RAICES DIFERENTES

a) $x(s) = \frac{s+2}{s^2+7s+12}$

$$\frac{s+2}{s^2+7s+12} = \frac{s+2}{(s+3)(s+4)} = \frac{A}{s+3} + \frac{B}{s+4}$$

Si: $A|_{s=-3} = \frac{-3+2}{-3+4} = \frac{-1}{1} = 1 \quad \therefore A = -1$

Si: $B|_{s=-4} = \frac{-4+2}{-4+3} = \frac{-2}{-1} = 2 \quad \therefore B = 2$

$$\therefore x(s) = -\frac{1}{s+3} + \frac{2}{s+4}$$

$$x(t) = e^{-3t} + 2e^{-4t}$$

B) $\frac{s+2}{s^3+4s^2+3s}$

$$\frac{s+2}{s^3+4s^2+3s} = \frac{s+2}{s(s+3)(s+1)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+1}$$

$A|_{s=0} = \frac{0+2}{3} = \frac{2}{3} \quad \therefore A = 2/3$

$B|_{s=-3} = \frac{-3+2}{-3(-3+1)} = \frac{-1}{6} = \frac{-1}{6} \quad B = -1/6$

$C|_{s=-1} = \frac{-1+2}{-1(-1+3)} = \frac{1}{-2} = \frac{-1}{2} \quad C = -1/2$

$x(s) = \frac{2}{3}\frac{1}{s} - 1/6\ \frac{1}{s+3} - \frac{1}{2}\frac{1}{s+1}$

$$\therefore x(t) = \frac{2}{3} - \frac{1}{6}e^{-3t} - \frac{1}{2}e^{-t}$$

# RAICES REPETIDAS

$$\frac{5s-1}{s^3+3s-2}$$

$$\frac{5s-1}{(s-2)(s+1)^2} = \frac{A}{s-2} + \frac{B}{(s+1)^2} + \frac{C}{s+1}$$

Para $A|_{s=2} = \frac{5(2)-1}{(2+1)^2} = \frac{9}{9} = 1 \quad \therefore A = 1$

Para $B|_{s=-1} = \frac{5(-1)-1}{(-1-2)} = \frac{-6}{-3} = 2 \quad \therefore B = -2$

Para C $\frac{dB}{ds} = \frac{-9}{(s-2)^2} = \frac{-9}{9} = -1 \quad C = -1$

$$X(s) = \frac{1}{s-2} + \frac{2}{(s+1)^2} - \frac{1}{s+1}$$

$$\therefore x(t) = e^{2t} + 2e^{-t}r(t) - e^{-t}$$

b) $$\frac{s^2+16s+2}{s^3+14s^2+65s+100}$$

$$X(s) = \frac{s^2+16s+2}{(s+5)^2(s+4)} = \frac{A}{(s+5)^2} + \frac{B}{s+5} + \frac{C}{s+4}$$

$$A|_{s=-5} = \frac{(-5)^2+16(-5)+2}{(-5+4)} = 53 \quad \therefore A = 53$$

$$B = \frac{dA}{ds} = \frac{x^2+8x+62}{(x+4)^2} = 47 \quad \therefore B = 47$$

$$C|_{s=-4} = \frac{(-4)^2+16(-4)+2}{(-4+5)^2} = 14 \quad \therefore C = 14$$

$$\therefore x(t) = 53e^{-5t}r(t) + 47e^{-5t} + 14e^{-4t}$$

# RAICES COMPLEJAS

$$X(s) = \frac{s^2 - 2s + 1}{s^3 + 3s^2 + 4s + 2} = \frac{as + b}{s^2 + 2s + 2} + \frac{c}{s+1}$$

Para $C|_{s=-1} = \frac{(-1)^2 - 2(-1) + 1}{(-1)^2 + 2(-1) + 2} = \frac{4}{1} = 4$ $\quad c = 4$

