Dynamics and Gravitational Wave Signatures of Magnetized Neutron Stars

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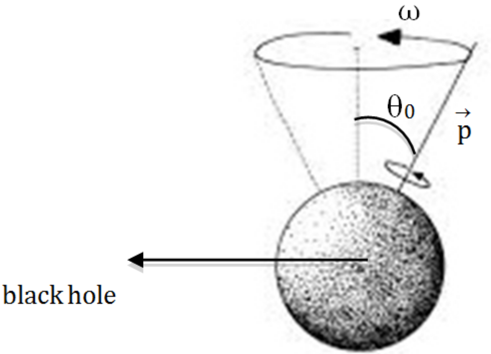
Overview

- As neutron star falls into black hole, precession of magnetic dipole creates EM waves.
- The induced electric field drives a current, establishing a circuit between the neutron star, black hole, and the plasma surrounding the black hole.
- Electromagnetic waves are emitted that can be detected.
- Black-hole neutron star binary can serve as source of electromagnetic waves.

Stationary, precessing magnetic dipole in Schwarzschild space-time.
Metric:

$$ds^2 = -\left(1 - \frac{2}{r}\right) dt^2 + \left(1 - \frac{2}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$



Electric dipole:

$$\vec{p} = (p_r, p_\theta, p_\phi) = p\left(\frac{\sin\theta_0\cos(\omega t)}{\sqrt{g_{rr}}}, -\frac{\cos\theta_0}{r_0}, \frac{\sin\theta_0\sin(\omega t)}{r_0}\right).$$

Use EM duality:

$$E \to B,\ B \to -E,\ p \to m$$

Dipole tensor:

$$Q^{\alpha\mu}(\tau) = V^\alpha p^\mu - p^\alpha V^\mu$$

Four-current:

$$J^\alpha = \nabla_\mu \int \frac{Q^{\alpha\mu}(\tau)\delta^{(4)}[x - x_S(\tau)]}{\sqrt{-g}} d\tau = \nabla_\mu \left(\frac{(\frac{dx^\alpha}{dt}p^\mu - p^\alpha\frac{dx^\mu}{dt})\delta^{(3)}[\mathbf{x} - \mathbf{x}_S(t)]}{\sqrt{-g}}\right)$$

Vector harmonics expansions

- Similar to solving the hydrogen atom in quantum mechanics, separate solution into angular and radial part.
- Spherical harmonics are convenient basis for angular part.
- Vector harmonics are generalization to vectors.
- Vector harmonics have two parities: odd, which transform like $(-1)^{\ell}$, and even, which transform like $(-)^{\ell+1}$.

$$4\pi J_\mu = \sum_{\ell,m} \left(\begin{bmatrix} 0 \\ 0 \\ \frac{\alpha^{\ell m}(r,t)}{\sin\theta} \frac{\partial Y^{\ell m}}{\partial \phi} \\ -\alpha^{\ell m}(r,t) \sin\theta \frac{\partial Y^{\ell m}}{\partial \theta} \end{bmatrix} + \begin{bmatrix} \Psi^{\ell m}(r,t) Y^{\ell m} \\ \eta^{\ell m}(r,t) Y^{\ell m} \\ \chi^{\ell m}(r,t) \frac{\partial Y^{\ell m}}{\partial \theta} \\ \chi^{\ell m}(r,t) \frac{\partial Y^{\ell m}}{\partial \phi} \end{bmatrix} \right)$$

$$\psi = p \sin\theta_0 \frac{g_{00}}{r^2} \left[\partial_r \left(\frac{\delta[r-R]}{\sqrt{g_{rr}}} \right) Y^* - i \frac{1}{r} \partial_\phi Y^* \delta(r-R) \right] e^{-i\omega t}$$

$$\eta = ip \sin\theta_0 \frac{\sqrt{g_{rr}}}{r^2} \omega \delta(r-R) Y^* e^{-i\omega t}$$

$$\alpha = p \sin\theta_0 \frac{1}{\ell(\ell+1)} \frac{1}{r} \omega \frac{\partial Y^*}{\partial \theta} \delta(r-R) e^{-i\omega t}$$

$$A_\mu = \sum_{\ell,m} \left(\begin{bmatrix} 0 \\ 0 \\ \frac{a^{\ell m}(r,t)}{\sin\theta} \frac{\partial Y^{\ell m}}{\partial \phi} \\ -a^{\ell m}(r,t) \sin\theta \frac{\partial Y^{\ell m}}{\partial \theta} \end{bmatrix} + \begin{bmatrix} f^{\ell m}(r,t) Y^{\ell m} \\ h^{\ell m}(r,t) Y^{\ell m} \\ \chi^{\ell m}(r,t) \frac{\partial Y^{\ell m}}{\partial \theta} \\ \chi^{\ell m}(r,t) \frac{\partial Y^{\ell m}}{\partial \phi} \end{bmatrix} \right)$$

