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THE MOON:

HER MOTIONS, ASPECT, SCENERY, AND PHYSICAL CONDITION.

BY

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"With how sad steps, O Moon, thou climb'st the sky,—
How silently and with how wan a face!"—WORDSWORTH.

"Art thou pale for weariness
Of climbing heaven and gazing on the earth,
Wandering companionless
Among the stars that have a different birth,—
And ever changing, like a joyless eye
That finds no object worth its constancy?"—SHELLEY.

WITH THREE LUNAR PHOTOGRAPHS BY RUTHERFURD
(ENLARGED BY BROTHERS)
AND MANY PLATES, CHARTS, ETC.

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TO

WARREN DE LA RUE, ESQ.

D.C.L., F.R.S., F.R.A.S., &c.

IN RECOGNITION OF THOSE IMPORTANT ADDITIONS TO OUR KNOWLEDGE

OF THE CELESTIAL BODIES,

AND ESPECIALLY OF THE SUN AND MOON,

WHICH HAVE RESULTED FROM HIS PHOTOGRAPHIC AND OTHER

SCIENTIFIC RESEARCHES,

This Work is respectfully Dedicated

BY

THE AUTHOR.

PREFACE.

Although I had long purposed to draw up a treatise on the Moon, to form part of the series of volumes to which my works on Saturn and the Sun appertain, I originally proposed that this treatise should be the last instead of the third of that series. But Mr. Brothers being desirous of publishing three of Mr. Rutherfurd's magnificent lunar photographs, asked me to prepare some accompanying letterpress, and the present work thus had its origin; for I soon found that if I supplied the required quantity of letterpress, I should to some extent injure the prospects of the more complete work which I had in view. Moreover, it seemed to me desirable not to take up the subject partially and resume it at some distant epoch, but to deal with it in a single effort. This will serve to explain the delay which ensued; for certain parts of my subject-matter required much time and close application, for their thorough and independent investigation. I ought also (for reasons which will be understood by subscribers to the large volume) to explain that Mr. Brothers and I found it convenient to separate our interests; so that while I provide the octavo volume (without photographs) to accompany his large volume of photographs (see p. 400), he provides the smaller photographs for the complete octavo treatise, on pre-arranged terms.

I take this opportunity of thanking Mr. Rutherfurd, on Mr. Brothers's behalf and my own, for his liberality in permitting the publication of his admirable photographs. I feel that the prospects of this work's success have been greatly enhanced, also, by the skill with which Mr. Brothers has enlarged these beautiful pictures of our satellite. (See advertisement leaf, p. 400.)

While this treatise has been in progress, I have heard that a work on the Moon has been commenced under the supervision of a well-known student of the Moon; and my friend Mr. Webb has mentioned his own intention of writing one day a book upon the same subject. It seemed to me, therefore, desirable, while presenting so much as was necessary for the completeness of the present work on the parts of the subject which the promised works are severally likely to treat with special fulness, to devote my chief attention to those departments to which my own tastes chiefly invited me. It is perhaps hardly necessary to remark that in any case some portions of the subject must have been less fully treated of than others, and some omitted altogether, since, in fact, ten such volumes as the present would be insufficient to deal satisfactorily with all the matters of interest connected with the Moon.

The only chapters in this treatise which require comment here, are the second and third, relating to the motions of the Moon, and to her changes of aspect :—

In Chapter II. I have given a very full account of the peculiarities of the Moon's motions ; and notwithstanding the acknowledged difficulty of the subject, I think my account is sufficiently clear and simple to be understood by any one (even though not acquainted with the elements of mathematics) who will be at the pains to read it attentively through. I have sought to make the subject clear to a far wider range of readers than the class for which Sir G. Airy's treatise on gravitation was written, while yet not omitting any essential points in the argument. In order to combine independence of treatment with exactness and completeness, I first wrote the chapter without consulting any other work. Then I went through it afresh, carefully comparing each section with the corresponding part of Sir G. Airy's Gravitation, and Sir J. Herschel's chapters on the lunar motions in his "Outlines of Astronomy." I was thus able to correct any errors in my own work, while in turn I detected a few (mentioned in the notes) in the works referred to.

I have adopted a much more complete and exact system of illustration in dealing with the Moon's motions than either of my predecessors in the explanation of this subject. I attach great importance to this feature of my explanation, experience having satisfied me not only that such matters should be very freely illustrated, but that the illustrations should aim at correctness of detail, and (wherever practicable) of scale also. Some features, as the advance of the perigee and the retreat of the nodes, have, I believe, never before been illustrated at all.

In Chapter III. I give, amongst other matters, a full explanation of the effects due to the lunar librations. I have been surprised to find how imperfectly this interesting and important subject has been dealt with hitherto. In fact, I have sought in vain for any discussion of the subject with which to compare my own results. I have, however, in various ways sufficiently tested these results.

The table of lunar elements will be found more complete than that usually given. In fact, in this table, and throughout the work, my aim has been to help the student of the subject by supplying information not given, or not so completely given, elsewhere. It has always seemed to me that although in works on scientific subjects much of what is written must be common property and many facts must be compiled from the writings of other authors, the main purpose of the writer should be to present results which he has himself worked out and which are calculated to be of use to others. I doubt, indeed, whether any one is justified in writing a treatise on science unless he has such a purpose chiefly in view.

RICHARD A. PROCTOR.

LONDON: *June* 1873.

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PHOTOGRAPHS BY RUTHERFURD.*

PLATES.

* These small photographs are not given in the subscribers' copies.

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* These plates will be found in the subscribers' folio volume, and are there given to avoid folding.

Subscribers' copies contain, also, a photograph of the moon, first quarter, taken by Mr. Brothers on Dec. 27, 1865; and a series of small photographs showing the progress of the lunar eclipse of Oct. 4, 1865. These are intended to show what may be done with a refracting telescope (5 inches in aperture), not like Rutherfurd's, corrected for the chemical rays, but of the ordinary construction. See pp. 230, 231.

SPECIAL DIRECTIONS TO BINDER.

Plates XVII. and XVIII. should be so placed that when both are unfolded they will be side by side for comparison.

Plates XXI. and XXII. should lie the same way, the top of each towards the left, so as to admit of being studied simultaneously.

ERRATA.

Plate VIII. fig. 30. – For "inwards," *read* "outwards."
fig. 31.—For "outwards," *read* "inwards."

Plate X. fig. 39.—The arrow near arc N′M′ should be below the I, not above as shown, and the arrow near the arc NM should be above the I, instead of below.

Plate XIII. fig. 55.—M_1, near top, should be M_2.
For fig. 53*a* *read* 56*a*.

Plate XV. fig. 77.—The left-hand O should be O′.

At p. 28, line 9 from bottom of page, for "more" *read* "less."
" p. 208, " 3 " " " for "that her, *read* "than that her."
" p. 349, " 3 " top " for "dusty," *read* "dusky."

THE MOON.

CHAPTER I.

THE MOON: DISTANCE, SIZE, AND MASS.

Although the sun must undoubtedly have been the first celestial object whose movements or aspect attracted the attention of men, yet it can scarcely be questioned that the science of astronomy had its real origin in the study of the moon. Her comparatively rapid motion in her circuit around the earth afforded in very early ages a convenient measure of time. The *month* was, of course, in the first place, a lunar time-measure. The *week*, the earliest division of time (except the day alone) of which we have any record, had also its origin, most probably, in the lunar motions. Then the changes in the moon's appearance as she circles round the earth must have led men in very early times to recognize a distinction between the moon and all other celestial objects. While inquiring into the nature of these changes, and perhaps speculating on their cause, the first students of the moon must have soon begun to recognize the fact that she traverses the stellar vault so as to be seen night after night among different star-groups. To the recognition of this circumstance must be ascribed the origin of astronomy properly so called. Until the varying position of the moon among the stars had been noticed, men must certainly have failed to notice the changes in the aspect of the stellar heavens night after night throughout the year. In examining the moon's motions among the stars, they must have been led to study the annual motion of the stellar sphere. Thence presently they must have learned to distinguish between the fixed stars and the planets. And gradually, as the study of the stars, the moon, and the planets continued, the fundamental problems of astronomy must have presented themselves with increasing distinctness, to be for centuries the object of ingenious speculation, more or less based on the actual results of observation.

It would be difficult to form just ideas respecting the order in which the various facts respecting the moon and her motions were ascertained by ancient astronomers. Indeed, it seems probable that among the various nations to whom the origin of astronomy has been attributed, the moon's changes of appearance and position were studied independently, the order of discovery not being necessarily alike in any two cases. We are free, therefore, in considering the knowledge of the ancients respecting the moon, to choose that arrangement of the various facts which seems best suited to the requirements of the student.

The first, as the most obvious peculiarity of the moon, is that continually varying aspect which has led men in all ages to select the lunar orb as the emblem of change. "The inconstant moon, that nightly changes in her circled orb," must, in the first place, have appeared as a body capable of assuming really different shapes; and it is far from unlikely that this apparent evidence of power, associated with the moon's rapid change of place among the stars, may have led to the earliest forms of Sabæanism. Yet in very early times the true explanation of the peculiarity must have been obtained. The Chaldæan astronomer undoubtedly recognized the moon as an opaque orb, shining only because reflecting the sun's light; for otherwise we should be unable to explain the care with which they studied the moon's motions in connection with the recurrence of lunar and solar eclipses. Their famous cycle, the Saros (of which I shall have occasion to speak more particularly farther on), shows that they must have paid very close attention to the moon's movements for a long period before the Saros was determined, and for a much longer period before the cycle was made known to other astronomers of ancient times. Moreover, as they recognized in the moon the occasion of solar eclipses, though they could see her waning as she approached the sun's place, and waxing from the finest crescent of light after passing him, it is clear that they must have understood that the lunar phases indicated no actual change of shape. Nor can we imagine that reasoners so acute as the Chaldæan astronomers failed to recognize how all the phases could be explained by the varying amount of the moon's illuminated hemisphere turned at different times towards the earth.*

Quite early, then, the moon must have been recognized as an opaque globe illuminated by the sun. It would be understood that only one half of her surface can be in light. And apart from the fact that the moon was early recognized as causing solar eclipses by coming between the earth and the sun, it would be understood by the fineness of her sickle when near the sun's place on the celestial vault, that she travels in a path lying within the sun's. That fine sickle of light shows that at such times the illuminated half is turned almost directly away from the earth; and therefore the illuminating sun must at such times lie not far from the prolongation of a line carried from the earth's centre to the moon's.

It is not improbable, indeed, that the acute Chaldæans deduced similar inferences respecting the moon's nature from a careful study of her face; for the features of the moon when horned or gibbous

* It is remarkable, however, that Aratus, writing about 230 B.C., long after the time when the Chaldæans established their system of astronomy, refers to the lunar phases in a way which implies either ignorance or forgetfulness of their real cause; for he speaks of the significance of the position in which the horns of the new moon are seen, regarding this position, though obviously a necessary consequence of the position of the sun and moon, as in itself a weather portent.

obviously correspond with those presented by the full moon, in such sort that no one who considers the phenomenon attentively can doubt for a moment that the moon undergoes no real change when passing through her phases. It may also be imagined that the same astronomers who recognized the fact that Mercury is a planet, though he is never visible except in strong twilight, must have repeatedly observed that the whole orb of the moon can be seen when the bright part is a mere sickle of light. Nay, it is even possible that in the clear skies of ancient Chaldæa * the chief lunar features might be discerned when the dark half of the moon is thus seen.

The comparative nearness of the moon was probably inferred very early from her rapid motion of revolution around the earth. Almost as soon as observers noticed that the celestial bodies have different apparent motions, they must have learned that the moon's daily change of place among the stars is much greater than that of any other orb in the heavens. It would seem almost, from the distinction drawn in Job between the sun and the moon, that for some time the moon was regarded as the only body

* It is not very easy to determine what was the true site of the region spoken of in Judith (v. 6), as the land of the Chaldæans. The verse here referred to shows clearly that the region was not in Mesopotamia. From astronomical considerations I have been led to suppose that the first Chaldæan observers occupied a region extending from Mount Ararat northward as far as the Caucasian range. See Appendix A to "Saturn and its System," and the Introduction to my Gnomonic Star-atlas.

which actually moves over the celestial vault; for he says, "If I beheld the sun when it shined or the moon walking in brightness" (Job xxxi. 26); and the recognition of the sun's annual circuit of the heavens most probably preceded the discovery of the motions of the planets. Be this, however, as it may, astronomers must quite early have ascertained that among the more conspicuous orbs not one travels so quickly over the celestial vault as the moon. Accordingly, we find that even in the very earliest ages of astronomy the moon was regarded as the orb which travels nearest to the earth; and in the system of Pythagoras, in which musical tones were supposed to be produced by the revolution of the spheres bearing the planets, we find the *neate*, or highest tone of the celestial harmonies, assigned to the moon.

Whether the Chaldæan astronomers ever ascertained the moon's distance observationally, is a question we have no means of answering satisfactorily. If they did, it is probable that the determination arose from the careful study of the moon's peculiarities of motion,—undertaken with the object of rendering the prediction of eclipses more trustworthy. So far as is known, however, the first actual determination of the moon's distance (as compared with the dimensions of the earth's globe) must be ascribed to the astronomers of the Alexandrian school. Aristarchus of Samos (B.C. 280) had attempted to compare the distances of the sun and moon by a method of observation altogether inadequate to the requirements of that immensely difficult problem.* But he does not appear to have investigated the subject of the moon's distance. Somewhat more than a century and a quarter later, Hipparchus attacked both problems; the first with no better success than had rewarded Aristarchus, but the second by a method which was probably very successful in his hands, though it is from his successor Ptolemy that we learn the actual results of observations applied according to the ideas of Hipparchus.

It would appear that the scrutiny of the moon's motions,—with the object of determining her path among the stars, and the exact laws according to which she traverses that path,—led Hipparchus to attack the problem of determining the moon's distance. We know that his observations were so carefully pursued that he determined the eccentricity of the moon's path, and its inclination to the sun's annual path on the star-vault. It is also highly probable that he detected a certain peculiarity of the moon's motion, called the *evection*, which will be described further on. Whether this is so, or whether the discovery should be ascribed to Ptolemy, it is certain that the labours of Hipparchus could not have led to the results actually obtained, without his having noticed certain effects due to the relative nearness of the moon as compared with the other celestial bodies. The study of these effects probably enabled him to form a fair estimate of the moon's distance.

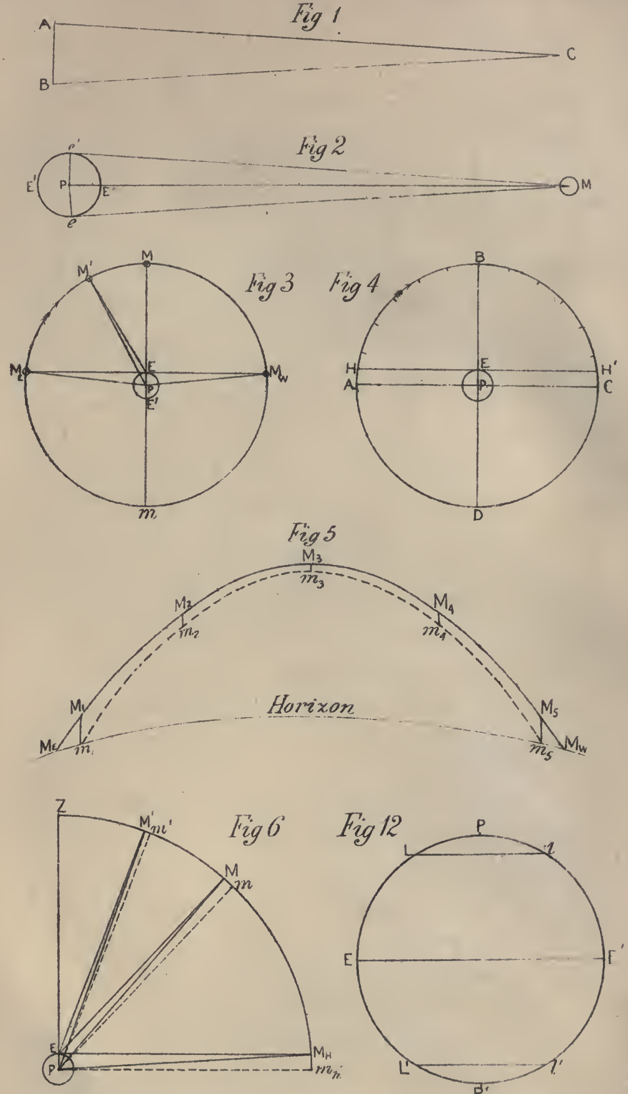
* His method is described in my treatise on the Sun (p. 7).

We have, however, no record of the results actually obtained by Hipparchus, and we must turn to the pages of the great work, the *Almagest*, written by Ptolemy about two centuries and a half later, for the first exact statement respecting the moon's distance, and the means used for determining it by the astronomers of old times.

The fundamental principle on which the measurement of the distance of any inaccessible object depends, is a very simple one. If a base-line (A B, fig. 1, Plate I.) be measured, and the bearing of the inaccessible object C from A and B (that is, the direction of the lines A C, B C, as compared with the line A B) be carefully estimated, then the distances A C and B C can, under ordinary circumstances, be determined. For, in the triangle A B C, we know the base-line A B, and the two base angles at A and B; so that the triangle itself is completely determined. Therefore, the ordinary formulæ of trigonometrical calculation,—or even a careful construction,—will give us the sides A C and B C.

If in all such cases we could determine A B and the base angles at A and B *exactly*, we should know the exact lengths of A C and B C. But even in ordinary cases, each observation must be to some extent, greater or less, inexact. Accordingly, the estimated distance of the object must be regarded as only an approximation to the truth. Setting aside mistakes in the measurement of the base-line, mistakes in determining the angles at A and B will obviously

PLATE I.



Illustrating the Measurement of the Moon's Distance, &c.

R. A. Proctor Delt.

affect more or less seriously the estimate of either A C or B C. And a very brief consideration of the matter will show that the greater is the distance of C as compared with the base-line A B,—in other words, the smaller the angle C,—the more serious will be the effect of any error in the observation of the angles A and B.

Now, the difficulty experienced by the astronomer in the application of this direct method to the determination of the distances of celestial objects, consists chiefly in this: that his base-line must always be exceedingly small compared with the distance which he wishes to determine. It is, indeed, only in the case of the moon that the astronomer can apply this method with the least chance of success; and even in her case the problem is by no means an easy one. We shall see presently that the distance of the moon exceeds the earth's diameter in round numbers some thirty times. If the reader draw a figure, as in fig. 1, but so that each of the lines A C and B C is about sixty times as long as A B, he will see that the angle at C is exceedingly minute, insomuch that a very slight error in the determination of either of the base angles at A and B would lead to a serious error in the estimate of the distance of C, even supposing a full diameter of the earth could be taken as the base-line.

Now, when we remember that the ancient astronomers were unable to undertake long voyages for the purpose of determining the moon's distance, and that, even though they could have set observers at widely distant stations, they had not the requisite acquaintance with the geographical position of different places to know what base-line they were making use of, it may appear surprising that Hipparchus or Ptolemy should have been able to form any satisfactory estimate of the moon's distance. But Hipparchus showed how the astronomer could deal with this problem without leaving his observatory. The earth's daily rotation carries the astronomer's station each day round a vast circle, and he has but to notice the effect of this motion on the moon's position, tó be enabled to form almost as satisfactory an estimate of her distance as by observations made at stations far apart. It is true that Hipparchus probably (and Ptolemy certainly) regarded the earth as fixed. But it is a matter of no importance (so far as the problem of determining the moon's distance is concerned) whether we regard the daily rotation of the moon with the celestial vault as due to the motion of the heavens themselves around the fixed globe of the earth, or as brought about by the rotation of the earth upon her axis.

Let us now consider the features of this method attentively:—

In the first place, let us conceive the moon (fig. 2, Plate I.) to be at rest on the celestial equator, e E e' being the earth's equator, and P the earth's pole. Then, a place at e' is carried by the diurnal rotation round the circle e E e'. If M e and M e' touch the circle e E e' E$'$, then, when the place is at e, the moon is seen on the horizon, due east; and when the place has been carried to *e′*, the moon is again on the horizon, but due west. When the place is at E, midway between *e* and *e′*, the moon (under the imagined conditions) is immediately overhead. Thus, the moon rising due east, passes to the point overhead and onwards to the west, where she sets. But it is clear that the moon's apparent motion in passing across the sky, from the eastern to the western horizon, is not uniform, as seen from the globe E E′. The arc *e* E *e′* is obviously less than a semicircle; in other words, the moon, under the imagined conditions, completes her course athwart the heavens—a seeming half-circle —in less than half a day, while she is below the horizon (completing the other seeming half of the circle) in more than half a day.

But as some find a difficulty in forming a clear conception of the apparent motion of a body placed as M in fig. 2, while a point is carried round such a circle as E E′, I will at this stage introduce a slight change in the method of considering the matter. It is of course obvious that the apparent motion of the moon is precisely the same as though the moon went round the earth, while the earth's globe remained at rest. Let us then suppose this to be the actual state of things. In fig. 3 the earth is supposed to be at rest, P being the pole, as in fig. 2; and the moon is supposed to be carried round uniformly about the earth's centre, in the direction shown by the arrow. Comparing figs. 2 and 3, the student will at once see how one illustrates the real, the other the apparent motion of the moon under the assumed conditions. Now, the line $M_E M_W$ is the horizon line from east to west; M_E the moon's place of rising in the east; M_W her place of setting in the west. Then (always under the assumed conditions, which regard her as not moving on her orbital path) she appears to traverse the arc $M_E M M_W$ while above the horizon, and while below the horizon she traverses the arc $M_W m M_E$. But, clearly $M_E M M_W$ is less than a semicircle, and its difference from a semicircle depends entirely on the fact that the globe E E′ has dimensions comparable with those of the circle M *m*; in other words, that E P is comparable with M P. If we suppose the circle E E′ drawn very much smaller, then the arc $M_E M M_W$ becomes very nearly a semicircle. If, on the other hand, we suppose the circle M *m* drawn very much larger, then again the arc $M_E M M_W$ becomes nearly a semicircle. So that, if observation shows the arc $M_E M M_W$ to differ appreciably from a semicircle, we have at once a means of determining the moon's distance as compared with the earth's radius.

Suppose, for instance, that instead of taking twelve hours in passing from M_E to M_W, the moon was observed to take only eleven hours, or 5½ in passing from horizon to zenith, then we have only to draw a circle such as A B C D in fig. 4, Plate I.; to divide the semicircle A B C into 12 parts, as shown, and to take B H, B H′, each equal to 5½ such parts; then the line H H′ cuts off for us P E, which represents the earth's radius, where P B represents the distance of the moon. Such a construction—or, if preferred, the corresponding calculation—would thus at once show what relation the moon's distance bears to the earth's diameter.

It is obvious that although atmospheric refraction causes the moon's apparent place, when she is near the horizon, to be somewhat higher than the place she would have if the atmosphere did not exist, yet this is a circumstance which the astronomer can take fully into account; since it is in his power, by observing the stars, to determine the exact value of atmospheric refraction on celestial bodies at different altitudes.

This method of determining the moon's distance is not the less available, that the moon is not at rest. Thus, suppose the moon to be travelling in the circle M *m*, fig. 3; then, if the rate of the moon's motion be known,—that is, the length of time in which the moon completes the circuit of the stars,—the observer can apply to the moving moon precisely the same considerations which he would apply to the moon regarded as at rest. He would still be able to compare together the periods during which the moon is above and below the horizon, since her own motion would cause *both* these periods to be correspondingly affected. He would thus obtain the two unequal arcs M_E M M_W and M_W *m* M_E (fig. 3), which would give him the cross line M_E E M_W, as before, and therefore the relative magnitude of E P and P M_E.

The actual problem is rendered somewhat less simple by the fact that the moon's motion does not take place in the circle M *m*, but in a path inclined to that circle. But it is obviously in the power of mathematics to take into consideration all the effects due to the moon's real motion, and thus, as in the simpler case imagined, to deduce the relation between E P and M_E P.

But we may now look at the problem in a somewhat different light. Hitherto we have only considered the effect of the earth's size in causing an apparent want of uniformity in the moon's rate of motion. We can see, however, from fig. 2, that what in reality happens is that the moon is not seen in the same direction from points on the earth's surface as from the centre of the earth; and that the apparent displacement is greater or less according as the moon is nearer to or farther from the horizon. If we suppose M to represent the moon's place when she is overhead, we see that she is seen in the same direction from E as from P. But when she is on the horizon at M_E, she is seen as though ninety degrees from the point overhead; whereas, as seen from P, she would be less than ninety degrees from that point: that is, she is seen from E lower down than she is in reality. In any intermediate position, as M′, she would be seen lower down from E than from P; but not so much depressed as when she is near the horizon.

But it is clear that this is equally true, wherever the station of the observer may be. The moon is always seen below the place she would occupy if she could be observed from the earth's centre, except when she is actually overhead; and she is more depressed the nearer she is to the horizon.

It follows that wheresoever the observer may be stationed on the earth, the moon cannot appear to move as she would if she could be watched from the centre of the earth. If M_E M_3 M_W (fig. 5, Plate I.) represent her path as supposed to be seen from the centre of the earth, then the actual path she follows is as shown in the dotted line m_1 m_3 m_5, her observed place being always vertically below her true place (for we may consider her place as supposed to be viewed from the earth's centre, her *true* place, since it is only as so viewed that her motions could show their true uniformity). This apparent displacement of the moon is called her *parallax*.

Hence, for any observer not placed at a station where the moon rises actually to the zenith, it is not her total displacement when on the horizon, called her *horizontal parallax*, which is to be compared with her true placing as she is seen on the zenith; but the former displacement is to be measured against the displacement which she shows when highest in the heavens. It is seen from fig. 6, Plate I., that when the moon rises high above the horizon this difference will be appreciable if the moon's horizontal parallax is appreciable. For let M represent the moon's place when she is 50 degrees above the horizon; then, as seen from P, she would lie in the direction P M; but from E she is seen in the direction E M, which is the same as P m (drawing P m parallel to E M). Thus the actual parallax, measured as an arc in the heavens, is represented by the arc M m. The horizontal parallax is represented by the arc M_H m_h, which is clearly greater than M m. If M′ represents the moon's place when she is 70 degrees above the horizon, then M′ m', her parallax, is less again than M m.*

Now the moon's apparent diurnal path at any station on the earth would precisely resemble the apparent diurnal path of a star at the same distance from the pole, if it were not, *first*, for the moon's actual motion amongst the stars, and *secondly*, for this effect, by which she is depressed below her true place more or less according as she is nearer to or farther from the horizon. The first circumstance could be taken into account so soon as the general course of the moon's motion came to be known. Her true path among the stars at any particular time could be ascertained. And *then* it would only remain to determine how much she seemed to depart from that path when on the horizon, and again when high above it. This could be done by means of any contrivance which would enable the observer to follow the moon

* The argument here relates to the actual construction of figures such as fig. 6; and the student should repeat the construction to satisfy himself on the point. The general mathematical determination of the displacement is as follows:—The arc M_H m_h is equal to E P (appreciably), and the arc M m is equal to a perpendicular from E on P m. Hence the parallax at M is to the horizontal parallax as the last-named perpendicular to E P, or as the sine of the angle M P Z. It follows that if the moon's horizontal parallax is H, her parallax when her true altitude is λ, is H cos λ.

in the same way that the sun or a star can be followed, by means of a suitable pointer carried round the axis on which the celestial vault seems to rotate in what is called the diurnal motion; that is, around an axis directed to the true pole of the heavens. Such a pointer directed once upon a star would follow the star from rising to setting (neglecting the effects of atmospheric refraction); but directed on the moon, and corrected from time to time, so that the moon's actual motions among the stars should be taken into account, the pointer would not follow the moon by a mere rotation around its polar axis. If pointed on the moon when she first rose above the horizon, it would be found to point *below* the moon when carried (round its axis) towards the place occupied by the moon when high above the horizon; for it would have to be depressed by the full amount of the horizontal parallax when the moon was on the horizon, and this depression would be too great when the moon was high above the horizon. In like manner, if the pointer were directed upon the moon when she was high above the horizon, it would be carried to a place above that occupied by the moon when setting beyond the western horizon.

It was in this way that the moon's distance was first ascertained. The reader will recognize in the description just given the principle of the equatorial telescope, which, turning around a polar axis, follows a star by a single motion. But the astronomical principle of this instrument was understood and applied long before the telescope itself was invented. Ptolemy, who is usually credited with the invention of the equatorially mounted pointer, was the first to apply the instrument to the determination of the moon's displacement or parallax.* The result contrasts strikingly with the ill success which he and other ancient astronomers experienced when they attempted to apply this and other methods to the determination of the sun's distance. He assigned 57′ as the moon's parallax when she is on the horizon,—in other words, his observations led him to the conclusion that the angle $E M_{II} P$ (fig. 6, Plate I.) is one of 57′, a value which would set the moon's distance at almost exactly sixty times the earth's radius. We shall see presently that this is very close to the true value.† Other observations were made by this method; and it is probable that the value given for the lunar

* A trace of this early application of the principle remains in the name *parallactic instrument* still sometimes given to the equatorial. The principle of the instrument is given in the Almagest, and the instrument, as made before the telescope was invented, was sometimes called *Ptolemy's Rule.*

† Before this Aristarchus of Samos had set the moon's distance at two million *stadia*, which, according to Buchotte's estimate of the length of the Greek stadium, would be equal to about 230,000 miles. The method by which he deduced this result is not well known; but it is believed to have been based on the consideration of the length of time occupied by the moon in passing from horizon to horizon; in fact, it would seem to have been a modification of the method hypothetically considered in pp. 10—12. If so, it corresponded to a certain degree with the method he applied to determine the sun's distance. (*See* "The Sun," p. 25.) Hipparchus considered that the moon's distance lay between 62 and 72½ times the radius of the earth. The above evaluation of Ptolemy is inferred from the numbers given at p. 211 of Prof. Grant's "History of Physical Astronomy."

parallax in the Alphonsine Tables, viz. 58′, was deduced from a comparison of many such observations. This would give a distance somewhat exceeding 59 times the earth's radius, or more exactly, with the present estimate of the earth's dimensions, 235,000 miles.

Tycho Brahé, from his own observations, based on the same principle, found for the moon's horizontal parallax 61′, corresponding to a distance somewhat less than 223,000 miles.*

But a more satisfactory method of determining the moon's distance is that which is based simply on the considerations discussed at pp. 8, 9,—in other words, the method of observing the moon from two distant stations whose exact position on the earth's globe has been ascertained.

Let us suppose, for convenience of illustration, that one station is the Greenwich Observatory, and the other the Observatory at the Cape of Good Hope.

* Before passing from the consideration of the method of determining the moon's distance by observations made at a single station, it may be mentioned that, as applied in later times, it depends on the moon's apparent displacement from her path, calculated for the earth's centre. Now since the moon's parallax always causes her to appear vertically below her true place, it is obvious that the *whole* of this displacement will operate to displace her from her calculated path, *only* when the part of the path which she is at the moment traversing is horizontal,—in other words, when she is on the highest part of that path at the moment above the horizon. Although her actual parallax would then not be a maximum, it would act solely to shift her from her calculated path. According to the old astronomical systems, such occasions were held to be particularly favourable for lunar observations. The highest part of the moon's path was called its *nonagesimal degree*,—a term also applied to the highest part of the elliptic.

These two stations are not on the same meridian, as will be seen from fig. 7, Plate II., which shows Cape Town more than 18° of longitude east of Greenwich.* At present, however, we shall not take into account the difference of longitude.

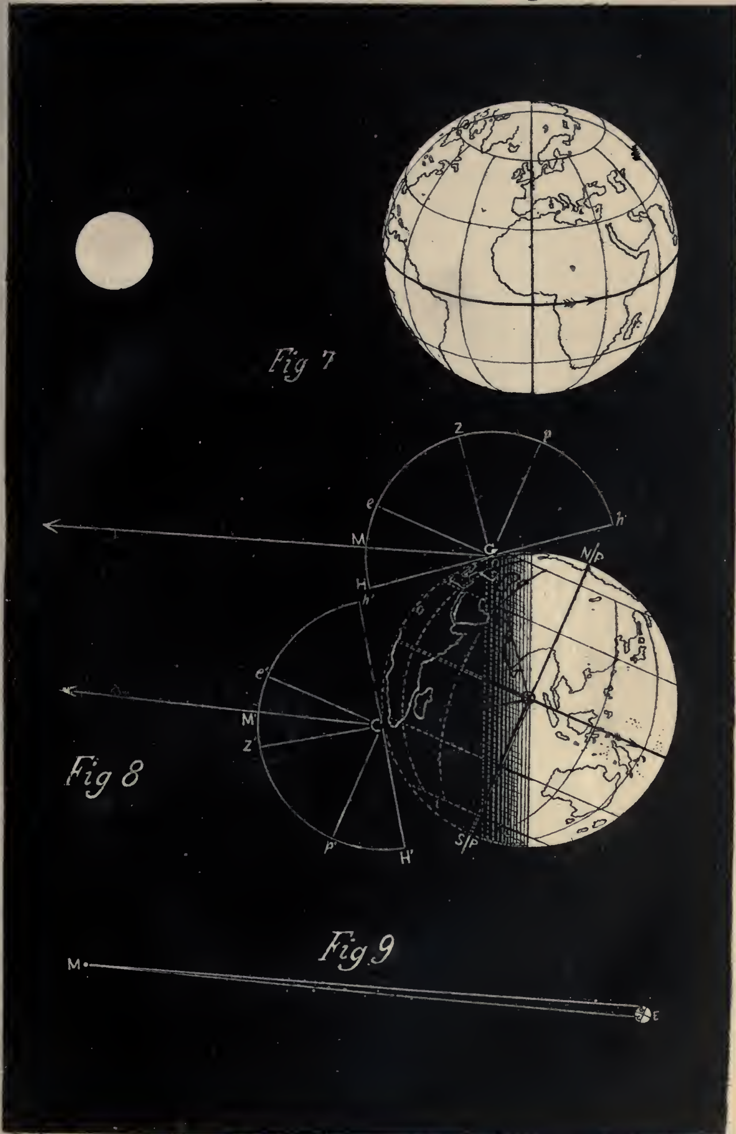
Let fig. 8, Plate II., represent a side view of the earth at night, when Greenwich is at the place marked G. Let H h be a north and south horizontal line at Greenwich, G Z the vertical, G p (parallel to the earth's polar axis) the polar axis of the heavens; and let us suppose that the moon, when crossing the meridian, is seen in the direction G M; then the angle p G M is the moon's north polar distance.

Again, let us suppose C to be the Cape Town Observatory, which has at the moment passed from the edge of the disc shown in fig. 8, by nearly $1\frac{1}{4}$ hours' rotation; but let us for the moment neglect this, and suppose the station C to be at the edge of the disc. Let H′ C h' be the north and south horizontal line at C, C Z′ the vertical, C p' (parallel to the earth's polar axis) the polar axis of the heavens (directed necessarily towards the south pole); and let us suppose that the moon, when crossing the meridian, is seen in the direction C M′. Then, since the lines G M and C M′ are both pointed towards the moon's centre, they are not parallel lines, but meet, when produced, at that point.

Let fig. 9, Plate II., represent this state of things

* This figure is reduced from one of the four summer pictures forming Plate VII. of my "Sun-views of the Earth."

Determining Moon's Distance by PLATE II.



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Observations at Greenwich & Capetown

on a smaller scale, M being the moon, G Greenwich, and C the Cape of Good Hope; then G C M is just such a triangle as we considered at page 8. The base-line G C is of course known; and it is very easily seen that the angles at G and C are known from the observations pictured in fig. 8.* Thus M C and M G can be calculated.

Such is the general nature of the method for determining the moon's distance by observations made at different stations, and either simultaneously or so nearly simultaneously that the correction for the moon's motion in the interval can be readily made.†

* The distance from Greenwich to Cape Town is not in question, but the distance between Greenwich and the point C on the meridian of Greenwich; for any effects due to the difference of longitude of Cape Town and Greenwich are readily taken into account astronomically. Now the distance C G is the chord of a known arc of a great circle of the earth, if we neglect the earth's ellipticity, or is a known chord of the elliptic section of the earth through her axis if we take the ellipticity into account (as we must of course do in exact measurement). Thus C G is known, and the angles O G C, O C G, are equally known. Now the angle M G C is the sum of the angles M G H and H G C; and of these M G H is the moon's observed meridian altitude at Greenwich, while H G C is the complement of the known angle O G C. Hence M G C is known. In like manner M′ C G is known. So that we have the base-line and the two base angles of the triangle M C G known, and therefore M C and M G can be calculated. In reality the angle M C G is about 1½ degrees.

† If such an instrument as the equatorial were as trustworthy as a meridional instrument, it would be easy to make the observations simultaneously, determining the polar distances of the moon at Greenwich and Cape Town respectively. But as a matter of fact, it is absolutely necessary to observe the moon when she is on the

One of the earliest series of observations directed to the determination of the moon's distance was that undertaken by Lacaille when he visited the Cape of Good Hope in 1750. From a comparison of his results with observations made in Europe, he deduced the value 57′ 13″·1 for the moon's mean equatorial horizontal parallax. This corresponds to a mean distance of 238,096 miles. But it is to be noticed that Lacaille was not acquainted with the true shape of the earth. He supposed the earth's compression to be greater than it really is; in fact, he supposed the equatorial to exceed the polar diameter in the proportion of 201 to 199, whereas in reality the proportion is approximately 300 to 299; in other words, the compression is $\frac{1}{300}$. If this correction is taken into account, Lacaille's results give for the lunar parallax 57′ 4″·6, corresponding to a distance of 238,679 miles. Lalande, by comparing Lacaille's observations with his own, made simultaneously at Berlin,* found for the lunar parallax the value 57′ 3″·7, corresponding to a distance of 238,749 miles. It will be noticed

meridian. What then is done is to deduce from the observed north polar distance of the moon when on the meridian at Cape Town (or from the moon's place at that time, with respect to some known star) her position at the moment when she is on the meridian of Greenwich.

* Lacaille was born on March 15, 1713, and Lalande on July 11, 1732, so that Lalande was nineteen years younger than Lacaille, who was himself but a young man when he made his observations. In fact, Lalande was but nineteen years old when he was sent to Berlin for the purpose of observing the moon simultaneously with Lacaille at the Cape of Good Hope.

that as Berlin is more than 13 degrees east of Greenwich, observations made on the moon when in the meridian, at Cape Town and at Berlin, are more nearly simultaneous than corresponding observations at Cape Town and Greenwich.

Bürg, by comparing Lacaille's observations with those made at Greenwich, deduced for the moon's parallax the value 57′ 1″, corresponding to a distance of 238,937 miles.

Henderson, the first who determined the distance of the celebrated star Alpha Centauri, made a series of lunar observations at the Cape of Good Hope in 1832 and 1833, with very imperfect instrumental means. From a comparison of these observations with others made at Greenwich and Cambridge, he deduced 57′ 1″·8 for the value of the moon's parallax. The corresponding distance amounts to 238,881 miles.

The Astronomer Royal, from a discussion of the whole series of Greenwich observations, deduced the value 57′ 4″·94, corresponding to a distance of 238,656 miles.

But probably the most accurate value is that which has been deduced by Professor Adams from a comparison of Mr. Breen's observations at the Cape of Good Hope, with others made at Greenwich and Cambridge. Professor Adams deduces for the lunar parallax the value 57′ 2″·7, corresponding to a distance of 238,818 miles.

One other method of determining the moon's distance remains to be mentioned. It cannot, however, be called a strictly independent method, since it is based on the theory of gravity, which could not have been established without an accurate determination of the moon's distance.

In showing that the earth's attraction keeps the moon in her observed orbit, Newton had to take into account the moon's distance. He reasoned that the earth's attraction reduced as the square of the distance would be competent at the moon's distance to cause the observed deflection of the moon from the tangent to her path. He assumed the lunar parallax to be 57′ 30″, corresponding to a distance of 237,000 miles; and he found that the terrestrial attraction calculated for that distance corresponded very closely with the observed lunar motions, so closely as to leave no doubt of the truth of the theory he was dealing with. But now, when once the theory of gravity is admitted, we have in the observed lunar motions the means of forming an exact estimate of the earth's attraction at the moon's distance, and as we know her attraction at the earth's surface, we are enabled to infer the moon's distance. And in passing it may be observed that this process is not, as it might seem at a first view, mere arguing in a circle. Observation had already given a sufficiently accurate estimate of the moon's distance to supply an initial test of the theory that it is the earth's attraction reduced as the square of the distance which retains the moon in her orbit. This theory being accepted, and other tests applied, we may fairly reason back from it in such sort as to deduce the exact distance of the moon.*

In this process, however, the mass of the moon would have to be taken into account. In fact, as will be seen in the next chapter, we must add the moon's mass to the earth's in considering the actual tendency of the moon towards the earth; so that, if we know the moon's mass, the earth's size, and the moon's period, we can deduce the moon's distance.†

Burckhardt applying this method, on the assumption that the moon's mass is $\frac{1}{75}$ of the earth's, deduced the parallax 57′ 0″, corresponding to a distance of 239,007 miles. Damoiseau, taking the moon's mass at $\frac{1}{74}$ of the earth's, deduced a parallax of 57′ 1″, corresponding to a distance of 238,937 miles. Plana,

* The case may be compared to the following: In determining the rotation period of Mars (*see* Appendix A to my "Essays on Astronomy"), I had certain dates, separated by long intervals, on which the planet presented a certain aspect. Now, knowing pretty accurately the rotation period, I could divide one of these long intervals by this pretty accurate period, to get the total number of rotations in the interval: I could be certain that I should not get a full rotation too many or too few, but only a small fraction of a rotation, which could very well be neglected. Then, having the number of rotations, I could reverse the process, dividing the interval by this number to obtain the rotation period more exactly,—to obtain, in fact, a period which, used as a divisor instead of the former rougher determination, would leave no small fraction over or above.

† The following is the treatment of the problem, on the assumption that the moon moves in a circle round the earth:—

Let P be the number of seconds in the moon's periodic time round the earth (the sidereal month); D, the distance of the moon in feet; g, the measure of the force of gravity at the earth's surface assuming the moon's mass to be $\frac{1}{87}$, found for the mean lunar parallax the value 57′ 3″·1, corresponding to a distance of 238,792 miles.

We shall throughout the rest of this work assume that the moon's mean equatorial horizontal parallax is 57′ 2″·7, and her distance, therefore, 238,818 miles, the earth's equatorial diameter being assumed equal to 7,925·8 miles.

Now it follows from this that, as seen from the moon at her mean distance, the earth's equatorial radius subtends an angle of 57′ 2″·7; that is, the equatorial diameter of the earth covers on the heavens an arc of 1° 54′ 5″·4, as seen from the moon at her mean distance. If the moon's orbit were circular,

(in other words, with the assumed units of time and space, $g = 32{\cdot}2$). Then the moon's velocity in her orbit

$$= \frac{2\pi D}{P};$$

and the accelerating force of gravity exerted by the earth on the moon, is therefore

$$= \frac{1}{D}\left(\frac{2\pi D}{P}\right)^2,$$

$$= \frac{4\pi^2 D}{P^2} \qquad \text{(i)}$$

But the attraction g, first increased so as to take the moon's mass into account, and then reduced according to the law of the inverse square

$$= g\,\frac{M + m}{M} \cdot \frac{r^2}{D^2} \qquad \text{(ii)}$$

where M is the earth's mass, m the moon's, and r the earth's radius. Hence, equating the expressions (i) and (ii) we find

$$D = \left\{\frac{g\ (M + m)\ P^2\ r^2}{4M\pi^2}\right\}^{\frac{1}{3}}$$

Fig 10. The Moon's Orbit. (Mean Eccentricity.)

In this Figure, MM', Mμ, & M'μ' shew the Moon's mean, least, & greatest Disc.

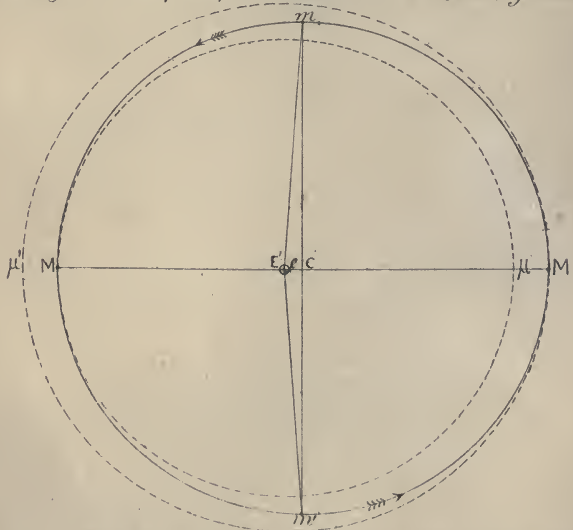

Fig 11: The Moon's Orbit. (Maximum Eccentricity.)

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the earth's equatorial diameter would always cover such an arc. But the moon traverses a path of considerable eccentricity. Its mean shape (for it varies in shape) is exhibited in fig. 10, Plate III., where C is the centre of the orbit, E the earth, M the place of the moon when nearest to the earth, or in perigee, M′ her place when farthest from the earth, or in apogee, m and m' her positions when she is at her mean distance (in other words, $m\ m'$ is the minor axis of the moon's orbit). Thus E C is the linear eccentricity of the orbit.* E C is about the eighteenth part of C M, and is thus not at all an evanescent quantity even on the small scale of fig. 10. The distance E C is equal to about 13,113 miles. It will be observed, however, that though the *eccentricity* of the orbit is shown in fig. 10, the *ellipticity*, that is the departure from the circular shape, is not indicated. In reality, it would not be discernible on the scale of fig. 10.†

But the eccentricity of the moon's orbit is not

* The true eccentricity is represented by the ratio of E C to E M; that is, in the case of the lunar orbit, it is about $\frac{1}{18}$ when the orbit is in its mean condition. When the orbit has its maximum eccentricity, the ratio rises to about $\frac{1}{15}$, and when the eccentricity is at its minimum, the value is about $\frac{1}{22}$.

† By a well-known property of the ellipse, the distances E m and E m' are equal to C M and C M′. Hence C m is easily found. If, for convenience, we represent C M or E m by the number 18, E C will be represented by unity. Hence C m will be represented by $\sqrt{(18)^2-1}$, or by $\sqrt{323}$, or by 17·9722. The semi-arcs C M and C m may be approximately represented by the numbers 1,800 and 1,797; that is, by the numbers 600 and 599; or C m is less than C M by less than 1-600th part of either.

constant. Owing to the perturbations which the moon undergoes (as explained in the next chapter), her path changes in shape, the mean distance remaining throughout nearly constant. The shape of her path when it is most eccentric, as well as when it is least eccentric, would not differ appreciably from fig. 10, and therefore, so far as this relation is concerned, no new figure is required. But for another purpose, presently to be explained, it is convenient to have a picture exhibiting the moon's path around the earth when the eccentricity is a maximum. It is therefore shown in fig. 11, Plate II., the centre being at C and the earth at E′, and M M′ the moon's path. The point *e* shows the position occupied by the earth's centre when the eccentricity is a minimum. The distance E′ C is 15,760 miles, while *e* C is 10,510 miles. Thus the difference, E′ *e*, is 5,250 miles, or about two-thirds of the earth's diameter. Owing to the peculiarities of the lunar perturbations, however, these numbers are not to be strictly applied in dealing with the lunar orbit. In fact, her distance from the earth is somewhat more increased, owing to perturbations, than it is reduced—when the maximum effects either way are compared.

The apparent diameter of the moon when she is at her mean distance is found by telescopic observation (at night) to be 31′ 9″, or 1,869″ (when reduced to correspond to the distance of the earth's centre; or, approximately, when supposed to be made on the moon in the horizon). But this value is partly increased by the effects of irradiation. When the moon's diameter is deduced from observations made during solar eclipses (at which time irradiation tends to reduce her apparent diameter, because she is then seen as a dark body on a light ground), the value depends partly on the telescope employed. With instruments of average power it is about 30′ 55″, or 1,855″. From a careful discussion of the occultations of stars by the moon, as observed at Greenwich and at Cambridge, the Astronomer Royal has inferred that the length of the moon's mean apparent diameter is 31′ 5″·1, or 1,865″·1.* This is the value assumed throughout the present work. (It is a useful aid to the memory to notice that the number of seconds of arc in this value gives the number of the year in which the Astronomer Royal announced his results.)

* As inconvenience is often experienced from the absence of all explanations of estimates such as these, I here state how the above value has been inferred; for I am unable to point to any passage in which the Astronomer Royal has distinctly stated it. In Mädler's "Der Mond" it is stated, in § 14, that Burckhardt assigns as the moon's semi-diameter 15′ 31″·95. In the Monthly Notices of the Astronomical Society for 1864–65, the Astronomer Royal assigns 2″ as the excess of the telescopic diameter of the moon over that inferred from stellar occultations; and speaking of the eclipse of 1833, he says that the observations of the moon gave −4″·2 as the correction on Burckhardt's semi-diameter, and −6″·8 as the correction on the telescopic semi-diameter. It follows that the telescopic semi-diameter exceeds Burckhardt's by 2″·6, and therefore that Burckhardt's estimate is less than Airy's estimate from occultations by 0″·6. Hence Airy's estimate from occultations (*nowhere stated in his paper*) must be 15′ 32″·55, corresponding to an apparent mean diameter of 31′ 5″·1.

There is no apparent flattening of the lunar orb as seen from the earth; the most careful measurement presents it as circular. Since the earth's semi-diameter subtends from the moon an angle or arc of 57′ 2″·7, or 3,422″·7, while the moon's diameter subtends from the earth an angle of 1,865″·1, it follows that the moon's diameter is less than the earth's radius (or 3,962·9 miles) in the proportion of 18,651 to 34,227. Thus it is readily calculated (by mere rule of three) that the moon's real diameter (or at least any diameter square to the line of sight from the earth) is 2,159·6 miles. It chances that this is the exact value adopted by Mädler, though obtained by employing a different value of the lunar parallax, of the lunar apparent diameter, and lastly of the earth's real diameter.

It follows that the earth's equatorial diameter exceeds the moon's in the proportion of about 3,670 to 1,000; or, if we represent the earth's equatorial diameter by 10,000, then the moon's would be represented by 2,725. Assuming the moon's shape to be globular, and the earth's compression $\frac{1}{300}$, it follows that the earth's surface exceeds the moon's in the proportion of about 13,435 to 1,000; or, if we represent the earth's surface by 10,000, the moon's will be represented by 744. Lastly, on the same assumption as to the moon's shape, the earth's volume exceeds the moon's in the proportion of about 49,263 to 1,000; or, if the earth's volume be represented by 10,000, the moon's will be represented by 209.

Roughly, we may take the moon's diameter as two-sevenths of the earth's, her surface as two twenty-sevenths, her volume as two ninety-ninths. Of these proportions, the most interesting is that between the moon's surface and the earth's; for neither the diameter nor the volume of the moon is specially related to her condition as a globe comparable with our earth as respects those features which affect our own requirements. But the surface of the moon's globe obviously affects her fitness, in one important respect, to be the abode of living creatures. Now the actual surface of the moon is rather more than two twenty-sevenths of the earth's, and the surface of the earth is about 196,870,000 square miles: hence the moon's surface is about 14,600,000 square miles. This is about the same as the area of Europe and Africa together (exclusive of the islands usually included with these continents). It is almost exactly equal to the areas of North and South America, exclusive of their islands. The portion of either hemisphere of the earth, lying on the polar side of latitude 58° 23′, is equal to the whole surface of the moon: that is, if E E′, fig. 12, Plate I., represent the earth, P and P′ being the poles, L l and L′ l' latitude parallels 58° 23′ north and south of the equator E E′, then either of the spaces of which L P l, L′ P′ l' are the visible halves, has an area equal to the moon's. The arctic and antarctic regions together exceed the moon in area in about the proportion of 10 to 9. Lastly, it may be noticed that, reckoning the Russian empire (in Europe and Asia) at 7,900,000 square miles, and the British dominions at 6,700,000, these two empires together are almost exactly equal in area to the whole surface of the moon: the part of the moon actually visible to us (taking her librations into account) is somewhat more extensive than the Russian empire, while the part totally concealed from us is somewhat less extensive than the British empire.

It is important to notice that, under all circumstances, whether the moon is at her mean distance, or nearer to or farther from the earth (in fact, whatever the size of her disc may be), the earth's disc, as supposed to be seen at the moment from the moon, is nearly $13\frac{1}{2}$ times larger. The actual proportion between the two discs is shown in fig. 7, Plate II.

But the variation of the moon's apparent size, according to her varying distance, must also be carefully taken into account. It is much greater than is commonly supposed. The observed telescopic mean diameter of the moon is, as already stated, $31' 9''$, while $31' 5''\cdot1$ is taken as the true mean diameter,—that is, the telescopic diameter reduced for the effects of irradiation. Now, the telescopic semi-diameter when the moon is at her nearest to the earth,—that is to say not merely in perigee, but in perigee at a time when her orbit has its greatest eccentricity—is found to be $33' 32''\cdot1$, while, when the moon is farthest from the earth, the observed diameter is $29' 22''\cdot9$. These values reduced for the effects of irradiation, give for the diameter,—

(1) When the moon is nearest to the earth,	33′	30·1″	or	2010·1″
(2) „ „ at her mean distance,	31	5·1	or	1865·1
(3) „ „ farthest from the earth,	29	20·9	or	1760·9

It has been already mentioned (p. 28) that the mean distance is not the arithmetic mean between the greatest and least distance; it necessarily follows that the mean apparent diameter is not the arithmetic mean between the greatest and least apparent diameters.

Now, the apparent surface of the lunar disc varies, not as these diameters, but as the squares of these diameters. It is easily calculated that if the size of the lunar disc, when the moon is at her mean distance, is represented by the number 10,000, then, when she is nearest to the earth, her disc shows a surface of 11,615; while, when she is farthest, the apparent surface is but 8,914. Or, if we call the surface of the moon's disc when nearest to us 10,000, then, when she is farthest from us, the surface of her disc would be represented by the number 7,674. We may very nearly represent the apparent size of the moon's disc when she is nearest to us, and when she is farthest from us, by the numbers 4 and 3; in other words, when the moon is full and farthest from the earth, she gives only three-fourths of the amount of light which she gives when full and at her nearest to the earth. But there is a very convenient way of representing the relative dimensions of the moon's disc when she is at her nearest and farthest. It is very easily shown that if we describe circles $M\mu$ and $M'\mu'$ about E′ as centre (fig. 11, Plate III.), and passing through the points M and M′, then the circles $M\mu$ and $M'\mu'$ represent the dimensions of the lunar disc when the moon is at M′ or M respectively. In like manner we could compare the dimensions of the lunar disc when the moon is in perigee and apogee, and the eccentricity has its least value (*i.e.* the earth as at *e*, fig. 11); or when the eccentricity has its mean value (the earth as at E, fig. 10).*

It remains only that we should consider the subject of the moon's mass,—that is, of the quantity of matter contained in her globe, whose volume or size is already known to us.

There are four different ways in which the moon's mass may be determined.

First, since we have already mentioned (and shall explain further in the next chapter) that the moon's motion under the earth's attraction is calculable when the size of the earth, the value of terrestrial gravity, and the moon's distance and mass are known, it follows that as the size of the earth, the earth's gravity,

* This is a very convenient method of comparing the apparent dimensions of the same orb seen at different distances. We take these distances, and with them describe circles; then these circles represent the relative apparent dimensions,—the largest, of course, corresponding to the appearance of the globe as seen at the least distance, and *vice versâ*. Thus suppose that we wish to compare the size of the sun as seen from two planets, which we may call, for convenience, P and P′, and that we have a chart of orbits including the orbits of these planets; then if the orbit of P represent the size of the sun as seen from P′, the orbit of P′ represents the size of the sun as seen from P.

and the moon's period are very accurately known, and as the moon's distance has been determined by independent observations, her mass may be inferred by the consideration of her observed motions; in fact, precisely as, in the method for determining the moon's distance, described at page 24, we infer the distance when the mass is known; so, if the distance be independently determined, we can infer the mass.* And it is to be observed that although, if these two methods alone existed for determining the mass and distance, they would leave both problems indeterminate; yet, as other methods exist, these two afford very useful tests of the accuracy of the results deduced by the other methods.

Laplace, adopting the value 57′ 12″·03 for the lunar parallax, deduced for the moon's mass, by this method, the value $\frac{1}{74 \cdot 2}$; the earth's mass being unity.

Another method for determining the moon's mass is based on the theory of the tides. If the height of the tides at any place be observed carefully for a long period of time, and then the mean height of the spring tides be compared with the mean height of the neap tides, we can infer the relative efficiency of the sun and moon when acting together to raise the tidal wave, and when their actions are opposed.

* It is easily seen that, on the assumptions made in the note at pp. 25, 26, the equations (i) and (ii) can either be used to give the result there stated, or to give the result

$$\frac{m}{M} = \frac{4\pi^2 D^3}{P^2 r^2 g} - 1$$

The problem is indeed rendered difficult by theoretical and practical considerations of much complexity. But presenting the problem roughly, we may say that, after careful attention to the observations, we obtain L+S and L−S, where L is the lunar action and S the sun's; the first at spring tides, the second at neap tides. Now, the sum of these compound actions is 2L, and the difference 2S; so that we can infer L the lunar action, and S the solar action. These enable us to infer the relation between the moon's mass and the sun's. Newton was led by comparing the results of his theory with the observed height of the tides, to the conclusion that the moon's mass is $\frac{1}{39{\cdot}788}$, the earth's being represented by unity. Laplace was led by the observation of the tides at Brest to the theory that the moon's mass is $\frac{1}{58{\cdot}6}$ of the earth's. He considered, however, that this result, although less than Newton's, might still be considerably too large, since he judged that the height of the tides at Brest might be influenced by several local circumstances. It seems obvious that this method cannot be susceptible of very great accuracy, since the figures of the ocean masses, as well with respect to their horizontal as to their vertical proportions, render the direct application of the theory of the tides impracticable.

Another method depends on the circumstance that the earth circuits once in each lunation around the centre of gravity of the earth and moon. Owing to this circumstance, the earth is sometimes slightly in advance of, and sometimes slightly behind, her mean place in longitude. In fact we know that the moon, circling around the same centre of gravity, but in a much wider orbit, is sometimes in advance of the earth and sometimes behind the earth,—regarding these orbs as two planets severally pursuing their courses round the sun; and if we look upon the earth's motion as representing very nearly the motion of a planet, at her distance and undisturbed by a satellite (which is not far from being the case), then we see that the moon, owing to her motion in an orbit 477,600 miles in diameter round the earth, is alternately 238,800 miles in advance of, and as many behind, her mean place in longitude. So that, since the earth circuits round the common centre of gravity of the two bodies, in a smaller orbit, she will be alternately in advance of and behind her mean place * by the radius of that orbit. Obviously the effect of this will be that the sun, round which the earth is thus moving, will seem to be alternately in advance of and behind the mean place due to his apparent annual motion round the heavens. His apparent place will obviously not be affected at all when the moon is on a line with the sun and earth, or *in syzygy*, as it is called (that is, when it is either *new* moon or *full*), for then the earth's displacement is on the same line, and the only effect is that the sun appears either very slightly larger (when the moon is "full" and the

* The mean place here referred to is that place which the earth would have if she were travelling alone round the sun,—not, as is actually the case, under the perturbing influence of a satellite.

earth most displaced *towards* the sun), or very slightly smaller (when the moon is "new" and the earth most displaced *from* the sun). Both effects would be quite inappreciable. But when the moon is at her first quarter, the earth is displaced towards the side occupied by the moon at her third quarter; that is, she is at her maximum displacement *in advance* of her mean place, and the sun also appears accordingly at his maximum displacement *in advance* of his mean place in his apparent annual motion round the heavens. In like manner, when the moon is at her third quarter, the sun appears at his maximum displacement *behind* his mean place. It is easy to ascertain what the sun's displacement should be, on any given assumption as to the moon's mass. Suppose the moon's mass, for example, to be $\frac{1}{80}$th of the earth's, then the centre of gravity of the earth and moon lies eighty times farther from the moon's centre than from the earth's. Hence the distance of this centre of gravity from the earth is $\frac{1}{81}$st part of 238,818 miles, or 2,949 miles. Thus the sun may be displaced from his mean place by the angle which a line 2,949 miles long subtends at the earth's distance from the sun. Since the equatorial diameter of the earth is 3,963 miles, this displacement of the sun is equal to about $\frac{3}{4}$ths of the small arc called the solar parallax, or is rather more than 6″·6, if we assume 8″·9 to be the mean value of the solar parallax. This quantity is about $\frac{1}{290}$th part of the sun's apparent diameter.

But obviously if the exact amount of the maximum displacement can be ascertained, we can infer precisely what proportion the distance of the earth's centre from the centre of gravity of the earth and moon bears to the earth's mean diameter. We shall have to make an assumption as to the value of the solar parallax (that is, in effect, as to the sun's distance); but that is an element which has been determined with a satisfactory degree of accuracy in many different ways. Hence the moon's mass can be determined with a corresponding degree of accuracy, if only the observations of the sun's displacement are accurately made.

From a great number of observations of the moon, Delambre deduced for the sun's maximum displacement (called the *sun's parallactic inequality*), the value $7''\cdot5$. Hence Laplace deduced the value $\frac{1}{69\cdot2}$ for the moon's mass. With the values at present adopted for the distances of the sun and moon, he would have deduced $\frac{1}{72}$ as the value of the moon's mass.

In recent times the meridional observations of the sun have been so numerous and exact, that the means of determining the moon's mass by this method are much more satisfactory. Thus we can place very great reliance on Leverrier's estimate of the parallactic inequality, viz. $6''\cdot50$. Professor Newcomb, of America, deduces from a yet wider range of observations the value $6''\cdot52$. These values lie so close together as to show that the observations on which they have been based suffice for the very accurate determination of this quantity.

Now the value of the moon's mass which we should infer from the mean (6″·51) of these two estimates, will depend on the value we assign to the solar parallax. If we estimate the mean equatorial horizontal solar parallax at 8″·91, it would follow that the distance of the centre of gravity of the earth and moon from the earth's centre is $\frac{651}{891}$ths of the earth's equatorial semi-diameter, or $\frac{651}{891}$ths of 3,963 miles; that is, about 2,895 miles. Thence it follows that the moon's mass is to the sum of the masses of the earth and moon as 2,895 to 238,818, or

$$\text{Moon's mass} : \text{Earth's mass} :: 2895 : 235923$$
$$:: 1 : 81{\cdot}5^{*}$$

that is, the earth's mass exceeds the moon's $81\frac{1}{2}$ times.

In calculating the sun's distance from the solar parallactic inequality, Mr. Stone adopted $\frac{1}{81{\cdot}45}$ for the moon's mass. Leverrier adopted the value $\frac{1}{81{\cdot}48}$ (originally, owing to an error of calculation which Mr.

* The actual relation may be given approximately thus:—Let R be the earth's equatorial radius, D the moon's distance, P the sun's parallactic inequality, and Π the sun's mean equatorial horizontal parallax, μ being the moon's mass when the earth's is represented by unity; then

$$\frac{\mu}{\mu+1} = \frac{PR}{\Pi D} : \text{ or } \mu = \frac{PR}{\Pi D - PR}$$

But the former form is more convenient for calculation.

Leverrier takes $\frac{R}{D}$ as 0·016620; Newcomb adopts the value 0·016461. The value resulting from the equatorial radius and the moon's distance adopted in the present work is 0·016593.

Stone detected, Leverrier adopted the value $\frac{1}{81\cdot84}$). Professor Newcomb adopted the value $\frac{1}{81\cdot08}$.

These values were deduced by an independent method, the last remaining to be described, and on the whole perhaps the most satisfactory. Owing to the attraction of the sun and moon on the bulging equatorial parts of the earth, the axis of the earth undergoes the disturbance called precession. Now this disturbance, whose period is about 25,868 years, depends on the inclination of the earth's equator-plane to lines drawn from the sun and moon. The portion due to the moon's action depends on the inclination of the equator-plane to a line from the moon. Now of course this inclination varies during the moon's circuit of the earth, because she twice crosses the celestial equator in such a circuit, and at these times the moon's action vanishes. But *these* changes are comparatively unimportant so far as the progress of the displacement of the earth's axis is concerned, simply because the displacement during a month is exceedingly small. There is, however, a change which, having a much longer period, is clearly recognizable. The moon's orbit is inclined to the ecliptic by rather more than five degrees. If the orbit thus inclined had a constant position, its inclination to the earth's equator (assumed also to have a constant position, which is approximately the case), would also be constant. But we shall see in the next chapter that the direction of the line in which the moon's plane intersects the ecliptic, makes a complete revolution once in about

$18\frac{1}{2}$ years. Hence the inclination of the moon's orbit to the equator is affected by an oscillation of rather more than five degrees on either side of the mean inclination, which is the same as that of the ecliptic to the equator, or about $23\frac{1}{2}$ degrees. Thus the inclination passes in the course of rather more than $18\frac{1}{2}$ years from about $18\frac{1}{2}$ degrees to about $28\frac{1}{2}$ degrees, and thence to about $18\frac{1}{2}$ degrees again. Obviously the lunar action varies accordingly; and, moreover, it is to be remembered that if the lunar action were alone in question, the pole of the equator would circle, not about the pole of the ecliptic, but about the pole of the moon's orbit-plane; and as this pole is itself circling about the pole of the ecliptic in a period of rather more than $18\frac{1}{2}$ years, it is readily seen that there will be a fluctuation in the motion of the pole of the heavens, having the same period. This fluctuation is necessarily small, because in $18\frac{1}{2}$ years the whole motion due to precession is small,* and this fluctuation is only a minute portion of the whole motion. It is found to amount in fact to about $9''\cdot2$, by which amount the pole of the heavens, and with it the apparent position of every star in the heavens, is at a maximum displaced from the mean position estimated for a perfectly uniform precessional motion. Now, since this displacement (called *nutation*) is solely dependent

* The 1360th part of the complete circuit made by the pole of the heavens round the pole of the ecliptic (less than $16'$ of a small circle of the heavens having an arc-radius of $23\frac{1}{2}$ degrees), or about $6\frac{1}{4}'$ of arc.

on the moon's mass, it follows that when its observed value is compared with the formula deduced by theory, a means of determining the moon's mass must necessarily be obtained.

