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MATHEMATICAL FUNDAMENTALS OF AERIAL PHOTO-INTERPRETATION OF FORESTS

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MATHEMATICAL FUNDAMENTALS OF AERIAL PHOTO-INTERPRETATION OF FORESTS

by

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TABLE OF CONTENTS

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

TABLE OF CONTENTS (cont'd)

FOREWORD

The contents of this book can be divided into three parts for convenience. In the first part (Chapter 1), we characterize the present status of measurement photo-interpretation in forest inventory and the extent to which correlations in the crown cover of forest stands have been studied. In the second part (Chapters 2, 3, 4, 5, 6, 7, and 8), we present results of theoretical and experimental studies on use of mathematical modeling and point systems and sets in investigating tree stands, discovering approximational relationships and methods of determining indicators in appraising the value of stands as applied to problems in forestry and topographic photo-interpretation. The problem of studying statistical correlations of the distributional and correlational relationships among stand indicators is taken up more fully. Elements of the theory and measurement methods of determining topographic and appraisal information from aerial photographs are set forth. The third part (Chapters 9 and 10) outlines ways of using measurement photo-interpretation in forest management and aerial evaluation of forests and looks at the prospects for automating the collection of information from aerial photographs and the use of electronic computers in forest management.

The book is the first to deal with this subject and treat it in this approach and so it obviously is not free of shortcomings. Some problems call for further exploration and can become subjects for independent monographs.

A large number of studies have been published on interpretation of aerial photographs of forests and on aerial methods in the U.S.S.R., the United States, Canada, and other countries. Sizable successes have been gained in theory and practice, although the theoretical fundamentals and objective methods of measurement interpretation of aerial photographs of forests are in need of further development. The practical bearing of reaching this goal was stressed by the International Photogrammetric Society, which in 1956 recommended that all countries engage in scientific research aimed at developing in the near future the theory and methods of determining information about forests by using aerial photographs. The task was even more fully posed by the All-Union Conference on the Theory and Practice of Aerial Photographic Interpretation, which in 1961 took note of the underdeveloped state of theory and recognized as a paramount problem the formulation of general theoretical fundamentals and objective methods of interpreting aerial photographs.

The expression "interpretation of aerial photographs" in our view does not fully measure up to what the work amounts to. Under today's conditions it is better to talk about the theory and methods of gathering information from aerial photographs. This is the sense in which the term is used in this book. Therefore, the book's title does not fully describe its content.

This book is intended for engineering-technical and scientific personnel in forest management, aerial photography, land management, geobotany, geography, cartography, and other specialists who rely on aerial methods.

The chapters written by M. K. ?Bcharov have been read by G. G. Samoylovich.

The authors express their gratitude to Docent B. A. Kozlovskiy, and also to the following professors -- V. I. Sukhov, A. V. Mazlov, N. D. Il'inskiy, Z. P. Morozov, A. N. Lobanov, and V. F. Deyneko, and to the following Candidates of Sciences -- M. I. Malykh, S. D. Dubov, S. F. Bogatov, V. M. Zaytsev, V. A. Zakharov, G. F. Panin, P. A. Yakovlev, N. A. Kornilov, and Z. I. Tolmacheva for valuable counsel and comments.

Chapters 2, 3, 4, 5, 6, 7, 8, and 10 were written by M. K. Bocharov and Chapters 1 and 9 -- by G. G. Samoylovich.

CHAPTER I

PRESENT STATE OF AERIAL PHOTO-INTERPRETATION IN FOREST INVENTORY AND STUDY OF CROWN CANOPY STRUCTURE

1. Present State of Aerial Photo-Interpretation and Use of Correlations of Crown Canopy Structure in Forest Inventory

Under today's conditions materials of aerial photography are the basis for forest inventories. Their use has led to a considerable improvement both in field as well as laboratory work. Consonantly, the technological process of inventorying forests based on use of aerial photographs has also changed.

At present, contour interpretation of aerial photographs and the visual estimation method of field evaluation of forests using aerial photographs has undergone the greatest development in forest management. Laboratory evaluational interpretation as well as measurement methods of determining evaluation indexes of forest stands from aerial photographs have not yet seen marked development in practical use. Also going unused are correlational ties linking indexes of stands and correlations of stand composition.

Still, investigations conducted in this area over the past 40 years both in our country and abroad have demonstrated wholly beneficial results. We will cite a number of them.

Based on the studies of Ristov (1924), Krutsh published the results of experiments on interrelationships between crown diameters and diameters at breast height (b.h.) in determining stand reserves from aerial photographs. Subsequently, these experiments were continued in Germany by Tsiger and

Neyman. We know of studies in this area in the United States [81], Canada, Finland, Sweden, and other countries. In our country, the first experiments on interrelationships between appraisal indexes of forest stands were begun by A. Ye. Novosel'skiy in 1923 and by G. G. Samoylovich since 1926. Their results were published in 1940 [46].

N. I. Baranov was responsible for a considerable number of test plots for exploring interrelationships between tree crowns and appraisal characteristics of tree stands (published in 1948 in the collection of the Central Scientific Research Institute of Forestry, Voprosy Lesnoy Taksatsii [Problems of Forest Appraisal] and in the article [4]).

The study of V. I. Levin and V. I. Kalinin validated the existence of interrelationships between crown diameters and cross-sectional areas at breast height, species numbers of crowns, and trunk volumes (Trudy Arkhangel' Lesotekhnicheskogo Instituta [Proceedings of the Arkhangel' Forest Technology Institute], No 15, 1954).

M. K. Bocharov derived relationships between appraisal indexes of stands in the light of commitments in forest cartography in a long series of studies [7-16] making use of aerial photographs.

V. S. Moiseyev, relying on materials of investigations in 1949-1951 at the Vokhomskiy Tree Farm Kostromskaya Oblast in collection No 1, Uchet Lesosyr'yevykh Resursov i Ustroystvo Lesov [Evaluating Forest Raw Material Resources and Forest Management], published in 1957, derives correlational equations relating mean tree stand diameters to the maximum and mean diameters of crown projections.

A. N. Polyakov in the journal Lesoinzhenernoye Delo [Forestry Engineering], No 1, 1958, published some new data on correlations in the structure of simple pure and even-aged stands and brought to light characteristerics in the distribution of trees by their crown diameters.

Relying on material obtained in different rayons of the Soviet Union, A. M. Berezin and I. A. Trunov [6] established that the correlations between mean crown diameters and d.b.h. correspond only to tree stands grown under specific physical-geographic conditions and cannot be extrapolated to other geographical regions.

In the collection 58 of the Trudy Instituta Lesa i Drevesiny Sibirskogo Otdeleniya Akademii Nauk SSSR [Proceedings of the Institute of Forests and Wood of the Siberian Division of the USSR Academy of Sciences] (1962), an article by N. G. Kharin was published on the study of relationships between several appraisal indexes of tree stands for interpretation of large-scale aerial photographs.

Based on the forest management instruction of 1964 to the lower organizational levels of forest management, laboratory and office appraisal interpretation of aerial photographs is recommended for sections located in mid-sighting expanses. However, it is also based on visual estimations of photo-images of tree stands visible in the stereoscope. Experience of practitioners and training play a decisive role in this procedure. Here the use by some only of visual estimation observations cannot guarantee uniformity in work and high quality, since visual impressions are subjective. The need has arisen to advance to new, objective methods of work based on the proper use of appraisal correlations of tree stand structure, on measurements of appraisal indexes of crown canopy based on aerial photographs, and on mathematical validation of forest interpretation of aerial photographs.

Below we will state which prerequisites for this are already in effect at the present time.

What then has held up the wide advancement of office interpretation and of measurement methods of securing appraisal characteristics of forest stands?

First of all, the use of aerial photographs of relatively small scales for this purpose (1:25,000 and smaller). The volume of information obtained by using such aerial photographs is limited. Many details of stands are lost on such aerial photographs because of the small image, and not more than 20-30 percent of the trees compared to their actual number show up. The discernability of the canopy in depth is sharply reduced, which hampers use of measurement instruments. As many years of experience have shown, scales of 1:10,000 and larger and not smaller than 1:15,000 are the most optimal for appraisal interpretation. Smaller scales of aerial photographs are the most suitable for cartographic purposes, in fact, and contour interpretation of aerial photographs. Therefore, at present the problem of the ends served by aerial photography using simultaneously two aerial cameras with a focal-length ratio of approximately 1:2 has been properly posed.

Extensive use of office interpretation as part of a unified technological process of forest inventory has become possible with the use of larger scales in aerial photography. Office interpretation of aerial photographs will be based not only on recognition of the objects in the photographed locality, but also on determination of qualitative and quantitative characteristics of the stands with the broad use of interrelationships between appraisal indexes of the stands and the corresponding measurement instruments.

The process of office interpretation of aerial photographs is not purely one of laboratory work. It commences in the period of preparatory work for forest management with study of the tree stands in sample plots selected out of sections that differ in appraisal characteristics. During this period the necessary interrelationships between appraisal indexes of stands must be uncovered for their subsequent employment in the various operational steps.

In the office period preceding field work contour interpretation of aerial photographs must be carried out, and depending on the area of operations and the level of forest management, the determination and measurement of several appraisal indexes of the stands that will be verified and supplemented through field work, given the essential minimum of steps for this purpose based on the rational placement of the appraiser's survey lines.

Only if there is the far-ranging incorporation and upgrading of the method of office interpretation of aerial photographs can several procedures in forest inventory be automated in forest management, above all at its lower levels.

Enlarging the scales of aerial photography and increasing the volume of office interpretation of aerial photographs under certain sets of conditions are compensated by reduced volume of time-consuming and costly field work.

How thoroughly several correlations in stand structure have been studied for purposes of aerial photo-interpretation, measurement methods of determining appraisal indexes, and mathematical fundamentals of measurement interpretation will be propounded in the following chapters.

The extensive advancement and use of measurement methods of interpretation and the transition from the aerial photograph to compiling appraisal indexes of stands based on the study of how appraisal indexes are interrelated requires, on the one hand, a concept of the imaging properties of aerial photographs, knowing what can be measured on aerial photographs of different dimensions, and on the other -- study of interrelationships between measured indexes of tree stand canopies and those appraisal indexes that are required in drawing up appraisal descriptions of tree stands.

To determine the essentials and content of this kind of interpretation requires first of all that we dwell on a number of features of tree stand structure and the nature of their photographic imaging on flat aerial photographs. Knowing what and how objects show up on aerial photographs, we can more confidently not only draw up measurement methods, but also determine, by starting from correlations of the stand structure, appraisal indexes of tree stands called for in practical work.

2. Present State of Studies on the Structure of Tree Stand Canopies in Relation to Aerial Photo-Interpretation

The concept of stand canopy. Thus far study of the structure of stand canopies has not been given autonomous prominence, therefore concepts of it have been treated in different ways in handbooks and textbooks.

Expanded research in sylviculture and progress in forest interpretation of aerial photographs have pushed to the fore the need to study the structure of the canopy of simple and mixed stands, and accordingly the interrelationship between canopy indexes and other appraisal indexes of tree stands.

It appears necessary to study forest cover by stand levels. Correlations inherent to tree stands (elements of a forest) comprising growths [canopy levels] must be appropriately expressed and find reflection in the makeup of the stand canopy.

The arboreal canopy of any tree stand is made up of the totality of the tree crowns. The latter differ in form and size both within the limits of a single tree species as well as among several species. In natural stands, depending on tree spacing and unevenness in their arrangement in the tract, the stand canopy will correspondingly be made up of differences in tree heights.

The composition of the tree crowns directly constituting the stand canopy is intimately related to the biological properties of each tree species considered separately, the species makeup of the stands, characteristics of tree growth and development, and on the conditions of the habitat.

Sylvicultural-biological properties of tree species, correlations in the makeup of tree stands of forest elements, and a long series of other factors have an effect on the composition of the canopy of each stand of trees.

In any more or less closed stand there are trees whose crowns are located in isolation in the canopy (free) or only touch at their branches; crowns whose lateral portions to some extent extend under the crowns of neighboring trees and, finally, crowns located wholly under the crowns of other trees are also constituents. Within the limits of a growth level the crowns of all these trees will then constitute the canopy of the stand growth level.

Consequently, we take the term canopy of a pure or mixed stand to denote that totality of crowns of trees differing in shape and size that within the limits of a growth story stand apart from others, are in contact with others, or are located in part, to some extent, or altogether under the crowns of neighboring trees.

Tree stands that are complex in configuration can include, in addition to the principal canopy, another one lying beneath it made up generally of other tree species whose crowns correspondingly mingle with each other within the limits of this growth story. For example, a thick spruce story underneath a birch story, or underneath oak, a growth story made up of its associates: linden, maple, or others.

In infrequent tree stands with closures of 0.3-0.4, the canopy can be made up of crowns of trees that are not predominantly in contact with each other but stand considerable distances apart. However, when trees are arranged in group fashion, some of the crowns can have their branches entering into the crowns of neighboring trees, and less commonly can be located under these neighboring trees. In tree groups this will characterize the closedness of the crowns, which is not one and the same thing as the closedness of a canopy.

Usually, with increase in stand density the extent of canopy closure also rises, as does the number of trees with crowns that are partly or completely covered by crowns of neighboring trees under which the former trees are situated.

The horizontal projection of a canopy will be made up of trees with crowns in the following situations: a) located in isolation in the canopy, that is, free-standing, b) touching the crowns of other trees, and c) partially overlapped by neighboring crowns.

The following main features characterize the horizontal projection of a stand canopy and its appearance from the top view: the forms and dimensions of the projections of tree crowns, the disposition of the crown arrangement in the canopy, the horizontal extent of canopy closure, and in mixed stands also the species makeup of the stand canopy. In addition, the following accompanying features are directly associated with these characteristics already listed: the forms and dimensions of intervals between crowns, distances between crowns, and several other characteristics (Figure 1).

The following main features characterize the vertical projection of a stand canopy to one side, its profile: the heights of trees, the forms and dimensions of their crowns, the heights up to the greatest width of crowns, the extent (length) of crowns, the arrangement of trees, the extent of vertical closure of the canopy, the extent or depth of the canopy, and also a number of other features (Figure 2).

The appearance of one canopy projection can intergrade into another. By knowing the structure of the stand canopy as viewed from the side, it is easier to imagine to oneself what the appearance would be viewed from the top, in the horizontal projection.

We note that thus far features of the form and dimensions of tree crowns that make up the stand canopy, the arrangement they occupy in the canopy, the extent of crown overlap, the variation in the size and nature of intervals between crowns, and the variation in incremental growth of crowns with increase in age in different forest types have not yet been adequately studied.

In the study of the pattern of stand growth, attention has also not been paid to variation in indexes characterizing crown dimensions, with the exception of data published, for example, by Gerkhardt in tables on stand growth patterns. Knowledge and provision for the above-listed indexes of stand canopies will prove of unquestioned value, for example, in sylviculture, in particular in studying forest upkeep fellings, for improvement and refinement in studies on interpreting aerial photographs and describing forests from aircraft. We

Diameters of tree crowns in a permanent sample area in quadrant 73 of the Derzhavinskiy forest tract of the Buzulunskiy Pine Forest. Area: 0.5 hectare

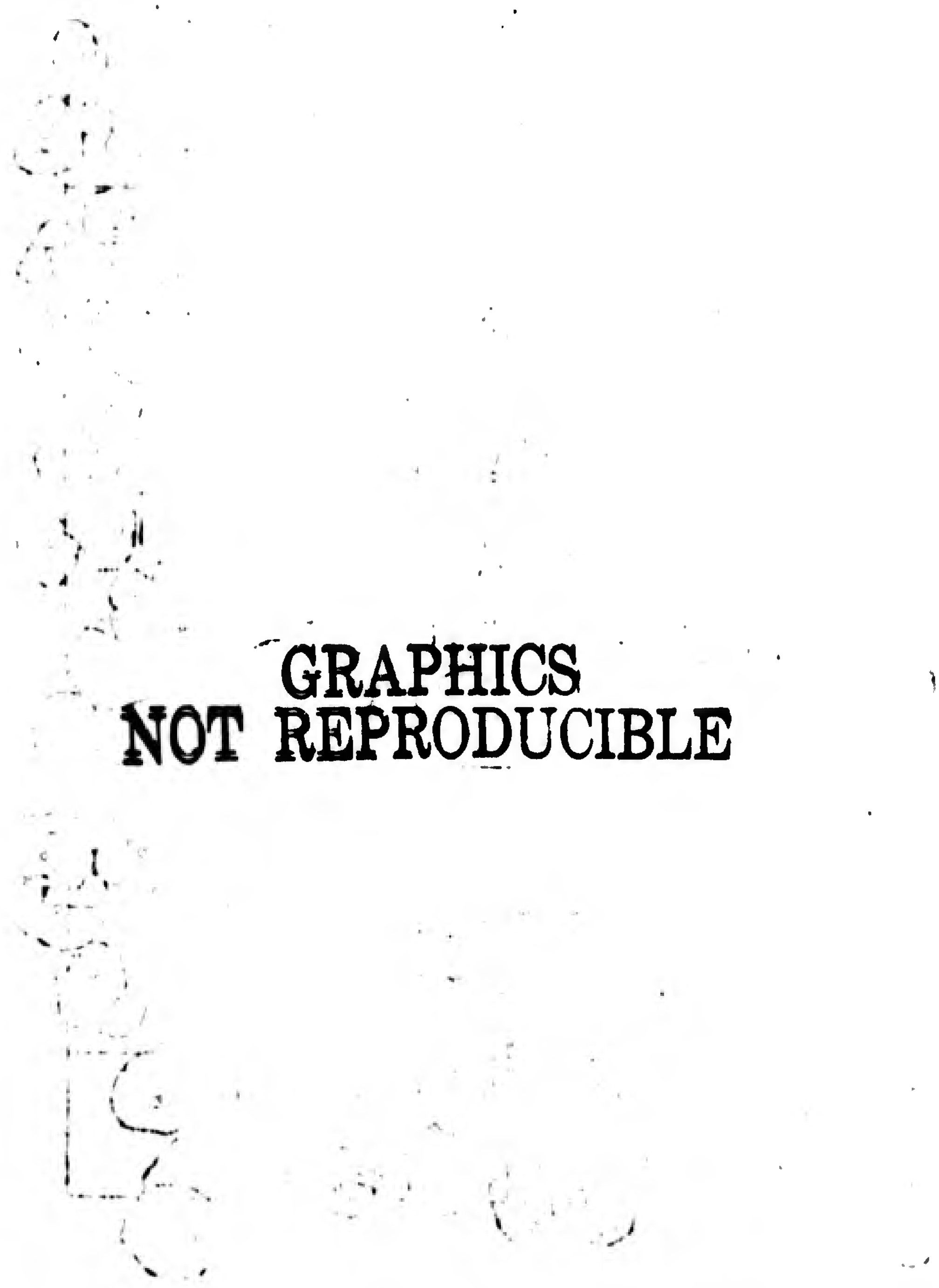

Figure 1. Horizontal projection of a stand of pine

began the study of this problem as far back as 1926 in managing the forests of the Mari ASSR, continued it in the Buzulunskiy pine forest (now Orenburgskaya Oblast) of Bashkirskaya ASSR, Leningradskaya Oblast, and then -- in other areas of the Soviet Union. These investigations were conducted by N. N. Mazhutin [39, 40], A. M. Berezin [6], and I. A. Trunov [56].

V. S. Moiseyev published results of work under conditions prevailing at the Vokhomskiy Tree Farm, Kostromskaya Oblast in his author's abstract of his candidate's dissertation, Deshifrirovaniye po Aersnimkam Smeshannykh Lesov pri Lesoustroystve v Sredney Chasti Tayezhnoy Zony [Interpretation of Aerial Photographs of Mixed Forests in Forest Management in the Central Part of the Taiga Zone] in 1952; A. Ya. Zhukov published work done in the same area in "Study of a Stand of Deciduous and Cedar Plantings Under Mountain Conditions for the Purpose of Interpretation of Aerial Photographs and Aerial Appraisal of Forests" in Trudy Lesotekhnicheskoy Akademii [Proceedings of the Forestry Technology Academy], No 82, Part II, 1957.

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Figure 2. Schematic view of a profile of a mixed stand and its representation in an aerial photograph.

A. N. Polyakov studied interrelationships between full occupancy, crown closure, and density of stands, at 25 sample areas of pine plantings of site class I located in Vladimirskaya Oblast, which was published in Nauchnyye Doklady Vysshey Shkoly [Scientific Reports of the Higher Schools] (Lesoinzhenernoye Delo [Forestry Engineering], No 1, 1959).

N. G. Kharin divided trees only into two categories in laying the groundwork for a method of forest interpretation of large-scale aerial photographs and counting trees on sample plots based on the percentage of crown participation in the canopy, using our original assumptions: trees emerging into the upper canopy and trees covered by the crowns of other trees. (Trudy Instituta Lesa i Drevesiny [Proceedings of the Institute of Forest and Wood] Vol 58, No 1, 1962).

At the present time there is a need for a new type of table giving the growth pattern to which the following canopy indicators have been added: composition of upper canopy (projected), crown closure, crown diameters D_c and crown length l_c.

Initially, individual considerations on a method of studying canopy cover were published incidentally in a presentation of various problems of forest interpretation of aerial photographs. When, however, the need for a more purposeful study of canopy cover cropped up, the first considerations found generalization in section VIII of Spravochnik Taksatora [Appraiser's Handbook] [54], and then in our booklet [50].

Types of canopies. Tree stands can be quite divergent structurally in different forest-growing zones. The composition of timber stands can include a variety of forest-forming tree species. Exhibiting biological properties intrinsic to trees and occupying specific interrelationships under given site conditions, they produce under natural settings particular structures of forest cover. Man, entering into the stand and intervening with economic activities, can alter it in the direction he needs, especially by forest maintenance fellings or gradual-sampling fellings. We cannot touch on all of these problems. We will focus our attention only on several general schemes of the structure of forest cover growing under conditions of the northern Taiga zone. We can conventionally single out three most widely distributed and typical schemes of forest cover structure.

In the single-story stand (typical forest element), the tree cover, especially in its upper section that shows up on aerial photographs, will be made up of crowns of trees more or less similar to each other in shape and size of crown, with small differences in tree heights and crown lengths. The crowns of trees predominant in this part of the forest canopy overlap at approximately the same height (h_{Dc}), producing the so-called horizontal closure (Figure 3, a).

The depth of a canopy of such stands, that is, the distance from the tip of the tallest tree to the beginning of the active crown of the lowest tree that is part of the canopy will be the greatest. On large-scale aerial photographs the ground surface shows up well in such stands, especially for average occupancies of 0.7 and lower.

Another type of canopy will have two-story stands. In these the tips of the crowns of the second-story trees will either overlap with the lower portions of the crowns of the first-story trees or partially enter into the upper part of the canopy of its first-story stand. This type of stand shows a clearly pronounced vertical closure (Figure 3, b).

Finally, there can be multistory stands consisting of several tree species in which some trees are located under others, have different heights, shapes, and lengths of crown, and as a consequence produce vertically stepped canopy closure (Figure 4). The depth of the canopy in such stands will be the greatest and their discernability in the stereoscope will also be slight.

The first description of this scheme of stand canopy structure is to be found in a study by Professor Dengler.

For a graphic estimation of the way the stand canopy looks from the side and its structure in the cross-section of vertical plane, we can sketch the stand profiles on different scales along the vertical and along the horizontal (Figure 5). In these we can depict at the desired scale typical crown shapes and sizes of different species (with an indication in percentages of the representation of particular crown shapes), the nature of the tree arrangement in the section relative to each other, the presence of a second story and its distribution in the stand, the extent of uniformity in the admixture of different species, and other most characteristic features of the stand. When colored pencils are used for different trees and stories, a vivid representation of the appearance and structure of the canopy of stands results. The first

Figure 3. Schematic view of the profile of a canopy of stands: a -- with horizontal closure of tree crowns; b -- with vertical closure of tree crowns.

experiments in this direction were conducted in 1926 in work done in the Mari ASSR, and in 1937-1938 appraisers sketched in profiles in repeated forest management work in the Bashkir ASSR, which proved useful both in the period of training appraisers studying interpretation clues as well as in office appraisal interpretation of aerial photographs.

If we project a bundle of parallel sun rays to this profile of the stand at an angle equal to the height of the sun at the moment of aerial photography, then we can grasp some, though approximate, idea of how it will show up in aerial photographs (Figure 2).

Figure 4. Schematic view of a profile of a stand canopy with stepped closure of tree crowns.

Features of forest cover in aerial photographs. On flat aerial photographs of moderate and large scale viewed stereoscopically the observer can, depending on their scale, differentiate not only features of the horizontal, but also of the vertical projection of stand cover, that is, can discern the visible model of the stand.

The following can be distinguished in a canopy of each of these stand types: totality of tree crowns that commonly have different shapes, sizes, tones, or colors; the arrangement of trees and crown spacings; the intervals between them; and the extent of crown closure. Depending on the types of aerial photographic film used, the species composition of the stands can be differentiated (interpretation composition). The latter, based on black and white tones, can best be discerned from spring and autumn panchromatic and summer infrachromatic aerial photographs, and when color film is used -- from spectrozonal aerial color photographs.

In addition, differences in tree heights and canopy depth can be distinguished in forest cover in the stereoscopic viewer and as a consequence the discernability of the ground surface.

Depending on the illumination conditions at the moment of aerial photography, the height of the stand, and the sun's

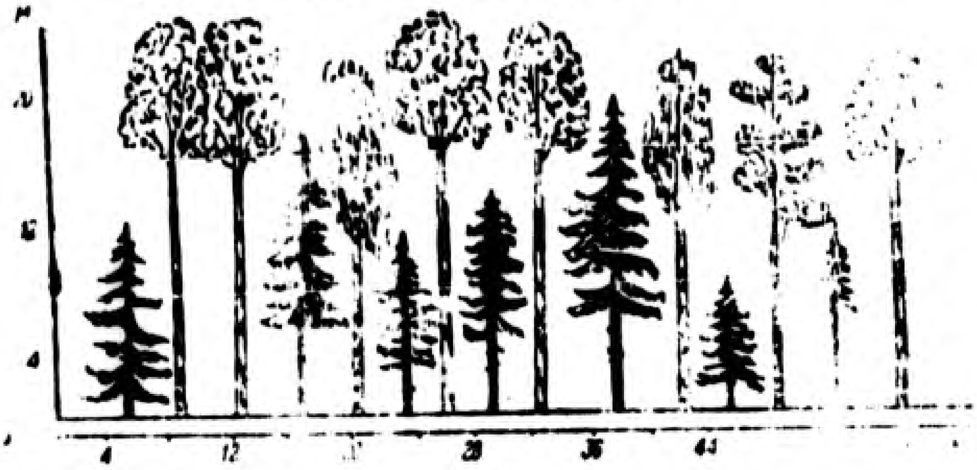

Figure 5. Profile of the canopy cover of an aspen-chernichnik stand with horizontal and vertical scales (in meters).

azimuth, imaging of illuminated parts of tree crowns shows up on aerial photographs. But trees whose crowns are wholly in shadow produced by the crowns of neighboring trees cannot produce images, especially when the aerial photography is done on a bright, sunny day.

All visible indexes of forest cover can be measured from aerial photographs using different methods and instruments, namely: crown sizes, distances between crowns, intervals between crowns, crown closure, number of trees, species composition, tree heights, and canopy depth.

The main roles in using moderate-scale aerial photographs are as follows: finding interrelationships among natural forest cover indexes and those that show up on aerial photographs, studying interrelationships between stand canopy indexes susceptible to measurement and those not discernible in aerial photographs but needed in compiling appraisal descriptions of forest tracts.

Mathematical interrelationships studied in this direction are set forth in the following chapters. They have already laid a reliable foundation for measurement interpretation and determination of several appraisal indexes of forest stands.

<u>Structure of the horizontal projection of forest cover.</u> This problem is most crucial in the theory and practice of

forest interpretation of aerial photographs. Therefore, first of all we must deal with the question of the structure of the horizontal projection of forest cover directly perceptible in the stereoscope.

All trees forming the horizontal projection of the natural forest cover can be schematically placed in three categories by how fully they participate in the projection.

