Application of Quasi-Static Modal Analysis to an Orion Multi-Purpose Crew Vehicle Test

Matthew S. Allen[1,2], Joe Schoneman[2], Wesley Scott[2], and Joel Sills[3]

[1]Department of Engineering Physics
University of Wisconsin – Madison

[2]ATA Engineering

[3]ESD/Cross Program Integration
NASA Johnson Space Center

ABSTRACT
Bolted structural joints often exhibit load-dependent stiffness and energy dissipation that leads to nonlinear, amplitude dependent frequency and damping in the structure. As an alternative to direct integration of the nonlinear equations of motion, quasi-static modal analysis (QSMA) determines the dependence of frequency and damping on response amplitude using loading behavior from nonlinear static analyses. QSMA has previously been demonstrated to substantially reduce computational cost and maintain accuracy relative to full nonlinear dynamic simulation. This work explores the applicability of QSMA to a complex, large-scale aerospace structure. QSMA is employed to analyze a nonlinear model of test hardware developed to support the Orion Multi-Purpose Crew Vehicle program, which exhibited nonlinear behavior during dynamic testing at flight-like load levels. In addition to the extraction of amplitude-dependent frequency and damping curves, a Bouc-Wen hysteresis model was used in conjunction with the quasi-static results to develop nonlinear, uncoupled, time-domain modal equations of motion for the structure. Excellent agreement was observed between the reduced and full-order nonlinear models, encouraging future employment of QSMA to support accurate and efficient model reduction of structures with bolted joint nonlinearities.

Keywords: QSMA, Bolted Joint Nonlinearity, Nonlinear Dynamics, Bouc-Wen

INTRODUCTION

Quasi-Static Modal Analysis (QSMA) estimates the effective natural frequency, damping ratio and mode shape of a nonlinear finite element using nonlinear static analyses. The derivation of QSMA reveals the loading to a structure required so that the resulting static response approximates the dynamic response of the. The result of the nonlinear static simulation can then be used to infer the way that the structure vibrates in a single mode of vibration over a range of amplitudes. This methodology provides additional insights into the dynamics of the structure, beyond what can typically be gleaned from a transient dynamic analysis, and when the goal is to update the finite element model, QSMA can make that process much more efficient.

While the basic concept of QSMA has been used for many years in one form or another, a formal framework appears to have been presented only recently. The method was first presented by Festjens, Chevallier & Dion [1][2]. They presented a derivation of the method including the concept of uncoupled nonlinear modes, and demonstrated it on a finite element model of a structure with a bolted joint. Allen & Lacayo were simultaneously working on a method inspired by the Implicit Condensation and Expansion model reduction approach [3][4] and independently came up with an approach that they dubbed

QSMA [5][6], which they found to be a special case of the algorithm by Festjens et al. Lacayo & Allen's implementation of Festjens et al.'s approach has gained considerable traction recently for a few reasons. First, it is simpler than the approach by Festjens et al.; their approach partitioned the structure into linear and nonlinear parts and required iterating on the solution, whereas Allen & Lacayo's QSMA is a single nonlinear step. Second, Lacayo and Allen were able to show that the algorithm agrees very well with dynamic response predictions, but with about three orders of magnitude reduction in the computation time, making the method very appealing for model updating [5]. Finally, they were able to correlate the method with recently developed test methods, which use a Hilbert transform to decompose the response of a weakly nonlinear system into the response of uncoupled or weakly coupled modes, and there is now a relatively large base of experimental data to which the method has been shown to apply, e.g. [7]-[11].

Advantages of QSMA are in the substantial computational cost reduction with retention of predictive accuracy relative to larger order models. The aforementioned studies all focused on bench-scale applications in academic settings. This work demonstrates the application of QSMA to the Orion Multi-Purpose Crew Vehicle (MPCV), a full-scale, real-world application, and discusses the challenges and opportunities presented by its use. Furthermore, the nonlinearities employed in the MPCV model are different than any to which QSMA has previously been applied—Coulomb friction sliders and bi-linear springs rather than Iwan joints—and so the application here provides unique challenges.

Additionally, this work explores the relationship between the modal model obtained by QSMA and the response to an arbitrary transient input. In particular, the nonlinearities observed in the model of interest do not follow the form of the Iwan joints that were used previously, and so a new methodology is presented here wherein a single-degree-of-freedom (SDOF) system with a Bouc-Wen element is fit to each mode using the QSMA results. The resulting collection of uncoupled, nonlinear oscillators is found to reproduce the dynamic response of the full order FEM with surprising accuracy.

BACKGROUND

Consider an otherwise linear structure that contains nonlinearities f_J due to the joints that may depend on the displacements x as well as on internal states $\boldsymbol{\theta}$ within the joint model.

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} + \boldsymbol{f}_J(\boldsymbol{x}, \boldsymbol{\theta}) = \boldsymbol{f}_{ext}(t) \tag{1}$$

For small vibrations, one can define a linear system that approximates the response about some equilibrium,

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \left(\boldsymbol{K} + \nabla\boldsymbol{f}_J\big|_{x_0}\right)\boldsymbol{x} = \boldsymbol{f}_{ext}(t) \tag{2}$$

where $\nabla\boldsymbol{f}_J\big|_{x_0}$ denotes the Jacobian of the nonlinear forces at the equilibrium state $\boldsymbol{x_0}$. One can now solve an eigenvalue problem to find the system's natural frequencies, ω_{0r}, and mass-normalized mode shapes $\boldsymbol{\varphi}_{0r}$ about this reference state:

$$\left[\left(\boldsymbol{K} + \nabla\boldsymbol{f}_J\big|_{x_0}\right) - \omega_{0r}{}^2\boldsymbol{M}\right]\boldsymbol{\varphi}_{0r} = 0 \tag{3}$$

Eq. (3) provides the modal properties of the system at very low response amplitudes when the nonlinearities are inactive (i.e. the joints are stuck).

