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Neutrino Masses and Magnetic Moments*

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Abstract

After a short review of popular mechanisms to generate small neutrino masses and mixings in the context of gauge theories, I discuss their relevance to the solar neutrino puzzle. The apparent anti-correlation of the observed solar neutrino flux with sun spots is suggestive of a large magnetic moment of the neutrino. This, however, would seem to be in conflict with a small neutrino mass. New symmetries which decouple the mass from the magnetic moment are introduced and their theoretical and phenomenological implications are discussed.

1. Introduction

Neutrinos have played an important role in the development of the theory of electro-weak interactions. Whether or not they have tiny masses is a key question which is a subject of intense scrutiny, both experimentally and theoretically. Any positive evidence for a neutrino mass would have profound impact on our understanding of nuclear and particle physics, as well as cosmology and astrophysics. Along with the small masses come small mass differences, making feasible macroscopic oscillation phenomena. If the masses are lepton number violating, neutrinoless double beta decay may be observable. Neutrino properties such as its mass, magnetic moment, number of species etc., are significant in cosmology, in connection with the question of dark matter, primordial nucleosynthesis and the formation of large-scale structures in the universe. Models of stellar cooling and evolution rely on the known weak interaction properties of the neutrinos. The signal of neutrinos reaching the earth from the sun may be drastically modified in presence of neutrino flavor oscillations.

In this set of lectures, I wish to explore the consequences of non-standard neutrino properties such as its mass, mixing and magnetic moment. A prime motivation for discussing these issues is the so-called solar neutrino puzzle, the resolution of which would seem to require one or more of these properties. There are several excellent reviews on this subject, in the form of text books[1-3] as well as review articles.[4-6] The aim of these lectures is to give a flavor of the various aspects of the problem. After noting some theoretical arguments for non-zero neutrino masses, I

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review popular mechanisms that generate small neutrino masses in the context of gauge theories. A brief introduction to the solar neutrino puzzle and its resolution is then given. Resolving the puzzle via a large magnetic moment of the neutrino runs into theoretical problems, since naively a large magnetic moment would also imply a large neutrino mass. New symmetries which decouple the magnetic moment from the mass are then introduced and their theoretical and phenomenological implications are outlined. Some recent proposals to reconcile the time dependence of the Chlorine and Kamiokande solar neutrino experiments are critically analyzed.

Arguments for non-zero neutrino mass

In the standard model of electro-weak interactions, neutrinos are assumed to be exactly massless. However, there is no fundamental symmetry associated with the masslessness of the neutrino. This is unlike the masslessness of the photon, which follows directly from a fundamental symmetry – electromagnetic gauge invariance. Although one can 'invent' symmetries to explain the zero mass of the neutrino, for eg., the γ_5 invariance, such symmetries are not viewed as 'fundamental'. In the absence of any convincing symmetry argument, it seems worthwhile to explore the consequences of a non-zero m_ν.

It is widely believed that the standard model of strong and electro-weak interaction, despite its enormous success in confronting a variety of experimental data, is only an effective low energy theory. At higher energies, it is expected to succumb to a more complete theory which (hopefully) would explain the many issues not addressed by the standard model, such as the origin of flavors, the gauge hierarchy problem, proliferation of couplings etc. In most theories which go beyond the standard model to address one or more of these shortcomings, it is natural to have non-zero neutrino masses. Examples are left-right symmetry, grand unified SO(10), supersymmetry, horizontal symmetry etc.

There may already be some indications that neutrinos have non-standard properties. The strongest evidence comes from the solar neutrino experiments. There is a deficit in the observed neutrino flux[7,8] compared to the theoretical expectations based on the standard solar model.[9] Barring any serious flaw in our understanding of the solar interior, non-standard neutrino properties would be required in order to resolve this discrepancy. One of the popular solutions is the oscillation between different neutrino flavors, which requires $m_\nu \neq 0$. An alternate solution, on which I will have more to say in these lectures, is motivated by an apparent time-variation of the neutrino flux in anti-correlation with the sun spots. This feature can be explained if the neutrino is endowed with a sizable magnetic moment. Such an interaction would violate the γ_5 invariance and would lead to a non-zero m_ν as well.

A host of particles have been proposed, neutrinos included, as candidates for the cosmological dark matter. Among these particles, neutrinos are unique in that they are the only ones known for sure to exist.

Although electrically neutral, neutrinos may play a role in explaining the question of charge quantization, one of the profound mysteries of nature. Within the minimal standard model, even with its explicit $U(1)$ factor, it is possible to understand charge quantization from the cancellation of triangle gauge anomalies and from the invariance of the Lagrangian which generates charged fermion masses.[10] A mild perturbation in the particle content, say the addition of a right-handed neutrino, will however, spoil this nice feature. This is due to the appearance of a 'hidden local symmetry' (viz., a symmetry which can be gauged)[11,12] associated with $U(1)_{B-L}$ in the presence of ν_R. Charge quantization can, however, be recovered by explicitly breaking this hidden symmetry. The simplest way is to give the neutrino a Majorana mass[11]. It has also been noted that even without the addition of ν_R, lepton number differences such as $l_e - l_\mu$ are 'gaugable' symmetries within the three generation standard model, which would spoil charge quantization.[13] The obvious way to preserv : quantization is to assume neutrino flavor mixing.

Limits on neutrino masses

We know now that neutrinos come in three flavors, each associated with its own charged lepton partner: ν_e, ν_μ and ν_τ. Precision measurement of the Z boson width excludes a fourth light neutrino. Primordial nucleosynthesis calculations based on the big bang hypothesis would also seem to be in conflict with observations if there is a fourth light neutrino. Experimentally, there are only upper limits on the masses of the three neutrinos:

$$m_{\nu_e} \leq 10\ eV\ ; \quad m_{\nu_\mu} \leq 250\ keV\ ; \quad m_{\nu_\tau} \leq 35\ MeV\ . \tag{1}$$

This leaves a whole range of masses for which neutrino oscillation can be observed both in terrestial and astrophysical experiments.

Astrophysical and cosmological considerations can be used to place limits on neutrino properties. A limit of $m_{\nu_e} \leq 10\ eV$ has been inferred from the observation of neutrinos from the supernova SN1987A. This bound arises from the clustering of the observed events to within 10-12 seconds. If neutrino has a mass, it would travel slower than light. The low energy neutrinos would arrive the detector at a later time than the high energy ones, creating a time delay. The clustering of events then puts the above quoted limit.

If neutrinos are stable, there should be relic neutrinos from the big bang, which are expected to be almost as abundant as the photon, their number density being $110/cm^3$. If they have mass, it will contribute to the energy density of the universe. Demanding that this contribution not exceed the critical density gives the limit $m_\nu h^2 \leq 100\ eV$, where $1/2 \leq h \leq 1$ reflects the observational uncertainty in the Hubble parameter. If the neutrino decays with a life-time shorter than the age of the universe, this bound is altered to $m_\nu \left(\tau/t_0\right)^{1/2} \leq 100 h^2\ eV$, where t_0 is the present age of the universe.

Dirac or Majorana Neutrinos?

For a charged fermion, such as the electron, the Lagrangian which describes the mass can be written as

$$\mathcal{L}_D = m\left(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L\right) = m\overline{\psi}\psi\ . \tag{2}$$

This Lagrangian is invariant under the global symmetry transformation $\psi \rightarrow e^{i\alpha}\psi$. The conserved charge associated with this transformation is the fermion number.

For the neutrino, since it is charge neutral and a color singlet, in addition to Eq. (2), one can write a term

$$\mathcal{L}_M = M\left(\nu_L^T C \nu_L + H.C.\right) \tag{3}$$

Here C is the Charge conjugation matrix with the properties

$$C^T = C^\dagger = -C\ ; \qquad C\gamma_\mu^T C = \gamma_\mu\ . \tag{4}$$

(I shall work with four-component spinors, and define the γ_5 matrix to project out the left and right helicity states.) Defining a conjugate field in the usual way as $\psi^c = C\overline{\psi}^T$, it follows that the conjugate of a left-chiral field is right-chiral and vice versa. That is, $(\psi_L)^c = (\psi^c)_R$. Using this identity, we can rewrite Eq. (3) as

$$\begin{aligned}\mathcal{L}_M &= M\left[\overline{(\nu_L)^c}\nu_L + H.C.\right]\\ &= M\left[\overline{(\nu^c)}_R\nu_L + H.C.\right]\end{aligned} \tag{5}$$

This differs from Eq. (2) in that the right helicity state forming a Dirac mass is obtained from the conjugate of the left-helicity state itself. Note that $\mathcal{L}_M$ breaks all additive quantum numbers, including lepton number. In general, neutrinos could have both the Dirac mass and the Majorana mass. The neutrino is then called a Majorana particle. If lepton number is a conserved quantum number (i.e., $\mathcal{L}_M = 0$), then the neutrino, like the electron, will be a Dirac particle. Most extensions of the standard model tend to yield Majorana neutrinos, rather than Dirac neutrinos.

Neutrino Oscillations

If neutrinos have mass, the eigenstates of weak interaction, (ν_e, ν_μ, ν_τ), will, in general, not be simultaneous eigenstates of the mass matrix, which we denote by (ν_1, ν_2, ν_3). This is due to mixing of flavors that can occur in the mass matrix, analogous to the Cabibbo-Kobayashi-Maskawa mixing in the quark sector. These mixings will lead to the phenomenon of neutrino oscillation. Consider for simplicity, the case of two flavors ν_e and ν_μ. The mass eigenstates are related to the gauge eigenstates via a 2×2 orthogonal matrix

$$\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}\ . \tag{6}$$

A ν_e beam produced in weak interaction, with a definite energy E, will propagate according to

$$\begin{aligned}|\nu_e(x,t)\rangle &= e^{-iEt}\left[\cos\theta\,|\nu_1(x,0)\rangle + \sin\theta\,|\nu_2(x,0)\rangle\right]\\ &= e^{-iEt}\left[\cos\theta e^{ip_1x}\,|\nu_1(0,0)\rangle + \sin\theta e^{ip_2x}\,|\nu_2(0,0)\rangle\right]\ ,\end{aligned} \tag{7}$$

where p_1 and p_2 are the momentum eigenvalues, which in the relativistic approximation can be written as

$$p_1 \simeq E - \frac{m_1^2}{2E} \; ; \quad p_2 \simeq E - \frac{m_2^2}{2E} \; . \tag{8}$$

The probability of detecting the neutrino in the ν_μ state at a distance L is then

$$P_{\nu_e \to \nu_\mu} = \sin^2 2\theta \sin^2 \left(\frac{\pi L}{\lambda} \right) \tag{9}$$

where

$$\lambda = \frac{4\pi E}{m_2^2 - m_1^2} \tag{10}$$

is the oscillation length. If due to lack of spatial overlap of the wave packet, or due to non-coherent scattering, the phase information is lost, the oscillating term in Eq. (9) will average to a 1/2. For maximal mixing, present accelerator experiments are sensitive to $\Delta m^2 = (m_2^2 - m_1^2)$ of order 1 eV^2. For $\Delta m^2 \gg 1\ eV^2$, the upper limit on $\sin^2 2\theta$ is at the level of 10^{-3}, depending on the nature of flavors being considered. As we shall see later, the picture presented above will be drastically modified when the neutrino propagates inside a dense medium, such as the interior of the sun.

2. Neutrino masses in gauge theories

In the standard model based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, the fermions are assigned to the following representation of the gauge group:

$$\begin{aligned} Q_L(3,2,1/6) &= \begin{pmatrix} u \\ d \end{pmatrix}_L \; ; \quad u_R(3,1,2/3) \; ; \quad d_R(3,1,-1/3) \\ \psi_L(1,2,-1/2) &= \begin{pmatrix} \nu \\ e \end{pmatrix}_L \; ; \quad e_R(1,1,-1) \; . \end{aligned} \tag{11}$$

We have exhibited only one family of fermions, three such families exist, the e, μ and τ families. The model assumes the existence of a single doublet of Higgs bosons

$$\phi(1,2,1/2) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \; . \tag{12}$$

In addition to the gauge covariant kinetic terms of the fermions and the bosons, the following Yukawa Lagrangian is allowed by the symmetries of the model.

$$\mathcal{L}_Y = h_d \overline{Q}_L \phi d_R + h_u \overline{Q}_L \tilde{\phi} u_R + h_e \overline{\psi}_L \phi e_R + H.C. \tag{13}$$

Here $\tilde{\phi} = i\tau_2 \phi^*$, h_d, h_u and h_e are the 3×3 Yukawa coupling matrices in generation space, corresponding to the down-quarks, up-quarks and charged leptons respectively. Upon spontaneous symmetry breaking via $\langle \phi^0 \rangle \neq 0$ they generate masses for quarks and charged leptons.

An interesting feature worth noting is the absence of the right-handed neutrino ν_R from the spectrum of the standard model. As a result, the Dirac masses of the neutrinos are not allowed. Due to the simplicity of the Higgs sector, lepton number turns out to be an exact global symmetry of the Lagrangian, so that the Majorana masses are also forbidden. Thus, in the minimal standard model, all neutrinos are massless, there is no room for neutrino oscillation or a magnetic moment. This situation is not at all exciting from the experimental point of view. In what follows, I shall always assume that the structure of the underlying theory is somewhat richer, so that interesting phenomena can occur in the neutrino sector.

Addition of the right-handed neutrino

A simple extension of the standard model is obtained just by adding the right-handed neutrino ν_R (one per family) into the fermionic spectrum. Since it is a singlet under the gauge symmetry, none of the successes of the gauge sector of the standard model will be spoiled by doing so. Now we can write a new term in the Yukawa Lagrangian of Eq. (13):

$$\mathcal{L}_\nu = h_\nu \overline{\psi}_L \tilde{\phi} \nu_R + H.C. \tag{14}$$

This term will lead to non-zero Dirac neutrino masses. However, with this term alone, the mass of the neutrino will be expected to be of the same order as its charged lepton counterpart, unless the coupling h_ν is fine-tuned to be much smaller than h_e of Eq. (13). This unpleasant feature will disappear, however, if one notes that ν_R transforms as $(1,1,0)$ under the gauge symmetry. A bare Majorana mass term is then permissible by gauge invariance.

$$\mathcal{L}_M = M \left(\nu_R^T C \nu_R + H.C. \right) \tag{15}$$

The full mass 6×6 mass matrix for the neutrinos can be written as

$$[\overline{(\nu_L)^c} \quad \overline{\nu_R}] \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \begin{bmatrix} \nu_L \\ (\nu_R)^c \end{bmatrix} , \tag{16}$$

where m_D is the 3×3 Dirac mass matrix obtained from Eq. (14). Since the Majorana mass term M is a gauge singlet, it is generally expected that it will be much larger than m_D, which breaks $SU(2)_L \times U(1)$ symmetry. Concentrating on one generation for the moment, the mass eigen-values of the above matrix are (with $M \gg m_D$)

$$m_{\nu_L} \simeq \frac{m_D^2}{M} \; ; \quad m_{\nu_R} \simeq M \; . \tag{17}$$

This is the well-known 'see-saw mechanism'.[14] If the scale of lepton number breaking M is made larger and larger, the mass of the lighter neutrino becomes smaller and smaller. There is a tiny mixing between the light and heavy state given by $\theta \approx m_D/M \simeq \sqrt{m_{\nu_L}/m_{\nu_R}}$. For $m_{\nu_L} \simeq 1\ MeV$ and $m_{\nu_R} \simeq 1\ TeV$, this angle is 10^{-3}.