$$s^2 - 2s + 1 = (as + b)(s+1) + 4(s^2 + 2s + 2)$$

$$s^2 - 2s + 1 = (a+4)s^2 + (b + a + 8)s + b + 8$$

$a + 4 = 1 \qquad b + 8 = 1 \qquad \therefore a = -3 \quad b = -7$

$$X(s) = \frac{-3s - 7}{s^2 + 2s + 2} + \frac{4}{s+1}$$

$$\frac{-3(s+1)}{(s+1)^2 + 1} - \frac{4}{(s+1)^2 + 1} + \frac{4}{s+1} =$$

$$x(t) = -3e^{-t}\cos(t) - 4e^{-t}\operatorname{sen}(t) + 4e^{-t}$$

b) $\frac{s^2 - 2s + 1}{s(s^2+4)} = \frac{A}{s} + \frac{bs + c}{(s^2+4)}$

$A|_{s=0} = \frac{1}{4} \quad \therefore A = 1/4$

$$s^2 - 2s + 1 = \frac{1}{4}(s^2+4) + s(bs + c)$$

$$s^2 - 2s + 1 = \left(\frac{1}{4} + b\right)s^2 + cs + 1$$

$\frac{1}{4} + b = 1 \qquad c = -2$

$b = 3/4 \qquad c = -2$

$$\frac{1}{4}\frac{1}{s} + \frac{\frac{3}{4}s - 2}{s^2 + 4}$$

racionalizando

$$\frac{1}{4}\frac{1}{s} + \frac{\frac{3}{4}(s+4) - 2}{s^2 + 4}$$

$$x(t) = e^{-4t}\frac{3}{4}\cos 4t - \operatorname{sen} 4t + \frac{1}{4}$$

RAICES DIFERENTES

$$Y(z) = \frac{z(z+1)}{(z+2)(z-1)(z+3)} = A_0 + \frac{A_1 z}{z+2} + \frac{A_2 z}{z-1} + \frac{A_3 z}{z+3}$$

$$A_0/_{z=0} = 0$$

$$A_1/_{z=-2} = \frac{z(z+1)}{(z-1)(z+3)z} = \frac{2}{-3(1)(-2)} = \frac{2}{6} = \frac{1}{3}$$

$$A_2/_{z=1} = \frac{z+1}{(z+2)(z+3)} = \frac{2}{12} = \frac{1}{6}$$

$$A_3/_{z=-3} = \frac{z+1}{(z+2)(z-1)} = \frac{-2}{(-1)(-4)} = -\frac{1}{2}$$

$$Y(z) = 0 + \frac{1}{3}\frac{z}{z+2} + \frac{1}{6}\frac{z}{z-1} - \frac{1}{2}\frac{z}{z+3}$$

$$Y(k) = \frac{1}{3}(-2)^k + \frac{1}{6}(1)^k - \frac{1}{2}(-3)^k$$

B) $$Y(z) = \frac{2}{z^2 - 5z + 6} = A_0 + \frac{A_1 z}{z-3} + \frac{A_2 z}{z-2}$$

$$A_0 = \frac{1}{3}$$

$$A_1 = \frac{2}{(z-2)z} = \frac{2}{(1)(3)} = \frac{2}{3}$$

$$A_2 = \frac{2}{(z-3)z} = \frac{2}{(-1)(2)} = \frac{2}{-2} = -1$$

$$Y(z) = \frac{1}{3} + \frac{2}{3}\frac{z}{z-3} - 1\frac{z}{z-2}$$

$$Y(k) = \frac{1}{3}\delta(k) + \frac{2}{3}(3)^k - (2)^k$$

RAICES REPETIDAS

$$X(z) = \frac{6z^3 + 2z^2 - z}{z^3 - z^2 - z + 1} = A_0 + \frac{A_1 z}{(z-1)^2} + \frac{A_2 z}{(z-1)} + \frac{A_3 z}{z+1}$$

Para $A_0 = 0$

Para $A_1/_{z=1} = \frac{6z^2 + 2z - 1}{(z+1)} = \frac{6+2-1}{2} = \frac{7}{2}$

Para $A_2 \frac{dA_1}{dz} = \frac{6z^2 + 12z + 3}{(z+1)^2} = \frac{6+12+3}{9} = \frac{21}{9}$

Para $A_3/_{z=-1} = \frac{6z^2 + 2z - 1}{(z-1)^2} = \frac{6-2-1}{4} = \frac{3}{4}$

$$X(z) = \frac{7}{2}\frac{z}{(z-1)^2} + \frac{21}{9}\frac{z}{z-1} + \frac{3}{4}\frac{z}{z+1}$$

$$X(k) = \frac{7}{2}(1)^{k-1} + \frac{21}{5}(1)^k + \frac{3}{4}(-1)^k$$

$$X(z) = \frac{4z^2 - 6z}{z^2 - 6z + 9} = \frac{z(4z-6)}{z^2 - 6z + 9} = \frac{z(4z-6)}{(z-3)z} =$$

$$= A_0 + \frac{A_1 z}{(z-3)^2} + \frac{A_2 z}{z-3}$$

$A_0 = 0$

$A_1 = 6$

$A_2 = \frac{dA_1}{dz} = 4$

$$Y(z) = 6\frac{z}{(z-3)^2} + 4\frac{z}{z-3}$$

$$Y(k) = 6k(3)^k + 4(3)^k$$

## RAICES COMPLEJAS

$z_{1,2} = 1 \pm 3j \begin{cases} a^k \cos(\omega k) \\ a^k \operatorname{sen}(\omega k) \end{cases}$

$$\frac{z^2 - 2z}{z^2 - 2z + 10} = \frac{z(z-2)}{z^2 - 2z + 10}$$

$$y(z) = \frac{z(z-2)}{z^2 - 2z + 10} = A_0 + \frac{A_1 z (z - a\cos\omega) + A_2 z a \operatorname{sen}\omega}{z^2 - 2az\cos\omega + a^2} + \frac{A_2 z a \operatorname{sen}\omega}{z^2 - 2az\cos\omega + a^2}$$

$$y(z) = \frac{z(z-2)}{z^2 - 2z + 10} = A_0 + \frac{A_1 z (z - a\cos\omega) + A_2 z a \operatorname{sen}\omega}{z^2 - 2az\cos\omega + a^2}$$