I. Trees standing apart in the canopy with crowns separate from the crowns of other trees or only in contact with them at their branches. This category of trees is called: trees with "free" crowns.

Figure 6. Typical categories of trees participating in the horizontal projection of a canopy.

II. Trees whose crown branches partially penetrate the crowns of neighboring trees. Their peaks are exposed and their crowns are not more than 50 percent covered by the crowns of neighboring trees. This category of trees is called: trees with "partially covered" crowns.

III. Trees whose crowns stand completely under the crowns of neighboring trees and are completely covered by them. These are chiefly trees of the lower or less often of the moderate thickness classes (Figure 6). They are called: trees with "covered" crowns.

If trees can be counted by species from the extent their crowns participate in the canopy and from their thickness class, as a result we can establish what the number of trees is and what thickness classes are projected in the forest cover [50].

knowing what the number of trees of various species that enter into the upper canopy cover and are projected into it, we can more validly proceed to determining the makeup of the upper stand canopy (interpretation composition). This latter term was introduced by us because there can be a difference between it and the canopy calculated from ground cruising data and this difference will be all the greater the larger the number of trees under the crowns of trees in the stand canopy.

The makeup of stands is usually determined from the proportion of the stock of each particular species in the overall stand. A similar method is applicable also in determining the interpretation composition of the stand. The latter index is more validly determined from the number of trees producing a projection of the stand canopy onto aerial photographs. The role of this feature takes on greatest importance when using moderate- and large-scale aerial photographs.

Stand canopy indexes. In looking at the stand canopy in the overall view, we can assume that any stand in aerial photographs is made up of images of the projections of tree crowns and intervals between them. In the overall, however, they will produce an image of the projection of the stand canopy. In order to make clear to oneself what will show up of the stand canopy in a visible form on the aerial photograph, we must study characteristics of the stands themselves, their structure, and above all the shape and size of tree crowns.

The scale of the aerial photograph, the resolving power of the photographic system, the extent of image displacement, the height of the sun, the time of day of the aerial photography, and appraisal characteristics of the stands affect the imaging of crown shapes and their dimensions.

The model of a stand made up of trees of different heights, of different crown shapes and sizes, with intercrown intervals of different forms and values, and with different degrees of ground surface exposure is especially graphically visible in the stereoscope used with large-scale aerial photographs.

In any stand, even those made up of the same tree species, both crown shapes and dimensions vary.

But depending on the species, age, degree of occupancy, crown canopy closure, density of tree stands, and admixtures of other species particular types or forms of crowns predominate in the stand canopy, and other crown shapes are less pronounced.

Free-standing trees as a rule have different crown shapes and sizes than do trees of the same age in stands.

The shapes of tree crowns are characterized by the following indicators in the light of goals in interpreting aerial photographs: crown diameter (D_c), crown length (l_c), height to greatest crown width (h_{D_c}); and h_{0_c}, height up to start of crown (Figure 7).

For the same tree height, crown length, and crown diameter, but for different heights up to greatest crown width, crown shapes vay. The greater the height up to the greatest crown width, the more rounded and bushier will be the crown (III). Such crowns (III) will be larger in size on aerial photographs than crowns with acute, conical (I) or paraboloid (II) shape (Figure 8).

In the central part of a flat aerial photograph, when there is perpendicular crown projection, in spite of the presence of the same tree height, the same D_c and l_c of the crowns, they will differ in shapes of upper crown sections. To be specific, the crowns of the first type will show up in greater depth in the stereoscope, and they will have the most clearly pronounced cone of their cast shadow. The crowns of the second type owing to the smoother transition from the illuminated side to the side in shadow will show up more convex than the crowns of other types. Finally, crowns of type III with the most bushy tip will appear planar, without clearly pronounced shadows cast. Such crowns in stand canopy and in aerial photographs of different scales will be the most distinctly imaged and look shallower in the stereoscope.

The nature of the photographic image of crowns in aerial photographs will depend on the external appearance of the crowns, their habit, or their architectonics.

Starting with these premises, it is first of all necessary both in pure and especially in mixed stands to arrive at a clear idea of the crown shapes of tree species predominating in the given stand and constituting the upper stand canopy.

In aerial photographs trees most fully illuminated at the moment of photography, with the broadest peak, and the greatest height predominate in producing images, as well as

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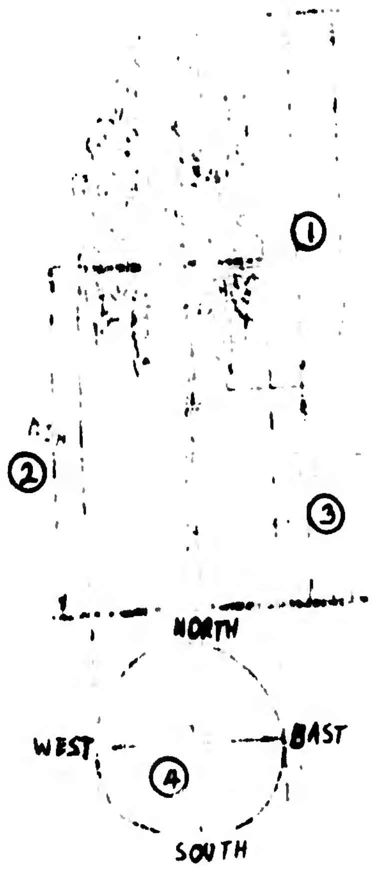

Figure 7. Indexes of crown shape and size.

LEGEND: 1 -- l_c 2 -- hD_c 3 -- ho_c 4 -- D_c

trees of moderate height that are not shaded by the shadows cast from the crowns of neighboring trees whose dimensions are greater than the resolving power of the photographic system. Therefore, we must first pay attention to crown shapes and sizes of such trees in each pure stand.

In some mature and pure stands the shapes and sizes of tree crowns in the closed section of the stand canopy differ but slightly from each other.

In mixed pine-birch stands crown shapes maturing and sometimes at mature ages are close to each other if they are located in the same stand canopy. Often the upper sections of the crowns are paraboloid. Sometimes the height to the greatest crown width in birch is lower than in pine. Crown length commonly is greater for birch than for pine. Therefore

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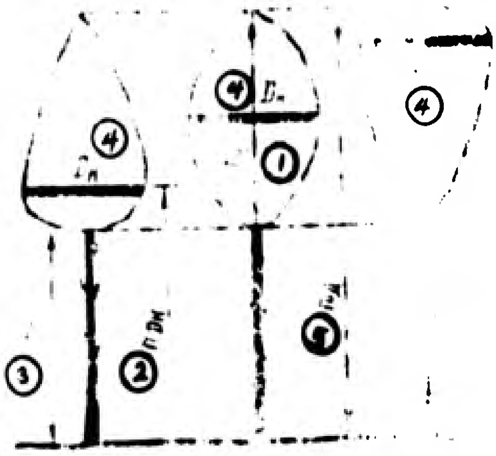

Figure 8. Variation in crown shape.

LEGEND: 1 -- l_c 2 -- h_{D_c} 3 -- h_{0_c} 4 -- D_c
5 -- h_D

they "sit deeper" in the canopy. When there is group arrangement of trees birch crowns stand closer to each other than do the crowns of pine, therefore the discernability in the stereoscope of canopies at these locations deteriorates. Sometimes birch heights are lower than pine heights and their crown projections are finer. These characteristics of stands at times are so representative that one can determine without error the composition of such stands on the basis of these features.

Groups of mature-aged aspen located in admixtures to pine and spruce always differ from these species in characteristic crown shapes. They are rounder and flatter in the upper crown section. In groups and curtains the crowns are bunched tightly together. In aerial photographs they appear irregularly rounded, of the lightest grey tone, and almost without openings between crowns.

Pure spruce forests under northern conditions are made up chiefly of different generations, therefore both crown shapes as well as tree heights vary widely here. Distances between trees are also not the same, as the result of which the intercrown intervals can be different in size on aerial photographs. An impression is given of a disorderly, non-uniform arrangement of these trees in the canopy with different degrees of discernability "in depth".

In mixed stands when they are studied for interpretation we must first of all take into account crown shapes and sizes within the limits of each individual species. Here it is obligatory to conduct a comparative evaluation among species.

Setting up a stable classification of crown shapes for different tree species under different site conditions is one of the most urgent tasks in forest interpretation of aerial photographs.

We note that those trees whose crowns are imaged in the canopy projection will be of the greatest importance in interpretation of aerial photographs. Trees lagging in growth and in development, standing under the crowns of other trees, in most cases will not be imaged in aerial photographs.

Tree crown sizes are determined by the crown length and by its width (D_c). We take crown length (l_c) to refer to the extension of crown from the outermost live twigs to its apex. Individual branches separate from the total aggregation of branches not participating in crown formation and not determining its shape are not taken into consideration.

Crown width refers to its main diameter (D_c). It is determined as the arithmetic mean between two or four measurements of the crown cross-sectional area: С - Ю, СВ - ЮЗ, В - З, ЮВ - СЗ.

In each stand there can be variations in tree crown sizes within the limits even of the same species (Figure 1).

However, in nature in a mixed stand there can be such cases in which the D_c of the crowns (greatest width) are the same for different species, but in crown shape are different, which depends on the height to the greatest crown width. For example, spruce are conical and pine are rounded.

Spruce crowns appear in aerial photographs to be 3 to 4 times smaller in terms of projection sizes than do aspen owing to the fact that at the moment of aerial photography in the case of the spruce only the upper and not the wide sections of the crowns are illuminated, while in aspen the opposite is the case, the upper being the widest and flattest crown peaks. In terms of difference in crown sizes and, consequently, in the appearance of their projections on aerial photographs we can more confidently proceed to determining the makeup of timber stands. In general, the lower the height up to the greatest crown width, the smaller will be the value of its diameter in the aerial photograph. This is especially important when the sun is not high in the sky during morning aerial photography. A different ratio in crown sizes can be the case when the aerial photography is done at midday when the sun is the highest in the sky.

On the number of trees producing the horizontal projection of timber canopy. Processing counts by the nature of crown participation in horizontal projection of stand canopies vividly showed that the distribution of tree crowns by the extent of thickness is expressed by a normal distribution curve, just as in the case for the total number of trees (Figures 9 and 10). However, trees standing in the canopy and the crowns of the trees are distributed along a descending curve. Of these, the greatest number of trees is recorded along the lower degrees of thickness. As the results of counts have shown, not only can the lowest and thinnest trees be located under the crowns of other trees, but also some of the trees of moderate and even high degrees of thickness. Data in this direction are to be found in the earlier indicated studies of G. G. Samoylovich, N. I. Baranov, V. S. Moiseyev, A. Ya. Zhukova, I. N. Machugina, A. I. Berezina, and I. A. Trunov.

In stands not differing in appraisal characteristics, the number of trees of different degrees of thickness that are part of the horizontal projection of the canopy does not remain the same. Let us give several examples from experience we gathered in working in the northern Taiga zone of the European part of the USSR. In an 80-year-old pure birch stand, occupancy 0.8, site class II, the canopy projection was made up of the following: all trees ranging in thickness from 20 cm and higher, 96 percent of the 16-cm-thick trees, 64 percent of the 12-cm-thick trees (in terms of the total number of trees of each thickness class), and the rest of the trees stood under the crowns of neighboring trees.

In a pure aspen 100-year-old stand, occupancy 0.5, and site class II, the canopy projection was represented by all trees ranging in thickness from 20 to 40 cm (seven thickness classes), 80 percent of the 16-cm-thick trees, 50 percent of the 12-cm-thick trees, and 28 percent of the 8-cm-thick trees, in terms of the total number of trees of each thickness class.

As a rule, aspen in the canopy projection were more numerous than birch in birch-aspen stands independently of

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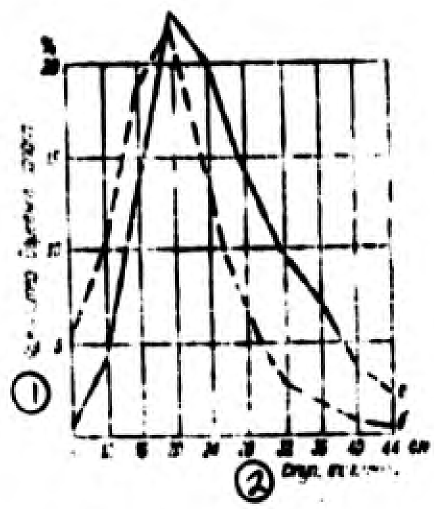


Figure 9. Distribution of number and stock of trees by thickness class in a stand of maturing larch (according to A. Ya. Zhukov): a -- in terms of stock; b -- in terms of number of trees.

LEGEND: 1 -- number of stock trees; 2 -- thickness class.

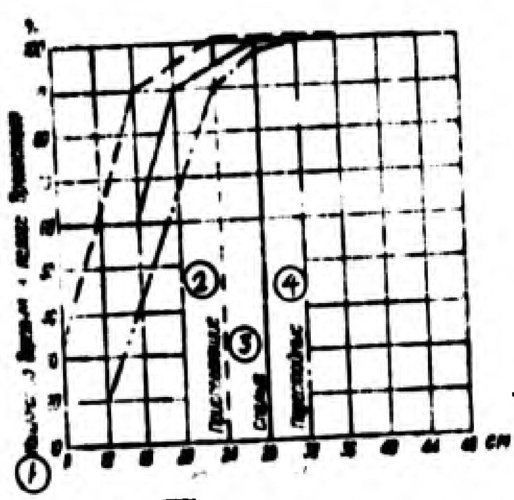


Figure 10. Distribution of the number of trees by thickness class, as standing under crowns of trees in a canopy of all-aged pure stands (according to A. Ya. Zhukov).

LEGEND: 1 -- number of trees in stand canopy; 2 -- maturing; 3 -- mature; 4 -- over-mature.

the proportion of aspen admixture for different degrees of occupancy and site classes. Aspen was present in the mature age class from 88 to 100 percent, but birch only from 64 to 90 percent.

In stands of pure 20-year-old pine saplings, occupancy 0.9 and site class II, 70-75 percent of the total number of trees showed up in the canopy projection. This number included up to 65 percent of the 8-cm-thick trees, some of the 12-cm-thick trees, and almost all of the 16-cm-thick trees and thicker. If there were admixtures in these stands of spruce and birch and a reduction in occupancy down to 0.7, the number of pine in the canopy projection rose to 80 percent. Admixture of spruce in the canopy projection of the stand showed up only with the 16-cm thickness class, and birch admixture only with 12-cm thickness.

From 60 to 80 percent of trees participate in the projection of the canopy of pine stands 40-60 years old, with occupancies 0.7-0.9, and site classes I-III. It was noted that in the case of the admixture to the pine of one spruce unit the number of pine in the canopy projection rose to 84 percent, which doubtless affects the appearance of the stand canopy in aerial photographs.

In pure pine stands 100-110 years in age and with occupancy of 0.6 and site classes II-III, and forest types -- chernichnik and dolgomoshnik, the canopy projection is made up of 85 to 95 percent of the trees. In these same stands, when occupancy is raised to 0.9, only 55-60 percent of the trees are represented.

In pine stands with admixture of spruce and birch up to five units, aged 80-100 years, site classes II-III, and for occupancies of 0.6-0.9, only 70-73 percent of the total number of trees remained in the canopy projection, some of the 16-cm-thick trees, and all of the 20-cm and thicker trees (6-7 thickness classes). Spruce and birch show up in the canopy projection for thicknesses of 20-24 cm.

In closed spruce stands of 100-120 years in age, site class II, the canopy projection is made up of trees ranging in thickness from 20 to 24 cm, and less often is this true of stands with 28-cm-thick trees (that is, 6-8 thickness classes).

In spruce groves 160-240 years in age, site classes III-IV, occupancy 0.7-1.0, and with pine and birch admixture up to 3-4 units, almost all trees 24-32 cm thick are part of the canopy projection, not more than 40 percent of the 12-cm-thick trees, up to 65 percent of the 16-cm-thick trees, up to 70 percent of the 20-cm trees, up to 84 percent of the 24-cm trees, and up to 95 percent of the 28-cm-thick trees, in terms of the total number of trees in each thickness class. The entire pine admixture appears in the canopy projection, but of the birch admixture -- trees 12 cm thick are represented up to 75 percent, and those 16 cm thick up to 90 percent of the total number of trees in each thickness class.

It is clear from the examples cited that depending on the appraisal characteristics of stands the projection of the stand canopy is made up of trees of different thickness classes. With increase in age and reduced occupancy, a greater number of trees of different thickness classes then show up in the projection of the stand canopy. At mature age, in pure and mixed stands tree species are most fully represented in the stand projection in terms of the total number of trees in the following order: aspen, pine, birch, and least of all spruce. This conclusion bears specific practical value in forest interpretation of aerial photographs.

On the number of trees standing under the crowns of other trees in the stand canopy. Let us cite examples based on counts made of trees from the participation of their crowns in the projection of the stand canopy. These will help us to answer the question as to what trees stand under the crowns of nearby trees and what trees do not participate in forming the horizontal projection of the canopy.

In mature birch stands of site class II, aged 80 years, with occupancy 0.8, up to 12 percent of trees of all thickness classes are located under crowns. But, under an analogous stand 100 years in age, under otherwise equal conditions only 2 percent of trees are located under crowns.

In a mixed stand of 50 percent pine and 5 percent birch, aged 60 years, with 0.7 occupancy, and site class II 29 percent of the birch were found under crowns, and 10 percent of the pines. If the total number of birch were 296 trees, and the number of aspen 242, then it turned out that, in contrast, more aspen (220 trees) than birch (210 trees) appeared in the canopy projection. This can already have an effect on precision of interpretation of forest composition from aerial photographs.

In a birch-spruce stand (5 birch-5 spruce, 70 years of age, occupancy 0.7, and site class III), almost three times more spruce were found under tree crowns than birch, while the birch was one-half as numerous as an analogous pure stand.

When comparing birch-spruce stands identical in composition and occupancy (7 birch-3 spruce, with occupancies of 1.0), but at different ages from growth classes V to X, it was established that the number of spruce under the crowns, respectively, was decreased from 61 to 47 percent, and that of birch from 36 to 3 percent. The greater the age, the fewer birch were found under the crowns of stands mixed with spruce. In all other analogous stands there proved to be considerably more spruce under crowns than birch.

In mature mixed birch-aspen stands of site class III, when the occupancy was reduced from 0.7 to 0.5, that is, by 0.2 under otherwise identical appraisal indexes of the stands, the number of birch under crowns was reduced from 29 to 23 percent, while the number of aspen remained the same (ten percent).

It is clear from the foregoing that in mixed stands spruce, birch, and least of all aspen, in that order, are found under crowns. In young and moderate-aged larch stands, it is chiefly trees of the lower thickness classes that are located under crowns -- up to 12 cm thick, and less often up to 16 cm. But in mature stands trees of higher thickness classes are to be found under crowns -- up to 24 cm thickness, and less often higher thickness classes.

Pine stands. In pure pine saplings 10 pine, 20 years of age, site class I, from 24 to 30 percent of the pine were found under crowns for different stand occupancies. In the case of the admixture to the stand of one unit spruce, it was almost completely located under pine crowns (from available material, up to 82 percent of the total number).

In pine stands 40-60 years of age with occupancy rates from 0.6 to 0.9 and site classes I-III, the pine under crowns proved to amount to 17 to 30 percent of the trees. In the event of spruce or birch admixture of not more than one unit, on the average 68 percent of the spruce proved to be located under crowns, and 59 percent of the birch.

It is characteristic that when the occupancy rate is brought down to 0.2 under the same appraisal characteristics in other respects for stands, the number of trees under crowns of the same stands is reduced on the average by 15-16 percent, but when the age of the stands is increased by 20 years the total number of trees under the crowns is reduced on the average by 10-12 percent.

In pure pine stands 100-110 years of age, occupancy 0.6, and site class III, for the dolgomoshnik type forest, from 4 to 14 percent of the trees are found under crowns, chiefly those 8-16 cm in thickness, and only here and there those 20 and 24 cm thick.

In a mixed pine stand 5 pine, 3 spruce, 2 birch, occupancy 0.65, and site class III, for pine aged 80 years, spruce aged 70 years, and birch aged 70 years, and for a dolgomoshnik forest type, 5 percent of the pine stood under crowns (in thickness classes 12-16 cm), 52 percent of spruce (thickness classes from 8 to 20 cm), and 21 percent birch (thickness classes from 8 to 20 cm). Under this same occupancy, but for 110 years of age, in the stand 8 pine, 2 spruce 42 percent of the spruce were under pine crowns (thickness classes from 8 to 20 cm, and infrequently 24 and 28 cm).

In pine plantings of 170-190 years of age, at occupancy rates of 0.7-0.8, and for site class III, for a slight admixture of birch or spruce from 1 to 5 percent of the pine stood under crowns, from 5 to 22 percent of the birch, and 50 percent of the spruce. Of the second spruce generation up to 60 percent of the trees were located under crowns.

In stands of site class IV with the same admixture of spruce and birch, for the same occupancy rates from 1 to 15 percent of the pine stood under crowns, from 20 to 42 percent of the spruce (of its second generation up to 71 percent), and from 3 to 15 percent of the birch. It is clear from these examples that as the site class is reduced, the number of spruce situated beneath crowns is reduced.

Spruce stands. In spruce stands mixed with birch and pine, of composition 6 spruce, 2 pine, 2 birch, aged 80-90 years (for all species), for occupancy rates of 0.6-0.9, site class III, and dolgomoshnik type forest, from 14 to 21 percent of spruce were located under crowns, from 3 to 5 percent of pine, and up to 10 percent of the birch. The latter proved to be found under crowns more than pine, which once again confirms its high shade tolerance.

In plantings of the same composition and age, 5 spruce, 4 pine, 1 birch (spruce aged 100 years, pine and birch also 100 years in age), site class III, but for different occupancy rates, spruce located under crowns proved to be in the following numbers:

For occupancy 0.49 -- 12 percent (thickness from 16 to 24 cm),
for occupancy 0.56 -- 28 percent (thickness from 8 to 24 cm), and
for occupancy 0.65 -- 33 percent (thickness from 8 to 16 cm),

that is, as the occupancy rises, the number of spruce trees under crowns also rises. With increase in occupancy the presence of a large number of thin spruce was noted. Pine, except for trees here and there (thicknesses of 8, 20, and 24 cm), all showed up in the projection of the stand canopy.

The number of birch standing beneath crowns differed depending on occupancy:

For occupancy 0.49 -- 7 percent (thickness from 18 to 20 cm),
for occupancy of 0.56 -- 9 percent (thickness from 12 to 25 cm), and
for occupancy of 0.65 -- 16 percent (thickness from 8 to 16 cm).

Thus, as stand occupancy increases, the number of spruce and birch situated under crowns rises (chiefly owing to the most slender trees). Let us take as an example a stand made up of the three species with almost identical participation of spruce, pine, and birch, 4 spruce, 3 pine, 3 birch (spruce and pine aged 100 years, birch aged 90 years), and occupancy 0.9, therefore we find that the following proved to be the percentages of trees situated under crowns -- 39 percent of spruce (thickness from 8 to 24 cm), 9 percent of pine (thickness from 12 to 20 cm), and 21 percent of birch (from 8 to 16 cm in thickness).

In stands of this same forest type, but with a smaller proportion of pine and birch admixture -- 8 spruce, 1 pine, 1 birch (spruce and pine aged 100 years, birch aged 90 years), and with 0.6 occupancy the following proved to be the crown-covered values: 22 percent of spruce (from 8 to 28 cm in thickness), 6 percent of birch (from 16 to 28 cm in thickness), and there were no pine at all covered by crowns.

Analysis of mature mixed stands showed that as the stand occupancy is increased more trees of lower thickness classes (8-12 cm) were found under the crowns of other trees. For example, in the stand 4 spruce, 3 pine, and 3 birch (spruce and pine aged 100 years, birch aged 90 years), with occupancy 0.9, site class III, and dolgomoshnik type forest the trees were located under crowns, in relation to thickness classes, in the following order:

Thickness class	8	12	16	20	24
Number of trees under crowns:					
spruce	20	20	31	17	2
pine	--	4	2	2	--
birch	16	21	2	--	--

In mixed spruce stands aged from 80 to 120 years, for different occupancy rates on the average there were 23 percent of spruce, 3-5 percent of pine, and 9 percent of birch under crowns.

In over-mature mixed spruce stands of site class IV the admixture of pine and birch did not exceed 3 units. Here, pine independently of the proportion of its admixture was not at all to be found under crowns. But the admixture of birch aged from 100 to 120 years in stands with occupancy rates from 0.8 to 1.0 were partly located under crowns (from 5 to 12 percent). Spruce predominating in the stand composition was located under crowns to the extent of from 16 and in some cases to 40 percent. In accordance with the foregoing, when interpreting the composition of stands from aerial photographs the coefficients for spruce and for birch can be understated in the stand makeup.

Thus, based on a count of trees from their crown participation in the stand canopy, it can be stated that of the predominant tree species growing in the Taiga zone the most represented in the horizontal projection of the canopy out of all the total number of trees is aspen followed by pine, birch, and least of all spruce.

In interpreting aerial photographs the very same proportionality is retained in the makeup of species in the stand canopy projection.

On the composition of stand canopy in horizontal projection. With a decrease in the total number of trees making up the projection of a stand canopy, there is also a change in the number of trees in individual tree species constituting the stand. From this, consequently, the composition can also vary for that portion of the stand which accounts for the horizontal projection of the canopy. In fact, these variations are of exceptional importance in interpreting aerial photographs, therefore we will cite a number of examples based on data of sampling areas.

In mature and over-mature pine stands in the northern Taiga zone, for example, of the site class III we can encounter stands of the two-story type or stands with several forest generations. Spruce most commonly predominates in the second story. It sprouts under the crowns of pine, birch or aspen or occupies "windows" in the horizontal projections of the stand canopy. Individual spruce showing up in the first story occupy the free space between crowns in the canopy.

When deciphering aerial photographs, especially those of the large-scale category, without knowing biological characteristics of stands we can allow substantial errors in determining stand makeup. Calculating the composition of the horizontal projection of a stand canopy (by the number of trees it contains) has revealed the following:

[Table on following page]

/S = spruce; P = pine; B = birch/

Sample plot number	Composition and age of stand by stories	Occupancy	Site class	Composition of stand canopy in horizontal projection
1	I. 10 P (200 yrs) + S II. 9 S (160 yrs) 1 B (100 yr	0.65 0.3	III	9P1S+B
2	I. 9 P (165 yrs) 1 B (95 yrs) + S II. 10 S (150 yrs)	0.85 0.3	III	7P2S1B
3	10 P (155 yrs) + B (75 yrs) + S (80 yrs)	0.8	IV	8P1S1B
4	5 P (85 yrs) 3 S (75 yrs) 2 B (65 yrs)	0.7	IV	6P2S2B
5	8 P (190 yrs) 1 S (115 yrs 1 B (70 yrs)	0.6	V	9P1B+S
6	8 S (155 yrs) 1 B (125 yrs 1 P (150 yrs)	0.73	IV	8S2B+P
7	6 S (210 yrs) 2 P (230 yrs 2 B (100 yrs)	0.8	IV	7S1P2B
8	I. 7 P (130 yrs) 2 S (230 yrs 1 B (70 yrs) II. 10 S (150 yrs)	0.9 0.5	III	5S4P1B
9	I. 7 S (130 yrs) 2 S (230 yrs 1 B (70 yrs) II. 10 S (140 yrs)	0.7 0.5	III	6P3S1B
10	I. 9 P (140 yrs) 1 B (90 yrs) II. 10 S (130 yrs)	0.5 0.4	IV	5P4S1B

Second-story trees (II) differing only in height (by 15 percent) and entering into the intervals between canopy crowns, vary the ratios in the proportion of species representation in the makeup of the stand (sample plots 1, 2, 8, 9, and 10), which is correspondingly reflected in the aerial photographs.

The most stable species in the horizontal projection of the canopy proved to be birch. The proportion of its admixture remained unchanged or was added to by one unit. In addition, in single-story stands the admixture of spruce diminished since some of it was situated under tree crowns (sample plots 4 and 5). In some birch or aspen stands mixed with spruce, instead of 4 spruce units in the horizontal canopy projection, no more than 1-2 units were left. The rest of the spruce trees were located under the crowns of deciduous trees or pine.