Majorana neutrinos without ν_R

In order to generate non-zero neutrino masses, it is not essential to introduce ν_R. If the Higgs sector is sufficiently complicated, lepton number violation would occur, which would lead to Majorana masses for the left-handed neutrinos. In the standard spectrum, since ψ_L transforms as $(1,2,-1/2)$ and e_R as $(1,1,-1)$, from the bilinear products of these fields, it is clear that the following Higgs particles can couple to the leptons (besides the standard doublet):[15]

$$\Delta(1,3,1) \; ; \quad h^+(1,1,1) \; ; \quad k^{++}(1,1,2) \; . \tag{18}$$

Introducing one or more of these fields into the scalar spectrum would yield different models of neutrino masses.

Consider first the Higgs triplet model. The new scalar is denoted by $\Delta(1,3,1)$. From the electric charge formula $Q = T_3 + Y$, it follows that Δ contains one neutral, one singly charged and one double charged member. It can be written in a matrix form as

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} . \tag{19}$$

A new term in the Yukawa Lagrangian is now allowed:

$$\begin{aligned} \mathcal{L}_Y^{(1)} &= h\psi_L^T C i\tau_2 \Delta \psi_L + H.C. \\ &= h\left[\overline{(\nu_L)^c}\nu_L\Delta^0 + \overline{(e_L)^c}e_L\Delta^{++} - \{\overline{(\nu_L)^c}e_L + \overline{(e_L)^c}\nu_L\}\Delta^+\right] + H.C. \end{aligned} \tag{20}$$

Here we have suppressed the family indices. Once the neutral member of Δ acquires a vacuum expectation value (vev) denoted by $\langle \Delta^0 \rangle = v_3$, a non-zero neutrino mass will be generated given by $m_\nu = hv_3$. v_3 contributes to the masses of the W and Z as well, which are now altered to (v_2 is the doublet vev)

$$M_W^2 = \frac{1}{2} g^2 \left(v_2^2 + 2v_3^2\right) \; ; \; M_Z^2 = \frac{1}{2}\left(g^2 + g'^2\right)\left(v_2^2 + 4v_3^2\right) . \tag{21}$$

The experimental bound on the electro-weak ρ parameter defined as

$$\rho = \frac{M_W^2}{M_Z^2 cos^2\theta_W} \tag{22}$$

viz., $\rho_{exp} = 0.998 \pm 0.0086$ limits $v_3/v_2 \leq 0.07$.[16] Furthermore, in order to explain the observed smallness of the neutrino masses, one should assume $hv_3 \ll m_e$, which is not very pleasing aesthetically. In other words, there is no good explanation for the smallness of the neutrino mass within this scheme.

Radiative neutrino masses

It is clear from the preceeding discussion that in order to explain the smallness of the neutrino masses in the absence of ν_R, a new mechanism is needed. One way is to generate the masses at the loop level, so that they are naturally small. Here we shall discuss two rather simple schemes based on the standard gauge group which achieve this goal.

Consider the introduction of a singly charged Higgs boson h^+ which has the quantum numbers $(1, 1, 1)$ into the standard model.[17] One can write an invariant Yukawa term

$$\mathcal{L}_Y^{(2)} = f_{ab}\psi_{La}^T C i\tau_2 \psi_{Lb} h^+ + H.C. \tag{23}$$

Here a, b are generation indices. It follows from Fermi statistics that $f_{ab} = -f_{ba}$. The proof of this is simple. One writes the above term in terms of the spinor indices and makes use of the anti-commutativity of the spinors:

$$\begin{aligned} f_{ab}\psi_{La}^T C i\tau_2 \psi_{Lb} &= f_{ab}\psi_{La}^\alpha i\tau_2 \psi_{Lb}^\beta C_{\alpha\beta} \\ &= +f_{ab}\psi_{Lb}^\beta i\tau_2 \psi_{La}^\alpha C_{\alpha\beta} \\ &= +f_{ba}\psi_{La}^\alpha i\tau_2 \psi_{Lb}^\beta C_{\beta\alpha} \\ &= -f_{ba}\psi_{La}^\alpha i\tau_2 \psi_{Lb}^\beta C_{\alpha\beta} . \end{aligned} \tag{24}$$

Here, in the last line, we made use of the anti-symmetry of the Charge conjugation matrix C. Explicitly, the Lagrangian of Eq. (23) can be written as

$$\mathcal{L}_Y^{(2)} = 2\left[f_{e\mu}\left(\overline{\nu_e^c}\mu - \overline{\nu_\mu^c}e\right) + f_{e\tau}\left(\overline{\nu_e^c}\tau - \overline{\nu_\tau^c}e\right) + f_{\mu\tau}\left(\overline{\nu_\mu^c}\tau - \overline{\nu_\tau^c}\mu\right)\right] h^+ + H.C. \tag{25}$$

where all the fields are left-handed.

The above Lagrangian conserves lepton number, which can be verified by assigning 2 units of leptonic charge to h^+. This means that Eq. (23) by itself cannot be responsible for neutrino masses. If there are terms in the Higgs potential which violate lepton number, in combination with Eq. (23), it could generate neutrino masses. This would be the case as in the Zee model,[17] where the scalar potential contains a term

$$V = \mu \left(\phi^T i\tau_2 \phi' h^- + H.C.\right) \tag{26}$$

Here, ϕ is the standard doublet. If $\phi' = \phi$, Bose symmetry would imply that the above term is identically zero. Hence, it is necessary to have at least two Higgs doublets in the Zee model.

The scalar coupling of Eq. (26) would result in mixing of the h^+ field with the ϕ^+ and ϕ'^+ fields. At the one loop, this will result in a finite neutrino mass via the

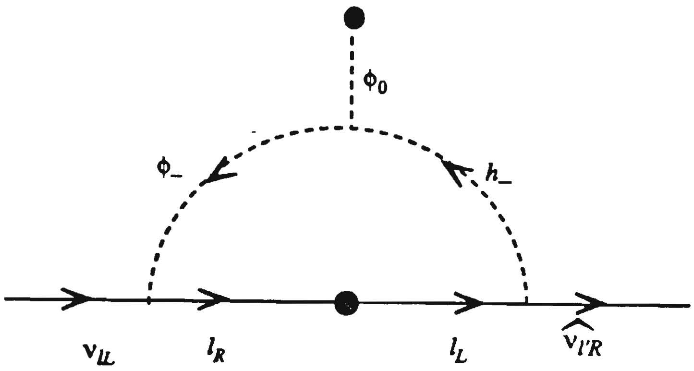

Fig. 1. Diagram generating neutrino masses in the Zee model.

diagram shown in Fig. 1. The entries of the mass matrix are given by

$$m_{ab} \simeq \frac{f_{ab} g}{16\pi^2} \left(\frac{\mu v'}{M_h^2}\right) \frac{(m_a^2 - m_b^2)}{M_W} \; . \tag{27}$$

Note that due to the anti-symmetry of the coupling matrix f_{ab}, at the one-loop level, all the diagonal elements of the mass matrix are zero. The above mass matrix can be diagonalized in the limit of neglecting the electron mass.[18] Defining

$$\tan\alpha = \frac{f_{\mu\tau}}{f_{e\tau}} \left(1 - m_\mu^2/m_\tau^2\right) \; ; \quad \sigma = \frac{f_{e\mu}}{f_{e\tau}} \frac{m_\mu^2}{m_\tau^2} \cos\alpha \quad , \tag{28}$$

and assuming $\sigma \ll 1$, the mass matrix can be written as

$$M_\nu = m_0 \begin{pmatrix} 0 & \sigma & \cos\alpha \\ \sigma & 0 & \sin\alpha \\ \cos\alpha & \sin\alpha & 0 \end{pmatrix} . \tag{29}$$

The neutrino mass eigen-values are then

$$\begin{aligned} m_1 &\simeq -m_0 \sigma \sin 2\alpha \\ m_2 &\simeq -m_0 \left[1 - \frac{1}{2}\sigma \sin 2\alpha\right] \\ m_3 &\simeq m_0 \left[1 + \frac{1}{2}\sigma \sin 2\alpha\right] . \end{aligned} \tag{30}$$

Two of the states are nearly degenerate with a common mass m_0, while the other state has a much smaller mass. If the Yukawa couplings f_{ab} are in the range of 10^{-2}, the model is consistent with cosmological mass limits, with ν_μ and ν_τ having masses in the eV range. However, mass parameters in the range required to resolve the solar neutrino puzzle do not emerge naturally.

Neutrino-less double beta decay

If lepton number is violated, double beta decay without the emission of neutrinos ($\beta\beta_{0\nu}$) can occur. The relevant diagram is shown in Fig. 2. The amplitude

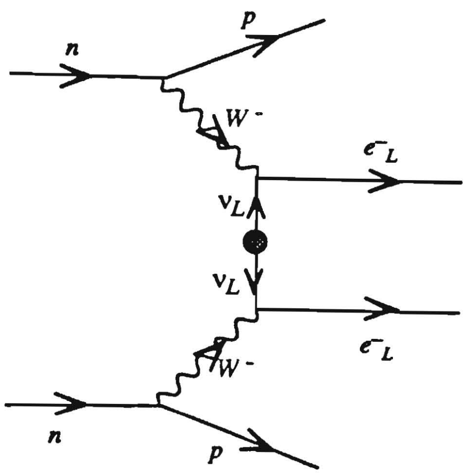

Fig. 2. Neutrino-less double beta decay graph.

is proportional the $\nu_e \nu_e$ Majorana mass. Current experimental limit on this mass is of order 2 eV. In the Zee model, $(\beta\beta)_{0\nu}$ decay is naturally suppressed. This is due to the vanishing of the $\nu_e \nu_e$ entry in Eq. (29). In fact, in the limit where any one of the couplings $f_{e\mu}$, $f_{e\tau}$ or $f_{\mu\tau}$ vanishes, the model has an exact symmetry (eg., $l_e - l_\mu + l_\tau$ symmetry if $f_{e\tau}$ is zero) which forbids neutrino-less double beta decay. In the presence of all three couplings, an effective $\nu_e \nu_e$ Majorana mass will be induced, proportional to the product of all three, which is well within the present experimental limits.

Doubly charged Higgs model

Consider now the inclusion of a doubly charged scalar k^{++} along with the singly charged h^+.[19] Lepton number violation will result in this case even with one doublet of Higgs bosons. k^{++} has the following Yukawa coupling to the right-handed charged leptons:

$$\mathcal{L}_Y^{(3)} = g_{ab}\left(e_{Ra}^T C e_{Rb} k^{++} + H.C.\right) , \tag{31}$$

where $g_{ab} = g_{ba}$. This is in addition to the couplings of h^+ of Eq. (23). As before, these Yukawa couplings themselves conserve lepton number, as can be seen by assigning 2 units of L to k^{++} and η^+. The scalar potential now contains a term

$$V = \mu\left(h^+ h^+ k^{--} + H.C.\right) \tag{32}$$

which violates lepton number. As a result, finite neutrino masses will arise at the

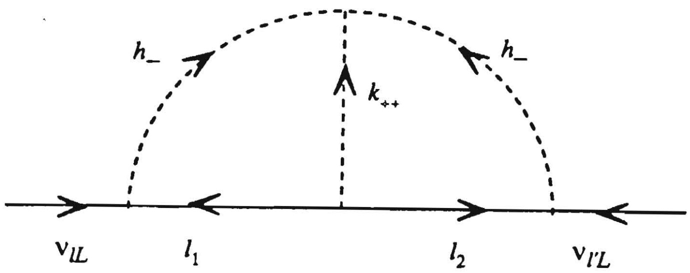

Fig. 3. Two-loop neutrino mass generation via doubly charged Higgs.

two loop level via the diagram of Fig. 3. The neutrino mass matrix has the structure given by[19]

$$M_{ab}^{\nu} = \frac{1}{16\pi^2}\frac{1}{M}\left(fDgDf^T\right)_{ab} \tag{33}$$

where $D = \text{diagonal}\left(m_e,\ m_\mu,\ m_\tau\right)$ and M is the mass of the heaviest scalar running inside the loop, which is assumed to be of the same order as the electro-weak scale. An interesting feature of this mass matrix is that for the case of three generations of neutrinos, due to the anti-symmetry of the matrix f, the determinant of M^ν turns out to be zero. This implies that one neutrino is massless at the two-loop level. At higher orders, this zero will be corrected. Thus the spectrum, for the couplings in the range $(10^{-2} - 10^{-1})$ can be

$$m_{\nu_e} \simeq 0\ ;\quad m_{\nu_\mu} \simeq 10^{-3}\ eV\ ;\quad m_{\nu_\tau} \simeq 1\ eV\ . \tag{34}$$

Such a pattern is within the required range to resolve the solar neutrino puzzle.

Spontaneous lepton number violation

The models discussed so far assume explicit violation of lepton number. It is also possible to break this symmetry spontaneously. Since lepton number is a global symmetry in the standard model, its spontaneous breaking will lead to a physical Goldstone particle, which is dubbed the Majoron. Such particles were thought to be dangerous phenomenologically, as they could mediate long range forces. It was shown[20] in 1980 that the force mediated by the Goldstone particle is spin-dependent, for which the laboratory limits are very weak. The existence of the Majoron was shown to be consistent with all known phenomenological data. Furthermore, it proved to be very useful in by-passing many of the constraints on the neutrinos implied by big bang cosmology.