Maxwell's equations

We are interested in solving Maxwell's equations in curved space-time

$$(\sqrt{-g}F^{\mu\nu}),\nu = \sqrt{-g}4\pi J^{\mu},$$

where $g = \det g_{\alpha\beta}$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, which reduces to solving

$$(g^{rr}a')' - g_{rr}\ddot{a} - \frac{\ell(\ell+1)}{r^2}a = \alpha$$

$$(g^{rr}b')' - g_{rr}\ddot{b} - \frac{\ell(\ell+1)}{r^2}b = \frac{1}{\ell(\ell+1)}[(r^2\Psi)' - r^2\dot{\eta}],$$

where $b = \frac{r^2}{\ell(\ell+1)}(\dot{h} - f')$. Working in frequency space,

$$(g^{rr}a')' + \left(g_{rr}\omega^2 - \frac{\ell(\ell+1)}{r^2}\right)a = \alpha$$

$$(g^{rr}b')' + \left(g_{rr}\omega^2 - \frac{\ell(\ell+1)}{r^2}\right)b = \frac{1}{\ell(\ell+1)}[(r^2\Psi)' + i\omega r^2\eta]$$

Parameters of simulation

In units of $G = M_{BH} = c = 1$:

- $r_0 = 25$
- $\omega = .2$
- $p \sin \theta_0 = 1$

Method for solving

1. Due to delta function nature of source, separate region into two regions, $r < r_0$ and $r > r_0$. Let u_L be solution for $r < r_0$, and u_R be solution for $r > r_0$.
2. Numerically solve, applying approprate boundary conditions for both a and b.
3. Apply junction conditions at $r = r_0$ (taking into account delta function source terms).

Boundary conditions

Tortoise coordinate $r_* \equiv r + 2\log(r/2 - 1)$.
Ingoing wave conditions:

$$\lim_{r_* \to -\infty} u_L(r_*) \sim e^{-i\omega r_*}$$

$$\lim_{r_* \to -\infty} u'_L(r_*) \sim -i\omega e^{-i\omega r_*}$$

Outgoing wave conditions:

$$\lim_{r_* \to \infty} u_R(r_*) \sim e^{i\omega r_*}$$

$$\lim_{r_* \to \infty} u'_R(r_*) \sim i\omega e^{i\omega r_*}$$

Junction conditions

For a:

$$u_R(r_0) - u_L(r_0) = 0$$
$$u'_R(r_0) - u'_L(r_0) = \frac{1}{\ell(\ell+1)} \frac{p\omega \sin\theta_0}{r_0} \frac{\partial Y^*}{\partial \theta},$$

For b:

$$u_R(r_0) - u_L(r_0) = -i \frac{p \sin\theta_0}{\ell(\ell+1)} \frac{g_{rr}}{r_0} \partial_\phi Y^*$$
$$u'_R(r_0) - u'_L(r_0) = -\frac{1}{r_0^2} p \sin\theta_0 \sqrt{g_{rr}}\, Y^*$$

Odd parity:

$$E_\theta = -\frac{1}{r^2 \sin\theta} g^{00} \dot{a} \frac{\partial Y}{\partial \phi} \qquad B_\theta = -\frac{1}{r^2 \sin\theta} g^{rr} a' \frac{\partial Y}{\partial \theta}$$

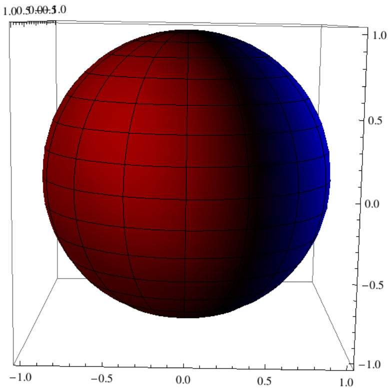
$$E_\phi = \frac{1}{r^2 \sin\theta} g^{00} \dot{a} \frac{\partial Y}{\partial \theta} \qquad B_\phi = -\frac{1}{r^2 \sin\theta} g^{rr} a' \frac{\partial Y}{\partial \phi}$$

Even parity:

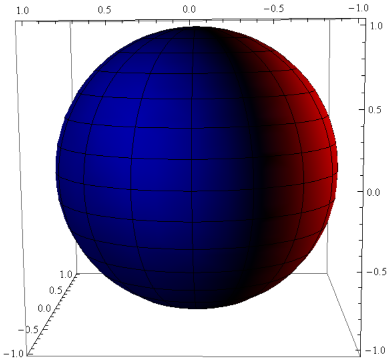
$$E_\theta = \frac{1}{r^2} b' \frac{\partial Y}{\partial \theta} \qquad B_\theta = \frac{1}{r^2 \sin^2\theta} \dot{b} \frac{\partial Y}{\partial \phi}$$

$$E_\phi = \frac{1}{r^2 \sin^2\theta} b' \frac{\partial Y}{\partial \phi} \qquad B_\phi = -\frac{1}{r^2} \dot{b} \frac{\partial Y}{\partial \theta}$$