Laplace, adopting Maskelyne's value of the maximum nutation,—namely, 9″·6, inferred for the moon's mass $\frac{1}{72}$ (the earth's being regarded as unity). Professer Newcomb adopting 9″·223 for the lunar nutation, and 50″·378 for the annual luni-solar precession, deduces the value $\frac{1}{81\cdot08}$. Leverrier with the same values deduces $\frac{1}{81\cdot48}$. Mr. Stone, in his latest calculation, with the same values, deduces for the moon's mass $\frac{1}{81\cdot36}$.*

In the present work we adopt $\frac{1}{81\cdot40}$ (or 0·01228) as the moon's mass, the earth's being regarded as unity. Taking the moon's volume as $\frac{1}{49\cdot26}$ (the earth's as unity), it follows that the moon's mass bears a smaller proportion to the earth's than her volume bears to the earth's volume, in the ratio of 4,926 to 8,140. Hence the moon's mean density must be less than the earth's in this ratio. So that if we express the earth's density by unity, the moon's will be expressed by 0·6052. If the earth's mean density be held to be 5·7 times that of water, the moon's mean density is rather less than 3½ times the density of water.

Such are the main circumstances of that long process of research by which astronomers have been enabled to pass from the first simple notions sug-

* To these values may be added Lindenau's estimate $\frac{1}{87\cdot7}$, and the estimate obtained by MM. Peters and Schidlowski, $\frac{1}{81}$.

gested by the moon's aspect and movements, to their present accurate knowledge of the distance, diameter, surface, volume, and weight of this beautiful orb, the companion of our earth in her motion around the sun.

CHAPTER II.

THE MOON'S MOTIONS.

ALTOGETHER the most important circumstance in what may be called the history of the moon, is the part which she has played in assisting the progress of modern exact astronomy. It is not saying too much to assert that if the earth had had no satellite the law of gravitation would never have been discovered. *Now* indeed that the law has been established, we can see amid the movements of the planets the clearest evidence respecting it,—insomuch that if we could conceive all that has been learned respecting the moon blotted out of memory, and the moon herself annihilated, astronomers would yet be able to demonstrate the law of gravity in the most complete manner. But this circumstance is solely due to the wonderful perfection to which observational astronomy on the one hand, and mathematical research on the other, have been brought, since the law of gravitation was established, and *through* the establishment of that law. It needs but little acquaintance with the history of Newton's great discovery, to see that only the overwhelming evidence he was able to adduce from the moon's movements, could have enabled him to compel the scientific world to hearken to his reasoning, and to accept his conclusions. We can scarcely doubt that he himself would never have attacked the subject as he actually did, with the whole force of his stupendous intellect, had he not recognized in the moon's movements the means of at once testing and demonstrating the law of the universe. Had the evidence been one whit less striking, the attention of his contemporaries would soon have been diverted from his theories, which indeed could barely have risen above the level of speculations but for the lunar motions. Astronomy would never have attained its present position had this happened. It would have seemed vain to track the moon and the planets with continually increasing care, if there had been no prospect of explaining the peculiarities of motion exhibited by these bodies. Kepler had already done all that could be done to represent the planetary *motions* by empirical laws,—the planetary *perturbations* could be explained in no such manner. The application of mathematical calculations to the subject would have been simply useless; and there would have been nothing to suggest the invention of new modes of mathematical research, and therefore nothing to lead to those masterpieces of analysis by which Laplace and Lagrange, Euler and Clairaut, Adams, Airy, and Leverrier, have elucidated the motions of the heavenly bodies.

The history of the progress of investigation by which Newton established the law of gravitation is full of interest. And although a high degree of mathematical training is requisite, in order fully to apprehend its significance, yet a good general idea of the subject may readily be obtained even by those who are not profoundly versed in mathematics. I propose to endeavour, in this place, to present the subject in a purely popular, yet exact manner. I wish the reader to see not merely how the law of gravity accounts for the more obvious features of the moon's motion, but also how her peculiarities of motion—her perturbations—are explained by the law of attraction. On the one hand the Scylla of too great simplicity is to be avoided, lest the reader should be left with the impression that the evidence for the law of gravity is not so complete as it actually is; on the other, the Charybdis of complexity must be escaped from, lest the general reader be deterred altogether from the investigation of a subject which is not only extremely important but in reality full of interest.

I invite the general student to notice, in the first instance, that the whole of the following line of argument must be attentively followed. If a single paragraph be omitted or slurred over, what follows will forthwith become perplexing. But I believe I can promise him that, with this sole *proviso*, he will meet with no difficulties of an important nature. On the other hand, should the more advanced student by chance peruse these pages, I invite him to consider that the account here presented is intended only as a sketch, and that if certain details are but lightly treated, or omitted altogether, this has not been done without a purpose.

It had been recognized long before Newton's time that this globe on which we live possesses a power of drawing to itself objects left unsupported at any distance above the earth's surface. It is, indeed, very common to find the recognition of this fact ascribed to Newton, who is popularly supposed to have asked himself *why* a certain apple fell in his orchard. But the fact was thoroughly recognized long before his time. Galileo, Newton's great predecessor, had instituted a series of researches into the law of this terrestrial attraction. He had found that all bodies are equally affected by it, so far as his experimental inquiries extended; and he established the important law that the velocity communicated to falling bodies by the earth's attraction increases uniformly with the time of falling; so that whatever velocity is acquired at the end of one second, a twofold velocity is acquired at the end of the next, a triple velocity at the end of the third, and so on.

In order to estimate the actual velocity which gravity communicates to falling bodies, Galileo caused bodies to descend slightly inclined planes. He showed that the action of gravity was diminished in the proportion which the height of the plane's summit bears to the sloped face; and by making the slope very slight, he caused the velocity acquired in any given short time to be correspondingly reduced. To reduce friction as much as possible, he mounted the descending bodies on wheels, and made the inclined planes of hard substances perfectly polished. But other and better methods were devised; and when Newton's labours began, men of science were already familiar with the fact that a falling body, if unretarded by atmospheric resistance or other cause, passes in the first second over $16\frac{1}{10}$ feet, and has acquired at the end of the second a velocity of $32\frac{1}{5}$ feet per second; by the end of the second second it has passed over $64\frac{2}{5}$ feet in all, and has acquired a velocity of $64\frac{2}{5}$ feet per second; at the end of the third it has passed over $144\frac{9}{10}$, and has acquired a velocity of $96\frac{3}{5}$ feet per second; and so on,—the law being that the space fallen varies as the square of the number of elapsed seconds,* while the velocity varies as this number directly.

So much, as I have said, was known before Newton began to inquire into the laws influencing the celestial bodies; so that, if there is any truth in the story of the apple, Newton certainly did not inquire *why* the apple fell to the earth. It is not impossible that on some occasion, when he was pondering over the motions of the celestial bodies,—and perhaps thinking of those inviting speculations by which Borelli, Kepler, and others had been led to regard the celestial motions as due to attraction,—the fall of an apple may have suggested to Newton that terrestrial gravity afforded a clue which, rightly followed up, might lead to an explanation of the mystery. If the attraction of

* The spaces traversed in successive seconds are proportional to the numbers 1, 3, 5, 7, &c.

the sun rules the planets, the attraction of the earth must rule the moon. *What if the very force which drew the apple to the ground be the same which keeps the distant moon from passing away into space on a tangent to her actual orbit!*

Whether the idea was suggested in this particular way or otherwise, it is certain that in 1665, at the age of only 23 years, Newton was engaged in the inquiry whether the earth may not retain the moon in her orbit by the very same inherent virtue or attractive energy whereby she draws bodies to her surface when they are left unsupported.

In order to deal with this question, he required to know the law according to which the attractive force diminishes with distance. Assuming it to be identical in quality with the force by which the sun retains the several planets in their orbits, he had, in the observed motions of the planets, the means of determining the law very readily. The reasoning he actually employed is not quite suited to these pages. I substitute the following, which the reader may if he please omit (passing to the next paragraph), but it is not difficult to grasp. Let us call the distance of a planet (the earth, suppose), unity or 1, its period 1, its velocity 1. Let the distance of a planet farther from the sun be called D; then the third law of Kepler tells us that its period will be the square root of $D \times D \times D$, or will be $D\sqrt{D}$. But regarding the orbits as circles around the sun as centre, the circumference of the larger orbit will exceed that of the smaller in the proportion of D to 1; hence, if the velocity of the outer planet were equal to that of the inner, the period of the outer planet would be D. But it is greater, being $D\sqrt{D}$ (that is, it is greater in the proportion of $\sqrt{D}$ to 1); hence the velocity of the outer planet must be less, in the proportion of 1 to $\sqrt{D}$. Now the sun's energy causes the direction of the earth's motion to be changed through four right angles in the time 1; that of the outer planet being similarly deflected in the time $D\sqrt{D}$; and we know that a moving body is more easily deflected in exact proportion as its velocity is less; so that the outer planet, moving $\sqrt{D}$ times more slowly, ought to be deflected $\sqrt{D}$ times more quickly if the sun influenced it as much as he does the nearer one. Since the outer planet, instead of being deflected $\sqrt{D}$ times more quickly, is deflected $D\sqrt{D}$ times less quickly, the influence of the sun on the outer planet must be less than on the earth, $\sqrt{D} \times D\sqrt{D}$ times,—that is, $D \times D$ (or D^2) times less. In other words, the attraction of the sun diminishes inversely as the square of the distance.

Newton had therefore only to determine whether the force continually deflecting the moon from the tangent to her path is equal in amount to the force of terrestrial gravity reduced in accordance with this law of inverse squares, in order to obtain at least a first test of the correctness of the theory which had suggested itself to his mind. Let us consider how this was to be done; and in order that the account may agree as closely as possible with the actual history of the discovery, let us employ the elements actually adopted by Newton at this stage of his labours.

Newton adopted for the moon's distance in terms of the earth's radius a value very closely corresponding to that now in use. We may, for our present purpose, regard this estimate as placing the moon at a distance equal to sixty terrestrial radii. Thus the attraction of the earth is reduced at the moon's distance in the proportion of the square of sixty, or 3,600, to unity. Now, let us suppose the moon's orbit circular, and let $m\ m'$, fig. 13, Plate IV., be the arc traversed by the moon in a second around the earth at E ($m\ m'$ is of course much larger in proportion than the arc really traversed by the moon in a second), then when at m the moon's course was such, that if the earth had not attracted her, she would have been carried along the tangent line $m\ t$; and if t be the place she would have reached in a second, then $m\ t$ is equal to $m\ m'$, and E t will pass almost exactly through the point m'. Thus $t\ m'$, which represents the amount of fall towards the earth in one second, may be regarded as lying on the line t E.* Now m' E is equal to m E, and therefore $t\ m'$ represents the difference between the two sides m E and t E of the

* In the account ordinarily given, $t\ m'$ is taken as lying parallel to m E. This is *also* approximately true. As a matter of fact the point m' lies a little outside t E (that is on the side away from m) and a little within the parallel to m E, through t. But the angle t E m is exceedingly minute; this angle as drawn representing the moon's motion for about half a day instead of a single second of time.

right-angled triangle *m* E *t*. Newton adopted the measure of the earth in vogue at the time, according to which a degree of arc on the equator was supposed equal in length to 60 miles, or the earth's equatorial circumference equal to 21,600 miles. This gave for the circumference of the moon's orbit 1,296,000 miles, and for the moon's motion in one second rather less than half a mile. Thus *t m* and *m* E are known, for *m* E is equal to thirty terrestrial diameters; and thus it is easy to determine *t* E.* Now Newton found, that with the estimate he had adopted for the earth's dimensions, *t* E exceeded *m* E by an amount which, increased 3,600-fold, only gave about 14 feet,—instead of $16\frac{1}{10}$ feet, the actual fall in a second at the earth's surface.

This discordance appeared to Newton to be too great to admit of being reconciled in any way with the theory he had conceived. If the deflection of the moon's path had given a result *greater* than the actual value of gravity, he could have explained the discrepancy as due to the circumstance that the moon's own mass adds to the attraction between the earth and herself. But a *less* value was quite inexplicable. He therefore laid aside the investigation.

Fourteen years later Newton's attention was again attracted to the subject, by a remark in a letter addressed to him by Dr. Hooke, to the effect that a body attracted by a force varying inversely as the

* By Euc. I. 47 the square on *t* E is equal to the squares on *t* M and M E.

square of the distance, would travel in an elliptic orbit, having the centre of force in one of the *foci*. I do not at present pause to explain this remark, which is indeed only introduced here to indicate the sequence of Newton's researches. It is to be noted that Hooke gave no proof of the truth of his remark; nor was there anything in his letter to show that he had established the relation. He was not, indeed, endowed with such mathematical abilities as would have been needed (in his day) to master the problem in question. Newton, however, grappled with it at once, and before long the idea suggested by Hooke had been mathematically demonstrated by Newton. Yet, even in ascribing the idea to Hooke's suggestion at this epoch, we must not forget that Newton, in the very circumstance that he had discussed the moon's motion as possibly ruled by the earth's attraction, had implicitly entertained the idea now first explicitly enunciated by Hooke: for the moon does not move in a circle around the earth, but in an ellipse.

In studying this particular problem, Newton's attention was naturally drawn again to the long-abandoned theory that the earth's attraction governs the moon's motions. But he was still unable to remove the discrepancy which had foiled him in 1665.

At length, however, in 1684, news reached him that Picard* had measured a meridional arc with great

* Picard died at Paris in 1682, two years before the news of his labours had reached the ears of Newton.

care, and with instrumental appliances superior to any which had been hitherto employed. The new estimate of the earth's dimensions differed considerably from the estimate employed by Newton before. Instead of a degree of arc at the equator being but 60 miles in length, it now appeared that there are rather more than 69 miles in each degree. The effect of this change will be at once apparent. The earth's attractive energy at the moon's distance remains unaffected, simply because the proportion of the moon's distance to the earth's diameter had alone been in question. Newton, therefore, still estimated the earth's attraction at the moon's distance as less than her attraction at her own surface, in the proportion of 1 to 3,600. But now all the real dimensions, as well of the earth as of the moon's orbit, were enlarged linearly in the proportion of $69\frac{1}{2}$ to 60. Therefore the fall of the moon per second towards the earth, increased in the proportion of 3,600 to 1, was enlarged from rather less than 14 feet to rather more than 16 feet,—agreeing, therefore, quite as closely as could be expected with the observed fall of $16\frac{1}{10}$ feet per second in a body acted upon by gravity and starting from rest.

It is said that as Newton found his figures tending to the desired end, he was so agitated that he was compelled to ask a friend to complete the calculations. The story is probably apocryphal, because the calculations actually required were of extreme simplicity. Yet if any circumstance could have rendered Newton unable to proceed with a few simple processes of multiplication and division, undoubtedly the great discovery which was now being revealed to him might have led to such a result. For he clearly recognized the fact that the interpretation of the moon's motions was not what was in reality in question, nor even the explanation of the movements of all the bodies of the solar system; but that the law he was inquiring into must be, if once established, the law of the universe itself.

If we consider the position in which matters now stood, we shall see that in reality the law of gravitation had already been placed on a somewhat firm and stable basis. Newton had shown that the motions of the planets are conformable to the theory that the sun attracts each planet with a force inversely proportional to the square of the planet's distance. The motions of Jupiter's satellites (the only scheme known to Newton) agreed similarly with this law of attraction. And now he had shown that in the case of our own moon, the attraction exerted by the central body round which the moon moves, is related to the attraction exerted by this body, the earth, on objects at her surface, according to precisely the same law. Furthermore, it was known that all bodies are attracted in the same way by the earth, let their condition or elementary constitution be what it may. The inference seemed abundantly clear that the law of attraction,—with effects proportional to the attracting masses, and inversely proportional to the distances separating them, is the general law of matter, and prevails, as far as matter prevails,—throughout the universe.

But Newton was sensible that a law of this nature could not be established unless some special evidence, suited to attract the attention of scientific men to the subject, were adduced and insisted upon. The discovery must throw light on some facts hitherto unexplained,—must in effect achieve some striking success, — before men could be expected to look favourably upon it.

What Newton determined to do, then, was this. The law had been shown to accord with the general features of the lunar motions. But the moon's motion is characterized by many peculiarities. At one time she takes a longer, at another a shorter time in circling around the earth, than that average period called the sidereal lunar month. At one time she is in advance of her mean place, calculated on the supposition of a simple elliptic orbit; at another time she is behind her mean place. The inclination of her path is variable, as is the position of its plane; so also the eccentricity of her path and the position of her perigee are variable. Newton saw that if the law of gravitation be true, the moon's motion around the earth must necessarily be disturbed by the sun's attraction. If he could show that the peculiarities of the moon's motion vary in accordance with the varying effects of the sun's perturbing influence, and, still more, if he could show that the extent of the lunar perturbations corresponds with the actual amount of the sun's perturbing action, the law of gravitation would be established in a manner there could be no disputing.

In presenting so much of the history of this inquiry as is necessary for my present purpose, it is necessary, in the first instance, to show generally how the motion of a body in an elliptic orbit accords with the action of a force like gravity. Absolute proof of the fact requires in the learner an amount of mathematical knowledge, which the general reader cannot be supposed to possess. But the difficulties which at a first view surround the idea of elliptic motion, or of motion in any non-circular orbit, described under attractive influences, can be removed without dealing with mathematical considerations. I think the most salient difficulties are the following:—

Suppose A B *a b*, fig. 14, Plate IV., to be an elliptic path described about an attracting body S, placed at one focus of the ellipse,—then the learner finds some difficulty in understanding how the change of distance from the small distance S A to the great distance S *a*, and, *vice versâ*, can proceed in regular alternation. *Because*, if the attracting force, greatly reduced at the distance S *a*, can nevertheless compel the body to approach from that distance until its distance is reduced to S A, how much more, it would seem, should the much greater attraction exerted at this reduced distance S A, continue to cause the approach of the body, until finally the latter is brought to rest at S. Or again, if when the attracting orb is exerting its greatest influence on the moving body at

Fig 13

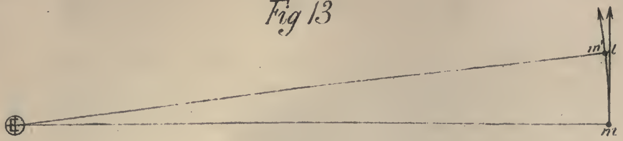


Fig 14

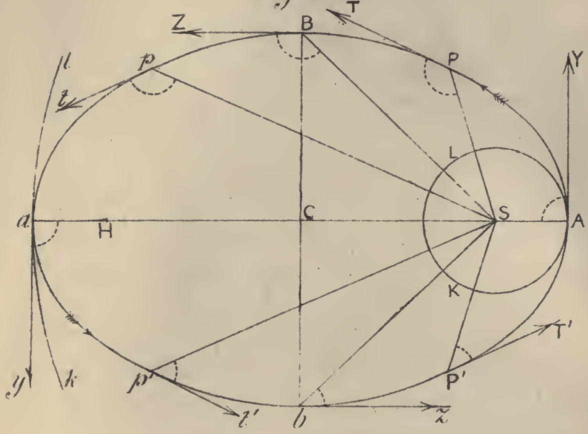
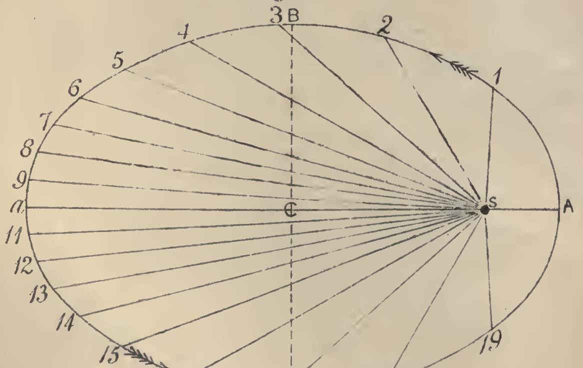


Fig 15



Illustrating the motion of a body around an attracting Orb.

A, this body is still able to move in such a way as continually to increase its distance until it is as far off as *a* from S, how much more is it to be expected that, having reached this distance, where the sun's force is so greatly reduced, the body should be able yet farther to increase its distance, and so to travel for ever away from S.

And I think that another difficulty, which is very commonly experienced, is this :—The curve A B *a b* is quite symmetrical, both as respects the line A *a* and the line B *b*. Thus the part near A is exactly like the part near *a* ; yet these perfectly similar parts are described under quite dissimilar circumstances, — the attraction on the body being different, the velocity of the body being different, and all the circumstances in fine at a maximum of dissimilarity. Nay, the very circumstance that a symmetrical orbit should be described about an eccentrically-placed point, seems at a first view inexplicable.

Let S, fig. 14, Plate IV., be the attracting orb around which a body is moving in the elliptical orbit A B *a b* ; and let us consider the motion of the moving body from the point A, where the velocity is greatest. At this point the velocity is greater than that with which a body would describe the circle A L K around S : the tendency to travel on the tangent line A Y is therefore stronger than in the case of such a body. Thus an intermediate course, A P, is pursued, the sun's influence deflecting the moving body from the tangent-line A Y ; but not being strong enough to deflect it into the circular course A L K. Now, the distance of the body from S is increasing throughout this process, and this amounts to saying that a tangent-line, as P T, makes an obtuse angle with the line S P drawn to the body at the moment. But this being so, it is obvious from the figure that the orb at S must exert a retarding influence. At A there was no retardation (for the moment), because the pull was square to the body's course; but so soon as the body's distance begins to increase, the pull is partly backwards (as at P) with reference to the body's motion; and thus there is retardation. Now two opposing influences are at work when the body is in such a position as P: *one*, the tendency of the body to move in the direction P T, tends to enlarge the angle S P T; *the other*, the pull of the orb at S, tends to reduce this angle. So long as the velocity exceeds a certain value, the former influence prevails. But the velocity is being continually reduced; and though the distance of the body is increasing, and therefore the pull from S diminishing, yet the power of S to deflect the body does not diminish so rapidly as the absolute power of S on the body, for deflection becomes so much the easier as the velocity of the body is reduced. At length, when at B, the body has reached a position where the two forces counterbalance each other in this respect, the angle S B Z between the line of the body's motion and the line from S having here its maximum value. At this point the body is travelling on a course square to the direction it had had when at A.* And let this be noted as to the present condition of the body. It has increased its distance from S, and has thus far asserted, in a sense, the power inherent in it when at A, by virtue of its high velocity there; it has also increased its angular rate of escape, the angle S B Z exceeding the angle S A Y. But in effecting this it has sacrificed a portion of its velocity, the influence of the orb at S having acted retardingly throughout the whole of this portion of the body's course; and, as a matter of fact, the velocity at B is less than the velocity at A, in the proportion that A S is less than B C.

As the body passes onwards from B, the sun's action continually reduces the angle corresponding to S P T, S B Z, continually reducing also the velocity of the body so long as this angle remains obtuse. This process is *in this respect* the reverse of the process passed through as the body moved from A to B, and ends in the restoration of the rectangular thwart motion when the body has arrived at *a*, directly opposite to A. Yet the part B *p a* of the body's course is described under circumstances wholly different from those operating while the body was moving from A to B. The time from A to B is much less than the time from B to *a*,—in the same proportion, in fact,

* It will be remembered that in the above paragraph explanation of what actually happens, and not a proof that it *must* happen, is attempted. To show that when the angle corresponding to S P T ceases to increase, the body must be travelling on a course at right angles to A Y, is impossible without introducing mathematical considerations much more fully than is proper in this place.

that the area A S B is less than the area B S *a*; and as the body moves more sluggishly from B to *a*, so also it is more sluggishly retarded by the orb at S. But it is precisely because of these opposite differences that the course of the body from B to *a* resembles in shape the course from A to B. To show that this is in accordance with the facts,—that is, to explain the relation without undertaking to prove that it must hold,—let us consider the state of the body at *p*, a point symmetrically placed with respect to P (that is, as far from B C, *a* C, as P is from B C, C A). At *p* the course of the body is for the moment in direction *p t*, and by the properties of the ellipse the angle S *p t* is equal to the angle S P T; so that as far as direction is concerned the retarding influence of the orb at S is as effective on the body when at *p* as when at P. In magnitude, however, the pull is less in the proportion of the square of S P to the square of S *p*. But to make up for this,—*exactly*,—the velocity at *p* is less than the velocity at P, in the same proportion that S P is less than S *p*.* This deficiency acts doubly (or, as it were, squares itself): for the reduced velocity causes the

* The velocities in an elliptic orbit are not *generally* proportional inversely to the distances from the attracting centre, but they are in the case of two such positions as *p* and P. In reality, because of the eqnal description of areas the velocitics at different points are inversely proportional to the perpendiculars from the point S to the tangent through the respective points. But if tangents were drawn from S to P T and *p t*, these perpendiculars would clearly be proportional to S P and S *p*, simply because S P and S *p* are equally inclined to P T and *p t*.

body to remain proportionately longer under the influence of the body at S while describing any given small arc at p than when describing a corresponding one at P; and it also causes the deflecting influence of the body at S to be proportionately more effective. Thus the actual curvature of the path at p is exactly equal to the curvature at P.

At a, then, the curvature is the same as at A, or the path lies *within* a circular arc, as $l\,a\,k$, about S as centre. The distance of the body from S begins now therefore to diminish. Nor is it difficult to see that the course now pursued by the body must be the exact counterpart of that already traversed, only pursued in a reverse order; for all the circumstances are symmetrically reversed, so to speak. The distance of the body diminishing, the course of the body must be inclined at an acute angle to the line from S, and the influence of S must therefore act to accelerate the motion of the body. Thus when the body is at p', (as far from S as p is), its course lies for the moment in the direction $p'\,t'$, and the pull of the orb at S, acting in direction p' S, must needs accelerate the body's motion. Also, as the body starts from a with the same velocity that it had when it reached a, and moving at the same angle with S a, it is clear that the reduction of its velocity and distance from S, after it has passed a, must be affected in a manner precisely corresponding (point for point of its course) to the increase of its velocity and distance from S before it reached a. Up to the point b (corresponding to the point B) the angle between the course of the body and the line drawn from S continues to diminish, and at *b* this angle has its minimum value, S *b z*. At this point *b*, the body has recovered a portion of the velocity it had lost, but its distance has diminished, and its course is now directed as nearly towards the body at S as it can possibly be. After passing *b* the continual access of velocity, owing to the sun's attracting force, causes the body to travel on a course inclined at a continually increasing angle to the line from S, but the distance of the body continues to diminish, until at A, where the angle between the course of the body and the line from S is again a right angle, the distance is reduced to the minimum value S A, as at first. All the circumstances are now the same as when the motion began.

It is to be noticed of the above explanation, that though it does not prove that an ellipse *must* be described, it shows that the description of an ellipse corresponds with the circumstances of the case,—that, in fact, in each quadrant of the ellipse forces tending to produce motion in a curve of such a shape, are in operation. This is all that can be done by way of popularly explaining a proposition whose inherent difficulty is such that eminent mathematicians like Wren and Halley failed to solve it.* But the above

* It has been objected even that Newton's demonstration is imperfect inasmuch as it only shows that the curvature at any point of a conic section corresponds with that due to the law of force according to the inverse squares of the distances. But taken

explanation removes in reality the real difficulties experienced by the learner; for it shows that the equal curvatures at corresponding points, P, p, p', and P′, in the four quadrants A B, B a, a b, and b A, is a relation according with the amount of force exerted by the orb at S on the moving body at these four points. This has been already indicated as respects the points P and p, and holds in like manner as respects the points p' and P′; while it needs no demonstration to show that at p' the curvature must be the same as at p, since the velocities at these points are equal, the forces on the moving body also equal, and the retarding action at p precisely accordant with the accelerating action at p', so far as the production of curvature is concerned; and lastly, it follows in like manner that the curvature at P′ is equal to the curvature at P.

Before passing from this investigation of elliptic motion, it may be well to notice in what respect the points A and a, B and b are critical points of the body's motion:—

(i.) At A the velocity is at a maximum, the distance at a minimum, and the direction of the body's motion has a mean value, being at right angles to the line from S.

(ii.) At a the velocity is at a minimum, the distance at a maximum, and the direction of the body's motion

in its proper place,—and in conjunction with what precedes and follows,—the demonstration is in reality complete has again a mean value, being again at right angles to the line from S.

(iii.) At B the direction of the body's motion is inclined at a maximum angle to the line from S, the distance has its mean value B S, being the arithmetical mean between A S and S a; and the velocity has what may be entitled its mean value, being the geometrical mean between the velocities at a and A.

(iv.) At b the same conditions prevail as respects distance and velocity as at B, but the direction of the body's motion is inclined at a minimum angle to the line from S.*

The relation which we have been considering corresponds to the first law which Kepler recognized in the planetary motions; viz., that each planet travels in an ellipse, the sun being situated at one focus of the curve. This law is not strictly true for the planets, or indeed for any known case in nature, since no orb is free to revolve around another quite independently of extraneous attractions. The law is, however, approximately true when any orb is subject almost wholly to the attraction of a single body; or else, though sub-

* Since any point of the orbit may be regarded as a starting-point, we notice that the same shaped curve is described whether a body is projected as at B on a course making the obtuse angle S B Z with the line from S, or with the same velocity from the equidistant point b, on a course making the acute angle S b Z with the line from S. The more general proposition also holds, that in whatever direction a body be propelled from a given point and with a given velocity, its orbit will have a major axis of constant length.

ject to other attractions, yet so shares these attractions with another orb that in its motions round this orb it may be regarded as almost wholly under its influence. For instance, the law approximately holds in the case of a planet's motion around the sun: and it is also true of the motion of the moon around the earth, though the moon is chiefly under the sun's influence; for the earth and moon are both swayed almost equally by the sun.

The second law of Kepler, as applied to the moon, also concerns us here very importantly. It was thus presented by Kepler:—The line drawn from the sun to a planet sweeps over equal areas in equal times.

Thus if S (fig. 15, Plate IV.) be the centre around which a body is revolving in the path A B *a* *b* under the influence of gravity, and if in any given equal intervals of time the body passes from A to 1, thence to 2, thence to 3, and so on, then the spaces A S 1, 1 S 2, 2 S 3, 3 S 4, and so on, are equal in area. For example, if the path were carefully drawn on paper according to true scale, then, if the spaces just named were cut out and carefully weighed, it would be found that they were exactly equal in *weight*.

The third law of Kepler does not directly concern us here, because it deals with the relation between the mean distances and periods of different bodies travelling around one and the same centre. Nevertheless, as the moon's motions are subject to changes of velocity, direction, and so on, while the attraction actually drawing the moon towards the centre of the earth is variable (because partly depending on the sun, and therefore on the moon's position), it is desirable to have clear ideas at the outset as to the effects of such changes. The third law of Kepler bears directly on this subject. It is as follows :—

The squares of the periods in which the planets travel around the sun vary as the cubes of the mean distances.*

This law would be strictly true if the planets were infinitely minute compared with the sun; but the masses of the planets, though very small, bear yet definite relations to the sun, and, as a matter of fact, instead of considering each planet as swayed by the sun's mass, we must regard each as though swayed by the sum of its own mass and the sun's, supposed to be gathered at the sun's centre. This at least is a sufficient rule as regards the period of a planet and the dimensions of its orbit with respect to the sun; though of course to determine the actual orbit around the common centre of gravity, we should have to take into account the actual disposal of the masses forming this sum. So that, in effect, to obtain the exact law for the periods and mean distances of the planets, we have to regard them, not as bodies circling around the same centre, but as so many different bodies revolving

* More exactly thus :—Fixed units of time and space being chosen, the square of the number expressing the periodic time of a planet bears a constant ratio to the cube of the number expressing the mean distance of the planet.

The mean distance is equal to half the major axis of the orbit.

around centres slightly differing in attractive energy; Jupiter, for instance, around a centre equal in mass to Jupiter and the sun; Saturn round a centre equal in mass to Saturn and the sun; and so on. The result of this consideration is that, instead of finding the fraction $\frac{\text{(mean distance)}^3}{\text{(period)}^2}$ *constant* for the solar system, we find that this fraction calculated for the different planets (1) Mercury, (2) Venus, (3) Earth, and so on, gives results respectively proportional to—(1) the sun's mass added to Mercury's, (2) the sun's mass added to Venus's, (3) the sun's mass added to the earth's, and so on.*

* The law thus interpreted is applicable to all cases where different bodies revolve around a common centre. But it also admits of being generalized for different bodies travelling round different centres. Thus extended, it runs as follows :—

If a body of mass m revolves round a centre of mass M in time P, and at a mean distance D, and another body of mass m' revolves round another centre of mass M′ in time P′, and at a mean distance D′, then

$$\frac{D^3}{P^2(M+m)} = \frac{D'^3}{P'^2(M'+m')}$$

This general law, almost as simple, be it observed, as Kepler's third law, is extremely important. It may be regarded as the fundamental law of the celestial motions. It presents the influence of gravity as a bond associating the motions of all the orbs in the universe, whether of double suns around each other, or of primary planets around suns, or of secondary planets around their primaries. It is a law absolutely universal (so far as is known), and strictly exact, excepting in so far as *perturbations* come into operation to affect it; and as perturbations have very little effect on *mean* periods of revolution, the exactness of the law is scarcely affected in this way. It is a wonderful thought that we can by means of such a law associate the motions of bodies, which to ordinary apprehen-

Let us now pass on to the subject of the moon's perturbations caused by the sun's attraction.

Here, in the first place, I may mention a fact which will perhaps seem surprising to many. Though the sun's disturbing influence on the moon is such that the moon's course around the earth is not very different in any single revolution from that which she would have if the sun's attraction had no existence; yet the sun actually exerts a far more powerful influence on the moon than the earth does. As we shall have to consider the relation between the two forces, we may as well proceed at once to prove this excess of power on the sun's part.

The law of gravitation enables us at once to com-

sion have nothing in common; that, for instance, such a relation as this can be affirmed :—

$$\frac{\left[\begin{matrix}\text{Moon's mean distance}\\ \text{from earth.}\end{matrix}\right]^3}{\left[\begin{matrix}\text{Moon's mean}\\ \text{period.}\end{matrix}\right]^2 \left[\begin{matrix}\text{Sum of moon's}\\ \text{mass and earth's.}\end{matrix}\right]} = \frac{\left[\begin{matrix}\text{Mean distance between}\\ \text{components of } \alpha \text{ Centauri.}\end{matrix}\right]^3}{\left[\begin{matrix}\text{Their period}\\ \text{of revolution.}\end{matrix}\right]^2 \left[\begin{matrix}\text{Sum of}\\ \text{their masses.}\end{matrix}\right]}$$

It will be observed how the law enables us at once to compare the sums of the masses, when we know the mean distances and periods. For it may be written

$$\frac{M + m}{M' + m'} = \frac{D^3 P'^2}{D'^3 P^2}$$

Also where m and m' are both small, compared with M and M' respectively, the law becomes simplified into

$$\frac{M}{M'} = \frac{D^3 P'^2}{D'^3 P^2}.$$

This law is in effect applied, in what immediately follows in the main text, to the determination of the moon's mass. It is there also independently established, at least in the case of circular orbits.

pare the sun's mass with the earth's. For precisely as we have been able to show that under the influence of terrestrial gravity the moon, at her distance, should follow such a path as she actually traverses, so we can determine how much a body should be deflected per second at the earth's distance from the sun, if his mass were equal to the earth's; and by comparing this amount with the actual deflection, we can compare the sun's mass with the earth's.

Or we may proceed in this way:—

The earth, at a distance of 238,800 miles from the moon, has power to deflect the direction of the moon's motion through four right angles in 27·322 days, the moon moving with a velocity which we may represent by $\frac{238,800}{27·322}$.* Now the sun at a distance from the earth equal to about 91,500,000 miles, has power to deflect the direction of her motion through four right angles in 365·256 days, the earth moving with a velocity which we may represent by $\frac{91,500,000}{365·256}$. Now, *first*, since gravity varies inversely as the square of the distance, the sun would require (if other things were equal) to have an attractive power exceeding the earth's in the ratio $\left(\frac{91,500,000}{238,800}\right)^2$ to produce the same effect on her that she produces on the moon; and *secondly*, since the deflection of a body's line of motion is a work which will be

* We need not consider the velocity in miles per hour, or the like; because, throughout the paragraph, relative and not absolute velocities are in question. Hence we can represent the moon's velocity by the radius of her orbit divided by her period, provided we represent the earth's velocity round the sun in like manner.

done at a rate proportional to the force which operates, the sun's power (if other things were equal) should be less than the earth's in the ratio $\frac{27 \cdot 322}{365 \cdot 256}$, to accomplish in one year what the earth accomplishes in a month; and, *lastly*, since the faster a body moves the greater is the force necessary to deflect its course through a given angle in a given time, it is obvious that the sun's attractive power should exceed the earth's in the proportion of $\frac{91,500,000}{365 \cdot 256}$ to $\frac{238,800}{27 \cdot 322}$,— that is, in the ratio $\frac{91,500,000 \times 27 \cdot 322}{238,800 \times 365 \cdot 256}$ to produce a given change of direction in the case of the quickly-moving earth in the same time that the earth produces such a change in the case of the less-swiftly-moving moon. Now, we have only to combine these three proportions,* which take into account every circumstance in which the sun's action on the earth differs from the earth's action on the moon, in order to deduce the relation between the sun's attractive energy and the earth's,—at equal distances from the centre of either. This gives the proportion $\left(\frac{91,500,000}{238,800}\right)^3 \times \left(\frac{27 \cdot 322}{365 \cdot 256}\right)^2$,— which reduces to 314,798, — in which proportion the sun's attractive energy exceeds the earth's. We may take 315,000 as representing this proportion in round numbers, with an accuracy at least equal to that with which the sun's distance has been determined.

Now in order to see whether the sun or the earth has the greater influence on the moon, we have only to compare the masses of the first-named two orbs

* The whole process corresponds exactly to an ordinary problem in double (or rather multiple) rule of three.

and the influence of their respective distances from the moon. We thus have, *first*, the proportion 315,000 to 1, in which the sun's attraction exceeds the earth's at equal distances; and *secondly*, the proportion $(238,800)^2$ to $(91,500,000)^2$ in which the attraction due to the sun's distance falls short of that due to the earth's. Thus we have this relation,—the sun's actual influence on the moon bears to the earth's the proportion which $314,500 \times (238,800)^2$ bears to $(91,500,000)^2$, or approximately a proportion of 15 to 7.* Thus the sun's influence on the moon is more than twice as great as the earth's.

It may be asked, then, how it is that the moon does not leave the earth's company to obey the sun's superior influence? In particular it might seem that when the moon is between the earth and the sun (or as placed at the time of a total eclipse), our satellite being then drawn more than twice as forcibly from the earth towards the sun as she is drawn towards the earth from the sun, ought incontinently to pass away sunwards and leave the earth moonless.

The answer to this enigma is, simply, that the sun attracts the earth as well as the moon, and with almost the same degree of force, his pull on the earth sometimes slightly exceeding, at others slightly falling short, of his pull on the moon, according as the distance of the moon or earth from him is greater at the

* The actual proportion, is 2·1421 correct to the fourth decimal place. The proportion 15 to 7 is equal to 2·1429, which for ordinary purposes is sufficiently near.

moment. Thus the earth, in order to prevent the escape of her satellite, has not to overcome the sun's pull upon the moon, but only the excess of that pull over the pull he exerts upon the earth herself. This excess, as will presently appear, is always far less than the earth's own influence on the moon.

But it may be noticed, that in considering the moon's course round the sun we recognize the inferiority of the earth's influence in a very evident manner. The moon seems well under the earth's control when we consider only the nature of the lunar orbit round the earth; but if for a moment we forget the fact that the moon is circling round the earth, and consider only the fact that the moon travels as a planet round the sun,—with perturbations produced by the attractions of another planet,—our own earth,—we can readily test the extent of these perturbations. Now let the circle M M′ (fig. 16, Plate V.) represent the moon's path round the sun S, and let us suppose that at 0 the moon is between the earth and sun, and again similarly placed at 1, 2, 3 . . . . 11, and 12,—being therefore on the side away from the sun at the intermediate stations marked with a small line outside the circle M M′; then the moon's orbital course is a serpentine or waved curve, having its minima of distance from the sun at 0, 1, 2, 3 . . . . 11, 12, and its maxima of distance at the intermediate points. But on the scale of fig. 16, the whole of this serpentine curve would lie within the breadth of the fine circular line M M′. Thus it will readily be understood that the curvature of the moon's path remains throughout concave towards S, even when, as at the points 0, 1, 2, 3, &c., the convexity of the orbital path round the earth is turned directly towards the sun. In other words, as the moon travels in her orbit round the sun her course is continually being deflected inwards from the tangent line, or always towards the sun. It is to be noticed, however, that the earth's perturbing influence is an important element in determining the moon's real orbit. For when the earth and sun are on the same side of the moon, or at the time of full moon, the pull on the moon is the sum of the pulls of the earth and sun, or exceeds the sun's pull alone in the ratio 22 to 15; and on the other hand, when the earth and sun are on opposite sides of the moon, or at the time of new moon, the pull on the moon is the difference of the pulls of the sun and earth, or is less than the sun's pull alone in the proportion of 8 to 15. Thus at the time of full moon the moon is acted on by a force which exceeds that acting on her at the time of new moon in the ratio of 22 to 8 or 11 to 4. And though at the time of full moon the moon's actual velocity (that is, her velocity in her orbit round the sun) is at a maximum, being then the sum of her mean orbital velocity round the sun and of her velocity round the earth; yet this by no means counterbalances the effects of the greatly increased pull on the moon:* so

* The earth's velocity in her orbit being about 65,000 miles per hour, and the moon's about 2,000 miles per hour, the extreme variation of the moon's motion in her orbit round the sun lies between the values 67,000 and 63,000 miles (roughly), or about four

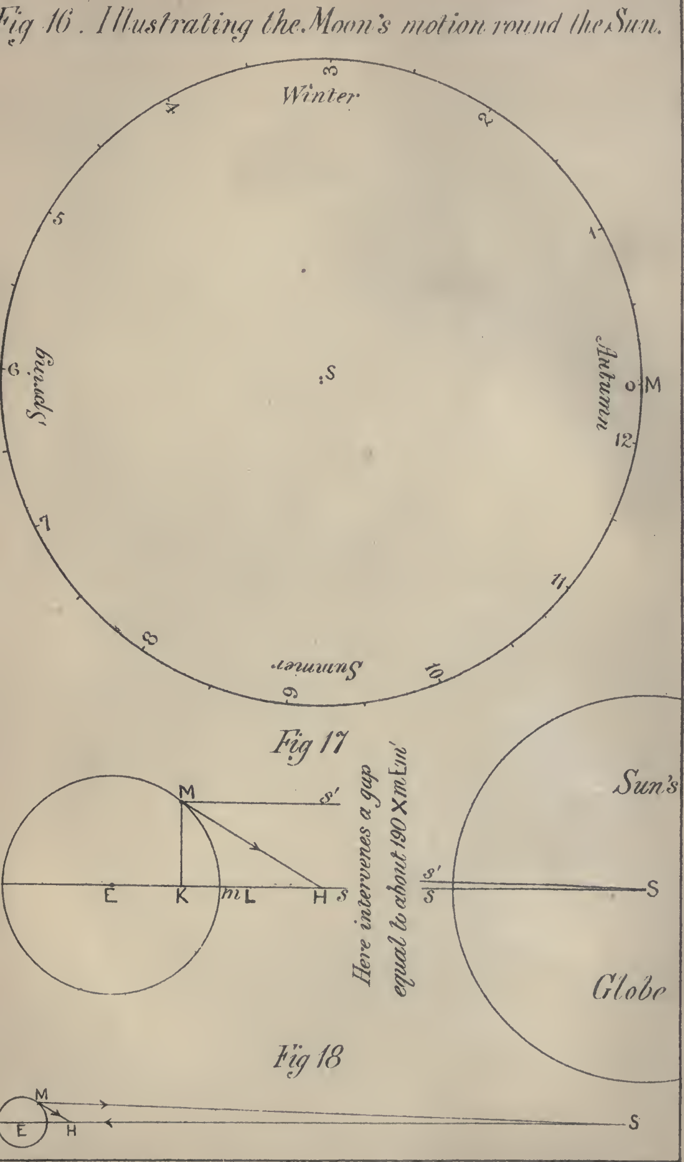
that the curvature of her path when she is "full" greatly exceeds the curvature at the time of new moon.

It was necessary to say so much about the moon's path round the sun, and the sun's real influence upon our satellite, because a great deal of confusion very commonly prevails in the student's mind on this subject. He is exceedingly apt, when his attention is chiefly (and in the first instance) directed to the sun's perturbing influence, to suppose that our earth plays the chief part in ruling the motions of the moon, whereas the sun's influence is in reality paramount at all times.

In considering the moon's motion around the earth, however, we may leave out of consideration the common influence of the sun upon both these orbs, and need consider only the difference of his influence upon the earth and moon, since this difference can alone affect the moon's motion around the earth.

Now we are enabled to deal somewhat more readily with this case than with the general problem of three bodies, because the moon is always very close to the earth as compared with the distance of either from the sun. On this account lines drawn to the sun from the earth and moon enclose so small an angle that they may be regarded as appreciably parallel. Again, these lines are at all times so nearly equal, that in determining the relative pull on the earth and moon we

times, or in the ratio of 110 to 103. But the attractive force on the moon varies in the ratio of 110 to 40, as above shown.



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may employ a simple method available with quantities that are nearly equal. Thus, suppose two bodies placed at distances represented by 100 and 101 respectively from a certain centre of force, then the attractions in the two bodies are inversely proportional to the squares of 100 and 101, or are in the ratio 10,201 to 10,000; but this ratio is appreciably the same as the ratio of 102 to 100. Therefore in this case, and in all such cases * where the distance from one body exceeds the distance from the other by a relatively minute quantity, we can obtain the relative *forces* by representing them as lines having a relative difference *twice* as great.

Now let us apply this principle to the moon and earth. Suppose E (fig. 17, Plate V.) to be the earth, M the moon, and that the lines E *s*, M *s′* (appreciably parallel) are directed towards the distant sun. We may suppose the globe S to represent the sun, and we may regard S *s′* and S *s* as the prolongations of M *s′* and E *s*, if we recognize the fact that the gap at *s s′*, *s s′*, would, on the scale of our figure be some ten yards across. Now suppose that the sun's attraction on a unit of the moon's mass † is

* The student should test this assertion by a few calculations. Thus he can take the numbers 45,681 and 45,682, and show that the ratio of the squares of these numbers is approximately represented by the ratio 45,681 to 45,683; and therefore the inverse ratio of the squares by the ratio 45,683 to 45,681. We may equally well take 45,680 to 45,682 for the ratio of the squares, and 45,682 to 45,680 for that of the inverse squares.

† Throughout the explanation it must be carefully borne in mind that when the attraction on the moon or earth is spoken of,

represented by the line joining S and M, then the line joining S and E will be too large to represent the sun's attraction on a unit of the earth's mass, for E is farther away from S than M is (in the state of things represented by the figure), so that the attraction on E is less than the attraction on M. If we draw M K square to E *s*, we have the distance of K from S appreciably equal to the distance of M from S. K E is then the excess of the distance of E from S over the distance of M from S. If the sun's attraction diminished as the distance increased,—that is, if it were simply as the inverse distance,—we need only take off K L equal to this excess E K, in order to get the line from S to L representing the attraction of the sun on the earth at E. But as the force is inversely as the square of the distance, we must (from what was shown in the preceding paragraph) take K H equal to twice the excess E K, in order to have the distance from S to H representing the sun's attraction on the earth at E.

what is really to be considered is the attraction on each unit of the mass of either body. The attraction of the sun on the whole mass of the earth is always far larger than his attraction on the whole mass of the moon: but this circumstance in no way concerns us in studying the lunar perturbations. For that excess of attraction which depends on the earth's greater mass is strictly compensated by the circumstance that the mass affected by it is corrrespondingly great. The case may be compared to that of two unequal masses let fall at the same moment from the same height above the earth. Here the earth's attraction on the greater mass is greater than her attraction on the less. Yet the greater mass falls at no greater rate; because that greater attraction is employed to move a correspondingly greater mass.

Now, let us make a separate figure to indicate the actual state of things in such a case as we have considered. There is the sun at S (fig. 18, Plate V.) pulling at the moon with a force which we have represented by M S; and he is pulling at the earth with a force which we represent on the same scale by S H. This last force, so far as the moon's place with respect to the earth is concerned, is clearly a force tending to keep the moon and earth together. It may be represented then, *in this sense,* by the line S H, or as a force tending to thrust the moon from the sun (almost as strongly and directly as the direct action on M tends to draw the moon *towards* the sun).* Thus the moon is virtually acted on by the two forces represented by M S and S H, and therefore, by the well-known proposition called the triangle of forces, we have as the resultant perturbing action on the moon, a force represented by the line M H.†

* Of course the sun's action on the earth does not really amount to a force thrusting or repelling the moon from the sun. But in determining the sun's perturbing action on the moon, we have in effect to take the excess or defect of the sun's full action on the moon, as compared with his full action on the earth, so that the latter action necessarily comes to be viewed in a sense contrary to its real nature, precisely as in ordinary arithmetic a sum which is positive in itself comes to be viewed as negative when it is to be subtracted.

† If the student is more familiar with the parallelogram of forces than with the same property under the form called the triangle of forces, he should draw a line from M parallel and equal to S H; he will find that M H is the diagonal of a parallelogram having this line and M S as adjacent sides.

Thus we have an exceedingly simple construction for determining the sun's perturbing action on the moon (as compared with his direct action) when she is in any given position. We have merely to draw M K square to the line joining E and S, to take K H equal to twice E K, and to join M H; then M H is the perturbing force, where the line joining M and S represents the sun's direct action on the moon.*

Let us now figure the various degrees of perturbing force exerted on the moon when she is in different parts of her orbit, neglecting for the present the inclination of her path to the ecliptic; in other words, regarding all such lines as M H (fig. 18) as lying in one plane. The ellipticity of the moon's orbit is also for the moment neglected. In fig. 19, Plate VI., this has been done. To avoid confusion, the different points where the action of the perturbing force is indicated have not been all lettered. Nor has the construction for obtaining the lines indicating the perturbing force been indicated in any instance. The student will, however, have no difficulty in interpreting the figure. $M_1 M_2 M_3 M_4$ is the moon's orbit around the earth at E. The sun is supposed to lie on the right in the prolongation of E A. At M_1 the perturbing force is outwards towards the sun, and is represented in magnitude and direction by the line M_1 A, which is

* Practically M H may be taken to represent the sun's perturbing action on the moon when the line joining E and S represents the sun's direct action on the earth; for the proportion of M S to either E S or H S, is very nearly unity under all circumstances.

Fig 20

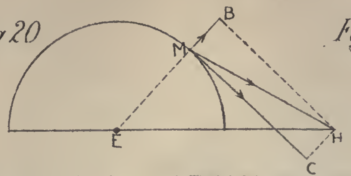


Fig 21

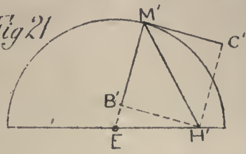


Fig 19. Shewing the total forces perturbing the Moon.

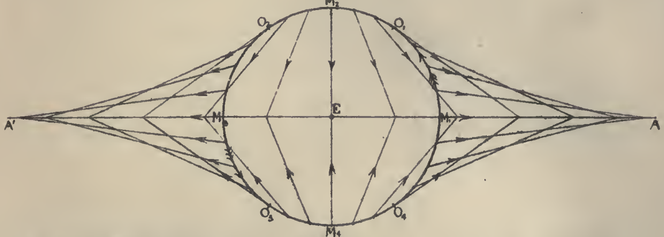


Fig 22: The Radial parts of the same perturbing forces.

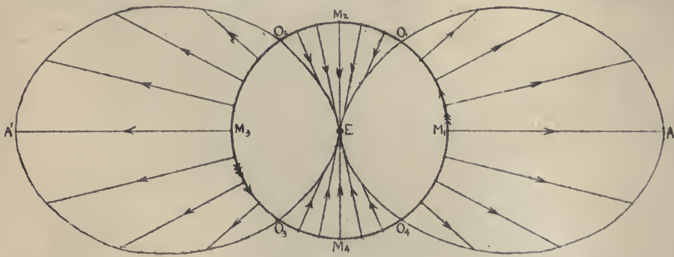
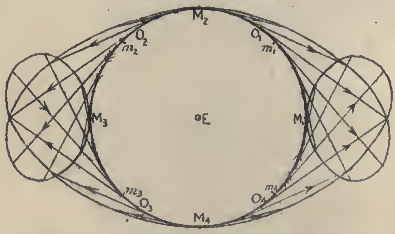


Fig 23. The Tangential parts of the same perturbing forces.



On the scale of Figs. 19, 22, & 23, the Earth's attraction on the Moon would be represented by 60 times AA′, the Sun's by 128 times AA′.

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equal to twice $E M_1$. As the moon passes from M_1 to O_1 the perturbing force gradually becomes more and more inclined to the line $E A$, but continues to act outwards with respect to the orbit $M_1 M_2 M_3 M_4$. At O_1,* however, the perturbing force is for the moment tangential to the orbit, and after the moon has passed O_1, the force acts inwards. This continues until the moon has passed to O_2, a point corresponding in position to O_1 but on the left of $M_2 E$. At M_2 it is clear that the force is represented by the line $M_2 E$, and is simply radial. Also, in actual amount the perturbing force is less at M_2 than at any other point in the semicircle $M_1 M_2 M_3$. After passing O_2 the force is again exerted outwards, becoming wholly outwards at M_3, when it is represented by the line $M_3 A'$ equal to $M_1 A$, or to the diameter of the circle $M_1 M_2 M_3 M_4$. Passing from M_3 to M_4, and thence to M_1, the moon is subjected to

* O_1 is determined by the circumstance, that when $O_1 K$ is drawn square to $E A$ (the student should pencil in the lines and letters here mentioned), and $K L$ taken equal to twice $E K$, $O_1 L$ is a tangent to the circle $M_1 M_2 M_3 M_4$. Since the square on line $E O_1$ is equal to the rectangle under $E K$, $E L$, or to three times the square of $E K$, we obviously have the cosine of the angle $O_1 E K$ equal to $\frac{1}{\sqrt{3}}$, whence $O_1 M_1$ is an arc of $54° 44'$; and $O_2 M_3$, $M_3 O_3$, and $O_4 M_1$ are also arcs of $54° 44'$. In Herschel's *Outlines of Astronomy* these arcs are given as $64° 14'$, and the figure to art. 676 is correspondingly proportioned. But $54° 44'$ is the correct value. Indeed it will be obvious in a moment that an arc of $60°$ would give a perturbing force lying within the tangent, since the tangent at the extremity of an arc of $60°$ clearly cuts the line $E A$ at a distance from E four times as great as the distance of the foot of a perpendicular let fall from the same extremity.

corresponding perturbing forces, varying in the reverse way. At O_3 the force is wholly tangential, at M_4 it is wholly radial, and represented by the line $M_4 E$. At O_4 it is again wholly tangential; and, lastly, at M_1, the force is again wholly radial as at first, and represented by the line $M_1 A$.

It will be very obvious that, on the whole, the perturbing action tends to diminish the earth's influence on the moon, since the forces acting outwards are greater in amount, and act over larger arcs than those acting inwards. We see that $M_1 A$ and $M_3 A'$, the maximum outward forces, are twice as great as $M_2 E$ and $M_4 E$, the maximum inward forces; while the arcs $O_4 O_1$ and $O_2 O_3$ each contain 109° 28′, or in all nearly 219° out of 360°,—that is, more than three-fifths of the complete circumference. Hence we can infer that there is a considerable balance of force exerted outwards.

But it will be well to picture the radial forces separately.

Let M H, fig. 20, represent the disturbing force on the moon at M, and draw E M B radially and M C tangentially. Then complete the rectangle B C by drawing the perpendiculars H B and H C. By the well-known rule for the resolution of forces, the force M H is equivalent to the two forces represented by M B and M C, one radial, the other tangential. Similarly, if we had commenced by considering the force M′ H′, fig. 21, exerted on the moon at M′, we should have found by a similar construction that the radial and tangential forces at M′ are represented by the lines M′ B′ and M′ C′; and so on, for all positions.

Leaving the tangential forces for subsequent consideration, let us suppose the above construction extended so as to give the radial forces exerted at points all around the moon's orbit. We should then have the result pictured in fig. 22,—the radial forces all exerted outwards from O_4 to O_1, and from O_2 to O_3, while they are all exerted inwards from O_1 to O_2, and from O_3 to O_4. We see that the former forces largely exceed the latter.

The first great result, then, from the consideration of the moon's perturbing action, is this,—it tends to draw the moon on the whole outwards from the earth, reducing the earth's influence to a certain extent.

We can compare the actual amount of the radial force (or of the perturbing force generally) with the amount of the earth's attraction; and it is important that we should do so in order that we may judge how the forces acting on the moon are related as respects magnitude.

For it will be remembered that the construction for obtaining fig. 19 is based on the supposition that the line from E to the sun represents the sun's direct attraction on the earth or moon. Now, the line from the earth to the sun is about 91,500,000 miles long; while the line M_1 A is equal to the diameter of the earth's orbit, or to 238,800 miles. So that the sun's maximum perturbing action on the moon is less than his direct action, in the proportion of 2,388 to 915,000, or is about one-383rd part of the latter. But the earth's direct action on the moon is, as we have seen, equivalent to about 7-15ths of the sun's. Hence the sun's maximum perturbing influence is less than the earth's mean attraction on the moon, in the proportion of 15 to 7 × 383, or is about one-179th part of the latter. Thus the force pulling the moon at M_1 towards the sun, would be represented by a line 383 times as long as M_1 A, while the force pulling the moon towards E would be represented by a line 179 times as long as M_1 A. The relations for the perturbing forces exerted on the moon in other positions, as well for the whole forces as for their radial and tangential portions, are indicated by the proportions of the lines in figs. 19, 22, and 23. When the perturbing force has its least value, or when the moon is at M_2 or M_4, this force, now wholly radial, is about one-776th of the sun's direct action, and about one-358th of the earth's.

But now we have to consider the circumstance that the earth's path around the sun is eccentric, and that thus the sun's perturbing influence on the moon necessarily varies in amount. It will be obvious that the perturbing forces must all be greater when the earth and her satellite are nearer to the sun. Let us inquire in what degree they will increase.

This question is readily answered. Fig. 19 indicates the magnitude of the perturbing forces when the line from the sun to E indicates the sun's direct action. Now to simplify matters let us take an illustrative case, in order to determine the law according to which the magnitude of the perturbing forces are affected. We have hitherto supposed the earth at her mean distance from the sun, or about 91,500,000 miles from him. Let us now take the case when she is in perihelion, or about 90,000,000 miles from him. The moon's distance, 238,800, is contained a smaller number of times in the smaller distance, in the proportion of 900 to 915; in other words, the perturbing force represented by $M_1 A$ is a larger aliquot part of the sun's direct influence, in the proportion of 915 to 900. But the sun's direct influence is itself increased by the approach of the earth and her satellite, in the proportion of the squares of these numbers; or as $(915)^2$ to $(900)^2$. Hence the actual amount of the perturbing force is increased in the proportion of the cubes of these numbers, or as $(915)^3$ to $(900)^3$. Similarly when the earth is in aphelion, or 93,000,000 miles from the sun, the sun's perturbing influence is less than when the earth is at her mean distance, in the proportion of $(900)^3$ to $(930)^3$.

There is, however, a simpler method, sufficiently accurate for our purposes, of indicating these relations. When we cube two numbers which are nearly equal, we triple the proportional difference (approximately). Thus if we cube 100 and 101 (whose difference is 1-100th of the former) we obtain the numbers 1,000,000 and 1,030,301, which are to each other very nearly as 100 to 103; so that their difference is about 3-100ths of the former. Now the earth's greatest, mean, and least distances from the sun are approximately as the numbers 62, 61, and 60; and therefore the perturbing influences on the moon when the earth is in aphelion, at mean distance, and in perihelion, are respectively as the numbers 64, 61, and 58 (obtained by leaving the middle number 61 unaltered, and making the first and last differ three times as much as before from the middle number).

There is, then, an appreciable difference between the perturbing forces exerted by the sun when the earth is in perihelion, or at about the beginning of January, and when the earth is in aphelion, or at about the beginning of July. The earth's power over the moon is more considerably diminished in the former case than in the latter. Now the partial release of the moon from the earth's influence results in a slight increase of her mean distance and a lengthening of the moon's period of revolution (we refer of course to her sidereal revolution) around the earth. This will be evident when we consider that the earth's attraction is always tending, though the tendency may not actually operate, to reduce the moon's distance; so that any cause diminishing the total force towards the earth must enable the moon to resist this tendency more effectually than she otherwise would. In winter, then, when the earth is near perihelion, the moon's mean distance and her period of revolution are somewhat in excess of the average; for the sun's releasing effect is then at a maximum. In summer, on the contrary, the earth being near aphelion, the moon's mean distance and her period of revolution are reduced slightly below their mean values; for the sun's releasing effect is then at a minimum. Thus the moon lags somewhat during the winter months, and regains her place by slightly hastening during the summer months. She is farthest behind her mean place, so far as this circumstance is concerned, in spring and autumn (at those epochs when she is at her mean distance), for it is at these times that the loss begins to change into gain, or *vice versâ*. The greatest possible amount of lagging accruing in spring is such that the moon is behind her mean place by about a third of her own diameter. In autumn she gets in advance of her mean place by about the same amount.

This peculiarity of the moon's motion is called the *annual equation*, and was discovered by Tycho Brahé.

Associated with this variation is another of much greater delicacy, and having a period of much greater length. We have seen that the eccentricity of the earth's orbit affects the amount of the sun's perturbing influence, insomuch that this influence is sometimes greater and sometimes less than when the earth is at her mean distance. It might appear that as there is thus an excess at one season and a defect at another, the general result for the year would be the same as though the earth travelled in a circular orbit at her present mean distance from the sun. This, however, is not the case. If we consider that, supposing the earth to revolve always at her mean distance she would describe a circle having a diameter as great as the major axis of her actual orbit, we see that the elliptical area of her real path is less than that of the supposed circular orbit. Hence, on the whole, she is nearer to the sun than if she described a circular orbit in a year instead of her elliptical path. It is true that she moves more slowly when in aphelion, and thus her virtual yearly distance (so to speak) from the sun is increased; but this does not compensate for the actual reduction of her orbit-area due to the eccentricity of her orbit.* Hence the sun's perturbing influence on the moon is somewhat greater, owing to the ellipticity of the earth's orbit. Now this ellipticity is subject to slow variation, due to the influences of planetary attraction. At present it is slowly diminishing. The earth's orbit is slowly becoming more and more nearly circular, without, however, any change (or any corresponding change) in the period of revolution. Thus the area swept out by the earth each year is slowly increasing, and the total of the sun's perturbing influence on the moon in each year is slowly diminishing. The moon then is somewhat less retarded year after year; so that in effect she travels somewhat more quickly year after year. This change is called the *secular acceleration of the moon's mean motion*, or rather an acceleration which is partially accounted for

* The reasoning by which this may be demonstrated corresponds precisely with that in pp. 166, 167 of my treatise on Saturn, where I show that a planet receives more heat during a complete revolution in an elliptical orbit, than it would receive in revolving round a circular orbit in the same period.

by the above reasoning has received this name. As a matter of fact, the moon's mean motion is subject to an acceleration nearly twice as great as the change in the ellipticity of the terrestrial orbit will account for; and astronomers have been led to suspect that a portion of this acceleration may be only apparent and due to a real retardation of the earth's rotation,—that is, a slight increase in the sidereal day, the unit by which we measure astronomical time. With this circumstance, however, we are not at present concerned, save in so far as it relates to the history of that interesting cause of acceleration which has been described above. Halley had been led to suspect that the moon had advanced somewhat farther in her orbit than was consistent with the accounts of certain ancient eclipses.* Further inquiries confirmed the

* I quote here some remarks on Halley's researches by Mr. J. M. Wilson, of Rugby, from a valuable paper contributed to *The Eagle*, a magazine supported by members of St. John's College, Cambridge (No. xxvi. vol. v.). "Halley," he says, "seems to have been the first who considered this question. With astonishing clearness he seized the conditions of the question, saw that the knowledge of the elements, on which the solution was to be founded, was as yet incomplete, and saw also the probability that when the accurate knowledge was obtained, it would appear that there was a peculiarity in the moon's motion entirely unforeseen by others, that it was now moving faster and performing its revolution in a shorter time than it did in past time. If the longitudes of Bagdad, Antioch, and other places, were accurately known, 'I could then,' he says, 'pronounce in what proportion the moon's motion does accelerate; which that it does, I think I can demonstrate, and shall (God willing) one day make it appear to the public.' Newton adds to his second edition of the *Principia* the words,—'Halleius

suspicion. The moon's advance was slight, it is true, but to the astronomer it was as real as though it had taken place under his very eyes. The theory of gravitation seemed to give no account of this acceleration of the moon's motion. At length, however, Laplace was led to turn his attention to the variation of the earth's eccentricity as a probable cause of the peculiarity. His calculation of the effects due to this variation accorded very closely with the observed amount of the acceleration. Yet, although this agreement might have appeared convincing, and although a portion of the acceleration is undoubtedly due to the cause in question, the inquiries of Professor Adams (confirmed by the researches of Delaunay and others, and now universally admitted) show that in reality only half the observed acceleration can be explained by the change in the earth's orbital eccentricity.