Consider the standard model with the inclusion of the right-handed neutrinos ν_R. If lepton number is a global symmetry of the Lagrangian, the Majorana mass terms of Eq. (15) for ν_R will not be permitted. The see-saw mass matrix can still be generated by writing a coupling of the ν_R to a complex scalar field S carrying two units of lepton number.[20]

$$\mathcal{L}_Y^{(4)} = h_{ab}\nu_{Ra}^T C \nu_{Rb} S + H.C. \tag{35}$$

where $h_{ab} = h_{ba}$. The Higgs potential is also assumed to respect lepton number. This is to say that it depends only S^*S. The vacuum expectation value of S would generate a Majorana mass for ν_R, yielding the see-saw mass matrix. $J = Im(S)$ is the massless Goldstone particle, the Majoron. It couples to ν_R with full strength, given by the Yukawa couplings of Eq. (35). The coupling of J to light neutrinos

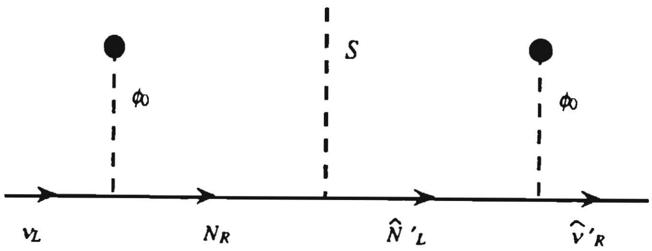

Fig. 4. The coupling of the Majoron to light neutrinos.

arises via the diagram of Fig. 4. It can be estimated to be

$$f_{\nu_L \nu_L J} \simeq \frac{m_D^2}{M^2} \simeq \frac{m_\nu}{M}\ . \tag{36}$$

Although very small numerically, such an off-diagonal coupling would enable the neutrino to decay fast into a light neutrino and J, avoiding cosmological mass density constraints.

How about the coupling of the Majoron to the charged fermions? Since S has no direct coupling to the charged fermions, such couplings can only be induced at the loop level. The relevant diagrams are shown in Fig. 5. The eeJ coupling can

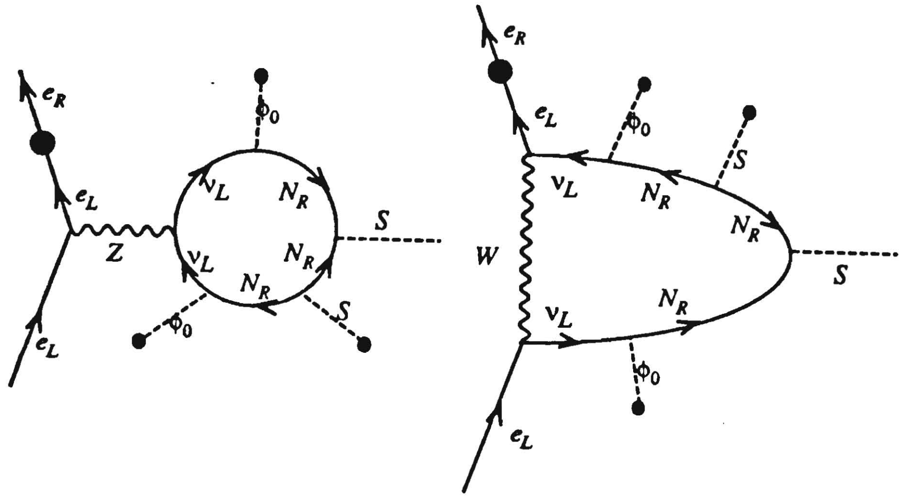

Fig. 5. Induced coupling of the Majoron to electron.

be estimated to be

$$f_{eeJ} \sim \frac{1}{16\pi^2} G_F m_e m_\nu \; . \tag{37}$$

The proportionality to the electron mass is a direct consequence of the derivative coupling of the Goldstone particle. For $m_\nu \simeq 10\ MeV$, this coupling is 10^{-12}.

There are astrophysical bounds on the coupling of electrons and nucleons to the Majoron. Compton like scattering processes $\gamma + e \rightarrow e + J$ (shown in Fig. 6) can create the Majoron inside the star, which, once produced, would escape

Fig. 6. Compton-like process producing Majoron inside stars.

freely, carrying some energy of the star. This could lead to rapid cooling of the star. Bounds on the Majoron couplings to electron and nucleons can be obtained by demanding that the energy loss not be excessive so as to contradict observations.[21] From the sun, one obtains $f_{eeJ} \leq 10^{-10}$. A more stringent bound arises from red giants: $f_{eeJ} \leq 10^{-12}$. The couplings of the singlet Majoron satisfy these constraints.

Suppose the m_{ν_τ} is near its present experimental limit of 35 MeV. Can its decay be fast enough in the singlet Majoron model to avoid the cosmological constraints? It turns out the answer to this is rather subtle.[22] Consider the 6×6 Majorana mass matrix M_ν, and the corresponding Majoron coupling matrix denoted by Y_J:

$$M_\nu = \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix} \; ; \; Y_J = \begin{pmatrix} 0 & 0 \\ 0 & M \end{pmatrix} \frac{1}{\langle S \rangle} \, . \tag{38}$$

Since the entries of m are much smaller than those of M, the mass matrix M_ν can be diagonalized perturbatively. The transformation $U^T M_\nu U$, where U is a unitary matrix given by

$$U = \begin{pmatrix} 1 - \rho\rho^T/2 & \rho \\ -\rho^T & 1 - \rho^T\rho/2 \end{pmatrix} + \mathcal{O}(\rho^3) \, , \tag{39}$$

brings M_ν to a block-diagonal form. Here $\rho = mM^{-1}$ is a 3×3 matrix with its entries all smaller than 1. The transformed matrices have the block form

$$\begin{aligned} U^T M_\nu U &= \begin{pmatrix} -mM^{-1}m^T & 0 \\ 0 & M \end{pmatrix} + \mathcal{O}(\rho^3) \\ U^T Y_J U &= \begin{pmatrix} mM^{-1}m^T & -m \\ -m & M \end{pmatrix} \frac{1}{\langle S \rangle} + \mathcal{O}(\rho^3) \, . \end{aligned} \tag{40}$$

Notice that the light 3×3 block of the transformed mass matrix is proportional to that of the Majoron coupling matrix. A subsequent diagonalization of the light block in the mass matrix will also diagonalize the Majoron coupling matrix of the light neutrinos. Thus the off-diagonal coupling will be zero to order ρ^2. They can arise at order ρ^4. However, once the loop effects are included, this line of reasoning will not work any more, there will be $\mathcal{O}(\rho^2)$ terms, suppressed by loop factors.[23] With its inclusion, it is possible that ν_τ with its mass near the limit could decay fast, avoiding cosmological constraints.

3. The solar neutrino puzzle

The sun is constantly burning Hydrogen into Helium via nuclear fusion reactions. Neutrinos are emitted in these reactions. These neutrinos, known as the solar neutrinos, have been detected on earth in two experiments using widely different techniques. The observations seem to indicate a deficit in their flux compared to theoretical expectations. This has come to be known as the solar neutrino puzzle. In this section, I shall briefly review the puzzle and discuss its various resolutions.

It will be useful to recall some of the vital statistics of the sun.[9] It is roughly a spheroid, as shown in Fig. 7. The radius of the sun is $R = 7 \times 10^{10}\ cm$. The inner

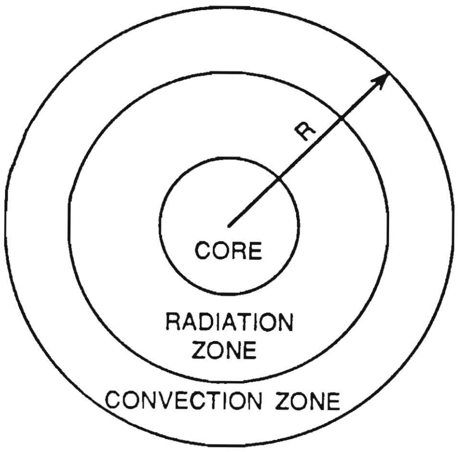

Fig. 7. Schematic diagram of the sun.

25% of the radius constitutes the core, where the density is as high as 150 gm/cc and the Temperature is about $1.5 \times 10^7 K$. Most of the nuclear fusion reactions producing neutrinos take place within the core. Just outside of the core is the radiation zone, where the primary mode of energy transport is radiation. The outer 25% of the radius is the convective zone, which is characterized by the turbulent flow of the plasma. This generates a magnetic field inside the convective zone, which is believed to be in the 10 kG range. Observationally this corresponds to the sun spots, which are the lines of the magnetic field. The convective magnetic field has a 11 year quasi-cycle: its magnitude goes through a maximum and minimum in 11 years. The strength of the field varies at least by an order of magnitude between the solar minimum and maximum. The absence of sun spots at low solar altitude is taken to mean that the field is nearly zero at the solar equator. The earth to sun distance is $1.5 \times 10^{13}\ cm$.

The dominant mode of energy production inside the sun is the fusion of protons to form He nuclei (the pp chain). The relevant reactions are listed below. In reactions generating a neutrino, the maximum neutrino energy is also given in parenthesis.

pp I:

$$p + p \rightarrow {}^2H + e^+ + \nu_e \quad (0.42\ MeV)$$

$$p + e + p \rightarrow {}^2H + \nu_e \quad (1.44\ MeV)$$

$${}^2H + p \rightarrow {}^3He + \gamma$$

$$^3He + {}^3He \rightarrow {}^4He + 2p \tag{41}$$

pp II:

$$^3He + {}^4He \rightarrow {}^7Be + \gamma$$
$$^7Be + e^- \rightarrow {}^7Li + \nu_e \quad (0.861\ MeV)$$
$$^7Li + p \rightarrow 2\ {}^4He \tag{42}$$

pp III:

$$^7Be + p \rightarrow {}^8B + \gamma$$
$$^8B \rightarrow {}^8Be^* + e^+ + \nu_e \quad (15\ MeV)$$
$$^8Be^* \rightarrow 2\ {}^4He \tag{43}$$

The *pp* neutrinos of Chain *I* are the most abundant of solar neutrinos. However, they are low energy neutrinos, which are not easily detectable. Ernest search for these neutrinos have only begun recently.

Two of the production reactions generate monochromatic neutrinos, the *pep* reaction with an energy of 1.44 MeV and the electron capture of ^7Be with an energy of 0.861 MeV. The highest energy neutrinos come from the beta decay of ^8B, which has a continuous distribution from $0 - 15\ MeV$. Present experiments are most sensitive to the ^8B neutrinos. Besides the *pp* chain, there is also the CNO cycle, which generates approximately 1.5 % of the solar neutrinos.

Solar neutrino experiments

Two experiments have given positive evidence for solar neutrino detection. The Chlorine experiment of Davis and collaborators has the longest running history, it has been running for more than 20 years.[7] It is a radio-chemical experiment which utilizes the reaction

$$\nu_e + {}^{37}Cl \rightarrow e + {}^{37}Ar\ . \tag{44}$$

The experiment uses 10^5 gallons of the compound C_2Cl_4. The radioactive ^{37}Ar are chemically extracted and counted (its half life is 35 days). The threshold of the reaction is 0.81 MeV, which means that the most abundant *pp* neutrinos will go undetected. The experiment is most sensitive to the ^8B neutrinos, which have the highest energy. The average count rate of Davis, expressed in the Solar Neutrino Unit (SNU), where 1 $SNU = 10^{-36}$ events/s/target atom is

$$\Phi_{\rm observed} = (2.1 \pm 0.9)\ SNU\ . \tag{45}$$

The theoretically expected rate, based on the standard solar model is[9]

$$\Phi_{\rm theory} = (7.9 \pm 2.6)\ SNU \tag{46}$$

where the error quoted, which involves both theoretical and experimental uncertainties, is an effective 3σ error. The discrepancy between Eq. (45) and (46) constitutes the solar neutrino puzzle.

In addition to the deficit in the observed signal, data taken over the past twenty years in the *Cl* experiment shows a time variation in the flux in anti-correlation with the sun spots. This is demonstrated in Fig. 8. As will be discussed later, this behavior can be understood if the neutrino has a sizable magnetic moment.

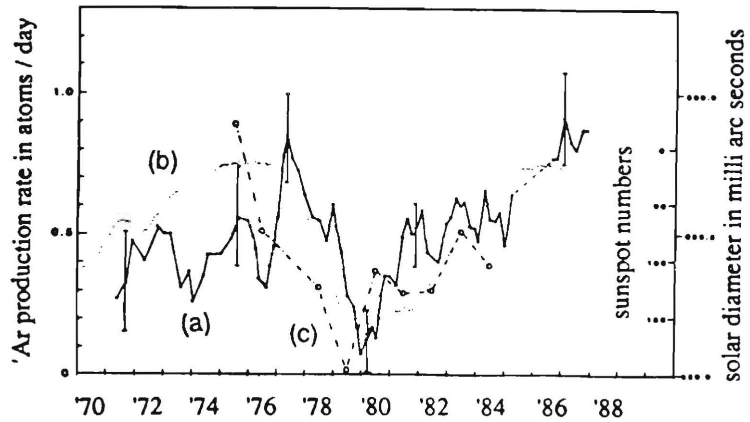

Fig. 8. Chlorine solar neutrino flux as a function of time. Also plotted are the sun spot numbers.

More recently, the Kamiokande experiment in Japan has reported the observation of solar neutrinos using a water Cherenkov detector.[8] This experiment makes use of the elastic scattering of ν_e on electrons, which has a forward peaked angular distribution, so that from the directionality of the events solar neutrinos can be identified. The reaction threshold is 7.5 MeV. So the experiment is sensitive only to the ^8B neutrinos. The observed flux is expressed normalized to the standard solar model flux,

$$\frac{\Phi_{\rm obs}}{\Phi_{\rm SSM}} = 0.46 \pm 0.05 \pm 0.06 \tag{47}$$

for the first 1040 days. This confirms the deficit reported in the *Cl* experiment.

Two other experiments are under way, primarily to look for the low energy *pp* neutrinos. These are the *Ga* experiments, one in the Soviet Union (SAGE), which uses 60 tons of metallic Gallium and the other in Italy (GALLEX), which uses 30 tons of liquid Gallium. Both of these make use of the reaction

$$\nu_e + {}^{71}Ga \rightarrow e + {}^{71}Ge \ . \tag{48}$$

The half-life of ^{71}Ge is 11.4 days. The reaction threshold is 0.23 MeV, facilitating the detection of *pp* neutrinos. The preliminary report of the SAGE experiment is that their results are consistent with a deficit.[24]

Resolving the puzzle

Before discussing the possible resolution of the puzzle, it is useful to recall the contribution of the various nuclear reactions to the total flux for each of these experiments. They are listed in SNU units in Table. 1.

Table. 1. Standard solar model prediction for the neutrino flux for the Chlorine, Kamiokande and Gallium experiments (in SNU).