The system in Eq. (1) can be transformed to modal coordinates using the low-amplitude modal basis, $\boldsymbol{x} = \boldsymbol{\Phi_0 q}$, where $\boldsymbol{\Phi_0}$ is the matrix of mode shape vectors $\boldsymbol{\varphi}_{0r}$. This produces the following equation of motion for mode r:

$$\ddot{q}_r + 2\zeta_{0r}\omega_{0r}\dot{q}_r + \boldsymbol{\varphi}_{0r}^T\boldsymbol{K}\boldsymbol{\varphi}_{0r}q_r + \boldsymbol{\varphi}_{0r}^T\boldsymbol{f}_J(\boldsymbol{\Phi}_0\boldsymbol{q}, \boldsymbol{\theta}) = \boldsymbol{\varphi}_{0r}^T\boldsymbol{f}_{ext}(t) \tag{4}$$

Equation (4) assumes the modes diagonalize the linear damping so the low-amplitude linear damping ratio is defined in terms of the low-amplitude mode shape and frequency using

$$\zeta_{0r} \triangleq \boldsymbol{\varphi}_{0r}{}^T\boldsymbol{C}\boldsymbol{\varphi}_{0r}/(2\omega_{0r}).$$

Note that in general the nonlinearities can cause all of the modes to influence the rth mode. However, empirical evidence ([10], [12], [13]) suggests that the linear modes tend to be preserved in the microslip regime for a structure with typical nonlinearities due to bolted interfaces, so that one can approximate the response effectively with a superposition of weakly nonlinear oscillators. The theoretical foundation for such an occurrence is detailed in [14]. Whether the modes remain uncoupled or not, the QSMA approach estimates the response assuming the structure oscillates purely in a single mode, so the equation of motion is approximated as:

$$\boldsymbol{\varphi}_{0r}^T \boldsymbol{f}_J(\boldsymbol{\Phi}_0 \boldsymbol{q}, \boldsymbol{\theta}) \approx g(q_r) \tag{5}$$

However, we recognize that the other modal displacements, q_j for $j \neq r$ are generally not zero. So, we assume that they are slaved to the rth DOF in the same ratios that are observed in the static response.

Specifically, we excite each mode in turn with a force applied over the entire structure in the shape of that linear mode by solving the following quasi-static problem,

$$\boldsymbol{Kx} + \boldsymbol{f}_J(\boldsymbol{x}, \boldsymbol{\theta}) = \boldsymbol{M}\boldsymbol{\varphi}_{0_r}\alpha \tag{6}$$

where α is the load amplitude. This produces the quasi-static response $\boldsymbol{x}(\alpha)$ of the structure up to some maximum load amplitude. The response is computed at several intermediate steps yielding the entire load displacement curve. The resulting displacements are then mapped to each of the modes using: $q_r(\alpha) = \boldsymbol{\varphi}_{0r}{}^T \boldsymbol{M}\boldsymbol{x}(\alpha)$. The modes that were not directly excited are also retained and used to assess the degree to which the modes are statically coupled at each load amplitude. It is also important to note that we have obtained a single valued relationship between $\boldsymbol{x}(\alpha)$ and α by requiring that the load always ramps from an unloaded state to a maximum load α_{max}. As a result, it is no longer necessary to track any internal states, $\boldsymbol{\theta}$, e.g. slider states for the Iwan joint.
Using the curve of the modal force $f_r(\alpha) = \boldsymbol{\varphi}_{0_r}{}^T \boldsymbol{M}\boldsymbol{\varphi}_{0_r}\alpha = \alpha$ versus modal displacement, q_r, the full hysteresis curve is constructed using Masing's rules [15], [16], i.e. $\hat{f}_1(q_r) = 2f_r\left(\frac{q_r + q_r(\alpha)}{2}\right) - \alpha$ and $\hat{f}_2(q_r) = \alpha - 2f_r\left(\frac{q_r(\alpha) - q_r}{2}\right)$ as illustrated in Figure 1.

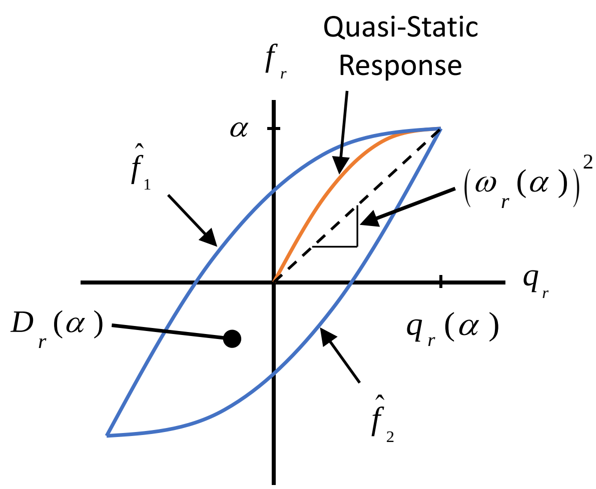

Figure 1. Sample loading curve obtained by solving Eq. (6). (blue) Hysteresis curve (forward $\hat{\boldsymbol{f}}_1(\boldsymbol{q}_r)$ and reverse $\hat{\boldsymbol{f}}_2(\boldsymbol{q}_r)$) computed using Masing's rules.

Hysteresis curves such as these can now be used to estimate the instantaneous stiffness and damping for each mode. The secant to the curve is used to estimate the effective natural frequency, as follows.

$$\omega_r(\alpha) \triangleq \sqrt{\frac{\alpha}{q_r(\alpha)}} \tag{7}$$

The energy dissipated per cycle of vibration is the area enclosed by the full hysteresis curve

$$D_r(\alpha) = \int_{-q_r(\alpha)}^{q_r(\alpha)} \left(\hat{f}_1(q_r) - \hat{f}_2(q_r)\right) dq_r \tag{8}$$

where α is the constant maximum load at the level of interest, and f_r is a function of displacement q_r.