But it is to be noted that the variation itself is exceedingly small, as is also the discrepancy between observation and theory. We have seen that the *annual equation* causes the moon to be displaced by about one-third of its diameter in opposite directions in spring and autumn, the actual range of this oscillatory variation being therefore equal to about two-

noster motum medium Lunæ, cum motu diurno terræ collatum paulatim accelerari primus omnium quod sciam deprehendit.' "

I have given an account of the subject in the *Quarterly Journal of Science* for October, 1866, in an essay entitled "Prof. Adams's Recent Discoveries," and a more popular account appears in my *Light Science for Leisure Hours*, in a paper called "Our Chief Timepiece losing Time."

thirds of the moon's diameter. But the theoretical *secular acceleration,* though its effects are accumulative, and in geometrical progression, yet in a century would only cause the moon to be in advance of the place which she would have had, if the acceleration had not operated during the century, by one-300th part of her diameter; and the actual secular acceleration only causes the moon to gain about twice this distance, or about one-150th part of her diameter, in a century.*

We have next to consider one of the most important perturbations to which the moon is subjected so far as the rate of her motion in her orbit is concerned.

We have hitherto considered chiefly the radial part of the perturbing force. We must now discuss the variations in the tangential force. We have already seen how this force can be separated from the radial force (see p. 82). Let us suppose the method applied to give a figure of the tangential forces corresponding to that already given (fig. 22) for the radial forces. To do this, we have to draw a number of lines obtained as M C and M′ A′ were obtained in figs. 20, 21. When this is done (and the reader is recommended not to be satisfied until he has effected the construction for himself independently), the force-lines are found to arrange themselves as shown in fig. 23. It will be

* In two hundred years the gain is four times as great, in three hundred years nine times as much, and so on. For the above illustration I am indebted to Mr. Wilson's paper mentioned in the preceding note.

seen that each loop springs from one of the points M_1, M_2, M_3, M_4 (where the tangential force vanishes, and the radial forces have their unequal maxima), and bends round so as to end at another of those four points; and we see that at the four points m_1, m_2, m_3, m_4 (midway between the former, and not far from those where the radial force vanishes) the tangential force has its maximum value.*

Now as the moon is passing over the arc $M_1 M_2$, the tangential force, acting in the direction shown by the curves, is retardative, and most effectually so when the moon is in the middle of this arc, or at the point m_1. As the moon passes from M_2 to M_3, the tangential force is accelerative, and most effectually so when the moon is at the middle of the arc $M_2 M_3$, or at the point m_2. As the moon passes over the arc $M_3 M_4$, the tangential force is again retardative; and it is again accelerative as the moon traverses the arc $M_4 M_1$, attaining its greatest value when the moon is at the middle of these respective arcs, or at m_3 and m_4. Since, then, retardation ceases to act when the moon is at M_2, the moon is moving there with minimum velocity, so far as this cause of disturbance is concerned. In like manner the moon is moving with maximum velocity at M_3, with minimum velocity at

* The tangential force attains its maximum midway between the points M_1, M_2, M_3, M_4, and not, as is sometimes stated, at the points where the radial force vanishes. It will be obvious from fig. 20 that if we call the angle H E M, θ, we have $HB = 3 \cos \theta \sin \theta = \frac{3}{2} \sin 2\theta$; and this expression has its greatest value when $\sin 2\theta = 1$, or $\theta = 45°$.

M_4, and lastly with maximum velocity again at M_1. It will be clear, then, that near the points m_1, m_2, m_3, and m_4, the moon moves with mean velocity; the arcs $m_4\,m_1$ and $m_2\,m_3$ are traversed with a velocity exceeding the mean; and the arcs $m_1\,m_2$ and $m_3\,m_4$ with a velocity falling short of the mean. Thus at or near m_1 the moon ceases to gain, and therefore the amount by which she is in advance of her mean place has attained its maximum * when the moon is at or near m_1. Similarly the amount by which the moon is behind her mean place has attained its maximum when the moon is at or near m_2.

* It is singular how frequently the very simple principles on which the attainment of a maximum, mean, or minimum value depend are misunderstood; and how commonly the mistake is made of supposing that a maximum or minimum value is attained when the increasing or diminishing *cause* is most effective. It is precisely when an increasing cause is most effective that the *rate of increase* is greatest, and therefore the maximum value is then clearly not attained. And so of a minimum value; there can clearly be no minimum while the decreasing cause is still effective. It is when a cause neither tends to increase nor diminish,—that is, when it has a mean value,—that the maximum or minimum of effect is attained. Thus in spring the sun's daily elevation is increasing more rapidly than at any other time, and in autumn the daily elevation is diminishing most rapidly; but it is not at these seasons that the sun attains his maximum or minimum degree of elevation; this happens in the summer and winter, when his daily elevation changes least. In an illustrative case such as this there can be no mistake; yet very often, when less familiar instances are dealt with, the mistake to which I have referred is made. Thus in Mr. Lockyer's *Elementary Lessons in Astronomy*, we have the seasons when the equation of time is zero described as those when the real sun's motion is the same as the mean sun's; the fact really being that it is precisely at these seasons that the real sun's motion attains either its maximum or minimum value.

At m_3 she is again in advance of her mean place by a maximum amount, and at m_4 she is again behind her mean place by a maximum amount.

This inequality of the moon's motion is called the *Variation*. It is so marked that at the points corresponding to m_1 and m_3 the moon is in advance of her mean place by an amount equal to about her own diameter, while at m_2 and m_4 she is by a similar amount behind her mean place. The range of the variation is thus equal to about twice the moon's diameter. The period of the variation is on the average half a lunation, since in that time the moon passes from her greatest retardation (due to this cause) to her greatest advance, and so back to her greatest retardation. We owe to Tycho Brahé the discovery of this inequality in the moon's motion.*

And now, precisely as we had, after considering the *annual equation*, to discuss an associated but much less considerable inequality, so there is an inequality associated with the variation, but much smaller in amount. It is, however, more interesting in many respects, precisely as the *secular acceleration* of the moon is a more interesting inequality than her *annual equation*.

We have hitherto not taken into account the circumstance that though the sun's distance enormously

* It will be evident that the ancients, who trusted chiefly to eclipses to determine the laws of the moon's motion, were precluded from recognizing the remarkable displacement due to the *variation*; since eclipses necessarily occur when the moon is on the line passing through the earth and sun, or when the moon is at M_1 or M_3, at which points the variation vanishes.

exceeds the radius of the moon's orbit, it is nevertheless not so great but that there is an appreciable relative difference between the moon's distance from the sun when in conjunction with him (or at the time of new moon), and when in opposition (or at the time of full moon). When the earth is at her mean distance from the sun (or 91,500,000 miles from him), the moon's distance from him when she is new is 91,738,800 miles, and when she is full it is only 91,261,200 miles,—so that these extreme distances are proportioned as the numbers 917,388 and 912,612, or, nearly enough for our purposes, they are as the numbers 201 and 200. Hence, by what has been already shown, the perturbing forces on the moon in these two positions are as the numbers 203 and 200. Thus the difference, though slight, is perceptible. Yet again, the points where lines drawn from the sun touch the moon's orbit are not quite coincident with the points M_2 and M_4 (fig. 19), but are slightly displaced from these positions towards M_1. Here, again, the amount of either displacement, though slight, is appreciable. It amounts, in fact, to an arc of about $8\frac{3}{4}$ minutes; so that the points in question divide the moon's orbit into two unequal arcs, whereof one, the farthest from the sun, exceeds a semicircle by $17\frac{1}{2}$ minutes, the other falling short of a semicircle by the same amount,—the larger thus exceeding the smaller by 35′, or more than half a degree.

It necessarily follows that the direct effect of the tangential force in increasing or diminishing the moon's mean motion, is not equal in the two halves $M_4 M_1 M_2$ and $M_2 M_3 M_4$ (fig. 19). It is greater in the former semicircle, on the whole, than in the latter; but the points where the tangential force vanishes lie outside the extremities of this latter semicircle. Thus the points where the variation attains its maximum value lie on the sides of m_1 and m_4 *towards* M_1, and on the sides of m_2 and m_3 *away* from M_3; and the amount of the maxima at the two former stations is greater than the amount at the two latter. Moreover, when the earth is in perihelion these effects are greater, while when she is in aphelion they are less than when she is at her mean distance. The maximum inequality thus produced, a variation of the variation as it were, amounts to about two minutes, or about the sixteenth part of the moon's apparent diameter. It is called the *parallactic inequality*, because of its dependence on the sun's distance, which, as we know, is usually expressed by a reference to the solar parallax. And as the inequality depends on the sun's distance, so its observed amount obviously supplies a means of determining the sun's distance. It was, in fact, a determination of the sun's distance, deduced by Hansen from the observed amount of the moon's maximum parallactic inequality, which recently led astronomers to question a value of the distance, based on observations of Venus in transit, which had been for many years adopted in our text-books and national ephemerides.

Before passing from the consideration of the direct action of the tangential force, it is to be noticed that this force affects the secular acceleration of the moon. It had long been held that only the radial force can really be effective in long intervals of time, because the tangential force is self-compensating,—if not in each lunation,* yet at least in the course of many successive lunations. But as a matter of fact, inasmuch as the eccentricity of the earth's orbit is undergoing a continual though very gradual diminution, an element is introduced which renders this compensation incomplete,—not merely in many successive lunations or in many successive years, but in many successive centuries. So long as the eccentricity of the earth's orbit continues to diminish, there can in fact be no tendency to exact compensation so far as this particular element is concerned. Now this circumstance had not escaped Laplace when he discussed the moon's secular acceleration; but he was led to believe that its effects would be wholly insignificant. Professor Adams, however, in re-examining the whole subject, was led to inquire how far this view of the matter is justified. The experience of past inquirers had shown that no cause of variation, and particularly no cause having effects

* Of course, in a thorough analysis of the action of the tangential force, it has to be remembered that, apart from the ellipticity of the moon's orbit, and the consequent inequality of the sun's perturbing action in different quadrants as well as in different halves, the orbit is undergoing a process of continual change, even under the action of the tangential and radial forces themselves.

cumulative for many successive years, can safely be neglected. Professor Adams remarked, "In a great problem of approximation, such as that presented to us by the investigation of the moon's motion, experience shows that nothing is more easy than to neglect, on account of their apparent insignificance, considerations which ultimately prove to be of the greatest importance." We shall see presently how Newton himself fell into an error precisely resembling that of Laplace, in fact so far identical in its nature that it was the tangential force that Newton, like Laplace, held to be self-compensatory, though the instances to which this erroneous consideration was applied were altogether distinct in their nature.*

* It is a somewhat curious circumstance, that while the correction applied by Adams to Laplace's labours resulted in reducing the theoretical secular acceleration by one-half, so the correction applied by Clairaut to Newton's inquiry into the motion of the moon's perigee resulted in doubling the theoretical amount of that motion. (Clairaut had himself, in the first instance, obtained by analytical researches the same erroneous value which Newton had obtained from geometrical considerations.) It is to be remarked also that Adams's labours set theory and observation at apparent discordance after they had been brought into agreement, while Clairaut's labours brought theory and observation, which had long seemed discordant, into perfect agreement. Nothing, perhaps, could more thoroughly demonstrate Adams's mastery of the lunar theory than his maintaining his views against the great reputation of Laplace, seemingly also against observation, and actually against the concurrent opinion of nearly all the greatest continental mathematicians. How slowly his views made ground will be seen from this, that the paper from which I have quoted was read before the Royal Society in 1853, and that it was not until the year 1866 that

Fig 24

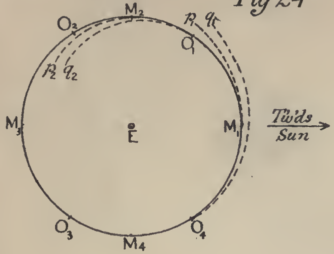
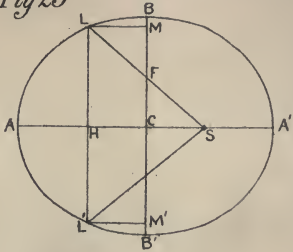


Fig 25



Illustrating effects of Radial perturbing force in Perigee & Apogee.

Fig 26

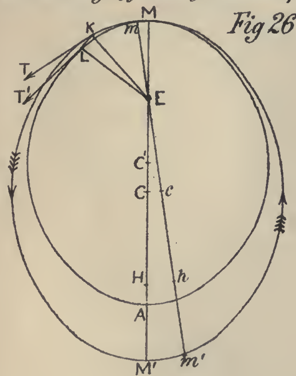
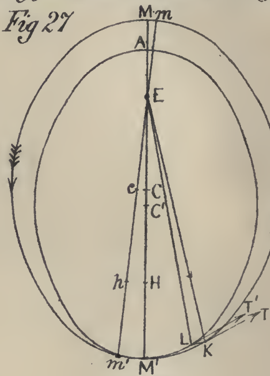


Fig 27



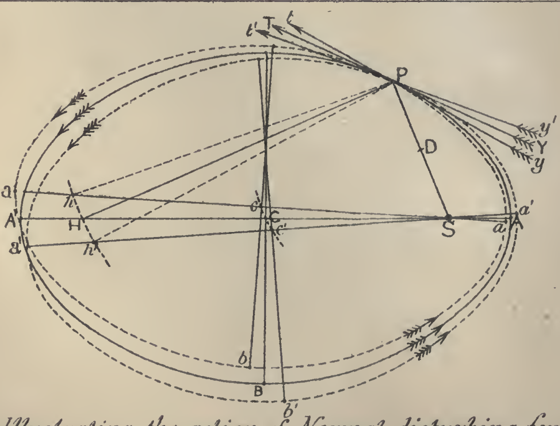


Fig 28. Illustrating the action of Normal disturbing forces.

R. A. Proctor del^t.

Thus far we have considered those general effects only which could be adequately discussed without any reference to the ellipticity and inclination of the lunar orbit. Now we have to examine the effects of the perturbing forces on the figure and position of the moon's orbit, as well with respect to its ellipticity as to its inclination.

In the first place it will be well to prevent a misapprehension which is very commonly entertained by those who approach this subject for the first time. Looking at fig. 24, Plate VII., and noticing the nature of the forces which are exerted upon the moon at different parts of her orbit, it seems natural to infer that since the moon is drawn outwards when at or near M_1 and M_3, while she is drawn inwards when at or near M_2 and M_4, her orbit must necessarily be lengthened along the line $M_1 M_3$ and narrowed alone the line $M_2 M_4$. In reality, the contrary happens. The forces exerted on the moon tend to diminish the curvature of the orbit at M_1 and M_3, and to increase the curvature at M_2 and M_4. This is easily shown. We have seen that at M_1 there is a radial perturbing force acting outwards, or resisting the earth's attraction on the moon. Hence if we suppose the moon to arrive at M_1 as if moving on our hypothetical circular orbit, then instead of moving onwards to O_1 on this circle,

the matter was so definitely settled in his favour, that the gold medal of the Astronomical Society was awarded to him. So late as 1861, we find his great rival Leverrier saying that "very certainly the truth lay with Adams's opponents."

she would move on a path such as $M_1 p_1$, being less strongly drawn towards the earth. Or if we suppose that she had arrived at O_4 (the place where the radial force vanishes) on the circular orbit, she would follow such a course as $O_4 q_1$, on a curve less strongly curved than the circle $O_4 M_1 O_1$. Again at M_2 the radial action reinforces the earth's attraction. Hence if the moon had arrived at M_2 on the hypothetical circular path, then under the increased action due to the radial force, she would follow a path such as $M_2 p_2$; or such a path as $O_1 q_2$, if she had arrived at O_1 on the circular orbit: and it is obvious that both $M_2 p_2$ and $O_1 q_2$ are more strongly curved than the circle $O_1 M_2 O_2$. So that, partially freed from control over the arcs $O_4 O_1$ and $O_2 O_3$, the moon there tends to follow an arc of less curvature, a more flattened arc, so to speak, than in traversing the arcs $O_1 O_2$ and $O_3 O_4$, where she is subjected to an increased radial force.

Now let us inquire into the various circumstances which affect the position of the moon's perigee.

We know that the radial force acts sometimes inwards and sometimes outwards, and that the tangential force is sometimes accelerative and sometimes retardative. Before proceeding to the actual relations of the lunar orbit, it will be well to consider the effect of radial and tangential forces thus acting. We ought, indeed, now that the ellipticity of the moon's orbit is considered, to distinguish between a radial force and a force acting square to the tangential force; but the moon's orbit is not so eccentric as to render the distinction important in an inquiry such as the present. For the sake of brevity, and because in effect a complete inquiry would require a volume instead of a section of a chapter, I take only the action on the moon when in perigee and apogee.

Let us suppose that when the moon is at her perigee M, fig. 26, she is exposed to a radial perturbing force acting towards E, and let M K M′ be the path which she would follow but for this perturbing force. Now if the attraction of the earth were suddenly but permanently increased when the moon was at M, the path subsequently pursued by our satellite would be (so far as this cause is concerned) an ellipse, such as M L A, still having its perigee and apogee on the line M M′. But if, after the increased radial force had acted for awhile,—say till the moon had reached L,—it ceased to act, the moon's orbit would as it were recover from its temporary contraction. The motion at L would be nearly the same as the motion at a point K on the undisturbed orbit (K being as far from E as L is, and the tangent K T making appreciably the same angle with E K that the tangent L T makes with E L). The velocity at L will also be appreciably the same as the velocity at K, for the arcs M L and M K are small, and the main effect of the radial disturbing force during the short time of its action, is that which it has had in drawing the moon inwards. Thus the line E K in the original orbit may be regarded as having advanced to the position E L. The moon passes on from L under the same conditions as those under which she would have passed from K if undisturbed. The orbit, therefore, which the moon would traverse if thenceforth undisturbed is identical in shape with the orbit M K M′, but differs in being shifted forwards so that E K has taken the position E L. E M then has taken the position E *m*—in other words, the perigee has advanced to the position *m*, and *m* E *m′* is the new position of the major axis.

An increase of the radial force, then, acting when the moon is in or near perigee causes the perigee to advance. And by like reasoning it may be shown that a diminution of the radial force acting near perigee, causes the perigee to regress.*

Next let us take the case where the disturbing radial force acts on the moon when she is in or near apogee. Here, supposing that M′ is the apogee of the lunar orbit and M′ K M the undisturbed path, we have for the path which would result from a permanent increase of radial force, such an orbit as M′ L A. But the disturbing increase ceasing when the moon is at L, we have the same conditions at L as at a point K on the original orbit (as far from E as L is, and having the tangent K T inclined at appreciably the same angle to E K as L T′ to E L). Thus the position of the moon at L corresponds to a more advanced

* We can regard M *m* L as the original orbit in this case, and M K as the orbit traversed under the reduced radial action. Then E K corresponds to E L, a more advanced position in the former orbit ; in other words, in this case each corresponding radial line in the new orbit (and therefore the line to the perigee) is farther back than in the old orbit.

position at K in the original orbit. All other lines in the original orbit are similarly thrown backwards or caused to regrede. Thus the apogee is thrown to m' and the perigee to m, and the new orbit has its major axis in the position m E m'.

An increase of the radial force, then, acting when the moon is in or near apogee, causes the perigee to regress. And by like reasoning it may be shown that a diminution of the radial force acting near apogee causes the perigee to advance.*

But a consideration of the reasoning in the cases just considered will show us how we may infer the effects of a change in the radial force when the moon is in other parts of her orbit. We shall, however, in what follows speak of the *normal* † force, because in fig. 28, and in figs. 29, 30, 31, &c., it is convenient to have an elliptic orbit of such a figure that there is a considerable difference between the direction of the radial line and of the normal (or perpendicular to the tangent). We remind the student, however, that in the actual case of the moon's motions the radial and normal lines are always very nearly coincident. Now supposing the moon to be at P, fig. 28, when an

* We may regard M′ L as the original orbit in this case, and M′ K as the disturbed orbit; then E K in the latter corresponds to a less advanced line, E L, in the former, and thus every corresponding line in the new orbit takes up an advanced position.

† By the normal force is here understood the force acting perpendicularly to the tangent. The actual normal force, of course, acts always inwards; it is only the perturbation which acts sometimes outwards, diminishing the normal force, or inwards, increasing it.

increase of the radial force is experienced, and to be travelling in the path P A′ A, she would travel on a course touching her former path at the point P, but forming an ellipse smaller than P A A′, if the radial force were permanently increased, and a path still touching the former path in P, but larger than the ellipse P A A′, if the radial force were permanently diminished. But as the increase or diminution of the radial, and therefore of the normal force, acts but for a short time, we have to consider that when the moon has traversed some small distance from P on the new path, the normal force is restored to its original value. In reality of course the normal force passes above and below its mean value with a continuous process of change, not starting suddenly from its mean to its maximum or minimum value: and in any thorough investigation intended to determine the quantitative effects of such changes, this circumstance must be taken into account. But in an inquiry such as the one we are upon, it is sufficient to consider the effects of an increase or diminution continuing to act during some short but definite time. Now it will be obvious that if exposed to an increased normal force, the moon, after travelling a short distance from P, would be moving on a course making a larger acute angle with P T than the course she would have had at the same epoch if undisturbed; whereas, if the normal force were diminished at P, the moon would be travelling on a course making a smaller acute angle with P T than the course she would have had at the same epoch if undisturbed. The alteration of the direction does not take place at P, or at any definite point on the moon's course; but is the sum of the effects resulting either from increased or diminished radial action as the moon moves onwards from P. But, *this remembered,* we shall not err greatly if, to simplify the illustration, we suppose the alteration of direction to take place, *per saltum,* at the point P, even though P is the precise point where a *permanent* increase or diminution of the radial force would leave the tangency absolutely unaltered. A temporary change in the normal force does actually produce an alteration in the position of the orbit, which, if no further changes took place after the moon had travelled some distance from P, would affect the tangency of the orbit close by P.* The angle corresponding to S P T would be diminished if the normal force were increased; while that angle would be increased if the normal force were diminished; and the period of the orbit (or the major axis) would be unaffected, because the normal force would resume its original value, while the main effect produced on the moon's motion would be merely a change of direction. So that since the major axis of the orbit is equal to the sum of the lines S P, P H (H being the other focus), the new orbit would have its focus as far from P as H is, and therefore on the circular arc h H h' about P as centre. And obviously, if the tangent P T took up the position P t, owing to

*. The moon's new orbit would not pass through P in that case.

a diminished normal force, the new position of the other focus would be as at h, and S h would be the direction of the new axis; while, an increased normal force causing the tangent at P to assume the position P t', would bring the other focus to such a position as at h', S h' being the new direction of the axis.*

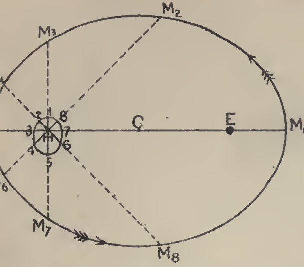
It is easily seen how the complete orbits corresponding to the new direction of the axis can be constructed. (The points c C c' lie on a circular arc about D, the middle point of S P.)

Now, if the reader have carefully followed the preceding reasoning, he will readily see that a decrease of the normal force acting at the point M_1, fig. 30, Plate VIII, will shift the other focus from H to 1; if the decrease acts at M_2, the other focus will be shifted to 2; and so with the other points marked round the orbit, a decrease acting at M_3, M_4, M_5, M_6, M_7, or M_8, causing the focus to shift to 3, 4, 5, 6, 7, or 8, respectively.†

* It is easily seen that the angle h P H, fig. 28, must be twice the angle T P t, and h' P H equal to twice the angle T P t'. To prove the first relation, we have the angle T P S equal to the angle T′ P H; and clearly, when T P T′ assumes the position t P y, the angle T P S is less and the angle y P H is greater than the angle T P S, by the angle t P T. Thus the angle y P H exceeds the angle t P S by twice the angle t P T; and as y P H is equal to the angle t P S, we have the angle H P h equal to twice the angle t P T. So with the other case.

† The student will readily see why an oval shape is given to the curve through the points 1, 2, 3, ... 8. For the disturbing radial force, as is evident from fig. 17, is greater as the distance of the moon is greater. Thus it is greater when it acts at M_5 than when it acts at M_1; and, moreover, the moon is moving more slowly when at M_5 than when at M_1. Each circumstance tends of itself

llustrating the action of normal forces at various points of an orbit.



ig 30. Normal force inwards.

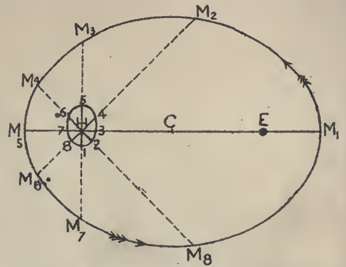
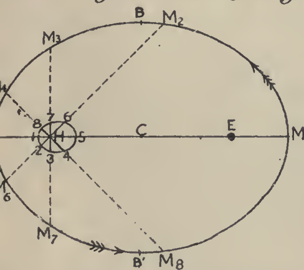


Fig 31. Normal force outwards.

llustrating the action of tangential forces at various points of an orbit.



ig 32. Tangential Acceleration.

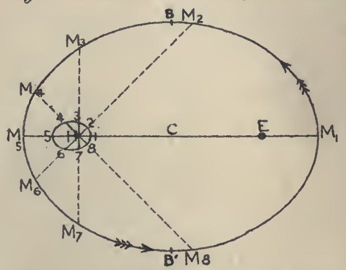


Fig 33. Tangential Retardation.

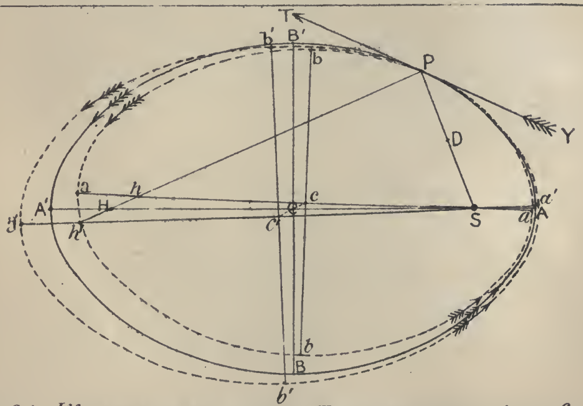


Fig 29. Illustrating the action of Tangential disturbing forces.

A. Proctor del.t

It thus appears that if the moon is anywhere on the arc $M_1 M_3 M_5$, a decrease of the normal force causes the eccentricity to increase, the other focus being thrown farther from E, while if the moon is anywhere on the arc $M_5 M_7 M_1$, a decrease of the normal force causes the eccentricity to decrease, the other focus being brought nearer to E. And again, if the moon is anywhere on the arc $M_7 M_1 M_3$, a decrease of the normal force causes the perigee to regrede, while if she is anywhere on the arc $M_3 M_5 M_7$, a decrease of the normal force causes the perigee to advance. The latter arc is considerably smaller than the former, but it is not described in a much shorter time, because it is at the apogeal end of the lunar orbit.* On the

to make H 5 greater than H 1, in the proportion of $H M_5$ to $H M_1$, so that both together would make H_5 greater than H_1 in the square of this proportion. But, on the other hand, there is a circumstance acting with contrary effect in such sort as to leave the increase only in the direct proportion of $H M_5$ to $H M_1$,—the fact, namely, that $H M_5$, which sways, as it were, around the point M_5, is shorter than $H M_1$. Thus, on the whole, H 5 exceeds H 1 in the proportion of $H M_5$ to $H M_1$. In intermediate positions corresponding effects accrue, the disturbance of the other focus from the position H being always directly proportional to the distance of the moon from E. Hence the curve through the points 1, 2, 3, 4, 5, 6, 7, and 8 is an ellipse resembling the ellipse $M_1 M_2 M_3 \ldots M_8$, but placed as shown in the figure. Similar remarks apply when the radial force is increased. See fig. 31.

* If in an ellipse having axes A C A′, B C B′, fig. 25, and foci S, H, we draw L H L′ (the *latus rectum*) square to A A′, and join S L, S L′, the time in the arc L′ A′ L is to the time in the arc L A L′ as the sectorial area L S L′ to the remainder of the ellipse. Now if we draw L M, L′ M′ square to B B′, it is obvious that the areas A S L, A S L′ severally exceed the areas B C A, B′ C A by the

other hand, the normal action is considerably more effective when it is exerted on the moon at any point in the arc $M_3\,M_5\,M_7$. Hence, since in the course of many lunar revolutions, a decrease of radial force must have acted at every part of the lunar orbit, there will be, on the whole, a balance in favour of those effects which accrue when the moon is near her apogee,—or, so far as disturbances decreasing the radial force are concerned, the perigee will, on the whole, tend to advance, while (on the average of many lunar revolutions) the eccentricity will remain unchanged. Yet the eccentricity will not be constant, though it will undergo no permanent alteration.

In like manner, an increase of the normal force, acting at the respective points M_1, M_2, M_3 . . . . M_8 (fig. 31), causes the other focus to shift from H to the respective points 1, 2, 3, &c.* We see that if the moon is anywhere on the arc $M_1\,M_3\,M_5$, the eccentricity is decreased by an increase of the normal force; while if the moon is anywhere on the arc $M_5\,M_7\,M_1$, an increase of the normal force causes the eccentricity to increase. Again, if the moon is anywhere on the arc $M_7\,M_1\,M_3$, an increase of the

small spaces B M L, B′ M′ L′, for the two triangles M F L, S F C are equal; thus the area L S L′ falls short of one half of the ellipse by the sum of the small areas B M L, B′ M′ L′. Hence the defect of the time in L′ A′ L from one half the periodic time bears to said time the small ratio which twice the area B M L bears to the whole area of the ellipse.

* See note (†) at page 106. Exactly the same reasoning applies in this case as in the case there considered.

normal force causes the perigee to advance, while if she is anywhere on the arc $M_3 M_5 M_7$, the perigee is caused to regrede. And by reasoning precisely resembling that in the preceding paragraph, we see that on the average of many revolutions, during which increase of normal force must have acted on the moon at every part of her orbit, the perigee will have been caused to regrede by such increased radial action. Again also, the eccentricity, though not remaining constant, will undergo no permanent change through the change of the normal force.

Now, since as we have shown (see fig. 19, Plate VI.) the sun's perturbing action tends, on the whole, to diminish the radial action on the moon, the maximum diminution being twice as great as the maximum increase, and diminution prevailing over a much larger portion of the moon's orbit, it follows that the balance of perigeal advance due to the diminution of the radial force, in any given number of revolutions, must exceed the balance of perigeal regression due to increase of the radial force. On the whole, then, the radial perturbations must leave a balance of perigeal advance.

Next let us consider the tangential perturbing force. Here we have simpler preliminary considerations to deal with. We know that when the tangential force acts to accelerate the moon's motion, it tends to increase the major axis of the orbit, while when it retards the moon's motion, it tends to diminish the major axis. Now it is obvious that where there is no change in the direction of the moon's motion, the direction in which the other focus lies with respect to the moon cannot be altering (see note, p. 106), but that the other focus will be thrown farther away or brought nearer, according as the axis is increasing or diminishing, or, in other words, according as the tangential force is accelerative or retardative.

This is illustrated for a particular position of the moving body, in fig. 29; but, as in the case of normal forces, the student would do well to repeat the construction for a variety of cases.

In fig. 29, S is the attracting centre, round which the body is moving in the orbit A B′ A′B, the other focus being at H. If, when the body is at P, its motion is accelerated, the other focus still lies towards H,—but farther away than H, viz., at a position such as *h*′, so that the new major axis, which, by the properties of the ellipse, is equal to the sum of the focal distances of P, is equal to S P and P *h*′ together, or greater than S P and P H together. Then if we join S *h*′, *c*′, the middle point of S *h*′, is the new centre of the orbit. We must take each of the lines *c*′ S *a*′ and *c*′ *h*′ a′, equal to the half of S P and P *h*′ together; or, which is the same thing, we must make each of these lines *c*′ S *a*′ and *c*′ *h*′ a′ exceed CA′ by half H *h*′. The rest of the construction for determining the complete figure of the changed orbit is obvious.*

* We draw b′ *c*′ *b*′ square to a′ *a*′, and with centre *h*′ or S and distance a′ *c*′ describe a circular arc cutting b′ *c*′ *b*′ in b′, *b*′, the

In like manner the construction proceeds, when the body is retarded at P, the new orbit having a focus at h and centre at c. The three positions of the centres at c, C, and c', obviously lie on a straight line through D, the bisection of S P.

Hence, when the tangential force is accelerative, fig. 32 shows the change in the position of the focus H for different positions of the moon in her orbit. When she is at M_1, an acceleration shifts the farther focus to 1; when she is at M_2, the farther focus shifts to 2; and when she is at M_3, M_4, M_5, &c., to M_8, the farther focus shifts to 3, 4, 5, &c., to 8, respectively.*

extremities of the new minor axis. It is to be noticed that the new eccentricity is not $c' h'$ but $\frac{c' h'}{c' \text{a}'}$; so that it is not necessarily increased when the distance S h' is greater than S H. It can readily be shown that the eccentricity increases when the body is on the arc $b' a'$ b′, and decreases when the body is on the arc b′ a′ b'.

* It will be easily seen that H 1 should be greater than H 5. For when the moon is in perigee the accelerative force is less than when she is in apogee, and acts for a shorter time, because she moves more quickly; while, also, given accelerations increase the velocity in a smaller relative degree, since it is already large. Against this is to be set the circumstance that for a given increase of velocity the increase of the major axis is proportional to the velocity (as is easily shown from the relation $\frac{1}{a} = \frac{2}{r} - \frac{v^2}{\mu}$), and therefore inversely proportional to the distances at apogee and perigee. There still remains an excess of increase of the major axis when the moon is at her apogee. It is easily seen, also, that the maximum and minimum effects accrue when the moon is at her apogee and perigee respectively, the effects when she is at intermediate parts of her orbit being greater or less according as she is nearer to apogee or to perigee.

Thus when the moon is on the arc M_7 M_1 M_3, fig. 32, the distance of the other focus from S is increased by an accelerating tangential force; whereas, if she is on the arc M_3 M_5 M_7, the distance of the other focus from S is diminished. The actual eccentricity is increased or diminished, according as the body is on the arc B′ M_1 B or B M_5 B′. Again, when the moon is on the arc M_1 M_3 M_5, an accelerating tangential force causes the perigee to advance; while when she is on the arc M_5 M_7 M_1, such a force causes the perigee to regrede. Thus, in any considerable number of revolutions, tangential acceleration will cause (directly) neither advance nor regression of the perigee, the effects in one direction being, in the long run, exactly counterbalanced by those in the other.

When the tangential force is retardative, fig. 33 shows the change in the position of the focus H for different positions of the moon in her orbit. If she is at M_1, M_2, M_3, M_4, &c. . . . . . M_8, when the retardative action is exerted, the shift of the focus H is as towards 1, 2, 3, &c., 7 and 8,* respectively.

Thus we see that when the moon is on any part of the arc M_7 M_1 M_3, the distance of the farther focus from S is diminished by retardative tangential force, while when she is on any part of the arc M_3 M_5 M_7, the distance of the other focus from S is increased by such retardation. The actual eccentricity is diminished or increased according as the body is on the arc B′ M_1 B or B M_5 B′. Again, when she is on

* See preceding note.

the arc $M_1 M_3 M_{51}$ retardation causes the perigee to regrede, while when she is on the arc $M_5 M_7 M_1$ retardation causes the perigee to advance. In this case, as in the former, the perigee neither advances nor regredes, on the average of many revolutions, so far as the direct action of the tangential force is concerned.

It appears, then, that the radial disturbing force causes on the whole a progression of the perigee, while the tangential force does not directly produce any permanent effect on the position of the perigee. It was to have been expected that a difference of this sort should assert itself, since fig. 22 shows that the radial disturbing force inwards is not equivalent to the outward disturbing radial force. On the other hand, the tangential force is altogether self-compensatory (see fig. 23), its action being alternately accelerative and retardative in the four quadrants, and equal in each, save on account of the eccentricity of the moon's orbit, which, however, favours permanently neither the accelerative nor the retardative effect. This does not prevent, however, a temporary advance or recession of the perigee through the direct action of the tangential force, nor a temporary increase or diminution of eccentricity.

But although the tangential force does not directly produce any permanent effect on the position of the perigee, it is indirectly as effective as the radial force. In showing how this happens, I shall consider four special cases of the operation of the radial and tangential forces in causing the advance of the perigee; but

I would invite the student who wishes really to master the subject to consider intermediate cases, carefully making the requisite constructions and applying to them considerations resembling those which will now be applied to the four selected cases :—

Let the major axis of the lunar orbit (or, as it is called, the "line of apsides,") be directed as in fig. 34, Plate IX., towards the sun, the perigee being nearest to the sun as at *p*. Then the perturbing forces, for certain parts of the orbit, are indicated in the figure. At *p* and *a* the radial force exerts its maximum outward actions; at M and M_4 it exerts its maximum inward actions. Near O_1, O_2, O_3, and O_4 the tangential force has its maximum values.*

Now, from fig. 30 combined with fig. 34, we see that the outward action of the radial force over the perigeal arc $O_4\,p\,O_1$ results in a regression of the perigee; †

* The student should make tracings from figs. 34, 35, 36, and 37, and draw the radial and tangential resolved parts of the forces, precisely as in Plate VI. This will be found to be a most instructive exercise.

† To prevent misconception I will go through the reasoning leading to this result, leaving to the student to deal in like manner with other cases as they arise. Fig. 34 shows that we have outward radial perturbing action as the moon is passing her perigee. Now, fig. 30 illustrates the effect of outward radial (or normal) perturbations. In this figure M_8 M_1 M_2 is the perigeal arc, and we see that H is shifted towards 8 or 1 or 2, according as the body is at M_8 or M_1 or M_2 when the perturbation takes place; and in intermediate positions towards intermediate points. The perigee then is shifted from M_1 backwards; i. e. in the direction contrary to that indicated by the arrow on the orbit. With very

Fig. 34.

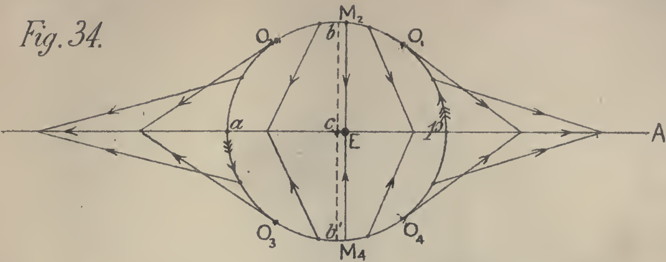


Fig. 35.

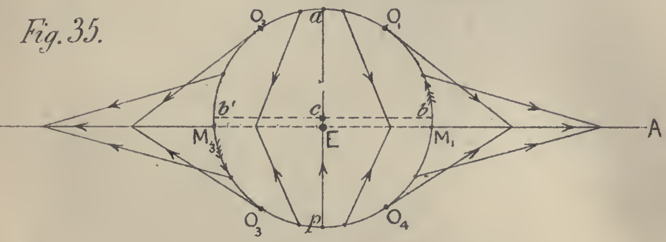


Fig. 36.

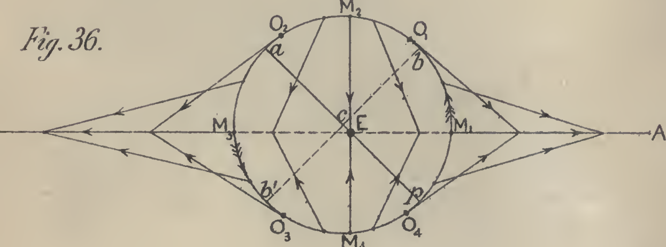
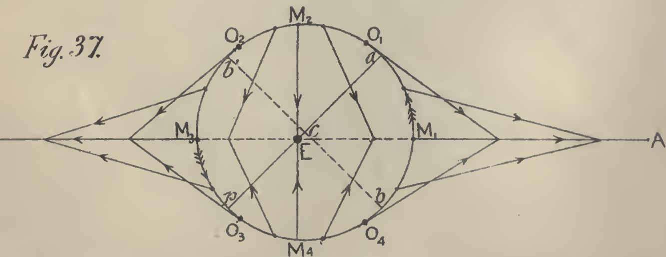


Fig. 37.



Illustrating the Motion of the Perigee of the Moon's Orbit.

A. Proctor del.

while the reduction of the radial force over the apogeal arc $O_2\,a\,O_3$ results in an advance of the perigee. It is obvious also that the inward action of the radial force over the arcs $O_1\,O_2$ and $O_3\,O_4$ will have opposite and exactly counterbalancing effects. We have only to inquire then whether the regression just mentioned is less or greater than the advance. It is obvious that the outward action over the arc $O_4\,p\,O_1$ is less than the outward action over the arc $O_2\,a\,O_3$; moreover, the moon, moving more swiftly in perigee than in apogee, is exposed for a shorter time to the former smaller action than to the latter larger action. Accordingly, on both accounts, the perigeal regression is less than the perigeal advance, so far as the radial force is concerned. There is therefore a balance of advance due to the radial action in a complete lunar evolution, when the perigee is as in fig. 34.

Next as to the tangential action. In moving from b' to p, the moon is accelerated. Hence, from fig. 32 we see that the perigee recedes. In moving from p to b the moon is retarded; hence, from fig. 33 we see that the perigee still recedes. In like manner, in moving from b to a the moon is accelerated; hence, from fig. 32 the perigee advances. And in moving from a to b' the moon is retarded; hence, from fig. 33 the perigee still advances. Now, in order to ascertain whether

little practice the student will be able to deduce such results in an instant; and with a little more he will have no occasion to refer to the figures 30, 31, 32, and 33.

the advance or recession is greater, we have only to notice that,—(1) the moon moves more rapidly over the arc $b' p\, b$ than over the arc $b\, a\, b'$, so that any given acceleration or retardation will produce a smaller *proportionate* increase or decrease of velocity in the former than in the latter arc; (2) fig. 34 shows that the actual forces are less in the former than in the latter arc; and (3) the forces act for a shorter time over the former than over the latter arc, because the moon moves over the former arc more quickly. On all three accounts the perigeal advance exceeds the perigeal recession. Thus there is a balance of advance due to the tangential force. But there is also a balance of advance due to the radial force. Hence, there is a total balance of advance when the moon is traversing her orbit placed as shown in fig. 34.

It is perfectly obvious that precisely the same result would have followed if the apogee had been turned directly towards the sun instead of the perigee.

Next, let the major axis of the moon's orbit be placed at right angles to the line from the sun, as in fig. 35; and let similar constructions be employed in this case as in the former. Now, here it is obvious that the radial forces acting outwards on the moon as she traverses the arcs $O_4\, b\, O_1$ and $O_2\, b'\, O_3$, produce opposite and exactly counterbalancing effects. But the radial forces acting inwards on the moon when traversing her apogeal arc $O_1\, a\, O_2$ produce a regression of the perigee (see fig. 31), which exceeds the advance of the perigee produced by the radial forces acting inwards on the moon when traversing her perigeal arc $O_3\ p\ O_4$. For the former forces exceed the latter (see fig. 35), and act during a longer time, owing to the moon's slower motion near her apogee. Hence, so far as the radial force is concerned, there is a balance of perigeal recession in a complete lunar revolution when the orbit is placed as in fig. 35. As regards the tangential force, there is retardation as the moon moves from M_1 to a, and therefore (see fig. 33) the perigee recedes. In moving from a to M_3, the moon is accelerated, and therefore (see fig. 32) the perigee still recedes; thence to p there is retardation, and (see fig. 33) the perigee advances; and, lastly, in moving to M_1, the moon is accelerated, and therefore (see fig. 32) the perigee still advances. But the recession over the apogeal arc $M_1\ a\ M_3$ is greater than the advance over the perigeal arc $M_3\ p\ M_1$, for like reasons to those urged in the corresponding case in the preceding position. Hence, on the whole, there is a balance of perigeal recession due to the tangential force. But we have seen that there is also a balance of perigeal recession due to the radial force. Hence, there is a total balance of perigeal recession when the moon's orbit is placed as shown in fig. 35.

Obviously the same result would have been obtained if the major axis had been placed as in fig. 35, but with the positions of the perigee and apogee interchanged.

Next, let us take such intermediate cases as are illustrated in figs. 36 and 37, where p and a, as in the two former figures, indicate the position of the perigee and apogee respectively. In the case of fig. 36, as the moon is moving from *p* to *b*, the outward action of the radial force causes the perigee to recede (see fig. 30), while, as the moon is moving over the arc *a b′*, the outward action of the radial force causes the perigee to advance. The advance exceeds the recession. Again, as the moon is moving over the arc *b a*, the action of the radial force (chiefly inward) causes the perigee to recede (see fig. 30); while, as the moon is moving over the arc *b′ p*, the chiefly inward action of the radial force causes the perigee to advance. The recession exceeds the advance. Thus there is a balance of recession to be set against the former balance of advance; and though it is easily seen that in the actual circumstances indicated in fig. 36 the balance of advance is somewhat the greater, so that there remains a final balance of advance due to the radial force; yet this balance is very small compared with the balances we had to deal with in the two preceding paragraphs. Again, as to the tangential action. It is retardative over the arc M_1 *b* M_2, and there (see fig. 33) causes recession; accelerative over the arc M_2 *a* M_3, and causes (fig. 32) an advance over M_2 *a*, almost exactly compensated by recession over *a* M_3; retardative over M_3 *b′* M_4, and there causes advance almost exactly compensating the recession over the arc M_1 *b* M_2; and, lastly, it is accelerative over the arc M_4 *p* M_1, and causes a recession over M_4 *p*, almost exactly compensated by advance over

p M_1. Hence, on the whole, the action of the tangential forces produces no effect; and, accordingly, when the moon's orbit is placed as in fig. 36, the combined action of the radial and tangential forces produces very little change in the position of the perigee.

It will be found that precisely similar reasoning applies to the position of the lunar orbit illustrated in fig. 37.

The perigeal advance attains its greatest rate when the lunar orbit has its major axis directed towards the sun (that is either as in fig. 34, or with the apogee nearest to the sun). The perigeal regression attains its greatest rate when the major axis is at right angles to the line from the sun (that is when the lunar orbit is either as in fig. 35, or with the major axis directly reversed). As the orbit changes from the former position to the latter (through the effect of the earth's motion round the sun), the advance of the perigee per lunar month gradually diminishes until it vanishes, and then changes into regression, which continually increases until it attains its maximum value. Thence as the orbit changes to the former position again, the regression gradually diminishes until it vanishes, and then changes into advance, which continually increases until it attains its maximum value. The maximum rate of perigeal advance is about 11° in a lunar revolution, the maximum rate of perigeal regression is about 9° in a lunar revolution. It is easy to see why the former exceeds the latter, for in the case illustrated by fig. 34, the advance of the perigee is due to the excess of a force which at its maximum is represented by a A′, over a force which at its maximum is represented by p A; while in the case illustrated by fig. 35, the regression of the perigee is due to the excess of a force which at its maximum is represented by the line E a over a force which at its maximum is represented by the line E p. Clearly the former excess must be double the latter, just as the former forces are double the latter. It is easy also to see that the moon's orbit in passing from the position where maximum advance prevails, to the position where maximum regression prevails, will be longer in a position involving advance than in a position involving regression,—simply because the degree of advance which is reduced to zero by such change of position, is greater than the degree of regression which is afterwards acquired. So that not only is the absolute maximum of perigeal advance greater than the maximum rate of perigeal regression, but advance continues during a longer period than regression.

Another circumstance causes the advance to be greater than it would otherwise be. The balance of advance really depends on the excess of the disturbing action at and near apogee over the action at and near perigee. It will therefore be increased by any cause tending to increase the time during which the apogeal action takes place. Such a cause is to be found in the fact that the moon's motion round the earth does not exceed the sun's apparent motion round the earth, so greatly, when the moon is near apogee as when she is near perigee. Thus when near apogee the moon lingers longer than she otherwise would under those disturbing influences, which (on the whole) cause the advance of the perigee. We must not confuse this circumstance with what has been already mentioned respecting the effects of the moon's slower motion in apogee; for when those effects were considered, the apparent motion of the sun was not taken into account. The sun's motion may be regarded as increasing the disproportion between the moon's motion in apogee and perigee. Thus let us represent the moon's angular motion round the earth by 14 when she is in perigee, and by 10 when she is in apogee; and the sun's apparent angular motion in the same direction by 1. Then the apparent motion of the moon from the sun will clearly be represented by 13 when she is in perigee, and by 9 when she is in apogee. Thus the ratio of 14 to 10 is changed to the ratio of 13 to 9, which is larger than the former in the proportion of 65 to 63; and in about this proportion the perigeal advance is increased, owing to this cause.

But another circumstance, the consideration of which will lead us to the recognition of the indirect effect of the tangential force already alluded to, is much more important in its effects.

When the perigee is advancing, it is moving in the same direction as the sun around the earth; thus its angular displacement from the sun is due to the difference of these two advances. Since the sun's mean advance during a lunar revolution is about 27°, and the maximum advance of the perigee in the same time is about 11°, the displacement of the perigee from the sun may amount to so little as 16° in a revolution. This happens when the perigee is advancing most rapidly, and tends to keep the perigee longer near that position with respect to the sun which is favourable to perigeal advance. Now when the perigee is regreding most rapidly, or at the rate of about 9° in a revolution, the displacement of the perigee from the sun is due to the sum of its regression and the sun's advance. It amounts therefore to 36° per revolution. Thus the perigee does not remain long in that position with respect to the sun which is favourable to perigeal regression. Hence the balance of perigeal advance is importantly increased. This reasoning is strengthened by the consideration that the rate of perigeal advance deduced from the maximum advance *per month*, is a less rate considerably than the rate deduced from the advance per hour, (say) when the apogee is advancing most rapidly (even in a month when *on the whole* the perigee regredes); and a similar consideration applies to the perigeal regression, (even in months, when, on the whole, the perigee advances).

But it will be obvious that any cause which tends to encourage either the lingering of the lunar orbit in positions favourable to perigeal advance, or the rapidity with which it shifts from positions favourable to perigeal regression, must tend to increase the effect here considered, even though it might exercise no direct influence on the motion of the perigee. Now we have seen that when the moon's orbit is so placed that the radial force causes the perigee either to advance or recede, the tangential force causes the perigee to move in the same way, thus reinforcing the effects due to the radial disturbing action. These effects due to the tangential action do indeed counterpoise each other, so far as they are directly concerned; in other words, their direct effect is *nil* in the long run. But insomuch as they reinforce those effects which (as we have seen) cause the lunar orbit to linger in positions favourable to perigeal advance, and to shift quickly from positions favourable to perigeal regression, they indirectly reinforce the perigeal advance. Surprising though it may seem, these indirect results of the tangential action,—these perturbations of perturbations as they may be called,—actually exert so important an influence as to double the mean rate of perigeal advance. Newton either overlooked this indirect action, or rather fell into the mistake of supposing it might safely be neglected. Accordingly the only striking feature of the lunar perturbations which he was unable to explain in full, was the advance of the perigee. He could account but for about one-half of the advance. Clairaut, who first applied analytical investigations (as distinguished from Newton's geometrical method) to this question, deduced a result agreeing very closely with Newton's, having fallen into the same mistake. For some time it seemed as though the theory of gravitation were endangered, Clairaut himself suggesting that a force acting according to some other law than that of the inverse squares of the distances, seemed to be in operation. This opinion of a mathematician on a strictly mathematical question was energetically opposed by the non-mathematician Buffon, who argued in favour of the simple Newtonian law. Clairaut was thus led to re-examine the subject, taking into account considerations which he had hitherto neglected, and which he did not expect to find importantly influencing the result. To his surprise he found in the hitherto neglected indirect effects of the tangential action, the explanation of the difficulty which had so long perplexed mathematicians. [See, however, note on p. 137.]

The actual motion of the perigee from conjunction to conjunction with the sun is indicated in fig. 42, Plate XI., except that no account is taken of the oscillations which occur *within* the periods of successive lunations. The circle e_1 e_2 e_3, &c., is the orbit of the earth, while the lines $p_1 e_1 a_1$, $p_2 e_2 a_2$, $p_3 e_3 a_3$, indicate successive positions of the major axis of the moon's orbit, p_1, p_2, p_3, &c., being the perigee. When the earth is at e_1, the perigee is at p_1, or in conjunction with the sun. Here the position corresponds to that illustrated in fig. 34, the advance is rapid, and accordingly we see that at the next station the perigee p_2 is no longer parallel to the line e_1 S, but has shifted in a direction agreeing with that indicated by the arrows on the moon's orbit,—that is, in the direction of the moon's *advance*. But when the earth has got to e_3 (somewhat more than a quarter of a revolution) the position of the major axis corresponds to that illustrated in fig. 35, and the recession is rapid: hence in passing to this position from e_2, and away from this position to e_4, there is regression of the perigee, insomuch that the line $p_4\,e_4\,a_4$ is shifted *back* even beyond parallelism with $p_1\,e_1\,a_1$. In the next two stages, however, there is corresponding advance; for at e_5 the earth is so placed that the major axis is directed towards the sun: and we see that p_6 is very much *advanced* round the point e_6. In the next two stages there is regression, so that p_8, though somewhat advanced as compared with p_1, is not so much advanced as p_6. Lastly, as the earth passes to the position e_9, there is advance. Matters are now as at first; and as the earth circles again round the sun, corresponding changes occur in the position of the perigee. The order of such changes is indicated in fig. 42 *a*, where E 1 represents the position of the perigee when at p_1: we see how it advances to 2 (corresponding to p_2), recedes to 3 and 4, even behind 1; then advances to 5 and 6, recedes to 7 and 8, and lastly advances to 9. If we had begun with the position $p_3\,e_3\,a_3$, we should have had, in a complete circuit, the oscillatory progression indicated in fig. 42 *b*. It will be observed that the two figures agree perfectly as respects the nature of the loops. Also the total angles of advance (1 E 9 in fig. 42 *a*, and 3 E 11 in 42 *b*) are equal. Another circuit would take the perigee to the position E 17 in one case, and E 19 in the other. The arc of the advance between successive conjunctions of the perigee with the sun, that is the arc corresponding to $e_1\,e_9$, fig. 42, is about 45° 51⅓′, the mean interval between such conjunctions being 411·767 days. The perigee performs a complete circuit in a mean interval of 3232·575 days.

Let it be particularly noticed that the point e_1 is no definite point of the earth's orbit, but is taken to represent the earth's position at any epoch when the perigee is in conjunction with the sun. Again, it is to be noticed that the advance of the perigee does not take place in reality after the comparatively simple manner indicated in figs. 42 *a* and 42 *b*, since in each lunation there are two periods of advance and two of regression; whereas these figures take into account only the *balance* of advance or regression. Moreover the periods of advance and regression are of various lengths in different lunations.

Fig. 42 aptly illustrates the circumstances mentioned with respect to the lingering of the perigee in positions favourable to advance. Let it be held so that S is on the right and $p_1\,e_1\,a_1$ on the left (or upside down), then the perigee is in the position favourable to advance, or as in fig. 34. Now let it be held with S on the right and $p_2\,e_2\,a_2$ on the left. Then, since the earth has moved halfway of the way towards the position e_3, where the major axis is favourably placed for regression, we should expect to have $p_2\,e_2\,a_2$ inclined halfway between the positions favourable for advance or regression, or as in fig. 36: but instead of this we find p_2 much nearer to the line joining S and e_2. And so with the positions e_4, e_6, and e_8.

We have seen that the eccentricity of the lunar orbit, though affected during any given revolution, as well by the radial as by the tangential disturbing forces, is yet not subject to permanent alteration. The range within which the eccentricity varies is, however, one of the most important of all the features of the lunar theory. I do not propose here to enter on the consideration of the circumstances producing these oscillations in the value of the eccentricity, though the materials for the inquiry are contained in the considerations illustrated by the four figures 30, 31, 32, 33, and those of Plate IX. The student will find no difficulty whatever in satisfying himself that when the axis is placed as in fig. 34 or 35,—or in either of these positions, but with perigee and apogee interchanged,—the eccentricity is not appreciably affected in a complete lunation. When the axis is placed as in fig. 36 (or exactly reversed), the eccentricity is on the whole diminished in a complete lunation; and when the axis is placed as in fig. 37 (or exactly reversed), the eccentricity is on the whole increased in a complete lunation. Thus it is easily seen that as the earth passes from the position e_1 to the position e_3 (fig. 42), the eccentricity passes from a maximum to a minimum value; at e_5 it is again at a maximum; at e_7 again at a minimum; and, lastly, at e_9 it is at a maximum as at first. The range of the eccentricity is so considerable as to exceed $\frac{2}{5}$ths of the mean value of the eccentricity. So that representing this mean value as 5, the maximum is about one-fifth greater, or 6; and the minimum about one-fifth less, or 4. Thus the greatest eccentricity exceeds the least in the proportion of 3 to 2. Since the mean eccentricity of the lunar orbit is 0·054908, the greatest and least values of the eccentricity are respectively about 0·066 and about 0·044.

The irregularity of the perigeal motion and the variation of the eccentricity are oscillatory disturbances; and their combined influence on the actual position of the moon in her orbit is therefore also oscillatory in its effects. It will be easily inferred that the moon's position is importantly modified at times by these causes, especially by the variation in the eccentricity, since the eccentricity causes the moon's motion in longitude to be unequal, and it is so much the more unequal as the eccentricity is greater. The moon, owing to these causes, may be in advance of, or behind, the place she would have if these perturbations had no existence, by no less than 1° 18′. This perturbation is called the *evection*, and is the only lunar perturbation which the ancient astronomers discovered. The discovery is commonly attributed to Ptolemy, though there are reasons for believing that it was actually made by Hipparchus.

Hitherto we have supposed the lunar orbit to lie in the plane of the ecliptic, since we have regarded the three lines joining the sun and moon, the sun and earth, and the earth and moon, as all lying in the plane of the moon's orbit. We know, however, that the moon's orbit is slightly inclined to the plane of the ecliptic; and although the inclination does not importantly affect the value of the radial and tangential forces, it produces a very important and interesting effect on the position of the lunar orbit. This effect we shall now proceed to examine.

In the first place, let us take the general case of a body moving on a path inclined to any plane. Let N M N′, fig. 43, be part of the path of the body about the centre E, and let N *m* N′ be the plane to which the motion is referred, so that N E N′ is the *line of nodes*, and the angle P N *m* the *inclination* of the path. Then if, when at P, or passing from a node to its greatest distance from the plane of reference, the body is disturbed by a force acting *towards* that plane, it will proceed to move as along P *k*,—the prolongation of this new path (backwards) setting the node as at *n*, or behind N, while the new inclination, or P *n m*, is obviously less than the former inclination P N *m*. It is equally clear that if the body is at Q when it is deflected towards the plane of reference, the new path placed as Q *n*′, has its node *n*′ behind the former position N′, but the inclination Q *n*′ *m* is greater than the former inclination Q N′ *m*.

Similarly if the disturbing force acts *from* the plane of reference and the body is at P, or anywhere on the arc N M (fig. 44), the node advances as to *n*, and the inclination increases; while if the body is at Q, or anywhere on the arc M N′, the node advances as to *n′* and the inclination diminishes.

It will be observed that as the motion of the node is here referred to the direction of the body's motion, the result applies equally whether N be the ascending or the descending node.

We have, then, these general rules:—Force *towards* the plane of reference,—inclination diminishes while the body's distance from plane of reference is increasing, and *vice versâ;* but nodal line regredes throughout. Force *from* the plane of reference,—inclination increases while the body's distance from the plane of reference is increasing, and *vice versâ;* but nodal line advances throughout.

Now let us apply these results to the moon's motion round the earth, the plane of reference in this case being the ecliptic.

Let us suppose, first, that the line of the moon's nodes is placed (and remains during a complete revolution of the moon) as is shown in fig. 38, Plate X., N being the ascending node, and N′ the descending node.* Then the line A′A, to points in which all the perturbing forces act, lies in the plane of the moon's orbit, being coincident with N N′ in direction and situation. It

* The small lines surmounted by arrow-heads in this and succeeding figures are intended to indicate the amount of the distance of the corresponding points of the lunar orbit, above or below the plane of the ecliptic,—this last plane being supposed to be represented by the plane of the paper. They are drawn to scale.

Fig. 38.

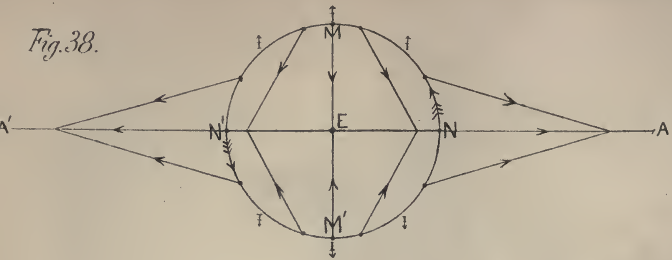


Fig. 39.

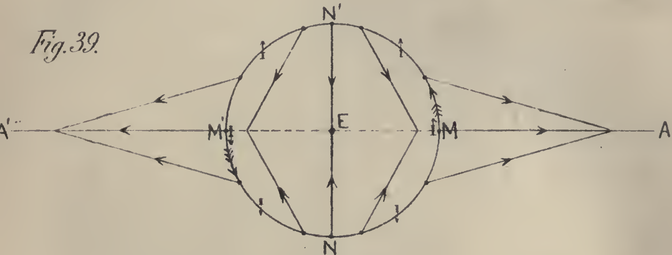


Fig. 40.

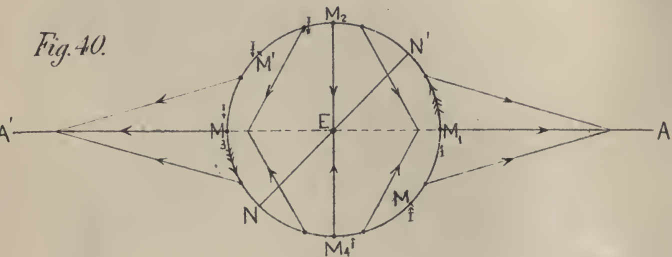
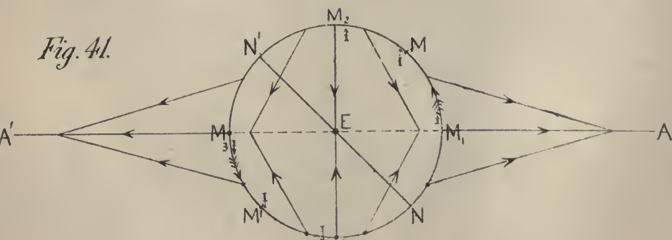


Fig. 41.



Illustrating the Motion of the Nodes of the Moon's Orbit.

R. A. Proctor del^t

is obvious, then, that in this configuration these forces can exercise no effect whatever in shifting the moon from the plane in which she is travelling at the time; for each line of force lies in that plane.* The same will hold, of course, if the line of nodes has the same position but with the ascending and descending nodes interchanged.

Next, let us suppose that the line of nodes remains during a complete revolution in the position shown in fig. 39, N being, as before, the rising node, and N′ the descending node. In this case E A lies below the plane of the moon's orbit, while E A′ lies above that plane.

Since the moon in passing over the half-orbit N M N′ is above the plane of the ecliptic, while the disturbing forces draw the moon towards points in the line E A below the plane of the lunar orbit, it is clear that throughout this part of the orbit the moon is drawn *towards* the plane of the ecliptic. In like manner, since in passing over the half-orbit N′ M′ N the moon is below the plane of the ecliptic, while the forces draw her to points in the line E A′ above the plane of the lunar orbit, it is clear that throughout

* It is clear that, instead of resolving the force represented by such lines as M H, M′ H′ (figs. 20, 21, Plate VI.) into the tangential and radial forces only, we must begin (when we take the inclination of the moon's orbit into account) by dropping a perpendicular from H, &c., upon the plane of the moon's orbit, and taking this perpendicular to represent the force drawing the moon from her plane of motion, we must take a line from its foot to M M to represent the resultant of the tangential and radial forces.

this part of the orbit the moon is also drawn *towards* the plane of the ecliptic. Throughout the whole of her revolution, then, under the imagined condition, the moon is drawn towards the plane.