Source	E_ν (MeV)	^{37}Cl	K-II	^{71}Ga
^8B	$0 - 15$	6.1	0.05	14.0
^{15}O	$0 - 1.73$	0.3	-	6.1
pep	1.442	0.2	-	3.0
^{13}N	$0 - 1.199$	0.1	-	3.8
^7Be	0.863	1.1	-	34.3
pp	$0 - 0.42$	-	-	70.8
Total	-	7.9	0.05	132

Although it may not be impossible to resolve the puzzle by modifying the standard solar model, here I shall consider the possibility of explaining it via modification of the neutrino properties. Some popular solutions are (i) long wave length vacuum oscillation, (ii) matter enhanced resonant oscillation (MSW effect) and (iii) magnetic moment of the neutrino. In all these scenarios, something happens to the neutrinos on its way from the solar core to the detector on earth.

Let us define following Ref. 25,

$$R = \Phi_{\rm obs}/\Phi_{SSM} \ , \tag{49}$$

so that the average R for the Cl and K-II experiments are

$$\begin{aligned} R_{Cl} &= 0.27 \pm 0.05 \\ R_{\rm K-II} &= 0.46 \pm 0.09 \ . \end{aligned} \tag{50}$$

One can now define the survival probability for the high energy ^8B neutrinos as $P_H(\nu_e \to \nu_e)$ and the probability for intermediate energy neutrinos (^7Be, pep, ^{13}N, ^{15}O) as P_I. Then, from Table 1, we have

$$R_{Cl} = 0.78 P_H + 0.22 P_I \ . \tag{51}$$

If ν_e converts into another flavor, say ν_μ, the converted ν_μ will go undetected in the Cl detector. However, it has a small cross section via neutral current interactions in the water detector, given by $\sigma(\nu_\mu e \to \nu_\mu e) \simeq \frac{1}{7}\sigma(\nu_e e \to \nu_e e)$. Hence, for the K-II experiment, we have

$$R_{\rm K-II} = \frac{1}{7} + \frac{6}{7} P_H \ . \tag{52}$$

The best fit obtained from Eq. (51) and (52) is

$$P_H = 0.36 \; ; \quad P_I = 0 \; . \tag{53}$$

That is, the intermediate energy neutrinos are completely suppressed, while the high energy ^8B neutrinos are suppressed by a factor 0.36.

This analysis can be extended to Ga experiment. We define a ν_e survival probability P_L for the low energy pp neutrinos. For $P_H = 0.36$ and $P_I = 0$, we get

$$R_{Ga} = 0.04 \pm 0.54 P_L \; . \tag{54}$$

If $P_L = 0$, the expected flux in Ga will be 5 SNU. For $P_L = 1$, the flux is 71 SNU, which will be the maximum allowed value. I should caution the reader that these numbers are derived based on the best fit, and do not take into account the allowed deviations.

Long wave length vacuum oscillation

Consider the oscillation of two flavors of neutrinos ν_e and ν_μ as given in Eq. (6). The ν_e survival probability is

$$P_{\nu_e \to \nu_e} = 1 - \sin^2\theta \sin^2 \left(\frac{\pi L}{\lambda}\right) \; , \tag{55}$$

where the wave length of oscillation λ is given by

$$\lambda = \frac{4\pi E_\nu}{\Delta m^2} = 2.5 \times 10^{10} \; m \; \left(\frac{E_\nu}{1 \; MeV}\right) \left(\frac{10^{-10} eV^2}{\Delta m^2}\right) \; . \tag{56}$$

For MeV solar neutrinos, the oscillation length will be of the order of the earth-sun distance if $\Delta m^2 \approx 10^{-10} \; eV^2$.

The survival probability goes through minima whenever

$$L = \left(n + \frac{1}{2}\right) \lambda \; , \tag{57}$$

where $n = 0, 1, 2, ...$For $P_I \approx 0$, ^7Be neutrinos should be near its minimum at earth. This requires

$$\Delta m^2 = \left(n + \frac{1}{2}\right) \frac{4\pi E_\nu}{L} = \left(n + \frac{1}{2}\right) 1.4 \times 10^{-11} \; eV^2 \; , \tag{58}$$

where we have substituted the energy of ^7Be in the second step. Similarly, $P_H = 0.36$ requires

$$0.36 = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4}\frac{L}{E_\nu}\right) \tag{59}$$

with E_ν corresponding to the ^8B neutrinos. One can now look for a simultaneous fit treating δm^2 and n as the free parameters. It is found that[25] there is no solution for $n = 0, 1, 2$. Solutions exist for $n = 3 - 20$. For large n, the oscillating term averages to a 1/2, which does not give a combined fit.

It should be emphasized that long wave length oscillation can explain the puzzle only if the mixing angle is near maximal. One interesting feature of this solution is that the low energy pp neutrinos, have smaller oscillation lengths. They would oscillate rapidly, averaging to a 1/2. Thus $P_L = 1/2$ and therefore $R_{Ga} = 0.31$ (corresponding to 40 SNU).

MSW mechanism

The properties of the neutrinos which traverse through a dense medium, such as the interior of the sun, can be drastically modified due to the coherent forward scattering by the medium, as discussed first by Wolfenstein.[26] It was shown by Mikhevey and Smirnov[27] that for a wide range of oscillation parameters, the conversion probability goes through a resonant enhancement, which could explain naturally the deficit in solar neutrinos. Here I shall outline the main features of the MSW mechanism.

Let us reconsider the oscillation of two neutrino flavors in vacuum. The mass eigen-states ν_1 and ν_2, with energies E_1 and E_2 evolve with time according to

$$\begin{aligned} |\nu_1(t)\rangle &= e^{-iE_1 t}\,|\nu_1(0)\rangle \\ |\nu_2(t)\rangle &= e^{-iE_2 t}\,|\nu_2(0)\rangle \ . \end{aligned} \tag{60}$$

The time evolution can then be written in terms of a Hamiltonian equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = H\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} , \tag{61}$$

where H is given by

$$H = \begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix} = |\vec{p}| + \begin{bmatrix}\frac{m_1^2}{2|\vec{p}|} & 0\\ 0 & \frac{m_2^2}{2|\vec{p}|}\end{bmatrix} . \tag{62}$$

The second equality uses the relativistic approximation. In terms of the weak interaction eigen-states ν_e and ν_μ, the corresponding equation is

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H'\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \tag{63}$$

with

$$H' = \left(|\vec{p}| + \frac{m_1^2 + m_2^2}{4|\vec{p}|} \right) + \frac{\Delta m^2}{4|\vec{p}|} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} . \tag{64}$$

In our notation, $\theta \leq 45^0$, $\Delta m^2 = m_2^2 - m_1^2$ is either positive or negative.

Neutrinos propagating through a medium will feel an extra potential due to the coherent forward scattering off the medium. In the solar interior, such scattering can occur via neutral current as well as by charged current interactions. The extra potential due to neutral current scattering will be identical for ν_e and ν_μ. However, the charged current processes distinguish the two, as there are electrons inside the medium, but not muons. The potential difference created by this is obtained from the effective interaction

$$\begin{aligned} V_{eff}^{cc} &= \frac{G_F}{\sqrt{2}} [\bar{e}\gamma_\mu(1-\gamma_5)\nu_e] [\overline{\nu_e}\gamma^\mu(1-\gamma_5)e] \\ &= \frac{G_F}{\sqrt{2}} [\overline{\nu_e}\gamma_\mu(1-\gamma_5)\nu_e] [\bar{e}\gamma^\mu(1-\gamma_5)e] \end{aligned} \tag{65}$$

where we made a Fierze rearrangement in the second line. For neutrino propagation, one should average over the medium, which gives $\langle \bar{e}\gamma^\mu(1-\gamma_5)e \rangle = \langle \bar{e}\gamma^0 e \rangle = N_e$, the electron number density. (Note that the γ_5 term is spin-dependent and averages to zero.) The effective potential seen by ν_e is then (both the 1 and γ_5 terms contribute equally for a relativistic neutrino)

$$V = \sqrt{2} G_F N_e . \tag{66}$$

(For a more rigorous derivation of the potential, see J. Nieves, these proceedings.) The Hamiltonian H' of Eq. (64) is now modified to

$$H' = \left(|\vec{p}| + \frac{m_1^2 + m_2^2}{4|\vec{p}|} \right) + \begin{bmatrix} -\frac{\Delta m^2}{4|\vec{p}|}\cos 2\theta + \sqrt{2} G_F N_e & \frac{\Delta m^2}{4|\vec{p}|}\sin 2\theta \\ \frac{\Delta m^2}{4|\vec{p}|}\sin 2\theta & \frac{\Delta m^2}{4|\vec{p}|}\cos 2\theta \end{bmatrix} . \tag{67}$$

The effective mixing angle in matter is

$$\tan 2\theta_m = \frac{\tan 2\theta}{1 - A/A_0} \tag{68}$$

where

$$A = \sqrt{2} G_F N_e |\vec{p}| \; ; \qquad A_0 = \Delta m^2 \cos 2\theta . \tag{69}$$

The effective mixing angle becomes maximal ($\theta_m = 45^0$) when $A = A_0$, regardless of how small the vacuum mixing angle was. This phenomenon of resonance occurs when the two diagonal elements of the matrix in Eq. (67) become equal. The resonance density is

$$N_e = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2} G_F |\vec{p}|} \ . \tag{70}$$

The condition for resonance clearly requires $m_2^2 \geq m_1^2$, i.e., ν_μ should be heavier than ν_e, which conforms to theoretical expectations.

The effective mass-squared eigenvalues in matter, obtained from Eq. 67, are

$$M_{1,2}^2 = \frac{1}{2}\left[\left(m_1^2 + m_2^2 + A\right) \mp \left\{\left(\Delta m^2 \cos 2\theta - A\right)^2 + \left(\Delta m^2\right)^2 \sin^2 2\theta\right\}^{1/2}\right] \ . \tag{71}$$

The behavior of the two eigen-values are depicted in Fig. 9. When the density ρ is

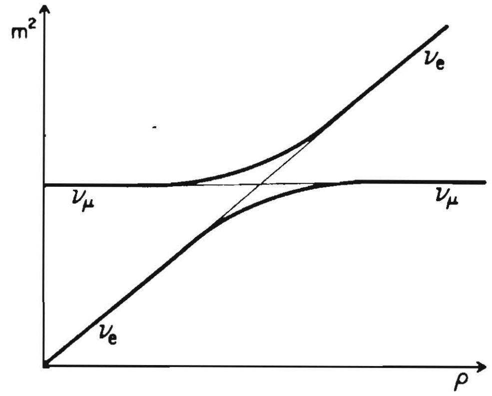

Fig. 9. The effective masses of ν_e and ν_μ in matter as a function of density.

very large, say inside the solar core, ν_e state is heavier than ν_μ. At a critical density the two states become degenerate, where level crossing occurs. For lower densities, ν_e will be the lighter state.

ν_e of a given energy will go through complete conversion into ν_μ if the neutrino propagation is adiabatic. Quantitatively, this means that the rate of change of effective mixing angle should be smaller than the oscillation frequency. From Eq. (68), the adiabaticity condition is

$$\frac{d}{dt}(\ln \rho)|_{res} \ll \frac{\Delta m^2 \sin^2 \theta}{|\vec{p}| \cos 2\theta} \ . \tag{72}$$

Even if the adiabaticity condition, Eq. (72), is not satisfied, there is a finite probability of jumping from one eigen-state to the other. This can be analytically described (for linearly changing density) by the Lndau-Zener exponent:

$$P_{\nu_e \to \nu_e} = exp\left[-\frac{\pi}{4}\frac{\Delta m^2}{|\vec{p}|}\frac{\sin^2 2\theta}{\cos 2\theta}\left(\frac{d}{dt}\ln\rho\right)^{-1}_{res}\right] . \tag{73}$$

Detailed numerical calculation have been carried out using the parameters of the standard solar model by several groups.[28] The deficiency in Cl experiment can be understood in terms of the MSW resonance for a wide range of values of the parameters:

$$10^{-8}\ eV^2 \leq \Delta m^2 \leq 10^{-4}\ eV^2\ ; \quad \sin^2 2\theta \geq 10^{-3}\ . \tag{74}$$

The allowed parameter range when the Cl data is combined with the K-II observations is shown in Fig. 10 (adapted from Ref. 25). The long-wavelength solutions are also shown there.

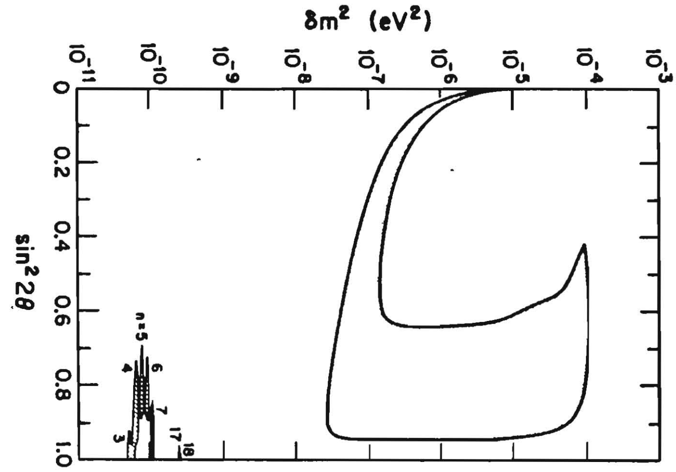

Fig. 10. Combined fit of the Cl and K-II flux in the MSW scenario (from Ref. 25). Also shown is the long wave length oscillation fits for various n.

Neutrino magnetic moment to resolve the solar puzzle

An old idea (originally due to Cisneros[29]) that the solar neutrino deficit may be explained if the neutrino has a large magnetic moment has recently been revived by Voloshin, Vysotski and Okun.[30] The motivation for this revival was the apparent anti-correlation of the neutrino flux seen in the Cl experiment with the sun spots (see Fig. 8). Such an anti-correlation can be understood in the magnetic moment picture.

The solar convective zone has a large magnetic field, in the range of few kilo-Gauss, which exhibits a 11 year quasi-periodicity. The time of maximal sun spots corresponds to large convective magnetic field. If ν_e has a magnetic moment, the transition $\nu_{eL} \to \nu_{eR}$ can occur inside the convective zone. The right-handed species, being sterile with respect to the Cl experiment, will go undetected. This will explain the deficit. More importantly, the rate of conversion into the sterile state depends on the strength of the convective magnetic field, being more efficient during solar maxima. This would account for the anti-correlation of the neutrino flux with sun spots.

