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> A PROGRESS REPORT FOR JANIARY 1,1976 to JUNE 30,1976

\title{
DATA ACQUISITION AND PATH SELECTION DECISION MAKING FOR AN AUTONOMOUS ROVING VEHICLE
}

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION GRANT 7184 NSG
D. K. Frederick
C. N. Shen
S. W. Yerazunis

\begin{abstract}
Problems related to the guidance of an autonomous rover for unmanned planetary exploration have been investigated. Included in these studies were: simulation on an interactive graphics computer system of the Rapid Estimation Technique for detection of discrete obstacles; incorporation of a simultaneous Bayesian estimate of states and inputs in the Rapid Estimation Scheme; development of methods for estimating actual laser rangefinder errors and their application to date provided by Jet Propulsion Laboratory; and modification of a path selection system simulation computer code for evaluation of a hazard detection system based on laser rangefinder data.
\end{abstract}

\section*{INTRODUCTION}

A thorough exploration of planets or other extraterrestrial bodies by unmanned data acquisition systems must deal with formidable control problems posed by the long ( 9 to 45 minutes for Mars and more in other cases) round-trip communications delay. Time delays of these orders of magnitude preclude all but the most rudimentary and short range missions unless a high degree of automation is achieved. An augmented Viking mission or a sophisticated sample return program aspiring to the exploration of a region of \(500-1000 / \mathrm{cm}\) diameter would require the availability of an autonomous rover requiring only occasional earth control.

In looking forward to detailed unmanned exploration of the planets, there are two prerequisites. First is the requirement that a high mobility rover or relocatable device be available. The higher the mobility (or capability of the device to deal with irregular and difficult terrain features), the more available will be the paths to the desired targets. A rover of low mobility may well find itself with no acceptable paths in many instances.

Second, a path selection system of capability comparable to the mobility must be available. By path selection system are implied: (a) the terrain sensor(s) which are to acquire the necessary terrain data, (b) the terrain modeler which is to process the acquired data with the goal of describing the terrain features, and (c) the path selection algorithm to determine the specific path to be followed towards the target. Unless the path selection system can take full advantage of the rover's terrain negotiating capabilities, such a mission will be made unnecessarily lengthy in time, or desired objectives will not be achieved, or, in the worst case, the rover will be immobilized when in fact passable paths are available. On the other hand, if the path selection system is insufficiently discriminating, the rover may follow a dangerous path with the risk of a catastrophic accident.

The research program described herein has been directed towards problems of path selection including data acquisition and processing, path selection algorithms and the corresponding integrated path selection systems.

The program involves two major tasks: (a) terrain and obstacle detection and interpretation and (b) path selection system simulation.

The task of terrain and obstacle detection and interpretation is aimed at investigating alternative procedures for using range-point'ing angle data such as might be obtained from a laser rangefinder with the goal of dotecting discrete obstacle hazards and/or impassable terrain contours. Included in the investigation are the effects of data density in space and time as well as of measurement errors inherent to the sensor and oo the dynamical motion of the rover. A major emphasis is being directed \(t\) (, the mathematical processing of the data to reduce computational requirfments and to increase the amount of useful information derived from the meazurements. Although the study is aimed at the mid-range of 3 to 30 meters, it can be extended with very modest effort to longer ranges. Thus it could be used as the basis for optimization of long range path segments. The sub-tasks whirh have been
completed to date include implementation of the simulation of the Rapid Estimation Scheme on an interactive graphics computer system, development of the mathematical procedures for using a Bayesian Estimate of States and Inputs for obstacle detection and estimates of laser rangefinder accuracy for Mars rover applicetinn mese studies are summarized briefly below and are described in more detai in the reprints of three papers which are included. Future work will be directed to the application and evaluation of these methods with respect to reducing data rec uirenents while retaining performance levels.

The second major task, namely, path selection systems simulation and evaluation, has the objective of appraising the effectiveness of alternative integrated path selection systems including the sensor(s), terrain modeler and path selection systems with data errors due to the sensor and vehicle dynamics. The simulator is a digital computer code capable of generating three-dimensional terrains of a broad variety on which can be superimposed a wide range of discrete hazards. The simulation affords the opportunity to screen, evaluate and modify alternative concepts prior to implementation in hardware. To date, the simulation has been modified to reflect a laser range finder of the type required to employ the Rapid Estimation Scheme and the mathematics of the edge detection procedure. Consideration has been given to a path selection algorithm suitable to this type of data. The progress which has been made is detailed in a following section. Also appended is a paper describing the use of the simulation in evaluating a short range hazard detection system based on a triangulation concept which suggests the effectiveness of the procedure. This paper is based on work supported by NASA Grant NGL 33-018-091.

Task A. Terrain and Obstacle Detection and Interpretation
Task A.1. Simulation of Edge Enhancement Technique - K. L. Leung Faculty Advisor: Prof. C. N. Shen

An edge enhancement technique for the detection of discrete obstacles such as boulders and craters using a Rapid Estimation Scheme in conjunction with a Kalman Filter has been developed. In order to increase the researcher's effectiveness in perceiving the effects of orientation, form of hazard, data spacing and sensor error on the obstacle detection concept, the technique has been programmed on the IDIIOM/Varian Interactive Graphics Computer System. The Rapid Estimation Scheme not only is shown to be capable of detecting the top edges of boulders and the near edges of craters but along the bottom edges of boulders and the far edges of craters. The simulation is now an operational tool which will be used to explore the detection concept in detail.. It is described in detail in a reprint of a paper which follows below.

\title{
PRESENTED AT THE SEVENTH ANNUAL CONFERENCE ON MODELING AND SIMULATION, PITTSBURGH, PA., April 1976 and Published in the Proceedings of the Conference
}

\section*{SIMUATION OF CBETACLE DETECTION SCHEME FOR MARS TE!RAIM USING MMICOMPUTER*}

\author{
ORIGINAL PAGE i:
}

K L. Leung and C. N. Shen**
Renssileer Polytechnic Institute Troy, New York 12181

\section*{ABSTRAC'}

Simulation for detection of obstacles on Martien Terrain is achicued by a rapid estiration scheme on Varian Data 620/i computer with IDIIO: display. Sufficient information can be obtained to recognize the near and far edges of pyramids, hemispherical craters or boulders, and the combination thereof.

\section*{IMTRODUCTIO:}

An autonomous roring vapicle is to be sent to Mers for future exploration'. A laser ranefinder having an aecuracy of \(\pm 10\) centimeters, will scan an area ahead of the rover. An obstacie detection scheme will therefore be needed to process these range data to obtain complete cutlines of distinct objects.

One detection scheme that has been ceveloped by Reed, Sanyal and Shen [2] wakes use of the fourdirectional Laplacian method. This scheme works well in detecting top edge of a boulder and near edge of a crater but it fails to detect the botton edge of a boulder and the far edge of a crater. The schere can only detect large jurps in ranges, but not large jumps in slopes.

The ebove scheme was simulated on both the IEM 360/ 67 computer and the Varian Dats \(620 / 1\) computer by Sher and Shen[3]. on both computers, the range data were generated by a searching scheme and then Gaussien notse vas added to the ranfe data to simulate the actual range. In the si-ralation with the Varien Data computer, en IDIIOM graphic cathod-rey tube was employed to display the para-eters, the topvier of costacle leyout and the results for edge irage. A light pen was aiso used by the operator to interrupt the displey and to change the parameters.

Another obstacle detection schere wes developed by R. V. Sonalkar and C. Nt Shen[4] which uses a Rapid Estimation Scheme (5,5)(7.E.S.) in conjunction with Kalmon rilter. In the simulation on the TBM \(360 / 67\) computer, their scheme wes shown to work extremely well in detecting corplete edges of

\footnotetext{
* Research sponsored by :ASA Grant, NSG-7184
}
** Professor-Electricel \& Systems Engtreering Degt.
boulders, craters, and other obstacles.

\section*{OBJECTIVE}

The objective is to evaluate the performance of the obstacle detection scheme on a minicomputer. This paper simulates the Sonalkar-Shen Obstacle Detection Scheme on the Varian Data 620/i corputer, zaking use of Sher and Shen's display subroutines for parameter, topview, and edge image displays.

The reason for this simulation is two-fold. Firstly, this enables easy parameter changing with light pen and teletype. Topvieus of obstacies and the resultin edge imares can be seen and the differences observei. The operator will then change the parameters accordingly end repest the simulation. This means that the simulation can be cone adeptively.

Secondly, the possibility of implerenting this scheme on a small computer such es the Varien Deta must be ascerteined.

EQUATIONS FOR OBSTACLE DETECTIO:
The range data are given by the natrix \(R\) with the following elements:


Where \(r_{i f}\) is the renge data for the terrsin, the length of the laser been from the fround. The indices 1 end \(I\) indicate the location in the argle of elevation, \(B_{i}\), end ezimuth angle, \(\partial_{j}\). The quantity \(r_{\text {f }}\) is the sum of the ectuel distance between the trans-itter and the point on the eround
and the Gaussian noise tue to the inaceuracy of the laser rangefinder. Hence
\[
\begin{equation*}
r_{i j}=d_{i j}+v_{i j} \tag{2}
\end{equation*}
\]

The range matrix \(R\) is processed both along all columns and across all rows. When processed vertically up a column, it is expected to detect jumps in both range and in slope, a two-dimensional state vector is defined. The first component is the range \(d_{1}\) and the second is the difference in successive \({ }^{1}\) range data \(\varepsilon_{i}\) (slope) where
\[
\begin{equation*}
\varepsilon_{i}=d_{i+1}-d_{i} \tag{3}
\end{equation*}
\]

The quantities \(\varepsilon_{i}, d_{i}, \beta\) ere illustrated in Figure \(1, \theta\) is illustrated in Figure 2.

Along a column, the \(g\) index is held constant. The state equation is given as follows by omitting the index 1 .
\[
\begin{equation*}
x_{i+1}=F_{i} x_{i}+w_{i}+B u_{k} \delta_{i k} \tag{4}
\end{equation*}
\]

The measurement model is:
\[
\begin{equation*}
r_{i+1}=H x_{i+1}+v_{i+1} \tag{5}
\end{equation*}
\]
where
\[
\begin{align*}
& x_{i}=\left[\begin{array}{l}
d_{i} \\
\varepsilon_{i}
\end{array}\right] ; \quad F_{i}=\left[\begin{array}{ll}
1 & 1 \\
0 & f_{i}
\end{array}\right] ; \quad H=\left[\begin{array}{ll}
1 & 0
\end{array}\right] \\
& B=\left[\begin{array}{l}
0 \\
1
\end{array}\right] ; u_{k}=j u m p \text { in } \varepsilon_{k} \text { due to an edge } \tag{6}
\end{align*}
\]

It can be shown that for a flat plane,

Also
\[
\begin{align*}
& f_{i}=\frac{1+\Delta \beta \tan \beta_{i}}{1-2 \Delta \beta} \cot \beta_{i}  \tag{7}\\
& \delta_{i k}= \begin{cases}1 & \text { for } 1=k \\
0 & \text { for } i \neq k\end{cases} \tag{8}
\end{align*}
\]
which is a K-oenecker delta function.
Vectors \(v_{i}\) and \(v_{i}\) represent plant and measurement noise respectively. The quantity \(w_{i}\) accounts for the deviation of the model from the actual stete equation. The quantity \(v_{i}\) eccounts for error in the rangefinder.

\section*{RANGE DATA AND NOISE GENEPATIO:}

The surface of each obstacle within the scanned area is described by its own geometric equation
\[
\begin{equation*}
f(x, y, z)=0 \tag{3}
\end{equation*}
\]

If the point on the ground ripht underneath the laser mast is taken as the oricin, then a level plane with no obstacle is described by \(z=0\).

To determine the range reading with azimuth angle \(\theta\), and anple of elevation \(\beta_{\text {, }}\), imarine a laser beam at these angles. The height \(H\) and \((x, y)\) coordinates of a point on the beam a small distance away from the laser rangefinder is calculated and this height is compared with the height \(z\) of the terrain with the same ( \(x, y\) ) coordinates. If \(H\) is creater than \(z\), then \(H\) of the point one increment down the beam is calculated and compared \(w^{t}\) th \(z\) of the correspondin \((x, y)\). The procedure is repeated until \(z\) is rreater than H. Using a bisection
method, the \((x, y)\) coordinates of the point where the beam hits the terrain is determined. If the height of this point is \(z^{\prime \prime}\) and the height of the rangefinder is \(D\), then the true range reading \(d_{i j}\) is given by
\[
\begin{equation*}
d_{1 j}=\left(D-z^{\prime \prime}\right) / \sin \left(\beta_{i}\right) \tag{10}
\end{equation*}
\]

Note that if the beam does not hit any obstacle,
\(z^{*}=0\) and
\[
\begin{equation*}
d_{i j}=D / \sin \beta_{i} \tag{11}
\end{equation*}
\]

These are illustrated in Figures 2 and 3.

\section*{CAFACITY OF MACHINE}

The computing machine used in this simulation is the Varian Data 620/i. The memory of this corputer when expanded, is a limiting 32767 words, or roughly 32 K . The operating system used in this computer takes up about 4 K . Hence the arailable memory is only 28 K . For comparison, the memories required by the various programs are tabulated on Table 1.
\begin{tabular}{|c|c|c|}
\hline Sonalkar
Shen on
IBM \(360 / 67\) & \begin{tabular}{l}
Sher- \\
Shen on Varien
\end{tabular} & LewngShen on Varian \\
\hline Fapide Estimation Scheme -- \(\overline{8} K\) 4-directional w/O Display 4-directional w/ Display & \[
\begin{aligned}
& 22 \mathrm{~K} \\
& 28 \mathrm{~K}
\end{aligned}
\] & \\
\hline R.E.S. w/Displey by combining Sonalkar and Sher & & 34 K \\
\hline R.E.S. w/Display after Improvement & & 27.5K \\
\hline
\end{tabular}

\section*{TEMTNIQUES USED IN COMBINING THE PROGRAUS}

The original Sonalkar-Shen program that simulates the Rapid Estination Scheme was written in Fortran suitable for the IBM corputer, which has a great deal of convenient features not found in the Varian computer. In order to run this program on the Verian machine, these convenient features must be changed to standard FORTRAN.

After the two programs are made compatible, the four-directional Laplacien obsiacle detection part in the Sher-Shen propram is taken out, and the Sonalkar-Shen program, i.e., The Rapid Estimation Schere, is put it..

Each of the two originel progroms is vers long by itself. After they are combined, the resultant program is too long and requires too much memory, 34 K . Hence it must be improved by reusing some memory spaces after the numbers that are stored in these spaces are uscd. That is to say, two or more variables share the same memory space because they are stored in that space at different non-overlapping instances of time during the execution. This only accounts for a small amount of nemory saved. The major reduction in the remory requirement is achieved by storing the noiseless rance matrix and noisy range matrix in the same memory spaces.
Moreover, the opaces set aside for the noise catrix
in the original program is eliminated. Hence the noise is added to the noiscless rance readinc; as it is generated and the resultine noisy range reading is put back in the same space, therefore erasing the noiseless readine. Here memory savinf does not come at no expense because the nolseless rance matrix and noise ratrix can be retrieved as two separated matrices from the noisy range matrix.

