# 11-th Korean Mathematical Olympiad 1997/98

## First Round – November 16, 1997

1. For a positive integer $n$ let $f(n)$ be the number of different ways to decompose $n$ into a sum of positive odd integers (two decompositions that differ only in order are considered different). Find an expression for $f(n)$ in terms of $n$.

2. For a positive integer $n$ let
$$a_n = \sum_{k=0}^{[n/2]} \binom{n-k}{k} \left(-\frac{1}{4}\right)^k.$$
Compute $a_{1997}$.

3. Let $ABCDEF$ be a convex hexagon with $AB = BC$, $CD = DE$ and $EF = FA$. Prove the inequality
$$\frac{BC}{BE} + \frac{DE}{DA} + \frac{FA}{FC} \geq \frac{3}{2}$$
and find when equality occurs.

4. Let $p$ be an odd prime and $a,b$ be positive integers such that
$$1 + \frac{1}{2^3} + \cdots + \frac{1}{(p-1)^3} = \frac{a}{b}.$$
Prove that $p$ divides $a$.

5. In a triangle $ABC$ of area $T$, $a,b,c$ are the lengths of sides $BC,CA,AB$, and $x,y,z$ the lengths of the medians from $A,B,C$, respectively. Prove the inequality
$$\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} \geq 4\sqrt{T\sqrt{3}}$$
and find the cases of equality.

6. Find all polynomials $p(x,y)$ with the following properties:
   (i) $x^{100} + y^{100} \leq p(x,y) \leq 101(x^{100} + y^{100})$ for all $x,y$;
   (ii) $(x-y)p(x,y) = (x-1)p(x,1) + (1-y)p(1,y)$ for all $x,y$.

7. Let $X,Y,Z$ be points outside triangle $ABC$ such that $\angle BAZ = \angle CAY$, $\angle CBX = \angle ABZ$, $\angle ACY = \angle BCX$. Show that the lines $AX,BY,CZ$ are concurrent.

8. Prove that there are no different positive integers $x,y,z,w$ such that $x^2,y^2,z^2,w^2$ form an arithmetic progression.



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