In order for this spin-flip mechanism to be efficient, the precession angle should be of order 1. This requires

$$\mu B L \approx \mathcal{O}(1) \ . \tag{75}$$

Using the length of the convective zone for L and a few kilo-Gauss for B, we find that the required magnetic moment is

$$\mu \approx \left(10^{-11} - 10^{-10}\right)\mu_B \qquad \left(\mu_B = \frac{e}{2m_e}\right) \ . \tag{76}$$

The neutrino can have two different types of magnetic moments. One is the familiar Dirac type moment, parametrized by the interaction Lagrangian

$$\mathcal{L}_D = \mu_D \overline{\nu_L} \sigma_{\mu\nu} \nu_R F^{\mu\nu} \ . \tag{77}$$

The other is the lepton number violating Majorana (or transition) magnetic moment:

$$\mathcal{L}_M = \mu_M \nu_L^T C \sigma_{\mu\nu} \nu_L' F^{\mu\nu} \ . \tag{78}$$

If $\nu_L = \nu_L'$, the Majorana moment will be identically zero. This is due to CPT invariance. The magnetic moment of a particle should be equal and opposite to that of its anti-particle, but a Majorana particle is its own anti-particle, so its diagonal moments must vanish. This argument does not preclude off-diagonal transition moments. Such moments can lead to $\nu_e \to \bar{\nu}_\mu$ transition, which is a $\Delta L = 2$ process, in the solar convective zone. In the Dirac scenario, $\nu_{eL} \to \nu_{eR}$ conversion occurs inside the sun.

Consider the Dirac magnetic moment scenario first. The evolution of the 2×2 neutrino system is governed by

$$i\frac{d}{dt}\begin{pmatrix} \nu_{eR} \\ \nu_{eL} \end{pmatrix} = \begin{bmatrix} 0 & \mu B \\ \mu B & a_{\nu_e} \end{bmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{eL} \end{pmatrix} \tag{79}$$

where

$$a_{\nu_e} = \sqrt{2} G_F \left(N_e - \frac{N_n}{2}\right) \tag{80}$$

is the matter potential seen by ν_e. Here, we had to include the neutral current contribution as well, since ν_{eR} does not see that potential. N_n is the neutron number density. For constant magnetic field and constant density, one can write the conversion probability as

$$P_{\nu_{eL}\rightarrow\nu_{eR}} = \left[\frac{(2\mu B)^2}{a_{\nu_e}^2 + (2\mu B)^2}\right] \sin^2\left[\sqrt{a_{\nu_e}^2 + (2\mu B)^2}\,\frac{t}{2}\right] \ . \tag{81}$$

Note that the effect of matter is to suppress precession, as the pre-factor to the sin in Eq. (81) is always less than 1.

For the case of transition magnetic moments, analogous equations can be derived. Since the vacuum masses are not the same to begin with, precession will be suppressed unless[30]

$$\frac{\Delta m^2}{2E} \leq \mu B \qquad \Rightarrow \qquad \Delta m^2 \leq 10^{-7}\ eV^2\ . \tag{82}$$

However, as shown by Lim and Marciano and by Akhmedov[31], there could also be a resonant enhancement of the oscillation in the presence of a finite Δm^2 and matter effects. To appreciate this point, let us consider the 2×2 matrix equation for $\nu_e - \bar{\nu}_\mu$ transition in the limit of zero flavor mixing.

$$H = \begin{bmatrix} a_{\nu_e} & \mu B \\ \mu B & \frac{\Delta m^2}{2E} - a_{\nu_\mu} \end{bmatrix} \tag{83}$$

where $a_{\nu_\mu} = -G_F N_n/\sqrt{2}$. Resonant $\nu_e \rightarrow \bar{\nu}_\mu$ conversion will occur when the two diagonals become equal (analogous to the MSW mechanism). Using the approximate relation that $N_n \approx N_e/6$, the resonance condition can be written as

$$N_e = \frac{3\sqrt{2}}{10}\frac{\Delta m^2}{G_F E}\ . \tag{84}$$

This is the resonant spin-flavor precession ($\nu_e \rightarrow \bar{\nu}_\mu$).

In the presence of non-zero flavor mixing, the Hamiltonian of the system is somewhat more complicated. For two flavors, in the basis (ν_e, ν_μ, $\bar{\nu}_e$, $\bar{\nu}_\mu$), it is

$$H = \begin{bmatrix} a_{\nu_e} & \frac{\Delta m^2}{4E}\sin2\theta & 0 & \mu B \\ \frac{\Delta m^2}{4E}\sin2\theta & \frac{\Delta m^2}{2E}\cos2\theta + a_{\nu_\mu} & -\mu B & 0 \\ 0 & -\mu B & -a_{\nu_e} & \frac{\Delta m^2}{4E}\sin2\theta \\ \mu B & 0 & \frac{\Delta m^2}{4E}\sin2\theta & \frac{\Delta m^2}{2E}\cos2\theta - a_{\nu_\mu} \end{bmatrix}\ . \tag{85}$$

In this case, more than one resonance can occur:

$$\frac{\Delta m^2}{2E}\cos 2\theta = a_{\nu_e} - a_{\nu_\mu} \qquad (\nu_e \rightarrow \nu_\mu)$$

$$\frac{\Delta m^2}{2E}\cos 2\theta = a_{\nu_e} + a_{\nu_\mu} \qquad (\nu_e \rightarrow \bar{\nu}_\mu)\ . \tag{86}$$

4. Magnetic moment of the neutrino

It was shown in the previous section that a sizable magnetic moment of the neutrino in the range $(10^{-11} - 10^{-10})\ \mu_B$ can explain the solar neutrino puzzle, in particular, the apparent anti-correlation of the flux with the sun spots. Here I shall discuss the theoretical and phenomenological implications of such a large neutrino magnetic moment.

Bounds on the neutrino magnetic moment

As noted earlier, the neutrino can have two types of magnetic moments: the Dirac moment and the Majorana transition moment. There are constraints on these moments both from direct lab experiments as well as from cosmology and astrophysics. The laboratory bound, which applies to both Dirac and transition magnetic moments, arises from the scattering of reactor neutrinos on electrons. There would be a new contribution to the process via photon exchange in the presence of a neutrino magnetic moment. This amplitude does not interfere with the standard weak interaction amplitude, due to the difference in the final state helicity. The bound derived, based on consistency with the standard model is[32]

$$\mu_\nu \leq 4 \times 10^{-10} \mu_B\ . \tag{87}$$

A large magnetic moment of the neutrino (either Dirac or Majorana) can lead to plasmon decay 'γ' $\rightarrow \nu\bar{\nu}$ inside a star, where 'γ' stands for the photon which has an effective mass (of the order of the plasma frequency). Such processes could drastically affect stellar evolution, and can be used to place bounds on μ_ν.[33] The bound when applied to the sun is $\mu_\nu \leq 13 \times 10^{-10}\mu_B$. From white dwarfs, the bound derived is $\mu_\nu \leq 0.4 \times 10^{-10}\mu_B$ and from red giants a somewhat more stringent bound emerges: $\mu_\nu \leq 0.14 \times 10^{-10}\mu_B$. These indirect limits are more stringent than the direct laboratory bound, but would still allow a range of magnetic moment that is relevant for solar neutrinos.

If neutrinos are Dirac particles, their magnetic moments are constrained by primordial nucleosynthesis calculations. Most recent calculations quote a limit of 3.3 on the number of effective neutrino species that are in equilibrium with the plasma during the epoch of nucleosynthesis.[34] If the Dirac type magnetic moment is larger than $2 \times 10^{-11}\mu_B$, the ν_R would be in equilibrium via the magnetic moment scattering off electrons.[33] This would contribute as one extra species, violating the bound.

Dirac type moments are also constrained severely by neutrino observation from the supernova SN1987A. The argument is that ν_R can be produced within the core of the supernova via the reaction $\nu_L e \to \nu_R e$, mediated by the magnetic moment interaction of the photon. Once produced, ν_R would escape freely from the supernova, draining its energy. This is because the mean free path of the ν_R, which has no weak interactions, would be much longer than the radius of the supernova. If the magnetic moment interaction itself should trap the ν_R, $\mu_\nu \geq 10^{-9}\mu_B$ is required, which would violate the laboratory limit. Demanding that the energy lost via ν_R emission is smaller than the binding energy released in the supernova collapse leads to the bound $\mu_\nu \leq 10^{-12}\mu_B$.[35] Although it is possible to evade this bound by postulating new interactions of the ν_R, which may be needed anyway to generate a large magnetic moment, the same interaction that would trap ν_R inside the core of the supernova would also keep it in equilibrium during nucleosynthesis. This would violate the bound $N_\nu \leq 3.3$ derived from nucleosynthesis calculations. Thus if both bounds are taken seriously, it would appear that the Dirac type magnetic moment is unlikely to be relevant for the solar neutrino puzzle. Neither the supernova bound nor the nucleosynthesis bound of course apply to transition magnetic moments. As shown below, from theoretical viewpoint as well, the transition moment preferable.

Neutrino magnetic moment in gauge theories

In the minimal standard model, the absence of ν_R from the fermionic spectrum and the conservation of lepton number imply that the neutrino has no mass and no magnetic moment. The simplest extension to generate a non-zero magnetic moment would be to introduce the ν_R into the spectrum. A magnetic moment would be generated via the diagrams of Fig. 11. Evaluation of these graphs yield[36]

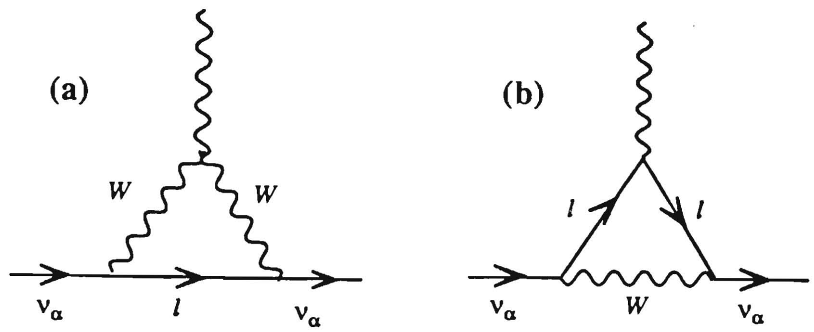

Fig. 11. Diagrams generating neutrino magnetic moment in the standard model with ν_R.

$$\mu_{\nu_e} = \frac{3eG_F m_{\nu_e}}{8\sqrt{2}\pi^2} = 3 \times 10^{-19}\mu_B \left(\frac{m_{\nu_e}}{eV}\right) . \tag{88}$$

The proportionality to the neutrino mass is due to the fact that ν_R has no gauge interactions. Therefore chirality flip should occur on the external leg in Fig. 11, which brings in a factor m_ν. Given that $m_{\nu_e} \leq 10\ eV$, the magnetic moment that can be generated is many orders of magnitude below the required value to resolve the solar neutrino puzzle.

In left-right symmetric models, based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$,[37] the fermions are assigned to the two $SU(2)$ groups in a symmetric manner. This requires the introduction of ν_R into the spectrum, which forms an $SU(2)_R$ doublet with e_R. Thus ν_R will have gauge interactions with the right-handed W_R boson. It is then conceivable that the induced magnetic moment is larger. It is indeed the case, μ_{ν_e} is proportional to the electron mass, rather than the neutrino mass and is given by

$$\mu_{\nu_e} = \frac{G_F}{\sqrt{2}\pi^2} m_e^2 \mu_B \sin 2\xi \ , \tag{89}$$

where ξ is the $W_L - W_R$ mixing angle. Present experimental constraints from polarized μ decay require $\sin\xi \leq 0.035$. As a result, $\mu_{\nu_e} \leq 10^{-14}\mu_B$, still far too short.

In a variety of popular models, such as supersymmetric models, E_6-based models etc., the magnetic moment of the neutrino generated by gauge boson interactions turns out to be too small to be relevant for solar neutrinos.[38] The maximum value attainable in this class of models is $10^{-12}\mu_B$, which brings us to the next topic.

Higgs model of large magnetic moment

Although a large neutrino magnetic moment is difficult to realize theoretically via gauge boson interactions, it can be achieved by a proper extension of the Higgs sector. Here I shall discuss such an extension of the standard model, proposed in Ref. 39.

Consider the inclusion of the scalar h^+ (the same particle encountered in the Zee model of neutrino masses) into the standard spectrum, in addition to the ν_R. The following Yukawa couplings are then allowed.

$$\mathcal{L}_Y^{(5)} = f_{ab}\psi_{La}^T C i\tau_2 \psi_{Lb} h^+ + g_{ab} e_{Ra}^T C \nu_{Rb} h^+ + H.C. \tag{90}$$

Here, $f_{ab} = -f_{ba}$. Lepton number is assumed to be conserved, so that neutrinos are Dirac particles. The above Yukawa couplings generate a non-zero magnetic moment

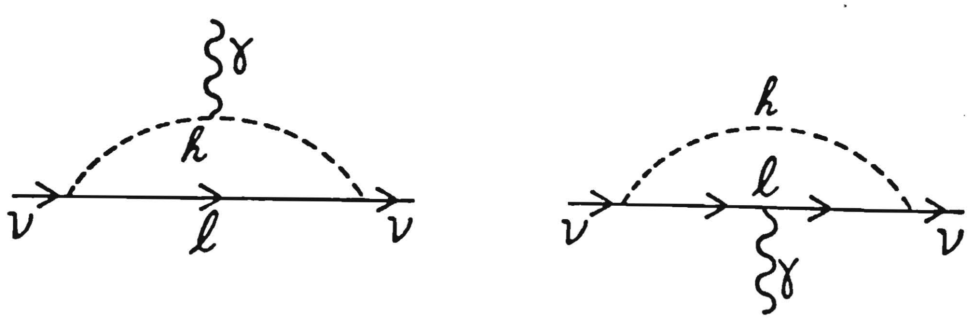

Fig. 12. Neutrino magnetic moment via charged Higgs exchange (Ref. 39).

via the graphs of Fig. 12 which evaluates to

$$\mu_{\nu_i} = e \sum_{j \neq i} \left(\frac{g_{ij} f^{\dagger}_{ji} + f_{ij} g^{\dagger}_{ji}}{32\pi^2} \right) \frac{m_j}{M^2} \left(\ln \frac{M^2}{m_j^2} - 1 \right) , \quad (91)$$

where m_j denotes the jth charged lepton mass and M is the scalar mass. Note that due to the anti-symmetry of the matrix f, μ_{ν_e} and μ_{ν_μ} become proportional to the τ lepton mass, and are thus expected to be larger than μ_{ν_τ} (which is proportional to m_μ). For $f_{13} g_{31}/M^2 \simeq (0.4 - 1.2) \times 10^{-6} GeV^{-2}$, $\mu_{\nu_e} \simeq 10^{-10}$ can be obtained.