Then, using the relationship between energy dissipated per cycle and the damping ratio for a linear system, the effective damping ratio can then be calculated from the following.

$$\zeta_r(\alpha) = \frac{D(\alpha)}{2\pi(q_r(\alpha)\omega_r(\alpha))^2} \tag{9}$$

It is important to note that these effective "modal properties" are not a linearization of the structure about some state. Indeed, we are considering the effective behavior of the structure over a typical cycle of vibration. Such a model is typically called a quasi-linearization or a "describing function" model ([17], [18]).

A comprehensive verification of QSMA is presented by Lacayo & Allen [5] and is not repeated here for brevity.

SDOF SYSTEM WITH BOUC-WEN JOINT MODEL

In the application presented in this work, the goal was not simply to understand the nonlinear characteristics of each mode but to use QSMA to predict the dynamic response. As a result, the QSMA results had to be used to estimate a nonlinear dynamic model that could capture the observed hysteresis. In this process, the numerical QSMA curves of modal force vs. displacement are translated into a single degree of freedom system model that includes the hysteresis and which can be integrated in time to obtain the dynamic response. This requires some method to implement the load-displacement hysteresis in the system. In this work, the Bouc-Wen joint model was used; another candidate is a generalization of the modal Iwan model concept, which is typically applied using a four- or five-parameter form but in theory can be used to represent any symmetric, hysteretic modal force/displacement curve [16].

Originating in the civil engineering community, one of the most prominent available models for simulation of hysteretic systems is the Bouc-Wen (BW) model [20], which augments a linear, second-order, single degree-of-freedom system to include a hysteretic force. With modal displacement q, modal velocity $\dot{q}$, linear natural frequency ω, and critical damping ratio ζ, the equations of motion for an individual mode are:

$$\begin{gathered} \ddot{q} + 2\zeta\omega\dot{q} + f(q,z) = f_{ext}(t) \\ \dot{z} = \dot{q} - \beta|\dot{q}||z|^{n-1} - \gamma\dot{q}|z|^n \end{gathered} \tag{10}$$

The additional state z corresponds to the hysteretic behavior of the system, with parameters β, γ, and the exponent n governing the behavior of z. In the modal equation of motion, the function $f(q,z)$ is

$$f(q,z) = \alpha k_i q + (1-\alpha)k_i z \tag{11}$$

Where the initial stiffness k_i and stiffness loss ratio α are related to the pre- and post-yield stiffnesses of the hysteresis loop.

The Bouc-Wen parameters can be identified using an iterative least-squares process, as explained in reference [21] and improved upon for this work. The authors of [21] do not set an initial stiffness, leaving it as an independent variable in the fit. This significantly reduced accuracy at low amplitude excitation. The initial stiffness can instead be set by specifying $k_i = \omega^2$ and $\alpha = k_f/\omega^2$ with k_f the high-amplitude, post-yield stiffness value associated with the hysteresis loop. The procedure works best if a constant, final value of k_f is achieved from QSMA, although not all models need exhibit such behavior.

The remaining parameters are n, β, and γ. The latter two are linearly related to the evolution of z, but, an exponent, n cannot be obtained by least squares calculation. Instead, n is specified with $n = 2$ and updated if needed as discussed below. Then, we consider the set of QSMA results as a series of measurements f_j associated with modal displacements q_j. A difference equation can be written in terms of the nonlinear $f(q,z)$:

$$f_j - f_{j-1} = \Delta f_j = \alpha k_i \Delta q_j + (1-\alpha)k_i \Delta z_j \tag{12}$$

If a fictitious Δt is considered, then fictitious time derivatives $\dot{z}_j \approx \Delta z_j/\Delta t$ and $\dot{q}_j \approx \Delta q_j/\Delta t$ can be defined. Performing these substitutions and referring to Eq. (10) leads to

$$f_j - k_i \Delta q_j = \alpha - (1-\alpha)k_i \left[\beta|\Delta q_j||z_j|^{n-1} z_j + \gamma \Delta q_j |z_j|^{n}\right] \tag{13}$$

or

$$y_j = [\alpha - (1-\alpha)k_i\,] \cdot \left[|\Delta q_j||z_j|^{n-1} z_j, \quad \Delta q_j |z_j|^{n}\right] \begin{Bmatrix} \beta \\ \gamma \end{Bmatrix} \tag{14}$$

Building up matrices associated with all j measurements leads to a least-squares problem $\boldsymbol{y} = \boldsymbol{\Phi}\mathbf{x}$ to identify β and γ for a given exponent n. Repeating for multiple values of n ultimately leads to an optimal set of parameters for the BW model. In general, these will not match the hysteresis loop obtained from QSMA exactly. However, as shown in the comparisons that will be discussed later, the hysteresis loops were very well approximated by the BW model for this application.

QSMA IMPLEMENTATION

The main steps used to apply QSMA to the MPCV are summarized graphically in Figure 2. Starting from a multi-level dynamic test of the MPCV configuration, a nonlinear model calibration was performed. The calibrated nonlinear FEM was constructed using Abaqus software and consisted of several linear direct matrix inputs along with a collection of nonlinear connector elements at the matrix interfaces. Both friction and bilinear elements were used in the model, with joint properties calibrated to match dynamic test results. The QSMA procedure was applied to the nonlinear FEM to obtain the quasi-static modal hysteresis curves, from which effective frequency and damping were computed as functions of modal amplitude using Eq. (7) and Eq. (9). Finally, transient simulations of a QSMA-derived dynamic model were performed by fitting a Bouc-Wen model to the QSMA hysteresis curves, which is described in greater detail in a subsequent section. Each step of the process presents certain challenges, as elaborated in the next few paragraphs.

