A Logical Analysis of Meaning

by

Susanne Knauth Langer

A Dissertation Presented at Radcliffe College
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
in the Department of Philosophy

1926

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Donald Dryden
February 8, 2021

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A logical Analysis of Meaning

presented by Susanne Knauth Langer

candidate for the degree of Doctor of Philosophy,
and hereby certify that it is worthy of acceptance.

C. I. Lewis
A. N. Whitehead

Henry M. Sheffer

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(Abstract)

The controversies which have arisen in recent years over the "meaning of meaning" have failed so far to yield either agreement among the disputants, or even well-defined opposed positions. The several points of view are so completely disparate that they do not offer any common ground, even as a mere starting point for argument.

The subject of Meaning has so many different aspects, and ~~in~~ its several perspectives look so utterly various, that one may be led to wonder whether they are really perspectives of one and the same thing. Meaning in the "dictionary sense" seems indeed to have nothing in common with the meaning of art or religion. There appear to be any number of perfectly discrete meanings of Meaning, dependent upon the context wherein our definition is made, -- whether in practical life, in psychology, or in semantics, or with an eye to some metaphysic of value. It is certainly not easy to discover the locus where all these perspectives meet. Yet the fact that the one word Meaning covers this div_ersity of instances points to the fact that there is such a common element.

The only method whereby such a universal factor can be abstracted from the multitude of particular cases is the <u>method of logical analysis</u>. Thereby we shall not give an exhaustive rendering of any one phase of Meaning, but a generalized, therefore non-exhaustive, account which (if it is correct) is perfectly true of <u>every</u> phase. In order to justify all the approved uses of the word Meaning we must be able to show that some essential charac-

ter is always involved as the basis for such usage.

This essay attempts to show that the common element is the _logical form_ of all _meaning-situations,_ and that the allogical or "interpretational" elements which enter into every particular case are responsible for its individual character. To this purpose I have drawn the distinction between _abstraction_ and _interpretation_ (Ch 3). The common form is often so deeply obscured by the non-logical factors that it has been necessary to introduce the highest possible degrees of abstraction, going somewhat beyond the familiar use of variables and values which have hitherto served the needs of logic and mathematics.

The course of this analysis has sent me seeking for a logic more general even than the formal treatment of _propositions,_ exemplified in Principia Mathematica. In defense of this "spectral dance of bloodless categories", I have interpolated the chapter on "propositional logic" (Ch. 5). The method here advocated is not the study of related propositions or propostional functions, but the _analysis of systems,_ disregarding entirely the nature of their elements.

The forms which appear, at the present stage of analysis, to be fundamental to every system have been tentatively denoted as "logical data". Between these and the concrete system lie any number of possible levels of abstraction, every level giving rise to a logical form or in "configuration".

The logical basis of Meaning is here defined as correspondence of configuration between systems. Some systems show a

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similarity of logical structure upon comparatively superficial analysis; these,--such as three counters representing three objects,--are said to stand in a "natural"meaning-relation. In other cases the analysis must be pressed to further levels of abstraction. It stands to reason that, if there are logical data common to all systems, there must always be some level where a correspondence can be established.

The meaning-relation, commonly held (as by Husserl) to be assymetrical,is here found to be ~~convertible~~ symetrical. The fundamental types of Meaning are presented as denotation, connotation, signification, and description, and their respective converses. These,in all possible combinations, appear to exhaust the field of Meaning, from the dictionary to the sacred mysteries.

A phenomenology of Meaning, originally compiled by Messrs. Ogden and Richards (in "The Meaning of Meaning), has been used to check up the results of this analysis, and to demonstrate how the most various and discrepant uses of the word coincide in the single point at issue,the logical basis which is correspondence of configurations.

TABLE OF CONTENTS.

PREFACE

The following pages contain an attempt to show (1) that meaning is a fundamental element in logic, even of the non-epistemological sort; (2) that a logical analysis of meaning entails a logical analysis of <u>systems</u>; (3) that the <u>logical condition for meaning is the presence of any structure whatever</u>; and (4) that the logical framework is similar for all meaning-situations, from the word that "means" a table or a chair, to the Moral Ego that "means" the self-realization of the Absolute.

This <u>analysis of meaning is, essentially, a philosophical application of the purely formal work done in recent years by Dr. Sheffer</u>, which is at present recorded only in a brief unpublished paper. If I have not acknowledged my debt to him in every instance, it is because of the dearth of printed passages for reference.

The logical foundation of meaning is a point whereupon there is practically no literature. <u>Kempe, Royce and Sheffer</u>--none of them voluminous writers--are its casual discoverers. But there is a vast literature just beside the point, which sometimes throws valuable lights in our direction. Hence the motley appearance of my bibliography, which takes in such diverse material as Couturat's "The Algebra of Logic" and Ogden's and Richards' "The Meaning of Meaning." For my purpose I have

had to avail myself of those works which deal with meaning, in its logical or non-logical aspects, and also of the literature of modern logistic.

The great emphasis which I have given to the question of the nature of logic needs, perhaps, some justification. I have had unusual opportunity to discuss the question of logical systems with Prof. Whitehead, who has taken a very kindly interest in this bit of research, yet to my surprise, I have found the logic of Principia inapplicable to the analysis in hand. In view of the fact that our greatest classic in modern logic is his and Mr. Russell's system of "propositional" logic, I may be pardoned for investigating at such length the reasons why an analysis of meaning could not be based upon it. I have come to the conclusion that the universe of possible meaning-situations includes the meaning of propositions, exactly as the universe of possible structures includes deductive systems.

The results of the following study, once they are expressed, are almost too obvious to carry with them any appearance of novelty. But if they have been generally known to philosophers who have argued the problem of meaning, it is remarkable that they have not previously been employed to supply a common footing in the general and hopeless confusion of standpoints. To point out a unifying principle in virtue of which "Meaning,"--the word of many meanings,--may be legitimately applied in all its diverse instances, has been the whole object of this somewhat tedious analysis.

I   INTRODUCTORY PART

Chapter 1.

Metaphysical Riddles and Logistic Solutions.

Anyone who has ever tried to establish precisely the philosophical concept of meaning, can appreciate the complicated muddle to which attempts at definition have led. Psychologists, metaphysicians, and epistemologists have been rampant on this subject, but in reading over the literature which has resulted from the rampage, one is struck by the futility of the discussion--in fact, such a symposium as that which appeared several years ago in Mind, can hardly be called a discussion (1). We cannot say that people are "discussing" anything when each holds forth on a separate subject. They are merely orating simultaneously. And this is the impression one is apt to gain from the above symposium. Prof. Schiller tells us how the relation of signs to their objects is established; Mr. Russell tells us how a sign will function in a causal system; Prof. Joachim contributes a burlesque of the previous proceedings, and tries to tell us what it feels like to mean anything. Now, none of these gentlemen has actually contradicted either of his opponents on any point, in the triangular debate; one has taken the psychological, another the empirical, the third the aesthetic aspect of

(1) Symposium--"The Meaning of Meaning", by F. C. S. Schiller, B. Russell, H. Joachim. "Mind" N.S. Vol. 29, pp. 385-414; Vol. 30, pp. 185-190, 444-447. (See also--C. A. Strong, "The Meaning of Meaning," Vol. 30, pp. 313-316; A. Sidgwick, "Statements and Meaning," Ibid., pp. 271-286; and B. Russell, "On Propositions; what they are and how they mean," Proc. Arist. Soc. Supl. Vol. II "Problems of Science and Phil." (pp. 1-43).

the subject, and each appears to be somewhat peeved that his view has not been considered by the others, and feels in a vague way that he has been pooh-poohed, if not contradicted. There is such a complete lack of common ground that the discussion can find no starting place.

One of the chief difficulties lies in the fact that all these authors tacitly assume the word "meaning" to have one and only one legitimate application. This appears in the title of the symposium--"The Meaning of Meaning." We find this somewhat risky assumption recurring in a book of the same title (2), which appeared subsequently and adds a little more psychology and some ethnology and philology to the material in hand (3). Strange to say, Messrs. Ogden and Richards cling to the naive assumption that there can be but one correct use of the word, although they are quite aware of the fact that it is actually used in the most various ways. Unfortunately their list of these ways (4) confuses uses of the word and theories about it; but barring the theories (such as: that it is a relation, that it is a character inherent in words, etc.), we may gather from their account that "meaning" is used in the sense of connotation,

(2) C. F. Ogden and I. A. Richards, "The Meaning of Meaning," London, 1923.

(3) It might, incidentally, be remarked that the meaning of "Meaning" is rarely the subject of debate; in the papers by Profs. Schiller and Joachim, as also in the above-mentioned book, it is rather the origins of certain meanings which are under discussion.

(4) cf. loc. cit. p. 306

denotation, value, consequence, and other apparently disconnect-
ed notions. This whole Babel of synonyms, taken from the works
of philosophers, psychologists (5), propagandists and hackwriters
(6), is disparagingly labelled "the meaning of Philosophers."
(This slight confusion between philosophers who try to define
the term and other writers who merely employ it has led to the
ambiguity in the list referred to above.) But, although an in-
ventory of all possible uses of "meaning" by all sorts of users
is not properly described as "The Meaning of Philosophers," it
may well be taken as material for the philosophical analysis of
"Meaning" in the broadest sense. Prof. Hoernlé (7) has called
our attention to the need of a "patient and exhaustive phenome-
nology" in this storm-ravaged field. A few writers, notably
Frege (8), Pierce (9), Lady Welby (10), Husserl (11), and Meinong
(12)

          (5). cf. The long excerpts from the works of H. Muen-
sterberg, W. McDougall, J. J. Putnam, E. B. Titchener and J. M.
Baldwin, who are all primarily if not exclusively psychologists.

          (6) cf. The footnotes on pp. 296, 300, and 301.

          (7) R. F. A. Hoernlé, "A Plea for a Phenomenology of
Meaning," Proc. of the Arist. Soc. N. S. Vol. XXI, pp. 71-89.

          (8) Notably--"Grundgesetze der Arithmetik," "Über
Sinn. und Bedeutung," "Über Begriff and Gegenstand," "Funktion
und Begriff," "Begriffsschrift."

          (9) Articles in Baldwin's "Dictionary of Philosophy
Psychology."

          (10) "What is meaning?" "Sense, Meaning and Interpre-
tation." (Mind, N. S. Vol. 5) Article, "Signifies" in Baldwin's
"Dictionary of Phil. and Psych."

          (11) "Logische Untersuchungen."

          (12) "Über Annahmen."

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have made painstaking efforts to divide the sheep from the
goats; but the result has not been very happy. For the obstrep-
erous flock, instead of dividing neatly into two or three clas-
ses each according to its kind, have divided into the most ter-
rifying hierarchy of icons, qualisigns, legisigns, semes, phemes,
and delomes and there is little comfort in Pierce's joyous dis-
covery that instead of 59,049 such monstrosities we need only to
deal with sixty-six (13).

Husserl, distinguishing each type of "meaning" as a
separate notion, has as many theories as he has specific "mean-
ings." (14). But this has brought us no nearer to our goal,
which is the definition of meaning in the most general sense.
We have still the sheep and the goats and all their several
relatives; what we are seeking is a generalized form, an Urtier,
which shall exhibit the essential traits of all the various
species. For even if we elaborate the phenomenology indefinite-
ly, and accounted separately for each new kind of "meaning," we
could never be sure that the inventory was exhaustive (15). The
phenomenological treatment is always inductive; and we should
have no criterion of a perfect induction.

(13) From two unpublished letters to Lady Welby, 1904
and 1908 respectively, quoted by Ogden and Richards, "The Mean-
ing of Meaning," Appendix D,§ 6, pp. 435-444.

(14) E. Husserl, "Logische Untersuchungen," Vol. II,
Chs. 1 and 3: Vol. III, Ch. 3

(15) cf. Whitehead and Russell, Principia Mathematica Vol. I
(2d Ed.) p. XV: "In practice, generality is not obtained by
the method of complete enumeration, because this method requires
more knowledge than we possess."

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Phenomenologically, we are no more likely to discover this generalized form of "meaning" than to find Goethe's _Urtier_ in the Zoo; as one of the special kinds, it does not exist. Herein lies the principal error of philosophers, the error we are most anxious to avoid-taking _one_ kind of meaning as the fundamental or only legitimate one. What we are looking for is the _formal possibility of meaning_ which must attach even to the most abstract mathematical symbolism (16). Words, phrases, gestures, images, etc. may have any amount and sort (or sorts) of actual meaning that satisfies the formal condition; but it is interesting to note what is really given in a so-called "mathematics without meaning," or "string of meaningless marks"; the word "meaningless" here refers to the fact that _none of the specific kinds of meaning_ is in question, whereas any specific kind may be read into the symbolism (17). And it is due to this

(16) R. F. A. Hoernlé, loc. cit., p. 77: "But is it not an exaggeration to say, or imply that for the cognitive interest of the mathematician the signs are just meaningless marks? If not, what of the variable marks and of their mathematical concepts? The symbols eliminate irrelevant diversity but retain a minimum of meaning defined by the rules of operation." "Minimum" is not quite the right word; it is not a matter of _degree_ of meaning, but of _condition for it._

(17) This fact has been recognized, though in a very different context, by Prof. J. M. Baldwin; it is probably due to his psychological theories, or to his exceedingly utilitarian evaluation of the sciences, that he regards this kind of abstractness as belonging essentially to "play."
cf. "Thought and Things, or Genetic Logic," Vol. I, p. 165: "The play object does not mean an individual in the same sense that a memory object proper does. It is--to use a term now made familiar--an "experimental" object. It is held and controlled with the express psychic proviso, or reservation that its meaning is yet to be made up. It is constructed, but not assigned...the and so individuation, therefore....holds it as an object fit for alterna- stand tive meanings. It is this construction, essentially characteristic for, of the play-mode, and of the higher semblant of art-consciousness, that I propose to call Schema."

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peculiar situation that none of the meanings of meaning phenom-
enologically discovered has been the "meaning" of pure logic.

Just as all our thinking, or for that matter, all ex-
perience, is logical, so we may say that all our experience is
meaningful. The fact seems to be that meaning in its profound-
est sense is one of the fundamental notions of logic. The only
way we can express logical facts is through the employment of
symbols; and it is the formal condition for meaning that makes
the distinction between a mark on paper which is a symbol and
one which is due to spilled ink or the murder of a mosquito.
Whitehead and Russell would have us believe that such a symbol
as "$\frac{d}{dx}$" being an "incomplete symbol," i. e. not expressive of a
true or false assertion is "meaningless"(18); yet we feel it
makes a considerable difference whether they write "$\frac{d}{dx}$" or "$\Lambda\Lambda\Lambda$".
The latter is meaningless in a much fuller sense, namely that
we do not recognize it as a symbol at all. It may be shown,
even without recourse to a theory of types, that certain symbols
such as $\phi[\phi\hat{x}]$ express nonsense; but it could not be shown in
the symbolism of Principia that $\Lambda\Lambda\Lambda$ expresses nonsense, because
(always within the symbolism) $\Lambda\Lambda\Lambda$ does not express.

The fact that meaning is one of the fundamental notions
of logic, together with the general haziness that characterizes
the concept in philosophical thought, has led to a peculiar and

(18) cf. "Principia Mathematica," pp. 69-71.

unexpected strain of mysticism in the most highly rationalized
systems of thought, the "scientific" philosophy based upon
mathematical logic. Mr. Russell generally uses the word "sig-
nificance" in place of our "meaning," because the latter had
been reserved as a synonym for denotation (19) (and is employed
as the English equivalent of Frege's "Sinn," which rather con-
fuses the issue (20)). Now, significance attaches to anything
that is true or false, and to such things only (21). But the
logical principles involving truth and falsehood defy symbolic
expression, and mark "The essential limitations of formalism."
(22) It is here that the essential mysticism comes to light;
truth and falsehood, i. e. significance, cannot be symbolized
but must be taken as a sort of logical surd, an allogical factor
in every system (23). This mysticism finds its heroic expres-
sion in the pages of Ludwig Wittgenstein, to which Mr. Russell

(19) Principles of Mathematics, 47: "Even among concepts.
It is only those that denote that have meaning."

(20) cf. "Princ. of Math." App. A, "The logical and
arithmetical doctrines of Frege," p. 502: "The distinction
between meaning (Sinn) and indication (Bedeutung) is roughly,
though not exactly, equivalent to my distinction between con-
cept as such and what the concept denotes."

(21) "Princ. Math." p. 48: "Here X...may have any value
with which ('X is a man' implies 'X is a mortal') is significant,
i. e. either true or false."

(22) This surrender to the supposed intangibility of
meaning is found also in Husserl's "Logische Untersuchungen,"
(Vol. II, p. 183): "Was 'Bedeutung' ist, das kann uns so un-
mittelbar gegeben sein, wie uns gegeben ist, was Farbe und,
Ton ist. Es lässt sich nicht weiter definieren, es ist ein
deskriptiv Letztes." cf. Chapter 9.

(23) B. Russell, "Princ. of Math.", p. 16

in the main assents (24). Here meaning is taken as something essentially inexpressible, truth as the Unknowable, and all philosophizing becomes a Hunting of the Snark (25).

Thus the problem of meaning in the broadest sense merges into the problem of the fundamental notions of every formal logic. What are the indispensible assumptions of all logic? Can they be expressed at all, or are they insoluble Wittgensteinian mysteries? These are the questions we must answer before we shall be able to catch our Urtier, the "possibility of meaning" that attaches to all experience because it is contained in every symbolism (26). Therefore I shall not

(24) B. Russell, Introduction to "Tractatus Logico Philosophicus," by L. Wittgenstein, p. 23: "To have constructed a theory logic of which is not at any point obviously wrong is to have achieved a work of extraordinary difficulty and importance. This merit, in my opinion, belongs to Mr. Wittgenstein's work."

(25) Wittgenstein, "Tractatus Log. Phil.": "6.1 The propositions of logic are tautologies.
6.13 Logic is not a theory but a reflexion of the world. Logic is transcendental.
6.41 The sense of the world must lie outside the world.
6.522 There is indeed the inexpressible. This shows itself; it is the mystical.
6.523 The right method of philosophy would be this: To say nothing except what can be said, i. e. the propositions of natural science, i. e. something that has nothing to do with philosophy, and then always when someone else wished to say something metaphysical, to demonstrate to him that he has given no meaning to certain signs in his propositions.
6.54 My propositions are elucidatory in this way: He who understands me finally recognizes them as senseless, when he has climbed out through them, on them, over them."

(26) Hoernlé, "Phenomenology of Meaning," p. 71: "It is remarkable that the prominence of signs and symbols in our lives...
(next page)

begin with the sort of phenomenology of meaning for which Prof.
Hoernlé pleaded before the Aristotelian Society, but rather, I
shall begin with an analysis of logic, and if this yields the
desired result, i. e.. a discovery of the logical basis of any
and every kind of meaning, I hope to use the specific instances
of meaning, the findings of a conscientious phenomenology, to
check up the hypothesis by various applications (27).

Here I wish to guard against a misunderstanding which
is almost inevitable for a reader of pragmatic leanings or other
anti-rationalist sentiments; this is the error of supposing that
I consider meaning essentially as a logical relation, and hope
by logical analysis to exhaust all its constituent factors. I
do not believe it to be ever a purely logical affair, any more
than judgment, or empirical existence; only, like these, it has
logical form, and I believe that certain logical forms, or rela-

(note 26)
has not attracted attention more persistently in this field of
inquiry. Yet 'brain-work' is eminently sign-work, sign-using,
sign-interpreting; if necessary, sign-inventing."
        Also Aars, K. B. R., "La nature de la pensee logique,"
(Rev. de Met. et Nov., Vol. 17, pp. 808-823) p. 809: "Toutes
nos pensees sont symboliques; sur ce point, les philosophes
tombent de nos jours, de plus en plus d'accord."

        (27) E. Cassirer, "Philosophie der symbolischen Formen."
p. 12: "Diese Form bedeutet nicht lediglich die Summe oder die
nachträgliche Zusammenfassung der Einzelerscheinungen dieser
Gebiete, sondern das bedingende Gesetz ihres Aufbaus. Freilich
giebt es zuletzt keinen anderen Weg, sich dieses Gesetzes zu
versichern, als dass wir es an den Erscheinungen selbst aufzeichen
und es von ihnen 'abstrahieren'; aber eben diese Abstraktion erweist
es zugleich als ein notwendiges u. konstitutives Moment für den
inhaltlichen Bestand des Einzelnen."

tions, are its prerequisites (28); it is these integral but abstractable _forms_ with which the present analysis shall concern itself--expecting thus to discover certain _necessary_ _though not sufficient_ conditions for the phenomena of meaning.

At first, the fundamentals of logic will appear to be quite unrelated to the nature of meaning; only as the picture nears completion, the connection may be expected to emerge. Therefore, throughout the greater part of this study, the part entitled "logistic," we will perhaps do best to forget the problem of meaning altogether, until we return to it in the final, philosophical part. If it seems uncertain at the outset how an analysis of such an epistemological concept can follow from general logistic consideration, let me but point to the closely analogous structure of Wittgenstein's treatise (29). As his mysticism in regard to meaning, truth, reality and knowledge follows (psychologically if not logically)

(28) B. Russell, Introduction to "Tractatus Logico Philosophicus," by L. Wittgenstein, p. 8: "What relation must one fact (such as a sentence) have to another in order to be _capable_ of being a symbol for that other? This last is a logical question...."

(29) B. Russell, loc. cit., p. 7: "The logical structure of propositions and the nature of logical inference are first dealt with. Thence we pass successively to Theory of Knowledge, Principles of Physics, Ethics, and finally the Mystical."

upon the analysis of propositions, so the somewhat different philosophical conclusions which I have reached have resulted from a very similar study of logical systems (with, incidentally, very similar findings). There is, furthermore, some precedent for this indirect approach to the subject. For centuries, metaphysics has been ridden with certain apparently insoluble difficulties--problems of infinity, continuity, change, the One-and-the-Many, and so forth. The greatest minds of the ages have vainly constructed one metaphysical theory after another to answer these persistent questions; the theories simply would not give a definitive and adequate answer (30). It was not until the perfectly independent developments of mathematical logic happened to touch these traditional mysteries that a little illumination was cast over the old darkness; infinity has ceased to be a magic word, continuity and change have escaped from the bondage of Eleatic theory. The One-and-the-Many has assumed the more manageable form of theory of classes. The problem of meaning is well on the way toward becoming such a persistent enigma. Philosophers have usually treated it from

(30) cf. B. Russell, "On Scientific Method in Philosophy," In "Mysticism and Logic." (London, 1917) p. 112: "In some problems, for example,--the analysis of space and time, the nature of perception, or the theory of judgment--the discovery of the logical form of the facts involved is the hardest part of the work, involved and the part whose performance had been most lacking hitherto. It is chiefly for want of the right logical hypothesis that such problems have hitherto been treated in such an unsatisfactory manner, and have given rise to those contradictions or antinomies in which the enemies of reason among philosophers have at all times delighted."

the point of view of psychology or epistemology; the former
kind of inquiry has yielded interesting results, but only--
for psychology. We cannot fairly expect one science to estab-
lish facts for another, and it is quite natural and proper
that the definitions of psychologists should be inadequate for
the more general purposes of the metaphysician. Epistemology,
on the other hand, is a hybrid science, composed of half a
dozen other disciplines and is therefore not likely to deal
simply and clearly with any subject, let alone one which is
already, in its own nature, so complex as the subject of mean-
ing. It has led us so far, only to an incomplete and rather
unpromising phenomenology. Psychology is too limited, episte-
mology too ambitious to solve this latest riddle of the Sphinx.
But following the example of those men who have unravelled some
of the older philosophical snarls, I would first of all inquire
what are the logical relations underlying any situation wherein
meaning is manifest; and having these clearly before us, we may
find with greater ease the extra-logical factors that determine
the various actual meaning-situations. It is for this purpose
that I shall begin with an analysis of logic itself.

Chapter 2.

The Nature of Logic and Logistic.

If a logical analysis of meaning seems like a some-
what ambitious undertaking, due to the existing confusion of
definitions and terminologies relating to it, surely a logistic
analysis of logic must offend even more by the sin of reckless-
ness. There may be many conceptions of meaning, but there are
even more of logic. But whereas the former are usually too
narrow to be acceptable, the latter are apt to be too great—to
contain all sorts of notions which go beyond the indispensible
needs of "mere" logic. To go from a superabundance of elements
to a relatively small nucleus is an easier task than to begin
with a haphazard insufficient number and add strictly relevant
material thereto until the deficit is made up; analysis is usually
a simpler and safer process than synthesis. Moreover, opinions
on the nature of logic are far better established than those
which concerned us in the last section; we are dealing with
more or less familiar and time-sanctioned views instead of pre-
liminary gropings.

It must be admitted that, however well-expounded and
philosophically respectable these opinions may be, their number
is certainly legion. A glance at the definitions in the dic-
tionary is enough to convince us that Logic in its present state
suffers from multiple personality. There we read that it is
"the science or art of exact reasoning, or of pure and formal
thought, or of the laws according to which the processes of
pure thinking should be conducted, the science of the formation

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and application of general notions; the science of general-
ization, judgment, classification, reasoning, and systematic
arrangement." (1) And this generous definition is not even
exhaustive. From standard works on logic, where cursory defi-
nitions usually figure by way of introduction, as also from
numerous minor writings dealing more particularly with the
nature of the disputed discipline, we may gather that logic
is anything and everything. Now it is psychology, now epis-
temology, and now it walks in the image of metaphysics; (2)
it has even been identified with professional etiquette (3).

The differences of opinion in this field are largely
due to the fact that logic, besides being theoretically inter-
esting as a complex structure in itself, is also the most

(1) Webster's Standard Dictionary, article--"Logic."

(2) cf. Couturat's review of the 2nd philosophical
congress at Geneva, published in the Rev. de Met. et Mor. 1904
(p. 1063)--"Il est facheux que ce mot (Logique) soit employé de
nos jours dans un sens 'lache,' pour designer toutes sortes
d'etudes qui resortissent en realité à la psychologie, à la théorie
de la connaissance, à la théorie de la connaissance ou même à la
métaphysique. Cet emploi confus du mot est unes des causes pour
lesquelles la plupart des philosophes négligent ou ignorent si
complètement la vrai logique ou du moins sa partie essentielle,
qui est la Logique formelle."

(3) R. Arms,   ., "The Relation of Logic to Mathematics"
Monist 1919, p. 151: "Logic does not contain an 'ought,' nor
yet an 'is'; it is an accepted code of validity, a kind of
gentlemen's agreement the violation of which should lead to
scientific ostracism...we guarantee the acceptability of the
code by defining it as what is accepted."

catholic science in its applications (4). Many of our sciences, such as physics, mathematics, psychology and others, are commonly divided into "pure" and "applied." The modern tendency--(especially in this country)-- is to exalt the "applied" science, and to esteem the "pure" system only in so far as it may be expected soon to yield practical results. To the average layman, "radioactivity" is not a branch of physics, but is the science of developing and improving the wireless telephone. Psychology is the science that teaches us how to control our psychic states, and reckon with the psychic states of others. Mathematics, to most non-mathematicians, is the science of "figuring" and solving practical puzzles. Logic, being older than most sciences and more easily applied, has met with this fate long ago and in every school (5). Its bearing upon the practical processes of

(4) cf. J.S. Mill, "System of Logic" (1st edition) p. 10: "As the far greatest portion of our knowledge, whether of general truths or of particular facts, is avowedly matter of inference, nearly the whole, not only science, but of human conduct, is amenable to the authority of logic... Our definition of logic, therefore, will be in danger of including the whole field of knowledge, unless we qualify it by some further limitation."

