsilon_{1x3} + \delta_{2x2}\varepsilon_{2x3} + \delta_{3x2}\varepsilon_{3x3})$$

$$A_2 = \frac{1}{3}(\delta_{1x3} + \delta_{2x3} + \delta_{3x3})$$

$$+ \frac{1}{3}(\delta_{1x3}\varepsilon_{1x1} + \delta_{2x3}\varepsilon_{2x1} + \delta_{3x3}\varepsilon_{3x1})$$

$$- \frac{a\sqrt{3}}{6}(\varepsilon_{1x3} - \varepsilon_{3x3})$$

$$- \frac{1}{3}(\delta_{1x1}\varepsilon_{1x3} + \delta_{2x1}\varepsilon_{2x3} + \delta_{3x1}\varepsilon_{3x3})$$

$$A_3 = -\frac{1}{3}(\delta_{1x2} + \delta_{2x2} + \delta_{3x2})$$

$$- \frac{1}{3}\,a\,(-\frac{1}{2}\varepsilon_{1x1} + \varepsilon_{2x1} - \frac{1}{2}\varepsilon_{3x1})$$

$$- \frac{1}{3}(\delta_{1x2}\varepsilon_{1x1} + \delta_{2x2}\varepsilon_{2x1} + \delta_{3x2}\varepsilon_{3x1})$$

$$+ \frac{a\sqrt{3}}{6}\,(\varepsilon_{1x2} - \varepsilon_{3x2})$$

$$+ \frac{1}{3}(\delta_{1x1}\varepsilon_{1x2} + \delta_{2x1}\varepsilon_{2x2} + \delta_{3x1}\varepsilon_{3x2})$$

$$A_4 = -\frac{1}{3}a(-\frac{1}{2}\varepsilon_{1x2} + \varepsilon_{2x2} - \frac{1}{2}\varepsilon_{3x2})$$

$$-\frac{1}{3}(\delta_{1x2}\varepsilon_{1x2} + \delta_{2x2}\varepsilon_{2x2} + \delta_{3x2}\varepsilon_{3x2})$$

$$-\frac{1}{3}(\delta_{1x3}\varepsilon_{1x3} + \delta_{2x3}\varepsilon_{2x3} + \delta_{3x3}\varepsilon_{3x3})$$

$$A_5 = -\frac{1}{3}(\delta_{1x1} + \delta_{2x1} + \delta_{3x1})$$

$$-\frac{a\sqrt{3}}{6}(\varepsilon_{1x1} - \varepsilon_{3x1})$$

$$-\frac{1}{3}(\delta_{1x1}\varepsilon_{1x1} + \delta_{2x1}\varepsilon_{2x1} + \delta_{3x1}\varepsilon_{3x1})$$

$$-\frac{1}{3}(\delta_{1x3}\varepsilon_{1x3} + \delta_{2x3}\varepsilon_{2x3} + \delta_{3x3}\varepsilon_{3x3})$$

$$A_6 = -\frac{1}{3}(\delta_{1x1} + \delta_{2x1} + \delta_{3x1})$$

$$-\frac{a\sqrt{3}}{6}(\varepsilon_{1x1} - \varepsilon_{3x1})$$

$$-\frac{1}{3}(\delta_{1x1}\varepsilon_{1x1} + \delta_{2x1}\varepsilon_{2x1} + \delta_{3x1}\varepsilon_{3x1})$$

$$-\frac{1}{3}a(-\frac{1}{2}\varepsilon_{1x2} + \varepsilon_{2x2} - \frac{1}{2}\varepsilon_{3x2})$$

$$-\frac{1}{3}(\delta_{1x2}\varepsilon_{1x2} + \delta_{2x2}\varepsilon_{2x2} + \delta_{3x2}\varepsilon_{3x2})$$

$$A_7 = \frac{1}{3}(\delta_{1x2} + \delta_{2x2} + \delta_{3x2})$$

$$+ \frac{1}{3}\ a\ (-\frac{1}{2}\ \varepsilon_{1x1} + \varepsilon_{2x1} - \frac{1}{2}\ \varepsilon_{3x1})$$

$$+ \frac{1}{3}(\delta_{1x2}\varepsilon_{1x1} + \delta_{2x2}\varepsilon_{2x1} + \delta_{3x2}\varepsilon_{3x1})$$

$$+ \frac{a\sqrt{3}}{6}\ (\varepsilon_{1x2} - \varepsilon_{3x2})$$

$$+ \frac{1}{3}(\delta_{1x1}\varepsilon_{1x2} + \delta_{2x1}\varepsilon_{2x2} + \delta_{3x1}\varepsilon_{3x2})$$

$$A_8 = \frac{1}{3}(\delta_{1x3} + \delta_{2x3} + \delta_{3x3})$$

$$+ \frac{1}{3}(\delta_{1x3}\varepsilon_{1x1} + \delta_{2x3}\varepsilon_{2x1} + \delta_{3x3}\varepsilon_{3x1})$$

$$+ \frac{a\sqrt{3}}{6}(\varepsilon_{1x3} - \varepsilon_{3x3})$$

$$+ \frac{1}{3}(\delta_{1x1}\varepsilon_{1x3} + \delta_{2x1}\varepsilon_{2x3} + \delta_{3x1}\varepsilon_{3x3})$$

$$A_9 = \frac{1}{3}(\delta_{1x3}\varepsilon_{1x2} + \delta_{2x3}\varepsilon_{2x2} + \delta_{3x3}\varepsilon_{3x2})$$

$$+ \frac{1}{3}\ a\ (-\frac{1}{2}\ \varepsilon_{1x3} + \varepsilon_{2x3} - \frac{1}{2}\ \varepsilon_{3x3})$$

$$+ \frac{1}{3}(\delta_{1x2}\varepsilon_{1x3} + \delta_{2x2}\varepsilon_{2x3} + \delta_{3x2}\varepsilon_{3x3})$$

$$A_{10} = \frac{1}{3}\ (\varepsilon_{1x1} + \varepsilon_{2x1} + \varepsilon_{3x1})$$

$$A_{11} = \frac{1}{3}(\varepsilon_{1x2} + \varepsilon_{2x2} + \varepsilon_{3x2})$$

$$A_{12} = \frac{1}{3}(\varepsilon_{1x3} + \varepsilon_{2x3} + \varepsilon_{3x3})$$

$$B_0 = \frac{1}{3}(\varepsilon_{1y0} + \varepsilon_{2y0} + \varepsilon_{3y0})$$

$$B_1 = -\frac{1}{3}(\delta_{1y3} + \delta_{2y3} + \delta_{3y3})$$

$$-\frac{1}{3}(\delta_{1y3}\varepsilon_{1y2} + \delta_{2y3}\varepsilon_{2y2} + \delta_{3y3}\varepsilon_{3y2})$$

$$+\frac{1}{3}a\left(-\frac{1}{2}\varepsilon_{1y3} + \varepsilon_{2y3} - \frac{1}{2}\varepsilon_{3y3}\right)$$

$$+\frac{1}{3}(\delta_{1y2}\varepsilon_{1y3} + \delta_{2y2}\varepsilon_{2y3} + \delta_{3y2}\varepsilon_{3y3})$$

$$B_2 = \frac{1}{3}(\delta_{1y3}\varepsilon_{1y1} + \delta_{2y3}\varepsilon_{2y1} + \delta_{3y3}\varepsilon_{3y1})$$

$$-\frac{a\sqrt{3}}{6}(\varepsilon_{1y3} - \varepsilon_{3y3})$$

$$-\frac{1}{3}(\delta_{1y1}\varepsilon_{1y3} + \delta_{2y1}\varepsilon_{2y3} + \delta_{3y1}\varepsilon_{3y3})$$

$$B_3 = \frac{1}{3}(\delta_{1y1} + \delta_{2y1} + \delta_{3y1})$$

$$-\frac{1}{3}a\left(-\frac{1}{2}\varepsilon_{1y1} + \varepsilon_{2y1} - \frac{1}{2}\varepsilon_{3y1}\right)$$

$$-\frac{1}{3}(\delta_{1y2}\varepsilon_{1y1} + \delta_{2y2}\varepsilon_{2y1} + \delta_{3y2}\varepsilon_{3y1})$$

$$+\frac{a\sqrt{3}}{6}(\varepsilon_{1y2} - \varepsilon_{3y2})$$

$$+\frac{1}{3}(\delta_{1y1}\varepsilon_{1y2} + \delta_{2y1}\varepsilon_{2y2} + \delta_{3y1}\varepsilon_{3y2})$$

$$B_4 = -\frac{1}{3}(\delta_{1y2} + \delta_{2y2} + \delta_{3y2})$$

$$-\frac{1}{3}\, a(-\frac{1}{2}\,\varepsilon_{1y2} + \varepsilon_{2y2} - \frac{1}{2}\,\varepsilon_{3y2})$$

$$-\frac{1}{3}(\delta_{1y2}\varepsilon_{1y2} + \delta_{2y2}\varepsilon_{2y2} + \delta_{3y2}\varepsilon_{3y2})$$

$$-\frac{1}{3}(\delta_{1y3}\varepsilon_{1y3} + \delta_{2y3}\varepsilon_{2y3} + \delta_{3y3}\varepsilon_{3y3})$$

$$B_5 = -\frac{a\sqrt{3}}{6}\,(\frac{1}{2}\,\varepsilon_{1y1} - \frac{1}{2}\,\varepsilon_{3y1})$$

$$-\frac{1}{3}(\delta_{1y1}\varepsilon_{1y1} + \delta_{2y1}\varepsilon_{2y1} + \delta_{3y1}\varepsilon_{3y1})$$

$$-\frac{1}{3}(\delta_{1y3}\varepsilon_{1y3} + \delta_{2y3}\varepsilon_{2y3} + \delta_{3y3}\varepsilon_{3y3})$$

$$B_6 = -\frac{1}{3}(\delta_{1y2} + \delta_{2y2} + \delta_{3y2})$$

$$-\frac{a\sqrt{3}}{6}\,(\varepsilon_{1y1} - \varepsilon_{3y1})$$

$$-\frac{1}{3}(\delta_{1y1}\varepsilon_{1y1} + \delta_{2y1}\varepsilon_{2y1} + \delta_{3y1}\varepsilon_{3y1})$$

$$-\frac{1}{3}\, a\, (-\frac{1}{2}\,\varepsilon_{1y2} + \varepsilon_{2y2} - \frac{1}{2}\,\varepsilon_{3y2})$$

$$-\frac{1}{3}(\delta_{1y2}\varepsilon_{1y2} + \delta_{2y2}\varepsilon_{2y2} + \delta_{3y2}\varepsilon_{3y2})$$

$$B_7 = \frac{1}{3}(\delta_{1y1} + \delta_{2y1} + \delta_{3y1})$$

$$+ \frac{1}{3} a(- \frac{1}{2} \varepsilon_{1y1} + \varepsilon_{2y1} - \frac{1}{2} \varepsilon_{3y1})$$

$$+ \frac{1}{3}(\delta_{1y2}\varepsilon_{1y1} + \delta_{2y2}\varepsilon_{2y1} + \delta_{3y2}\varepsilon_{3y1})$$

$$+ \frac{a\sqrt{3}}{6} (\varepsilon_{1y2} - \varepsilon_{3y2})$$

$$+ \frac{1}{3}(\delta_{1y1}\varepsilon_{1y2} + \delta_{2y1}\varepsilon_{2y2} + \delta_{3y1}\varepsilon_{3y2})$$

$$B_8 = \frac{1}{3}(\delta_{1y3}\varepsilon_{1y1} + \delta_{2y3}\varepsilon_{2y1} + \delta_{3y3}\varepsilon_{3y1})$$

$$+ \frac{a\sqrt{3}}{6}(\varepsilon_{1y3} - \varepsilon_{3y3})$$

$$+ \frac{1}{3}(\delta_{1y1}\varepsilon_{1y3} + \delta_{2y1}\varepsilon_{2y3} + \delta_{3y1}\varepsilon_{3y3})$$

$$B_9 = \frac{1}{3}(\delta_{1y3} + \delta_{2y3} + \delta_{3y3})$$

$$+ \frac{1}{3}(\delta_{1y3}\varepsilon_{1y2} + \delta_{2y3}\varepsilon_{2y2} + \delta_{3y3}\varepsilon_{3y2})$$

$$+ \frac{1}{3} a(- \frac{1}{2} \varepsilon_{1y3} + \varepsilon_{2y3} - \frac{1}{2} \varepsilon_{3y3})$$

$$+ \frac{1}{3}(\delta_{1y2}\varepsilon_{1y3} + \delta_{2y2}\varepsilon_{2y3} + \delta_{3y2}\varepsilon_{3y3})$$

$$B_{10} = \frac{1}{3}(\varepsilon_{1y1} + \varepsilon_{2y1} + \varepsilon_{3y1})$$

$$B_{11} = \frac{1}{3}(\varepsilon_{1y2} + \varepsilon_{2y2} + \varepsilon_{3y2})$$

$$B_{12} = \frac{1}{3}(\varepsilon_{1y3} + \varepsilon_{2y3} + \varepsilon_{3y3})$$

$$C_0 = \frac{1}{3}(\varepsilon_{1z0} + \varepsilon_{2z0} + \varepsilon_{3z0})$$

$$C_1 = \frac{1}{3}(\delta_{1z2} + \delta_{2z2} + \delta_{3z2})$$

$$- \frac{1}{3}(\delta_{1z3}\varepsilon_{1z2} + \delta_{2z3}\varepsilon_{2z2} + \delta_{3z3}\varepsilon_{3z2})$$

$$+ \frac{1}{3}\, a(- \frac{1}{2}\, \varepsilon_{1z3} + \varepsilon_{2z3} - \frac{1}{2}\, \varepsilon_{3z3})$$

$$+ \frac{1}{3}\, (\delta_{1z2}\varepsilon_{1z3} + \delta_{2z2}\varepsilon_{2z3} + \delta_{3z2}\varepsilon_{3z3})$$

$$C_2 = - \frac{1}{3}(\delta_{1z1} + \delta_{2z1} + \delta_{3z1})$$

$$+ \frac{1}{3}(\delta_{1z3}\varepsilon_{1z1} + \delta_{2z3}\varepsilon_{2z1} + \delta_{3z3}\varepsilon_{3z1})$$

$$- \frac{a\sqrt{3}}{6}(\varepsilon_{1z3} - \varepsilon_{3z3})$$

$$- \frac{1}{3}(\delta_{1z1}\varepsilon_{1z3} + \delta_{2z1}\varepsilon_{2z3} + \delta_{3z1}\varepsilon_{3z3})$$

$$C_3 = - \frac{1}{3}\, a(- \frac{1}{2}\, \varepsilon_{1z1} + \varepsilon_{2z1} - \frac{1}{2}\, \varepsilon_{3z1})$$

$$- \frac{1}{3}(\delta_{1z2}\varepsilon_{1z1} + \delta_{2z2}\varepsilon_{2z1} + \delta_{3z2}\delta_{3z1})$$

$$+ \frac{a\sqrt{3}}{6}(\varepsilon_{1z2} - \varepsilon_{3z2})$$

$$+ \frac{1}{3}(\delta_{1z1}\varepsilon_{1z2} + \delta_{2z1}\varepsilon_{2z2} + \delta_{3z1}\varepsilon_{3z2})$$

$$C_4 = -\frac{1}{3}(\delta_{1z3} + \delta_{2z3} + \delta_{3z3})$$

$$- \frac{1}{3} a(-\frac{1}{2} \varepsilon_{1z2} + \varepsilon_{2z2} - \frac{1}{2} \varepsilon_{3z2})$$

$$- \frac{1}{3}(\delta_{1z2}\varepsilon_{1z2} + \delta_{2z2}\varepsilon_{2z2} + \delta_{3z2}\varepsilon_{3z2})$$

$$- \frac{1}{3}(\delta_{1z3}\varepsilon_{1z3} + \delta_{2z3}\varepsilon_{2z3} + \delta_{3z3}\varepsilon_{3z3})$$

$$C_5 = -\frac{1}{3}(\delta_{1z3} + \delta_{2z3} + \delta_{3z3})$$

$$- \frac{a\sqrt{3}}{6}(\varepsilon_{1z1} - \varepsilon_{3z1})$$

$$- \frac{1}{3}(\delta_{1z1}\varepsilon_{1z1} + \delta_{2z1}\varepsilon_{2z1} + \delta_{3z1}\varepsilon_{3z1})$$

$$- \frac{1}{3}(\delta_{1z3}\varepsilon_{1z3} + \delta_{2z3}\varepsilon_{2z3} + \delta_{3z3}\varepsilon_{3z3})$$

$$C_6 = -\frac{a\sqrt{3}}{6}(\varepsilon_{1z1} - \varepsilon_{3z1})$$

$$- \frac{1}{3}(\delta_{1z1}\varepsilon_{1z1} + \delta_{2z1}\varepsilon_{2z1} + \delta_{3z1}\varepsilon_{3z1})$$

$$- \frac{1}{3} a(-\frac{1}{2} \varepsilon_{1z2} + \varepsilon_{2z2} - \frac{1}{2} \varepsilon_{3z2})$$

$$- \frac{1}{3}(\delta_{1z2}\varepsilon_{1z2} + \delta_{2z2}\varepsilon_{2z2} + \delta_{3z2}\varepsilon_{3z2})$$

$$C_7 = \frac{1}{3} a(-\frac{1}{2} \varepsilon_{1z1} + \varepsilon_{2z1} - \frac{1}{2} \varepsilon_{3z1})$$

$$+ \frac{1}{3}(\delta_{1z2}\varepsilon_{1z1} + \delta_{2z2}\varepsilon_{2z1} + \delta_{3z2}\varepsilon_{3z1})$$

$$+ \frac{a\sqrt{3}}{6}(\varepsilon_{1z2} - \varepsilon_{3z2})$$

$$+ \frac{1}{3}(\delta_{1z1}\varepsilon_{1z2} + \delta_{2z1}\varepsilon_{2z2} + \delta_{3z1}\varepsilon_{3z2})$$

$$C_8 = \frac{1}{3}(\delta_{1z1} + \delta_{2z1} + \delta_{3z1})$$

$$+ \frac{1}{3}(\delta_{1z3}\varepsilon_{1z1} + \delta_{2z3}\varepsilon_{2z1} + \delta_{3z3}\varepsilon_{3z1})$$

$$+ \frac{a\sqrt{3}}{6}(\varepsilon_{1z3} - \varepsilon_{3z3})$$

$$+ \frac{1}{3}(\delta_{1z1}\varepsilon_{1z3} + \delta_{2z1}\varepsilon_{2z3} + \delta_{3z1}\varepsilon_{3z3})$$

$$C_9 = \frac{1}{3}(\delta_{1z2} + \delta_{2z2} + \delta_{3z2})$$

$$+ \frac{1}{3}(\delta_{1z3}\varepsilon_{1z2} + \delta_{2z3}\varepsilon_{2z2} + \delta_{3z3}\varepsilon_{3z2})$$

$$+ \frac{1}{3}\, a(- \frac{1}{2}\, \varepsilon_{1z3} + \varepsilon_{2z3} - \frac{1}{2}\, \varepsilon_{3z3})$$

$$+ \frac{1}{3}(\delta_{1z2}\varepsilon_{1z3} + \delta_{2z2}\varepsilon_{2z3} + \delta_{3z2}\varepsilon_{3z3})$$

$$C_{10} = \frac{1}{3}(\varepsilon_{1z1} + \varepsilon_{2z1} + \varepsilon_{3z1})$$

$$C_{11} = \frac{1}{3}(\varepsilon_{1z2} + \varepsilon_{2z2} + \varepsilon_{3z2})$$

$$C_{12} = \frac{1}{3}(\varepsilon_{1z3} + \varepsilon_{2z3} + \varepsilon_{3z3})$$

APPENDIX B

SIMULATION AND CALIBRATION OF NINE-ACCELEROMETER PACKAGE (NAP) FOR ANTHROPOMORPHIC DUMMIES

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

APPENDIX B1

COMPUTER PROGRAM LISTING

1. INTRODUCTION

1.1 BACKGROUND

In support of NHTSA requirements to expand the data base of biomechanical information available for use in automotive safety assessment, a project to measure human and dummy head acceleration is being conducted. Instrumentation for the measurement of both linear and angular acceleration will be developed for this purpose. The accurate measurement of angular acceleration is particularly difficult to achieve. Currently, the most commonly used technique consists of an array of nine linear accelerometers. This technique is discussed in detail in References 1 and 2. Reference 1 describes a system which places the accelerometers in a 3-2-2-2 configuration, while Reference 2 discusses a 3-3-3 configuration. The 3-2-2-2 arrangement, shown in Figure 1.1, has been found to be more reliable than the 3-3-3 configuration in most cases.

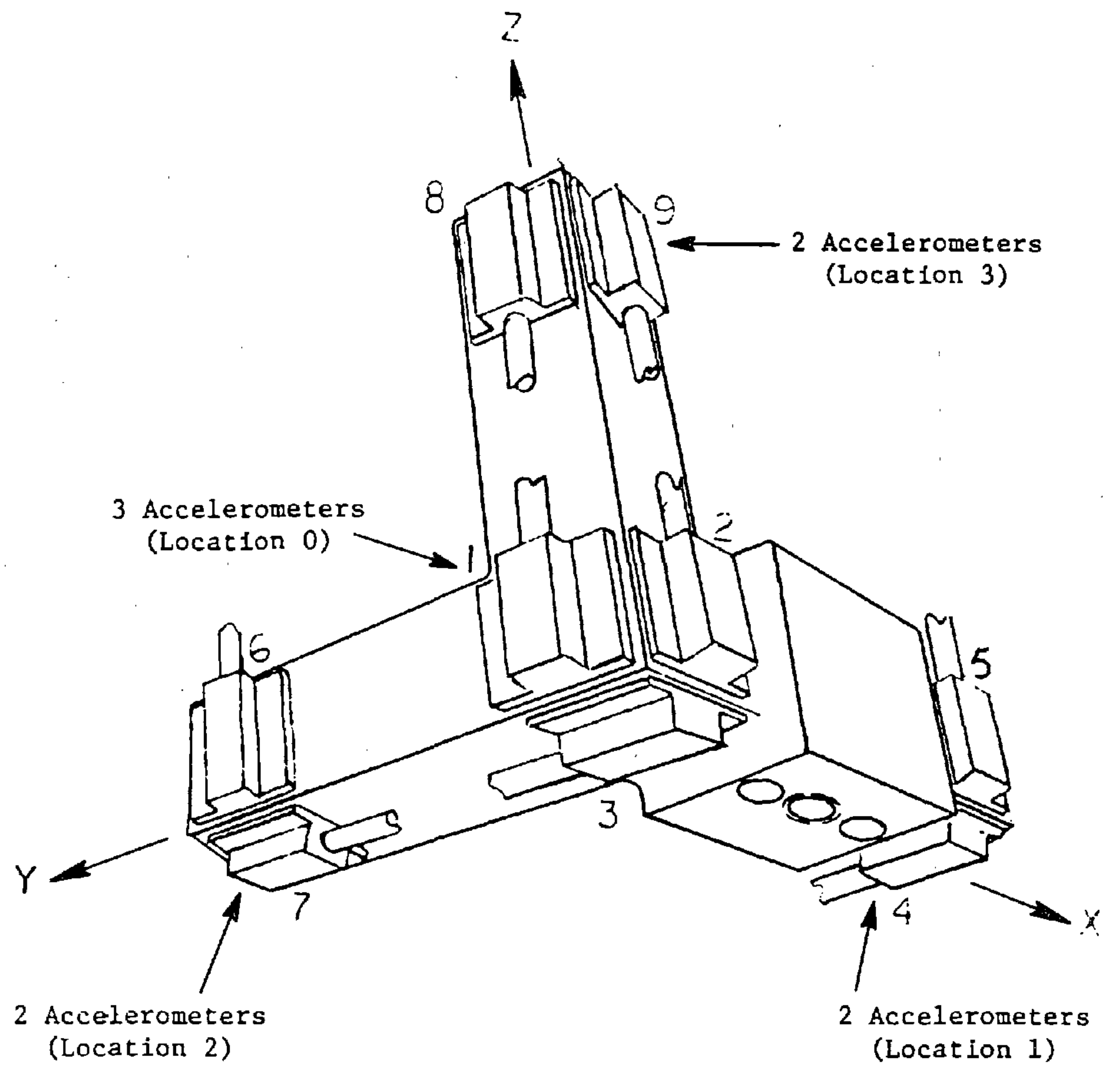
The work discussed in this report deals with two specific issues related to a 3-2-2-2 configuration Nine Accelerometer Package (NAP):

1. Develop a computer program which can be used to perform error sensitivity analyses for the package.

2. Develop a test procedure for determining the error coefficients of the package and calibrating it.

The equations required to perform either task were already derived by TSC and available in Reference 3.