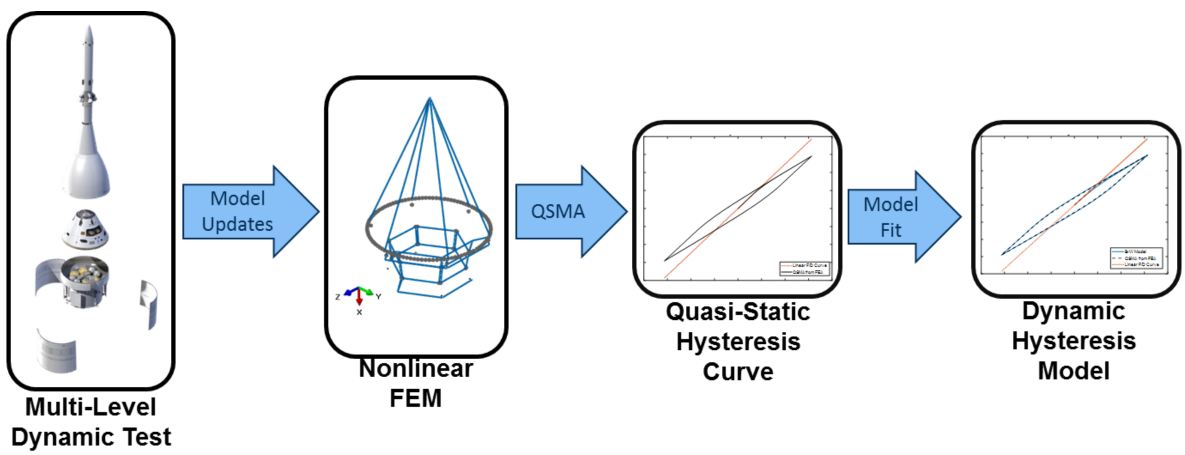

Figure 2. Key steps for construction of uncoupled hysteretic modal equations of motion using the MPCV nonlinear FEM.

During multi-level dynamic testing, forcing must be at a high enough level and applied in such a way that it will adequately excite the important nonlinearities that are present in the loading envelope. Repeatability is a major concern for structures dominated by joint dynamics. Furthermore, the forcing must also not be too high in amplitude, otherwise it could induce additional nonlinearity–hard-stops, geometric effects, or even material plasticity–that are not of interest in the actual environment and would require much more complicated methods for simulating the nonlinear joints.

Nonlinear FEMs are expensive to construct, debug, simulate, and calibrate. It can be challenging to select nonlinear elements and tune their parameters such that they produce the behavior observed in the measured response. Unlike linear models, response sensitivities of a commercial nonlinear FEM can only be obtained by finite-difference approaches in general.

Uncertainty quantification methods are similarly difficult to apply to nonlinear models, which presents additional challenges due to the aforementioned repeatability issues presented by structural joints.

Quasi-static hysteresis curves are, in theory, straightforward to obtain, but one must use care when deriving them from commercial FEMs. Since QSMA is not a standard procedure implemented within current Finite Element Analysis (FEA) codes, it is up to the analyst to ensure correct coordinate transformations are applied during every step of the process: Modal extraction, force generation, force application, displacement extraction, and modal filtering. QSMA requires access to the structural mass matrix, which is typically available but can grow quite large for commercial-scale models. Modes can be obtained either through the FEA modal solve routine, or by extracting system matrices and computing modes in an external sparse eigenvalue solver.

Once the QSMA procedure has been implemented, required analyst inputs are fairly minimal. The analyst must first determine which modes to probe for nonlinearity; one could perform QSMA on every mode in the frequency range of interest, but often there is prior knowledge which reveals which modes are most likely to exhibit nonlinearity. Next an appropriate load scaling factor must be selected for each mode. This requires some amount of iteration, but an informed decision can be made by examining the hysteresis curves that have been computed for each mode. Too low a factor will be seen to not sufficiently excite nonlinearity. However, if the factor is too high, the static analysis may fail to converge or the nonlinearity may be so strong that the low-level response cannot be well approximated. In the latter case, if the load is reasonable for the structure then this simply reveals that the joints will essentially always be in their slipping state and is not a fault of the analysis. As mentioned above, an optimal value can typically be found after several iterations of the QSMA process, and for this application, simple trial and error took only a few minutes to find each value.

The result of QSMA to the first mode of the MPCV configuration is demonstrated in Figure 3. Starting from the origin, the model force initially follows the linear stiffness at low amplitudes before rolling off in a bilinear fashion. As the load is reduced and reversed, a symmetric hysteresis loop is evident in the response. In Figure 3, the QSMA output from FEA is shown with a dashed black curve with a Bouc-Wen model fit in blue. The red dashed line shows the linear stiffness of this mode.

The QSMA results can be used to compute effective frequency and damping ratios as a function of modal amplitude, as shown in Figure 4. Note that the plotted damping ratio is purely nonlinear, and should be considered in addition to any baseline viscous damping present in the system. In Figure 4, the damping increases to a maximum value before beginning to decrease. This behavior is frequently observed in systems with Coulomb friction sliders. In contrast, the frequency drops monotonically from its linear value down to a minimum decrease of roughly 20%. This reveals that this mode loses a substantial fraction of its stiffness as the load increases.