Thus the line of nodes regredes throughout. The inclination diminishes while the moon is moving from N to M, increases as she moves to N′, decreases as she moves to M′, and increases as she moves to N. It is therefore, on the whole, very little affected during the complete revolution.

A like result obviously follows if the line of nodes is situated as in fig. 39, but the places of the nodes interchanged.

Thirdly, let the line of nodes be supposed to remain throughout a complete revolution of the moon in a position intermediate to those just considered, as at N N′ (figs. 40, 41). First let the moon pass a node in moving from M_1 to M_2, fig. 40. Then it will readily be seen (from considerations precisely like those in the preceding cases) that over the arcs M_4 M_1 N′ and M_2 M_3 N the moon is drawn towards the plane of the ecliptic, whereas over the two shorter arcs, N′ M_2 and N M_4, she is drawn from that plane. Hence, on the whole, there is a balance of nodal regression in the complete revolution. I do not trace out the change of inclination, for the same reasons that I did not trace out the change of eccentricity,—viz., first, to avoid prolixity, and secondly, because the change is an oscillatory one, producing no permanent effects. It will be found, however, that in the case indicated in fig. 40, the inclination is on the whole undergoing increase.*

Lastly, let the moon pass a node in moving from M_4 to M_1, fig. 41. Then the node is regreding as the moon moves from N to M_2, and from N′ to M_4, and advancing throughout the rest of the moon's motion; hence, on the whole, the line of nodes regredes. The inclination decreases on the whole during a complete revolution.†

It is obvious that in all the four cases here considered, matters will not be altered if the nodes be interchanged.‡

Thus we see that in all positions of the lunar orbit, except at the moment when the line of nodes is directed towards the sun, the nodal line regredes on the whole during each lunation, the regression being obviously most rapid when the line of nodes is at right angles to the line joining the earth and sun (or in the position shown in fig. 39).

If the line of nodes remained fixed, it would be carried round the sun once in the year. But

* It is only decreasing from M_2 to M′, and from M_4 to M; increasing everywhere else. The student will readily see this.

† The inclination will be found to decrease everywhere except between M and M_2, and between M′ and M_4.

‡ Excepting, of course, that there will be a slight change due to the fact that the sun's distance does not indefinitely exceed the moon's; in other words, the perturbing forces on the side $M_4 M_1 M_2$ are slightly greater than those on the side $M_2 M_3 M_4$. A similar reservation applies to the corresponding cases in which the perigee and apogee were interchanged.

under the actual circumstances the line of nodes is carried round in the manner represented in fig. 45, Plate XI. As the earth moves from e_1, where the line of nodes $n_1 e_1 n'_1$ is directed towards the sun, the nodes regrede, slowly at first, and with alternations of advance, but more rapidly, and with shorter periods of advance, until the earth is in position e_3, not quite one-fourth of a revolution from e_1. At this time the line of nodes is square to the line from the sun. As the earth passes on to e_5 the line of nodes still regredes on the whole, but with longer and longer alternations of advance, until when the earth is at e_5 the line of nodes has the position $n_5 e_5 n'_5$, or is again directed towards the sun. The regression recommences, and is continued with increasing effect to the position e_7, and thence with diminishing effect to the position e_9.* The actual nature of the nodal regression, as the earth passes through the nine stages, e_1, e_2, e_3, &c., is indicated in the figure 45*a*, except that no account is here taken of the intermittence of the regression. Instead of a complete year being occupied by the moon's return in this way to a position where the rising node is again in conjunction with the sun as at first only 346·607 days are so occu-

* The inclination decreases as the earth moves from e_1 to e_3, thence increases as the earth moves to e_5, decreases as the earth moves to e_7, and finally increases as the earth moves to e_9.

It is to be particularly noticed that e_1 is not a fixed point on the earth's orbit, as the vernal equinox or the like. It is simply the point where, at the particular time illustrated, the nodal line $n n'$ is directed towards the sun.

Fig 42. Illustrating the Advance of the Moon's Perigee.

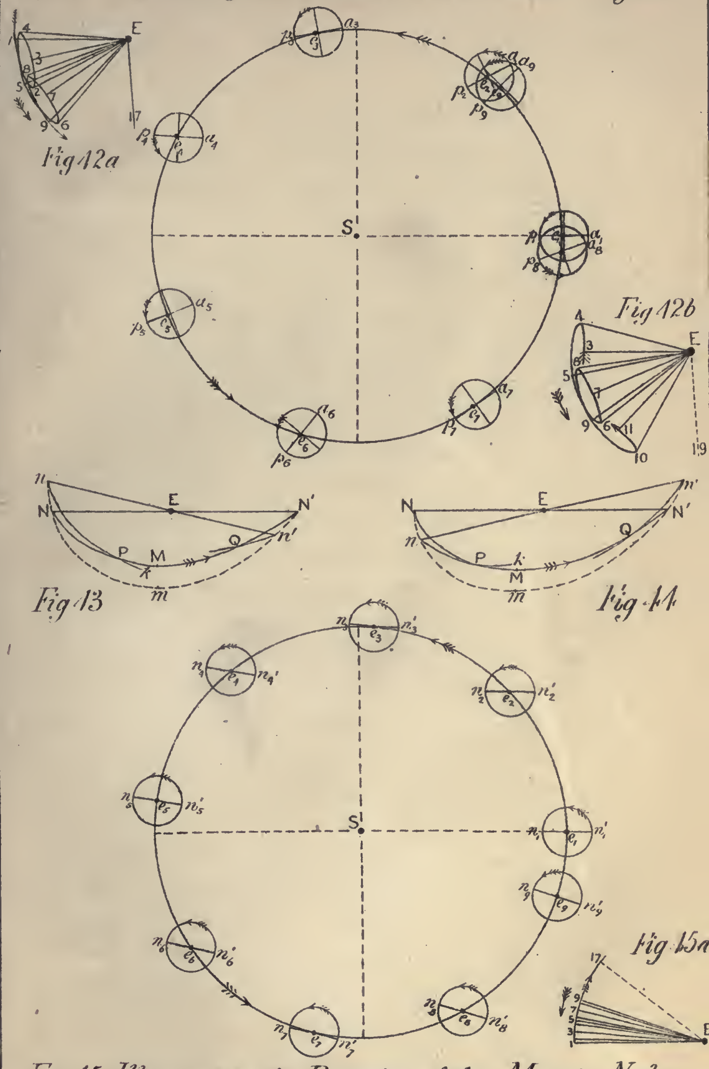


Fig 45. Illustrating the Recession of the Moon's Nodes.

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the moon's line of nodes regredes, but that the regression is most rapid when the plane of the lunar orbit is most inclined to the line from the sun.

The plane's inclination to the ecliptic changes in an oscillatory manner in a mean period equal to one-half the interval of 346·6 days, between successive returns of the line of nodes to such a position as is indicated in fig. 45 at $n\,e_1\,n'$ and $n\,e_5\,n'$, or in 173·3 days. The variation amounts to about 8′, by which the inclination is alternately greater and less than the mean value,—rather less than 5° 9′.

Such are the chief perturbations to which the moon is subject. Others of lesser importance need not here occupy our attention, because their discussion would introduce no new principles to our notice, at least none which could be discussed in such a work as the present.

This chapter cannot properly be drawn to a conclusion, however, without dwelling on the singular interest of the history of the researches made by astronomers into the subject of the lunar motions. The whole progress of the inquiry has been attended by difficulties only to be mastered by the most wonderful exercise of skill and patience. It was only the unique combination of powers possessed by Newton that permitted the problem to be grappled with in the first instance; and even Newton would have failed but for certain fortunate circumstances by which he was assisted. Since his day the problem has been dealt with by the most acute mathematicians, by the most skilful observers. Mathematical analysis has been carried to an unhoped-for degree of perfection to account for peculiarities of lunar motion revealed by observation. Observation has been pushed to the utmost point of delicacy to detect peculiarities of lunar motion predicted by mathematical analysis. The history of the contest is adorned by the names of nearly all the leading observers and mathematicians of the last century and a half;—Laplace and Lagrange; Euler, Clairaut, and d'Alembert; Airy, Leverrier, Adams, and Cayley; Hansen, Delaunay, Peirce, and Newcomb; a host, in fine, of names so distinguished, that it becomes almost invidious to particularize any among them. In the whole history of the researches by which men have endeavoured to master the secrets of nature, no chapter is more encouraging than that which relates to the interpretation of the lunar motions.

Note.—Since this chapter was in type, I have found that Prof. Grant, in an appendix to his "History of Physical Astronomy," records how Newton, in the original edition of the "Principia" (1687), gives very satisfactory values of the progression of the perigee in sizygy and its regression in quadrature. Thus he found 11° 21′ for the monthly progression, and 8° 1′ for the monthly regression; and a mean annual advance of 40°. Modern tables assign 11°, 9°, and 40° 40′ 32″ for these quantities respectively. Newton refrained from publishing the details of his researches, but as Prof. Grant remarks, whatever Newton's method may have been, "it was manifestly one which was capable of grappling with the main difficulties of the question."

CHAPTER III.

THE MOON'S CHANGES OF ASPECT, ROTATION, LIBRATION, ETC.

The moon's motions in the heavens, as seen from the earth, are readily understood from what is known of her actual motions. I propose now to enter into a general consideration of these apparent motions of the moon, and of the varying aspect which she accordingly presents to us. It would be possible to fill a much larger volume than the present with the detailed discussion of these matters; nor would such a volume be wanting in interest, at least to those having mathematical tastes. I do not indeed know of any subject which a geometrician could better wish to examine. It is full of neat and interesting problems, and might worthily occupy many years of labour. But in this volume such researches would be out of place. We must be content with such a consideration of the subject as shall leave none of its salient features unexplained. In passing it may be remarked that even such a treatment of the moon's apparent motions has long been a desideratum, inasmuch as our text-books of astronomy have hitherto left these matters almost untouched.

In the first place, then, it is to be noticed that the moon completes the circuit of the heavens on the average in 27·322 days, that is in $27^{d.}\ 7^{h.}\ 43{\cdot}7^{m.}$ If we watched her motion from the time when she was in conjunction with any given star until the next conjunction, and the next again, and so on, for many successive conjunctions, we should find that the mean interval is that just stated. This is called the *sidereal month.*

If, however, instead of taking a star, we took the point on the heavens where the ecliptic crosses to the north of the equator, we should not find the interval exactly the same as the sidereal month; because this point on the heavens is constantly, though slowly, moving backwards, or so as to *meet* the moon's motion. This point—called, as all know, "the first point of Aries"—makes the complete circuit of the heavens in 25,868 years; and therefore in a sidereal month travels over a very minute arc indeed, less in fact than 4″. So that the difference between this new kind of month, called the *tropical month,* and the sidereal month, is very minute. The mean tropical month is necessarily slightly less than the sidereal. The latter is, with great exactness, 27·32166 days, the tropical month is 27·32156 days, or about 6⅓ seconds shorter.

Now let us in the first instance consider this motion as though it took place in the ecliptic, and uniformly, so that in fact we are supposing the moon to move apparently in the same course among the stars as the sun, only that instead of taking about $365\frac{1}{4}$ days in completing the circuit she takes about $27\frac{1}{3}$ days.

Let E E′, fig. 46, Plate XII., represent a part of the earth's path round the sun S, and let M_1 M_2 M_3 M_4 be the path of the moon, and suppose that the moon is at M_1 when the earth is at E. Then it is the time of "new moon;" the moon lies towards the sun's place, and if she could be seen, would be at the same part of the ecliptic, or in conjunction with the same star *s*. Let E E′ be the arc traversed by the earth in 27·322 days, or in a sidereal month. Then the moon has gone once round, and is in conjunction with the same star,—in other words, the line $E' m_1 s'$ directed towards the moon, is in the same direction as E *s*,—that is, $E' m_1 s'$ is parallel to E *s*. But the moon has not come up to the line $E' M_1' S$, joining the sun and earth. Some time has still to elapse, therefore, before it is again new moon. In like manner, if the moon had been at M_2 when the earth was at E, it was the time of "first quarter,"—she would be at m_2 when the earth is at E′,—in other words, she would not yet have reached M_2', the place of "first quarter." And similarly if it had been "full moon," "third quarter," or any other lunar epoch, when the earth was at E, the corresponding epoch would not have arrived, when a sidereal month had elapsed.

We see then that the *lunation*, or the time in which the moon goes through her phases, is longer than the

Fig. 46.

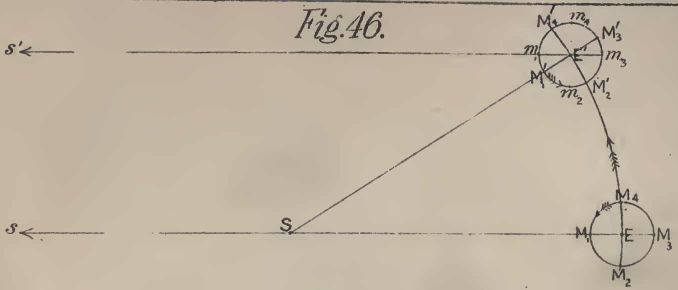


Fig. 47.

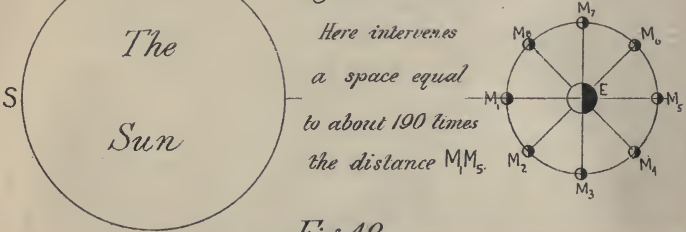


Fig. 48.

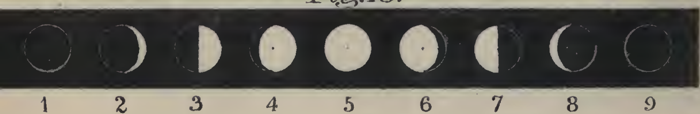


Fig. 49.

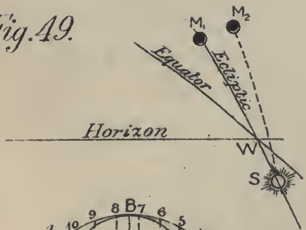
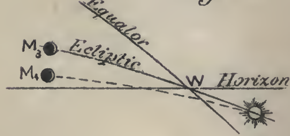


Fig. 50.



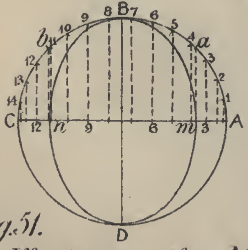


Fig. 51.

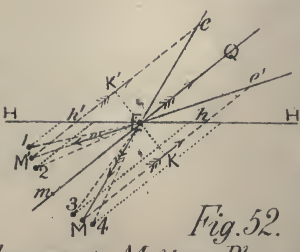


Fig. 52.

Illustrating the Moon's Apparent Motions, Phases, &c.

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sidereal or than the tropical month. And it is very easy to calculate the exact length of the lunation, or, as it is called, the *synodical month.* In 27·322 days, the moon has not completed the whole cycle of her phases, but only the portion $M_1' M_3' m_1$ out of the whole cycle,—that is, she has completed the whole cycle, less the portion $m_1 M_1'$. Now, the angle $m_1 E' M_1'$ is obviously the same as the angle E′ S E; hence the part wanting from the complete cycle bears to the whole cycle the same ratio that E E′ bears to the complete orbit of the earth, or that 27·322 days bears to 365·242 days. The moon, then, in 27·322 days, completes only $\frac{337 \cdot 920}{365 \cdot 242}$ths of a lunation (the numerator being obtained by taking 27·322 from 365·242). So that a mean synodical month exceeds a mean sidereal month (or 27·322 days) in the same proportion that 365·242 exceeds 337·920. Increasing 27·322 in this proportion (a mere rule-of-three sum), we obtain 29·531,* which is the length of a lunation.

The phases of the moon are explained in text-books of astronomy. But a few remarks on the subject may be useful.

Let $M_1 M_2 M_3$ . . . . fig. 47, Plate XII., represent the moon's orbit, the sun being at S, only many times farther away than in the figure. The earth and moon are relatively much exaggerated in dimensions; and the moon is shown in eight equidistant positions, as though she performed a complete circuit, while the

* More exact values are given in the tables.

earth remained at E. Now obviously, when the moon is at M_1, her darkened side is turned towards the earth, and she cannot be seen. She is as at 1, fig. 48. As she advances towards M_2, the observer on the earth E, and supposed to be standing on the half of the earth shown in the figure, sees the moon on the left of the sun,—that is, towards the east,* and he would clearly see the right or western side of the moon partly illuminated. The case, so far as this illumination is concerned, is exactly the same as though the moon at M_1 had turned an eighth round on an axis upright to the plane of her motion, in such a way as to bring into view the parts beyond her eastern edge. Thus, the aspect of the moon is as shown at 2, fig. 48. It is readily seen that when she is at M_3, fig. 47, her aspect is as at 3, fig. 48; and so on.

All this is as explained in the text-books. But there are two points, even in this elementary matter, which may need a word or two of explanation.

First, as to the position of the lunar crescent. We see the moon in varying positions on the sky; and at first sight there appears to be no definite relation between her position and the position of her cusps or horns. Indeed, this feature of her aspect has seemed so changeful and capricious that it has even been regarded as a weather-token. In reality, however, there is a simple relation always fulfilled by the moon's cusps. The line joining them is always at right angles to the

* The reader should here hold the plate so as to have E towards him, and S and M_2 from him.

great circle passing through the sun and moon.* As the moon is always near the ecliptic, this amounts to saying that the line joining the cusps is always nearly at right angles to the ecliptic. It follows, of course, that as the angle at which the ecliptic is inclined to the horizon is variable, so the position of the line joining the cusps varies with respect to the horizon. As respects the gibbous moon (or moon more than half-full), these variations are not much noticed; but in the case of the crescent moon, generally observed rather near the horizon, they are very noteworthy. For instance, let the time of year be such that the part of the ecliptic near the western horizon, soon after sunset, is inclined at nearly the greatest possible angle to the horizon,—that is, let the season somewhat precede the vernal equinox,—the time, as we know, when the zodiacal light is most conspicuous in the evening. Then in our latitudes, the inclination of the ecliptic to the horizon is about sixty-two degrees, and supposing the moon on the ecliptic, and *young*, as shown at M_1, in fig. 49, Plate XII., the line

* This will perhaps seem obvious to most readers. The proof of the proposition is comprehended in the following considerations:—The circle bounding the illuminated half of the moon necessarily has its plane at right angles to the line joining the centres of the sun and moon; the circle bounding the moon's visible hemisphere necessarily has its plane at right angles to the line joining the centres of the earth and moon: thus the intersection of these circles or the lunar cusps, must lie on a line at right angles to the plane containing the three centres,—that is, to the plane of the great circle through the sun and moon.

joining the cusps will only be inclined about twenty-eight degrees to the horizon. But next, suppose that the moon at this time is at her greatest distance north of the ecliptic, or at M_2, five degrees from the position M_1, and about the same distance as in the former case from the sun. Then the great arc-circle $S\,M_2$ from the sun to the moon is inclined ten or more degrees (according to the moon's age) to $S\,M_1$, and the line joining the cusps is, in this case, inclined less than 18° to the horizon. Indeed, when the moon is *very* young, the angle $M_1\,S\,M_2$ is considerable. Hence $S\,M_2$ makes a considerably larger angle with the horizon than $S\,M_1$, and the line joining the cusps is, as shown in the figure, much more nearly horizontal. A very young moon seen soon after sunset, under these circumstances, may have the line joining its cusps quite horizontal, or even have the northern cusp lower than the southern.* Like considerations apply to the case of the old crescent moon,† before sunrise, soon after the autumnal equinox.

Next, however, suppose the western part of the

* It is hardly necessary to say that the exact angles for any position can be quite readily calculated; but the matter is not of such a nature as to require the introduction of such calculations here. The student acquainted with the elements of spherical trigonometry may find interesting and not uninstructive occupation for a leisure hour or so in considering a few cases. The angle $M_2\,S\,M_1$ is more than 10° when the moon is less than one-eighth full, or halfway to the first quarter.

† The word crescent here means merely crescent-shaped, not crescent in the sense of increasing.

ecliptic, at its least inclination to the horizon, soon after sunset, or the time of year shortly before the autumnal equinox. The state of things is that illustrated in fig. 50, Plate XII. Then in our latitudes the inclination of the ecliptic to the horizon is about 15 degrees, and supposing the moon on the ecliptic and *young*, as at M_3, the line joining the cusps will be inclined about 75 degrees to the horizon. But suppose the moon, as at M_4, at her greatest distance south of the ecliptic, or five degrees from the position M_3, and about the same distance from the sun, then the great circle $S\,M_4$ from the sun to the moon is inclined more than ten degrees to $S\,M_3$; and the line joining the cusps may be much more nearly upright than when the moon is as at M_3. But this line cannot be actually upright when the sun is below the horizon, for the line must always be square to the great circle through the sun and moon, and, of course, when the moon is above and the sun below the horizon, this great circle is inclined to the horizon, and a line perpendicular to it is correspondingly inclined from the vertical. Similar considerations apply to the case of the old crescent moon before sunrise, soon after the vernal equinox.

We see, from these extreme cases, that the line joining the moon's cusps can have every inclination, from being nearly vertical to a horizontal position, and even that the northern cusp may be below the southern, according to the season of the year and the moon's position in her orbit. So that, to assert that there will be such and such weather when the line joining the cusps is seen (for instance) nearly horizontal, the moon being new, is the same as asserting that there must be such and such weather at the time of new moon in February and March, if the moon is then nearly at her maximum distance from the ecliptic. And so with all such cases. If there were any value at all in such predictions, they would imply the strictly cyclic return of such and such weather.

Secondly, as to the rate at which the moon changes in shape.

Let us suppose that A B C D, fig. 51, Plate XII., represents the moon's disc (dark in the first instance), and that when the illumination begins on the right, B D is the line joining the cusps. Now, from what has just been shown, it is seen that the position of B D must vary during the progress of the lunar month, unless we suppose the moon to be moving in the ecliptic. As, however, we may wish to know the rate at which the moon *fills*, we may make this assumption for convenience. Now, the variation of phase obviously corresponds exactly to the supposition that the semicircle B A D, which separates on the right the dark from the illuminated hemispheres, rotates round the axis B D, the point A travelling apparently straight across to C, but in reality, of course, traversing a semicircle, which is seen projected into the straight line A C. Now, to find what point of A C will have been reached by the advancing boundary of the illuminated hemisphere, we have only to imagine a point traversing the semicircle A B C uniformly in $14\frac{3}{4}$ days. From whatever position as *a*, this moving point would have reached in so many days, we must let fall a perpendicular *a m* on A C. Then *m* will obviously be the position of the advancing edge at the time in question; for A *m* is obviously the projected view of an arc exactly equal to A *a*. Hence the semi-ellipse B *m* D indicates the concave outline of the illuminated portion at this epoch. Thus, in the figure, A *a* is one-fourth of the semicircle A B C, and, therefore, A B *m* D is the shape of the moon's crescent when she is an eighth of a lunation old, or nearly $3\frac{7}{10}$ days old. In like manner, if *b* be midway between B and C, *b n* perpendicular to A C gives us B *n* D, the elliptical outline of the gibbous moon, at the time when she is gibbous, midway between first quarter and full; and A B *n* D is the phase of the moon at this time, when she is about $11\frac{3}{10}$ days old. It is readily seen that B C D *m* is the figure of the gibbous moon at a time midway between "full" and third quarter; while, lastly, B C D *n* is the figure of the waning moon at a time midway between third quarter and new.

Now, as the lunar month contains about $29\frac{1}{2}$ days, if we divide A C into $14\frac{3}{4}$ equal parts, as shown by the numbered division-lines, we obtain, by letting fall perpendiculars, the daily progress of the advancing rim of light from new to full, as shown by the numbered division-marks on C A. We have only to invert the figure to have the daily progress of the receding rim of light from "full" to "new." Or we may construct such a figure on a larger scale, and divide the semicircle A B C into 59 equal parts, then the fact of perpendiculars let fall from the division-points upon A C will correspond very nearly indeed to the progression and retreat of the advancing illuminated rim from "new" to "full," and thence to "new" again, at six-hourly intervals.

Let us next consider the actual motions of the moon in the heavens at different times. We shall have, in so doing, to take into account the inclination of the moon's path to the ecliptic, as well as the eccentricity of the lunar orbit.

So long as we regard the moon as moving in the ecliptic, we can at once determine the nature of the moon's movements during any month of the year, by considering where the sun is placed on the ecliptic during that month. Thus in March the sun crosses the equator ascendingly. Hence, at the time of new moon, the moon is near the equator, and, like the sun, is about as many hours above as below the horizon. As the moon passes to the first quarter, she traverses the ascending part of the ecliptic, and at the time of first quarter is near the place occupied by the sun at the midsummer solstice. In other words (for we cannot too directly refer these motions to the stellar heavens) the moon is near the place where the constellations Taurus and Gemini meet together. Thus the first-quarter moon in spring is a long time above the horizon, and is high when in the south, like the sun in midsummer. She passes on to full, when she is again near the equator,—or rather when she is "full" *in March* (which may be earlier than the date when she is at her first quarter) she is near the equator where the ecliptic crosses it, or in Virgo. So that the full moon in spring is about twelve hours above the horizon, and as high when due south as the sun in spring. The "third-quarter moon" in March is, in like manner, nearly in the part of the ecliptic occupied by the sun in winter, or where the ecliptic crosses the equator in Sagittarius. She is therefore but a short time above the horizon, and low down when due south, like the winter sun. And it is easily seen how at intermediate phases she occupies intermediate positions.

By similar reasoning, we find that in midsummer—(i) the new moon is in Taurus or Gemini,* and long above the horizon; (ii) the first-quarter moon is in Virgo, and about twelve hours above the horizon; (iii) the full moon in Sagittarius, and a short time above the horizon; (iv) the third-quarter moon in Pisces, and about twelve hours above the horizon. In mid-autumn,—(i) the new moon is in Virgo, and about twelve hours above the horizon; (ii) the first-quarter moon, in Scorpio or Sagittarius, and only a short time above the horizon; (iii) the full moon in Pisces, and about twelve hours above the horizon;

* The reference throughout is to the *constellations*, not to the *signs*.

(iv) the three-quarter moon in Taurus or Gemini, and a long time above the horizon. And, lastly, in mid-winter (i) the new moon is in Scorpio or Sagittarius, and only a short time above the horizon; (ii) the first-quarter moon in Pisces, and about twelve hours above the horizon; (iii) the full moon in Taurus or Gemini, and a long time above the horizon; and (iv) the third-quarter moon in Virgo, and about twelve hours above the horizon.

The student will find no difficulty whatever in extending these considerations to other months, or in applying much more exact considerations to special cases. For he will notice that what has just been stated presents only the rougher features of the matter. But nothing can be easier than to apply the first rough corrections for such an inquiry. Supposing, for example, that we wish to know generally what will be the moon's diurnal path (that is her course round the heavens during the twenty-four hours) when she is at her first quarter on the 10th of April: we know that on the 10th of April the sun is some twenty degrees past the vernal equinox, which he had crossed on or about the 20th of March; the moon at her first quarter is 90° farther forward, or some twenty degrees past the place of the summer solstice; corresponding to a position on the ecliptic, about equidistant from the two stars κ and δ Geminorum. Her course above the horizon will correspond to the sun's course about twenty-one days after the summer solstice,—that is, on or about

July 11th.* Similarly any other case can be dealt with.

Before passing from this part of our subject, we may here conveniently consider the phenomena of the *harvest moon* and of the *hunter's moon*.

If the moon moved in the equator, she would rise later night after night by a nearly constant interval; or, in other words, the actual number of hours between successive risings (or settings) would be constant. But as she moves on a path considerably inclined to the equator, this does not happen with her any more than it does with the sun; moreover, as she moves much more rapidly along the equator than the sun does, the difference is much more perceptible. If we consider two extreme cases, we shall see the reason of this. Let H H′, fig. 52, Plate XII., be a portion of the eastern horizon, E the true east point, E Q the equator; and let us suppose that when the moon rises on a certain night she is on the equator at E. She is then carried by the diurnal motion along E Q to her culmination in the south, and so to her setting place in the west. Now if her orbital motion were on the equator, she would be on the next night at the same hour at a point such as *m* on the equator (E*m* being an arc of about 12° 12′), and would be carried

* In a work now out of print, called the "Constellation Seasons," I introduced a map showing the sun's diurnal course at different dates, in such sort that his elevation and bearing at any time could be at once ascertained. Such a map serves many useful purposes besides those for which it is primarily intended.

by the diurnal motion to E, where she would rise about 50½ minutes later than on the former day (and about 13° in advance of her former place). But her actual motion is nearly on the ecliptic; and when she was at E on the first day the ecliptic must have been in one of the two positions *e* E or *e′* E. (In other words, E must be the point where the ecliptic crosses the equator, either descendingly or ascendingly.)* Now in the former case, the moon on the second night will be as at M, and will be carried by the diurnal motion to the point *h* on the horizon; in the latter she will be as at M′, and will be carried to the point *h′*; and obviously M *h* is a much longer arc than M′ *h′*. In fact, if K E K′ be part of the equinoctial colure (or circle square to the equator through the equinoctial point E), the two arcs M K and M′ K′ are obviously equal,† and we see that M *h* exceeds, while M′ *h′* falls short of the common length of those equal arcs by the very appreciable equal arcs K *h* and K′ *h′*. Thus the hour of rising in the former case will be later than in the latter, by the time corresponding to twice the diurnal arc K *h* or K′ *h′*, as well as by a not inconsiderable increment of time due to the fact that the moon is all the while moving on her orbit, and moves farther, of course, the longer she is delayed. The hour of rising will in both cases be later than the

* The direction in which we follow the ecliptic is contrary to that of the diurnal motion, because the sun's annual motion in the ecliptic is from west to east.

† They are also each very nearly equal to E *m*.

hour at which the moon rose on the preceding night (at least in our latitudes, and everywhere save in very high latitudes), but the difference will be much greater in one case than in the other.

Now these are the extreme cases: the ecliptic can never cross the horizon at a greater angle than *e* E H′, or at a less angle than *e′* E H′. Accordingly—still assuming that the moon moves in the ecliptic—we shall have the greatest possible difference between the hours of rising when the moon is on the ecliptic placed as at *e* E M, and the least possible difference when she is on the ecliptic placed as *e′* E M′; and if the moon is "full" or nearly so, when in one or other of these positions, the peculiarity will be very noteworthy. In one case, we shall have a remarkable retardation in the hours of rising on successive days, and in the other as remarkably small a difference. Now the full moon is in or near the former position in spring, for then the new moon is, with the sun, at or near the ascending node of the ecliptic, and therefore the full moon at or near the descending node. Accordingly in spring the difference between the hours at which the full moon rises on successive nights is considerable. It amounts, in fact, on the average, in our latitudes to about an hour and twenty minutes,*

* There is a table in Ferguson's Astronomy which seems to imply differently, since he gets 1 h. 16 m. as the greatest possible difference between the hours of successive rising or setting of the moon, when the inclination of her orbit to the ecliptic is taken into account; and this value has been carefully reproduced in our text-

the mean interval being only about $50\frac{1}{2}$ minutes. And the full moon is near the ascending node of the

books of astronomy. But it should be noticed that Ferguson did not compute the values in this table, but only estimated the values "as near as could be done from a common globe, on which the moon's orbit was delineated with a black-lead pencil," and he was not successful even in his application of this very rough method, by which, or by a simple method of projection, it may readily be shown that the maximum difference is greater and the minimum difference less than Ferguson supposed. If the eccentricity of the moon's orbit and her consequently variable motion be taken into account, a yet greater difference results. It is easy to obtain equations whence we can calculate the difference in the hour of rising under the circumstances in question. They are as follows, the assumption being made that the moon is crossing the equator at rising :—Let α be the inclination of the moon's path to the equator (α ranging in value between 28° 44′ and 18° 10′), l the latitude of the station. Then let h be the moon's mean hourly motion on the ecliptic (about $30\frac{1}{2}$ minutes of arc), x the time in hours between her rising on the day when she is on the equator and on the next day. Then her motion on the ecliptic is $x\,h$. Put $x\,h = \theta$. Take then

$$\sin\psi = \sin\alpha\sin\theta \qquad \text{(i)}$$

$$\text{and } \sin\phi = \tan l\tan\psi \qquad \text{(ii)}$$

Then ϕ is approximately the hour-angle by which the interval between successive risings exceeds or falls short of the mean interval (1 d. $50\frac{1}{2}$ m.). So that

$$x = 24{\cdot}84 \pm \frac{\phi}{h}; \text{ that is } \theta = 24{\cdot}84\,h \pm \phi$$
$$= 373^\circ \pm \phi \quad \text{approximately.}$$

These equations are theoretically sufficient to determine θ (or x); but practically, it is sufficient to adopt a value of $\frac{\phi}{h}$ (half an hour is near enough), giving $x = 24{\cdot}34$; $\theta = 12^\circ\ 22\frac{1}{3}'$ about. Then use (i) and (ii) to calculate ϕ, and repeat the process, using in it the value of ϕ thus deduced.

ecliptic in autumn, for then the new moon is, with the sun, at or near the descending node of the ecliptic. Accordingly, in autumn, the difference between the hours at which the full moon rises on successive nights is small. It amounts, in fact, on the average, in our latitudes to rather more than twenty minutes (or about half an hour less than the mean interval).

But the inclination of the moon's orbit and the moon's variable motion due to the eccentricity of her orbit cause these results to be considerably modified. We can at once consider this feature (proposing presently to discuss more particularly the moon's motion on her inclined eccentric orbit). Let us suppose that when at E, fig. 52, Plate XII., the moon is crossing the equator, ascendingly or towards M′, and is also at the rising node of her orbit. Then, instead of following the course E M′, she will travel along such a course as is shown by the dotted line E 1, or will be yet nearer than M′ to the horizon at the end of the twenty-four hours,—in other words, the interval between successive risings at this season will be yet more shortened than we have found it to be on the assumption that the moon moves on the ecliptic. In like manner if when at E, and crossing the equator descendingly, the moon is at her descending node (which will obviously correspond to the period when she crosses the equator ascendingly while near her ascending node) then, instead of following the course E M, she will follow the course E 4, or will be yet farther than M from the horizon at the end of the twenty-four hours,—in other words, the interval between successive risings will be yet further lengthened than we have found it to be on the assumption that the moon moved in the ecliptic. On the contrary, if the moon, when crossing the ecliptic ascendingly, is at her descending node (so following the course E 2), while when crossing the ecliptic descendingly she is at her ascending node (so following the course E 3), the intervals between successive risings and settings will be less markedly affected than on the assumption that the moon moves in the ecliptic. These are the extreme cases either way. It is readily seen, however, that the position of the moon as to the perigee and apogee of her orbit must also have an effect, since her motion from E will be greater or less according as she is nearer or farther from her perigee, and the interval between successive risings will be diminished or increased respectively.

Taking all these considerations into account, it is found that instead of the moon rising about 20 minutes later night after night for several successive days at the time of harvest moon, she at times rises only nine or ten minutes later on successive nights; while at other times, at the same season, the difference exceeds half an hour. As regards the maximum difference between the hours of rising of the full moon in spring, it varies from about an hour and ten minutes to about an hour and a half.

It is to be noticed that in every lunation corresponding variations occur, because the moon necessarily passes through Pisces and Aries, and through Virgo and Libra in each lunation. But it is only in spring that the full moon is in Libra and Virgo, and in autumn that the full moon is in Pisces and Aries. The autumn phenomena are the more important, since they result in nights almost completely moonlit for four or five days in succession. We have at, and near the time of full moon in September, the moon rising not far on either side of six in the evening, and though the hour of setting varies considerably, yet this is obviously a matter of small importance, since the moon sets in the morning hours. The operations of harvesting can thus be continued far on into the night, or all night if need be. This relates, however (at least in England), to the full moon preceding the middle of September, for harvesting operations are nearly always completed throughout England before that time. The full moon following September, which partakes to about an equal degree with that preceding the autumnal equinox, in the peculiarity we have been dealing with, is sometimes called the *hunter's moon*.

In latitudes higher than ours the phenomena of the *harvest moon* and *hunter's moon* are more marked, because the angle H E M′ (fig. 52) grows smaller and smaller as the arctic circle is approached. At the arctic circle this angle vanishes, and the moon, when moving parallel to the ecliptic, rises night after night (for a time in each lunation) at the same sidereal time, or nearly four minutes earlier on successive nights. However, into such peculiarities as these we do not here enter, because the subject would thus become an exceedingly wide one, while in reality there is little importance in the relations thus involved, since in the arctic regions there are no harvesters to be benefited, nor is hunting there pursued in the night hours.

But we must now take into account the circumstance that the moon moves on an orbit somewhat inclined to the ecliptic. It will, in the first place, be manifest that if the *position** of the plane in which

* I use this word to indicate not the actual place of the plane in question, but the manner in which it is posed in space. Thus the position of the earth's equator-plane would, according to this usage of the word, be described as identical (neglecting precession) throughout the year, the position of the earth's orbit-plane identical year after year as the sun moves onward with his family of dependent orbs through space, the position of the plane of the Saturnian rings identical throughout the Saturnian year, and so on. A discussion occurred a year or two ago, in the pages of a weekly journal, as to the proper word to indicate this particular relation, and I advocated then the use of the word "position" as on the whole the most suitable. The question is one to which my attention has been particularly drawn, because it has chanced that repeatedly in my writings I have had to deal with this feature; and I have found no word so readily understood in this particular sense as the word "position." At the same time I must admit, first, that the word is not wholly free from objection, and secondly, that several mathematicians, to whose opinion I feel bound to attach great weight, are opposed to its use in this sense. Unfortunately they suggest no other term. It appears to me that the objections to the use of the word "position" in the sense in question are precisely parallel to those which may be used against the word "direction" as applied to lines. I find, moreover, that Herschel, Grant, and other writers, use the word position as I have done, being apparently forced so to use it

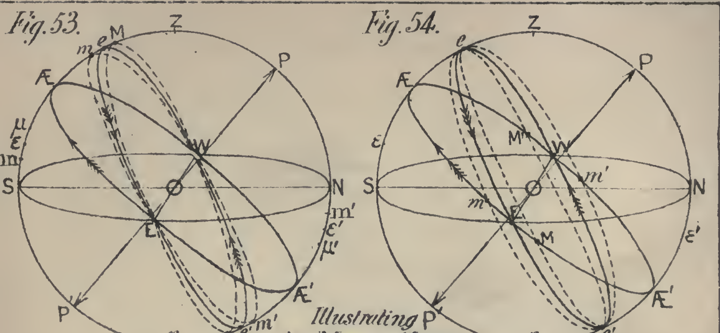
the moon travels were invariable, she would cross the ecliptic at the two fixed points which would be her nodes. During any single revolution of the moon this is not far from the actual case; so that we may say without gross error that in a sidereal month the moon is twice on the ecliptic, and twice at her greatest distance north and south of the ecliptic, that is, about 5° 8′ (on the average) north and south of that circle. Viewing the matter in this way for the moment, let us inquire in what way the moon's range north and south of the equator, and her motions generally, as seen from the earth, are affected, according as her nodes lie in different parts of the ecliptic.

Let S E N W (fig. 53, Plate XIII.) represent the plane of the horizon, N being the north point, and let S P N be the visible celestial sphere. Let E Æ W Æ′ be the celestial equator, the arrow on this circle showing the direction of the diurnal motion, and let W *e* E *e*′ be the ecliptic, the arrow showing the direction of the sun's annual motion. The student will understand of course that the ecliptic is only placed, for convenience of drawing, in such a position as to cross the equator on the horizon at E and W. Twice in each day it occupies that position, as it is

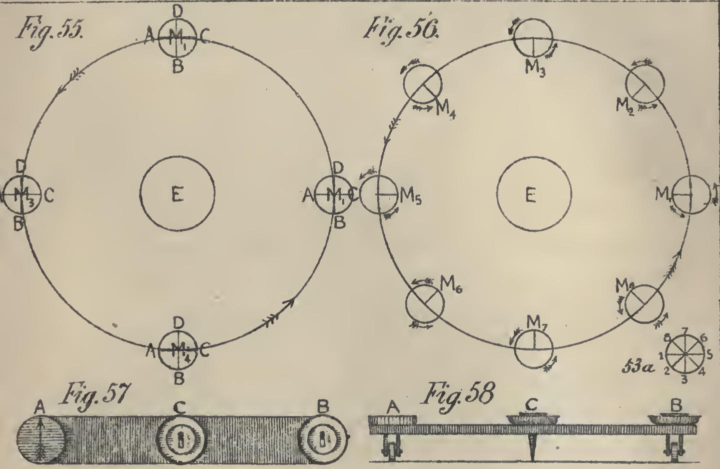
for want of any better word. Accordingly I retain the use of the word, and would suggest, as the best remedy against its defects, that writers should carefully avoid the use of the word to indicate *place*, adopting instead the word *situation*. I give, then, this definition :—Planes are said to have the same *position* when lines normal to them have the same *direction*.

carried round by the diurnal motion, and once in each day it is in the exact position indicated in fig. 53; that is, with its ascending node (or the first point of Aries) just setting in the west.

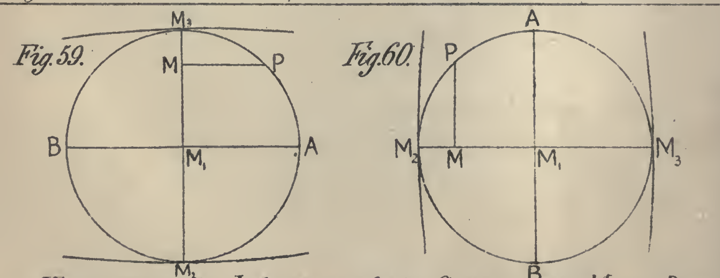
Now let us suppose that the rising node of the moon's orbit is at W, the place of the vernal equinox. Then W M E M′ is the moon's orbit, *e* M and *e*′ M′ are arcs of about 5° 9′; and we see that the range of the moon north and south of the equator exceeds the range of the ecliptic (that is, of the sun) by these equal arcs. In other words, the moon when at M is about 28° 36′ north of the equator instead of being only about 23° 27′ north, as she would be if she moved on the ecliptic, while when at M′ she is about 28° 36′ south of the equator: she moves throughout the sidereal month as the sun moves throughout the sidereal year, passing alternately north and south of the equator, but with a greater range, due to the greater inclination of her orbit. Accordingly, she remains a longer time above the horizon when at any given stage of the northern half of her orbit, and she remains a shorter time above the horizon when at any given stage of the southern half of her orbit than she would be if she moved on the ecliptic. She also passes higher than the sun above the horizon when at her greatest northerly range, attaining at this time (in our latitudes) a height of more than 66°, as at M, instead of less than 61°; and she is correspondingly nearer the horizon in southing when at her greatest southerly range from the equator, attaining in fact a southerly elevation of less than



Illustrating the Moon's Motions.



Figs. 55, 56, 57, and 58, illustrate the Moon's Axial Rotation.



Illustrating the Libration of the Centre of the Moon's Disc.

R. A. Proctor del^t.

10° as at *m*, instead of more than 15°, as is the case with the sun.

Next let us suppose that the descending node of the moon's orbit is at W (fig. 53, Plate XIII.), the place of the vernal equinox; then W *m* E *m′* is the moon's orbit; *e m* and *e m′* are arcs of about 5° 9′; and we see that the range of the moon north and south of the ecliptic is less than the range of the sun by these equal arcs. Thus the moon when at *m* is about 18° 18′ north of the equator instead of 23° 27′, and she is about 18° 18′ south of the equator when at *m′*. Thus she has a smaller range than the sun north and south of the equator. She never attains a greater elevation above the southern horizon than about 56° as at *m*; but, on the other hand, her least elevation when due south exceeds 20°, as at μ (the sun's greatest and least southing elevations, as at *e* and ε, being respectively about 61° and about 15°).

Thirdly, let the rising node of the moon's orbit be near *e*, the place of the summer solstice (fig. 54, Plate XIII.); then *e* M *e′* M′ is the moon's orbit, which crosses the equator at two points, M and M′, in advance of the equinoctial points W and E.* We see

* These points and the points *m* and *m′* are about 12½ degrees from the points E and W, being determined by the relation that they are points on the equator about 5° 9′ north of the ecliptic. If great nicety were required in the above explanation, we should have to take into account the fact that the moon's orbit has not exactly its mean inclination to the equator when the nodes are on the solsticial colure; for the angle *e* M Æ is not equal to the angle *e* E Æ, the mean inclination in question. But considerations of

that its greatest range from the ecliptic is attained nearly at the points *e* and *e′*, and is therefore appreciably equal to the sun's range. The circumstances of the moon's motion must therefore resemble very closely those of the sun's, the chief difference resulting from the fact that the nodes of the moon's orbit in the equator are some twelve or thirteen degrees in advance of the equinoctial points.

Lastly, similar considerations apply when the descending node of the moon's orbit is near *e*, the moon's path being in this case *e m e′ m′*, and its nodes on the equator some twelve or thirteen degrees behind the equinoctial points.

Now let it be noticed that the moon's orbit passes through the complete cycle of changes (of which the above four cases are the *quarter* changes) in about 18·6 years, the lunar node moving on the whole backwards on the ecliptic. Thus, if such a cycle of years begin with the moon's orbit in the position W M E M′ (fig. 53, Plate XIII.), then in about a fourth of the cycle (that is, in about 4·65 years), the moon's orbit is in or near the position *e′ m′ e m*, fig. 54, the node having moved backwards from W to near *e′*, or one quarter of a revolution. One fourth of the cycle later,—that is, about 9·3 years from the beginning of the cycle, the moon's orbit is in or near the position E *m′* W *m*, fig. 53, the node having moved still backwards from *e′* to near E. Yet another fourth of the

this kind need not detain us in a general explanation such as that we are now upon.

cycle later, or about 13·95 years from its commencement, the moon's orbit is in or near the position *e* M *e*′M′, fig. 54, the rising node having shifted backwards from E to near *e*; and, lastly, at the end of the complete cycle of 18·6 years, the moon's orbit is in or near its original position.

It is obvious that since, on the whole, the lunar nodes thus regrede, or, as it were, meet the advancing moon, she must cross her nodes at intervals somewhat shorter than a sidereal month. In fact, supposing her to start from her rising node at the beginning of a sidereal month of 27·322 days, then at the end she has returned to the part of the ecliptic she had occupied at the beginning, while the node has regreded on the average by that amount which is due to a period of 27·322 days. This amount is easily calculated, since the node regredes through the complete circuit of the ecliptic in 6793·391 days: it is rather less than 1° 27′. So that, estimating her motion with reference to her rising node, the moon completes a circuit and nearly a degree and a half *over*, in 27·322 days; hence she completes a nodal circuit in a period less than 27·322 in the proportion very nearly of 360 to 361½.* This period, called the

* Or another and more exact way of viewing the matter is as follows:—The moon advances at a mean rate of $\frac{360}{27 \cdot 322}$ degrees per day, the node regredes at a mean rate of $\frac{360}{6793 \cdot 39}$ degrees per day. Thus the diurnal advance of the moon with respect to the node is the sum of these two quantities, and we have only to calculate how often this sum is contained in 360 degrees to find the exact number

nodical month, amounts to 27·212 days. It follows that the mean interval between successive passages of the lunar nodes is about 13⅗ days. Accordingly, the moon must always be twice at a node in every lunar month of 29½ days, and may be three times at a node; since, if she is at a node within the first 2·3 days from new moon, she is again at a node within 15·9 days from new moon, and yet again within 29·5 days,—that is, before the next new moon.

The effects of the eccentricity of the lunar orbit are too obvious to need any special discussion. The moon moves more quickly (in miles per hour) when in perigee than when in apogee, in the proportion of about 19 to 17 on the average; but as she is nearer in the same degree when in perigee, her apparent rate of motion along her orbit is yet farther increased, and in the same degree, so that her motion in her orbit is greater when she is in perigee than when she is in apogee, in about the proportion of the square of 19 to the square of 17, or about as 5 to 4.* (We

of days in a mean nodical month. This number is obviously the reciprocal of $\frac{1}{27\cdot322}+\frac{1}{6793\cdot39}$. This method is clearly the correct method to pursue in all such cases. The rule may be thus expressed:—Let P, P′ be the periods in which two objects—which may be planets, nodes, perigee-points, and so on—make a circuit of the same celestial circle, P′ being greater than P: then the interval between successive conjunctions is the reciprocal of $\frac{1}{P}-\frac{1}{P'}$ if the objects move in the same direction, and the reciprocal of $\frac{1}{P}+\frac{1}{P'}$ if they move in different directions.

* When we wish to obtain a fair approach to the ratio of the squares of two large numbers differing by two, we have a ready

note, in passing, that 19 to 17 is about the ratio in which the moon's apparent linear dimensions are greater when she is in perigee than when she is in apogee, while 5 to 4 is the apparent ratio in which her disc when she is in perigee exceeds her disc when she is in apogee.) As the eccentricity of her orbit is variable, its mean value being about 0·055, while its greatest and least values are about 0·066 and 0·044, there is a different range in her rates of real and apparent motion, according to the amount of eccentricity when she is in perigee or apogee respectively. The actual maximum rate of the moon's motion is attained when she is in perigee and her eccentricity has its maximum value 0·066, while the actual minimum is attained when she is in apogee at such a time. The ratio between her real motions, under these circumstances, is that of 1,066 to 934, or about 8 to 7; the ratio between her apparent motions in her orbit being rather greater than 13 to 10.

These variations are sufficiently great to modify, in a remarkable degree, the movements of the moon when considered with reference to the change from day to day in her apparent place in the heavens, and therefore, in her apparent course from horizon to horizon. We saw that this must be so, when we inquired

means in the following considerations:—The ratio $(a + 2)^2 : a^2$ is nearly the same as the ratio $(a + 1)(a + 3) : (a - 1)(a + 1)$; that is, is nearly the same as the ratio $(a + 3) : (a - 1)$. In the above case this gives 20 to 16, or 5 to 4. The real value of the ratio $(17)^2 : (19)^2$ is not 4-fifths or ·8, but ·80056, which differs from ·8 by less than the fourteen-hundredth part.

into the phenomenon called the harvest moon. It is manifest also that all the circumstances of eclipses, solar as well as lunar, must be importantly modified by the remarkable variations which take place in the moon's distance from the earth, and in her real and apparent motions. The eccentricity of the moon's orbit also produces very interesting effects in relation to her librations. If the perigee and apogee always held a fixed position with respect to the nodes of the lunar orbit, the peculiarities thus arising would be less remarkable; but the continual shifting of the relative positions of the nodes and apses (as the perigee and apogee are called) causes a continual variation, as we shall see hereafter, in the circumstances of the lunar librations.

Speaking generally it may be said that the lunar perigee advances at the mean rate mentioned in the preceding chapter, that is in such a way as make a complete circuit in about 3232·575 days. Accordingly, applying considerations resembling those applied to her motion with respect to her nodes, we see that the period of her motion from perigee to perigee must exceed a sidereal month. Its actual length is found to be about 27·555 days. This is the mean *anomalistic* month;* it exceeds the mean nodical month by rather more than the third part of a day; or more exactly by 0·342 of a day.

The actual motion of the perigee and apogee with

* The actual interval between the moon's passages of her perigee varies during the course of a year from about 25 days to about 28½ days.

respect to the nodes is very variable. As shown in the preceding chapter, the apses are sometimes advancing rapidly—and they advance on the whole or regrede on the whole for several successive months—while at others they are almost as rapidly regreding, and the node itself, though on the whole regreding in every lunation, yet sometimes advances slowly for several successive days. Thus the perigee and rising node are sometimes moving the same way, at others in opposite ways; they may be both advancing or both receding, or the perigee may be advancing and the apogee receding, or the perigee receding and the apogee advancing. We can see from figs. 42 and 45 (Plate XI.) how variable these relations are even when no account is taken of the advance and recession taking place during the course of individual lunations. However, so far as the mean advance of the perigee from the node is concerned, the case is sufficiently simple; for the perigee advances so as to complete a revolution on the average in 3232·575 days, or 8·8505 years, while the node recedes so as to complete a revolution on the average in 18·5997 days. Thus the mean annual advance of the perigee is $\frac{1}{8\cdot8505}$ of a revolution, while the mean annual regression of the perigee is $\frac{1}{18\cdot5997}$ of a revolution. Adding these together we find the mean motion of the perigee with respect to the node equal to $\frac{1}{5\cdot997}$ of a revolution.* In other words, the

* The agreement of the figures in the denominator of this fraction with the last four in the fraction representing the motion of the node is of course a merely accidental coincidence.

mean interval between successive conjunctions of the perigee and rising node is very nearly six years, falling short of six years in fact by but about three thousandths of a year, or almost exactly $1\frac{1}{10}$ days.* The mean interval between successive conjunctions of the apses and nodes (without regard to the distinction between apogee and perigee, rising node and descending node) is three years, wanting only about half a day, or more exactly wanting 13 h. 18·5 m.

We are now in a position to discuss the effects of the moon's rotation.

If the moon as she went round the earth turned several times round upon an axis nearly square to the level of her path, she would present every part of her surface several times successively towards the earth, precisely as the earth turns every part of her surface towards the sun in the course of a year. On the other hand, if the moon did not *turn* round at all as she went round the earth, we should see in turn every part of her surface, since at opposite sides of the earth she would necessarily present two opposite faces towards the earth. Since as a matter of fact it may be said (as a first rough account of the moon's appearance) that she turns always the same face towards the earth, it follows that she must turn *once* on an axis nearly square to the level of her path as she performs one complete circuit.

* The mean interval between successive conjunctions of the perigee and the rising node is 2190·343 days, and in six years there are 2191·452 days; so that the mean difference is 1·109 days.

Thus let us suppose that the globe M_1 (fig. 55, Plate XIII.) circuits round the globe E without any change of *position*. Then when the moving globe has completed one-fourth of a revolution, A B C D, and is at M_2, the points A, B, C, D will be in the position shown, B instead of A being towards E. When the moving globe is at M_3, C will be towards E; when the globe is at M_4, D will be towards E; and lastly, when a complete revolution has been effected, A will again be turned towards E. Obviously, to keep A always directed towards E, the line M_2 A should be shifted through a quarter-revolution to the position M_2 B; M_3 A should be shifted through half a revolution to the position M_3 C; and M_4 A through three-quarters of a revolution to the position M_4 D,—all these shiftings being made in the same direction, viz. in the direction A B C D, which is the same as that in which the body itself is moving.

This is shown again in fig. 56, Plate XIII., where we see that if the middle point of the disc of the moving globe is the same real point on this globe, as it travels through the positions M_1, M_2, M_3, M_4 . . . . to M_8, this globe must have turned in the manner shown in fig. 56 *a*, the radii in which to the points 1, 2, 3, 4, &c., are respectively parallel to the radii to M_1, M_2, M_3, M_4, &c., all of which are directed upon the central orb E.

But it may occur to some readers that although undoubtedly if a globe were carried from the position

M_1 to M_2, fig. 55, and A C forcibly kept in the position indicated, there would be the change of face we have described, yet that in the nature of things if a body were set without rotation travelling round a central globe, it would as it went round turn itself *also*, as if upon an axis, and so keep always the same face directed towards the central globe. For example, if a rod extending from E and rigidly attached to M_1, carried that globe round E in the manner indicated, then the face A would remain constantly turned towards E: may it not be, it might be asked, that as the globe moved under gravity round E the same thing would happen? If the globe M_1, initially at rest, were propelled by a blow directed exactly on the line B M_1 with precisely the velocity corresponding to the circular orbit $M_1 M_2 M_3 M_4$ under gravity, might not the result of the attractions exerted by E be to cause the globe M_1 not only to go round E, but to turn itself always so as to have the same face directed towards E?

Now it is mathematically demonstrable that the attraction of E can have no effect whatever in causing the direction of the line A M to *change* as the body (supposed to be spherical *) circles around E. But the considerations on which such a demonstration would be based are by no means so obvious as is com-

* If the body be not spherical, forces tending to produce a rotation come into play; but if the body has even only a roughly globular form, such forces are altogether too small to produce any appreciable amount of rotation during a single revolution.

monly supposed. We shall not, therefore, present them here,* but proceed at once to mention two experimental proofs of the fact in question. The first experiment is very simple. Let a tolerably heavy ball be suspended by a long fine cord. Let it be left hanging until all signs of twisting have passed away; then, having placed a mark upon the ball anywhere except near the top or bottom, cause it to swing in a circle, communicating this motion by means of the string held at a point high above the ball, so that no rotational movement can be imparted. It will be found that the mark continues always to be directed towards the same point of the compass, *not* turning so as always to bear in the same direction with respect to the centre of motion. The second was suggested by Galileo, who pointed out that if a body be set to float in a basin of water, and this basin be held out at arm's length while the holder turns round, it will be found that the floating body does not partake in the turning motion; so that the side turned towards the holder of the basin at the beginning is turned directly away from him when he has made half a turn. It is, however, by no means easy to carry out this experiment in a satisfactory manner, the most striking phenomenon

* It may suffice to remark that if a body circuits round E in the manner shown in fig. 55, the total quantity of *work done* accords exactly with that due to the imparted velocity; but if it moves in the manner shown in fig. 56, the amount of work done exceeds that due to the imparted velocity by the amount corresponding to one complete rotation of the body.

under ordinary conditions being the spilling of three-fourths of the water, or thereabouts.

But a very effective experiment for those who feel doubts respecting the moon's rotation may be conducted as follows :—Let A B (fig. 57, Plate XIII.) be a flat wooden bar of any convenient dimensions (according to the circumstances under which the experiment is to be conducted). Let fig. 58 present a side view of the same bar, which, it will be observed, is arranged to run on casters at A and B, and to turn on a pivot at C. At A let a small circle and arrow be marked on the bar; at C and B let small basins of water be placed, in which let small wooden rods float,—or preferably, let the rods float in half-filled saucers, themselves floating in the basins. If now the experimenter wait until the water is still, the floating rods being central and parallel to the arrow at A, and if he then gently turn the wooden bar round on its pivot at C, the casters rolling on a smooth table or floor, he will see that the rods floating at B and C both retain a direction almost wholly unchanged throughout the motion; and thus while continuing parallel to each other and also to any line on the table or floor to which they were parallel in the first instance, they no longer continue parallel to the arrow at A, whose direction changes throughout the motion. The slight change of position they undergo is obviously referable to friction between the water and the basins and saucers. Of course the basin C is not essential in this experiment, nor the fixed arrow at A. If the basin B were simply carried round the end A as a centre, a similar result would follow. But it is interesting to show that so far as the rotation of the water within the basin is concerned, the condition of the basin B is exactly the same as that of a basin at C turning simply on a pivot immediately beneath it.

Another experiment may be tried with the same apparatus. The water in C and B may, without much trouble, be set rotating at the same rate. If this be done, and then the rod be carried round at the same rate, so that the floating rod in C retains an unchanged position with respect to the bar A B or to the arrow at A, it will be found that the water in B behaves precisely as the moon's globe behaves (so far at least as the general relation we are dealing with is concerned), turning always the same portion towards the centre C. Thus we learn that it is only by an *additional* rotational movement that such a relation can be preserved.

The moon then turns once upon her axis as she completes the circuit of her orbit. Yet it is not strictly the case that the moon turns always the same face towards the earth. We see somewhat more than one half of the moon's surface. Let us inquire how this is brought about.

In the first place, the moon's axis is not at right angles to the plane of the path in which she travels round the earth. (Let it be noticed, in passing, that it is the inclination of the moon's axis to this plane, and not to the plane of the ecliptic, which affects her appearance as seen from the earth. This will appear obvious as we proceed.)

The moon's equator-plane is inclined 1° 30′ 11″ to the plane of the ecliptic, and is always so placed that when the moon is at the ascending or descending node of her orbit, the equator-plane is turned edgewise towards the earth, and is inclined descendingly or ascendingly (respectively) to the ecliptic. In other words, if *e* M *e*′ (fig. 61, Plate XIV.) represent the ecliptic, M being the rising node of the moon's orbit, M M′, then the moon's equator is in the position E E′; while, if M is the descending node (fig. 63), then the equator is in the position E E′ (fig. 63); the angle *e* M E in both cases being one of 1° 30′ 11″. Since the average inclination of the moon's orbit to the ecliptic is nearly 5° 9′, it follows that the angle E M M′ has a mean value of about 6° 39′; but this angle varies as the inclination of the moon's orbit varies, and is sometimes as great as 6° 44′,* sometimes as small as 6° 34′.

Now the effect of this inclination of the moon's axis to the plane of her orbit about the earth corresponds precisely to the seasonal variations of the earth's presentation towards the sun. Thus we see that as the moon passes away from the position shown in fig. 61,

* I find commonly 6° 47′ set as the value of this angle. This seems to be obtained by adding the moon's maximum orbit inclination 5° 17′ to the inclination of her axis to the ecliptic. But the moon is always near a node when her orbit attains its maximum inclination, whereas the maximum opening due to her inclination is attained when she is farthest from her nodes.

THE LIBRATION IN LATITUDE.

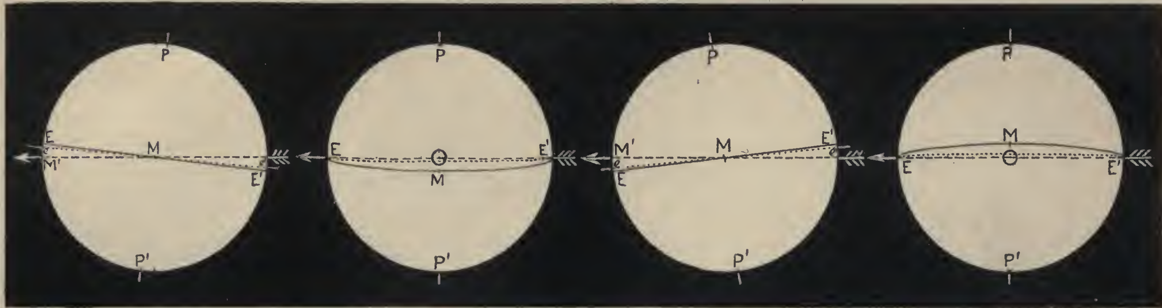


Fig. 61. At Descending Node. Fig. 62. Greatest S. Lat^{de}. Fig. 63. At Ascending Node. Fig. 64. Greatest N. Lat.

THE LIBRATION IN LONGITUDE.

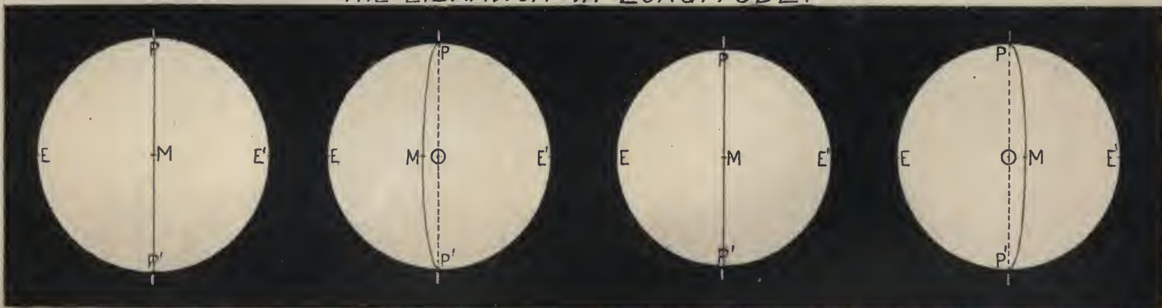


Fig. 65. In Perigee. Fig. 66. At Mean Distance after Perigee. Fig. 67. In Apogee. Fig. 68. At Mean Dist^{ce}. after Apogee.

Plate XIV., moving towards the left, the pole P will be brought into view, and the moon's equator will open out with its convexity downwards, so that at the end of a quarter of a revolution (from rising node to rising node) the aspect of the moon will be as shown in fig. 62. At the end of another quarter of a revolution, when the moon will again be at a node, her aspect will be as in fig. 63; at the end of the third quarter as at fig. 64; and when the revolution is completed she will again be as at fig. 61. We see, then, that her face varies on account of her inclination, the middle of her visible disc lying about 6° 39′ north of the equator, when she presents the aspect shown in fig. 62, and as far south of the equator when she presents the aspect shown in fig. 64. Here no account is taken of rotation, precisely as in dealing with the terrestrial seasons we consider separately the seasonal changes of the earth's aspect and the effects of her rotation.