(5) Mario Vecchi: "La logica secondo le vedute di F.Enrigues" (Riv. di Fil., III, p. 500): "La questione se la Logica abbia un valore reale o puramente formale,---conduce a una netta partitione della logica; si hanno così: una 'logica' nel senso formale, che concerne i problemi che si riferiscono all'aspetto logico del processo conoscitivo,---e una 'applicazione della logica,' che comprende l'induzione scientifica...."

thinking is so obvious that the association of <u>logical forms</u> with <u>actual judgments</u> is well-nigh inevitable; in fact, it takes a considerable degree of sophistication to think of the former as separable from the latter (6). That is why the most disinterested intellectuals, the scholastic formulators of logic, have considered it as the study of judgments. In an age when the sciences in general were not advanced enough to be clearly divided into "pure" and "applied," it was natural that no fine distinction should be drawn between the study of <u>judgments</u> and <u>judging</u>, i. e., between the process and its results. Hence the disjunctive character of Webster's definition: "The <u>science or art</u> of exact reasoning <u>or of pure and formal thought</u>." The two alternative notions are somewhat scrambled in this elucidation; the disjunction would have been better stated as: "The science of pure and formal thought, or the art of exact reasoning." This shows plainly the "pure" versus the "applied" aspect--the concept of logic as science

(6) The first writer, to my knowledge, who has drawn the distinction is G. Frege, who writes in the preface to his "Grundgesetze der Arithmetik"p. XV: "Dass die logischen Gesetze Richtschnuren für das Denken sein sollen zur Erreichung der Wahrheit, wird zwar vorweg. allgemein zugegeben;aber es geräth nur zu leicht in Vergessenheit. Der Doppelsinn des Wortes 'Gesetz' ist hier verhängnissvoll.....das Wort 'Denkgesetz' verleitet zu der Meinung, diese Gesetze regierten in derselben Weise das Denken, wie die Naturgesetze die Vorgänge in der Aussenwelt."

cf. also the commentary on the title of G. Boole's work, "The Laws of Thought," in Russell's article on "Mathematics and the Metaphysicians."(in "Mysticism and Logic"). "He was... mistaken in supposing that he was dealing with the laws of thought. The question how people actually think was quite irrelevant to him, and if his book had really contained the laws of thought, it was curious that no one should ever have thought in such a way before."

and as art. The little confusion in the lexicographer's wording is, perhaps, a fair index to the gradual amalgamation of the two viewpoints which has taken place. We have here the source of Prof. J. M. Baldwin's assertion that "the treatment of the operations of thinking, that is, of the discursive or reasoning faculty generally, under the term logic, is so conventional and established, that further defence of it is not necessary." (7) The asserted conventionality is certainly not to be denied, whatever we may think of its implications for scientific procedure.

Now since logic is so naturally and in fact unconsciously applied to thought in general, it cannot escape the consequent application to the particular systems of thought whereupon individual interests happen to centre. A philosopher with predelictions for psychology sees judgments essentially as psychological facts; for him the study of judgments is "that inquiry which, pursuing genetic and functional methods, investigates thinking with a view to tracing the derivation,

(7) J. M. Baldwin, "Thought and Things; or, Genetic Logic." Vol. 2, p. 3: Compare the dictum of an otherwise radically different-minded logician, namely Hegel: "Dass das Denken der Gegenstand der Logik sey, darüber ist man allgemein ein verstanden." "Encyklopädie der philosophischen Wissenschaften", I Theil: Logik. (2nd. Ed. 1843 p. 31)

development, and embodiments of Belief." (8) A scientist

whose eye is directed chiefly toward the outer world of objects,

which are "sets of properties," sees in the arrangement of this

world an instance of logical relations, and decides that "the

subject matter of logic is therefore the relations of sets of

properties." (9) (Logic in this case is not limited to the

actual sets; it deals with all possible such sets.) To an

idealist, who finds the Law of Reality in the law of his own

thinking, the study of pure thought is identical with the

study of actual being. "Die Logik fällt daher mit der Meta-

physik zusammen.....Betrachten wir das Denken als das wahrhaft

(8) J. M. Baldwin, Op. cit., Vol I, p. 7. Compare
with this the definition of J. S. Mill, "Logic," p. 13 (1st ed.):
"Logic, then, is the science of the operations of the under-
standing, which are subservient to the estimation of evidence;
both the process itself of advancing from known truths to un-
known, and all other intellectual operations in so far as
auxiliary to this....The analysis of the instruments we employ
in the investigation of truth, is part of the investigation
itself; since no art is complete, unless another art, that of
constructing the tools and fitting them for the purposes of the
art, is embodied in it." (This would imply that the manufacture
of canvas and brushes is an integral part of the painter's art.)
        Also G. Uphues, "Erkenntnis-kritische Logik," p. 10:
"In der Philosophie der neueren Zeit erhält die Logik eine
andere und neue Aufgabe. In ihr soll nicht mehr die bereits
erworbene Erkenntnis erwiesen, sondern es soll der Weg gezeigt
werden, wie die Erkenntnis erworben wird; aus der ars demonstrandi
wird jetzt die ars inveniendi."
        Psychological research is not the only task which has
latterly been laid upon logic, the poor maid-of-all-work in
philosophy; E. Kieseritzy, in an article "Die Emanzipierung von
der Folgestrenge" absolves her from all her usual duties, and
ordains, instead: "Die Logik hat...die Aufgabe, den Wertgründen
des Gewissens nachzuspüren."

(9) Wrinch, D., "On the Structure of Scientific
Inquiry," Proc. Arist. Soc. N. S. Vol XXI, pp. 181-210. See p. 201

Allgemeinen alles Natürlichen und auch alles Geistigen, so greift dasselbe über alles dieses über und ist die Grundlage von Allem." (10)

Logic has even been conceived as a branch of Anthropology (11).

These arbitrary limitations of logic to some specific material which exemplifies its laws, has given rise to the beliefs (a) that there can be several kinds of logic (12), (b) that there can be experience which does not conform to logical principles, and (c) that logic itself may be an unfinished thing, a product of evolution. The second supposition is based upon the first; for if there can be more than one logic, then no logic is strictly universal, and if we have not one universal logic, it may be doubted whether the

(10) G. W. F. Hegel., Op. cit. pp. 45-46.

(11) J. F. Fries, "System d. Logik," p. 1: "Der Ausdruck 'reine Logik' ist unbestimmt. Wir können darunter eben so wol eine anthropologische als eine philosophische Lehre von den Denkgesetzen verstehen."

(12) Prof. Baldwin (Op. cit. Vol. I. p. 3) holds that there are three kinds of logic--the Metaphysician's logic, the Logicians logic, and the Knower's logic. This sort of distinction, when applied to other sciences, gives such results as: the ground-hog's meteorology, the meteorologist's meteorology, the motorist's meteorology; or again, the Christian Scientist's pathology, the physician's pathology, the sick man's pathology. It is hard to understand why Prof. Baldwin, in the case of logic, casts his vote for the third type; in the instances of meteorology and medicine I presume that he consults respectively the reports from Washington and the verdict of his physician.

sum of all the logics really covers all experience, or whether
there be a residue of genuinely "alogical," "mystical,"
"irrational" reality. The third proposition is likewise
dependent on (a); for our world is conceivable minus any one
of its special constituents; we can postulate a materialistic
world in which no "Geist," no teleological "νοῦς" is operative,
or a monadic order which does not contain "Sets of properties,"
or a dead astronomical universe whose members register no mental
processes. Every such curtailment of the actual contingent
world would make void one of the traditional conceptions of
logic. And if we can postulate a world without logic, then
we can imagine that logic is something superadded to our en-
vironment, and like anything else in it, a product of evolution.
If logic is the laws of thought, it may hold only for human,
not for angelic or martian or amoeboid thought. If it is a
"gentleman's agreement" or a "determination to be rational," (13)
it is conceivable that different cliques of gentlemen might
sign different agreements, or that in a moment of weakness they
might waver in their determination.

But after all, it seems hardly plausible that validity
of thought is determined by a social contract, subject to change
through a majority-vote among scientists, or by age-long habit;
that in the days of the Primaeval Slime the law of contradiction

(13) R. Arms, loc. cit. p. 153: "Logic is the
articulate development of the determination to be rational."

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did not hold, so that the Slime was both slimy and not slimy, and at the same time was <u>neither</u> slimy nor not-slimy, and the creatures who formed the useful habits of logical law were <u>not</u> the creatures who formed them. It is not necessary to dwell on the fallacies besetting such doctrines, especially as other writers have stated them in all their absurdity (14). The relevant factor for us is, that they all point back to the classic confusion of logical principles with the "laws of thought" (15). That the latter did not operate before thought had "evolved," nor govern mental processes other than thought, must certainly be granted.

The mystic's assertion, that there is an irrational factor in Reality, is less obviously untenable. But the basis of this contention is, again, epistemological rather

(14) W. P. Montague;"The Antinomy and its Implications for Logical Theory," p. 234: "When Heraclitus proclaimed his doctrine of universal flux, he made no exception of any single thing in the world; everything changed.....But the first and greatest of dynamists did not hesitate to set a certain kind of limit to his universal flux.
All <u>things</u> change, but the laws according to which all things changed were themselves changeless. They were changeless because they were the measure and condition of the change in things."

(15) Ibid. p. 237: "The laws of gasoline engines were just the same in the days of the ancient Athenians as now. We know them and <u>they</u> did not. Not physics, but man's knowledge of physics has changed."

than logical; the term "irrational" is used to assert the independence of Ultimate Reality from the forms of human reason, not from Form as such (16). Pure Content is as much an abstraction as pure Form; and the true mystic has a deep-seated "horror abstractionis." He would have us believe that the forms of Ultimate Reality are incommensurable with those of thought--and incidentally, that he is gifted with a special epistemological talent which allows him to appre-hend without thinking, so that he can appreciate both the Reality and its incommensurability with our vulgar logic. Unfortunately the mystic is usually under the impression that logic is the art of twisting the syllogism, or worse yet, the habit of drawing inductive conclusions from sense-experience; so that, in most instances, his claim to "irra-tional" knowledge means nothing more than non-syllogistic thinking. In a symposium held by the Aristotelian Society a few years ago, Evelyn Underhill gives a general exposition

(16) Symposium: "Can the New Idealism dispense with Mysticism?" I--by Evelyn Underhill. Proc. Arist. Soc. Suppl. Vol. III (1923) pp. 148-160.
cf. p. 160: "The insistence on the prime im-portance of humility.....appears to be the emotional expres-sion of the profoundly felt truth that these experiences are supra-rational; and that attempts to square them with a reason developed through the frictions and demands of the here-and-now will inevitably frustrate themselves. This is declared by mysticism to be a statement of fact....and it adds.....that the world into which the self thus obtains entrance is above reason and beyond reason."

of the mystic's attitude, wherein she tries to avoid what
she considers "the excesses to which it has always been
liable; the tendency to vapour off into an ecstatic feeling-
state without communicable content, to reject the time-world,
to de-humanize experience, to confuse a blank placidity with
'pure immediacy.'" (17) To Miss Underhill, mysticism does
not really connote much more than a religiously-tinted Monism.
It is as much a system as any philosophical doctrine (18),
and being a system, it must employ some form of thinking.
Her claim for "mystical" reality is merely that it does not
conform to the categories of thinking. "It is notorious that
in all these supreme ways of knowing and feeling, only a part
of what is apprehended can be expressed;.....Yet in all, a
reality exceeding our analytic and descriptive powers 'shows
itself', as Wittgenstein says, to transcendental feeling;...
since propositions cannot express anything higher than them-
selves." (19) (I must confess that the last assertion leaves
me thoroughly mystified, as I do not know what is the "height"
of p, provided p is a proposition, and why, if p expresses,
e. g., a relation between Socrates and the rest of mankind,
that relation cannot be "higher" than p.)

(17) E. Underhill, loc. cit., p. 152

(18) Ibid. p. 153: "It is enough if we insist that
it (mystical apprehension) gives a higher synthesis, a closer
approximation to ultimates, than those other systems by which
man seeks to apprehend the universe."

(19) Ibid. p. 153 infra. See also the book entitled
"Mysticism" by the same author.

It has, of course, often enough been remarked that even mystics try to convince us by logical argument, by an appeal to our understanding, rather than by hypnotism or magic (20). If reason is entirely wrong, these philosophers are simply lying; if it is merely insufficient, then they are telling us a part of the truth--namely that part which we also, who are not mystics, can reach by the exercise of reason. In neither case are the disparagers of reason making a great contribution to our wisdom. Their feelings, which reveal the mystery to them, are unfortunately as private as my toothache would be in an otherwise toothless and painless society. They cannot even evoke sympathy through analogy, much less communicate the revelation (21).

(20) B. Russell, Introd. to Wittgenstein's Tract. log.-phil., p. 22: "What causes hesitation is the fact that, after all, Mr. Wittgenstein manages to say a good deal about what cannot be said.....The whole subject of ethics, for example, is placed by Mr. Wittgenstein in the mystical, inexpressible region. Nevertheless he is capable of conveying his ethical opinions.

(21) Bergson confesses to this inability of expression, despite the fact that he writes a book to express his doctrine. "By intuition," he says, "is meant the kind of intellectual sympathy by which one places oneself within an object in order to coincide with what is unique in it and consequently inexpressible."......."We may sympathize intellectually with nothing else, but we certainly sympathize with our own selves."(H.Bergson, ., "An Introduction to Metaphysics," transl. by T. E. Hulme, N. Y. and London, 1912, pp. 7-9,) Since this "intellectual sympathy" is identification of subject and object, the upshot of this passage seems to be that we certainly are identical with ourselves, and probably not with anything else--a conclusion which cannot be said to contradict ordinary logic.

Thus we are driven back to the orthodox view that there is one great and only Logic, and everybody is its prophet. Without _some_ fundamental agreement, we could not even begin a dispute. And without some basis of forms common to all experience, we would not so much as exist for each other. Now anything that, we say, _may exist_, is such that it _may exist for us_, although we may be forever debarred from any knowledge of it; but if we can make any assertion about it, true or false, i. e., _if it exists at all, then it has some form, though this may not be one with which we are familiar._ It may be a form which is not commensurable with the forms of thought; this is what Kant postulates for his "Ding-an-sich," which consequently is relegated to the "unknowable." A universal logic must be more general than the "laws of thought," or the laws of the real world, or of any other thing; and therefore I propose to treat logic as the science of _forms as such_. It is in this spirit that Royce has defined Logic--in his words, "_Logic is the General Science of Order_, the Theory of the Forms of any Orderly Realm of Objects, real or ideal." (22) This will involve _the analysis of systems_

(22) J. Royce," The Principles of Logic," in Windelband and Ruge's "Encyclopaedia of the Phil. Sciences" (Eng. transl. 1913) Vol. I, p. 69.
           cf. Also a passage wherein this highly generalized view of logic is approached by G. Mouret ('l'Egalite Mathematique,' Rev. Phil. Vol. 32, p. 116). Although Mouret still deems it necessary to conceive pure forms as applicable to _all reality_ rather than as _independent of reality_: "Dans le sens où je prends la logique, c'est une science spéciale, qui n'est ni la psychologie, ni la grammaire, ni la mathématique. C'est une science objective et abstraite qui a son domain propre, et ce domain....ne
(next page)

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qua systems, the discovery of relations which hold for all
possible worlds (or parts of worlds), i. e. for any and every
universe of discourse. The _terms_ of pure logic thus conceived
are neither concepts, nor "sets of properties," nor thoughts,
nor things; they are variables, capable of denoting any of
these entities disjunctively. The "system" of the mystic, in
so far as it has any form, is just as truly expressible by the
symbolisms of logic as the system of the rationalist, idealist
or pragmatist. _Logic is the study_, not of specific systems,
but of _that by virtue of which a system is a system_, i. e. a
study of _patterns_--conceptual, physical, psychical or what-not.
The generality of logic, in this sense, is foreshadowed in an
analogy drawn by B. Russell in his little essay on "The Study
of Mathematics," where he tells us that "the rules of logic are
to mathematics what those of structure are to architecture...
Literature embodies what is general in particular circumstances
whose universal significance shines through their individual
dress; but mathematics endeavors to present whatever is most
general in its purity, without any irrelevant trappings." (23)
Now it seems to me that a _system of logic which is universal is_

(note 22)
peut être que l'ensemble des objets extérieurs de la connaissance,
considérés indépendamment de leur nature particulière, c'est-à-
dire les relations et les concepts généraux. Les lois de la
logique sont, par conséquent., les lois communes à toutes les
sciences, et la logique est la science abstraite par excellence..."

(23) B. Russell, "The Study of Mathematics" In "Mys-
ticism and Logic," p. 61.

one which, carrying out these analogies, gives us a skeleton, a pattern, which expresses disjunctively either the laws of mathematics, of structure, of literary art, of thought (syllogistic or non-syllogistic according to their abstract form), or of anything else whatever. This ideal has been stated and initiated by Dr. H. M. Sheffer (24), who calls such a variable pattern, which yields a variety of systems by interpretation, a "system-function." It is interesting to note the gradual and groping development of this notion; for the "system-function" is really the same thing as Russell's "relation-number." (25) Both versions of the idea seem to encounter difficulties of language, and to bear a name that does not readily suggest it, but requires considerable elucidation. The term "relation-number" is probably derived through a certain similarity between

(24) H. M. Sheffer, "General Theory of Notational Relativity," 1921. Unpublished paper in the Library of Harvard University, p. 2: "Deductive systems, it is well known, may be determined by means of postulate sets in various ways.....These distinct determinations are all 'equivalent'--any two of the postulate sets are uniquely interchangeable. May there not be, then, a set of 'superpostulates,' of which Hilbert's, Vebleu's, Huntington's, and other postulate sets are special cases? There is. And as a matter of fact, the 'invariant' of these postulate sets turns out to be of an extraordinarily simple character."

(25) With this possible distinction, that Dr. Sheffer does not explicitly define it, or treat it, as a class of systems; I am inclined to believe that he is talking about the pattern as such, not the class of things which conform to the pattern; whereas Mr. Russell treats relation-number (like all other numbers) as a class.
See his "Introduction to Mathematical Philosophy", p56 : "The 'relation-number' of a given relation is the class of all those relations that are similar to the given relation."

the new concept and the somewhat narrower but more familiar
one of "ordinal number" (26); "system-function" is an expan-
sion of the notion which Russell has christened "propositional
function." Personally, I prefer the simpler expression "log-
ical pattern," although I shall often have occasion to refer
to system-functions, especially where my thesis is a direct
application of Dr. Sheffer's more general theory.

The following analysis of logic claims, then, to be
a simultaneous analysis of all possible things, given in
abstract terms (27), and therefore not an analysis of any
particular system or thing. It is therefore an analysis of
logic in the most universal sense; and this I consider the
correct (though, as Prof. Lewis calls it, the "heterodox")
meaning of the disputed term "logistic."

(26) B. Russell, "Introduction to Mathematical Philosophy,"
p. 57: "If n is a finite cardinal number, the relation-number
of a series which has n terms is called the 'ordinal' number n."

(27) This implies the possibility of rendering the
whole subject in variable terms, thus including tacitly every
actual system. For the relation of abstract to variable terms,
see Chapter 4.

II   ANALYTIC PART
Chapter 3.
Abstraction and Interpretation.

Abstraction has played a major part in philosophy ever since the day of Plato and Aristotle. Metaphysicians have a tendency to consider it a falsification of any subject matter under discussion; an "abstract" description is an incorrect one, and a science which deals in abstractions is "vicious." Logic has frequently been condemned as "merely abstract", hence as giving an untrue account of its material. It seems impossible for certain philosophers to believe that any science could be deliberately abstract.

Now, as a matter of fact, this deliberate abstractness of logic is just the feature which distinguishes it from metaphysics; for the subject of metaphysics is the world in its entirety, the synthesis of all aspects of all things (1); of logic, it is the world under its formal aspect only. In metaphysics, abstractness is indeed "vicious," because any purely formal treatment necessarily omits what the philosopher claims to include: the content of experience (or Reality, or what-not). But in logic it is highly commendable, because it omits what the logician avowedly would not include--namely,

(1)   cf. R. F. A. Hoernlé, "Matter, Mind, Life, and God" p. 34: "It is just because science abstracts that it cannot be 'synoptic', or take the place of philosophy, which seeks to transcend abstract points of view in order to see things together and to comprehend the world as a whole."

anything other than the forms of experience (2) (or Reality, or what-not). That is why metaphysicians have always been prone to confront the logicians with a charge of "mere" abstractness, and logicians have failed to feel the sting of the reproach, and have returned an equally irrelevant verdict to the effect that the metaphysicians are "vague."

The actual quality of any experience is necessarily and strictly private. What is communicable is only the formal aspect--the conceptual element wherein A's experience and B's correspond. Therefore it is true that an ideal metaphysics is impossible--unless we are to take it in the Bergsonian sense, in which it is an activity, a personal feeling, and is ex hypothesi inexpressible. In this case, of course, no two persons can have the same metaphysic, any more than they can feel the same tooth-ache; and it is no more a discipline to be "pursued" or taught at the universities than the local and momentary ache of my tooth. Whether metaphysicians in general would agree with such a definition is a matter open to grave doubt. Taken in the ordinary sense, i. e. as an attempt to describe the "given" world under a synthesis of all its

(2) "Princ. Math". 2d Edition, p. XXX: "Constants do not occur in logic; that is to say, the a, b, c which we have been supposing constant are to be regarded as obtained by an extra-logical assignment of values to variables."

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aspects (3), metaphysics is irremediably limited by the impossibility of communicating at least one of these aspects, namely the purely qualitative, or subjective. It can at best describe the forms of things with the constant avowal that they have such a specific and unique content. This appreciation of the concrete is not required in logic.

All our thinking is more -or less abstract, more or less concrete (4). We are constantly recognizing "patterns" or logical forms which are displayed in our experience, or recalling such empty forms and fitting the data of experience into them. The former process is abstraction and the latter is interpretation. In metaphysics we aim at the maximal amount of interpretation; we are limited in this by the fact that the subjective element cannot be defined and thus conveyed to other minds, but we strive by "hints and suggestions" to transmit as much as possible of the indefinable content, to make our description of the world as concrete as possible. In logic, on the other hand, it is

(3) cf R. F. A. Hoernlé, "Studies in Contemporary Metaphysics," p. 27: "Philosophical argument of the best sort is material, not formal. It seeks to use the very stuff and substance of actual experience as its datum."

(4) This fact has been pointed out and treated at length by Fr. Paulhan in his series of essays, "L'Abstraction et les Idées Abstraites" (Rev. Philosophique, Vol. 27, pp. 26 - 57, 171 - 188, 545 - 565, & Vol. 28, pp. 69-93).

our ideal to treat of the form alone (5); this also is impossible, as we shall see, because we are inevitably bound to some medium of expression which lends an irrelevant content to our structure. We may know that a proposition is logically "the same" whether it be stated in German, English or Hottentot; but the fact remains that in stating it we are bound to some one language, with its idiosyncrasies of grammar and vocabulary. There is no "Interlingua" which is an abstraction from languages; the later writings of Prof. Peano, as most of us discover to our sorrow, do not escape the provincialism of existing tongues, but are simply bound to an (n + 1)th specific version of Aryan syntax. We may regard "language" in a broader sense, to include all modes of expression--gesture, tone, mathematical symbolism or what-not--and yet we are caught in the same predicament; we can use only one language on each occasion, and we must use one at least (6).

(5) cf. Russell, B. "Logic as the Essence of Philosophy" (In "Our Knowledge of the External World") p. 53: "Pure logic and atomic facts are the two poles, the wholly a priori and the wholly empirical. But between the two lies a vast intermediate region..."

(6) cf. A. B. Kempe, "A Memoir on the Theory of Mathematical Form," p. 4: "In order to put form in evidence some "accidental" clothing is of course necessary; if, however, we employ more than one species of clothing, each species being uniform and united to forms of every kind, the likelihood of its accidental nature being overlooked will be reduced to a minimum."

But in spite of this barrier between our ideal logic and any actual system, it seems certain that _every_ logic abstracts from experience. The nature of abstraction, which is a psychological process, and of _abstractions,_ which are supposedly its result, would fill many volumes of the world's collected philosophical muddles. Of course the possibilities for confusion are in this case almost endless, and logicians have not failed to avail themselves thereof. The psychology of abstraction and the character of abstract entities have spent centuries together in the psychologistic melting-pot--until the aforesaid entities, which the old logic honored as "concepts," emerge as "general ideas," "composite images," "Vorstellungen," and fictions of every sort (7).

I do not propose to venture on any theory in regard to the process of abstraction. That is a topic for psychologists; for our purposes, one "genetic" doctrine is as good as another. The upshot of the process is the same upon any theory--it is the fact that we postulate, or assume, or imagine, or recognize certain _constant_ elements in the world. Without

-(7) The abuse of concepts is well criticized by A. Voigt in his article "Was ist Logik?" (Vierteljahrsch. fur wissensch. Phil.", 16) where we read on p. 305: "Wäre der Begriff aus einer Reihe von Vorstellungen durch Abstraction von deren Unterschieden entstanden, so müsste er selbst eine Vorstellung sein. Man beachtete nicht dass die Thätigkeit der Abstraction auf Vorstellungen überhaupt nicht amwendbar ist. Man kann wohl _Theile_ einer Vorstellung weglassen, nicht aber von _Eigenschaften_ einer Vorstellung abstrahieren. Eine Vorstellung ist und bleibt immer konkret."

constants of some sort logic is not possible; that is the
fundamental meaning of the Aristotelian dictum that "A= A."
Whether these constants really exist in nature, or whether
they are "mere ideas" or "Annahmen" or Bergsonian "falsifi-
cations," does not affect their logical importance in the
least. Bergson may condemn logic as an immoral pursuit be-
cause it employs constant entities, since anything constant .
is of shady character in his estimation; but he cannot give us
a logic of inconstant entities. He can only dispense with logic
altogether and put his faith in stranger things. Platonists and
Hegelians, for whom all constants are Ideas, may raise logic to
the exalted station of "Geistesphilosophie"; empiricists and
Pragmatists, more frankly and prosaically psychologistic, treat
it as a branch of "mental science." But all schools seem to be
in agreement upon this one point: that logic deals with constant
communicable entities which are somehow abstracted from our hete-
rogeneous, private, mundane experience.

These constant elements are the "concepts" of the
classical logic, and the various "images" and ideas" of the new.
The word "concept" has been kept, but has had to submit to many
definitions. In the 19th century it practically became a truism
that a concept must be an idea in somebody's mind, involving the
act of judgment (8). (That is why logic, the study of relations

(8) This ingrained tradition finds its echo even in
Russell's opinion, (Mysticism and Logic, p. 212) where "a univer-
sal of which we are aware is called a concept." Without a his-
torical explanation, it would seem enigmatic that whereas Mr.
(next page)

between concepts, has been so generally characterized as the "laws of thought.")

Now unfortunately, an idea in somebody's mind, even if it may be "constant" throughout the whole series of its appearances, is just as private as the ache of somebody's tooth (9). Perhaps the tooth-ache of yesterday is the same as that of today and tomorrow; that alone does not make it communicable. The usual plea of the mentalists is that A's conception is enough like B's that they tally in certain respects and thus to a degree are "held in common." Here we have come to the point where the non-psychologistic logician would draw a distinction: the concept, to him is just that and only that common element in A's and B's conceptions in virtue of which they are "similar"--this only is public, as neither A's nor B's, but is the abstract element of both. A's notion may be "general," it may be compounded of past images, and so forth, but that does not make it abstract. The fact that it can be "similar" to B's implies that the two have an element in common; for all similarity rests upon partial identity, and only this identical portion is the object

(note 8)
Russell deals with "hard data" as not essentially subjective, although they depend on sensing for their being known, he still regards concepts as necessarily conceived, i. e. as "universals of which we are aware".

(9) cf. Frege, "Grundgesetze," p. XIX: "Wenn wir nichts erfassen könnten, als was in uns selbst ist, so wäre ein Widerstreit der Meinungen, eine gegenseitige Verständigung unmöglich, weil ein gemeinsammer Boden fehlte."

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of logical abstraction (10).

This notion of the "pure concept," which is not an object and not an idea, has caused great ontological difficulties. In what sense may it be said to "exist," if it is neither physical nor mental?