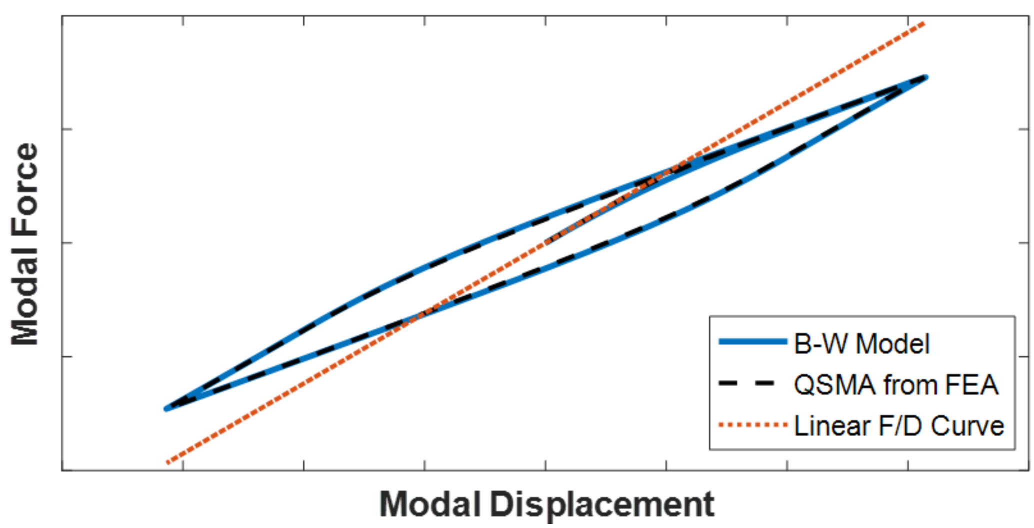

Figure 3. Modal force vs. displacement curve for primary Y-bending mode of the MPCV.

QSMA relies on the assumption that modes remain decoupled even as the amplitude of excitation increases. This assumption can be checked, statically at least, by assessing the relative displacement of each mode as forcing is applied in the shape of just the mode of interest. Figure 5 shows a graphical check of the independence of mode 10, the first bending mode, as a function of modal displacement. It is seen that the activation of all other modes is much lower than mode 10, with mode 12, the second bending mode, being the most activated at no more than 10% the level of mode 10.

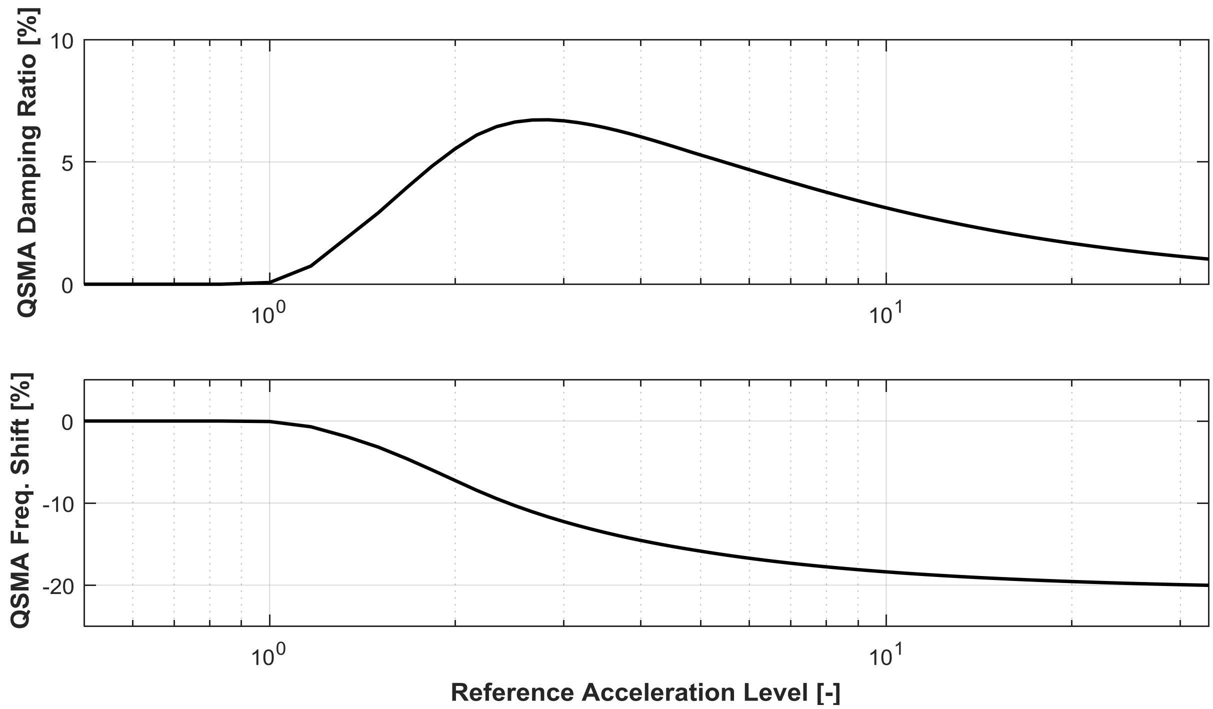

Figure 4. QSMA frequency and damping ratio as a function of modal acceleration for the primary Y bending mode with acceleration normalized such that the frequency shift begins at roughly a reference amplitude of 1.0.