\section*{PROCEDURE OF SI!NLATIO:}

The IDIIOM dispiays three separnte pictures, the specification, the top view of the layout and the edge enhancement. If no interruption occurs in the display process, the Varian 620/i will use the initial parameters for all calculations. The lifht pen is used to interrupt the processink by pointing at the parameter that one desires to chance. The new value of this parameter is entered through the teletype. The Varian 620/i will then use the new dats and continue the processing. The picture on d: play can be replaced ry nother picture by pointing the light pen a: the appropriate character.

The inputs to the computer ann the dimensions and locations of the obstacles (the dimensions of the obstacles not used will be zero), the height and location of the laser ranrefinder, the maximum and minimum azimuth ancles, the maximum and minimum angles of elevation, and the standard deviation of the Gaussian noise used. The above input parameters have initial values as specified in Figure \(4(a)\). They can be chanced through the use of the light pen and the teletype.

With the input parameters, the program then displevs the top view of the obstacle leyout on the IDIIC\%. When the displey is interrupted by the light pen, the program then calculates the range data using Eq.'s (10) end (11).

With the noisy range data and the noise covariance, the program processes the rancc data to detect the complete edres of the obstacles, using the Rapid Estimation Schem, i.e., Eqं's (1) throuph (8). Eleven cases are simulated. The values of the inputs for each case are shom in Table 2.

\section*{NUMERICAL RESULTS AND DISCUSSICH:}

Eleven cases were simulated. Each case is a different conbination of obstacle sizes, locations, and notse parameters. Some have sinple obstacle and sche have inultiple obstacles. For each case, the parameters, topview, and edre enhancerent are displayed. The values of differcnt parameters are shown in the parareter display. The layouts of the obstacles are shom in the topview displays and the edge enhencement results, i.e., the edge imares, are shown in the edge enhancerent displays. The parameters are chosen to illustrate the effects of the change of noise covariance, obstacle sizes, and the distance between obstacles and laser for use in the Rapid Estimation Scheme. Some of these eleven cases are shown in Figures \(4-10\).

In Figure \(4(\mathrm{c})\), case No. 1, only the top edge of the 1 meter radius hemispherical boulder at 10 meters from the leser rangefinder (L.R.F.) can be
detected because the ohstacie is too cluse to the L.R.F. and hence the oignal-to-noise ratio (SRR) is small with 5 cm ; noise standard deviation. In case No. 2, the noise standard deviation is increased to 10 cm , and everything else is the same as in casc No. 1.

In Figure 5, case No. 3, the conplete edge of the boulder at 20 m . from L.R.F. is detected for noise with standard deviation equal to 5 cm . In case No. 4 and case No. 5, the simulation is repeated for case No. 3, except that the nolse standard deviation is now 10 cm . and 20 cm . respectively. The performance is very good. In Figure 6, case No. 6 , the complete edge of a 1 m . radius hemisphericnl boulder at 30 m . from L.R.F. with a noise standard devintion of 5 cm . is detected. The edge enhancement result is better than that of the seme boulder with the seme noise at 20 m . from L.R.F. In case No. 7, the noise standard deviation \(\sigma_{y}\) is increased to 20 cm . In this case, the bottom edge is not so vell defined. Case number 9 is a hemisphericnl crater with 1.5 m . radius at 12 m . from the rover, with \(\sigma_{n}\) equal to 5 cm . The far and near edges of the crater are detected but some noisy edges are picked up in the backeround.

Figure 7, case No. 8, is an equilateral pyramid of height 1 m . at 20 m . from the rover, with \(\sigma_{y}\) equal to 5 cm . and 10 cm . respectively. Here R.E.S. does not work too well. The top of the pyramid gets truncated.

Case No. 10 (Fig. 8) and case No. 11 respectively are multiple obstacles having a herispherical boulder, a hemispherical crater, and a pyrarid with their sizes and locations shown in the topview and parameter displays. The \(\sigma_{v}\) in case No. 10 and 11 are 5 cm . and 10 cm . respectively. Complete edges of all three obstacles are detected for the case with a smaller \(\sigma_{N}\). For the case with a larger \(\sigma_{N}\), the far edge of the crater and the bottom edge of the fyramid are not detected.

\section*{CONCLUSION}

It can be concluded that the Rapid Estimation Scheme works very well in detecting complete edges of obstacles if the obstacles are far enouch from the laser rangefinder ( \(>\) about 15 meters) and the noise standard deviation is not too large (<ebout 10 cm . This scheme can still detect top edge of boulder and near edce of crater if the obstecles are too close and noise standard deviation is larger than 10 cm . Therefore, the Rapid Estimation Scheme is equavalent to or better than the fourdirectional Laplacian scheme in performance. However, the former scheme needs sliphtly rore computation than the latter. Therefore, the R.E.S. extracts more information from the noisy range matrix if the slifintly larger number of computation is allowable.

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Proceedings of the Milwaukee Symposium on Automatic Control, Mar. 1974, Milwaukee, Wis.
3. J. S. Sher end C.N.Shen, "Simulntion of The Range Data and The Ficture Enhoncement Scheme For The Mars Rover", 1975 Computer Science Conference, University of Naryiand.
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5. P. Sanyal and C.N. Shen, "Bayes Decision Rule for Rapid Detection and Adeptive Estimntion Scheme with Spnce Aprlications", IEFE Transact ions on Automntic Control Vol. AC-19, No. 3, June 1974, pp 228-231.
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TABLE 2 Values of Input Parazeters
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline 8 & \multicolumn{3}{|l|}{Location ( \(\mathrm{X}, \mathrm{y}\) ) in Netces} & \multicolumn{2}{|l|}{Refeht of in Vatans} & \multicolumn{2}{|l|}{Radins of
in Vaters} & \multicolumn{4}{|c|}{Angle in Degrees} & \multirow[t]{2}{*}{\[
\begin{aligned}
& \text { Etend. } \\
& \text { fevie- } \\
& \text { fion or } \\
& \text { nolse } \\
& \text { in cr. }
\end{aligned}
\]} & \multirow[t]{2}{*}{\[
\begin{gathered}
\hline 8 \\
0 \\
0 \\
15 \\
F!5 .
\end{gathered}
\]} \\
\hline N & \[
\begin{gathered}
\text { of } \\
\text { Pyraz1d }
\end{gathered}
\] & or & \[
\begin{aligned}
& \text { of } \\
& \text { Crater }
\end{aligned}
\] & \[
\operatorname{ran} 62-
\]
f:nder & Pyra-id & Boulder & Crater & \[
\begin{gathered}
\text { Nax. } \\
A=\text { inuth }
\end{gathered}
\] & \[
\begin{aligned}
& \text { Nin. } \\
& \text { Aztruth }
\end{aligned}
\] & \[
\begin{array}{|c|}
\text { Max. } \\
\text { Elevat!or }
\end{array}
\] & Ein. & & \\
\hline 1 & \((-1,0)\) & \((0,10)\) & \((1,0)\) & 3.0 & 0.0 & 1.0 & 0.0 & 7.0 & -7.0 & 19.6 & 10.0 & 5.0 & 4 \\
\hline 2 & \((-1,0)\) & \((0,10)\) & \((1,0)\) & 3.0 & 0.0 & 1.0 & 0.0 & 7.0 & -7.0 & 19.6 & 20.0 & 20.0 & 5.s. \\
\hline 3 & \((-1,0)\) & \((0,20)\) & \((1,0)\) & 3.0 & 0.0 & 1.0 & 0.0 & 4.0 & -4.0 & 9.0 & 5.3 & 5.0 & 5 \\
\hline 4 & \((-1,0)\) & \((0,20)\) & \((1,0)\) & 3.0 & 0.0 & 1.0 & 0.0 & 4.0 & -4.0 & 9.0 & 5.0 & 10.0 & n.s. \\
\hline 5 & \((-1,0)\) & \((0,20)\) & \((1,0)\) & 3.0 & 0.0 & 1.0 & 0.0 & 4.0 & -4.0 & 9.0 & 5.0 & 20.0 & n. 3. \\
\hline 6 & \((-1,0)\) & \((0,30)\) & \((1,0)\) & 3.0 & 0.0 & 1.0 & 0.0 & 3.0 & -3.0 & 6.6 & 3.0 & 5.0 & 6 \\
\hline 7 & \((-1,0)\) & \((0,30)\) & \((1,0)\) & 3.0 & 0.0 & 1.0 & 0.0 & 3.0 & -3.0 & 6.6 & 3.0 & 10.0 & n.s. \\
\hline 6 & \((0,20)\) & \((-2,0)\) & \((1,0)\) & 3.0 & 1.0 & 0.0 & 0.0 & 4.3 & -4.3 & 9.0 & 5.0 & 5.0 & 7 \\
\hline 9 & \((1,26)\) & \((-1,18)\) & \((0,12)\) & 2.0 & 0.0 & 0.0 & 1.5 & 7.5 & -7.5 & 11.3 & 2.3 & 5.0 & n.s. \\
\hline 10 & \((1,26)\) & \((-2,28)\) & \((0,22)\) & 2.0 & 1.0 & 1.0 & 1.5 & 7.5 & -7.5 & 11.3 & 2.3 & 5.0 & 8 \\
\hline 12 & \((1,26)\) & \((-1,28)\) & \((0,12)\) & 2.0 & 1.0 & 1.0 & 2.5 & 7.5 & -7.5 & 11.3 & 2.3 & 10.0 & n. 5. \\
\hline
\end{tabular}
n.s. mears 'not shown'


Figure 1 - Side View of Rover and Terrain


Figure 3- Side View of the Plane with \(\theta=0^{\circ}\)

\section*{RECOCHITIOM OF 3-D ODSTRCLES \(\ldots\) A SIHULIIIOH
}
 USED at the rillice fllide of the kirs rover
 aswar Pemtent:


cowate tu:nt mi ist





WESEI TOPY!EX RAICE ERIA EDCE EETEGIION

\section*{2. \(51 \%\)}








Figure 5
Display for
    Case \#
            PAEAREIE IUPY:CN RAKCE ZAIA




Figure 7
Display for Cese " 8

PARFAETER TOPYIEU RAMCE ZAIA




Task A. 2 Detection of Discrete Obstacles - R.V. Sonalkar Faculty Advisor: Professor C. N. Shen

There are many potential methods which could conceivably be used to detect discrete obstacles on a terrain. The major objective of this task is to enhance the outlines of obstacles in a two-dimensional perspective view from a set of laser rangefinder data contaminated by measurement noise. A large discrete jump of the first differences of noisy rangefinder data will outline certain edges on obstacles (i.e. top and sides of a boulder and near edges of craters or crevasses). The Rapid Estimation Scheme can determine this jump of input under a sub-optimal condition by erasing all of the previous memory of the covariances of the Kalman Filter at the stage where the jump occurs. The present task has been aimed at retaining the past memory in the Kalman filter at the time during the estimation of the jump of the input and thus improve the estimate. The mathematical procedures required to achieve this objective have been developed and are described in detail in the reprint of the paper entitled "Simultaneous Bayesian Estimate of States and Inputs" which follows.

PRESENTED AT THE JOINT AUTOMATIC CONTROL CONFERENCE, July 1976, at Purdue University. Proceedings of the same conference.

SImULTANEOUS bAYESTA: ESTIMATL F STATES AND IMFUTS

\section*{R. V. Sonalkar, Member IEE:}

PATTERN AMALYSIS AND RECOCNIT:O: CORP.
On the :'all
Rome, New York 13440
C. N, Shen, Professor

Electrical and Systems Engineering Dept. RENSSFLAER POLYTECHMICA: I:SSTITUTE Troy, New York 12181
\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|l|}{ABSTRACT} \\
\hline \multicolumn{2}{|l|}{This paper describes a Simultancous Eaycsian} \\
\hline \multicolumn{2}{|l|}{Estimate (S3E) of otates and inp:ts. .he dascreve} \\
\hline \multicolumn{2}{|l|}{to unkno:n inputs, isolated on repoated. The Ste} \\
\hline \multicolumn{2}{|l|}{oceurs. It is mainly desicnet to ifipreve the} \\
\hline \multicolumn{2}{|l|}{upon get-irg a minima variance (i! int inp:t ez:imate.} \\
\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{rovides a proper correction te the existing state otimate Es oposed to the ejurent RES withh}} \\
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\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{merely oitains a \(: 1 /\) input estimnte. The approach}} \\
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\hline \multicolumn{2}{|l|}{which were impesed for ob:uini:z, the :V inp:t} \\
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estimate; :.e., it loss not imeco any more restric- \\

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\hline \multicolumn{2}{|l|}{the oncs that alreasy eyjsten. Since the spe ts a} \\
\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{to deter:ire if the zaknown inputs are present. A Bayes desision rule uscil for such a purfose has been}} \\
\hline & \\
\hline \multicolumn{2}{|l|}{described in Ref. [1] and is not repested in this papor.} \\
\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{NOMERCLATU:}} \\
\hline & \\
\hline \multicolumn{2}{|l|}{Alphabeti=0: s-mbels} \\
\hline \multirow[t]{2}{*}{3} & \multirow[t]{2}{*}{(rxq) Input distribution matrix} \\
\hline & \\
\hline \(\operatorname{Cov}(x)\) & Covariance metrix of the random variable \(x\). \\
\hline \(E(x)\) & Statistisal expestution of \(x\) \\
\hline \multirow[t]{2}{*}{\(F_{i}\)} & (rxn) State transition or system Ajnamice \\
\hline & matrix \\
\hline \(\mathrm{H}_{1}\) & (nxn) Meazurenent mats: \\
\hline in & Hypothesis \\
\hline \(I_{n}\) & (nxn) Idensity mas: \\
\hline \multirow[t]{2}{*}{k} & First stage as which trjut oczurs in a \\
\hline & sequence \\
\hline \(\mathrm{n}_{1}\) & Covartance of the predieted state voctor \(\bar{x}_{L}\) \\
\hline
\end{tabular}
\(n\) Dimension of the state vestor \(x_{i}\)
\(N(m, I)\) Normal distribution with mean vecto: and covariance matrix
r.- null vector
if Covariance of the coatc estimate \(\hat{x}_{i}\)
\(p(x / h)\) pdf of \(x\) condivicred on \(h\)
Dimersion of the infut vector \(u_{i}\)
dast stige at which an input occurs in
a seguence
(sxs) Cbecwation notse covartance mitr:
Dimansion of the obecrva:ion vestor z:
(quı) inpu: vector
(sx1) WQ: seçucnce vecte:
(nx1) "M plant or systen :oise secuenec
vector
(n\%1) S:ave reeter
\((n+q) \times:\) Augmensed state vez:or
nxl Cempenen: vecter
exi Compenent vecter

(ex:) Chservation vecter

\section*{Greek Sy-bols}
\(\delta_{i j}\) Kroenecinen del:a function \(=1\) tf i=j znd =0 if ifj
- Matrix ceflece by tq. (24b), frverse of
\(\rho\) (A) Rank of matrix \(\wedge\)
* Natrix defined by Eq. (0)

\section*{Subscripts}

\section*{i,j Time stage subscripts \\ \(n, n+q\) Dimensions of null-vectors 0 .}

\section*{Superseripts}
\(T\) Matrix transpese
* Quantity related to augmented system

1,0 Hypotheses

\section*{Special Symbol3}
\begin{tabular}{ll}
\(\dot{x}_{i}\) & Best estimate of \(x_{i}\) \\
\(\bar{x}_{i}\) & Fredicted estimate of \(x_{i}\)
\end{tabular}

\section*{Abbreviations}

Kr Kaman filter
Map Haximum A Posteriori
av Minimun variance
pdf Prolability density function
RES Rapid Estimation Sct.eme
SBE Sinultaneous Eaycsian Estimate
WGM White Caussian : Noisc

\section*{InTROLUCTIUN}

The applicatioti, of the ralman Eilter have been extre:nely varied in nuncrous fields such as: satellite, aireraft, ard ground vehicle tracking; fectask process control; inage processing; edce detection; ete. The extcnsive usc of the filter to obtain estimates of the statas of a systom has also nanifested its shortec-i-5e. The filter is optirsi only if the systen paramesers are known correctij, On the other hand, uncertaintias in the system dymaniez and noise statistics often cause the filter to diverge, if no safcruards are introdjeed to detect and prevent such a runavay condition. Sudden changes in the states of a syste-, which may be encountared whlle tracking a nanewer'ng target, also result in throwing the filter off-track. These difficultles are incrcased then tracking multiple targets with multíle redars.