This work was performed by Arthur D. Little under TTD No. 16, contract DTRS-57-80-C-00132.



Source: Endevco

FIGURE 1-1. THE 3-2-2-2 CONFIGURATION OF THE NAP

1.2 ORGANIZATION OF THE REPORT

Chapter 2 describes a computer program developed to help determine what accelerometer system parameters and characteristics are most important in determining the angular acceleration measurement errors, and what tolerance in these parameters and characteristics are required to achieve specified accuracies in the measurement of angular accelerations. This chapter also provides instructions to potential users of the computer program.

Chapter 3 describes a methodology which can be used to calibrate the package and derive all error coefficients. The emphasis here is on obtaining system error coefficients rather than errors and performance characteristics associated with individual accelerometers in the package.

Chapter 4 provides a summary and recommendations. Appendix B-1 includes a listing of computer programs.

2. COMPUTER MODEL OF 3-2-2-2 CONFIGURATION

2.1 SET OF EQUATIONS

In order to perform an error sensitivity analysis of the NAP, we need equations which relate the actual accelerations and error coefficients to measured accelerations. These equations have been derived by TSC and reported in Reference 3. Figure 2.1 provides a summary of the equations. In these equations:

$\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$, are actual angular accelerations,
ω_x, ω_y, ω_z, are actual angular velocities
$\ddot{x}$, $\ddot{y}$, $\ddot{z}$, are actual linear accelerations
AAO...AA12, are error coefficients
BAO...BA12
CAO...CA12
ALO...AL12
BLO...BL12
CLO...CL12
$\dot{\gamma}_x$, $\dot{\gamma}_y$, $\dot{\gamma}_z$ are measured angular accelerations
$\ddot{X}$, $\ddot{Y}$, and $\ddot{Z}$ are measured linear accelerations

Here x, y, z are three axes fixed to the NAP. The relationships between the error coefficients and errors of the individual accelerometers in the package are provided in Figure 2.2. The details of how these equations were derived can be found in Reference 3.

2.2 COMPUTER PROGRAM STRUCTURE

The computer program essentially takes the error terms (δ_{ijk} and ε_{ijk}) plus user selected actual accelerations as inputs and calculates measured accelerations. This way the effects of each error term on measurement errors can be studied, and a sensitivity analysis can be performed.

$$\dot{\gamma}_x = \dot{\omega}_x + AA0 + AA1\,\dot{\omega}_x + AA2\,\dot{\omega}_y + AA3\,\dot{\omega}_z + AA4\,\omega_x^2 + AA5\,\omega_y^2 + AA6\,\omega_z^2 + AA7\,\omega_x\omega_y + AA8\,\omega_x\omega_z + AA9\,\omega_y\omega_z + AA10\,\ddot{x} + AA11\,\ddot{y} + AA12\,\ddot{z} \quad \ldots\ (1)$$

$$\dot{\gamma}_y = \dot{\omega}_y + BA0 + BA1\,\dot{\omega}_x + BA2\,\dot{\omega}_y + BA3\,\dot{\omega}_z + BA4\,\omega_x^2 + BA5\,\omega_y^2 + BA6\,\omega_z^2 + BA7\,\omega_x\omega_y + BA8\,\omega_x\omega_z + BA9\,\omega_y\omega_z + BA10\,\ddot{x} + BA11\,\ddot{y} + BA12\,\ddot{z} \quad \ldots\ (2)$$

$$\dot{\gamma}_z = \dot{\omega}_z + CA0 + CA1\,\dot{\omega}_x + CA2\,\dot{\omega}_y + CA3\,\dot{\omega}_z + CA4\,\omega_x^2 + CA5\,\omega_y^2 + CA6\,\omega_z^2 + CA7\,\omega_x\omega_y + CA8\,\omega_x\omega_z + CA9\,\omega_y\omega_z + CA10\,\ddot{x} + CA11\,\ddot{y} + CA12\,\ddot{z} \quad \ldots\ (3)$$

$$\ddot{X} = \ddot{x} + AL0 + AL1\,\dot{\omega}_x + AL2\,\dot{\omega}_y + AL3\,\dot{\omega}_z + AL4\,\omega_x^2 + AL5\,\omega_y^2 + AL6\,\omega_z^2 + AL7\,\omega_x\omega_y + AL8\,\omega_x\omega_z + AL9\,\omega_y\omega_z + AL10\,\ddot{x} + AL11\,\ddot{y} + AL12\,\ddot{z} \quad \ldots\ (4)$$

$$\ddot{Y} = \ddot{y} + BL0 + BL1\,\dot{\omega}_x + BL2\,\dot{\omega}_y + BL3\,\dot{\omega}_z + BL4\,\omega_x^2 + BL5\,\omega_y^2 + BL6\,\omega_z^2 + BL7\,\omega_x\omega_y + BL8\,\omega_x\omega_z + BL9\,\omega_y\omega_z + BL10\,\ddot{x} + BL11\,\ddot{y} + BL12\,\ddot{z} \quad \ldots\ (5)$$

$$\ddot{Z} = \ddot{z} + CL0 + CL1\,\dot{\omega}_x + CL2\,\dot{\omega}_y + CL3\,\dot{\omega}_z + CL4\,\omega_x^2 + CL5\,\omega_y^2 + CL6\,\omega_z^2 + CL7\,\omega_x\omega_y + CL8\,\omega_x\omega_z + CL9\,\omega_y\omega_z + CL10\,\ddot{x} + CL11\,\ddot{y} + CL12\,\ddot{z} \quad \ldots\ (6)$$

FIGURE 2-1. SET OF EQUATIONS FOR CONVERTING ACTUAL ACCELERATIONS TO MEASURED ACCELERATIONS

```
AA0=(E230-E030-E320+E020)/(2*R)
AA1=(E233+E322)/2+(S232-S032+S323-S023)/(2*R)
AA2=-E321/2+((S231-S031)/(2*R))
AA3=-E231/2-((S321-S021)/(2*R))
AA4=(E323-E232)/2+((S322-S022+S033-S233)/(2*R))
AA5=E323/2+(S033-S233)/(2*R)
AA6=-E232/2+(S322-S022)/(2*R)
AA7=E231/2+(S021-S321)/(2*R)
AA8=-E321/2+(S231-S031)/(2*R)
AA9=(E233-E322)/2+(S232-S032-S323+S023)/(2*R)
AA10=(E231-E031-E321+E021)/(2*R)
AA11=(E232-E032-E322+E022)/(2*R)
AA12=(E233-E033-E323+E023)/(2*R)

BA0=(E310-E010-E130+E030)/(2*R)
BA1=E312/2+(S032-S132)/(2*R)
BA2=(E311+E133)/2+(S313-S013+S131-S031)/(2*R)
BA3=-E132/2+(S012-S312)/(2*R)
BA4=-E313/2+(S133-S033)/(2*R)
BA5=(E131-E313)/2+(S011-S311+S133-S033)/(2*R)
BA6=E131/2+(S011-S311)/(2*R)
BA7=-E132/2+(S312-S012)/(2*R)
BA8=(E311-E133)/2+(S313-S013-S131+S031)/(2*R)
BA9=E312/2+(S032-S132)/(2*R)
BA10=(E311-E011-E131+E031)/(2*R)
BA11=(E312-E012-E132+E032)/(2*R)
BA12=(E313-E013-E133+E033)/(2*R)

CA0=(E120-E010-E210+E010)/(2*R)
CA1=-E213/2+(S023-S123)/(2*R)
CA2=-E123/2+(S013-S213)/(2*R)
CA3=(E122+E211)/2+(S121-S021+S212-S012)/(2*R)
CA4=E212/2+(S022-S122)/(2*R)
CA5=-E121/2+(S211-S011)/(2*R)
CA6=(E212-E121)/2+(S022-S122+S211-S011)/(2*R)
CA7=(E122-E211)/2+(S121-S021-S212+S012)/(2*R)
CA8=E123/2+(S013-S213)/(2*R)
CA9=-E213/2+(S123-S023)/(2*R)
CA10=(E121-E021-E211+E011)/(2*R)
CA11=(E122-E022-E212+E012)/(2*R)
CA12=(E123-E023-E213+E013)/(2*R)
```

EIJK = ϵ_{ijk} which is error type k (0 - bias plus noise, 1 = uncertainty in scale factor, 2,3 = cross axis sensitivity) for accelerometer measuring in the j direction (1=x, 2=y, 3=z) situated at location i (see Figure 1.1).

SIJK = δ_{ijk} which is location error in k direction for accelerometer measuring in j direction situated at location i.

FIGURE 2-2. ERROR COEFFICIENT EQUATIONS

```
AL0=E010
AL1=-S013*E012+S012*E013
AL2=S013+S013*E011-S011*E013
AL3=-S012-S012*E011+S011*E012
AL4=-S012*E012-S013*E013
AL5=-S011-S011*E011-S013*E013
AL6=-S011-S011*E011-S012*E012


AL7=S012+S012*E011+S011*E012
AL8=S013+S013*E011+S011*E013
AL9=S013*E012+S012*E013
AL10=E011
AL11=E012
AL12=E013

BL0=E020
BL1=-S023-S023*E022+S022*E023
BL2=S023*E021-S021*E023
BL3=-S022*E021+S021+S021*E022
BL4=-S022-S022*E022-S023*E023
BL5=-S021*E021-S023*E023
BL6=-S021*E021-S022-S022*E022
BL7=S022*E021+S021+S021*E022
BL8=S023*E021+S021*E023
BL9=S023+S023*E022+S022*E023
BL10=E021
BL11=E022
BL12=E023

CL0=E030
CL1=-S033*E032+S032+S032*E033
CL2=S033*E031-S031-S031*E033
CL3=-S032*E031+S031*E032
CL4=-S032*E032-S033-S033*E033
CL5=-S031*E031-S033-S033*E033
CL6=-S031*E031-S032*E032
CL7=S032*E031+S031*E032
CL8=S033*E031+S031+S031*E033
CL9=S033*E032+S032+S032*E033
CL10=E031
CL11=E032
CL12=E033
```

FIGURE 2-2. ERROR COEFFICIENT EQUATIONS (cont.)

There are a couple of issues which add complexities to this simple program.

- Usually, angular velocities are not provided as inputs, yet they are required to calculate measured accelerations. Thus an algorithm which can integrate angular accelerations is required.

- The program should have an option of accepting acceleration values either in the form of analytical expressions or in the form of experimental data provided as series of acceleration measurements at definite time intervals. Since the time interval of input values, in the second case, may not coincide with the time step ideally suited for the integration routine, we have to incorporate an interpolation scheme.

The structure of the computer program is shown in Figure 2.3. As can be seen, the user supplies error terms, time step, initial time, final time, and actual acceleration values. If the accelerations are in analytical form, equations of type:

Acceleration 1 = f_1 (time), and
Acceleration 2 = f_2 (time),

are needed. If they are in a time series form, as what experimentally obtained accelerations are likely to be, then the following type of arrays are needed:

Time	acceleration 1	acceleration 2
.	.	.
.	.	.
.	.	.

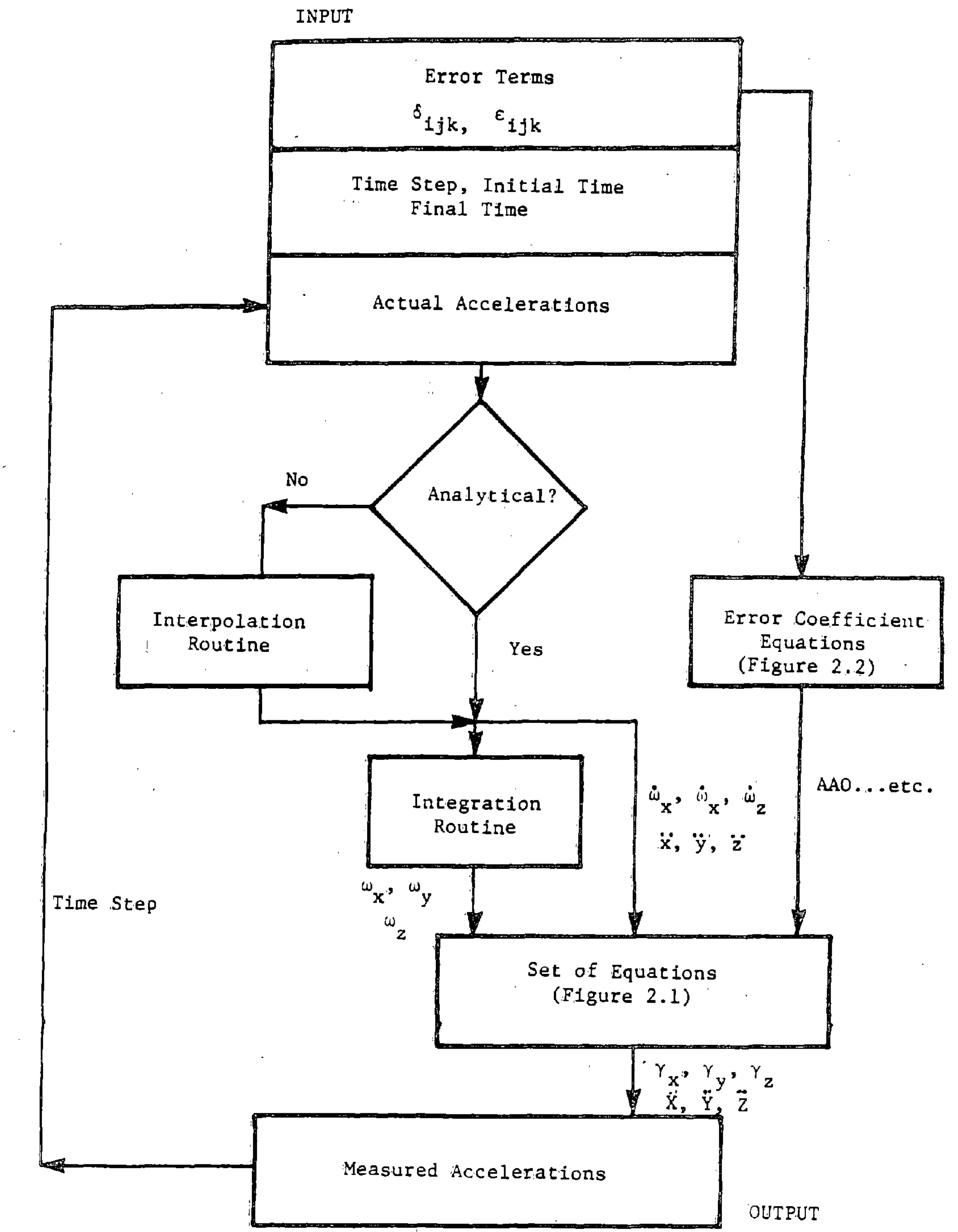


FIGURE 2-3. STRUCTURE OF THE COMPUTER PROGRAM

The integration routine (a fourth order Runge-Kutta routine) integrates the angular acceleration terms to produce angular velocities. The error coefficients are calculated from the error terms using equations given in Figure 2.2. Then, using the set of equations shown in Figure 2.1, we calculate measured accelerations and print them as output.

The actual acceleration values are obtained at every time step, either from the analytical expression or through interpolation of values provided in the array format. The process of integration, calculating the measured accelerations and printing them is also repeated at every time step until the user-specified final time value is reached.

2.3 INPUT FORMAT

A listing of the program is given in Appendix B. This program needs several inputs from the user:

- Error terms δ_{ijk}, ϵ_{ijk}

  These are to be provided in the following format:

  ϵ_{010}, ϵ_{011}, ϵ_{012}, ϵ_{013}, δ_{011}, δ_{012}, δ_{013},
  ϵ_{020}, ...
  ϵ_{030}, ...
  ϵ_{120}, ...
  ϵ_{130}, ...
  ϵ_{210}, ...
  ϵ_{230}, ...
  ϵ_{310}, ...
  ϵ_{320}, ...

- Value R, which represents distance of locations, 1, 2 and 3 from location 0.

- Input Index, IX, which should be set to zero if the acceleration terms are analytical, and to one, if they need to be interpolated.

- Time values: Initial time (STIME), time step (DTIME), Final time (FTIME).

All these values are read by the main program. The information on actual accelerations is obtained by subroutine STEQU (see Appendix B) or by subroutine EXTRAP, depending on whether the accelerations are analytical in form or provided as time series. If they are in analytical form, the expression has to be provided under comment "Analytical Inputs" in subroutine STEQU.

An example of such an expression is:

$\dot{\omega}_x$ = 5 Sin 3t, which becomes

```
WDOTX = 5.0 * SIN (3.0 * TIME)
```

In the other case, IN equals the number of data points in the time series. The series itself is of the format:

Time, $\ddot{x}$, $\ddot{y}$, $\ddot{z}$, $\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$,

.

.

.

2.4 OUTPUT FORMAT

The output of the program is a printout of time, the actual accelerations ($\ddot{x}$, $\ddot{y}$, $\ddot{z}$, $\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$), the actual angular velocities (ω_x, ω_y, ω_z), and the measured accelerations ($\ddot{X}$, $\ddot{Y}$, $\ddot{Z}$, $\dot{\gamma}_x$, $\dot{\gamma}_y$, $\dot{\gamma}_z$) at every time step. By comparing the actual accelerations with the measured ones, we can estimate the effects of a set of error terms on NAP output.

2.5 SUGGESTIONS ON INPUT

There are several points which need to be discussed to facilitate the use of the program.

The time step has to be judiciously selected. A time step (DTIME in the program) which is too large will lead to numerical instability, while one which is too small will cause large numerical errors to accumulate. In general, the time step should be significantly smaller than the smallest period of acceleration inputs. Thus, if the acceleration inputs incorporate a frequency of 100 Hz, the smallest period will be 0.01 second and thus the time step should be, say, 0.001 second.

It is very difficult to handle frequency dependent error terms for any but the unlikely situation of sinusoidal acceleration input. Presumably, one would need to perform a fast fourier transform at every few time steps to determine the dominant frequency in that time span and then adjust the error terms accordingly. The presence of more than one dominant frequency would, however, make even this method ineffective. In such a case, there seems to be no clear cut analytical approach available.

If an error term is dependent on magnitude of acceleration, it should be defined in the main program, just above the set of equations, instead of read from a data file. Thus, if ϵ_{120} is dependent on $\dot{\omega}_x$, then term of the form:

$$\varepsilon_{120} = K\,\dot{\omega}_x \text{ or}$$

```
E120 = AK * WDOTX
```

should be inserted above the set of equations which convert actual accelerations to measured accelerations. Also, each error coefficient term which includes E120 should be transferred from its present position to under this equation for E120, so that it also changes according to acceleration magnitude.

3. METHODOLOGY FOR DETERMINING ERROR COEFFICIENTS

This section describes a methodology developed to determine the error coefficients identified in equations (1) through (6) in Figure 2.1.

This methodology is somewhat different from that proposed by the Central Inertial Guidance Test Facility, of the U.S. Air Force which is discussed in Reference 4. The primary difference is that we treat the NAP as a package and therefore our methodology produces error coefficients corresponding to the package and not error terms for each accelerometer within the package. The methodology discussed in Reference 4 deals with identifying scale factors, null bias, misalignment angle, one-g bias, and temperature sensitivity for each accelerometer in the NAP. Also, the model used in Reference 4 for describing the errors in acceleration measurement is different from that used in Reference 3 on which our methodology is based.

The methodology incorporates six sets of tests, and requires three types of equipment:

1. A linear shaker (for sinusoidal linear input).
2. A rotational shaker (for sinusoidal angular input).
3. A turntable (for constant rotational input).

The tests are summarized in Table 3.1 and described in the following subsection. Some thoughts on fixture design and mounting are provided in subsection 3.2.

TABLE 3-1. TESTS REQUIRED TO DETERMINE ERROR COEFFICIENTS

Test	NAP Orientation	Input	Unknowns Identified	Remarks
1. Steady state	x axis (both ways)	$\ddot{x} = \pm g$	AAO, AA10 BAO, BA10 CAO, CA10 ALO, AL10 BLO, BL10 CLO, CL10	Zero frequency values only
	y axis (both ways)	$\ddot{y} = \pm g$	AA11, BA11, CA11, AL11, BL11, CL11 (plus confirm AAO, BAO, CAO, ALO, BLO, CLO)	
	z axis (both ways)	$\ddot{z} = \pm g$	AA12, BA12, CA12, AL12, BL12, CL12, (plus confirm AAO, BAO, CAO, ALO, BLO, CLO)	
2. Linear Sinusoidal at different frequencies	x axis	$\ddot{x}=A \sin \omega t$	Same as those for Test 1, except for different frequencies	
	y axis	$\ddot{y}=A \sin \omega t$	Same as those for Test 1 except for different frequencies	
	z axis	$\ddot{z}=A \sin \omega t$	Same as those for Test 1, except for different frequencies.	

TABLE 3-1. TESTS REQUIRED TO DETERMINE ERROR COEFFICIENTS (cont.)

Test	NAP Orientation	Input	Unknowns Identified	Remarks
3. Constant angular velocity with aligned axis	x axis	ω_x = const.	AA4, BA4, CA4, AL4, BL4, CL4	Zero frequency values only
	y axis	ω_y = const.	AA5, BA5, CA5, AL5, BL5, CL5	
	y axis	ω_z = const.	AA6, BA6, CA6, AL6, BL6, CL6	Assumes AAO, etc. are known. Otherwise, test at two different angular velocities.
4. Sinusoidal Rotation at different frequencies	x axis	$\omega_x = a \sin \omega t$, $\dot{\omega}_x = a\omega \cos \omega t$	AA1, BA1, CA1, AL1, BL1, CL1 plus unknowns identified in Test 3 for different frequencies (plus confirm AAO...etc. at different frequencies)	
	y axis	$\omega_y = a \sin \omega t$, $\dot{\omega}_y = a\omega \cos \omega t$	AA2, BA2, CA2, AL2, BL2, CL2 plus those in Test 3 for different frequencies	
	z axis	$\omega_z = a \sin \omega t$, $\dot{\omega}_z = a\omega \cos \omega t$	AA3, BA3, CA3, AL3, BL3, CL3 plus those in Test 3 for different frequencies	
5. Constant Angular velocity with Misaligned Axis	Rotation axis perpendicular to x axis, making angle δ with y axis, and passing through origin	$\omega_y = \omega \cos \delta$ $\omega_z = \omega \sin \delta$	AA9, BA9, CA9, AL9, BL9, CL9	If rotation axis does not pass through the origin, add centripetal acceleration. Also, zero frequency values only.

TABLE 3-1. TESTS REQUIRED TO DETERMINE ERROR COEFFICIENTS (cont.)

Test	NAP Orientation	Input	Unknowns Identified	Remarks
5. (Continued)	Rotation axis perpendicular to y axis, making angle δ with z axis, and passing through origin.	$\omega_z = \omega \cos \delta$ $\omega_x = \omega \sin \delta$	AA8, BA8, CA8, AL8, BL8, CL8	
	Rotation axis perpendicular to z axis, making angle δ with x axis, and passing through origin	$\omega_x = \omega \cos \delta$ $\omega_y = \omega \sin \delta$	AA7, BA7, CA7, AL7, BL7, CL7	
6. Sinusoidal Rotation with Misaligned Axis at Different Frequencies	Rotation axis perpendicular to x axis, making angle δ with y axis and passing through origin	$\omega_y = a \sin \omega t \cos \delta$ $\omega_z = a \sin \omega t \sin \delta$ $\dot{\omega}_y = a\omega \cos \omega t \cos \delta$ $\dot{\omega}_z = a\omega \cos \omega t \sin \delta$	Same as in Test 5 except for different frequencies	If rotation axis does not pass through the origin, add centripetal acceleration

TABLE 3-1. TESTS REQUIRED TO DETERMINE ERROR COEFFICIENTS (cont.)

Test	NAP Orientation	Input	Unknowns Identified	Remarks
6. (continued)	Rotation axis perpendicular to y axis, making angle δ with z axis and passing through origin	$\omega_z = a \sin \omega t \cos \delta$ $\omega_x = a \sin \omega t \sin \delta$ $\dot{\omega}_z = a\omega \cos \omega t \cos \delta$ $\dot{\omega}_x = a\omega \cos \omega t \sin \delta$	Same as in Test 5 except for different frequencies.	
	Rotation axis perpendicular to z axis, making angle δ with x axis and passing through origin	$\omega_x = a \sin \omega t \cos \delta$ $\omega_y = a \sin \omega t \sin \delta$ $\dot{\omega}_x = a\omega \cos \omega t \cos \delta$ $\dot{\omega}_y = a\omega \cos \omega t \sin \delta$	Same as in Test 5 except for different frequencies.	