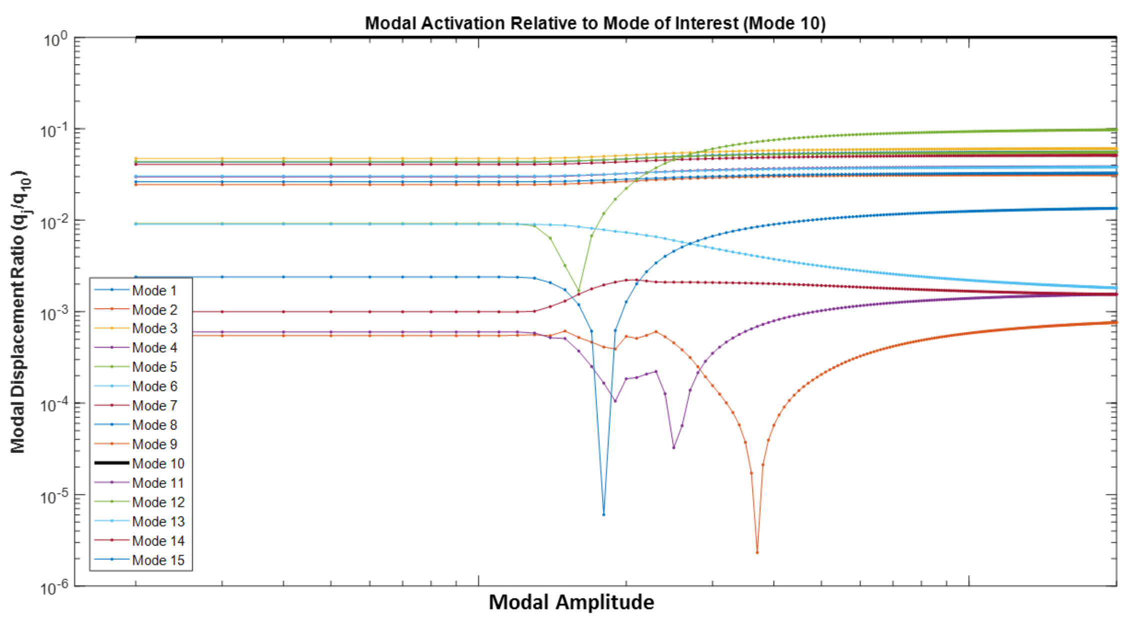

Figure 5. The importance of modal coupling can be inferred from static loading done in the QSMA framework. Here relative to mode 10, all other modes maintain much lower displacement for all amplitudes.

RESULTS AND COMPARISONS

The QSMA results, BW model fit, and implementation of transient dynamic equations of motion were verified through numerical comparison to results from the nonlinear Abaqus FEM. The response of the MPCV was computed to reference base-excitation inputs for three simulations:

- Abaqus/Standard nonlinear simulation, using the HHT-α integrator with default $\alpha = -0.05$ and 2 kHz integration rate. Each integration took on the order of several hours to complete.

- Abaqus modal dynamics (linear) solution. The Abaqus modal dynamics solver uses an explicit central difference solver to integrate the modal coordinates using a linear model. The same 2 kHz integration was again specified. Each integration took on the order of several minutes to complete.

- A MATLAB-based integration of the QSMA/BW model implemented using the standard ODE45 solver, with accuracy tolerances decreased from the default 10^{-6} to a more restrictive 10^{-9}. ODE45 is an adaptive timestep solver but an output frequency of 2 kHz was specified. Each integration took on the order of several seconds to complete.

The modal models included 14 fixed-base modes of the MPCV model. A fixed viscous damping ratio of 1.5% was applied to the MATLAB equations of motion, while damping of the Abaqus models was obtained directly from the supplied damping matrix inputs. The QSMA/BW model was simulated in a completely uncoupled fashion; a small amount of modal coupling may have been present in the Abaqus modal dynamics simulation due to projection of the modes through the damping matrix.

Two separate sets of base excitation inputs were used for verification. The first input, here called Event 1, is a load case which features moderate amplitudes and excitations across the entire modal bandwidth, with little variation during the integration period. A second input, here called Event 2, provided high-amplitude excitations at lower frequencies, with a large reduction in amplitude during the period of integration. The time histories were supplied as 6-DOF base excitations. An additional scaled version of Event 1 reduced to 10% of its nominal value was used to verify correct implementation of the base excitation loading in MATLAB.

The Pearson correlation coefficient, $\rho_{x,y} = \frac{E[(x-\mu_x)(y-\mu_y)]}{\sigma_x \sigma_y}$, was used to evaluate agreement between time histories computed from each simulation method. This coefficient tends to punish errors in phase much more harshly than errors in amplitude, but still provides a useful metric to summarize agreement between multiple signals. Values for the coefficient range between 0 and 1, with values close to 1 showing a high level of correlation between signals.

Results for each case are shown in the titles of each of Figure 6 through Figure 8. In first of these figures, the input has been scaled to 10% of nominal to limit excitation of nonlinear behavior. Here good agreement between all methods is evident, as expected for a low-amplitude excitation. The Abaqus linear modal model performs somewhat better than the QSMA/BW model due to limitations of the BW model fit, which begins rolling off into hysteresis earlier than the FEM and, as a result, displays a somewhat higher effective damping ratio at low amplitudes.