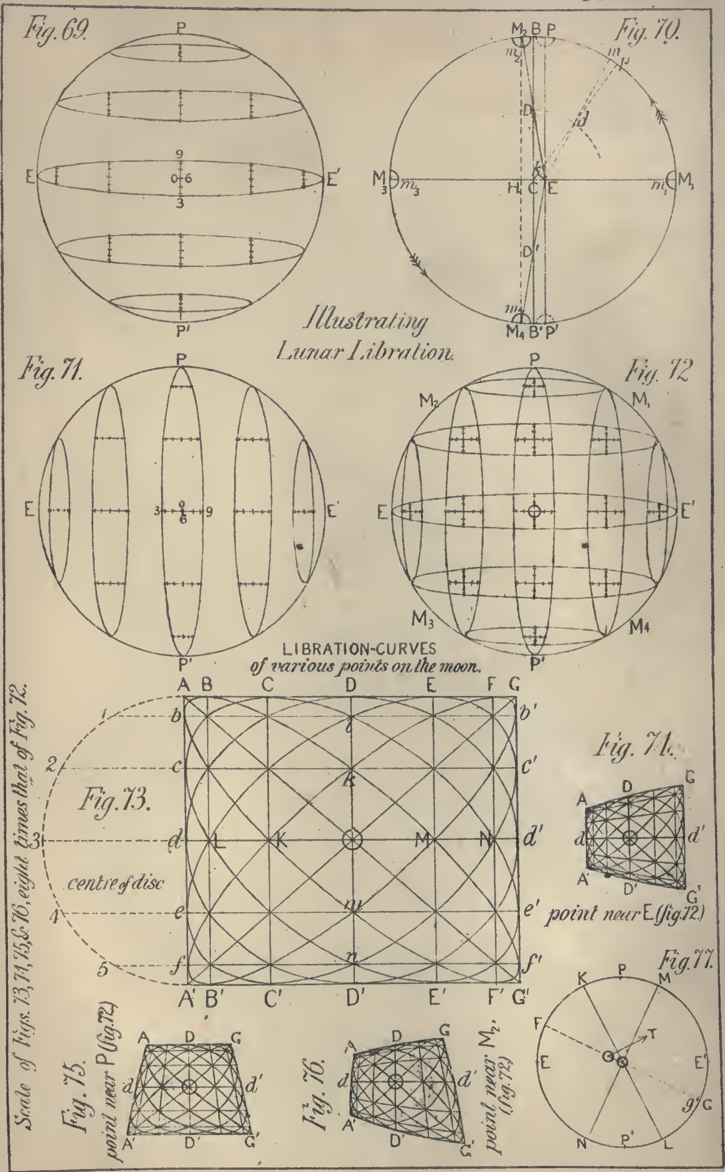
Seeing that the middle of the disc passes alternately north and south of the moon's equator, or, which is the same thing, that M, the middle point of the visible half of the equator, passes alternately south and north of the centre of the disc, we are led to inquire at what rate this oscillatory motion takes place at different parts of the nodical month. The mathematician will find no difficulty in proving the following relation: *

* See note on pp. 80 and 81 of my treatise on Saturn for considerations rendering the solution of all such problems exceedingly easy.

Let M_1, fig. 59, Plate XIII., represent the middle of the disc when the moon shows the aspect indicated in figs. 61 and 63, Plate XIV.; M_2 the middle of the visible half of the equator under the aspect shown in fig. 62; and M_3 the same point under the aspect shown in fig. 64. Then if a circle, A M_2 B M_3, be described about M_1 as centre, and a point be supposed to traverse this circle at a uniform rate, in a nodical month, starting from A when the moon is at a rising node; and if P be the position of this point at any part of such a month, then P M drawn perpendicular to M_2 M_3 gives M, the position occupied by the middle of the visible half of the moon's equator at that moment. Thus we see that the middle point of the moon's equator oscillates northwards and southwards (along the apparent projections of the moon's polar axis), moving very slowly near M_2 and M_3, and most quickly in crossing the point M_1, the middle of the moon's disc.

It will be easily seen that, considering only the effects due to the moon's inclination, fig. 69, Plate XV., represents the changes affecting points which, in the moon's mean state (as in figs. 61 and 63, Plate XIV.), occupy the middle of the short vertical lines of that figure. Supposing the nodical month divided into twelve equal parts (each, therefore, about $2\frac{1}{4}$ days in length), these points would travel upwards and downwards along the vertical lines of fig. 69 (which lines are, of course, not really vertical on the moon's disc as we see it), traversing the points there marked in

PLATE XV.



R. A. Proctor Delt

the order indicated by the numbers on the central short line.

But now let us take into consideration the effect of the want of perfect accordance between the moon's motions of revolution and rotation. She rotates uniformly on her axis, or very nearly so, while she moves with varying velocity round the earth. But fig. 56, Plate XIII., shows, that in order that the same face should always be shown, there should be perfect agreement between the motions of rotation and revolution.

Let $M_1M_2M_3M_4$ (fig. 70, Plate XV.) be the lunar orbit about E, the earth, M_1 being the perigee of the orbit and C its centre, the shape of the orbit (see note, p. 27) being appreciably circular. Then the moon moves more rapidly over the arc M_1 B than over the arc B M_3; and therefore, at the end of a quarter of a revolution she is not at B, but in advance of that point. To find how much, we have only to consider that in a quarter of a revolution a line from the earth to the moon sweeps over a quarter of the area $M_1M_2M_3M_4$. Hence, if M_2 be the moon's place at the end of a quarter of a revolution, the area M_2 E M_1 is equal to the quarter B C M_1 of the complete orbit. So that the triangle C D E must be equal to the space B D M_2. This will obviously be *very* nearly the case if D is the middle point of the line C B (see further the note on p. 107). And in like manner, we obtain the point M_4, reached by the moon after three-quarters of a revolution, by drawing E D′ M_4 through the middle point D′ of E B′. Now, it will be readily conceived that since the moon when at B is at her mean distance, she is travelling nearly at her mean rate in the neighbourhood of this point (her orbit being nearly circular in shape), so that at M_2 she is no longer getting in advance of her mean place, and has, therefore, attained her maximum displacement in advance. In like manner, when she is at M_4, she has attained (approximately) her maximum displacement behind her mean place. And it is very easy to find the effects (necessarily maximum effects, at the points M_2 and M_4) of the non-accordance between the motions of rotation and revolution. If the moon swept at a uniform rate round the point E, she would be at P and P′ at the times when, in reality, she is at M_2 and M_4 (P E P′ being drawn at right angles to C E). This is obvious, since the four angles P E M_1, P E M_3, P′E M_3, and P′ E M_1 are all equal, and the moon occupies equal times in going from M_1 to M_2, thence to M_3, thence to M_4, and thence, finally, to M_1 again.* So

* In fact, we are assuming that P E M_2, P′ E M_4, represent the maximum values of the difference between the true and the mean anomaly, or, with ordinary notation, the maximum values of $(\theta - n\,t)$. Now it is obvious that the circular measure of the angle P E M_2 is very nearly represented by $\frac{2\,CE}{EB}$, or by $2\,e$. Hence we are assuming that the maximum value of $\theta - n\,t$ is very nearly equal to $2\,e$. In reality, this value is represented by an infinite series, beginning

$$2\,e + \frac{11\,e^3}{3.2^4} + \frac{599\,e^5}{5.2^{10}} + \&c.$$

For the mean value of the lunar eccentricity, the term involving e^3 amounts only to 0·00003792, or less than the 2,895th part of $2\,e$.

that, if we draw magnified pictures of the moon at M_1, M_2, M_3, and M_4, and put m_1 as the point nearest to the earth (or the middle of the moon's visible disc) when the moon is at M_1, the line $M_1 m_1$ will have shifted so as to be parallel to P E when the moon is at M_2,—in other words, it will have the position $M_2 m_2$, and instead of being the middle of the visible lunar disc, m_2 will be displaced in the direction in which the moon is moving, or towards the east. When the moon is at M_3, the same line will have made half a turn, or be in the position $M_3 m_3$; directed therefore, as at first, towards the earth. When the moon is at M_4, the same line will be parallel to P′E, or have the position $M_4 m_4$, and instead of being the middle of the moon's visible disc, m_4 will be displaced towards the west. When the eccentricity of the moon's orbit has its mean value, the value of the angle P E M_2 or P′E M_4 is about 6° 17′ 19″·04. But when the eccentricity has its maximum value, the angle P E M_2 or P′ E M_4 amounts to 7° 20′, and owing to lunar perturbations, it may be increased to so much as 7° 45′.*

It follows then that the effect of the want of accordance between the moon's rotation and revolution is to sway the lunar meridian through the middle point of

* This is the result of my own calculations. I find 7° 53′ and 7° 55′ set by different authorities as the greatest value of the angle in question. It appears to me that the circumstance has been overlooked that the moon's orbit never has its maximum eccentricity when the moon is at her mean distance.

the disc, when the moon is in perigee or apogee, in the manner indicated in figs. 65, 66, 67, and 68. Here no account is taken of the change of aspect due to the moon's inclination, but the polar axis is supposed to be throughout in an unchanged position. When the moon is in perigee, this meridian has the position shown in fig. 65, Plate XIV. A quarter of a revolution after perigee it has the position shown at fig. 66 on the east of the mean position. When the moon is in apogee, the meridian is again central, or as in fig. 67; a quarter of a revolution later, it has the position shown at fig. 68, to the west of the mean position; and lastly, when she is again at her perigee, the meridian is again as shown at fig. 65.

In this case, also, seeing that the point M passes alternately east and west of the middle of the disc, we are led to inquire at what rate this oscillatory motion takes place in different parts of the anomalistic month. The mathematician will find no difficulty in proving that, approximately, the law of this oscillatory motion is similar to that of the libration due to the moon's inclination: *—

* If M (fig. 70, Plate XV.) be the position of the moon in any part of her orbit, it is easily shown that C *d p*, having *d* as its middle point, shows the direction (approximately) in which the moon would have been at the moment if she had circled at a uniform rate around C. Thus, whereas C D E (measured by E C) gives the maximum libration, C *d* E (measured on the same scale by E *k*) gives the libration when the moon is at *m*. Now the ratio E *k* by E C is the sine of the angle p C M_1, which proves the above proposition.

Thus, let M_1 fig. 60, Plate XIII., represent the middle of the disc when the moon is in perigee or apogee, or as shown in figs. 65 and 67, Plate XIV.; M_2, the position of the same point (corresponding to M) when the moon is as shown in fig. 66; and M_3, the position of the same point when the moon is as shown in fig. 68. Then if a circle A M_2 B M_3 be described about M_1 as centre, and a point be supposed to traverse this circle at a uniform rate in an anomalistic month, starting from A when the moon is in perigee; and if P be the position of this point at any part of such a month, P M drawn perpendicular to M_2 M_3 gives M the position occupied at that moment by the point which had been at the middle of the visible disc when the moon was in perigee. Thus, this point oscillates eastwards and westwards (in a direction at right angles to the apparent projection of the moon's polar axis), moving very slowly near M_2 and M_3, and most quickly in crossing the point M_1, the middle of the moon's disc.

It will easily be seen that considering only the effects due to the moon's variable motion in her orbit, fig. 71, Plate XV., represents the changes affecting points which, in the moon's mean state, occupy the middle of the short horizontal lines of the figure. Supposing the anomalistic month divided into twelve equal parts (each therefore about $2\frac{1}{4}$ days in length), these points would travel forwards and backwards along the horizontal lines of fig. 71 (which lines are of course not really horizontal on the moon's disc as we see it), traversing the points thus marked in the order indicated by the numbers on the central short line.

It remains for us to determine the combined effects of the movements here separately dealt with. Fully to treat the matter in this general aspect would require much more space than can here be given; moreover, the problems involved are not quite suitable for these pages. But a sufficient idea of the general effects of libration can be obtained by attending to the following considerations:—

In the first place, let it be noticed that we need not concern ourselves at all about the varying slope of P P′, as illustrated in figs. 61, 62, &c., but need attend only to the consideration that the moon librates, owing to her inclination, precisely as though swaying on an axis through the points E E′ on the edge of her disc. In like manner, owing to her varying rate of rotation, she librates precisely as though swaying on an axis through the points P P′, on the edge of her disc.

In the next place, let it be noticed that everywhere over the moon's disc, except at points so near to the edge that the libration actually carries them at times out of sight, the effects of the two forms of libration can be obtained by combining the two figs. 69 and 71, as in fig. 72, and noting that the small crosses indicate the double oscillation of the several points at the intersection of the cross-lines, and that such double oscillation is combined into a single oscillation, whose nature at any instant depends on the relation existing at the moment between the moon's motion from rising node to rising node, and from perigee to perigee. If we consider these effects for the middle point of the disc, we shall be able to infer their nature for points anywhere on the disc.

Let A G G′A′, fig. 73, Plate XV., represent a magnified view of the small space at the centre of P E P′E. Divide A A′ in the manner indicated (describing a semicircle on this line and dividing the semicircle into—say—six equal parts, drawing perpendiculars on A A′ from the points of division) and A G similarly. This division corresponds to that illustrated by figs. 69, 70, and 71. Draw parallels through the division-points so as to make the complete series of rectangular divisions shown in fig. 73.

Suppose that the moon is at a rising node and also in perigee, so that there is no libration either in longitude or latitude. Then the centre of the moon's disc is the true centre of the portion of the moon's surface discernible from the earth. This point is at O, the centre of the rectangular space A G G′ A′.

Now as the moon advances on her orbit this central point (which for convenience we may call the *mean centre*) is carried eastwards of O, because the moon has just passed her perigee, and southwards of O because she has just passed her rising node. The first motion would carry it to K, L, *d*, at intervals each equal to a twelfth part of the anomalistic month; the second would carry it to *m*, *n*, D′, at intervals each equal to a twelfth part of a nodical month. Assuming for the moment that these months are equal, which we may do without important error so far as a single revolution of the moon is concerned, we see that the moon will be carried along the line O A′, reaching the points or stages indicated along that line at intervals each equal to about 2¼ days. It will be carried by the continuance of the same combined librations back to O, which it will reach when the moon is in apogee and at her descending node, or half a month (mean nodical and anomalistic) from the beginning of the motion; then it will pass northwards and westwards to G, and so back to O at the end of the month.

Thus, in the imagined state of things, the mean centre sways backwards and forwards along the line A O G.

Now, as a matter of fact, the mean nodical month is shorter than the mean anomalistic month. Therefore the moon, starting under the conditions just described, will presently so move as to reach her rising node some time before she reaches her perigee. Let us suppose the node to have separated 30° (a twelfth part of a complete circuit) from the perigee, which will happen on the average almost exactly half a year from the time when these points coincided.

Now, therefore, the moon comes to her rising node when the mean centre is still west of the centre of the disc by the amount due to one-twelfth of the anomalistic month; whereas, when she is at her perigee, the mean centre is south of the centre of the disc by the amount due to a twelfth of a nodical month. Accordingly, when she is at the rising node, the mean centre is at M, and when she is at her perigee the mean centre is at *m*. And it is very easy to see that, supposing the node and perigee to retain this position throughout the month, the mean centre traverses the oval M B′ K *b*′ in that direction.

By like reasoning it is obvious that when the rising node is 60° behind the perigee, the mean centre traverses the oval N C′ L *c*′ in that direction.

When the rising node is 90° behind the perigee, the mean centre traverses the oval *d*′ D′ *d* D in that direction. At this time there is no libration in longitude when the libration in latitude is at a maximum, and no libration in latitude when the libration in longitude is at a maximum. On the average, almost exactly a year and a half has now passed from the time when the mean centre librated along the line A O G.

When the rising node has regreded 120° from the perigee, it is clear that when the moon is at a node the westerly libration has not reached its maximum. The mean centre is then at N, and moving southwards and westwards. It traverses then the oval N *e*′ E′ L *c* C in that direction.

When the rising node has regreded 150° from the perigee, the mean centre traverses the oval M *f*′ K *b* in that direction.

When the rising node has regreded 180° from the perigee, or coincides with the apogee, the mean centre again librates linearly over O, but on the line

A O G′. This happens almost exactly three years on the average from the time when the rising node and perigee were in conjunction.

Still regreding, the node passes 30° behind the apogee, at which time the mean centre traverses the oval K F′M B in that direction. It will be noticed that throughout the former series of changes, the direction of its motion around O was the same as that of the hands of a watch. Now the direction is reversed, and continues so during the series of changes taking place as the rising node passes from coincidence with the apogee to coincidence with the perigee.

When the rising node is 60° behind the apogee, the mean centre traverses the oval L E′N C in that direction.

When the rising node is 90° behind the apogee, the mean centre traverses the oval *d* D′*d*′D in that direction. At this time the state of things corresponds with that which prevailed when the rising node was 90° behind the perigee, except that the oval having axes D D and *d d*′ is traversed in the opposite direction. An interval of almost exactly 4½ years has (on the average) now passed since the rising node and perigee were in conjunction.

When the rising node is 120° behind the apogee, the mean centre traverses the oval L C′N E in that direction.

When the rising node is 150° behind the apogee, the mean centre traverses the oval K B′M b' in that direction.

And lastly, when the rising node is again coincident with the perigee, the mean centre moves backwards and forwards along the line A′O G, as at the beginning of the period. This period is on the average almost exactly six years.

It will be easily seen how changes corresponding with those just described take place for every point on the moon's disc. If we call the intersection of any of the small cross-lines in fig. 72, Plate XV., a mean point, this mean point sways over and round its mean position precisely as the mean centre sways over and round O, only that the ovals described differ in shape from A G G′ A, and are also less symmetrical in figure. Fig. 74, Plate XV., illustrates the motions for a point on P P′, and not far from P; fig. 75, for a point on E E′, and not far from E; while fig. 76 illustrates the motions for a point not far from the point M_2. It will be understood that the letters in all these figures correspond with those in fig. 73; so that when the mean centre is at A, D, or G (fig. 73) —for example, the mean points corresponding to figs. 74, 75, and 76, are at A, D, or G, on those several figures respectively.

Now since, as a matter of fact, the rising node does not move from the perigee by sudden shiftings of 30°, it follows that the path traversed by the mean centre shifts gradually from one oval to another of fig. 73;

not completing any one of these ovals, but so moving that one oval merges into the next in a continuous manner. But since the node is not always regreding nor the perigee always advancing, there is not a steady advance from one shape of loop to the next, but an alternation of advance and retrogression as respects the completion of the series of changes. In traversing a single circuit, indeed, there is always a double alternation, and sometimes a more complex series of alternations of this sort; because the perigee alternately advances and regredes twice in each lunar revolution, the node doing likewise, though in a less marked degree. But these effects are insignificant compared with those due to the regression of the perigee for several successive months, as explained in the preceding chapter, and illustrated by fig. 42, Plate XI. This causes for the time being a reversal of the effects we have been considering; so that we have in every interval between successive conjunctions of the perigee and sun (or in every period of 411 days) two periods when the processes of change in the loops of fig. 73, Plate XV. (as well of course as in those of figs. 74, 75, and 76) are reversed for the time being. Adding to this consideration the circumstance that the eccentricity and inclination both undergo alterations (so that the length and breadth of A G G′A′ are variable), and that nearly 80½ nodical months occur between successive conjunctions of rising node and perigee, we see that the path actually traced out by the mean centre is exceedingly complicated. *The* *motion of this point involves implicitly the whole theory of the moon's motions.*

We have not considered thus far the effects of libration on those parts of the moon's disc which lie so near to the edge that they pass at times out of sight. These movements might be dealt with like those we have just been considering, by regarding them as due to two distinct libratory motions taking place about P P′ and E E′ at known rates. But the matter may be simplified by noting that where (as in the present instance) such small arcs as from 6° to 10° are concerned, the libratory motions of points near the rim of the mean lunar disc may be regarded as virtually carrying those points backwards and forwards at right angles to the rim. And it is very easy to see what will be the extent of this libratory motion at any given part of the rim when the libration of the mean centre is known. Thus, take the point P (fig. 72, Plate XV.). Here there is *always* an alternate swaying at right angles to the rim equal in range to the libration in latitude; for whatever the oval traversed by the mean centre, it always ranges in latitude from A G to A′ G′. In like manner at E and E′ there is *always* an alternate swaying at right angles to the limb equal in range to the libration in longitude; for the oval traversed by the mean centre always ranges in longitude from A A′ to G G′. But take the part M_2 of the disc's edge. Here, when the mean centre is librating along A O G′, points near the edge sway backwards and forwards at right angles to the edge over a range equal to the maximum libratory swing A O G′ (foreshortened of course, so as never to bring such points far within the edge in appearance). But when the mean centre is librating along A′O G, points near M are scarcely shifted at all. In intermediate cases, points near M_2 have an intermediate range. Thus when the mean centre is traversing the oval M F K B′, the range of points near M_2 is equal to the breadth of this oval measured parallel to M_2 M_4. These remarks apply unchanged to points near M. At points near M_1 and M_3 corresponding changes take place; only it is when the mean centre is librating along A′O G that points near M_1 and M_3 sway over the largest arc across the rim of the disc, and when the mean centre is librating along A O G′ that these points remain nearly at rest. No point has any libratory motion along the rim of the disc.*

Such are the chief features of the lunar libra-

* By the principles of rotation, we know that since under all circumstances the librations in latitude and longitude take place about the axes E E′ and P P′, the actual libratory motion at any moment must always be about an axis in the plane P E P′E′. And it is very easy to determine the momentary position of that axis, as well as the actual circumstances of the displacement of the moon from its mean position. Thus let O, fig. 77, Plate XV., be the centre of the moon's disc, O′ the position of the mean centre at the moment, O′ T a tangent to the direction in which the mean centre is at the moment moving. Then K O L at right angles to P T is the momentary axis of rotation, and the actual displacement of points on the moon's surface at the moment is the same as would have been produced by rotating the moon from its mean position about an axis M N at right angles to O O′, so that the mean centre

tions in latitude and longitude. It remains that we should consider what is the actual extent of the moon's surface which these librations bring into view in addition to that which is seen when the mean centre is at the actual centre of the lunar disc. In making the inquiry, we must take into account another libration, called the *diurnal* libration, which depends on the circumstance, that owing to the earth's rotation, the place of the observer is shifted with respect to the line joining the centres of the earth and moon. This form of libration might very well be made the subject of a separate investigation, which would, however, be more tedious than profitable, because the extent and nature of the diurnal libration varies in different latitudes and at different seasons. On this point, I shall content myself with remarking that if we imagine an observer placed at the centre of the moon's visible disc, a line drawn from him to any station on the earth would be carried by the earth's rotation along a latitude-parallel, and the angle which it made at any moment with a line joining the centres of the earth and moon would correspond to the

was carried from O to O′. Thus points M and N are at their mean place, and points F and G are shifted by an arc equal to O O′ from their mean position. The point which in its mean position would be at F is behind the disc, and the point which in its mean position would be at G has advanced on the disc as to *g* (G *g* being an arc equal to O O′, but foreshortened). It is obvious that wherever O′ may be (except at O), two points only, and those both on the edge of the disc, are in their mean positions (as M and N in the case illustrated by fig. 77).

diurnal displacement of the moon's centre, as seen from the station at that moment. This consideration, combined with what will hereafter be stated respecting the aspect of the earth as seen from the moon, will suffice to show the exact nature of the diurnal libration at any given station, and at any season. Here, however, all that is necessary to be noticed is that, since the earth's radius, as supposed to be seen from the moon, subtends nearly a degree when the moon is at her mean distance, and more than a degree when the moon is in perigee, we may obviously add an arc of about a degree on the moon's surface to any libratory displacement in any direction whatever, estimated for the centre of the earth, if we wish to determine the maximum displacement in that direction *for any part of the earth*. For, if we suppose an observer on the moon to shift his place, in any direction, by one lunar degree (corresponding to a distance of nearly twenty miles), he would see the earth's centre shifted one degree on the heavens; and, therefore, the point on the heavens formerly occupied by the earth's centre would now be occupied by a point on or very close to the circumference of the earth's disc. Therefore, when we have determined the fringe of extra surface brought into view by the moon's maximum librations, we can widen this fringe all round by a breadth of about one degree. We must not indeed widen it everywhere by a breadth of 1° 1′ 24″, the maximum apparent semi-diameter of the earth as seen from the moon, simply because this apparent semi-diameter is only presented when the moon is in perigee, while the moon attains her greatest total libration (corresponding to the displacement of the mean centre from O to A, or A′, or G, or G′, fig. 73), as well as her greatest libration in longitude (corresponding to the displacement of the mean centre from O to D, or *d*, or D′, or *d*′, fig. 73), only when she is at her mean distance. We may, however, employ even this maximum value of the horizontal parallax when the moon has her maximum libration in latitude, since there is nothing to prevent her from attaining this libration when she is at her nearest to the earth. These considerations, however, are unimportant, compared with those depending on the moon's librations in longitude and latitude, simply because the diurnal libration—or, as it may more fitly be termed, the parallactic libration—attains its maximum only when the moon is on the horizon, and therefore very ill-placed for telescopic observation.

In considering the actual extent of the moon's surface, which her librations carry into and out of view alternately, we need not trouble ourselves about the varying nature of the combined libration. It might seem, at first sight, as though certain parts of the moon would only be brought into view while the libration in latitude attains its maximum value,—that is, when the libration in longitude vanishes; and *vice versâ*. But as a matter of fact, if we consider the four cases where the total libration has its absolute maximum value—viz., when the mean centre is at the four points A, G, A′, and G′ (fig. 73, Plate XV.), we take into account every portion of the moon's surface which libration can possibly bring into view.

Thus in fig. 78, Plate XVI., P E P′E′ represents the outline of the moon's disc, A G A′ G′ the space over which the mean centre shifts owing to librations. Assume that when the mean centre is at A, G, A′, and G′, successively, the lune-shaped spaces brought into view are represented by the four crescents $a' M_2 g m_2$, $a M_1 g' m_1$, $a M_3 g' m_3$, and $a' M_4 g m_4$ (in reality, on the globe itself, these lunes have at the points M_1, P, M_2, &c., the breadth indicated in fig. 78). Now it is easy to determine the breadth of these lunes at any distance from the points M_1, M_2, M_3, and M_4, where they are widest. Thus we know that the ratio of P p to $M_1 m_1$ is the cosine of the angle P O M_1. But $M_1 m_1$ is equal to O G, and the ratio of O D to O G is as the cosine of the same angle P O M_1.* Hence P p is equal to O D—that is, we get the same libratory displacement (on the sphere, of course) of the point P, by taking the maximum libration of the mean centre (either to G or A) as though we took the greatest libration in latitude, or O D *alone*. And similarly with the greatest libration in

* If we circumscribe the figure D *d*, as shown in fig. 78, it can readily be seen that *any* great circle taken as O D P p is taken, will have the portion within the small circle corresponding to the portion O D equal to the portion within the lune $a m_1 g' M_1$ (corresponding to P p). A similar remark applies to circles circumscribing the figures D *d*, *d* D′, and D′ *d*′.

Fig. 78.

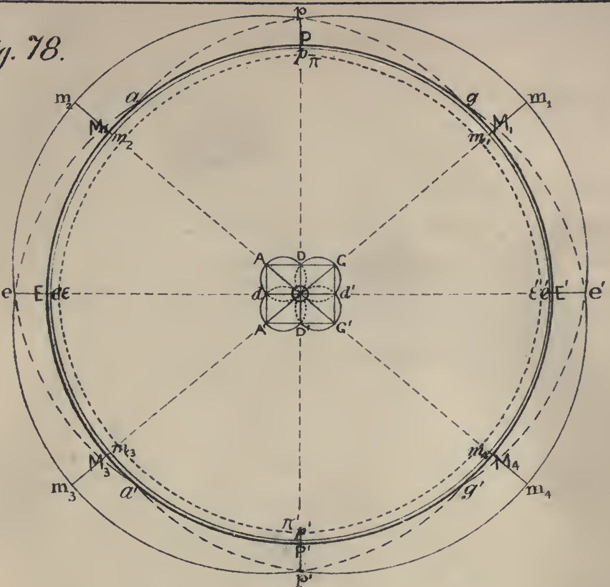
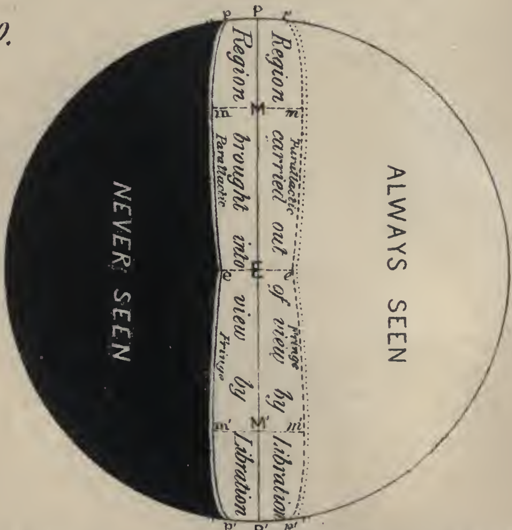


Fig. 79.



Figs. 78 and 79 shew the exact extent of the Moon's surface affected by Libration.

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longitude: it cannot shift E or E′ more than they are shifted when O is carried either to A or A′ on one side, or to G and G′ on the other.

So that all we have to ascertain is the area of the space on the sphere corresponding to that, in fig. 78, between the circle P E P′E′ and curves p e, e p′, p′ e′, and e′ p. This is easily effected,* and we learn that

* We know that the maximum breadth of the four lunes—viz., the breadth at M_1, M_2, M_3, and M_4—is 10° 16′. So the area of each opposite lune bears to the area of the whole sphere the ratio which 10° 16′ bears to 360°. Now, by a well-known property of the sphere, the space $P\,p\,M_1\,m_1$ bears to the half-lune $a\,m_1\,M_1\,a$ ratio equal to the sine of the angle $P\,O\,M_1$, equal therefore to G D by O G. But G D represents an arc of 7° 45′ on the sphere, while O G represents an arc of 10° 16′. Thus the area $P\,m_1$ bears to the whole sphere the ratio which 7° 45′ bears to twice 360°. In like manner, the area $E'\,m_1$ bears to the whole sphere the ratio which 6° 44′ bears to twice 360°. This gives us the area of the space P p E′ e′, which is one-fourth of the total area brought into view by libration. Thus this total area bears to the whole sphere the ratio

$$
\begin{aligned}
& 2\,(7^\circ\,45' + 6^\circ\,44') : 360^\circ \\
= {} & 14^\circ\,29' : 180 \\
= {} & 869 : 10800 \\
= {} & \tfrac{1}{12{\cdot}43}, \text{ or } {\cdot}08046
\end{aligned}
$$

The total area brought into view by libration bears to the hemisphere invisible at the time of mean libration the ratio of about 100 to 621. (Arago makes the ratio 1 to 7, though using 10° 24′ as the absolute maximum of libration.) It is not easy to understand how an error crept into his treatment of a problem so simple. The proportion of the part of this hemisphere never seen to the whole hemisphere is therefore about 521 to 621; or if we represent the whole sphere by 1, the area of the part absolutely invisible will be represented by ·4198. (Klein, in his "Sonnensystem," gives ·4243, which is nearer to the truth than the value resulting from Arago's the area thus brought into view by libration is between one-twelfth and one-thirteenth of the whole area of the moon, or nearly one-sixth part of the hemisphere turned away from the earth when the moon is at her state of mean libration. Of course a precisely equal portion of the hemisphere turned towards us during mean libration is carried out of view by the lunar librations.

If we add to each of these areas a fringe about 1° wide, due to the diurnal libration,—a fringe which we may call the parallactic fringe, since it is brought into view through the same cause which produces the lunar parallax,—we shall find that the total brought into view is almost exactly one-eleventh part of the whole surface of the moon; a similar area is carried out of view: so that the whole region thus swayed

estimate, namely, ·4286, yet still considerably in error, particularly as Klein also names the value 10° 24′ for the maximum libration.)

If, however, we take into account the effects of the diurnal libration, it can readily be shown that the portion of the moon which is never seen under any circumstances bears to the area of the whole moon almost exactly the proportion which 148 bears to 360, or 37 to 90,—that is, it is equal to 0·4111 of the whole area. The part which can be carried out of view or into view by the libration, including the parallactic libration, amounts to $\frac{8}{45}$ths of the whole surface, or $\frac{1}{5\frac{5}{8}}$ if the whole area is represented by unity.

The above numerical results have been carefully tested, and can be relied on as strictly accurate. It is easy for the reader to re-examine them. It may be noted, that instead of the above method for determining the area brought into view by libration we may simply add to the two lunes $a\,M_1\,g'$ and $a\,M_3\,g'$, the spherical triangles $e\,m_2\,P\,M_2$ and $e'\,m_4\,P'\,M_4$.

out of and into view amounts to $\frac{2}{11}$ths of the moon's surface.

In fig. 79, Plate XVI., a side-view of the moon is given. It is supposed to be obtained by rotating the moon from the position P E′ P′ E of fig. 78, about its axis P P′ (E approaching), or by the observer travelling round P P′ until he is in the prolongation of O E. The figure is self-explanatory: but it is to be observed that m M *m* and m′ M′ *m′* are arcs of 20° 32′, corresponding to the absolute maximum libratory swayings, A G′ and A′ G of fig. 78: p P *p* is an arc of 13° 28′, corresponding to the maximum libratory swaying in latitude (D p D′ of fig. 78); and e E *e* is an arc of 15° 30′, corresponding to the maximum swaying in longitude (*d* O *d′* of fig. 78).

It must always be remembered, however, that although such regions as p E p′ (fig. 79, Plate XVI.) are brought into view by libration, they are always seen very much foreshortened, not as presented in fig. 79. In fig. 78 the space between the circles *p e p′ e′* and P E P′ E′ represents the portion of the lunar disc within which these regions are always seen; and it is easy to see that since their real area is represented by the space between the latter circle and the curves p e, e p′, p′ e′, and e′ p, we can obtain very little insight into the configuration of these portions of the lunar surface. A more important effect of the libration is to be recognized in the changed aspect under which parts within the disc at mean libration are seen at the times of maximum libration. Thus the region which at mean libration occupies the portion P E′ e' p (fig. 78) of the disc, is by maximum libration carried to the position $p\ e'\ \varepsilon'\ \pi$, with its apparent area three times as great, owing to the reduction of foreshortening. Thus it can be studied much more favourably. Similarly of the four other quadrants. But for this circumstance very little value could be attached to the portions of many maps representing regions near the edge of the disc at the time of mean libration.

If it is remembered that the time of mean libration is also the time when the libratory range is greatest—for it is only when the mean centre crosses O that it sways along the arcs AG′ or A′G (fig. 73, Plate XV., and fig. 78, Plate XVI.)—we see that at such times we have the best means for studying the general effects of libration. The last occasion of the kind occurred in October, 1871, and at any time within three or four months on either side of that epoch libratory effects could be studied under favourable conditions. The next occasion of the kind will occur in October, 1874, when the perigee and rising node will be nearly in conjunction at the time when the moon is passing either.* From what has been already stated it will be

* The moon will be in perigee on October 25th, at about 6 A.M., and at her rising node on October 24th, at about 11½ P.M., or about 6½ hours earlier. Thus, at about 3 A.M., on October 25th, she will be very nearly at her mean libration, as she will be "full" at 7.21 A.M. of the same day. A very favourable opportunity of studying her "mean" aspect will be afforded during the early morning hours of October 24th. At 4 h. 36 m. A.M. she enters the penumbra of the

seen that the moon will be again near her mean libration in October 1877, October 1880, and so on, for many successive three-yearly intervals.

This chapter would be incomplete without some reference to what has been called the physical libration of the moon.

We have assumed throughout the preceding pages that the moon rotates with perfect uniformity on her axis, while revolving around the earth. This, however, is not strictly the case. In the first place it is manifest that since the moon's mean sidereal revolution is undergoing at present a process of diminution (see pp. 87—90), owing to what is termed her secular acceleration, her rotation must either undergo a corresponding acceleration, or she would in the course of time so turn round with respect to the earth that the regions now unseen would be revealed to terrestrial observers. She would, in fact, thus have turned round by the time when, owing to her acceleration, she had gained half a revolution. It has been shown, however, by Laplace, that the attractions to which she is subject suffice to prevent such a change, and that her rate of rotation changes *pari passu* with her rate of revolution. It must, therefore, be to this slight extent variable. A similar remark applies to all secular perturbations affecting the moon's motions. So that it is impossible that the further side of the

earth, but does not reach the true shadow until 5 h. 41·5 m. She will be totally eclipsed from 7 till 7 h. 32 m., but sets at Greenwich before the total phase begins.

moon should ever be turned towards the earth unless under the action of some extraneous influence, as the shock of a mass comparable with her own.

But a real libration much more considerable in amount and possibly recognisable by observation, must affect the moon's rotation. Newton was the first to point out, that if the moon was originally in a fluid state, the earth's attraction would draw her into the form of a spheroid, the longer axis of which, when produced, would pass through the earth's centre. "Comparing this phenomenon," says Professor Grant, "with the tidal spheroid occasioned by the action of the moon upon the earth, he found that the diameter of the lunar spheroid which is directed towards the earth, would exceed the diameter at right angles to it by 186 feet. He discovered in this elongation of the moon the cause why she always turns the same side towards the earth, for he remarked that in any other position the action of the earth would not maintain her in equilibrium, but would constantly draw her back, until the elongated axis coincided in direction with the line joining the earth and moon. Now, in consequence of the inequalities of the moon in longitude, the elongated axis would not always be directed exactly to the earth. Newton, therefore, concluded that a real libration of the moon would ensue, in consequence of which the elongated axis would oscillate perpetually on each side of its mean place." Thus, if we consider figs. 65, 66, and 67, Plate XIV., we see that throughout the motion from perigee to apogee the longer axis of the moon is so placed that its mean extremity M is on the east of the line joining the centre of the earth and moon; and we see from figs. 67, 68, and 65, that it is on the west of that line throughout the motion from apogee to perigee. Hence, during the moon's motion from perigee to apogee, the earth's attraction tends to restore M to its mean position, or to pull M towards E′, whereas during the motion from perigee to apogee the earth's attraction tends to pull M towards E. As the rotation carries M continually in the direction E E′ (though the revolution prevents us from recognizing the movement), it follows that as the moon moves from perigee to apogee her rotation is accelerated, and as she moves from apogee to perigee her rotation is retarded. Thus her rotation rate is at a maximum when she is in apogee, and at a minimum when she is in perigee. Moreover, her rotation rate is above the mean value when she is moving from mean distance after apogee to mean distance, and below its mean rate as she completes her revolution from mean distance after perigee to mean distance again. Hence it follows that the greatest real displacement of M (or of any given point on the moon's equator) occurs when the moon is at her mean distance, and is towards E′ when the moon is passing to her apogee, and towards E when she is moving towards her perigee. In other words, the apparent maximum libration in longitude is always reduced by this real libration.

Lagrange, in dealing with this relation, noticed further, what had apparently escaped Newton's attention, that owing to the moon's rotation on her polar axis, her globe must be, to some slight degree, compressed in the direction of this axis. "Lagrange," says Professor Grant, "found that both effects were of the same order, and that the moon would, in reality, acquire the form of an ellipsoid, the greatest axis being directed towards the earth, and the least being perpendicular to the plane of the equator. The greatest and the mean axes will both lie in the last-mentioned plane."

Proceeding to consider the effect of the earth's attraction upon the rotating moon, Lagrange found that the mean rotation would be affected by a series of changes corresponding to those affecting the moon's mean motion round the earth. In effect, all the perturbations affecting the moon's motion of revolution would, as it were, be reflected, or represented in miniature, in these variations of her motion of rotation.

While dealing with this matter, Lagrange noted a circumstance to which Newton had not referred, though, as Professor Grant well remarks, it is a natural corollary to Newton's reasoning. He showed that it was not necessary to suppose that the motions of revolution and rotation were equal in the beginning. If the moon's true rotation once took place in a period not absolutely coincident with that of her revolution, the attraction of the earth would have sufficed to force the rotation-period into *mean* coincidence with the period of revolution. The rotation would, in that case, however, no longer be strictly uniform, apart from the real libration we have hitherto considered. The moon would librate on either side of her mean position, independently of her variable motion in her orbit. This libration would depend, like the other, on the circumstance that the orb of the moon must be somewhat elongated in the direction of the line joining the centres of the earth and moon.

Now the form of real libration last mentioned has not been observationally recognized; but the real libration, theoretically predicted by Newton, and confirmed by the analytical researches of Lagrange, has been detected by observers. I have said, that in this libration every feature of the moon's motions is reflected. Now it might seem, at first sight, that this libration would be most noticeable as depending on the moon's varying motion in a single revolution, since she may be so much as 7° 45′ before or behind her mean place. But, as a matter of fact, the extent of the real libration depends much more on the length of time during which the earth's action is exerted, than on the actual displacement of the moon's longer axis from its mean position. Accordingly, the lunar irregularity called the annual equation, although (as we have seen at page 90) it only affects the earth's place by a small amount at the maximum, yet, as its period is a long one, enables the earth to affect the mean rotation rate more effectually than do any of the other lunar perturbations. "Bouvard and Nicollet undertook," says Professor Grant, "a series of careful observations of the moon's librations in longitude, at the Royal Observatory of Paris. The *Connaissance des Temps* for 1822 contains a beautiful paper by Nicollet, in which he submitted these observations, amounting in number to 174, to a searching discussion. The only sensible inequality was that corresponding to the *annual equation.* It appeared by observation to have a maximum value equal to 4′ 45″. The results at which he arrives relative to the ratios of the axes do not accord with the generally admitted opinion respecting the primitive condition of the moon. He found, in fact, that the difference between the least and greatest axes was greater than it would be on the supposition that the moon was originally a fluid mass." It would, however, be rash to base any opinion respecting the latter hypothesis upon observations so very delicate in their nature.

It is important to notice that the ellipsoidal form of the moon is not only demonstrated by the existence of a recognizable real libration, but also by the continuance of that singular relation between the position of the moon's equator and orbit referred to at p. 174, and illustrated in figs. 61—64, Plate XIV. It is manifest that since the *position* of the plane of the orbit is continually shifting, this plane would depart from coincidence with the plane of the moon's equator, unless some extraneous force acted to preserve the coincidence. If the moon were a perfect sphere, the earth would have no grasp upon her, so to speak, whereby to maintain the observed relation between the equator plane and the orbit plane. But Lagrange has shown that the action of the earth on an ellipsoidal moon would constantly maintain the coincidence. As the coincidence is maintained, we must conclude that the moon is necessarily an ellipsoid, and not a sphere.

However, it need hardly be said that no instrumental means at present in our possession could show the ellipticity of the lunar disc. Assuming Newton's estimate to be correct, and that the longest axis, directed (in its mean position) exactly towards the earth, is 186 feet longer than the mean axis E E′ of the figures in Plate XIV.; and adopting Lagrange's estimate of the polar compression (as one-fourth of the extension of the longest axis), we have the polar axis 46½ feet shorter than the mean axis. Since the moon's mean diameter is 2159·6 miles and 46½ feet is less than one-113th part of a mile, it follows that P P′ is less than E E′ by less than one-244,000th part of either diameter, a quantity altogether inappreciable, even independently of the fact that the least of the lunar mountains is many times higher than the calculated difference between P P′ and E E′.

The ellipsoidal figure of the moon remains none the less, however, a demonstrated fact.

CHAPTER IV.

STUDY OF THE MOON'S SURFACE.

ALTHOUGH the study of the moon's surface can scarcely be said to have been fairly commenced before the invention of the telescope, yet in very early ages men began to form opinions respecting the moon based on the appearances presented by her disc. Doubtless the ancient Chaldæan, Chinese, Indian, Egyptian, and Persian astronomers theorized about the moon's physical constitution; but of their views no record has reached us. We know only that they studied the moon's movements so carefully as to recognize the principal features of her orbital motion, but what ideas they formed as to the condition of her surface we do not know.

The earliest recorded opinion as to the moon's condition is the theory of Thales (B.C. 640), that a portion of the moon's lustre is inherent. He recognized the faint light from the illuminated part of the moon's globe at the time of new moon, or rather at the time before and after new moon, when the illuminated portion forms a narrow crescent; and it was also known to him that the moon does not disappear wholly when totally eclipsed. He therefore inferred that she shines in part by native light. It is somewhat singular that he did not perceive the remarkable contrast which exists between the two kinds of light which he regarded as belonging to the moon. The deep ruddy colour of the totally eclipsed moon differs so completely from the ashy pale light of "the old moon in the new moon's arms," that one can hardly understand how both could be referred to one and the same cause. Nevertheless, there have not been wanting those who, in comparatively recent times, have maintained a similar theory.

Anaxagoras (B.C. 500) was the next of the ancient philosophers who theorized respecting the moon. We learn from Diogenes Laertius that Anaxagoras regarded the moon as an inhabited world, and taught that the varieties of tint perceived on her surface are due to mountains and valleys. He held—and was ridiculed for holding—the opinion that the moon may be as large as the Peloponnesus.

Some of the Pythagorean philosophers, on the contrary, taught that the moon is a body altogether unlike the earth. They regarded her as a smooth, crystalline body, having the power of reflecting light like a mirror; and they supposed the spots upon her disc to be the reflection of the oceans and continents of our earth. But others believed the moon to be an inhabited world like the earth, and since daylight on the moon continues for about fifteen terrestrial days, they concluded somewhat boldly that the creatures inhabiting the moon must be fifteen times as large as corresponding terrestrial beings. Heraclitus supposed the moon to be of the same nature as the sun, but darker, because involved in the denser part of the earth-surrounding ether. Origenes, also maintaining the moon to be a self-luminous body, considered her surface to be uneven, and regarded the dark spots as the shadows of the regions lying higher.

Passing over many less distinguished names, we come to Aristotle, who adopted the theory that the light and dark regions in the moon are the reflected images of the continents and oceans of our own earth. It is worthy of notice that the maintenance of this opinion indicates either complete ignorance or a very remarkable forgetfulness respecting the laws of reflexion on the one hand, and those relative motions of the moon and earth on the other hand respecting which even the Ptolemaists held accurate ideas. Whether the earth is fixed or in motion, whether she rotates or the heavens rotate around her, it is certain that her continents and seas are presented in a continually varying manner towards the moon. It is obvious, then, that if the moon were a mirror reflecting the features of the earth, the moon's aspect must necessarily change from hour to hour, and from day to day. Yet nothing is more certain, even to those who only study the moon with the unaided eye, that her aspect, so far as the spots on her disc are concerned, remains very nearly constant. Her phases cause a greater or less portion of her spotted disc to be visible to the observer on earth, but the part which is seen belongs always to one and the same face.

The Stoics maintained for the most part that the moon is a mixture of fire, earth, and air, but spherical, like the earth and sun.

Lastly,—for it would be idle to devote any considerable portion of our space to the vague fancies which the ancients formed respecting the moon,—we find that Plutarch strenuously supported the views which Anaxagoras had maintained six hundred years earlier. He even recognized the indications of mountains in the moon, in the irregularities of the lunar terminator, noting that the lunar mountains would necessarily throw vast shadows, precisely as Mount Athos, at the time of the summer solstice, cast a shadow towards evening which reached across the Thracian sea as far as the market-place of Myrina, in Lemnos, a distance of eighty-seven miles.

But it was only after the invention of the telescope that just ideas began to be formed as to the condition of the moon's surface.

In May, 1609, Galileo directed towards the moon the first telescope of his own construction. His first observations showed him that the moon's surface is covered with irregularities; but it was not until he applied his largest telescope—magnifying only thirty times—that he recognized the true conformation of the lunar surface. He found that the lunar mountains are for the most part circular in shape, forming rings around depressed regions, and in some respects resembling the mountain-chains which surround Bohemia. He could perceive bright points of light separated by dark spaces from the terminator of the crescent or gibbous moon, and he recognized the fact that these points are the tops of mountains, illuminated by sunlight, while the surrounding valleys are in darkness. He traced at once the analogy between this circumstance and terrestrial phenomena. Those who have watched the rising of the sun from the summit of a lofty mountain know that when the summit of the mountain is in the full glory of sunlight, the sides of the mountain are still in shadow, and that the neighbouring valleys are plunged in a yet deeper gloom. Corresponding appearances are seen when the sun is setting. Long before the mountain-tops are darkened the level country around is shadowed over, and the obscurity of night has already settled over ravines and passes. The only difference which Galileo perceived in the phenomena of sunrise and sunset on the lunar mountains and what is observed on our earth, was that no half-lights could be seen, nothing but the full blaze of sunlight on the mountain-tops and intense blackness in the valleys. Here was the first indication of a circumstance on which I shall presently have to descant at greater length,—the absence of any lunar atmosphere, or at least the extreme tenuity of whatever air there may be on the moon. For it is readily seen that the faint light which illuminates the valleys of a mountain-region while as yet only the mountain-tops are in sunlight, comes from the sky, and the light of the sky is due to the existence of an atmosphere.

The reader will find illustrations of the illumination of lunar mountain-tops in the accompanying photographs of the moon near her first and third quarter.

Galileo perceived that in the phenomenon here described he possessed the means of measuring the altitude of the lunar mountains. Without entering into details, it may be remarked that in the case of a mountain standing alone on a wide plain, the distance of the peak, when just touched by the light, from the boundary of light and darkness on the plain, depends obviously on the height of the mountain. For, in fact, if a person is on the summit of the mountain at the moment, he will see the sun on the horizon, and the point on his horizon where he sees the sun is in reality a point on the plain where also the sun is rising at the moment. Now the distance of this point, or of the observer's horizon, depends on the height of the mountain, as is shown in all our text-books of astronomy. Hence, if this distance is known, the height of the mountain can be determined, and what is true of a mountain on our earth is true with certain changes as to details for a mountain on the moon. Now it was in Galileo's power to estimate the apparent distance of a lunar mountain-peak in sunlight from the neighbouring terminator, and to determine thence the real distance in miles. This done, he could estimate the height of the mountain, always supposing that the mountain was isolated and the surrounding region fairly level. Proceeding on this assumption, Galileo was led to the conclusion that several of the lunar mountains are nearly five miles in height.

It will be obvious, however, from a study of the moon at her quarters, that this method cannot be depended upon alone to give trustworthy results; and this will be yet more manifest to any who will examine the moon, when not full, with a telescope of even moderate power. It is seen that as a rule not only are the lunar mountains not isolated, but the surrounding regions are so uneven as to be thrown into light or shadow, confusedly intermixed, when the sun is low down, that is, when they lie near the terminator. There is no means of judging exactly where the mean terminator lies,—that is, where the boundary between light and darkness would lie if the moon were a smooth orb. Accordingly very little reliance could be placed in the measurements of Galileo, or in any estimate of the height of a lunar mountain not based on a long and careful study of the region surrounding the mountain.

It is worthy of notice, in passing, that the recognition of lunar mountains by Galileo was regarded by some of his contemporaries as not his least offence against the Aristotelian philosophy. Even those who admitted that his telescope showed objects which appeared like mountains, maintained that in reality the surface of the moon is smooth. Over the irregularities perceived by Galileo, they argued, there exists a transparent or crystalline shell, filling up the cavities and having an outer surface perfectly smooth, as Aristotle taught. To this argument Galileo gave an answer precisely suited to the value of the objection. "Let them be careful," he replied; "for if they provoke me too far, I will erect on their crystalline shell invisible crystalline mountains, ten times as high as any I have yet described."

Galileo was the first to recognize the great number of craters which exist on certain parts of the moon's surface. He compared the craters in the south-western quadrant of the moon (*see* the accompanying lunar chart) to the "eyes" in a peacock's tail.

Galileo's chart of the moon, though creditable to him considering his imperfect telescopic means, has very little value except as a curiosity. A similar remark applies to the researches of Scheiner, Schirlaus, and others.

At this early stage of lunar research the darker portions of the moon's surface were considered to be seas, the brighter parts being looked upon as land regions. Thus we find Kepler saying: "Do maculas esse maria, do lucidas esse terras." Galileo himself seems to have been better satisfied with his recognition of mountains and valleys on the moon than with the supposed distinction between land and sea regions. It is worthy of notice that in Milton's brief references to the Florentine astronomer, based undoubtedly on the poet's recollection of his interviews with Galileo (see the "Areopagitica"), there is no mention of seas.

Thus in Book I. of "Paradise Lost" Milton compares the shield of Satan to

> "the moon, whose orb
> Through optic glass the Tuscan artist views
> At evening, from the top of Fesolé,
> Or in Val d'Arno, to descry new lands,
> Rivers, or mountains, on her spotty globe."

Again, in the fifth book, Raphael sees the earth

> "as when by night the glass
> Of Galileo, less assured, observes
> Imagined lands and regions in the moon."

It is difficult to suppose that Milton would not have said "oceans" instead of "regions" if Galileo had entertained the opinion that the dark lunar regions are seas.

Hevelius, who next made any considerable advance in the study of the moon's surface, adopted Kepler's opinion as to the distinction between the dark and bright regions of the moon. He constructed a chart which contained more detail and was more correct than Galileo's, and adopted a system of nomenclature indicating his belief in the existence of analogies between lunar and terrestrial regions. Thus we find in his list of names,—mountains, deserts, marshes, seas, lakes, islands, bays, promontories, and straits. In some cases he named these from their imagined resemblance to terrestrial regions; in others, he indicated their appearance as seen in his telescope. Thus, the great crater now called Copernicus, was by Hevelius called Mount Etna; while the dark enclosed surface

called Plato was named by Hevelius "the greater black lake." The chart by Hevelius was necessarily imperfect compared with those now in existence. The telescopic power he employed was very little greater than that used by Galileo; and he had to trust, like Galileo, to mere estimation of the proportions of the different lunar regions, not possessing even the roughest appliances for micrometrical measurement.

Hevelius, following Galileo's method of determining the height of the lunar mountains, deduced three miles as the maximum height.

We owe to Hevelius the recognition of the most important of the lunar librations. Galileo had detected the libration in latitude, and had shown that there must also be a small diurnal libration (see last chapter). Hevelius perceived that spots near the eastern and western edge of the lunar disc were sometimes farther on the disc than at others. He not only showed that this is due to the libration in longitude, but was able to prove that this libration depends on the varying motion of the moon and her (appreciably) uniform rotation.

Hevelius's "Selenographia," which contains his chart (engraved on metal by himself), appeared in 1647. At this time Peyresl and Gassendi were engaged in the construction of a lunar chart; but when they heard that Hevelius had completed such a chart they ceased from their labour, having drawn only one sheet of their chart.

Father Riccioli, of Bologna, published in 1651 a much less valuable chart than that of Hevelius. He adopted a new system of nomenclature, replacing the terrestrial names of Hevelius by the names of astronomers and philosophers. Mädler says, indeed, that Riccioli's work would have been forgotten had he not been led by vanity to find a place for his own name on the moon, an arrangement only to be achieved by displacing all the names used by Hevelius, at the risk of causing perplexity and confusion to later astronomers. The charge is rather a serious one.

Riccioli's estimates of the altitude of the lunar mountains were altogether unsatisfactory.

Dominic Cassini constructed a chart of the moon 12 Paris feet in diameter, but not showing many details. So far as the method of construction was concerned, this map should have been an important improvement on its predecessors. The places of the chief lunar spots were determined by measurement, the other spots were placed by eye-estimates corrected for the effects of libration. In 1680 Cassini constructed a chart 20 Paris inches in diameter, respecting which Mädler remarks that it surpassed Hevel's in fulness of detail but not in correctness. All the copies of this chart were soon sold, and Mädler considers it likely that the chart was unknown in Germany until a new edition was published by Lalande in 1787.

The first really reliable chart of the moon was constructed by Tobias Mayer. During a lunar eclipse in the year 1748, Mayer wished to note the passage of the earth's shadow over the principal lunar features, and he recognized the want of an exact chart of the moon. It would appear from Lichtenberg's account, that Mayer proposed to himself the construction of a chart on a large scale, showing the places of the chief lunar spots determined micrometrically. This plan he was prevented from carrying out by a pressure of other engagements. A small chart, however (7½ Paris inches in diameter), was found among his papers, and was published at Göttingen in 1775, thirteen years after his death, among his "Opera inedita," and remained until 1824 the only trustworthy map of our satellite.

Schröter of Lilienthal studied the moon with great care and patience, using first a 7-feet reflector, then one of 18 feet, and lastly one of 27 feet in focal length. The labours of Schröter as a selenographist were not altogether successful, because of his want of skill in delineating what he saw. Beer and Mädler consider that the accuracy of Schröter's work was further affected by his desire to recognize signs of change in the moon. But Webb, than whom no better authority exists on the subject, says, respecting Schröter's "Selenographische Fragmente," "I have never closed the simple and candid record of his most zealous labours with any feeling approaching to contempt," and he adds that possibly Beer and Mädler were not themselves free from a prepossession opposite to that which they condemned in Schröter.

The work of Lohrmann must be regarded as the first really scientific attempt to delineate the moon's surface in detail. Lohrmann was a land surveyor of Dresden. He planned the construction of a lunar chart on a large scale in 25 sections, and in 1824 the first four sections were published. But failing sight compelled him to desist from his arduous attempt. In 1838 he published an excellent general chart of the moon, 15¼ inches in diameter.*

MM. Beer and Mädler began their selenographic work in 1830, and their 3-feet chart, together with their fine work on the moon,† appeared in 1837. The telescope employed by them was only four inches in diameter, and the chart does not show every feature which can be recognized with a telescope of that aperture. Yet the amount of detail is remarkable, and the labour actually bestowed upon the work will appear incredible to those who are unfamiliar with the telescopic aspect of the moon. In "Der Mond," Beer and Mädler give their measurements of the positions of no less than 919 lunar spots, and 1,095 determinations of the height of lunar mountains.‡ The map which accompanies the present work was reduced by Mr. Webb from the large chart of MM. Beer and Mädler, and owes no small part

* Lohrmann died in 1840.

† "Der Mond, nach seinen kosmischen und individuellen Verhältnissen."

‡ The heights are given in toises, a toise being about 6·3946 English feet. The highest mountain of all is very appropriately named Newton, and according to the measures of Beer and Mädler, its summit is 3,727 toises, or about 23,800 feet above the level of the floor of the crater.

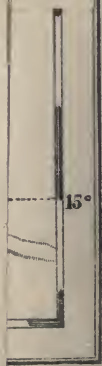
of its value to the fact that the reduction has been made by one who is himself so skilful a student of the moon's surface. The following is Mr. Webb's very modest account of a map which has long been recognized as a most important contribution to selenography:—"It professes to be merely a guide to such of the more interesting features as common telescopes will reach. It has been carefully reduced from the 'Mappa Selenographica' of Beer and Mädler, omitting an immense amount of detail accumulated by their diligent perseverance, which would only serve to perplex the learner. Selection was difficult in such a crowd. On the whole, it seemed best to include every object distinguished by an *independent name;* many of little interest thus creep in, and many sufficiently remarkable ones drop out; but the line must have been drawn somewhere, and perhaps would have been nowhere better chosen for the student. Other spots, however, have been admitted, from their conspicuousness, to which Beer and Mädler have given only a *subordinate* name; minuter details come in, in places, for ready identification; elsewhere larger objects are passed by, as less useful for the purpose of the map."

Two lists of the lunar objects in Mr. Webb's map—first, in the order of the number; and secondly, in alphabetical order—will be found at the end of this volume. They are identical with those in Mr. Webb's "Celestial Objects," third edition.

I may add that in the year 1869 I carefully examined every object included in Webb's map, with a telescope

$2\frac{1}{4}$ inches in aperture, using low powers, and satisfied myself that the map fulfils in every respect the object aimed at by its designer.*

* I cannot indicate a more pleasing occupation for the possessor of a telescope of that size (or any larger size up to four inches) than to go over the moon's disc, examining each object *seriatim*, and carefully comparing what is seen with the account given by Mr. Webb. In particular, it is a most useful and instructive exercise to observe the varying appearance of particular objects as they come into sunlight, as sunlight grows fuller upon them, and afterwards, as sunlight passes away from them, until at length they are in darkness. The most convenient objects to select for this purpose (though it need hardly be said that the true lunarian astronomer will not be content with observing these only) are those which lie near the terminator of the moon rather early during her first quarter, for these will be again on the terminator rather early in the third quarter. Thus they can be observed first in the early evening, and then later and later, until, when the terminator is just leaving them, they must be observed after midnight, but not very late; whereas those objects which are first reached by the advancing terminator during the moon's second quarter are left by the receding terminator during the fourth quarter, and to be well studied at this time must be observed in the early morning hours. Those students of astronomy, however, who are ready to observe at any hour of the night from twilight to dawn, can study any part of the moon from sunrise to sunset at that part. It will be obvious that thoroughly to examine any spot on the moon, it must be observed during many lunations. Apart from the circumstance that unfavourable weather breaks the continuity of the observations, the interval of many hours necessarily elapsing between successive observations suffices to render the study of any spot during any single lunation imperfect. This is especially the case with objects near the eastern and western limbs, because the moon must be nearly new (either before or after conjunction with the sun) when sunrise or sunset occurs at such points, and the moon can only be observed a short time in the morning when she is approaching conjunction, and a short time in the evening soon after


15°

The stereographic map has been constructed by myself from Mr. Webb's map (as it originally appeared); it will be found useful for determining the effects of foreshortening near the edge of the moon's disc.

The labours of Schmidt, of Athens, although not as yet fully published, must be regarded as altogether the most important contribution yet made to selenography. The observations on which the construction of his chart has been based were commenced in 1839, and in 1865 Schmidt began to combine these observations together into chart-form. He proposed at that time to have a chart with a diameter of 6 Paris feet, and divided into four quadrants, like Mädler's chart. The telescope employed for reviewing the observations was the refractor of the Athens Observatory, having

conjunction. But even for other parts of the moon the difficulty exists. An observer may watch the progress of sunrise at any spot near the terminator of the half-moon, hour after hour, for several hours in succession; but he must be interrupted for a much longer period, after the moon has approached the horizon too low for useful study, until she is again at a fair elevation. Now in the interval —say sixteen or seventeen hours—sunrise or sunset at the spot will have made great progress, notwithstanding the great length of the lunar day. For sixteen hours on the moon (about a forty-fourth part of the lunar day) correspond to more than half an hour on the earth, and we know that in every part of the earth the sun's place on the heavens alters considerably in half an hour. In fact, in sixteen hours, the sun, as seen from the moon, changes his place by about eight degrees, and this must importantly affects the position and dimensions of the shadows thrown by any lunar heights, especially near the time of sunrise and sunset. It is further to be considered that the circumstances under which a lunar spot is studied vary markedly during the progress of a lunation.

an aperture of 6 Paris inches. In April, 1868, the work had progressed so far that Schmidt was able to form an opinion as to the probable value of a chart completed on the adopted plan. He was dissatisfied with the result. The work was not exact enough nor sufficiently delicate in drawing for his purposes. He determined therefore to begin the charting afresh. Retaining the original diameter of 6 Paris feet, he divided the chart into 25 sections, adopting Lohrmann's arrangement. Each section forming a much smaller map than the former quadrants, it was possible to adopt a much finer and more exact method of drawing. He began this work in April, 1868, and it is now ready for publication. I believe there are difficulties on the score of expense, but these will surely be surmounted. When a man has given the labours of a life, or the best part of a life, to a scientific work of such great difficulty, and with results so valuable, it is not asking too much that means should be found for publishing the work in a way securing to its author a just reward for his untiring exertions.

The map of Bullialdus and the surrounding region affords an idea of Schmidt's method of delineation. It has been reduced, however, considerably from the original. The reader should seek out Bullialdus in Mr. Webb's map (it is numbered 213, and is in the third quadrant). The comparison of the two maps will afford an excellent idea of the scale on which Schmidt has carried out his processes of charting.

It remains to be mentioned that a chart of the moon is in course of preparation under the supervision of Mr. Birt, and in accordance with a scheme projected by the British Association.

The application of photography to the moon, closely associated with the subject of lunar charts, has next to be considered.

So early as 1840 Arago dwelt on the possibility that the moon might be persuaded to take her own portrait,—speaking of the hope that instead of those long and wearisome labours by which men had hitherto sought to chart the moon, a few minutes might suffice to bring her image on Daguerre's prepared plates. However, in the very year when Arago made this remark, Dr. Draper, of New York, had succeeded in photographing the moon. The following history of photographic work on the moon is abridged from a chapter on the subject contributed by Mr. Brothers to Chambers' "Handbook of Descriptive Astronomy":—

"It appears from a paper by Professor H. Draper, of New York, published in April, 1864, that in the year 1840 his father, Dr. J. W. Draper, was the first who succeeded in photographing the moon. Dr. Draper states that at the time named (1840) 'it was generally supposed the moon's light contained no actinic rays, and was entirely without effect on the sensitive silver compounds used in daguerreotyping.' With a telescope of 5 inches aperture Dr. Draper obtained pictures on silver plates, and presented them to the Lyceum of Natural History of New

York. Daguerre is stated to have made an unsuccessful attempt to photograph the moon, but I have been unable to ascertain when this experiment was made.

"Bond's photographs of the moon were made in 1850. The telescope used by him was the Cambridge (U.S.) refractor of 15 inches aperture, which gave an image of the moon at the focus of the object-glass 2 inches in diameter. Daguerreotypes and pictures on glass mounted for the stereoscope were thus obtained, and some of them were shown at the Great Exhibition of 1851, in London.

"Between the years 1850 and 1857 we find Secchi in Rome, and Bertch and Arnauld in France, and in England Phillips, Hartnup, Crookes, De La Rue, Fry, and Huggins, appearing as astronomical photographers. To these may be added the name of Dancer, of Manchester, who in February, 1852, made some negatives of the moon with a 4¼-inch object-glass. They were small, but of such excellence that they would bear examination under the microscope with a 3-inch objective, and they are believed to be the first ever taken in this country. Baxendell and Williamson, also of Manchester, were engaged about the same time in producing photographs of the moon.