The pure concept--that part of our world (ideas, things, sensations or anything else) which is truly public, i. e., abstractable, can exist in any literal sense of the word, only in connection with some content (11). The concept is the form of a content. B. Russell, in the course of analyzing propositions discovered the impossibility of treating forms as constituents in things. "Form," he says, "is not another constituent, but is the way the constituents are put together. It is forms, in this sense, that are the proper

(10) The dependence of similarity upon a common element is emphasized by Russell in the "Introd. to Math. Phil." p. 61: "Two relations have the same 'structure,' we shall say, when the same map will do for both.....And that, as a moment's reflection shows, is the very same thing as what we have called 'likeness.'"
    et infra: "It is often said that space and time are subjective, but they have objective counterparts;--Where such hypotheses are made, it is generally supposed that we can know very little about the objective counterparts. In actual fact, however,--the objective counterparts would form a world having the same structure as the phenomenal world......the only difference must lie in just that essence of individuality which always eludes words and baffles description, but which, for that very reason, is irrelevant to science."

(11) cf. E. Cassirer,"Substanzbegriff u. Funktionsbegriff" p. 6:- "Der Begriff tritt der sinnlichen Wirklichkeit nicht als ein Fremdartiges gegenüber, sondern er bildet einen Teil eben dieser Wirklichkeit selbst; einen Auszug dessen, was in ihr unmittelbar enthalten ist."

object of philosophical logic." (12) Empiricists have re-
peatedly denied that such an abstraction could be known,
since it is neither a sense-datum nor an "idea"; which argu-
ment entails, to my mind, not a summary dismissal of con-
cepts -- the fact remaining that we _do_ apprehend such common
elements, else we could not communicate with one another --
but an indictment of our traditional psychology, which has
failed so far to account for our acquaintance with pure
concepts. So, although we may not say that concepts exist
independently of content, we may yet safely maintain that
they are objective and apprehensible and can therefore be
studied apart from the special medium that happens to give
them metaphysical support. If logic is the study of _forms_
as such, as we have defined it in the previous chapter, it
is the study of concepts in this purely abstract sense.

I have so far retained this much abused word
"concept"- perhaps unwisely -- because it is the word employed
by Frege (13), who has given the clearest expression I have yet
found to the position here advocated. In his "Grundgesetze der
Arithmetik" he thus confesses his philosophical faith: "Ich

(12) B. Russell, "Logic as the Essence of Philosophy,"
in "Our Knowledge of the External World". R 43) r. 43.

(13) Frege uses "Begriff." "Concept" is our most
accurate translation of that word.

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erkenne ein Gebiet des Objectiven, Nichtwirklichen an, während die psychologischen Logiker das Nichtwirkliche ohne weiteres für subjectiv halten."(14).

"Wir können zwischen physischen u. logischen Gegenständen unterscheiden........Jene sind im eigentlichen Sinne wirklich; diese sind es nicht, aber darum nicht minder objectiv; sie können zwar nicht auf unsere Sinne wirken, aber durch unsere logischen Fähigkeiten erfasst werden." (15)

"Begriff u. Beziehung sind die Grundsteine, auf denen ich meinen Bau aufführe." (16)

A very similar view is expressed by G. E. Moore, who leaves us in no doubt as to the objectivity and constancy of concepts.

"The concept is not a mental fact, nor any part of a mental fact.....Concepts are possible objects of thought; but.....it is indifferent to their nature whether anybody thinks them or not." (17)....."Just as concepts are themselves

(14) Preface, p. XVIII
(15) Vol. II, p. 86
(16) Vol. I, p. 3
(17) "The Nature of Judgment," Mind, N S. 8. p. 179

immutably what they are, so they stand in infinite relations
to one another equally immutable." (18)

Dr. Moore is not content to build merely the edifice
of _logic_ upon Frege's corner-stones of concept and relation;
he would have us believe that the real world of things, ideas,
etc., is "composed of nothing but concepts." (19) To these
metaphysical lengths we need not and will not follow him.
It is at this point, notably, that the greatest of English
logicians, who so far had been in agreement with Dr. Moore,
departs from his doctrines. For Bertrand Russell, the actual
world is not composed entirely of concepts--in fact, there
are "hard" data, more indubitable than any concept, which are
essentially private and momentary. And out of these, with
the aid of "universals" (Dr. Moore's "concepts") Mr. Russell
constructs the world of things and ideas.

Now just as it seems impossible for Dr. Moore to
forego the short step from logic to metaphysic, so vice versa,

(18) loc. cit. p. 180.

(19) p. 182. It is hard to discover just what is
the status of _relations_ in Dr. Moore's system; we are never
told explicably that relations are concepts, yet his sweeping
statement that concepts are "the only objects of knowledge"
would seem to imply that relations, which certainly can be
known to us, are included among such objects. How concepts
can be related by further concepts is at first sight very puzzling,
but as the subject is not even touched upon in the paper under
consideration there is no room to debate the question.

Mr. Russell cannot resist the converse temptation, to bring his outfit of actual _things_ into logic (20). His symbols are supposed to denote "particulars," existents in the actual world, as freely as universals. Instead of naked concepts, shorn and shivering on their Platonic heights, Mr. Russell would use _individuals_, classes of individuals, classes of classes of individuals, etc., as the terms of his logic. This makes his system appear more applicable to metaphysical problems than a structure of pure abstractions, and undoubtedly lends it much of its peculiar charm and interest for the philosopher; but it also involves him in well-nigh insuperable metaphysical difficulties--problems of truth, of existence, of denotation, even of logical material itself--so that Prof. Lewis is not unjustified in remarking that "one wonders whether the last state of this philosopher is not worse than his first."

The introduction of _individuals_ into logic involves us in a grave paradox. For in the recent new edition of _Principia Mathematica_ we read, (p. XXX) that "constants do not occur in logic," but "are to be regarded as obtained by an extra-logical assignment of values to variables"; yet "individ-

(20) "Principles of Mathematics," p47: "A proposition, unless it happens to be linguistic, does not contain words; it contains the entities indicated by words."
cf. B Russell: Principles of Mathematics, pVIII: "On fundamental questions of philosophy, my position, in all its chief features, is derived from Mr. G. E. Moore."

uals" are "constants" in this sense--i. e. are the several values for the lowest type of variable--and every atomic proposition concerns an individual. Now logic is built up on atomic propositions; therefore logic is built up on things that cannot occur in logic. If it dealt with concepts, we might suppose that these are inherent in individuals, and thus build our logic on things that are more than logical, and of which the more-than-logical aspects may be disregarded. But Mr. Russell will have the individual, the whole individual--in fact, his use of the word "individual" suggests that he will have no abstraction therefrom. Thus he would really base logic on things that cannot figure in logic--like building a house of materials that cannot enter into house-building.

It may be the better part for philosophy, but it certainly is a very dangerous one for logic, to traffic in real poets, lovers, unicorns and non-existent bald potentates. Logic, after all, can deal with these entities only in so far as they are definable; Mr. Russell would have us believe that "when I say 'I met a man', what I met was not a concept of a man...." and that therefore it is a real man, not a concept, that figures in the proposition. But this argument is misleading. There may be a concrete, allogical fact wherein the real man "with a tailor and a bank-account or a public-house and a drunken wife" was a constituent; in the proposition we find only the communicable abstraction, the concept of our man.

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If he himself, with his concrete indefinable particularity, were to figure in the proposition, we should have to be personally acquainted with him--in fact, he would have to be physically present--if the proposition were to have any meaning for us. The confusion is due to Mr. Russell's theory that "only those propositions have meaning which denote." What is denoted is certainly a man, but what is conveyed is a concept; and so long as this conveyance takes place, it need not trouble us--as logicians--whether anything is denoted or not.

In trying to introduce "particulars" into logic we have, in fact, trespassed upon the field of interpretations. Interpretation is the converse of abstraction; given a purely formal system of concepts, we seek some concrete situation which shall exemplify this form. In other words, we are now in search of the form with a content. This content can never be supplied from our logical outfit--no infinity of concepts; no complexity of relations will bridge the gap between the pattern and the thing; we may, as Dr. Moore desires, analyze the world into simpler and simpler concepts ad infinitum, but "all the king's horses and all the king's men" couldn't put it together again. Interpretation seems to involve a going outside of logic for part of our material. When we interpret an abstract form, we are no longer dealing with this form alone, but with some situation where the form appears in conjunction

with some non-logical element, a content.

The relation of form and content raises a very interesting and difficult problem. At first sight it seems obvious that there can be such a relation; but if there is, then it can be expressed symbolically, as R(fc); and thereby we have transformed our content into a logical term, i. e. we have formalized it, and are no longer dealing with a <u>content</u>. So it appears that <u>there can be no such thing as the relation between form and content</u>. Just what is the nature of concrete experience, if not the recognition of such a relation, and how we are to apprehend the <u>formed content</u> or <u>realized form</u> in any sense other than that of a relation R(fc), I am at present quite unable to imagine (21). Perhaps a clearer analysis of what is meant by a "relation" would reveal that certain relations cannot be established, just as Mr. Russell's analysis of classes revealed the fact that certain apparently possible classes could not be formed; but whether the inadequacy lies with our notion of "content" or of "relation" I must at present leave undecided, merely pointing out for astuter minds that here is a real philosophical difficulty, and hoping that the solution may shortly follow from some broader point of view.

(21) Josiah Royce appears to have found himself nonplussed at just this point of logical inquiry, as he suggests in his article on "The Principles of Logic" (Windelband and Ruge's "Encyclopædia of the Philosophical Sciences," English transl.) p. 58: "This relation between validity and existence, which is no other than that of <u>form</u> and <u>content</u>, is the ultimate and irreducible point, beyond which the analysis of knowledge cannot pass."

Chapter 4.

Variables, and Levels of Abstraction.

Mathematics, and that type of logic which is some-
times called "mathematical," are generally considered the most
abstract of all sciences. Prof. Whitehead in his little
"Introduction to Mathematics" tells his readers repeatedly
that every sort of mathematics is "abstract." (1) Now in a
sense this is certainly true. But accepting the word "abstract"
in the precise sense here employed, we may find reason to doubt
whether it is the only sense, or whether there is another legit-
imate use of the word--something we might designate as "philo-
sophical" abstraction in contradistinction from the "mathemat-
ical" type. This question, of course, becomes important only
if the "mathematical" meaning does not appear adequate for all
the purposes of philosophy. So long as it seems to meet our
needs, we have no reason to seek different meanings.

The philosophical purpose which throws some suspicion
upon the adequacy of mathematical abstraction, is the analysis
of logic. This subject has been designated by Mr. Russell as
a proper realm for formal logic,--"and this," he says, "is
the same thing as mathematics." (2) If, however, the sort of

(1) See Ch. I "The Abstract Nature of Mathematics", and
the statement on p. 236: "Geometry, like the rest of mathe-
matics, is abstract."

(2) "Mathematics + Metaphysicians," in "Mysticism & Logic";
p. 74

"abstractness" which belongs to mathematics proves to be irrelevant or insufficient for the extremest sort of logistic analysis, we may have to reject, after all, Mr. Russell's definition of logic, and reinstate some principle of abstraction which he has at present relegated to the burning-pile of "metaphysical lumber." (3) (We should not, of course, have to drag any such discarded notion back into mathematics.)

The sort of abstractness which belongs to mathematics is all summed up in the notion of the variable (4). On this hang all the law and the prophets. The logic of Principia Mathematica, being inspired by mathematical ways of thinking, is based on the principle of the variable. Now this principle does not, as a matter of fact, lead to any high degree of abstractness, but only to generality (5). It is perfectly possible to ennunciate a propositional function--i. e. an

(3) "Logic as the Essence of Philosophy," p. 42

(4) A. N. Whitehead, "Introduction to Mathematics," p. 15: "Mathematics as a science commenced when first someone, probably a Greek, proved propositions about any things or about some things, without specification of definite particular things."
Also Russell, B., "Mathematics and Metaphysicians," (In "Mysticism and Logic") p. 75: "If our hypothesis is about anything, and not about some one or more particular things, then our deductions constitute mathematics."
cf. Also his "Principles of Mathematics," p. 5: "Mathematical propositions are not only characterized by the fact that they assert implications, but also by the fact that they contain variables."

(5) Carlyle, in "Sartor Resartus," facetiously gives Prof. Teufelsdröckh the title of "Professor of Things in General." He probably was not aware of the implication that the German sage was a Professor of Mathematics.

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assertion involving a variable--and regard it as an assertion

about Smith, Jones or Brown who are supposed to be living

individuals, in no wise abstract. What we derive from the use

of variables is ambiguity. Ambiguity is abstractness in a very

special sense--it is the abstractness which belongs strictly

to the symbol, not to the thing symbolized. But a symbol, if

it is to communicate a true or false proposition, must denote

something; it is the thing denoted, not the symbol, which, in

Mr. Russell's opinion, figures in the actual proposition--and

he regards logic as a system of propositions, not of symbols.

Logic is a special way of asserting propositions. Its way of

asserting them is abstract; but its subject matter, the body

of propositions, may be perfectly concrete.

This symbolic abstractness is the only kind desig-

nated by Russell, in one of his Lowell lectures, under the

"principle of abstraction" (it is there not given in terms of

variables and values, but of class and membership in a class,

chiefly, I presume, in consideration of the audience and the

allotted time); and he is evidently quite aware of its special-

ized character, for he remarks that his notion "might equally

well be called 'the principle which dispenses with abstraction'." (6)

When, however, we come to deal with the fundamental

notions of logic itself, it becomes evident that these notions

(6) "Logic as the Essence of Philosophy," p. 42
     (cf. reference above, p.45, Note 3).

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are themselves truly abstract. We are not talking in ambiguous terms about any one of a class of objects, i. e. about any value of an ambiguous denoting symbol; we are talking about the ultimate common factor without which two members could not belong to the same class. Russell discards this common element on the ground that "we do not know whether the property exists"; to this we might reply that we do not know just what he means by a property, because he has not told us, but we do know that, whether the common element be correctly defined as a property or not, there is something common to our two terms. It may be a property or relation or what-not; at any rate, it unites the two terms in a class, or under a concept (the ambiguously denoting function). Now for all mathematical purposes, the fact that something may be asserted of an unspecified one of the several terms will serve to insure generality; but when as in logistic, we are looking for the condition in virtue of which such an ambiguous assertion is possible, we are looking for a perfectly definite and unambiguous notion. Such a condition might be said to lie upon a further "level of abstraction" than the terms which exhibit it (7).

(7) Nevertheless, even Wittgenstein, whose interests are philosophical rather than mathematical, perpetuates this confusion: "The concept number," he says, "is nothing else than that which is common to all numbers, the general form of number.
    The concept number is the variable number." ("Tractatus," 6.022).

I have borrowed the term, "level of abstraction,"
from Dr. Sheffer, whose short unpublished paper in the Harvard
Library (8) (to which I have already had occasion to refer)
is the only treatise known to me which definitely sets forth
the notion of what I have called real abstraction, to dis-
tinguish it from the symbolic abstraction exemplified in the
logic of mathematics. A little study of this paper, (which
unfortunately is too brief in relation to its novelty and
importance to be comprehensible without further ellucidation)
is indispensible at this point, in order to explain the mean-
ing and derivation of such "levels."

The process by which Dr. Sheffer proceeds from a
given class of entities to a "class-function," from a relation
to a "relation-function," from a particular system to a "system-
function" is not divergent from Russell's process of mathematical
generalization; in fact, such names as "class-function," "system-
function," are coined upon the analogy of the familiar "propo-
sitional function" of Principia Mathematica. When we deal with
a logical base ($K^n R_m$), we have before us a complex variable
structure--variable not only in respect of a certain class of sys-
tems, as it would be were we to write, e. g., ($K^3 R_2$), which
would ambiguously denote any of a class of systems having three
elements and a dyadic relation--but in respect of certain

(8) "General Theory of Notational Relativity"
        Cambridge(Mass.) 1921

classes of classes of systems, namely the class of systems having 3, 4, 5,....n elements and a dyadic relation, the class of systems of 3, 4, 5.....n elements and a triadic relation, etc. etc. Thus our symbol expressing the logical base depends upon four independently variable factors. And yet we are operating upon precisely the same "level of abstraction" as the logic of propositional functions.

We now come to the notion of validation--a strictly logical equivalent for Russell's dangerously metaphysical notion of truth (respectively falsehood)--and, as we might expect, the respective generalization, the validation-function or "validand"; and we are told that "a validation-function, α, represents, therefore, ambiguously, either the positive validatum, '+', or the negative validatum, '-.'" (9) This validand characterizes any and every possible combination of the elements of K, such as: "R a b c....," meaning either "R a b c...+," or "R a b c...-." Any such specific combination is a postulate on the logical base KR, and "R a b c... " denotes ambiguously two possible KR-postulates, namely the positive and negative determinations for "R a b c...."

But on the following page we find that combinations, such as R a a, R a b, R a b c, etc. may be characterized by

(9) H. M. Sheffer, loc. cit. p. 7

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alphas whose values are determined not with respect to their possible interpretations, but only with respect to each other. Thus we have "R aa$\alpha_{(a\ a)}$" and "R a b $\alpha_{(ab)}$" which may be determined with respect only to each other--i. e., so that either αa a = αa b, or αa a ≠ αa b, without introducing the range of ambiguous denotation of either alpha. Such an expression as "R a a$\alpha_{(aa)}$" Dr. Sheffer calls a "superpostulate" (10). Now when we establish a relation, like the one cited above, between superpostulates, we are dealing with the notion unambiguously denoted by "α", not with either value of α ambiguously. The ambiguity here belongs not to the symbol but to the concept symbolized. This concept, α, is not either "+" or "-"; it is the concept "either-plus-or-minus." And at this point I think we have proceeded from symbolic abstraction to conceptual abstraction--from the disjunctively given values of a variable to the concept of a disjunction (11).

Since the paper on notational relativity was written, Dr. Sheffer has, I believe, worked out certain further generalizations, such as that from an $R_{(m)}$ (an R of any degree) to

(10) loc. cit. p. 8

(11) This view may be contrasted with that of L. Wittgenstein, who holds that the abstracted concept is the variable. (See footnote 7, p.47)
cf. Tractatus Logico-philosophicus, 6.022: "The concept number is nothing else than that which is common to all numbers, the general form of number." The concept number is the variable number."

one truly abstract R-concept (12), which he denotes by the Greek letter "ρ", and which no longer involves the notion of degree. This is the sort of abstract concept which seems to me indispensible for the study of the fundamentals of logic; if fundamentals can be reached at all, they are presumably such "super-elements" (to coin a word on the analogy of "super-postulate") as the notion designated by "ρ" (which I think may be said to mean something very close to relatedness per se) or by a possible--though yet uninvented--abstraction therefrom. But as this part of Dr. Sheffer's researches is still unpublished and inaccessible I have no source whereupon to base such (probably premature) speculations. My purpose is served if the case of the super-postulate, involving inter-related though uninterpreted validands, illustrates the possibility of further "levels of abstraction" than the usual level of ambiguous denotation.

Mathematics, which is based on the use of variables, could probably be constructed partially on an alternative system of abstract concepts. The choice, in this case, of the "variable" level is a wise and natural one though perhaps not the only one possible. The fact is that <u>every abstract concept may be replaced by a variable</u>, though the converse does not

(12) It should be said in fairness to the author of that paper that (so far, at least, as the paper is concerned) he is innocent of all metaphysical notions about the status of R, R-concepts, etc.--"My sins upon my head!"

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necessarily hold (13). We may always pass from a superpostu-
late to a postulate, from this to an assumption--from a rela-
tion-concept (which Dr. Sheffer calls a "relation-function-
function") ("ρ") to a generalized relation-function ("R_m")
and from this to a special relation-function (such as R_2) and
finally to a specific relation (such as "beside," "within,"
etc.). In other words, although abstraction is difficult
enough, once the abstract idea is familiar, interpretation
is comparatively easy. And it certainly would be folly to
go further in the direction of psychological darkness and
metaphysical abstruseness than the subject-matter in question
demands. If as mathematicians we are able to work with a
"principle that dispenses with abstraction," we do not need
William of Occam to convince us of our good fortune.

It is interesting to note, however, that even
Mr. Russell may become involved in the subtleties of "real
abstraction." Readers of the first edition of <u>Principia</u> have
ever been puzzled about the exact meaning of the symbol $\phi\hat{x}$ (14).
The <u>variable</u> proposition, or propositional function, is rendered
by ϕx; in a recent talk with Prof. Whitehead I came to the con-
clusion that the meaning of $\phi\hat{x}$ cannot be rendered verbally,
at least under the grammatical restrictions of any language

(13) It has, for example, so far proved impossible
to abstract from "K" to a significant notion "K_1".

(14) Compare p. 40 (second edition).

known to me. But the new edition of <u>Principia</u> gives, if not
an interpretation of the symbol, at least a definition of its
meaning: "We may define a function $\phi\,\hat{x}$ as that kind of sim-
ilarity between propositions which exists when one results
from the other by the substitution of one individual for
another." (15) This seems to me to make $\phi\hat{x}$ a true abstraction,
for the similarity of propositions (more properly--that
structural element, common to several propositions, whereon
such similarity rests) certainly is not a variable, having
propositions or propositional functions for its values.

(15) "Princ. Math." p. XXX

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Chapter 5.

Propositional Logic

There is among modern logicians of almost every
school a tendency to treat the proposition as the fundamental
element in logic (1). Whether, like Frege, Russell and
McColl, we call this basic unanalyzable element by the objec-
tive name proposition, or regard it, with F. H. Bradley, as
a judgment, or with Husserl as an "act," we are talking about
the same thing--something which is either true or false.
Truth and falsehood belong essentially and exclusively to
propositions (or, to generalize our terms, truth-value belongs to
propositions or propositional functions(2) The old logic, which regarded
concepts as its ultimate constituents, yielded true or false
statements, just as any science does, but it was not a science
of truth and falsehood, any more than physics or geography.
Like every science, it was expressed in a body of propositions;
but these were its medium and not its matter. But of late it
appears to be the general concensus of opinions that proposi-

(1) cf. Louis Conturat, "Sur les Rapports Logiques
des Concepts et des Propositions" ("Rev. de Met. et Moral,"
Vol. 24) p. 15: "Tandis que la Logique classique reposait
tout entière sur la notion fondamentale de concept, la Logis-
tique considère cette notion comme complexe et derivée, et...
la subordonne à la notion de jugement, qui est beaucoup plus
générale et vraiment primordiale."
Also Hugh McColl, "La Logique Symbolique et
ses Applications" (Bibl. du Congrès Internat. de Phil." III)
p. 135: "Il est certain que les proposition, quelles soit simples
et indivisibles, ou complexes et divisibles, sont les unités
sur lesquelles nous basons, et avec lesquelles nous exprimons,
tous nos raisonnements."

(2) Ibid. p. 17: "Ainsi une fonction proposition-
elle est dans chaque cas particulier, susceptible des deux
valeurs: vrai et faux, et de celles-là seulement (comme
toute proposition.).

tions are logically prior to concepts (3.), and are thus the content as well as the form of logic; that therefore truth and falsehood are constitutive elements in its field, belonging to the material of the science as well as to its conclusions; and implication, which is the most important relation obtaining between propositions, must be a basic unanalyzed notion. The proposition must be taken as a whole, not broken up into concepts; for in the latter process it loses its character of being true or false, and can no longer figure as a term of an implication-relation (4).

This glorification of the implicative function of

(3.) W. M. Kozlowski, in a review of Kazimir Fwardowski's book, "Representations et Concepts," (Rev. Phil. Vol. 49 pp. 435-6) treats this general opinion as a definite conclusion: "Ce n'est que dernièrement qu'on est parvenu à la conclusion que les jugements participent à la formation des concepts. Quant aux relations qui existent entre les jugements et les concepts, il y a trois opinions diverses. Les uns, comme M M Suppé et Erdmann, identifient les concepts avec les jugements; les autres (M M Bergmann, Jerusalem, Wundt) considèrent les concepts comme résultats des jugements. Les troisièmes (MM. Rickert, Lipps, Bosanquet, Ribot) voient dans les concepts des jugements potentiels."

(4). cf. G. Frege, "Ueber Begriff u. Gegenstand," p. 205: "Nun haben wir bei der Beziehung dieselbe Schwierigkeit, die wir beim Begriffe vermeiden wollten; denn mit den Worten 'die Beziehung des Fallens eines Gegenstandes unter einen Begriff' bezeichnen wir keine Beziehung, sondern einen Gegenstand, und die drei Eigennamen 'die Zahl 2,' 'der Begriff Primzahl,' 'die Beziehung des Fallens eines Gegenstandes unter einen Begriff,' verhalten sich eben so spröde zu einander wie die beiden ersten allein; wie wir sie auch zusammenstellen, wir erhalten keinen Satz.'"

propositions is historically explicable, because logic has recently passed through the utilitarian stage (5), when its chief duty was supposed to lie in imparting new knowledge to scientists; in fact, this stage can hardly be considered altogether outgrown--Pragmatists still turn to the supremely impractical and intellectual discipline as to an oracle, and we have not entirely lived down the scandal called "induction." Now the old logic of concepts made implication appear as an analyzable relation dependent upon the inclusion of one concept within the other; this made obvious the fact that the conclusion was entirely contained in the premises.(6) Hamilton, by explicitly "quantifying the predicate," destroyed every illusion of novelty in the conclusion--he pulled the white rabbit out of the conjurer's sleeve and showed plainly that syllogistic reasoning was mere analysis.(7) Consequently the old logic of concepts sank in the esteem of the utilitarians, who were holding the philosophical field; not the concept, which is "static" and "sterile," but the proposition, which conveys real information, should henceforth be exalted.

(5) J. S. Mill, "System of Logic," (1st. ed.) p. 12: "If a science of logic exists, or is capable of existing, it must be useful."

(6) This viewpoint is defended by A. Sidgwick in his article on "Concepts and Meaning" (Mind, N. S. 4): "When we have two 'premises' and do not yet know exactly what conclusion they support, that fact implies some failure in our apprehension of the meaning of the 'premises' themselves. Their joint meaning really contains the conclusion." (p. 298)

(7) The "quantification of the predicate" is an old notion, dating probably from Theophrastus; but its recognition in modern logic is due to Hamilton.

While Hamilton and Jevons proved that conceptual logic was analytic, the algebraists reduced the logic of classes to a mechanical scheme, and arrived at the same results for extensional terms and relations which the conceptualists had reached upon the intensional view. Here, again, a calculus of propositions had to be brought to the rescue, and implication, with its promise of yielding new information, became the logical relation par excellence (8).

These are the developments which have set the stage for the logic of propositions—for Boole's "propositional interpretation" of his logical Algebra, McColl's "Calculus of Equivalent Statements," Frege's "Urtheilskalkül," and finally the elaborate masterpiece of Whitehead and Russell, "Principia Mathematica." There is now a well-established view that the study of propositions and of the relations which obtain between propositions is the only legitimate claimant to the title of "logistic," and is, in fact, formal logic itself. Prof. Lewis has called this the "orthodox" view of logistic (9).

(8) cf. Lewis, C. I. "La Logique et la Methode Mathematique" (Rev. de Met. et Mor., 29) p. 469: "Les travaux de l'école italienne, comme ceux qui s'inspirent de Whitehead et Russell fondent toute la logistique sur la rélation d'implication."
        Also B. Russell, "Princ. of Math", p. 11: "Symbolic Logic is essentially concerned with inference in general."

(9) C. I. Lewis, "Survey of Symbolic Logic," p. 354

Now if it be true that <u>logic deals essentially</u>
<u>with propositions and their interrelations</u>, it certainly is a
peculiar fact that the calculus of propositions was originally
nothing but an alternative interpretation for a system of
marks capable of other, more obvious and natural applications
(10). Conturat thought to fix the desired propositional mean-
ings, by adding a postulate which would rule out all other
contexts, the much debated "postulate 10" of his "L'Algèbre
de la Logique"; "X $\neq$ 1 < X = 0, X $\neq$ 0 < X = 1." (11)

At first it appears strange that this formula, which
sets forth a purely structural trait in an otherwise widely
adaptable system, should really <u>limit</u> the possible adaptations
of that system to a single one. As a matter of fact, every
other interpretation of Boolean Algebra that has as yet been
found, breaks down on the addition of "postulate 10" (12).
But the reason for this lies not in the unique character of

(10) A. Voigt, op. cit., p. 307: "So wie die
Objekte können nun auch die Urtheile und Begriffe zu einander
in Beziehung gesetzt werden, d. h. Urtheile gebildet werden,
deren Objekte Urtheile u. Begriffe sind."