3.1 DESCRIPTION OF TESTS

Test 1: Steady State

(a) Input to x axis

By placing the NAP with the x axis in the vertical direction, we can provide $\ddot{x} = + g$ input. All other input terms will be zero.

Then from Eqn. (1)

$$\dot{\gamma}_x = AAO + AA10\ g$$

and by inverting the NAP, $\ddot{x} = - g$ will be applied.

Then,

$$\dot{\gamma}_x = AAO - AA10\ g$$

We can solve the above equations simultaneously to get AAO and AA10. Similarly, from equations (2), (3), (5), and (6), we can obtain BAO, BA10, CAO, CA10, BLO, BL10, CLO and CL10.

To obtain ALO and AL10, we need to solve the following two equations:

for

$\ddot{x} = + g$

$$\ddot{X} = ALO + (1 + AL10)\ g$$

for $\ddot{x} = -g$

$$\ddot{X} = ALO - (1 + AL10)\ g.$$

(b) Input to y axis

For input to y axis, $\ddot{y} = + g$ or $\ddot{y} = -g$.

Then solving equations (1) through (6), we can get

AA11, BA11, CA11, AL11, BL11, and CL11, plus we can confirm values of AA0, BA0, CA0, AL0, BL0 and CL0 obtained above.

(c) Input to z axis

In a similar manner, we can get AA12, BA12, CA12, AL12, BL12, and CL12, plus we can confirm values of AA0...etc.

Note: In the rest of the tests, acceleration due to gravity is not taken into account because:

(i) the acceleration input may be much greater than g, therefore g can be neglected, and
(ii) we are not sure of the direction of the input relative to the g vector.

Here we should note that similar difficulties exist in incorporating the g term in the computer program described in Chapter 2. Here, the direction of g vector with respect to the NAP axes will generally change as a function of time, as the package rotates. Thus, we will need to keep track of rotational positon of the package with respect to the g vector and assign components of g to each acceleration measurement depending on the position of individual accelerometers. While this is not a very difficult task, it may create unnecessary complications in the program, which will continue providing accurate results as long as the measured accleration levels are substantially higher than g.

Test 2: Linear Sinusoidal at Different Frequencies

(a) Input to x axis.

If the NAP is mounted with x axis receiving linear sinusoidal inputs, of the form $\ddot{x}$ = A sin ωt (the other inputs are zero), then, from equation (1):

$$\dot{\gamma}_x = AAO + AA10(A \sin \omega t)$$

The output, $\dot{\gamma}_x$, will be a sinusoidal signal, with an offset of AAO and amplitude (peak to peak) of 2 AA10(A). Thus, both AAO and AA10 can be obtained. This is the same as for Test 1, except now we can change the frequency, ω, and plot AAO and AA10 values at different frequencies.

In a similar manner, we can obtain BAO, BA10, CAO, CA10, BLO, BL10, CLO, and CL10, each as function of frequency.

To obtain ALO and AL10, we need to solve a slightly different equation:

$$\ddot{X} = ALO + (1 + AL10) A \sin \omega t.$$

Here, the peak to peak amplitude of the output sine wave will be 2(1+AL10)A. This will give a value of AL10 as function of frequency.

(b) Input to y axis

In a similar manner, by providing sinusoidal input to the y axis, we can get AA11, BA11, CA11, AL11, BL11, and CL11, each as a function of frequency. In addition, we can confirm values of AA0...etc.

(c) Input to z axis

Finally, by mounting the NAP on the z axis, we can get AA12, BA12, CA12, AL12, BL12, and CL12 as functions of frequency. Also, values of AA0...etc. can be further confirmed.

Test 3: Constant Angular Velocity with Aligned Axis

(a) Input to x axis

By rotating the NAP mounted along its x axis on a turntable, we provide it the following input:

$$\omega_x = \text{constant}.$$

All other inputs will be zero. Then, from equation (1):

$$\dot{Y}_x = AA0 + AA4\ \omega_x^2.$$

Since we know all the terms, except AA4, we can find AA4. (If AA0 is not known, test at two different angular velocities and solve two simultaneous equations.)

Similarly, BA4, CA4, AL4, BL4, and CL4 can be found using equations (2) through (6).

(b) Input to y axis

Similarly, by mounting the NAP along its y axis and rotating it at a constant angular velocity, we can find AA5, BA5, CA5, AL5, BL5 and CL5.

(c) Input to z axis

Finally, by rotating the NAP on its z axis, we can find AA6, BA6, CA6, AL6, BL6, and CL6.

Test 4: Sinusoidal Rotation at Different Frequencies

(a) Input to x axis

By rotating the NAP in a sinusoidal manner on its x axis using a rotational shaker, we provide it two inputs:

$$\omega_x = a \sin \omega t.$$
$$\dot{\omega}_x = a\omega \cos \omega t.$$

All other inputs will be zero. Substituting these inputs in equation (1), we get:

$$\dot{\gamma}_x = AA0 + (1+AA1)\ a\omega \cos \omega t + AA4\ a^2 \sin^2 \omega t.$$

However, $\sin^2 \omega t = 1/2-1/2 \cos 2\omega t$.

So,

$$\dot{\gamma}_x = AA0 + (1 + AA1)\ a\omega \cos \omega t + AA4\ a^2/2 - AA4(a^2/2)\cos 2\omega t.$$

Thus, $\dot{\gamma}_x$ will have three components:

Constant bias $= AA0 + a^2/2$

Sinewave at ω frequency $= (1 + AA1)\, a\, \omega \cos \omega t$.

Sinewave at 2ω frequency $= - AA4(a^2/2)\cos 2\omega t$.

By using simple filtering techniques, it is easy to separate these three components and then obtain values of AA4, AA0, and AA1. Then, by varying frequency we can obtain these values at different frequencies. Since the values of AA0 at different frequencies are already known (from Test 2), this test would serve to confirm those values. The value of AA4 at zero frequency is known from Test 3. Thus, the results of this test will indicate frequency dependence of AA4. The error coefficient which has not been obtained previously is AA1. The value of AA1 can be obtained in this test across the frequency range. The value at zero frequency can however, be obtained only through extrapolation.

Using equations (2) through (6), we can get values of BA1, CA1, AL1, BL1, and CL1, plus BA4, CA4, AL4, BL4 and CL4, all at different frequencies. In addition we will be able to confirm the values of BA0...etc., at different frequencies.

(b) Input to y axis

Similarly, by rotating the NAP in a sinusoidal manner on its y axis, we provide it with two inputs:

$\omega_y = a \sin \omega t$.

$\dot{\omega}_y = a\, \omega \cos \omega t$.

Then, using a procedure similar to that described earlier, we can get values of AA2, BA2, CA2, AL2, BL2, and CL2, plus AA5, BA5, CA5, AL5, BL5, and CL5. These values can be obtained for different frequencies. This test would give yet another confirmation to the values of AA0..etc.

(c) Input to z axis

Finally, we can obtain values of AA3, BA3, CA3, AL3, BL3 and CL3, plus, AA6, BA6, CA6, AL6, BL6, and CL6 (for different frequencies) by rotating the NAP in a sinusoidal manner on its z axis.

Test 5: Constant Angular Velocity with Misaligned Axis

(a) Simultaneous Inputs to y and z axis

By mounting the NAP on the turntable, so that the rotation axis is perpendicular to the NAP's x axis, while making angle δ with its y axis, we can provide simultaneous rotation along y and z axes. This is shown in Figure 3.1. In addition, we should make the rotation axis pass through the origin of the NAP, otherwise we will need to account for centripetal acceleration, which will give a non-zero value to $\ddot{x}$.

Assuming that the axis does pass through the origin, the two inputs to the NAP will be:

$\omega_y = \omega \cos \delta$

$\omega_z = \omega \sin \delta$ All other inputs will be zero.

Then, from equation (1),

$$\dot{\gamma}_x = AA0 + AA5\ \omega^2 \cos^2 \delta + AA6\ \omega^2 \sin^2 \delta + AA9\ \omega^2 \sin \delta \cos \delta$$

Knowing, AA0, AA5, AA6, ω, and δ, we can then find AA9. Similarly, using equations (2) through (6), we can determine BA9, CA9, AL9, BL9, and CL9.

(b) Simultaneous Inputs to x and z axis

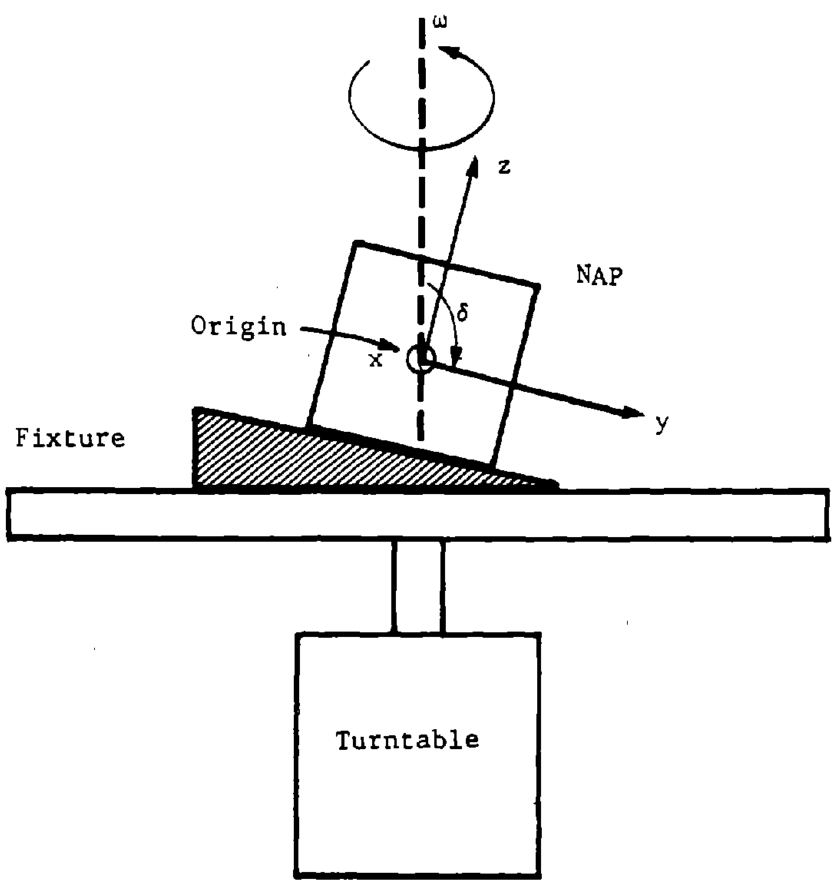


FIGURE 3-1. MOUNTING FOR TESTS 5 AND 6: MISALIGNED AXIS

Similarly, by mounting the NAP so that the rotation axis is perpendicular to y axis, while making angle δ with z axis, we can obtain AA8, BA8, CA8, AL8, BL8, and CL8.

(c) Simultaneous inputs to x and y axes

Finally, by mounting the NAP so that the rotation axis is perpendicular to z axis while making angle δ with x axis, we can measure AA7, BA7, CA7, AL7, BL7 and CL7.

Test 6: Sinusoidal Rotation with Misaligned Axis at Different Frequencies

One final test set needs to be done if we want to determine the error coefficients found in Test 5, for different frequencies. In this test, the NAP is to be mounted in a manner similar to that for Test 5, except, instead of constant angular velocity, we will provide sinusoidal rotation using the rotational shaker.

(a) Simultaneous Inputs to y and z axis

The inputs to the NAP will be:

$$\omega_y = a \sin \omega t \cos \delta$$
$$\omega_z = a \sin \omega t \sin \delta$$
$$\dot{\omega}_y = a \omega \cos \omega t \cos \delta$$
$$\dot{\omega}_z = a \omega \cos \omega t \sin \delta$$

All other inputs will be zero.

The way of analyzing the output signal will be similar to that discussed under Test 4. For example $\dot{\gamma}_x$ will include constant terms, those at frequency ω and those at frequency 2ω. Without going into details, the 2ω signal will be:

$$- a^2/2 \cos 2\omega t\,(AA5 \cos^2 \delta + AA6 \sin^2 \delta + AA9 \sin \delta \cos \delta).$$

All the terms in this equation except AA9 are known as functions of frequency. Thus, the value of AA9 can be determined. Similarly, using equations (2) through (6), we can find values of BA9, CA9, AL9, BL9, and CL9, as functions of frequency.

(b) Simultaneous Inputs to x and z Axes

Similarly, the values of AA8, BA8, AL8, BL8, and CL8 can be found as functions of frequency by providing sinusoidal rotation to the NAP mounted with its rotation axis perpendicular to the y axis, making angle δ with the z axis, and passing through the origin.

(c) Simultaneous Inputs to x and y Axes

Finally, the values of AA7, BA7, CA7, AL7, BL7 and CL7 can be found as functions of frequency by mounting the NAP with its rotation axis perpendicular to the z axis, making angle δ with the x axis, and passing through the origin.

3.2 FIXTURE DESIGN AND MOUNTING

While performing tests outlined in Table 3.1, extreme care will have to be taken in designing an appropriate fixture and mounting the NAP on the fixture, otherwise it will be impossible to differentiate between errors which are inherent in the package being calibrated and those which are due to inaccuracies in mounting.

As we understand, a NAP can be procured as a box which can be mounted inside a dummy's head. Alternatively, the inside surfaces of the dummy head can be machined to mount nine accelerometers in appropriate locations to create a NAP.

In general, it will be easier to calibrate the NAP which comes as a separable box than the one created inside a dummy's head and thereby not separable.

The NAP of either type will have an origin and three axes associated with the package. The linear accelerations we get out of the package will be referenced to the origin of the package and the angular accelerations will be around the three axes thus defined. Any deviation of an individual accelerometer from its theoretical locations along one of the axes will give rise to an error term which will make one or more terms in the equation set non-zero. Some such errors are bound to be there since linear accelerometers have a finite volume and thus more than one accelerometer cannot be placed exactly at the same location. Similarly, any difference between actual and theoretical angular positions will give rise to errors. Finally, the errors inherent in each accelerometer will give rise to errors in the NAP. The calibration scheme provided in this report will identify each error coefficient and provide the basis for corrections of the measurements.

The above discussion provides a framework for understanding the need for mounting accurately. Effectively, mounting will define an origin and a set of axes which, if the mounting is accurate, will coincide with the package's origin and set of axes. The error coefficients produced by a calibration test will refer to the set of axes defined by mounting and <u>not</u> to those defined by the package. This is fine, as long as the package need be mounted only once and all tests can be performed without any changes in the mounting. In that case, the output of the package, corrected by the error coefficients (found from calibration tests) will give accurate linear and angular accelerations, only they will refer to the axes and origin defined by mounting, and not those defined by the package. As long as the two axes sets are reasonably close to each other, this will be acceptable.

The problem arises when the package needs to be mounted more than one time. In that case a new set of axes may be defined each time it is mounted. The error coefficients obtained then will be inconsistent and there will be no way of obtaining accurate linear and angular accelerations--not for any set of axes. This is the situation with the test plan we have proposed. Unfortunately, there is only one recourse--mount the package as accurately as possible so that all the error coefficients refer to one set of axes, even if it does not coincide exactly with the package-defined set of axes.

For an NAP enclosed in a cube or a rectangular box with accurately machined sides, it will be possible to design fixtures to mount it on the test tables. However, in case of the package machined inside the dummy's head, and thereby inseparable, it will be extremely difficult to mount the head on the tables accurately, unless the head is rigidly packaged in a cube or rectangular box, or a rigid set of axes is fixed on the head. Neither option is easily implementable. Another advantage of the separable package is: once it is calibrated, it can be used in different dummies without requiring recalibration.

4. SUMMARY AND RECOMMENDATIONS

This report discusses two specific issues related to a 3-2-2-2 configuration NAP:

- A computer program which can be used to perform error sensitivity analyses of the package.

- A test procedure which can be used for determining the error coefficients of the package and calibrating it.

The test procedure described requires a linear shaker, a rotational shaker, and a turntable. Six sets of tests are required to obtain every error coefficient. Fewer tests will be needed if the error coefficients are not frequency dependent.

The principal recommendations for further work are:

- The computer program should be used to determine the effects of each error term on the measurement errors.

- A procedure to convert measured accelerations to actual accelerations, knowing error coefficients, should be developed. This requires essentially inverting the set of equations provided in Reference 3 and used in this report.

- Fixtures for mounting the NAP on the test shakers and turntable have to be designed and fabricated, keeping in mind the tolerance levels required in positioning the package so that meaningful values of error coefficients can be derived.

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      DIMENSION Y(3),DY(3)

      DIMENSION A(1000),B(1000),C(1000),D(1000),E(1000),F(1000)
      COMMON/STATE/Y
      COMMON/DYNM/TIME,DTIME,FTIME,DY,IX,IJ,IL,IN
      COMMON/ACCL/RX,RY,RZ,WDOTX,WDOTY,WDOTZ
      COMMON/EXT/A,B,C,D,E,F

C     INPUT FILE
      OPEN(UNIT=1,FILE='IN',STATUS='OLD')
C     OUTPUT FILE
      OPEN(UNIT=2,FILE='OUTDATA',STATUS='NEW')

      READ(1,*) STIME,DTIME,FTIME,R
      WRITE(2,1)
1     FORMAT(1X,'STIME,DTIME,FTIME,R   ')
      WRITE(2,100) STIME,DTIME,FTIME,R
      WRITE(6,4)
 4    FORMAT(1X,'INPUT INDEX ',$)
      READ(5,*)   IX
      IF (IX.NE.1) GO TO 23
2     WRITE (6,113)
113   format(1x,'input number of observations    ',$)
      read (5,*) in
23    continue
  100 FORMAT(1X,9(F9.4,1X))
C
C   ERROR TERMS
C
      READ(1,*) E010,E011,E012,E013,S011,S012,S013
      READ(1,*) E020,E021,E022,E023,S021,S022,S023
      READ(1,*) E030,E031,E032,E033,S031,S032,S033
      READ(1,*) E120,E121,E122,E123,S121,S122,S123
      READ(1,*) E130,E131,E132,E133,S131,S132,S133
      READ(1,*) E210,E211,E212,E213,S211,S212,S213
      READ(1,*) E230,E231,E232,E233,S231,S232,S233
      READ(1,*) E310,E311,E312,E313,S311,S312,S313
      READ(1,*) E320,E321,E322,E323,S321,S322,S323

      WRITE(2,100) E010,E011,E012,E013,S011,S012,S013
      WRITE(2,100) E020,E021,E022,E023,S021,S022,S023
      WRITE(2,100) E030,E031,E032,E033,S031,S032,S033
      WRITE(2,100) E120,E121,E122,E123,S121,S122,S123
      WRITE(2,100) E130,E131,E132,E133,S131,S132,S133
      WRITE(2,100) E210,E211,E212,E213,S211,S212,S213
      WRITE(2,100) E230,E231,E232,E233,S231,S232,S233
      WRITE(2,100) E310,E311,E312,E313,S311,S312,S313
      WRITE(2,100) E320,E321,E322,E323,S321,S322,S323
```

```
            WRITE(2,105)
            WRITE(2,101)
            WRITE(2,102)
            WRITE(2,103)
            WRITE(2,104)
  105       FORMAT(1X,' ')
  101       FORMAT(1X,'TIME')
  102       FORMAT(1X,'  WX           WY           WZ')
  103       FORMAT(1X,'  RX           RY           RZ        WDOTX        WDOTY
     &      WDOTZ')
  104       FORMAT(1X,'  GDOTX        GDOTY        GDOTZ     GDOTX        GDOTY
     &      GDOTZ')
C
C   CALCULATE THE A,B,C VALUES FOR ANGULAR ACCELERATION
C
            AA0=(E230-E030-E320+E020)/(2*R)
            AA1=(E233+E322)/2+(S232-S032+S323-S023)/(2*R)
            AA2=-E321/2+((S231-S031)/(2*R))
            AA3=-E231/2-((S321-S021)/(2*R))
            AA4=(E323-E232)/2+((S322-S022+S033-S233)/(2*R))
            AA5=E323/2+(S033-S233)/(2*R)
            AA6=-E232/2+(S322-S022)/(2*R)
            AA7=E231/2+(S021-S321)/(2*R)
            AA8=-E321/2+(S231-S031)/(2*R)
            AA9=(E233-E322)/2+(S232-S032-S323+S023)/(2*R)
            AA10=(E231-E031-E321+E021)/(2*R)
            AA11=(E232-E032-E322+E022)/(2*R)
            AA12=(E233-E033-E323+E023)/(2*R)
C
            BA0=(E310-E010-E130+E030)/(2*R)
            BA1=E312/2+(S032-S132)/(2*R)
            BA2=(E311+E133)/2+(S313-S013+S131-S031)/(2*R)
            BA3=-E132/2+(S012-S312)/(2*R)
            BA4=-E313/2+(S133-S033)/(2*R)
            BA5=(E131-E313)/2+(S011-S311+S133-S033)/(2*R)
            BA6=E131/2+(S011-S311)/(2*R)
            BA7=-E132/2+(S312-S012)/(2*R)
            BA8=(E311-E133)/2+(S313-S013-S131+S031)/(2*R)
            BA9=E312/2+(S032-S132)/(2*R)
            BA10=(E311-E011-E131+E031)/(2*R)
            BA11=(E312-E012-E132+E032)/(2*R)
            BA12=(E313-E013-E133+E033)/(2*R)
C
            CA0=(E120-E010-E210+E010)/(2*R)
            CA1=-E213/2+(S023-S123)/(2*R)
            CA2=-E123/2+(S013-S213)/(2*R)
            CA3=(E122+E211)/2+(S121-S021+S212-S012)/(2*R)
            CA4=E212/2+(S022-S122)/(2*R)
            CA5=-E121/2+(S211-S011)/(2*R)
            CA6=(E212-E121)/2+(S022-S122+S211-S011)/(2*R)
            CA7=(E122-E211)/2+(S121-S021-S212+S012)/(2*R)
            CA8=E123/2+(S013-S213)/(2*R)
            CA9=-E213/2+(S123-S023)/(2*R)
            CA10=(E121-E021-E211+E011)/(2*R)
            CA11=(E122-E022-E212+E012)/(2*R)
```

```
C
C   CALCULATE THE A,B,C VALUES FOR LINEAR ACCELERATION
C            AL0-AL12,BL0-BL12,CL0-CL12
C
          AL0=E010
          AL1=-S013*E012+S012*E013
          AL2=S013+S013*E011-S011*E013
          AL3=-S012-S012*E011+S011*E012
          AL4=-S012*E012-S013*E013
          AL5=-S011-S011*E011-S013*E013
          AL6=-S011-S011*E011-S012*E012
          AL7=S012+S012*E011+S011*E012
          AL8=S013+S013*E011+S011*E013
          AL9=S013*E012+S012*E013
          AL10=E011
          AL11=E012
          AL12=E013
C
          BL0=E020
          BL1=-S023-S023*E022+S022*E023
          BL2=S023*E021-S021*E023
          BL3=-S022*E021+S021+S021*E022
          BL4=-S022-S022*E022-S023*E023
          BL5=-S021*E021-S023*E023
          BL6=-S021*E021-S022-S022*E022
          BL7=S022*E021+S021+S021*E022
          BL8=S023*E021+S021*E023
          BL9=S023+S023*E022+S022*E023
          BL10=E021
          BL11=E022
          BL12=E023
C
          CL0=E030
          CL1=-S033*E032+S032+S032*E033
          CL2=S033*E031-S031-S031*E033
          CL3=-S032*E031+S031*E032
          CL4=-S032*E032-S033-S033*E033
          CL5=-S031*E031-S033-S033*E033
          CL6=-S031*E031-S032*E032
          CL7=S032*E031+S031*E032
          CL8=S033*E031+S031+S031*E033
          CL9=S033*E032+S032+S032*E033
          CL10=E031
          CL11=E032
          CL12=E033
          CA12=(E123-E023-E213+E013)/(2*R)
```

```
C
C   INITIALIZE ANGULAR VELOCITY TERMS, TIME
C
      Y(1)=0.0
      Y(2)=0.0
      Y(3)=0.0
      TIME=STIME
      IJ=2
      IL=0
      CALL STEQU
      GO TO 11
10    IF(TIME.GT.FTIME) GO TO 40