Igualando denominadores

$10 = a^2 \qquad a = \sqrt{10}$

$-2 = -2\sqrt{10}\cos\omega \Rightarrow \cos\omega = \frac{2}{2\sqrt{10}} = \frac{1}{\sqrt{10}}$

$\omega = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right) = 1.24$

$\operatorname{sen}\omega = 0.94$

Para $A_0 / z=10 = 0$

Para $A_1$ y $A_2$

$z(z-2) = A_1 z (z - \sqrt{10}\left(\frac{1}{\sqrt{10}}\right) + A_2 z \sqrt{10}\ (0.94))$

Sust a $z = 1 + 3j$

$1 + 3j - 2 = A_1(1 + 3j - 1) + A_2 3$

$-1 + 3j = 3j A_1 + 3A_2$

Por igualación de complejos

$A_1 = 1 \qquad A_2 = -\frac{1}{3}$

$$y(z) = \frac{(1)z(z - \sqrt{10}\cos\omega) - \frac{1}{3} z\sqrt{10}\operatorname{sen}\omega}{z^2 - 2\sqrt{10}\, z\cos\omega + (\sqrt{10})^2}$$

$$\therefore y(k) = (\sqrt{10})^k \cos(1.24k) - \frac{1}{3}(\sqrt{10})^k \operatorname{sen}(1.24k)$$

RAICES CONJUGADAS

a) $X(j\omega) = \frac{j\omega+2}{j\omega^2+7j\omega+12}$

$$\frac{j\omega+2}{(j\omega+3)(j\omega+4)} = \frac{A}{j\omega+3} + \frac{B}{j\omega+4}$$

Si $A/_{j\omega=-3} = \frac{-3+2}{-3+4} = \frac{-1}{1} = -1 \quad \therefore A = -1$

Si $B/_{j\omega=-4} = \frac{-4+2}{-4+3} = \frac{-2}{-1} = 2 \quad \therefore B = 2$

$$\therefore X(s) = -\frac{1}{j\omega+3} + \frac{2}{j\omega+4}$$

$$x(t) = e^{-3t} + 2e^{-4t}$$

b) $\frac{j\omega+2}{j\omega^3+4j\omega^2+3j\omega} = \frac{j\omega+2}{j\omega(j\omega+3)(j\omega+1)} = \frac{A}{j\omega} + \frac{B}{j\omega+3} + \frac{C}{j\omega+1}$

$A/_{j\omega=0} = \frac{0+2}{3} = \frac{2}{3} \quad \therefore A = 2/3$

$B/_{j\omega=-3} = \frac{-3+2}{-3(-3+1)} = \frac{-1}{6} = -\frac{1}{6} \quad B = 1/6$

$C/_{j\omega=-1} = \frac{-1+2}{-1(-1+3)} = \frac{1}{-2} = -\frac{1}{2} \quad C = -1/2$

$$X(j\omega) = \frac{2}{3}\frac{1}{j\omega} - \frac{1}{6}\frac{1}{j\omega+3} - \frac{1}{2}\frac{1}{j\omega+1}$$

$$\therefore x(t) = \frac{2}{3} - \frac{1}{6}e^{-3t} - \frac{1}{2}e^{-t}$$

RAICES REPETIDAS

$$\frac{5j\omega - 1}{j\omega^3 + 3j\omega - 2}$$

$$\frac{5j\omega - 1}{(j\omega - 2)(j\omega + 1)^2} = \frac{A}{j\omega - 2} + \frac{B}{(j\omega + 1)^2} + \frac{C}{j\omega + 1}$$

Para $A/_{j\omega = 2} = \frac{5(2) - 1}{(2+1)^2} = \frac{9}{9} = 1 \quad \therefore A = 1$

Para $B/_{j\omega = -1} = \frac{5(-1) - 1}{(-1 - 2)} = \frac{-6}{-3} = 2 \quad B = -2$

Para $C \; \frac{dB}{dj\omega} = \frac{-9}{(j\omega - 2)^2} = -\frac{9}{9} = -1 \quad C = -1$

$$\therefore x(t) = e^{-2t} + 2e^{t}r(t) - e^{-t}$$

c) $$\frac{j\omega^2 + 16j\omega + 2}{j\omega^3 + 14j\omega^2 + 65j\omega + 100}$$

$$x(s) = \frac{j\omega^2 + 16j\omega + 2}{(j\omega + 5)^2 (j\omega + 4)} = \frac{A}{(j\omega + 5)^2} + \frac{B}{j\omega + 5} + \frac{C}{j\omega + 4}$$

$A/_{j\omega = 5} = 53 \quad A = 53$

$B = \frac{dA}{dt} = \frac{j\omega^2 + 8j\omega + 62}{(j\omega + 4)^2} = 47 \quad B = 47$

$C/_{j\omega = -4} = 14 \quad \therefore C = 14$

$$\therefore x(t) = 53e^{-5t}r(t) + 47e^{-5t} + 14e^{-4t}$$