(11) p. 85

(12) cf. C. I. Lewis, "Survey of Symbolic Logic,"
p. 66: "It is a curious fact that the one obvious law of an
algebra of 0 and 1 which Boole does <u>not</u> assume is exactly the
law which would have <u>limited</u> the logical interpretation of his
algebra to propositions. The law if X $\neq$ 1, X = 0 and if X $\neq$ 0,
X = 1 is exactly the principle which his successors added to
his system when it is to be considered as a calculus of propo-
sitions. This principle would have made his system completely
inapplicable to logical classes."

propositions and their relations, but in the fact that Conturat's axiom introduces into the system a structural fallacy, which the customary wording of the propositional interpretation happens to conceal, but which any other interpretation makes immediately obvious (13). We are then persuaded that the application cannot be carried out simply on account of some difference of form between the two systems we sought to co-ordinate.

The fallacy, which is hidden by the "particular" reading in terms of propositions and _truth-values_, shows itself upon a more precise and literal reading of the symbols than has so far been customary. Let us take Conturat's rule together with one of the most important theorems of the

Algebra: (a) $0 < X < 1$ (Theorem)

          (b) $X \neq 1 < X = 0$)

              $X \neq 0 < X = 1$) (Postulate 10)

The interpretation of (a) is usually written, "A false proposition implies any proposition and a true proposition is implied by any proposition." (14) The symmetrical appearance of the symbolic expression and the assymetry in the form of the rendering should make one suspicious of some inaccuracy in the trans-

(13) An attempt to construct another interpretation which should commit the same fallacies and thereby appear equally plausible, will be found in the appendix to the present chapter. (App. A.)

(14) This is the verbal interpretation given in "Principia Mathematica", where the symbolic expression reads: ✳2.02"⊢ : q .⊃. p ⊃ q" ("a true proposition is implied by any proposition") ✳ 2.21. "⊢ : ∼p . ⊃ . p ⊃ q" ("a false proposition implies any proposition").

lation. As a matter of fact, the correct wording is--"any false proposition implies any proposition, and any proposition implies any true proposition." This emphasizes the _variable_ nature of 0 and 1. In the Calculus of Classes this is not paralleled. The empty class and the universe class are constant entities. The second expression, however, cannot be read in like manner--we do not mean, "any proposition is any true or any false proposition," but--any proposition _is true or false._ The fallacy lies in using 0 and 1 both as indices of truth-value and as _terms_ in a calculus where every term has a truth-value. Prof. Lewis, following Frege, translates "0 $<$ X" and "X $<$ 1" as, respectively, "_the false_ implies any proposition" and "any proposition implies _the true._" This is, of course, a slightly ambiguous way of stating that a (or any, or every) false proposition implies any proposition, etc. But we cannot use "the false" and "the true" in the sense of the class of all false (respectively true) propositions, and then write "X $=$ 1." If 0 and 1 may really be used as _terms_ in the calculus, and we accept the postulate that any term X either $=$ 0 or $=$ 1, then we have not only a _two-valued_ algebra, but a _two-termed_ algebra, which would be perfectly valid but could hardly escape the charge of triviality (15).

(15) Frege regarded logical algebra as, strictly speaking, two-termed; every proposition being a proper name whose denotation is "the True" or "the False."
" ^{R} ^{f} $\underline{0}$ $_a$ – a' (nenne ich) einen _Namen_, weil es das Wahre bedeutet; es ist ein Eigennahme." ("Grundgesetze," p. 43)

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Thus it appears that Conturat's tenth postulate does not limit the possible interpretations of Boolean Algebra, but introduces a fallacy into the system which the propositional treatment happens to hide, but which becomes apparent by any other application. The calculus of propositions is, in fact, only one among several meanings that may be given to the abstract form developed by the algebraists. That it can thus be treated, is evident from the application of the algebra to propositions as Boole suggested it in his "Laws of Thought"; for he does not employ any postulate here which has not its counterpart in the calculus of classes or of relations (16).

Principia Mathematica makes another attempt to formulate a scheme which should lend itself only to the propositional rendering. Whether any other interpretation is possible I do not know; I have as yet never seen one that seemed entirely unobjectionable, but that is no proof to the contrary. It is interesting to note, however, that in so

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(16) G. Boole, "An Investigation of the Laws of Thought," p. 162; "Instead of classes of things, we shall have to substitute propositions, and for the relations of classes and individuals, we shall have to consider the connexions of propositions and events. Still, between the two systems, however differing in purport and interpretation, there will be seen to exist a pervading harmonious relation, an analogy which.....is of itself an interesting subject of study, and a conclusive proof of that unity of character which marks the constitution of the human faculties."

far as the calculus has any claim to uniqueness, this rests upon the same "two-valued" character as Conturat's; and the notions of truth and falsehood play the same equivocal parts that they played in the guise of 0 and 1. Only their machinations are much subtler here, for there are no symbols for "true" and "false." (17) If p is true, $\sim$p is false, and vice versa; but which is true and which false cannot be indicated. Sometimes the assertion-sign is taken as the index for truth--"if ' $\vdash$ (p $\supset$ p)' occurs, it is to be taken as a complete assertion convicting the authors of error unless the proposition 'p $\supset$ p' is true (as it is)." (18) Elsewhere, however, we are told that 'p is true' is not the exact philosophical meaning of " $\vdash$ p." (19) But granted, even that

(17) The same criticism applies to the works of G. Frege, upon which, in large part, Russell's logic of propositions was modelled. I have chosen Whitehead and Russell's "Principia Mathematica" for my "household example" simply because it gives the latest version of the idea suggested by Frege, and because its symbolism is incomparably more convenient than Frege's. That he too, however, was confronted with the problems of truth-value is obvious from passages like the following ("Grundgesetze der Arithmetik," p. 43): "Den Urtheilsstrich rechne ich weder zu den Namen noch zu den Marken; er ist ein Zeichen eigener Art.....(p. 50) So haben wir nun den Fall, dass der Satz ans dem Urtheilsstriche und einen Namen eines Wahrheitswerthes zusammengesetzt ist. Durch einen solchen Satz wird nun behauptet, dass dieser Name das Wahre bedeutet."

(18) Principia Mathematica, p. 9

(19) Principia Mathematica, p. 92: "The sign ' $\vdash$ ' is called the assertion-sign; it may be read 'it is true that' (although philosophically that is not exactly what it means)."

truth is expressed by the assertion-sign, there is no corres-
ponding symbol for falsehood; for if we translate "∼p" as
"not-p," or the proposition which contradicts p, then we may
write " ⊢ ∴ ∼p"--which, in fact, is a perfectly legitimate
expression according to <u>Principia</u>. Now the assertion of
"not-p" is not the same proposition as "p is false"; the lat-
ter is a proposition <u>about p</u>, and its contradictory is not
"p," but "p is true." <u>Neither in "p is false" nor "p is true"</u>
<u>is "p" asserted</u>; it figures in both these assertions as an
unasserted proposition, "as a subordinate part of an asserted
proposition." (20) The confusion arising from the use of
"∼p" to express the complement of "p," which is a term in the
calculus and may have either positive or negative truth value,
and the negative truth-value for "p", is evident when we try
to symbolize the statement that "∼p is true implies that p
is false." This can be stated only, " ⊢ ∴ ∼p ⊃ ∼p." (21)
Now this statement may be read in several ways--(a) "'p is
false' implies 'p is false,'" (b) "not-p implies not-p,"
(c) "'p is false' implies 'not-p is true,'" "'not-p is true'
implies 'p is false.'" The reason why this sort of equivo-

(20) Loc. cit. infra: In this respect, the symbolism
of the "two-valued Algebra" is superior to the compact econom-
ical symbolism of Principia; for its great variety of forms,
generally criticised as useless redundancy, allows us to draw
the distinction, thus: "∼p = 1 < p = 0."

(21) "Principia Mathematica" p. 92: "The sign '⊢'
is called the assertion-sign; it may be read 'it is true that'
(although philosophically that is not exactly what it means).

cation is not immediately apparent lies, of course, in the fact that all four readings yield valid propositions. Yet it is possible to construct a genuine contradiction due to the double meaning of the negating function, e. g., $\vdash : p \supset q$. This is, by definition, "$\vdash : \sim p \lor q$." The terms "p" and "q" are real variables, restricted only to the field of propositions of a certain type. Therefore "q" may take the value "$\sim p$." (22) Now if we read "$\vdash : \sim p \lor q$" as, "either p is false or q is true," and give "q" the value "$\sim p$", then if p is, in fact, not false, we must conclude that "if p is true, then not-p is true." According to the formal properties of the calculus, "$\vdash = \sim p \lor \sim p$" should be perfectly valid if "$\sim p$" can be asserted at all, i. e., if "$\sim p$" is a proposition (23). As long as we treat "$\sim p$" as the complement (contradictory) of "p", such contradictions cannot arise; it is only with the introduction of truth-value as something expressed by the symbols of the calculus, that contradictions (which always rest on unperceived changes of meaning) can arise in an otherwise consistent system.

Truth-value, however, is just the element that seems to set propositions apart from all other kinds of entity. Truth-value is indispensible if we are to have inference,

(22) p and $\sim$p are of the same type.
          cf. Principia:*1.71, "if p is an elementary proposition, $\sim$p is an elementary proposition."

(23) "Princ. Math."4.24

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and inference is deemed necessary for any system deserving the name of logic. It has been sufficiently pointed out that the calculus of implications given by <u>Principia</u> does not deal with "real" implication, but substitutes for it a relation which might just as well, if not better, carry another name (24). "Real" implication, expressed in the form--"a, therefore b," remains in the "al/ogical" background, and escapes symbolic fixation in logistic systems as well as in any other (25).

Boole in his propositional calculus does not use the notion of implication. He calls attention to the fact that "<u>a</u> never, or <u>b</u> always" is true whenever the relation of implication holds, and that in his calculus of truth-values it appears <u>instead</u> of the implicative relation. In his system, not every proposition equates to 0 or 1; hence the propositional interpretation does not require any unique postulates. The germs of "material implication" and "formal implication" are all well started in the Boolean system, but he does not confuse the conditions for an impli-

(24) Lewis, C. I., "La Logique et la méthode mathématique," p. 470: "Cette relation n'est pas celle que nous avons ordinairement dans l'esprit quand nous disons 'A implique B' ou 'B peut être inferé de A.'"

(25) "Princ. of Math." p. 35: "We need, in fact, the notion of <u>therefore</u>, which is quite different from the notion of <u>implies</u>, and holds between different entities....When we say <u>therefore</u>, we state a relation which can only hold between asserted propositions, and which thus differs from implication. Wherever <u>therefore</u> occurs, the hypothesis may be dropped, and the conclusion asserted by itself."

cation with the occurrence of it (26). Couturat, who prefers to deal with actual implication, takes it as a fundamental notion. He does not attempt to define it. But if we know that a implies b, we may deduce the theorem that if a is true, b is true--in his symbolism: "(a < b) < (a.b' = 0)," or "(a < b) < (a' + b = 1)." As soon as implication is defined-- as Russell and Whitehead define it in terms of negation and disjunction--it has lost its power, so to speak; it is no longer the relation which lets us infer that this year is the year 1925 because last year was 1924. What has taken its place is merely an ordering relation--transitive, non-symmetrical, and so forth.

The uniqueness and importance of "real" implication rests upon the fact that it allows us to infer something. There is, in the conclusion of an inference, nothing "new" except in relation to some apprehending mind; what is new is an idea (contained in the premises) which we had not separately recognized before (27). Psychologically, implication is

(26) G. Boole, "An Investigation of the Laws of Thought," p. 170: "To express the conditional proposition, 'If the proposition Y is true, the proposition X is true.'" "Since whenever the proposition Y is true, the proposition X is true, it is necessary and sufficient here to express, that the time in which the proposition Y is true is the time in which the proposition X is true."

(27) "Princ. of Math.", p. 33: "In the discussion of inference, it is common to permit the intrusion of a psychological element, and to consider our acquisition of new knowledge by its means. But it is plain that wherever we validly infer one proposition from another, we do so in virtue of a relation which holds between the two propositions, whether we perceive it or not."

the all-important, fundamental relation between assertions;
logically, it is a complicated, specialized and secondary one.

It is interesting to note how the logic of proposi-
tions, once it has been perfected to the degree of yielding
an almost faultless calculus, returns to the status which it
had at the beginning of its ambitious career--being one of
several interpretations of some purely formal system. The
psychological element of _assertion,_ which makes propositions
appear indefinably different from any other objects, notably
from propositional concepts, and the "true" implicative func-
tion which depends upon assertion (28) are both characteris-
tics which will not be caught in any calculus; but this is
not due to the shortcomings of formalism, but to the fact that
these are interpretational elements, which cannot be rendered
in abstracto any more than the sound of "one-lined C#" or the
feel of velvet. _Assertion is related to belief,_ despite Mr.
Russell's somewhat vague allegation that there is a non-psy-
chological sense of assertion; in this "non-psychological sense,"
"$\vdash = p$" simply means that p has some place in the system, and
the assertion-sign is an item of punctuation. In ordinary
writing, we take "assertion" in this sense for granted, and
instead of indicating it by any special device, we have a
symbol for non-assertion--the inverted commas which denote that
the word or phrase they enclose is either not to be believed,

(28) cf. "Princ. Math." pp. 34-35.

or is to be treated as a _term_ of the discourse. As for
implication, it loses all its peculiar significance if we
divorce it from the psychological element which Husserl has
distinguished as „Urtheilsgefühl"; what remains, in purely
formal terms, is a relation between terms (of any sort)
which validates or invalidates assertions, and several com-
plex relations between concepts, which underlie implication.
So these attempts which have been made at a unique _proposi-_
tional system have all been made at the expense of formalism;
and it seems rather unfortunate that logic should be char-
acterized by certain arbitrary alogical elements.

The search for "logical fundamentals" reveals the
inefficiency of a propositional system to furnish the neces-
sary analysis. The attempt made by Wittgenstein to discover
the philosophical basis of logic, ending as it does in per-
fect alogicism, or mysticism, is a fair illustration of this
fact. So long as we do not separate form and content to the
best of our ability, we cannot hope to find the _maximally_
abstract forms which are really indispensible to any and every
system. Russell himself has defined the "logical constants"
as the _forms_ which propositions (or classes of propositions)
have in common (29); now these forms _are not peculiar to_

(29) "Logic as the Essence of Philosophy."

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propositions. In so far as they are exemplified in the
propositional system only, they are special cases of more
generalized forms belonging to wider and wider systems.
And a process of true abstraction, which is inapplicable
to the propositional calculus without very quickly destroying
the nature of propositions and the implicative function, is
the only process that is likely to reveal the ultimate (or
at least provisionally ultimate) "data" of logic.

Chapter 6.

Logical Data.

In view of the preceding chapter, logic appears no longer as the study of the relations that hold between propositions and enable us to draw inferences, but as the study of forms regardless of their content. The material of logic, which is structure, is inherent in every possible thing, fact, situation--"orderliness and system are much the same in their most general characters, whether they appear in a Platonic dialogue, or in a modern text-book of botany, or in the commercial conduct of a business firm, or in the arrangement and discipline of an army, or in a legal code, or in a work of art, or even in a dance or in the planning of a dinner. Order is order. System is system. Amidst all the variations of systems and of orders, certain general types and characteristic relations can be traced." (1) It is through this approach that we shall find logic a deeper and broader science than the study of related propositions; the latter may interest the epistemologist, but the study of order, or as I prefer to call it, the study of forms (as I am not convinced that order is at all points an indispensible notion) is of primary importance in all philosophy-- ontology, cosmology, aesthetics, epistemology, and is, incidentally, the guiding principle of every empirical science (2).

(1) J. Royce, "Principles of Logic" (Eng. Transl.) p. 73

(2) cf. A. B. Kempe, "A Memoir on the Theory of Mathematical Form," p. 29: "Two systems which are of the same form have precisely the same properties, although the garbs in which they are severally clothed may, by their dissimilarity, lead us to place the systems under very different categories, and even to regard them as belonging to 'different branches of science.'"

In pure analysis we have no prescriptive proposi-
tions, such as Axiom 4 of Russell's "Principles of Mathemat-
ics": "A true hypothesis in an implication may be dropped,
and the consequent asserted," (3) or the rule for the con-
struction of classes and relations given in Principia Mathe-
matica: "Just as a class must not be capable of being or
not being a member of itself, so a relation must not be ref-
erent or relatum with respect to itself." (4) We arrive
only at descriptive propositions, expressing the character
of the system in question. The psychological or practical
effects of this character, e. g. that it "yields information,"
"allows us to infer," etc., are quite accidental and irrel-
evant so far as logical theory is concerned. Thus in a purely
analytic logic we do not take operations as fundamental, but
express in descriptive terms the relations which underlie and
validate them. Operations always give rise to a calculus; but
not every system is such that a calculus may be derived from it.
The relations which justify operations are always rather spec-
ialized and complex (being, in fact, such that an M-adic relation
corresponds to an (M + 1)-adic operation), (5) and therefore it
is an arbitrary curtailment of any analysis to regard an operation
as "undefinable." This is the principal limitation of Boolean

(3) p. 16
(4) p. 86
(5) Royce, "The Relation of Logic to the Foundations
     of Geometry" Trans. Amer. Math. Soc. Vol. 6

Algebras and Propositional Calculus. The limitation is justified by their purpose, which is to set up a system of inferences, inclusions, subsumptions or what-not, but it debars them from any application to the analysis of logic.

Prof. C. I. Lewis, in his "Survey of Symbolic Logic," maintains that the logic which he calls "heterodox" is distinguished from the "orthodox" type in that the latter depends for validity upon the meaning of its symbols and the <u>truth</u> of some of its own propositions, whereas the former kind depends purely on the observance of certain mechanical rules of operation, such as substitution. The rules always represent an arbitrary, "alogical" element (6). Now it is just the resort to "alogical" principles which seems to me to confine propositional ("orthodox") logic in unnecessarily narrow limits; is it true that "structural" logic also must face this metaphysical dead-stop as soon as it would reflect upon its own validity--must recognize some deus ex machina that governs its fundamental operations?.

Strictly speaking, <u>a truly</u> <u>descriptive logic has no</u> <u>rules of operation</u>. <u>Its character may be stated entirely in</u> <u>terms of relation</u>, and any reference to such notions as substi-

(6) "Survey," p. 361: "In fact, symbolic logic....<u>must</u> be developed by operations the validity of which is presumed apart from the logic so developed." (An interesting discussion follows of the circularity of justifying procedure by the postulates of the system itself.)

tution is only "mental shorthand," based upon our pragmatic
habits of thought. Kempe, to whom we owe the descriptive
method, uses the notion of substitution, but it is perfectly
feasible to dispense with this notion, as Dr. Sheffer has, I
think, conclusively shown. Both Kempe and Royce have suggested
rather than carried out the "descriptive" program. The idea
as they propounded it was still vague and controversial enough
to need a certain amount of justification, and the three or
four articles wherein they offer it are largely taken up with
the problem of mediating between the new view and the estab-
lished logistic. That is why they translate their notions as
quickly as possible into terms of logical operation--of con-
junction, disjunction, etc., The notion of substitution seems
fundamental as yet (7); but that is a weakness of Kempe's presen-
tation rather than of his system. We may do away with the pre-
scription that certain forms may be substituted for others, e. g.
(ab·c) for (xy·z), and recognize, instead, the fact that one is
here a replica of the other in respect of some relation (8).

(7) A.B.Kempe, "A Memoir on the Theory of Mathematical
Form," p. 48: "Each row of the tabular representation of a system
is derived from each of the others by definite substitutions.
Instead of writing down the various rows, we may give one only,
and state the law or laws according to which the other rows are
derived." The analysis of the system, however, is made without
the operation of substitution, and from the tabulated results of
this analysis the rules for substitutions are derived.

(8) A.B.Kempe, op. cit. p.29:"If a, b, c, d......and ⍺,
β, ɣ, δ ....be two systems of units such that a and ⍺ are unique,
with respect to each other, as also b and β, c and ɣ, etc., and
and if, a and b being any two units of the first system, when
a ⟩———⟨ b we have also a⍺ ⟩———⟨ bβ, then ⍺, δ, ɣ, ∂,....may be
called a replica of a, b, c, d,....."

If they are not distinct in the system, they may of course
be interchanged; but there is no occasion to interchange them
so long as we are merely analyzing the system, not aiming at
the deduction of special theorems; so the question of "sub-
stitution" becomes entirely irrelevant.

What holds for substitution holds also for other
operations--in fact, substitution is by far the simplest of
these, and is indispensible for all processes of proof. But
in analyzing a system we are not proving theorems; deduction
is always relevant to ignorance and special interest. Logi-
cally, a theorem which can be deduced from a set of premises
is simply a redundancy--a statement already given in the
premises, but more conveniently worded for us in the form of
the "theorem." To analyze the system means simply to determine
what entities it involves and what are all their interrelations.
These relations constitute the specific character of every
term involved; therefore a complete statement of them is a
description of the system.

The al/ogical rules of operation, then, are not fund-
amental elements of logic from the "heterodox" point of view.
But that does not finally settle our account with the mystics
of logical theory--those who insist that the basis of logic is

"al*l*ogical." (9) We still may hold, with L. Wittgenstein,

that there are relations which cannot be expressed by any

symbolism--the fundamental relations of any system whatever.

Mr. Wittgenstein's mysticism seems to me the result

of one great difficulty--that he attempted to discover the

fundamentals of logic through propositional logic. The first

thing he encountered was, naturally enough, the destruction

by analysis of what he considered his systematic unit--the

proposition (10). Then he encountered the notion of meaning,

which seemed again like an elementary constituent, but was

not to be found among such elements as analysis revealed. It

had followed its lord the proposition, into some metaphysical

limbo of the Inexpressible.

(9) See, for example, O. Hazay's essay, „Die Struktur
des logischen Gegenstandes" (Kantstudien, Ergänzungsheft 35)
p. 10: "Jedes System ist alogisch unterbaut, das eine mehr,
das andere weniger. Auf je mehr inhaltlichen Voraussetzungen
es beruht, um somehr Alogisches ist in ihm enthalten, aber auch
das relativ am reinsten formale System muss sich schon in seinen
Relationsgrundlagen an Alogisches klammern."

(10) The evils of regarding the proposition as the unit
of logical analysis is well pointed out by Miss Wrinch in her pa-
per, "On the Nature of Judgment." (Mind, N.S.4 4) On p. 325 she
says: "In a judgment, it is thought that the verb of the propo-
sition must function as a verb and not as an ordinary constituent.
Now there is a definite point in this criticism, and in bringing
forward any theory of judgment the verb of the proposition must
either function in a special way or some answer must be made to
this criticism. In the propositional theory of judgment the verb
functions in a special way...It seems to me that it is only as a
deduction from the assumption that propositions are unities that
one can hold that the verb must function in a peculiar way. Func-
tioning as a verb and not as an ordinary constituent means, it
appears, acting as a binder. Acting as a binder of certain con-
stituents means making them a unity. Thus the criticism seems to
be reducible to the criticism that the verb binds the elements of
the proposition together into a unity."

The analysis of logical System as such can only be
carried out through a series of abstractions from all possible
varieties of systems. The method first suggested by Kempe is
ideally suited to this purpose, because the purely descriptive
attitude puts us at the best vantage-point for viewing the
forms of systems and comparing one with the other. In this
way, their accidental discrepancies are most easily seen,
and by a process of constant elimination of unessential no-
tions we may arrive at those fundamental ones which have been
thought to evade formal expression.

The most formidable difficulty in analyzing a given
system lies in the fact that systems are usually given us as a
heterogeneous jumble of simple and complex relations, whose
terms now are irreducible entities, now combinations of such
entities, arranged according to several, and sometimes very
many, different relations, in a pattern as fantastic and confus-
ing as an old-fashioned wall-paper. To arrange and classify
the terms and relations in a system of any complexity is well-
nigh hopeless, and usually unprofitable. The surest way to
analyze any and every type of system is to select one relation,
dyadic, triadic, ----m-adic, which holds between some "elements"
of the system, and then array all the elements in such a way
that they may be tabulated in all their possible dyadic, or
triadic,---- or m-adic combinations; thus it will appear upon
inspection whether the relation in question holds or fails for

each individual combination. The relation in question may thus be taken as an _ordering_ relation, R, in the class of entities, K; and having determined (KR) we have a primitive fully-determined system (11). There may, of course, be several ordering relations, R, S, T, etc. of different degrees, so that the _given_ system may be described in dyadic, triadic, -----m-adic combinations. Thus our given system may be $(K^n \bar{R}_l, S_m, T_n, -----etc)$. Often it is possible to describe the same system entirely in alternative terms, i. e. in terms now of a dyadic, now a tryadic, tetradic, -----m-adic relation.

(11) For this method I am entirely indebted to Dr. Sheffer. The special application made of it by Kempe in the tabulation of elements of the "base-system," being restricted entirely to a triadic relation, or by Royce in his "Analysis of the System $\mathcal{E}$" in terms of tetradic O-relations and F-relations, have ex- hibited but not generalized this universal type of logical ar- rangement. On the other hand, such a logical procedure has been suggested, but not applied by Vittorio Benini in an article published in 1888, in the "Rivista Ital. di Filosofia." ("Dell' Analogia considerata dal punto di vista logico"). Benini's idea is too casual and sketchy to pass for an anticipation of the "an- alytic method," but it certainly contains the essence of the "heterodox" logistic, in its grandiose projects for an Analysis of the World. The whole passage (pp. 26-28) is well worth con- sideration, but for lack of room I quote only a fragment to set forth the theory: "In questa serie possiamo imaginare tanti gruppi distinti, ai quali si può applicare la legge suindicata, untandola secundo ill gruppo che si considera. ---Ora mettendo a raffronto tutti questi gruppi possianno calcolare il grado delle somiglianze e delle differenze, in ciò procedendo per analogia. Qualora si potesse stabilire questa serie suddistinta in gruppi e rigorosamente determinare simili gruppi, si potrebbe anche costruire una vera classificazione delle scienze fondata sulla essenza delle cose e non su criterj arbitrarj. Anche questa classificazione sarebbe frutto del metodo analitico."

We may then abstract from these several relations (in the case of an ambiguously expressed system such as "KR" I should say: relation-functions) to what Dr. Sheffer calls a relation-function-function, and designates by the Greek letter ρ. This may be imagined to stand for the notion of "Ordering Relation-Function" without reference to degree (12). Wherever we have a pattern, i. e. a system-function, we have some determinate form, and where such form is assumed but is ambiguously given, we are dealing with a system-function-function (Kρ).

From the system-function-function, or as I call it: the logical pattern as such, we may abstract one step further to the (K) where we do not assume the existence of a determinate form, i. e. where ρ has disappeared. Here, I think, we reach the minimum notion of relatedness, which not only every system,

(12) The generalization of a form, or true abstraction seems to be a very elusive idea. Mr. F. C. Russell expressed the extreme difficulty of the remoter levels of abstraction in a comment on E. T. Dixon's "The Foundations of Geometry": "A three-fold infinity of right lines differing in direction can be drawn in ordinary space, to each of which pertain two corresponding universals of direction, one converse to the other. Now the conception of direction that Mr. Dixon proposes for service as an elementary geometrical datum is the universal that subsumes all these lower ranking universals as particulars. Of course he has difficulty in even trying to explain what he means..... Owing to its excessive abstractness his conception is wholly unfit for service in elementary geometry. One has to become a good geometer before the conception can even be approached." ("Logic as Relation-lore," The Monist, Vol. 3, p. 279) Dixon, so far as we may judge from Russell's article (unfortunately I have not been able to procure Dixon's book) is exploring the remoter levels of abstraction which make the analysis of logic possible.

but any material for a system, that is, every logical material, must have--the entirely indeterminate notion of relatedness contained in the assumption of K--the "togetherness" of things in a universe of discourse. I call this primitive togetherness of elements, coessence. It may be argued that this is one of the notions which cannot be symbolically expressed; to which I reply, it is expressed in the symbol (K). (13) We have here gone beyond the notion of any ordering relation, Γ; "coessence" cannot strictly be called a relation, for it is implicit in every relation or relation-function, and its presence, being universal and necessary, does not contribute to the "pattern." We may consider it as the modulus of Relation.