      CALL RKDIF
11    WX=Y(1)
      WY=Y(2)
      WZ=Y(3)
C
C  CALCULATE THE ESTIMATED ANGULAR ACCELERATIONS
C
      GDOTX=WDOTX+AA0+AA1*WDOTX+AA2*WDOTY+AA3*WDOTZ+
     2    AA4*WX**2+AA5*WY**2+AA6*WZ**2+AA7*WX*WY+
     3    AA8*WX*WZ+AA9*WY*WZ+AA10*RX+AA11*RY+AA12*RZ
      GDOTY=WDOTY+BA0+BA1*WDOTX+BA2*WDOTY+BA3*WDOTZ+
     2    BA4*WX**2+BA5*WY**2+BA6*WZ**2+BA7*WX*WY+
     3    BA8*WX*WZ+BA9*WY*WZ+BA10*RX+BA11*RY+BA12*RZ
      GDOTZ=WDOTZ+CA0+CA1*WDOTX+CA2*WDOTY+CA3*WDOTZ+
     2    CA4*WX**2+CA5*WY**2+CA6*WZ**2+CA7*WX*WY+
     3    CA8*WX*WZ+CA9*WY*WZ+CA10*RX+CA11*RY+CA12*RZ
C
C  CALCULATE THE ESTIMATED LINEAR ACCELERATIONS
C
      GDOTX=RX+AL0+AL1*WDOTX+AL2*WDOTY+AL3*WDOTZ+
     2    AL4*WX**2+AL5*WY**2+AL6*WZ**2+AL7*WX*WY+
     3    AL8*WX*WZ+AL9*WY*WZ+AL10*RX+AL11*RY+AL12*RZ
      GDOTY=RY+BL0+BL1*WDOTX+BL2*WDOTY+BL3*WDOTZ+
     2    BL4*WX**2+BL5*WY**2+BL6*WZ**2+BL7*WX*WY+
     3    BL8*WX*WZ+BL9*WY*WZ+BL10*RX+BL11*RY+BL12*RZ
      GDOTZ=RZ+CL0+CL1*WDOTX+CL2*WDOTY+CL3*WDOTZ+
     2    CL4*WX**2+CL5*WY**2+CL6*WZ**2+CL7*WX*WY+
     3    CL8*WX*WZ+CL9*WY*WZ+CL10*RX+CL11*RY+CL12*RZ
      WRITE(2,100)TIME
      WRITE(2,100)WX,WY,WZ
      WRITE(2,100)RX,RY,RZ,WDOTX,WDOTY,WDOTZ
      WRITE(2,100)GDOTX,GDOTY,GDOTZ,GDOTX,GDOTY,GDOTZ
      GOTO 10
40    END
```

```
C       ******************************************************************
C
        SUBROUTINE EXTRAP
        DIMENSION DY(3),T(1000),A(1000),B(1000),C(1000),D(1000),E(1000),
     &  F(1000)
        COMMON/ACCL/RX,RY,RZ,WDOTX,WDOTY,WDOTZ
        COMMON/DYNM/TIME,DTIME,FTIME,DY,IX,IJ,IL,IN
        COMMON/EXT/A,B,C,D,E,F
C       LOAD EXPERIMENTAL DATA
        OPEN (UNIT=3,FILE='EXPERVAL',STATUS = 'OLD')
        IF (il.eq.1)  GO TO 23
        DO 25 II = 1,IN
25      READ(3,*) T(II),A(II),B(II),C(II),D(II),E(II),F(II)
        IL=1
23      IF (T(IJ).GT.TIME) GO TO 21
        IJ=IJ+1
        GO TO 23
21      IK = IJ - 1
        DTX = (TIME-T(IK))/(T(IJ)-T(IK))
        RX=A(IK)+(A(IJ)-A(IK))*DTx
        RY=B(IK)+(B(IJ)-B(IK))*DTX
        RZ=C(IK)+(C(IJ)-C(IK))*DTX
        WDOTX=D(IK)+(D(IJ)-D(IK))*DTX
        WDOTY=E(IK)+(E(IJ)-E(IK))*DTX
        WDOTZ=F(IK)+(F(IJ)-F(IK))*DTX

        RETURN
        END
```

```
C************************************************************
C************************************************************

      SUBROUTINE STEQU
      DIMENSION Y(3),DY(3)
      COMMON/STATE/Y
      COMMON/DYNM/TIME,DTIME,FTIME,DY,IX,IJ,IL,IN
      COMMON/ACCL/RX,RY,RZ,WDOTX,WDOTY,WDOTZ
C
      IF (IX.EQ.1) GO TO 17
C
C     ANALYTICAL INPUTS
C
      WDOTX=5*SIN(31.4*TIME)
      WDOTY=0.0
      WDOTZ = 0.0
      RX=0.0
      RY=0.0
      RZ=0.0
      GOTO 12

17    CONTINUE
C     EXPERIMENTAL INPUTS
      CALL EXTRAP
12    DY(1)=WDOTX
      DY(2)=WDOTY
      DY(3)=WDOTZ
      RETURN
      END
```

```
C*************************************************************
C*************************************************************
      SUBROUTINE RKDIF
C  FOURTH ORDER RUNGE KUTTA NUMERICAL INTEGRATION SUBROUTINE

      DIMENSION Y(3),SY(3),YO(3),Y1(3),Y2(3),DY(3)
      COMMON/DYNM/TIME,DTIME,FTIME,DY,IX,IJ,IL,IN
      COMMON/STATE/Y
C
      H=DTIME/2
      DO  I=1,3
        SY(I)=Y(I)
        YO(I)=DY(I)
        Y(I)=H*DY(I)+Y(I)
      END DO

      TIME=TIME+H
      CALL STEQU
      DO I=1,3
        Y1(I)=DY(I)
        Y(I)=SY(I)+H*DY(I)
        Y2(I)=DY(I)
        Y(I)=SY(I)+DTIME*DY(I)
      END DO

      TIME=TIME+H
      H=H/3.0
      DO  I=1,3
        PRT1=2.0*(Y1(I)+Y2(I))
        PRT2=YO(I)+DY(I)
        Y(I)=SY(I)+H*PRT1+H*PRT2
      END DO

      CALL STEQU
      RETURN
      END
```

APPENDIX C

REVISED COMPUTER PROGRAM FOR THE LABORATORY MODEL

```
************************************************************************************
*
*       NAPLABG.FOR
*
*       PROGRAM TO CALCULATE THE ESTIMATED LINEAR AND ANGULAR
*       ACCELERATIONS OF A NINE-ACCELEROMETER HEAD IMPACT MEASUREMENT
*       SYSTEM WITH A 3-2-2-2 CONFIGURATION.
*
*       YOU ARE REQUIRED TO INPUT AN ARRAY FOR THE ACCELEROMETER AND
*       GEOMETRY ERROR TERMS.  YOU ARE ALSO REQUIRED TO INPUT A DATA
*       ARRAY FOR AN EXPERIMENTAL INPUT PULSE OR CHOOSE THE CHARACTER-
*       ISTICS OF AN ANALYTICAL PULSE.  IF YOU INPUT AN EXPERIMENTAL
*       PULSE, THE LINEAR ACCELERATION TERMS SHOULD BE IN INCHES/S*S
*       AND THE ANGULAR ACCELERATION TERMS IN RAD/S*S.
*
*       THE OUTPUT WILL BE IN G-FORMAT AND CONSIST OF THE FOLLOWING
*       PARAMETERS:
*
*       STIME = START TIME (SECONDS)
*       DTIME = TIME PER SAMPLE (SECONDS)
*       FTIME = FINISH TIME (SECONDS)
*       TIME = TIME (SECONDS)
*       WX = ESTIMATED ANGULAR VELOCITY ABOUT THE X-AXIS (R/S)
*       WY = ESTIMATED ANGULAR VELOCITY ABOUT THE Y-AXIS (R/S)
*       WZ = ESTIMATED ANGULAR VELOCITY ABOUT THE Z-AXIS (R/S)
*       RX = ACTUAL LINEAR ACCELERATION ALONG THE X-AXIS (G'S)
*       RY = ACTUAL LINEAR ACCELERATION ALONG THE Y-AXIS (G'S)
*       RZ = ACTUAL LINEAR ACCELERATION ALONG THE Z-AXIS (G'S)
*       QDOTX = ESTIMATED LINEAR ACCELERATION ALONG THE X-AXIS (G'S)       *
*       QDOTY = ESTIMATED LINEAR ACCELERATION ALONG THE Y-AXIS (G'S)       *
*       QDOTZ = ESTIMATED LINEAR ACCELERATION ALONG THE Z-AXIS (G'S)       *
*       WDOTX = ACTUAL ANGULAR ACCELERATION ABOUT THE X-AXIS (R/(S*S))     *
*       WDOTY = ACTUAL ANGULAR ACCELERATION ABOUT THE Y-AXIS (R/(S*S))     *
*       WDOTZ = ACTUAL ANGULAR ACCELERATION ABOUT THE Z-AXIS (R/(S*S))     *
*       GDOTX = ESTIMATED ANGULAR ACCELERATION ABOUT THE X-AXIS (R/(S*S))  *
*       GDOTY = ESTIMATED ANGULAR ACCELERATION ABOUT THE Y-AXIS (R/(S*S))  *
*       GDOTZ = ESTIMATED ANGULAR ACCELERATION ABOUT THE Z-AXIS (R/(S*S))  *
*                                                                          *
************************************************************************************

        DIMENSION Y(3),DY(3)
        DIMENSION T(5000),A(5000),B(5000),C(5000),D(5000),E(5000),F(5000)

        CHARACTER*16 ERFIL,OUTFIL,ANS*1,INFIL1

        COMMON/STATE/Y
        COMMON/DYNM/TIME,DTIME,FTIME,DY,ANS,IJ,IL,NS
        COMMON/ACCL/RX,RY,RZ,WDOTX,WDOTY,WDOTZ,INMAGX,INMAGY,INMAGZ,
     *  DURAX,DURAY,DURAZ,RMAGX,RMAGY,RMAGZ,DURRX,DURRY,DURRZ
        COMMON/EXT/T,A,B,C,D,E,F

C       ERROR FILE
        TYPE 9
    9   FORMAT(/////)
        TYPE 1
    1   FORMAT(1X,'            ******************************************
     &****************')
```

```
      TYPE 2
 2    FORMAT(1X,'        *
 &                *')
      TYPE 3
 3    FORMAT(1X,'        *                    NOTICE
 &                *')
      TYPE 2
      TYPE 4
 4    FORMAT(1X,'        *  This program is disseminated under the
 & sponsorship      *')
      TYPE 5
 5    FORMAT(1X,'        *  of the Transportation Systems Center in the
 & interest   *')
      TYPE 6
 6    FORMAT(1X,'        *  of information exchange. The United States
 & Government  *')
      TYPE 7
 7    FORMAT(1X,'        *  assumes no liability for its contents or
 & use thereof.  *')
      TYPE 2
      TYPE 1
      TYPE 9

      TYPE 10
 10   FORMAT(/,1X,'ENTER NAME OF ERROR FILE DESIRED.')
      ACCEPT 20,ERFIL
 20   FORMAT(A16)
      OPEN(UNIT=1,FILE=ERFIL,STATUS='OLD')
 30   TYPE 40
 40   FORMAT(/,1X,'WILL INPUT PULSE DATA BE EXPERIMENTAL
   * OR ANALYTICAL? (E OR A)')
      ACCEPT 20,ANS
      IF (ANS.EQ.'A') GO TO 50
      IF (ANS.EQ.'E') GO TO 280
      GO TO 30
 50   TYPE 60
 60   FORMAT(/,1X,'ENTER NUMBER OF OBSERVATIONS (SAMPLES)')
      ACCEPT 70,NS
 70   FORMAT(I)
      TYPE 80
 80   FORMAT(/,1X,'ENTER PULSE START AND FINISH TIMES IN SECONDS (2F)')
      ACCEPT *,STIME,FTIME
      TYPE 90
 90   FORMAT(/,1X,'ALL ANALYTICAL INPUTS WILL BE POSITIVE
   * HALF SINE PULSES.')
      TYPE 100
100   FORMAT(/,1X,'ENTER PEAK MAGNITUDE(RAD/(S*S)) AND')
      TYPE 110
110   FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR ANGULAR')
      TYPE 120
120   FORMAT(1X,'ACCELERATION ABOUT THE X-AXIS. (2F)')
      ACCEPT *,INMAGX,DURAX
      IF (INMAGX.EQ.0.)DURAX=0.1      !AVOID DIVIDE BY ZERO
      TYPE 130
130   FORMAT(/,1X,'ENTER PEAK MAGNITUDE(RAD/(S*S)) AND')
      TYPE 140
140   FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR ANGULAR')
      TYPE 150
150   FORMAT(1X,'ACCELERATION ABOUT THE Y-AXIS. (2F)')
      ACCEPT *,INMAGY,DURAY
```

```
      IF (INMAGY.EQ.0.)DURAY=0.1      !AVOID DIVIDE BY ZERO
      TYPE 160
160   FORMAT(/,1X,'ENTER PEAK MAGNITUDE(RAD/(S*S)) AND')
      TYPE 170
170   FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR ANGULAR')
      TYPE 180
180   FORMAT(1X,'ACCELERATION ABOUT THE Z-AXIS. (2F)')
      ACCEPT *,INMAGZ,DURAZ
      IF (INMAGZ.EQ.0.)DURAZ=0.1      !AVOID DIVIDE BY ZERO
      TYPE 190
190   FORMAT(/,1X,'ENTER PEAK MAGNITUDE(G''S) AND')
      TYPE 200
200   FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR LINEAR')
      TYPE 210
210   FORMAT(1X,'ACCELERATION ALONG THE X-AXIS. (2F)')
      ACCEPT *,RMAGX,DURRX
      IF (RMAGX.EQ.0.)DURRX=0.1      !AVOID DIVIDE BY ZERO
      TYPE 220
220   FORMAT(/,1X,'ENTER PEAK MAGNITUDE(G''S) AND')
      TYPE 230
230   FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR LINEAR')
      TYPE 240
240   FORMAT(1X,'ACCELERATION ALONG THE Y-AXIS. (2F)')
      ACCEPT *,RMAGY,DURRY
      IF (RMAGY.EQ.0.)DURRY=0.1      !AVOID DIVIDE BY ZERO
      TYPE 250
250   FORMAT(/,1X,'ENTER PEAK MAGNITUDE(G''S) AND')
      TYPE 260
260   FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR LINEAR')
      TYPE 270
270   FORMAT(1X,'ACCELERATION ALONG THE Z-AXIS. (2F)')
      ACCEPT *,RMAGZ,DURRZ
      IF (RMAGZ.EQ.0.)DURRZ=0.1      !AVOID DIVIDE BY ZERO
      GO TO 310

280   TYPE 290
290   FORMAT(/,1X,'ENTER NAME OF EXPERIMENTAL DATA FILE')
      ACCEPT 20,INFIL1
      OPEN(UNIT=14,STATUS='OLD',FILE=INFIL1)
      TYPE 60
      ACCEPT 70,NS
      DO II=1,NS
      READ(14,300)T(II),A(II),B(II),C(II),D(II),E(II),F(II)
      END DO
300   FORMAT(1X,7G12.5)
      STIME=T(1)
      FTIME=T(NS)
310   TYPE 320
320   FORMAT(/,1X,'ENTER INTEGRATION TIME STEP (SEC)')
      ACCEPT *,DTIME
      TYPE330
330   FORMAT(/,1X,'ENTER NAME OF OUTPUT FILE TO BE CREATED.')
      ACCEPT 20,OUTFIL
      OPEN(UNIT=2,FILE=OUTFIL,STATUS='NEW')

340   READ(1,*) STIMEL,DTIMEL,FTIMEL,R
350   WRITE(2,360)
360   FORMAT(1X,'     STIME       DTIME       FTIME             R   ')
      WRITE(2,370) STIME,DTIME,FTIME,R
370   FORMAT(1X,9(G12.4,1X))
```

```
C
C     ERROR TERMS
C
         READ(1,*) E010,E011,E012,E013,S011,S012,S013
         READ(1,*) E020,E021,E022,E023,S021,S022,S023
         READ(1,*) E030,E031,E032,E033,S031,S032,S033
         READ(1,*) E120,E121,E122,E123,S121,S122,S123
         READ(1,*) E130,E131,E132,E133,S131,S132,S133
         READ(1,*) E210,E211,E212,E213,S211,S212,S213
         READ(1,*) E230,E231,E232,E233,S231,S232,S233
         READ(1,*) E310,E311,E312,E313,S311,S312,S313
         READ(1,*) E320,E321,E322,E323,S321,S322,S323

         WRITE(2,370) E010,E011,E012,E013,S011,S012,S013
         WRITE(2,370) E020,E021,E022,E023,S021,S022,S023
         WRITE(2,370) E030,E031,E032,E033,S031,S032,S033
         WRITE(2,370) E120,E121,E122,E123,S121,S122,S123
         WRITE(2,370) E130,E131,E132,E133,S131,S132,S133
         WRITE(2,370) E210,E211,E212,E213,S211,S212,S213
         WRITE(2,370) E230,E231,E232,E233,S231,S232,S233
         WRITE(2,370) E310,E311,E312,E313,S311,S312,S313
         WRITE(2,370) E320,E321,E322,E323,S321,S322,S323
         WRITE(2,380)
         WRITE(2,390)
         WRITE(2,400)
         WRITE(2,410)
         WRITE(2,420)
  380    FORMAT(1X,' ')
  390    FORMAT(1X,'     TIME')
  400    FORMAT(1X,'     WX            WY            WZ')
  410    FORMAT(1X,'     RX            RY            RZ            WDOTX         WDOTY
     &      WDOTZ')
  420    FORMAT(1X,'     QDOTX         QDOTY         QDOTZ         GDOTX         GDOTY
     &      GDOTZ')
C
C   CALCULATE THE A,B,C VALUES FOR ANGULAR ACCELERATION
C
         AA0=(E230-E030-E320+E020)/(2*R)
         AA1=(E233+E322)/2+(S232-S032+S323-S023)/(2*R)
         AA2=-E321/2+((S231-S031)/(2*R))
         AA3=-E231/2-((S321-S021)/(2*R))
         AA4=(E323-E232)/2+((S322-S022+S033-S233)/(2*R))
         AA5=E323/2+(S033-S233)/(2*R)
         AA6=-E232/2+(S322-S022)/(2*R)
         AA7=E231/2+(S021-S321)/(2*R)
         AA8=-E321/2+(S231-S031)/(2*R)
         AA9=(E233-E322)/2+(S232-S032-S323+S023)/(2*R)
         AA10=(E231-E031-E321+E021)/(2*R)
         AA11=(E232-E032-E322+E022)/(2*R)
         AA12=(E233-E033-E323+E023)/(2*R)
C
         BA0=(E310-E010-E130+E030)/(2*R)
         BA1=E312/2+(S032-S132)/(2*R)
         BA2=(E311+E133)/2+(S313-S013+S131-S031)/(2*R)
         BA3=-E132/2+(S012-S312)/(2*R)
         BA4=-E313/2+(S133-S033)/(2*R)
         BA5=(E131-E313)/2+(S011-S311+S133-S033)/(2*R)
         BA6=E131/2+(S011-S311)/(2*R)
         BA7=-E132/2+(S312-S012)/(2*R)
         BA8=(E311-E133)/2+(S313-S013-S131+S031)/(2*R)
```

```
      BA9=E312/2+(S032-S132)/(2*R)
      BA10=(E311-E011-E131+E031)/(2*R)
      BA11=(E312-E012-E132+E032)/(2*R)
      BA12=(E313-E013-E133+E033)/(2*R)
C
      CA0=(E120-E010-E210+E010)/(2*R)
      CA1=-E213/2+(S023-S123)/(2*R)
      CA2=-E123/2+(S013-S213)/(2*R)
      CA3=(E122+E211)/2+(S121-S021+S212-S012)/(2*R)
      CA4=E212/2+(S022-S122)/(2*R)
      CA5=-E121/2+(S211-S011)/(2*R)
      CA6=(E212-E121)/2+(S022-S122+S211-S011)/(2*R)
      CA7=(E122-E211)/2+(S121-S021-S212+S012)/(2*R)
      CA8=E123/2+(S013-S213)/(2*R)
      CA9=-E213/2+(S123-S023)/(2*R)
      CA10=(E121-E021-E211+E011)/(2*R)
      CA11=(E122-E022-E212+E012)/(2*R)
C
C  CALCULATE THE A,B,C VALUES FOR LINEAR ACCELERATION
C          AL0-AL12,BL0-BL12,CL0-CL12
C
      AL0=E010
      AL1=-S013*E012+S012*E013
      AL2=S013+S013*E011-S011*E013
      AL3=-S012-S012*E011+S011*E012
      AL4=-S012*E012-S013*E013
      AL5=-S011-S011*E011-S013*E013
      AL6=-S011-S011*E011-S012*E012
      AL7=S012+S012*E011+S011*E012
      AL8=S013+S013*E011+S011*E013
      AL9=S013*E012+S012*E013
      AL10=E011
      AL11=E012
      AL12=E013
C
      BL0=E020
      BL1=-S023-S023*E022+S022*E023
      BL2=S023*E021-S021*E023
      BL3=-S022*E021+S021+S021*E022
      BL4=-S022-S022*E022-S023*E023
      BL5=-S021*E021-S023*E023
      BL6=-S021*E021-S022-S022*E022
      BL7=S022*E021+S021+S021*E022
      BL8=S023*E021+S021*E023
      BL9=S023+S023*E022+S022*E023
      BL10=E021
      BL11=E022
      BL12=E023
C
      CL0=E030
      CL1=-S033*E032+S032+S032*E033
      CL2=S033*E031-S031-S031*E033
      CL3=-S032*E031+S031*E032
      CL4=-S032*E032-S033-S033*E033
      CL5=-S031*E031-S033-S033*E033
      CL6=-S031*E031-S032*E032
      CL7=S032*E031+S031*E032
      CL8=S033*E031+S031+S031*E033
      CL9=S033*E032+S032+S032*E033
      CL10=E031
```

```
      CL11=E032
      CL12=E033
      CA12=(E123-E023-E213+E013)/(2*R)
C
C   INITIALIZE ANGULAR VELOCITY TERMS, TIME
C
      Y(1)=0.0
      Y(2)=0.0
      Y(3)=0.0
      TIME=STIME
      GC=386.089      !G CONVERSION FACTOR
      IJ=2
      IL=0
      CALL STEQU
      GO TO 440
 430  IF(TIME.GT.FTIME) GO TO 450

      CALL RKDIF
 440  WX=Y(1)
      WY=Y(2)
      WZ=Y(3)
C
C  CALCULATE THE ESTIMATED ANGULAR ACCELERATIONS
C
      GDOTX=WDOTX+AA0+AA1*WDOTX+AA2*WDOTY+AA3*WDOTZ+
     2    AA4*WX**2+AA5*WY**2+AA6*WZ**2+AA7*WX*WY+
     3    AA8*WX*WZ+AA9*WY*WZ+AA10*RX+AA11*RY+AA12*RZ
      GDOTY=WDOTY+BA0+BA1*WDOTX+BA2*WDOTY+BA3*WDOTZ+
     2    BA4*WX**2+BA5*WY**2+BA6*WZ**2+BA7*WX*WY+
     3    BA8*WX*WZ+BA9*WY*WZ+BA10*RX+BA11*RY+BA12*RZ
      GDOTZ=WDOTZ+CA0+CA1*WDOTX+CA2*WDOTY+CA3*WDOTZ+
     2    CA4*WX**2+CA5*WY**2+CA6*WZ**2+CA7*WX*WY+
     3    CA8*WX*WZ+CA9*WY*WZ+CA10*RX+CA11*RY+CA12*RZ
C
C  CALCULATE THE ESTIMATED LINEAR ACCELERATIONS
C
      QDOTX=RX+AL0+AL1*WDOTX+AL2*WDOTY+AL3*WDOTZ+
     2    AL4*WX**2+AL5*WY**2+AL6*WZ**2+AL7*WX*WY+
     3    AL8*WX*WZ+AL9*WY*WZ+AL10*RX+AL11*RY+AL12*RZ
      QDOTY=RY+BL0+BL1*WDOTX+BL2*WDOTY+BL3*WDOTZ+
     2    BL4*WX**2+BL5*WY**2+BL6*WZ**2+BL7*WX*WY+
     3    BL8*WX*WZ+BL9*WY*WZ+BL10*RX+BL11*RY+BL12*RZ
      QDOTZ=RZ+CL0+CL1*WDOTX+CL2*WDOTY+CL3*WDOTZ+
     2    CL4*WX**2+CL5*WY**2+CL6*WZ**2+CL7*WX*WY+
     3    CL8*WX*WZ+CL9*WY*WZ+CL10*RX+CL11*RY+CL12*RZ
      WRITE(2,370)TIME
      WRITE(2,370)WX,WY,WZ