Such changes can te modezled es iselated inputn to the systen [1, 3] or segunnees of infuts [2] added to the state. In genaral, the difference equation for the states of a system subiect to unknown infyts can be written as follows: Syctem dynamics:
\(x_{i+1}=r_{i} x_{i}+w_{i}+\sum_{j=k}^{r} B u_{j}{ }^{\delta}{ }_{i j}\)
Obscrvation equation:
\(z_{i+1}=H_{i+1} x_{i+1}+v_{i+1}\)
where \(x_{i}\) and \(w_{i}\) are \(n\)-vectors; \(u_{j}\) is an unknown \(c\) vector; \(z_{i+1}\) and \(v_{i+1}\) are s-vectors; \(H_{i+1}, 3\), and \(r_{i}\) are known matrices of compatible dimensions. Vectors \(w_{i}\) and \(v_{i}\) are WG: sequence vectors. The plant noise vestor \(w_{1}\), the observation noise vector \(v_{i}\), and the initial state \(x_{0}\) are all serumed to be uncorrelated and of known distributions; the distributicns being \(N\left(O_{n}, Q_{i}{ }^{\delta}{ }_{i j}\right), ~ H\left(O_{z}, R_{i} \sigma_{i j}\right)\), and \(N\left(\dot{x}_{0}, P_{0}\right)\) respectiveiy. Covariance \(Q_{i}\) is positive semi-definite while \(F_{o}\) and \(r_{i}\) are postive definit matrices. llere, and in the subsequent derivations, \(O_{n}\) denotes a null vector of diturnsion ( \(n \times 1\) ), i.e., an n-vector with all zero elements.

In the abscnce of the input term, the state \(x_{i}\) could have becn estimated optimally by the Kalman :ilter [4]. However, the presence of the unkrowns \(u_{i}, k\), and \(r\) in Eq. (1) pose the problem to be solved.

To detect such an input, it nust be estinated at Eirst. RES has used a suboptimal ninimum varianco input esti"nte which has been shown to exist if the following condtions are satisfled.
a. \(q \leq n \leq s\)
(3a)
b. Katrix (HD) is of fuil rank (3b)

The implications of thete two condttions will be discussed in a later section.

It has also been shown in Ref. [5] that the nv input esti-ate i. suboptimal si.ce? is based on certain afproximations. UUdei certain condisions the XV input estimato also causes the filter to lose its "memory" by basing the current state estirate entirely on the current observation.

Therefore, the advantige of havins a smooth entimate until the input detection takes place, is lost and subsequent estimutas are noisier.

In an effort to determinc a better and possibly optimal joint state and input estimate, the method described in this paper was devoloped. The estimate retains the menory of the filter as opposed to the MV estimate which, under certain conditions, became a linited memory open-filter. The estimate has not been proved to be optinal, or cven better than the previous MV estimate. However, because it has been derived from the basics without approximations, and because it retains the memory of the filter, it is expected that the simultaneous Bayesian estimate would provid- a better filter performance.

The problem of detection is not described. The interested reader will find it in Refis. [1, 2, 5, 6]. The SBE is derived, conditioned on the presence of the input. A decision schene will have to be used in conjunction with this to determine whether this conditional estimate is correct.

\section*{bayesian tstimate}

Two hypotheses are defined -
\(h^{\prime}\) : Irput occurs at stage i.
\(h^{0}\) : Input does not eccur at stage \(i\).
These definitions assume that the decisions
between stages 0 and \(i-1\) have alredty been made.
\[
\begin{align*}
& \therefore \quad h_{1}: x_{i+1}^{1}=1_{i} x_{i}+w_{i}+\mid j u_{i}  \tag{it}\\
& h_{0}: x_{i+1}^{0}=\Gamma_{i} x_{i}+w_{i}  \tag{5}\\
& \therefore x_{i+1}^{1}=x_{i+1}^{0}+B u_{i} \tag{6}
\end{align*}
\]

Thus \(x_{i+1}^{0}\) is the partial state without the input. Defining a ne: aucmented statc \(X_{i+1}\), Eq (5) van be rowrikten in a nen notation.
\[
x_{i+1}^{2}=\left[\begin{array}{ll}
I_{n} & B
\end{array}\right]\left[\begin{array}{l}
x_{i+1}^{0}  \tag{7}\\
u_{i}
\end{array}\right]^{0}=\Psi x_{i+1}
\]
where is \(\left[\begin{array}{ll}I_{n} & 3\end{array}\right] ; x_{i+1}\left[\begin{array}{c}x_{i+1}^{0} \\ u_{i}\end{array}\right]\)
\(I_{n}\) is a ( \(n \times n\) ) unity matrix and \(X_{i+1}\) is the augmented \((n+q)\) vector. A priori information about \(x_{i+1}^{0}\) exists in the form of the Kalman filter estimate at stage i.
\(\therefore \dot{E}\left(x_{i}^{0}, z_{i}\right)=\bar{x}_{i+1}^{0}=F_{i} \hat{x}_{i}\)
where \(\hat{x}_{i}\) is the best unbiased estimate at nitace \(i\) obtained from the obecrvation \(z_{i}\).

The error covariance of the unbiased a priori estimate \(\bar{x}_{i+1}^{0}\) is obtained from the system dynamies, Eq. (1).
\[
\begin{gather*}
M_{i+1}^{0} \Delta E\left[\left(x_{i+1}-\bar{x}_{i+1}^{0}\right)\left(x_{i+1}-\bar{x}_{i+1}^{-0}\right)^{T} / h^{0}\right] \\
=F_{i} P_{i} F_{i}^{T}+Q_{i} \tag{10}
\end{gather*}
\]
where
\[
\begin{equation*}
p_{i} \Delta E\left[\left(x_{i}-\hat{x}_{i}\right)\left(x_{i}-\hat{x}_{i}\right)^{T}\right] \tag{11}
\end{equation*}
\]
is the error covariarce of the estimate \(x_{i}\).

Substituting Eq. (7) in (2), the observation equation conditioned on \(h^{1}\) can be rewititen in a different form.
\[
\begin{align*}
h^{1}: z_{i+1} & =H_{i+1} x_{i+1}^{1}+v_{i+1} \\
& =H_{i+1}^{*} x_{i+1}+v_{i+1} \tag{12}
\end{align*}
\]
whe:o \(\quad\left\|_{i+1}^{*} \quad \wedge_{i+1}\right\|_{i}^{\psi}\)
and \(\psi\) is defined by Eq. (0).

To obtain a MAP (raximum a posterioni) estimate of \(x_{i+1}\), the probability censity function (pdf) of \(x_{i+1}\), conditioned on \(z_{i+1}\), must be maximizod. Using the Bayes rule for conditional probabilitics [4]:
\[
\begin{equation*}
p\left(x_{i+1} / z_{i+1}\right)=\frac{p\left(z_{i+1} / x_{i+1}\right) p\left(x_{i+1}\right)}{p\left(z_{i+1}\right)} \tag{14}
\end{equation*}
\]

Since, the state \(x_{i+1}^{0}\) and input \(u_{i}\) are independent of each other, \(p\left(X_{i+1}\right)\) hin. be wricten as a product of \(p\left(x_{i+1}^{0}\right)\) and \(p\left(u_{i}\right)\).
\(\therefore p\left(x_{i+1} / z_{i+1}\right)=\frac{p\left(z_{i+1} / x_{i+1}\right) p\left(x_{i+1}^{0}\right) p\left(u_{i}\right)}{p\left(z_{i+1}\right)}\)

We may consider \(u_{i}\) to be normally distributed, \(\because\left(\bar{u}_{i}\right.\), \(\left.P_{u}\right)\) so that the derivative of \(\ln p\left(u_{i}\right)\) with respoct to \(X_{i}\) gives.
\(\frac{\partial 1 n}{\partial x_{i}} p\left(u_{i}\right)=-\left(\frac{\partial u_{i}}{\partial x_{i}}\right)^{T} p_{u}^{-1}\left(u_{i}-\bar{u}_{i}\right)\)

The input vector \(u_{i}\) is assumed to be deterministic and of unknown magnitude. Therefore, it may be considered as a random variable with covariance tending to infinity. Allowing \(P_{u} \rightarrow \infty\) causes \(\frac{\partial \ln p\left(u_{i}\right)}{\partial x_{i}} \rightarrow o_{n+q},[7]\). This fact allows the maximization of \(p\left(X_{i+1} / z_{i+1}\right)\) to be expressed in the following form.
\[
\begin{align*}
& \frac{\partial \ln }{\partial x_{i+1}} p\left(x_{i+1} / z_{i+1}\right) \left\lvert\, \dot{x}_{i+1}=\left[\frac{\partial 1 n}{\partial x_{i+1}} p\left(z_{i+1} / x_{i+1}\right)\right.\right. \\
& \left.+\frac{\partial \ln }{\partial x_{i+1}} p\left(x_{i+1}\right)\right]_{-x_{i+1}}=0_{n+1} \tag{17}
\end{align*}
\]

As is well' \(r: s\), the solution of this equation leade to the MAP estim?te \(\hat{X}_{i+1}\) of the augmented state \(X_{i+1}\)

It is understood that all the puf's in F. (17) are conditioned on the previcus observation \(z_{i}\), which permits the palf of \(x_{i}\) to be completely specified by its mean, Eq. (9), and covariance, Eq. (10). The conditional pdep( \(\left.z_{i+1} / x_{i+1}\right)\) requires the following quantitics.
\[
E\left(z_{i+1} / X_{i+1}\right)=H_{i+1}^{*} x_{i+1}
\]
\[
\begin{equation*}
\operatorname{cov} .\left(z_{i+1} / x_{i+1}\right)=R_{i+1} \tag{18}
\end{equation*}
\]

Before proceeding any further, let le suppress the subscripts for convenience and use the following notation.
\[
\begin{aligned}
& x_{i+1}^{0}=x, \quad x_{i+1}=x{s_{i+1}}^{0} \quad M_{i+1}=n \\
& u_{i}=u, \quad z_{i+1}=z, \quad H_{i+1}^{*}=H^{n}, \quad H_{i+1}=H
\end{aligned}
\]

For normally distributed variables, [q. (17) c. casily be shown to be reducible to the followine minimization.
\(\frac{\partial}{\partial X}\left[\left(z-H^{*} X\right)^{T} R^{-1}\left(z-H^{*} X\right)+(x-\bar{x})^{T} M^{-1}(x-\bar{x})\right]_{\hat{X}}=0_{n+q}\) (20)

Since \(x=\left[\frac{x}{u}\right]\), the mininization of the second term inside the parenthesis can be performed as follows:
\[
\begin{align*}
\frac{\partial}{\partial X^{2}}\left[(x-\bar{x})^{T} M^{-1}(x-\bar{x})\right] & =\left[\begin{array}{l}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial u}
\end{array}\right] \quad\left[(x-\bar{x})^{T} M^{-1}(x-\bar{x})\right] \\
& =\left[\begin{array}{c}
M^{-1}(x-\bar{x}) \\
{ }_{q}
\end{array}\right] \tag{21}
\end{align*}
\]

The last equality is due to the fact that \(x\) is independent of the input \(u\). As a resuit, the terin \(\frac{\partial}{\partial u}(x-\bar{x})^{T} M^{-1}(x-\bar{x})\) is equal to the \(q\)-dimensional. null vector, \(\mathrm{O}_{\mathrm{q}}\). Subutituting Eq. (21) in (20)
Eives:
\(-H * *^{T_{R}}\left(z-11^{-1}: \hat{X}\right)+\left[\begin{array}{c}\therefore^{-1}(\hat{x}-\bar{x}) \\ 0_{6}\end{array}\right]=0_{n+q}\)

By further manipulations, an expression for the Baycsian estimate \(X\) is obtained.
\[
\begin{align*}
\hat{x} & =\bar{x}+q^{-1} H^{T_{R}}-1  \tag{23}\\
\bar{x} & =\left[0_{q}^{x}\right]=\left[\begin{array}{c}
H_{i} \\
r_{i}^{0} \\
0_{i}^{0}
\end{array}\right] \tag{24a}
\end{align*}
\]
where
\[
\phi=\left\{H * *^{T} R^{-1} H^{*}+\left[\begin{array}{cc}
u^{-1} & 0_{n q}  \tag{24b}\\
0_{q n} & o_{q q}
\end{array}\right]\right\}
\]
and
\[
\hat{x}=\left[\begin{array}{l}
\hat{x}_{i+1}^{0}  \tag{24c}\\
\hat{u}_{i}
\end{array}\right]
\]

Even though similar in appearance, Eq. (23) bears differences in two striking details with the Kalman filter estimate. First, \(\bar{X}\) given by Eq. (24a) is not equal to \(E(X)=\left[\begin{array}{c}E(x)\end{array}\right]\) as is generaily true in the Kalman filtor updating equation. Secondly, the matrix \(p\) is invertible oniy undor
certain conditions to be derived below. In the \(K r\) equations, the error covariance \(P\), which generally oceurs in place of \(\phi^{-1}\), is positive definite by definition. On the other hand, SBE can be obtained only when \(\rho\) is invertible when certain conditions are met.