This appears to be the ultimate--or at least provision-ally ultimate--substratum of all relations. But relations are not the whole material of logic (14); relations demand relata, or as Kempe has expressed it, units. Every system is based upon some collection, which I indicate by "K" (to signify that no ordering relation is here considered--a system, which in-volves more than a collection does, is expressed in its most general form by "(KΓ)". Now the units of a system may or may

(13) If it should prove possible to abstract further from the notion of K, then the symbol denoting the abstracted concept would express the notion of coessence.

(14) I am quite aware that a contrary point of view may be held, as for example, that of Olivér Hazay, expressed in his essay, "Die Struktur des logischen Gegenstandes."

not be such that we can distinguish one from another; i. e.
in a system (Ka, b, c ---n, R_m) we may have a = b or a $\neq$ b;
this is determined by the ordering relation R. If a (KR) func-
tion, say ϕ, that holds (or fails) with argument a holds (respec-
tively fails) likewise with argument b, then a and b are not
distinguishable in (KR). They are what Dr. Sheffer has
called "isotropic" in the system.

But the fact that we deal with two entities, a and
b, indicates that a and b must be in some sense distinguishable,
else we should write simply "a". That is why I have taken
care, above, to say: "not distinguishable in (KR);" "isotrop-
ic in the system." The sign "=" expressed equivalence, not
identity; equivalence, as the word implies, is a value-notion,
and value is always relative to some parameter. In this case
the parameter is the ordering relation (or relations) of the
system. In what sense, then, are a and b distinguishable?
In the sense that all their possible relations are not ex-
hausted in any system where "a = b" is significant. If (KR) is
such that two K-elements, a and b, are not distinguishable,
then there is some system (K'ρ) including the same two elements,
wherein a $\neq$ b. Thus we can derive a hierarchy of systems,
beginning with any (KR) which is not entirely "allotropic"
(or, in Kempe's terminology, not a perfectly "discrete heap"),
such that every successive system shall contain the same ele-
ments as the first (and possibly more besides), but each system

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yields more information about these elements than the one below it in the hierarchy. Ideally such a series of systems must reach a point at which every element is distinguishable from every other; this condition is implicit in the statement "a ≈ b," or else we are writing nonsense. When we speak of elements a and b, such that a = b, we are really transcending the limits of our system when we denote these elements by different letters. One of the difficulties Wittgenstein encounters is that of staying in the bounds of one system, rather than of staying in the bounds of logic.

But supposing that we write: "a = a.", Is not this a statement of identity rather than of equivalence? No, because the difference of sign which distinguishes a from b is not the only symbolization of distinctness. The numerical distinctness of a, a, a, a is just as clearly indicative of the diversity of the four elements named, as a, b, c, d. (15). In a relation "aRa" we are not dealing with <u>one</u> a, but with two, one being referent and the other relatum; we might properly write "a$_{ref.}$" and "a$_{rel.}$"--the use of "a" in both cases connoting, here, that the two terms are not otherwise distin-

(15) cf. Kempe, "On the Relation between the Logical Theory of Classes and the Geometrical Theory of Points" (Proc. London Math. Soc. Vol. XXI) p. 148: "The equivalent entities may be represented by like letters, a, a, a,...; or we may represent them by different letters and denote their equivalence by an equation, thus: a = b.",

guishable in the system than by their respective positions. This principle, which is the converse of the Identity of Indiscernibles, might be termed the principle of the Distinctness of Discernibles. Every logical system is such that either a = b $\underline{or}$ a $\neq$ b; I will indicate this disjunctive notion (which lies on the same "logical level" as the notion "$\neq$") by the symbol, $(\overline{\neq})$. Thus for any a and b in any system, we may write: "a$(\overline{\neq})$b." But when we have exhausted our hierarchy of systems we must come to the level where Ordering Relation is no longer assumed, and with this disappearance of the value-parameter the notion of equivalence-inequivalence also disappears, so that "$(\overline{\neq})$" becomes meaningless. We have reached the fundamental distinctness of units which justifies the use of just so many distinct marks on the paper. This primitive distinctness may be thought of as the "modulus" of Individuation, just as coessence is the "modulus" of Relation. Like coessence, it is given with every system; the distinctness of at least two terms (these may be a and a, where a = a) is expressed by the symbol R (or its derivatives by abstraction).

If there is any notion metaphysically prior to these "logical data," (as we may term the immediately given concepts that underlie all logical structure), it is the metaphysical datum of "some thing," of a "this," or perhaps of "Pure Being"-- whatever our philosophical creed commands. I will designate it as generally as possible, by the term "Entity." It will be

observed that any one of these logical data is indivorcible

from the others (16). Even "entity," unless it be thought

of as a sort of Schellingesque "Absolute" (which Hegel, who

steered perilously near the same Inferno, chose to call "die

Nacht in der alle Kühe schwarz sind"), must possess the rud-

iments of logical form--true self-identity and individuality.

--But I may be pardoned for consigning this to the metaphysi-

cian's science, of which I am at present innocent.--Certain

it is that coessence and distinctness are mutually implied (17).

        If there be any "alχogical" factors in logic, which

escape symbolic expression, these are the guilty ones, or at

        (16) Such indivorcibility is the criterion for the
really integral parts (what Husserl calls "Momente" in con-
trast to "Teile") of a situation. This criterion has been
previously employed by Schmitz-Dumont in his analysis of
Thought, where he distinguishes ten elements which are irre-
ducible one to another: "Jene zehn Begriffe sind correlativ
zu einander, d. h. ein jeder von ihnen erhält erst dadurch
seinen Sinn, das die neun anderen stillschweigend mitgedacht
werden. Betrachten wir z. B. die einfachste Formel derselben.
Dieselbigkeit = (a = a), a hat nur dadurch eine bestimte
Bedeutung, dass es von allem Anderen als a ...unterschieden
wird; ....die Formel a = a ist also sinnlos, wenn nicht die
Formel O(a,b) mitgedacht wird."

        (17) cf. F. C. Russell, "Logic as Relation Lore,"
(Monist Vol. III) p. 277: "This rigorously prime operation
of distinction is not only pure relation-ing but it is that
sort of relation-ing that is at once a distinguishing and a
conjoining."

least are among them. But I can see no reason for calling such
fundamental factors "allogical." They are the very warp and
woof of logic. Their expression, as Wittgenstein has pointed
out (only to add, thereupon, that they are "inexpressible")
lies not in any specific symbol, but in the structure of sym-
bolism.(18) They are not represented; they are presented (19).
All specific terms and relations may be individually lacking,
but where there is any logical structure there must be Entity,
Coessence, and Distinctness. Their symbols cannot be with-
drawn; because their symbols are the positions of all the
accidental (i. e. variable) symbols of the system. I shall
call this sort of expression "positional symbolization."

The level at which "positional symbolization" is
considered, at which the "logical data" are in evidence, is,
of course, prior to any system that allows of "operations."
We have here no relations complex enough to give rise to de-
ductions, substitutions, etc.; what there is can be known by
inspection. There is no process. So the question naturally

(18) Tractatus logico-philosophicus," 6.124:
"in logic it is not we who express, by means of signs, what
we want, but in logic the nature of the essentially necessary
signs itself asserts."

(19) Ibid, 4.0312: "My fundamental thought is that
the 'logical constants' do not represent; that the logic of the
facts cannot be represented."
         also 4.121: "Propositions cannot represent the
logical form: this mirrors itself in the props. "That which
mirrors itself in language, language cannot express."------
"The props. show the logical form of reality."-"They represent
it."

arises: how do we derive from this material the indispensible "laws of logic"? Must we not add arbitrarily at least Aristotles rules of identity, of contradiction, and of excluded middle? The answer is, that these are formulated for us in the logical data--they, like the moduli of form, are empirically given in the material. It is empirically impossible to write "a" and in the same position to write something other than "a." A thing is what it is; every symbol expresses this fact directly. It remains what it is, for if we write another symbol we are not writing "it," but another. A thing is or is not; we cannot--empirically--escape this condition. The explicit statement of these "laws" may be "allogical" and date from Aristotle, but their exemplification is present wherever anything is, and is older than the mind of man. They are positionally symbolized, i. e. presented, in every symbolism.

Pure analysis, then, brings us to no "allogical" basis of logical systems, but to a level which we might call "infra-logical" only to distinguish it from the levels at which we have specific patterns. Logic proper may be said to comprise every situation that has specific form, i. e. all systems and patterns of systems ("system functions"). Thus it includes any "level" at which ($\overline{\overline{7}}$) is significant. The "infra-logical" level is that level at which ($\overline{\overline{7}}$) is not significant. At the "infra-logical" level we have our primitive material, but no

possibility of "reasoning"; everything is obvious. With
the study of specific patterns begins what is usually called,
the study of Logic. .

III PHILOSOPHICAL PART

Chapter 7.

The Logical Basis of Meaning.

Since the analysis of logical systems has not revealed any "al/ogical" elements, we may dismiss the hypothesis that meaning is one of these supposed mystical ingredients of logic. Yet it certainly is a fundamental notion. A logic or mathematics without meaning, i. e. strictly a <u>mere</u> "<u>string of marks</u>," would be without theoretical interest of any kind. It would be "particular," and trivial into the bargain. The reason such a "string" is interesting even when it is not assumed to have any specific meaning, is that it <u>exemplifies things which are true of many systems</u>, any of which systems it may "mean." (1)

The "string," in short, is a pattern. And the systems it is capable of signifying are all cut by that same <u>pattern</u>, <u>though often of very diverse cloth</u>. Thus the calculus of classes, of concepts, and of propositions are all systems whereof Boolean Algebra is the pattern. Since any such calculus is itself ambiguously expressed, i. e. since its terms are

(1) cf. Frege, „Grundgesetze der Arithmetik," Vol. II p. 100-101: "Warum kann man von arithmetischen Gleichungen Anwendung machen? Nur weil sie Gedanken ansdrücken. Wie könnten wir eine Gleichung anwenden, die nichts ansdrückte, nichts wäre als eine Figurengruppe, die nach gewissen Regeln in eine andere Figurengruppe umgewandelt werden könnte!....Darum ist es billig, vom Arithmetiker diese Arbeit soweit zu fordern, als er sie leisten kann, ohne in jene besonderen Wissensgebiete (der Anwendungen) überzugreifen. Dazu gehört vor allen Dingen, dass dann so allgemein sein, dass er mit Hülfe der geometrischen Axiome, der physikalischen und astronomischen Beobachtungen und Hypothesen mannigfache Anwendungen in diesen Wissenschaften finden kann."

er mit sein Formeln ein Sinn verbind und dieser wi

variables, it is in itself a pattern rather than an actual
system; it is a "system-function", and the Algebra, in which
not only the terms but also the ordering relations are var-
iables, is a "system function function," a pattern whose
copies are patterns. The structural element they have in
common is what Mr. Russell has called (rather unfortunately)
"relation-number."

We have several terms to denote this relation--Dr.
Sheffer has called it "intervalence" between two or more
systems or system functions; Kempe calls one system a "repli-
ca" of any other system with which it is intervalent (2);
but for the structural character itself, by virtue of which
"intervalence" exists, by virtue of which systems can be
"replicas" of other systems, we have only Russell's term
"relation-number," which sounds rather formidable to the
unmathematical logician (3). I shall therefore, more simply
if less subtly, speak of it as "Configuration." (4)

(2) "Theory of Mathematical Form," p. 29

(3) B. Russell, "Introduction to Mathematical Philosophy, p. 56 ff

(4) Correspondence of configurations is the real
meaning of analogy. Thus we may gather from the foregoing
chapter that a logic of analogy, which certain writers (not-
ably A. Chide in his essay "La Logique de l'Analogie" and
V. Benini in his "dell' Analogia considerata dal punto di vista
logico") hail as the logic of the future, is not--as they be-
lieve--another method of inference; it cannot transcend the
given information, but can only discover the relations between given
materials. Only when we withdraw the demand that logic should
give new propositions for old, can we credit the words of Benini (op.
cit., "Rivista Italiana di Filosofia" Vol III, p. 26): "tre,....
sono le parti della logica-la deduttiva, l'induttiva, l'analogica;
la prima è gloria di Aristotele, la seconda di Frandesco Bacone,
la terza sarà di qualche filosofo dell'avenire."

Intervalence, or correspondence between configurations, may be simple or complex. Two systems, e. g. S and T, may be such that (a) for every element a in S there is one and just one element a' in T, such that a = a', and that (b) any relation R which holds between a and b in S, holds between a' and b' in T. The two are then replicas one of the other in the sense that two garments of the same style, material and size are replicas. Such systems are not distinguishable by any internal factors, but only by reference to something outside the systems; just as the two garments, if they were ideally perfected so that neither showed any accidental traits, would be distinguishable only by their spatial and temporal relations. Here the one system is a _duplicate_ of the other.

Or the two systems, S and T, may be such that (a) for any element a in S there is one and just one element a' in T, such that a = a' and that (b) for any relation R that holds between a and b in S, there is just one relation R' that holds between a' and b' in T. This sort of correspondence between configurations is exemplified by two garments of the same cut but of different sizes (or materials; for the molecular structure of materials enters into the "configuration" of the object, though it is easier for our purpose of illustrating "patterns" to ignore minute relations). Likewise we may have S and T such that (a) for every element, a, in S there is just one element a', _not equal to_ a, in T, and that (b) every relation R that holds between a and b

in S, holds between a' and b' in T. A good example of this situation is a melody rendered in different keys. Here every tone of the first rendering is different from the one which corresponds to it in the second; yet the relations are the same, and the melody is "the same," although it may sound different in the diverse ranges--pleasing perhaps in one, and not in another.

These are the simplest types of replicas--where both elements and relations of the two intervalent systems have mutually a one-one correspondence, and some feature is common to both systems--either the elements are actually equivalent, or precisely the same ordering relations enter into the two situations, or both. The case where no common relation or equivalent element exists, but the elements and relations of both show a perfect one-one correspondence, is a little more complicated, but still belongs into the same class, which I shall call the class of likenesses.

But there are intervalent systems where one element of the S corresponds to some group of elements in T, or where for every relation R in S there are several relations X, Y, Z, in T; or for a group of elements or relations in S there is but one class or relation in T, etc. etc. We may have any degree of complication. The two systems will then be very different in appearance, i. e. they will not appear as like-

nesses; only logical analysis will show their intervalence.
A highly complicated example of such correspondence between
configurations may be found by comparing the world of (actual
or possible) sense-experience with the world of physics. We
can hardly recognize the one in the other upon mere inspec-
tion; yet it is the world of sights, sounds, etc. which the
physicist describes in terms of infinitesimal motions, and
B. Russell in his essay, "Sense Data and Physics," has made
a brilliant though sketchy attempt to reverse the process (5).
In fact, it has been said that the great task of the metaphy-
sician is that of mediating between the various systems which
command our belief (6).

It is interesting to note that, by integrating units
and groups of units more and more, any system, however complex,
may be brought into correspondence with any other system, no
matter how simple. This rests upon the fact that there are
certain fundamental characters--which I have (without any claim
to finality or exhaustiveness in the matter) here determined

(5)   The method here suggested has been utilized by
subsequent writers--compare C. D. Broad's "Perception, Physics
and Reality."
        The most thorough-going research on the problem
of intervalence between the worlds of physics and of sense, is
Prof. Whitehead's "The Concept of Nature."

(6)   R. F. A. Hoernlé, "Studies in Contemporary Meta-
physics," p. 15:   "The task of philosophy is to point out by
analysis of the bearing upon each other of conflicting types
of experience, how their conflict is actually overcome, and
how, therefore, in principle it admits of solution."

as <u>coessence</u> and <u>distinctness</u> of the material, or <u>entity</u>, in question--fundamental characters common to all systems. We can always invent some classification of units, some general-ization of relations that will make the more complex system intervalent with the simpler one, though we should be forced to the utmost degree of abstraction--even to the "logical data" which constitute the <u>universal basis of Structure</u>. If (p. 84) we limit ourselves to finite systems, this is obvious, since we may bring collections of any finite cardinality into one-one or one-many correspondence; or if we consider one infinite and one or more finite systems, for then we can divide the infinite system into a finite number of infinite groups and correlate these with elements or finite groups of the other system or systems. Whether two or more infinities can always (or even sometimes) be brought into correspondence I would not venture to say (7).

Thus it appears that, whether we consider the universe as finite or infinite, we may <u>legitimately regard any finite structure as a microcosm</u>, if our philosophical taste calls for such an analogy.

(7) The discovery of configurations and correspon-dences between configurations has had a very gradual develop-ment. It seems to present peculiar psychological difficulties. Its classical expression is, of course, the "ambiguity" of Boolean Algebra. This is not, however, its first appearance. In 1847--the year of publication of Boole's first book ('The Mathematical Analysis of Logic'), too early for possible plagiarism, A. A. Cournot published his work "De L'origine et des Limites de la Correspondance entre L'Algèbre et la Géométrie," wherein we find passages like the following:

(p. 138) "Il (Descartes) voulait appliquer l'algèbre à la géométrie; mais il est clair que par cela même il donnait aussi une méthode uniforme pour appliquer la géométrie a l'algèbre."

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(p. 145) "Non-seulement l'idée de Descartes etait féconde en consequences importantes, soit qu'il sagit de l'application de l'algèbre à la géométrie ou de celle de la géométrie à l'algèbre, mais elle contenait le germe d'une abstraction plus élevée, d'une théorie qui domine par sa généralité l'algèbre comme la géométrie."

(p. 364) "Toutes ces prétendues conventions ne sont-elles que l'expression necessaire de rapports que l'esprit est obligé sans doute de representer des signes de forme arbitraire, mais qu'il saisit seulement, en vertu de la puissance qu'il a de généraliser et d'abstraire? Voilà ce qui partage les géométres en diverses écoles; voilà le fond de la philosophie des mathématiques, et c'est aussi le fond de toute philosophie."

(p. 397) "Aussi, non-seulement les concordances et les discordances peuvent se rattacher a des chefs généraux, se classer et s'enchaîner logiquement, mais au fond la distinction même des chefs de concordance et de discordance se rattache à une corrélation fondamentale dont nous pouvous entrevoir la raison philosophique, tenant à ce que les nombres et l'etendue figurée manifestent à leur manière les mêmes idées fondamentales dont le type est dans la théorie générale de l'ordre; ce qui nous met sur la voie du sens voilé, mais profond, de ce mot de Pascal: 'La nature s'imite---Les nombres imitent l'éspace, qui sont de nature si différente.'"

It is interesting to find that Cournot takes his first suggestion from Descartes, who, however, was quite unaware of the philosophical implication of his mathematical stunt.

Intervalence, besides being practically valuable
because it allows us to classify large numbers of correspond-
ing systems together, and describe innumerable situations by
the formulae of one mathematical structure, is metaphysically
important because it is the entire logical basis of meaning.
In so far as meaning contains a formal, or strictly "relation-
al" ingredient, this ingredient is the common configuration
of symbol and object. It is by virtue of this relation that
the system of physics "describes" the world we know through
sense; its formulae mean our world.

A strictly logical analysis such as this professes
to be, can never determine the empirical existence of its
object, but only its possibility. This is true of "meaning"
as it is of anything else. Intervalence determines the pos-
sibility of meaning, not its actual existence; the latter
depends upon extra-logical factors--upon cognitive ability
and pragmatic interests. Meaning never exists apart from a
mind. It always involves a psychological act. Unfortunately
our language has only the one word, Meaning, for both the re-
lation between symbol and object, and the mental act. The
Germans have reserved the word "meinen" for the subjective
sense and generally use "bedeuten" for the part played by the
symbol (8). But that character by virtue of which anything

(8) The use of „meinen" for „bedeuten" is sometimes
found in German literature, but is strictly speaking a vul-
garism; so that the philosophical distinction may be fixed
by mere observance of the better usage.

can be a symbol of something else, is intervalence between the two things in question.

Intervalence is a symmetrical relation. Therefore we have no logical reason to regard A as a symbol for B rather than B as a symbol for A (9). Our choice in this matter depends upon conditions of expediency. For instance, we find in almost every Sunday newspaper a little diagram consisting of a square field equally divided into 64 smaller squares, and dotted with various fantastic little marks. To a devotee of the chessboard, this symbolizes a chess-situation and suggests certain moves by which the game advances; to the uninitiate it is a highly intricate disposition of little marks on paper, quite impossible for his mind to retain. If, however, we explain to him the process of the game, introduce him to the figures, etc. and show him the chess-situation on the board which corresponds to the hieroglyphics in the picture, the latter will become clear at once, and its relations be comprehensible; for we have explained it through symbolizing it on the chess-board. To a blind person, the picture can be known only through this or

(9) Husserl does not subscribe to this view, as we may judge from the following passage in his "Logische Untersuchungen" „Jedermann bekannt ist aus seiner inneren Erfahrung die Ungleichwertigkeit der beiderseitigen Bestandstücke, worin sich die Ungleichseitigkeit der Relation zu dem Ausdruck und dem mittels der Bedeutung ausgedrückten (genannten) Gegenstand spiegelt."

some equivalent symbolism (10). Similarly, we usually re-
gard a stage-drama as a picture of real life--as symbolic
of a real situation; the motions of the actor who slides to
the ground at the touch of a dull rapier-point, mean the
death of the person whom he represents. We know that he,
the man on the stage, is not Prince of Denmark, is (we hope)
not melancholy, and does probably ~~does~~ not love the lady
strewing flowers on the floor; but he symbolizes, he means
Hamlet. The whole play means serious, human life. Yet such
a meaning-relation may be reversed, as when one observes,

> "All the world's a stage,

> And all the men and women merely players;

> They have their exits and their entrances."

Yet, ordinarily, we are not in doubt as to which,
of two situations, is referent and which relatum in the mean-
ing-relation. What determines their respective parts? The
answer is, value and perceptibility. We use symbols, usually
of no intrinsic value whatever, such as a string of marks or
a succession of noises, which are easily perceived in all
their relations to one another, to symbolize things of real
value which are temporally or spatially too remote, or too big,

(10) The chess-game itself is a piece of symbolism,
as a little reflection upon the names of the pieces and their
characteristic moves reveals. The pieces represent a mediaeval
social order, and the progress of the game is planned so that
it symbolizes the progress of a political conflict.
    A cynic, reflecting upon a battle, might just as
well say that "it is all a mere game of chess." We would then
say that "a game of chess" is all the battle means to him.

or in themselves too complicated to be apprehended and analyzed by direct inspection (11). Logically we are simply considering one of two intervalent systems (12); we might as well have chosen to contemplate the other. But psychologically we are using that which is uninteresting but easy to grasp, to symbolize that which is interesting but evasive.

The relation between symbol and object, or of intervalence between two systems, presupposes a system which comprises the two in question. There is nothing about this intervalence-relation, or meaning-relation, that is not purely logical--a relation of forms, considered as pure forms, in a system. The meaning-relation does not offer the difficulty of form and content touched upon in Chapter II. Interpretation is not an act of importing a content into a given form; it is the act of relating a form with one specific content, to the same or another form, with another given content. Either or both forms may be as "abstract" as we like; as we may interpret two

(11) cf. John Laird, "A Study in Realism," p. 34: "The usefulness of a sign lies in the ease and celerity with which it can be used as a substitute....we use a sign because it has the same logical properties as the things it signifies."

(12) In the case of a single entity serving as a symbol, we may view it as the simplest type of "system"--a system of one element and the ordering relation of self-equivalence.
cf, C.I.Lewis, "Survey," p. 355: "There is no theoretical reason why a single mark may not, in some cases, be recognized as a 'string.'"

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symbolisms to "mean" a highly abstract idea, and thus indirect-
ly to "mean" one another.

Speech and Notation are collections of very uninterest-
ing things (noises or marks), but capable of various and highly
complex relations. Considered apart from their meanings they
are, of course, utterly trivial; yet even a valueless collec-
tion is a system--it has configuration. Therefore it follows
that anything we can symbolize by speech or notation or any
other device is not unique in its configuration, and converse-
ly, if there be a situation which has a unique relation-number,
it is indescribable. But it seems almost certain that ideally,
at least, some symbolism could be invented for every configu-
ration; so it is highly probable that there is no situation
having a unique configuration.

Perhaps the apprehension of a unique system is the
basis of mysticism; perhaps Mr. Wittgenstein's inability to
express his experience of the world as a whole is due to the
fact that this object, of whose existence he seems convinced,
has (of course) no replica. Certain it is that if there were
a unique system, our knowledge of it (if we could have knowl-
edge, i. e. more than momentary experience, of it) would re-
quire "imageless thought," and would at all events be incom-
municable.

Chapter 8.

Denotation, Connotation, Signification and Description.

Before venturing upon a phenomenology of Meaning, we may do well to remark the most general division in the universe of our subject--the division into <u>meaning as denotation</u> and <u>meaning as connotation.</u> This division is obvious enough to have impressed even the unphilosophical writers of text-books--perhaps because it was observed by the schoolmen. In the classical logic it appears as the distinction between the <u>extension</u> and <u>intension</u> of terms; Frege distinguished between "Bedeutung" and "Sinn" (1); Husserl between the <u>content</u> (Inhalt) and the <u>object</u> (Gegenstand) of a word (2). The last-named distinction will presently prove very useful because its unsymmetrical terminology indicates the fundamental discrepancy between the two senses of "meaning." The one kind, namely "Inhalt," intention, seems somehow to belong to

(1) "Grundgesetze der Arithmetik." "Rechtmässig gebildete Namen müssen immer etwas bedeuten." (p. 45): „Aber nicht nur eine Bedeutung, sondern auch ein Sinn kommt allen rechtmässig aus unsern Zeichen gebildeten Namen zu. Jeder solche Name eines Wahrheitswerthes <u>drückt</u> einen Sinn, einen <u>Gedanken aus</u>... Die einfachen oder selbst schon zusammengesetzten Namen nun, aus denen der Name eines Wahrheitswerthes besteht, tragen dazu bei, den Gedanken auszudrücken, und dieser Beitrag des einzelnen ist sein <u>Sinn</u>. Wenn ein Name Theil des Namens eines Wahrheitswerthes ist, so ist der Sinn jenes Namens Theil des Gedankens, den dieser ausdrückt." (p. 50)

(2) "Log. Untersuch." Vol II, p. 37: „Sehen wir nun von den Erlebnissen, die speziell zur Kundgebung gehören, ab.... so scheint zweierlei übrig zu bleiben: der Ausdruck selbst und das, was er als seine Bedeutung (als seinen Sinn) ausdrückt. Indessen hier sind mehrfältige Relationen miteinander verflochten, und die Rede von dem <u>was ausgedrückt ist</u> und von <u>Bedeutung</u> ist dementsprechend eine vieldeutige."

the symbol itself (3)--to be the more interesting part of "meaning," not dependent upon the accident of existence of some external object. It is this aspect which gives rise to the theory that meaning is a _quality_ of symbols, not a relation. The very words signifying it suggest that it is entirely contained in the symbol. The other kind--„Gegenstand," extension--seems, on the contrary, to belong somewhere beyond the symbol; it depends upon the existence of external objects, and has an arbitrary appearance--giving rise to the belief that a rose by another name were just as sweet (a proposition doubtful to psychologists), and to the theory that meaning is always a relation between symbols and their objects (4). Now the two sorts of meaning are hard to keep apart, since some words which are not mere babble seem to denote no objects, others (strictly proper names) _are_ mere babble unless coupled with their objects, and still others which are significant in their own right also happen to denote objects. To the first

(3) cf. F. C. Russell, "Logic as Relation Lore," (p. 281): "Relations are attributive predicates of terms and each one of them pertains strictly to its proper term or combination of terms, in the same sense for this turn (_pro hac vice_) that qualities are held to pertain to their so-called substances."

(4) cf. B. Russell, "Principles of Mathematics," p. 53: "The fact that description is possible--that we are able, by the employment of concepts, to designate a thing which is not a concept--is due to a logical relation between some concepts and some terms, in virtue of which such concepts inherently and logically _denote_ such terms."