      RX=RX/GC                !CONVERT TO G'S
      RY=RY/GC                !      "
      RZ=RZ/GC                !      "
      WRITE(2,370)RX,RY,RZ,WDOTX,WDOTY,WDOTZ

      QDOTX=QDOTX/GC          !CONVERT TO G'S
      QDOTY=QDOTY/GC          !      "
      QDOTZ=QDOTZ/GC          !      "
      WRITE(2,370)QDOTX,QDOTY,QDOTZ,GDOTX,GDOTY,GDOTZ

      GOTO 430
 450  END
```

```
C        ******************************************************************
C
         SUBROUTINE EXTRAP
         DIMENSION DY(3),T(5000),A(5000),B(5000),C(5000),D(5000),E(5000),
     &   F(5000)
         COMMON/ACCL/RX,RY,RZ,WDOTX,WDOTY,WDOTZ,INMAGX,INMAGY,INMAGZ,
     *   DURAX,DURAY,DURAZ,RMAGX,RMAGY,RMAGZ,DURRX,DURRY,DURRZ
         COMMON/DYNM/TIME,DTIME,FTIME,DY,ANS,IJ,IL,NS
         COMMON/EXT/T,A,B,C,D,E,F
C        LOAD EXPERIMENTAL DATA
         IF (IL.EQ.1) GO TO 20
         DO 10 II = 1,NS
   10    READ(3,*) T(II),A(II),B(II),C(II),D(II),E(II),F(II)
         IL=1
   20    IF (T(IJ).GT.TIME) GO TO 30
         IJ=IJ+1
         GO TO 20
   30    IK = IJ - 1
         DTX = (TIME-T(IK))/(T(IJ)-T(IK))
         RX=A(IK)+(A(IJ)-A(IK))*DTx
         RY=B(IK)+(B(IJ)-B(IK))*DTX
         RZ=C(IK)+(C(IJ)-C(IK))*DTX
         WDOTX=D(IK)+(D(IJ)-D(IK))*DTX
         WDOTY=E(IK)+(E(IJ)-E(IK))*DTX
         WDOTZ=F(IK)+(F(IJ)-F(IK))*DTX

         RETURN
         END
C*****************************************************************
C*****************************************************************

         SUBROUTINE STEQU

         CHARACTER*16 ERFIL,OUTFIL,ANS*1,INFIL1

         DIMENSION Y(3),DY(3)

         COMMON/STATE/Y
         COMMON/DYNM/TIME,DTIME,FTIME,DY,ANS,IJ,IL,NS
         COMMON/ACCL/RX,RY,RZ,WDOTX,WDOTY,WDOTZ,INMAGX,INMAGY,INMAGZ,
     *   DURAX,DURAY,DURAZ,RMAGX,RMAGY,RMAGZ,DURRX,DURRY,DURRZ

         GC=386.089
C
         IF (ANS.EQ.'E') GO TO 10
C
C        ANALYTICAL INPUTS
C
         WDOTX=INMAGX*SIN(2*3.1416*(1/(2*DURAX))*TIME)
         WDOTY=INMAGY*SIN(2*3.1416*(1/(2*DURAY))*TIME)
         WDOTZ=INMAGZ*SIN(2*3.1416*(1/(2*DURAZ))*TIME)
         RX=RMAGX*GC*SIN(2*3.1416*(1/(2*DURRX))*TIME)
         RY=RMAGY*GC*SIN(2*3.1416*(1/(2*DURRY))*TIME)
         RZ=RMAGZ*GC*SIN(2*3.1416*(1/(2*DURRZ))*TIME)

         IF(TIME.GT.DURAX)WDOTX=0.0
         IF(TIME.GT.DURAY)WDOTY=0.0
         IF(TIME.GT.DURAZ)WDOTZ=0.0
         IF(TIME.GT.DURRX)RX=0.0
         IF(TIME.GT.DURRY)RY=0.0
```

```
      IF(TIME.GT.DURRZ)RZ=0.0

      GOTO 20

   10 CONTINUE
C     EXPERIMENTAL INPUTS
      CALL EXTRAP
   20 DY(1)=WDOTX
      DY(2)=WDOTY
      DY(3)=WDOTZ
      RETURN
      END
C**********************************************************************
C**********************************************************************
      SUBROUTINE RKDIF
C  FOURTH ORDER RUNGE KUTTA NUMERICAL INTEGRATION SUBROUTINE

      DIMENSION Y(3),SY(3),Y0(3),Y1(3),Y2(3),DY(3)
      COMMON/DYNM/TIME,DTIME,FTIME,DY,ANS,IJ,IL,NS
      COMMON/STATE/Y
C
      H=DTIME/2
      DO  I=1,3
        SY(I)=Y(I)
        Y0(I)=DY(I)
        Y(I)=H*DY(I)+Y(I)
      END DO

      TIME=TIME+H
      CALL STEQU
      DO I=1,3
        Y1(I)=DY(I)
        Y(I)=SY(I)+H*DY(I)
        Y2(I)=DY(I)
        Y(I)=SY(I)+DTIME*DY(I)
      END DO

      TIME=TIME+H
      H=H/3.0
      DO  I=1,3
        PRT1=2.0*(Y1(I)+Y2(I))
        PRT2=Y0(I)+DY(I)
        Y(I)=SY(I)+H*PRT1+H*PRT2
      END DO

      CALL STEQU
      RETURN
      END
```

APPENDIX D

COMPUTER PROGRAMS FOR THE

FIELD MODELS

APPENDIX D1

COMPUTER PROGRAM FOR THE

FIELD MODEL

(EULER INTERGRATION)

APPENDIX D2

COMPUTER PROGRAM FOR THE

FIELD MODEL

(RUNGE-KUTTA METHOD)

APPENDIX D1

COMPUTER PROGRAM FOR THE FIELD MODEL (EULER INTEGRATION)

```
****************************************************************************
*                                                                          *
*       NAPFLDEUL.FOR                                                      *
*                                                                          *
*       PROGRAM TO CORRECT THE FIELD MEASURED LINEAR AND ANGULAR           *
*       ACCELERATIONS OF A NINE-ACCELEROMETER HEAD IMPACT MEASUREMENT     *
*       SYSTEM WITH A 3-2-2-2 CONFIGURATION.                               *
*                                                                          *
*       YOU ARE REQUIRED TO INPUT AN ARRAY FOR THE NAPLAB SYSTEM           *
*       COEFFICIENTS (THE [A+I], [B] AND [BI] MATRICES ).  YOU ARE THEN    *
*       REQUIRED TO SELECT AN ANALYTICAL, NAPLAB OR EXPERIMENTAL INPUT     *
*       PULSE. THE PROGRAM WILL CREATE THE [C], [D] AND [EM] MATRICES FOR  *
*       THE NAPFLD CALCULATIONS.                                           *
*                                                                          *
*       PROGRAM USES EULER INTEGRATION                                     *
*                                                                          *
*       THE OUTPUT CONSISTS OF THE FOLLOWING PARAMETERS:                   *
*                                                                          *
*       STIME = START TIME (SECONDS)                                       *
*       DTIME = INTEGRATION STEP TIME (SECONDS)                            *
*       FTIME = FINISH TIME (SECONDS)                                      *
*       TIME = TIME (SECONDS)                                              *
*       WX = ESTIMATED ANGULAR VELOCITY ABOUT THE X-AXIS (R/S)             *
*       WY = ESTIMSTED ANGULAR VELOCITY ABOUT THE Y-AXIS (R/S)             *
*       WZ = ESTIMATED ANGULAR VELOCITY ABOUT THE Z-AXIS (R/S)             *
*       RX = CORRECTED LINEAR ACCELERATION ALONG THE X-AXIS (G'S)          *
*       RY = CORRECTED LINEAR ACCELERATION ALONG THE Y-AXIS (G'S)          *
*       RZ = CORRECTED LINEAR ACCELERATION ALONG THE Z-AXIS (G'S)          *
*       QDOTX = MEASURED LINEAR ACCELERATION ALONG THE X-AXIS (G'S)        *
*       QDOTY = MEASURED LINEAR ACCELERATION ALONG THE Y-AXIS (G'S)        *
*       QDOTZ = MEASURED LINEAR ACCELERATION ALONG THE Z-AXIS (G'S)        *
*       WDOTX = CORRECTED ANGULAR ACCELERATION ABOUT THE X-AXIS (R/(S*S))  *
*       WDOTY = CORRECTED ANGULAR ACCELERATION ABOUT THE Y-AXIS (R/(S*S))  *
*       WDOTZ = CORRECTED ANGULAR ACCELERATION ABOUT THE Z-AXIS (R/(S*S))  *
*       GDOTX = MEASURED ANGULAR ACCELERATION ABOUT THE X-AXIS (R/(S*S))   *
*       GDOTY = MEASURED ANGULAR ACCELERATION ABOUT THE Y-AXIS (R/(S*S))   *
*       GDOTZ = MEASURED ANGULAR ACCELERATION ABOUT THE Z-AXIS (R/(S*S))   *
*                                                                          *
****************************************************************************


      CHARACTER*16 INFIL,INFIL2,INFIL3,OUTFIL,ANS1*1,QLAB*21,
     *  RLAB(5)*60


      DOUBLE PRECISION RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,
     *  INMAGX,INMAGY,INMAGZ,DURAX,DURAY,DURAZ,RMAGX,RMAGY,RMAGZ,DURRX,
     *  DURRY,DURRZ,GDOTX,GDOTY,GDOTZ,QDOTX,QDOTY,QDOTZ,A,B,C,D,EM,BI,
     *  TIME,STIME,DTIME,FTIME,ETIME,T,AX,BX,CX,DX,EX,FX,AA0,BA0,CA0,
     *  AL0,BL0,CL0,V,R


      DIMENSION A(6,6),B(6,6),C(6,6),D(6,6),EM(6,1),BI(6,1),Q(100),
     *  T(4000),AX(4000),BX(4000),CX(4000),DX(4000),
     *  EX(4000),FX(4000)


      COMMON/BLK1/IJ
```

```
      COMMON/BLK2/TIME,DTIME
      COMMON/BLK3/RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,GDOTX,GDOTY,
     * GDOTZ,QDOTX,QDOTY,QDOTZ
      COMMON/BLK4/INMAGX,INMAGY,INMAGZ,DURAX,DURAY,DURAZ,RMAGX,
     * RMAGY,RMAGZ,DURRX,DURRY,DURRZ
      COMMON/BLK5/C,D,EM
      COMMON/BLK6/ANS1
      COMMON/BLK7/T,AX,BX,CX,DX,EX,FX


      TYPE 9
    9 FORMAT(/////)
      TYPE 1
    1 FORMAT(1X,'          *********************************************
     &*****************')
      TYPE 2
    2 FORMAT(1X,'          *
     &                 *')
      TYPE 3
    3 FORMAT(1X,'          *                          NOTICE
     &                 *')
      TYPE 2
      TYPE 4
    4 FORMAT(1X,'          *  This program is disseminated under the
     & sponsorship      *')
      TYPE 5
    5 FORMAT(1X,'          *  of the Transportation Systems Center in the
     & interest    *')
      TYPE 6
    6 FORMAT(1X,'          *  of information exchange. The United States
     & Government   *')
      TYPE 7
    7 FORMAT(1X,'          *  assumes no liability for its contents or
     & use thereof.   *')
      TYPE 2
      TYPE 1
      TYPE 9

      TYPE 20
20    FORMAT(/,1X,'ENTER NAPLAB SYSTEM COEFFICIENT FILE NAME (A16)')
      ACCEPT 21,INFIL
21    FORMAT(A16)
      OPEN(UNIT=1,STATUS='OLD',FILE=INFIL)

      DO I=1,6
      READ(1,*)(A(I,J),J=1,6)                !READ [A+I] MATRIX
      END DO

      CALL MATINVSUB(A,INFIL)

      DO I=1,6
      DO J=1,6
      C(I,J)=A(I,J)                          ![C] = INVERSE OF [A+I] MATRIX
      END DO
      END DO

      READ(1,10)
10    FORMAT(1X)

      DO I=1,6
```

```
      READ(1,*)(B(I,J),J=1,6)                   !READ [B] MATRIX
      END DO

      READ(1,10)

      READ(1,*)AA0,BA0,CA0,AL0,BL0,CL0          !READ BIAS VALUES

      BI(1,1)=AA0                               !CREATE [BI]
      BI(2,1)=BA0
      BI(3,1)=CA0
      BI(4,1)=AL0
      BI(5,1)=BL0
      BI(6,1)=CL0

************************************************************
*     FIND THE PRODUCT [D] = [C]*[B]                       *
************************************************************

      DO I=1,6
      DO J=1,6
      D(I,J)=0.0
      DO K=1,6
      D(I,J)=D(I,J)+C(I,K)*B(K,J)
      END DO
      END DO
      END DO

************************************************************
*     FIND THE PRODUCT [EM] = [C]*[BI]                     *
************************************************************

      DO I=1,6
      EM(I,1)=0.0
      DO K=1,6
      EM(I,1)=EM(I,1)+C(I,K)*BI(K,1)
      END DO
      END DO

************************************************************


30    TYPE 31
31    FORMAT(/,1X,'ANALYTIC, EXPERIMENTAL OR NAPLAB INPUT? (A, E OR N)')
      ACCEPT 32,ANS1
32    FORMAT(A1)
      IF(ANS1.EQ.'A')GO TO 43
      IF(ANS1.EQ.'N')GO TO 35
      IF(ANS1.EQ.'E')GO TO 60
      GO TO 30
35    TYPE 36
36    FORMAT(/,1X,'ENTER NAME OF NAPLAB FILE')
      ACCEPT 21,INFIL2
      OPEN(UNIT=3,STATUS='OLD',FILE=INFIL2)
      TYPE 44
      ACCEPT 45,NS
      TYPE 47
      ACCEPT *,STIME,FTIME
      TYPE 46
      ACCEPT *,DTIME
```

```
        READ(3,55)QLAB
55      FORMAT(A21)
        READ(3,*)(Q(I),I=1,67)
        READ(3,57)(RLAB(I),I=1,5)
57      FORMAT(A60)

        GC=386.089D0

        DO II=1,NS
        READ(3,*)T(II)
        READ(3,*)WXL,WYL,WZL
        READ(3,*)RXL,RYL,RZL,WDOTXL,WDOTYL,WDOTZL
        READ(3,*)AX(II),BX(II),CX(II),DX(II),EX(II),FX(II)

        AX(II)=AX(II)*GC                !CHANGE TO INCHES/S*S
        BX(II)=BX(II)*GC                !          "
        CX(II)=CX(II)*GC                !          "

        END DO

        GO TO 50

****************************************************************************

43      TYPE 44
44      FORMAT(/,1X,'ENTER NUMBER OF OBSERVATIONS (SAMPLES)')
        ACCEPT 45,NS
45      FORMAT(I)
        TYPE 47
47      FORMAT(/,1X,'ENTER PULSE START AND FINISH TIMES IN SECONDS (2F)')
        ACCEPT *,STIME,FTIME
        TYPE 46
46      FORMAT(/,1X,'ENTER INTEGRATION TIME STEP (SEC)')
        ACCEPT *,DTIME
        TYPE 72
72      FORMAT(/,1X,'ENTER PEAK MAGNITUDE(RAD/(S*S)) AND')
        TYPE 73
73      FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR ANGULAR')
        TYPE 74
74      FORMAT(1X,'ACCELERATION ABOUT THE X-AXIS. (2F)')
        ACCEPT *,INMAGX,DURAX
        IF (INMAGX.EQ.0.)DURAX=0.1      !AVOID DIVIDE BY ZERO
        TYPE 76
76      FORMAT(/,1X,'ENTER PEAK MAGNITUDE(RAD/(S*S)) AND')
        TYPE 77
77      FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR ANGULAR')
        TYPE 78
78      FORMAT(1X,'ACCELERATION ABOUT THE Y-AXIS. (2F)')
        ACCEPT *,INMAGY,DURAY
        IF (INMAGY.EQ.0.)DURAY=0.1      !AVOID DIVIDE BY ZERO
        TYPE 80
80      FORMAT(/,1X,'ENTER PEAK MAGNITUDE(RAD/(S*S)) AND')
        TYPE 81
81      FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR ANGULAR')
        TYPE 82
82      FORMAT(1X,'ACCELERATION ABOUT THE Z-AXIS. (2F)')
        ACCEPT *,INMAGZ,DURAZ
        IF (INMAGZ.EQ.0.)DURAZ=0.1      !AVOID DIVIDE BY ZERO
        TYPE 84
84      FORMAT(/,1X,'ENTER PEAK MAGNITUDE(G''S) AND')
```

```
        TYPE 85
85      FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR LINEAR')
        TYPE 86
86      FORMAT(1X,'ACCELERATION ALONG THE X-AXIS. (2F)')
        ACCEPT *,RMAGX,DURRX
        IF (RMAGX.EQ.0.)DURRX=0.1      !AVOID DIVIDE BY ZERO
        TYPE 88
88      FORMAT(/,1X,'ENTER PEAK MAGNITUDE(G''S) AND')
        TYPE 89
89      FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR LINEAR')
        TYPE 90
90      FORMAT(1X,'ACCELERATION ALONG THE Y-AXIS. (2F)')
        ACCEPT *,RMAGY,DURRY
        IF (RMAGY.EQ.0.)DURRY=0.1      !AVOID DIVIDE BY ZERO
        TYPE 91
91      FORMAT(/,1X,'ENTER PEAK MAGNITUDE(G''S) AND')
        TYPE 92
92      FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR LINEAR')
        TYPE 93
93      FORMAT(1X,'ACCELERATION ALONG THE Z-AXIS. (2F)')
        ACCEPT *,RMAGZ,DURRZ
        IF (RMAGZ.EQ.0.)DURRZ=0.1      !AVOID DIVIDE BY ZERO


        GO TO 50

************************************************************************

60      TYPE 61
61      FORMAT(/,1X,'ENTER NAME OF EXPERIMENTAL DATA FILE')
        ACCEPT 21,INFIL3
        TYPE 44
        ACCEPT 45,NS
        TYPE 46
        ACCEPT *,DTIME
        OPEN(UNIT=14,STATUS='OLD',FILE=INFIL3)
        DO 300 II=1,NS
300     READ(14,*)T(II),AX(II),BX(II),CX(II),DX(II),EX(II),FX(II)
        STIME=T(1)
        FTIME=T(NS)

************************************************************************

50      TYPE51
51      FORMAT(/,1X,'ENTER NAME OF OUTPUT FILE TO BE CREATED.')
        ACCEPT 21,OUTFIL
        OPEN(UNIT=2,STATUS='NEW',FILE=OUTFIL)

C
        WRITE(2,98)
98      FORMAT(1X,'    STIME      DTIME      FTIME          R')
        V=0.D0
        R=1.5D0

        WRITE(2,100)STIME,DTIME,FTIME,R

        DO I=1,9
        WRITE(2,100)V,V,V,V,V,V,V
        END DO
100     FORMAT(1X,9(G12.4,1X))
```

```
      WRITE(2,105)
      WRITE(2,101)
      WRITE(2,102)
      WRITE(2,103)
      WRITE(2,104)
105   FORMAT(1X,' ')
101   FORMAT(1X,'      TIME')
102   FORMAT(1X,'      WX          WY          WZ')
103   FORMAT(1X,'      RX          RY          RZ          WDOTX       WDOTY
     &     WDOTZ')
104   FORMAT(1X,'      QDOTX       QDOTY       QDOTZ       GDOTX       GDOTY
     &     GDOTZ')
C
C
C  INITIALIZE ANGULAR VELOCITY TERMS, TIME, OTHER CONSTANTS
C
      WX=0.D0
      WY=0.D0
      WZ=0.D0

      TIME=STIME
      IJ=2


200   CALL FCT

      CALL OUTP

      TIME=TIME+DTIME                  !UPDATE TIME
      ETIME=FTIME+DTIME/1000.

      IF(TIME.GT.ETIME)GO TO 210

      WX=WX+DTIME*WDOTX                !UPDATE ESTIMATES OF W
      WY=WY+DTIME*WDOTY                !         "
      WZ=WZ+DTIME*WDOTZ                !         "

      GOTO 200

210   STOP
      END

****************************************************************
****************************************************************
*      CALCULATE THE ESTIMATED ANGULAR ACCELERATIONS           *
****************************************************************
****************************************************************


      SUBROUTINE FCT

      CHARACTER ANS1*1

      DOUBLE PRECISION RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,
     *  INMAGX,INMAGY,INMAGZ,DURAX,DURAY,DURAZ,RMAGX,RMAGY,RMAGZ,DURRX,
     *  DURRY,DURRZ,GDOTX,GDOTY,GDOTZ,QDOTX,QDOTY,QDOTZ,C,D,EM,
     *  TIME,T,AX,BX,CX,DX,EX,FX,GC,TMOD
```

```
      DIMENSION C(6,6),D(6,6),EM(6,1),T(4000),AX(4000),BX(4000),
     *   CX(4000),DX(4000),EX(4000),FX(4000)

      COMMON/BLK1/IJ
      COMMON/BLK2/TIME,DTIME
      COMMON/BLK3/RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,GDOTX,GDOTY,
     *   GDOTZ,QDOTX,QDOTY,QDOTZ
      COMMON/BLK4/INMAGX,INMAGY,INMAGZ,DURAX,DURAY,DURAZ,RMAGX,
     *   RMAGY,RMAGZ,DURRX,DURRY,DURRZ
      COMMON/BLK5/C,D,EM
      COMMON/BLK6/ANS1
      COMMON/BLK7/T,AX,BX,CX,DX,EX,FX

      GC=386.089D0

      IF(ANS1.NE.'A')GO TO 18

***********************************************************
*        CREATE HALF SINE PULSE INPUT FUNCTIONS           *
***********************************************************

      GDOTX=INMAGX*SIN(2.*3.1416*(1./(2.*DURAX))*TIME)
      GDOTY=INMAGY*SIN(2.*3.1416*(1./(2.*DURAY))*TIME)
      GDOTZ=INMAGZ*SIN(2.*3.1416*(1./(2.*DURAZ))*TIME)
      QDOTX=RMAGX*GC*SIN(2.*3.1416*(1./(2.*DURRX))*TIME)
      QDOTY=RMAGY*GC*SIN(2.*3.1416*(1./(2.*DURRY))*TIME)
      QDOTZ=RMAGZ*GC*SIN(2.*3.1416*(1./(2.*DURRZ))*TIME)

      IF(TIME.GT.DURAX)GDOTX=0.0
      IF(TIME.GT.DURAY)GDOTY=0.0
      IF(TIME.GT.DURAZ)GDOTZ=0.0
      IF(TIME.GT.DURRX)QDOTX=0.0
      IF(TIME.GT.DURRY)QDOTY=0.0
      IF(TIME.GT.DURRZ)QDOTZ=0.0

      GO TO 12


*****************************************************************************
*        EXTRAPOLATION ROUTINE FOR HANDLING EXPERIMENTAL AND NAPLAB DATA    *
*****************************************************************************

18    TMOD=TIME-DTIME/1000.             !TMOD WAS ADDED IN CASE TIME PICKED
      IF(T(IJ).GE.TMOD)GO TO 320        !UP A BIT ALONG THE WAY
      IJ=IJ+1
      GO TO 18
320   IK=IJ-1
      DTX=(TIME-T(IK))/(T(IJ)-T(IK))
      QDOTX=AX(IK)+(AX(IJ)-AX(IK))*DTX
      QDOTY=BX(IK)+(BX(IJ)-BX(IK))*DTX
      QDOTZ=CX(IK)+(CX(IJ)-CX(IK))*DTX
      GDOTX=DX(IK)+(DX(IJ)-DX(IK))*DTX
      GDOTY=EX(IK)+(EX(IJ)-EX(IK))*DTX
      GDOTZ=FX(IK)+(FX(IJ)-FX(IK))*DTX

