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2-D PROBLEMS IN STRESS ANALYSIS

In stress or strain analysis we can model most the problems as 2-D, instead of 3-D analysis it is oftentimes better to use 1-D or 2-D representations of the actual stress or strain case.We can categorize the models as in the following diagram and we apply can employ these models under appropriate conditions.

PLANE STRESS PROBLEMS

$\sigma_{zz} = 0$

$\sigma_{yz} = 0$

$\sigma_{xz} = 0$

PLANE STRAIN PROBLEMS

$\epsilon_{zz} = 0$

$\epsilon_{yz} = 0$

$\epsilon_{xz} = 0$

$\partial/\partial z = 0$

AXISYMMETRIC PROBLEMS

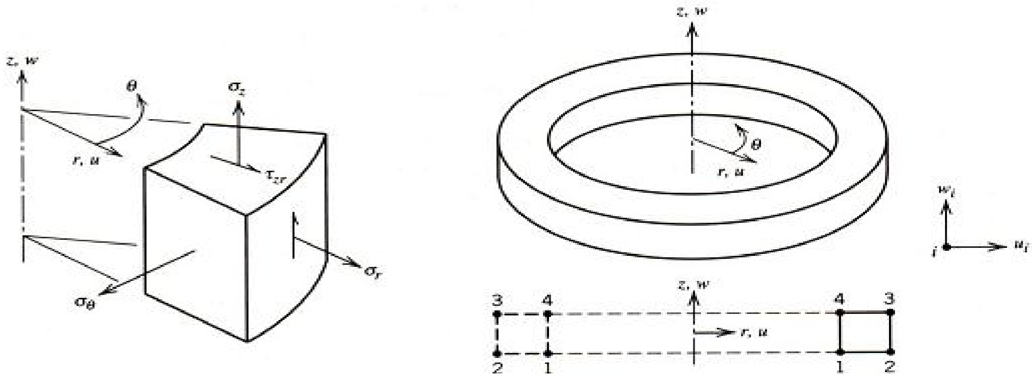
$\partial/\partial\theta = 0$

Fundamental 2-D Stress-Strain Models

Plane stress: Assume infinitely thin plate (x-y plane), loading is uniformly distrubuted over the thickness parallel to the plane of stress.

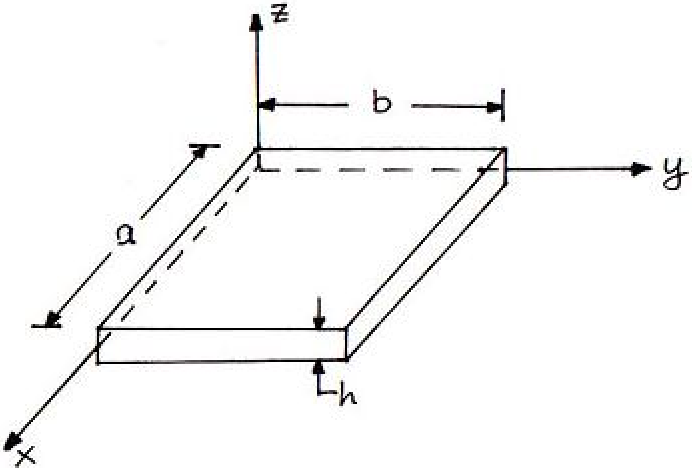
Plane strain: Assume element infinitely thin along z (x-y plane),all cross sections experience same deformation.

Axisymetric: Axisymmetric geometry and loads (r-z plane),at a constant radii load or strain conditions are the same along the circumference.In this problems polar coordinates are used.



(Axisymmetric figure)

PLANE STRESS PROBLEMS



The body has dimensions are such that

$$h << a, b$$

Thus, the plate is thin enough such that there is no variation of displacement (and temperature) with respect to z, it depends on x and y.Also the stresses in z direction are zero.

Equilibrium Equations

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \cancel{\frac{\partial \tau_{xz}}{\partial z}} + B_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \cancel{\frac{\partial \tau_{yz}}{\partial z}} + B_y = 0$$

$$\cancel{\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + B_z = 0}$$

Then we have plane stress equations as in the following manner:

1. $\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + B_x = 0$

2. $\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + B_y = 0$

3. $\epsilon_{xx} = \frac{\partial u}{\partial x}$

4. $\epsilon_{yy} = \frac{\partial v}{\partial y}$

5. $\epsilon_{xy} = \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$,

6. $\epsilon_{xx} = \frac{\sigma_{xx}}{E} - v \times \frac{\sigma_{yy}}{E}$

7. $\epsilon_{yy} = \frac{\sigma_{yy}}{E} - v \times \frac{\sigma_{xx}}{E}$

8. $\epsilon_{xy} = \frac{\tau_{xy}}{G}$

Note that although we have

$$\epsilon_{zz} = \frac{\partial w}{\partial z}$$

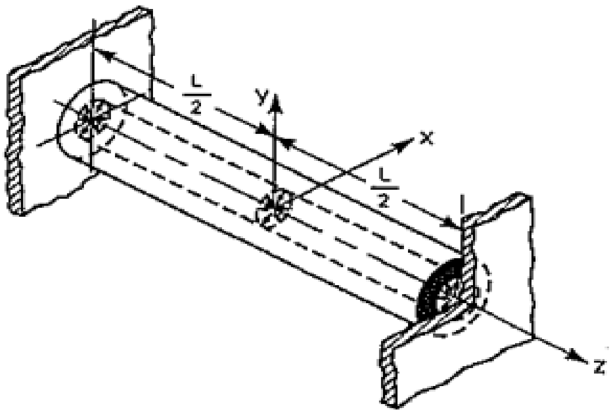
$$\epsilon_{zz} = \frac{-v}{E}(\sigma_{xx} + \sigma_{yy})$$

these equations,since they are uncoupled they are not used.

Solving equations (6) and (7) together for $(\sigma_{xx} + \sigma_{yy})$ and substituting the result into ϵ_{zz} we get:

$$\epsilon_{zz} = -\frac{v}{1-v}(\epsilon_{xx} + \epsilon_{yy})$$

PLANE STRAIN PROBLEM



Dimension in z direction is much larger than in x and y directions

L>>x,y

We consider a cylinder rod and assume it is subjected to lateral loading such as the pressure. Since the body is infinitely long important loads are in the x-y plane and external force is only a function of x and y, therefore we expect all cross sections to experience identical deformation.

$\frac{\partial}{\partial z} = 0 \Longrightarrow$ w=0 (no displacement gradient in z direction)

Since $\frac{\partial}{\partial z} = 0$ elasticity equations reduce to (1) and (2),here we note that σ_{xz} and σ_{yz} also exist,but not primarily used.

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + B_{y=0}$$

1. $\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + B_x = 0$
2. $\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + B_y = 0$
3. $\epsilon_{xx} = \frac{\partial u}{\partial x}$

4. $\epsilon_{yy} = \frac{\partial v}{\partial y}$

5. $\epsilon_{xy} = \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$

Note that :Our assumptions ($\frac{\partial}{\partial z} = 0$) and w=0 yield;

$\epsilon_{xz}=\epsilon_{yz}=\epsilon_{zz}=0$.

For the plane strain $\epsilon_{zz}=0$, substituting this in Hooke's law we can find σ_{zz} ;

$$\sigma_{zz} = \lambda(\epsilon_{xx} + \epsilon_{yy}) = v \times (\sigma_{xx} + \sigma_{yy})$$

Substituting σ_{zz} in Hookes law to obtain ϵ_{xx} and ϵ_{yy} we obtain the following equations:

6. $\epsilon_{xx} = \frac{1-v^2}{E}\left(\sigma_{xx} - \frac{v}{1-v} \times \sigma_{yy}\right)$

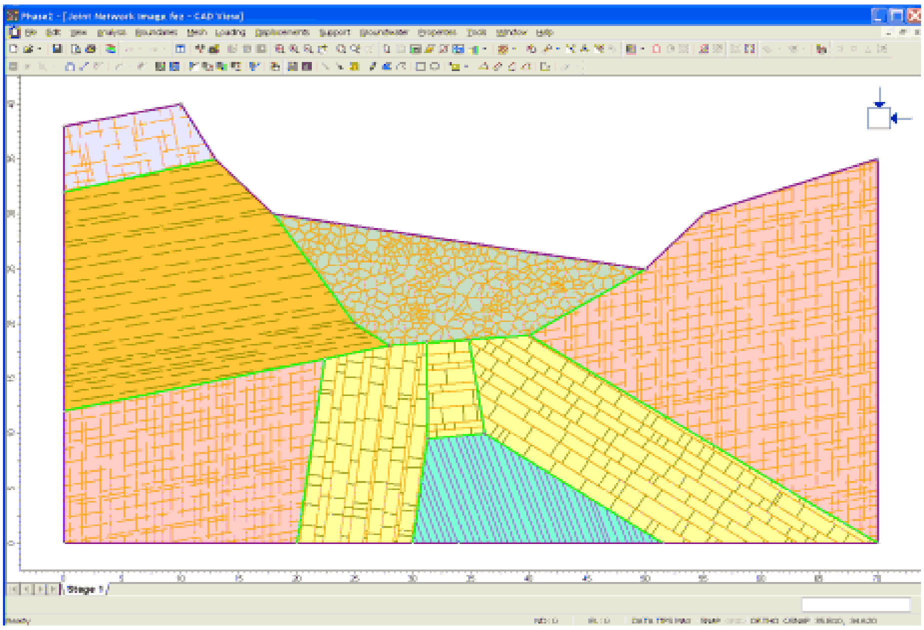
7. $\epsilon_{yy} = \frac{1-v^2}{E}\left(\sigma_{yy} - \frac{v}{1-v} \times \sigma_{xx}\right)$