Using Eqs. (8) and (13), the first term in \(Y\) is seen to be of the following form:
\(H *{ }^{T} R^{-1} H A=\psi^{T} H^{T} R^{-1} H \psi=\left[\begin{array}{l:c}H^{T} R^{-1} H & H^{T} R^{-1} H B \\ \hdashline B_{H} T_{R} R^{-1} & B^{T} H_{R} T^{-1} H B\end{array}\right]\)
The matrix in Eq. (25) is of iimensiens \([(n+q) \times(n+q)]\), but is of rank \(n\). Since we have assuned \(s>n, R\) positive definite, and S to be of full rank, \(\mathrm{H}^{\mathrm{T}} \mathrm{R}^{-1} \mathrm{H}\) is positive cefinite. Moreover, \(\psi\) is a \([n \times(n+q)]\) matrix, given by [q. (8), and is of full rank. Therefore, \(\psi T_{11} T_{R}^{-1}{ }_{11} \psi\) camot be of any hipher rank than \(n\).
\[
\triangleright=\left[\begin{array}{l:l}
H^{T} R^{-1} H & H^{T} R^{-1} H B \\
\hdashline B^{T} H^{T_{R}} R_{H}^{-1} & B^{T_{H}} H^{\top} R^{-1}
\end{array}\right]+\left[\begin{array}{c:c}
n^{-1} & 0_{n q} \\
\hdashline 0 & 0_{q n} \\
0_{q} & 0_{q}
\end{array}\right](26)
\]

In Eq. (26), both matrices on the right sidc are positive scri-definite and of rank \(n\) each. Hence the rank of o cannot be greater thar 2 n . Let us see under what conditions, a can be a positive definite matrix. If \(\phi\) is to be positive definite, \(y^{T} \emptyset \mathrm{y}\) must be positive for any \((\mathrm{n}+\mathrm{q})\) dimensional vector \(y\), which does not have all zero elements.

If \(y\) is defined in terms of its \(n\) and \(q\) dimensional component vectors \(y_{1}\) and \(y_{2}\) respectively, i.e., if
\[
\left.y=\left[\begin{array}{l}
y_{1}  \tag{27}\\
y_{2}
\end{array}\right]\right\} \begin{aligned}
& \mathrm{n} \\
& \mathrm{q}
\end{aligned}
\]
then, using Eq. (24b), \(y^{T}\) y may be written as follows:
\[
\begin{equation*}
y^{T} \odot y=y^{T} H^{T} R^{-1} H^{n} y+y_{1}^{T} H^{-1} y_{1} \tag{28}
\end{equation*}
\]

Therefore, if 1 is to be positive definite, then
\[
\begin{equation*}
y^{T} H^{-} T^{-1} H^{\prime} A y+y_{1}^{T} H^{-1} y_{1}>0 \tag{29}
\end{equation*}
\]
must be satisfied for all ( \(n+q\) ) dimensional vectors \(y \neq 0_{n+q}\). Here \(y \neq 0_{n+q}\) means that \(y\) is not equal to the nuil vector of ( \(n+q\) ) dimensions, i.e., \(y\) has ut least one non-zero element.

Any non-null vector \(y\), defined by F.q. (27) can have either \(y_{1}=0_{n}\) or \(y_{1} \neq O_{n}\). Puttine these two cases together, we cover all vectors \(y\) which are non-mull, i.c., \(y \neq 0_{n+q^{\circ}}\)
i) If \(y_{1}=o_{n}\), then \(y_{2} \neq o_{q}\); since \(y_{1}=o_{n}\) and \(y_{2}=0_{q}\) simultaneousiy will make \(y=0_{n+q}\), which must be eycluded for provirig (29). :low, tnec̣ality (29) reduces to the following after usi:ag fq. (25).
\[
\begin{equation*}
Y_{2}^{T} B^{T} H^{T} R^{-1} H B y_{2}>0 \tag{30}
\end{equation*}
\]

If this inequality is to be true for all \(y_{2} \neq 0_{Q}\) then (HB) \({ }^{T} R^{-1}(H B)\), which is a ( \(q \times q\) ) matrix, must be positive definite. This condition is satisfied if \(q \leq n \leq s\) in acdition to (HB) being a matrix of full rank. Matrices H and B are of dimensions (sxn) and ( \(n \times q\) ) respectively, so that the ( \(s \times q\) ) matrix (HB) must be of rank q. Therefore, inequality (30) is true for all \(y_{2} \neq 0_{q}\) if:
a) \(q \leq n \leq 5\) and b) Rank (HB) \(=q\)
ii) When \(y_{1} \neq o_{n}, y_{2}\) may take any value, since \(y \neq 0_{n+q}\) is satisfied by virtue of \(y_{1} \neq 0_{n}\). Under this condition, inequality (29) remains unchanged. However, \(\mathrm{N}^{-1}\) is positive definite, which makes \(y_{1}^{T} M^{-1} y_{1}>0\) for all \(y_{1} \neq 0_{n}\). Therefore, the sum of two quantities, one of which is positive ( \(y_{1}^{T} M^{-1} y_{1}>C\) ) and the other non-negative ( \(y^{T} \|:^{T} R^{-1} H: ~ V \geq 0\) ), must be positive. As a result, under this case also, inequality (20) is satisfied.

Putting these t:o cases tozether, the conditions to be satisfied for the positive definiteness of \(\$\) are:
a) \(q \leq n \leq s: \quad \operatorname{rank}(H 1 P)=q\)
b) \(\mathrm{M}^{-1}\) must be positive definite

Conditions (31a) are the same as (3a) and (3b) which were the necessary conditions for obtainire the MV estimate in FIS. Also, II is the error covariance of the Kr prodicted state \(\bar{x}\) and i: therefore poesitive definitu. Therefore, S3E does not inpose any additional constraiate on the "Elere for the purpose of obtaining the additional input estinate.

\section*{implications or the hecessary compitions}

Herc, we are tryinz to nake a 'one-shot' estimate of the input vector without having ,he benefit of 'input-dynamics.' Therefore, the filter necessitates the availability of sufiticient observations by imposing the condition ( \(q \leq n \leq s\) ).

The requirement that (HB) be of full rark indicates that all tio q inputs must apper in the observations. If rank ( \(: 33\) ) \(=0(H B)<q\) then it would mean that all of \(q\) input components do t.ut dircetly affect the observatiens in any way. However, they may do so through the statcs at subsequent stages ard may be estimated later [8, 9]; Therefore, \((n(H B)=q\) ) may be interpreted as the 'observability' condition for the input.

Condition (31b) is not a new one and merely requires that a reliable prior estimate must exist. It is always true in case of a properly formulated problem, as long as the systern hoise covariance \(Q\) is not taken to be a null matrix. .

PROPERTEES OE THE ESTIMATE
If the estimate (23) is \(t=\) bo unbiased, it must satisfy the requirement
\[
\begin{equation*}
E(x)=E(X) \tag{32}
\end{equation*}
\]
where
\[
E(x)=E\left[\begin{array}{l}
x_{i+1}^{0}  \tag{33}\\
u_{i}
\end{array}\right]=\left[\begin{array}{l}
E\left(x_{i+1}^{0}\right) \\
E\left(u_{i}\right)
\end{array}\right]=\left[\begin{array}{l}
E\left(x_{i+1}^{0}\right) \\
u_{i}
\end{array}\right]
\]

Taking the expectation of Eq. (23) we get
\[
\begin{align*}
E(\hat{X}) & =E(\bar{X})+\phi^{-1} H *^{T} R^{-1} E\left(z-H^{*} \bar{x}\right) \\
& =E(\bar{X})+\phi^{-1} H *^{\top} R^{-1} H^{*} E(X-\bar{X}) \tag{34}
\end{align*}
\]

The last equality in (34) is obtained from Eq. (12) and from the fact that \(v_{i+1}\) is a zero mean voctor. Because \(\bar{x}\) is an unbiased estimate of \(x\) :
\[
E(\bar{X})=\left[\begin{array}{l}
E(\bar{x})  \tag{35}\\
Q_{q}
\end{array}\right]=\left[\begin{array}{l}
E(x) \\
Q_{q}
\end{array}\right]
\]

Using Eq̧. (33) and (35) it is possible to write the expectation of \(x-\bar{x}\) in the following form.
\[
E(X-\bar{x})=E\left[\begin{array}{l}
x \\
u
\end{array}\right]-E\left[\begin{array}{l}
x \\
0 \\
q_{Z}
\end{array}\right]=\left[\begin{array}{c}
E(x-\bar{x}) \\
u
\end{array}\right]=\left[\begin{array}{l}
0 \\
u \\
u
\end{array}\right]_{(36)}
\]

Due to the particular form of \(\mathrm{E}(\mathrm{X}-\overline{\mathrm{X}})\) in the above equation, it can be easily confirmed that the following icentity is true.
\[
\phi^{-1}\left[\begin{array}{ll}
n^{-1} & 0_{n q}  \tag{37}\\
0_{q n} & 0_{q q}
\end{array}\right] E(x-\bar{x})=0_{n+q}
\]

Adding this null vector to the right side of Eq. (34), we get
\(E(\hat{X})=E(\bar{x})+\phi^{-1}\left\{H^{T} R^{-1} H^{*}+\left[\begin{array}{ll}M^{-1} & 0_{n q} \\ 0_{q n} & 0_{q q}\end{array}\right]\right\}\)
\[
\begin{equation*}
E(x-\bar{x}) \tag{38}
\end{equation*}
\]

But, the quantity in the brackets in this equation is itself, Eq. (24b). Therefore, the required property emerges imnicdiately.
\[
\begin{equation*}
E(x)=E(\bar{x})+E(x-\bar{x})=E(x) \tag{39}
\end{equation*}
\]

By drawing analogies with the Kalman filter described in Rof. [4], the fc lowing quantities can be defined:
\[
\begin{align*}
& \text { Gain }= K_{i+1}^{*}=  \tag{40}\\
& Q^{-1}{ }_{i+1}^{T} R_{i+1}^{-1} \\
& \text { Covariance }= P_{i=1}^{*}=  \tag{41}\\
&\left.\left(x_{i+1}-X_{i+1}\right)^{T}\right\}
\end{align*}
\]

Using Eq. (7), corresponding quantities for the ndimensional system of interest can be deduced.

\section*{DISCUSSIO:IS}

The Baycsian estimate derived in this part can be calculated under conditions which can easily be satisficd, (31). In last, these conditions were assuned to be true cven for obtaining the minimum variarce input estimate in Ref. [1]. Therefore, the Bayesian estimste dene not inpose any new restric:ions, and at the same time, it makes use of the 'a priori' information in the form of \(\bar{x}_{i+1}\) and \("_{i+1}\). Under cortain onnditions, this information was lost in the RES, [5].

No approvirations were made and the Fq. (23) represents a valid and a cerrect Biyesian cotimate. Even if neyesian estimation is not a now tecmique, its application to simul vancous state and mput estimation is. Also, state abmentation has been used often in the literature for e:timatire binses and unknom time-distributed inputs, but not for one-stage estitnates. The schence proposed here is to be employed in conjunction with a decision rule to estimate and detect any inp:ts, as they occur. Therefore, it is suitable for inproving the performance of the RES where irmociate detection of sudkicn changes in state (inputs) is considered equally as important as obtaining a good overall stace estimate.

SBE still remains to be investigated further. The optimality on sub-optirality properties have to be deter-mined under the orti:ogenality condition. The fact that \(S E E\) retains the menory of the filter, gives a strong indication that it will give a much better state estimate in the presence of inputs. Further research concerning this approach and corputer simulations of the RES incorporating the SBE are necessary.

\section*{ACKIONLEDGEMENT}

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\author{
Task A. 3 Estimation of Laser Rangefinder Errors - T. Ostroski Faculty Advisor: Prof. C. N. Shen
}

The detection of discrete obstacles using laser rangefinder data involves the mathematical processing of range data contaminated by instrument measurement noise. The instrument noise should be determined in advance of specifying parameters to be used in evaluating a system for enhancing the images of potential obstacles. In the past the mean and variance of the noise were assumed and employed in computer simulations of the edge enhancement procedures. The actual determination of the statistics of the noise for an actual instrument was not attempted.

The purpose of this task was to establish the actual noise statistics of an existing laser rangefinder. Methods cor determining the noise were developed and applied to a set of such data provided by Mr. Dobrotin of the Jet Propulsion Laboratory. Details of the procedure and the results are provided in the reprint of the paper which follows.

PRESENTED AT THE SEVENTH ANNUAL MODELING AND SIMULATION CONFERENCE, PITTSBURGH, PA., April 1976 and published in the Proceedings of the Conference.

ACCURACY ESTIMAIE OF THE LASER RAMCEFTMDER FOR MARS ROVER*

\author{
T. Ostroski and C. N. Shen \({ }^{-}\) \\ Rensselaer Polytechnic Instizute \\ Troy, New York \\ 12181
}

\section*{ORIGINAL PAGE is \\ OF POOR QUALITY}

\section*{ABSTRACT}

A method was developed to determine the accuracy of a rangefinder when errors exist due to the test setup. An analytic procedure was developed to eveluate actual data from a test perforred on the leser rangeflnder for the Mars rover project.

\section*{INTRODCCTION}

The laser rangefinder and a telerision cemera are two elternatives being considered to supply the
data for the terrain zodelire, obstacle detection, and path selection systems for the Nars eutonomous rover. The advantage of the laser rangefinder is that actual distance to obstacles can be found. Therefore, it is important to know the accuracy of the rangefinder. The actual data, which is used In this paper, was taken from a test on a prototype rangefinder constructed at the jet Propulsion Lab as part of the Mars Rover : oject.

\section*{STATEVETT OF TEE PROBLEM}

The problem is to estimate the eccuracy of a rangefinder given the data from an actual test run. The Field of scen for this test is a flat plane with no obstecies. If obstacies exist they must be removed from the range data set. The inaccuracies involved in the esst scen core from many sourzes of error. These sources of error ere in the device itself and possibly in the test setup also. Any errors due to the test setup must be eliminated in the estimate of the error for the device ftself.

\section*{the pange data mazrix}

A diagram of the scaming device relative to the scenned ares is shom in Figure 1 . Point \(A\) is the scanning derice itself. Point \(B\) is the point on the inne, \(\overline{A B}\), wish intersects the horizontal plane that includes the somned erea. Line \(\overline{A B}\) is perpendiculur to this plane. \(Z_{0}\) is the distance from A to 3 .