************************************************
*        CALCULATE CORRECTED OUTPUTS           *
```

```
*****************************************************

12      WDOTX=-EM(1,1)+C(1,1)*GDOTX+C(1,2)*GDOTY+C(1,3)*GDOTZ-
     2      D(1,1)*WX**2-D(1,2)*WY**2-D(1,3)*WZ**2-D(1,4)*WX*WY-
     3      D(1,5)*WX*WZ-D(1,6)*WY*WZ+C(1,4)*QDOTX+C(1,5)*QDOTY+
     4      C(1,6)*QDOTZ
        WDOTY=-EM(2,1)+C(2,1)*GDOTX+C(2,2)*GDOTY+C(2,3)*GDOTZ-
     2      D(2,1)*WX**2-D(2,2)*WY**2-D(2,3)*WZ**2-D(2,4)*WX*WY-
     3      D(2,5)*WX*WZ-D(2,6)*WY*WZ+C(2,4)*QDOTX+C(2,5)*QDOTY+
     4      C(2,6)*QDOTZ
        WDOTZ=-EM(3,1)+C(3,1)*GDOTX+C(3,2)*GDOTY+C(3,3)*GDOTZ-
     2      D(3,1)*WX**2-D(3,2)*WY**2-D(3,3)*WZ**2-D(3,4)*WX*WY-
     3      D(3,5)*WX*WZ-D(3,6)*WY*WZ+C(3,4)*QDOTX+C(3,5)*QDOTY+
     4      C(3,6)*QDOTZ
C
C   CALCULATE THE ESTIMATED LINEAR ACCELERATIONS
C
        RX=-EM(4,1)+C(4,1)*GDOTX+C(4,2)*GDOTY+C(4,3)*GDOTZ-
     2      D(4,1)*WX**2-D(4,2)*WY**2-D(4,3)*WZ**2-D(4,4)*WX*WY-
     3      D(4,5)*WX*WZ-D(4,6)*WY*WZ+C(4,4)*QDOTX+C(4,5)*QDOTY+
     4      C(4,6)*QDOTZ
        RY=-EM(5,1)+C(5,1)*GDOTX+C(5,2)*GDOTY+C(5,3)*GDOTZ-
     2      D(5,1)*WX**2-D(5,2)*WY**2-D(5,3)*WZ**2-D(5,4)*WX*WY-
     3      D(5,5)*WX*WZ-D(5,6)*WY*WZ+C(5,4)*QDOTX+C(5,5)*QDOTY+
     4      C(5,6)*QDOTZ
        RZ=-EM(6,1)+C(6,1)*GDOTX+C(6,2)*GDOTY+C(6,3)*GDOTZ-
     2      D(6,1)*WX**2-D(6,2)*WY**2-D(6,3)*WZ**2-D(6,4)*WX*WY-
     3      D(6,5)*WX*WZ-D(6,6)*WY*WZ+C(6,4)*QDOTX+C(6,5)*QDOTY+
     4      C(6,6)*QDOTZ


        RETURN
        END


*****************************************************
*****************************************************
*       WRITE OUTPUT FILE                           *
*****************************************************
*****************************************************


        SUBROUTINE OUTP


        DOUBLE PRECISION RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,
     *  GDOTX,GDOTY,GDOTZ,QDOTX,QDOTY,QDOTZ,TIME,GC


        COMMON/BLK2/TIME,DTIME
        COMMON/BLK3/RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,GDOTX,GDOTY,
     *  GDOTZ,QDOTX,QDOTY,QDOTZ


        WRITE(2,100)TIME
        WRITE(2,100)WX,WY,WZ
```

```
      GC=386.089D0

      RX=RX/GC                  !CONVERT TO G'S
      RY=RY/GC                  !      "
      RZ=RZ/GC                  !      "
      WRITE(2,100)RX,RY,RZ,WDOTX,WDOTY,WDOTZ

100   FORMAT(1X,9(G12.4,1X))

      QDOTX=QDOTX/GC            !CONVERT TO G'S
      QDOTY=QDOTY/GC            !      "
      QDOTZ=QDOTZ/GC            !      "
      WRITE(2,100)QDOTX,QDOTY,QDOTZ,GDOTX,GDOTY,GDOTZ

      RETURN
      END
```

APPENDIX D-2

COMPUTER PROGRAM FOR THE

FIELD MODEL

(RUNGE-KUTTA METHOD)

```
**************************************************************************
*                                                                        *
*                                                                        *
*       NAPFLDRK.FOR                                                     *
*                                                                        *
*       PROGRAM TO CORRECT THE FIELD MEASURED LINEAR AND ANGULAR         *
*       ACCELERATIONS OF A NINE-ACCELEROMETER HEAD IMPACT MEASUREMENT    *
*       SYSTEM WITH A 3-2-2-2 CONFIGURATION.                             *
*                                                                        *
*       YOU ARE REQUIRED TO INPUT AN ARRAY FOR THE NAPLAB SYSTEM         *
*       COEFFICIENTS (THE [A+I], [B] AND [BI] MATRICES ).  YOU ARE THEN  *
*       REQUIRED TO SELECT AN ANALYTICAL, NAPLAB OR EXPERIMENTAL INPUT   *
*       PULSE. THE PROGRAM WILL CREATE THE [C], [D] AND [EM] MATRICES FOR *
*       THE NAPFLD CALCULATIONS.                                         *
*                                                                        *
*       PROGRAM USES RUNGE-KUTTA METHOD                                  *
*                                                                        *
*       THE OUTPUT CONSISTS OF THE FOLLOWING PARAMETERS:                 *
*                                                                        *
*       STIME = START TIME (SECONDS)                                     *
*       DTIME = INTEGRATION STEP TIME (SECONDS)                          *
*       FTIME = FINISH TIME (SECONDS)                                    *
*       TIME = TIME (SECONDS)                                            *
*       WX = ESTIMATED ANGULAR VELOCITY ABOUT THE X-AXIS (R/S)           *
*       WY = ESTIMSTED ANGULAR VELOCITY ABOUT THE Y-AXIS (R/S)           *
*       WZ = ESTIMATED ANGULAR VELOCITY ABOUT THE Z-AXIS (R/S)           *
*       RX = CORRECTED LINEAR ACCELERATION ALONG THE X-AXIS (G'S)        *
*       RY = CORRECTED LINEAR ACCELERATION ALONG THE Y-AXIS (G'S)        *
*       RZ = CORRECTED LINEAR ACCELERATION ALONG THE Z-AXIS (G'S)        *
*       QDOTX = MEASURED LINEAR ACCELERATION ALONG THE X-AXIS (G'S)      *
*       QDOTY = MEASURED LINEAR ACCELERATION ALONG THE Y-AXIS (G'S)      *
*       QDOTZ = MEASURED LINEAR ACCELERATION ALONG THE Z-AXIS (G'S)      *
*       WDOTX = CORRECTED ANGULAR ACCELERATION ABOUT THE X-AXIS (R/(S*S)) *
*       WDOTY = CORRECTED ANGULAR ACCELERATION ABOUT THE Y-AXIS (R/(S*S)) *
*       WDOTZ = CORRECTED ANGULAR ACCELERATION ABOUT THE Z-AXIS (R/(S*S)) *
*       GDOTX = MEASURED ANGULAR ACCELERATION ABOUT THE X-AXIS (R/(S*S)) *
*       GDOTY = MEASURED ANGULAR ACCELERATION ABOUT THE Y-AXIS (R/(S*S)) *
*       GDOTZ = MEASURED ANGULAR ACCELERATION ABOUT THE Z-AXIS (R/(S*S)) *
*                                                                        *
**************************************************************************

      EXTERNAL FCN1
      INTEGER N,IND,NW,IER
      REAL*8 Y(6),C(24),W(6,9),X,TOL,XEND,YPRIME(6)


      CHARACTER*16 INFIL,INFIL2,INFIL3,OUTFIL,ANS1*1,QLAB*21,
     * RLAB(5)*60


      DOUBLE PRECISION RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,
     * INMAGX,INMAGY,INMAGZ,DURAX,DURAY,DURAZ,RMAGX,RMAGY,RMAGZ,DURRX,
     * DURRY,DURRZ,GDOTX,GDOTY,GDOTZ,QDOTX,QDOTY,QDOTZ,A,B,CI,D,EM,BI,
     * TIME,STIME,DTIME,FTIME,ETIME,T,AX,BX,CX,DX,EX,FX,AA0,BA0,CA0,
     * AL0,BL0,CL0,V,R


      DIMENSION A(6,6),B(6,6),CI(6,6),D(6,6),EM(6,1),BI(6,1),Q(100),
     * T(4000),AX(4000),BX(4000),CX(4000),DX(4000),
     * EX(4000),FX(4000)
```

```
      COMMON/BLK1/IJ
      COMMON/BLK2/TIME,DTIME
      COMMON/BLK3/RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,GDOTX,GDOTY,
     * GDOTZ,QDOTX,QDOTY,QDOTZ
      COMMON/BLK4/INMAGX,INMAGY,INMAGZ,DURAX,DURAY,DURAZ,RMAGX,
     * RMAGY,RMAGZ,DURRX,DURRY,DURRZ
      COMMON/BLK5/CI,D,EM
      COMMON/BLK6/ANS1
      COMMON/BLK7/T,AX,BX,CX,DX,EX,FX


      TYPE 9
9     FORMAT(/////)
      TYPE 1
1     FORMAT(1X,'          *********************************************
     &****************')
      TYPE 2
2     FORMAT(1X,'          *
     &                *')
      TYPE 3
3     FORMAT(1X,'          *                            NOTICE
     &               *')
      TYPE 2
      TYPE 4
4     FORMAT(1X,'          *  This program is disseminated under the
     & sponsorship      *')
      TYPE 5
5     FORMAT(1X,'          *  of the Transportation Systems Center in the
     & interest   *')
      TYPE 6
6     FORMAT(1X,'          *  of information exchange. The United States
     & Government  *')
      TYPE 7
7     FORMAT(1X,'          *  assumes no liability for its contents or
     & use thereof.  *')
      TYPE 2
      TYPE 1
      TYPE 9

      TYPE 20
20    FORMAT(/,1X,'ENTER NAPLAB SYSTEM COEFFICIENT FILE NAME (A16)')
      ACCEPT 21,INFIL
21    FORMAT(A16)
      OPEN(UNIT=1,STATUS='OLD',FILE=INFIL)

      DO I=1,6
      READ(1,*)(A(I,J),J=1,6)                  !READ [A+I] MATRIX
      END DO

      CALL MATINVSUB(A,INFIL)

      DO I=1,6
      DO J=1,6
      CI(I,J)=A(I,J)                           ![CI] = INVERSE OF [A+I] MATRIX
      END DO
      END DO

      READ(1,10)
```

```
10      FORMAT(1X)

        DO I=1,6
        READ(1,*)(B(I,J),J=1,6)              !READ [B] MATRIX
        END DO

        READ(1,10)

        READ(1,*)AA0,BA0,CA0,AL0,BL0,CL0     !READ BIAS VALUES

        BI(1,1)=AA0                          !CREATE [BI]
        BI(2,1)=BA0
        BI(3,1)=CA0
        BI(4,1)=AL0
        BI(5,1)=BL0
        BI(6,1)=CL0

*********************************************************
*       FIND THE PRODUCT [D] = [CI]*[B]                 *
*********************************************************

        DO I=1,6
        DO J=1,6
        D(I,J)=0.0
        DO K=1,6
        D(I,J)=D(I,J)+CI(I,K)*B(K,J)
        END DO
        END DO
        END DO

**********************************************************
*       FIND THE PRODUCT [EM] = [CI]*[BI]                *
**********************************************************

        DO I=1,6
        EM(I,1)=0.0
        DO K=1,6
        EM(I,1)=EM(I,1)+CI(I,K)*BI(K,1)
        END DO
        END DO

*********************************************************


30      TYPE 31
31      FORMAT(/,1X,'ANALYTIC, EXPERIMENTAL OR NAPLAB INPUT? (A, E OR N)')
        ACCEPT 32,ANS1
32      FORMAT(A1)
        IF(ANS1.EQ.'A')GO TO 43
        IF(ANS1.EQ.'N')GO TO 35
        IF(ANS1.EQ.'E')GO TO 60
        GO TO 30
35      TYPE 36
36      FORMAT(/,1X,'ENTER NAME OF NAPLAB FILE')
        ACCEPT 21,INFIL2
        OPEN(UNIT=3,STATUS='OLD',FILE=INFIL2)
        TYPE 44
        ACCEPT 45,NS
        TYPE 47
```

```
      ACCEPT *,STIME,FTIME
      TYPE 46
      ACCEPT *,DTIME
      READ(3,55)QLAB
55    FORMAT(A21)
      READ(3,*)(Q(I),I=1,67)
      READ(3,57)(RLAB(I),I=1,5)
57    FORMAT(A60)

      GC=386.089D0

      DO II=1,NS
      READ(3,*)T(II)
      READ(3,*)WXL,WYL,WZL
      READ(3,*)RXL,RYL,RZL,WDOTXL,WDOTYL,WDOTZL
      READ(3,*)AX(II),BX(II),CX(II),DX(II),EX(II),FX(II)

      AX(II)=AX(II)*GC                  !CHANGE TO INCHES/S*S
      BX(II)=BX(II)*GC                  !          "
      CX(II)=CX(II)*GC                  !          "

      END DO

      GO TO 50

**************************************************************************

43    TYPE 44
44    FORMAT(/,1X,'ENTER NUMBER OF OBSERVATIONS (SAMPLES)')
      ACCEPT 45,NS
45    FORMAT(I)
      TYPE 47
47    FORMAT(/,1X,'ENTER PULSE START AND FINISH TIMES IN SECONDS (2F)')
      ACCEPT *,STIME,FTIME
      TYPE 46
46    FORMAT(/,1X,'ENTER INTEGRATION TIME STEP (SEC)')
      ACCEPT *,DTIME
      TYPE 72
72    FORMAT(/,1X,'ENTER PEAK MAGNITUDE(RAD/(S*S)) AND')
      TYPE 73
73    FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR ANGULAR')
      TYPE 74
74    FORMAT(1X,'ACCELERATION ABOUT THE X-AXIS. (2F)')
      ACCEPT *,INMAGX,DURAX
      IF (INMAGX.EQ.0.)DURAX=0.1      !AVOID DIVIDE BY ZERO
      TYPE 76
76    FORMAT(/,1X,'ENTER PEAK MAGNITUDE(RAD/(S*S)) AND')
      TYPE 77
77    FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR ANGULAR')
      TYPE 78
78    FORMAT(1X,'ACCELERATION ABOUT THE Y-AXIS. (2F)')
      ACCEPT *,INMAGY,DURAY
      IF (INMAGY.EQ.0.)DURAY=0.1      !AVOID DIVIDE BY ZERO
      TYPE 80
80    FORMAT(/,1X,'ENTER PEAK MAGNITUDE(RAD/(S*S)) AND')
      TYPE 81
81    FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR ANGULAR')
      TYPE 82
82    FORMAT(1X,'ACCELERATION ABOUT THE Z-AXIS. (2F)')
      ACCEPT *,INMAGZ,DURAZ
```

```
        IF (INMAGZ.EQ.0.)DURAZ=0.1      !AVOID DIVIDE BY ZERO
        TYPE 84
84      FORMAT(/,1X,'ENTER PEAK MAGNITUDE(G''S) AND')
        TYPE 85
85      FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR LINEAR')
        TYPE 86
86      FORMAT(1X,'ACCELERATION ALONG THE X-AXIS. (2F)')
        ACCEPT *,RMAGX,DURRX
        IF (RMAGX.EQ.0.)DURRX=0.1      !AVOID DIVIDE BY ZERO
        TYPE 88
88      FORMAT(/,1X,'ENTER PEAK MAGNITUDE(G''S) AND')
        TYPE 89
89      FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR LINEAR')
        TYPE 90
90      FORMAT(1X,'ACCELERATION ALONG THE Y-AXIS. (2F)')
        ACCEPT *,RMAGY,DURRY
        IF (RMAGY.EQ.0.)DURRY=0.1      !AVOID DIVIDE BY ZERO
        TYPE 91
91      FORMAT(/,1X,'ENTER PEAK MAGNITUDE(G''S) AND')
        TYPE 92
92      FORMAT(1X,'DURATION(SECONDS) OF HALF SINE PULSE FOR LINEAR')
        TYPE 93
93      FORMAT(1X,'ACCELERATION ALONG THE Z-AXIS. (2F)')
        ACCEPT *,RMAGZ,DURRZ
        IF (RMAGZ.EQ.0.)DURRZ=0.1      !AVOID DIVIDE BY ZERO


        GO TO 50

*************************************************************************

60      TYPE 61
61      FORMAT(/,1X,'ENTER NAME OF EXPERIMENTAL DATA FILE')
        ACCEPT 21,INFIL3
        TYPE 44
        ACCEPT 45,NS
        TYPE 46
        ACCEPT *,DTIME
        OPEN(UNIT=14,STATUS='OLD',FILE=INFIL3)
        DO 300 II=1,NS
300     READ(14,*)T(II),AX(II),BX(II),CX(II),DX(II),EX(II),FX(II)
        STIME=T(1)
        FTIME=T(NS)

*************************************************************************

50      TYPE51
51      FORMAT(/,1X,'ENTER NAME OF OUTPUT FILE TO BE CREATED.')
        ACCEPT 21,OUTFIL
        OPEN(UNIT=2,STATUS='NEW',FILE=OUTFIL)

C
        WRITE(2,98)
98      FORMAT(1X,'     STIME      DTIME      FTIME           R')
        V=0.D0
        R=1.5D0

        WRITE(2,100)STIME,DTIME,FTIME,R

        DO I=1,9
```

```
      WRITE(2,100)V,V,V,V,V,V,V
      END DO
100   FORMAT(1X,9(F9.4,1X))

      WRITE(2,105)
      WRITE(2,101)
      WRITE(2,102)
      WRITE(2,103)
      WRITE(2,104)
105   FORMAT(1X,' ')
101   FORMAT(1X,'     TIME')
102   FORMAT(1X,'     WX          WY          WZ')
103   FORMAT(1X,'     RX          RY          RZ          WDOTX       WDOTY
    &    WDOTZ')
104   FORMAT(1X,'     QDOTX       QDOTY       QDOTZ       GDOTX       GDOTY
    &    GDOTZ')
C
C
C   INITIALIZE ANGULAR VELOCITY TERMS, TIME, OTHER CONSTANTS
C
      WX=0.D0
      WY=0.D0
      WZ=0.D0

      TIME=STIME
      IJ=2
*****************************************************************

      NW=6
      N=6
      X=STIME
      XEND=STIME
      Y(1)=0.D0
      Y(2)=0.D0
      Y(3)=0.D0
      Y(4)=0.D0
      Y(5)=0.D0
      Y(6)=0.D0
      TOL=.0001
      IND=1


200   CALL DVERK(N,FCN1,X,Y,XEND,TOL,IND,C,NW,W,IER)

      IF(IND.LT.0.OR.IER.GT.0) GO TO 210

      WX=Y(1)
      WY=Y(2)
      WZ=Y(3)


      CALL OUTP

      TIME=TIME+DTIME                   !UPDATE TIME
      XEND=TIME
      ETIME=FTIME+DTIME/1000.

      IF(TIME.GT.ETIME)GO TO 210
```

```
      GOTO 200

210   STOP
      END

*****************************************************************
*****************************************************************
*     CALCULATE THE ESTIMATED ANGULAR ACCELERATIONS             *
*****************************************************************
*****************************************************************


      SUBROUTINE FCN1(N,X,Y,YPRIME)

      INTEGER N
      REAL*8 Y(N),YPRIME(N),X

      CHARACTER ANS1*1

      DOUBLE PRECISION RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,
     * INMAGX,INMAGY,INMAGZ,DURAX,DURAY,DURAZ,RMAGX,RMAGY,RMAGZ,DURRX,
     * DURRY,DURRZ,GDOTX,GDOTY,GDOTZ,QDOTX,QDOTY,QDOTZ,CI,D,EM,
     * TIME,T,AX,BX,CX,DX,EX,FX,GC,TMOD

      DIMENSION CI(6,6),D(6,6),EM(6,1),T(4000),AX(4000),BX(4000),
     * CX(4000),DX(4000),EX(4000),FX(4000)

      COMMON/BLK1/IJ
      COMMON/BLK2/TIME,DTIME
      COMMON/BLK3/RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,GDOTX,GDOTY,
     * GDOTZ,QDOTX,QDOTY,QDOTZ
      COMMON/BLK4/INMAGX,INMAGY,INMAGZ,DURAX,DURAY,DURAZ,RMAGX,
     * RMAGY,RMAGZ,DURRX,DURRY,DURRZ
      COMMON/BLK5/CI,D,EM
      COMMON/BLK6/ANS1
      COMMON/BLK7/T,AX,BX,CX,DX,EX,FX

      WX=Y(1)
      WY=Y(2)
      WZ=Y(3)

      GC=386.089D0

      IF(ANS1.NE.'A')GO TO 18

*************************************************************
*     CREATE HALF SINE PULSE INPUT FUNCTIONS                *
*************************************************************

      GDOTX=INMAGX*SIN(2.*3.1416*(1./(2.*DURAX))*TIME)
      GDOTY=INMAGY*SIN(2.*3.1416*(1./(2.*DURAY))*TIME)
      GDOTZ=INMAGZ*SIN(2.*3.1416*(1./(2.*DURAZ))*TIME)
      QDOTX=RMAGX*GC*SIN(2.*3.1416*(1./(2.*DURRX))*TIME)
      QDOTY=RMAGY*GC*SIN(2.*3.1416*(1./(2.*DURRY))*TIME)
      QDOTZ=RMAGZ*GC*SIN(2.*3.1416*(1./(2.*DURRZ))*TIME)

      IF(TIME.GT.DURAX)GDOTX=0.0
      IF(TIME.GT.DURAY)GDOTY=0.0
      IF(TIME.GT.DURAZ)GDOTZ=0.0
      IF(TIME.GT.DURRX)QDOTX=0.0
```

```
      IF(TIME.GT.DURRY)QDOTY=0.0
      IF(TIME.GT.DURRZ)QDOTZ=0.0

      GO TO 12


*************************************************************************
*     EXTRAPOLATION ROUTINE FOR HANDLING EXPERIMENTAL AND NAPLAB DATA   *
*************************************************************************

18    TMOD=TIME-DTIME/1000.           !TMOD WAS ADDED IN CASE TIME PICKED
      IF(T(IJ).GE.TMOD)GO TO 320      !UP A BIT ALONG THE WAY
      IJ=IJ+1
      GO TO 18
320   IK=IJ-1
      DTX=(TIME-T(IK))/(T(IJ)-T(IK))
      QDOTX=AX(IK)+(AX(IJ)-AX(IK))*DTX
      QDOTY=BX(IK)+(BX(IJ)-BX(IK))*DTX
      QDOTZ=CX(IK)+(CX(IJ)-CX(IK))*DTX
      GDOTX=DX(IK)+(DX(IJ)-DX(IK))*DTX
      GDOTY=EX(IK)+(EX(IJ)-EX(IK))*DTX
      GDOTZ=FX(IK)+(FX(IJ)-FX(IK))*DTX


***********************************************
*     CALCULATE CORRECTED OUTPUTS             *
***********************************************

12    WDOTX=-EM(1,1)+CI(1,1)*GDOTX+CI(1,2)*GDOTY+CI(1,3)*GDOTZ-
     2    D(1,1)*WX**2-D(1,2)*WY**2-D(1,3)*WZ**2-D(1,4)*WX*WY-
     3    D(1,5)*WX*WZ-D(1,6)*WY*WZ+CI(1,4)*QDOTX+CI(1,5)*QDOTY+
     4    CI(1,6)*QDOTZ
      WDOTY=-EM(2,1)+CI(2,1)*GDOTX+CI(2,2)*GDOTY+CI(2,3)*GDOTZ-
     2    D(2,1)*WX**2-D(2,2)*WY**2-D(2,3)*WZ**2-D(2,4)*WX*WY-
     3    D(2,5)*WX*WZ-D(2,6)*WY*WZ+CI(2,4)*QDOTX+CI(2,5)*QDOTY+
     4    CI(2,6)*QDOTZ
      WDOTZ=-EM(3,1)+CI(3,1)*GDOTX+CI(3,2)*GDOTY+CI(3,3)*GDOTZ-
     2    D(3,1)*WX**2-D(3,2)*WY**2-D(3,3)*WZ**2-D(3,4)*WX*WY-
     3    D(3,5)*WX*WZ-D(3,6)*WY*WZ+CI(3,4)*QDOTX+CI(3,5)*QDOTY+
     4    CI(3,6)*QDOTZ
C
C  CALCULATE THE ESTIMATED LINEAR ACCELERATIONS
C
      RX=-EM(4,1)+CI(4,1)*GDOTX+CI(4,2)*GDOTY+CI(4,3)*GDOTZ-
     2    D(4,1)*WX**2-D(4,2)*WY**2-D(4,3)*WZ**2-D(4,4)*WX*WY-
     3    D(4,5)*WX*WZ-D(4,6)*WY*WZ+CI(4,4)*QDOTX+CI(4,5)*QDOTY+
     4    CI(4,6)*QDOTZ
      RY=-EM(5,1)+CI(5,1)*GDOTX+CI(5,2)*GDOTY+CI(5,3)*GDOTZ-
     2    D(5,1)*WX**2-D(5,2)*WY**2-D(5,3)*WZ**2-D(5,4)*WX*WY-
     3    D(5,5)*WX*WZ-D(5,6)*WY*WZ+CI(5,4)*QDOTX+CI(5,5)*QDOTY+
     4    CI(5,6)*QDOTZ
      RZ=-EM(6,1)+CI(6,1)*GDOTX+CI(6,2)*GDOTY+CI(6,3)*GDOTZ-
     2    D(6,1)*WX**2-D(6,2)*WY**2-D(6,3)*WZ**2-D(6,4)*WX*WY-
     3    D(6,5)*WX*WZ-D(6,6)*WY*WZ+CI(6,4)*QDOTX+CI(6,5)*QDOTY+
     4    CI(6,6)*QDOTZ