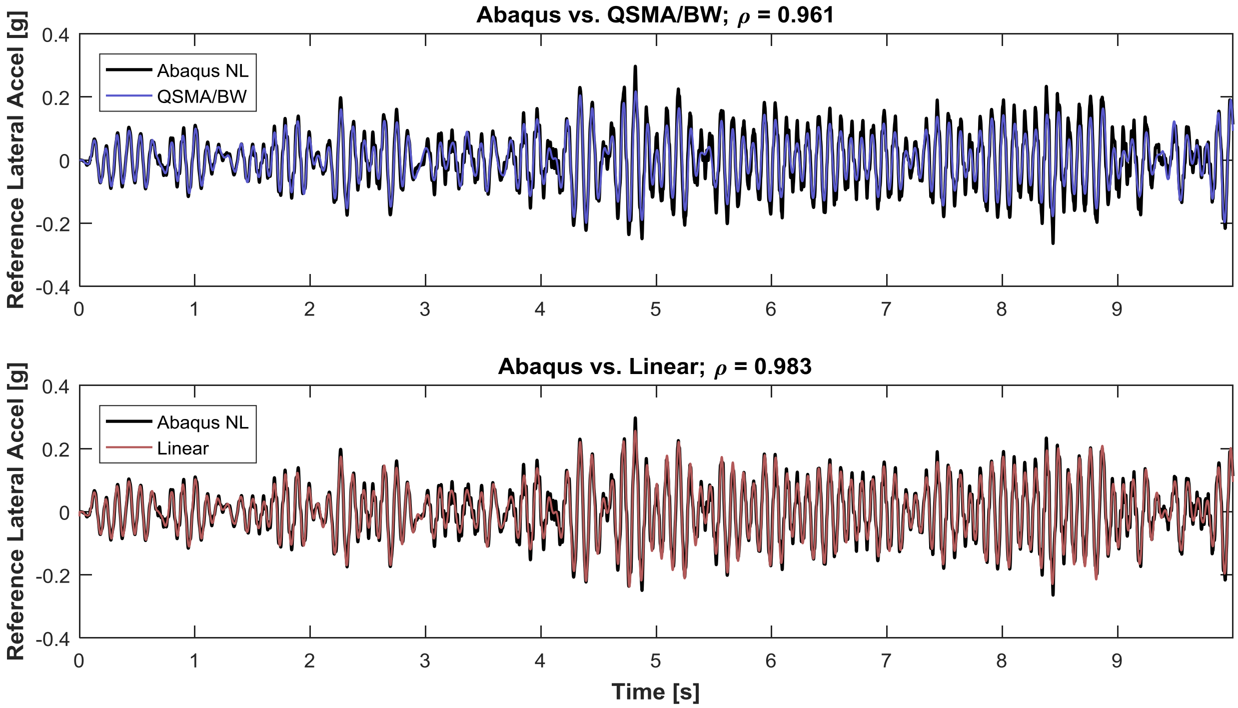

Figure 6. Time history comparisons between (top): Abaqus/Standard and QSMA/BW; (bottom): Abaqus/Standard and Abaqus linear modal dynamics for a 10% scaled Event 1.

For the excitations at nominal amplitude levels, the QSMA/BW model performs significantly better than the linear model. While the QSMA/BW model does exhibit noticeable amplitude errors for the Event 1 response in Figure 7, phase errors are minimal. The linear model, on the other hand, dramatically overpredicts the response by a factor of nearly three. As shown in Figure 8, QSMA/BW performs very well in simulating the Event 2 response, whereas the linear model initially overpredicts, then underpredicts, the actual response, and exhibits high-frequency dynamics that are not present in the late-time nonlinear responses.

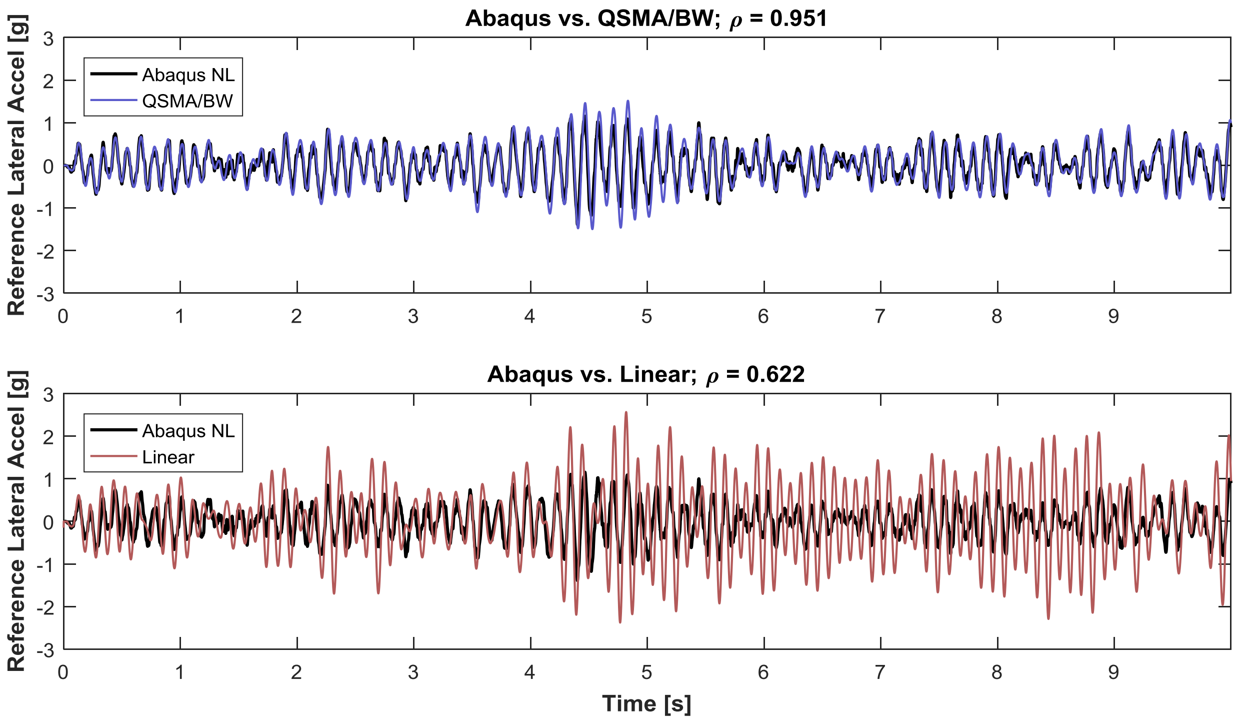

Figure 7. Time history comparisons between (top): Abaqus/Standard and QSMA/BW; (bottom): Abaqus/Standard and Abaqus linear modal dynamics for nominal Event 1 base excitation.