"The first detailed account of experiments in celestial photography which I have met with is by Professor Phillips, who read a paper on the subject at the meeting of the British Association at Hull in 1853. In it he says: 'If photography can ever succeed in portraying as much of the moon as the eye can see and discriminate, we shall be able to leave to future times monuments by which the secular changes of the moon's physical aspect may be determined. And if this be impracticable—if the utmost success of the photographer should only produce a picture of the larger features of the moon, this will be a gift of the highest value, since it will be a basis, an accurate and practical foundation of the minuter details, which, with such aid, the artist may confidently sketch.' The pictures of the moon taken by Professor Phillips were made with a 6¼-inch refractor, by Cooke, of 11 feet focus: this produced a negative of 1¼ inch diameter in 30 seconds. Professor Phillips does not enter very minutely into the photographic part of the subject, but he gives some very useful details of calculations as to what may be expected to be seen in photographs taken with such a splendid instrument as that of Lord Rosse. It is assumed that an image of the moon may be obtained *direct* of 12 inches diameter, and this, when again magnified sufficiently, would show 'black bands 12 yards across.' What may be done remains to be seen, but up to the present time these anticipations have not been realized.

"We have next, from the pen of Crookes, a paper communicated to the Royal Society of London in December, 1856, but which was not read before that Society until February in the following year. Mr. Crookes appears to have obtained good results as early as 1855, and, assisted by a grant from the

Donation Fund of the Royal Society, he was enabled to give attention to the subject during the greater part of the year following. The details of the process employed are given with much minuteness. The telescope used was the equatorial refractor at the Liverpool Observatory, of 8 inches aperture and $12\frac{1}{2}$ feet focal length, which produced an image of the moon 1·35 inch diameter. *The body of a small camera* was fixed in the place of the eye-piece, so that the image of the moon was received in the usual way on the ground glass. The chemical focus of the object-glass was found to be $\frac{8}{10}$ths of an inch beyond the optical focus, being over-corrected for the actinic rays. Although a good clock movement, driven by water-power, was applied to the telescope, it was found necessary to follow the moon's motion by means of the slow-motion handles attached to the right ascension and declination circles, and this was effected by using an eye-piece, with a power of 200 on the finder, keeping the cross-wires steadily on one spot. With this instrument Hartnup had taken a large number of negatives, but owing to the long exposure required he was not successful; but with more suitable collodion and chemical solutions, and although the temperature of the Observatory was below the freezing-point, Mr. Crookes obtained dense negatives in about 4 seconds. Crookes afterwards enlarged his negatives 20 diameters, and he expresses his opinion that the magnifying should be conducted simultaneously with the photography by having a proper arrangement of lenses, so as to throw an enlarged image of the moon at once on the collodion plate; and he states that the want of light could be no objection, as an exposure of from 2 to 10 *minutes* would not be 'too severe a tax upon a steady and skilful hand and eye.'

"In an appendix to his paper Mr. Crookes gives some particulars as to the time required to obtain negatives of the moon with different telescopes, from which it appears that the time varied from 6 minutes to 6 seconds. The different results named must, I conclude, have been caused not so much by the differences in the instruments as in the various processes employed, and in the manipulation. I must observe, also, that it is not stated whether all the experiments were tried upon the *full* moon—a point materially affecting the time.

"In 1858 De La Rue read an important paper before the Royal Astronomical Society, from which it appears that the light of the moon is from 2 to 3 times brighter than that of Jupiter,* while its actinic power is only as 6 to 5, or 6 to 4. On Dec. 7, 1857, Jupiter was photographed in 5 seconds and Saturn in 1 minute, and on another occasion the moon and Saturn were photographed in 15 seconds just after an occultation of the planet.

"The report of the Council of the Royal Astrono-

* Theoretically the light of the moon should be nearly 27 times as bright as Jupiter's, since Jupiter is $5\frac{1}{5}$ times farther from the sun.

mical Society for 1858 contains the following remarks :—'A very curious result, since to some extent confirmed by Professor Secchi, has been pointed out by De La Rue, namely, that those portions of the moon's surface which are illumined by a very oblique ray from the sun possess so little photogenic power that, although to the eye they appear as bright as other portions of the moon illumined by a more direct ray, the latter will produce the effect, called by photographers solarization, before the former (the obliquely-illumined portions) can produce the faintest image.' And the report also suggests that the moon may have a comparatively dense atmosphere, and that there may be vegetation on those parts called seas.

"At the meeting of the British Association at Aberdeen, in 1859, De La Rue read a very valuable paper on Celestial Photography. An abstract of it was published at the time in the *British Journal of Photography*, and in August and September of the following year further details of this gentleman's method of working were given in the same journal. The processes and machinery employed are so minutely described that it is unnecessary here to say more than that he commenced his experiments about the end of 1852, and that he used a reflecting telescope* of his

* "The advantage of the reflecting over the refracting telescope is very great, owing to the coincidence of the visual and actinic foci; but it will presently appear that the refractor can be made to equal, if not excel, the work of the reflector."

own manufacture of 13 inches aperture and 10 feet focal length, which gives a negative of the moon averaging about $\frac{1}{10}$th of an inch in diameter. The photographs were at first taken at the side of the tube after the image had been twice reflected. This was afterwards altered so as to allow the image to pass direct to the collodion plate, but the advantage gained by this method was not so satisfactory as was expected. In taking pictures at the side of the tube, *a small camera box* was fixed in the place of the eye-piece, and at the back a small compound microscope was attached, so that the edge of a broad wire was always kept in contact with one of the craters on the moon's surface, the image being seen through the collodion film at the same time with the wire in the focus of the microscope. This ingenious contrivance, in the absence of a driving-clock, was found to be very effectual, and some very sharp and beautiful negatives were thus obtained. De La Rue afterwards applied a clockwork motion to the telescope, and his negatives taken with the same instrument are as yet the best ever obtained in this country.

"Nearly a quarter of a century has elapsed since the moon was first photographed in America, and a good deal has been done since on that side of the Atlantic. To an American we are indebted for the best pictures of our satellite yet produced, and it is difficult to conceive that anything superior can ever be obtained; and yet with the fact before us that De La Rue's are better than any others taken in this country, so it may prove that even the marvellous pictures of Mr. Rutherfurd may be surpassed.

"Mr. Rutherfurd appears, from a paper in Silliman's *American Journal of Science* for May, 1865, to have begun his work in lunar photography in 1858 with an equatorial of $11\frac{1}{4}$ inches aperture and 14 feet focal length, and corrected in the usual way for the visual focus only. The actinic focus was found to be $\frac{7}{10}$ths of an inch longer than the visual. The instrument gave pictures of the moon, and of the stars down to the fifth magnitude, satisfactory when compared with what had previously been done, but not sufficiently so to satisfy Mr. Rutherfurd, who, after trying to correct for the photographic ray by working with combinations of lenses inserted in the tube between the object-glass and sensitive plate, commenced some experiments in 1861 with a silvered mirror of 13 inches diameter, which was mounted in a frame and strapped to the tube of the refractor. Mr. Rutherfurd enumerates several objections to the reflector for this kind of work, but admits the advantage of the coincidence of foci. The reflector was abandoned for a refractor specially constructed, of the same size as the first one, and nearly of the same focal length, but corrected only for the chemical rays. This glass was completed in December, 1864, but it was not until March 6th of the following year that a sufficiently clear atmosphere occurred, and on that night the negative was taken from which the prints were made."

Mr. Brothers has himself taken many photographs

of the moon with great success, though using a telescope (refracting) only five inches in aperture. The convenience and simplicity of the arrangements he employed will be recognized when it is mentioned that on the evening of the partial eclipse of the moon, Oct. 4, 1865, he succeeded, "with the help of two assistants, in taking no less than 20 negatives, though the telescope was several times disturbed to oblige friends who desired to see the progress of the eclipse through the instrument."

Before passing to the description of the general results which have followed from the telescopic observation of the moon, as well as from processes of charting and photographing, it will be well to discuss the observations which have been made on the moon's light—viz., first on the total quantity of light which she reflects, when full, towards the earth; secondly, on the varying proportion of light so reflected when she is at her phases; and thirdly, on the different light-reflecting qualities of different portions of her surface.

The consideration of the total quantity of light reflected by the moon implies in reality the question what degree of *whiteness* she possesses. For a perfectly white object * would reflect all the light it received, but a coloured object reflects only a portion,

* There is no such thing as perfect whiteness in nature (referring to opaque objects). Even new-fallen snow does not reflect so much as four-fifths of the incident light. The following table (resulting from the observations of Zöllner) is useful for purposes

while a perfectly black object would reflect none. An object of many colours—and the moon unquestionably is such an object—may be said to tend (as a whole) towards blackness or whiteness, according as it reflects less or more of the light which shines upon it.

Let us first consider the comparison between the moon's light and the sun's, according to the best observations hitherto made :—

The observations of Bouguer assigned to the moon a total brightness equal to one 300,000th part of the sun's. The method he employed was the direct comparison between sunlight and candlelight, and between moonlight and candlelight. Wollaston also took candlelight as the means of comparison, but determined the relative brightness of the sources of light

of comparison, the total light incident on a surface being represented by unity :—

Snow just fallen reflects	0·783
White paper	0·700
White sandstone	0·237
Clay marl	0·156
Quartz porphyry	0·108
Moist soil	0·079
Dark grey syenite	0·078

These objects shine by diffused reflected light. For light regularly reflected the following table is useful :—

Mercury reflects	0·648
Speculum metal	0·535
Glass	0·040
Obsidian	0·032
Water	0·021

by the method of equalizing the shadows. He obtained the result that the moon's light is but one 801,070th part of the sun's.

We owe, however, to Zöllner the most satisfactory determination of the moon's total brightness. He employed two distinct methods. In one he determined the illumination by comparing surface-brightness; in the other he obtained point-like images of the sun and moon for comparison with corresponding images of candle-flames. The results obtained by these two methods were in close agreement,—according to one, the light of the full moon is one 618,000th part of the sun's light, while, according to the other, the proportion is as 1 to 619,000.

It would be easy to determine from this result the exact proportion of the incident light which the moon's surface, regarded as a whole, is capable of reflecting, if the moon were a smooth but unpolished sphere: for we know exactly what proportion of the sun's light the moon intercepts, and it is also known that a smooth half-sphere seen under full illumination, reflects two-thirds of the light which a flat round disc of the same diameter would reflect. But the problem is complicated in the present instance by the unevenness of the moon's surface, which causes the light to fall upon various parts of the lunar surface at angles very different from those in the case of a smooth sphere. In fact, it is perfectly manifest from the aspect of the full moon, that we have to deal with a case very different from that of a smooth, or, as it is called, *mat* surface. For such a surface, seen as a disc under full solar illumination, would be brightest at the centre, and shade off gradually to the edge; whereas it is patent to observation that the disc of the full moon is as bright near the edge as near the centre.* Before we can undertake the inquiry, therefore, into the moon's average brightness, we must endeavour to ascertain what effect should be ascribed to the inequalities upon her surface.

This has been accomplished by Zöllner in a sufficiently satisfactory manner, by comparing the total quantity of the moon's light at her various phases, with what would be obtained if the moon were a smooth sphere. It is obvious that as the different parts of the moon's disc, when she is full, do not shine with the brightness due to a smooth surface, we might expect to find her total brightness at any given phase markedly different from the value estimated for the case of a smooth sphere. This Zöllner found to be the case. The 'full' moon is far brighter by comparison with the gibbous moon (especially when little more than half full), than would happen if she were smooth. Now the considerations on which Zöllner based his interpretation of this peculiarity are not suited to

* It is necessary to exercise some caution, however, in adopting a result of this kind, since the eye is very readily deceived. We see the full moon on a dark background, and this certainly tends to add to the apparent brightness of the edge of the disc. As a case illustrating this effect of contrast, it may be mentioned that Jupiter appears to the eye to be brighter near the edge than near the middle of the disc, and yet when his disc is examined with a graduated darkening glass, it is found to be brighter near the middle than near the edge.

these pages, involving analytical considerations of some complexity. The result, therefore, is all that need here be stated. Zöllner's conclusion is, that the average slope of the lunar inequalities amounts to about 52 degrees.* Be it noticed, that this result is

* The following table will show how closely the results obtained by Zöllner agreed with the empirical formula which he deduced from his estimate of the mean slope of the lunar irregularities. The first column gives the distance of the moon from full, the distance being regarded as positive when the observation was made after the time of full moon, and as negative when the observation preceded full moon :—

Arc from Moon's place to point opposite the Sun.	Theoretical Brightness. Full Moon's as 100.		Observed Brightness.
	Moon regarded as smooth.	By Zöllner's formu'a.	Zöllner.
+1°	99·98	98·60	98·60
5	99·63	92·79	87·20
8	99'06	88·41	92·19
11	98·24	84·04	88·76
−13	97·57	81·21	82·60
+19	94·93	72·29	68·41
24	92·13	65·15	71·38
27	90·18	61·00	57·90
−27	90·18	61·00	63·47
+28	89·50	59·60	56·15
−28	89·50	59·60	57·00
+23	85·82	52·90	48·60
−39	80·87	45·00	41·70
+40	80·04	43·70	47·10
41	77·78	42·50	43·95
−42	76·27	41·40	38·00
+46	74·61	36·70	36 10
−52	68·87	27·63	29·11
+58	62·91	24·30	27·10
−62	58·89	20·60	20·40
−69	51·82	15·20	14·60

in no degree affected by observations of the apparent slope of lunar mountains and craters, because irregularities much smaller than any which the telescope can detect, would suffice to explain the observed variations of brilliancy. If the whole surface of the moon were covered with conical hills only a foot, or even only an inch, in height, the same general result would be produced as though there were mountains of the same form a mile, or several miles, in height.

It appears from this result, that the brightness of the full moon is considerably greater than it would be if the moon were a smooth sphere; and, in fact, Zöllner would seem to regard the brightness of the full moon as very nearly equal to that of a flat disc of equal diameter. I do not enter here into a calculation of the quantity of light which such a disc would reflect; but the following result may be accepted as sufficiently near the truth. A perfectly white disc of the same diameter as the moon's, and under direct solar illumination, would have a total brightness equal to about one 92,600th part of the sun's. Now we have seen that the actual quantity received from the moon is about one 618,000th part of the sun's light; and taking into account the smaller mean disc of the moon, as compared with the sun, we find that the moon's light is rather more than one-sixth part of that of a disc of perfect whiteness, under direct solar illumination, and looking as large as the moon's disc. Zöllner deduces from his estimate of the mean irregularity of the moon's surface, a result so near to this as to imply what I have just stated,—viz., that he regards the brightness of the full moon as not much less than that of a flat disc of equal size, and having a surface of the same average reflective power. For he sets the light of the full moon as rather less than a sixth part of that which it would have if the moon were made of a perfectly white substance. The exact proportion assigned by him is that of 1,736 to 10,000. This is what, following Lambert, he calls the *albedo,* or whiteness of the moon, and he justly remarks that, considering her whole brightness, she must be regarded as more nearly black than white. Nevertheless, he adds that from his estimates of the moon's brighter parts he is satisfied that their whiteness can be compared with that of the whitest of terrestrial substances.*

It is worthy of notice that Sir John Herschel had already in a far simpler way deduced a result closely agreeing with Zöllner's. It will be seen from the table in the note at p. 232 that white sandstone reflects about 0·237 of the incident light; and it may be inferred from other values in that table that weathered sandstone rock would have an *albedo* of about 0·150. Now Herschel remarks that the actual illumination of the lunar surface is not much superior to that of weathered sandstone rock in full sunshine. "I have frequently," he proceeds, "compared the moon

* His words are: "Dass der Mond an seinen *helleren* und hellsten Stellen aus einem Stoffe besteht, der, auf die Erde gebracht zu dem weisesten der uns bekannten Körper gezählt werden würd."

setting behind the grey perpendicular façade of the Table Mauntain, illuminated by the sun just risen in the opposite quarter of the horizon, when it has been scarcely distinguishable in brightness from the rock in contact with it. The sun and moon being nearly at equal altitudes, and the atmosphere perfectly free from cloud or vapour, its effect is alike on both luminaries."*

A difficulty will present itself to most readers on a first view of Zöllner's result. The full moon, taken as a whole, appears white when high above the horizon on a dark clear night; and it appears quite impossible to regard her as more nearly black than white. Again, as another form of the same difficulty, it appears obvious to any one who regards ordinary sandstone or any substance of like reflective power in full daylight, that the brightness of the substance is markedly inferior to that of the full moon at midnight. Herein is illustrated one of those effects of contrast which are so deceptive in all questions of relative brightness. We see the full moon in a dark

* It is to be noted, however, that the illumination of the sandstone would be reduced by atmospheric absorption, which would not happen, of course, with the moon. The effect of atmospheric absorption in reducing the apparent brightness of the moon thus fully illuminated, and of the sandstone thus not quite fully illuminated, would not be equal, because the sandstone was seen through only a portion of the atmospheric strata interposed between the eye and the moon. Hence would result a near approach to equalization so far as atmospheric effects are concerned.

background, and with no other object comparable to her in brightness, and the eye accordingly overestimates her light,—a comparison is made between her real but not obvious partial blackness, and the very obvious and much greater blackness of the sky, and thus the idea of whiteness is suggested. On the contrary, when we look at stone or rock illuminated by full sunlight, objects as brightly, or even more brightly, illuminated are all around, and the eye accordingly estimates fairly, or perhaps even underestimates, the whiteness of the illuminated rock.*

* Amongst hundreds of illustrations of the effect of contrast in deceiving the eye in such cases (a subject of the utmost importance in astronomical observations) may be mentioned our estimate of the brightness of the old moon in the new moon's arms. Nothing can be more certain than that in reality the light of the old moon in this case is due to illumination by the earth, and at a moderate computation this illumination exceeds full moonlight twelve times. (The only doubtful point is the average light-reflecting quality of the earth's surface, which I am here assuming to be rather less than that of the moon's surface.) Now we know how bright a landscape appears when bathed in full moonlight, and we can infer that under twelve times that amount of light the brightness would be very considerable. Assuredly an object as large in appearance as the moon would under such light appear very conspicuous, and *white.* Yet the old moon in the new moon's arms, though illuminated to this degree, can scarcely be perceived at all until twilight has made some progress. The light of the early evening sky is quite sufficient to render the considerable light of the old moon quite imperceptible. To this may be added the fact that the disc of the moon during total eclipses, although it appears so dark to the eye, is nevertheless illuminated by nearly full earth-light, and certainly with ten times the lustre of a terrestrial landscape under full moonlight.

Returning to Zöllner's remark that the brightest parts of the moon are comparable in whiteness with the whitest terrestrial substances, it follows obviously that the darker portions of the moon are very much less bright than the average. Thus the bright summit of Aristarchus, whose reflective power is so great that, as seen on the dark part of the moon (when therefore only illuminated by earth-light), it has been mistaken for a volcano in eruption, has probably a reflective power equal to that of new-fallen snow, or 0·783, which exceeds the average whiteness of the moon about 4½ times. And we may assume that the dark floor of Plato and the yet darker Grimaldi are as far below the average of brightness. But even dark-grey syenite, the lowest in reflective power of all the substances in Zöllner's table (see note, p. 235), reflects 0·078 of the incident light, which indicates a whiteness nearly half the average whiteness of the moon's surface. We may safely assume that the darkest parts of the moon are blacker than this.

A question of much greater difficulty is suggested by observations which appear to indicate changes in the brightness of certain lunar regions. Some observations of this kind are referred to in a subsequent chapter. At present I shall merely remark that such observations do not appear to have been hitherto made in such a way as to afford convincing evidence that change takes place with the progress of the lunar day. In particular, it seems to me that the readiness with which the eye may be deceived by the effect of contrast has not been duly taken into account. Therefore, while recognizing, in the observations directed to the recognition of tint-changes or colour-changes, a possible means of advancing in a very marked manner our knowledge of the moon's condition, I find myself at present unable to regard as demonstrated any of the phenomena which are described by those who have made researches in this department of selenography.*

In considering the general results of the telescopic scrutiny of the moon, it is well to remember the circumstances under which such scrutiny has been made.

The highest power yet applied to the moon (a power of about six thousand) brings her, so to speak, to a distance of forty miles,—a distance far too great for objects of moderate size to become visible. Many of my readers have probably seen Mont Blanc from the

* I have been supplied by Mr. Neison, F.R.A.S., with a series of very interesting observations made by him, which tend to show that the Floor of Plato darkens with the advance of the lunar day and grows lighter as the day wanes, confirming observations of a similar character made and collected by Mr. Birt. But a careful study of these observations, as well as of all the observations of the same kind that I have had access to, has not satisfied me that the whole series of phenomena may not be subjective merely. No sufficient precautions have been hitherto taken to eliminate as far as possible the effect of contrast, and till this has been done it would be unsafe to adopt any conclusion as demonstrated. Moreover, the assumption that the floor is smooth seems to me altogether unsafe. We cannot possibly be certain that it is not covered with irregularities too small to be individually discernible.

neighbourhood of Geneva, a distance of about forty miles. At this distance the proportions of vast snow-covered hills and rocks are dwarfed almost to nothingness, extensive glaciers are quite imperceptible, and any attempt to recognize the presence of living creatures or of their dwellings (with the unaided eye) is utterly useless. But even this comparison does not present the full extent of the difficulties attending the examination of the moon's surface with our highest powers. The circumstances under which such powers are applied are such as to render the view much less perfect than the mere value of the magnifying power employed might seem to imply. We view celestial objects through tubes placed at the bottom of a vast aërial ocean, never at rest through any portion of its depth; and the atmospheric undulations which even the naked eye is able to detect are magnified just in proportion to the power employed. These undulations are the bane of the telescopist. What could be done with telescopes, if it were not for these obstructions to perfect vision may be gathered from the results of Professor Smyth's observations from the summit of Teneriffe. Raised here above the densest and most disturbed strata, he found the powers of his telescope increased to a marvellous extent. Stars which he had looked for in vain with the same instrument in Edinburgh now shone with admirable distinctness and brilliancy. Those delicate stipplings of the discs of Jupiter and Saturn, which require in England the powers of the largest telescopes, were clearly seen in the excellent but small telescope he employed in his researches. It is probably not too much to say that even if the Rosse telescope were perfect in defining power, which unfortunately is very far indeed from being the case, yet on account of atmospheric disturbance, instead of reducing the moon's distance to forty miles, it would in fact not be really effective enough to reduce that distance to less than 150 miles.

Accordingly, though we recognize in the grey plains or seas on the moon the appearance of smoothness, it is very far from being certain that these regions may not in reality be covered with irregularities of very considerable slope. The assumption that they resemble old sea-bottoms, or that in smoothness they are analogous to deserts and prairies on our own earth, seems an unsafe one. The uniformity of curvature which marks their surfaces as a whole does indeed afford an argument in favour of their having once been in a liquid condition; but that their solidification should have resulted in a smooth surface is far from being certain. On the contrary, it seems not unlikely that the true surface may be marked with corrugations, or crystalline formations, or other uniform unevennesses, if one may so speak.

It is a noteworthy circumstance that the lunar plains do not form portions of the same sphere, some lying deeper than others, that is, belonging to a sphere of smaller radius.

Again, it is to be remembered that the mountain-chains on the moon are seen under circumstances which enable us to recognize none but the boldest features of these formations. It is as unsafe to theorize as to their geological or selenological conformation, as it would be to speculate on the structure of a mountain-range on earth which had only been seen from a distance of two or three hundred miles. The following description by Mr. Webb must be read with this consideration carefully held in remembrance:—"The mountain-chains," he remarks, "are of very various kind: some are of vast continuous height and extent, some flattened into plateaux intersected by ravines, some rough with crowds of hillocks, some sharpened into detached and precipitous peaks. The common feature of the mountain-chains on the earth—a greater steepness along one side—is very perceptible here, as though the strata had been tilted in a similar manner. Detached masses and solitary pyramids are scattered here and there upon the plains, frequently of a height and abruptness paralleled only in the most craggy regions of the earth. Every gradation of cliff and ridge and hillock succeeds; among them a large number of narrow banks" (that is, of banks which look narrow at the enormous distance from which they are seen), "of slight elevation but surprising length, extending for vast distances through level surfaces: these so frequently form lines of communication between more important objects, uniting distant craters or mountains, and crowned at intervals by insulated hills, that Schröter formerly, and Beer and Mädler in modern times, have ascribed them to the horizontal working of an elastic force, which, when it reached a weaker portion of the surface, issued forth in a vertical upheaval or explosion. The fact of the communication," he justly adds, "is more obvious than the probability of the explanation."

But although, as will be manifest from the photographs which illustrate this work, the lunar mountain-ranges form by no means an unimportant feature of the moon's surface, the crateriform mountains must be regarded as the more characteristic feature. If we adopt Mallet's theory of the formation of surface-irregularities on a planet, we must assume that the intermediate stage between the formation of great elevated and depressed regions corresponding to our continents and oceans, and the epoch of volcanic activity, lasted but a relatively short time. If the crateriform mountains were due to volcanic action, then that action must have lasted longer, must have been more widespread, and must have been also far more intense than on our own earth. The considerations thus suggested are discussed in a subsequent chapter. Here I shall consider only the classification of the crateriform mountains. They may be conveniently divided, after Webb, into walled or bulwarked plains, ring mountains, craters, and saucer-shaped depressions or pits. "The second and third," he remarks, "differ chiefly in size; but the first have a character of their own, in the perfect resemblance of their interiors to the grey plains, as though they had been originally deeper, but filled in subsequently with the same material, many of them, in fact, bearing evident marks of having been broken down and overflowed from the outside. Their colour is often suggestive of some kind of vegetation, though it is difficult to remark this with the apparent deficiency of air and water. It has been ingeniously suggested that a shallow stratum of carbonic acid gas, the frequent product of volcanoes and long surviving their activity (for instance, among the ancient craters of Auvergne, where it exists in great quantity), may in such situations support the life of some kind of plants; and the idea deserves to be borne in mind in studying the changes of relative brightness in some of these spots. The deeper are usually the more concave craters; but the bottom is often flat, sometimes convex, and frequently shows subsequent disturbance in ridges, hillocks, minute craters, or more generally, as the last effect of eruption, central hills of various heights, but seldom attaining that of the wall, or even, according to Schmidt, the external level. The ring is usually steepest within, as in terrestrial craters, and many times built up in vast terraces, frequently lying, Schmidt says, in pairs divided by narrow ravines. Nasmyth refers these — not very probably—to successively decreasing explosions; in other cases he more reasonably ascribes them to the slipping down of materials upheaved too steeply to stand, and undermined by lava at their base, leaving visible breaches in the wall above. They would be well explained on the supposition of fluctuating levels in a molten surface. Small transverse ridges occasionally descend from the ring, chiefly on the outside; great peaks often spring up like towers upon the wall; gateways at times break through the rampart, and in some cases are multiplied till the remaining piers of wall resemble the stones of a huge megalithic circle."

The accompanying picture of Copernicus,* taken by Father Secchi with the fine refractor of the Roman Observatory, aptly illustrates the appearance of large craters when seen with powerful telescopes. I give Mr. Webb's description of the crater in full, as showing his method of dealing with lunar details, in the admirable work to which I have already invited the reader's attention at p. 219. "Copernicus," he says, "is one of the grandest craters, 56 m. in diameter. It has a central mountain (2,400 feet in height, according to Schmidt), two of whose six heads are conspicuous; and a noble ring composed not only of terraces, but distinct heights separated by ravines; the summit, a narrow ridge, not quite insular, rises 11,000 feet above the bottom, the height of Etna, after which Hevel named it. Schmidt gives it nearly 12,800 feet, with a peak of 13,500 feet, west; and an inclination in some places of 60°. Piazzi Smyth observed remarkable resemblances

* The cut is one of the large number of engravings illustrating Fr. Secchi's book on the sun now in my hands, and under process of translation. It has been kindly lent to me by Messrs. Longmans for the illustration of the present work.

between the interior conchoidal cliffs and those of the great crater of Teneriffe. A mass of ridges leans

The Lunar Crater Copernicus (Secchi).

upon the wall, partly concentric, partly radiating: the latter are compared to lava. The whole is beautifully, though anonymously, figured in Sir J. Herschel's 'Outlines of Astronomy.' There is also a large drawing by Secchi" (from which the accompanying picture has been reduced); "but this grand object requires, and would well reward, still closer study. It comes into sight a day or two after the first quarter. Vertical illumination brings out a singular cloud of white streaks related to it as a centre. It is then very brilliant, and the ring sometimes resembles a string of pearls. Beer and Mädler once counted more than fifty specks."

Schmidt's map of Bullialdus and the neighbourhood also well illustrates the nature of the lunar crateriform mountains of various dimensions. But yet further insight into the characteristics of the more disturbed and uneven portions of the moon's surface will be obtained from the study of Plate XX., which represents a very rough and volcanic portion of the moon's surface, as modelled from telescopic observations by Mr. Nasmyth. The engraving was taken from a photograph of the original model furnished to Sir J. Herschel by Mr. Nasmyth; and I am indebted to Messrs. Longmans for permission to use this admirable engraving in the present work.

"A succession of eruptions may be constantly traced," Mr. Webb remarks, "in the repeated encroachment of rings on each other, where, as Schmidt says, the ejected materials seem to have been disturbed before they had time to harden, and the largest are thus pointed out as the oldest craters, and the gradual decay of the explosive force, like that of many terrestrial volcanoes, becomes unquestionable. The peculiar whiteness of the smaller craters may indicate something analogous to the difference between the earlier and later lavas of the earth, or to the decomposition caused, as at Teneriffe, by acid vapours in the grey levels. We thus perhaps obtain an indication of the superficial character of their colouring."

The lunar valleys include formations as remarkable as the long banks described above,—viz., the *clefts* or *rills*, furrows extending with perfect straightness for long distances, and changing in direction (if at all) suddenly, thereafter continuing their course in a straight line. These were first noticed by Schröter, and a few were discovered by Gruithuisen and Lohrman; but Beer and Mädler added greatly to the known number, which was raised by their labours to 150. Schmidt has discovered nearly 300 more. Mr. Webb makes the following remarks on the rills:—"These most singular furrows pass chiefly through levels, intersect craters (proving a more recent date), reappear beyond obstructing mountains, as though carried through by a tunnel, and commence and terminate with little reference to any conspicuous feature of the neighbourhood. The idea of artificial formation is negatived by their magnitude (Schmidt gives them 18 to 92 miles long, $\frac{1}{2}$ to $2\frac{4}{10}$ miles broad): they have been more probably referred to cracks in a shrinking

Pl XX.

H.Adlard sc.

London: Longmans & Co.

surface. The observations of Kunowsky, confirmed by Mädler at Dorpat, seem in some instances to point to a less intelligible origin in rows of minute contiguous craters; but a more rigorous scrutiny with the highest optical aid is yet required."

A feature which is well seen in the illustrative photographs, but best in the view of the full moon, is the existence of radiating streaks from certain craters. The most remarkable system of rays is that which has the great crater Tycho as its centre. It will be seen from the photograph of the full moon that this system can be recognized over a very large proportion of the visible hemisphere, and doubtless extends on the south (that is, the uppermost part of the picture) far upon the unseen hemisphere. The same picture well exhibits the radiating bright streaks from Copernicus, Kepler, and Aristarchus; and three other less striking systems can be recognized in this view of the moon. The telescope shows others. "In some cases," Webb remarks, "the streaks proceed from a circular grey border surrounding the crater; in others they cross irregularly at its centre. They pass alike over mountain and valley, and even through the rings and cavities of craters, and seem to defy all scrutiny" (query, interpretation). Nichol makes the following suggestive comments on this peculiarity, though in quoting his remarks I would not be understood to imply assent to them in all respects:—"They consist of broad brilliant bands (seen in their proper splendour only when the moon is full) issuing from all sides of the crater, and stretching to various distances from their origin,—one of them can be traced along a reach of 1,700 miles. There are several *defining* characteristics of these bands. *First,* It is only when the moon is full that we see them in their entire clearness. They may be traced, although very faintly, when the moon is not full: their splendour at full moon is very great. This cannot wholly be attributed to the effect of direct instead of oblique light, because at the edges of the moon's apparent disc, on which the solar ray falls very obliquely at full moon, their brilliancy is the same. No rational explanation whatever has been proposed regarding this remarkable peculiarity. *Secondly,* The light thrown towards us by the rays from Tycho is of the same kind as that reflected from the edge and centre of the crater itself; so that the matter of which they are composed had probably the same origin as those other portions of Tycho. *Thirdly,* These rays pass onward in thorough disregard of the general contour of the moon's surface; nowhere being turned from their predetermined course by valley, crater, or mountain-ridge. Now, this critical fact quite discredits the hypothesis that they are akin to lava, or that they are merely superficial. A stream of lava spreads out on meeting a valley or low-land, and forms a lake; nor can it ever overpass a mountain barrier. The question remains then, are these rays composed of matter that has been shot up from the *interior* of the moon? It may seem incredible that we can solve this problem by virtually digging pits of vast depth down through those singular bands, and thus ascertaining practically that the matter composing them certainly descends towards the interior of our satellite, and that in all probability it has been forced up from that interior. The telescope, which in this instance is our *labourer,* has discovered numerous small craters of varying depth in the midst of many of the rays, and it reveals the fact, that these small craters, however deep, do not penetrate through the matter we are examining, inasmuch as there comes from their bases always the same kind of light that characterizes the ray. There is one remarkable case in point. A large crater named Saussure, and not far from Tycho, lies directly in the line of a ray, and of course appears to interrupt it; but at the bottom of Saussure, notwithstanding the great depth of that crater, the ray from Tycho may be traced. Nay, there is reason to believe that in favourable circumstances the same ray might be seen rising *up the sides* of Saussure, just as a vein of trap or of volcanic rock pierces the sedimentary strata upon earth. What, then, can we make of such phenomena? Are not our terrestrial trap dykes or veins their fitting similitudes? Piercing the other rocks, as if shot up from below, these singular veins pass onward across valley and over mountain; their direction *their own*—independent for the most part of the rocks they have cut; they appear, too, in *systems,* some limited in magnitude, and evidently radiating from a known source; others of vast extent, and usually considered parallel, but probably owing their apparent parallelism to the fact that we trace them only through a brief portion of their course. Accept this analogy,—and none other appears within reach,—and the rays or bright lines of the moon assume an import quite unexpected,—they become *indices to those successive dislocations that constitute epochs in the progress of our satellite.*" Elsewhere Nichol indicates in what sense he uses these words: where any system of radiations is intersected by another, it is manifest that the later formation will alone have its radiations unbroken at the places of intersection. Then Nichol assigns as the result of the telescopic scrutiny of the radiations from Copernicus, Aristarchus, and Kepler, that the three systems were formed in the order in which they are here named. He also assigns to the radiations from Tycho (manifestly with good reason) a great antiquity. "Another indication," he proceeds, "furnished by the rays demands notice. Reflect on the course, as to *continuous visibility,* of any stream of lava or any trap dyke upon the surface of the earth. No lava current from Etna could be traced to any great distance by a spectator in the moon, however powerful his telescope; and it would be the same with regard to those lines or dykes of trap, even supposing them endowed with an excessive power to reflect light. The reason is that they soon enter forest regions, and are concealed there, or become overspread by grass or other vegetable carpetings. But not even a lichen stains the brightness of the bands issuing from Tycho; they preserve, not their visibility merely, but one invariable brightness through their entire courses. The inference is but too clear; and we are glad to find a refuge from it, in the certainty that arrangements must be different on the other face of our satellite. The existence of a rocky desert, devoid of life or living thing, of the extent of even one lunar hemisphere, is startling enough."

Nasmyth is of opinion that the radiations "are cracks divergent from a central region of explosion, and filled up with molten matter from beneath." But Webb objects that this theory is irreconcilable with the fact that the radiations seldom, if ever, cause any deviation in the superficial level. "Trap dykes on the earth are indeed apt to assume the form of the surface, but the chances against so general and exact a restoration of level all along such multiplied and most irregular lines of explosion, would be incalculable; many of the rays are also far too long and broad for this supposition, or for that of Beer and Mädler, that they may be stains arising from highly heated subterranean vapour on its way to the point of its escape." It appears to me impossible to refer these phenomena to any general cause but the reaction of the moon's interior overcoming the tension of the crust; and to this degree Nasmyth's theory seems correct; but it appears manifest also, that the crust cannot have been fractured in the ordinary sense of the word. Since, however, it results from Mallet's investigations that the tension of the crust is called into play in the earlier stages of contraction, and its power to resist pressure in the later stages,—in other words, since the crust at first contracts faster than the nucleus, and afterwards not so fast as the nucleus, we may assume that the radiating systems were formed in so early an era that the crust was plastic. And it seems reasonable to conclude that the outflowing matter would retain its liquid condition long enough (the crust itself being intensely hot) to spread widely, a circumstance which would account at once for the breadth of many of the rays, and for the restoration of level to such a degree that no shadows are thrown. It appears probable also, that not only (which is manifest) were the craters formed later which are seen around and upon the radiations, but that the central crater itself acquired its actual form long after the epoch when the rays were formed. In the chapter on the moon's physical condition, considerations will be dealt with which bear upon this point. At the moment I need only remind the reader that the processes of cooling must have proceeded much more rapidly in the moon's case than in the earth's, and that this circumstance serves to account for phenomena indicative of a widely extended bursting of the lunar crust. I am disposed to believe, moreover, that although the radiating systems are manifestly not contemporaneous, they were all formed during a period of no great duration—possibly, indeed, not lasting for more than a few years, if so long.

The following peculiarities of arrangement noted by

Mr. Webb should be carefully noted in connection with the considerations dealt with in Chapter VI. "The remarkable tendency to circular forms, even where explosive action seems not to have been concerned, as in the bays of the so-called seas, is very obvious; and so are the horizontal lines of communication. The gigantic craters, or walled plains, often affect a meridional arrangement: three huge rows of this kind are very conspicuous near the centre, and the east and west limbs. A tendency to parallel direction has often a curious influence on the position of smaller objects: in many regions these chiefly point to the same quarter, usually north and south, or north-east and south-west; thus in one vicinity (between G, L, and M, in the map of the moon), Beer and Mädler speak of thirty objects following a parallel arrangement for one turned any other way; even small craters entangled in such general pressures (as round L) have been squeezed into an oval form, and the effect is like that of an oblique strain upon the pattern of a loosely-woven fabric: an instance (near 27, 28 on the map) of double parallelism, like that of a net, is mentioned, with crossing lines from south-south-west and south-east. Local repetitions frequently occur; one region (between 290 and 292) is characterized by exaggerated central hills of craters; another (A) is without them; in another (185) the walls themselves fail. Incomplete rings are much more common towards the north than the south pole; the defect is usually in the north, seldom in the west part of the circle; sometimes a cluster of craters are all breached on the same side (near 23, 32). Two similar craters often lie north and south of each other, and near them is frequently a corresponding duplicate. Two large craters occasionally lie north and south, of greatly resembling character—the southern usually three-fourths of the northern in size—from 18 to 36 miles apart, and connected by ridges pointing in a south-west direction (20, 19; 78, 77; 83, 84; 102, 103; 208, 207, 204; 239, 242; 261, 260; 260, 263; 340, 345). Several of these arrangements are the more remarkable, as we know of nothing similar on the earth."

But, interesting as these observations may be, it has not been for such discoveries as these that astronomers have examined the lunar surface. The examination of mere peculiarities of physical condition is, after all, but barren labour, if it lead to no discovery of physical condition. The principal charm of astronomy, as indeed of all observational science, lies in the study of change,—of progress, development, and decay, and specially of systematic variations taking place in regularly-recurring cycles. The rings of Saturn, for instance, have been regarded with a new interest, since the younger Struve first started the theory of their gradual change of figure. The "snowy poles of moonless Mars," in like manner, have been examined with much more attention and interest by modern astronomers than they were by Cassini or Maraldi, precisely because they are now recognized as snow-covered regions, increasing in the Martial winter and diminishing in the Martial summer. In this relation the moon has been a most disappointing object of astronomical observation. For two centuries and a half, her face has been scanned with the closest possible scrutiny; her features have been portrayed in elaborate maps; many an astronomer has given a large portion of his life to the work of examining craters, plains, mountains, and valleys for the signs of change; but hitherto no certain evidence—or rather no evidence save of the most doubtful character—has been afforded that the moon is other than "a dead and useless waste of extinct volcanoes."

Early telescopic observations of the moon were conducted with the confident expectation that the moon would be found to be an inhabited world, and that much would soon be learned of the appearance and manners of the Lunarians. With each increase of telescopic power a new examination was conducted, and it was only when the elder Herschel's great reflector had been applied in vain to the search, that men began to look on the examination as nearly hopeless. Herschel himself, who was too well acquainted, however, with the real difficulties of the question to share the hopes of the inexperienced, was strongly of opinion that the moon is inhabited. After describing the relations, physical and seasonal, prevailing on the lunar surface, he adds, "there only seems wanting, in order to complete the analogy, that it should be inhabited like the earth."

When Sir John Herschel conveyed a powerful reflector to Cape Town, the hope was renewed that something might yet be learned of the lunar inhabitants, through observations conducted in the pure skies of the southern hemisphere. So confidently was this hope entertained and expressed, that the opportunity seemed a good one to some American wits to play off a hoax on those who were anxiously awaiting the result of Sir John's observations. Accordingly an elaborate account was prepared of a series of discoveries respecting the appearance and behaviour of certain strange and not very well-conducted creatures inhabiting the moon. The readiness with which the story was believed in many quarters was a sufficient indication of the prevalence of the opinion that the moon is inhabited.

Lord Rosse's giant reflector has been applied, as we have said, to the examination of the moon's surface, without any results differing in character from those already obtained.

The considerations discussed at p. 242 are sufficient to show that it is not only hopeless to look on the moon's surface for the presence of living creatures, but even to look for constructions erected by such supposed inhabitants of the moon, unless these works were far greater than the largest yet constructed by man. Large cities, indeed, might be visible, but not separate edifices; nor could variations in the dimensions of cities be easily detected. It has been argued, indeed, that since gravitation, which gives weight to living creatures as well as to the objects around them, is so much less at the moon's surface than at the earth's, lunar inhabitants might, without being cumbrous or unwieldy, be very much larger than the races subsisting on our earth; they might also easily erect buildings far exceeding in magnitude the noblest works of man. Nor is the argument wholly fanciful. A man of average strength and agility placed on the lunar surface (and supposed to preserve his usual powers under the somewhat inconvenient circumstances in which he would there find himself) could easily spring four or five times his own height, and could lift with ease a mass which, on the earth, would weigh half a ton. Thus it would not only be possible for a race of lunarians, equal in strength to terrestrial races, to erect buildings much larger than those erected by man, but it would be *necessary* to the stability of lunar dwellings that they should be built on a massive and stupendous scale. Further, it would be convenient that the lunarians, by increased dimensions and more solid proportions, should lose a portion of the superabundant agility above indicated. Thus we have at once the necessity and the power for the erection of edifices far exceeding those erected by man.

But having thus shown that lunar structures might very possibly be of such vast dimensions as to become visible in our largest telescopes, it remains only to add, that no object that could, with the slightest appearance of probability, be ascribed to the labours of intelligent creatures, has ever been detected on the moon's surface.

Failing the discovery of living creatures, or of their works, it was hoped that at least the telescope might reveal the progress of natural processes taking place on a sufficiently important scale. There can hardly be a doubt that our earth, examined from the moon's distance, would exhibit (in telescopes of considerable power) a variety of interesting changes. It would be easy to trace the slow alternate increase and diminution of the polar snow-caps. The vast llanos, savannahs, and prairies would exhibit with the changing seasons very easily distinguishable changes of colour; the occasional covering of large districts by heavy snow-falls would also be a readily recognizable phenomenon.

Now the moon's surface exhibits distinctly-marked *varieties* of colour. We see regions of the purest white—regions which one would be apt to speak of as *snow-covered*, if one could conceive the possibility that snow should have fallen where (now, at least) there is neither air nor water. Then there are the so-called seas, grey or neutral-tinted regions, differing from the former not merely in colour and in tone, but in the photographic quality of the light they reflect towards the earth. Some of the seas exhibit a greenish tint, as the Sea of Serenity and the Sea of Humours. Where there is a central mountain, within a circular depression, the surrounding plain is generally of a bluish steel-grey colour. The region called the Marsh of Sleep exhibits a pale red tint, a colour seen also near the Hercynian mountains, within a circumvallation called Lichtenburg.

But although there are varieties, there has never yet been detected any *variation* of colour. Nothing has been seen which could be ascribed, with any appearance of probability, to the effects of seasonal change.

Failing evidence of the existence of living creatures, or of processes of vegetation, there only remains one form of variation to be looked for: I refer to changes such as, on our own earth, are produced by volcanic eruptions or by earthquakes.

It is evident, in the first place, that the inquiry must be one of extreme delicacy. Indeed, if the volcanic changes taking place on the moon were no greater than those observed on our own earth, it would be almost hopeless to seek for traces of their existence. The light proceeding from a burning mountain could never be detected at the moon's distance. It would also be extremely difficult to detect such small new craters as have been formed on our earth. It is the overspreading of extensive tracts with the materials ejected from volcanoes that would form the most readily detected feature of change. There have been instances in which, for many miles around a volcano, the country has been covered with ashes, and there can be little doubt that the change of appearance thus produced might be detected even at the moon's distance. There have also been cases in which, during an earthquake, the features of an extensive region have been entirely altered. Instances such as these, however, are so few and far between, that if we supposed the moon's surface correspondingly altered, the chances would be great against the detection of such change.

Assuming the probable, or, at least, the possible, existence of active volcanoes upon the moon, it remains to be seen how the operation of such volcanoes is to be detected from our earth. The colours seen in different parts of the moon's surface are little marked, and grey or neutral-tinted regions are so prevalent that it would be very difficult to note the change of colour produced by the downfall, over large tracts, of matter ejected from erupting volcanoes. Differences of elevation produced by such downfalls afford a much more favourable object of examination.

One of the earliest to record the supposed occurrence of volcanic action upon the moon was the elder Herschel. He observed luminous appearances, which he attributed to the presence of active volcanoes on the dark part of the moon's disc. The cause of these (which had been noticed also, but less satisfactorily, by Bianchini and Short) has now been shown to be the greater brilliancy of the light reflected under particular circumstances from our own earth upon the moon.*

* The following statements by Prof. Shaler, of the Harvard University, afford interesting evidence respecting the degree of illumination of the "old moon in the new moon's arms":—"With the 15-inch Merz of the Observatory of this university it is possible, under favourable conditions, to see all the principal features of the topography on the dark region illuminated only by this earth-shine. In the course of some years of study upon the

Schröter, who devoted a large part of his time to the study of the moon, imagined that he had detected signs of change, which, singularly enough, he seems

geology, if we may so call it, of the moon, I have had several opportunities of seeing under these conditions all the great features of the dark surface shine out with amazing distinctness. The curious point, however, is that the eye is not enabled to recognize the craters by light and shade, for the light is too feeble for that, besides being too vertical for such a result ; but the relief is solely due to the difference in the light-reflecting power of the various features of the topography. Whatever becomes very brilliant under the vertical illumination of the full moon (the edges and floors of many craters, certain isolated hills, and the radiating bands of light) shines out with a singular distinctness when lit by our earth's light. This is important, inasmuch as it shows pretty conclusively that the difference in the brightness of various parts of the surface of the moon is not due to the effects of the heating of the surface during the long lunar day, but is dependent upon difference in the light-reflecting power. There are several degrees of brightness observable in the different objects which shine out by the earth-light. In this climate there are not over three or four nights in the year when the moon can be caught in favourable conditions for this observation. The moon should not be over twenty-four hours old (the newer the better), and the region near the horizon should be reasonably clear. Under these conditions I have twice been able to recognize nearly all the craters on the dark part, over 15 miles in diameter, and probably one-half the bands, which show with a power of 100 when the moon is full. That this partial illumination of the dark part of the moon is in no way connected with the action of an atmosphere, is clearly shown by the fact that the light is evenly distributed over the whole surface, and does not diminish as we go away from the part which is lit by direct sunlight, as it should do if an atmosphere were in question. It will be noticed that this fact probably explains the greater part of the perplexing statements concerning the illumination of certain craters before the terminator came to them. It certainly accounts for the volcanic to have been disposed to attribute rather to changes in a lunar atmosphere of small extent than to volcanic action. He was not able to assert positively, however, that appreciable changes had taken place. One observation of his, however, deserves special notice, as will presently appear. In November, 1788, he noted that the place of the crater Linnæus, in the Sea of Serenity, was occupied by a dark spot, instead of appearing, as usual, somewhat brighter than the neighbouring regions. Assuming that he made no mistake, we have evidence here of activity in this particular crater.

Since the time of Schröter, other observers have been led to suspect the occurrence of change. Mr. Webb pointed out in 1865 eight noteworthy instances. Several of these seem easily explained by the well-known effects of difference in telescopic powers, observational skill, keenness of vision, and the like; but there are one or two which seem to deserve a closer scrutiny:—

On February 8, 1862, the south-south-west slope of Copernicus was seen to be studded with a number of minute craters not seen in Beer and Mädler's map. These seemed to form a continuation of a region crowded with craters between Copernicus and Era-

activity which has so often been supposed to be manifested by Aristarchus. Under the illumination of the earth-light this is by far the brightest object on the dark part of the moon's face, and is visible much longer and with poorer glasses than any other object there."

tosthenes. And it is singular that this last-named region exhibits a honeycombed appearance, which appears not to have existed in Schröter's time, since it is not recorded in his maps, and could hardly have escaped his persevering scrutiny.

Another instance of supposed change is well worthy of attention, as showing the difficulty of the whole subject. There is a ring-mountain, called Mersenius, which has attracted the close attention of lunar observers, in consequence of its *convex* interior—a very uncommon feature. This bubble-like convexity is represented by Schröter, and also by Beer and Mädler, as perfectly smooth. Not only is this the case, but we have distinct evidence that Beer and Mädler paid particular attention to this spot. Now, in 1836, only a year or two after the publication of Beer and Mädler's map, Mr. Webb detected a minute crater on the summit of the convexity within Mersenius; he also saw several delicate markings, resembling long irregular ravines, "formed by the dropping-in of part of an inflated and hollow crust." Here one would feel satisfied that a change had taken place, were it not that in Lohrman's map a minute crater had already been inserted on the convexity in question, while from the dates (1822 and 1836) between which Lohrman constructed his map, the probability is that the crater had been observed by him at or before the time when Beer and Mädler failed to detect its existence.

I have already referred to Schröter's observations of the crater Linné on the Sea of Serenity. Whether Schröter had been deceived or not, when he asserted that a dark spot hid the place of this crater in 1788, it is certain that during the last half-century the crater had been distinctly visible. When the sun is high upon Linné, it appears as a small bright spot, but when the spot is near the terminator, the crater has been recognizable through the appearance of a shadow within and without its wall. Now, in October, 1866, Schmidt observed that the crater Linné had disappeared. When the spot was close to the terminator no shadow could be seen, as usual, either within or without the crater. In November he again looked in vain for Linné.

It is to be noted that the crater is no insignificant formation, but fully five and a half miles wide, and very deep. It is, in fact, one of the largest craters within the Sea of Serenity; (H in Webb's map, where Linné is numbered 74).

The crater is represented in Riccioli's map (published in 1653). We have seen, also, that in 1788 Schröter recorded the appearance of a dark spot, instead of a crater, in Linné's place. Lohrman, in 1823, observed Linné to be one of the brightest spots on the whole surface of the moon. His map shows Linné as a distinct crater, and he describes it as more than one (German) mile in diameter, very deep, and visible in every illumination. In Beer and Mädler's map the crater is also distinctly noted; they measured its position no less than seven times; and they describe it as very deep and very bright. In photographs by De La Rue and Rutherfurd, Linné appears as a very bright spot; but singularly enough, we have also photographs taken during the month in which Schmidt looked in vain for the crater, and in these photographs (taken by Mr. Buckingham, of Walworth) Linné, though discernible as a light spot, has scarcely one-third of the brilliancy observed in De La Rue's and Rutherfurd's photographs, taken between the years 1858 and 1865.

Mr. Webb, one of our most careful observers, examined the Mare Serenitatis on December 13, 1866, for confirmation or disproof of Schmidt's views. The following is extracted from his notes of this observation:—"About one-third of the way from a marked high mountain on the northern shore of the Sea of Serenity, is a minute darkish-looking crater. This I presume to be Linné, *as I can trace no crater anywhere else. At some little distance south-east, there is an ill-defined whitishness on the floor of the sea.*" When Mr. Webb tested the results of his observations by means of a lunar map, he found that the first-named crater was not Linné, and that the "ill-defined whitishness" occupied the exact spot on which Linné is depicted. Subsequent observations fully confirmed the existence of this spot, which, singularly enough, is found, on careful measurement, to be twice as large as the crater whose place it conceals.

Many other observers who carefully examined Linné agreed in confirming the results of Schmidt's observation. One of the most satisfactory observations of Linné was effected by Father Secchi at Rome. On the evening of February 10, 1867, he watched Linné as it entered into the sun's light, and on the 11th he renewed his observations. In place of the large crater figured in lunar maps, he could just detect—with the powerful instrumental means at his command—a very small crater, smaller even than those craters which have received no names. "There is no doubt," he said, "that a change has occurred." Schmidt, it may be mentioned, independently detected the small crater described by Secchi.

The evidence thus far was as follows:—Where there used to be a ring-mountain surrounding a deep crater — so that, under suitable illuminations, the shadow of the mountain could be seen within and without the crater—no shadow could be traced; a space, considerably larger than that originally surrounded by the ring-mountain, appeared somewhat brighter than the neighbouring parts of the Sea of Serenity; in very powerful telescopes a minute black spot could be seen in place of the original wide and deep crater. It seemed clear, then, first, that there had not been a mere eruption of ashes filling up the crater, because then we should still see the shadow of the ring-mountain. Nor could the whole region have sunk, because then a large shadow would appear when the spot was near the terminator. The ring-mountain had not been destroyed, because its fragments and their shadows would remain visible. The only explanation available, therefore, appeared to be this,—that a mass of matter had been poured into the crater from below, and had overflowed the barrier formed by the ring-mountain, so as to cover the steep outer sides of the ring. Instead, therefore, of an outer declivity which could throw a shadow, there appeared to be an inclination sloping so gradually that no shadow could be detected, the whole surface thus covered with erupted matter shining with the same sort of light, so that a spot was seen somewhat lighter than the Sea of Serenity, and larger than the original crater.

Not only did the above explanation account for all the observed appearances, but it corresponded to phenomena of eruption presented on our own earth. Mud volcanoes (or *salsen*), as distinguished from volcanoes proper, present a very close analogy to the process of change just described. "Mud volcanoes," says Humboldt, "continue in a state of repose for centuries. When they burst forth, they are accompanied by earthquakes, subterranean thunder, the elevation of a whole district, and (for a short time) by the eruption of lofty flames. After the first forcible outburst, mud volcanoes present to us the picture of an incessant but feeble activity."

Yet subsequent observations have not confirmed the interpretation thus placed on the apparent changes in Linné. It has been shown by several observers, and notably by Mr. Browning in 1867, that Linné changes remarkably in aspect in a very short space of time, under changing solar illumination; and the inference would seem to be, that the supposed changes have been merely optical. Many observers of experience still retain the opinion, however, that there has been a real change in this region.

In Chapter VI. reasons will be suggested for believing that, owing to the changes of temperature of the moon's surface, as the long lunar day and night succeed each other, gradual processes of change must take place in the surface-contour.

The history of the inquiries which have been made as to the actual heating of the moon's surface during the lunar day is full of interest, but in this place I must be content with a brief account of the matter.

There are two ways in which the moon's surface sends out heat towards the earth. First, a portion of the sun's heat must necessarily be reflected precisely as the sun's light is reflected. But the moon's surface must also be heated by the sun's rays, and this heat is radiated into space. Thus at and near the time of full moon, the moon's surface is reflecting sun-heat towards us, and it is also giving out the heat which it has itself acquired under the sun's rays. Now the distinction between these two forms of heat is recognizable by instrumental means. The reflected heat is of the same quality as direct solar heat, and accordingly passes readily, like sun-heat, through absorbing media, such as glass, moist air, and others, which have the power of preventing the passage of heat which is merely radiated from bodies not so far heated as to become highly luminous.* We see this fact illustrated in our greenhouses. The sun's heat passes freely through the glass (at least only a small proportion is prevented from passing), but the warmed interior of the greenhouse does not part thus freely with its heat, the glass preventing the heat from passing away. Accordingly, when evening comes on, the interior of the greenhouse becomes considerably warmer than the surrounding air. In like manner, the heat reflected by the moon will pass freely through glass, while the heat which she radiates will not so pass; and in this circumstance we recognize the means of comparing the quantity of heat which the moon reflects and radiates, and thus of determining the degree to which the moon's surface is actually heated at any given time.

The first inquiries made into this subject did not, however, deal with relations so delicate as these. "Probably," says the anonymous writer of a fine essay on the subject in *Fraser's Magazine* for January, 1870, "the old observers had exaggerated notions of the moon's warmth and thought they could measure it by an ordinary thermometer. This was the tool employed

* We may state the matter thus: the shorter heat-waves pass through the media in question, the longer heat-waves are absorbed. From researches by Dr. Draper, it may be inferred that heat is produced, not merely or chiefly by waves from the red end and beyond the red end of the spectrum, but by waves from all parts of the visible spectrum and from beyond both ends of the spectrum. His researches, as also those of Sorby of Sheffield, demonstrate also that chemical action is produced by æther-waves of all orders of length.

by one Tschirnausen, who condensed the moon-ligh by means of burning glasses, in hope of getting measurable warmth, somewhere about the year 1699. Of course he got nothing. The famous La Hire followed suit some half a dozen years after, using a three-foot burning mirror and the most delicate thermometer then known; he, too, could obtain no indication, though his mirror condensed the light, and any heat with it, some 300 times; that is to say, the quantity of light falling upon the reflector was concentrated upon a spot one-three-hundredth of its area. After these failures, a century elapsed, and then Howard, and subsequently Prevost, attempted to gain direct evidence of lunar caloric, but since they had only expansion thermometers at their command, their results were valueless; for one, from some accidental circumstance, brought out a temperature obviously too high, while the other found negative heat!"

The much more effective heat-measuring instrument called the thermopile, was first brought into action by Melloni. Space does not permit me to describe here at length the nature of this instrument, for a full description of which I would refer the reader to Prof. Tyndall's "Heat considered as a Mode of Motion." Suffice it to say that the heat to be measured is suffered to fall on the place of junction of plates of bismuth and antimony, and that the electric current thus established is measured by the movement of a delicately poised magnetic needle. Melloni "concentrated the lunar rays" (says the account from which

I have already quoted) "by means of a metallic mirror, upon the face of his thermopile, in the hope of seeing the needle swing in the direction indicating heat; but it turned the opposite way, proving that the anterior and exposed surface of the pile was colder than its posterior face. Here was an anomaly. Did the moon, then, shed cold? No, the reverse action was due to the frigorific effect of a clear sky: the pile cooled more rapidly on one side than on the other, and a current was generated by this disturbance of the thermal equilibrium; a current, however, of opposite character to that which would have been produced if the moon had rendered the exposed face of the pile warmer than that which was turned away from the sky. Melloni's experiments were made about the year 1831.

"Two or three years after this the late Professor Forbes set about some investigations upon the polarization of heat, which involved the use of a very sensitive thermopile, and he was tempted to repeat Melloni's moon-test, with the substitution of a lens for a mirror as a condenser. The diameter of this lens was 30 inches, and its focus about 40 inches; of course it was of the polygonal construction familiar to lighthouse-keepers and their visitors, the grinding of a thirty-inch lens of continuous surface not having been contemplated in those days. Allowing for possible losses from surface-reflection or absorption by the glass, it was estimated that the lunar light and heat would be concentrated three thousand times.

One fine night in 1834, near the time of full moon, the lens and thermopile were put to the test. First the condensed beam of moon-rays was allowed to fall upon the pile and then it was screened by an interposed board. The exposures and screenings were repeated many times; but Professor Forbes was always disappointed with the effect, for it was nearly *nil*. There was a suspicion of movement in the galvanometer needle, but the amplitude of the swing was microscopic, possibly not greater than a quarter of a degree. Assuming that this deflection may have resulted, Professor Forbes subsequently proceeded to estimate the amount of heat that it represented. By exposing his pile and a thermometer to one and the same source of artificial heat, he was enabled to institute a comparison between the indications of each, and when he had done this and made all allowances for the condensing power of his lens, he concluded that the warming effect of the full moon upon our lower atmosphere was only equal to about the two hundred thousandth part of a centigrade degree!

"From what has since been learned, it appears strange that, with such a condensing power, such an insignificant result should have come out; but there was one point to which Forbes does not appear to have given the consideration it demanded. The sky was covered, he tells us, with a thin haze. Here was the secret, no doubt, of his comparative failure: this haze entirely cut off the little heat the moon had to give. When Melloni, using a similar lens, repeated his experiments under the pure sky of Naples, he saw his galvanometer swerve three or four degrees whenever the moon's condensed light fell upon the pile; from which he concluded that the moon gave warmth by no means insignificant, though he did not take the pains to infer the actual degree upon any known scale.

"This last essay of Melloni's was made in 1846. Ten years elapsed before it was repeated, and then Professor Piazzi Smyth, who was about to test the advantages of a lofty astronomical station by carrying instruments to the summit of Teneriffe, placed this subject upon his programme, thinking reasonably, that in higher regions of the atmosphere he might catch some of the warmth that is intercepted in its passage through these to the earth. He furnished himself with a pile and thermomultiplier, as the sensitive galvanometer has been termed; but he used no lens, contenting himself with a polished metal cone in front of the pile to collect and reflect the lunar heat upon its face. There was no mistaking the effect at this elevation of 10,000 feet: when the cone was turned towards the moon, the needle swung towards the heat side of the scale through a perceptible angle, and when it was turned towards the sky opposite to the moon, the needle returned to zero. By repeating this alternation of exposures an average deflection was obtained which was free from the effects of slight disturbing causes. Then it became of interest to learn what this average deflection meant in terms of any terrestrial source of warmth, and Professor Smyth found that it was equivalent to one seventeenth part of that which his warm hand produced when it was held three feet from the pile, or about twice that of a Price's candle fifteen feet distant. He left as an afterwork the conversion of this warmth into its equivalent on a known scale. The translation was quite recently made in France by M. Marié-Davy, and the result showed that the moon-heat experienced upon the mountain-top amounted to 750 millionths of a centigrade degree.

"We come now to touch upon the recent more conclusive experiments of the Earl of Rosse. When we look back upon the old trials, it is easy to see that the instruments employed, sensitive as they were, were yet not sufficiently so for the purpose. It seems that the want of delicacy was not in the thermopiles that converted the heat into weak electric currents, but in the galvanometers by which the weak currents were sought to be measured. Now these were formed of ordinary magnetic needles, poised upon points or turning upon pivots, the motion of the needle in each case being impeded to some extent by friction at its bearings. Then again, upon small, that is, short needles, feeble deflections are with difficulty seen, and those caused by the weak currents generated by moon-heat were, perhaps, too small to be seen at all. But it will be remembered that the requirements of sub-atlantic telegraphy brought about the invention of an exceedingly delicate galvanometer, in which the needle is suspended by a hair, and its most minute deflections are rendered visible by a small mirror which reflects a beam from an adjacent lamp on to a distant scale, so that an almost imperceptible twist of the needle causes a large displacement of the reflected light-spot. Here, then, was an indicator capable of rendering visible the most feeble of electric currents generated in a thermopile. It was not invented long before it was turned to use by the astronomers. The Earl of Rosse was the first to test its capabilities upon the moon.

"Lord Rosse using a reflecting telescope of three feet aperture, set about measuring the lunar warmth, with a view to estimating, first what proportion of it comes from the interior of the moon itself, and is not due to solar heating; second, that which falls from the sun upon the lunar surface, and is then reflected to us; and third, that which falling from the sun upon the moon, is first absorbed by the latter and then radiated from it. We need not follow the instrumental details of the processes employed for the various determinations; suffice it for us to know that the moon-heat was clearly felt, and that the quantity of warmth varied with the phase of the moon—greatest at the time of full and least towards the period of new. From this it was evident that little or no heat pertains to the moon *per se;* that our satellite has no proper or internal heat of its own, or at least that it does not radiate any such into space; if it did, there would probably have been found evidence of a continuity of warming, independent of the change of phase. Of the heat which came with the light only a small portion would pass through a glass screen in front of the pile; from this it was evident that the greater part of the whole consisted of heat-rays of low refrangibility; from which Lord Rosse concludes that the major portion of the lunar warmth does consist of that solar heat which has first been absorbed by the moon and then radiated from it.

"By the aid of a vessel containing hot water, subtending the same angle at his pile as the reflector employed to condense the moon's light and heat, he was enabled to judge of the actual temperature which the lunar surface must have to produce the effect that it does; and this was found to be about 500 degrees of Fahrenheit's scale. In this result we have a striking verification of a philosophical deduction reasoned out by Sir John Herschel, many years ago, that 'the surface of the full moon exposed to us must necessarily be very much heated, possibly to a degree much exceeding that of boiling water.'*

* These observations have recently been renewed under more favourable conditions. The result has been to show that a larger proportion of the moon's heat than had been supposed is reflected sun-heat. The difference in the radiation from the full moon and from the new indicates, according to these later observations, a difference of about 200 degrees in temperature. Moreover, during a partial eclipse of the moon on November 14, 1872, it was found that "the heat and light diminished nearly, if not quite, proportionally, the minimum for both occurring at or very near the middle of the eclipse, when they were reduced to about half their amounts before and after contact with the penumbra."