Although a rectanguar coordinate system is given, the scanning schere is in a sphericel coordinote system. In figure 1 , the elevation orgle, \(\theta\), and the azimuthal angle, f, are dirided into steps. The size of the steps are \(\Delta \theta\) and \(\Delta s\) for the elevation step end astmuthel step, respectively. The
actual position of the rectangular coordinete system is arbitrary (the coordinate system hes min equel to \(0^{\circ}\) from the \(y\) axis for convenience). \({ }^{\text {min }}\)

The data metrix, \(R_{i}\), is an \(m \times n\) matrix, where \(m\) is the number of eIevation steps and \(n\) is the number of azimuthal steps. The quentity, \(R_{i}\), corresponds to the \(i^{\text {th }}\) elevation angle step and the \(j\) th azimuthel encle step. Therefore, \(R\), corresponds to \(\theta_{\min }\) and \(\phi_{\min }\) and \(R_{m n}\) correspond \({ }^{-1}\) to \(\theta_{\max }\) and \({ }^{\circ}\) mex

\section*{SETHOD OF ANALYSIS}

Tro arproeches are given in this section. The choice of the method depends upc: how well the test setup hes been celibrated.
1. Tro-Dimensional Ancuracy Analysis - For an ideel test setup, all range data values for a given elevation angle should be equal. That is, if the exis of the rangefinder is perpendicular to the horizontel plane, varying the azimuth angle with a constent elevation angle will sweep an are of a circle on the horizontal surface with the center at point \(B\).
Under this condition, the data for all ezimuthel angle steps at a given elevation angle cen be aver- \(^{\text {a }}\) aged for each of melevation steps. Fiese mavaraged values are called \(\overline{3}_{i}\) and are corputed by the following equation:
\[
\begin{equation*}
\bar{R}_{i}=\frac{1}{n_{i}} \sum_{j=1}^{n_{i}} R_{i j} \tag{1e}
\end{equation*}
\]
where \(n_{i}\) is the number of range data points in rew. The \(\bar{R}_{i}\) can be plotted to create a two dimensional profite of the range deta. Mis plot is useful for a visual inspection of the linearity of the elevation mechanism. The standard deviation, \(S_{i}\), cen then be calculated for the data points in eech row, i, using the \(\bar{R}_{2}\), in the formule:
\[
\begin{equation*}
S_{1}=\sqrt{\frac{\sum_{j=1}^{n_{1}}\left(R_{i j}-\bar{R}_{i}\right)^{2}}{n_{i}-1}} \tag{2b}
\end{equation*}
\]

A leest-squares fit to the twu-dimensional plot is valuable for this problem. It can provide a better
analysis of the non-linearity than a visual inspecition. Also,-values of \(Z\) and \(\theta_{\text {in }}\), which are shown In Figure 1 , can be calcilated from the fitted ine and compared to the values measured for the test.

In the least-squares flt, the estirated equation is fof the form:
\[
\begin{equation*}
z-v=a u+b \tag{2}
\end{equation*}
\]

To facilitate the estimate, another coordinate system was used. This transformation is shown in Figure 2. In the \(u v\) coordinate system, the rangeFinder (point A) is the origin; and, to eliminate \(\theta_{\text {min }}\) from the analysis, the mean value for the first elevation angle value is placed on the lu axds.

The values of \(u_{i}\) and \(v_{1}\) can be obtained from the mean values, \(\bar{X}_{i}\), of the rows of \(\mathrm{R}_{i 1}\) by the following equations, where \(\Delta 0\) is the eievation step size:
\[
\begin{align*}
\text { Segin text of fiu } & \begin{aligned}
-t & =-\bar{R}_{1} \cdot \cos [(1-1) \Delta \theta] \\
v_{i} & =\bar{R}_{i} \sin [(i-1) \Delta \theta]
\end{aligned} \tag{3a}
\end{align*}
\]

The equations used to calculate the values of a and b in equation (2) can be obtained from a probability [1] or computing methods text under the head!ings, "linear regression". They are as follows:
\[
\begin{align*}
& b=\frac{m \sum_{i=1}^{m} u_{i} v_{i}-\left(\sum_{i=1}^{m} u_{i}\right)\left(\sum_{i=1}^{m} v_{i}\right)}{n \sum_{i=1}^{m}\left(u_{i}\right)^{2}-\left(\sum_{i=1}^{m} u_{i}\right)^{2}}  \tag{4}\\
& a=\frac{1}{m_{i=1}^{m} v_{i}-\frac{b}{m} \sum_{i=1}^{m} u_{i}} \tag{5}
\end{align*}
\]
:The estimated value of \(z\) is computed using the fact that slope of the line in Figure \(2, \overline{A B}\), is the negative of the reciprocal of the slope, \(\varepsilon\), of the estimated line. The equation for the line \(\overline{A B}\) :1s
\[
\begin{equation*}
v=-\frac{1}{8} u \tag{6}
\end{equation*}
\]

Solving equations (2) ard (6) simultaneously, the :coordinates of point \(B, u_{0}\) and \(v_{0}\), ere obtained:
\[
\begin{align*}
& u_{0}=-\frac{a b}{1+a^{2}}  \tag{7}\\
& v_{0}=\frac{b}{1+a^{2}} \tag{8}
\end{align*}
\]

The length of \(z\) can then be calculated by taking the square root of the sum of the squares of \(u_{0}\) and \(\nu_{0}\) from equations (7) and (8):
\[
\begin{equation*}
z_{0}=\sqrt{\left(-\frac{a b}{1+a^{2}}\right)^{2}+\left(\frac{b}{1+a^{2}}\right)^{2}} \tag{9}
\end{equation*}
\]

The \(\theta_{\text {min }}\) can be celculated using the value of \(Z\) o
and \(b\) The angle between the \(v\) axis and \(\overline{A B}\) in Figure 2 is equal to \(\theta_{\text {min }}\). Therefore, \(\theta_{\text {min }}\) can be calculated as follows: \({ }^{\text {min }}\)
\[
\begin{equation*}
\theta_{m 1 n}=\arccos \frac{z_{0}}{b} \tag{10}
\end{equation*}
\]

The differences, \(e_{i}\), between the actual \(\bar{R}_{i}\) values and the estirated line shown in Figure 3 are very userul for analysis. To calculate, \(e_{i}\), the equaition of the line \(\overline{\mathbb{R}}_{\text {, }}\) that passes throukh the origin is determined. Thê equation for it is:
\[
\begin{equation*}
v=u[\tan \Delta \theta(1-1)] \tag{11}
\end{equation*}
\]

If equations (2) and (11) are solved simultaneously for \(u_{\text {, the }}\) value is cailed \(u_{i}^{+}\)and is \(E\) ven in the following equation:
I \(\because\) rin this \(00 u_{1}^{\prime}=\frac{b}{\tan (\Delta \theta(1-1)]-a}\)
(12)
and then the horizontal projection of the error, \(\left(e_{u}\right)^{\text {is }}\) is:
\[
\begin{equation*}
\left(e_{u}\right)_{1}=u_{i}-u_{i}^{\prime} \tag{13}
\end{equation*}
\]

Finally, the errors, \(e_{i}\), between the \(\bar{R}_{i}\) and the fitted line, are calcutated by dividing \({ }^{\frac{1}{g}}\left(e_{u}\right)_{i}\) by the cosine of (1-1) \(\Delta \theta\) :
\[
\begin{equation*}
e_{i}=\left\{u_{i}-\frac{b}{\operatorname{ten}[\Delta \theta(i-1)]-a}\right] / \cos [\Delta \theta(i-1)] \tag{14}
\end{equation*}
\]
2. Three Dimensional Accuracy Analysis - The threedimensional accuracy analysis is desired when there is a "tilt" in the test setup. In the two dimensional analysis, eny such "tilt" will show up as an apparent increase in the error for each row.
The methods in this section also compute the "tilt" so that the rangefinder can be aligned to be perpendicular to the horizontal plane. A visuel inspection of the range data points can reveal whether or not there is a "tilt". General trends of increasing or decreasing values of \(R_{1}\), as \(I\) goes from 1 to \(n\), indicate that a "tilt" exists.
The range data is given in a polar ccordinate system so the data must first be transformed into the \(x\), \(y, z\) coordinate system shown in Figire 1 . The conversion formulae are:
\[
\begin{align*}
& x_{1 j}=R_{i j}\left(\cos \theta_{i}\right) \cos \phi_{j}  \tag{15}\\
& y_{i j}=R_{i j}\left(\cos \theta_{i}\right) \sin \phi_{j}  \tag{16}\\
& z_{1 j}=R_{i j}\left(\sin \theta_{i}\right)  \tag{17}\\
& \theta_{i}=\theta_{\min }+(i-1) \Delta \theta  \tag{28}\\
& \theta_{j}=(j-1) \Delta \theta \tag{19}
\end{align*}
\]
where

The plane which contains the range data points cen also be estimated by a least-squares estimate. The equation of the plane will be:
\[
\begin{equation*}
z=w_{1} x+w_{2} y+w_{3} \tag{20}
\end{equation*}
\]

The objective function that must be minimized is:
\[
\begin{equation*}
J=\sum_{i=1}^{m} \sum_{j=1}^{n}\left[z_{i j}-\left(w_{1} x_{i j}+w_{2} y_{i j}+w_{3}\right)\right]^{2} \tag{21}
\end{equation*}
\]

Taking partial derivetives with respect to the coefficients, \(\mathrm{w}_{1}, w_{2}, w_{3}\), and setting them equal to zero:
we obtein;
\[
\begin{equation*}
W=0^{-1} P \tag{22}
\end{equation*}
\]

The matrices are defined es:
\[
\begin{align*}
& Q=\left[\begin{array}{lll}
\Sigma \Sigma x_{1 j}^{2} & \Sigma \Sigma x_{i j} y_{i j} & \Sigma \Sigma x_{i j} \\
\Sigma \Sigma x_{1 j} y_{i j} & \Sigma \Sigma y_{1 j}^{2} & \Sigma \Sigma u_{1 j} \\
\Sigma \Sigma x_{1 j} & \Sigma \Sigma y_{i j} & =n
\end{array}\right]  \tag{23}\\
& P=\left[\begin{array}{l}
\Sigma \Sigma x_{1 j} z_{1 j} \\
\Sigma \Sigma y_{1 j} z_{i j} \\
\Sigma \Sigma z_{i j}
\end{array}\right]
\end{align*}
\]
(24)


These equations can be solved using a matrix solution routine to find the coefficient vector \(\underline{w}\), if \(Q\) is nonsingular.
The actual range data can be compared to the est1mated plane by generating the values of range data that correspond to the estimated plane. These range values are called \(R_{i j}\). The equation for \(\hat{R}_{i}\) is derived by substituting equations (15), (16) ig and (17) Into equation (20) and solving for \(R_{i j}\) :
\[
\begin{equation*}
\hat{R}_{1 j}=\frac{w_{3}}{\sin \theta_{1}-\cos \theta_{i}\left(w_{1} \cos \phi_{g}+w_{2} \sin \phi_{j}\right)} \tag{26}
\end{equation*}
\]
where \(\theta_{1}\) and \(\phi_{1}\) are given in equations ( 18 ) and
(19). 1 This primed range data matrix, \(\mathrm{R}_{\mathrm{i}}\), can be subtracted from the original range data matrix, \(R_{i j}\), to form an error metrix, \(R_{1 j}^{*}\) :
\[
\begin{equation*}
R_{i j}^{*}=R_{i j}-\hat{R}_{i j} \tag{27}
\end{equation*}
\]

The \(R_{1}^{*}\) matrix shows the difference between the estimated plane and the original data.
Using equetions (26) and (27) the matrices, \(\hat{R}_{i}\), and \(R_{1,}^{*}\) values were calculated. The \(R_{i,}^{*}\) values wête matipulated to provide more useful data. First, the values were averaged in each rov and called \(r_{1}^{\prime \prime}\) :
\[
\begin{equation*}
r_{1}^{*}=\frac{1}{n_{1}} \sum_{j=1}^{n_{1}} R_{1 j}^{*} \tag{28}
\end{equation*}
\]
where \(n_{1}\) is the number of data points in row 1 .
Next, the standard deviation of the values in each row, si, were calculated using the following
equation:
\[
\begin{equation*}
S_{1}^{*}=\sqrt{\frac{\sum_{j=1}^{n_{1}} R_{i j}^{*}}{n_{1}-1}} \tag{29}
\end{equation*}
\]

The values for \(r_{i}^{*}\) and \(S_{1}^{*}\) are ohown in Table 1. The means and standard deviations, calculated in the three-dimensional analysis, correspond to \(\overline{R_{i}}\), and \(S_{1}\), calculated in the two-dimensional analyșis. The starred values, however, are computed teking the "tilt" into consideration.
The direction of the "tilt", as grojected on the \(x, y\) axes, is needed to allm the rangefinder. This direction, d, show in Fir. 1 , can be computed by taking the gradient of Fqua. (20). The vector dis given by
\[
\underline{d}=\left[\begin{array}{ll}
w_{1} & w_{2} \tag{30}
\end{array}\right]^{T}
\]

If no tilt exists, \(\mathbb{d} w 11\) be a zero vector.

\section*{THE MAPS ROVER: AN APPLICATICN}

The techniques shown in this paper were developed to analyze actual test data from the laser rancefinder developed for the Mars Rover from tests performed at Jet Propulsion Lab., Pasedena, CA. and obtained from Mr. B. Dobrotin.
The rangefinder, consisting of a laser photodetector, and an electronic circuit, measures time of flight by a time-to-pulse-heipht conversion technique. It uses a 10 bit \(A / D\) converter to obtain
a digital output simal. Six values are mensured for each position; the final five are averaged to get the range value for that position. The beam orientation is changed by iving a mirror that positions the laser beam with two stepping motors. For the test date, there vere 30 elevation steps and 50 azimuthal steps. The range data set for the laser ranperinder is shown in Fif. 3. The range points that fell on an obstacle were set eaual to zero so the computer programs for the enalysis would skip these points.
The parameters for the test setup [2] for this data are: \(Z_{0}=636.0 \mathrm{~mm}, \Delta \ominus=.3307\) degrees \(/ \mathrm{step}, \Delta \phi=.3059\) degrees/step, \(\theta_{\mathrm{min}}=35.04\) degrees.
The Results - The values of the mean, \(\bar{R}_{i}\), ead standard deviation, \(S_{i}\), computed from equatiōns (1a) and (1b) and number of data points for each row of the ranke date are shown in Table 1 . Using these mean values, \(\overline{\mathrm{R}}_{j}\), end equations (2) thru (5), the leastsquares estimate of the line passing through these points was made:
\[
v=.62859 \quad u+792.95
\]
(31)

The estimated value of \(Z\), using Equa. (9), is 671.3 mi: From Equa. (10), the estimated velue of \(\theta\) min is calculated to be \(32.15^{\circ}\).
The values of \(\dot{e}_{\text {i }}\) in Table 1 show the difference between the \(\bar{p}\), and the estimated line in Eque. (2) as shown in Fi \(\frac{1}{R}\). 2. These values vere calculated using Equa. (14). Any trends in the \(\overline{\bar{p}}\), velues can be observed in the chanzes of the values of \(e_{i}\).
This analysis was given only es an illustration. There is a "tilt" in this range data, so the standard deviations, \(S_{i}\), are not zood estimates of the performance of the laser rangefinder.
The "tilt" in the data can be observed in Fig. 2 by comparink the left- and ripht-most columns of the range data. In 22 of the 30 rows, the first element is smaller than the last; and in the other rows, they are equal. This indicates that "tilt" probably exists and that the three-aimensional analysis should be applied.
For the least-square estimate of the plene of the date, the matrix equations, (22) thru (25) were solved for the vector, \(\underline{y}\). Equation (20) for this estimate is:
\[
\begin{equation*}
z=.03946 x+.03383 y+678.0 \tag{32}
\end{equation*}
\]

The direction of the tilt, \(d\), was calculated to be [.03046 .03383]T usinp Equa. (28). In the top view of Fif. 1, the position of \(d\) is shown relative to the plane of the scan in the \(\bar{x}, \mathrm{y}\) plane. This direction of the 'tilt' is in aqreement with the earlier observations of the ranze data.