It can be verified that this choice of parameters is consistent with all phenomenological constraints. For example, the coupling of h^+ would mediate the rare decay $\mu \to e\gamma$ at the one loop. The decay rate is given by

$$\Gamma(\mu \to e\gamma) = \frac{\alpha}{2} \left[\frac{1}{32\pi^2} \frac{m_\mu}{12M^2} \left(f f^{\dagger} + g g^{\dagger} \right)_{12} \right]^2 m_\mu^3 . \quad (92)$$

Consistency with the limit $B(\mu \to e\gamma) \leq 10^{-11}$ requires $\left(f f^{\dagger} + g g^{\dagger} \right)_{12} \leq 2 \times 10^{-4}$. This can be satisfied if we choose the couplings $(g_{23}, f_{23}) \approx 10^{-2} (g_{13}, f_{13})$. That is, all couplings cannot be assumed to be of the same order.

The above example shows that it is not impossible to generate a large magnetic moment of the neutrino in gauge theory context. However, the model suffers from naturalness problems. This is because the mass of the neutrino is arbitrary in the model. It may be argued that it is not a real problem, since m_ν, being a dimension 3 operator, undergoes infinite renormalization, while the magnetic moment, being a dimension 5 operator is a finite quantity. One could adjust the mass counter-term to fit the observed value of the neutrino mass. This procedure, however, does not provide any theoretical understanding of the smallness of the neutrino mass. Moreover, the diagram which generates the magnetic moment, will also generate a neutrino mass (once the external photon line is removed). Naively, if one evaluates the contribution to the mass with an ultra-violet cut-off, one finds it will be of order 10 keV. This one-loop contribution should be cancelled by the mass counter-term, to obtain a physical mass of less than 10 eV. The associated fine-tuning is the naturalness problem.

Is it possible to generate a large neutrino magnetic moment and at the same time explain the smallness of the neutrino mass? We shall explore here new symmetries which achieve this desirable goal. The neutrino mass is necessarily a loop induced effect in this scheme.

One immediately encounters a problem with small neutrino mass and large magnetic moment. Consider an arbitrary diagram which generates a large magnetic moment of order $10^{-11}\mu_B$. Once the photon line is removed from that graph, it would also contribute to the neutrino mass. From naive dimensional arguments, one finds a relation between μ_ν and m_ν.

$$\mu_\nu \simeq \frac{m_\nu e}{M^2} , \tag{93}$$

where M is the mass of the heaviest particle circulating inside the loop. This should be charged, since it has to couple to the photon. For $\mu_\nu \geq 10^{-11}\mu_B$ and $m_\nu \leq 10\ eV$, one needs $M \leq 1\ GeV$. But we know experimentally that no charged bosons that couples to fermions exist in this mass range. If we choose the present lower limit on charged bosons ($\sim 30\ GeV$), the magnetic moment that can be attained is at most $\simeq 10^{-14}\mu_B$. Alternatively, a magnetic moment of $10^{-11}\mu_B$ would imply $m_\nu \simeq 10\ keV$.

In the presence of additional symmetries, it is possible that the naive dimensional arguments given above are no longer valid. I now turn to the discussion of such symmetries.

$SU(2)_\nu$ symmetry

In 1988, Voloshin made an important observation[40] which enables one to decouple the neutrino mass from the magnetic moment. Making use of the symmetry property of the neutrino mass operator, viz.,

$$\nu^T C \nu^c = +\nu^{c^T} C \nu , \tag{94}$$

where ν^c denotes the conjugate of the right-handed neutrino, and the anti-symmetry of the magnetic moment operator

$$\nu^T C \sigma_{\mu\nu} \nu^c = -\nu^{c^T} C \sigma_{\mu\nu} \nu \tag{95}$$

he observed that if (ν, ν^c) form a doublet of an $SU(2)_\nu$ symmetry, the anti-symmetric combination, being a singlet of this $SU(2)_\nu$, would be allowed, but the symmetric mass term, being a triplet, would be forbidden. That is, in the limit of exact $SU(2)_\nu$, neutrino mass is forbidden, while the magnetic moment is allowed. Thus there is a symmetry that decouples the mass and magnetic moment.

I should caution the reader that this argument works only in the limit of $SU(2)_\nu$ symmetry. But in nature, there is no shred of evidence for such a symmetry transforming ν to ν^c. In fact, ν transforms as a doublet of weak $SU(2)_L$, while ν^c is a singlet. So the new $SU(2)_\nu$ symmetry, it it exists, does not even commute with the standard model. If this symmetry is broken at a high scale, we are back to square one. Models which realize Voloshin symmetry was explored by Barbieri and Mohapatra, who pointed out several difficulties of implementing this idea.[41]

Horizontal $SU(2)_H$ symmetry

A simple way out of this dilemma was proposed in Ref. 42. There it was noted that rather than Dirac type magnetic moment, if one constructs transition magnetic moment, a simple symmetry, viz., the horizontal $SU(2)_H$ symmetry between the electron and muon generations, would forbid the neutrino mass, while allowing the magnetic moment. Since this horizontal symmetry is already an approximate symmetry of the standard model (it is only broken by the electron and muon masses), it c n easily be implemented in realistic models. The symmetry is broken by terms of order

$$\frac{m_e - m_\mu}{2M_W} \simeq 5 \times 10^{-4} . \tag{96}$$

This would provide an extra suppression factor for the neutrino mass relative to the magnetic moment. The naive estimate of $m_\nu \sim 10\ keV$, corresponding to $\mu_\nu \sim 10^{-11}\mu_B$ can be brought below $m_\nu \leq 10\ eV$.

Several models have been constructed based on the idea of $SU(2)_H$ symmetry for a large μ_ν.[42-50] Here I shall describe a minimal version,[41,42] which keeps the neutrino mass essentially zero, while leading to a large magnetic moment.

$SU(2)_H$ model of large neutrino magnetic moment

Consider the following $SU(2)_H$ symmetric extension of the standards model where the leptons are assigned under $SU(2)_L \times U(1)_Y \times SU(2)_H$ as follows[42,43]:

$$\begin{aligned}
\psi_L &= \begin{pmatrix} \nu_e & \nu_\mu \\ e & \mu \end{pmatrix}_L & &(2, -\tfrac{1}{2}, 2) \\
\psi_R &= (\, e \quad \mu\,)_R & &(1, -1, 2) \\
\psi_{3L} &= \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L & &(2, -\tfrac{1}{2}, 1) \\
&\tau_R & &(1, -1, 1)
\end{aligned} \tag{97}$$

The first two families form a doublet of $SU(2)_H$ while the τ family is a singlet. Quarks have standard weak interaction properties and are assumed to singlets under the horizontal $SU(2)_H$.

The horizontal $SU(2)_H$ may be either a global symmetry or a genuine gauge symmetry. The fermionic assignment given above is free of triangle anomalies, and does not require new fermions, so $SU(2)_H$ can be gauged. If $SU(2)_H$ is a global symmetry, it may be broken either spontaneously or softly. I shall not commit to either one while discussing the magnetic moment. When $SU(2)_H$ breaking effects are involved, for definiteness, I shall choose the local symmetry, which will be spontaneously broken.

The Higgs spectrum of the model consists of the following multiplets:

$$\phi_S = \begin{pmatrix} \phi_S^+ \\ \phi_S^0 \end{pmatrix} \quad (2, \tfrac{1}{2}, 1)$$

$$\Phi = \begin{pmatrix} \varphi_1^+ & \varphi_2^+ \\ \varphi_1^0 & \varphi_2^0 \end{pmatrix} \quad (2, \tfrac{1}{2}, 2)$$

$$\eta = (\, \eta_1^+ \quad \eta_2^+ \,) \quad (1, 1, 2) \tag{98}$$

The leptonic Yukawa couplings allowed by the symmetry can be written as

$$\begin{aligned} \mathcal{L}_Y^{(6)} &= h_1 \mathrm{Tr}\left(\overline{\psi}_L \phi_S \psi_R\right) + h_2 \overline{\psi}_{3L} \phi_S \tau_R + h_3 \overline{\psi}_{3L} \Phi i \tau_2 \psi_R^T + \\ & f \eta \tau_2 \psi_L^T \tau_2 C \psi_{3L} + f' \mathrm{Tr}\left(\overline{\psi}_L \Phi\right) \tau_R + H.C. \end{aligned} \tag{99}$$

The h_1 term gives equal mass for the electron and the muon. The h_2 term generates τ lepton mass. If $h_3 = 0$, τ lepton number will be a good symmetry of the Lagrangian. The terms f and f' are crucial for the generation of the magnetic moment of the neutrino.

The scalar potential contains, among other terms, the following term.

$$V = \mu \eta \Phi^\dagger i \tau_2 \phi_S^* + H.C. \tag{100}$$

Consider the limit of unbroken $SU(2)_H$ symmetry, but broken $SU(2)_L \times U(1)_Y$ via $\langle \phi_S^0 \rangle = \kappa_S$. The above term in the Higgs potential, Eq. (100), will lead to the mixing of η and φ fields.

$$V_{\eta-\varphi} = \mu \kappa_S \left(\eta_1^+ \varphi_1^- + \eta_2^+ \varphi_2^-\right) + H.C. \tag{101}$$

In the $SU(2)_H$ limit, (η_1^+, η_2^+) are degenerate, as are $(\varphi_1^+, \varphi_2^+)$. Hence the 2×2 mass-squared matrices in the (η_1^+, φ_1^+) and (η_2^+, φ_2^+) sectors will be identical.

Neutrino masses and magnetic moments will be generated via the graphs of Fig. 13. There are two such graphs, one with the $(\eta_1^+ - \varphi_1^+)$ exchange and the

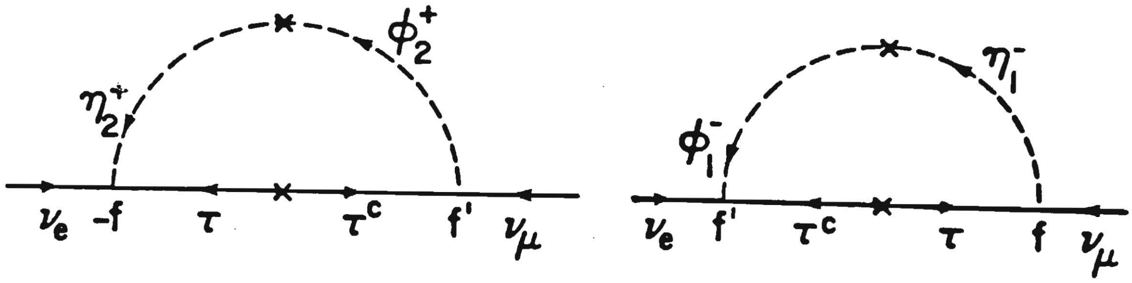

Fig. 13. Feynman diagrams generating neutrino masses in the $SU(2)_H$ model. The same graphs contribute to the magnetic moment if a photon line is attached to the internal lines.

other with $(\eta_2^+ - \varphi_2^+)$ exchange. Since the masses of the particles inside the loop are the same, the magnitudes of these two graphs are identical. However, they have a relative minus sign at one of the vertices. While computing the contribution to the neutrino mass, the two graphs, due to this relative minus sign, add to zero. For the magnetic moment, we have to attach a photon line on the internal loop. This brings in another relative minus sign (note the direction of charge flow is opposite for the two graphs). As a result, the two graphs add to give a finite magnetic moment:

$$\mu_\nu = \frac{ff'}{8\pi^2} m_\tau \sin 2\alpha \left[\frac{1}{m_1^2} \left\{ \ln \frac{m_1^2}{m_\tau^2} - 1 \right\} - \frac{1}{m_2^2} \left\{ \ln \frac{m_2^2}{m_\tau^2} - 1 \right\} \right] . \qquad (102)$$

Here (m_1^2, m_2^2, α) are the mass-squared eigen-values and the mixing angle between (φ_1^+, η_1^+), which is identical to that of (φ_2^+, η_2^+) in the limit of exact $SU(2)_H$. For reasonable values of the couplings and masses of the scalars, it is possible to have $\mu_\nu \approx (10^{-11} - 10^{-10}) \mu_B$.

Several comments are now in order.

(i) $SU(2)_H$ symmetry must break, otherwise e and μ will remain degenerate. Once $SU(2)_H$ breaks, the neutrino mass will not remain zero. This is because η_1^+ will not be degenerate with η_2^+ any longer (similarly $(\varphi_1^+, \varphi_2^+)$), so that the cancellation between the two graphs of Fig. 13 will not be exact.

(ii) In order for the spin-precession to be efficient in the solar convective zone, $\Delta m^2 \leq 10^{-7}\ eV^2$ is required. Once $m_\nu \neq 0$, one has to ensure that this limit is not violated. This can be satisfied by an unbroken $l_e - l_\mu$ symmetry. If this symmetry is exact, the 2×2 neutrino mass matrix will have the form

$$M_\nu = \begin{pmatrix} 0 & A \\ A & 0 \end{pmatrix} . \tag{103}$$

Such a matrix will lead to $\Delta m^2 = 0$. This type of unbroken leptonic symmetries were first considered by Zeldovich, Konopinski and Mahmoud (ZKM).[51] Such neutrinos are called ZKM neutrinos.

(iii) An immediate concern is whether the $l_e - l_\mu$ symmetry can be maintained if the $SU(2)_H$ is a local symmetry of the model. $l_e - l_\mu$ being the diagonal subgroup of $SU(2)_H$, if left unbroken, will lead to a massless gauge particle, in conflict with observations. Here I shall describe a way to keep $l_e - l_\mu$ and still break $SU(2)_H$ completely.[43]

Consider the breaking of $SU(2)_H$ symmetry by two triplets of Higgs bosons (σ_1, σ_2), which are assumed to be singlets under the standard group. One can now define a global $O(2)$ symmetry operating on (σ_1, σ_2). Clearly, the gauge interactions respect this global symmetry. Since the triplets have no couplings to the fermions, it is sufficient to ensure that this symmetry is respected by the scalar potential. Then the full symmetry of the Lagrangian is $SU(2)_H \times O(2)_G$. Once σ_1 and σ_2 acquire vev's, this symmetry will break to a global $O(2)_G$, which is a diagonal sum of the two parent symmetries. This residual global $O(2)_G$ will act on fermions as $l_e - l_\mu$. Note that $SU(2)_H$ is completely broken, so there is no new massless gauge particle. One interesting aspect of the gauge sector will then be that the horizontal gauge bosons V_3 and $V_\pm$ (which are electrically neutral) will not mix. (This arises since the global $O(2)$ imposed on the Higgs potential implies that only the first component of σ_1 and the second component of σ_2 can have vev's which are equal).