      YPRIME(1)=WDOTX
      YPRIME(2)=WDOTY
```

```
      YPRIME(3)=WDOTZ
      YPRIME(4)=RX
      YPRIME(5)=RY
      YPRIME(6)=RZ

      RETURN
      END


**************************************************
**************************************************
*       WRITE OUTPUT FILE                        *
**************************************************
**************************************************


      SUBROUTINE OUTP

      INTEGER N,IND,NW,IER
      REAL*8 Y(6),C(24),W(6,9),X,TOL,XEND,YPRIME(6)


      DOUBLE PRECISION RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,
     *   GDOTX,GDOTY,GDOTZ,QDOTX,QDOTY,QDOTZ,TIME,GC


      COMMON/BLK2/TIME,DTIME
      COMMON/BLK3/RX,RY,RZ,WDOTX,WDOTY,WDOTZ,WX,WY,WZ,GDOTX,GDOTY,
     *   GDOTZ,QDOTX,QDOTY,QDOTZ



      WRITE(2,100)TIME
      WRITE(2,100)WX,WY,WZ

      GC=386.089D0


      RX=RX/GC                 !CONVERT TO G'S
      RY=RY/GC                 !      "
      RZ=RZ/GC                 !      "
      WRITE(2,100)RX,RY,RZ,WDOTX,WDOTY,WDOTZ

100   FORMAT(1X,9(G12.4,1X))

      QDOTX=QDOTX/GC           !CONVERT TO G'S
      QDOTY=QDOTY/GC           !      "
      QDOTZ=QDOTZ/GC           !      "
      WRITE(2,100)QDOTX,QDOTY,QDOTZ,GDOTX,GDOTY,GDOTZ

      RETURN
      END
```

APPENDIX E

ERROR TERM MATRICES

Error matrices have the following format:

```
STIME,DTIME,FTIME,R
E010,E011,E012,E013,S011,S012,S013
E020,E021,E022,E023,S021,S022,S023
E030,E031,E032,E033,S031,S032,S033
E120,E121,E122,E123,S121,S122,S123
E130,E131,E132,E133,S131,S132,S133
E210,E211,E212,E213,S211,S212,S213
E230,E231,E232,E233,S231,S232,S233
E310,E311,E312,E313,S311,S312,S313
E320,E321,E322,E323,S321,S322,S323
```

Where:

STIME = The start time of the input pulse.

DTIME = The sample interval of the input pulse.

FTIME = The finish time of the input pulse.

EIJK = ε_{ijk} which is error type k (0 - bias plus noise, 1 = uncertainty in scale factor, 2,3 = cross axis sensitivity) for accelerometer measuring in the j direction (1=x, 2=y, 3=z) situated at location i (see Figure 1.1).

SIJK = δ_{ijk} which is location error in k direction for accelerometer measuring in j direction situated at location i.

```
0,0.001,0.1,1.5
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0

* ERTERM000.DAT
* NULL SET ASSUMES PERFECT MOUNT AND PERFECT TRANSDUCERS
```

```
0,0.001,0.1,1.5
0,0,0,0,-.09,.25,.20
0,0,0,0,.25,-.09,.20
0,0,0,0,.25,.25,-.09
0,0,0,0,.25,-.09,.20
0,0,0,0,.25,.25,-.09
0,0,0,0,-.09,.25,.20
0,0,0,0,.25,.25,-.09
0,0,0,0,-.09,.25,.20
0,0,0,0,.25,-.09,.20

* ERTERM001.DAT
* STANDARD ENDEVCO MOUNT
* PRINCIPAL AXES COINCIDE WITH EDGES OF MOUNT
* ORIGIN IS AT APEX OF MOUNT
* R=1.5"
* ASSUMES PERFECT ACCELEROMETERS
```

```
0,0.001,0.1,1.5
0,0,0,0,-.34,0,0
0,0,0,0,0,-.34,0
0,0,0,0,0,0,-.29
0,0,0,0,0,-.34,0
0,0,0,0,0,0,-.29
0,0,0,0,-.34,0,0
0,0,0,0,0,0,-.29
0,0,0,0,-.34,0,0
0,0,0,0,0,-.34,0

* ERTERM002.DAT
* STANDARD ENDEVCO MOUNT
* PRINCIPAL AXES ARE ALONG THE LINES OF ACTION OF THE THREE
  TRANSDUCERS AT THE ORIGIN.  THE ORIGIN IS AT THE INTERSECTION
  OF THESE LINES OF ACTION.
* R=1.5
* ASSUMES PERFECT ACCELEROMETERS
```

```
0,0.001,0.1,1.5
0,0,0,0,-.025,-.025,0
0,0,0,0,-.025,-.025,0
0,0,0,0,-.025,0,-.025
0,0,0,0,.025,-.025,0
0,0,0,0,.025,0,-.025
0,0,0,0,-.025,.025,0
0,0,0,0,-.025,0,-.025
0,0,0,0,-.025,-.025,0
0,0,0,0,-.025,-.025,0

* ERTERM003.DAT
* MATRIX WITH UNCERTAINTY IN SEISMIC CENTER LOCATIONS
  FROM ENDEVCO BLUEPRINT
* ALL DIMENSIONS ARE INCREASED FOR MAXIMUM UNCERTAINTY
* THE UNCERTAINTY IS WITH RESPECT TO THE NULL SET
* R=1.5"
* ASSUMES PERFECT TRANSDUCERS


0,0.001,0.1,1.5
0,0,0,0,-.115,.225,.20
0,0,0,0,.225,-.115,.20
0,0,0,0,.225,.25,-.115
0,0,0,0,.275,-.115,.20
0,0,0,0,.275,.25,-.115
0,0,0,0,-.115,.275,.20
0,0,0,0,.225,.25,-.115
0,0,0,0,-.115,.225,.20
0,0,0,0,.225,-.115,.20

* ERTERM004.DAT
* MATRIX WITH UNCERTAINTY IN SEISMIC CENTER LOCATIONS
  FROM ENDEVCO BLUEPRINT
* MATRICES ERTERM001 + ERTERM003
* R=1.5"
* ASSUMES PERFECT TRANSDUCERS


0,0.001,0.1,1.5
0,0,0,0,-.115,.225,.20
386.089,0,0,0,.225,-.115,.20
0,0,0,0,.225,.25,-.115
0,0,0,0,.275,-.115,.20
0,0,0,0,.275,.25,-.115
0,0,0,0,-.115,.275,.20
0,0,0,0,.225,.25,-.115
0,0,0,0,-.115,.225,.20
0,0,0,0,.225,-.115,.20

* ERTERM005.DAT
* MATRIX WITH UNCERTAINTY IN SEISMIC CENTER LOCATIONS
  FROM ENDEVCO BLUEPRINT
* MATRICES ERTERM001 + ERTERM003 WITH THE ADDITION OF
  E020 = 386.089 = 1G
* R=1.5"
* ASSUMES PERFECT TRANSDUCERS
```

```
0,0.001,0.1,1.5
0,0,0,0,0,0,0
386.089,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0

* ERTERM006.DAT
* NULL SET ASSUMES PERFECT MOUNT AND PERFECT TRANSDUCERS
  WITH THE EXCEPTION OF A 1G BIAS ON THE Y-TRANSDUCER AT THE ORIGIN.
```

```
0,0.001,0.1,1.5
0,0,0,0,-.365,-.025,0
0,0,0,0,-.025,-.365,0
0,0,0,0,-.025,0,-.315
0,0,0,0,.025,-.365,0
0,0,0,0,.025,0,-.315
0,0,0,0,-.365,.025,0
0,0,0,0,-.025,0,-.315
0,0,0,0,-.365,-.025,0
0,0,0,0,-.025,-.365,0

* ERTERM007.DAT
* MATRIX WITH UNCERTAINTY IN SEISMIC CENTER LOCATIONS
  FROM ENDEVCO BLUEPRINT
* ALL DIMENSIONS ARE INCREASED FOR MAXIMUM UNCERTAINTY
* MATRICES ERTERM002 + ERTERM003
* R=1.5"
* ASSUMES PERFECT TRANSDUCERS
```

```
0,0.001,0.1,1.5
0,0,0,0,0,0,.29
0,0,0,0,.34,-.34,.29
0,0,0,0,.34,0,0
0,0,0,0,.34,-.34,.29
0,0,0,0,.34,0,0
0,0,0,0,0,0,.29
0,0,0,0,.34,0,0
0,0,0,0,0,0,.29
0,0,0,0,.34,-.34,.29

* ERTERM008.DAT
* MATRIX WITH AXES PASSING THROUGH THE SEIZMIC CENTERS IN THE X-Z PLANE
  FROM ENDEVCO BLUEPRINT
* R=1.5"
* ASSUMES PERFECT TRANSDUCERS
```

```
0,0.001,0.1,1.5
0,0,0,0,0,.34,.29
0,0,0,0,.34,0,.29
0,0,0,0,.34,.34,0
0,0,0,0,.34,0,.29
0,0,0,0,.34,.34,0
0,0,0,0,0,.34,.29
0,0,0,0,.34,.34,0
0,0,0,0,0,.34,.29
0,0,0,0,.34,0,.29

* ERTERM009.DAT
* MATRIX WITH ALL THREE PLANES PASSING THROUGH SEIZMIC CENTERS
  FROM ENDEVCO BLUEPRINT
* R=1.5"
* ASSUMES PERFECT TRANSDUCERS


0,0.001,0.1,1.5
0,0,0,0,.203,0,0
0,0,0,0,0,.203,0
0,0,0,0,0,0,.203
0,0,0,0,0,.203,0
0,0,0,0,0,0,.203
0,0,0,0,.203,0,0
0,0,0,0,0,0,.203
0,0,0,0,.203,0,0
0,0,0,0,0,.203,0

* ERTERM010.DAT
* PRINCIPAL AXES ARE ALONG THE LINES OF ACTION OF THE THREE
  TRANSDUCERS AT THE ORIGIN.  THE ORIGIN IS AT THE INTERSECTION
  OF THESE LINES OF ACTION.
  DEFAULT GEOMETRY FROM ROBERT DENTON BLUEPRINT
* R=1.5" (FOR THE SAKE OF COMPARISON WITH OTHER MATRICES)
* ASSUMES PERFECT TRANSDUCERS



0,0.001,0.1,1.5
0,0,0,0,0,-.203,-.203
0,0,0,0,-.203,0,-.203
0,0,0,0,-.203,-.203,0
0,0,0,0,-.203,0,-.203
0,0,0,0,-.203,-.203,0
0,0,0,0,0,-.203,-.203
0,0,0,0,-.203,-.203,0
0,0,0,0,0,-.203,-.203
0,0,0,0,-.203,0,-.203

* ERTERM011.DAT
* MATRIX WITH ORIGEN OF THE SYSTEM MOVED +.203X, +.203Y AND +.203Z
  FROM ROBERT DENTON BLUEPRINT
* R=1.5" (FOR THE SAKE OF COMPARISON WITH OTHER MATRICES)
* ASSUMES PERFECT TRANSDUCERS
```

```
0,0.001,0.1,1.5
0,0,.05,.05,0,0,0
0,0,0,0,0,0,0
0,0,.05,.05,0,0,0
0,0,0,0,0,0,0
0,0,.05,.05,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,.05,-.05,0,0,0
0,0,0,0,0,0,0

* ERTERM012.DAT
* ASSUMES PERFECT MOUNT GEOMETRY
* FOUR TRANSDUCERS IN THE X-Z PLANE HAVE CROSS AXIS SENSITIVITIES
  OF 5% IN BOTH DIRECTIONS - SIGNS SET FOR MAXIMUM ERROR
* R = 1.5"
```

```
0,0.001,0.1,1.5
0,0,.04,.04,0,0,0
0,0,0,0,0,0,0
0,0,.04,.04,0,0,0
0,0,0,0,0,0,0
0,0,.04,.04,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,.04,-.04,0,0,0
0,0,0,0,0,0,0

* ERTERM013.DAT
* ASSUMES PERFECT MOUNT GEOMETRY
* FOUR TRANSDUCERS IN THE X-Z PLANE HAVE CROSS AXIS SENSITIVITIES
  OF 4% IN BOTH DIRECTIONS - SIGNS SET FOR MAXIMUM ERROR
* R = 1.5"
```

```
0,0.001,0.1,1.5
0,0,.03,.03,0,0,0
0,0,0,0,0,0,0
0,0,.03,.03,0,0,0
0,0,0,0,0,0,0
0,0,.03,.03,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,.03,-.03,0,0,0
0,0,0,0,0,0,0

* ERTERM014.DAT
* ASSUMES PERFECT MOUNT GEOMETRY
* FOUR TRANSDUCERS IN THE X-Z PLANE HAVE CROSS AXIS SENSITIVITIES
  OF 3% IN BOTH DIRECTIONS - SIGNS SET FOR MAXIMUM ERROR
* R = 1.5"
```

```
0,0.001,0.1,1.5
0,0,.02,.02,0,0,0
0,0,0,0,0,0,0
0,0,.02,.02,0,0,0
0,0,0,0,0,0,0
0,0,.02,.02,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,.02,-.02,0,0,0
0,0,0,0,0,0,0

* ERTERM015.DAT
* ASSUMES PERFECT MOUNT GEOMETRY
* FOUR TRANSDUCERS IN THE X-Z PLANE HAVE CROSS AXIS SENSITIVITIES
  OF 2% IN BOTH DIRECTIONS - SIGNS SET FOR MAXIMUM ERROR
* R = 1.5"
```

```
0,0.001,0.1,1.5
0,0,.01,.01,0,0,0
0,0,0,0,0,0,0
0,0,.01,.01,0,0,0
0,0,0,0,0,0,0
0,0,.01,.01,0,0,0
0,0,0,0,0,0,0
0,0,0,0,0,0,0
0,0,.01,-.01,0,0,0
0,0,0,0,0,0,0

* ERTERM016.DAT
* ASSUMES PERFECT MOUNT GEOMETRY
* FOUR TRANSDUCERS IN THE X-Z PLANE HAVE CROSS AXIS SENSITIVITIES
  OF 1% IN BOTH DIRECTIONS - SIGNS SET FOR MAXIMUM ERROR
* R = 1.5"
```

```
0.,.0005,.01,2.5
0.,0.,0.,0.,.203,0.,0.
0.,0.,0.,0.,0.,.203,0.
0.,0.,0.,0.,0.,0.,.203
0.,0.,0.,0.,0.,.203,0.
0.,0.,0.,0.,0.,0.,.203
0.,0.,0.,0.,.203,0.,.04
0.,0.,0.,0.,0.,0.,.243
0.,0.,0.,0.,.203,.04,0.
0.,0.,0.,0.,0.,.243,0.

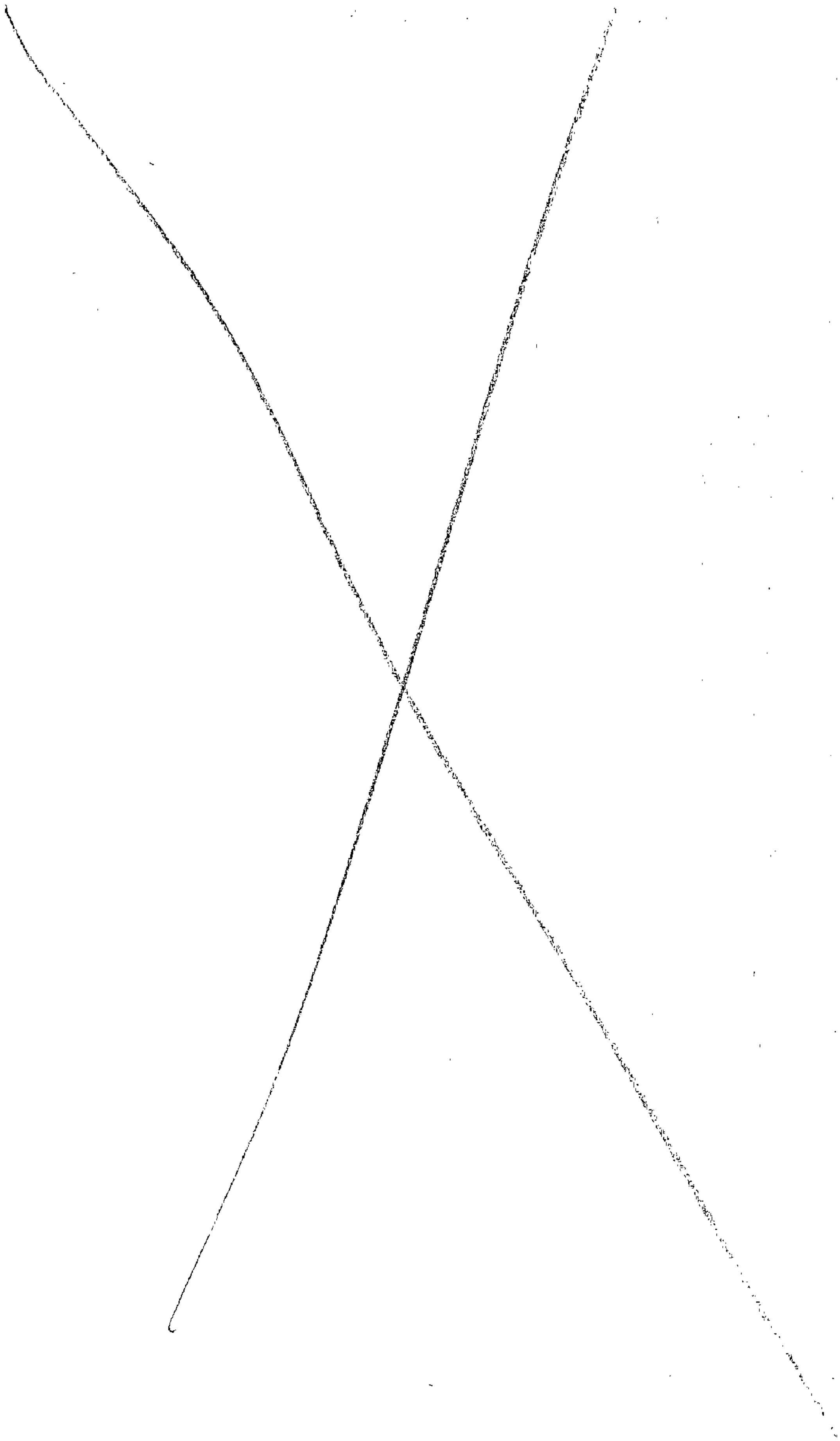
* ERTERMA.DAT
* Experimental matrix
* Rotation corresponding to .04 inch at one end of 2.5 inch arm
  (0.92 degrees)
```

```
0.,.0005,.01,2.5
0.,0.,0.,0.,0.,0.,0.
0.,0.,0.,0.,0.,0.,0.
0.,0.,0.,0.,0.,0.,0.
0.,0.,0.,0.,0.,0.,0.
0.,0.,0.,0.,0.,0.,0.
0.,0.,0.,0.,0.,0.,.04
0.,0.,0.,0.,0.,0.,.04
0.,0.,0.,0.,0.,.04,0.
0.,0.,0.,0.,0.,.04,0.
```

* ERTERMB.DAT
* Experimental matrix
* Rotation corresponding to .04 inch at one end of 2.5 inch arm (0.92 degrees)
* Otherwize presumes perfect locations for transducers (no geometric offsets)

APPENDIX F

MATRIX INVERSION SUBROUTINE



```
************************************************************************
*       MATINVSUB.FOR                                                  *
*                                                                      *
*       SUBROUTINE TO CALCULATE THE INVERSE OF THE 6 BY 6              *
*       [A+I] MATRIX.                                                  *
*                                                                      *
************************************************************************

        SUBROUTINE MATINVSUB(A,INFIL)

        DIMENSION A(6,6)
        DOUBLE PRECISION A,X

        DO L=1,6
        X=A(L,L)
        IF(X.NE.0.0)GO TO 200
        TYPE 170
 170    FORMAT(///,1X,'SINGULAR MATRIX!')
        STOP
 200    A(L,L)=1.0
        DO J=1,6
        A(L,J)=A(L,J)/X
        END DO
        DO I=1,6
        IF(I.EQ.L)GO TO 220
        X=A(I,L)
        A(I,L)=0.0
        DO J=1,6
        A(I,J)=A(I,J)-X*A(L,J)
        END DO
 220    END DO
        END DO

        RETURN

        END
```

APPENDIX G

COMPUTER PROGRAM FOR CREATING LABORATORY SYSTEM MATRICES FROM ERROR TERM MATRICES

```
************************************************************************
* CREATMAT.FOR                                                         *
*                                                                      *
* PROGRAM TO CREATE NAP SYSTEM MATRICES [A+I], [B] AND [BI]            *
* AS WELL AS [C], [D] AND [E] FROM AN ERROR FILE.                      *
*                                                                      *
* THE [A+I], [B], [BI], [C], [D] AND [E] MATRICES WILL BE              *
* WRITTEN TO A SINGLE FILE IN THAT ORDER.                              *
************************************************************************

        CHARACTER*16 ERFIL,OUTFIL
        DIMENSION A(6,6),B(6,6),BI(6,6),C(6,6),D(6,6),E(6,6)
        DOUBLE PRECISION A,B,BI,C,D,E,X



C       ERROR FILE
        TYPE 30
30      FORMAT(1X,'ENTER NAME OF ERROR FILE DESIRED.')
        ACCEPT 31,ERFIL
31      FORMAT(A16)
        OPEN(UNIT=1,FILE=ERFIL,STATUS='OLD')
        TYPE 32
32      FORMAT(1X,' ')
        TYPE37
37      FORMAT(1X,'ENTER NAME OF OUTPUT FILE TO BE CREATED.')
        ACCEPT 31,OUTFIL
        OPEN(UNIT=2,FILE=OUTFIL,STATUS='NEW')

        READ(1,*) STIME,DTIME,FTIME,R
C
C     ERROR TERMS
C
        READ(1,*) E010,E011,E012,E013,S011,S012,S013
        READ(1,*) E020,E021,E022,E023,S021,S022,S023
        READ(1,*) E030,E031,E032,E033,S031,S032,S033
        READ(1,*) E120,E121,E122,E123,S121,S122,S123
        READ(1,*) E130,E131,E132,E133,S131,S132,S133
        READ(1,*) E210,E211,E212,E213,S211,S212,S213
        READ(1,*) E230,E231,E232,E233,S231,S232,S233
        READ(1,*) E310,E311,E312,E313,S311,S312,S313
        READ(1,*) E320,E321,E322,E323,S321,S322,S323

C
C  CALCULATE THE A,B,C VALUES FOR ANGULAR ACCELERATION
C
        AA0=(E230-E030-E320+E020)/(2*R)
        AA1=(E233+E322)/2+(S232-S032+S323-S023)/(2*R)
        AA2=-E321/2+((S231-S031)/(2*R))
        AA3=-E231/2-((S321-S021)/(2*R))
        AA4=(E323-E232)/2+((S322-S022+S033-S233)/(2*R))
        AA5=E323/2+(S033-S233)/(2*R)
        AA6=-E232/2+(S322-S022)/(2*R)
        AA7=E231/2+(S021-S321)/(2*R)
        AA8=-E321/2+(S231-S031)/(2*R)
        AA9=(E233-E322)/2+(S232-S032-S323+S023)/(2*R)
        AA10=(E231-E031-E321+E021)/(2*R)
        AA11=(E232-E032-E322+E022)/(2*R)
        AA12=(E233-E033-E323+E023)/(2*R)
```