\section*{INTERPRETATION OF THF PESULTS}

A comparison of these statistical results of values provides useful information. Both the \(e_{i}\) and the F shown in Table l indicate a trend of हैurvature in the data. Because it appears in the three-dimensional enalysis, the trend is not due to the tilt of the plane. This non lincarity is from the device itself and orizinates either in the elevation mechanism, the range measurement apparatus, or both.

The standard deviations of the three-dimensional analysis are in general smaller than those of the two-dimensional enalysis. This is expected because the three dimensional analysis removes the added errors due to the tilt in the test. The standard deviations, \(S_{\text {" }}^{\text {, rance from approximately }}\) 2 to 7 mm . Studying these \(S_{1}^{\text {n }}\), one will notice a trend of increasing error as the distance of the range value increases (Row 1 has the largest range values). Because the change is not significant, hovever, an actual value of the error cannot be established. The standard deviations \(\mathrm{C}_{6}\) and \(\mathrm{S}_{3}\) are 7.74 vom and 5.12 mm . They are larger by about 4 mm than the other values of \(\mathrm{s}^{*}\) around them. In subsequent tests, the standard deviations of these rows should be checked to see if this problem reoccurs.
Before the data in Table 1 is interpreted further, the quantization error must be estimated. The standard deviation, \(S\), of the quantization error can be found indepencently of the scan by the following formula, where \(\Delta\) is the difference between successive quantization levels:
\[
\begin{equation*}
s_{q}=\sqrt{\frac{\Delta^{2}}{12}} \tag{33}
\end{equation*}
\]

For this data set, \(\Delta\) equals 9 mm : therefore, \(S_{q}\) equals 2.6 mm . Except for rows 6 and 30 , the \(S_{1}^{*}\) ranke from 2.9 to 4.7 mm . Most of the inaccuracy therefore is due to quantization error. The next largest source of error is the nonlinearity of the range readings. The inaccuracy due to this non linearity makes the performance of the rangefinder poor for the ranke of approximately 1 to 1.3 meters. Modifications to the rangefinder should be made
if it is to be used in this ranke.

\section*{CONCLUSIO:}

This paper has presented two methods to analyze the accuracy of a rangefinding device.
The two-dimensional analysis is useful when the rangefinding device has been aligned so that its vertical axis is perpendicular to the scanned plane. The errors, \(e_{i}\), introduced from the rancefinding device or othêr sources can be observed in Table 1 of the means \(\bar{F}_{i}\), and standard deviations, \(S_{i}\). \(A\) visual inspection of the \(e_{\text {, vill reveal any }}{ }^{1}\) noninearities of the data. These nonlinearites of the system are show at different elevation angles and, hence, at different ranae distances.
The three-dimensional analysis is used when a "tilt" exists between the scanned plane and the verticel exis of the rengefindine device. In this method, the plane of the scan is determined using a leastsquares estimate. From the equation of the estimated plane, the verticni axis enn be realigned, for the actual device and the test can be run eapain. Also, values of rance data can be corputed from the equation of the eatimated plane. The difference of these values and the actual range data can be used to give a rough estimate of the accuracy when "tilt" exists.
If it is possible to make rultiple tests of the rankefinder, the best approach is to first correct the alimment of the ranpefinder using the second method, and then use the first method to analyze the data from the ranzefinder test.

The methods Just described show the errors of the - rangefinding device as the elevation anple, \(\theta\), changes or, equivalently, as the distance of the measurement increases. The source of these errors are the elevetion angle positioning mechanism or the distance measuring device itself. Other sources of error that are included in the error estimate are quantization error and error due to noise in the environment. The quantization error can be estimated independently and compared to the finel results.

The change of range reading with the intensity of the reflected pulse is also important [3]. The intensity of the relfected pulse can be chanped by changing the reflecting surface or by changing the angle of the reflecting surface with the incident beam. The errors due to the latter are measured by the methods in this paper; because as the distance of a range data increases, the angle that the surface forms with the beam elso increases [4]. However, these errors and the errors due to increasing distance of the measurement cannot be separated in the analysis. It is recommended that a testing scheme be constructed that could measure the errors due to changes of intensity.

Table 1. Statistical Results
\begin{tabular}{|c|c|c|c|c|c|c|}
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rwn & \({ }_{\text {e }}\) & \(r_{1}^{\prime \prime}\)
mun & & /row \\
\hline 1 & 1264.00 & 5.71 & 2.23 & 2.73 & 4.45 & 24 \\
\hline 2 & 1252.52 & 5.14 & 2.23 & 2.35 & 3.69 & 23 \\
\hline 3 & 1239.96 & 5.87 & 0.90 & 1.43 & 3.81 & 25 \\
\hline 4 & 1230.58 & 4.56 & 2.50 & 3.05 & 4.34 & 26 \\
\hline 5 & 1218.97 & 4.27 & 1.65 & 2.72 & 4.30 & 29 \\
\hline 6 & 1210.18 & 5.65 & 3.39 & 4.90 & 7.74 & 33 \\
\hline 7 & 1196.57 & 2.63 & 0.09 & 1.93 & 4.65 & 37 \\
\hline 8 & 1187.27 & 4.17 & 0.88 & 2.55 & 4.66 & 37 \\
\hline 9 & 1177.19 & 3.93 & 0.69 & 2.18 & 3.74 & 37 \\
\hline 10 & 1166.54 & 4.78 & -0.27 & 1.05 & 4.33 & 37 \\
\hline 11 & 1154.62 & 5.12 & -2.71 & -1.65 & 3.51 & 50 \\
\hline 12 & 1145.52 & 4.85 & -2.51 & -1.61 & 3.39 & 50 \\
\hline 13 & 1135.46 & 5.44 & -3.46 & -2.71 & 4.14 & 50 \\
\hline 14 & 1127.28 & 4.99 & -2.71 & -2.11 & 3.33 & 50 \\
\hline 15 & 1118.58 & 5.17 & -2.65 & -2.20 & 3.27 & 50 \\
\hline 16 & 1110.62 & 4.58 & -2.03 & -1.71 & 3.23 & 50 \\
\hline 17 & 1101.02 & 5.81 & -3.21 & -3.03 & 4.14 & 50 \\
\hline 18 & 1093.48 & 5.17 & -2.50 & -2.45 & 4.01 & 50 \\
\hline 19 & 1085.44 & 4.79 & -2.44 & -2.52 & 3.93 & 50 \\
\hline 20 & 1078.72 & 3.79 & -1.22 & -1.42 & 3.36 & 50 \\
\hline 21 & 1070.44 & 4.44 & -1.71 & -2.03 & 3.84 & 50 \\
\hline 22 & 1063.80 & 3.77 & -0.71 & -1.14 & 3.17 & 50 \\
\hline 23 & 1055.94 & 3. 32 & -1.06 & -1.61 & 3.40 & 50 \\
\hline 24 & 1040.96 & 4.52 & 0.32 & -0.34 & 3.20 & 50 \\
\hline 25 & 1043.60 & 3.90 & 1.18 & 0.42 & 2.88 & 50 \\
\hline 26 & 1036.86 & 4.46 & 1.53 & 0.66 & 3.44 & 50 \\
\hline 27 & 1029.24 & 3.36 & 0.87 & -0.10 & 3.31 & 50 \\
\hline 28 & 1023.72 & 4.43 & 2.19 & 1.12 & 3.02 & 50 \\
\hline 20 & 1017.80 & 4.99 & 2.98 & 1.81 & 3.40 & 50 \\
\hline 30 & 1013.80 & 6.15 & 5.57 & 4.31 & 5.12 & 50 \\
\hline
\end{tabular}
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}
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Figure 1-Diegram of the test setup

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 \(\begin{array}{lllllllllllllllllllll}0 & 1231 & 1231 & 1211 & 1231 & 1231 & 1211 & 1231 & 1231 & 1231 & 1231 & 1239 & 1231 & 1231 & 1231 & 1231 & 1231 & 1239 & 1231 \\ 0 & 1222 & 1222 & 1216 & 1216 & 1222 & 1214 & 1222 & 1222 & 1222 & 1222 & 1222 & 1222 & 1222 & 1222 & 1222 & 1222 & 1222 & 1227\end{array}\)









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 Figure 3 - The Range Data Matrix of \(R_{1 j}\)

Task B. Path Selection System Simulation - B. Longendorfer Faculty Advisor: Prof. D. K. Frederick

The overall objective of this task is to define a path selection system for the exploration of the surface of a planet using Mars as a case. Because of the delay introduced by one-way communication to Earth from Mars (on the order of twenty minutes), the vehicle must be able to sense its environment and direct its own movements without human assistance. Accordingly, algorithms are sought which will organize the collected data into an appropriate "world model", and utilize it in choosing an optimal safe path.

The paticular methods investigated are edge-detection techniques. Edge detection techniques require a matrix of range/pointing angle data. Sudden changes in magnitude between adjacent elements, or differences in the rate of change of these magnitudes signal the existence of "edges".

The application of these techniques to the Mars Rover requires much supporting simulation work. For example, the matrix of input data consists of laser range readings from a specified field of view, requiring the simulation of a range-finding sensor.

Practical problems arise when considering the multitude of situations that occur in the physical world. Flat field, eingle obstacle encounters (with which most of the theoretical work has been done) are rare. Problems such as what initialization procedures shall be used, which side of an edge (or both) constitutes an obstacle, what threshold values yield the "best" performance, etc. must be considered. At times, additional processing might be required to obtain a more complete picture.

The vehicle then must relate the resulting matrix of edges to the actual environment, through a Terrain Modeller. The Terrain Model must calculate the actual location of the obstacles sensed, their approximate size and height, their relationship to the surrounding terrain, and the location of scanned (known) and unscanned (unknown) regions. Work on the Terrain Model has reached the debugging and concept testing stage.

Once information about its surroundings is available to the vehicle, it must select a safe optimal path to its target. The method employed is called a path selection algorithm. The algorithm is necessarily influenced by the data-acquisition method in such factors as the conservatism of the method (because of the susceptibility to noise or the lack of it), and the type of obstacles which the sensor is intended to detect (discrete obstacles, slopes, positive or negative obstacles). Work on the path selection algorithm is in the planning stage.

\section*{Sensor}

A laser rangefinder sensor was specified for a nominal range of three to thirty meters. Previous sensor simulations were not suited to this particular task. They allowed for only a small number of elevation angles (one to four), and these were constrained to certain values (for example, only horizontal). Azimuth
angles were similarly constrained. Clearly, the sensor used in edge detection needed to be able to generate a large matrix of readings at one time, at any angle coordinates. The sensor now programed overcomes these defects, as well as providing for time lapse while scanning. Noise affecting the range readings is taken into account in two ways:
(1) noise of specified type, mean, variance, maximum amplitude, natural frequency, and damping constant may be added directly to the range readings, and
(2) the specified accuracy of the sensor may be changed by the user.

Plans for further work on the sensor include the modification of the added noise by a factor proportional to the range squared, in order to better simulate errors in time of flight measurement.

The sensor simulation uses the following method to compute range readings. It first calculates the position of the laser and the position of the location where the laser would intersect ground, both with respect to the vehicle, assuming no slopes. It transforms both points to the planet reference frame, by its knowledge of vehicle heading, and actual in-path and cross-path slopes. It calculates the direction cosines of the line connecting the two points and steps along this beam with a user-specified increment (BEMSTP, usually set to five centimeters), until it finds itself below the actual terrain level. It enters a bisection scheme until the desired acruracy is reached. The accuracy is a user-specified input (SIMSTP) and is defined as the difference between exact measurement and the computed measurement (not as the difference between the height of the computed measurement and the terrain at that point, as in previous sensors). Efficiency of the sensor is vastly improved by the use of an initial guess at the range reading, and logic which will not recalculate iransformations unless the vehicle has moved from its previous position. The initial guess is taken to be zero for the elevatior angle nearest the vehicle. For other elevat. 1 angles, the guess is set equal to a value somewhat smaller than that for the elevation angle nearest to it but ctoser to the vehicle.

Testing of the sensor is now complete.

\section*{Scan Generator}

The scan generator serves as a preprocessor for input and output for the sensor. It allows input in a variety of convenient forms and calculates the necessary azimuth and elevation angles to cover the specified field of view while maintaining a certain data density. Because of the requirement that the Kalman filter must receive only angles spaced by a constant number of degrees, the data density (number of points per square meter) will vary, sometimes quite drastically.

Input may appear in the form of a list of angles, or maximum, minimum and incr mental angles, or in the form of field dimensions
such as leneth, width, center and maximum poiat spacing.
Testing is now complete.

Eine Detection Techniques
The edge detection programs had to be interfaced with the existing simulation package.

The edge detection routines employ a Kalman filter, which is performed separately on each row and each column. Row processing looks for changes in the magnitude of range readings (vertical edges), while column processing looks for both magnitude and slope changes (horizontal edges). The Kalman filter predicts values for each element of the array in succession, and computes the probability of an edge having occurred at that stage bas \({ }^{\text {d }}\) on factors such as the difference between actual and expected values, the probability that an edge will occur, the cost of making a wrong decision on the location of the edge, etc. Bayes' Risk calculations weighted by these considerations make the actual decision.

The performance of the edge detection routines has been terted by generating a terrain with the simulation package, measuring ranges with the sensor, and attempting to reconstruct the edges with the computer programs. Varying success has been attained, as shown in Figures 1 through 5. Problems identified here will be explored further in months to come.

The bottom edges of boulders and the far edges of craters have not been shown by the program. To remedy the situation, two courses are presently being iursued. One fossible solution is to replace some of the threshold values (originally programmed as constants) with variables, and re-test the performance. Another solution is to install an auxiliary test to look specifically for an important situation not now being identified. A "moving average" or constantly updated average value, will flag changes in slope greater than this average plus a tolerance or expected changes that fail to occur (i.e. changes less than this average minus a tolerance). The first case would signal the far edge of a negative obstacle while the second case is that of the near edge of a positive obstacle. At prenent, the "moving average" is able to be engaged or disengaged by a rlag so that its effect may be studied. The tolerance itself is selected as a fraction of the computed average difference \(: 2\) as the value computed assuming the maximum transversible slope.

Noise seems to have very little effect on the performance of the Kalman filter. Figures 4 and 5 show the effects of unfiltered white noise, which was added independently to eacn of the rarge readings. Range readings differ by as much as twenty centimeters along the same row (which should ideally have exactly the same readings, since they result fror identical elevation angles).

Another problem is the correct initialization procedure to follow. Until recently, initialization involved calculating expected range readings and changes in range readings given that the surrounding terrain was a flat plane. Changes in slopes between elements of a coilumn were done the same way. However, a new initialization procedure is being tried at present. Rows and columns are initiated to the range reading actually obtained. If an edge is actually present there, it will occur at the next stage instead. The small inherent error will be more than compensated for by the lack of initialization error on slopes, which could possibly block off an entire forward path.

