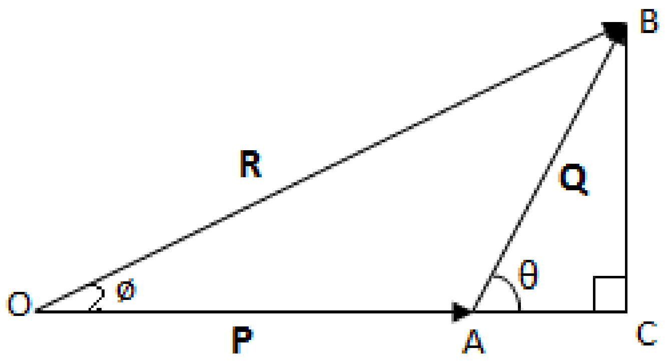
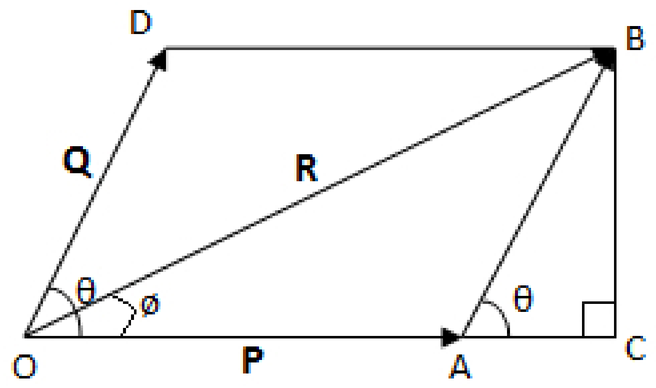
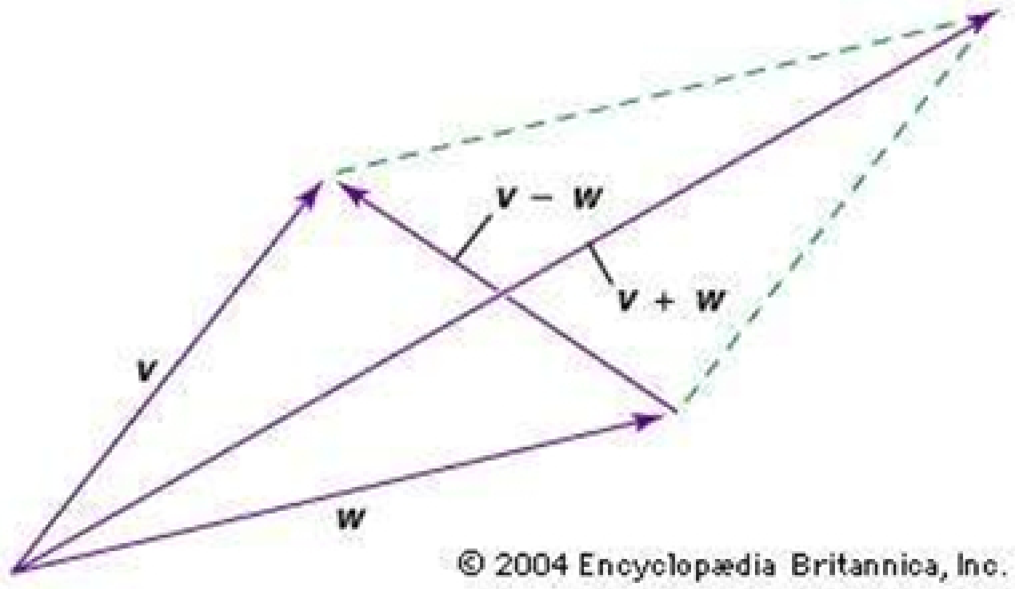
COURSE SUMMARY

Addition of vectors







Analytical method

$AC = Q\cos\theta$

$BC = Q\sin\theta$

$OB^2 = R^2 = OC^2 + BC^2$

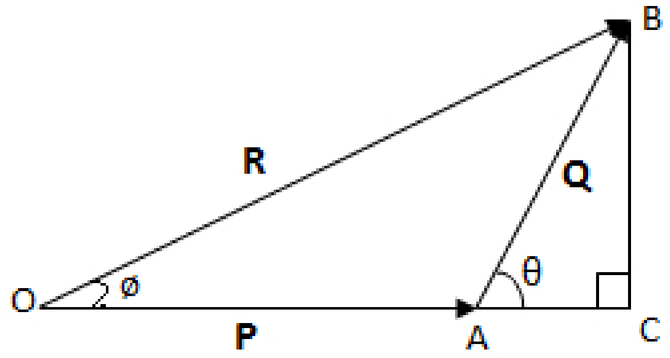
$= (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$

