Enter your name and date as MuPad Text at the top of your MuPad worksheet. Staple your pages in their proper order.

USE MuPad TO DO EACH STEP. NOTHING SHOULD BE DONE BY HAND.

Set `DIGITS` to 20 at the top of your MuPad session to obtain greater precision.

1. We first we seek a special number — it is a positive number x whose reciprocal is 1 less than x. Write the equation for x that satisfies this statement, and use MuPad's `solve` command to determine the value of x. You will get two results, but we seek the positive value. Express it in decimal form, and store it as Phi.

2. Now consider the Fibonacci sequence, whose first members are

$$F_0 \;=\; 0\,, \quad F_1 \;=\; 1\,,$$

   and the subsequent members are determined by

$$F_k \;=\; F_{k-1} + F_{k-2}\,, \quad k = 2, 3, 4, \ldots \tag{1}$$

   Use a `for...do` loop to determine the terms $\{F_2, F_3, F_4, \ldots, F_{30}\}$ of the Fibonacci sequence.

   NOTE: In MuPad, you may denote a subscripted term like F_0 as `F[0]`, F_1 as `F[1]`,..., F_k as `F[k]`, etc.

3. Use a `for...do` loop to construct a new sequence $\{a_k\}$ by taking the ratio of successive pairs of Fibonacci numbers **in decimal form**:

$$a_k \;=\; \frac{F_k}{F_{k-1}}\,, \quad k = 2, 3, \ldots, 30\,. \tag{2}$$

   To what value do the ratios appear to converge? (State your answer in Text mode.)

The number $\Phi = 1.6180339887498948482045868343 65\ldots$ is called the *golden ratio*, *golden mean*, and *divine ratio*. Its occurrence in nature is abundant. The ratios of pairs of Fibonacci numbers closely measure the fraction of a turn between successive leaves on the stalk of a plant (called *phyllotaxis*): 1/2 for elm and linden, 1/3 for beech and hazel, 2/5 for oak and apple, 3/8 for poplar and rose, 5/13 for willow and almond, etc. (Coxeter 1969, Ball and Coxeter 1987). The Fibonacci numbers are sometimes called pine cone numbers (Pappas 1989, p. 224). The role of the Fibonacci numbers in botany is sometimes called Ludwig's law (Szymkiewicz 1928; Wells 1986, p. 66; Steinhaus 1999, p. 299).