"Lord Rosse's conclusion that the heat increases with the extent of illumination has been confirmed by Marié-Davy, who has even measured the actual warmth day by day of a semi-lunation, and given the results in parts of the centigrade scale. He finds that the moon at first quarter warms the lower air by 17 millionths of a single degree, and that a regular increase takes place till about the time of full moon, when the calorific effect reaches 94 millionths of a degree! These insignificant figures refer only to the heat which can penetrate our atmosphere. The greater part of the whole lunar caloric must be absorbed in the high aërial regions."*

Here I must conclude my brief and necessarily imperfect sketch of the researches which have been made into the aspect and condition of the moon's surface. Those who are desirous of extending their acquaintance with the subject should carefully study all the observations which are recorded in the Proceedings of the Astronomical and Royal Societies, and the British Association, in this country, the leading Astronomical Societies on the Continent and in America, and the works in which Schröter, Gruithuisen, Mädler, and Schmidt have dealt with lunar phenomena. But after all, no course of reading can prove so instructive or interesting as a thorough study of the moon's surface with a telescope, even though the telescope be of

* *Fraser's Magazine* for January, 1870.

moderate power; and I cannot better close this chapter than by earnestly recommending every student of astronomy to survey the lunar details as completely and systematically as his leisure and his instrumental means may permit.

CHAPTER V.

LUNAR CELESTIAL PHENOMENA.

In discussing the nature of the celestial phenomena presented to lunarians, if such there be, we have considerations of two classes to deal with. In the first place, there are demonstrable facts respecting the apparent motions of the sun, earth, stars, and planets, the progress of the lunar seasons, year, and so on; in the second, we have other points to consider respecting which we can only form opinions more or less probable,—as the possible existence of a lunar atmosphere of small extent, the nature and effects of such an atmosphere, the question whether life—animal or vegetable—exists on the moon, with other matters of a similar nature.

But the only point of a doubtful nature respecting which I propose to speak at any length in this chapter, is the possible existence of a lunar atmosphere. All celestial phenomena must be so importantly affected by the presence or absence of an atmosphere that it is desirable to inquire carefully into the evidence bearing on the subject.

Remembering that our air is a mixture of oxygen and nitrogen (in the main), not a chemical compound of these gases, we see that there is no absolute necessity for the proportion in which these gases appear in our atmosphere. In the atmosphere of another body they might be differently proportioned. Moreover, carbonic acid gas, which forms a comparatively small part of the terrestrial atmosphere, might form a much larger proportion of the atmosphere of another planet. It is also conceivable that other and denser gases might be present in other atmospheres.

But even when all such considerations as these have been taken into account, it remains certain that unless we assume the existence on the moon of gases unknown on earth, a lunar atmosphere would have a specific gravity, under like conditions of pressure, differing in no marked degree from that of our earth's atmosphere. It would be a somewhat bold assumption to take for the average specific gravity of the lunar atmosphere that of carbonic acid gas, which, as we know, is almost exactly half as great again as that of air. But even if we supposed the lunar atmosphere composed of a gas as heavy as chlorine (which has a specific gravity nearly 2½ times as great as that of air), or like phosgene gas, which is nearly 3¾ times as heavy as air, the argument which follows would not be seriously affected.

Our air is sufficient in quantity to form a layer about 5½ miles in depth over the whole surface of the earth, and as dense throughout as air at the sea-level. This air, according to the laws of gaseous pressure, adjusts itself so that at any given height the density corresponds to the quantity of air above that height. The air above any height acts, in fact, as a weight pressing upon the air at that height, and compressing its elastic substance until it has a density proportional to the pressure so produced. Obviously, therefore, the density of the air at any given level depends on the amount of the earth's attraction. For every weight on the earth would be doubled if the earth's attraction were doubled, and halved if the earth's attraction were halved, and so on; and this applies as fully to the air as to any other matter having weight. Accordingly, if the earth's gravity were reduced to the value of gravity at the moon's surface (0·16 where the earth's gravity is represented by unity), the pressure of the air at the sea-level, and consequently the density of the air there, would be reduced to less than one-sixth of its present value. Of course, a given quantity of air at the sea-level would then occupy more space; and the whole atmosphere would expand correspondingly. Instead of having to attain a height of about 3½ miles, as at present, before the pressure would be reduced to one-half that at the sea-level (or to $\frac{1}{12}$th that at present existing at the sea-level), it would be necessary to attain a height more than six times as great, or nearly 22 miles. In other words, instead of one half of the whole atmosphere lying as at present below the height of 3½ miles, the lower half of the atmosphere would then extend to a height of nearly 22 miles.

Accordingly, if on the moon there were an atmosphere constituted like ours, and sufficient in quantity to cover the moon's surface to a depth of about 5½ miles of uniform specific gravity equal to that of our air at the sea-level, then such an atmosphere under the moon's smaller attracting power would expand so greatly that the half nearest the moon would extend to a height of about 22 miles.* At the mean level of the moon's surface,—that is, a level corresponding pretty nearly to our sea-level, so as to be as much above the greatest lunar depressions as below the greatest lunar heights,—the pressure would be about one-sixth that at our sea-level. Thus it is seen that even though the lunarians had as much air per mile of surface as we have on the earth, they would have a much rarer atmosphere. At a height of seven miles from the earth, a greater height than has ever yet been attained, or than could be attained by man,† the

* Here I take no account of the reduction of the moon's attracting power at this height from the surface. The consideration of such reduction would be important, however, in estimating the height to which the rarer strata would extend.

† "In the celebrated ascent by Messrs. Glaisher and Coxwell, in which the greatest height yet reached by man was attained, Mr. Glaisher became insensible before the balloon had attained a height of six miles. Mr. Coxwell, after endeavouring to rouse Mr. Glaisher, found that he was himself losing his strength. Indeed, he was unable to use his hands, and had he not succeeded in pulling the valve-string with his teeth, he and his companion must inevitably have perished. The height attained before the string was pulled would seem, from an observation made by Mr. Coxwell, to have been about 6½ miles. At this time the temperature was 12° below

air is still one-fourth as dense as at the sea-level. So that, even though the lunarians had so large a quantity of air as I have supposed, they must still be constituted very differently from men, since men would perish at once if placed in an atmosphere so attenuated.

But there is a more important point to be considered. We see that an atmosphere of a given quantity per square mile of lunar surface would reach much higher than a similar atmosphere on the earth. One half of it would lie above a height of 22 miles, that is, enormously above the summits of the highest lunar mountains. Far the greater portion of the atmosphere would ile above the lunar high lands. Supposing the atmosphere differently constituted, and of specific gravity six times as great as our air under the same

zero, and the neck of the balloon was covered with hoar frost."—(From my article on the balloon in Rodwell's "Science Dictionary.") "It is worth noticing, however," I proceed, "that although it would seem from this experience that no man accustomed to breathe the air at ordinary levels, can hope to attain a greater height than 6½ miles, it is not impossible that those who pass their lives at a great height, as the inhabitants of Potosi, Bogota, and Quito, might safely ascend to a far greater height. We know that De Saussure was unable to consult his instruments when he was at no higher level than these towns, and that even his guides fainted in trying to dig a small hole in the snow; whereas the inhabitants of the towns thus exceptionally placed, are able to undergo violent exercise. We may assume, therefore, that their powers are exceptionally suited to such voyages as those in which Glaisher and Coxwell so nearly lost their lives." Nevertheless it may be regarded as certain that no race of men could exist even for a few minutes in an atmosphere having a specific gravity less than one-sixth that of our own air.

circumstances of pressure, yet even then we should have only the same density at the moon's level as at the earth's. That density could only be due to the pressure of the superincumbent parts of the atmosphere. Diminishing with height above the moon's mean surface, according to the laws of gaseous pressure, it would extend as high above the moon's surface as our air above the earth's, even on the supposition of its having so remarkable a specific gravity compared with that of common air.

We see, then, that if we were to suppose the atmospheric pressure at the moon's surface equal to that at the earth's, we should have to suppose either that this atmosphere is composed of gases of very great specific gravity, or else that it extends to a much greater height than our own atmosphere. In either case, it is obvious that we should expect to find very marked effects produced by such an atmosphere.

In the first place, when the moon was carried by her motion over a star, the place of the star would be affected by refraction, not only when the moon's edge was very close to the star, but for some considerable time before. If the lunar atmosphere were actually as dense near the moon's mean surface as our air is at the sea-level, then a star would not be occulted at all, even though the moon passed so directly over the star's true place on the heavens that the geometrical line joining the star and the observer's eye passed through the moon's centre. This is easily seen. For the moon's semidiameter subtends an angle of less than 16′. Now the sun appears wholly in view when in reality he is below the level of the horizon, our atmosphere having sufficient refractive power to raise the sun's image by about 34′ (his diameter is about 31′). And this action is produced on rays which have only passed through the atmosphere to reach the earth tangentially. In passing out again, such rays would be deflected through 34′ more, or in all by about 68′. Accordingly, since 16′ is less than a quarter of 68′, if the moon's atmosphere possessed only a fourth part of the refractive power of our own atmosphere, a star in reality behind the centre of the moon's disc would appear as a ring of light. Nor would this ring be very faint. The light of the star would not be diluted or spread over the ring and therefore reduced in corresponding degree. On the contrary, the moon's atmosphere would act the part of an enormous lens, increasing the total quantity of light received from the star, in the same way that the lens of a telescope's object-glass increases the quantity of light received from any celestial object.*

* An effect, indeed, somewhat similar to that here considered, may be produced by covering all but the outer ring of an object-glass with a black disc, and removing the eye-piece; if then, the telescope be directed nearly towards a bright star, and shifted from that position until exactly directed on the star, the light from the star will be presented in the form of an arc, gradually extending farther and farther round until it forms a complete circular ring.

In the case supposed, as the moon really passed over a star, we should see the star changing in appearance into an arc, this arc gradually increasing in length and span, until at length, when the star was centrally behind the moon, it would appear as a ring around her disc.

The actual circumstances of an occultation of a star by the moon are very markedly contrasted with those here mentioned. In nearly all cases a star disappears instantly, when the moon's edge reaches the star's place. There is no perceptible displacement of the star, no change of colour, no effect whatever such as a refractive atmosphere would produce. In certain instances, the brightness of a star has been observed to diminish just before disappearance; but we cannot be sure that, where this has happened, the star may not be really multiple, or perhaps nebulous. In the case of the star κ Cancri, according to some observers, the star has seemed suddenly reduced by about one half of its light, and almost instantly after to vanish; but these phenomena, only noticed in the case of this star, may be fairly explained by supposing the star to be a close binary. Again, there have been instances where a star has seemed to advance for some distance upon the moon's disc before vanishing; but it is by no means unlikely that the star has in such a case chanced to cross the moon's limb where a valley or ravine has caused a notch or depression which is too small to be indicated by any ordinary method of observation.* There is every reason to believe that when a single star is occulted opposite a smooth part of the moon's limb, the disappearance of the star is absolutely instantaneous.

Moreover, the evidence thus obtained has been

* It is to be remembered that such disappearances as these always take place opposite the bright limb of the moon, for the dark limb, even when the moon is nearly new, cannot be properly seen. Accordingly, irradiation comes into play, as well, of course, as the ordinary optical diffraction of the images of points forming the lunar limb, both these causes tending to remove all trace of minute notches really existing on the limb. But when a star is occulted at such a notch, it of course remains visible, despite the irradiation of the moon's limb ; so that it seems to be shining *through* the moon's substance. That this explanation is sound, seems to be confirmed by the circumstance that observers at stations not very wide apart recognize different appearances. Take, for instance, the following passage from Smyth's "Celestial Cycle":—"One of the most remarkable projections of a star on the moon's disc which I ever observed, was that recorded in the fifth volume of the Astronomical Society's Memoirs, p. 363, of 119 Tauri, on the 18th of December, 1831. On that occasion the night was beautiful, the moon nearly full, and the telescope adjusted to the star which passed over the lunar disc, and did not disappear till it arrived between two protuberances on the moon's bright edge. This was also noted by Mr. Snow, p. 373 of the same volume ; but Sir James South saw nothing remarkable, although in a few minutes afterwards he observed the star 120 Tauri perform a similar feat." "Such anomalies," adds Smyth, "are truly singular." I cannot but think, however, that they are to be expected as a natural consequence of the unevennesses which certainly characterize certain parts of the lunar limb. Such unevennesses on the limb must be minute to escape detection through the effects of irradiation ; and accordingly a very slight difference in the position of two observers would suffice to render the observed phenomena at their two stations altogether different.

strengthened by spectroscopic evidence. Dr. Huggins has watched the occultation of the spectrum of a star,—that is to say, he has watched the spectrum of a star until the moment when the star itself has been occulted. He has found that the spectrum disappears as instantaneously as the star itself. Now this is well worth noticing; for it might be supposed that any atmosphere existing round the moon would affect the red rays more than the other; as our atmosphere, for example, refracts the red light of the sun more fully than the rest. Hence it might be expected that the blue end of the spectrum would disappear a moment or two before the red end. But this did not happen.

The spectroscope has also afforded direct evidence of the non-existence of a lunar atmosphere of any considerable extent. For when the spectrum of the lunar light has been observed (by Dr. Huggins first, and later by others) it has been found to be absolutely similar to the solar spectrum,—that is, there is no trace whatever of absorptive action exerted by a lunar atmosphere upon the solar rays which are reflected by her to the earth. This evidence is, of course, not demonstrative of the absolute want of air of any sort on the moon, because a very rare and shallow atmosphere would produce no appreciable absorptive effect; but it confirms the other evidence showing that any lunar atmosphere must not only be extremely shallow but extremely rare. That is, there is not, as had been suggested by a well-known physicist, a dense atmosphere so shallow as not to rise above the summit of the lunar mountains. It is difficult, indeed, to conceive how such an atmosphere could be supposed to exist, since, as we have seen above, a gas six times as dense (under the same conditions) as our air, would on the moon only be as dense as our air, if so great in quantity as to reach as high as our air. An atmosphere sufficient in quantity to give traces of its presence in lunar shallows, but not extending higher than the summits of the lunar mountains, must be of a specific gravity so greatly exceeding (under the same conditions) that of common air, or indeed of any gas known to us on earth, that we are justified in regarding the theory of its existence as altogether unsupported by evidence.

But perhaps the strongest evidence we have to show that the moon has either no atmosphere or so little that she may be regarded as practically airless, is to be found in the phenomena of solar eclipses. It is certain, in the first place, that if the moon had an atmosphere resembling the earth's, the sun would not disappear at all, even at the moment of central eclipse, and when the sun was at his smallest and the moon at her largest. The moon's atmosphere would act as a lens (or as part of a lens) and reveal the sun to our view as a ring of blazing lustre—as really sunlight as the light of our setting sun. If the moon's atmosphere were at her mean surface but about one-fourth as dense as ours at the sea-level, the central part even of the sun's disc would be transmuted into a ring of light close to the moon's edge, while the parts nearer the sun's edge would form outer and brighter parts of the ring of glory round the moon. A very shallow lunar atmosphere indeed would suffice to bring the parts close to the edge of the sun's disc into view. It was, indeed, once supposed that the sierra of red light seen round the moon's disc during total eclipse (that ring of red light which Mr. Lockyer so strangely supposed that he had *discovered* in 1868) was produced by the refraction of the sun's light by the moon.* We now know that no part of the

* Thus Admiral Smyth wrote in 1844 :—"The red flames or protuberances of light, observed during total eclipses, and so correctly noted by the Astronomer Royal and Mr. Baily during that of July 1842, seemed to be attributable to an atmospheric effect, albeit there may be no distinguishable atmosphere. So long ago as 1706, Captain Stannyan, at Berne, observed of the sun, 'that his getting out of the eclipse was preceded by blood-red streaks of light from the left limb, which continued not longer than six or seven seconds of time.' On this Flamsteed remarks in a letter to the Royal Society : 'The Captain is the first man I ever heard of that took notice of a red streak of light preceding the emersion of the sun's body from a total eclipse ; and I take notice of it to you, because it infers *that the moon has an atmosphere ;* and its short continuation of only six or seven seconds of time tells us that its height is not more than the five- or six-hundredth part of her diameter.' This phenomenon was again noted during the total eclipse of the sun in April 1715, by Charles Hayes, the author of *A treatise on Fluxions*, who states in his philosophical dialogue *Of the Moon* that there was a streak of 'dusky but strong red light' preceding the sun's reappearance. There is much uncertainty, however, in all these observations, from their being liable to so many conditions of place, weather, instrument, and wind." I quote the remainder of Admiral Smyth's remarks as bearing importantly on our subject :—"From more than

light outside of the moon during totality is sunlight refracted by the moon, simply because the part where

one observation, I had worked myself up to a belief that the globes of Saturn and Jupiter were more affected under occultation than could be assigned to the inflection of their light in passing by the lunar surface; and I also thought that I had seen the satellites of Jupiter change their figure at the instant of immersion. Thus prejudiced, so to say, I prepared to establish the point by the occultation of the 1st of June, 1831, and certainly observed it under a train of favouring circumstances; but my result, as stated in the second volume of the Astronomical Society's Memoirs, p. 37, is this: Although the emersions of the satellites were perfectly distinct, they were certainly not so instantaneous as those of the small stars, which I think was owing more to light than disc. Jupiter entered into contact rather sluggishly; but though the lunar limb was tremulous from haze, there was not the slightest loss of light. Faint scintillating rays preceded the emersion, which was so gradual, that, as the planet reappeared, the edge of the moon covered it with a perfectly *even* and black segment, which cut the belts distinctly, and formed clear sharp cusps, slowly altering until the whole body was clear. There was no appearance of raggedness from lunar mountains, and Jupiter's belts were superbly plain while emerging; but there was not the slightest distortion of figure, diminution of light, or change of colour. . . . Schröter concluded that there existed a lunar atmosphere, but he estimated it to be only 5,742 feet high; and Laplace considered it as being more attenuated than what is termed the vacuum in an air-pump. The slowness of the moon's motion on its axis may account for such result." (There is, however, no basis for this supposition.) . . . "MM. Mädler and Beer, whose selenographical researches have been carried to an unprecedented extent, arrive at the conclusion that the moon is not without an atmosphere, but that the smallness of her mass incapacitates her from holding an extensive covering of gas, and they add, 'it is possible that this weak envelope may sometimes, through local causes, in some measure dim or condense itself,' the which would explain some of the conflicting details of occultation phenomena."

such refracted light would be strongest gives its own proper spectrum quite distinct from the spectrum of sunlight. But strong as this evidence is, there is yet stronger evidence. It has been discovered by Prof. Young that the sun has a relatively shallow atmosphere (say from two hundred to five hundred miles in height), whose existence is only rendered discernible by spectroscopic analysis, *aided by the moon.* As the moon passes over the face of the sun, the visible sickle of the sun's disc grows narrower and narrower, until at last it vanishes; at that moment the shallow solar atmosphere is not yet covered, but is just about to be covered. For a moment or two the spectroscope gives the spectrum of this atmosphere, and this spectrum is found to consist of myriads of bright lines,—the reversed Fraunhofer lines in fact. These are visible only for a second or two, and in the ordinary condition of the shallow atmosphere they vanish so suddenly that their disappearance has been compared to the vanishing of rocket stars.*

* During the annular eclipse of June 1872, the lines were seen by Mr. Pogson, Government Astronomer at Madras, for about 2 or 3 seconds when the annulus was completed, and for about 6 or 7 seconds when the annulus broke, showing a variable condition of the solar atmosphere. Moreover, the lines did not vanish suddenly in the latter case, as when the phenomenon was observed by Young in December 1870, and by Tennant, Maclear, and others, in December 1871. These peculiarities have no bearing on the question of the moon's atmosphere, but I thought it desirable to mention them, lest the reader should derive erroneous impressions from the account given above. The general subject of the sun's complex shallow

Now if the moon had an atmosphere comparable even with what is called the vacuum of an air-pump, the recognition of the delicate phenomena attesting the existence of the shallow solar atmosphere would be wholly impossible. The slightest residue of sunlight brought into action by the refractive power of such an atmosphere would suffice to obliterate the beautiful but delicate spectrum of the complex solar atmospheric envelope.

The evidence derived from the non-existence of any twilight circle on the moon, or the extreme narrowness of any such zone which may exist, need not here be closely considered. The only observations yet made which appear to indicate the existence of lunar twilight, seem explicable as due to the fact that the sun is not a point of light illuminating the moon's surface, but presents, as seen from the moon, a disc as large as she shows to us. Thus there would be in the case of a smooth moon, a penumbral fringe bordering the illuminated hemisphere, and about 32′ of the arc of a lunar great circle in width. This would correspond to a breadth of nearly ten miles, and would be readily discernible from the earth. In the case of a rough body like the moon, there would be no regular penumbral fringe, but along some parts of the border-line between light and darkness the effect would be

atmosphere is fully discussed in my treatise on 'The Sun,' in the first edition of which I adopted the theory that such an atmosphere must exist, while as yet the decisive observations remained to be effected.

reduced, while along other parts it would be exaggerated. On the whole, there would result appearances closely resembling those due to a twilight circle of small extent; and we can reasonably ascribe the supposed twilight effects hitherto recognized to this cause,—that is, to the fact that the sun as seen from the moon is not a point of light but a disc.

The conclusion to which we seem forced by all the evidence obtainable, is that either the moon has no atmosphere at all (which scarcely seems possible), or that her atmosphere is of such extreme tenuity as not to be perceptible by any means of observation we can apply. I must, however, make some remarks here on a theory which has been advocated by astronomers of repute, and even discussed by Sir John Herschel as not wholly incredible,—the theory, namely, that a lunar atmosphere (and lunar oceans) may possibly exist on the hemisphere of the moon which is turned directly away from the earth.

This theory is based on another,—the theory, namely, that the moon's centre of gravity is nearer to us than her centre of figure. Thus Professor Hansen considers that an observed discrepancy between the actual lunar motions and the results of the theoretical examination of the moon's inequalities, is removed if the centre of gravity of the moon is assumed to be 33½ miles farther from the earth than her centre of figure. This result —which, however, Professor Newcomb questions—appears to have been confirmed by the comparison of photographic pictures of the moon, taken at the times of her extreme eastern and western librations. In the year 1862, M. Gussew, Director of the Imperial Observatory at Wilna, carefully examined two such pictures taken by Dr. De la Rue. The result of the examination may be thus stated:—The outer parts of the visible lunar disc belong to a sphere having a radius of 1,082 miles, the central parts to a sphere having a radius of 1,063 miles; the centre of the smaller sphere is about 79 miles nearer to us than the centre of the larger; the line joining the centres is inclined at an angle of about 5° to the line from the earth at the epoch of mean libration: thus the central part of the moon's disc is about 60 miles nearer to us than it would be if the moon were a sphere of the dimensions indicated by the disc's outline. If we suppose the invisible part of the moon's surface to belong to the larger sphere, and the density of the moon's substance uniform, it would follow from this conformation that the centre of gravity of the moon is about 30 miles farther from the earth than is the middle point of the lunar diameter directed towards the earth, that is, than is the centre of the moon's apparent figure. This result accords sufficiently well with Hansen's theoretical conclusion.

On this Sir John Herschel remarks: "Let us now consider what may be expected to be the distribution of air, water, or other fluid on the surface of such a globe, supposing its quantity not sufficient to cover and drown the whole mass. It will run towards the lowest place, that is to say, not the nearest to the centre of figure, or to the central point of the mere space occupied by the moon, but to the centre of the mass, or the centre of gravity. There will be formed there an ocean of more or less extent, according to the quantity of fluid directly over the heavier nucleus, while the lighter portion of the solid material will stand out as a continent on the opposite side....In what regards its assumption of a definite level, air obeys precisely the same hydrostatical laws as water. The lunar atmosphere would rest upon the lunar ocean, and form in its basin a *lake of air*, whose upper portions, at an altitude such as we are now contemplating, would be of excessive tenuity, especially should the lunar provision of air be less abundant in proportion than our own. It by no means follows, then, from the absence of visible indications of water or air on this side of the moon, that the other is equally destitute of them, and equally unfitted for maintaining animal or vegetable life. Some slight approach to such a state of things actually obtains on the earth itself. Nearly all the land is collected in one of its hemispheres, and much the larger portion of the sea in the opposite. There is evidently an excess of heavy material vertically beneath the middle of the Pacific; while not very remote from the point of the globe diametrically opposite rises the great table-land of India and the Himalaya chain, on the summits of which the air has not more than a third of the density it has on the sea-level, and from which animated existence is for ever excluded."

But pleasing though the idea may be that on the farther hemisphere of the moon there may be oceans and an atmosphere, it appears to me impossible to accept this theory. In the first place, it has not been demonstrated, and is in fact not in accordance with theoretical considerations, that the moon is egg-shaped, or bispherical, according to Gussew's view. The farther part may also project as the nearer part does (supposing Gussew's measurements and inferences to be trustworthy). But even if we assume the moon to have the figure assigned to it by Gussew, the invisible part is not that towards which the atmosphere would tend. The part of the surface opposite the centre of the visible disc is in fact not nearest to the centre of gravity, but (assuming the unseen part spherical, and of the radius indicated by the visible disc) is 30 miles farther from the centre of gravity than are points on the edge of the visible disc. The band or zone of the moon's surface lying on this edge is the region where oceans and an atmosphere should be collected* (if water and air existed in appreciable quantity) on the moon's surface.

* The argument is presented in another form in a paper contributed by me to the Monthly Notices of the Astronomical Society, as follows :—" Let us assume, with Hansen, that the moon's surface is formed of two spherical surfaces, the part nearest to us having the least radius, so that in fact the moon is shaped like a sphere to which a meniscus is added, said meniscus lying on the visible hemisphere. If we imagine the meniscus removed, the lunar atmosphere would dispose itself symmetrically round the moon's spherical sur-

We seem justified in considering the phenomena presented to an observer supposed to be stationed on the moon, as practically those which would be seen if the moon had no atmosphere at all.

face. Now, suppose that while this state of things exists, the lunar air within the region now occupied by the meniscus of solid matter is suddenly changed to matter of the moon's mean density, what could be the effect of this change, by which new matter would be added on the side of the moon towards the earth? Surely not that the remaining atmosphere would tend to the further side of the moon, but on the contrary that it would be attracted towards the nearer side by the new matter there added. The lunar air would be shallower on this nearer side, no doubt, because the air thus drawn to it would not make up for the air supposed to be changed into the solid form; but at the parts which form the edge of the disc there would be an access of air, without this diminishing cause, and the air would therefore be denser there than elsewhere. But in this final state of things there would be equilibrium; we learn then what are the conditions of equilibrium for a lunar atmosphere, assuming the moon's globe to have the figure supposed by Hansen. There would be a shallow region in the middle of the visible disc, and a region slightly shallow directly opposite, while the mid-zone would have the deepest atmosphere. But it is around this zone precisely that no signs of a lunar atmosphere have as yet been recognized. I may remark that this reasoning may be extended to the earth. Assuming the waters of the earth drawn towards the South Pole because of a displacement in the earth's centre of gravity, we may regard the surface of the sea in the southern hemisphere as standing above the mean surface of the globe, and a part of the southern seas as therefore constituting a meniscus like that conceived by Hansen to exist in the case of the moon. It would follow, then, if my reasoning be correct, that we should have the atmosphere shallowest in high southern latitudes—shallow, but only slightly so, in high northern latitudes, and densest between the tropics; but this, as is well known, is precisely the observed arrangement."

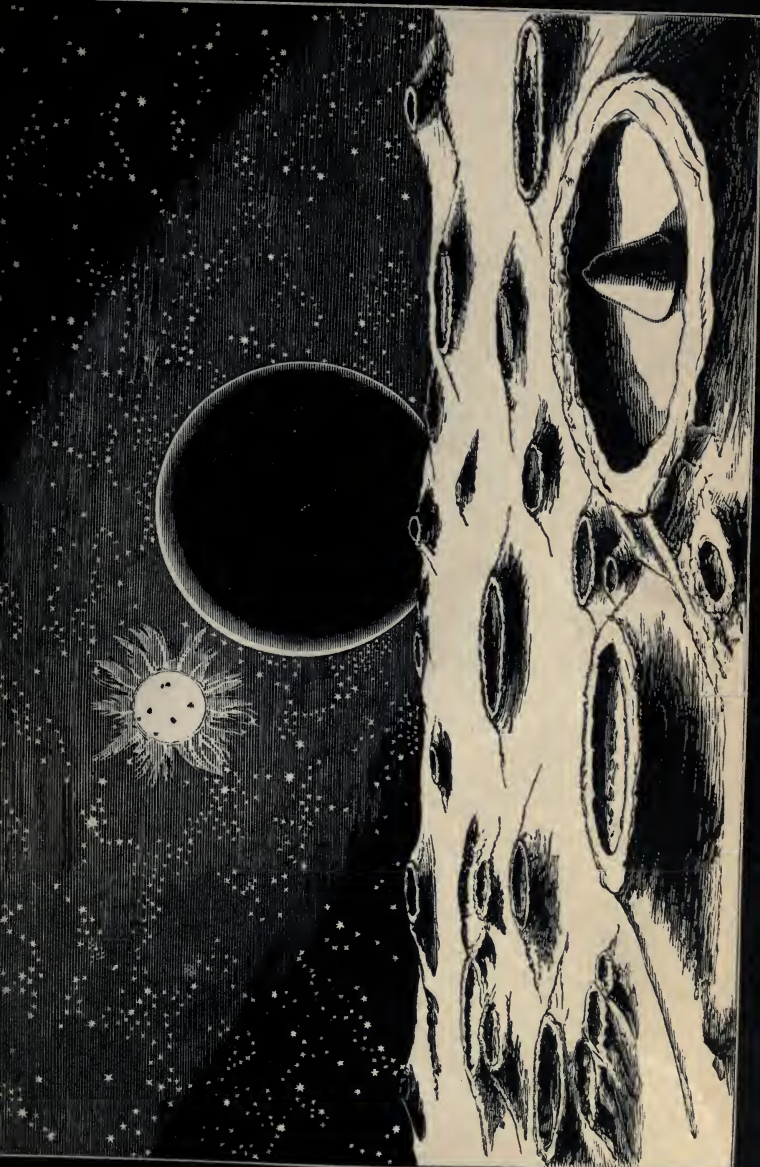
These phenomena may be divided into celestial and lunarian.

Of lunarian phenomena,—that is, of the appearance presented by lunar landscapes, I shall say little; because, in point of fact, we know far too little respecting the real details of lunar scenery to form any satisfactory opinion on the subject. If a landscape-painter were invited to draw a picture presenting his conceptions of the scenery of a region which he had only viewed from a distance of a hundred miles, he would be under no greater difficulties than the astronomer who undertakes to draw a lunar landscape, as it would actually appear to any one placed on the surface of the moon. We know certain facts,—we know that there are striking forms of irregularity, that the shadows must be much darker as well during the lunar day as during an earthlit lunar night, than on our own earth in sunlight or moonlight, and we know that whatever features of our own landscapes are certainly due to the action of water in river, rain, or flood, to the action of wind and weather, or to the growth of forms of vegetation with which we are familiar, ought assuredly not to be shown in any lunar landscape. But a multitude of details absolutely necessary for the due presentation of lunar scenery are absolutely unknown to us. Nor is it so easy as many imagine to draw a landscape which shall be correct even as respects the circumstances known to us. For instance, though I have seen many pictures called lunar landscapes, I have never seen one in which there have not been features manifestly due to weathering and to the action of running water. The shadows again are never shown as they would be actually seen if regions of the indicated configuration were illuminated by a sun but not by a sky of light. Again, aërial perspective is never totally abandoned, as it ought to be in any delineation of lunar scenery.

I do not profess to have done better, myself, in the so-called lunar landscapes which illustrate this chapter. I have, in fact, cared rather to indicate the celestial than the lunarian features shown in these drawings. Still, I have selected a class of lunar objects which may be regarded as on the whole more characteristic than the mountain scenery usually exhibited. And by picturing the greater part of the landscape as at a considerable distance, I have been freer to reproduce what the telescope actually reveals. In looking at one of these views, the observer must suppose himself stationed at the summit of some very lofty peak, and that the view shows only a very small portion of what would really be seen under such circumstances in any particular direction. The portion of the sky shown in either picture extends only a few degrees from the horizon, as is manifest from the dimensions of the earth's disc; and thus it is shown that only a few degrees of the horizon are included in the landscape.

Now, as respects the celestial phenomena visible from the moon's surface, much more might be said. The subject would afford a mathematician who had sufficient leisure a fine field for very unprofitable

PLATE XX



labour. He could determine the exact course of every star in the sky of every celestial hemisphere for every latitude in the moon. He could discuss the seeming motions of the sun with great completeness, and lastly he could indicate the exact nature of the varying curves traced out by the centre of the earth's disc as it librated responsively to the moon's varying motions in her orbit, until he had shown that the minutest detail of the lunar theory as already mastered, as well as yet minuter details of the lunar theory of the future, have their analogues in the apparent motions of the earth as seen from the moon. All this he might do, and the work, to be properly achieved, would try to their utmost the powers of a Newton. But when the work was finished it would be of very little worth; a new Diamond might set the new Newton's manuscript in flames without deserving even the mild reproach "Oh, Diamond, Diamond! thou little knowest the mischief thou hast done."

These matters, then, must here be more summarily disposed of.

As to the stars, we have these general considerations:—

First, the stars must be visible day and night, since the lunar sky in the daytime must be perfectly black, except where the sun's corona and the zodiacal light spread a faint light over it; and even where this light is, the stars must be quite clearly visible. Secondly, many orders of stars below the faintest discernible by our vision must be visible in the lunar heavens to eyesight such as ours, by day as well as by night. The Milky Way, in particular, must present a magnificent spectacle.*

The apparent motions of the stars correspond to the moon's rotation. Since she turns on her axis once in 27·322 of our days, and in the same direction that our earth turns, it follows that the star-sphere turns round from east to west as with us, but at a rate more than twenty-seven times slower. The pole of the lunar heavens lies close to the pole of the ecliptic, since the inclination of the moon's axis is only 1½ degrees. But the pole shifts more quickly than the pole of our heavens, completing its circuit around the pole of the ecliptic, in a circle 3 degrees in diameter, in 18·6 years. Thus in the course of a lunar day the pole of the heavens shifts appreciably in position, and therefore the stars do not travel in true circles, nor remain at a constant distance from the pole of the heavens (as our stars appreciably do). This noticed, the motion of the star-sphere, except as to rate, corresponds latitude for latitude with that of our star-sphere. The northern and southern poles of the heavens are overhead to observers placed respectively at the northern and southern poles of the moon; and as the lunarian travels towards the equator from a northern or southern station, the pole descends along a northerly

* I have not ventured to include any part of the Milky Way in the pictures illustrating this chapter,—for this reason simply, that no ordinary engraving could give the slightest idea of the splendour of the galaxy as seen from an airless planet.

or southerly meridian respectively, until at the lunar equator the two poles are both on the horizon. The equator of the lunar star-sphere lies always close to the ecliptic, the points corresponding to those parts of our celestial equator which lie farthest from the ecliptic, being only 1½ degrees instead of about 23½ degrees, as with us, from the ecliptic. These points and the nodes of the equator shift round so as to perform a complete circuit of the ecliptic in 18·6 years.

The motions of the sun bear the same relation to the star-motions as in the case of our own celestial phenomena. As our solar day exceeds the sidereal day, on account of the sun's advance on the ecliptic, so the solar day on the moon exceeds the sidereal lunar day, amounting to 29·531 of our terrestrial days, instead of 27·322 days.

But while the lunar day is much longer than ours, the lunar year is considerably shorter. For the precession of the nodes is, as we have seen, much more rapid in the moon's case than in the earth's; and the lunar tropical year, which is of course the year of seasons, is correspondingly shortened by the rapid motion of the vernal and autumnal equinox-points to meet the sun as he advances along the ecliptic. We know precisely what the lunar tropical year is, from the result stated at p. 134. It lasts 346·607 of our days, or 11·737 lunar days. Thus on the average each lunar season — spring, summer, autumn, or winter—lasts 2·934 lunar days, or nearly three days. But the seasons are not very marked, since the sun's range is only from $1\frac{1}{2}°$ north to $1\frac{1}{2}°$ south of the ecliptic, which is rather less than the range of our sun during four days before and after either equinox, vernal or autumnal. It appears to me that this state of things scarcely warrants the statement of Sir W. Herschel that "the moon's situation with respect to the sun is much like that of the earth, and by a rotation on its axis it enjoys an agreeable variety of seasons and of day and night."

Differences of climate exist, however, on the moon; and the circumstance is one to be carefully borne in mind in discussing the physical condition of our satellite. Day and night are nearly equal everywhere on the moon's surface, and during all the year of twelve long days. Moreover, the sun everywhere and at all times rises nearly due east and sets nearly due west. But his meridian altitude varies with latitude precisely as the meridian altitude of our spring or autumn sun varies with latitude. Along the lunar equator he rises to the point overhead, or very near to it, at mid-day; and the same may be said of all places within the lunar tropical zone (three degrees only in width). Near the lunar poles, on the contrary, the mid-day sun is close to the horizon. And in mid latitudes the mid-day sun has an intermediate altitude, which is greater or less according as the place is nearer or farther from the equator.

The motions of the planets as seen from the moon need not be fully discussed. It may be noted that all the motions of advance and retrogression observed from the earth can be seen from the moon also. The principal difference in the view of the planets obtained from the lunar station consists first in the visibility of Venus and Mercury when close to the sun, so that the varying illumination of these planets can be traced during their complete circuit around the sun, and secondly, in the visibility not only of Uranus, Neptune, and whatever other planets may travel beyond Neptune, but of many hundreds, and perhaps thousands, of the asteroids. If any planet or planets travel within the orbit of Mercury, lunarian astronomers, if such there be, must be well aware of the fact, supposing their powers of vision equal to ours.

The solar surroundings, as the prominences, corona, zodiacal, meteor systems, comet-families, and so on, must be perfectly visible from the moon; and in particular, before sunrise and after sunset these objects must form a very striking feature of the lunar heavens. I shall presently venture to give a brief ideal sketch of some of the more remarkable circumstances of the scene presented to a supposed lunar observer, as these and other phenomena pass in review before him. The reader will find in this sketch a description of the probable appearances presented during an eclipse of the sun by the earth.

But it is in the phenomena presented by our earth herself that our imagined lunarians must find their most interesting and difficult subject of study. On her they have an object of contemplation utterly unlike any known to our astronomers.

Of course on the farther side of the moon, at least on those parts which are never brought into view by libration, the lunarians never see the earth at all. On the hither side she is at all times visible, though under very varying conditions of illumination. On the zone including places on the moon which alternately pass into view and out of view, she is alternately seen and concealed, but to varying degrees.

Thus let us begin with the 'parallactic fringe' $pmem'p'$, next to the illuminated region, shown in fig. 79, Plate XVI. The inner edge of this fringe (the left-hand edge in the figure) indicates a line on the moon where the earth's centre in extreme librations just descends to the horizon, but never below the horizon. The outer edge marks a line where the earth's whole disc just disappears in extreme librations. Thus on places within this fringe-region the earth sometimes descends so low as to show less than half her disc above the horizon. If a fringe equally wide were drawn just within the inner edge of this fringe, it would include all places where the earth in extreme librations descends so low that some part of her disc (less than half) is concealed below the horizon.

Next let us take the space marked as the 'Region carried out of view by libration.' This is the lunar zone where the earth passes out of view in libration, but is for the greater part of the time in view, wholly or partially. The next zone, marked as the 'Region brought into view by libration,' is the lunar zone where the earth passes into view in libration, but is for the greater part of the time wholly or partially concealed.

Lastly, let us take the parallactic fringe p m e m′ p′. Here the edge next to the unseen region indicates a line on the moon where, in extreme librations, the earth's edge just touches the horizon, no part of the earth becoming visible. The other edge indicates a line on the moon where, in extreme librations, the earth's centre just reaches the horizon. On any place, therefore, within the fringe, a part of the earth's disc, but always less than a half, becomes visible in extreme librations. A fringe as wide as the other side of the line p m e m′ p′, includes places on the moon where, in extreme librations, more than the half of the earth's disc becomes visible, but not the whole disc, and its right-hand edge is a line on the moon where, in extreme librations, the whole disc of the earth *just* becomes visible, touching the horizon at its lower edge.

Now as to the actual motions of the earth's disc as seen either at places on these several zones, or at any part of the region where the earth is always above the lunar horizon, we have, as I have mentioned above, a problem which, in its entirety, is of the most complicated kind. But all that is useful to be known can be inferred from a few simple considerations, which I proceed now to indicate.

In fig. 72, Plate XV., the moon's disc is supposed to be indicated, and the range of libration at thirteen points on that disc, the actual libration-curves being illustrated in figs. 73, 74, 75, and 76. Now at a point close to P, the north pole of the moon, and on P O P′, the earth is seen close to the south point of the horizon. (This is manifest, if we consider that a line drawn from the eye to a point near P, is nearly tangential to the half-sphere P P′, and extends from near P in a direction which, estimated by a lunarian, would lie on the meridian P O P′.) Again, at O, the earth must be seen overhead. And it is clear that, to a lunarian travelling uniformly from P to O, and thence to P′, the earth would be seen to pass from the southern horizon * to the point overhead, and thence to the northern horizon, by an equally uniform motion.

* Before our lunarian left the point P he would have no determined southern point on the horizon. In fact, to a person at the pole of a globe like the moon or earth, every point of the horizon is towards the south; as soon as he moves from the pole, be it only by a single step, the point of the horizon towards which he has thus moved becomes the south point; and then, of course, the point behind him is the north point, the point on his right is the west point, and the point on his left the east point.

It seems perplexing to many, viewing the problem dealt with above, that a lunarian near P seeing the earth towards the south, while a lunarian at E sees the earth towards the east, a lunarian at M_2 should see the earth towards the east, and not towards the south-east. But this depends on precisely the same considerations which show how the sun, in spring or autumn, rises due east for all parts of the earth. It will be observed that the geometrical explanation consists in the fact, that if we draw lines M_2 O M_4, M_2 O M_1 in fig. 72, and regard these as the orthogonal projections of circles, both these circles cross the arc P E at M_2 at right angles. It should be noted, that whereas we call E the eastern and E′ the

Again, suppose a lunarian to travel from M_2 to M_1 on a lunar latitude-parallel (which seen from the earth would appear as a straight line from M_2 to M_1). It is manifest that as he thus travelled, the earth would change in apparent position precisely as though he were being carried round by a rotation of the moon on the axis $P P'$, which would necessarily produce the same effects as we on earth recognize in the sun's daily motion. The earth, to our travelling lunarian, would seem to pass from the eastern horizon to the south, where her elevation (from what was shown in the preceding paragraph) would be equal to the arc $P M_2$ or $P M_1$.

Accordingly, the apparent place of the earth, as seen from any point of a latitude-parallel $M_2 M_1$, on the moon's disc, is at once determinable by the consideration that the earth lies on the half-circle from the eastern to the western horizon of the given place on the moon, this half-circle being inclined to the horizon by as many degrees as there are in the arc $P M_2$, and the earth being as many degrees (measured along this circle) from the eastern point as the given point on the moon is from M_2 (these degrees being measured along the latitude-parallel $M_2 M_1$).

We can thus at once determine on what part of the lunar sky the earth is seen from any given place on the moon. Thus taking the intersections of the cross

western edge of the moon's disc (because so seen on the sky), a lunarian at O would have his east towards E′ and his west towards E.

lines in fig. 72, as conveniently and sufficiently illustrating the above reasoning, we have, as the parts of the sky where the earth is seen, the following results, placed so as to correspond to the position of the thirteen intersection-points in fig. 72 :—

			S.			
			S. 30°			
		Abt. S.E. 23°	S. 60°	S.W. 23° abt.		
E.	E. 30°	E. 60°	Overhead.	W. 60°	W. 30°	W.
		Abt. N.E. 25°	N. 60°	N.W. 23° abt.		
			N. 30°			
			N.			

Having thus ascertained the mean position of the earth on the lunar sky for any given lunar station, we can infer the nature of the libratory motions of the earth about this mean position. We see that the libration in longitude, if it acted alone, would necessarily sway the earth backwards and forwards along an arc of about $14\frac{2}{3}$ degrees (that is $7\frac{1}{3}$ degrees on either side of her mean position) upon the circle through the east and west horizon-points. This is obvious, because the libration in longitude necessarily takes place along a lunar latitude-parallel. We can also readily determine the position of the line on the heavens along which the earth is shifted by the libration in longitude in any given case. Thus, take the libration-cross near M_2, fig. 72, Plate XV. We see that its upper end, corresponding to the time when the earth is lowest down, is separated by a smaller latitude-arc from the eastern edge of the disc than the mean libration-point. This shows that the earth, in leaving her mean position southwards, or descendingly, shifts also eastwards; and, of course, in leaving her mean position northwards, she shifts westwards. Thus we have a libration-arc inclined at an acute angle to the libration-arc before considered. The corresponding motions of the earth's centre are therefore not like fig. 76, but as this figure would appear if D D′ were inclined to $d\,d'$ at an acute angle.

It would be idle, however, to enter into further details on these points, simply because the result would have no value. It is indeed instructive to consider the general features of the heavens as seen from any celestial body, and the general fact that the earth, as seen from each lunar station on the visible hemisphere, has such and such a mean position, and sways libratingly around or across that position, is sufficiently interesting. But the special circumstances of these librations have no interest, because in no sense affecting the physical habitudes of the different lunar regions. Moreover, a volume much larger than the present would be required for their adequate discussion.

It is manifest that at each lunar station the earth changes in phase precisely as the moon changes with us. When we see the moon full, the lunarians have the earth 'new,' that is, wholly dark; when we see the moon at her third quarter, the earth, as seen from the moon, is at her first quarter; when the moon is new, the earth is 'full'; and, lastly, when the moon is at her first quarter, the earth is at her third quarter. But in the case of the earth seen from the moon, the changes are all gone through while she is in one and the same part of the heavens; and though they necessarily depend on the sun's distance from the earth, this distance changes by the sun's apparent motion around the lunar heavens, and not, as in the case of the moon, by the motions chiefly of the lesser luminary. Moreover it is manifest that the earth's phases occur at different hours of the lunar day at different stations. Where the earth is seen on the meridian, 'new earth' necessarily occurs at noon-day, 'first quarter' at sunset, 'full earth' at midnight, and 'third quarter' at sunrise. Where the earth is seen on the east of the meridian, 'new earth' occurs in the forenoon, 'first quarter' in the afternoon, 'full earth' between sunset and midnight, and 'third quarter' between midnight and sunrise. Where the earth is seen on the west of the meridian, 'new earth' occurs in the afternoon, 'first quarter' between sunset and midnight, 'full earth' between midnight and sunrise, and third quarter in the forenoon.

Again, the earth changes in aspect to the lunarians on account of the inclination of her axis. When the moon is north of the equator, the lunarians see the north polar regions, or have a view of the earth resembling a summer sun-view of the earth;* when the moon is south of the equator, the lunarians see the south polar regions, or a view resembling a winter sun-view of the earth. These changes correspond exactly, in sequence, with the varying sun-views of the earth during a year, since the moon, like the sun, passes alternately north and south of the equator as she travels towards the east on the heavens. But the period of these changes, in the case of the moon, is of course the period occupied by the moon in passing from the equator to her greatest northerly declination, thence to the equator, again to her greatest southerly declination, and finally to the equator once more, and this period has a mean value equal to a nodical month. It will be manifest from fig. 54, Plate XIII., and the explanation in pp. 158—164, that the range of the earth's apparent sway, by which her north and south poles are brought alternately into view, varies from 18° 18′ to 28° 35′ on either side of the mean position (when both poles are on the edge of her visible disc). The period in which these changes are completely passed through is of course that of the revolution of the moon's nodes, or 18·6 years.

In the considerations here dealt with, the student who has sufficient leisure will find the necessary

* See my "Sunviews of the Earth."

materials for the complete discussion of the varying aspect and position of the earth as supposed to be seen from any lunar station.

Before drawing this chapter to a conclusion, however, I shall venture to attempt the description of some of the chief events of a lunar month, as they might be supposed actually to present themselves if an inhabitant of earth could visit the moon and observe them for himself. I select time and place so as to include in the description the phenomena of an eclipse of the sun by the earth. The reader will perceive that neither of the views illustrating this chapter corresponds with the relations considered in the following paragraphs,—in fact, it was absolutely necessary to select for pictorial illustration a lunar station where the earth would be low down, whereas for descriptive illustration it was manifestly better to take a station having the earth high above the lunar horizon.

To an observer stationed upon a summit of the lunar Apennines on the evening of November 1, 1872, a scene was presented unlike any known to the inhabitants of earth. It was near the middle of the long lunar night. On a sky of inky blackness stars innumerable were spread, amongst which the orbs forming our constellations could be recognized by their superior lustre, but yet were almost lost amidst myriads of stars unseen by the inhabitants of earth. Nearly overhead shone the Pleiads, closely girt round by hundreds of lesser lights. From them towards Aldebaran and the clustering Hyads, and onwards to the belted Orion, streams and convolutions of stars, interwoven as in fantastic garlands, marked the presence of that mysterious branch-like extension of the Milky Way which the observer on earth can with unaided vision trace no farther than the winged foot of Perseus. High overhead, and towards the north, the Milky Way shone resplendent, like a vast inclined arch, full "thick inlaid with patines of bright gold." Instead of that faint cloud-like zone known to terrestrial astronomers, the galaxy presented itself as an infinitely complicated star region,

> "With isles of light and silvery streams,
> And gloomy griefs of mystic shade."

On all sides, this mighty star-belt spread its outlying bands of stars, far away on the one hand towards Lyra and Bootes, where on earth we see no traces of milky lustre, and on the other towards the Twins and the clustering glories of Cancer,—the 'dark constellation' of the ancients, but full of telescopic splendours. Most marvellous too appeared the great dark gap which lies between the Milky Way and Taurus; here, in the very heart of the richest region of the heavens,—with Orion and the Hyades and Pleiades blazing on one side, and on the other the splendid stream laving the feet of the Twins,—there lay a deep black gulf which seemed like an opening through our star system into starless depths beyond.

Yet, though the sky was thus aglow with star-light, though stars far fainter than the least we see on the clearest and darkest night were shining in countless myriads, an orb was above the horizon whose light would pale the lustre of our brightest stars. This orb occupied a space on the heavens more than twelve times larger than is occupied by the full moon as we see her. Its light, unlike the moon's, was tinted with beautiful and well-marked colours. At the border, the light of this globe was white, while somewhat to the left of the uppermost point, and as much to the right of the lowest, a white light of peculiar purity and brilliancy extended for some distance upon the disc. But whereas the upper passed farthest round the disc's edge, and seemed on the whole to be the most extensive, the lower spread farther in upon the disc, and appeared rounded into an oval shape. Corresponding to this peculiarity was the circumstance that the greater part of the disc's upper half was occupied by a misty and generally whitish light, amidst which spots of blue could be seen on the right and left, and brownish and yellowish streaks near the middle; while, on the contrary, the lower half of the disc was nearly free from misty light, and occupied on the sides by widely-extended blue regions, and in the middle by green tracts on a somewhat yellowish background. To an inhabitant of earth it would not have been difficult to recognize in this last-named region the continent of South America bathed in the full light of a southern summer sun.

The globe which thus adorned the lunar sky and illuminated the lunar lands with a light far exceeding that of the full moon was our earth. The scene was not unlike that shown to Satan when Uriel,

> "one of the seven
> Who in God's presence, nearest to the throne,
> Stand ready at command,"—

pointing earthwards from his station amid the splendour of the sun, said to the archfiend,—

> "Look downward on that globe whose hither side
> With light from hence, though but reflected, shines:
> That place is earth, the seat of man; that light
> His day, which else, as th' other hemisphere,
> Night would invade."

In all other respects the scene presented to the spectator on the moon was similar; but as seen from the lunar Apennines the glorious orb of earth shone high in the heavens; and the sun, source of the light then bathing her oceans and continents, lay far down below the level of the lunar horizon.

And now, as hour passed after hour, a series of changes took place in the scene, which were unlike any that are known to our astronomers on earth. The stars passed, indeed, athwart the heavens on a course not differing from that followed by the stars which illumine our skies, but so slowly that in an hour of lunar time they shifted no more than our stars do in about two minutes. And marvellous to see, the great orb of earth did not partake in this motion. Hour by hour passed away, the stars slowly moved on their course westwards, but they left the earth still suspended as a vast orb of light high above the southern horizon. She changed, indeed, in aspect. The two Americas passed away towards the right, and the broad Pacific was presented to view. Then Asia and Australia appeared on the left, and as they passed onwards the East Indies came centrally upon the disc. Then the whole breadth of Asia could be recognized, but partly lost in the misty light of the northern half, while the blue of the Indian Ocean was conspicuous in the south. And as the hours passed on, Europe and Africa came into view, and our own England, foreshortened and barely visible, near the snow-covered northern region of the disc.

But although such changes as these took place in the aspect of the earth, her globe remained almost unchanged in position. It was indeed traversing the ecliptical zone, along which the sun and moon and planets pursue their course; and this star-zone was itself being carried slowly round the lunar sky: but these motions were so adjusted that the earth herself appeared at rest. The zone of the ecliptic was carried round from east to west behind the almost unmoving globe of the earth. When South America was in view, she had been close to the eastern border of Aries; and now Aries had passed away westwards, and Taurus was behind the earth. And yet it could not be said that the earth by advancing along the ecliptic was hiding the stars of the zodiacal constellations; rather it appeared as though these stars were hiding themselves in turn behind the earth.

But the stars were not hidden as they are when the moon passes over them. The terrestrial astronomer in such a case observes that a star vanishes instantly, and reappears with equal suddenness when the due time has arrived. But the passage of the multitudinous stars of the lunar sky behind the earth was accomplished in a different manner. The border of the earth's disc was seen to be full of a light far more resplendent than that of the disc itself. As the stars on their passage to the region behind the earth approached this border, their light was seen to be merged in the ring of splendour. This ring was, in fact, produced by the mingled lustre of all the stars which were behind the earth's disc; and speaking correctly these stars did not vanish at all. The earth's atmosphere, like a gigantic lens, brought all these stars into view, and became filled with their diffused light, just as the object-glass of a telescope is seen to be filled with a star's light when we remove the eyeglass.

Here then was another feature in which the earth, seen as a celestial body from the moon, differed wholly from any celestial orb visible to terrestrial astronomers. Her orb, beautiful from its size and splendour, beautiful also in its variegated colours, was girt around with a ring of star-light, a ring infinitely fine as seen from the moon by vision such as ours, yet conspicuous because of the quality of the light which produced it.

It may seem surprising that though the orb of earth was shining so splendidly above the lunar horizon, stars could be seen which the far fainter lustre of the full moon obliterates from our skies; and not these alone, but countless thousands of other stars, which only the telescopist can see from a terrestrial station. But the observer on the moon has no sky, properly so called. Above and around him is the vault of heaven, while the atmosphere which forms our sky, not only in the splendour of day, but in the darkest night, when the stars seem to shine as on a background of intense blackness, is wanting on the moon. The blackness of our darkest skies is as the light of day by comparison with the darkness of space on which the stars of the lunar heavens are seen projected. The glorious orb of the earth was there at the time we speak of, and her light would have lit up an atmosphere like our own, so that the whole sky would have been aglow; but on the moon there was no atmosphere to illuminate, so that above and around the observer there was no sky.

Yet the lunar lands were lit up with the splendour of earth-light. The mountainous region around shone far more brightly than a similar terrestrial scene under the full moon, and the glory of the earth-lit portions was rendered so much the more remarkable by the amazing blackness of the parts which were in shadow. But the lustre of the stars was not dimmed. There was no veil of light to hide the stars, as when the full moon pours her rays upon the terrestrial air. Homer's famous description of a moonlit night corresponds far better with the lunar scene than with night on the earth. For whereas on earth the glory of the moon hides the heaven of stars from our view, on the moon, in the far greater splendour of the full earth,

> "the stars about the *Earth*
> Look beautiful. . . .
> And every height comes out, and jutting peak
> And valley, and the immeasurable heavens
> Break open to their highest, and all the stars
> Shine."

The long hours passed, measured by the stately motion of the stars behind the scarce moving earth, and by the changing aspect of her globe, as continents and oceans were carried from left to right across her face by her rotation. And gradually her orb lost its roundness. The ring of brilliant star-light which encircled her disc remained perfectly round indeed, but within this ring on the right a dark sickle began to be seen, and, slowly spreading, invaded the disc on that side. The earth was no longer full, but had assumed a gibbous phase, like the moon a day or two after full. Yet her aspect was wholly unlike that of the gibbous moon. The ring of light surrounding her true orb would of itself have made her appear unlike the moon; but besides this peculiarity, there was a marked contrast in the appearance of the darkened portion. Instead of that sharply-defined edge presented by the gibbous moon, there was in the case of the earth a softening off of the light by gradations so gentle that no eye could perceive where the enlightened hemisphere terminated and the dark hemisphere began. As night on our earth comes on with stealthy pace, the shades of evening closing in so gradually that we can hardly say when day ends and night begins, so from the station of the lunar observer the shading on the earth's darkened side which showed where night was coming on presented no recognizable outlines. One familiar with the earth and with the ways of her inhabitants could not but picture to himself how, as country after country was carried by the earth's rotation into that darkened region, the labours of men were being drawn towards a close for the day.

When about a week had passed, the earth had become a half-earth. The shape of the darkened half of the disc could still be recognized by the ring of star-light, which always surrounds her as she is seen from the moon, and remains nearly always bright and conspicuous, though sometimes, when many and bright stars are behind the earth, the ring is brighter than at others. This in fact had been the case when the Milky Way, where it crosses Gemini, had been carried behind the earth, which now, however, had passed beyond that region, and was entering the constellation of the Lion.

The aspect of the earth had in the meanwhile altered in another respect. The southern polar regions had been turned more fully than before towards the moon,—more fully than towards the sun.

We may pass, however, from the further consideration of changes in the earth's aspect, to describe a far more interesting series of phenomena which had already commenced to be discernible in the eastern part of the heavens.

At all times the zodiacal light is visible in the lunar heavens, forming a zone completely round the zodiac, and perfectly distinct in appearance from the Milky Way. It is far more brilliant, even when faintest, than the zodiacal light we recognize through our air, at once dense enough to conceal, and sufficiently illuminated, whether by twilight, moonlight, or starlight, to spread a veil over the delicate light of the zodiacal. But near the sun's place the zodiacal has an aspect utterly unlike that of even the brightest portions seen by us. Its complicated structure becomes discernible, and its colour indicates its community of nature with the outer parts of the solar corona. At the epoch we are considering, the corona itself was rising in the east, and its outer streamers could be seen extending along the ecliptical zone far into the bright core of the zodiacal.

Infinitely more wonderful, however, and transcending in sublimity all that the heavens display to the contemplation of the inhabitants of earth, was the scene presented when the sun himself had risen. I shall venture here to borrow some passages from an essay entitled "A Voyage to the Sun," in which a friend of mine has described the aspect of the sun as seen from a station outside that atmosphere of ours which veils the chief glories of the luminary of day. "The sun's orb was more brilliantly white than when seen through the air, but close scrutiny revealed a diminution of brilliancy towards the edge of the disc, which, when fully recognized, presented him at once as the globe he really is. On this globe could be distinguished the spots and the bright streaks called faculæ. This globe was surrounded with the most amazingly complex halo of glory. Close around the bright whiteness of the disc, and shining far more beautiful by contrast with that whiteness than as seen against the black disc of the moon in total eclipses—stood the coloured region called the chromatosphere, not *red*, as it appears during eclipses, but gleaming with a mixed lustre of pink and green, through which, from time to time, passed the most startlingly brilliant coruscations of orange and golden yellow light. Above this delicate circle of colour towered tall prominences and multitudes of smaller ones. These, like the chromatosphere, were not red, but beautifully variegated. In parts of the prominences colours appeared which were not seen in the chromatosphere,—and in particular, certain blue and purple points of light which were charmingly contrasted with the orange aud yellow flashes continually passing along the whole length of even the loftiest of these amazing objects. The prominences round different parts of the sun's orb presented very different appearances; for those near the sun's equatorial zone and opposite his polar regions differed very little in their colour and degree of light from the chromatosphere. They also presented shapes resembling rather those of clouds moving in a perturbed atmosphere, than those which would result from the tremendous processes of disturbance which astronomers have lately shown to be in progress in the sun. But opposite the spot-zones the prominences presented a totally different appearance. They resembled jets of molten matter, intensely bright, and seemingly moving with immense velocity. They formed and vanished with amazing rapidity, as when in terrestrial conflagrations a flame leaps suddenly to a great height and presently disappears."... "Around the sun a brightly luminous envelope extended to about twice the height of the loftiest prominences, while above even the faintest signs of an atmosphere, as well as through and amidst both the inner bright envelope and the fainter light surrounding it, there were the most complex sprays and streams and filaments of whitish light, here appearing as streamers, elsewhere as a network of bright streaks, and yet elsewhere clustered into aggregations which can be compared to nothing so fitly, though the comparison may seem commonplace, as to hanks of glittering thread. All these streaks and sprays of light appeared perfectly white, and only differed among themselves in that whereas some appeared like fine streaks of a uniform silvery lustre, others seemed to shine with a curdled light. The faint light outside the glowing atmosphere surrounding the prominences was also whitish; but the glowing atmosphere itself shone with a light resembling that of the chromatosphere, only not so brilliant. The pink and green lustre,—continually shifting, so that a region which appeared pink at one time would shine a short time after with a greenish light,—might aptly be compared in appearance to mother-of-pearl. The real extension of the white streaks and streamers was not distinguishable, for they became less and less distinct at a greater and greater distance from the sun, and finally became imperceptible."

Much more might be said on this inviting subject, only that the requirements of space forbid, obliging me to remember that the moon and not the sun is the subject of this treatise. The reader, therefore, must picture to himself the advance of the sun with his splendid and complicated surroundings towards the earth, suspended almost unchangingly in the heavens, but changing gradually into crescent form as the sun drew slowly near. He must imagine also, how, in the meantime, the star-sphere was slowly moving westwards, the constellations of the ecliptic in orderly succession passing behind the earth at a rate slightly exceeding that of the sun's approach, so that he, like the earth, only more slowly, was moving eastwards so far as the star-sphere was concerned, even while the moon's slow diurnal rotation was carrying him westwards towards the earth.

At the station we are considering, the lunar eclipse which took place on November 15, 1872, was only partial. Here, therefore, though the sun actually passed in part behind the earth, a portion of his orb remained unconcealed. But owing to the refractive power of the earth's atmosphere the rest of his disc was also brought into view, amazingly distorted, and forming a widely-extended crescent of red light—true sun-light—around a large arc of the earth's edge, the visible portion of the solar disc being at the middle of this crescent.

To an observer near the north pole of the moon, the eclipse was total, at least in our terrestrial mode of considering lunar eclipses: the true shadow of the earth fell on that portion of the moon. From a station so placed then, no part of the sun's disc could be seen by the lunarian; nevertheless a crescent of sun-light was visible in this case also, the crescent extending farther round the earth's disc than in the former case, and in fact round considerably more than a semicircle, the brightest part of the crescent being opposite the part of the earth's disc behind which the sun's disc was in reality placed.

I must, however, leave the reader to conceive the slow processes of change by which, as the sun advanced to the position here indicated, his disc became gradually modified into this crescent of true sun-light, this distorted image of the *whole* sun, thus seen through the spherical shell of the earth's atmosphere, and how, passing onwards towards the west, he gradually reappeared. Space will not permit me to dwell as I should wish on the multitude of interesting relations presented as the solar surroundings passed in their turn behind the earth, either before the sun as he approached the earth, or after the sun as he moved on westwards. Enough has been said to indicate to the thoughtful reader the general nature of the phenomena presented during the whole course of the sun's passage from the eastern to the western horizon, as well as those which followed after he had set, until the lunar month was complete, and the earth again seen, on November 30, 1872, with fully illuminated orb upon the lunar sky.

CHAPTER VI.