E_r Plots

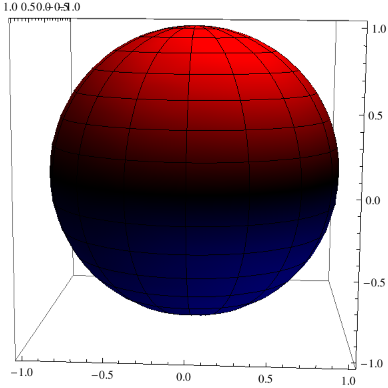


(a) front

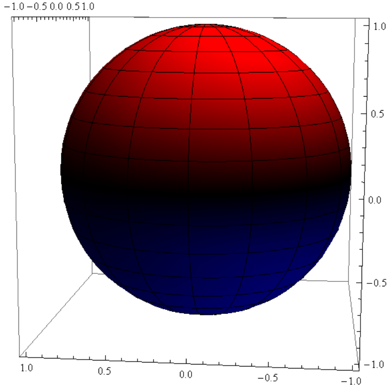


(b) back

B_r Plots

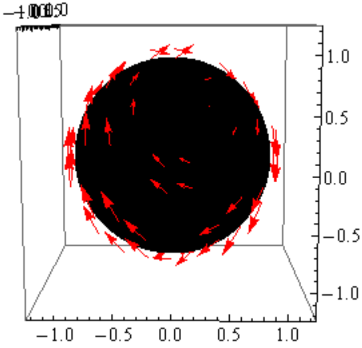


(c) front

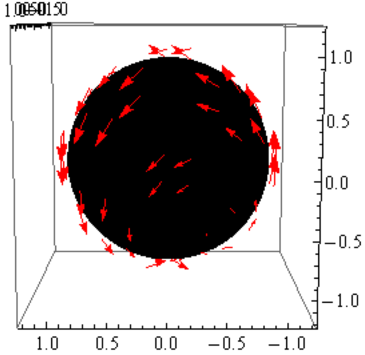


(d) back

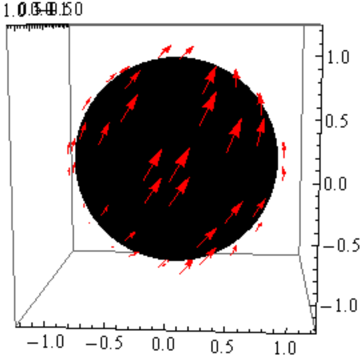
E tangential Plots



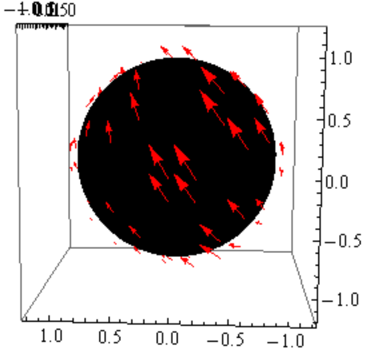
(e) front



(f) back



(g) front

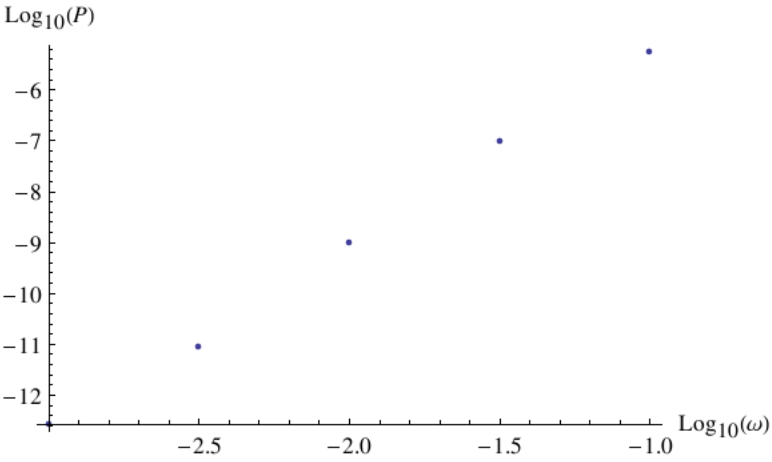


(h) back

$$\vec{S} = \mathrm{Re}\left[\frac{1}{8\pi}\vec{E} \times \vec{B}^*\right]$$

$$P = \int \vec{S} \cdot d\vec{A} = \frac{1}{8\pi}\mathrm{Re}\left[\sum_{\ell,m} \ell(\ell+1)(\dot{a}\bar{a}' - \dot{b}\bar{b}')\right]$$

Power from precessing, magnetic dipole: $P = \frac{(\sin\theta_0 p)^2 \omega^4}{3}$.



Slope of line ≈ 4.

Poynting flux

- Flux at infinity is 5.3×10^{-4}
- Flux through horizon is 8.6×10^{-7}.
- Flux at infinity in flat space-time is 5.3×10^{-4}.

Further avenues of inquiry

- Plunging dipole
- Introduction of plasma
- Kerr geometry

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