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class belong all words signifying abstractions, such as "goodness," "meaning," etc.--things which never exist in toto as individuals; names of non-existent things, "Venus," "Atlantis," "the Millenium"; also propositional concepts which do not denote a fact--i. e. such propositional concepts as correspond to false propositions. (This class includes Meinong's "impossible objects"--the round square, wooden iron, etc; since these concepts are expressible by false propositions, e. g. " $\vdash$; $(\exists x) \cdot x$ is square and x is round.") To the second class belong proper names without connotation, i. e. "Jack," "William," etc.--names arbitrarily given. Note that names signifying just one person, "Socrates," "William II," "Napoleon I," etc., are in this function connotative. They are interchangeable with "the philosopher who drank the hemlock," "The last German Emperor," "The prisoner of Elba." When "Jack" is given definite denotation, it also acquires connotation; but without denotation it is meaningless. The third class contains the vast majority of symbols--having both connotation (i. e. being capable of definition) and denotation (i. e. functioning as names of things). So the division of terms into connotative and denotative does not yield a dichotomy, and consequently will not serve for classification. The two sorts of meaning seem to be incommensurable, one depending upon a quality, the other upon a relation.

But upon closer inspection we find that both are
dependent upon relations, that the connotation of a term
has one relation to the term, and another to the denota-
tion (object) of the term, and that the latter relation
is not precisely that of the term itself to its object.
To make this variety of relations evident we must invoke
the old distinction of a class as one and a class as many;
of a system as a unit and a system as a complex (5).

The content or intension of a term is its defin-
iens. (6) If dictionaries were perfect, the content of
a term would be its "dictionary meaning." Now a definition
is a set of symbols interchangeable with the word defined;
definiens and definiendum stand to each other in the rela-
tion of an "ordered heap" regarded essentially as ordered,
i. e. as having terms with some sort of arrangement, and an
"ordered heap" regarded essentially as heap--as a unit, un-
analyzed. Therefore, wherever the set of symbols is used as a

(5) For the distinction between class as many and
a complex--cf. Husserl's distinction between "Stücke" and
"Momente." (Log. Unt. II (I)) p. 266: "Wir fixieren zunächst
eine fundamentale Einteilung des Begriffes Teil, nämlich in
Stücke oder Teile im engsten Sinne, und in Momente oder
abstrakte Teile des Ganzen."

(6) This relation of intension holds equally among
symbols and among concepts. The symbol itself is usually
ignored in favor of the concept; the first writer to my knowl-
edge who has recognized the possibility of regarding intension
as a characteristic of symbols is William Thompson, who in his
"Outline of the Laws of Thought" distinguishes between three
aspects of propositions, instead of the older-fashioned two--
namely, extension, intension and denomination. The first of
these concerns the actual things denoted by the terms of prop-
ositions; the second, the concepts constituting propositions;
the third, the symbols expressing them.
cf. Part III, Paragraph 105.

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whole, it may be dispensed with and the term it defines may

be substituted for it. Thus connotation regarded as a rela-

tion is the exact converse of definition--the _definiendum_

connotes the _definiens_, and the _definiens_ defines the _defin-

iendum_. Both relations hold between symbols (7); they may

occur in a formal system, which is to say that they hold

within any set of concepts which is an interpretation of the

symbolism.

Denotation, on the other hand, is a direct one-one

relation between a symbol and an object (" Gegenstand"). Both

the symbol and the object are capable of analysis, but are

taken, in this relation, as units. It is not correct to say

that a denoting symbol "points to" an object; for where the

symbol figures the object is dispensed with, and vice versa.

In answer to a question such as "who did this deed?" we may

name the person, or point him out, but if the questioner un-

derstands that the name belongs to that person, our pointing

out is unnecessary, or if he has seen us point, the name is

irrelevant. We say "John did this deed," when it is not con-

venient to let John himself figure in the situation we would

(7) Wittgenstein, "Tractatus", 3.24: "The combination
of the symbols of a complex in a simple symbol can be expressed
by a definition."
See also "Princ. Math.", p. 11: "A definition
is concerned wholly with the symbols, not with what they sym-
bolize."

create. Words such as _this_, _that_, _here_, _there_,--in short, all demonstratives--have the special function of filling up the formal gap in a symbolic system, such as a proposition, when some symbol is omitted and its object substituted for it. "This is a splendid view," is not really a proposition; for "this" by itself is meaningless--it simply takes the verbal place left vacant by substituting an actual _thing_ for a name, and we would express the same situation by indicating a vista by some eloquent gesture and exclaiming: "A splendid view!" (8) In such cases only does the object of a proposition figure in the proposition itself. But if we say, "Vesuvius, from Sorrento, is a splendid view," Vesuvius (from Sorrento, or per se) does not, as Mr. Russell maintains, figure in the proposition. It is the phrase "Vesuvius, from Sorrento" that figures in the word-structure, and any interpretation of this word-structure must contain parts in such relation that the phrase finds therein its analogue; this may be an interpretation in terms of pure concepts, in which case there must be a concept of Vesuvius and a concept of Sorrento, etc. in the prescribed relations; or in terms of images, in which case we must have images answering to the words of the formal prop-

(8) "This" has the same "unfulfilled" character as Frege's symbol "ξ" or the "X" of mathematical functions; "Der Buchstabe 'X' dient nur dazu, Stellen offen zu halten für ein Zahlzeichen.....Ich nenne diese Stellen _Argument-stellen_." (Frege, "Grundgesetze" p. 6.)

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osition (there may be an infinite number of such sets of images); or the situation itself, namely, the splendid view of Vesuvius from Sorrento, may be one such interpretation. But never is the interpretation a part of the logical form-- where the object is made to figure in the proposition, the proposition is not capable of complete interpretation. That is what I meant by saying, above, that such a structure is not a complete symbolism, since one of its terms is not a symbol. Therefore we put a syllable, such as "this," in place of the object, to preserve the symbolic character of the whole proposition. ●

Denotation, then, is the relation by virtue of which one entity may be substituted for another. It is a relation between two terms, a name and a thing. This is the simplest sort of "meaning," where the symbol serves as a "handle" to its object for purposes of mental manipulation. In the case of names having no connotation in themselves, this relation may be set up with any object whatever for relatum; but where a connotation exists--i. e. where the symbol is itself definable--the scope of possible denotations is limited (9). Here we come to the third type of meaning-relation, which holds between the intension of a term, and its object.

(9) This reciprocity between intension and extension has been clearly pointed out by E. T. Dixon in an article "On the Distinction between real and verbal propositions." (Mind, N. S. Vol. II, p. 341): "We may lay it down arbitrarily that

Just as an analyzable term in a symbolism may be taken as a single entity or as a set of definitely related other terms, so an object may be taken as a single entity or as a complex of characteristics in definite relations. Thus where a name connotes a content and denotes an object, both symbol and object are systems; and in order to establish a valid meaning-relation, a "correct" use of the symbol to name that object, these two systems must be likenesses. A direct denoting relation must hold between their elements, i. e. between the term as defined and the object as a complex, or a "class as many," in order that the term as a unit may fitly denote the object as a unit, or a "class as one." When we argue about the proper use of words for certain things, we rarely disagree about the acceptability of arbitrary names, known as "good usage"; it is practically always a question of the relation between the "content" of a word, and the structure of its object; that is to say, whether--the intension being fixed---the extension of the term covers the object.

(note 9, preceding page)
a given term shall denote any given set of things, or connote any given set of attributes, but having done so it is no longer in our power to determine what attributes it shall connote, in the first place, and what things it shall denote in the second. The arbitrary part of the meaning of a term I shall call its definition, and the remaining part of its whole meaning its import. Thus by its definition I mean either the extension of the term or its intension, whichever is laid down arbitrarily, including....every item which may be formally shown to be implied by that assertion."

in question, or not. The relation of the intension of a term to its object is called description. In this sense, not only one symbol may "mean" one object, as in the case of names, but a system of symbols may mean one object--that is, in the sense that a descriptive phrase "means" the single object it describes (10).

There is a further relation which holds between an element of the intension of a symbol, and a constituent event in the event which is the object of that symbol. This relation is the non-psychological basis of suggestion, and might be called "signification." It is a reversible relation and is, in fact, often reversed for practical purposes (11), as we shall later have occasion to note in observing specific phenomena of meaning. Signification is the sort of meaning which attaches to <u>signs</u> (as distinguished from symbols). The reversibility is more marked in non-verbal meaning-situations than description, because in such situations the one system is less categorically marked

(10) L. Wittgenstein, Tractatus: 3.24 "A proposition about a complex stands in internal rel. to the prop. about its constituent parts."

(11) The sign-function, which may attach to words but more often attaches to other things, makes the reversibility of the meaning-relation more evident than any other type of meaning, because in non-verbal meaning-situations the symbolic character is not so obviously stamped upon one of the systems in question as it is in situations where one such system is verbal.

as the symbol and the other as the object, than in those where words are employed. It also depends upon intervalence; but with the distinction that the intervalent systems are here not contained in the term (as its intension and its object-as-complex) but the term and its object are contained respectively in the two intervalent systems. Take, for example, a sign of low air-pressure--the fall of the barometric index to 28 in.. This sign-function of the barometer depends upon the fact that the system of air-pressures and the system of barometric behaviour are intervalent, i. e. that for every event, a, in the pressure-system, there is a correlated event a' in the system of barometry. The sign-relation holds not between the two systems, but between a of one and a' of the other.

Thus we can see that many meanings of Meaning are contained in connotation and denotation, which are indeed complex relations, though by no means as artificial and dangerous as Messrs. Ogden and Richards would have us believe. They are, in fact, very interesting ones, giving rise to several types of meaning, and are confusing only to superficial and inexact contemplation. On the basis of these relations, a term "means" (a) its intension; (b) an object it arbitrarily names; (c) an object it describes; (d) an object it signifies (correctly, suggests, allows us to infer). (When I say that a symbol or sign means an object, I am of course

taking object in the broadest sense, of "space-time event," (12) to include facts, notions, feelings, qualities, etc. as well as things.)

Meaning always involves a psychological factor. A word (or other symbol) may be capable of meaning a certain object, or a certain intension,--all this possibility requires is the logical structure--but it does not actually mean it until a mind apprehends the existing relation and adds the arbitrary act of reference which establishes actual meaning (13). The two aspects are best distinguished in the German terminology which separates "meinen" and "bedeuten". A „Bedeutung" is possible wherever correlation exists, and to just the extent to which this correlation is feasible; but only where Meaning in the sense of „meinen" is superadded, does the „Bedeutung" become actual. There are, then, three relevant factors in an actual meaning-situation: (a) the type of logical relation making it possible, (b) the mental act of recognizing or using the relation, (c) the nature of the terms between which relation holds. Obviously, when any one of these is altered, the actual

(12) I take this notion from Prof. Whitehead. cf. His "Enquiry" and "The Concept of Nature".

(13) cf. J. Laird, "A Study in Realism", p. 209: "When symbols express facts.....this expressiveness can be recognized and known only if symbol, fact, and the relation between them is recognized and known."

situation is altered. Now I do not assert that there are only four "meanings of Meaning,"--in fact, there are certainly more, perhaps nearer forty than four--but I do maintain that the four abstract situations adduced above, with their converses and possible hybrids, exhaust the logical types of meaning and consequently supply the formal basis for every empirical class. (14)

(14) See Appendix B, p.160, for diagram of meaning-relations.

Chapter 9.

The Catholicity of "Meaning."

To formulate an even fairly exhaustive phenomenology of meaning is not an easy task; perhaps the most successful attempt is, so far, that made by Ogden and Richards in "The Meaning of Meaning." The inventory of definitions they give in the chapter which bears the name of the book distinguishes between sixteen different meanings of meaning. It is rather unfortunate for serious phenomenology that this inventory is designed largely in a facetious spirit, to exhibit the obvious inanity of logicians, and the gratuitous labor of writers who, like Prof. Urban, think too logically to produce a popular style. The lampooning interest is not always to be trusted for scientific candor. But despite this disadvantage, the list appears to be in the main what its authors claim for it:

"The following is a representative list of the main definitions which reputable students of meaning have favored. Meaning is--

A $\begin{cases} \text{I} & \text{An Intrinsic property (1).} \\ \text{II} & \text{A unique unanalyzable Relation to other things.} \end{cases}$

(1) I have so far been unable to discover the principle of capitalization here employed, but copy faithfully.

III   The other words annexed to a word in the Dictionary.

IV   The Connotation of a word.

V   An Essence.

VI   An activity Projected into an object.

VII   (a) An event Intended.

B   (b) A Volition.

VIII   The Place of anything in a system.

IX   The Practical Consequences of a thing in our future experience.

X   The Theoretical consequences involved in or implied by a statement.

XI   Emotion aroused by anything.

XII   That which is Actually related to a sign by a chosen relation.

XIII   (a)   The Mnemic effects of a stimulus. Associations acquired.

(b)   Some other occurrence to which the mnemic

C   effects of any occurrence are Appropriate.

That which a sign is Interpreted as being of.

What anything suggests.

In the case of Symbols.

That to which the user of a Symbol actually refers.

XIV   That to which the user of a symbol Ought to
be referring.

   XV   That to which the user of a symbol Believes
himself to be referring.

C   XVI   That to which the Interpreter of a symbol

         (a) Refers

         (b) Believes himself to be referring . :

         (o) Believes the User to be referring."

     The compilers of this list would convince us that
XIII is the _real_ meaning of meaning (there must be just one
"real" meaning, wherefore in this case the five acceptable
meanings are collectively given one index-number), and that
all the other definitions are due either to a metaphorical
use of language (2), or to "word-magic," the employment of
words for their own sake. Now the sin of "word-magic" is
charged against all logicians, because these deal with "prop-
erties or adjectives," which are "fictitious or nominal en-
tities which we are led to feign through the influence of the
bad analogy by which we treat certain parts of our symbols as
though they were self-complete symbols.....There is a _linguistic_

     (2) It may be noticed here that metaphor has also
its meaning, and that a metaphorical description may be correct
or incorrect as well as a literal one. Ogden and Richards, for
example, use the words "nomad" and "mendicant" as metaphorically
descriptive of certain types of expression.

necessity for such procedure but to exalt this into a logical necessity for the 'subsistence' of such elements is to forget what the world is like." (3)

A person charged with this form of forgetfulness-- Mr. Russell, for instance--having thought perhaps more deeply about metaphysical problems than either of his critics, might smile over the cock-sureness of their knowledge "what the world is like." Their extreme nominalism, based (as nominalism usually is) upon common sense, makes them impatient of such abstruse problems as require the uncommon sense of Mr. Russell, not only for their solution, but even for their recognition. Perhaps they have not even recognized the pertinent problems, (a) how properties may be "symbolically distinguished" which do not in any sense exist; or how things can be "propertied" if there really are no properties (4); or, (b) granting that properties exist in some sense, though not in the same sense as "propertied things," (i. e., physical components of the real world) who ever claimed for properties the same type of existence as for spatio-temporal objects; and furthermore, (c)

(3) p. 309-310

(4) p. 309: "The only entities in the real world are propertied things which are only symbolically distinguish- able into properties and things."

why a distinction between dependent things, such as properties, is not "an addition to our knowledge" (5); and (d) why the aspects (properties, relations) of a thing, just because they do not walk the streets in its absence, must not have names (6). Naive Nominalism has the power of making all abstraction, and therefore all logistic, look absurd; the trouble, however, lies in "what the world is like"--namely, in that it is not peopled entirely with Naive Nominalists.

The moral of this tale is that if we would discover "the meaning of meaning," it will not do to choose from among the sixteen or more possibilities the one which fits our metaphysical prejudices and reject as "incorrect" or "verbal" all other meanings of the term. If it is persistently used in ways our theory does not account for, then our theory is too narrow, not the usage of the word too wide. Now I believe that all the current meanings of the term are notions founded on the relations expounded in the previous chapter. Most of

(5) infra: "No convenient symbolic device is objectionable so long as we know that it is a device and do not suppose it to be an addition to our knowledge."

(6) cf. G. Santayanna, "Three Proofs of Realism," (In "Essays in Crit. Realism") p. 166: "If we regard things ideally and ontologically, we may say with Hume that whatsoever is distinguishable is separable. In this sense the events that common sense regards as interdependent are just as separable as those which it regards as disconnected."

these "definitions" are such that it is perfectly possible to accept several of them at once, as they do not claim to be exhaustive and are not mutually contradictory. Thus we may well enough hold, say, both V and XII to be true, VII and IX and X, V, VII b, and XI, and so forth. The only definitions which seem, by their metaphysical simplicity, to deny any variety of types of Meaning, are I and II. Messrs. Ogden and Richards consider these so foolish as to be negligible. But here, even group A of their list shall be taken seriously--especially since they adduce such eminent authorities for these definitions as John Laird and Bertrand Russell (7).

Their quotation from Russell is taken from an article in "Mind," a part of that memorable symposium of 1920 which unfortunately yielded little besides rather sharp-tongued banter. Our authors enter promptly into the spirit of that debate, by quoting Mr. Russell out of context, which--in this case--alters the effect of his words. They use his statement that "meaning is an observable property of observable entities" to bear out Foolish Definition I: "Meaning is an intrinsic property," when a fair reading of the "Mind" article shows that the emphasis is to be laid upon "observable," not "property,"

(7) "M. of M.", p. 295 ff

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and that Mr. Russell is using "property" as a convenient term, in a very wide sense, to mean anything pertaining to the entities that have meaning--a relation, a quality, a function or what-not--because precision in that particular context is irrelevant. This misquotation is all the more unpardonable, as Mr. Russell commits himself fully and explicitly in his subsequent pages (8). "Meaning," he writes, (p. 402) "in my view, is a characteristic of 'signs', and 'signs' are sensible (or imaginal) phenomena which cause actions appropriate, not to themselves, but to something else with which they are associated."

And again: "We find sometimes that, in mnemic causation, an image or word, as stimulus, has the same effect (or very nearly the same effect) as would belong to some object, say a certain dog.....In that case, we say that the image or word 'means' that object."....

"We may therefore lay down the following definitions:--

"(a) A 'sign' is an occurrence which, through mnemic causation, has mnemic effects (not, in general, other effects)

(8) It may appear gratuitous to criticise at such length a breach of professional honor which is below criticism; I do it here not in the interest of Mr. Russell, who needs no apologist, but in order to justify the tedious checking-up to which I have subjected all other references given by Messrs. Ogden and Richards, and the adduction of further examples wherever I feel that their statements can really be substantiated.

appropriate (from the point of view of the animal's instincts and desires) to some other occurrence or set of occurrences with which it is apt to be associated.

"(b) In such a case, the other occurrence or set of occurrences is the 'meaning' of the occurrence which is a sign."

Certainly this more ample and fairly chosen quotation serves to class its author as a subscriber, not to Foolish Definition I, but to XIII a. The passage quoted, from Prof. Laird's "A Study in Realism," "meaning is directly perceptible just like colour and sound," is, I believe, really expressive of that writer's belief, as other passages of the book suggest (9). His conception of Meaning, however, is something between I and II, for meaning is both an intrinsic property, and a relation (though not an unanalyzable one) between other properties. The first assertion, that it is an intrinsic property, merely states that some relations are "internal "; this has nothing to do with our present problem. As for the nature of the relations which constitute this

(9) For example, on p. 36: "Perception is the awareness of a sensory complex.....which has meaning or significance as well as extension, duration, color or tone, and this meaning or significance belongs to it quite as indefeasibly as hardness, extension and the rest."

property, they are clearly of the sort we have analyzed in
the last chapter--i. e., _correlations_ between entities, or
systems, or elements of systems. (p. 34) There is a very
similar passage in Husserl's „Logische Untersuchungen,"
which serves to substantiate the assertion that Meaning is
"an intrinsic property":

"Was Bedeutung ist, das kann uns so unmittelbar
gegeben sein, wie uns gegeben ist, was Farbe und Ton ist.
Es lässt sich nicht weiter definieren, es ist ein deskriptiv
Letztes."

This definition may be classed not as I or II but
as "A," since its author tells us not whether it is a relation
or a property (in fact, it may belong under B V). All we are
told is that Meaning is indefinable, unanalyzable. (We must
not be misled by the analogy to color and sound, which are
qualities; it is quite possible that something which is _not_
a quality may yet be perceived "in the same way.") As a
definition the passage really tells us nothing at all, since
it simply equates two kinds of alleged data--what meaning is,
and what color or sound is--and the latter (and, through the
equation, also the former) _is not a datum_. A specific color
or tone itself is a datum, but _what color (or tone) is_, is a
legitimate scientific question, to be answered in terms of
some system of concepts. Thus by analogy the most that could

possibly be "given" to our meaning-perceiving sense would
be a_Meaning--that is to say, by this sense we could detect
the presence of Meaning, not determine what Meaning is.
About the presence of a directly perceived property there
can, of course, be no argument. It is a "this," and the
only problem it presents (besides the scientific one of its
relations to other things, i. e., of its nature in terms of
some system) is the problem of giving it a (denotative) name.
But granting that Prof. Husserl has a sense-organ for the
perception of Meaning, which I gravely doubt, let us consider
his uses of the word "Meaning," and see whether these apply
to situations, which exhibit the logical relations we have
discussed. Meaning in its actual, i. e., concrete and partic-
ular manifestations („das konkrete Phänomen des sinn belebten
Ausdrucks") may be an unanalyzable property, like red or C#;
perhaps every concrete event is of this nature--this is a
question for the metaphysician. But that the occurrence of
Meaning depends upon the presence of certain relations, Prof.
Husserl readily admits.

        "Sehen wir nun von den Erlebnissen, die speziell zur
Kundgebung gehören, ab......so scheint zweierlei übrig zu
bleiben: der Ausdruck selbst und das, was er als seine Bedeu-
tung (als seinen Sinn) ausdrückt. Indessen hier sind mehr-
fältige Relationen miteinander verflochten, und die Rede von
dem was ausgedrückt ist und von Bedeutung ist dementsprechend

eine vieldeutige."

"Stellen wir uns auf den Boden der reinen Deskrip-
tion, so gliedert sich das konkrete Phänomen des sinnbelebten
Ausdrucks einerseits in das physische Phenomen, in welchem
sich der Ausdruck nach seiner physischen Seits konstituiert,
und andererseits in die Akte, welche ihm die Bedeutung und
eventuell die anschauliche Fülle geben, und in welchen sich
sich die Beziehung auf eine ansgedrückte Gegenständlichkeit
konstituiert. Vermöge dieser letzteren Akte ist der Ausdruck
mehr als ein blosser Wortlaut. Er meint etwas, und indem er
es meint, bezieht er sich auf ein Gegenständliches."

Here we have the general statement that meaning implies
a relation between a symbol and its object--i. e. the event
which is the symbol, and some other event. In the case of names,
this is unmistakable. A name always refers to an object, and
this is the sense in which it "means" the object (10). So far,
the relation between a name and an object is the simple one-one
correspondence of units we have called "denotation." This is
taken by Husserl as the real sense of Bedeutung (this word means,
quite literally, "denotation"; "deuten" = "to point." It has

(10) p. 37: Der Name beispielsweise nennt unter
allen Umständen seinen Gegenstand, nämlich indem er ihn meint."
(Note that this employs "meinen" as a synonym for "bedeuten,"
not in the original, psychological sense--a procedure to be
lamented in the interests of clear terminology. Compare the
footnote on p. 73 above.)

a secondary meaning, namely _interpretation_--as in „Traum-
deutung"--based etymologically also on the idea of _pointing
out_.) That the nature of this relation, aside from the for-
mal property it shares with many others (i. e. being a one-
one relation between two entities of usually very diverse
character) is unanalyzable, „ein deskription Letztes," and
that its actual occurrence depends upon an act (or acts) of
some mind, („die Akte, welche ihm die _Bedeutung_ und event-
uell die _anschauliche Fülle_ geben"), I am perfectly ready to
accept.

         But denotation, taken as the arch-type of „Bedeu-
tung" in the broader sense where „Bedeutung" = _Meaning_, leads
to notorious difficulties. There seem, indeed, to be cases
where this one-one correspondence does not apply. In a pas-
sage quoted above, Husserl remarks, „die Rede von dem _was
ausgedruckt ist_ und von _Bedeutung_ ist dementsprechend eine
vieldeutige." Thus _signs_ do not function in the former,[11] nor
subjective expressions (gestures, ejaculations) in the latter
sense of Meaning (11). The fact is that "was ausgedrückt ist"
is intension, the object of connotation, and Bedeutung (as a
pure, not a verbal noun) the extension, or object of denotation.[12]

         (11) p. 23: „Zeichen im Sinne von _Anzeichen_ (Kenn-
zeichen, Merkzeichen u. dgl.) _drücken nichts aus._"
         (12) p. 31: derartige 'Ausdrücke' (unwillkürliche
Gesten, Mienenspiel) haben _eigentlich keine Bedeutung._"

Signs such as marks are non-connotative (the meaning of marks
will be discussed under the addenda to Ogden and Richards'
list), and unconscious expression is neither denotative nor
connotative--in fact, it has no Meaning for its producer since
he is not conscious of it, whereas for the observer it is
significant and its logical basis is the complex relation be-
tween analogous terms of two systems which we have called
"signification." This relation, which is quite distinct from
connotation as well as denotation, falsifies the term "expres-
sion" here used so that Husserl feels obliged to use it
("Ausdruck") in inverted commas, to distinguish it from
"Ausdruck"--intension. In fact, like "Anzeichen," (marks),
he rules it out of the discussion of Meanings entirely.

His next difficulty arises in regard to simple sym-
bols for complex objects, and vice versa. This is a rather
subtile affair and requires more detailed consideration of the
one-and-the many problem than our last chapter has furnished,
although the way out of the dilemma is, I believe, suggested
there. Husserl chooses for his example the expression "simple
object"--which, he claims, is a complex symbol denoting a
simple object (13). Now this difficulty seems to me to be

(13) · Husserl, Loc. cit. II, (1) p. 296: "Zusammen-
gesetzte Bedeutungen können einfache Gegenstände 'vorstellen'.
Ein ebenso klares als entscheidendes Beispiel liefert unser
Ausdruck einfacher Gegenstand selbst."

based upon two confusions: (a) the confusion of a term with its definition, and (b) of the object as a unit with the object as a complex. (Note that the two fallacies are correlative.--term and definition being analogous to unit and complex.) And the relation of the symbol "simple object" to said object is a "hybrid" relation, because the symbol in this case both denotes and describes--i. e. it is related at once to the object as one and to the object as a complex. It will be retorted that the object here in question is never a complex; to which I reply that there can be no such object (14) (even using "object" in the broad sense in which Husserl, following Meinong, uses "Gegenstand"). For an object may always be viewed (a) as a locus of all its relations, in which case it is simple, or (b) as a product of those relations, in which case it is complex. Any object is simple or complex according to the system wherein it functions--even the hypothetical entities at the end of an exhaustive analysis of a system, as such an object is essentially "that which has such-and-such relations." That is to say, an object may always be defined in terms of other objects in some system wherein it is capable of figuring. Now in so far as the words "simple object" denote an object per se, they function

(14) p. 297: "Es ist dabei ganz gleichgültig, ob es solch einen (einfachen) Gegenstand gibt oder nicht." Yes; but it is not a matter of indifference whether or no there can be such an object.

together as a name, and are not divorcible,--i. e. in a lang-
uage where "simple object" is merely the name of something,
the symbol cannot be divided without losing all meaning (15).
It is just as much a single symbol as "Francis Joseph" or
"Marie Antoinette." But where it does not function purely as
a name, there its parts perform two discrete offices: "object"
denotes the object in question, and "simple" tells us something
about its (inevitable) relations in some system. The object
here figures in more than one system--the one we are talking
about, where it can only be named, being one of the indivisible
fundamentals, and the system of concepts we are talking in,
where it is not so and can be analyzed, i. e. "described."
Thus the symbol "single object" which denotes the object and
that which describes it are really, by their functions, dif-
ferent symbols, one being merely compound (like a double name,
cf. examples above,) and the other complex. Compound names
are not logically interesting. (Note that full names, under
our current naming system, such as: "Mary Stuart," are complex,
not compound.) Complex names, however, usually "mean" both the
single object and one of its features. They do not "mean" it
in the sense that a definition of its name "means" directly the
name it defines and indirectly the object it describes, for in

        (15) Wittgenstein,"Tractatus" 3.202: "The simple
signs employed in propositions are called names.
                3.26: "The name cannot be analyzed further by
any definition. It is a primitive sign."

such a _pure_ description the object _is not named._ A pure
description enumerates a set of attributes but mentions no
subject other than "that," "that which." Such a phrase as
"The Man in the Iron Mask" is not a pure description, since
"The Man" is taken to mean an object as a unit and the rest
of the phrase assumes the object as a complex. Thus the most
famous of logical relations, that of subject and predicate,
in so far as it figures (as it usually does) as a direct rela-
tion, is really a confusion of several relations, both direct
and indirect: (a) of a name to an object, (b) of a definition
to its definiendum, (c) of a description to its object (which
is a complex of attributes or of concepts), and (d) of a name
with the attributes of its object (i. e. its object as a com-
plex). It is really, to borrow a term from musical theory, a
"cross-relation," holding between terms of different systems.
It is not surprising that so complex a relationship should have
been productive of deep confusions wherever it has been viewed
as direct and unanalyzable.