\section*{Terrain Mode1}

The Terrain Mode 1 interprets the data obtained from the sensor and edge detection routines, and translates it into an organized, easily understandable form. The Path Selection Algorithm is then freed from the concern for which method was actually used to obtain the data, and it can be a much more generally applicable program.

The Terrain Model presently being developed divides the general region of the vehicle and target into a grid of squares, and uses them as a type of obstacle map to provide memory.

There are several stages preceding the construction of the final obstacle map. First, the edge matrix provided by the edge detection routines must be scanned for edges. The model must decide on which side of the edge (or both) the obstacle lies, and whether the obstacle is positive or negative. This is currently determined by taking an average range over the proposed obstacle (i.e. that terrain which is between one edge and another, where an edge is defined as the beginning or end of a row or column or as the edge identified by the detection programs). The average range is compared to a value plus or minus a threshold. The comparison value currently corresponds to that for flat ground, but a better comparison of the known surface area (such as prediction from orthogonal polynomial least squares) will soon be employed. This is an example of the interaction of the terrain model and the edge detection routines. Second, all matrix elements determined to be within the obstacle are marked according to type and blocked out. Third, all squares on the obstacle map of the environment which contain any part of the region scanned by an edge in the edge matrix are also marked with the obstacle type. This can be achieved by using attitude data and some geometry to or cain the planet ( \(x, y, z\) ) coordinates at the scan point. Projecting the obstacle location on to the \(x y-p l a n e\) yields the map location. Actually, this is done at any point of the entire region sampled by one data point which will cause a new ;quare to be blocked on the obstacle map. Further information may be obtained and included in the map such as a height code, which would be useful for computing and storing slope, and flagging hazardous terrain. However, this has not as yet beei incorporated in the model. Edge detection is designed to handle discrete obstacles, so that has been the main area of concentration of effort so far.

All terrain is initially labelled as unknown and is changed to another code only after it has been scanned at least once.

Certain problems remain to be dealt with. Storage and memory capabilities directly conflict in the choice of the size of the blocks on the obstacle map. However, if only a certain amount of memory is needed while a large amount of terrain is to be covered, a "rolling pointer" might be used, which would indicate the current farthest sensor reach, while old data would be overlaid with new data from a different region. Another problem is the conflicting declarations of obstacles. Depending on the conservatism of the algorithm, differences in successive measurements would make the algorithm more or less sensitive to noise.

Other assumptions include:
(1) The vehicle has been supplied with a map of its surroundings up to a certain radius at the beginning of the simulation (otherwise a fixed sensor could never scan the field).
(2) The sensor cannot change, add or delete angles from its scan once they are initialized.
(3) Primary effort should be given to the detection of discrete obstacles.

\section*{Path Selection Algorithm}

The problem of selecting a path given an obstacle map consisting of squares has E ien dealt with by C.Y. Lee, Reference 1 and F. Lallman, Reference 2.

The algorithm consists of:
(1) identifying the known (scanned) region
(2) finding a minimal length path to the target
( 3 ) following the portion of this path of maximum length until the target is reached or another scan is necessary.

The minimal length path is identifie' by placing numbers in ascending order in blocks adjacent to the target, adjacent to the block adjacent to the target. As long as the vehicle progresses from one number to the next lower number, it is following a minimal length path.

Problems not corsidered in the literature that are faced here are:
(1) The vehicle may be more than one square in dimension of length or width.
(2) The vehicle has inherent dynamical characteristics and annot turn abruptly.
(3) The vehicle scans only a smail portion of its surroundings.
(4) The vehicle cannot proceed within a certain radius of unknown terrain without losing the ability to sense obstacles that may be present.

Edge de eection appears to be able to overcome two of the basic problems other path selection algorithms could not easily deal with: noise, and confusion of slopes with discrete obstacles. The tradeoff seems to be memory space and computation time.

Since simulation efforts are only now giving way to proposed problem solutions, it is too early to draw definitive conclusions. A reprint of a paper describing the application of this simulation code to the evaluation of a triangulation-based short range hazard detection system follows. The reprint is included to illustrate some of the type of output which can be expected when complete simulations of the concepts described herein are implemented.

\section*{REFERENCES}
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\section*{ORIGINAL PAGE is OF POOR QUALITY}

\author{
S. Yerazunis, D. K. Frederick and J. Krajewski
}

\author{
Rensselaer Polytechnic Institute Troy, New York
}

A four-wheeled vehicle capable of dealing with irregular terrain features such as might be encountered in unmanned exploration of extraterrestrial bodies has been designed, constructed and tested. Its mobility and maneuverability are such that applications on earth requiring autonomous control may be served by it or a suitable variation. It could also serve as a test bed for artificial intelligence/robotics research. Alternative concepts for terrain sensing and modeling, and path selection algorithms have been investigated. A triangulation-based laser/photodetector path selection \(s^{\prime}\) 'stem developed by simulation is found to ve able to deal with a broad range of terrain features. The path selection system is now in the process of construction and implementation and will permit field evaluation of the overall vehicle system as in autonomous rover.

\section*{INTRODUCTION}

The very long term interests of man require that a thorough exploration of the planets of the solar system, their moons and the asteroids be undertaken eventually. Although it is likely that man himself will visit or eatablish himself ultimately on some of these extraterrestrial bodies, economic and technological considerations suggest that unmanned systems must be used during the shorter run.

Considerable knowledge of Mars, Venus, Mercury and Jupiter has been gained by fly-by or orbiter missions and more advanced remote sensor concepts will contribute further to man's understanding of the solar system. However, basic understanding of the chemical, biological, geological, meteorological and physical characteristics of extraterrestrial bodies requires surface exploration. The current Viking missions are intended to meet some of these requirements for the planet Mars.

Despite the historic implications of the Mars Viking program, Viking-type missions are severely limited in scope because of the
restricted sampling range of its ten foot boom with a \(120^{\circ}\) arc. Thus only a minute fraction of the martian surface will have been scrut inized and a larger but still very small fraction as limited by line of sight will be examined visually. A more thorough exploration is not likely to be effected by increasing the number of such stationary landers because of the large number which would be required and the problem of selecting and reaching precise landing sites where critical measurements can be made. A method for exploring the planet in question in detail and to do so adaptively on the basis of the knowledge being gained is essential.

The scenario for planetary exploration could involve either a stationary lander containing sophisticated and adaptable instrumentation for in-situ analysis or a sample return vehicle. In either case, the scenario must involve sample and/or data gathering devices capable of being relocated over much of the planetary surface, and in the case of sample gathering devices, capable of returning to the lander site. In order that such missions can be executed in a \(r \in a\) sonable time of the order of several months \(t\), perhaps a year, such sample or data gathering devices must have a high level of both mobility and automatism.

The mobility requirements as expressed in terms of characteristics such as the maximum slope which can be climbed or descended, the maximum boulder which can be negotiated without avoidance, the ability to transverse depressions and very rough terrains, etc., has a direct bearing on the availability of paths to the desired sites. A vehicle of limited mobility may require an inordinate length of time and distance travelled to reach the target. Indeed, in some circumstances, the vehicle may not be able to reach the target. As the vehicle's mobility is increased, it will be able to deal effectively with increasingly difficult terrains. More paths to target will be available and the opportunity fo. selecting more optimal paths will be increased. Thus, one major research objective should be aimed at developing and evaluating alternative concepts of relocatable devices capable of serving a stationary lander or sample return mission.

Of equal importance to advanced exploration missions is the level of autonatism which can be achieved. The decision for the vehicle to follow one path or another towards its destination must be provided by some path selertion system comprised of appropriate terrain data
sensors, terrain modeler and path selection algorithm. The character and capabilities of such a system must be appropri.te to the objectives of the mission. A low level path selection system will have to be biased conservatively to minimize the risk of an unperceived hazard in the vehicle's path. Thus many, perhaps all, acceptable paths towards the target may be excluded. The effect of a low level detection system is, in fact, to reduce the vehicle's mobility. As a minimum, the path taken towards the destination will be lengthier and the range of the exploration will be reduced. In the other extreme, the vehicle may become immobilized. On the other hand, a higher level, more sensitive, more perceptive path selection system will identify a larger fraction of the passable paths and will permit selection of those directions most compatible with the mission goals. Thus a second major research objective is the development of efficient path selection systems for the safe relocation of a rover.

The research program \({ }^{(1)}\) aimed at these two major objectives is applicable not only to extraterrestrial exploration but to robotics applications on earth where hostile environments and special circumstances may exclude either direct or remote manual control. The major emphasis of this paper is on the rover and the path selection system selected for immediate implementation.

\section*{THE RENSSELAER ROVER}

By virtue of its design the vehicle has a very high mobility and maneuverability, Ref. 1 and 2. In its deployed state, Figure 1, it has a stable stance allowing it to traverse very irregular terrain. The configuration of the struts serving the individually-driven wheels permits this vehicle to be collapsed into a compact space, Figure 2 , from which it can be deployed automatically. The struts, which are driven separately by torsion bars, can be used to raise or lower the payload compartment as required and to orient it fore-and-aft with respect to the horizontal. The front wheel wagon steering insures that all four wheels will be in contact with the terrain for all but the most severe surface irregularities. It also permits a sharp turning radius such that in the extreme case, the vehicle can turn about a center through the rear wheel axles. The vehicle is able to deal effectively with a broad range of slope situations and discrete obstacles.

Perhaps the most outstanding feature of the design is that this vehicle can recover from the situation in which both front wheels fall over the edge of a crevasse or a deep crater. For all other vehicles which have been proposed, such an event would be catastropic and would terminate the mission. That is not so with this concept.

\footnotetext{
(1) Faculty participating in the overall research program are: Profs. D.K. Frederick, D. Gisser, G.N. Sandor, C.N. Shen and S Yerazunis.
}

Because of the unequal lengths of the front and rear struts and the torsion bar system, the front and rear wheel assemblies can be reversed and the vehicle can withdraw safely from the hazard. This same maneuver can be used to extricate the vehicle from a "box canyon" or equivalent terrain feature in which there is insufficient room for the vehicle to turn itself around and from which it may not be able to back out. Previously proposed vehicles would also fail in this instance.

The vehicle has been refined to the point where it can serve as an exceptional test bed for the evaluation of alternative path selection systems. In this sense it is a valuable asset for exploring artificial intelligence or robotic aspects related to a machine seeking an acceptable path to a prescribed destination, an objective not necessarily restricted to extraterrestrial exploration.

In addition to the mechanical and propulsion systems, the rover is equipped with electronic four-wheel speed controls to permit effective steering, attitude and heading gyroscopes, position sensors for all major strut cumponents, and a two-way telemetry link to an off-board minicomputer to provide the necessary vehicle data for guidance and control. All that remains is the implementation of the hardware and software required for an operating path selection system.

\section*{PATH SELECTION SYSTEM SIMULATION}

The path selection system required to guide an autonomous vehicle must include: terrain sensor or sensors hardware, procedures for interpreting the data, and an algorithm for selecting safe paths on the basis of the data. The best combination of hardware and software will depend on the details of the mission and the dynamical and mobility characteristics of the vehicle or robot. An optimization of these requirements through an iterative process of constructing and evaluating specific hardware and software will be inordinately expensive and tedious and is not likely to be optimal. A digital computer simulation has been developed which can be used to screen all conceivable systems, Ref. 3 and 4. The simulation can "create" a very broad spectrum of terrains possessing such large and small scale details as desired. The pitching, heaving and rolling of the vehicle can be simulated and their effect on the terrain sensor data can be evaluated. The terrain sensor can be simulated to reflect both the error due to the motion of the sensor as a result of vehicular dynamics and the inherent sensor measurement errors. Proposed terrain modelers and path selection algorithms can be analyzed, evaluated and modified. When these considerations are taken all together, a first but meaningful appraisal of the strengths and weaknesses of proposed hardware systems can e obtained. Its application to a specific path selection system is described later in this paper.

ALTERNAIIVE TERRAIN DATA GATHERING CONCEPTS
Efforts in developing alternatives for the gathering of terrain data have been focused on:
(a) a range-pointing angle concept and (b) a triangulation concept.

An in-depth study of the use of range-angle pointing data for terrain modeling and hazard detection has been on-going for several years. This phase, which ha resulted in numerous publications, Ref. 5, 6, 7, 8, 9, 10, has focused on the midrange problem from \(3-30\) meters. Methods have been developed for interpreting these data to obtain estimates of the terrain gradients and to detect discrete objects whose contours deviate from the main terrain. Edge detection and edge enhancement techniques have veen developed which provide complete outlines of boulders, ridges, craters and crevasses provided that a sufficiently high data density is available. Current efforts are concentrating on the problem of reducing the data requirements without sacrificing the effectiveness of the interpretation. These studies are concerned with the data density in both time and space and to the implications of inherent sensor crrors and errors due to the motion of the sensor as a consequence of vehicular dynamics.

As an alternative, a short range (1-3 meter) system based on triangulation has also been investigated. Indeed the latter system has been selected for implementation on the Rensselaer Rover. The concept is illustrated in Figure 3 in which are shown a laser beam along one pointing angle and a photodetector along a second pointing angle. The existence of a reflecting surface, i.e., terrain within the zone of intersection, will be sensed by the photodetector. In principle, any number of lasers and photodetectors can be deployed to obtain terrain surface data to any desired density and discreteness. This information can then be used as the basis for modeling the terrain and making path selection decisions.

From a terrain modeling point of view, the range-pointing angle and triangulation concepts are equivalent. That is, both concepts provide the same type of data in that the terrain surface is concluded to lie along a line segment of some length and angular orientation. However, the two concepts involve different technical obstacles in their implementation. The uncertainty of locating the terrain surface along the line segment using the range-pointing angle concept is determined by the ability of instrumentation to measure time-offlight with possibly weak return signals and in the presence of undesired reflections. As one's objective in hazard detection focuses more on the short-range aspect, the time-of-flight measurement becomes increasingly difficult due to the need to measure extremely short time intervals. On the other hand, the uncertainty related with terrain location by triangulation is controlled by the geometrical relationship of the laser and the detector and the cone-of-vision of the latter. By using overlapping detectors and by scanning with a high frequency laser, the uncertainty can be made extremely small for the short range. However, the triangulation system begins to suffer inherent and significant uncertainty in long range applications. Thus, hardware systems based on range-pointing angle data look more attractive at longer ranges whereas the triangulation concept is far more favorable at short range.

Given the objective of demonstrating a minimal level of autonomous roving capability, it was concluded that a path selection system based on the triangulation concept would be implemented.

\section*{THE PROPOSED PATH SELECTION SYSTEM}

As noted earlier, a path selection system consists of a sensor(s) to gather the data, a terrain modeler to process the data into a proper form, and a path selection algorithm to employ the modeled results with the objective of defining guidance commands. The path selection system which has been chosen for hardware and software implementation consists of two lasers scanned azimuthally along fifteen positions equally spaced at \(10^{\circ}\) to provide a \(140^{\circ}\) field of view. One photodetector possessing a \(9.6^{\circ}\) cone of vision will also be scanned over the same azimuthal angles. The two lasers will be located at a height of 1.5 meters at elevation angles of \(43^{\circ}\) and \(46^{\circ}\) whereas the photodetector at the 0.75 meter height will have an elevation angle of \(62^{\circ}\), Figure 3 . These initial design paraneters are specified on the basis of the simulation studies completed to date. The mast on which the lasers and the photodetector are to be mounted has been constructed to provide up to 6 azimuthal \(140^{\circ}\) sweeps per second. The instrumentation mounting arrangements are such that the lasers can be scanned in elevation as well as azimuth and that additional photodetectors can be added as desired. Finally, the mast configuration will allow for significant adjustment of the geometric parameters so that the researchers will have considerable flexibility in studying nore sophistricated triangulation-based alternatives.