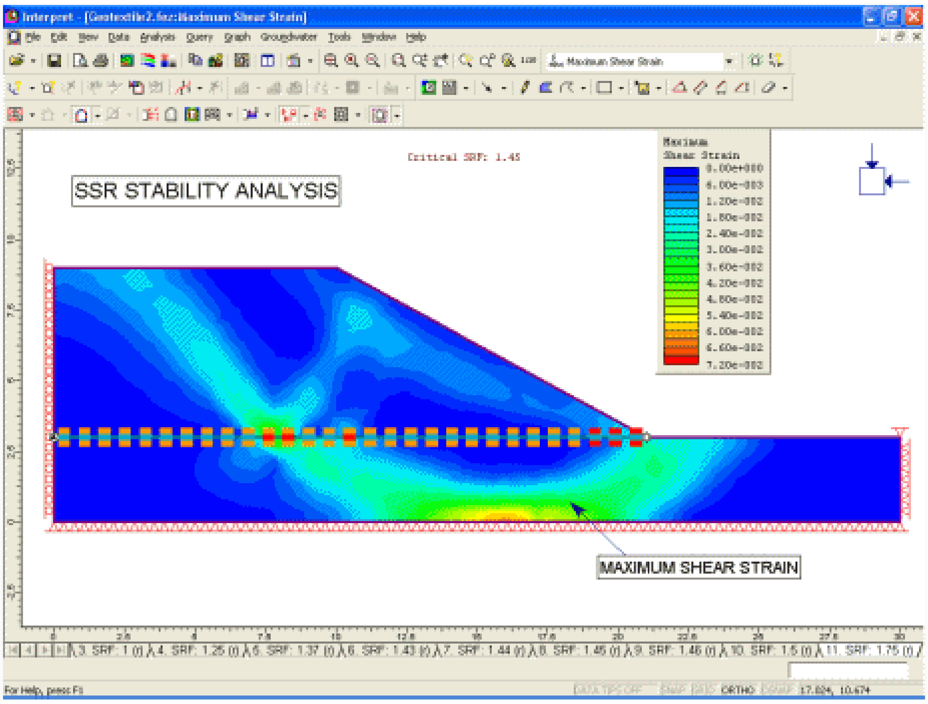
8. $\epsilon_{xy} = \frac{\tau_{xy}}{G}$

APPLICATION

A FINITE ELEMENT ANALYSIS SOFTWARE

Phase2 is a **2-dimensional (as the name implies)** elasto-plastic finite element program for calculating stresses and displacements around underground openings, and can be used to solve a wide range of mining, geotechnical and civil engineering problems.The following figures show 2-D applications using this software:





PLANE STRAIN ANALYSIS IN PHASE2

In the Project Settings dialog, two different Analysis Types are available - Plane Strain or Axisymmetric analysis.

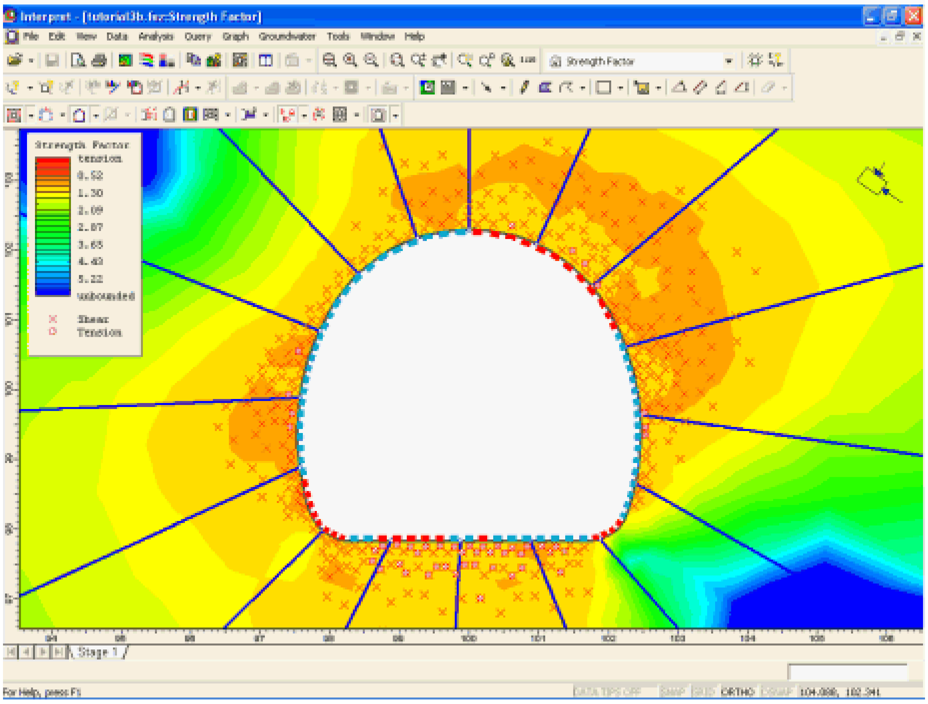
Plane Strain assumes that the excavation(s) are of infinite length normal to the plane section of the analysis. In most cases you will be performing a Plane Strain analysis. In a Plane Strain analysis Phase2 calculates:

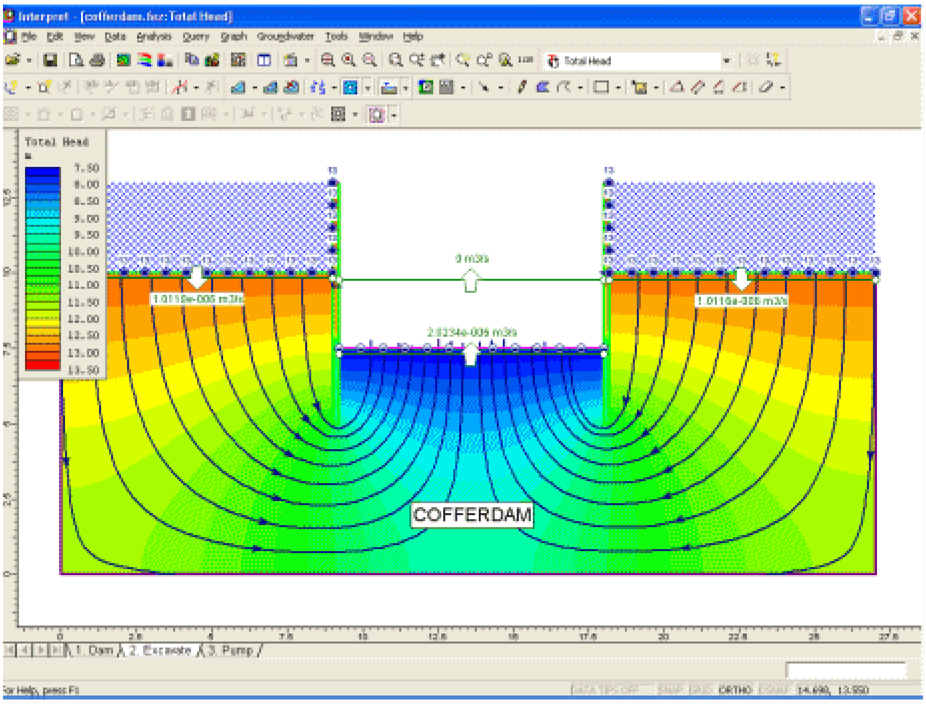
- the major and minor in-plane principal stresses (Sigma 1 and Sigma 3),
- the out-of-plane principal stress (Sigma Z)
- in-plane displacements and strains

By definition, the out-of-plane displacement (strain) is ZERO in a Plane Strain analysis.

In practice, as the out-of-plane excavation dimension becomes less than approximately five times the largest cross-sectional dimension, the stress changes calculated assuming Plane Strain conditions begin to show some exaggeration because the stress flow around the "ends" of the excavation is not taken into account. This exaggeration becomes more pronounced as the out-of-plane dimension approaches the same magnitude as the in-plane dimensions.

If you have an excavation which is rotationally symmetric about an axis, then you can use the Axisymmetric modeling option.





You can check manuals and tutorials to have an idea about FEM and 2-D analysis the following link:

http://www.rocscience.com/products/Phase2.asp

REFERENCES

1) Ugural and Fenster, Advanced Strength and Applied Elasticity, 4th Ed., Prentice Hall, PTR, 2003

2) MIT Structural Mechanics Lecture notes
Retrieved(15 November,2009) from
http://ocw.mit.edu/NR/rdonlyres/Aeronautics-and-Astronautics/16-20Structural-MechanicsFall2002/02E38870-4C45-4732-AD80-B0FE55FA6B0D/0/unit6.pdf

3) Lecture Notes in MTU
Retrieved(15 November,2009) from
http://www.me.mtu.edu/~bettig/MEEM4405/Lecture11.pdf

4) Class Notes