Differences in the heights of tree stands also are vitally important here.

By way of an example we present data for 4 sample plots:

/S = spruce; P = pine; B = birch; As = aspen/

Sample Plot Number	Composition	Age of Dominant Species	Site class	Occupancy	Species	Mean tree stand heights
1	4 P	70	I	0.87	P	23
	3 As				As	22
	2 S				S	19
	1 B				B	21
2	4 P	120	I	0.94	P	31
	4 S				S	27
	2 B				B	30
3	5 S	120	II	1.1	S	25.4
	4 B				B	28.3
	1 As				As	29.2
4	5 B	100	II	0.5	B	25.5
	4 S				S	20.7
	1 P				P	26.6

In the stands, as to average height spruce was below birch, aspen, and pine by 2-5 meters. Not all the spruce trees entered into the horizontal projections of the stand canopy, since some of them were shadowed or situated under the crowns of deciduous trees, which understated the proportion of their representation in the stand composition.

In general, the proportion of the spruce admixture to other species, for example, to pine, oak with linden, always as a rule understated the minimum by 1-2 units, and in dense stands of saplings or moderate-aged deciduous stands -- by up to 3 units. The same was noted also in respect to associates of oak -- linden, maple, and elm.

In order not to allow errors, when interpreting, in the stand composition, we must study in advance characteristics of tree stands in nature, establish the interrelationship between the stand composition and the composition of its horizontal canopy projection. Only timely and careful training can help the practitioner to avoid errors in determining stand composition.

On heights, lengths and widths of the crowns of trees making up the horizontal projection of the stand canopy. The amplitude of variations between heights of trees comprising the horizontal projection of the stand canopy is somewhat less than for the total number of trees forming tree stands of forest elements. The smallest fluctuations in the heights of trees comprising horizontal projections of a canopy are observed in closed aspen, and secondarily in birch stands. The same has been observed also in pine stands. The greatest fluctuations in tree heights in the horizontal projection of the canopy occur in spruce stands, particularly if they consist of several forest generations.

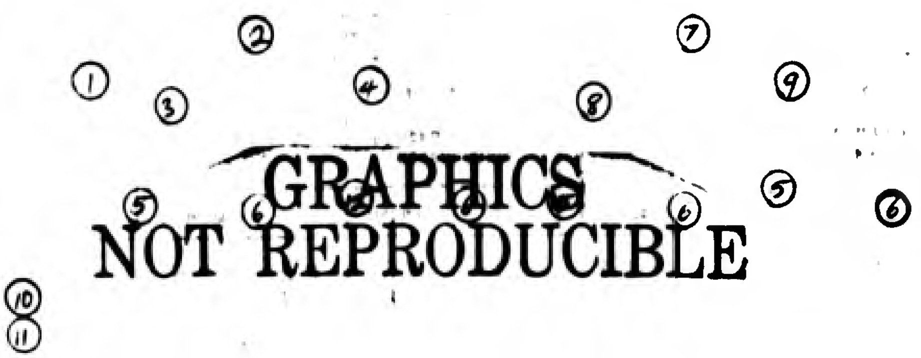
To illustrate the foregoing, let us give several examples. In a pure 80-year-old birch stand, site class II, and occupancy 0.8, the difference between average tree heights in the lower and higher thickness classes varied up to 10.5 meters, variations in crown length -- up to 9.3 meters, and in crown widths -- up to 2.9 meters.

If, however, we calculate the average height of trees comprising the horizontal projection of the stand canopy, in precisely the same way as we usually calculate the average heights of tree stands, forest elements, then the difference between the mean, greatest, and smallest tree heights will be 4 meters (for 68 percent), and the corresponding value for crown lengths -- 4.9 meters (47 percent), and crown widths -- 1.7 meters (41 percent).

In a 100-year-old birch stand, site class II, occupancy 0.8, the difference in average tree heights as between lower and higher thickness classes equals 11.7 meters, for l_c -- 7.0 meters, and for D_c -- 5.3 meters; but among the very same trees making up the horizontal projection of the canopy, the difference proved less: for height, 3.5 meters (70 percent), for l_c -- 3.7 meters (47 percent), and for D_c -- 3.2 meters (40 percent). These indexes (in relative values) prove to be similar to data given above for 80-year-old birch stands.

In these stands the average heights of trees comprising the horizontal projection of stand canopy proved to be higher than the general average stand heights, and this is already of specific importance in respect to the measurement interpretation of tree stands.

In order to discover whether the occupancy of stands affects amplitudes of fluctuations in tree heights, in l_c, and in D_c within the limits of forest elements in the horizontal projection of stand canopies, let us compare birch-spruce stands uniform in appraisal characteristics, but differing in occupancy rates (0.5 and 0.7).



LEGEND: 1 -- fluctuations in meter between; 2 -- occupancy 0.5; 3 -- for the entire stand; 4 -- for trees in the horizontal projection of the stand canopy; 5 -- birch; 6 -- spruce; 7 -- occupancy 0.7; 8 -- for the entire stand; 9 -- for trees in the horizontal projection of the stand canopy; 10 -- l_c; 11 -- D_c.

As the occupancy rate is increased from 0.5 to 0.7, the extent of difference in heights as between trees in the horizontal projection of the canopy for spruce decreases, but rises for birch. Difference in the crown length is retained almost at its former level, but the difference in crown width for spruce in an occupancy of 0.4 is increased, while that for birch is decreased. Thus, for an occupancy of 0.5, there will be more crowns of spruce trees differing in size in the horizontal projection of the canopy, and for an occupancy of 0.7, more of birch.

In a birch-aspen stand composition 5 birch, 5 aspen, age 40 years, occupancy 0.7, and site class III, the amplitude of variations were characterized by the following table.

LEGEND: 1 -- fluctuations in meters as between; 2 -- for the entire stand; 3 -- birch; 4 -- aspen; 5 -- for trees in the horizontal projection of the stand canopy; 6 -- l_c; 7 -- D_c.

In this stand fluctuations between appraisal indexes were slight as between aspen and birch.

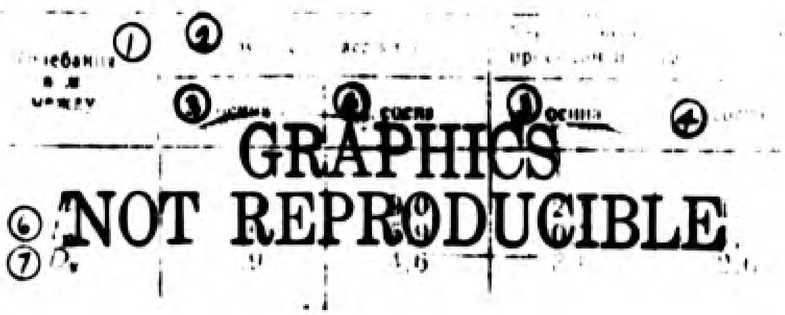
In this stand of site class III the difference between these indexes as a whole was less than in the site class I stand.

The average height for the stand (forest element) of aspen was 20.6 meters, but for the trees making up the composition of the canopy, 21.1 meters, and the difference amounted to +0.5 meter. The average height of a birch stand of trees of a forest element was 19.9 meters, but for trees entering into the projection of the canopy, 24.4 meters, and the difference also was +0.5 meter.

The height up to the greatest crown width was larger for the aspen than for the birch, by 1.5 meters, but the crown length was 1.1 meters less in the case of the aspen compared to the birch. The spread of crowns from the average height was 43 percent for the birch, and 33 percent for aspen.

In a horizontal projection of a canopy of site class III, variations in the average heights of birch and aspen tree stands were greater than in the case of a site class I stand, smaller for crown width for birch and aspen, and greater for aspen when crown length was compared.

Let us look further at an aspen-pine stand of composition 5 aspen, 5 pine, age 60 years, occupancy 0.6, and site class I.



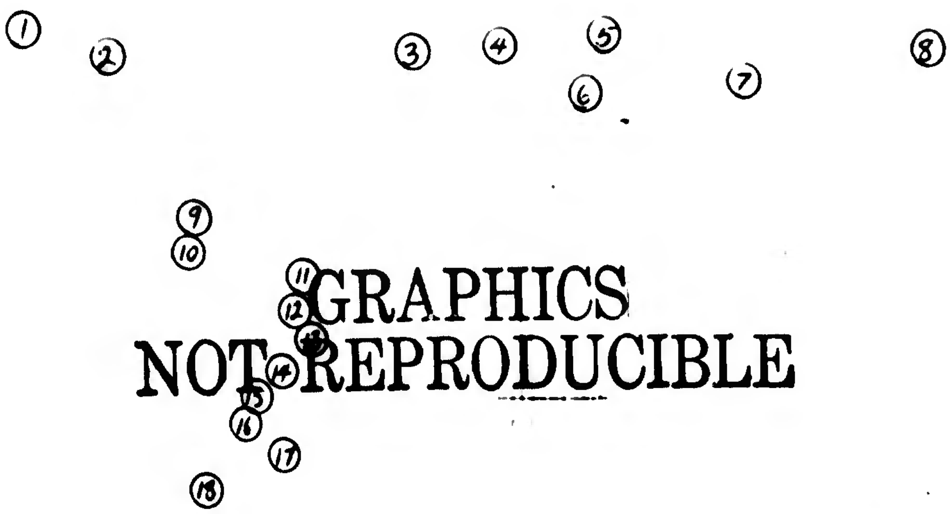
LEGEND: 1 -- variations in meters as between; 2 -- for the entire stand; 3 -- aspen; 4 -- pine; 5 -- for trees in the horizontal projection of the stand canopy; 6 -- l_c; 7 -- D_c.

In the horizontal projection of the stand canopy variations in height were 57 percent less in the case of pine compared to aspen, 13 percent less for D_c as between these two species, and twice as great for l_c as between these same two species.

The extent of crowns was 55 percent for aspen, and 36 percent for pine, with respect to the average height of tree stands (forest element). The average crown length in the case of pine was on the average 3 meters less than for aspen. But the average D_c values for aspen and pine were approximately the same, at 3.2 meters.

If we compare pure aspen stands with homogeneous mixed aspen-pine stands, then we will see that the pine admixture in the horizontal projection of the canopy of such stands reduces the variation in h and l_c for aspen.

The difference in average heights of saplings and average-aged pine stands compared with the average heights of trees comprising their canopy projection, based on the data of several sample plots, is listed below.



Legend: 1 -- sample plot number; 2 -- composition of stand and age; 3 -- occupancy; 4 -- site class; 5 -- average height in meters; 6 -- of stand; 7 -- of trees in projection of stand canopy; 8 -- difference in meters; 9 -- 10 pine (20); 10 -- 10 pine (50); 11 -- 8 pine (50) 2 spruce (50); 12 -- 9 pine (20) 1 birch (60); 13 -- 9 pine (35) 1 birch (40); 14 -- 10 pine (50) + spruce, birch; 15 -- 8 pine (20) 1 spruce, 1 birch; 16 -- 10 pine (35) + birch; 17 -- 9 pine (60) 1 spruce (60); 18 -- 10 pine (100).

In stands of young trees 20 years of age of different occupancy rates and site classes (I-III), differences in average heights prove to be about the same (+1.5 meters), and the greatest difference in heights was found for ages 50-60 years independently of occupancy rates, while in the period of the most pronounced differentiation of trees in site classes I and II, it was higher (+2.2 and +3.0) than in site class III (+1.8 meters).

In 100-year-old pine groves, occupancy 0.7, site class III, this same difference was considerably less (+0.7 meter) owing to a reduction in height increment. Consequently, the difference in average tree stand heights in the horizontal canopy projection of the stands rose up to the 60-year age mark, after which it dropped off considerably. The fluctuation in heights here was also less.

From the data considered it is clear that in horizontal projections of the canopy of stands differing in appraisal indexes, the range of fluctuations in h, l_c, and D_c is predominantly less compared with the mean indexes for stands as a whole, and less often remains the same as this latter class. The greatest crown length was observed for spruce, followed by birch and pine, and least of all for aspen. The lower boundary of the stand was most highly located in aspen tree stands, while in spruce tree stands it descended sometimes very shallowly, terminating close to the ground surface, while birch and pine occupied a position intermediate to these. The height up to the greatest crown width, of importance in interpreting appraisal features of stands, was highest of all for the aspen, followed by pine, birch, and spruce.

Spruce admixture breaks up the canopy of closed birch and aspen stands, increasing its visibility "in depth". In pure densely closed aspen groves the visibility below "in depth" is the lowest. As the age rises in pure deciduous and mixed stands, the range of variations in stand projection by height, D_c, and crown lengths is reduced. The range of variations is also reduced for tree stands made up of deciduous species in terms of h, l_c, and D_c as stand occupancy is reduced, but for spruce, in contrast, an increase is the rule.

The above-noted characteristics of stands must be taken into account, in particular, when measuring heights of tree stands for individual species.

Closure of canopy and stand occupancy. To determine occupancy of stands when interpreting flat aerial photographs, one must resort to using the index of canopy closure, previously establishing the interrelationships between these two quantities. Accumulating material allows us at present to report the following.

In pure birch stands 80-100 years of age, site class II, with occupancy from 0.8 and higher, canopy closure is commonly 0.1 less than occupancy. In general, in fact, as the stand age increases, canopy closure of such stands is reduced.

In pure aspen stands canopy closure is most commonly 0.1 higher and less often equal to stand occupancy. Aspen stands compared to stands of other species differ by higher occupancy and greater canopy closure. Intervals between crowns in such stands are covered by branches owing to the high light requirements of aspen, which considerably reduces the possibility of inspecting the stands "in depth."

In mixed birch-aspen stands of different ages, site classes, and occupancy rates, canopy closure is most commonly equal to occupancy, but in the case of group admixture is 0.1-0.2 higher than occupancy.

On sample plots in birch-spruce stands canopy closure proved to be lower than stand occupancy. Thus, spruce admixture to birch stands reduced the canopy closure and increased the discernability of such stands "in depth."

In an aspen-pine stand 5 aspen, 5 pine, aged 60 years, occupancy 0.6, and site class I, canopy closure was 0.1 higher than occupancy. In a pure aspen stand, with otherwise the same appraisal characteristics, stand closure was also 0.1 higher than occupancy. In nature and extent of closure aspen and pine stands are similar to each other, whence follows the finding that pine admixture to aspen does not introduce substantial changes into the canopy closure of such stands.

Pine and aspen admixture to birch and birch-spruce stands increases their canopy closure. In birch and birch-spruce stands per se, canopy closure is lower than occupancy rate.

In spruce stands aged 100-120 years with an admixture of up to 5 units of birch and 4 units of pine, and site class III, the following relationship was noted between canopy closure and occupancy. For occupancies of 0.3-0.4, canopy closure coincided with occupancy; in the same stands, but with occupancy of 0.5, canopy closure was 0.1 less than occupancy.

Of two spruce stands with occupancies of 0.9, in the one in which the proportion of birch and pine admixture was brought to 6 units, the canopy closure was 0.1 higher than occupancy, while in the other stand in which the proportion of pine and birch admixture was one-half as great, canopy closure proved to be 2 units less. This example is quite convincing.

In general, in spruce stands canopy closure in aerial photographs shows up to be less than stand occupancy, while in pine stands as a rule the opposite is the case.

In mature and over-mature cedar stands crown closure is from 0.2 to 0.5 less than occupancy. This is wholly understandable since in the stands trees of higher thickness classes predominate, and there are sizable openings between crowns in the canopy. The greatest difference between these indexes has been observed in mixed stands of cedar, pine, birch, and spruce. In addition, the larger the difference between occupancy and crown closure (0.2-0.3), the more the number of trees, especially spruce, that are found under the stand canopy.

In pure average-aged and maturing pine stands of site classes I-III, crown closure is close to occupancy, and with increase in age becomes less than occupancy. This difference usually does not exceed 0.1, and less often 0.2.

Spruce admixture to pine, birch, and aspen, even in mature age, reduces canopy closure of such stands from 1.2-0.2 to as much as 0.3. Research on this subject has been reported by V. S. Moiseyev, I. N. Machugin, A. Ya. Zhukov, A. N. Polyakov, A. M. Berezin, I. A. Trunov, and N. G. Iharin.

In conclusion we must note that the examples given above graphically demonstrate the role and importance of studying stand canopy for forest interpretation of aerial photographs. Accumulation of data in this direction allows us to establish more definite interrelationships both between appraisal indexes of stand canopies as well as between the nature of their imaging in aerial photographs.

The presence of such interrelationships will be the basis for setting up correlations in canopy structure and mathematical validation of measurement methods of determining appraisal indexes of stands.

The data cited graphically evidence the role that the study of stands -- a complex natural environment -- plays for their interpretation from aerial photographs.

CHAPTER 2

MATHEMATICAL MODELING OF TIMBER STANDS

By mathematical modeling we refer to a method of investigating timber stands. It must not be confused with stereoscopic models obtained from aerial photographs in stereophotogrammetric instruments. Mathematical modeling expands the theory of timber stand study, simplifies derivation of approximational relationships among appraisal indexes, and allows us to find new methods of measurement and determination of timber stand indexes from aerial photographs and on site.

In present-day methods of investigation mathematical modeling of a variety of phenomena is widely used not merely in the physical and mathematical sciences but also in biology. Essentially, modeling physical and biological processes consists of reproducing actual objects on artificially selected or constructed analogues. One of the problems of modeling is held to be reproduction of external similarity between the real-life object and its schematic analogue. Depending on the phenomena we are investigating, the models can be geometric analogues (figures), physical constructs, mathematical equations, and their representation on electronic computers. The advantages of models include the fact that they allow us to make quantitative measurements of complex phenomena and processes. We know that real-life objects are not always susceptible to direct measurement or require extremely laborious and costly work over a period of many years for this purpose. But with models there is the opportunity of varying all characteristics of the phenomenon under different sets of conditions and combinations specified from our inspection. In a number of cases the mathematical model allows us to digress from qualitative features of the object or process we are studying.

Use of principles of cybernetics and the theory of similarity has led to much easier compilation and investigation of animate nature. Advances in mathematical and physical modeling of physiological and biological processes have raised hopes for the successful application of mathematical modeling in studying phytocenoses.

Geometrical analogues of timber stands allow us to find new mathematical relationships between appraisal indexes and to explore variation of these indexes in the overall view -- on analogy with variation of indexes in actual tree stands over their entire growth. The mathematical functions derived from geometrical analogues will, of course, yield only a general representation of the actual timber stands. However, general mathematical equations can then be made specific and refined by sampling, real-life measurements in timber stands differing in conditions of habitat, composition of species, age, and forest types. Mathematical models also allow us to find new methods of measurement and determination of appraisal indexes. One method of mathematical modeling can be cited as the method of point systems and sets developed by the author for modeling timber stands and other discrete (discontinuous) phenomena that can be reduced to point sets. The models obtained by this method are being used in deriving mathematical functions and finding new methods of determining indexes of tree stands.

3. Mathematical Relationship Between Spacing and Number of Points in Point Systems

If we want to determine the density of a forest N and the mean spacing between trees l, we have to know the relationship between these quantities. The relationship between N and l is a particular case of the relationship between the number of discrete points (objects) and spacings between them in point systems and sets. Therefore, let us look at the solution of this problem in the general case.

Initially, the problem of finding a relationship between distance l and number of points N for a given area P arose in the course of investigating other material [7]. Based on the investigations made, it proved possible to find at first an approximational relationship, and then a precise formula for the relationship between N and l which later could be applied to the study of forests and other phenomena capable of being reduced to point systems and sets.

We can form a regular system of points on a plane by placing the points in the centers of regular contiguous geometric figures.

If we draw on a plane two mutually perpendicular systems of straight lines equally spaced from each other, then we get a network of squares. By placing the points in the centers of these squares, we get a regular square system of points. It is not difficult to construct rhombic, circular, rectangular, triangular and other point systems.

In an elementary way, the number of points N on a given area P can be determined approximately by dividing P by the area of a small figure in the center of which the points lie. We take as the distance between points the length 1, a square of which expresses the size of the area enclosed by the small figure.

If we take as the small area a square having the side 1, then the number of points in the area P will be

$$N_1 = \frac{P}{l_1^2}, \text{ a } l_1 = \sqrt{\frac{P}{N_1}} \qquad (1)$$

When P = 1 hectare, $N_1 = 10,000/l_1^2$, and $l_1 = 100/\sqrt{N_1}$.

Formula (1) has been used in forest appraisal to determine tree spacing 1 or forest density N [3, 54].

If we place the points in the centers of small areas that are mutually contiguous circles of diameters l_2, the number of points

$$N_2 = 1,2738 \frac{P}{l_2^2}, \text{ a } l_2 = 1.128 \sqrt{\frac{P}{N_2}} \qquad (2)$$

When P = 1 hectare, $N_2 = 12,738/l_2^2$, and $l_2 = 112.8/\sqrt{N_2}$.

Placement of points in the centers of rhombuses with a small diagonal equal to the side of the rhombus l_3 gives us

$$N_3 = 1,155 \frac{P}{l_3^2}, \text{ a } l_3 = 1,075 \sqrt{\frac{P}{N_3}} \qquad (3)$$

When P = 1 hectare, $N_3 = 11,550/l_3^2$, and $l_3 = 107.5/\sqrt{N_3}$.

It is clear formulas (1), (2), and (3) that the values of N and 1 for the same area are not the same when the points lie in the centers of different geometrical small figures. Consequently, different values of l for the same P and N will correspond to values of N calculated from the ratio of P and the size of the areas of small figures. This is to be expected since the small figures enclose different areas. The relationship between N and l in these cases is not established directly, but rather indirectly.

It is of the greatest practical and theoretical interest to establish a direct relationship between N and l, bypassing all intermediate constructions and procedures. It is also important to obtain such a system of points in which all spacings between neighboring points would be the same, and not different, as occurs in point systems and formulas (1), (2), and (3). Finally, it is desirable to find such a system of points for which we will obtain the largest number of points N for a given, large enough area P for a specified smallest spacing l between points. These properties are exhibited by a system of points located at the apexes of adjoining equilateral triangles or identical rhombuses with small diagonal equal to the rhombus side. This kind of system is also formed by points located at the centers of contiguous circles with a radius equal to half the distance l between (Figure 11).

We then find the formula of the function $N = F_{(l)}$ for this most remarkable system of points. Let us take a square of area P with side D. Within the square let us draw straight lines parallel to its vertical side at a distance equal to the large diagonal of the rhombus described above, that is, at the distance $l\sqrt{3}$. Then we will draw straight lines parallel to the horizontal side of the square at a distance equal to half the side of the rhombus, that is, $0.5 \cdot l$. By taking the upper left angle of the square as the location of the apex of the first equilateral triangle and using the points of intersection of the lines, we will construct a system of contiguous triangles or rhombuses for the entire square. As a result, we obtain a system of points located at the apexes of the equilateral triangles at the identical spacing l between all neighboring points. Let us now find a relationship between N and l under the condition that all points will lie within the square or on its sides and will not go beyond its bounds. Then the number of points on the horizontal straight line will be

$$n_1 = \frac{D}{l\sqrt{3}} + 1,$$

and the number of points on the vertical straight line, including

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Figure 11. Regular system of points.

points shifted to the right by half the large diagonal of the rhombus,

$$n_3 = \frac{D}{0.5l} + 1.$$

The product $n_{hor} \cdot n_{ver}$ will give the general maximum number of points in the square

$$N_1 = \left(\frac{D}{0.5l} + 1\right)\left(\frac{D}{l\sqrt{3}} + 1\right) \qquad (4)$$

Formula (4) is valid given the condition of equality between the number of points on all horizontal straight lines.

As a result of the periodic shifting of rhombus apexes from the left vertical side by half the distance of the large diagonal, the length of the horizontal straight lines will alternately (every other) take on the values of D or $D - 0.5 \cdot l \cdot \sqrt{3}$. Therefore, the number of points on the horizontal lines will be $n'_{hor} = \frac{D}{l\sqrt{3}} + 1$

or $n''_{hor} = \frac{D - 0.5l\sqrt{3}}{l\sqrt{3}} + 1.$

The number of horizontal lines with the number of points n''_{hor} is equal to $0.5\ n_{ver}$. Consequently, when $n'_{hor} = n''_{hor}$ the total number of the points in a square is calculated from formula (4), but when $n''_{hor} < n'_{hor}$ for one point, the total number of points will be

$$N_2 = \left(\frac{D}{0.5l} + 1\right)\left(\frac{D}{l\sqrt{3}} + 1\right) - 0.5\ \frac{D}{0.5l} + 1 \qquad (5)$$

Since the number of points can only be an integer, then in calculations from formulas (4) and (5), we must select only integers, and fractions (obtained as the result of division) are completely disregarded. Formulas (4) and (5) are kept for any values of D and P at the locality, map, or other plane, and only units of measurement will change, since in deriving the formulas neither the quantities D, P, and l nor their dimensions were involved.

When D = 1 hectare, l will be expressed in meters, and when P = 1 decimeter^2, l will be expressed in millimeters, and so on.

For practical use there is no need to perform on each occasion complex calculations of N and l from formulas (4) and (5).

Table 1 gives N and l calculated from formulas (4) and (5), where N denotes the largest number of points that is possible to place in the given area P for a given l. In this table the values of l are given at intervals of 0.5, but N can be calculated from formulas (4) and (5) for any fraction of l values.

In those cases when it is required to know the relationship between area P and the quantities N and l, we can use the approximational formulas (6), (7), and (8) obtained by establishing correlational ties between these three quantities. The original formulas (4) and (5) underlie the basis of the multiple correlation of N, l, and P.

Table 1

l	N	l	N	l	N
0,5	46 717	9,5	[illegible]	[illegible]	[illegible]
[illegible]	11 658	[illegible]	[illegible]	[illegible]	[illegible]
1,5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2,0	2 980	11,0	107	20,0	33
2,5	[illegible]	11,5	[illegible]	[illegible]	30
3,0	[illegible]	[illegible]	[illegible]	[illegible]	30
[illegible]	986	[illegible]	[illegible]	[illegible]	[illegible]
4,0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4,5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5,0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6,0	[illegible]	[illegible]	[illegible]	[illegible]	20
6,5	279	15,5	[illegible]	29	18
[illegible]	247	16,0	[illegible]	30	14
7,5	[illegible]	[illegible]	[illegible]	[illegible]	14
8,0	195	17,0	[illegible]	34	12
[illegible]	[illegible]	17,5	[illegible]	[illegible]	12
9,0	[illegible]	18,0	42	40	9

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When N < 1000

$$N = 1.22\frac{P}{l^2}; \quad P = 0.82Nl^2; \quad l = \sqrt{\frac{P}{0.82N}} \qquad (6)$$

When N = 1000 - 5000

$$N = 1.2\frac{P}{l^2}, \quad P = 0.83Nl^2 \quad l = \sqrt{\frac{P}{0.83N}} \qquad (7)$$

When N > 5000

$$N = 1,166\frac{P}{l^2}; \quad P = 0,857Nl^2 \quad l = \sqrt{\frac{P}{0,86N}} \qquad (8)$$

Formulas (6), (7), and (8) can be used in calculating l and n for any figures of the area P.

Formulas (4) and (5) give the maximum number of points for areas. If the squares are superimposed on a system of points formed by the apexes of adjoining equilateral triangles (or a system of points in the centers of rhombuses with a small diagonal equal to the side of the rhombus), in an arbitrary order (orientation), then the relationship between N, l, and P is determined by formula (9) derived by the author in the study [13]

$$N = 1{,}155 \frac{P}{l^2} \qquad (9)$$

Since the areas of forest plots in most cases are irregular in outline, it becomes necessary to determine N from l and P by the relationship of these quantities in formulas (4), (5), (6), (7), (8), and (9). In these cases it is useful to transform the irregular contour of the forest plot into an equidimensional square and then to calculate the total number of points using the above-indicated formulas. However, we must take into account the fact that in the equidimensional square it is probable that there will be a fuller placement of points compared to the irregular configuration of the forest plot.

As was remarked on earlier, the not wholly satisfactory formula (1) is used in forest appraisal to calculate forest density N, tree spacing l, and the size of sampling plots for the purpose of studying the pattern of stand growth, determining occupancy, and subsequent appraisal of timber stand reserves [3, 54].