$= P^2 + Q^2 + 2PQ\cos\theta$

$\tan\phi = BC/OC$

$= (Q\sin\theta)/(P + Q\cos\theta)$



Special cases

- If θ = 0; **A**.**B** = AB (maximum value)
- If θ = 180; **A**.**B** = -AB (maximum negative value)
- If θ = 90; **A**.**B** = 0 (minimum value)
- If θ is acute; **A**.**B** is +ve
- If θ is obtuse; **A**.**B** is -ve

In component form

- i.i = j.j = k.k = 1
- i.j = j.i = j.k = k.j = i.k = k.i = 0

- **A** = 3i + 4j – 2k ; **B** = 5i – 3j + 2k
- **A**.**B** = 3i(5i – 3j + 2k) + 4j(5i – 3j + 2k) – 2k(5i – 3j + 2k)

= 15 – 12 – 4

= -1

properties

- It is commutative :

A.**B** = **B**.**A**

- It is distributive :

A.(**B** + **C**) = **A**.**B** + **A**.**C**

- Dot product of perpendicular vectors is zero.

A.**B** = ABcosθ = 0 (when θ=90)

- Dot product of a vector with itself

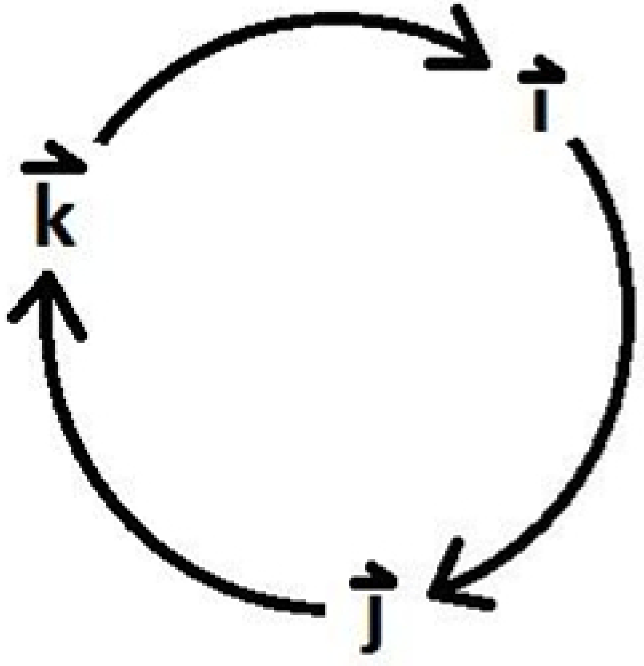
A.**A** = Aacosθ = A^2

Vector product

- $\mathbf{A} \times \mathbf{B} = AB \sin\theta\, n$
- $|\, \mathbf{A} \times \mathbf{B} \,| = AB \sin\theta$
- $n = (\mathbf{A} \times \mathbf{B}) / |\, \mathbf{A} \times \mathbf{B} \,|$

Cross product of unit vectors

- Multiplication of two clockwise gives third
- Multiplication of two anticlockwise gives negative of third
- i x j = k
- j x k = i
- k x i = j
- j x i = -k
- k x j = -i
- i x k = -j



Component form

A = 3i + 4j + 2k

B = 2i – 5j – 3k

A x **B** = (3i + 4j + 2k) x (2i – 5j – 3k)

= 3*2(i x i) – 3*5(i x j) – 3*3(i x k) + 4*2(j x i) – 4*5(j x j) – 4*3(j x k) + 2*2(k x i) – 2*5(k x j) – 2*3(k x k)

= 0 – 15k – 9(-j) + 8(-k) – 0 – 12i + 4j – 10(-i) – 0

= -2i + 13j – 23k

Determinant method

A = 3i + 4j + 2k

B = 2i – 5j – 3k

A x **B** = i(4*-3 – 2*-5) + j(2*2 – 3*-3) + k(3*-5 – 2*4)

= -2i + 13j – 23 k

$$\begin{vmatrix} i & j & k \\ 3 & 4 & 2 \\ 2 & -5 & -3 \end{vmatrix}$$

Properties

- Anti commutative property

B x **A** = -(**A** x **B**)

- Distributive property

A x (**B** + **C**) = **A** x **B** + **A** x **C**

- Associative property

(**A** + **B**) x (**C** + **D**) = **A** x **C** + **A** x **D** + **B** x **C** + **B** x **D**

- Cross product of two parallel vectors

angle = 0

so, **A** x **A** = 0