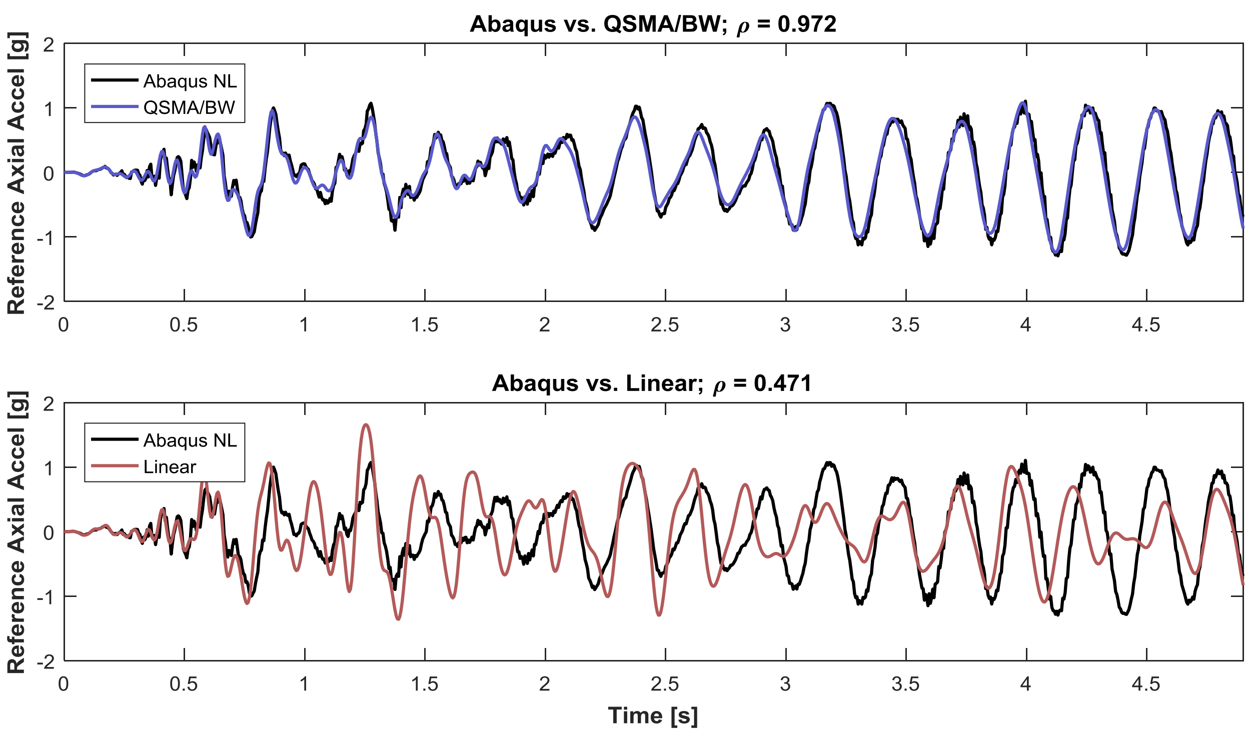

Figure 8. Time history comparisons between (top): Abaqus/Standard and QSMA/BW; (bottom): Abaqus/Standard and Abaqus linear modal dynamics for Event 2 base excitation.

Pearson correlation coefficients for each case are summarized in Table 1. While a much more comprehensive set of verification data, along with a model validation effort on flight-representative hardware, would be required to more rigorously verify the applicability of QSMA and associated dynamics models to nonlinear coupled loads analysis, the results of this study are promising and should encourage future efforts towards this goal.

Table 1. Summary of correlation coefficients for each QSMA/BW test case as compared to the fully nonlinear Abaqus FEM

Case	QSMA/BW	Linear Modal Dynamics
Event 1 (10% level)	0.961	0.983
Event 1 (100% level)	0.951	0.622
Event 2	0.971	0.471

A relevant short-term outcome of the QSMA/BW modeling effort is the finding that an uncoupled nonlinear modal model based on the linearized modes of the MPCV was able to accurately predict the nonlinear response of the fully-coupled nonlinear finite element model. This suggests that dynamic modal coupling is of limited importance in this model of the MPCV and at the load levels that are relevant for system dynamic analysis. (Static modal coupling *is* included in the QSMA model.) However, the changes in natural frequency and damping for each mode are significant.

This finding could have several important implications. First, when seeking to update the FEM to correlate with measurements, this finding reveals that one can consider each mode independently. For example the mode shape in question can be considered to determine which joints would cause a significant change in its properties, to help guide model updating. Any change that produces damping in that mode will bring the response near the associated resonance into better agreement. In other words, one can adapt linear modal methods to nonlinear model updating. Second, because the structure behaves as if it has modes, only with somewhat variable frequency and damping with amplitude, it may be reasonable to use a linear model that is tuned to the appropriate amplitude to perform coupled loads analysis or other life predictions.

CONCLUSION

Quasi-Static Modal Analysis (QSMA) was applied to an industrial-scale system with nonlinearities approximating bolted joints. The system was simulated in the time domain by fitting the QSMA hysteresis to a Bouc-Wen (BW) model form. Time integration of the equations of motion with the Bouc-Wen model was shown to match a full nonlinear solution response with greater than 95% accuracy for a few limited cases. Additionally, this came with a substantial cost reduction, with Bouc-Wen models running on the order of seconds, while Abaqus/Standard runs took on the order of a day. This cost savings could make it feasible to perform extensive parametric studies, model updating, and uncertainty quantification.

The ability of an uncoupled nonlinear modal model based on linearized modes to accurately predict the nonlinear response of a complicated and fully-coupled nonlinear finite element model indicates that modal coupling is of limited importance to the model studied. This alone could have several important implications. First, FEM updating can be performed on a modal basis; the alternative would be to consider whether each update to the FEM improves agreement across a potentially large range of potential loading scenarios. Second, because the structure behaves as if it has modes, only with somewhat variable frequency and damping with amplitude, it may be reasonable to use a linear model that is tuned to the appropriate amplitude to perform coupled loads analysis or other life predictions.

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