Formula (1) gives understated values of N for the same l values (cf Table 2); thus, when l = 5, formula (1) gives N = 400, but when formulas (5) and (4) are used, N = 492. Consequently, when using formula (1), we will always get a lower forest density N, which leads correspondingly to understating occupancy, and subsequently understating timber stand reserve for the same tree spacings, determined in forests or from aerial photographs.

In determining the size of sampling plots ensuring that they include only 200 trees, formula (1) results in their overstatement. Thus, when l = 8 meters, it is required to superimpose in the forest 1.3-hectare-sized sampling plots, while from formulas (4) and (5) it is sufficient to lay out plots of 1.0 hectare size for areas of the same 200 trees, that is, one-third less, which leads to a considerable reduction in field appraisal work, and savings in time and money. Table 2 gives the values of N and l and the sizes of sampling plots P when they include 200 trees, where N_1 denotes tree density per hectare from formula (1), N_4 -- from formulas (4) and (5), P_1 -- the value of the sampling plot in hectares from formula (1), and P_4 -- the same quantity from formulas (4) and (5).

From the foregoing it is clear that in forest appraisal it is best to use formulas (4) and (5) or Table 1.

Table 2

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4. Transformation of Point Sets to Point Systems

In practice we have to deal not only with regular system of points, but also with arbitrary arrangement of points (discrete objects) in which distances between neighboring points (objects) are not equal to each other, which in general is the case encountered in nature.

The problem of finding a mathematical relationship between N and l in the case of arbitrary variation in distances between points located chaotically apart from any visible order has proven to be very difficult. But establishing such a relationship is altogether necessary, since trees in forests are located unevenly, and distances between trees differ, which means that one obviously cannot apply to them a relationship between N and l that obtains for regular point systems. The task we face must be solved in such a way, if not directly, then by some indirect mathematical procedure.

In attempting to find approaches to the mathematical solution of this problem, the idea of looking at a forest, that is, trees, as a particular example of a point set arose. Actually, if we take as the point location the centers of trunks, then the totality of these discrete objects from the mathematical point of view can be regarded as a point set in an area P.

We will define point set to refer to any number of arbitrarily arranged points on an area P at different spacings between neighboring points.

Let there be a regular system of points with a relationship between N and l established for them by formula, and a point set in which the dependence between N and l is unknown to us and which we desire to find by an indirect approach. This case, in general, is very commonly encountered in science. For example, the irregular physical body Earth cannot be directly expressed in mathematical formulas. Accordingly, regular geometric analogues are used -- a sphere or an ellipsoid of revolution -- and for these figures all mathematical calculations in geodesy and cartography are carried out.

Essentially, from the premise point of view, when we have a point set and we cannot directly find a mathematical expression of the relationship of N and l in the set, then there remains the same method as applied in geodesy, that is, to attempt to find some regular geometrical analogue of the point set.

We can take by way of such an analogue a regular system of points located at the apexes of adjoining equilateral triangles.

A mathematical expression of the relationship between N and l has been found for this system of points. The problem here consists of relating the set and the system, and then using formulas (4) and (5) for a mathematical description of the set. What then can serve as the relating link between the set and the system of points?

The idea of converting the point set into a regular system of points was proposed by us to find the factor relating the set and system of points.

Let there be a set of points located at any suitable distances from each other on an area P (for example, a square), but necessarily over the entire square P, and not in any one part of it. We will carry out here deliberately a shifting of the points in the area in such a way that they occupy positions corresponding to the locations of the points in a regular system, that is, at the apexes of the adjoining equilateral triangles. Thus, the set of points is converted or transformed into a regular system. As a result of this deliberate or active displacement of points in the area P, an ordinary regular system of points is formed which, as shown earlier, is governed by equations (4) and (5) as to the relationship between N and l. Consequently, if there are N points on the area P, the distance between them in the formed system of points will be l obtained from equations (4) and (5) or from Table 1 based on N.

Thus, the first link in the relationship between the point set and the point system is given by the translation of the former into the latter. As a result of this transformation we obtain some l, but how this l is associated with distances in the point set is not known to us. And is there in general some relationship between l and the set of different l_1 among the set points? When we know N, then it is easy to obtain l from Table 2. In practice it is of importance to be able to determine l when N is unknown.

It follows from the foregoing that the main problem here boils down to looking for the relationship between l with the totality of distances l_1 in the point set. If we were able to find this relationship, then the entire problem of mathematically describing a set via a system of points would be solved.

Let us assume that a set of points N_1 is arranged on the area P, where N_1 and P remain constants. The points are placed in arbitrary fashion, but over the entire area P, and not in any one part of it, that is, just as trees are located in a forest plot.

Let us connect all neighboring points by straight lines in such a way that they form the totality of adjoining triangles. The straight lines are drawn between the nearest neighboring points, that is, the shortest straight lines are selected, not one of which must intersect with any other straight line outside the apex of the triangle within the bounds of the area P. In this way we get a set of points arranged at the apexes of non-equilateral triangles, but the regular system is formed by equilateral triangles.

We will now study the two aggregates of these triangles expressing the point set and the system of points.

Let us make measurements of all sides of the aggregate of non-equilateral triangles and determine the mean length of the side or the mean distance between the points in the set, that is,

$$l_0 = \frac{\Sigma l}{\beta}$$

where:

Σl = total length of all sides of the triangle in the area P,

β = number of sides of the aggregate of non-equilateral triangles.

As an example, we will examine a set of points of N = 30 in an area P = 1 decimeter^2.

Figure 12 shows the arbitrary placement of 30 points. After they were connected by straight lines according to the above-indicated rule, an aggregate of non-equilateral triangles was formed. All 72 sides of the triangles were precisely measured, and then the following were calculated: the mean distance l'_0 = 2.14 cm, the variance v_1 = 28.5%, and dispersion σ^2.

Figure 13 depicts a still more uneven placement of points, and we see here even their group arrangement which produces a quite decided difference in the length of the triangle sides. The values of l''_0, v_2, and σ^2_2 were calculated from the measured sides.

In spite of the great unevenness in point placement and the high variance (v_2 = 42.8%), the mean distance between points l''_0 = 2.16. Figure 11 shows a regular system of points with N_1 = 30 and l_1 = 2.15 cm obtained from N_1 = 30 from Table 1. In the regular system of points the variance v = 0.

Comparison of the mean differences for the set of points in Figures 11, 12, and 13 shows that they are all equal to each other, namely, $l'_0 = 2.14 \approx l''_0 = 2.16 \approx l_1 = 2.15$.

Therefore, when N = 30 and P = const the mean distance between points in different sets is equal to the distance l_1 obtained from N_1 = 30 from Table 1.

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Figure 12. Point set.

And thus, if we have a set of points N in an area P, then after transforming the point set into a regular system of points we obtain the same distance between the points l_1 equal to the distance taken from Table 1 with respect to N and at the same time equal to the mean distance between all points in the set. Consequently, any point set can be transformed via $l_0 = l_1$ to a regular system of points in which the deliberate or effective variation in point disposition does not upset the relationship between N and l, that is, it remains constant in spite of the fact that the point placement in the area P is changed.

The procedure of converting (or reducing) any point set to a regular system of points allows us to deal with the former just as with the latter.

Now we can, by determining the mean distance between points in a point set, find the number of points in this set from Table 1 and, vice versa, from the known number of points

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Figure 13. Irregular point set.

in the set determine the mean distance between points, taking this also from Table 1.

The quality $l_0 = l_1$ is obtained from the point sets in a volume $N = 30$ and for variances from 0 to 43 percent. To verify the conclusion about the equality $l_0 = l_1$ we took a point set of volume $N = 624$ points in a single hectare (or $N_1 = 262$ in 0.42 hectare). This set is the totality of trees in a large forest plot. The distances between all trees in this forest tract were measured on the spot. In all, these distances numbered 706 measurements.

Based on the data of the 706 distance measurements the following quantities were calculated: the mean distance $l_0 = 4.43$, variance $v = 44\%$, and dispersion. We get $l_1 = 4.4$ meters from Table 1 for $N = 624$. Consequently, in this case $l_0 = l_1$, which confirms the conclusion of the applicability of the relationship N and l in systems of points to point sets.

Measurements of 806 distances between trees in the second forest plot (0.6 hectare in area) gave the mean distance $l_0 = 5.17$ meters for a variance $v = 41\%$. The number of trees in the forest plot $N = 488$ (or $N_2 = 293$ for 0.6 hectare). We obtain $l_1 = 5.025$ meters from Table 1 for $N = 488$. Here also $l_0 \approx l_1$, but the deviation of only 14 cm can be more properly related to errors in measuring distances between trees in the forest than to the incorrectness of the conclusion about the equality $l_0 = l_1$. And thus, the experimental measurements confirm the equality $l_0 = l_1$. Small and practically negligible deviations of l_0 from l_1 can exist, but they can be neglected. Some presuppositions about the theoretical proof of the equality $l_0 = l_1$ are presented below.

Let P = const and N_1 = const. We obtain l_1 from Table 1 for N_1. After measuring all sides of the non-equilateral triangles in the point set, we determine the mean distance l_0.

Now we will reason from the principle of proof from the contrary well known in mathematics. Let us assume that $l_0 \neq l_1$. Then the inequalities $l_0 > l_1$ or $l_0 < l_1$ must be valid, but in this case the equality $N_1 \neq N_0$ or the inequalities $N_0 < N_1$ and $N_0 > N_1$ must be valid in this case as well. But this cannot be the case, since we earlier premised that in the area P the number of points N = const, that is, $N_1 = N_0$, and this is possible only for the equality $l_0 = l_1$. This proof would be correct if for the same value of N the distance l under formulas (4) and (5) would be unique. Since the number of points N is a discrete variable and can only be an integer, then for the same N there must be several values of l, though in rigorous, precisely known limits of variance of l and of variations Δl from the mean value of l for each given N. Thus, for example, when $N = 30$, the value of l can be equal to both 21 and 22 (Table 1), but in no way can be equal to 20 or 23, since for these l values the number of points P will be, respectively, 33 and 27, and not 30.

Consequently, when $N = 30$ the value of l_1 can take on values from 21 to 22. If we take the mean value $l'_0 = 21.5$, in the general case when N = const the value of $l_1 = l'_0 \pm \Delta l$ with decrease in l and increase in N the value Δl will decrease and in practical terms it can be neglected. But theoretically we must assume that for each such value $\pm \Delta l$ l_0 will differ from l_1. Therefore, a small difference between l_0 and l_1 observed in the experiment is wholly accountable and to be expected. In conclusion we note the general significance of the mathematical relationship of N and l in point sets, since it can be used for any discrete object and phenomenon that forms a point set.

Formulas (4) and (5) are applicable to any plant communities, or brush, low brush, subshrub, grassy and other vegetation consisting of discretely located plant individuals which can from the mathematical point of view be considered as point sets (just as trees in a forest).

The point method of investigation has been found possible to apply also to continuous phenomena. Thus, for example, this is done in respect to study of terrain [13, 14]. Without this method in geomorphology, various investigators of the same area would get non-comparable results on the disarticulation of terrain.

This method is also applicable in deriving formulas of the visibility range in forests and the protective depths of a forest formulas of the traversibility of a forest, canopy closure, classification of forests by density, and for methods of determining forest density and mean tree spacing.

In this example, in several other cases, the very same mathematical formula proved applicable to solving the most diverse problems. To illustrate, formulating and mathematically expressing only one regular relationship of two to three variables of a point set led to the solution of a long series of different practical problems.

We investigated point sets, but with equal validity we can also investigate any other sets, for example, sets of triangles, their sides and angles, etc.

In solving the last-named practical problems it was of interest to relate sides and apexes of the set of triangles.

Let us assume that we have any convenient set of neighboring (freely chosen) triangles (of volume Δ) with the number of sides (triangles) β and the number of apexes (triangles) N. The following relationship exists between the variables Δ, β, and N in any set of any number of triangles (including equilateral):

$$N = \beta - \Delta + 1.$$

We can be easily convinced of the validity of this equality if we construct any set of adjoining triangles.

The relationship between the number of apexes (points, objects) and the number of sides (distances) is necessary in practice in determining the number of trees between which we have to measure distances when we know the number of distances of a sampling aggregation necessary in determining the mean distance l_0 with desired precision and confidence interval.

Using the equations given above, we obtain an approximational formula to determine the overall length of the lines (triangle sides) in the area P.

Since

$$l_0 = \frac{\Sigma l}{m} \quad \text{a} \quad m = p = N + \Delta - 1$$

then

$$\Sigma l = m l_0 = l_0 (N + \Delta - 1)$$

From formula (7)

$$N \doteq 1\,2 \frac{P}{l_0^2}$$

Then

$$\Sigma l = 1\,22 \frac{P}{l_0} + l_0 (\Delta - 1) \qquad (10)$$

As we can see from formula (10), to determine the overall length of lines in the area P it is necessary to count the number of triangles Δ formed by these lines, and to determine by a sampling method the mean distance between points (apexes, objects) of the set. The equality (10) can be applied also in determining the overall length of roads on maps.

It is important to know the overall length of such systems of lines in an area P as horizontals or any other isolines on maps in solving many scientific and practical problems.

We can determine the overall length of isolines from the formula

$$D = n(2 + kk) \qquad (11)$$

where:

D = overall length of isolines in cm in an area of size 2 x 2 cm;

n = greatest number of isolines intersected by a straight line 2 cm in length drawn perpendicular to the isolines;
k = number of bends of isolines (depressions) in a square 2 x 2 cm in size;
h = average length of the camber of isolines along the depressions.

The overall length of isolines in an area 1 decimeter^2 in size will be

$$D_s = 25n(2 + kh) \qquad (12)$$

A sampling method is used to determine D_s, and as sampling plots -- squares 2 x 2 cm in size (for a low frequency of isolines the size of the square is increased to 4 x 4 cm and even to 10 x 10 cm).

Transformation of point sets into regular systems of points allows us to establish a relationship between them via the mean distance between points in the set and to obtain a mathematical characteristic of the totality of distances in any point set solely from one index -- from the mean distance. In practical terms it is important to know the mathematical characteristic of these distances in more detail, for example, how many and which distances can be encountered in a given set if we know the mean distance or any other variable expressing the general characteristic of the totality of distances.

The new problem essentially means discovering a function of the distribution of distances among points in a set. The solution of this problem is broken down into two stages. The first stage consists of the possibility of finding a general correlation of the distribution of distances intrinsic to any point set.

The second stage consists in discovering a specific and wholly satisfactory series (function) for the distribution of distances characteristic solely of the given natural point set, for example, the correlation of the distribution of distances among trees in a natural forest. Particular series and functions of distance distribution, their stability and regularity are determined by the specific nature of each point set, by the effect, for example, of a large number of factors on the arrangement, growth, and development of trees in the given geographical area. This specific detail will also govern the form of the function of the distribution of distances among trees or among other plants.

This rough outline is the relationship among general laws of distribution in a point set and particular functions of distribution of distances characteristic only of a given natural phenomenon which can be viewed as a particular case of a point set.

5. Mathematical Modeling of the Placement, Density, and Size of Tree Crowns in the Development Dynamics of Tree Stands and an Experimental Method of Determining Forest Density and Tree Spacings

We know that as tree stands increase in age, forest density is reduced, and crown sizes increased. Based on the study of the dynamics of how these variables change, we have constructed a geometrical model reflecting their status at the most critical periods of tree-stand development.

This model gives the first approximation to actuality, but it is wholly suitable in investigating variation in the structure of crown imaging on aerial photographs of various scales and for searching for approaches to a sampling method of determining forest density and mean tree spacings.

Figure 14 presents geometrical models in which the placement of crowns in the following scales is shown: 1:2,000, 1:6,000, 1:10,000, and 1:15,000. Four models of crown imaging a, b, c, and d are shown in each of the figures, corresponding to the four gradations of density N and crown diameters D_c that are close to the actual fact. Figure 14 presents an examination of variance of direct measurement of l and calculation of N within the limits of sampling plots.

Determination of l_0 can be done in two ways:

direct measurement of distances between crown images

and

direct count of the number of crown images with subsequent conversion to l_0 from Table 1.

We can measure l among a group of neighboring trees or among crowns arranged in the form of a strip.

Figure 14 shows along the boundaries b, c, and d a group of five to six crowns. The number of groups -- from 1 to 3, and the total number of measured distances are, respectively, 7, 14, and 21.

The deviation of the mean l_0 obtained by measurement in two groups from the true l does not exceed 0.2 meter (converted at the locality).

We take a narrow rectangle 5 x 0.5 cm on the scale 1: 2,000 as the strip sample. We measure l successively along the boundaries of the strip between crown centers. Experimental data show that the deviations of l_0 from l for a are close to 0.9 meter, and for b to 0.4 meter.

Determination of l_0 by the crown counting method can be done in two steps:

by counting crowns in the square or rectangle with subsequent conversion to N per hectare and obtaining of l_0 from Table 1;

by counting the crowns in the narrow strip (rectangle) with subsequent division of the number of crowns by the strip length.

Figure 14a shows a square 2 x 2 cm in size (0.16 hectare), Figure 14b, c -- 1 x 1 cm (0.04 hectare), and Figure 14d -- 0.5 x 0.5 cm (0.1 hectare). The number of such samples is two to three. The conversion from the number of crowns to N per hectare is achieved from the expression

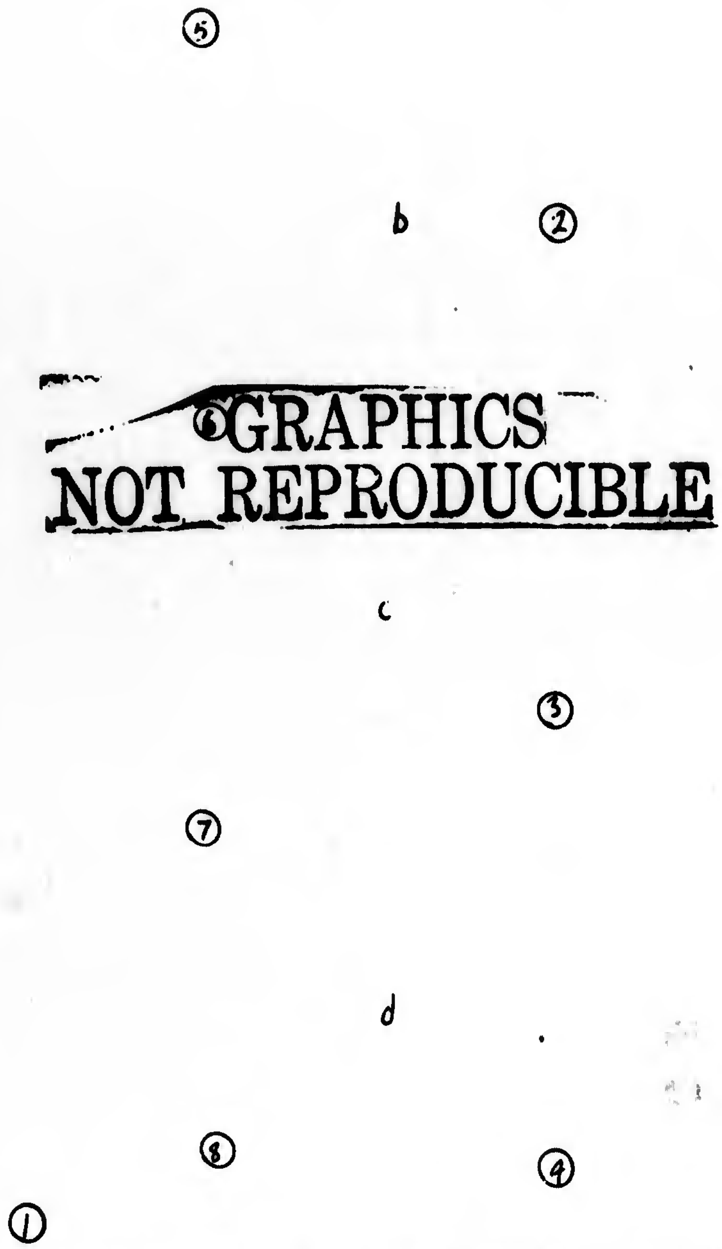
$$N = \frac{n}{P}$$

where:

n = number of crowns in square (or rectangle);
P = area of square (rectangle) expressed in hectares.

Under this method the deviation of l_0 from l proved to be equal to 0.2 meter. Instead of squares we can take samples in the form of narrow rectangles. Figure 14a shows three narrow strips 5 x 0.5 cm in size. Crowns entering into the rectangle for half and more of their area are counted in each strip. The mean l_0 was obtained by dividing the strip length by the number of crowns it contains. The value of l_0 was determined with an error of about 0.5 meter. In Figure 14c samples of the following sizes were tested: 2 x 0.5 cm (0.04 hectare) and 3 x 0.3 cm (0.036 hectare). The mean l_0 was determined with an error of about 0.2 meter.

As the scale of aerial photographs is reduced, narrow rectangles are converted into ordinary straight lines. Thus, a rectangle 100 meters in length and 5 meters wide at the



LEGEND: 1 -- 1:2000 scale; 2 -- 1:6000 scale; 3 -- 1:10,000 scale; 4 -- 1:15,000 scale; 5 -- D_c; 6 -- D_c; 7 -- D_c; 8 -- D_c

locality will be imaged in a 1:10,000-scale aerial photograph by a straight 1 cm long and 0.5 mm thick, and on a 1:20,000-scale aerial photograph it will be converted into a thin straight line 0.5 cm long and 0.25 mm thick. Transformation of narrow sample plots into linear plots allows us to use straight lines to determine l_0 in aerial photographs.

In Figure 14b and c we have taken three straight lines 4 cm in length and 0.2 mm thick. Then we counted the crowns intersected by the straight line and tangent to it. The mean l_0 was obtained by dividing the length of the lines by the number of crowns. The deviation of l_0 from l_1 proved to be about 0.2-0.6 meter (at the locality).

In Figure 14d we used three straight lines 2 cm long, 0.1 mm thick, and 1 cm long and 0.1 mm thick. In the first case, l_0 was determined with an error of 0.1 meter, and in the second -- 0.2 meter. Consequently, samples in the form of straight lines will give good precision of l_0 determination for a large enough density of crown images on aerial photographs. In this experiment we tested six methods and made 96 variants processing more than 600 measurements. The experimentally-derived methods of determining l_0 and N were tested and verified in the aerial photographs and in the field.

6. Mathematical Model of Canopy Closure and Approximational Relationship Between Closure, Density, Crown Diameter, and Inter-Crown Opening

In studying approximational quantitative relationships between canopy closure C, forest density N, crown diameter D_c, and inter-crown opening Δd, we can construct a simplified mathematical model of the tree stand without taking into account the story status and curvature of tree disposition. Quantitative relationships obtained by means of a model will give only a general picture, a method of investigating for subsequent refinement of interrelationships in specific tree stands.

Canopy closure can vary from C = 1 to C = 0.1 and less.

The effect of canopy closure depends on forest density N, mean distance between trees l, crown diameter D_c, and the nature of tree placement in the forest plot. In practice, covering properties of the forest are sometimes characterized by the value of the opening, interval, or distance between crowns

$$\Delta d = \frac{l - D_K}{D_K},$$

expressed in crown diameters.

We know that a specific interrelationship does exist between forest characteristics. In our case it is important to find even an approximational relationship between C, 1, N, D_c, and Δd.

Canopy closure increases as the forest becomes denser and with reduction in the mean tree spacing. Under average conditions this function is more or less justified if we do not take cognizance of other factors that affect the value C. Canopy closure increases with rise in density only up to a certain limit, after the attainment of which a further increase in N does not lead to an increase in C, for example, after formation of C = 1. The nature of tree placement also has a substantial bearing. In the event of a strongly pronounced sloping or grouped tree placement, with increase in N closure rises more slowly than for the case of uniform tree placement.

The value C depends to a greater extent on crown sizes. For the same density N, but for different D_c closure can vary within appreciable limits.

The question also arises as to what crown value we must take in determining C. In any forest plot we will encounter trees that have crowns of different sizes. Independently of species composition crown diameters of individual trees will not be the same.

Thus, the inequality of D_c of individual trees in the forest plot is a natural inevitability. Measurement of each tree is too laborious. Therefore, to determine C we have to pick an average D_c. This is all the more justified in that a regular distribution of trees with respect to D_c close to the normal distribution is observed in forests (section 20).

Let us take a forest plot with a large number of trees located uniformly over the entire area, and measure the D_c of each tree. Let us calculate the average D_c and obtain deviations $\Delta d_c = D_c - D^0_c$. In absolute value ΔD_c will be small, with either plus or minus sign. It is obvious that the greatest number of trees will have D_c close to D^0_c.

In the simplified model of a tree stand under consideration (the crowns are not overlapping), to determine D it is important to know not the value of the individual deviations ΔD_c, but the effect of the total of these deviations on the precision of calculating canopy closure when using the average $D^0{}_c$. Let us assume that by the measurements we obtain the actual overall area occupied by the crowns of all individual trees. If we divide this area by the number of crowns, then we get the average crown area. Obviously, independently of the value of crown deviations of individual trees from the mean crown area, the product of the latter by the number of crowns will give the same overall area. Since the crown area is taken as equal to the area of a circle with diameter $D^0{}_c$, then all the foregoing will retain its effect also for the mean diameter $D^0{}_c$.

Consequently, to determine C we have to establish as precisely as possible the size of the mean value $D^0{}_c$ which would be equal to the actual mean crown diameter obtained by measuring crowns of all trees in the forest plot. For given N and D_c the average crown area will be

$$\Delta p = \frac{\pi}{4} D_{c}^{2}$$

LEGEND: 1 -- $D^2{}_c$

and canopy closure

$$C = 785 \cdot 10^{-7} N D_{c}^{2} \qquad (13)$$

LEGEND: 1 -- $D^2{}_c$

where:

C = canopy closure in fractions of unity;

N = forest density (number of trees per hectare);

D_c = mean crown diameter in meters.

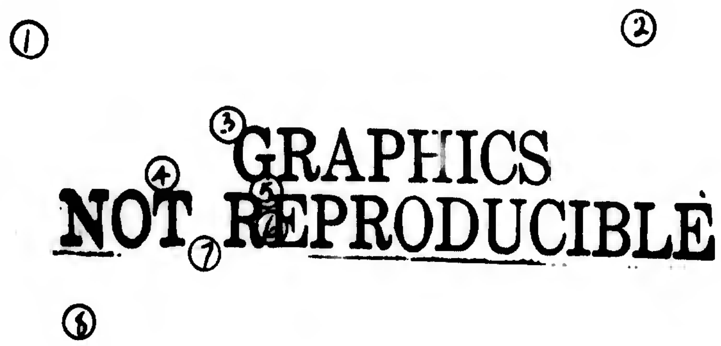
Example. For the simplified model (crowns do not overlap) for N = 407 and D_c = 4 meters, canopy closure from formula (13) will be

$$C = 785 \cdot 10^{-7} \cdot 407 \cdot 4^2 = 10^{-7} \cdot 785 \cdot 6512 = 10^{-7} \cdot 5111920 \approx 0{,}5.$$

For specific forest plots in which determination has been made of the mean 1 and N, canopy closure calculated from formula (13) will be quite close to the actual value of C. However, it is important for us to establish an approximate relationship between C, N, and D_c in the general case. This is possible only if the approximate relationship between forest density N and crown diameter value D_c will be known. Thus, a young forest aged 10-15 years will have 5,000-10,000 trees per hectare, and an old forest aged 130-150 years will have only 100-200 trunks per hectare. But with increase in age of forest the height and thickness of trees increases, as does also crown diameter D_c.

Consequently, with increasing forest aging an increase in D_c and a reduction in forest density N is observed. The approximational nature of the variation in the quantities N, D_c, d_t, and h in the schematic form is shown in Table 3 for mixed stands.

Table 3



LEGEND: 1 -- forest age; 2 -- N, individuals per hectare; 3 -- 10-15-year-old young forest; 4 -- 30-year-old pole-like forest [zherdnyak]; 5 -- moderate-aged forest, 50 years; 6 -- maturing forest 50-70 years of age; 7 -- mature forest, 90-110 years of age; 8 -- old (overmature) forest 100 years of age and older; 9 -- d_t; 10 -- D_c.