Central to the task of demonstrating and evaluating this concept during the late spring of this year has been the path selection system simulation effort. It has fallen on this group to screen a number of design concepts and to synthesize a specific alternative satisfying the basic requirement that the rover will not pursue a path which will abort the mission and that if a safe path to the specified target exists the rover will eventually detect it. Although the long-term objectives call for a high degree of discrimination on the part of the system to minimize the misinterpretation of safe alternatives as hazardous, the studies completed to date have defined a minimum but feasible system.

The parh selection system simulation, Ref. 4, has the capability of simulating the performance of a broad range of surface rover concepts, alternative data gathering concepts, data processors i.e. (terrain modelers) and path selection algorithms on a terrain selected by the user. The terrain can have large scale features as well as fine detail. Inherent sensors errors can be simulated along with those sensors errors due to the dynamic motion of the rover as it moves over highly irregular terrain and rubble. Thus, the simulation affords the users the opportunity to appraise alternative hardware and software concepts conveniently.

In the case at hand, the simulation was used to develop the design parameters specified above. The anticipated behavior of the selected system

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is show on Figures 4 through 8 . Note that the design parameters are set to detect a positive hazard (boulder or step) or negative hazard (crater or trench) in excess of approximately \(\pm 12^{\prime \prime}\) from the plane defined by the attitudinal status of the vehicle. With the single detector, this requirement is equivalent to setting the maximum gradient to \(\pm 12^{\circ}\). Parenthetically, it might be noted that the \(12^{\circ}\) gradient is far lower than the actual slope climbing capability of the Rensselaer Rover. Thus this system will rule out potentially passable paths because of its inability to discriminate between slopes and discrete obstacles.

Shown in Figure 4 are the results of three simulations as applied to a boulder-crater field superimposed on a flat base using the single laser-single detector concept described earlier assuming the absence of noise due either to inhe ient sensor limitations or the pitch or roll vehicle motions. The proposed path selection system is found to be quite effective in dealing with this problem. Also shown in Figure 4 is a simulation of this system with vehicle motion noise effects equivalent to fluctuations of \(\pm 5^{\circ}\) in pitch and \(\pm 10^{\circ}\) in roll at the contact points of the wheels. At these noise levels, the path selection was unaffected.

The noise parameter referred to above is intended to account for the vehicle motion caused by terrain features which are too small to model on an individual basis. Random fluctuations in the pitch and roll angles are generated and then added to the vehicle's attitude as determined by the slope of the deterministic terrain under the vehicle. These random fluctuations are computed by entering a sequence of uniformly distributed random numbers into a second-order low-pass digital filter whose damping ratio and undamped natural frequency are representative of the vehicle's dominant mode. The purpose of the filter is to simulate the smoothing effect of the vehicle's suspension system. Roll and pitch are treated separately, with different random sequences. For example, using an unfiltered randor sequence uniformly distributed over \(\pm 10\) degrees resulted in filtered angles having a standard deviation of \(\pm 3.74\) degrees and extreme values of \(\pm 9.0\) in the pitch direction. Hence, the excursions in the attitude of the mast and laser-detector combination are somewhat less than the unfiltered fluctuations which are assumed to exist at the wheel contact points. The standard deviation and the extreme values for other levels of vehicle dynamic noise would be proportional to these for 10 degrees.

Also shown in Figure 4 are simulations involving the two-laser system scheduled for implementation with noise levels of \(10^{\circ}\) and \(15^{\circ}\) in both pitch and roll. The performance of the system continues to be effective despite the high noise levels. However, it is clear that as the noise level due to vehicular dynamics and terrain irregularities is increased, a point will be reached eventually at which the effectiveness of the path selection system will be degraded. This question has not been fully explored yet because as will be described below noise effects are far more significant in terrain situations involving

Shown in Figure 5 are simuiations involving a terrain described by sinusoidal functions. It should be noted that for a sinusoidal terrain of 6 meter period and 0.20 metes amplitude in the absence of noise that the path selection system is able to direct the vohicle in a straight line fashion between the inftial point and the target destinacion. However, an increase of the amplitude to 6.3 meterf/orces the one laser hazard detection sypiem to depart significantly from the straight line path and to meander toward the targer. Although the target is reached eventually a considerably larger distance traveled is involved. The path selection system can be improved by adding a second laser at an incremental elevation of \(3^{\circ}\). This modification has the effect cf providing additional information which allows the path selection system to take a someviat more direct route toward the target. However neither system can achieve a direct route for an amplitude of this magniture. Increasing the amplitude to 0.33 metcrs while retaining the period at 6 meters, causes both the one and twolaser systems to prescribe an even more tortuous path. At an amplitude 0.4 meters, the system is unable to contend with the situa*ions anc fails to make progress towards the target. The simulations shown in Figure 5 highlight a major weakness in this simple hazard detection system, namely, its inability to distinguish between a slope and a discrete obstacle. The rover found itself obliged to turn from the direct path at critical points where the sensor was detecting the existence of a moderate slope in excess of the \(12^{\circ}\) limit imposed by the assumed sensor system. It had to take on a direction such that the gradient to be encountered would be within the specified tolerance limits. This is not necessarily a fatal flow since the rover was able to select a trajectory either up or down hill which would be satisfactory. But it did so at the penalty of taking an unnecessarily long trajectory.

The consequences of dynamic noise are far more significant with respect to rolling terrain situations than in the case of the flat terrain on which a variety of boulder and crater obstacles are superimp 'sed. This behavior is shown in Figure 6 in which the one-laser and the two-laser systems are tested in a . 3 meter amplitude, 6 meter priod sinusoidal terrain with noise due to vehicle dynamics equivalent to \(10^{\circ}\) in pitch and \(10^{\circ}\) in roll. In order to compare the two alternatives, the same seed was used to generate the random numbers which serve as the basis for calculating the noise. Thus, both systems were tested against the same exact sequence of dynamical otion. Both simulations failed to the extent that extreme meandering and confusion is observed. It cannot be concluded that the target would not have been reached ultimately because the simulations were autonatically terminated on the basis of a time constraint. However, what is important to note in this comparison is that the two-laser system had no observable superiority over the one laser system in the case of this rolling terrain at this level of dynamical noise. Also shown on Figure 6 is the performance of the two laser system with vehicle dynamics noise reduced to \(5^{\circ}\) in pitch. Although a meandering path was followed, the system would ultimately lead the rover to the target.

Shown in Figure 7 are the..\(:\) ils of three sifalations all involving a rolling terrain described by a .25 meter amplit..de, 6 meter period sinusoidal surface. Two of the simulations compare the effectiveness with which the one- and two-laser systems could deal with this rolling terrain situation in the presence of \(10^{\circ}\) pitch, \(10^{\circ}\) roll dynamical motion. Neither simulation proved to be particularly effective although the fallure to reach the target is again a consequence of terminating the simulation rather "han a fundamental inability to achieve the target ultimately. Nevertheless, in the case of this level of dynamical noise it is clear that the path selection system is foccing the rover into very substantial meandering and is therefore not considered effective for both the one- and two-laser systems although the latter appears stronger. However, the third simulation involving dynamical noise levels of only \(5^{\circ}\) in pitch but with the \(10^{\circ}\) roll retained showed that the two-laser system was able to take a relatively direct route from the starting point to the final destination. This suggests therefore that in the design of an autonomous rover a significant impact on the effectiveness of hazard detection and path selection systems can be obtained by careful vehicular design which minimizes the effect of dynamical noise upon the sensing devices. Another way of loolin: at the significance of these simulations is to note that it is the sum of the gradients representative of the physical terrain situation and of the nows effects which determine how well the path selection system will perform for a given vehicle. On the basis of the data at hand, it can be concluded that even with a \(10^{\circ}\) pitch noise due to dynamical motion that there would be some rolling terrain either with reduced amplitude and/or increased period which could be handled effectively by this system. In effect, the consequences of the noise associated with the dynamical motion is to reduce the mobility of the vehicle by applying a bias required to offset the impact of noise.

Sumarized in Figure 8 are the results of three simulations in \(t a\) absence of vehicle dynamics noise in which the boulder/crater field is superimposed on the s-nusoidal rolling terrain. The combined terrain features are sec to pose a more serious problem than for either case separately. Case 1 involving the vehicle origin in the upper right ind corner found the vehicle get\(t\) ing itself into an awkward position in which a considerable amount of maneuvering was required before a good trajectory toward the target could be defined. The discontinuities in path which are shown are a consequence of the simulation program which applies when the vehicle finds itself in an impossible situation. The algorithm calls for the vehicle to back up 1 meter. As far as the simulation is concerned this backup is instantaneous and discontinuous. Case 2 involving an initiation point in the upper left hand corner for a twolaser system proceeded rather well to the target whereas the corresponding path for the onc-laser system encountered difficulty near the target and a considerable meandering before reaching the target. Also shown for comparison in Figure 8 is the path selection process for the boulder/crater field on a flat plane. It can be seen that the effect of the rolling terrain is to deny to the vehicle the most direct route to the target. However, the final simulation shown involving a
\(5^{\circ}\) in pitch and \(10^{\circ}\) in roll vehicle dynamics noise closely parallels the path of the noiseless flat terrain case. It would appear that in this case, the noise led to a fortatous selection of path.

Other simulations not reported herein have been conducted which suggest ways in which the system could be modified to deal more effective? either with the discrete obstacle such as boulders, craters, trenches or with slope characteristics. The tendency to interpret passable slopes as impassable can be reduced increasing the cone of vision of the detector. However, the effect of this action is to increase the size of the positive or negative discrete hazard to be interpreted as passable. A decrease in the cone of vision of the detector will permit smaller discrete obstacles to be detected but at the penalty of defining lower acceptable slope thresholds. The design parameters which have been selected as of this date represent what is believed to be the best =ompromise for the purpose of demonstrating and v-luating this \(k\) ! of a system in hardware.

As noted earlier, the problem of noise due to whi le dynamics has the effect of biasing the path se:ection decisions conservatively. The pitchlag or rolling motion of the vehicle provides false information which is interpreted as a vehicle hazard. It is also possible for the dynamical noise to give a false indication that a hazard is not present even though it really is. The latter does not prove to be significant because subsequent scans will disclose the existence of a real hazard, even though it may be overlooked on an individual scan. The reverse is not true; when a path is deemed to be blocked because of the dynamical noise, the path selection algorithm treats it as being blocked and not only directs the vehicle otherwise but retains in its memory that that path is blocked. Consideration is being given to incorporating memory into the -irrain modeler and path selection algorithm so as to reduce the false identification of hazardous paths. Such a system should provide a basis for alieviating the type of erratic behavior hown in Figures 6 and 7.

The mresent path selection system has one additiona ..ajor fault to thich attention is being directed. Spricifically, past researchers. Reference 11, have shown that a particular terrain feature may or may not be a hazard depending upon the main terrain characteristics on which the feature is located. thus in the ultimate, it will be necessary to relate the state of the terrain, as defined by the attitude information provided by the attitude gyroscope mounted on the vehicle to the data being provided by the hazard detection sensors. One possible solution is to implement a higher level triangulationbased system which involves elevation as well as azimuthal scanning of the laser and an increased number of detectors with smaller cones or zones oi vision. The effect of this type of a system is of course to decrease the coarseness of the information. Provided that a sufficiently fine mesh is obtained, pattern recognition techniques can be used to obtain a much more detailed and informative impression of the nature of the terrain in the path of the vehicle. The information gained in such a system could be tied to the
vehicle attitude data through appropriate mathemacical relationships to provide a more sound decision as to whether the terrain feature is or is not a hazard for the vehicle's particular state.

Alternatively, one could =onsider using the basic system described herein but replicating it in sufficient number so that only one of the systens would be operational at a given time depending upon the vehicle's attitude. Thus one might have a multi-unit system of the type described herein with the central unit applying to all situations in which the vehicle's attitude is not far removed from horizontal, with the immediately adjacent two units applying when the vehicle's attitude is slightly upwardly or downwardly inclined, and with additional adjacent units applying at greater and greater attitudinal deviations from horizontal. The number of such units would depend on the need for refined terrain information.

Consideration is befing given to other systems of intermediate complexity, namely, five \(9^{\circ}\) detectors overlapped as to produce nine discrete signals in combination with from three to five lasers being scanned azimuthally but with each laser at a very specific elevation. Such an intermediate system can be implemented in hardware and software relatively easily and have the advantage of providing enough added information to give the path selection algorithm a more reliable basis for decision making.

As of this writing, the research program to produce the required hardware, (i.e. lasers, photodetectors, scanning mast, etc.) and to program the software in a IDIIOM Graphics-Varian Computer, which is to serve as the data processor, are proceeding vigorously. The rover itself and its on-board control systems and actuators are operational along with the telemetry systems required to transmit data from the vehicle to the computer and vice versa. It is anticipated that all systems will be active in the neighborhood of April 1, and that laboratory and field research will be undertaken shortly thereafter.

\section*{CONCLUS IONS}

A path selection system based on a two-laser/ one-detector terrain sensor can be effective in guiding an autonomous rover over terrains whose general slopes are less than \(\pm 12^{\circ}\) and on which are distributed discrete hazards larger than \(\pm 12\) inches in the presence of vehicle dynamics noise of the order of \(\pm 5^{\circ}\) in pitch and \(\pm 10^{\circ}\) in roll.

The performance of such a system can be increased provided that the state (i.e. the attitude) of the vehicle is taken into account in the interpretation of the sensed terrain data and that additional attitude dependent sensor units are added.

Terrain sensors based on triangulation provide a basis for developing short-range path selection systems capable of dealing with very complex terrain situations.

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Figure 1. The Rensselaer Rover in its Collapsed Configuration


Figure 3
Terrain Detection by Triangulation


Figure 4. Path Selection Simulation Through a Boulder/Crater Field on a Flat Base Terrain


Figure 5. Path Selection Simulation Over a 6.0 Meter Period Sinusoidal Terrain in the Absence of Vehicle Dynamics Noise


Figure 6. Path Selection Simulation for a Sinusoidal Terrain 0.3 Meter Amplitude and 6.0 Meter Period in the Presence of Vehicle Dynamics Noise


Figure 7. Path Selection Simulation for a Sinusoidal Terrain of 0.25 Meter Amplitude and b. 0 Meter Period in the Presence of Vehicle Dynamics Noise


Figure 8. Path Selection Simulation for a Boulder/Crater Field Superimposed on a Sinusoidal Terrain of 0.3 Meter Amplitude and 6.0 Meter Period```