Without going further through all the various distinc-
tions of Meaning--"die wesentlichen Bedeutungsarten and Bedeu-
tungsformen" (16), we certainly must recognize that Husserl
does not dismiss the _logical_ problem of meaning with his state-
ment that Meaning in actu is immediately perceptible and not
describable in any terms. Whether this latter statement is true

(16) Husserl, "Log. Unt." II (1) p. 183

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does not really concern the present inquiry, which deals with the logical situations in which meaning is possible rather than the actual forms in which it does occur. (I), for instance, does not preclude any of the definitions under B and C; (II) does not conflict with any of the propositions here asserted about the types of relations which may give rise to Meaning, since a relation per se may be unique and unanalyzable and yet belong to a perfectly recognizable type. A fact, thing, or other actual system is probably always unique and unanalyzable; this does not debar it from exemplifying an abstract form or system-function, which is always common and analyzable.

Thus it appears that even writers who claim that Meaning is a simple property or a special, indefinable relation, use the word, quite consciously, in several different ways. There seems no way out of the predicament that the word is "legitimately" used with various meanings, and that it may, in fact, have several meanings at once. This multiplicity of meanings is quite in keeping with the tremendous importance of the whole subject--the fact that experience is "shot through and through with Meanings," (17) that our adult life is concerned with comparatively little direct sensation, but is filled with the symbolic functions of that little. It is, in fact, such a net-work of meanings that every item of our actual experience must be charged with many of them at once. The analyses

(17) Sellars, "Essays in Critical Realism."

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undertaken in the previous chapter make it quite evident, I believe, how one term such as a word (or any event that is apprehended as a unit) can enter at once into several relations underlying respective meaning situations. This also accounts for the difficulty, often experienced, in ascertaining what a word means in a given context--a difficulty altogether too deep to be laid at the door of that scape-goat of scientific error, the personal equation.

Chapter 10.

Phenomenology, the Measure of Theory.

Phenomenology, as has been previously remarked (1), is always inductive, and therefore cannot yield universal principles. But given a theory, its plausibility is certainly increased if it fits all our available facts. The following test of my theory of Meaning is not expected to render a proof, but merely to support the boast that it is not yet vitiated by one negative instance. For this purpose I shall take the list quoted above from Ogden and Richards, and examine the logical foundation of each of the suggested meanings of meaning.

(I) and (II), as I maintained in the foregoing chapter, are "vacuously" satisfied by the intervalence-theory, because they are non-committal about the conditions for Meaning. Let us go on to III; Meaning = "the other words annexed to a word in the dictionary." By this assertion, Meaning is the relatum of the relation we called "definition" and the referent of the converse relation, "connotation." This bears out the doctrine that connotation is a relation strictly between symbols--between a complex symbol and a simple one, which are equivalent, i. e. may be substituted for one another in any system that contains them both (2).

(1) See Chapter I.

(2) This relation, of which Messrs. Ogden and Richards are probably not aware, is necessary before they are justified in speaking of "one symbol being the correct analysis of another" (italics mine). cf. The Meaning of Mind, p. 328.

IV; Meaning = "the connotation of a word," uses connotation, unfortunately, in a different sense than the one for which I had technically reserved it; it uses connotation as a term in a relation whose other term is the word. The sense in which Ogden and Richards use it, is, I think, perfectly clear at this point--and the relation looks very much like the relation between symbols given under III. It is, as a matter of fact, an analogous relation between analogous terms; but instead of mere symbols, its terms are concepts. III is a purely formal, IV a philosophical definition of Meaning. For every case of Meaning that is possible under IV there is a possible case under III such that the two show a term-for-term correspondence, i. e. one is an exact replica of the other. III is an abstract statement of IV, and IV an interpretation of III. In either case, the word (respectively concept) means something by virtue of Connotation; and by the relation I have called signification, i. e. "intervalence" of systems, III means IV (and conceivably, vice versa).

V is, I believe, what Messrs. Ogden and Richards assert it to be--a hypostatisation of connotation (or, according to the present viewpoint, of "dictionary-meaning"). (3) Even if it be all that the Critical Realists hope for it, we may regard the distinction between IV and V as a metaphysical one.

(3) "M. of M. p. 308 , .

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The "form" of the relation is the same whether we regard
the relatum as a complex of symbols, or of°concepts which
are "mental" or in some other sense "unreal," or of "sub-
sistent" universals.

Between types V and VI there should be a break in
the classification. It is hard to see on what principle
III--V (inclusive) should be classed together with VI, VII
and XI, if IX and XIII are unlike enough to be separated
(unless they be arranged according to their supposed degree
of error).

VI, "Meaning is an activity projected into an ob-
ject," is presumably based on the utterances of Prof. F. C. S.
Schiller in the "Mind" Symposium, and quoted by Ogden and
Richards, who consider the cited passage; "Meaning is an
activity taken up toward objects and energetically projected
into them like an α 'particle," as "most charitably regarded
as a metaphor." But in a controversial rather than a charit-
able mood, we may also take the account seriously. I do not
know how α-particles are projected; but I think that with
regard to Meaning, Dr. Schiller is simply desirous of empha-
sizing the concrete psychological element, and disregarding
the logical conditions which are the subject of this enquiry.
In his sentence, quoted on page 273 of the "Meaning of Meaning,"

(4) cf. Quotation adduced by Ogden and Richards,
p. 313

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"what anything means depends on who means it," is somewhat
trying to the hair-splitting mind of the logician, who would
probably ask whether "it" refers to "anything" or to the
meaning of the anything. I assume that the latter reference
is the one desired. In this case we are, of course, dealing
with two senses of meaning, (a) a function of a sign or sym-
bol, and (b) the mental activity of the person using that
sign or symbol. What is asserted by Dr. Schiller is, there-
fore, that a word (for example) is capable of several mean-
ings, perhaps even kinds of meaning, and the one which it is
actually to have in a given situation depends upon the mental
activity of some person whose interest happens to "realize"
one of the possibilities. I do not think Dr. Schiller would
deny the existence of logical structures which determine what
a word can mean--if he were ever patient enough of logic to
consider unrealized things. Whether "realization" is the pro-
jection of activity from a mind into an object I am not meta-
physician enough to discuss. I doubt the theory, but certainly
cannot substitute any other. The sort of meaning Dr. Schiller
would indicate by the phrase: "what anything means," is really
one aspect of the sort of Meaning that belongs to a thing--the
aspect which Husserl calls its "Bedeutungserfüllung," which is
here said to depend on a personal element--which the German
philosopher has distinguished as the „Bedeutungsakt." It is
this „akt," of course, which is supposed to be energetically

projected. Our subject here, however, is neither of these events; it is the bare formal aspect which Husserl, in his thorough analysis, has also accounted for and has termed "leere Bedeutungsintention." We may therefore say that VI, like I and II satisfies our conditions "vacuously."

This is not the case with VII (a). The definition of Meaning as "an event intended" does not simply tell us one thing which is true of all meanings, like "definitions" I and II, nor single out one aspect of any meaning-situation, like VI; it really characterizes one of the many current senses of meaning. If I send a letter to inform a friend of my arrival in the near future, the conveyance of that information is the meaning of my sending the letter. I have correlated two elements of respectively two systems—the system of my acts, which is a physical system, and of their expected consequences, which is an imaginal one. (Note that their actual consequences are irrelevant to the Meaning here discussed; the Meaning remains unaltered even if the letter does not reach its destination, or my friend cannot decipher my hasty script. The meaning relation holds between the action and its expected effect.) And as the word "Meaning" is, unfortunately, applied equally to the relation or the relatum, the "event intended" is a meaning of my act, considered from my point of view, or a meaning "for me." That my act may have other meanings from

other points of view, is a circumstance which does not annul the correctness of describing my purpose as the meaning of my act. We might, for example, convert the relation; to an outsider, the fact that my friend expects me means that I must have written about my prospective visit. Thus my act is to him the meaning of my friend's information. He has correlated the two events from the opposite point of view, from which the effect, being the immediate datum, "means" the cause, whereas to one for whom the cause is the immediately given, it receives its meaning from the effect. In both cases, the relation of the two events is that which in Chapter IX was characterized as "signification." Intention is a combination of this type of meaning with the peculiar circumstances that the cause is taken as referent (5), and is a deliberate act on the part of the person who supplies the psychological factor which "realizes" the meaning-relation. It is really a highly specialized case and the citation of this particular definition of (one kind of) Meaning is somewhat arbitrary on the part of Messrs. Ogden and Richards. As a matter of fact, in their chapter entitled "The Meaning of Philosophers,"

(5) "referent" is here used in the sense sanctioned by "Principia Mathematica"--not in the precisely opposite sense given the word by Ogden and Richards, where "referent" is the relatum in the meaning-relation. I think the usage of Whitehead and Russell is the older; anyway, it is the one adopted throughout all the foregoing chapters of this study.

they have adduced many other special cases which they have
not seen fit to incorporate in their list, but which resemble
"intention" in that they are special senses of the word de-
rived from signification by combination with special non-
logical conditions such as deliberate action or selective
interest.

VII (b) is the same case as that considered under
VII (a) save that the word "meaning" is taken to denote the
relation instead of the relatum. (To be consistent, this
division should be made under every heading except I and II,
which state the chosen application.)

VIII, "Meaning = the place of anything in a system,"
is not a definition of concrete Meaning but of the formal
aspect, the "leere Bedeutungsintention." It ought to read:
"Meaning depends on the place of anything in a system." This
inaccuracy of expression is due to the authors of the list,
not to the writers whose views they would present. They belie
their own rendering somewhat later in the same chapter where
they say: "the 'meaning' is the rest of the system within
which whatever has the 'meaning' is taken." Both statements
are misleading; such equivocations are apt to occur where
authors exalt common sense and readability above "hair split-
ting" exactness; but certainly it requires more than poetic
license to maintain that "the meaning of a man's life" = some.1

" its place among other lives," ===== " his fellowmen's
lives". The Meaning here may be taken to be one of two things
(as in most cases, due to the ambiguity of the term, noted
above): as the <u>relations</u> of his life to others, or the
<u>effects</u> produced through those relations in other lives.

Meaning which depends upon position in a system
can be found in any meaning-situation whatever. It is the
"minimum" that belongs to the "meaningless marks" of mathe-
matics (6); it is present in denotation, where the system is
the very simple one of two entities (name and object) where
the meaning of each term is (a) its relation to the other,
(b) the other. {In the case of <u>words</u> and the objects they
denote, the convertibility of the meaning-relation is not
obvious because words <u>as objects</u> are so supremely useless.
It may be grasped, however, in the case of words which are
taboo, where the system of language is suddenly interrupted
and some object substituted which <u>denotes</u> the missing word;
or in the case of ritual signs, such as the three signs of
the Cross which do duty for the words: "(In the name of) the
Father, the Son, the Holy Ghost."} In connotation or defini-

(6) cf. Chapter 7.

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tion, this sort of "meaning" belongs to the elements of the complex, each of which would indeed be "meaningless" without its peculiar relation to the others, i. e. apart from its place in the whole which is connoted or (conversely) is defined. On meaning in this "positional" sense depends the very important meaning-relation of signification, where the terms of two systems are coupled according to their respective places in the two complexes. Thus meanings determined by position in a system underlie the whole realm of expression by analogy--description, metaphor, allegory, and pictorial representation in all its forms (7). IX and X are, as Messrs. Ogden and Richards remark, instances of VIII; "various restrictions are commonly introduced by aid of which more specific senses of 'meaning,' as place within some systems, are obtained. Two of them are sufficiently important to rank as independent definitions of meaning, since each has been made the keystone of a metaphysical edifice, namely 'meaning' as the practical and as the theoretical consequences." (8)

(7) cf. Wittgenstein,"Tractatus", 4.01: "The proposition is a picture of reality."....
4.014: "The gramophone record, the score, the waves of sound, all stand to one another in that pictorial internal relation, which holds between language and the world. To all of them the logical structure is common."

(8) ~~Ogden and Richards~~, M. of M., p. 322

Note that the relation in virtue of which the practical consequences of an event constitute its meaning, is convertible; taking the consequences as relata, we establish the "Pragmatic" meaning, but taking the antecedents of an event for the relata, we find its historical meaning (this may be thought, also, to consist of both its antecedents and consequences--i. e. of all terms connected with it in the causal system which is, strictly speaking, its history).

As for the "theoretical consequences," these certainly include the practical, since any actual system may be viewed in abstracto, i. e. theoretically. But they supposedly go further. (Pragmatists would reject this supposition, but we must consider it here because not all philosophers are pragmatists.) They are the consequences of unreal as well as real events; the theorems contained in any postulate or set of postulates. Now meaning in this sense is based on implication, and implication is connotation on a wider scale: i. e. a concept connotes its constituent concepts, and a proposition implies its constituent propositions (9). {In a case of mutual implication it seems odd to speak of one proposition as a constituent

(9) cf. G. E. Moore, "The Nature of Judgment," Mind N. S. Vol. 8, p. 180: "It would seem....that a proposition is nothing other than a complex concept. The difference between a concept and a proposition, in virtue of which the latter alone can be called true or false, would seem to lie merely in the simplicity of the former."

of the other; as a matter of fact, we should say that the
two have the same constituents. In the case of such proposi-
tions as for example, "A B C is an equilateral triangle" and
"A B C is an equiangular triangle," the constituent proposi-
tions are of the type: $\angle A = \angle B$, A B = A C, etc., and these
are constituents of both propositions in question. The dis-
similar appearance of the two propositions is due to the fact
that the terms in which they are stated (i. e. "line" and
"angle" respectively) are not equally simple concepts in the
Euclidean system. The one term is needed for the definition
of the other (I would hazard a guess that either could be
made the foundation for the other, but both cannot be taken
as "primitives" without redundancy). Thus the notion, or
propositional concept, of the one is contained in the other--
or vice versa, according to the postulates of the system. But
whichever is chosen as the inclusive concept, its constituents
must be such that by some arrangement they yield the constitu-
ents of the other. Such concepts then as: "A B C is equilat-
eral implies A B C is equiangular," and vice versa, should
really read: "A B C is equilateral and A B C is equiangular
contain strictly analogous implications." This is the condi-
tion in virtue of which "mutual inclusion" is analogous to
"mutual implication" according to the classic interpretations
of Boolean Algebra, and both are used, in their respective
systems, to define "equivalence."})

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XI, "Meaning is the emotion aroused by anything,"
although our authors declare that it "requires little com-
ment," (10) and then go on to the rather interesting comment
that "The detailed examination of this sense of meaning is
almost equivalent to an investigation of values," (11) this
sense is really one of the special cases noted in connection
with VII. It refers to just those objects which are here
called "values," or--quoting Prof. Urban--"funded affective-
volitional meanings," or "emotional connotation." This lat-
ter phrase is not very fortunate, because "connotation" here
is used in place of "signification." The "meaning" here
referred to is not the connotation of the word, i. e. a part
of the complex of notions which defines it; it is, rather, its
meaning in the sense that an event causally connected with
another is sometimes said to be its "meaning," which we have
discussed under XI. There it appeared that IX was a special
case of VIII; and XI, instead of being "a definite sense of
meaning, which......if the writer is what is known as a stylist,
will have no substitute," (12) is a special case of IX. In
all cases where the logical relation is "positional" we must
distinguish between the causal relation, or coordination of

(10) M. of M., p. 323

(11) M. of M., p. 323 infra

(12) M. of M., p. 323

events _per se_, and that same nexus plus the psychological elements of _knowledge_ and _interest_. It is, of course, only in respect of a mind that the cause actually _means_ the effect or vice versa. And this, only when the effect is expected, or the cause known as such. But the causal relation as a correlation between (a) two events taken as units, or (b) two events taken as elements of two respective systems, serves as a sufficient formal basis for the relation here called "signification."

XII, The meaning that attaches to "natural signs," being, according to Ogden and Richards, "that which is Actually related to a sign by a chosen relation," is of course another instance of Meaning to be classed with VII and XI. It requires no more elucidation.

XIII, which subsumes five distinct definitions, is "the" definition accepted by the authors. The first version, which reads: "Meaning is the Mnemic effects of a stimulus," bases meaning again upon a causal relation (13); the second, "Meaning = associations acquired," is rather scantily worded, since we should like to know, "associations" with what, "acquired" by whom--speaker, hearer, etc. But taking it as gen-

(13) This type of causality, supposed to differ radically from that known to the physical sciences, was first expounded in Semon's book, "Die Mneme" (Leipzig, 1904) and is treated as the distinguishing feature of Mind and the basis of all Meaning in B. Russell's "The Analysis of Mind."

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erally as possible, as: "associations with the event that is the symbol or sign, acquired by _anyone_," we have here, and not under VII, a definition of that personal type of meaning which is "entirely psychological"; for naturally no two persons may be expected to acquire the same set of associations with any term. There undoubtedly _is_ such a sense of "meaning"; but for purposes of communication it could serve, at best, only in cases of soliloquy.

The third alternative is rather vague, since we are not told how "the mnemic effects of any occurrence" are supposed to be "appropriate" to still another occurrence. But we take it that in this case the "mnemic effects" are supposed to be actions or attitudes which are appropriate to something else than the stimulus which has produced them. This is not by any means the same as the first definition under XIII (a); for though it assumes the "mnemic effects" which there are said to _be_ the meaning of a term, it treats them as merely _determinant_ of the meaning--the latter being certain remoter things to which the effects are "appropriate." (14) Whatever may be the advantages or disadvantages of this definition, it is obvious that its logical structure is to some extent similar to XIII (a), though carried to greater lengths. It is the same _kind_ of structure, that of _causal connection_.

(14) This definition implies that "cat" does not mean anything unless I react to the word as I would to a real cat.

The second alternative under XIII (b), "Meaning is that which a sign is interpreted as being of," is a more satisfactory though less grammatical account. It leaves out of consideration the mechanism of interpretation, which is really the problem to be explained by "mnemic causation," and which is irrelevant here. One might modify the statement to the effect: Meaning is that which a sign is interpreted as being of, or a symbol as standing for. Unfortunately this does not answer our question--"what are the logical conditions under which any interpretation can be made?" It is another "vacuous" instance. It neither affirms nor denies that correspondence of configuration is the sine qua non of such an interpretation.

The third choice, "Meaning = what anything suggests," is a rather slip-shod attempt at restating the second defini-

(note 14)
If, for example, I detect my real cat in the act of stealing milk, and exclaim: "O you incorrigible cat!" as I flourish a broom, my reaction appropriate to the incorrigible one is not produced by the word--I am not venting on her the mnemic effects of a word, yet the word denotes a cat and nothing else; in fact, this very particular cat. Any case of apostrophe raises the same problem. Does a name, in direct address, "mean" the person? Or, when we are introduced to one "Mr. Tkaltchitch" or other such, whose name we probably have not entertained in our vocabulary before, does the name not denote its owner at the very first application? To say that the meaning of the name equals the (appropriate) object of its mnemic effects, implies that in this case, where there are as yet no such effects, either Mr. Tkaltchitch is not its meaning, or that he is meant properly enough, but does not exist.

tion under XIII (a). The word "meaning" is _not_ used in any

such general way. The proposition: "Your hat is green" may,

through some psychological trick, suggest grass or lettuce or

a green park bench, but neither your hat nor the statement

about it _means_ a vegetable or a bench to any sane speaker,

listener or philosophizing referee. To the extent that this

unguarded statement might be true, it coincides with "Meaning =

associations acquired."

The sixth definition classed under XIII, "That to

which the User of a Symbol actually refers," implies that

symbols have objective references, or relata, which are ex-

pressed willy-nilly wherever the symbol occurs. This objec-

tivity-theory rests not upon the logical foundation alone

(though this is, of course, objective--we cannot actually

express by a symbol something which the symbol cannot, in the

Wittgensteinian sense, "picture"), but upon the fact that the

non-logical element of the Meaning-situation is not a personal

interest, but a tribal habit of thought. Thus if it is agreed

that "John Smith" denotes a certain man, and someone, under

the delusion that this name denotes a different man, says:

"John Smith embezzled his brother's money," the sentence in the

sense of XIII b (3) "means" that the man commonly called John

Smith is the embezzler, though the speaker "meant" Alonzo Jones.

Here is an excellent case of two senses of meaning, both of

which certainly deserve the name, attaching to the same utterance by the same person under the same circumstances. The first of these senses is of vast importance, since it is the one we accept when we require consistency of a speaker and hold him to the meaning he first gives a term--expressed as a rule of scientific procedure, as: "let a = ..., let b = ..." The second sense is that which we employ when we read an author "sympathetically," i. e. when we let his supposed wish be the principle of choice between all the logically possible interpretations of a symbol. If it were practicable (which it is not) to hold to the first or "literal" meaning of meaning, we would have no need of all the commentaries, interpretations, and Doctor-of-Philosophy theses which have given certain German Universities the nickname, "Kantfabriken." Also we would not buy the classics in annotated editions.

Usually XIV and XV coincide (15); if they do not, it is hard to determine just what does constitute XIV; I for one cannot imagine a case where a man ought to be talking about what he does not think he is talking about.

XVI, which presents three alternatives,--"That to which the Interpreter of a symbol (a) Refers, (b) Believes

(15) "Meaning is:...XIV, That to which the user of a Symbol Ought to be referring. XV, That to which the user of a symbol Believes himself to be referring."
This list could be increased by a large number of combinations; the "meaning" of a statement as quoted from its author and expounded by a commentator, is "That to which the Interpreter believes the User to believe himself to be referring."

himself to be referring, (c) Believes the User to be refer-
ring," (16)--differs from the rest (XIII (b) (3) et infra)
merely by the difference of psychological view point. By
changing this purely contingent, i. e. non-formal ingredient
in an actual meaning-situation, we change the situation;
thus any of the four logical relations listed in the preceding
chapter may, by combination with "accidental" factors, give
rise to a large variety of actual "meanings." The variety is
so great that in reading a list like the one compiled by Ogden
and Richards, it is hard to find any common element that justi-
fies the use of the word "meaning" as a genus-name for the
whole conglomeration.

Correspondence of configuration is, of course, a
symmetrical relation; and at first sight it appears odd that
this should give rise to an essentially assymetrical relation
such as meaning. But the relation between the things involved
in the meaning situation, is in fact symmetrical. We have
seen that for every situation such that A means B, there is
another, from another point of view, wherein B means A. The
assymetry arises from the fact that actual meaning involves

(16) Husserl, Log. Unt.", Vol.II$_2$ p. 54: "Das Zeichen
als Objekt konstituiert sich uns im Akte des Erscheinens.
Dieser Akt ist noch kein bezeichnender, es bedarf im Sinne
unserer früheren Analysen der Anknüpfung einer neuen Intention,
einer neuen Auffassungsweise, durch welche statt des intuitiv
Erscheinenden ein Neues, das bezeichnete Objekt gemeint ist."

the further relation of the corresponding systems to a mind,
(16*) and this is never the same for A and B at the same time;
i. e. either A is "valued" and B is not, or A is known and B
is not, or vice versa, etc. etc.   That is why, although a
meaning-relation is never convertible, its converse is always
another possible case of meaning.   Thus in correspondence of
logical configurations or "patterns" we have caught the "Urtier"
of all existing species of Meaning; and, regarding Messrs. Ogden
and Richard's list as a fair representation, we may say that--
as we had predicted--it did not figure as a specimen in their
menagerie. In fact, like the "common Ancestor" of Adam and the
Ape,--it is a skeleton.

(16*)   (Note 16, preceding page.)

APPENDIX A

(Appendix to Chapter 5)

The "Ideational Calculus": an Attempt to Reinterpret Logical Algebra Inclusive of "Axiom 10."

The addition of Couturat's tenth postulate is supposed to restrict Boolean Algebra to one interpretation, namely the one in terms of propositions. But it seems to me at present that another interpretation is possible; that the Algebra is capable of expressing, for instance, the system of "ideas" advocated by the Empiricist "association-psychologists." Whether this system is a correct description of Mind, i. e., a good psychology, is beside the point, just as it is beside the point in logic to ask whether the geometries of Veblen, Hilbert, Huntington, etc. are correct descriptions of actual space. The fact remains that there is such a system and that it presents an "ideational" calculus analogous to the "propositional" calculus.

"Ideas" are to be taken in the Berkeleyan sense; "elementary ideas" being concrete, particular images, best indicated by the German word "Vorstellungen," which constitute the warp and woof of the system.

The fundamental relation of this calculus is the

binary relation denoted by $<$ (1) and here called <u>derivation</u>.
The derivation of ideas, according to our classical psychol-
ogists, may result from <u>association</u>, from <u>comparison</u>, (which
I shall call the <u>alternation</u> of ideas,) and <u>opposition</u>. Ideas
may be freely associated or compared, but every idea uniquely
determines its opposite, i. e. "red thing" determines "not-
red thing." (Here it will be noticed that the problem, whether
<u>contrast</u> is possible without a common element--in this case,
the "thing" which is red or not-red--is analogous to the prob-
lem of <u>negation</u>, whether "p" and "$\sim$p" do not involve a common
element, p, so that they really read, ϕ(p) and ψ(p); the dis-
tinction being in the <u>functions only</u>.) Thus we have our three
logical operations (2) of <u>association</u>, <u>alternation</u> and <u>opposi-
tion</u>, indicated by the operational signs: "$+$," "$\times$," and "$\rightarrow$".

Now every elementary idea ("Vorstellung") either cor-
responds to reality, or it does not; it is either <u>realistic</u> or
<u>fantastic</u>. And these two values are expressed by 1 and 0.
Thus our interpretation admits the axiom which is supposed to
coerce a propositional interpretation. The form under which

(1) Couturat, "The Algebra of Logic," p. 5: "The
fundamental rel. of this calculus is the binary relation which is
called <u>inclusion</u> (for classes), <u>subsumption</u> (for concepts),
or <u>implication</u> (for propositions).----In the system of principles
which we shall adopt, this relation is taken as a primitive
idea and is consequently indefinable."

(2) "Algebra of Logic," Axioms III & IV (p. 10), and
VIII (p.23).
The negative function here might be called that of
<u>deprivation</u>.

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the axiom is stated by Couturat, "(a = 1) = a," is obviously
senseless. It is only by a looseness of interpretation that
it can be thought expressive of a truth about propositions.
Couturat interprets it thus--("Algebra of Logic," p. 84)--"To
say that a proposition a is true is to state the proposition
itself. In other words, to state a proposition is to affirm
the truth of that proposition." If "a = 1" means "a is true,"
the symbol "=" is here abused, for 1 has a predicative character
and the "=" does not really express a symmetrical relation. If
we do treat it as symmetrical, our whole proposition may be
obverted, thus--"a = (a = 1)," which would mean (by the proposi-
tional interpretation) "everything is true." This defect is
partially due to the fact that assertion is an assumed but
unexpressed notion in the calculus. For without distinguishing
between "a" and " ⊦.a," we would have to conclude that every-
thing in the system is true, yet we are told that a uniquely
determines ∿a and that (a = 1) implies (∿a = 0).