When we take into account the approximational relationship of N and D_c and the fact that the area of the canopy projection (that is, the canopy closure C) is usually less than the overall area of the projections of crowns of all individual trees (owing to the unevenness of tree placement in the area and overlapping of crowns), closure was calculated from formula (13). In calculating C we took the following values of D_c: for N = 986 and higher D_c = 3 meters; for N ranging from 765 to 585, D_c = 3.15; for N ranging from 402 to 247, D_c = 4 meters; for N ranging from 195 to 105, D_c = 4.5; and for N ranging from 85 to 14, D_c = 5 meters.

A curve of closure was constructed from calculated values of C and N (Figure 15), and this curve was somewhat smoothed for the purpose of finding a smoother variation of D_c as a function of changing N. As a result, we arrived at Table 4.

GRAPHICS NOT REPRODUCIBLE

Figure 15. Interrelationship between canopy closure, tree spacings, and inter-crown openings. LEGEND: $D_K = D_c$.

In forest appraisal, in addition to canopy closure it is required to determine further stand occupancy. Stand occupancy is expressed just as is closure, in fractions of unity and is determined from the formula

①

②

LEGEND: 1 -- g_{given}; 2 -- g_{normal}

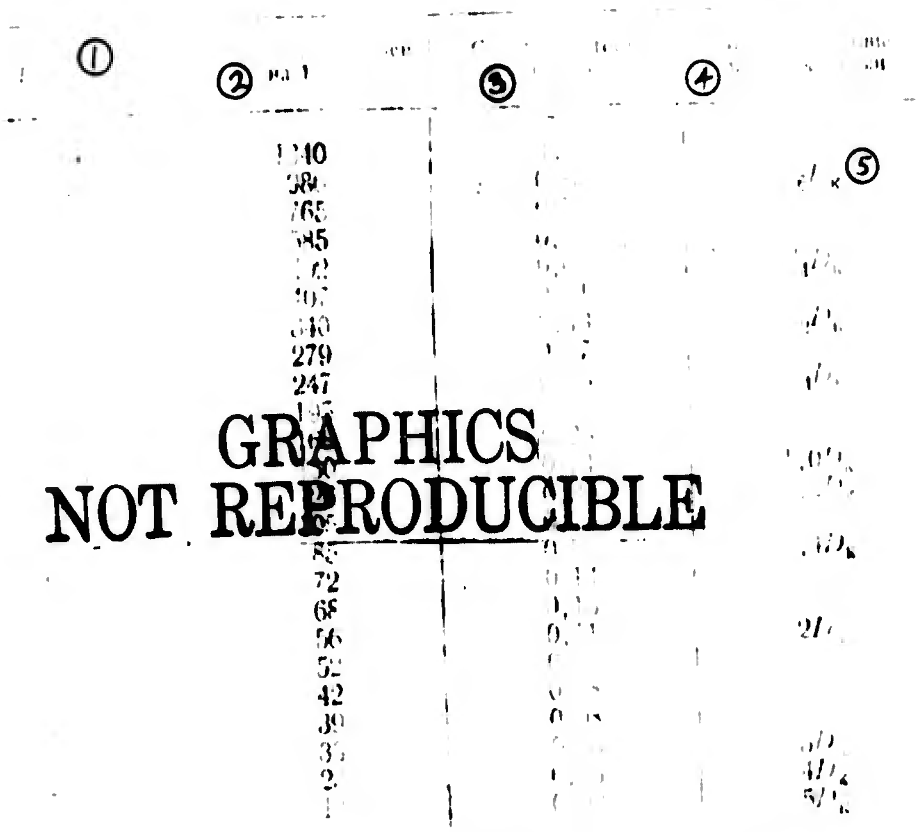
where:

Σg_{given} = total cross-sectional areas of trees in one hectare of the given (subscript) stand;

Σg_{normal} = total cross-sectional areas of normal (subscript) stand, that is, when p = 1.

Occupancy p and closure C are intimately related. Direct measurement of Σg_{given} in forests is very involved. Therefore, appraisers often determine p from C.

Table 4



LEGEND: 1 -- 1 in meters; 2 -- N = number of trees per hectare; 3 -- C = canopy closure; 4 -- Δ d = inter-crown spacing; 5 -- D_c.

7. Mathematical Modeling of Tree Stands to Determine Visibility Range

In visual estimation appraisal, the selected stands are examined usually from a clearing or with sighting rods.

The precision of the appraisal rests on the detail in the scan of the stands over the entire area between the sighting rods, and the detail of the scan is determined by the visibility range in the particular stands. Obviously, the distance between sighting rods to ensure a thorough scan of the appraised plot must equal twice the visibility range. The precision of visual estimation appraisal will be reduced as the difference in distance between sighting rods is increased and with the increase in doubled visibility range, since part of the forest lying outside the field of view will be estimated on analogy with the observed areas adjoining the sighting rods over the extent of the visibility range. Thus, preliminary knowledge even of the approximate visibility range in forests allows us to more properly take note of the distance between sighting rods, reduce their number, and thus speed up and lower the costs of field work in forest management.

A knowledge of the detail in what is visible in different forests is necessary in compiling topographic descriptions for the purpose of providing information to persons moving to a settled locality on foot or on vehicles.

Topographers determine visibility range in forests by the following method: a person is dispatched in the desired direction, he is kept in sight until he is barely visible with the naked eye, and then the distance up to him is measured along a straight line.

More or less reliable information about the visibility range in each forest tract can be obtained either by direct field observations in the majority of directions, but not by any single direction. However, in collecting information for topographic descriptions no one can determine range of visibility for most directions, since this would require too much time.

Range of visibility depends on many factors, and so it is very difficult to calculate in advance the actual range of visibility in a given expanse of forest. But for general judgment about the conditions of ground observation in the forest it is important to know even though approximately, the average range of visibility.

To determine the approximate visibility range, we can take a count of not all variable factors that have a bearing on the visibility range, but only the principal, more or less constant factors for any forest at the given period of time, that is, the density of the forest N and the mean tree thickness d_t.

GRAPHICS NOT REPRODUCIBLE

Figure 16. Geometric model of the profile of a stand of trees used in deriving the formula of the visibility range in forests.

Let it be required to determine the visibility range in a band of forest of width a and length L, density N, and mean tree thickness d_t.

Let us assume that all trees are more or less evenly distributed within the limits of the strip ABB'A' (Figure 16).

Let us now presuppose that we will project all trees onto the line A'B' of length a. As a result of this, all trees would appear to be laid in a single row. It is obvious that total non-visibility into the depths of the forest along the line A'B' will occur when the projected row of trees forms a continuous wall of trunks adjoining one another along the line A'B'.

The length of the projected series of trees is equal to

$$b = \frac{d_m \Sigma N}{100} \qquad (11)$$

If the length b will equal the length of the line A'B', then a continuous wall of contiguous trunks forms on the line A'B', which will evidence the total lack of visibility in the forest at a distance L.

Consequently, the condition of total non-visibility in the given forest can be expressed by the following equality:

The total of all trees in the forest strip is

$$a = b = \frac{d_m \Sigma N}{100} \quad (15)$$

and the number of hectares

$$\Sigma N = nN \quad (16)$$

from whence

$$n = \frac{aL}{10^4} \quad (17)$$

$$L = \frac{n \cdot 10^4}{a} \quad (18)$$

The quantity L characterizes range of visibility into the forest only in the case that the length a in equation (18) equals b according to formula (15).

Substituting in equation (18) the right-hand member of equation (15) instead of a and substituting $\Sigma N = nN$, we arrive at formula (19) expressing the range of visibility in the forest in meters

$$L_v = \frac{10^6}{Nd_m} \quad (19)$$

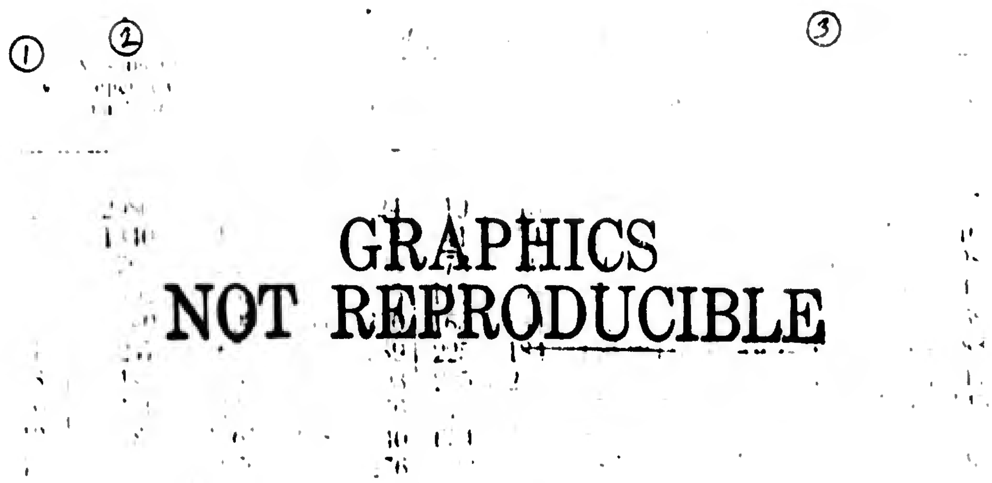
LEGEND: 1 -- L_{vis}

Thus, from known N and d_t we can calculate the range of visibility, that is, the distance L_{vis} at which total non-visibility of the object results. In order not to have to calculate L_{vis} each time from formula (19), Table 5 has been compiled, from which it is easy to determine L_{vis} in meters for the most widespread values of N and d_t in cm.

If in a forest the mean distance between trees 1 = 6 meters, and d_t = 22 cm, then from Table 5 the visibility range in such a forest L_{vis} = 134 meters. Using the relationship of N with 1 we can obtain L_{vis} from 1 and d_t which are determined from aerial photos using methods described in the following chapters. Consequently, the approximate visibility range in the forest can be determined also under office conditions from large-scale aerial photographs, and in any direction of interest to us and with any desired frequency.

For a level locality in the absence of brush, undergrowth, and species of trees with crowns more than 2 meters above the earth, the table will give, though an approximate, but still a quite satisfactory idea about the visibility range in the forest. In winter L_{vis} determined from formula (19) will be still closer to the actual L_{vis} at the locality. The effect of terrain on L_{vis} is taken into account by ordinary methods using topographic maps. For distances less than those

Table 5



LEGEND: 1 -- 1 in meters; 2 -- N = number of trees per hectare; 3 -- d_t = mean tree diameter in cm.

indicated in the table, the visibility range will depend on all other conditions and factors whose consideration is possible only by direct observation in the field.

In thick brush the very limited range of visibility is obvious even without calculations. In spruce forests with tree crowns beginning almost at the ground, the visibility range is sharply reduced by approximately as many times as the crown diameters D_c are thicker than the trunks d_t. The minimum visibility range would probably be noted for the case when the observation line will be at the level of the widest part of the tree crowns, which is most characteristic of spruce stands. To obtain the minimum visibility range, the table values of L_{vis} will have to be reduced by $k = D_c/d_t$ times or calculated from the following formula:

$$L'_{vis} = \frac{10^6}{ND_K} \qquad (20)$$

For d_t values from 15 to 34 cm, the coefficient $k \approx 17$, for $d_t < 15$ cm $k \approx 20$, and when $d_t > 34$ cm $k \approx 14$.

L_{vis} values calculated from formula (19) were verified with field determinations of visibility range in fairly dense forests. For example, in a mixed forest of pine, birch, and spruce with $l = 4$ meters and $d_m = 25$ cm, the visibility range on the spot was 50-60 meters, but according to the formula $L_{vis} = 53$ meters. In a birch-pine forest with $l = 2.5$-3 meters and d_t = 11 cm, the field L_{vis} was 50 meters. The calculated L_{vis} also proved to be 50 meters.

8. Principal Formulas in the Sampling Method of Investigation

Let us assume that we have a very large number of any kind of objects whatever and we decided to know the characteristic of the entire aggregate of these objects with respect to some feature they exhibit. This problem posed can be solved by two methods: by continuous examination of all the objects or only part of them.

The theory of the sampling method takes into account that organization of sampling under which non-continuous or individual observations, measurements, or counts would give characteristics close to the values of these characteristics in the entire aggregate of the objects under study.

As the basis of the sampling method we have the law of large numbers, which is quite well expressed by a theorem of the great Russian mathematician P. L. Chebyshev:

$$p(|\bar{x}-\bar{a}|<\varepsilon)\rightarrow 1 \text{ при } n\rightarrow\infty. \qquad (21)$$

Legend: 1 -- for

that is, for a large enough number n of variables with probability p as close as we like to unity, we can obtain deviations ε of the mathematical expectation $\bar{a}$ from the arithmetic mean $\bar{x}$, as small as we like in absolute value, of several independent random variables.

By way of an illustration of the importance of the sampling method in actual practice, let us consider only one example. Let us assume that on a 1:100,000-scale map it is required to show the mean distance between trees just as was done in respect to mean height and thickness of trees.

For an average forest density of 500 trees per hectare, to determine the mean distance between trees in the locality occupying only 1 decimeter^2 of the map, it would be necessary to measure more than ten million distances l.

If it takes a minute to measure a single distance, then for the complete measurement of all distances among these trees about 70 years would be required.

The advantages and the practical importance of the sampling method are plain to see.

The sampling method can be used as follows:

for plotting distribution series or series of percentage ratio of objects with respect to any characteristic they exhibit;

to determine mean values of a characteristic in an aggregate of objects;

to establish relationships between two and more characteristics of the objects.

We will introduce the following symbols for the subsequent exposition of the main concepts and formulas in the sampling method:

S = general population, that is, the total number of all objects that are to be investigated;

s = sample population, that is, the number of that portion of the objects to be subjected to observation, measurement, or counts;

p = probability, frequency, or proportion of objects exhibiting the given feature in the general or sample population;

t = normal deviation in the range of probabilities;

σ^2 = dispersion of the feature in the general or sample population;

v = variance of the general or sample population;

Δ = desired precision of sample determination of the proportion or average value of the feature studied in the population of objects;

x_0 = arithmetic mean value of the feature in the general or sample population.

In repeated sampling the sampled objects will return again once to the general population, but in non-repeated sampling they will not be returned. Non-repeated sampling is always more precise than repeated sampling, however the mean error of the former differs from the latter by the coefficient

$$c = \sqrt{\frac{S-s}{S-1}} \qquad (22)$$

which is less than unity, but for a large general population is very close to unity.

In determining the characteristics of a forest, non-repeated sampling will have the highest value.

Objects can be sampled out of a general population by two methods: regionalized and mechanical.

<u>In the regionalized ("typical") method of sampling</u> the examination is conducted by regions, parts, or groups exhibiting higher uniformity in the feature under study than occurs in the non-subdivided population.

Thus, in determining forest density the entire area of the forest tract is divided up on aerial photographs into individual areas or sections differing from each other in density of trees, and a separate sampling is taken in each region.

<u>Under the mechanical method</u> units of the sampling population are located uniformly over the entire general population. For example, the area of the entire region is divided into equal smaller subareas and the sampling population includes a small portion of these subareas chosen at random in chessboard fashion or in some other manner. This method ensures uniformity in selection of the necessary number of objects from the entire area of the forest tract.

The regionalized method of sampling is more precise than the mechanical, since its mean error is less than the error of the mechanical method.

In calculating the number of observations s, estimate of precision Δ, and its confidence interval in sampling determination of the proportion and the mean value of the features of the population of objects under study, it suffices to be able to use a small number of principal formulas.

The precision Δ or the mean error of determination of the proportion (percent ratio) of objects in non-repeated sampling is calculated from the formula

$$\Delta = \sqrt{\frac{p(1-p)}{s}\left(1-\frac{s}{S}\right)} \qquad (23)$$

The number of observations made in the sample population s that is necessary in determining the proportion with a desired precision Δ and with desired probability under the non-repeated sampling approach is determined by the formula

$$s = \frac{t^2 S p(1-p)}{S\Delta^2 + t^2 p(1-p)}. \qquad (24)$$

The precision, or the error, of the sampling determination of the mean value of the characteristic is calculated under the non-repeated sampling approach from formula (25), but under the repeated sampling approach from formula (26):

$$\Delta = \sqrt{\frac{t^2\sigma^2}{s}\left(1 - \frac{s}{S}\right)}, \qquad (25)$$

$$\Delta = \sqrt{\frac{t^2\sigma^2}{s}} \qquad (26)$$

The number of observations made in the sampling population s necessary in determining the mean value of the feature with desired probability and precision Δ is calculated from formulas (27) and (28) under the non-repeated sampling approach and from formulas (29) and (30) under the repeated sampling approach:

$$s = \frac{t^2\sigma^2 S}{(S-1)\Delta^2 + t^2\sigma^2}. \qquad (27)$$

$$s = \frac{t^2 v^2 S}{(S-1)\Delta^2 + t^2 v^2} \qquad (28)$$

$$s = \frac{t^2\sigma^2}{\Delta^2} \qquad (29)$$

$$s = \frac{t^2 v^2}{\Delta^2} \qquad (30)$$

In formulas (28) and (30) Δ is expressed in percentages.

In the case of large values of S, the number of observations s for non-repeated sampling can be determined from formulas (29) and (30), and the precision of the mean value from formula (26).

The variance is calculated from the expression

$$v = \frac{\sigma \cdot 100}{x_0}. \qquad (31)$$

It is clear from formulas (23) - (30) that the precision of the sampling method depends on the dispersion σ^2, variance v, and the variables S and p of the general population. But these characteristics of the general population are unknown to us. As a consequence we have to replace these characteristics by quantities closely similar to them that we can find either by a small sampling or by an indirect approach, or else we are forced to use known data established for similar populations.

Let us now look by way of an example at several typical problems associated with the use of the sampling method.

Calculation of the number of observations and the precision of the sampling determination of the mean thickness (diameter) of trees d_t is carried out from formulas (25) - (30). For this it is necessary that we know the variance of tree thickness. It is known that the variance of d_t in the case of pine v_{pine} = 20-25 percent, but for birch v_{birch} = 28.8 percent. The value of the variance decreases with increase in forest age. In complex and mixed stands the variance is greater than in homogeneous and single-story forests. On an average we can take v = 25 percent. Then the number of trees required for determining the mean d^o_t with a precision Δ = 10 percent and confidence interval p = 0.5 (t = 2) will be

$$s = \frac{4 \cdot 25^2}{10^2} \approx 25$$

The precision of determination of the mean d^o_t under specified sampling conditions is calculated from formulas (25) or (26). If we measure d_t for s = 10 trees, then when t = 2 and dispersion σ^2 = 25, the precision of determination of the mean d^o_t will be of the order of Δ = 3 cm.

Since there is a relationship between N and l for determination of forest thickness N and the mean tree spacing l_o, it is necessary to know only one of these quantities. However, l_o corresponds to l only in the case of the continuous measurement of distances between trees, which is practically impossible to carry out owing to the very laborious burden of this operation.

Table 6 shows the total number of trees at a locality corresponding to an area on the map of 1 decimeter², for a forest density N per hectare.

Table 6

(1) $N = 60$ $N = 100$ $N = 1000$

GRAPHICS NOT REPRODUCIBLE

Legend: 1 -- map scale

In practical terms, we will be forced to determine l_0 by measuring only a small number of distances between trees, that is, use the sampling method. In this case the problem will boil down to finding that sampling l which would be closer to l_0 arrived at from thorough measurements.

It is obvious that the precision of the sampling determination of l_0 will depend on the range of fluctuations of individual $l_1, l_2, \ldots l_n$ in the forest plot. Let us determine how many distances we must measure in order to obtain a l_0 with desired precision Δ.

Calculation of the number s of measured distances and the precision Δ of the sampling determination of l_0 is made from formulas (25) and (30). Let it now be required that we determine the mean distance between trees l_0 with a precision $\Delta = 20$ percent and confidence interval $p = 0.95$.

To determine the sampling population s from formula (30), it is necessary to know the variance v_1 and the normalized deviation of t. We will select the value of t for p from the table of probabilities given in [13, 25]. Let the variance of the distances v_1 be 35 percent.

When $P = 0.95$, $t = 2$. Then the number of measured distances

$$s = \frac{4 \cdot 35^2}{20^2} = 12$$

but the number of trees between it is necessary to measure l will be 7. For $p = 0.99$, $t = 3$, and the number $s = 27$. If we take $\Delta = 15$, then when $t = 2$ the value $s = 20$, but the number of trees = 10.

In practice we can determine the value of v_1 in an approximate way by the small sampling method or from standard l values in the aerial photographs.

Let us now assume that we have made a sampling determination and desire to know the precision Δ with which we obtained l_0 that under the giver sampling conditions is calculated from formula (25).

We can find the forest density N in an approximate way by the small sampling method, that is, by counting the number of trees in small subplots or by measuring l in them. From Table 1 we obtain S for l. Let us assume that S = 400, the number of measured distances s = 15, and the probability with which we will guarantee the quantity Δ is 0.95 percent [sic, should be 0.95] for t = 2. In forests of moderate density the dispersion $\sigma^2 = 4$. Since we estimate the precision of the already made determination of l_0, we will have a sampling σ^2 which we can use in calculating Δ. Under these sampling conditions, that is, for S = 400, s = 15, t = 2, and $\sigma^2 = 4$, the precision of determination of the mean distance l_0 will be

$$\Delta = \sqrt{\frac{4 \cdot 4}{15}\left(1 - \frac{15}{400}\right)} \approx 1 \text{ м}$$

Under those conditions when it is difficult to determine S, an estimate of the precision of the sampling l_0 can be made from formula (26).

The sampling determination of the mean distance from aerial photographs can be made mechanically as well as by the regionalized sampling methods.

The mechanical method of sampling consists in covering the forest tract evenly on the aerial photograph with small subareas, for example, in chessboard fashion. Direct measurement of l is made for a small number of these subareas. The mean of these areas is taken as the average distance between trees in the entire tract. This method can give satisfactory results if the trees are evenly enough distributed over the entire area of the tract. The method will find application in those cases when it is required to estimate the forest tract as a whole in terms of the number of trees it contains and in terms of the overall stock.

For large differences in forest density in individual parts of the tract it is advisable to use the regionalized sampling method instead of the mechanical method.

The regionalized sampling method consists of preliminary visual-estimation regionalization of the forest tract and individual plots differing from each other in forest density.

For this purpose first sections with the highest and smallest density of photographic images of tree crowns are marked off on the aerial photograph, and then intermediate areas readily discernible to the eye by the density of crown images therein are singled out. We know that young trees have very high density of tree stands and delicate crowns. With increase in age forests gradually thin out, but crown sizes increase. These differences in crown sizes produce easily noticeable differences in the structure of their photographic imaging on aerial photographs which must in fact be taken into account when marking off plots in the forest tract.

A separate sampling is taken within the bounds of the selected plots, for which the plot is more or less evenly covered with small subareas. Direct measurement of l or counts N are made within the limits of two or three of the most typical subareas of the plot.

CHAPTER 3

STATISTICAL CORRELATION OF THE DISTRIBUTION OF TREE SPACINGS IN TREE STANDS

9. Distribution of Distances Between Points in Point Sets

In Figures 12 and 13 two point sets are shown in their general form. To judge the distribution of distances between points in Figures 12 and 13 the following were determined: mean distance l_0, mean square deviation σ, and the variance of distances v_l for each set. Table 7 lists the results of a count made of the number of deviations expressed in terms of the mean square deviation for the point set in Figure 12. The percentage of deviations f percent and the accumulated frequencies w_{ac} were calculated from the number of deviations m; the accumulated frequencies were compared with the probabilities p in the integral law of normal distribution.

Table 7

[illegible]	[illegible]	m	f %	① f_{ac}	② w_{ac}	p
[illegible]	[illegible]	[illegible]	72.0	[illegible]	0.720	0.683
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.957	0.954
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.000	0.997
		72	100.0			

LEGEND: 1--f_{ac}; 2--w_{ac}

Table 8 gives analogous data on the distribution of distances in the point set shown in Figure 13.

It is clear from Tables 7 and 8 that more than 70 percent of the distances between the points lie within the limits $l_0 \pm \sigma$ or $l_0(1 + 0.01\ v)$, and not one distance exceeded the limits $l_0 \pm 3\sigma$ and, accordingly, finally the values of the deviations Δl (from the mean l_0) expressed in units of σ. Based on these (experimental) data, it can be assumed that the distribution of deviations Δl corresponds to the normal distribution, and that from l_0 and σ (or v) we can determine the percentage of those distances (that is, their distribution) in point sets. Of course, the distribution of l in units of $l_0 \pm k\sigma$ gives only a general idea of the probable distribution type and, as experimental measurements have shown, the accumulated frequencies w_{ac} prove to be even greater than the theoretical probabilities p of the normal law of distribution. Although investigation of the distribution of l in point sets more properly relates to interests of theory in mathematics, therefore we will limit ourselves to explaining the general nature of the distribution of l in point sets.

Small samplings -- this is a wholly essential stage of major statistical investigations. These afford at comparatively small outlays of time and effort discovering approaches for further exploration and establishing in preliminary form the probable distribution type and the extent of its stability. Sometimes small samplings provide the basis in general of not having to undertake subsequent large statistical efforts, since given hypotheses are clearly refuted with their data. Of the series of small samplings we will make only one, that relates to a forest tract in Moscow Oblast. Based on data of field measurements of distances between trees (with species composition 9 birch 1 pine and mean values h = 22 meters and d_t = 25 cm) the following were calculated: l_0 = 4.94 meters; $\sigma = \pm 1.98$; and v = 40 percent.

Table 9 gives cumulated frequencies w_{ac} of the empirical distribution of distances and the probability p of the normal distribution. As we can see from Table 9, w_{ac} and p are close to each other in value, and we can presuppose that in large sampling as well (made in the general population of distances) we will not encounter distances greater than $l_0 \pm 3\sigma$, with a probability almost equal to unity.

Distribution of distances can be expressed either in terms of σ or v. When l is expressed in terms of the variance v, the following simple relationships are used:

$$l_1 = l_0 \pm \sigma;$$

since $v = \frac{\sigma \cdot 100}{l_0}$, therefore $\sigma = \frac{v l_0}{100}$.

Then

$$l_1 = l_0 \pm \frac{v l_0}{100} = l_0(1 \pm 0.01v) = l_0 \pm \sigma$$

We obtain in a similar way

$$l_2 = l_0(1 \pm 0,02v) = l_0 \pm 2\sigma,$$
$$l_3 = l_0(1 \pm 0.03v) = l_0 \pm 3\sigma.$$

Table 8

① ②

LEGEND: 1-- f_{ac}; 2-- w_{ac}

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Table 9

Ko

① ②

LEGEND 1-- f_{ac}; 2-- w_{ac}

For all subsequent investigations unit intervals of the series of distance distribution expressed in fractions of the mean distance (at intervals of 0.2) were adopted. The practical and theoretical convenience of these intervals is that gradations of the distribution series remain unchanged, independently of change in mean distances, since for any mean distance the interval $0.2 \cdot l_0$ is retained. Taking intervals smaller than 0.2 does not make any sense, since it is not necessary to take into account variations in distances between trees that are less than 0.5 or 1 meter, for the tree thickness often amounts to 0.5 meter and in this case the minimum distance between two trees will be 1.0 meter.

Table 10 gives an empirical series of distance distributions in this forest tract. It gives a more detailed characteristic of the type of distance distribution compared with the characteristic of the distribution of l in terms of gradations of σ or v in Table 9. It is clear from Table 10 how distances are grouped to the right and to the left of l_0, and at what percentage different l values are encountered. The gradual decrease of f percent from the distribution center 1.0 or l_0, which is clearly suggestive of a curve close to the normal distribution, is plainly evident. Based on empirical series of distance distribution in small samplings the possible distribution type and the extent of stability of frequencies of the distribution series can be seen. However, experimental data of small samplings do not afford adequate ground for a decisive conclusion on the specific type, and more so on the theoretical distribution function reproducing the distribution of distances in the general population. For this we have to have large enough and independent samplings in different forests. Such large samplings were obtained later, and their mathematical treatment is conducted in subsequent sections of the text.