(iv) Once $SU(2)_H$ breaks, (η_1^+, η_2^+) will no longer be degenerate. This will lead to neutrino masses. If the mass splitting between η_1^+ and η_2^+ satisfies $\Delta m_\eta^2 \ll m_\eta^2$, neutrino mass will still be suppressed. This occurs in the present model since Δm_η^2 does not get any contribution from the large vev of σ. In order to break the $e - \mu$ mass degeneracy, one should introduce a field $\chi(2, 1/2, 3)$ into the spectrum. Its vev is assumed to be smaller than the standard vev. Δm_η^2 gets contribution proportional to the χ vev, which is small. The induced neutrino mass is

$$m_{\nu_e \nu_\mu} = \frac{ff'}{16\pi^2} m_\tau \sin 2\alpha \left(\frac{\delta m_2^2}{m_2^2} - \frac{\delta m_1^2}{m_1^2} \right) \tag{104}$$

For $m_{1,2} \approx 100\ GeV$, $m_\nu \leq 10\ eV$ and $\mu_\nu \geq 10^{-11}\mu_B$ can be satisfied provided $\delta m^2_{1,2} \leq 10\ GeV^2$. This can be naturally achieved in the model, as $\delta m^2_{1,2} \approx \lambda\kappa_t\kappa_S$, where λ is a quartic scalar coupling, κ_t and κ_S are the χ and ϕ_S vev's.

(v) ν_τ may or may not have a tiny mass in this model, depending on whether l_τ is imposed or not. $l_e - l_\mu$ conservation would prevent $\varphi^0_{1,2}$ from acquiring vev's.

Phenomenology of $SU(2)_H$ model

(i) The model generically predicts $m_\nu \simeq 1 - 10\ eV$. This is because the mass-splitting between the scalars cannot be made arbitrarily small. The natural suppression factor is $(m_e - m_\mu)/M_W \simeq 10^{-3}$.

(ii) The various charged scalars predicted by the model should be observable in the mass range of 100 GeV. The scalars cannot be much heavier than this, since that would make the magnetic moment too small. They will couple predominantly to leptons.

(iii) One obvious question is the constraints on the scale of horizontal symmetry breaking from flavor-changing rare processes. Due to the unbroken $l_e - l_\mu$ symmetry, rare processes such as $\mu \to 3e$, $\mu \to e\gamma$, μ^-+Nucleus $\to e^-$+Nucleus, $K_L \to \mu e$ and neutrino-less double beta decay are all forbidden in the model. So the scale of horizontal symmetry breaking cannot be constrained from these considerations.

(iv) There is a new contribution to the μ decay via the exchange of V_+ gauge

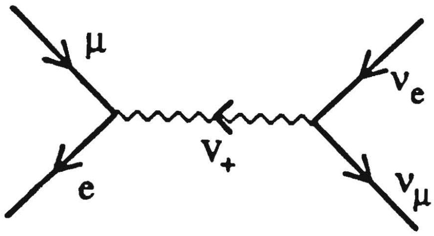

Fig. 14. Horizontal gauge boson contribution to μ decay.

bosons, shown in Fig. 14. The decay amplitude is now modified to[43]

$$A = \frac{G_F}{\sqrt{2}}\left[(1+\epsilon_\mu)\left\{\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\mu\right\} - 2\epsilon_\mu\left\{\bar{e}(1-\gamma_5)\nu_e\bar{\nu}_\mu(1+\gamma_5)\mu\right\}\right] \tag{105}$$

where

$$\epsilon_\mu = (g_H^2 M_W^2 / g^2 M_{V_\pm}^2) \ . \qquad (106)$$

This effective Hamiltonian leaves the $(V-A)$ prediction for the Michel parameter, $\rho = 3/4$. The asymmetry parameter η is now different, $\eta = -\epsilon_\mu$. This puts a bound on $\epsilon_\mu \leq 0.046$. A more stringent bound arises from demanding that G_β (G_F measured in β decay) be equal to G_μ to within 0.5%. This translates into $\epsilon_\mu \leq 0.25 \times 10^{-2}$. For $g_H = g$, this corresponds to a limit $M_{V_\pm} \geq 1.6\ TeV$.

(v) The process $e^+e^- \to \mu^+\mu^-$ receives new contribution from the V_3 and the $V_\pm$ gauge bosons via the diagram of Fig. 15. Consistency with forward-backward

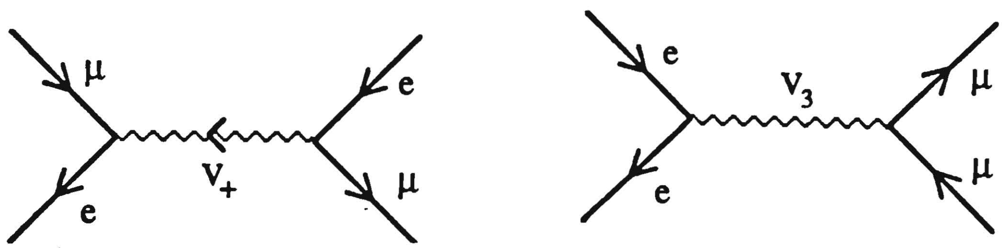

Fig. 15. $SU(2)_H$ gauge boson contribution to $e^+e^- \to \mu^+\mu^-$.

asymmetry measurements require $M_{V_3} \geq 600\ GeV$ (for $g_H = g$).

(vi) The scalars present in the model give rise to a new contribution in τ decay. The diagram is shown in Fig. 16. The amplitude is still of the $(V-A)$ form, but

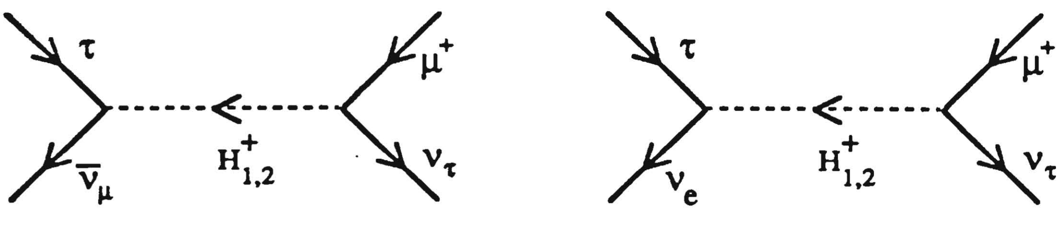

Fig. 16. Scalar contribution to τ decay in the $SU(2)_H$ model.

the strength is altered to $(1+\epsilon_\tau)$, where

$$\epsilon_\tau = \frac{f^2}{g^2}\left(\frac{\cos^2\alpha}{m_1^2} + \frac{\sin^2\alpha}{m_2^2}\right) M_W^2 \ . \tag{107}$$

Since the coupling f and the masses $m_{1,2}$ enter into the magnetic moment, they cannot be chosen arbitrarily. Demanding τ universality to within 5%, requires $f \leq 0.15$ for $m_{1,2} \sim 100\ GeV$.

(vi) The neutral components of the multiplet Φ, which are expected to be in the 100 GeV range (otherwise, the scalar contribution to electro-weak ρ parameter will be excessive) will contribute to the process $e^+e^- \longrightarrow \tau^+\tau^-$, and especially the forward-backward asymmetry. This puts a bound on $f' \leq 0.4$ for the neutral masses of 100 GeV.

It should be emphasized that all these experimental constraints are consistent with the parameters required for a large neutrino magnetic moment. Some or all of these processes should be accessible to future experiments.

Supersymmetric realization of $SU(2)_H$

Perhaps the simplest realization of the idea of $SU(2)_H$ symmetry is within the minimal supersymmetric standard model. A look at the scalar spectrum of the model described above, Eq. (98), reveals that they have the same gauge transformation properties as the fermionic spectrum, Eq. (97). This is suggestive of supersymmetry. In the following, I describe the SUSY generalization of the idea.[43]

Denote the chiral supermultiplets of the minimal standard SUSY model as $(L_e,\ L_\mu)$, L_τ, (e^c,μ^c), τ^c. The quark fields are denoted by Q, u^c, d^c. (The family indices on quarks will be suppressed.) Two Higgs superfields H_u and H_d are needed to give masses for up and down quarks. In the minimal version, R parity is assumed to be conserved to forbid rapid proton decay. This would also mean lepton number conservation, so that neutrino masses and magnetic moments are all zero. However, there is no reason to impose full R invariance to prevent proton decay. Baryon number conservation alone is sufficient for this purpose. Here I shall allow for explicit R violation. The most general superpotential, which respects $l_e - l_\mu$ symmetry and Baryon number is

$$\begin{aligned} W \ = \ & h_u Q H_u u^c + h_d Q h_d d^c + h'_d Q L_\tau d^c + f(L_e L_\tau e^c + L_\mu L_\tau \mu^c) + f' L_e L_\mu \tau^c + \\ & h_\tau L_\tau h_d \tau^c + h_\mu (L_e H_d e^c + L_\mu H_d \mu^c) + m_H H_u H_d + m'_H H_u L_\tau + \\ & \epsilon_1 (L_e H_d e^c - L_\mu H_d \mu^c) + \epsilon_2 (L_e L_\tau e^c - L_\mu L_\tau \mu^c) \ . \end{aligned} \tag{108}$$

I have written this in a suggestive form: the terms in the first two lines respect $SU(2)_H$ symmetry under which (L_e, L_μ) and (e^c, μ^c) form doublets, while the $\epsilon_{1,2}$

terms in the last line break it. The Yukawa couplings $\epsilon_{1,2}$ can be chosen to be small, still realistic e and μ masses can be generated. Once the neutral components of H_u and H_d acquire vev's, $\langle H_u^0 \rangle = v_u$, $\langle H_d^0 \rangle = v_d$, neutrino masses and magnetic moments will arise. $l_e - l_\mu$ symmetry prevents $\widetilde{\nu}_{e,\mu}$ from acquiring vev's. $\widetilde{\nu_\tau} = 0$ can be chosen without loss of generality by redefining L_τ and H_d fields. The situation here is very similar to the local $SU(2)_H$ symmetry discussed. One simply identifies (φ_1, φ_2) with $(\widetilde{L}_e, \widetilde{L}_\mu)$ and (η_1^+, η_2^+) with (e^c, μ^c). Most of the phenomenological discussions in the preceeding section will then apply. One difference however, is that ν_τ will necessarily have a tiny mass in the SUSY model. There is a tree-level contribution from m'_H of Eq. (108) which is see-saw suppressed by the gaugino masses, as well as a one-loop contribution to μ_{ν_τ}. This is because l_τ cannot be maintained as a global symmetry unlike in the previous model. The other difference is that here we are assuming global $SU(2)_H$, which means there are no gauge particles associated with it.

The lightest SUSY particle, which is generally assumed to be the photino $(\widetilde{\gamma})$ will not be stable in our scheme, due to R parity violation. It would decay via the diagram of Fig. 17 into the final states $\widetilde{\gamma} \rightarrow \mu^- \tau^+ \nu_e$, $e^- \tau^+ \nu_\mu$, $\mu^+ \tau^- \overline{\nu}_e$, $e^+ \tau^- \overline{\nu}_\mu$.

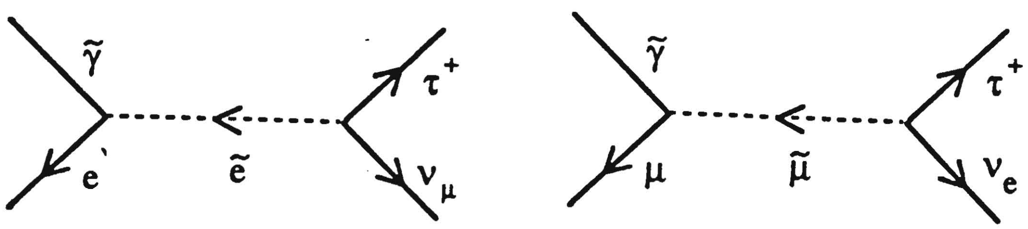

Fig. 17. Photino decay in the SUSY model with R parity violation.

The branching ratio to each of these four modes is 25%. The life-time of the photino, for $f' = 0.3$, is $\tau = 10^{-17} sec. \left(M_{\widetilde{\gamma}}/10\ GeV\right)^{-5}$. This means that GeV $\widetilde{\gamma}$ will decay within the detector, a signature drastically different from conventional SUSY.

Consider the pair production of photinos in e^+e^- collision. The cross section, in the limit of neglecting photino mass is

$$\sigma = \frac{\pi\alpha^2}{\sigma}\left[1 + \frac{p}{1+p} - 2p\ln\left(\frac{1+p}{p}\right)\right] \tag{109}$$

where $p = M_{\tilde{e}}^2/s$, s being the center of mass energy and $M_{\tilde{e}}$ the selectron mass. At $\sqrt{s} = 55\ GeV$, $M_{\tilde{e}} = 100\ GeV$, 4.4 photino pairs will be produced with an integrated luminosity of 10 pb^{-1}. The possible final states are $e^+e^-\tau^+\tau^-$, $e^\mp\tau^\pm e^\mp\tau^\pm$, $\mu^+\mu^-\tau^+\tau^-$, $\mu^\mp\tau^\pm\mu^\mp\tau^\pm$, $e^\mp\tau^\pm\mu^\mp\tau^\pm$ and $e^\mp\tau^\pm\mu^\pm\tau^\mp$ with missing energy associated with the neutrinos. These decay modes, especially the ones with same-sign dilepton, should provide spectacular signatures with essentially no standard model background.

Discrete subgroups of $SU(2)_H$

The full $SU(2)_H$ symmetry is not necessary to forbid the neutrino mass, while admitting a magnetic moment. Discrete subgroups of $SU(2)_H$ can also do the job.

Consider the following symmetry transformation[43-45]:

$$D: \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} . \tag{110}$$

If this symmetry D is unbroken, under D

$$\nu_e^T C \nu_\mu \rightarrow -\nu_\mu^T C \nu_e . \tag{111}$$

But from Fermi statistics, the mass term should be symmetric under $\nu_e \leftrightarrow \nu_\mu$, which means that $\nu_e \nu_\mu$ mass is forbidden by D. As for the magnetic moment,

$$\nu_e^T C \sigma_{\mu\nu} \nu_\mu = -\nu_\mu^T C \sigma_{\mu\nu} \nu_e \tag{112}$$

which is consistent with Fermi statistics. Hence D symmetry permits a magnetic moment while forbidding the off-diagonal mass. D by itself would allow diagonal masses $(\nu_e^T C \nu_e + \nu_\mu^T C \nu_\mu)$. If $l_e - l_\mu$ is imposed, this mass also would be zero.