The expression "(a = 1) = a" is, in fact, a glaring
example of the "vicious-circle fallacy" because (a = 1) is a
function (truth-function) of a, so that "(a = 1) = a" is of the form
ƒ a = a, wherefore we could legitimately write ƒ(a) = ƒ(ƒa).
Thus the whole uniqueness of the propositional calculus rests
upon a formula which violates the rules of the system. (Couturat
himself remarks that all formulae peculiar to the propositional
calculus hang upon this one formula which we have just condemned
as "vicious"; "all of them are derived from an axiom.....which

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may be called the 'principle of assertion.' This axiom is

as follows: (Ax. X.)   (a = 1) = a." (p.84)

The fallacy here involved is somewhat hidden in the corollary

which states the "two-valued" character of the algebra:

"(a ≠ 1) = (a = 0), (a ≠ 0)=(a = 1)". We interpret this to mean,

"Every idea is either fantastic or realistic". {Note that

"a ≠ 1" is not to be regarded here as a proposition, but as

an idea, whereas "(a ≠ 1) = (a = 0)" may legitimately be re-

garded as a proposition. Every calculus and every science is

expressed by a body of propositions; that does not mean that

it is a calculus (or a science) of propositions. Thus Physics

is expressed in terms of propositions, of words, spoken or

written, of graphs, diagrams, letters, numbers, etc., but it

is not a science of these things.})

We have now the necessary primitive ideas to check

up our interpretation of the remaining axioms. For convenience

I will tabulate all the axioms inclusive of those already men-

tioned: [The starred numbers refer to the corresponding axiom in Couturat.

Ax. I. a < a.*1 "Any idea is derivative from itself."

This is as much as to say that an idea once given may recur.

Ax. II. (a < b)(b < c) < (a < c)*2 "If a is derivative from

b and b from c, a is derivative from c."

*1 "A. of L." p 8
*2 "A. of L." p. 8

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The abuse of the symbolism is obvious here for a calculus of classes as well as of ideas. The principal "$<$" does not denote the fundamental relation of the calculus at all. Couturat himself was aware of this weakness (3). In the propositional calculus, where the ordering relation is _called_ implication, it is less obvious, and in fact escaped notice until ferretted out by Prof. Lewis (4). Since it is common to all interpretations we may consider it as an original fallacy of the calculus, not of our specific rendering (5). "Ax. III Given any two terms, a and b, then there is a term p such that

$$p < a, \ p < b,$$

and that for every value of x for which

$$x < a, \ x < b,$$

we have also

$$x < p." \quad {}^{*3}$$

"Given any two ideas, a and b, then there is an idea p such that p is derivative from a and from b, and any idea X derivative from a and from b is derivative from p."

(3)"Algebra of Logic", p. 9: "We see that in this formula the principal copula has always the sense of implication because the proposition is a secondary one."

(4)C.I. Lewis, "Survey", p. 225: "The framework of logical relations in terms of which theorems are stated must be distinguished from the _content_ of the system, even when that content is logic."

(5) Note that Couturat's statement also uses the notation for the _logical product_ (a < b) (b < c) to express the simultaneous validity of "a < b" and "b < c," although multiplication is not yet defined.

*3 "A. of L,"   p. 10

Thus p (in other terms "ab") in the ideational calculus is the <u>component</u> of a and b.

"Ax. IV   Given any two terms, a and b, then there exists a term s such that

$$a < s, \quad b < s,$$

and that, for any value of x for which

$$a < x, \quad b < x,$$

we have also

$$s < x."^{*4}$$

"Given any two ideas, a and b, then there is an idea s such that a and b are both derivative from s, and any other derivative of a and b is derivative from s."

Thus s (or "a + b") in the ideational calculus is the <u>comparison</u> of a and b.

{Note that a composite idea is always derivative from a comparison of ideas; it is by comparison of the ideas "round thing" and "square thing" that we try--and fail--to form the concept "round square"; by comparing "married man" and "soldier" that we derive "married soldier," etc. This does not cover the case of <u>generalization</u>, such as the derivation of "figure" from "round thing" and "square thing," or "human" from "man" and "woman." For the result of what we may, in this calculus, call a <u>psychological composition</u>, is

*4 " A. of L." p. 10

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not a generalization or abstraction; it is another genuine
"Vorstellung," an elementary idea. The abstract or general
idea of the present calculus corresponds to the propositional
function of the propositional calculus; at least one and at
most all of its components are <u>variable</u>, and the ideas that
are the values for such a variable are (for present purposes)
elementary ideas.}.

"Ax. V (a + b)c $<$ ac + bc"[*5]

The combination of an idea with either one or the
other of two different ideas is derivative from the comparison
of either composed therewith and the other similarly composed.
(i. e., The idea of a horse, composed with the alternate ideas of two other
animals--say a donkey or a rhinocerus--is derivative from the
comparison of a mule with a unicorn.)

"Ax. VI There is a term O such that whatever value may be
given to the term x, we have

$$O < x."\quad [*6]$$

This, upon our interpretation, means: "The Fantastic
may be derived from any idea."

"Ax. VII There is a term 1 such that whatever value may be

[*5] "A. of L.", p.16
[*6] "A. of L.", p.17

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given to the term X, we have

$$X < 1" \quad *7$$

This means that any idea is derivative from the Realistic--i. e. from "experience."

"Ax. VIII Whatever the term a may be, there is also a term a' such that we have at the same time:

$$aa' = 0, \quad a = a' = 1. \quad *8$$

Any idea combined with its opposite (i. e. not-round circle, etc.) is fantastic; any alternative between opposites is realistic; one or the other is realistic, or the idea of "something or its opposite" is realistic.

"Ax. IX $$1 \neq 0." \quad *9$$

The realistic is not derivative from the fantastic. This expresses the soul of empiricism--"reality first," so to speak. Every idea derives from experience and there can be no realistic ideas "a priori."

Ax. X does not really belong to the system of Boolean Algebra, but is separately given; therefore we have separately verified it here. But it may be added to the list:--

Ax. X $$(a \neq 1) = (a = 0), \quad (a \neq 0) = (a = 1).$$

*7 "A. of L." p. 17

*8 "A. of L." p. 22

*9 "A. of L." p. 37

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Its interpretation and discussion need not be restated here. Vide p III .

There are various points which bring out especially the parallelism between the propositional and the ideational interpretations; note, for example, that the construction of a' is such, in the first case, that if a is a proposition, a' is a function of a proposition; likewise in the second, if a is a concrete idea, a' is an abstraction, a function of a. Also, in the propositional system, 0 and 1 (truth and falsehood) apply to propositions, but not to propositional functions; i. e., "Jones is a butcher" is true or false, whereas "X is a butcher" is not. In the ideational calculus this is parallelled by the fact that reality or fantasy may be predicated only of concrete ideas--our basic terms--but not of abstractions. The idea "such and such a red book" is realistic or fantastic; the idea of "red" or of "book" apart from color, size, etc. is neither realistic nor fantastic. We cannot say "red" exists in the sense that books exist, which is the sense here intended; neither can we say "red" is fantastic, i. e., nothing corresponding to it exists. Just as a proposition, which is either true or false, is an instance (value) of a propositional function, which is not, so an idea, which is realistic or fantastic, is an instance of an abstraction, which is not.

(6) "Princ. Math". p. 6: " ∼p is the contradictory function with p as argument and means the negation of the proposition p."

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There is not room here to verify by interpretation
the theorems which follow from our axioms. It may be of
special interest, however, to trace the ideational rendering
through the postulate set which has become classic for propo-
sitions, and is supposed to apply to nothing else--the postu-
lates of Principia Mathematica. Note that the wording of our
fundamental relation, _derivation_, always necessitates a back-
ward reading, so to speak, of the symbolism; instead of read-
ing "a ⊃ b" "a implies b," we read "a is _derivative from_ b."
We begin with an important definition--"p ⊃ q = ∼p ∨ q Def."
To say that p is derivative from q is to say that "either p is
fantastic, or q is realistic." This states the obvious fact
that a realistic idea can be derived only from Reality (7).

1·2   ⊢: p ∨ p . ⊃ . p .   Pp (8)

    A comparison of an idea with itself is derivative
from that idea.

1·3   ⊢ = q . ⊃ : p ∨ q   Pp

    An idea q may be derived from the comparison of
itself and some other idea.

    (7) This corresponds to the "unsymbolizable" postu-
late of "Princ. Math.", that "Anything implied by a true elementary
proposition is true." The symbolic statements here are all
taken from Principia Mathematica, *1.

    (8) The assertion-sign, which has a somewhat doubtful
logical status, may here be taken to mean that what follows
is a possible idea or complex of ideas, and may be in somebody's
mind. Thus in this calculus, as in the propositional one, "⊢∼p·p"
cannot occur.

1·4   ⊢: p ∨q.⊃.q ∨p

The comparison of p with q is derivative from the comparison of q with p.

1·5   ⊢: p ∨(q ∨r) .⊃. q∨(p∨r) Pp

The comparison of an idea with two other, mutually compared ideas may be derived from the comparison of one of the others with the two other (now mutually compared) alternatives. For instance: the notion--"a horse, or a zebra-or-donkey" may be derived from a given notion--a zebra, or a horse-or-donkey."

1·6   ⊢:. q ⊃r.⊃: p ∨q.⊃. p ∨r Pp

If q is derivative from r, then the comparison of p and q is derivative from that of p and r. Example--if the idea of pain is derivative from that of heat, then the idea of pain-or-light is derivative from that of heat-or-light. Primitive propositions 1·7, 1·71, and 1·72 are not mere interpretations of the symbolism, as Principia Mathematica makes evident (p. 97). They presuppose the system of atomic and molecular--collectively, "elementary"--propositions and propositional functions. Now to the notion of "atomic" propositions corresponds that of such ideas as we find in Berkeley's system as the "thoughts of God"--i. e. our sense-experiences as immediate "ideas." They correspond to the "atomic propositions" of the other system, in that all "realistic" ideas can be derived from a complete collection

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of all possible "experiences."(9)In other words, natural sciences

are just as possible in a Berkleyan as in a "Positivist" world.

"Molecular" ideas would, by analogy, be ideas of ideas, i. e.

associations, derivations, ("mnemic" or what-not), comparisons,

oppositions. These are concrete ideas, "Vorstellungen," not

abstractions. Their concreteness they share with the "atomic"

ideas, or pure experiences; therefore the two classes of ideas

together may be termed "elementary." Having this notion we

may now render 1·7 and the subsequent postulates--

1·7   If p is an elementary idea, ∽p is an elementary idea. Pp.

1·71   If p and q are elementary ideas, p ∨q is an elementary

idea.   Pp.

1·72   If φp and ψp are elementary abstractions, whose "in-

stances" are elementary ideas, φp ∨ ψp is an elementary abstrac-

tion.   Pp.

(9) "Principia Mathematica," p. XV: "Given all true
atomic propositions, together with the fact that they are all,
every other true proposition can theoretically be deduced by
logical methods."
        {Why must the fact that they are all be specif-
ically given? How could all propositions be given without this
fact? Should not the passage read--"Together with the proposi-
tion that they are all"? But this proposition need not be given;
it is implied. For if, ex hypothesi, all true propositions are
given, then (whether we know it or not) "these are all atomic
propositions" is a true proposition, and as such it is implied
anyway (since a true proposition is implied by any proposition).
In fact, all true propositions are deducible anyway, from any-
thing.}
        cf. V. F. Lenzen, "Independence Proofs and the
Theory of Implication," Monist, Vol. XXIX.

Whether these last three postulates are needed for the development of empiricist psychology, I have not put to the test. The ideational system as it stands is a much simpler affair than the propositional calculus; what further relations can be deduced ~~is~~ would be a matter for psychologists to investigate, if the subject appears to be of more than historical or logical interest. But the rudiments of the calculus satisfy the demands of our symbolism.

The analogy which makes this alternative interpretation possible, holds especially between the _fallacious_ or _loose_ portions of both renderings; the general confusion of "true," "_the_ True" and "Truth" on the one hand, and of "realistic," "_the_ Real," and "Reality" on the other; the deviation from "dictionary-meaning" as soon as the fundamental relations-- _implication_ and _derivation_, respectively--are defined; and the difficulties, previously alluded to, of the negative function. A comparison of the two systems will bring out these structural weaknesses better than a study of propositions alone; and this is perhaps the only value we can claim for the "ideational" interpretation.

APPENDIX B [to Ch. 8]

A Diagram Illustrative of the Logical Relations
which Give Rise to Meaning.

The relations between 2 and 4 are almost always
indirect (through 1 or 3), and the types of meaning engen-
dered by their direct relations, although perfectly possible
"meanings," have no popular names.

1   2   3   4

Description*

Intension of   Name of Object   X (actual object)   Aspects
Name   (qualities,
                                                 relations,
Denotation*   etc.) of X

definition   subsumption

connotation   constitution

"X ε a" $\phi(X)$

"X ε b" $\psi(X)$

"X ε c" etc. ad libitum $\chi(X)$ etc.
                                             ad
} Signification {   infinitum
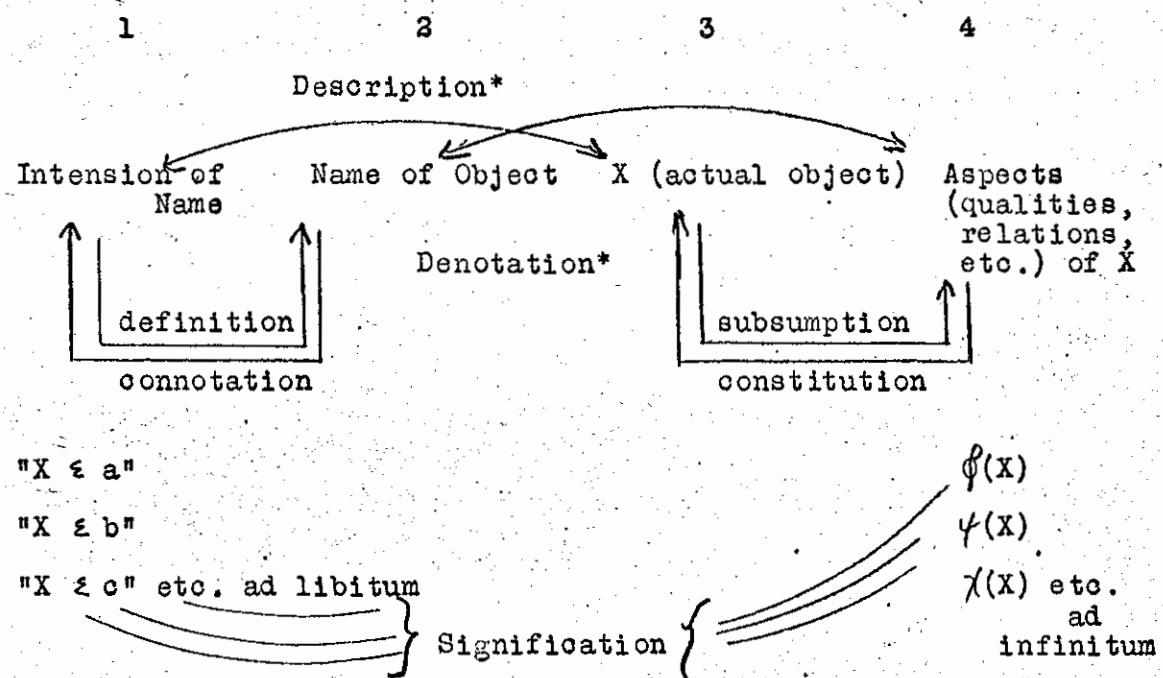
*The converses of description and denotation are
treated in Appendix C, for they are the relations called by
Husserl "Bedeutungserfüllungen."

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APPENDIX C.   (to Chapter 10)

Addenda to Ogden and Richards' Phenomenology.

1. Meaning as a "subsistent entity."

This conception of "Meaning" is a metaphysical one (1), like V,--Meaning as an essence of things. The difference, however, lies in the fact that not every event (treating words and phrases, as well as things, acts and happenings as "events") is necessarily correlated to a meaning, i. e. to such a subsistent entity, whereas certainly there is no reason to suppose that there is any aristocracy among events in respect of "essentiality." If any events have essence, presumably they are all equally endowed. But Meaning according to the "logical realists" is something over and above the object which "has" the meaning (2); an object is related to a meaning (or meanings), and there is no guarantee that this relation holds universally. It is, of course, a relation of denotation, i. e. a relation between a symbol and an object; the symbol belonging to an "existent," transient order, whereas the object is a member of a "subsistent," eternal universe.
(3)

(1) H. H. Parkhurst, "Recent Logical Realism," p. 39:
"The unity of meanings must, from the realists standpoint, be explained in terms not of function, relation, or effect, but of being."

(2) op. cit. p. 16: "Meanings, according to logical realism, are not shifting, relative, vanishing things, but independent subsistent entities."

(3) op. cit. p. 46 ff: "They (Meanings) form a system of closely inter-dependent elements, no one of which possesses a character conceivable in itself alone."

2. Meaning as a directive function.

This is a specialized case of denotation; it is the sort of meaning which attaches to marks--trade-marks, ear-marks, badges, etc.--such as Husserl has characterized as "Kenhzeichen." (4) They differ from natural signs or mnemonic signs („Merkzeichen") in that the latter are variations introduced into the symbolism, in order to correspond to distinctions in the objective (i. e. the denoted) order; whereas a mark is attached to the object to make it correspond to a distinction in the symbolism. For example: if we have three objects answering to the symbol: "Jones," we must introduce distinctions in the symbolism to correspond to the difference in the object; we therefore vary our symbolism and denote the three cases of "Jones" as Tom, Dick and Harry Jones. The distinction is inherent in the object and the symbol is made to conform to it. In the case of marks, on the other hand, we have a distinction in the symbolism which answers to a distinction in the conceptual order which we are trying to correlate with the existent. Take, for example, the descriptive phrase "the house of Ali Baba" as a variation of the general symbol (and concept) "house"; the object shows no such distinction. Every house on the street looks just like every other. Therefore, we put a mark, such as a chalked cross, on one house to make the distinction hold among the houses as well as among

(4) "Log. Unt". B. II, 1 p. 23

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our concepts and their respective symbols. Thus the meaning produced by a mark (5) is the converse of denotation; for the object, by virtue of the distinguishing mark upon it, answers to the concept (and the symbol). There is no English word for the converse of denotation; but Husserl has named this relation of an existent object to a denoting symbol, „Bedentungserfullung." (6) {Note that this relation holds between any object and the symbol that denotes it; but the mark or marks („Kennzeichen") may belong to the object naturally, as for example, Tom Jones' face is the "Kennzeichen" by virtue of which he "fulfills" the meaning-relation between him and his name.}

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(5) Note that the white cross chalked on the door does not answer to our concept,--has itself no meaning; it is the house with the white cross that answers to our concept, "house of Ali Baba."

(6) "Logische Untersuchungen," Vol. II (1), p. 38.

APPENDIX D

"Meaning" in Art.

"Lastly, there is the symbolism of art--a field of meaning and expression of meaning almost unexplored from the side of phenomenology. No one will say that the sounds of music are 'mere' sounds: no, they are expressive, they are charged with meaning, though what they mean can hardly be translated into or expressed in terms of any other language, unless it be dancing. Inference and association here leave us in the lurch as explanatory principles. Even the distinction of the indicative and expressive functions of signs becomes almost an irrelevant artificiality when applied to music..."

(1)

Meaning as it is found in art and religion has ever been the stumbling-block of any precise definition. It is probably this "unexplored" field, with its suggestion of vast complexity and ineffability, that has given rise to such "dynamic" concepts as Schiller's "energetically projected" activity or Strong's "unfathomed beyond which we cannot contemplate but can only intend." But a logical analysis of meaning should reveal the formal aspect of even such "higher" meanings as those of poetry and music, religion and mythology. And if the word "meaning" is properly applied to these, a "skeleton" theory, such as that of corresponding configurations, should cover these "higher" meaning-situations as well as the "lower" ones of

(1) R. F. A. Hoernlé, "A Plea for a Phenomenology of Meaning," p. 89.

simple indication of objects or conveyance of information (2).
In fact, if there can be a philosophical science of aesthetics
(i. e. if aesthetics is not purely a matter of psychology)
this science is a science of possible meanings. Psychology
is concerned with the question, how the "meanings" of art
actually are appreciated; philosophy may only question what
meanings might be conveyed; in other words: what possible
meanings exist, and under what conditions these are "artistic."

But (Prof. Hoernlé to the contrary) to begin aesthet-
ics with phenomenology, without some underlying principle, seems
indeed a hopeless undertaking. And I am not going to examine
a single phenomenon in itself. It is generally agreed that to
record in words the subtle meanings of a work of art is just
as bad as to explain a joke. These meanings must be "felt,"
"appreciated," "intuited"--they are not of the propositional
sort where something is first denoted as a whole and then some
assertion made about it (i. e. some of its constituent aspects
pointed out.); art is said to be connotative. In so far as it

(2) This ambition has also been voiced by E. Cassirer
("Philosophie der symbolischen Formen," p. 18): „Der Gehalt des Geis-
tes erschliesst sich nur in seiner Aeusserung; die ideelle Form
wird erkannt nur an und in dem Inbegriff der sinnlichen Zeichen,
deren sie sich zu ihrem Ausdruck bedient. Gelänge es, einen
systematischen Überblick über die verschiedenen Richtungen dieser
Art des Ausdrucks zu gewinnen.....so wäre damit das Ideal der
'allgemeinen Charakteristik," wie Leibnitz es für die Erkenntnis
aufstellte, für das Ganze des geistigen Schaffens erfüllt. Wir
besässen alsdann eine Art Grammatik der symbolischen Funktion
als solcher, durch welche deren besondere Ausdrücke und Idiome,
wie wir sie in der Sprache und in der Kunst, im Mythos und in der
Religion vor uns sehen, umfasst und generell mitbestimmt würden."

is denotative, it performs an extra-artistic function. But
in the sense we have given the word connotation, this is
hardly correct. Art does not connote--does not present some
symbol, such as a man, which is connected by the perfectly
artificial relation of definition with, say, a concept of
manly attributes, as the word "m a n" is thus connected.
Such a symbol would belong to picture-writing, not to art (3).

The meaning-relation that may belong to a work of
art as such is signification, not connotation. All art rests
upon a composition of experiences, sensations or (in the case
of literature) imaginal sights, sounds, events. These have
upon us either direct emotional (aesthetic) effects, or indi-
rect emotional effects (hopes, fears, etc.) through the situa-
tions they let us construe. (Note that where such construction
takes place we deal with yet another subordinate, extra-artistic
meaning-relation.). Thus either the sensory elements of such a
composition, or the factual elements suggested by it, produce a
feeling, or play of feelings. Now it is through relation to
our feelings that we make that correlation between elements of
different systems, known as "synaesthesia." And it is through

(3) cf. Schopenhauer," Welt als Wille und Vorstellung", BK III § 49.
"Allen unseren bisherigen Betrachtungen über die Kunst liegt
überall die Wahrheit zu Grunde, dass das Objekt der Kunst...
eine Idee, in Platons Sinne, ist und durchaus nichts Anderes:
nicht das einzelne Ding, das Objekt der gemeinen Auffassung;
auch nicht der Begriff, das Objekt des vernünftigen Denkens und
der Wissenschaft."

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the testimony of our feelings that we come to recognize the correspondence between two artistic compositions or between such a composition and a configuration of events. This correspondence between "synaesthetic" systems is the basis of meaning in art (4).

The perception of natural law, of the inevitable sequence of human affairs, and so forth, has a distinctly affective value. That is why natural law, fate, etc. are so often recognized as the meaning of artistic expression. But not only may we interpret art in terms of life or vice versa--we may also interpret one work of art in terms of another. In fact, this is apt to be the more direct and traceable process. Thus the dancers of the Russian Ballet interpret not only a story, e. g. the "Legend of Joseph," which they "represent" (an extra-artistic meaning) and the human laws this legend illustrates (the artistic "meaning" of it), but also they interpret the music which they dance. The analogies between art and art are probably based directly upon correspondence of synaesthetic sensations. A term borrowed from another form of art always has more force than one taken from the artist's proper technique--because the analogy is made through the medium of emotion, and thus the

(4) The importance of synaesthesia for the integration of our sensory experience (the "Gestaltung" of our world) was pointed out to me by Dr. Wolfgang Koehler.

borrowed term cannot fail to have, so to speak, an affective

content. This is the answer to a naive but intelligent ques-

tion that was recently put to me by an artist: "Why does it

mean so much more to speak of 'high tone' and 'low tone' in

painting, than to speak of 'pale paint' and 'dark paint'?"--

Because in the latter case we _describe_ the color-situation,

whereas in the former we describe a sound-situation which--

on the principle of synaesthetic correspondence--_signifies_

the paint-arrangement. Thus the latter case is always "charged

with meaning" in an emotional as well as an intellectual sense.

Upon this sort of analogy rests the symbolism of art, and by

this is it distinguished from other, "purely intellectual" types

of symbolism (5).

Artistic thought is rarely "discursive." _Analogies_

of synaesthetic configurations are usually obvious upon inspec-

tion, but intolerably complicated in verbal interpretation.

Speech, highly organized as it is, is really a rudimentary form

of expression. What can be said in words does not require the

(5) It has been pointed out, however, by Alphonse
Chide in his article "La Logique de l'Analogie" (Rev. Phil.,
Vol. 66) that the sciences, too, employ this method of mutual
plagiarism (cf. p. 629): "Notre langage psychologique est
empêtré dans la phraseologie de l'étendue et gate la fluidité
interieure par une logique du solide et du choc. Notre langage
cosmologique est, à rebours, obsédant de spiritualité."
The reason for this sort of analogy seems to lie,
however, in the preferance of familiar ideas over unfamiliar
for the sake of expounding or even apprehending difficult rela-
tionships.

painter's or composer's skill (6).

Schopenhauer's philosophy of art, and especially
of music, is an excellent instance of the non-verbal (non-
conceptual) apprehension of a situation (real or imagined)
through the <u>analogy</u> of a materially very different structure.
Toward the end of the third book of his „Welt als Wille und
Vorstellung" he tries to point out intellectually the par-
allelism of metaphysical and musical principles, and wisely
adds the warning:

"Man darf jedoch bei der Nachweisung aller dieser
vorgeführten Analogien nie vergessen, dass die Musik zu
ihnen kein direktes, sondern nur ein mittelbares verhältniss
hat;.....Sie drückt daher nicht diese oder jene einzelne und
bestimmte Freude, diese oder jene Betrübniss, oder Schmerz,
oder Entsetzen, oder Jubel, oder Lustigkeit, oder Gemüthsruhe
aus; sondern <u>die</u> Freude, <u>die</u> Betrübniss, <u>den</u> Schmerz, <u>das</u>
Entsetzen, <u>den</u> Jubel, <u>die</u> Lustigkeit, oder Gemüthsruhe selbst,
gewissermaassen <u>in abstracto</u>, das Wesentliche derselben, ohne
alles Beiwerk, also auch ohne die Motive dazu. Dennoch
verstehen wir sie, in dieser abgezogenen Quintessenz, vollkom-

(6)R.F.A.Hoernlé, "Phenomenology of Meaning," p. 89:
"The problem of the proverbial "thoughts too deep for words"
has all too rarely led philosophers and psychologists to ex-
plore the expressive value of non-verbal, symbolic actions,
indeed, the need of expressing in action what cannot be said
in words. Human life is full of such actions, especially
where deep affection is involved."

men. Hieraus entspringt es, dass unsere Phantasie so leicht
durch sie erregt wird und nun versucht, jene ganz unmittel-
bar zu uns redende, unsichtbare und doch so lebhaft bewegte
Geisterwelt zu gestalten und sie mit Fleisch und Bein zu
bekleiden, also dieselbe in einem analogen Beispiel zu verkör-
pern. Dies ist der Ursprung des Gesanges mit Worten und end-
lich der Oper........." (7)

        There is not space here for a theory of the Arts,
nor for an examination of those theories which exist. This
reflection upon artistic meaning is merely sketched to obviate
the conceivable charge that our doctrine covers the simple,
trivial, but not the "higher" cases in point. I hope that I
have forestalled this indictment and at least suggested the
directions in which the present theory of meaning might be
elaborated by ambitious phenomenologists--even to the ideal
heights suggested by Cassirer, "to compile a grammar of the
symbolic function as such," for language, art, mythology and
religion. And I hope it is apparent that the logical frame-
work which all meanings of meaning are here found to have in
common, and by virtue of which we may apply the word to so
many different phenomena, in no wise detracts from the extra-
logical or "interpretative" elements which give the various

        (7) "Die Welt als Wille und Vorstellung"
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cases their individuality, and raise the subject to its philosophical importance.

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