Table 10

Legend: A -- gradation of series with 0.2 interval; B -- series intervals when l_0 = 4.94; C -- frequency m in intervals of the series; D -- f percent -- frequency in percentages

Foresters and appraisers at the end of the last century and in the first decades of this century, especially in the Soviet period, investigated and discovered correlations in the distribution of trees by thickness and height, in form coefficients, in overall total of cross-sectional areas in thickness classes, and in wood reserves. The following statement of Professor N. V. Tret'yakov is valid: "If tree stands of a forest element have so regular a distribution of trees by thickness, then this means that the appraisal here will follow an authentic theory" [54]. All these correlations in the distribution of trees by given features relate to a statistical type of correlation, but this does not in any way diminish their importance in science and practical work, since they express those stable relationships intrinsic to the nature of the very phenomena.

A great deal of attention has been given to studying variations in density in relation to age of stands. The gradual decrease in density with age observed in forests was taken as the basis of compiling tables showing the growth pattern of normal stands in which the largest number of trees per hectare is found in young forests, and the smallest -- in the oldest (over-mature) forests. Actually, no precise data on forest density at any forest age have been established, but in tables of the growth pattern of stands the probable, though calculated, density is given by dividing the total cross-sectional areas of the normal stand for an occupancy equal to unity by the cross-sectional area of a single tree, also taken as normal.

Since the density of a forest is associated with tree spacing, then naturally as the forest age is increased tree spacing also rises. However, foresters and appraisers have been mainly engaged in studying variability of forest density, and not tree spacings.

In 1918 G. R. Eytingen investigated the question of what effect forest density has on growth of pine stands [61]. He noted that as pine increases in age from 3 to 18 years the variance of density is reduced from 44 to 23.5 percent. One of the first studies on variability of tree spacing in spruce forests was conducted by A. I. Leskov [38].

A. I. Leskov observed that there is an increase in mean tree spacing with increase in forest age. As a consequence of natural thinning in the forest specific values of l_0 and v are established under which an equilibrium of the tree stand is attained, associated with ensuring minimum feeding area for normal tree growth. Reducing this area leads to the death of some of the trees in the stand. The well-known Russian forest specialist Professor G. F. Morozov observed that for the highest variances v the most intensive thinning of forests is noted. These researchers concluded that tree spacing in stands is an important characteristic of the interplay between trees and the biologically caused forest development.

In 1952 N. P. Anuchin noted that the question of mean tree spacings was still little studied [3].

We have been unable to find any other studies by other authors aimed at discovering statistical correlation of the distribution of tree spacings in a natural forest either in the Soviet or in the foreign literature.

In geobotany Doctor of Geographical Sciences S. V. Viktorov formulated a theory of a geobotanical method of investigation in geology and hydrogeology [19]. He took as the basis of this method plant spacings in non-woody plant communities. He concluded that the mean distance between plants is a quantity summing up complex interrelationships of plant development in a given locality and that it is a highly important characteristic of plant communities. The promise and practical fertility of S. V. Viktorov's ideas was confirmed by the fact that types of plant distance distribution proved to be extremely good indicators of site conditions, presence of specific soils, depth of groundwater, its mineralization, salinity, and freshness, as well as the composition of rocks and geographical structure. Ye. A. Vostokovoa in 1952 employed the ratio of distances between chee grass individuals for indicating groundwater depth and its mineralization. In these studies, graphs of the ratio of distances were constructed in general without mathematical determination of the series and functions of distance distribution.

The author has had occasion to study distances between trees since 1944 in the interests of representing topographic information about forests on state topographic maps.

Since 1945, a representation of mean tree distances has been given on topographic maps. To develop calculation methods of determining traversibility and other properties of forests, we must know the percentage of given distances that we can encounter in moving through forests. In attempting to solve these practical problems it became necessary to provide theoretical validation for methods of determining mean tree spacings, which inevitably entailed arranging for studies on the possibility of establishing a law of distance distribution. Experimental measurements were made of distances in small forest areas for this purpose in 1946-1952. Sample measurements of distances allowed us to discover a certain degree of stability in the frequencies of distribution series and to sketch a hypothesis on the possible type of distance distribution. However, small samplings still did not yield adequate grounds for a final conclusion on the law of distance distribution, though they did point the way to highly interesting conclusions that were subsequently wholly confirmed.

With cooperation of co-workers at the All-Union Amalgamation Lesproyekt B. A. Kozlovskiy, P. A. Sergeyev, and A. F. Kruchinin, field measurements were made of distances between trees in large forest plots with sampling volume up to 700-1,000 distances. Precise maps were drawn on a 1:100 scale for these plots with the placement of each tree, projection of each crown, and so on.

The experimental materials allowed us to check theoretical premises on the function of tree spacing distribution for forests.

Theoretically, rigorous mathematical proof of the distance distribution function is required at the very least in the following stages of research:

verification of the hypothesis that two independent sampling populations belong to the same general population;

investigation of the excess and asymmetry of experimental distribution curves;

verification of the agreement of empirical and theoretical distribution functions with the aid of the appropriate criteria.

10. Checking the Hypothesis that Independent Sampling Distribution Functions Belong to the Same Type

We will consider all distances in large forest tracts as the general population of tree spacings. It is clearly infeasible to measure all distances in all forests of the temperate zone of the European part of the USSR. Therefore the law of the distribution of distances in a general population must be found by a sampling method. If we take at the very least even two large samplings independent of each other for two forest tracts, then the absence or presence of similarity in the distribution series in these samplings will testify either that these distribution series belong to the same distribution type characteristic of the distribution of distances in the general population, or that the series are not identical and, consequently, that the general population does not contain any stable and single type of distance distribution. To put it more simply, essentially the investigation boiled down to the situation that if distribution series in individual sampling tree stands are the same, then we can assume that this same distance distribution will hold for other tree stands. This is tantamount to the situation in which for other similar forests there are no grounds to anticipate any other distribution that differs sharply from the distribution obtained in the independent large samplings.

Large samplings 700-800 distances in volume in the two different forest tracts were obtained by measuring distances between all trees at the locality.

Table 11 and Figure 17 present composite data of an empirical series of distance distributions for the plot No 1, and Table 12 and Figure 18 -- for the plot No 2.

The mean distance l_0 = 4.4 meters, the mean square deviation σ = 1.9, and variance v = 43.2 percent were calculated from the variance series of plot No 1 (Table 11), and from the variance series of plot No 2 (Table 12) -- l_0 = 5.1, σ = 2.1, and v = 41.3 percent.

Now we have all the data we need for analytical verification of the hypothesis that two independent large samplings belong to the same general population.

Let us assume that we have two empirical distribution functions F_1 (l) and F_2 (l) for sampling volumes of n_1 and n_2. For continuous functions and for sampling volumes $n_1 > 50$ and $n_2 > 50$, criteria are developed that allow us to establish the identity or deviation of two independent functions F_1 (l) and F_2 (l). As a measure of the deviation of the two functions, we take the mathematical difference $D_{n_1 n_2}$, that is,

$$D_{n_1, n_2} = \max |\overline{F}_2(l) - \overline{F}_1(l)| \qquad (32)$$

Distribution of 1 in intervals of kl_o

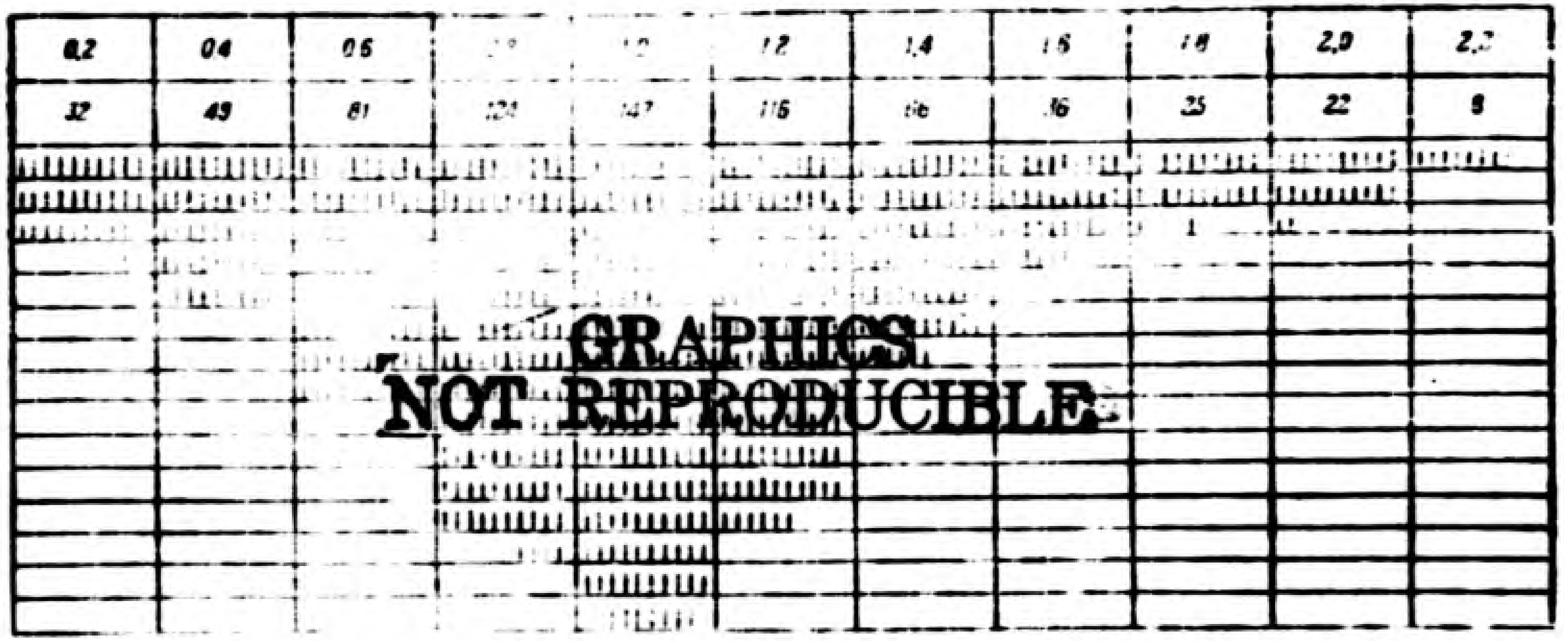

Figure 17. Empirical distribution of distances in plot No 1

Distribution of l in intervals of kl_o

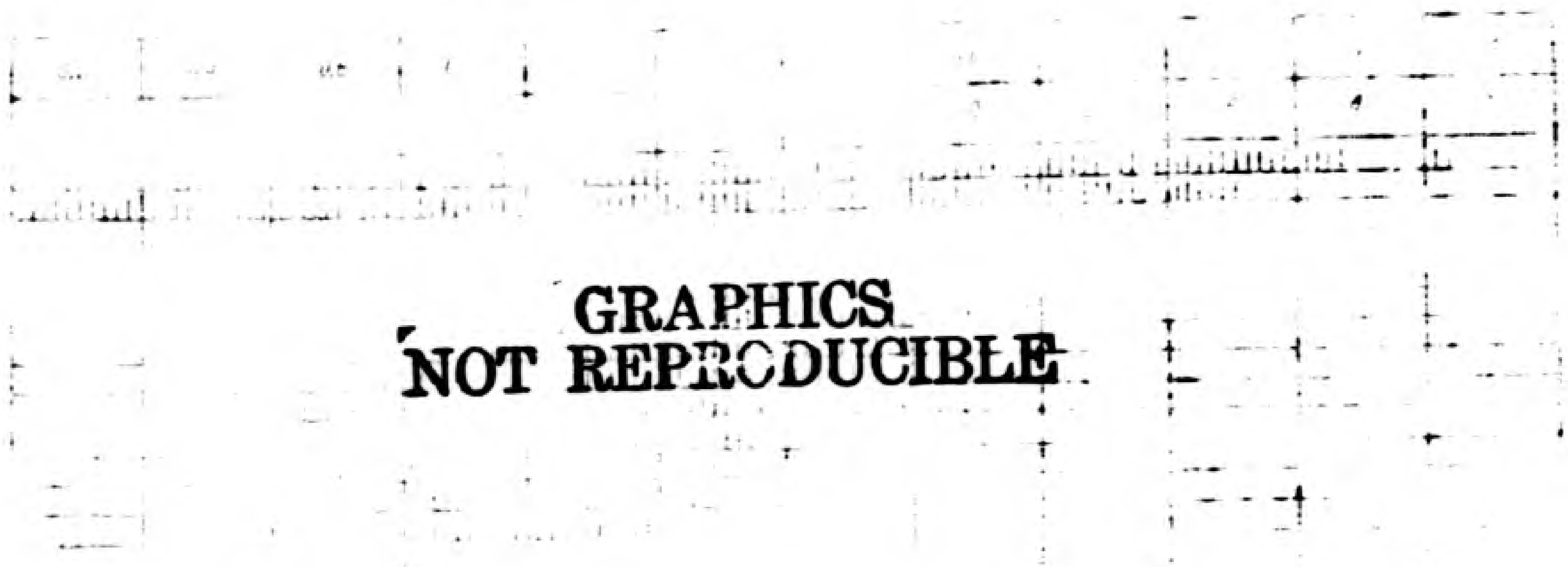

Figure 18. Empirical distribution of distances in plot No 2

Table 11

(A)

(B)

(C)

GRAPHICS
NOT REPRODUCIBLE

(D)

Legend: A -- intervals; B -- interval limits; C -- frequency m; D -- frequency fraction [chastost'] w

If the hypothesis on the identity of the two experimental distributions is valid, that is, $F_1(l) = F_2(l)$ or any l, then the quantity

$$D'_{n_1 n_2} = \sqrt{\frac{n_1+n_2}{n_1 n_2}} \qquad (33)$$

is governed in an approximate way by Kolmogorov's law of distribution $P(\lambda)$ independently of the type of function $F(x)$, that is,

$$P\left(\underset{\substack{n_1\to\infty \\ n_2\to\infty}}{D_{n_1 n_2}} < \lambda\sqrt{\frac{n_1+n_2}{n_1 n_2}}\right) \to P(\lambda) \qquad (34)$$

for any $\lambda > 0$.

If the hypothesis is valid, then

$$D_{n_1 n_2} < \lambda_\alpha\sqrt{\frac{n_1+n_2}{n_1 n_2}} \qquad (35)$$

for a given level of significance $\alpha = 0.05$.

If however

$$D_{n_1 n_2} > \lambda_\alpha\sqrt{\frac{n_1+n_2}{n_1 n_2}} \qquad (36)$$

Table 12

(A)

(B)

GRAPHICS NOT REPRODUCIBLE

(D)

Legend: A -- intervals; B -- interval limits; C -- frequency m; D -- frequency fraction [chastost'] w

then the hypothesis that the functions F_1 (l) and F_2 (l) belong to the same distribution type must be refuted.

Table 13 lists calculations of $D_{n_1 n_2}$ for two empirical distributions F_1 (l) and F_2 (l).

As we can see from Table 13, the maximum value of $D_{n_1 n_2}$ = 0.032.

The law of distribution of P(λ) and the values of were calculated and are given in courses on the theory of probability and mathematical statistics [25, 43].

The book [13] has a table of the values of P(λ) and λ . We take the value of λ for a fully adequate level of significance α = 0.05, which corresponds to the confidence interval p = 0.95. When P(λ) = 0.05, we find λ = 1.358 [13] from the table.

The volumes of our samplings were n_1 = 706 and n_2 = 806. We calculate the following quantity from n_1, n_2, and λ_α :

$$\lambda_\alpha \sqrt{\frac{n_1 + n_2}{n_1 n_2}} = 0{,}070. \qquad (37)$$

Table 13

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0,04[illegible]	[illegible]	[illegible],045		[illegible]
[illegible]	[illegible]	[illegible],114	10	0,[illegible]
[illegible]	[illegible]	[illegible]		0,[illegible]
0,[illegible]	[illegible],180	[illegible],405		0,01[illegible]
[illegible]	[illegible] 183	[illegible],612	[illegible]	0,[illegible]
[illegible]	[illegible]	[illegible]77		[illegible]
0,[illegible]	[illegible],119	[illegible],871		[illegible]
[illegible]	[illegible]	[illegible],922		[illegible]
[illegible]	[illegible]	[illegible]		[illegible]
[illegible]	[illegible]	[illegible]		[illegible]
[illegible]	[illegible]	[illegible]		[illegible]
	0 [illegible]		[illegible]	

Since

$$D_{n_1 n_2} = 0.032 < 0{,}070 = \lambda_\alpha \sqrt{\frac{n_1 + n_2}{n_1 n_2}}.$$

then the hypothesis that the distribution functions F_1 (1) and F_2 (1) belong to the same distribution type has been confirmed. The probability of the hypothesis is very high, equal to the confidence interval $p = 1 - \alpha = 0.95$, and the values of $D_{n_1 n_2}$ obtained from the experimental distribution series lie within the limits of the permissible values 0.07.

Thus, we can with a confidence of 0.95 assert that the distribution of distances in any other analogous forests will be identical F_1 (1) = F_2 (1) = F (1) and sufficiently stable, which evidences that there is a correlation between the investigated series of distance distributions. Naturally, this correlation is plainly statistical in nature owing to the effect of numerous factors that have a bearing on tree placement at the locality and the distribution of tree spacings. The verification made that the two independent experimental distribution series have a common origin is quite essential at the very outset of our investigation. If we had from experiment been convinced that the functions F_1 (1) and F_2 (1) were not identical, then no subsequent investigations would be of use, for we would not have a confirmation of the concept that the sought-for correlation does exist in nature, that is, in the forest. Then the direction of our investigations would have been changed, we would have shifted to a new hypothesis, set up new aims, and followed other approaches. But confirmation of the first hypothesis gives us quite adequate grounds for further investigation to find a specific form of the function of the distribution of tree spacings.

11. Investigation of Asymmetry of Experimental Distribution Functions

The monotypicity of the distribution series established above, F_1 (1) and F_2 (1), thus far does not afford a clear idea of the specific form of distribution function, since it in general can be any of a set of functions. Therefore, now our problem boils down to finding out of the set of probable distribution functions the unique, most precisely reproduced actual distribution of distances l in forests. We can estimate here in an approximate way about what the distribution type is from the nature and value of frequency fractions [chastosti] in empirical distribution series, and more graphically from histograms constructed on the basis of calculated distribution densities. The histograms are suggestive of bell-shaped curves, which affords grounds to presuppose that there is a close relationship between experimental distributions and some form of the normal distribution.

However, to be confident of this, we must conduct an analytical verification of the symmetry or asymmetry of the distribution curves. For this purpose, Table 14 lists calculations of the asymmetry coefficient of the variation series F_1 (1), and Table 15 does this for the series F_2 (1). The asymmetry coefficients were calculated from the following formula:

$$A = \frac{\sum (n\Delta^3)}{\sigma^3 \sum n} \qquad (38)$$

where $\Delta = l_i - l_0$.

The approximate value of the asymmetry coefficient can be calculated from the following expression:

$$A = \frac{l_0 - M_0}{\sigma}$$

where M_0 = mode.

We know that when A = 0, the series is always symmetrical.

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Table 14

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	42,7	[illegible]
[illegible]	[illegible]	-17,7	[illegible]
[illegible]	[illegible]	-5,7	461,7
[illegible]	[illegible]	0,7	[illegible]
[illegible]	[illegible]	0,0	0,0
[illegible]	[illegible]	0,7	[illegible]
[illegible]	[illegible]	5,7	[illegible]
[illegible]	[illegible]	+17,7	[illegible]
[illegible]	[illegible]	+42,7	[illegible]
[illegible]	[illegible]	+85,1	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

From data in Table 14, the asymmetry coefficient for the series F_1 (1) is

$$A_1 = +0.51.$$

that is, the curve of the distribution F_1 (1) has a very small right-handed (positive) asymmetry, which evidences that the mean l_0 lies to the right of the mode M_0. This is also confirmed by the fact that the theoretical mean obtained from formulas (4) and (5) is $l^t_0 = 4.4$, that is, somewhat less than the experimental mean $l_0 = 4.43$. Since $A_1 \approx 0$, then this distribution series can be assumed symmetrical.

From Table 15, the asymmetry coefficient of the series F_2 (1) is

that is, the series F_2 (1) also has a small right-sided (positive) asymmetry, which once more evidences that there is monotypicity of the investigated experimental distribution series.

Table 15

№	t_i	m	$t_i - t_0$	[illegible]	[illegible]	[illegible]
1	1,1	19	−4	16	[illegible]	[illegible]
2	2,1	70	−3	9	−27	[illegible]
3	3,1	106	−2	4	−8	[illegible]
4	4,1	145	−1	1	−1	[illegible]
5	5,1	[illegible]	0	0	0	[illegible]
6	6,1	113	+1	1	+1	[illegible]
7	7,1	[illegible]	+2	4	+8	[illegible]
8	8,1	[illegible]	+3	9	+27	[illegible]
9	9,1	[illegible]	+4	16	+64	[illegible]
10	10,1	[illegible]	+5	25	[illegible]	+2000
11	11,1	[illegible]	+6	36	+216	[illegible]
12	12,1	[illegible]	+7	49	+343	[illegible]
		80[illegible]				409[illegible] +78[illegible]

$\sigma = 2,[illegible]; \quad \sigma^2 = 4,41; \quad \sigma^3 = 9,3$

$$A_2 = \frac{\sum (mx^3)}{\sigma^3 \sum m} = + \frac{5744}{7495} = [illegible],49;$$

$$A_1 = \frac{\bar{t} - M_0}{\sigma} = \frac{5,1 - 5,0}{2,1} \approx [illegible],474.$$

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Since $A_1 \approx 0$ and $A_2 \approx 0$, we can confidently assert that there is no sharply pronounced asymmetry in the experimental series F_1 (1) and F_2 (1). This supports grounds to conclude that these series are not related to the distribution type that differs sharply from the normal distribution in which $A = 0$.

12. Investigation of Excess of Experimental Distribution Functions

To discover the type of the distribution curve it is important to know not only the absence of symmetry, but also the marked positive or negative excess, since the curve can be sharply skewed upward or by the canopy (low-lying). Verification of the presence or absence of a sharply positive or negative excess in two distribution series is necessary also to judge the similarity or difference in frequency fractions of two experimental functions F_1 (1) and F_2 (1). When there is a decided difference in the frequency fractions of equivalent intervals in the distribution F_1 (1) and F_2 (1), their inclusion in a single distribution type F (1) is in doubt.

The excess is calculated from the following formula:

$$K = \frac{\sum(\Delta^4 m)}{\sigma^4 \sum m} - 3, \qquad (39)$$

where $\Delta = l_i - l_0$.
At the central moments

$$K = \frac{\mu_4}{\mu_2^2} - 3. \qquad (40)$$

We know that in the normal distribution the excess K = 0.

Table 16 presents calculations of the excess K_1 of the experimental distribution curve F_1 (1), and Table 17 -- the excess K_2 for the curve F_2 (1). It is clear from these tables and the calculations made below that $K_1 = +0.06$, and $K_2 = -0.07$.

The excesses K_1 and K_2 are very small and, which is most remarkable, have different signs. The slight value of the excesses and their different signs evidence that both experimental distributions F_1 (1) and F_2 (1) are very similar to each other, and their small deviations upwards and downwards from some mean distribution curve are random in nature. Since in the normal distribution the excess K = 0, and in the investigated experimental distributions the excess is close to zero, then we can assume that along with the earlier noted symmetry the empirical distributions F_1 (1) and F_2 (1) refer to one of the forms of the normal distribution type.

However, for a final conclusion it is necessary to make an analytical verification of this assumption by using more powerful agreement criteria. We can use the chi^2 criterion (K. Pearson) and the criterion of Academician A. N. Kolmogorov as such criteria. But this requires that we first find the theoretical function of the distribution of distances F_1 (w).

13. Determination of the Theoretical Function of the Distance Distribution

Discovering and appraising the distribution law from sampling data represents one of the main scientific problems in mathematical statistics [25]. In those cases when it is found that the theoretical distribution law appertains to a specific family or type of distributions, then the problem of finding the distribution law of the quantity under investigation boils down to finding unknown parameters of the given type of distribution function. It is precisely with the aim of determining these parameters that in most cases the statistical study itself is conducted. If precise parametric values are known, the distribution law for the given quantity is determined fully. In our case, out of the set of possible forms of normal distribution functions it is necessary to find a unique function. This is possible only when we know the numerical expressions of the parameters of the sought-for function.

The theoretical distribution function F_1 (w) is usually calculated from experimental data using the function of the normalized normal distribution curve

$$Z_t = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \qquad (41)$$

where $t = \frac{l_i - l_0}{\sigma}$.

The values of Z_t are taken from special tables given in courses on mathematical statistics [43].

The book [13] contains a table of the values of Z_t for t from 0 to 3.59. Ordinates of the theoretical distribution curve are calculated from the formula:

$$F_1(w) = \frac{\Delta l}{\sigma} Z_t$$

where Δ l = interval of l_i values.

Table 18 lists calculations of the theoretical distribution function F_1 (w_1) from the data of the experimental distribution F_1 (l), and Table 19 -- calculations of the function F_1 (w_2) for the series F_2 (l).

Table 16

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Table 17

Table 18

[illegible]

$l_0 = 4,4$; $\sigma =$ [illegible]; $\Delta l =$ [illegible]; [illegible]

Table 19

l_i	w_i	$i - i_0$	$t = \frac{l_i - l_0}{\sigma}$	z_i	$F_i(w_i)$
1,1	0,023	—4	1,906	0,0651	0,0[illegible]6
2,1	0,067	—3	1,427	0,1442	0,070
3,[illegible]	0,132	—2	0,950	0,2541	0,121
4,[illegible]	0,180	—1	0,476	0,3565	0,170
5,1	0,183	—0	0,000	0,3989	0,191
6,1	0,140	1	0,476	0,3565	0,169
7,1	0,119	2	0,950	0,2441	0,121
8,1	0,074	3	1,427	0,1442	0,070
9,1	0,035	4	1,905	0,0651	0,036
10,1	0,020	5	2,380	0,0235	0,011
11,1	0,005	6	2,860	0,0067	0,004
12,1	0,002	7	3,340	0,0015	0,001

$l_0 = 5,1$; $\sigma = 2,1$; $\Delta l = 1,0$; $\frac{\Delta l}{\sigma} = 0,476$.

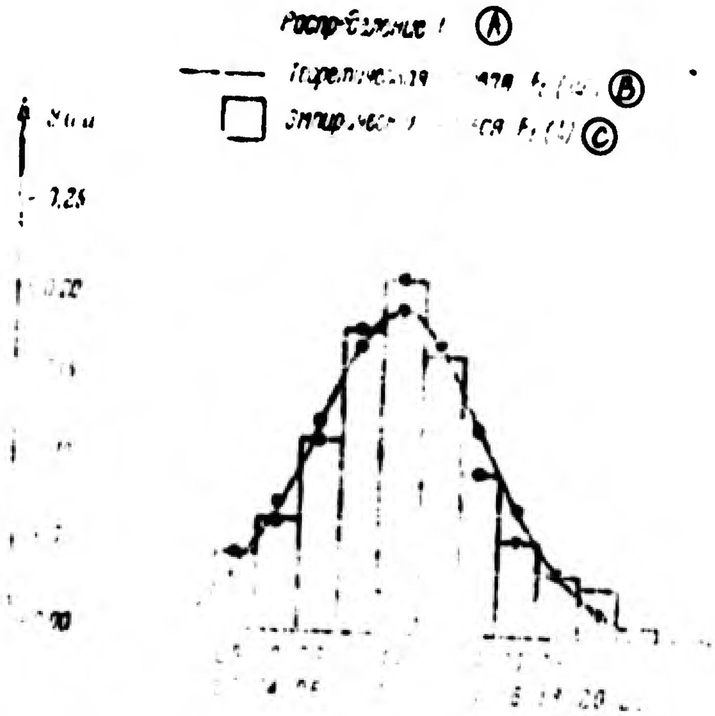

Figure 19. Theoretical and empirical distribution curves for tree spacings in plot No 1. Legend: A -- distribution t; B -- theoretical curve F_1 (w_1); C -- empirical curve F_1 (1).

Figure 19 shows the theoretical distribution curve F_1 (w_1) and the empirical distribution curve F_1 (1).

Figure 20 shows the theoretical distribution curve F_1 (w_2) and the empirical curve F_2 (1).