```
C
      BA0=(E310-E010-E130+E030)/(2*R)
      BA1=E312/2+(S032-S132)/(2*R)
      BA2=(E311+E133)/2+(S313-S013+S131-S031)/(2*R)
      BA3=-E132/2+(S012-S312)/(2*R)
      BA4=-E313/2+(S133-S033)/(2*R)
      BA5=(E131-E313)/2+(S011-S311+S133-S033)/(2*R)
      BA6=E131/2+(S011-S311)/(2*R)
      BA7=-E132/2+(S312-S012)/(2*R)
      BA8=(E311-E133)/2+(S313-S013-S131+S031)/(2*R)
      BA9=E312/2+(S032-S132)/(2*R)
      BA10=(E311-E011-E131+E031)/(2*R)
      BA11=(E312-E012-E132+E032)/(2*R)
      BA12=(E313-E013-E133+E033)/(2*R)
C
      CA0=(E120-E010-E210+E010)/(2*R)
      CA1=-E213/2+(S023-S123)/(2*R)
      CA2=-E123/2+(S013-S213)/(2*R)
      CA3=(E122+E211)/2+(S121-S021+S212-S012)/(2*R)
      CA4=E212/2+(S022-S122)/(2*R)
      CA5=-E121/2+(S211-S011)/(2*R)
      CA6=(E212-E121)/2+(S022-S122+S211-S011)/(2*R)
      CA7=(E122-E211)/2+(S121-S021-S212+S012)/(2*R)
      CA8=E123/2+(S013-S213)/(2*R)
      CA9=-E213/2+(S123-S023)/(2*R)
      CA10=(E121-E021-E211+E011)/(2*R)
      CA11=(E122-E022-E212+E012)/(2*R)
      CA12=(E123-E023-E213+E013)/(2*R)

C
C  CALCULATE THE A,B,C VALUES FOR LINEAR ACCELERATION
C           AL0-AL12,BL0-BL12,CL0-CL12
C
      AL0=E010
      AL1=-S013*E012+S012*E013
      AL2=S013+S013*E011-S011*E013
      AL3=-S012-S012*E011+S011*E012
      AL4=-S012*E012-S013*E013
      AL5=-S011-S011*E011-S013*E013
      AL6=-S011-S011*E011-S012*E012
      AL7=S012+S012*E011+S011*E012
      AL8=S013+S013*E011+S011*E013
      AL9=S013*E012+S012*E013
      AL10=E011
      AL11=E012
      AL12=E013
C
      BL0=E020
      BL1=-S023-S023*E022+S022*E023
      BL2=S023*E021-S021*E023
      BL3=-S022*E021+S021+S021*E022
      BL4=-S022-S022*E022-S023*E023
      BL5=-S021*E021-S023*E023
      BL6=-S021*E021-S022-S022*E022
      BL7=S022*E021+S021+S021*E022
      BL8=S023*E021+S021*E023
      BL9=S023+S023*E022+S022*E023
      BL10=E021
      BL11=E022
      BL12=E023
```

```
C
      CL0=E030
      CL1=-S033*E032+S032+S032*E033
      CL2=S033*E031-S031-S031*E033
      CL3=-S032*E031+S031*E032
      CL4=-S032*E032-S033-S033*E033
      CL5=-S031*E031-S033-S033*E033
      CL6=-S031*E031-S032*E032
      CL7=S032*E031+S031*E032
      CL8=S033*E031+S031+S031*E033
      CL9=S033*E032+S032+S032*E033
      CL10=E031
      CL11=E032
      CL12=E033

C
      AAI1=AA1+1.0
      BAI2=BA2+1.0
      CAI3=CA3+1.0
      ALI10=AL10+1.0
      BLI11=BL11+1.0
      CLI12=CL12+1.0


C     WRITE FILE FOR [A+I] MATRIX

      WRITE(2,100)AAI1,AA2,AA3,AA10,AA11,AA12
      WRITE(2,100)BA1,BAI2,BA3,BA10,BA11,BA12
      WRITE(2,100)CA1,CA2,CAI3,CA10,CA11,CA12
      WRITE(2,100)AL1,AL2,AL3,ALI10,AL11,AL12
      WRITE(2,100)BL1,BL2,BL3,BL10,BLI11,BL12
      WRITE(2,100)CL1,CL2,CL3,CL10,CL11,CLI12
      WRITE(2,110)
 110  FORMAT(1X)

C     WRITE FILE FOR [B] MATRIX

      WRITE(2,100)AA4,AA5,AA6,AA7,AA8,AA9
      WRITE(2,100)BA4,BA5,BA6,BA7,BA8,BA9
      WRITE(2,100)CA4,CA5,CA6,CA7,CA8,CA9
      WRITE(2,100)AL4,AL5,AL6,AL7,AL8,AL9
      WRITE(2,100)BL4,BL5,BL6,BL7,BL8,BL9
      WRITE(2,100)CL4,CL5,CL6,CL7,CL8,CL9
      WRITE(2,110)

      BI(1,1)=AA0
      BI(2,1)=BA0
      BI(3,1)=CA0
      BI(4,1)=AL0
      BI(5,1)=BL0
      BI(6,1)=CL0

C     WRITE  FILE FOR THE BIAS TERMS [BI]

      DO I=1,6
      WRITE(2,100)BI(I,1)
      END DO
      WRITE(2,110)
```

```
100     FORMAT(6F8.3)

        A(1,1)=AAI1
        A(1,2)=AA2
        A(1,3)=AA3
        A(1,4)=AA10
        A(1,5)=AA11
        A(1,6)=AA12

        A(2,1)=BA1
        A(2,2)=BAI2
        A(2,3)=BA3
        A(2,4)=BA10
        A(2,5)=BA11
        A(2,6)=BA12

        A(3,1)=CA1
        A(3,2)=CA2
        A(3,3)=CAI3
        A(3,4)=CA10
        A(3,5)=CA11
        A(3,6)=CA12

        A(4,1)=AL1
        A(4,2)=AL2
        A(4,3)=AL3
        A(4,4)=ALI10
        A(4,5)=AL11
        A(4,6)=AL12

        A(5,1)=BL1
        A(5,2)=BL2
        A(5,3)=BL3
        A(5,4)=BL10
        A(5,5)=BLI11
        A(5,6)=BL12

        A(6,1)=CL1
        A(6,2)=CL2
        A(6,3)=CL3
        A(6,4)=CL10
        A(6,5)=CL11
        A(6,6)=CLI12

        B(1,1)=AA4
        B(1,2)=AA5
        B(1,3)=AA6
        B(1,4)=AA7
        B(1,5)=AA8
        B(1,6)=AA9

        B(2,1)=BA4
        B(2,2)=BA5
        B(2,3)=BA6
        B(2,4)=BA7
        B(2,5)=BA8
        B(2,6)=BA9

        B(3,1)=CA4
        B(3,2)=CA5
```

```
      B(3,3)=CA6
      B(3,4)=CA7
      B(3,5)=CA8
      B(3,6)=CA9

      B(4,1)=AL4
      B(4,2)=AL5
      B(4,3)=AL6
      B(4,4)=AL7
      B(4,5)=AL8
      B(4,6)=AL9

      B(5,1)=BL4
      B(5,2)=BL5
      B(5,3)=BL6
      B(5,4)=BL7
      B(5,5)=BL8
      B(5,6)=BL9

      B(6,1)=CL4
      B(6,2)=CL5
      B(6,3)=CL6
      B(6,4)=CL7
      B(6,5)=CL8
      B(6,6)=CL9


*****************************************************************
*       CALCULATE THE INVERSE OF THE 6 BY 6 [A+I] MATRIX.       *
*****************************************************************

      DO L=1,6
      X=A(L,L)
      IF(X.NE.0.0)GO TO 200
      TYPE 170
 170  FORMAT(///,1X,'SINGULAR MATRIX!')
      STOP
 200  A(L,L)=1.0
      DO J=1,6
      A(L,J)=A(L,J)/X
      END DO
      DO I=1,6
      IF(I.EQ.L)GO TO 220
      X=A(I,L)
      A(I,L)=0.0
      DO J=1,6
      A(I,J)=A(I,J)-X*A(L,J)
      END DO
 220  END DO
      END DO

      DO I=1,6
      DO J=1,6
      C(I,J)=A(I,J)
      END DO
      END DO

C     WRITE FILE FOR [C] = INVERSE OF [A+I] MATRIX
```

```
      DO I=1,6
      WRITE(2,100)(C(I,J),J=1,6)
      END DO
      WRITE(2,110)
**********************************************************************


*************************************************************
*     FIND THE PRODUCT [D] = [C]*[B]                        *
*************************************************************

      DO I=1,6
      DO J=1,6
      D(I,J)=0.0
      DO K=1,6
      D(I,J)=D(I,J)+C(I,K)*B(K,J)
      END DO
      END DO
      END DO

C     WRITE A FILE FOR THE [D] MATRIX

      DO I=1,6
      WRITE(2,100)(D(I,J),J=1,6)
      END DO
      WRITE(2,110)

**************************************************************

***************************************************************
*     FIND THE PRODUCT [E] = [C]*[BI]                         *
***************************************************************


      DO I=1,6
      E(I,J)=0.0
      DO K=1,6
      E(I,1)=E(I,1)+C(I,K)*BI(K,1)
      END DO
      END DO

C     WRITE FILE FOR [E] MATRIX

      DO I=1,6
      WRITE(2,100)E(I,1)
      END DO

      STOP
      END
```

APPENDIX H

COMPUTER PROGRAM FOR CONVERTING LABORATORY AND FIELD SYSTEM OUTPUTS TO FILES CONTAINING SINGLE PARAMETER TIME HISTORIES

```
***************************************************************************
*                               CONV.FOR                                  *
*                                                                         *
*                                                                         *
*       PROGRAM TO CONVERT OUTPUT DATA FROM THE NAP PROGRAM INTO A FILE   *
*       FOR THE SPECIFIC PARAMETER DESIRED.  SAMPLE TIME AND THE          *
*       PARAMETER VALUES ARE WRITTEN TO THE NEW FILE.  THE NEW FILE       *
*       IS WRITTEN IN DOUBLE PRECISION AND MAY THEN BE USED AS INPUT      *
*       TO METAGRAPH.                                                     *
*                                                                         *
***************************************************************************

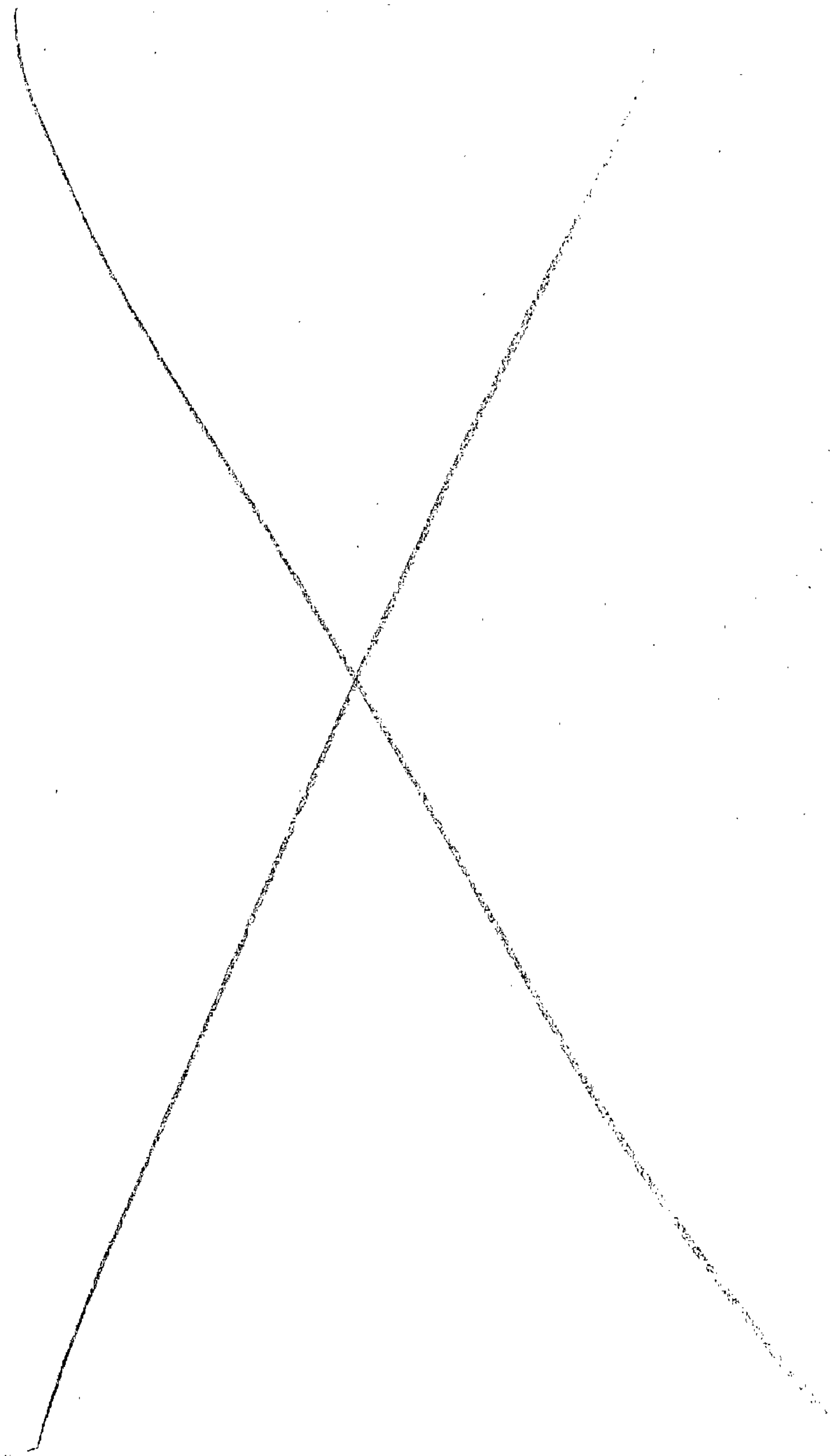

        DIMENSION Y(-40000:40000),C(-4000:4000)
        CHARACTER INFIL*16,OUTFIL*16,CLAB*21,YLAB(5)*60


        TYPE 100
100     FORMAT(1X,'ENTER NAME OF INPUT DATA FILE (A16)')
        ACCEPT 110,INFIL
110     FORMAT(A16)
        TYPE 120
120     FORMAT(1X,'ENTER NUMBER OF POINTS (SAMPLES IN TIME) (I)')
        ACCEPT 130,N
130     FORMAT(I)
        TYPE 175
175     FORMAT(1X,' ')
        TYPE 175
        TYPE 180
180     FORMAT(1X,'2  - WX')
        TYPE 181
181     FORMAT(1X,'3  - WY')
        TYPE 182
182     FORMAT(1X,'4  - WZ')
        TYPE 183
183     FORMAT(1X,'5  - RX')
        TYPE 184
184     FORMAT(1X,'6  - RY')
        TYPE 185
185     FORMAT(1X,'7  - RZ')
        TYPE 186
186     FORMAT(1X,'8  - WDOTX')
        TYPE 187
187     FORMAT(1X,'9  - WDOTY')
        TYPE 188
188     FORMAT(1X,'10 - WDOTZ')
        TYPE 189
189     FORMAT(1X,'11 - QDOTX')
        TYPE 190
190     FORMAT(1X,'12 - QDOTY')
        TYPE 191
191     FORMAT(1X,'13 - QDOTZ')
        TYPE 192
192     FORMAT(1X,'14 - GDOTX')
        TYPE 193
193     FORMAT(1X,'15 - GDOTY')
        TYPE 194
194     FORMAT(1X,'16 - GDOTZ')
        TYPE 175
        TYPE 196
```

```
196     FORMAT(1X,'ENTER A NUMBER FOR THE OUTPUT FILE DESIRED.')
        ACCEPT 130,L
        TYPE 140
140     FORMAT(1X,'ENTER NAME OF OUTPUT DATA FILE (A16)')
        ACCEPT 110,OUTFIL
C
C
        OPEN(UNIT=1,STATUS='OLD',FILE=INFIL)
        OPEN(UNIT=22,STATUS='NEW',FILE=OUTFIL)
C
C
        READ(1,150)CLAB
        READ(1,*)(C(I),I=1,67)
        READ(1,160)(YLAB(I),I=1,5)
        DO K=0,N-1
        READ(1,*)Y(K*16+1)
        READ(1,*)(Y(K*16+I),I=2,4)
        READ(1,*)(Y(K*16+I),I=5,10)
        READ(1,*)(Y(K*16+I),I=11,16)
        END DO
        DO K=0,N-1
        WRITE(22,172)Y(K*16+1),Y(K*16+L)
        END DO
150     FORMAT(A21)
160     FORMAT(A60)
172     FORMAT(1X,2(D23.16,1X))
        STOP
        END
```

APPENDIX I

COMPUTER PROGRAM FOR CONVERTING UDS FILES TO ASCII FILES



```
**************************************************************
*   PROGRAM UDS2ASCII          John E. Nickles               *
*                              Transportation Systems Center *
*                              20-March-1985                 *
*                                                            *
*   Short program to read UDS file and write an ASCII file.  *
*   Link with UDSIO.                                         *
**************************************************************

      DIMENSION Y(-1000:10000)
      CHARACTER*80 INFIL,OUTFIL
C
      INCLUDE '[ASGPROG]COMVAR.LIS'
C
      TYPE 100
      ACCEPT 110,INFIL
      TYPE 120
      ACCEPT 110,OUTFIL
C
      OPEN(UNIT=2,STATUS='NEW',FILE=OUTFIL)
C
      CALL UDSIO('R','F',INFIL,1,Y,IOS)
C
      TYPE *,'FORM=',FORM,', NFP=',NFP,', NLP=',NLP,', DEL=',DEL
      TYPE *,'PREF=',PREF,', FCUT=',FCUT,', FCOR=',FCOR,', FSTP=',FSTP
      TYPE *,'Y0=',Y0,', YD=',', YCAL=',YCAL
      WRITE(02,130)(Y(I),I=NFP,NLP)
      CLOSE(UNIT=1)
      CLOSE(UNIT=2)
      STOP
 100  FORMAT(' ENTER UDS INPUT FILE NAME. (A9.A3)')
 110  FORMAT(A)
 120  FORMAT(' ENTER ASCII OUTPUT FILE NAME. (A9.A3)')
 130  FORMAT(1X,6(G12.5,1X))
      END
```

APPENDIX J

COMPUTER PROGRAM FOR FORMATTING ASCII FILES

FOR PROCESSING WITH THE LABORATORY

AND FIELD MODELS

```
*****************************************************************************
*       EXPERFILE                                                           *
*                                                                           *
*       PROGRAM TO READ ASCII FORMAT OF UDS FILES AND CREATE A FILE THAT    *
*       CAN BE READ BY NAPLAB, NAPFLDEUL OR NAPFLDRK.                       *
*                                                                           *
*****************************************************************************

        CHARACTER*16 INFIL1,INFIL2,INFIL3,INFIL4,INFIL5,INFIL6,OUTFIL,ANS

        DIMENSION T(5000),A(5000),B(5000),C(5000),D(5000),E(5000),
     *  F(5000)

        GC=386.089


        TYPE 10
 10     FORMAT(/,1X,'ENTER MINIMUM NUMBER OF SAMPLES IN RECORDS (I)')
        ACCEPT 20,NS
 20     FORMAT(I)

 21     TYPE 22
 22     FORMAT(/,1X,'USE EVERY DATA POINT? (Y OR N)')
        ACCEPT 25,ANS
 25     FORMAT(A)

        IF(ANS.EQ.'Y')GO TO 37
        IF(ANS.EQ.'N')GO TO 27
        GO TO 21

 27     TYPE 28
 28     FORMAT(/,1X,'ENTER N FOR EVERY NTH POINT (I)')
        ACCEPT 20,N

        NSS=NS/N

 29     TYPE 30,NSS
 30     FORMAT(/,1X,'A FILE OF',I5,' POINTS WILL BE WRITTEN')

 37     TYPE 38
 38     FORMAT(/,1X,'ENTER SAMPLE TIME INTERVAL (SEC)')
        ACCEPT 40,DTIME
 40     FORMAT(F)

        TYPE 50
 50     FORMAT(/,1X,'ENTER NAME OF FILE FOR QDOTX(A)')
        ACCEPT 60,INFIL1
 60     FORMAT(A16)
        IF(INFIL1.EQ.' ')GO TO 80
        OPEN(UNIT=11,STATUS='OLD',FILE=INFIL1)

        READ(11,70)(A(I),I=1,NS)

        DO I=1,NS
        A(I)=GC*A(I)            !CHANGE TO INCHES/S*S
        END DO

 70     FORMAT(1X,6G12.5)
        GO TO 90
```

```
 80     DO I=1,NS
        A(I)=0.0
        END DO

 90     TYPE 100
100     FORMAT(/,1X,'ENTER NAME OF FILE FOR QDOTY(A)')
        ACCEPT 60,INFIL2
        IF(INFIL2.EQ.' ')GO TO 110
        OPEN(UNIT=12,STATUS='OLD',FILE=INFIL2)

        READ(12,70)(B(I),I=1,NS)

        DO I=1,NS
        B(I)=GC*B(I)              !CHANGE TO INCHES/S*S
        END DO

        GO TO 120
110     DO I=1,NS
        B(I)=0.0
        END DO

120     TYPE 130
130     FORMAT(/,1X,'ENTER NAME OF FILE FOR QDOTZ(A)')
        ACCEPT 60,INFIL3
        IF(INFIL3.EQ.' ')GO TO 140
        OPEN(UNIT=13,STATUS='OLD',FILE=INFIL3)

        READ(13,70)(C(I),I=1,NS)

        DO I=1,NS
        C(I)=GC*C(I)              !CHANGE TO INCHES/S*S
        END DO

        GO TO 150
140     DO I=1,NS
        C(I)=0.0
        END DO

150     TYPE 160
160     FORMAT(/,1X,'ENTER NAME OF FILE FOR GDOTX(A)')
        ACCEPT 60,INFIL4
        IF(INFIL4.EQ.' ')GO TO 170
        OPEN(UNIT=14,STATUS='OLD',FILE=INFIL4)
        READ(14,70)(D(I),I=1,NS)
        GO TO 180
170     DO I=1,NS
        D(I)=0.0
        END DO

180     TYPE 190
190     FORMAT(/,1X,'ENTER NAME OF FILE FOR GDOTY(A)')
        ACCEPT 60,INFIL5
        IF(INFIL5.EQ.' ')GO TO 200
        OPEN(UNIT=15,STATUS='OLD',FILE=INFIL5)
        READ(15,70)(E(I),I=1,NS)
        GO TO 210
200     DO I=1,NS
        E(I)=0.0
        END DO
```

```
210     TYPE 220
220     FORMAT(/,1X,'ENTER NAME OF FILE FOR GDOTZ(A)')
        ACCEPT 60,INFIL6
        IF(INFIL6.EQ.' ')GO TO 230
        OPEN(UNIT=16,STATUS='OLD',FILE=INFIL6)
        READ(16,70)(F(I),I=1,NS)
        GO TO 240
230     DO I=1,NS
        F(I)=0.0
        END DO


240     TYPE 250
250     FORMAT(/,1X,'ENTER DESIRED NAME OF OUTPUT FILE (A)')
        ACCEPT 60,OUTFIL
        OPEN(UNIT=20,STATUS='NEW',FILE=OUTFIL)

        TIME =0.0

        DO I=1,NS
        T(I)=TIME
        TIME=TIME+DTIME
        END DO

        DO I=1,NS

        D(I)=D(I)*1000.             ! SCALE ANGULAR ACCELERATION
        E(I)=E(I)*1000.             !           "
        F(I)=F(I)*1000.             !           "

        END DO

        IF(ANS.EQ.'N')GO TO 255

        DO I=1,NS
        WRITE(20,260)T(I),A(I),B(I),C(I),D(I),E(I),F(I)
        END DO

260     FORMAT(1X,7G13.6)

        STOP

255     DO K=0,NSS-1

        I=N*K+1

        WRITE(20,260)T(I),A(I),B(I),C(I),D(I),E(I),F(I)
        END DO


        STOP
        END
```

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