Observe that the two symmetries D and $l_e - l_\mu$ do not commute. Hence the full global symmetry of the model would be larger. The simplest symmetry group which realizes this idea has the following eight elements: $[\pm 1, \pm i\tau_1, \pm i\tau_2, \pm i\tau_3]$. This is the quarternionic group Q, which has the property that it contains a doublet representation and the singlet obtained from $2 \times 2 \rightarrow 1$ is the anti-symmetric combination.[45]

If $l_e - l_\mu$ is a conserved quantum number, ν_e and ν_μ^c would pair up to form a ZKM type Dirac neutrino. This means that $\Delta m^2 = 0$ in this case. Sometimes it is desirable to have $\Delta m^2 \neq 0$, so that there is energy dependence in the ν_e survival probability. Realistic models which generate a tiny mass-splitting can also be constructed. The interesting mass-squared difference of $\Delta m^2 = 10^{-8}\ eV^2 - 10^{-4}\ eV^2$, where matter effects assist spin-flavor conversion, can be generated in this case. One model which has been constructed has the $SU(2)_H$ symmetry acting on leptons as well as quarks.[50] There is no need to include exotic fermions in the model. Here μ_ν arises at the one-loop and $m_{\nu_e\nu_\mu}$ at the two-loop level. So the mass is naturally suppressed relative the magnetic moment. The diagonal masses $\nu_e^T C\nu_e$ and $\nu_\mu^T C\nu_\mu$ do not arise until the fifth loop level. $\Delta m^2 \approx 10^{-7}\ eV^2$ can thus naturally be realized.

Large magnetic moment from spin symmetry

Recently a very interesting observation has been made by Barr, Friere and Zee[52] for a large magnetic moment and small mass of the neutrino. Their mechanism does not make explicit use of the horizontal symmetry. Rather, it is based on a spin-symmetry argument.

In gauge theories, there is no direct γWS coupling, where S is a physical charged scalar. Such couplings, however, can be induced at the loop level. The Barr-Freire-Zee mechanism makes use of the symmetry properties of this induced vertex. Such a vertex will contribute to μ_ν. Consider now the mass generated by the same graph with the photon line removed. It is well known that for transversely polarized vector bosons, the transition from spin 1 to spin zero cannot occur. Only the longitudinal mode (the Goldstone mode) can then contribute to the neutrino mass via such a vertex. Such couplings are however, suppressed by the fermion masses. This will provide a natural suppression of m_ν relative to μ_ν.

The diagrams of Fig. 18 should illustrate these points. Fig. 18 (a) is the

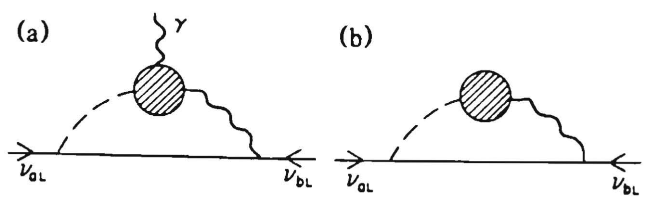

Fig. 18. Mechanism of Ref. 52 generating large neutrino magnetic moment and small neutrino mass.

contribution to the neutrino magnetic moment from the vertex γWS. It is necessarily a two-loop contribution. Fig. 18 (b) is the same graph with the photon line removed. This should contribute to the neutrino mass. The naive dimensional arguments have to be modified in this case.

Let us evaluate the mass contribution via the spin 0–spin 1 transition blob in the Landau gauge. The blob itself should be proportional to the four-momentum

k_μ of the scalar. Then one is left with evaluating

$$k^\mu D_{\mu\nu}\gamma_\nu = k^\mu \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \gamma_\nu$$
$$= k.\gamma - k.\gamma = 0 \; . \qquad (113)$$

In the Landau gauge, one should also evaluate the contribution from the unphysical scalars. However, their couplings to the fermions are proportional to the internal fermion mass. Chirality arguments require that the fermion mass should appear twice. Thus the naive argument that

$$\mu_\nu \approx \frac{m_\nu e}{M^2} \qquad (114)$$

where M is the largest mass running inside the loop is modified to

$$\mu_\nu \approx \frac{m_\nu e}{m_f^2} \qquad (115)$$

where m_f is the mass of the light fermion inside the loop, and not that of the heaviest particle. In other words, the neutrino magnetic moment is enhanced by a factor M^2/m_f^2 relative to the mass (compared to the naive dimensional argument).

This can yield large enough magnetic moment while keeping m_ν within experimental limits.

Zee model revisited: case for a large μ_ν

As an explicit realization of the spin symmetry argument that generates a large magnetic moment, let me reanalyze the Zee model of neutrino masses discussed in Section 2. (The model of Ref. 52 is similar, but differs slightly from the Zee model of Section 2.) The model contains two Higgs doublets (ϕ and ϕ') and a charged

Higgs singlet h^+. No ν_R is introduced. So the only allowed magnetic moment is the transition type. As in usual two Higgs doublet models, to suppress tree level flavor changing neutral currents, a discrete symmetry is imposed so that only ϕ couples to the fermions. For simplicity, let us also assume two generations of fermions. That is the tau family is decoupled. The Yukawa couplings of the scalar h^+ are then

$$\mathcal{L}_Y^{(7)} = f\,(\overline{\nu^c}_e \mu_L - \overline{\nu^c}_\mu e_L)\, h^+ + H.C. \qquad (116)$$

The Higgs potential contains a term

$$V = \left(\mu \phi^T i\tau_2 \phi' h^- + H.C. \right) \qquad (117)$$

Note that such a term is allowed by the discrete symmetry, provided both h^+, ϕ' and ψ_L transform under it. Upon symmetry breaking, h^+ will mix with ϕ and ϕ' This would generate a neutrino mass connecting ν_e and ν_μ. The diagram is shown in Fig. 1 (of Section 2). Its magnitude can be estimated to be

$$m_\nu \simeq \frac{fg}{16\pi^2}\sin\alpha \frac{m_e^2 - m_\mu^2}{M_W} \ln\left(\frac{M_h^2}{M_\phi^2}\right) , \tag{118}$$

where $\sin\alpha$ is the $h - \phi$ mixing angle. To be consistent with $m_\nu \leq 10\ eV$, we have to choose $f\sin\alpha \leq 0.02$. (M_h and m_ϕ are assumed to be of the same order.) The same graph also would generate a magnetic moment. But this is of order

$$\mu_\nu \simeq \frac{m_\nu e}{M_h^2} \leq 10^{-14}\mu_B . \tag{119}$$

At the two loop level the model predicts a large magnetic moment of the neutrino via the spin symmetry mechanism. To see this, let us neglect all the fermion masses, except the top mass. The smallness of the fermion masses makes this an excellent approximation. The model then has an $SU(2)_H$ symmetry. The neutrino mass is thus forbidden on symmetry grounds. The magnetic moment is allowed. At the one loop, there is no graph which generates μ_ν. The lowest order diagram is the two-loop graph of Fig. 18 which is the spin symmetry graph. Here, the h^+ can first mix with ϕ^+ via the μ term of Eq. 117. Then ϕ^+ has couplings to the top-quark, which is of order one. Indeed, the naive estimate of the magnetic moment is

$$\mu_\nu \simeq \frac{fg}{8\pi^2} e \sin\alpha \frac{gh_t}{16\pi^2}\frac{1}{M_W} . \tag{120}$$

Here h_t is the top quark Yukawa coupling, which is of order 1. Even with the constraint $f\sin\alpha \leq 0.02$, a magnetic moment as large as $10^{-11}\mu_B$ seems to be achievable. The mass arising from this graph is of order $10^{-3}\ eV$, so that the one-loop mass of Eq. (118) would dominate.

Time dependence of the Cl and K-II data

As discussed earlier, the Chlorine experiment of Davis and collaborators sees a clear time variation of the neutrino flux in anti-correlation with the sun spots. The K-II experiment now has collected data for sufficiently long time to look for possible anti-correlation with the solar activity. Their finding is that there was no substantial change in the flux between the solar minimum and maximum (see Fig.

19). This observation adds to the complexity of the solar puzzle. In the magnetic

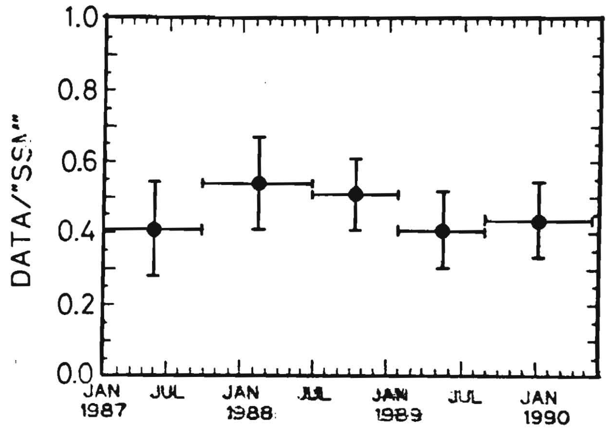

Fig. 19. Kamiokande II solar neutrino data (from Ref. 8) as a function of time.

moment scenario, it is possible to explain these differences. At least three ideas have been proposed to reconcile the time dependence. Let me briefly discuss these proposals.

A large Dirac neutrino magnetic moment

Suzuki et. al., in Ref. 53 explored the possibility that a large magnetic moment of the neutrino itself is responsible for the difference in time variation. The main point of this approach is that if the neutrino has a rather large Dirac magnetic moment, in the water detector, $\nu_R e \rightarrow \nu_L e$ could occur via the magnetic moment interaction. This process is not available for the Cl experiment. It would mean that even if part of ν_L converts into ν_R inside the solar convective field, the converted ν_R will not go completely undetected in the water detector. ν_R will however, be sterile with respect to the Cl detector. The magnetic moment will provide a constant background for K-II, regardless of what flavor of neutrino ν_L or ν_R arrives the detector. The net result is to have less of a time variation in K-II. A combined numerical fit was done in Ref. x. It was shown that if the magnetic moment is in the range $(4 - 7.9) \times 10^{-10}\mu_B$, the time variation in Cl and the apparent lack of variation in K-II can be explained. Such a large value of μ_ν is most probably ruled out by direct experimental limits. It certainly will run into trouble with cosmology and astrophysics. Nevertheless, this is a possibility, which seems to be marginally allowed, and should be confronted in future experiments.

Exotic ν_R interactions

In Section 3, a Higgs model of Dirac neutrino magnetic moment was discussed. Although there was difficulty in explaining the smallness of the neutrino mass, this model would predict new interactions of the ν_R. These new interaction could contribute to $\nu_R e$ scattering in K-II detector. The lack of time variation in K-II may be explained this way. Note that if ν_R interacts with the leptons (and not nucleons), the new interactions will not contribute to the Cl signal. So the time variation in Cl will be expected. Unlike in the previous scenario, exotic interactions will not give a constant signal in K-II, since only ν_R can have such interactions.

The Higgs model for neutrino magnetic moment discussed in Section 3 has been recently revived by Fukugita and Yanagida.[54] Since very little is known about ν_R, its coupling to the electron via the charged Higgs could be stronger than weak interaction. In the variant proposed in Ref. 54, instead of a singlet η^+, the authors prefer a second doublet ϕ which does not acquire a vev. Both the standard doublet and ϕ couple to leptons. (To suppress FCNC in the quark sector, one might use a third doublet that couples to quarks only.) The interaction Lagrangian of the model is

$$\mathcal{L}_Y^{(8)} = f_{ij}\bar{e}_{Ri}l_{Lj}\phi^* + g_{ij}\bar{\nu}_{Ri}l_{Lj}\phi + H.C. \tag{121}$$

Since ϕ is assumed to have zero vev, the couplings g_{ij} are not proportional to the neutrino mass, so they can be of order 1. As in the model of Section 3, large magnetic moment is possible, but the neutrino mass has to be fine-tuned.

Choosing a doublet of Higgs rather than the singlet has one advantage. The effective potential seen by ν_R inside the sun is now

$$V = +g_{11}^2 N_e/4M_\phi^2 \tag{122}$$

which is positive. (It would be negative if the interaction is mediated by η^+.) Then the matter suppression of the spin-flip transition via weak interaction can be cancelled by V of Eq. (122). The effective Hamiltonian is

$$H_{\text{eff}} = \begin{bmatrix} \sqrt{2}G_F(N_e - N_N/2) & \mu B \\ \mu B & g_{11}^2 N_e/4M_\phi^2 \end{bmatrix}. \tag{123}$$

The same effective potential V will contribute to $\nu_R e$ scattering in K-II. This would enhance the signal in K-II, even if ν_L is converted to ν_R. Thus the time variation in Cl and lack of time dependence in K-II can be explained. The price to pay of course is (i) no explanation of smallness of neutrino mass and (ii) seems to be inconsistent with nucleosynthesis and supernova constraints. Gallium experiment should see time variation in this scheme.

Combined matter and magnetic moment effects

Neutrino spin precession inside the sun can also be assisted by matter effects.[31,55] If $\Delta m^2 \neq 0$, energy dependence enters into the ν_e survival probability. Once energy dependence of the neutrino is brought in, since the K-II is sensitive to neutrinos above 7.5 MeV, while Cl sees all neutrinos down to 0.81 MeV, it is possible to explain the time dependence of both experiments. The idea is that low energy neutrinos are converted to another flavor, while the high energy ones are not. Moreover, in the transition magnetic moment picture advocated here, the converted ν_μ has a 15% cross section via neutral currents in K-II, which helps to weaken the time dependence in K-II. Recently a detailed numerical analysis was carried out using the parameters of the standard solar model and reasonable values of the magnetic field.[56] The result showed that a combined fit of the time variation is possible for

$\mu_\nu \approx 10^{-11}\mu_B$ and $\Delta m^2 \approx 10^{-8}\ eV^2$. Gallium experiment should show strong anti-correlation with the sun spots in this scenario.

5. Conclusions

The aim of these lectures was to give a flavor of the complexities of the problems of neutrino mass, magnetic moment etc. and their relevance to the solar neutrino puzzle. I hope this has given you sufficient motivation to think more along these lines. There are strong indications for non–standard neutrino properties. Perhaps the best one comes from the solar neutrino experiments. If the apparent time-variation in the Chlorine experiment is to be explained, the only consistent scenario is by giving a magnetic moment to the neutrino. This has the potential theoretical problem of generating too large a neutrino mass. The horizontal $SU(2)_H$ symmetry introduced enables one to decouple the mass from the magnetic moment. Realizations of this idea has definite predictions which can be tested in accelerator experiments as well as in on-going and future solar neutrino experiments. This include discovery of nearly degenerate charged and neutral Higgs particles with the mass around 100 GeV, deviations from universality in μ and τ decay etc. The scalar spectrum suggested by $SU(2)_H$ symmetry strongly suggests the existence of supersymmetry. In this case, R symmetry has to be broken, which would provide new signatures of discovering supersymmetry. The alternative to $SU(2)_H$ symmetry is the spin symmetry argument. This idea also has rich phenomenological structure. Reconciling the time dependence of Cl and K-II experiments is a more challenging issue. In the neutrino magnetic moment picture, it can be explained by taking into account the matter effects and energy dependence of the conversion probability. Future accelerator experiments and solar neutrino experiments should tell us more about all these. Let us wait and see.

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