Comparison of the frequency fractions of the empirical and theoretical distributions (in Tables 18 and 19), and also their curves in Figures 19 and 20 confirm that they are very close in agreement, and this evidences that there is identity between the distribution functions.

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Figure 20. Theoretical and empirical curves of the distribution of distances between trees in plot No 2

It is also important to note the fact that theoretical functions $F_1 (w_1)$ and $F_1 (w_2)$ for two independent empirical distribution series $F_1 (l)$ and $F_2 (l)$ are very similar to each other in values of frequency fractions in equivalent intervals, which confirms the stability of the distance distribution. However, simple comparison of curves and frequency fractions of four distribution series is not enough to corroborate the normality of the distance distribution. This requires an analytical verification of agreement between theoretical and empirical distribution functions using the appropriate criteria.

14. Verification of Agreement Between Empirical and Theoretical Distribution Functions Using the chi^2 Criterion

The chi^2 criterion is based on comparison of the frequencies of experimental and theoretical distribution series. Verification of the hypothesis of the distribution law using the K. Pearson criterion has several limitations, where interpretation of the verification results is not free of arbitrariness. The criterion is used for series that have large enough sampling volume and a sufficient value of frequencies in the extreme intervals of the distribution series, where the number of intervals in the series must not be less than five. The criterion can lead to erroneous conclusions if in the intervals at the ends of the series the frequencies prove to be very small. For example, when taking into account in the F_1 (1) series the two last intervals, we get $chi^2 = 34.72$. For this chi^2 value we can without adequate grounds conclude that there is disparity between experimental and theoretical distributions. In this case the lack of agreement more properly refers to the extreme intervals containing low frequencies, and agreement can prove to be very good in all of the rest of the main part of the series. Owing to these characteristics in the use of the criterion, it is recommended that the extreme intervals be combined into the single following interval.

To appraise observed and theoretical distributions, the criterion is expressed as follows:

$$\chi^2 = \sum \frac{[m - F(m)]^2}{F(m)} \quad (42)$$

where:

m = frequency of empirical series;
F(m) = frequency of theoretical distribution series.

In calculating F(m) tables of the normalized Laplace function Φ (t) are usually used. In our case we can use the previously calculated data of the theoretical distribution functions F_1 (w_1) and F_1 (w_2) and obtain theoretical frequencies from the expression F(m) = wM.

The rule of agreement verification consists in comparing the calculated chi^2 with the value of chi^2_q determined from Table 8 in the book [28] for the suitably chosen significance level α and the number of degrees of freedom k. The number of degrees of freedom k = i - c - 1, where i = number of intervals in the series, and c = number of parameters to be estimated in calculating F(w). If $chi^2 \leq chi^2_q$, then the deviation between the sampling and the presumed theoretical distribution is held not to be substantial and the hypothesis under verification is taken as valid. When $chi^2 > chi^2_q$, the proposed hypothesis is refuted.

We must note that the criterion chi^2 requires in each specific case careful selection of the significance level and the permissible values of chi^2_q corresponding to it. Essentially, the significance level (α = 2 percent, α = 1 percent, α = 0.1 percent, and α = 0.3 percent) is that the events corresponding to it are in practical terms regarded as impossible owing to their low probability ($p = 0.02$, $p = 0.01$, $p = 0.001$, and $p = 0.003$).

If we take the significance level $\alpha/100$, then we assume that the probability of the criterion chi^2 falling within the region of permissible values of chi^2_q is $1 - \alpha/100$. Therefore, the lower the significance level, the smaller is the probability of refuting the valid hypothesis. However, a low significance level affords grounds for the assertion that the calculated values of the criterion chi^2 do not contradict the hypothesis being tested, but only barely. But as the significance level is reduced the sensitivity of the criterion drops off.

In our case the problem is somewhat simplified since we wish to verify only the non-contradictoryness of experimental F_1 (l) and F_2 (l) with the theoretical functions F_1 (w_1) and F_1 (w_2) calculated earlier and qualitatively already verified as to several other criteria of distribution normality. Here we are interested in the significance level guaranteeing against rejection of the valid distribution hypothesis. If we take $\alpha = 2$ percent or $\alpha = 0.3$ percent, then we can assume that the probability that chi^2 exceeds the permissible value chi^2_q will be extremely small, that is, 0.02 or 0.003. This low probability affords grounds for regarding this event as practically impossible. The probability, in fact, that chi^2 falls within the region of permissible chi^2_q values is very great, equal to 0.98 or 0.997, that is, close to unity, which is tantamount to almost total certainty. Only in 2 percent or in 0.3 percent of all cases can we anticipate discrepancies between the observed facts and the adopted distribution hypothesis. It is clear from Table 20 that chi$^2_2 = 14.45$, and $k = i - 3 = 6$. For a significance level $\alpha = 2$ percent and $k = 6$, the permissible chi$^2_{q_2} = 15.0$. Since chi$^2_2 = 14.45 < 15.0 =$ chi$^2_{q_2}$, then we can assume that the experimental distribution of F_2 (l) does not contradict the theoretical F_1 (w_2).

For F_1 (l) and F_1 (w_1), Table 21 gives chi$^2_1 = 19.586$, and $k = 6$. The non-contradictoryness of the function F_1 (w_1) to the experimental distribution F_1 (l) corresponds to the significance level $\alpha = 0.3$ percent, since when $k = 6$ and $P(\chi^2 > \chi^2_q) = 0.003$, the value of chi$^2_{q_1} = 20.0$.

Table 20 gives calculations of the criterion chi$^2_2$ for the functions F_2 (l) and F_1 (w_2), and Table 21 -- for the

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Table 20

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	−10	100	[illegible]
[illegible]	[illegible]	[illegible]	11	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0,01
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	9	81	[illegible]
[illegible]		[illegible]			$\chi_2^2 =$ [illegible]

criterion chi^2_1 for the distribution functions F_1 (1) and F_1 (w_1).

Though the criterion chi^2 is often used, it still exhibits several drawbacks and points of arbitrariness in interpreting the results of verifying agreement between distributions. Therefore, it is always worth while to to check agreement between distributions by using other criteria. The criterion of Academician A. N. Kolmogorov is recognized as a more powerful criterion.

In the interests of theoretical rigor, we will verify agreement between distribution functions by using this criterion.

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Table 21

15. Verifying Agreement Between Empirical and Theoretical Distribution Functions by Using the A. N. Kolmogorov Criterion

The agreement criterion of Academician A. N. Kolmogorov can be used in appraising sampling distribution functions also in those cases when the law of distribution or the form of the function describing the distribution of the quantity in the general population is not known to us.

Only one continuity condition is imposed on the function. Use of the criterion also presupposes that the empirical function is formulated from quantities that are non-grouped in the intervals.

However, with some degree of approximation the confidence appraisal of the function F(1) will operative also in the case when the intervals of the distribution series will be small enough. In our case all these conditions are met. The intervals of 1 are taken very small, only 0.2 l_0, such that when l_0 = 5 this yields 0.5 meter. There is practically no necessity to take smaller-sized intervals, since tree spacings of 0.5 meter are encountered very infrequently and are not of importance from the practical and theoretical points of view.

The criterion of A. N. Kolmogorov is founded on a comparison of the accumulated frequency fractions of the empirical distribution series with the data of the integral function of the theoretical distribution by determining the maximum deviation between them and its estimate with respect to the distribution function $P(\lambda)$.

Let us denote the function of cumulative frequency fractions of the empirical distribution w^{ac}_{emp}, and the integral function of the proposed theoretical distribution w^{ac}_{theor}.

Then, for large samplings the probability $P(\lambda)$ is such that the maximum deviation of the frequency fractions $w^{ac}_{emp} - w^{ac}_{theor}$ exceeds a specified number $\lambda/\sqrt{n}$ can be determined in an approximate way from the function

$$P(\lambda) = 1 - \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2\lambda^2} \qquad (43)$$

The probabilities $P(\lambda)$ for different λ values have been calculated by Professor N. V. Smirnov from formula (43), and they can be found in courses on mathematical statistics. They are also in the book [13].

The maximum difference in the cumulative frequency fractions is denoted by

$$D = w^{ac}_{1} - w^{ac}_{2}. \qquad (44)$$

Legend: 1 -- w^{ac}_{emp}; 2 -- w^{ac}_{theor}

The value of λ in these calculations is determined from the expression

$$\lambda = D\sqrt{n} \qquad (45)$$

where n = scope of sampling.

We will write out the probability $P(\lambda)$ from a calculated λ from a special table. If $P(\lambda)$ proves to be very small, that is, smaller than 0.01 or 0.05, then the proposed theoretical distribution is not in agreement with the empirical distribution. When $P(\lambda)$ is greater than 0.01 (0.05) the agreement between the distributions under study is deemed to be corroborated.

Table 22 lists calculations of empirical and theoretical cumulative frequency fractions for determination of the maximum value of D in the distribution series F_2 (l) and F_1 (w_2).

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Table 22

i	l	(1)	(2)	(3) w_n	(4)	(5)
1	0,2	[illegible]	[illegible]	0,036	[illegible]	[illegible]0,013
2	0,4	[illegible]	[illegible]	0,070	[illegible]	[illegible]0,001
3	0,6	[illegible]	[illegible]	0,121	[illegible]	0,015
4	0,8	[illegible]	[illegible]	0,170	0,307	[illegible]
5	1,0	[illegible]	[illegible]	0,197	[illegible]	+0,017
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-0,012
7	1,4	[illegible]	[illegible]	0,121	[illegible]	-0,011
8	1,6	[illegible]	[illegible]	[illegible]	[illegible]	-0,010
9	1,8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]0,011
10	2,0	[illegible]	[illegible]	[illegible]	[illegible]	-0,002
11	[illegible]	[illegible]	[illegible]	0,004	[illegible]	-0,001
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Legend: 1 -- w_{emp}; 2 -- w^{ac}_{emp}; 3 -- w_{theor}; 4 -- w^{ac}_{theor}; 5 -- $w^{ac}_{emp} - w^{ac}_{theor}$

We can plainly see from Table 22 that the maximum $D = w^{ac}_{emp} - w^{ac}_{theor} = 0.025$. The scope of sampling $n = 806$. Then

For $\lambda = 0.71$ from the table of $P(\lambda)$ values we find $P_2(\lambda) = 0.69$. Since $P_2(\lambda) = 0.69 > 0.005$, then we can assert that the distribution of distances between trees agrees well with the theoretical normal distribution calculated for the appropriate values of the parameters l_0, σ, Δl.

An appraisal of agreement can be made using A. N. Kolmogorov's criterion also by stipulating the criterion of significance q or probability $p = 1 - q$. Let us assume that we take as a measure of confidence $p = 0.95$, which corresponds to the significance level $q = 0.05$, or 5 percent. When $q = 0.5$ the value $\lambda_q = 1.358$ (cf tables for $P(\lambda)$ and λ in the book [13]). Then the range of acceptable values will be:

$$D_q = \frac{\lambda_q}{\sqrt{n}} = \frac{1,358}{\sqrt{806}} = 0,048.$$

The value $D_q = 0.048$ is the region of permissible deviations for a 5-percent confidence probability. If the calculated D $= w^{ac}_{emp} - w^{ac}_{theor}$ proves to be less than the stipulated $D_q = \lambda_q/\sqrt{n}$, that is, will be found within the limits of the zone of permissible deviations D_q, then agreement between empirical and theoretical distributions is held quite good and trustworthy.

In our case $D = 0.025 < 0.048 = D_q$, therefore agreement between the distribution functions F_2 (1) and F_1 (w_2) is confirmed.

The A. N. Kolmogorov criterion can also be employed when using cumulative frequencies m of empirical and theoretical distributions.

In Table 23 the original data for the use of this criterion are calculated not from frequency fractions, but from the frequencies of the distribution series F_1 (1) and F_1 (w_1).

Table 23

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1	2	3	4	5	6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	81	[illegible]	[illegible]	76	[illegible]
[illegible]	16	[illegible]	[illegible]	161	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	280	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Legend: 1 -- m_{emp}; 2 -- m^{ac}_{emp}; 3 -- w_{theor}; 4 -- m_{theor}; 5 -- m^{ac}_{theor}; 6 -- $m^{ac}_{emp} - m^{ac}_{theor}$

It is clear from Table 23 that $D_{max} = m^{ac}_{emp} - m^{ac}_{tree} = 19$. In this case D is expressed in percentages, but to calcalculate λ it is necessary to express D in frequency fractions, that is, to divide D by the sampling scope n.

Then $\lambda = D\sqrt{n}/n = D/\sqrt{n}$.

The sampling size $n_1 = 706$. The calculated

$$\lambda_1 = \frac{19}{\sqrt{706}} = 0.713.$$

We find $P_1(\lambda) = 0.681$ from the table of $P(\lambda)$ values for $\lambda_1 = 0.713$. Since $P_1(\lambda) > 0.05$, then we must take the agreement between the second pair of empirical and theoretical distributions as good.

In this way the A. N. Kolmogorov criterion also confirms the agreement between empirical and theoretical distributions.

16. Conclusions on the Statistical Correlation of Tree Spacing Distribution in Tree Stands

This examination of empirical of empirical and theoretical distributions and the derivation of the function of theoretical distribution of spacings confirms the identity of the distribution series and their stability, which affords adequate grounds to speak about the existence of a natural correlation of tree spacing distribution in forests. We have been convinced that the empirical functions of two distribution series are identical to each other, that is, $F_1(l) = F_2(l)$. It has also proven to be the case that theoretical distribution functions agree well with empirical functions and with each other, that is, $F_1(w_1) = F_1(w_2) = F_1(l) = F_2(l)$. Small variations in frequency fractions of empirical distribution series about the theoretical distribution function are wholly expected for any statistical correlation, all the more so for sampling distribution series.

In the general population, the distribution of spacings will ultimately coincide with the theoretical distribution function $F(w)$ given the very essentials of investigating mass random phenomena.

Table 24 lists values of frequency fractions expressed in percentages for each interval of all four distribution series, that is, two empirical $F_1(l)$ and $F_2(l)$ and the two theoretical series $F_1(w_1)$ and $F_1(w_2)$ corresponding to these, and also the empirical $F_{samp}(l)$ obtained on the basis of small sampling (Table 10).

It is clear from Table 24 that even $F_{samp}(l)$ obtained from small sampling has good agreement with the rest of the distribution series for the main intervals.

Figure 21 shows the integral curve of the normalized normal distribution theoretical distribution $F_1(w_1)$ and the empirical distribution curves $F_1(l)$ and $F_2(l)$ constructed from cumulative frequency fractions. In this figure, the good agreement between all distribution curves is particularly graphic and distinctly evident.

Since the theoretical distribution functions $F_1(w_1)$ and $F_1(w_2)$ give identical distribution series, then we can select as the single function of distance distribution either of these two functions. Therefore, the function of the law of tree spacing distribution in a natural forest can in general form be written as follows:

$$F(l) = \frac{\Delta l}{\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{(l-l_0)^2}{2\sigma^2}}. \qquad (46)$$

After substituting numerical values for all parameters ($l_0 = 4.4$; $\sigma = 1.9$; $\Delta l = 0.9$), the final form of the distribution distribution function will be:

$$F(l) = \frac{0.473}{\sqrt{2\pi}} e^{-\frac{(l-4.4)^2}{7.22}}. \qquad (47)$$

The statistical function (47) actually acquires a single-valued appearance, since it has only a single variable l, that is, tree spacing. We can, of course, in actual practice form a unique distribution series by adding two theoretical distribution series, but their frequency fractions in each interval are so close that obtaining mean frequency fraction values would lead to their variation by only 0.1 or 0.2 percent. Therefore, we can altogether limit ourselves to the distribution series $F_1(w_1)$.

Since the function of the law of distance [tree spacing] distribution that we have found is one of the forms of the normal law of distribution, it is obvious that it retains all qualities of the distribution of l relative to σ and v.

Table 24

	1	2	3	4	5	6	7	8	9	10	11	12
k (l_0)	0,2	0,4	0,6	0,8	1,0	1,2	1,4	1,6	1,8	2,0	2,2	2,4
F_l (w_1)	3,5	7,3	12,0	16,9	19,0	16,9	12,0	7,3	3,5	1,3	0,3	
F_l (w_2)	3,5	7,0	12,1	17,0	19,1	16,9	12,1	7,0	3,6	1,1	0,4	0,1
F_1 (l)	4,5	6,9	11,5	17,6	20,8	16,4	9,4	5,1	3,6	3,1	1,1	
F_2 (l)	2,3	8,7	13,2	18,0	18,3	14,0	11,9	7,4	3,5	2,0	0,5	0,2
[illegible] (l)	4,9	4,9	16,4	18,1	19,7	13,1	11,5	6,5	3,3	1,6		

GRAPHICS NOT REPRODUCIBLE

Table 25

$l_0 \pm k\sigma$	l	P
$l_0 \pm 1\sigma$	$l_0(1 \pm$ [illegible]$)$	0,[illegible]3
$l_0 \pm 2\sigma$	$l_0(1 \pm$ [illegible]$)$	0,[illegible]4
$l_0 \pm 3\sigma$	$l_0(1 \pm$ [illegible]$)$	0,[illegible]7

Accordingly, the distribution of l for known σ and v retains the ratios shown in Table 25.

However, the distribution series in intervals of the mean tree spacing is more convenient, since to discover the percentage of given distances in a forest, it suffices to determine only the mean distance, then to use the ratios in Table 25 it is further necessary to determine or v, but this is too laborious, while in this case also we get from Table 25 vastly less detailed information on the percentage ratio of distances than from the distribution series of $F_1(w_1)$.

In conclusion, we note that the distribution series $F_1(w)$ will be valid for mature stands growing under ordinary, normal conditions and with variance $v = 40$ percent. There are as yet no grounds to assert definitively that this series will be retained with precision in young stands as well as in forests with habitat conditions sharply differing from ordinary (soil-climatic and geographic), since we do not have available to us large enough amounts of experimental data. Therefore, it is required to conduct additional experimental studies for these forests; these investigations will afford a refinement of the parameters of the statistical function of tree spacing distribution.

Since intervals of the distribution spacing series are taken independently of specific values of the mean spacing, then by setting any mean l_o we can calculate the distribution for any spacings l, which in fact has been done in Table 26 for mean spacings from 2 to 15 meters.

From Table 26 it is no trouble to determine the percentage of any spacings in the forest if we know l_o. Let $l_o = 5$ meters, then the distances $l = 8$ meters in the given forest will be only 7.3 percent of the total number of all spacings.

It is clear from the distribution series that 49.2 percent of all spacings are less than l_o, and 50.8 percent -- greater than l_o, that is, all spacings in the forest will be divided into two equal parts.

Distances equal to $1.2\ l_o$ and smaller prove to amount to about 75 percent, which is of weighty practical importance for estimating traversibility of the forest, since we can with a probability close to unity assert that in 75 cases out of 100 we will encounter precisely such distances in moving through the given forest.

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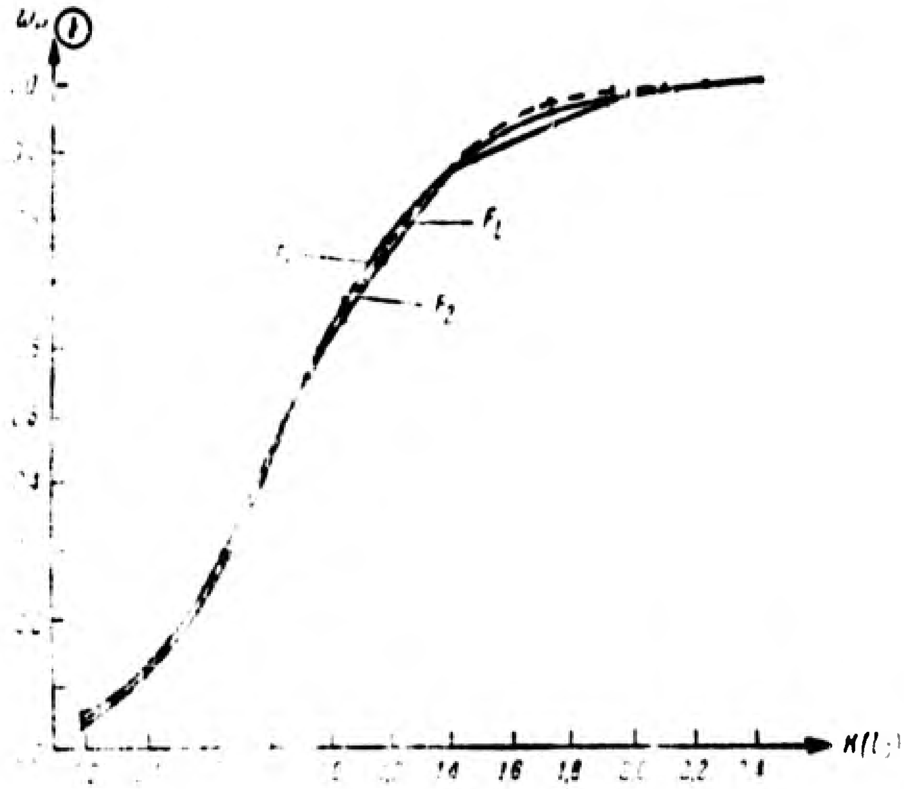

Figure 21. Integral curves of tree spacing distribution. F_1 and F_2 = empirical, F = theoretical.
Legend: 1 -- w_{acc}

Thus, for example, in a forest tract with mean l_0 = 6 meters, in 75 out of 100 cases we will encounter tree spacings equal to 7.2 meters and less, which gives a good idea about the traversibility of the particular forest.

Table 26, as was shown in section 38, is in need of refinement for small and large tree spacings.

The correlation of tree spacing distribution affords the possibility of solving many practical problems.

The scientific importance of discovering the statistical correlation of tree spacing distribution in natural forests needs no comment, for discovering any law of nature

GRAPHICS
NOT REPRODUCIBLE

Table 26

	f % / a / №	3,5 0,2 1	7,4 0,4 2	[illegible] 0,6 3	16,9 0,8 4	19,0 1,0 5	[illegible] 1,2 6	[illegible]	[illegible] 1,6 8	[illegible]	[illegible]	0,3 [illegible] 11
		0,4	0,3	1,2	1,6	2,0	2,1	[illegible]	3,2	3,6	4,0	[illegible]
		0,6	1,2	[illegible]	2,4	3,0	3,6	[illegible]	4,8	5,4	6,0	[illegible]
		0,8	1,6	2,4	3,2	4,0	4,8	[illegible]	6,4	[illegible]	[illegible]	8,8
		1,0	2,0	[illegible]	4,0	5,0	6,0	7,0	8,0	9,0	10,0	11,0
		1,2	2,1	[illegible]	4,8	6,0	7,2	[illegible]	9,6	[illegible]	12,0	13,2
		1,6	3,2	[illegible]	6,4	8,0	9,6	[illegible]	12,8	[illegible]	16,0	17,6
10		2,0	4,0	[illegible]	8,0	10,0	12,0	14,0	16,0	[illegible]	20,0	22,0
		2,4	4,8	[illegible]	9,6	12,0	14,4	[illegible]	19,0	[illegible]	[illegible]	26,2
		3,0	6,0	9,0	12,0	15,0	18,0	[illegible]	24,0	27,0	30,0	33,0

introduces a definite contribution to understanding correlations of our material world. Today a new correlation has been established in forestry (forest appraisal) and topography along with earlier discovered correlations of the distribution of trees by thickness, height, and other features. This once again confirms the principles of materialist dialectics on the universal interrelatedness of phenomena and the accessibility of the world to our cognition.

In topography, cartography, and forest appraisal the law of distance distribution affords a stable scientific and theoretical basis for practical methods of determining mean distances in forests and from aerial photographs.

In forest appraisal the correlation of distance distribution, along with solving purely scientific problems with respect to study of the forest as a plant community, can be employed also in arriving at methods of determining timber stand reserves. However, it appears worth while to conduct experimental studies on refining the parameters in the function of tree spacing distribution in several other zones and other forest age classes.

From the scientific and practical points of view the distribution of tree spacings and spacings between any other plants is intimately bound up with how the plants are arranged in the particular locality in their natural development under the effect of the many factors of the site.

Accordingly, several assumptions on the types of tree (plant) location and the probable types of distance distribution corresponding to them are of interest.

Theoretical treatment of this separate research area bears practical importance in elaborating principles for using the distribution curves and their main parameters as indicators of site conditions (for example, soils, groundwater mineralization, etc.). These problems are taken up in Chapter 10.

CHAPTER 4

STATISTICAL CORRELATIONS OF DISTRIBUTION OF TREES IN STANDS BY THICKNESS, HEIGHT, AND CROWN DIAMETERS

17. Correlation of Distribution of Trees by Thickness

Correlation of tree distribution by thickness has been studied by Russian and foreign foresters since the end of the last century.

The German professor Weisse studied the problem of mean tree diameter in forests. He found that the tree that is average in thickness will divide all trees in the forest into two approximately equal parts -- thinner (57.5 percent) and thicker (42.5 percent) than the mean tree. This correlation was then confirmed for almost all tree species.

At the end of the last century the Hungarian professor Fekete investigated the correlation of tree distribution by thickness in spruce stands and drew up a table of the percentage ratio of trees by thickness, starting with the thinnest tree in the forest.

The Austrian forester Schiffel expressed tree diameters in fractions of mean thickness and called these variables reduction numbers. He compiled a table of tree distribution by thickness with an interval of ten percent (from the thinnest to the thickest tree in the forest).

The most extensive theoretical generalization about the correlation of tree distribution by thickness were made by Professor N. V. Tret'yakov [55, 54] and by A. V. Tyurin [57, 58]. A. V. Tyurin examined distribution series relying on data of test stations in Switzerland, Sweden, Germany, Austria, Finland, and Russia for young and old forests. As a result of the experimental distribution series of trees by natural thickness classes that he drew up, Professor A. V. Tyurin concluded that the pattern of distribution of trees by thickness depends neither on the species nor on the site class, but on the stand occupancy and only in part does it depend on forest age, while depending strongly on maintenance fellings [58].

In his study [57], A. V. Tyurin observed that the law of distribution of trees by thickness applies to all forests he investigated within the USSR and within countries of western Europe. He also held that this law in general applies to young, mature, and over-mature forests [57].

In the study [58], A. V. Tyurin presents experimental distribution series for thin-stem, moderate-stem, and thick-stem stands.

A. V. Tyurin constructed a distribution curve from experimental distribution series. The graph of the curve is suggestive of the normal distribution curve. As a result, he concluded that distribution of trees by thickness is expressed by the normal Gauss-Laplace curve. We must, however, note that Professor A. V. Tyurin did not determine the theoretical curve of the function and in his works does not derive equations of this function. Therefore, any conclusion of the similarity of the experimental tree spacing distribution series with the normal distribution is essentially correct, but theoretically has not been proven. As far as we know, the literature on sylviculture and forest appraisal in general does not contain the derivation of the equation of the function of the distribution of trees by thickness. The correlated series of distribution of trees by thickness, validated by Professor A. V. Tyurin, is given in Table 27.

Table 27

k (d°_m)	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7
f %	0.7	3.5	9 5	16,1	18,4	18,1	13,1	8,9	6 3	3,3	1,5	0,5	0,1

It is found that in a uniform stand the highest percentage of trees are of a thickness close to the mean value. Trees thinner than the mean diameter are encountered in the amount of 57.25 percent, and those thicker -- 42.75 percent. The diameter of the thickest tree is 1.75 d^{o}tree, and the thinnest -- 0.5 d^{o}tree.

Further, Professor A. V. Tyurin calculated the sums of cross-sectional areas of trees and obtained the following percentage ratio of cross-sectional areas: trees thicker than the mean tree amounted to 40.35 percent, and those thinner -- 59.65 percent of the total of the cross-sectional area. This ratio is important in determining yield by approximate methods, since the basic yield is made up of trees close to d^{o}tree and larger than d^{o}tree, that is, trees of the main canopy, the projection of the upper part of which is imaged on aerial photographs.