CONDITION OF THE MOON'S SURFACE.*

If the study of our earth's crust—or the science of geology—is capable of throwing some degree of light on the past condition of other members of the solar system, the study of those other orbs seems capable of at least suggesting useful ideas concerning the past condition of our earth. There are members of the solar system respecting which it may reasonably be inferred that they are in an earlier stage of their existence than the earth. Jupiter and Saturn, for instance, would seem—so far as observation has extended—to be still in a condition of intense heat, and still the seat of forces such as were once probably at work within our earth. We see these planets enwrapped, to all appearance, within a double or triple coating of clouds, and we are compelled to infer, from the behaviour of these clouds, that they are generated by forces belonging to the orb which they envelope; we have, also, every reason which the nature of the

* This chapter is reprinted, with slight alterations, from the *Quarterly Journal of Science*, by the kind permission of the editor of that serial.

case can afford, to suppose that our own earth was once similarly cloud-enveloped. We can scarcely imagine that in the long-past ages, when the igneous rocks were in the primary stages of their existence, the air was not loaded heavily with clouds. We may, then, regard Jupiter and Saturn as to some degree indicating the state of our own earth at a long-past epoch of her existence. On the other hand, it has been held, and not without some degree of evidence in favour of the theory, that in our moon we have a picture of our earth as she will be at some far-distant future date, when her period of rotation has been forced into accordance with the period of the moon's revolution round the earth, when the internal heat of the earth's globe has been radiated almost wholly away into space, and when her oceans and atmosphere have disappeared through the action of the same circumstances (whatever they may be) which have caused the moon to be air-less and ocean-less. But whether we take this view of our earth's future, or whether we consider that her state has been from the beginning very different from that of the moon, it nevertheless remains probable that we see in our moon a globe which has passed through a much greater proportion of its history (so to speak) than our earth; and accordingly the study of the moon's condition seems capable of giving some degree of information as to the future (possibly also as to the past) of our earth.

I wish here to consider the moon's condition from a somewhat different point of view than has commonly been adopted. It appears to me that the study of the moon's surface with the telescope, and the consideration of the various phenomena which give evidence on the question whether air or water exist anywhere upon or within her, have not as yet led to any satisfactory inferences as to her past history. We see the traces of tremendous sublunarian disturbances (using the word 'sublunarian,' here and elsewhere, to correspond to the word 'subterranean' used with reference to the earth), and we find some features of resemblance between the effects of such disturbanccs and those produced by the subterranean forces of our earth; but we find also as marked signs of distinction between the features of the lunar and terrestrial crusts. Again, comparing the evidences of a lunar atmosphere with those which we should expect if an atmosphere like our own surrounded the moon, we are able to decide, with some degree of confidence, that the moon has either no atmosphere or one of very limited extent. But there our knowledge comes to an end; nor does it seem likely that, by any contrivances man can devise, the further questions which suggest themselves respecting the moon's condition can be answered by means of observation.

Yet there are certain considerations respecting the moon's past history which seem to me likely, if duly weighed, to throw some light on the difficult problems presented by the moon.

In the first place, it is to be noted that the peculiar relation between the moon's rotation and revolution possesses a meaning which has not hitherto, so far as I know, been attended to. We know that *now* there is an absolutely perfect agreement between the moon's rotation and revolution,—in this respect, that her mean period of rotation on her axis is exactly equal to her mean period of revolution. (Here either sidereal rotation and revolution or synodical rotation and revolution may be understood, so long as both revolution and rotation are understood to be of the same kind.) I say "mean period of rotation," for although as a matter of fact it is only the revolution which is subject to any considerable variation, the rotation also is not perfectly uniform. We know, furthermore, that if there had been, long ago, a *near* agreement between the mean rotation and revolution, the present exact agreement would have resulted, through the effects of the mutual attractions of the earth and moon. But so far as I know, astronomers have not yet carefully considered the question whether that close agreement existed from the beginning, or was the result of other forms of action than are at present at work. If it existed from the beginning, that is from the moon's first existence as a body independent of the earth, it is a matter requiring to be explained, as it implies a peculiar relation between the moon and earth before the present state of things existed. If, on the contrary, it has been brought about by the amount of action which is now gradually reducing the earth's rotation-period, we have first of all to consider that an enormous period of time has been required to bring the moon to her present condition in this respect, and moreover, that either an ocean existed on her surface or that her crust was once in so plastic a condition as to be traversed by a tidal wave resembling, in some respects, the tidal wave in our own ocean. This, at any rate, is what we must believe if we suppose, first, that the main cause of the lengthening of the terrestrial day is the action of the tidal wave as a sort of brake on the earth's rotating globe, and secondly, that a similar cause produced the lengthening of the moon's day to its present enormous duration. It may be, as we shall presently see, that other causes have to be taken into account in the moon's case.

Now we are thus, either way, brought to a consideration of that distant epoch when—according to the nebular theory, or any admissible modification thereof—the moon was as yet non-existent as an orb distinct from the earth. We must suppose, on one theory, that the moon was at that time enveloped in the nebulous rotating spheroid out of which the earth was to be formed, she herself (the moon) being a nebulous sub-spheroid within the other, and so far coerced by the motion of the other that her longer axis partook in its motion of rotation. Unquestionably in that case, as the terrestrial spheroid contracted and left the other as a separate body, this other, or lunar spheroid, would exhibit the kind of rotation which the moon actually possesses. On the other theory, we should be led to suppose that primarily the lunar spheroid rotated independently of its revolution; but that the earth's attraction acting on the outer shells, after they had become first fluid and then (probably) viscous, produced waves travelling in the same direction as the rotation, but with a continual brake-action, tending slowly to reduce the rotation until it had its present value, when dynamical equilibrium would be secured.

But, as I have said, in either case we must trace back the moon's history to an epoch when she was in a state of intense heat. And it seems to me that we are thus led to notice that the development of the present state of things in the moon must have taken place during an era in the history of the solar system differing essentially from that which prevailed during the later and better-known geological eras of our own earth. Our moon was *shaped*, so to speak, when the solar system itself was young, when the sun may have given out a much greater degree of heat than at present, when Saturn and Jupiter were brilliant suns, when even our earth and her fellow minor planets within the zone of asteroids were probably in a sun-like condition. Putting aside all hypothesis, it nevertheless remains clear that, to understand the moon's present condition, we must form some estimate of the probable condition of the solar system in distant eras of its existence; for it was in such eras, and not in an era like the present, that she was modelled to her present figure.

It appears to me that we are thus, to some extent, freed from a consideration which has proved a difficulty to many who have theorized respecting the moon. It has been said that the evidence of volcanic action implies the existence, at least when that action was in progress, of an atmosphere capable of supporting combustion,—in other words, an atmosphere containing oxygen, for other forms of combustion than those in which oxygen plays a part may here be dismissed from consideration. But the fiery heat of the moon's substance may have been maintained (in the distant eras to which we are now referring the formation of her crust) without combustion. Taking the nebular hypothesis as it is commonly presented, the moon's globe may have remained amid the intensely hot nebulous spheroid (which was one day to contract, and so form the globe of the earth) until the nebula left it to cool thenceforth rapidly to its present state. Whatever objections suggest themselves to such a view are precisely the objections which oppose themselves to the simple nebluar hypothesis, and may be disposed of by those who accept that hypothesis. But better, to my view, it may be reasoned, that the processes of contraction and of the gathering in of matter from without, which maintained the heat of the nebulous masses, operated to produce all the processes of disturbance which brought the moon to her present condition, and that thus there was not necessarily any combustion whatever. Indeed, in any case, combustion can only have commenced when the heat had been so far reduced that any oxygen existing in the lunar spheroid would enter into chemical combination with various components of the moon's glowing substance. If there were no oxygen (an unlikely supposition, however), the moon's heat would nevertheless have been maintained so long as meteoric impact on the one hand, and contraction of the moon's substance on the other, continued to supply the requisite mechanical sources of heat-generation. In this case there would not necessarily have been any gaseous or vaporous matter, other than the matter retained in the gaseous condition by intensity of heat, and becoming first liquid and afterwards solid so soon as the heat was sufficiently reduced.

It must here be considered how far we have reason to believe that the heat of the various members of the solar system—including the moon and other secondary bodies—was originally produced, and thereafter maintained, by collisions; because it is clear that, as regards the surface contour of these bodies, much would depend on this circumstance. There would be a considerable difference between the condition of a body which was maintained at a high temperature for a long period, and eventually cooled, but slowly, under a continual downfall of matter, and that of a body whose heat was maintained by a process of gradual contraction. It is true that in the case of a globe like the earth, whose surface was eventually modelled and re-modelled by processes of a totally different kind, by deposition and denudation, by wind and rain, river-action and the beating of seas, the signs of the original processes of cooling would to a great extent disappear; but if, as we are supposing in the case of the moon, there was neither water nor air (at least in sufficient quantity to produce any effect corresponding to those produced by air and water on the earth), the principal features of the surface would depend largely on the conditions under which the process of cooling began and proceeded.

Now here I must recall to the attention of the reader the reasoning which I have made use of in my "Other Worlds than Ours," to show that, in all probability, our solar system owed its origin rather to the gathering of matter together from outer space than to the contraction of a rotating nebulous mass. It is there shown, and I think that the consideration is one which should have weight in such an inquiry, that there is nothing in the nebular hypothesis of Laplace to account in any degree for the peculiarities of detail presented by the solar system. That theory explains the revolution of the members of the solar system in the same direction, their rotation in the same direction, the approach to circularity of the orbits, and their near coincidence with the mean plane of the system; but it leaves altogether unexplained the different dimensions of the primary members of the solar system, the apparent absence of law and order in their axial tilt and the inclination of the orbits of their satellite families. In particular, the remarkable difference which exists between the outer family of planets,—the giant orbs, Jupiter, Saturn, Uranus, and

Neptune,—and the inner family of small planets,—Mars, the Earth, Venus, and Mercury,—is left wholly unexplained. Nor can one recognize in the nebular hypothesis any reason whatever for the comparative exuberance of orb-forming activity in the outer family, and particularly in the two planets lying next to the zone of asteroids, and the poverty of material which is exhibited within the minor family of planets. All these circumstances appear to be explained satisfactorily when we regard the solar system as formed by the gathering in from outer space of materials once widely scattered. We can see that in the neighbourhood of the great primary centre there would be indeed a great abundance of gathered and gathering matter, but that, owing to the enormous velocities in that neighbourhood, subordinate centres of attraction would there form slowly, and acquire but moderate dimensions. Outside a certain distance there would be less matter, but a far greater freedom of aggregation; there we should find the giant secondary centres, and we should expect the chief of these to lie inwards, as Jupiter and Saturn, while beyond would be orbs vast indeed, but far inferior to these planets. And we can readily see that the border region between the family of minor planets and the family of major planets would be one where the formation of a planet would be rendered unlikely; here, therefore, we should look for the existence of a zone of small bodies like the asteroids. I touch on these points to show the kind of evidence (elsewhere given at length) on which I have based my opinion that the solar system had its birth, and long maintained its fires, under the impact and collisions of bodies gathered in from outer space.

According to this view, the moon, formed at a comparatively distant epoch in the history of the solar system, would have not merely had its heat originally generated for the most part by meteoric impact, but while still plastic would have been exposed to meteoric downfalls, compared with which all that we know, in the present day, of meteor-showers, aërolitic masses, and so on, must be regarded as altogether insignificant. It would be to such downfall mainly that the maintenance of the moon's heat would at that time be due, though, as we shall presently see, processes of contraction must have not only supplemented this source of heat-supply, but must have continued to maintain the moon's heat long after the meteoric source of heat had become comparatively ineffective.

Now, I would notice in passing that here we *may* find an explanation of the agreement between the moon's rotation-period and her period of revolution. It is clear that under the continuous downfall of meteoric matter in that distant era, the moon must have been in a process of actual growth. She is indeed growing *now* from the same cause; and so is the earth: but such growth must be regarded as infinitesimally small. In the earlier periods of the moon's history, on the contrary, the moon's growth must have progressed at a comparatively rapid rate. Now this influx of matter must have resulted in a gradual reduction of the moon's rate of rotation, if (as we must suppose) the moon gathered matter merely by chance collisions. In the case of a globe gathering in matter by its own attractive power, as the sun does for instance, the arriving matter may (owing to the manner in which the process is effected) serve to maintain and even to increase the rate of rotation; but in the case of a subordinate body like the moon we must suppose that all effects acting on the rotation would be about equally balanced, and that the sole really effective result would be the increase of the moon's bulk, and the consequent diminution of her rotation-rate. Now, if this process continued until the rotation-rate had nearly reached its present value, the earth's attraction would suffice not merely to bring the rate of rotation precisely to its present value, but to prevent its changing (by the continuance of the process) to a smaller value. It may be added that the increase in the moon's rate of revolution, as she herself and the earth both grew under meteoric downfall towards their present dimensions, would operate in a similar way,—it would tend to bring the moon's rate of revolution and her rate of rotation towards that agreement which at present exists.

If we attempt to picture the condition of the moon in that era of her history when first the process of downfall became so far reduced in activity as to permit of her cooling down, we shall be tempted, I believe, to consider that some of the more remarkable features of her globe had their origin in that period. It may seem, indeed, at a first view, too wild and fanciful an idea to suggest that the multitudinous craters on the moon, and especially the smaller craters revealed in countless numbers when telescopes of high power are employed, have been caused by the plash of meteoric rain,—and I should certainly not care to maintain that as the true theory of their origin; yet it must be remembered that no plausible theory has yet been urged respecting this remarkable feature of the moon's surface. It is impossible to recognize a real resemblance between any terrestrial feature and the crateriferous surface of the moon. As blowholes, so many openings cannot at any time have been necessary, whatever opinion we may form as to the condition of the moon's interior and its reaction upon the crust. Moreover, it should be remembered that our leading seismologists regard water as absolutely essential to the production of volcanic disturbance (the only form of disturbance which on our earth leads to the formation of cup-shaped openings). If we consider the explanation advanced by Hooke, that these numerous craters were produced in the same way that small cup-shaped depressions are formed when thick calcareous solutions are boiled and left to cool, we see that it is inadequate to account for lunar craters, the least of which (those to which Mr. Birt has given the name of craterlets) are at least half a mile in diameter.

The rings obtained by Hooke were formed by the breaking of surface bubbles or blisters,* and it is impossible for such bubbles to be formed on the scale of the lunar craters. Now so far as the smaller craters are concerned, there is nothing incredible in the supposition that they were due to meteoric rain falling when the moon was in a plastic condition. Indeed, it is somewhat remarkable how strikingly certain parts of the moon resemble a surface which has been rained upon while sufficiently plastic to receive the impressions, but not too soft to retain them. Nor is it any valid objection to this supposition, that the rings left by meteoric downfall would only be circular when the falling matter chanced to strike the moon's surface squarely; for it is far more probable that even when the surface was struck very obliquely and the opening first formed by the meteoric mass or cloud of bodies was therefore markedly elliptic, the plastic surface would close in round the place of impact until the impression actually formed had assumed a nearly circular shape.

Before passing from this part of my subject, I would invite attention to the aspect of the moon as presented in the photographs illustrating this work. It will be

* "Presently ceasing to boil," he says of alabaster, "the whole surface will appear covered all over with small pits, exactly shaped like those of the moon." "The earthy part of the moon has been undermined," he proceeds, "or heaved up by eruptions of vapour, and thrown into the same kind of figured holes as the powder of alabaster."

seen that the multitudinous craters near the top of these pictures (the southern part of the moon) are strongly suggestive of the kind of process I have referred to, and that, in fact, if one judged solely by appearances, one would be disposed to adopt somewhat confidently the theory that the moon had had her present surface contour chiefly formed by meteoric downfalls during the period of her existence when she was plastic to impressions from without. I am, however, sensible that the great craters under close telescopic scrutiny by no means correspond in appearance to what we should expect if *they* were formed by the downfall of great masses from without. The regular, and we may almost say battlemented, aspect of some of these craters, the level floor, and the central peaks so commonly recognized, seem altogether different from what we should expect if a great mass fell from outer space upon the moon's surface. It is indeed just possible that under the tremendous heat generated by the downfall, a vast circular region of the moon's surface would be rendered liquid, and that in rapidly solidifying while still traversed by the ring-waves resulting from the downfall, something like the present condition would result. Or we might suppose that the region liquefied through the effects of the shock was very much larger than the meteoric mass; and that while a wave of disturbance travelled outwards from the place of impact, to be solidified (owing to rapid radiation of heat) even as it travelled, a portion of the liquid interior of the moon forced its way through the opening formed by the falling mass. But such ideas as these require to be supported by much stronger evidence than we possess before they can be regarded as acceptable. I would remark, however, that nothing hitherto advanced has explained at all satifactorily the structure of the great crateriform mountain-ranges on the moon. The theory that there were once great lakes seems open to difficulties at least as grave as the one I have just considered, and to this further objection, that it affords no explanation of the circular shape of these lunar regions. On the other hand, Sir John Herschel's account of the appearance of these craters is not supported by any reasoning based on our knowledge of the actual circumstances under which volcanic action proceeds in the case of our own earth. "The generality of the lunar mountains," he says, "present a striking uniformity and singularity of aspect. They are wonderfully numerous, occupying by far the larger portion of the surface, and almost universally of an exact circular or cup-shaped form, foreshortened, however, into ellipses towards the limb; but the larger have for the most part flat bottoms within, from which rises centrally a small, steep, conical hill. They offer, in short, in its highest perfection, the true volcanic character, as it may be seen in the crater of Vesuvius; and in some of the principal ones, decisive marks of volcanic stratification, arising from successive deposits of ejected matter, may be clearly traced with powerful telescopes. What is, moreover, extremely singular in the geology of the moon is, that although nothing having the character of seas can be traced (for the dusty spots which are commonly called seas, when closely examined, present appearances incompatible with the supposition of deep water), yet there are large regions perfectly level, and apparently of a decided alluvial character."

It is obvious that in this description we have, besides those features of volcanic action which might, perhaps, be expected on the moon, a reference to features essentially terrestrial. Alluvial deposits can have no existence, for example, save where there are rivers and seas, as well as an atmosphere within which clouds may form, whence rain may be poured upon the surface of wide land regions. It is not going too far to say that we have the clearest evidence to show that in the moon none of these conditions are fulfilled. Whether in former ages lunar oceans and seas and a lunar atmosphere have existed, may be a doubtful point; but it is certain that all the evidence we have is negative, save only those extremely doubtful signs of glacier action recognized by Professor Frankland. I venture to quote from Guillemin's "Heavens" a statement of Frankland's views, in order that the reader may see on how slender a foundation hypotheses far more startling than the theory I have suggested have been based by a careful reasoner and able physicist. "Professor Frankland believes," says the account, "and his belief rests on a special study of the lunar surface, that our satellite has, like its primary, also passed through a glacial epoch, and that several, at least, of the *valleys, rills,* and *streaks* of the lunar surface are not improbably due to former glacial action. Notwithstanding the excellent definition of modern telescopes, it could not be expected that other than the most gigantic of the characteristic details of an ancient glacier-bed would be rendered visible. What, then, may we expect to see? Under favourable circumstances, the terminal moraine of a glacier attains enormous dimensions; and consequently, of all the marks of a glacier valley, this would be the one most likely to be first perceived. Two such terminal moraines, one of them a double one, have appeared to observers to be traceable upon the moon's surface. The first is situated near the termination of the remarkable streak which commences near the base of Tycho, and passing under the south-eastern wall of Bullialdus, into the ring of which it appears to cut, is gradually lost after passing Lubiniezky. Exactly opposite this last, and extending nearly across the streak in question, are two ridges forming the arcs of circles whose centres are not coincident, and whose external curvature is towards the north. Beyond the second ridge a talus slopes gradually down northwards to the general level of the lunar surface, the whole presenting an appearance reminding the observer of the concentric moraines of the Rhône glacier. These ridges are visible for the whole period during which that portion of the moon's surface is illuminated; but it is only about the third day after the first quarter, and at the corresponding phase of the waning moon, when the sun's rays, falling nearly horizontally, throw the details of this part of the surface into strong relief, and these appearances suggest this explanation of them. The other ridge answering to a terminal moraine, occurs at the northern extremity of that magnificent valley which runs past the eastern edge of Rheita."

Here are two lunar features of extreme delicacy, and certainly not incapable of being otherwise explained, referred by Frankland to glacier action. It need hardly be said that glacial action implies the existence of water and an atmosphere on the moon; and not only so, but there must have been extensive oceans, and an atmosphere nearly equal in density to that of our own earth, if the appearances commented upon by Frankland were due to glacial action. It is admitted by Frankland, of course, that there is now no evidence whatever of the presence of water, "but, on the contrary, all selenographical observations tend to prove its absence. Nevertheless," proceeds the account from which I have already quoted, "the idea of former aqueous agency in the moon *has received almost universal acceptation*" (the italics are mine). "It was entertained by Gruithuisen and others. But, if water at one time existed on the surface of the moon, whither has it disappeared? If we assume, in accordance with the nebular hypothesis, that the portions of matter composing respectively the earth and the moon once possessed an equally elevated temperature, it almost necessarily follows that the moon, owing to the comparative smallness of her mass, would cool more rapidly than the earth; for whilst the volume of the moon is only about 1-49th (and its mass, it might be added, only about 1-81st part), its surface is nearly 1-13th that of the earth. This cooling of the mass of the moon must, in accordance with all analogy, have been attended with contraction, which can scarcely be conceived as occurring without the development of a cavernous structure in the interior. Much of this cavernous structure would doubtless communicate, by means of fissures, with the surface, and thus there would be provided an internal receptacle for the ocean, from the depths of which even the burning sun of the long lunar day would be totally unable to dislodge more than traces of its vapour. Assuming the solid mass of the moon to contract on cooling at the same rate as granite, its refrigeration though only 180° F. would create cellular space equal to nearly fourteen and a half millions of cubic miles, which would be more than sufficient to engulf the whole of the lunar oceans, supposing them to bear the same proportion to the mass of the moon as our own oceans bear to that of the earth."

The great objection to this view of the moon's past history consists in the difficulty of accounting for the lunar atmosphere. It must be remembered that owing to the smallness of the moon's mass, an atmosphere composed in the same way as ours would have a much greater depth compared with its density at the mean level of the moon's surface than our atmosphere possesses compared with its pressure at the sea-level. If there were exactly the same quantity of air above each square mile of the moon's surface as there is above each square mile of the earth's surface, the lunar air would not only extend to a much greater height than ours, but would be much less dense at the moon's surface. The atmospheric pressure would in that case be about 1-6th that at our sea-level, and instead of the lower half of such an atmosphere (that is, the lower half in actual quantity of air) lying within a distance of about 3½ miles from the mean surface, as in the case of our earth, it would extend to a distance of about 22 miles from the surface. Now this reasoning applies with increased force to the case of an atmosphere contained within the cavernous interior of the moon, for there the pressure due to the attraction of the moon's mass would be reduced. It is very difficult to conceive that under such circumstances room would not only exist for lunar oceans, but for a lunar atmosphere occupying, one must suppose, a far greater amount of space even before its withdrawal into these lunar caverns, and partially freed from pressure so soon as such withdrawal had taken place. That the atmosphere should be withdrawn so completely that no trace of its existence could be recognized, does certainly appear very difficult to believe, to say the least.

Nevertheless, it is not to be forgotten that, so far as terrestrial experience is concerned, water is absolutely essential to the occurrence of volcanic action.* If we are to extend terrestrial analogies to the case of our

* Mr. Mattieu Williams, the author of that valuable and suggestive work, "The Fuel of the Sun," makes, however, the following remarks on this important subject in an essay in the Monthly Notices of the Astronomical Society, which I venture to quote nearly in full :—

"Many theoretical efforts, some of considerable violence, have been made to reconcile the supposed physical contradiction presented by the great magnitude and area of former volcanic activity of the moon, and the present absence of water on its surface. So long as we accept the generally received belief that water is a necessary agent in the evolution of volcanic forces, the difficulties presented by the lunar surface are rather increased than diminished by further examination and speculation.

"We know that the lava, scoriæ, dust, and other products of volcanic action on this earth are mainly composed of mixed silicates, those of alumina and lime preponderating. When we consider that the solid crust of the earth is chiefly composed of silicic acid, and of basic oxides and carbonates which combine with silicic acid when heated, a natural necessity for such a composition of volcanic products becomes evident.

"If the moon is composed of similar materials to those of the earth, the fusion of its crust must produce similar compounds, as they are formed independently of any atmospheric or aqueous agency.

"This being the case, the phenomena presented by the cooling of fused masses of mixed silicates in the absence of water become very interesting. Opportunities of studying such phenomena are offered at our great iron-works, where fused masses of iron cinder, composed mainly of mixed silicates, are continually to be seen in the act of cooling under a variety of circumstances.

"I have watched the cooling of such masses very frequently, and have seen abundant displays of miniature volcanic phenomena, especially marked where the cooling has occurred under conditions most nearly resembling those of a gradually cooling planet or moon, notwithstanding the signs that the conditions prevailing in her case have been very different from

satellite, that is, when the fused cinder has been inclosed by a solid resisting and contracting crust.

"The most remarkable that I have seen are those presented by the cooling of the 'tap cinder' from puddling furnaces. This, as it flows from the furnace, is received in stout iron boxes ('cinder bogies') of circular or rectangular horizontal section. The following phenomena are usually observable on the cooling of the fused cinder in a circular bogie.

"First a thin solid crust forms on the red-hot surface. This speedily cools sufficiently to blacken. If pierced by a slight thrust from an iron rod, the red-hot matter within is seen to be in a state of seething activity, and a considerable quantity exudes from the opening.

"If a bogie filled with fused cinder is left undisturbed, a veritable spontaneous volcanic eruption takes place through some portion, generally near the centre, of the solid crust. In some cases, this eruption is sufficiently violent to eject small spirts of molten cinder to a height equal to four or five diameters of the whole mass.

"The crust once broken, a regular crater is rapidly formed, and miniature streams of lava continue to pour from it, sometimes slowly and regularly, occasionally with jerks and spurts due to the bursting of bubbles of gas. The accumulation of these lava-streams forms a regular cone, the height of which goes on increasing. I have seen a bogie about 10 or 12 inches in diameter, and 9 or 10 inches deep, thus surmounted by a cone above 5 inches high, with a base equal to the whole diameter of the bogie. These cones and craters could be but little improved by a modeller desiring to represent a typical volcano in miniature.

"Similar craters and cones are formed on the surface of cinder which is not confined by the sides of the bogie. I have seen them beautifully displayed on the 'running-out beds' of refinery furnaces. These when filled form a small lake of molten iron covered with a layer of cinder. This cinder first skins over, as in the bogies, then small crevasses form in this crust, and through these the fused those existing in the case of our earth, we are bound to recognize at least the possibility that water once

cinder oozes from below. The outflow from this chasm soon becomes localized so as to form a single crater, or a small chain of craters; these gradually develop into cones by the accumulation of outflowing lava, so that when the whole mass has solidified it is covered more or less thickly with a number of such hillocks. These, however, are much smaller than in the former case, reaching to only one or two inches in height, with a proportionate base. It is evident that the dimensions of these miniature volcanoes are determined mainly by the depth of the molten matter from which they are formed. In the case of the bogies, they are exaggerated by the overpowering resistance of the solid iron bottom and sides, which force all the exudation in the one direction of least resistance, viz., towards the centre of the thin upper crust; and thus a single crater and a single cone of the large relative dimensions above described are commonly formed. The magnitude and perfection of these miniature volcanoes vary considerably with the quality of the pig iron and the treatment it has received, and the difference appears to depend upon the evolution of gases, such as carbonic oxide, volatile chlorides, fluorides, &c. I mention the fluorides particularly, having been recently engaged in making some experiments on Mr. Henderson's process for refining pig iron by exposing it when fused to the action of a mixture of fluoride of calcium and oxides of iron, alumina, manganese, &c. The cinder separated from this iron displayed the phenomena above described very remarkably, and jets of yellowish flame were thrown up from the craters while the lava was flowing. The flame was succeeded by dense white vapours as the temperature of the cinder lowered, and a deposit of snow-like flocculent crystals was left upon and around the mouth or crater of each cone. The miniature representation of conical eruptions was thus rendered still more striking, even to the white deposit of the haloid salts which Palmieri has described as remaining after the recent eruption of Vesuvius.

"The gases thus evolved have not yet been analytically examined, and the details of the powerful reactions displayed in this process still demand further study, but there can be no doubt existed on the moon. Moreover, it must be admitted that Professor Frankland's theory seems to accord far

that the combination of silicic acid with the base of the fluor spar is the fundamental reaction to which the evolution of the volatile fluorides, &c., is mainly due.

"A corresponding evolution of gases takes place in cosmical volcanic action, whenever silicic acid is fused in contact with limestone or other carbonate, and a still closer analogy is presented by the fusion of silicates in contact with chlorides and oxides, in the absence of water. If the composition of the moon is similar to that of the earth, chlorides of sodium, &c., must form an important part of its solid crust; they should correspond in quantity to the great deposit of such salts that would be left behind if the ocean of the earth were evaporated to dryness. The only assumptions demanded in applying these facts to the explanation of the surface-configuration of the moon are,—1st. That our satellite resembles its primary in chemical composition; 2nd. That it has cooled down from a state of fusion; and, 3rd. That the magnitude of the eruptions due to such fusion and cooling must bear some relation to the quantity of matter in action.

"The first and second are so commonly made and understood, that I need not here repeat the well-known arguments upon which they are supported, but may remark that the facts above described afford new and weighty evidence in their favour.

"If the correspondence between the form of a freely suspended and rotating drop of liquid and that of a planet or satellite is accepted as evidence of the exertion of the same forces of cohesion, &c., on both, the correspondence between the configuration of the lunar surface and that of small quantities of fused and freely cooled earth-crust matter, should at least afford material support to the otherwise indicated inference, that the materials of the moon's crust are similar to those of the earth's, and that they have been cooled from a state of fusion.

"I think I may safely generalize to the extent of saying that no considerable mass of fused earthy silicates can cool down under circumstances of free radiation, without first forming a heated solid crust, which, by further radiation, cooling, and contraction, will better with lunar facts than any of the others which have been advanced to account for the disappearance

assume a surface-configuration resembling more or less closely that of the moon. Evidence of this is afforded by a survey of the spoil-banks of blast-furnaces, where thousands of blocks of cinder are heaped together, all of which will be found to have their upper surfaces (that were freely exposed when cooling) corrugated with radiating miniature lava-streams that have flowed from one or more craters or openings that have been formed in the manner above described.

"The third assumption will, I think, be at once admitted, inasmuch as it is but the expression of a physical necessity.

"According to this, the earth, if it has cooled as the moon is supposed to have done, should have displayed corresponding irregularities, and generally, the magnitude of mountains of solidified planets and satellites should be on a scale proportionate to their whole mass. In comparing the mountains of the moon and Mercury with those of the earth, a large error is commonly made by taking the customary measurements of terrestrial mountain-heights from the sea-level. As those portions of the earth which rise above the waters are but its upper mountain-slopes, and the ocean bottom forms its lower plains and valleys, we must add the greatest ocean depths to our customary measurements, in order to state the full height of what remains of the original mountains of the earth. As all the stratified rocks have been formed by the wearing down of the original upper slopes and summits, we cannot expect to be able to recognize the original skeleton form of our water-washed globe.

"There is one peculiar feature presented by the cones of the cooling cinder which is especially interesting. The flow of fused cinder from the little crater is at first copious and continuous, then it diminishes and becomes alternating, by a rising and falling of the fused mass within the cone. Ultimately the flow ceases, and then the inner liquid sinks more or less below the level of the orifice. In some cases, where much gas is evolved, this sinking is so considerable as to leave the cone as a mere hollow shell, the inner liquid having settled down and solidified with a flat or slightly rounded surface at about the level of the base of the cone, of all traces of water or air. The theory that oceans and an atmosphere have been drawn to the farther

or even lower. These hollow cones were remarkably displayed in some of the cinder of the Henderson iron, and their formation was obviously promoted by the abundant evolution of gas.

"If such hollow cones were formed by the cooling of a mass like that of the moon, they would ultimately and gradually subside by their own weight. But how would they yield? Obviously by a gradual hinge-like bending at the base towards the axis of the cone. This would occur with or without fracture, according to the degree of viscosity of the crust and the amount of inclination. But the sides of the hollow cone-shell, in falling towards the axis, would be crushing into smaller circumferences. What would result from this? I think it must be the formation of fissures extending, for the most part, radially from the crater towards the base, and a crumpling up of the shell of the cone by foldings in the same direction. Am I venturing too far in suggesting that in this manner may have been formed the mysterious rays and rills that extend so abundantly from several of the lunar craters?

"The upturned edges or walls of the broken crust, and the chasms necessarily gaping between them, appear to satisfy the peculiar phenomena of reflection which these rays present. These edges of the fractured crust would lean towards each other and form angular chasms, while the foldings of the crust itself would form long concave troughs extending radially from the crater. These, when illuminated by rays falling upon them in the direction of the line of vision, would reflect more light towards the spectator than would the general convex lunar surface, and thus would become especially visible at the full moon.

"Such foldings and fractures would occur after the subsidence and solidification of the lava-forming liquid, that is, when the formation of new craters had ceased in any given region; hence they would extend across the minor lateral craters formed by outbursts from the sides of the main cone, in the manner actually observed.

"The fact that the bottom of the great walled craters of the moon is generally lower than the surrounding plains, must not be forgotten in connection with this explanation.

side of the moon cannot be entertained when due account is taken of the range of the lunar librations. Sir J. Herschel, indeed, once gave countenance to that somewhat *bizarre* theory; but he admitted, in a letter addressed to myself, that the objection I had based on the circumstances of libration was sufficient to dispose of the theory. The hypothesis that a comet had whisked away the lunar oceans and atmosphere does not need serious refutation; and it is difficult to see how the theory that lunar seas and lunar air have been solidified by intense cold can be maintained in presence of the fact that experiments made with the Rosse mirror indicate great intensity of heat in the substance of those parts of the moon which have been exposed to the full heat of the sun during the long lunar day.

If there ever existed a lunar atmosphere and lunar seas, then Prof. Frankland's theory seems the only available means of accounting for their disappearance. Accordingly we must recognize the extreme interest and importance of telescopic researches directed to the inquiry whether any features of the moon's surface indicate the action of processes of *weathering*, whether

"I will not venture further with the speculations suggested by the above-described resemblances, as my knowledge of the details of the telescopic appearances of the moon is but second-hand. I have little doubt, however, that observers who have the privilege of direct familiarity with such details, will find that the phenomena presented by the cooling of iron cinder or other fused silicates are worthy of further and more careful study."

the beds of lunar rivers can anywhere be traced, whether the shores of lunar seas can be recognized by any of those features which exist round the coast-lines of our own shores.

One circumstance may be remarked in passing. If the multitudinous lunar craters were formed before the withdrawal of lunar water and air into the moon's interior, it is somewhat remarkable that the only terrestrial features which can be in any way compared with them should be found in regions of the earth which geologists regard as among those which certainly have not been exposed to denudation by the action of water. Thus Sir John Herschel, speaking of the extinct volcanoes of the Puy de Dôme, remarks that here the observer sees "a magnificent series of volcanic cones, fields of ashes, streams of lava, and basaltic terraces or platforms, proving the volcanic action to have been continued for countless ages before the present surface of the earth was formed; here can be seen a configuration of surface quite resembling what telescopes show in the most volcanic districts of the moon; for half the moon's face is covered with unmistakable craters of extinct volcanoes." But Lyell, speaking of the same volcanic chains, describes them as regions "where the eruption of volcanic matter has taken place in the open air, and where the surface has never since been subjected to great aqueous denudation." If all the craters on the moon belonged to one epoch, or even to one era, we might regard them as produced during the withdrawal of the lunar oceans within the still heated substance of our satellite. But it is manifest that the processes which brought the moon's surface to its present condition must have occupied many ages, during which the craters formed earliest would be exposed to the effects of denudation, and to other processes of which no traces can be recognized. It is not likely, however, that the withdrawal of the lunar oceans into the moon's cavernous interior can have taken place suddenly. Up to a certain epoch the entry of the waters within the moon's mass would be impossible, owing to the intense heat, which, by maintaining the plasticity of the moon's substance, would prevent the formation of cavities and fissures, while any water brought into contact with the heated interior would at once be vaporized, and driven away. But when once a condition was attained which rendered the formation of cavities possible, the contraction of the moon's substance would lead to the gradual increase of such cavities, and so, as time proceeded, room would be found for all the lunar oceans.

We are next led to the inquiry whether the contraction of the moon's substance may not have played the most important part of all in producing those phenomena of disturbance which are presented by the moon's surface. Quite recently the eminent seismologist Mallet has propounded a theory of terrestrial volcanic energy, which not only appears to account—far more satisfactorily than any hitherto adopted—for the phenomena presented by the earth's crust, but suggests considerations which may be applied to the case of the moon, and in fact are so applied by Mallet himself. It behoves us to inquire very carefully into the bearing of this theory upon the subject of lunar seismology, and therefore to consider attentively the points in which the theory differs from those hitherto adopted.

Mallet dismisses first the chemical theory of volcanic energy, because all known facts tend to show that the chemical energies of the materials of our globe were almost wholly exhausted prior to the consolidation of its surface. This may be regarded as equally applicable to the case of the moon. It is difficult to see how the surface of the moon can have become consolidated while any considerable portion of the chemical activity of her materials remained unexhausted.

"The mechanical theory," proceeds Mallet, "which finds in a nucleus still in a state of liquid fusion a store of heat and of lava, &c., is only tenable on the admission of a very thin solid crust; and even through a crust but 30 miles thick, it is difficult to see how surface-water is to gain access to the fused nucleus; *yet without water there can be no volcano.* More recent investigation on the part of mathematicians has been supposed to prove that the earth's crust is not thin." He proceeds to show that, without attaching any great weight to these mathematical calculations, there are other grounds for believing that the solid crust of the earth is of great thickness, and that "although there is evidence of a nucleus much hotter than the crust, there is no certainty that any part of it remains liquid; but if so, it is in any case too deep to render it conceivable that surface-water should make its way down to it. The results of geological speculation and of physico-mathematical reasoning thus oppose each other; so that some source of volcanic heat closer to the surface remains to be sought. The hypothesis to supply this, proposed by Hopkins and adopted by some, viz., of isolated subterranean lakes of liquid matter, in fusion at no great depth from the surface, remaining fused for ages, surrounded by colder and solid rock, and with (by hypothesis) access of surface-water, seems feeble and unsustainable."

Now in some respects this reasoning is not applicable to the moon, at least so far as real evidence is concerned; though it is to be noticed that, if a case is made out for any cause of volcanic action on the earth, we are led by analogy to extend the reasoning (or at least its result) to the case of the moon. But it may be remarked that the solidification of the moon's crust must have proceeded at a more rapid rate than that of the earth's, while the proportion of its thickness to the volume of the fused nucleus would necessarily be greater for the same thickness of the crust. The question of the access of water brings us to the difficulty already considered,—the inquiry, namely, whether oceans originally existed on the moon. For the moment, however, we forbear from considering whether Mallet's reasoning must necessarily be regarded as inapplicable to the moon if it should be admitted that there never were any lunar oceans.

We come now to Mallet's solution of the problem of terrestrial volcanic energy.

We have been so long in the habit of regarding volcanoes and earthquakes as evidences of the earth's subterranean forces,—as due, in fact (to use Humboldt's expression), to the reaction of the earth's interior upon its crust,—that the idea presents itself at first sight as somewhat startling, that all volcanic and seismic phenomena, as well as the formation of mountain-ranges, have beeu due to a set of cosmical forces called into play by the *contraction* of our globe. According to the new theory, it is not the pressure of matter under the crust outwards, but the pressure of the earth's crust inwards, which produces volcanic energy. Nor is this merely substituting an action for reaction, or *vice versâ*. According to former views, it was the inability of the crust to resist pressure from within which led to volcanic explosions, or which produced earthquake-throes where the safety-valve provided by volcanoes was not supplied. The new theory teaches, in fact, that it is a deficiency of internal resistance, and not an excess, which causes these disturbances of the crust. "The contraction of our globe," says Mallet,* "has been met, from the

* I quote throughout from an abstract of Mallet's paper in the *Philosophical Magazine* for December, 1872. The words are probably, for the most part, Mallet's own ; but I have not the original paper by me for reference. I believe, however, that the abstract is from his own pen.

period of its fluidity to its present state,—first, by deformation of the spheroid, forming generally the ocean-basins and the land; afterwards by the foldings over and elevations of the thickened crust into mountain-ranges, &c.; and, lastly, by the mechanism which gives rise to volcanic actions. The theory of mountain-elevation proposed by C. Prévost was the only true one,—that which ascribes this to tangential pressures propagated through a solid crust of sufficient thickness to transmit them, these pressures being produced by the relative rate of contraction of the nucleus and of the crust; the former being at a higher temperature, and having a higher coefficient of contraction for equal loss of heat, tends to shrink away from beneath the crust, leaving the latter partially unsupported. This, which during a much more rapid rate of cooling from higher temperature of the whole globe, and from a thinner crust, gave rise in former epochs to mountain-elevation, in the present state of things gives rise to volcanic heat." By the application of a theorem of Lagrange, Mr. Mallet proves that the earth's solid crust, however great may be its thickness, "and even if of materials far more cohesive and rigid than those of which we must suppose it to consist, must, if even to a very small extent left unsupported by the shrinking away of the nucleus, crush up in places by its own gravity, and by the attraction of the nucleus. This is actually going on; and in this partial crushing," at places or depths dependent on the material and on conditions which

Mr. Mallet points out, he discerns "the true cause of volcanic heat.* As the solid crust sinks together to follow down after the shrinking nucleus, the *work* expended in mutual crushing and dislocation of its parts is *transformed into heat*, by which, at the places where the crushing sufficiently takes place, the ma-

* "In order to test the validity of his theory by contact with known facts" (says the *Philosophical Magazine*), "Mr. Mallet gives in detail two important series of experiments completed by him ;—the one on the actual amount of heat capable of being developed by the crushing of sixteen different species of rocks, chosen so as to be representative of the whole series of known rock-formations, from oolites down to the hardest crystalline rocks ; the other on the coefficients of total contraction between fusion and solidification, at existing mean temperature of the atmosphere, of basic and acid slags analogous to melted rocks. The latter experiments were conducted on a very large scale ; and the author points out the great errors of preceding experimenters, Bischoff and others, as to these coefficients. By the aid of these experimental data, he is enabled to test the theory produced when compared with such facts as we possess as to the rate of present cooling of our globe, and the total annual amount of volcanic action taking place upon its surface and within its crust. He shows, by estimates which allow an ample margin to the best data we possess as to the total annual vulcanicity, of all sorts, of our globe at present, that less than one-fourth of the total heat at present annually lost by our globe is upon his theory sufficient to account for it ; so that the secular cooling, small as it is, now going on, is a sufficient *primum mobile*, leaving the greater portion still to be dissipated by radiation. The author then brings his views into contact with known facts of vulcanology and seismology, showing their accordance. He also shows that to the heat developed by partial tangential thrusts within the solid crust are due those perturbations of hypogeal increment of temperature which Hopkins has shown cannot be referred to a cooling nucleus and to differences of conductivity alone."

terial of the rock so crushed and of that adjacent to it are heated even to fusion. The access of water to such points determines volcanic eruption. Volcanic heat, therefore, is one result of the secular cooling of a terraqueous globe subject to gravitation, and needs no strange or gratuitous hypothesis as to its origin."

It is readily seen how important a bearing these conclusions have upon the question of the moon's condition. So far, at any rate, as the processes of contraction and the consequent crushing and dislocation of the crust are concerned, we see at once that in the case of the moon these processes would take place far more actively than in the earth's case. For the cooling of the moon must have taken place far more rapidly, and the excess of the contraction of the nucleus over that of the crust must have been considerably greater. Moreover, although the force of gravity is much less on the moon than on our earth, and therefore the heat developed by any process of contraction correspondingly reduced, yet, on the one hand, this would probably be more than compensated by the greater activity of the lunar contraction (*i.e.* by the more rapid reduction of the moon's heat), and on the other, the resistance to be encountered in the formation of elevations by this process would be reduced precisely in the same proportion that gravity is less at the moon's surface. It is important to notice that, as Mr. Mallet himself points out, his view of the origin of volcanic heat "is independent of any particular thickness being assigned to the earth's solid crust, or to whether there is at present a liquid fused nucleus,—all that is necessary being a *hotter* nucleus than crust, so that the rate of contraction is greater for the former than for the latter." Moreover, "as the play of tangential pressures has elevated the mountain-chains in past epochs, the nature of the forces employed sets a limit" to the possible height of mountains on our globe. This brings Mr. Mallet's views into connection with "vulcanicity produced in like manner in other planets, or in our own satellite, and supplies an adequate solution of the singular, and so far unexplained fact, that the elevations upon our moon's surface and the evidences of former volcanic activity are upon a scale so vast when compared with those upon our globe."

All that seems wanted to make the explanation of the general condition of the moon's surface complete, according to this theory, is the presence of water in former ages, over a large extent of the moon's surface,—*unless* we combine with the theory of contraction the further supposition that the downfall of large masses on the moon produced that local fusion which is necessary to account for the crateriform surface-contour. It is impossible to contemplate the great mountain-ranges of the moon (as, for instance, the Apennines under favourable circumstances of illumination) without seeing that Mallet's theory accords perfectly with their peculiar corrugated aspect (the same aspect, doubtless, which terrestrial mountain-ranges would exhibit if they could be viewed as a whole from any suitable station). Again, the aspect of the regions surrounding the great lunar craters—and especially the well-studied crater Copernicus—accords closely, when sufficient telescopic power is employed, with the theory that there has been a general contraction of the outer crust of the moon, resulting in foldings and cross-foldings, wrinkles, corrugations, and nodules. But the multiplicity of smaller craters does not seem to be explained at all satisfactorily; while the present absence of water, as well as the want of any positive or direct evidence that water ever existed upon the moon, compels us to regard even the general condition of the moon's surface as a problem which has still to be explained. If, however, it be admitted that the processes of contraction proceeded with sufficient activity to produce fusion in the central part of a great region of contracting crust, and that the heat under the crust sufficed for the vaporization of a considerable portion of the underlying parts of the moon's substance, we might find an explanation of the great craters like Copernicus, as caused by true volcanic action. The masses of vapour which, according to that view, sought an outlet at craters like Copernicus must have been enormous however. Almost immediately after their escape they would be liquefied, and flow down outside the raised mouth of the crater. According to this view we should see, in the floor of the crater, the surface of what had formerly been the glowing nucleus of the moon: the masses near the centre of the floor (in so many cases) might be regarded as, in some instances, the *débris* left after the great outburst, and in others as the signs of a fresh outburst proceeding from a yet lower level; while the glistening matter which lies all round many of the monster craters would be regarded as the matter which had been poured out during the outburst.

We need not discuss in this connection the minor phenomena of the moon's surface. It seems evident that the *rilles*, and all forms of *faults* observable on the moon's surface, might be expected to result from such processes of contraction as Mallet's theory deals with.

It is, in fact, the striking features of the moon's disc—those which are seen when she is examined with comparatively low telescopic powers—which seem to tax most severely every theory which has yet been presented. The clustering craters, which were compared by Galileo to "eyes upon the peacock's tail," remain unaccounted for hitherto; and so do the great dark regions called seas. Mallet's theory explains, perhaps, the varieties of level observed in the moon's surface-contour, but the varieties of tint and colour remain seemingly inexplicable.

There is one feature of the lunar globe which presents itself to us under a wholly changed aspect if we adopt Mallet's theory. I refer to the radiations described at pp. 251-2. According to any theory which accounted for these features as due to internal forces acting outwards, it was exceedingly difficult to interpret the fact that along the whole length of these rays there can be observed a peculiar difference of brightness under direct illumination, while, nevertheless, such features of the surface as craters, mountain-ranges, plains, and so on, extend unbroken over the rays. I do not know that the theory of contraction serves to meet the difficulty completely; in fact, the difference of tint in the rays, and the circumstance that the rays can only be well seen under full illumination, appear to me to be among the most perplexing of the many perplexing phenomena presented by the moon's surface. But so far as the mere formation of radiations of enormous length is concerned, it seems to me that we have a far more promising interpretation in the theory of contraction than in any theory depending on the action of sublunarian forces. For whenever an outer crust is forced to contract upon an enclosed nucleus, a tendency can be recognized to the formation of radially-arranged corrugations. Nevertheless, it may be questioned whether—when this tendency is most clearly recognized—there is not always present some unyielding matter which forms a centre round which the radiations are formed; and it is somewhat difficult to see how or why such centres of resistance should exist in the case of the lunar crust. It is a little remarkable that here again we find ourselves led to entertain the notion that matter arriving from without has produced these sublunarian *knots*, if one may so speak, whose presence is not directly discernible, but is nevertheless strikingly indicated by these series of radiating streaks.

The circumstance already referred to, that these rays can only be well seen when the moon is full, has long and justly been regarded as among the most mysterious facts known respecting the moon. It is difficult to understand how the peculiarity is to be explained as due merely to a difference of surface-contour in the streaks; for it is as perplexing to understand how the neighbouring regions could darken from this cause just before full moon, and remain relatively dark during two or three days, as to explain the peculiarity by supposing that the rays themselves grow relatively bright. It is true that there are certain surfaces which appear less bright under a full than under an oblique illumination,—using the words 'full' and 'oblique' with reference to the general level of the surface. But the radiations occupy arcs of such enormous length upon the moon's surface, that the actual illumination of different parts of the radiations varies greatly, and of course there is a like variation in the illumination of different parts of the regions adjacent.

It is natural, under these circumstances, to inquire how far it is probable (1) that real processes of change take place month by month on the moon's surface, and (2) that it is to these processes that we owe the greater or lesser distinctness with which certain features present themselves.

We have seen that Dr. De la Rue was led, by his photographic researches into the moon's condition (for we may fairly thus describe his experience in lunar photography), to the conclusion that processes resembling vegetation take place on the moon, the period during which the vegetation passes through its series of changes being a lunar month, and that the moon may have an atmosphere of great density, but of small extent.

It is extremely important to notice that photography shows the light near the terminator to be less bright than it appears to the eye. It may be, of course, that the distinction resides mainly or entirely between the photographic power and the luminosity of these portions; there may, for example, be an excess of yellow light and a deficiency of green, while the greater photographic power of the parts under full solar illumination may indicate an increase of green light due to some process of vegetation. It is, however, important to inquire whether the greater part of the difference may not be due to a physiological cause; whether, in fact, the neighbourhood of the dark portion of the disc may not cause the illuminated parts near the terminator to appear, through contrast, brighter than they really are.

On the answer which may be given to this question depends, in a great degree (as it seems to me), the opinion we are to form of those recent researches by Mr. Birt which have appeared to indicate that the floor of Plato grows darker as the sun rises higher above it. Taking these researches in their general aspect, it cannot but be recognized that it is a matter of the utmost importance to determine whether they indicate a real change or one which is only apparent. If it is really the case that Plato grows darker under a rising sun, we should have to infer that in the case of Plato certainly, and probably in the case of other regions similarly placed, processes of change take place in each lunation which correspond (fairly) with what might be expected if these regions became covered with some sort of vegetation as the lunar month (or, which is the same thing, the lunar day) proceeds. Other explanations—meteorological, chemical, or mechanical—might indeed be available, yet in any case conclusions of the utmost interest would present themselves for consideration.

It must be remembered, however, that thus far Mr. Birt's observations (as well those made by himself as those which he has collected together) are based on eye-estimations. Nothing has yet been done to apply any photometric test to the matter; nor has the floor of Plato been brought alone under observation, but other light, of varying degrees of intensity, has always been in the field of view. Plato is seen bright when near the 'terminator,' and growing gradually darker as the sun rises higher and higher above the level of the floor of the crater. The point to be decided is, how far the brightness of Plato near the terminator is an effect of contrast. De la Rue's photographic observations go far to prove (they at least strongly suggest) that contrast has much to do with the matter. He has shown that, photographically, the parts near the terminator are not so bright as they look. May it not be that they look brighter than they are in reality? We have only to suppose that De la Rue's photographic results represent pretty accurately the true relative luminosity of different parts of the moon to answer this question at once in the affirmative.

It seems to accord with this view, that the greater darkness of the floor of Plato agrees, according to Mr. Birt's light-curves, with the time when the sun attains his greatest elevation above the level of the floor. For if the action of the sun were the cause of the darkening, we should expect the greatest effect to appear some considerable time after the sun had culminated (as supposed to be seen from the floor of Plato). We know that on our own earth all diurnal solar effects, except those which may be described as optical, attain their maximum after the sun has reached his highest point on the heavens, while all annual solar effects attain their maximum after midsummer. If an observer on Venus could watch the forests of our north temperate zones as they became clothed of vegetation, and were afterwards disrobed of their leafy garment during the progress of the year, it would not be on the 21st of June that he would recognize the most abundant signs of vegetation. In July and August vegetation most richly clothes the northern lands of our earth. It is then also that the heat is greatest; *that* is the time of true midsummer as distinguished from astronomical midsummer. And in like manner the true heat-noon is at about two o'clock in the afternoon, not at the epoch when the sun is highest, or at astronomical noon. The difference in either case amounts to about one-twelfth part of the complete period in question: in one case we find the maximum of heat a month or twelfth part of the year after the time of the sun's greatest northerly declination; in the other we find the time of greatest heat two hours, or one-twelfth part of a day, after the time of the sun's greatest elevation. If we take a corresponding portion of the lunar month, we find that the greatest effect of any solar action on the floor of Plato might be expected to take place about two and a half days after the sun had attained his greatest elevation. This differs to a sufficient degree from Mr. Birt's estimate to justify the suspicion that either the effect is physiological, or that it is purely an optical peculiarity, that is, due to the manner in which the light falls on a surface of peculiar configuration.

It does not appear to me, I may remark further, that Mr. Birt has *demonstrated* the occurrence of real variations in the condition of the spots upon the floor of Plato. He has ascertained that some of these are at times relatively darker or brighter than at others, and that this is not a mere physiological effect is proved by the fact that the result has been obtained by comparing the spots *inter se*. Nevertheless it must not be forgotten how largely the presentation of the floor of Plato towards the terrestrial observer is affected by libration, now tilting the floor more fully towards the observer and presently tilting it away from him; at one time tilting the floor eastwards, at another westwards, and at intermediate periods giving every intermediate variety of tilt; these changes, moreover, having their maximum in turn at all epochs of the lunation. Combining this consideration with the circumstance that very slight variations in the presentation of a flattish surface will cause certain portions to appear relatively dark or relatively light, it appears to me that a case has not yet been made out for those selenographical changes by which Mr. Birt has proposed to interpret these phenomena.

Nevertheless it cannot be insisted on too strongly that it is from the detailed examination of the moon's surface that we can now alone hope for exact information as to its present condition and past history. I would even urge, indeed, that the detailed examination at present being carried out is not sufficiently exact in method. I should be glad to hear of such processes of examination as were applied by Mr. Dawes to the solar spots. In particular it seems to me most important that the physiological effects which render ordinary telescopic observation and ordinary eye-estimates of size, brightness, and colour deceptive, should be as far as possible eliminated. This might be done by so arranging the observations that the conditions under which each part of the moon should be studied might be as far as possible equalized during the whole progress of the lunation. Thus, returning to the case of the floor of Plato: this region should not be examined when Plato is near the terminator as well as at the time of full moon, with the rest of the moon's disc or large portions thereof in the field of view; the eye of the observer should be protected from all light save that which comes from the floor itself; and, moreover, the artificial darkness produced for this purpose should be so obtained that the general light of the full moonlight should be excluded as well as the direct light from the disc. Then differences of tint should be carefully estimated either by means of graduated darkening-glasses, or by the introduction of artificially illuminated surfaces into the field of view for direct comparison with the lunar region whose brightness is to be determined.

When observations thus carefully conducted are made, and when the effects of libration as well as of the sun's altitude above the lunar regions studied are carefully taken into account, we should be better able than we are at present, as it appears to me, to determine whether the moon's surface is still undergoing changes of configuration. I cannot but think that such an inquiry would be made under more promising circumstances than those imagine who consider that the moon's surface has reached its ultimate condition, and that therefore the search for signs of change is a hopeless one. So far am I from considering it unlikely that the moon's surface is still undergoing change, that, on the contrary, it appears to me certain that the face of the moon must be undergoing changes of a somewhat remarkable nature, though not producing any results which are readily discerned by our imperfect telescopic means. It is not difficult to show reasons at least for believing that the face of the moon must be changing more rapidly than that of our earth. On the earth, indeed, we have active subterranean forces which may, perhaps, be wanting in the moon. On the earth again, we have a sea acting constantly upon the shore,—here removing great masses, there using the *débris* to beat down other parts of the coast, and by the mere effect of accumulated land-spoils acquiring power for fresh inroads. We have, moreover, wind and rain, river action, and glacier action, and, lastly, the work of living creatures by land and by sea, while most of these causes of change may be regarded as probably, and some as certainly, wanting in the case of our satellite. Nevertheless, there are processes at work out yonder which must be as active, one cannot but believe, as any of those which affect our earth. In each lunation, the moon's surface undergoes changes of temperature which should suffice to disintegrate large portions of her surface, and with time to crumble her loftiest mountains into shapeless heaps.* In the long lunar night of fourteen days, a cold far exceeding the intensest ever produced in terrestrial experiments must exist over the whole of the unilluminated hemisphere; and under the influence of this cold all the substances composing the moon's crust must shrink to their least dimensions—not all equally (in this we find a circumstance increasing the energy

* Nasmyth pointed this out long since.

of the disintegrating forces), but each according to the quality which our physicists denominate the coefficient of expansion. Then comes on the long lunar day, at first dissipating the intense cold, then gradually raising the substance of the lunar crust to a higher and higher degree of heat, until (if the inferences of our most skilful physicists, and the evidence obtained from our most powerful means of experiment can be trusted) the surface of the moon burns (one may almost say) with a heat of some 500° F. Under this tremendous heat all the substances which had shrunk to their least dimensions must expand according to their various degrees; not greatly, indeed, so far as any small quantity of matter is affected, but to an important amount when large areas of the moon's surface are considered. Remembering the effects which take place on our earth, in the mere change from the frost of winter to the moderate warmth of early spring, it is difficult to conceive that such remarkable contraction and expansion can take place in a surface presumably less coherent than the relatively moist and plastic substances comprising the terrestrial crust, without gradually effecting the demolition of the steeper lunar elevations. When we consider, further, that these processes are repeated not year by year, but month by month, and that all the circumstances attending them are calculated to render them most effective because so slow, steadfast, and uniform in their progression, it certainly does not seem wonderful that our telescopists should from time to time recognize signs of change in the moon's face. So far from rejecting these as incredible, we should consider the wonder rather to be that they are not more commonly seen, and more striking in their nature. Assuredly there is nothing which should lead our telescopists to turn from the study of the moon, as though it were hopeless to seek for signs of change on a surface so desolate. Rather they should increase the care with which they pursue their observations, holding confidently the assurance that there are signs of change to be detected, and that in all probability the recognition of such change may throw an instructive light on the moon's present condition, past history, and probable future.

INDEX TO THE MAP OF THE MOON.

TABLE I.

GREY PLAINS, USUALLY CALLED SEAS.

A. Mare Crisium
B. —— Humboldtianum
C. —— Frigoris
D. Lacus Mortis
E. —— Somniorum
F. Palus Somnii
G. Mare Tranquillitatis
H. Mare Serenitatis
I. Palus Nebularum
K. ——Putredinis
L. Mare Vaporum
M. Sinus Medii
N. —— Æstuum
O. Mare Imbrium
P. Sinus Iridum
Q. Oceanus Procellarum
R. Sinus Roris
S. Mare Nubium
T. —— Humorum
V. —— Nectaris
X. —— Fœcunditatis
Z. —— Australe

TABLE II.

CRATERS, MOUNTAINS, AND OTHER OBJECTS.

Numbered as in the Map.

1. Promontorium Agarum
2. Alhazen
3. Eimmart
4. Picard
5. Condorcet
6. Azout
7. Firmicus
8. Apollonius
9. Neper
10. Schubert
11. Hansen
12. Cleomedes
13. Tralles
14. Oriani
15. Plutarchus
16. Seneca
17. Hahn
18. Berosus
19. Burckhardt
20. Geminus

134. Laplace
135. Heraclides
136. Maupertuis
137. Condamine
138. Bianchini
139. Sharp
140. Mairan
141. Louville
142. Bouguer
143. Encke
144. Kepler
145. Bessarion
146. Reiner
147. Marius
148. Aristarchus
149. Herodotus
150. Wollaston
151. Lichtenberg
152. Harding
153. Lohrmann
154. Hevel
155. Cavalerius
156. Galileo
157. Cardanus
158. Krafft
159. Olbers
160. Vasco de Gama
161. Hercynian Mts.
162. Seleucus
163. Briggs
164. Ulugh Beigh
165. Lavoisier
166. Gérard
167. Repsold
168. Anaxagoras
169. Epigenes
170. Timæus
171. Fontenelle
172. Philolaus
173. Anaximenes
174. Anaximander
175. Horrebow
176. Pythagoras
177. Œnopides
178. Xenophanes
179. Cleostratus
180. Tycho
181. Pictet
182. Street
183. Sasserides
184. Hell
185. Gauricus
186. Pitatus
187. Hesiodus
188. Wurzelbauer
189. Cichus
190. Heinsius
191. Wilhelm I.
192. Longomontanus
193. Clavius
194. Deluc
195. Maginus
196. Saussure
197. Orontius
198. Nasireddin
199. Lexell
200. Walter
201. Regiomontanus
202. Purbach
203. Thebit
204. Arzachel
205. Alpetragius
206. Promontorium Ænarium
207. Alphonsus
208. Ptolemæus
209. Davy
210. Lalande
211. Mösting
212. Herschel
213. Bullialdus
214. Kies
215. Guerikė
216. Lubiniezky
217. Parry
218. Bonpland
219. Fra Mauro
220. Riphæan Mts.
221. Euclides
222. Landsberg
223. Flamsteed
224. Letronne
225. Hippalus
226. Campanus
227. Mercator
228. Ramsden
229. Vitello
230. Doppelmayer
231. Mersenius
232. Gassendi
233. Agatharchides
234. Schiller
235. Bayer
236. Rost
237. Hainzel
238. Capuanus
239. Schickard
240. Drebbel
241. Lehmann
242. Phocylides
243. Wargentin
244. Inghirami
245. Bailly
246. Dörfel Mts.

468. Blanchinus
469. Delaunay
470. Faye
471. Donati
472. Alexander
473. Janssen
474. Cassini, J. J.
475. Foucault
476. Lee
477. Piton
478. Herschel, Miss
479. Brayley
480. Lockyer
481. Daniell
482. Grove
483. Murchison
484. Lassell
485. Hermann
486. Manners
487. Schmidt
488. Secchi
489. Schiaparelli
490. Harbinger Mts.

TABLE III.

Alphabetical Table of Lunar Nomenclature.

TABLE IV.

ELEMENTS OF THE MOON. EPOCH, 1ST JANUARY, 1801.

(*Earth's equatorial diameter is taken as* 7925·8 *miles.*)

Mean longitude of moon at epoch	118° 17′ 8″·3
Ditto node	13° 53′ 17″·7
Ditto perigee	266° 10′ 7″·5
Mean distance from the earth (earth's radius 1) ...	60·2634
Same in miles	238,818
Maximum distance in miles	252,948
Minimum do. do.	221,593
Eccentricity of orbit	0·05490807
Mean equatorial horizontal lunar parallax ...	57′ 2″·7
Maximum do. do.	1° 1′ 28″·8
Minimum do. do.	53′ 51″·5
Moon's mean apparent diameter	31′ 5″·1
Moon's maximun do.	33′ 30″·1
Moon's minimum do.	29′ 20″·9
Moon's diameter in miles...	2159·6
Moon's surface in square miles	14,600,000
Moon's diameter (earth's equatorial diameter as 1)	0·2725
Earth's equatorial diameter (moon's as 1) ...	3·670
Moon's surface (earth's as 1)	0·0742
Earth's surface (moon's as 1)	13·471
Moon's volume (earth's as 1)	0·0202
Earth's volume (moon's as 1)	49·441
Moon's mass (earth's as 1)	0·01228
Earth's mass (moon's as 1)	81·40
Density (earth's as 1)	0·60736
Density (water's as 1, and earth's assumed = 5·7)	3·46
Gravity, or weight of one terrestrial pound ...	0·16547
Bodies fall in one second in feet	2·65
Mean inclination of orbit	5° 8′
Maximum do. do.	5° 13′
Minimum do. do.	5° 3′
Inclination of axis	1° 30′ 11″·3

Synodical revolution in days	29·53059
Sidereal do. do.	27·32166
Tropical do. do.	27·32156
Anomalistic do. do.	27·55460
Nodical do. do.	27·21222
Maximum evection	1° 20′ 29″·9
Maximum variation	35′ 42″·0
Maximum annual equation	11′ 12″·0
Maximum libration in latitude	6° 44′
Ditto do. in longitude	7° 45′
Maximum total libration (from earth's centre) ...	10° 16′
Maximum diurnal libration	1° 1′ 28″·8
Surface of moon never seen (whole as 10,000, and diurnal libration neglected)	4198
Surface seen at one time or other do. do. ...	5802
Ditto do. never seen if diurnal libration be taken into account	4111
Ditto do. seen at one time or other do.	5889
Mean revolution of nodes (retrograde) in days ...	6793·391
Ditto do. do. in years ...	18·5997
Mean regression of nodes per annum	19° 21′ 18″·3
Mean regression of node between successive conjunctions of sun and rising node	18° 22′ 3″·2
Mean interval between such conjunctions in days	346·607
Mean revolution of perigee (advancing) in days ...	3232·575
Ditto do. in years	8·8505
Mean advance of perigee per annum	40° 40′ 31″·1
Ditto do. between successive conjunctions of sun and perigee	45° 51′ 23″·7
Mean interval between such conjunctions in days	411·767

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