For topographic cartography, we must take cognizance of the distribution of trees by thickness in developing methods of determining mean tree thickness in forests and from aerial photographs, and also in determining correction coefficients in mean tree spacings measured on aerial photographs. Table 28 has been compiled on the basis of the correlation of distribution of trees by thickness. This table allows us to find the percentage of trees of a specified thickness d_{tree} from the known mean thickness d^{o}tree in the given forest tract for d^{o}tree values from 10 to 40 cm at intervals of 2 cm and with a certain amount of smoothing of d_{tree} to whole centimeters.

Let us assume that the mean tree thickness found in the field or from aerial photographs, d^{o}_{m} = 20 cm, and we wish to know the percentage of trees of thickness d_{tree} = 18 cm. It is clear from Table 28 that there will be about 18.4 percent of such trees in the stand.

18. Correlation of the Statistical Functions of Tree Distribution by Thickness and Tree Spacing

Careful inspection of the tree spacing distribution series reveals a closeness between the values of f percent in spacing intervals of 0.2 and in tree thickness intervals of 0.1 in the distribution series of Professor A. V. Tyurin. More detailed study of these two different distribution series revealed that the tree thickness distribution series expressed in fractions of the mean diameter shows a closeness

GRAPHICS NOT REPRODUCIBLE

Table 28

(2) (1)

Legend: 1 -- percentage of trees of a given thickness out of the total number of stems in the forest per hectare; 2 -- d^{o}_{tree}

in value of f percent with the tree spacing distribution series expressed in fractions of the mean spacing. Table 29 lists distribution series in d_{tree} and in 1. By comparing the values of $f_{d_{tree}}$ percent and f_1 percent, the similarity of the series shows up easily.

Table 29

(1)

(2)

Legend: 1 -- $k(d^{o}_{tree})$; 2 -- $f_{d_{tree}}$

The question naturally arises: is this coincidence of numerical values f percent in the tree distribution series by d_{tree} and by 1 a coincidence or a sign that there are natural correlations in the development of a forest as a plant community? Since each distribution series taken separately is orderly, confirmed by experimental data, then it appears probable to assume that the similarity in numerical values of f% is also not random, but a consequence of the manifestation of relationships between the main features of the tree community (by a logically caused process development and the stable existence of this community in nature).

The fact we discovered that there is agreement between values of f percent in intervals of 0.1 for d_m and in intervals of 0.2 for 1 has compelled specialists in forestry and botanists to reflect on the causes of this phenomenon.

The interrelationship of distribution series in d_{tree} and in 1 is of interest from the mathematical point of view. The issue is that if numerical values of f% are close to each other, then it is obvious that there must be a closeness between theoretical functions of these distribution series as well. Therefore, an attempt was made, even though approximate, to find the theoretical function of tree distribution by thickness and to compare it with the tree spacing distribution function.

The literature familiar to the author has not been found to contain data on the statistical distribution function $F(d_{tree})$. There is only a numerically expressed experimental distribution series of d_{tree}. Field data of the forest plot No 1 were used for the approximational determination of the function $F(d_{tree})$. In this plot $d^o{}_{tree} = 29.6 \approx 30$ cm, $h_o = 22.2$ meters, and species composition 6 pine 4 spruce. There were no experimental data about σ, therefore we have to determine σ from the mean variance of thickness for pines $v_{pine} = 20$ percent found by Docent V. I. Levin [3] and for birch -- from N. P. Anuchin [3] $v_{birch} = 28.8$ percent. The textbook on forest appraisal by Professor V. K. Zakharov contains data on this question.

Taking $v_c = 20$ percent, let us allow an error, for the species composition in the plot is mixed -- pine and spruce. But owing to the absence of precise data on v and σ, we use approximational data, since it was important to us to discover only a general pattern of the extent of similarity between theoretical functions F(1) and $F(d_{tree})$. Mean values of the intervals in d_{tree} were taken from Table 28 for $d^o{}_{tree} = 30$. The values of the intervals $k(d^o{}_{tree})$ and the frequency fraction w_i were adopted from the distribution series of Professor A. V. Tyurin.

Given this set of conditions, we get

$$\sigma_{c} = \frac{30 \cdot 10}{100} \approx 6,$$

Legend: 1 -- σ_{pine}

and $\Delta d/\sigma = 3/6 = 0.5$, since the intervals Δd^{i} are given every 3 cm (cf Table 28).

Now we have all the data we need to calculate an approximate theoretical distribution function

$$F(d_m) = \frac{\Delta d}{\sigma\sqrt{2\pi}} e^{-\frac{(d_m - d_m)^2}{2\sigma^2}} \quad (48)$$

Legend: 1 -- d_{tree}; 2 -- $d^2{}_{tree}$

Table 30 lists calculations of the function $F(d_{tree})$.

Table 30

[illegible] d_m	d_m	w_i	$d'_m - d_m$	t	[illegible]	$F(d_m)$
[illegible]	[illegible]	[illegible]	−15	[illegible]	[illegible]175	[illegible]
0.6	[illegible]	[illegible]035	−12	[illegible]	[illegible]0540	0.035[illegible]
[illegible]	[illegible]	[illegible]095	−9	[illegible]	[illegible]1295	0.06[illegible]
[illegible]	[illegible]	[illegible]	−6	1.0	[illegible]2420	0.121[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]521	0.17[illegible]0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.1760
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.06[illegible]
[illegible]	[illegible]	[illegible]	12	2.0	[illegible]540	[illegible]
[illegible]	[illegible]	[illegible]	15	[illegible]	0.017[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	18	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	21	[illegible]	[illegible]000	[illegible]

GRAPHICS NOT REPRODUCIBLE

Legend: 1 -- $d^{0}{}_{tree}$; 2 -- $d^{i}{}_{tree}$; 3 -- d_{tree}

After substituting parameters in formula (48) we obtain the theoretical function of the distribution of trees by thickness

$$F(d_m) \approx \frac{0.5}{\sqrt{2\pi}} e^{-\frac{(d_m - 30)^2}{72}} \quad (49)$$

Legend: m = [subscript] tree

Tables 30 and 26 were used to draw up Table 31 in which the f percent values are listed for theoretical functions F(1) and $F(d_{tree})$.

Table 31

$F(l)$	3,5	7,3	12 0	16 9	19 0	16 9	12 0	7 3	3 5	0 3
$F(d_m)$.	3 5	6 6	12 1	17 6	19 9	17 6	12 1	6 5	3 6	0 9

It is clear from Table 31 that theoretical functions F(1) and $F(d_{tree})$ are extremely close in value, in spite of the fact that $F(d_{tree})$ is determined from approximational distribution parameters. It is obvious that if there are precise parameters of the function of distribution of trees by thickness available, there will be an identical theoretical function of tree spacing distribution, that is, $F(d_{tree})$ F(1). Consequently, the assumption that there is a consistent similarity between the functions $F(d_{tree})$ and F(1) will have some basis. We must hold that further investigations of this important and interesting question will allow us to speak more confidently about similarity and interrelationships of the distribution functions F(1) and $F(d_{tree})$ and the factors underlying this similarity.

The theoretical function of the distribution of the trees by thickness (49) that we defined in Table 30 can serve as the first approximation to the mathematical expression of the experimental distribution series.

19. Correlation of Tree Distribution by Height

The correlation of tree distribution by height established by foresters for homogeneous stands is expressed by an asymmetrical distribution curve. Professors Schiffel, N. V. Tret'yakov, A. V. Tyurin, and M. V. Davidov, Docent V. I. Levin, and others have been engaged in clarifying and exploring this correlation.

Table 32 gives a series of tree distribution by height established by Professor M. V. Davidov.

Table 32

$k\ (h_0)$	0,725	0,819	0,870	0,910	0 945	0 970	1 000	1 020	1 050	1 100	1 140
f %	0	10	20	30	40	50	60	70	80	90	100

The intervals of the series $k(h_0)$ are expressed in fractions of mean height. In forest appraisal these intervals are called reduction numbers for height and are arrived at by dividing given heights by the average stand height. Setting any mean height values, we can obtain the values of all heights encountered in the given forest section by multiplying $k(h)$ by the given h_0.

In contrast to the series of tree distribution by height, M. V. Davidov gives the number of trees of a given height out of the total number of trees in the forest plot not for each interval $k(h_0)$ considered separately, but in the form of cumulative frequencies expressed then in percentages f percent.

Starting with the lowest trees to the highest, for each gradation of the series the percentage of trees is summed up and this overall percentage f percent is then listed in Table 32. Thus, for example, trees that are h_0 in height and shorter will represent 60 percent of all trees in the plot, but trees that have $h = 0.87\ h_0$ and smaller will account for only 20 percent. All trees from the lowest to the highest total 100 percent. Starting from the distribution series, the highest tree in the stand has $h_{max} \approx 1.15\ h_0$, that is, 15 percent above the mean tree height; and the shortest tree has $h_{min} \approx 0.7\ h_0$, that is, 30 percent less than the mean tree height.

The correlation of tree distribution by height has quite a bearing on the proper approach to determining average heights in forests and especially from aerial photographs in which it is primarily the upper section of the forest canopy that shows up. For these same purposes it is important to know the variance of tree heights. Based on data of Professor V. K. Zakharov [3] and Professor A. K. Kondrat'yev, the variance of the heights of pine forests v_{pine} = 6-8 percent, and the variance of the heights of birch stands, from data of Professor N. P. Anuchin [3], is v_{birch} = 8-10 percent. A relatively small variance of height allows us to determine the mean forest height from aerial photographs with adequate precision. However, we must always bear in mind that the earlier listed v values relate to homogeneous stands. Obviously, in non-homogeneous and mixed stands variance of height will be somewhat higher than 10 percent (v will increase in strongly pronounced double-story stands).

To solve practical problems it is important to know the distribution of heights for these different mean height values. For this purpose Table 33 has been drawn up, from which one can determine the percentage of trees of a given height for a known mean height h_0 of the forest plot.

Table 33

h	(1) [illegible] процен[illegible] [illegible]вев данной высоты от общего [illegible] стволов на участке леса										
	[illegible]	[illegible]	20	[illegible]	40	50	[illegible]	70	[illegible]	90	100
4	[illegible]	[illegible]	3,5	[illegible]	3,7	[illegible]	4	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	7,0	[illegible]	7,5	[illegible]	8	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	9,1	9,7	10	[illegible]	[illegible]	11,0	[illegible]
12	[illegible]	[illegible]	10,4	[illegible]	11,3	11,6	12	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	[illegible]	12,2	[illegible]	[illegible]	13,6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
16	[illegible]	[illegible]	13,9	[illegible]	15,0	15,5	16	[illegible]	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	15,7	[illegible]	16,9	17,5	18	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	17,4	[illegible]	18,8	19,4	20	[illegible]	[illegible]	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	23,2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	22,6	[illegible]	24,1	25,2	26	[illegible]	[illegible]	[illegible]	[illegible]
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	27,2	28	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	29,1	30	[illegible]	[illegible]	[illegible]	[illegible]
32	[illegible]	[illegible]	[illegible]	[illegible]	30,0	31,1	32	[illegible]	[illegible]	[illegible]	[illegible]
34	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	33,0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	35,0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
38	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

GRAPHICS NOT REPRODUCIBLE

Legend: 1 -- overall percentage of trees of a given height of the total number of stems in the forest plot

Scientifically, in our view it is necessary to conduct studies on determining the theoretical function of tree distribution by height and its correlation with theoretical functions of tree distribution by thickness and tree spacing.

If in approximate terms we take the normal distribution as the theoretical function of tree distribution by height, then we can obtain approximate relationships between h_{max}, h_0, σ_h, and v_h that are of practical importance for the sampling method of determining appraisal indexes in the forest and from aerial photographs. Given these assumptions

$$h_0 \approx h_{max} - 3\sigma_h \text{ или (1) } h_0 \approx h_{max} - 3{,}5\sigma_h. \quad (50)$$

Legend: 1 -- or

From the empirical distribution series

$$h_{max} \approx 1{,}15h_0 \text{ или } h_{max} \approx 1.2h_0. \qquad (51)$$

Legend: 1 -- or

Then

$$\sigma_h \approx 0{,}05h_0 \approx 0{,}01v_h h_0 \text{ или } \sigma_h \approx 0.057h_0. \qquad (52)$$

Legend: 1 -- or

If we determine h_0 in the forest or from aerial photographs, then we can obtain approximate values of σ_h and v_h of the stand from the expressions (52).

20. Correlation of Tree Distribution by Crown Diameters

Materials of research on sizes and forms of crowns of different tree species are presented in works by Professors G. G. Samoylovich [47, 48, 50, 51, and 54], N. I. Baranov [4], and other authors.

Study of the correlation of tree distribution by crown diameters is of practical interest in determining crown closure from aerial photographs with the aim of estimating occupancy and yield of stands as well as the concealing properties of forests.

Knowledge of the series of distribution of trees by crown size is necessary also to determine tree thickness with the aid of tables of the correlational tie between height, thickness, and crown diameter. It is also wholly possible to elucidate multiple correlation between values D_c, h, and d_{tree}, which will allow us to determine tree thickness by D_c and h measured on aerial photographs.

The study [14] presents the suggestion that tree distribution by crown size is close to the normal distribution. However, it has not subsequently been able to derive the function of crown distribution owing to the lack of materials on measurement of D_c in localities.

Obtaining experimental data that can be considered as large samplings with a volume of up to 300 measured crowns made it possible to determine a model series of tree distribution by crown size and the approximation of theoretical function of tree distribution by crown diameter.

Field measurements of the crowns of 262 trees on the forest plot No 1 (composition: first story 6 pine 4 spruce, second story 10 spruce, occupancy 0.81, age 60 years, and site class I) were used by us to determine the crown distribution function.

Table 34 lists counts made of crowns at intervals expressed in fractions of the mean crown diameter, calculations of main crown diameter, mean square deviation, and variance of crowns. Table 35 lists calculations of the theoretical function of tree distribution by crown size, that is

$$F(D_K) = \frac{\Delta d}{\sigma\sqrt{2\pi}} e^{-\frac{(D_K - D_K^0)^2}{2\sigma^2}} \quad (53)$$

Legend: K = [subscript] crown

The theoretical function of tree distribution by crown diameter is as follows:

$$F(D_K) = \frac{0.867}{\sqrt{2\pi}} e^{-\frac{(D_K - 3.5)^2}{2.2}} \quad (54)$$

Table 34

(1)

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(2)
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(3)

Legend: 1 -- intervals, D_c; 2 -- frequency, m; 3 -- frequency in percent

The calculated values are as follows: $D^0_c = 3.5$, $\sigma_c = \pm 1.05$, and $v_c = 30$ percent.

Figure 22 presents the curve of tree distribution by crown size.

From Table 36 it is clear that almost 70 percent of the crowns are of a diameter close to the mean crown diameter, that is, they are included in the limits from 0.8 D^0_c to 1.2 D^0_c.

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Table 35

(1) D_k	w_i	(1) $D_k^i - D_k^o$ (2)	t	z_t	$P(D_k)$ (3)
[illegible]	0,007	−2,8	2,66	0,0116	0,007
[illegible]	0,022	−2,1	2,00	0,0540	0,036
[illegible]	0,117	−[illegible],4	1,3[illegible]	0,1647	0,110
[illegible]	0,22	−0,7	0,67	0,3189	0,212
[illegible]	0,31	0,0	0,00	0,3989	0,270
[illegible]	0,19	0,7	0,67	0,3189	0,212
[illegible]	0,07	[illegible]	[illegible]	0,1647	0,110
[illegible]	0,026	[illegible]	2,00	0,0540	0,036
[illegible]	0,010	[illegible]	2,66	0,0116	0,007
[illegible]	0,00	[illegible]	3,33	0,0016	0,001
[illegible]	0,007	4,2	4,00	0,0001	0,000

Legend: 1 -- D^i_c; 2 -- D^o_c; 3 -- D_c

Table 36

k [illegible]	0,2	[illegible]	0,6	0,8	1,0	[illegible]	1[illegible]	1,6	1,8	2,0
f [illegible]	0,7	[illegible]	11,0	21,2	27,0	[illegible]	1[illegible],0	3,6	0,7	0,1

The variance of crowns was found equal to v_c = 30 percent, that is, somewhat greater than was assumed. However, this coefficient v_c was obtained when taking into account all crowns (trees in the first and second storys) for a mixed forest consisting of pine and spruce. In homogeneous stands the crown variance will evidently be less than 30 percent. In practical terms, the percentage of trees that have mean crown size is close to 30 percent. More than 90 percent of the crowns are concentrated within the limits from 0.6 to 1.4 D^o_c. The curve of distribution of crowns compared to the curve of distribution of distances proved to be more elongated upward and with a more abrupt descent toward the x axis (Figure 22).

The approximate series of tree distribution by crown size gives a theoretical foundation for developing practical methods of determining mean D^o_c from aerial photographs and allowing for characteristics of distribution of D_c in determining mean tree spacings and crown closure by the method of counting crowns along straight lines.

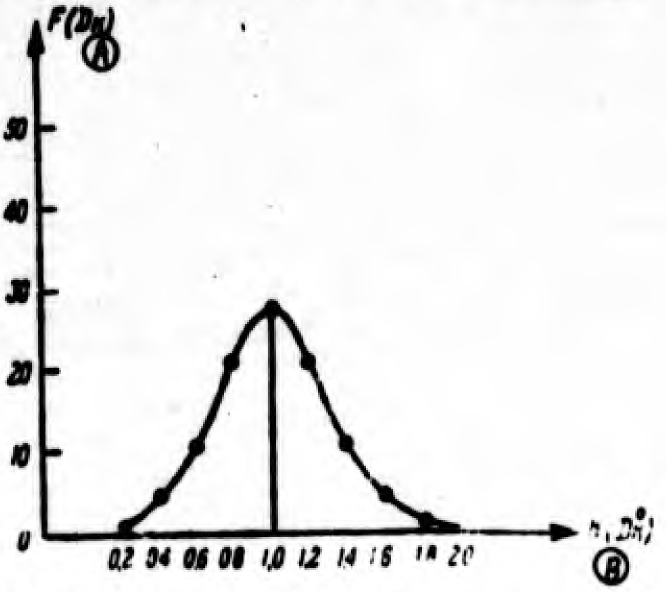

Figure 22. Theoretical curve of crown distribution.
Legend: A -- D_c; B -- D^o_c

Later studies on correlations of experimental crown distribution series were published by G. G. Samoylovich, A. Ya. Zhukov, and A. N. Polyakov (cf Chapter 1).

CHAPTER 5

CORRELATIONAL FUNCTIONS OF TREE STAND APPRAISAL INDICATORS

21. Remarks on Correlations and Precision of Statistical Variables

When we study the relationship between three or more variables, we are dealing with multiple correlation. The variables between which correlations can obtain in this case are taken as the mean quantities that serve to generalize representatives of tree sets.

In each stand there are trees of different height and thickness. In practice it is important to characterize the entire population of trees by height and thickness. To do this, the mean h_0 and d^0_{tree} are determined. Then, knowing the laws of distribution of h and d_m, we obtain a full idea of the heights and thicknesses of all trees in the plot.

The problem of whether there is a stable relationship between the mean h_0 and d^0_{tree} in different plot sections has arisen. If such a relationship holds in nature, then obviously we can find its quantitative expression in the form of mathematical equations, and from the equations calculate tables of numerical values of height and thickness. The presence of a quantitatively expressed relationship between two variables has valuable practical advantages, allowing us instead of measuring two variables to measure just one, and to take the second one from the table of correlation between the two.

Successful use of aerial photographs for forest appraisal, resumption of topographic maps, and the use of aerial methods in science and practice as a whole hinge on efforts to find correlations.

We know that a well-defined function has been observed between tree height and thickness. The first problem of research is to find a numerical expression of this function, and the second -- to appraise the closeness of the relationship and the precision that this relationship can yield when determining d_{tree} from measured h or D_c.

The third problem is determined by the practical value of drawing up the smallest number of correlation tables. The most convenient and simplest case is compiling a single table suitable in its precision for the largest number of forest types and for the greatest area in which forest tracts are located. If a single table does not ensure the desired precision, then we are compelled to prepare several tables for the biggest geographical zones, and then the subzones, oblasts, and forest-plant regions. By gradual approximation, we can find the smallest number of tables suitable for the largest territory of forest expanses.

This line of research at present is possible because in forest appraisal enormous experimental material has piled up over many decades and numerous tables of the growth pattern of normal stands have been compiled in which data on height and thickness of trees are given for pure homogeneous stands.

Tables of growth pattern afford probable heights and thicknesses for each species with a breakdown sometimes into seven site classes. They have been drawn up only for a number of oblasts of the USSR. Finally, foresters and appraisers are not wholly unanimous in appraising tables of stand growth pattern (local and universal), and all the more so in the methods by which they were compiled, for the concept of normal stands that underlies them has been placed in serious doubt scientifically and practically.

In forest appraisal appraisers determine height and thickness not fr : growth pattern tables, but from field measurements in the forest. In field topographic work the same thing is done. But now we can from aerial photographs measure just the height, and read off the thickness from correlation tables which we had to prepare earlier. This requires that we find a correlation equation of the type $h = F(d_m)$ from experimental data of measurement of mean tree heights and thicknesses in different forest plots.

However, before we find the correlational equation a number of remarks about the precision of the tables needed in practice and about the actual concept of precision of statistical variables are in order.

The concept precision of determination of mean statistical variables h_0 d^o_{tree} differs from the ordinary concept of measuring height or thickness of a given tree.

The mean h_0 and d^o_{tree} are statistical characteristics of a set of trees. They are only parameters of a consistent distribution series of trees by height and thickness. Therefore, precision of h_0 and d^o_{tree} determination is more correctly the precision of determination of the distribution series parameters.

We can measure any given tree with a high degree of precision down to millimeters, but if the selected tree differs sharply from the mean in height and thickness, then the highest precision in measuring this tree does not amount to anything, since it is not a parameter of the distribution series. Consequently, the main concern is to faithfully determine precisely the mean height and thickness that most fully reproduce the distribution series of the given set of trees with different heights and thicknesses. If the mean d^o_{tree} will be improperly selected, imprecisely, then the distribution series will deviate sharply from the actual. Thus, precision in determining the mean d^o_{tree}, paramountly and above all, must be estimated by the deviations of the two distribution series obtained for different mean d^o_{tree} values. For example, let the true (precise) mean d_{tree} = 20 cm, and the mean d^o_{tree} determined in the forest plot = 22 cm. We will try out here two distribution series of trees by thickness for d_{tree} = 20 cm and d^o_{tree} = 22 cm (Table 37).

Table 37

$k(d^o_m)$	0,7	3,6	[illegible]	16,1	18,4	18,1	13,1	8,9	6,3	3,3	1,5	0,5	0,1
d_m=20	10	12	14	16	18	20	22	24	26	28	30	32	34
d^o_m=22	11	13	15	18	20	22	24	26	29	31	33	35	37
Δd_m	+1	+1	+1	+2	+2	+2	+2	+2	+3	+3	+3	+3	+3

Legend: m = [subscript] tree

It is clear from Table 37 that a 2-cm error in determination of the mean thickness gives a different ratio of trees by thickness. When d^o_{tree} = 22 cm, the number of trees with greater thickness is increased. From the standpoint of determining the yield, this leads to its overstatement, and from the viewpoint of estimating traversibility, an impression is given of greater obstacles to vehicle passage than is the actual case.

Thus, the problem of precision in determining d^o_{tree} is bound up with permissible precision in solution of practical problems, for example, precision in determination of timber yield, etc.

In estimating permissible deviations we must also bear in mind the divisibility of dividing up plots by value of d^o_{tree}. In the course of ocular appraisal, it is customary to differentiate plots in which the difference between mean d^o_{tree} is greater than 4 cm.

Finally, in estimating the precision of correlations, we must start from the physical sense of the statistical variables and use stochastic methods of estimating precision. If, for example, the correlation table gives practically acceptable deviations for most forest plots with a probability close to unity, then this is sufficient for us to use the deviations in actual practice, for we cannot expect more from statistical variables since it is useless to require that nature itself not provide the relationships and correlations under study as being statistical in character.

Taking into account the foregoing, studies were made aiming at finding the correlation equation $h = F(d_{tree})$.

22. Determination of Correlation Equation Relating Thickness and Height of Trees

Actual measurements of mean heights and thicknesses of trees in 297 forest plots mainly in Moscow and Tul'skaya oblasts served as the basis for determining the correlation equation $h = F(d_{tree})$.

We will designate the original mean heights H^o and the mean thicknesses d^o.

Let us group H^o and d^o in distribution series with intervals ΔH = 2 meters and Δd = 2 cm. We denote by H and d mean interval values. To calculate the equation $H = F(d_{tree})$, we present the distribution series in the form of a correlation network given in Table 38.

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Table 38

№	H_1-H_2	H	H' \ d' (№)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	m_H
			d_1-d_2	3-5	5-7	7-9	9-11	11-13	13-15	15-17	17-19	19-21	21-23	23-25	25-27	27-29	29-31	31-33	33-35	35-37	37-39	39-41	
			d	4	6	8	10	12	14	16	18	20	22		26	28	30	32	34	36	38	40	
			d'	-9	8	7	6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	[illegible]	[illegible]	+9	m_H
1	3-5	4	-7	5																			[illegible]
2	5-7	6	-6		10		2																
3	7-9	8	-5		2	7	3																[illegible]
4	9-11	10	-4			2	16	2															[illegible]
5	11-13	12	-3				12	1	3		1												[illegible]
6	13-15	14	-2					11	14	3													25
7	15-17	16	-1					1	7	10													28
8	17-19	18	0								8	1											27
9	19-21	20	+1								[illegible]	6	1										34
10	21-23	22	+2								9	9	9	15	5	4							51
11	23-25	24	+3								2	3	2	20	12	9	1			·			72
12	25-27	26	+4											1		6	3	1		1			[illegible]
13	27-29	28	+5												1		8	8	2	4			23
14	29-31	30	+6																2		1		[illegible]
15	31-33	32	+7																	1			1
			m_d	5	12	11	33	21	24	20	38	19	14	26	16	21	12	9	4	7	0	3	237
			$\bar{H}_d$	4	6.3	9.1	10.3	13.0	14.3	16.4	19.1	19.5	19.9	20.9	21.7	23.1	25.2	25.8	27.0	[illegible]	0	30.7	
			$\bar{H}'_d$	-7	-5.8	4.5	3.8	-2.5	[illegible]	0.5	+0.05	+0.7	+0.9	+1.5	+1.8	+2.5	[illegible]	+3.9	[illegible]	[illegible]	0	[illegible]	

All the original data H^0 and d^0 are located in the cells of the network which are determined by the intervals H and d. As the result of summing up the units in each cell, we obtain the frequency of the distribution series in H and d. Thus, for example, in the intervals H = 4 and d = 4, the frequency m = 5. Since we are searching for the function H = F(d_{tree}), then first we obtain the conventional mean height values

$$\bar{H}_d = \frac{\Sigma H m}{m_d} \qquad (55)$$

for each interval d.

The overall frequency m_d for each interval d and also the calculated $\bar{H}_d$ and d are listed in Table 39.

The conventional mean $\bar{H}_d$ values calculated from formula (55) are listed in Table 40.

Table 39

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	0	[illegible]
[illegible]	[illegible]	[illegible]	30	[illegible]	[illegible]	6	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	12	9	[illegible]	7	[illegible]	

In the general case, the problem reduces to determining the equation

$$H = a_0 + a_1 d + a_2 d^2 \qquad (56)$$

The parameters a_0, a_1, and a_2 of equation (56) are usually sought for under the condition of minimum total of squares of the deviations of theoretical curve from empirical curve formed by the values $\bar{H}_d$ and d from Table 39.

Solving the equations for large values of $\bar{H}_d$ and d is extremely involved. To simplify the calculations, we replace the variables H and d by their deviations from the mean arithmetic values, that is, $H - H_0$ and $d - d_0$, which in turn we divide by the intervals $\Delta H = 2$ and $\Delta d = 2$ of the distribution series, which ultimately gives us small values

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Table 40

d	m_d	ΣHm	$\bar{H}_d$
4	5	4·5=20	4
6	12	6·10+8·2=76	6,3
8	11	8·7+10·2+12·2=100	9,1
10	33	6·2+8·3+10·16+12·12=340	10,3
12	21	10·2+12·7+14·11+16·1=274	13,0
11	24	12·3+14·14+16·7=344	14,3
16	20	14·3+16·10+18·7=328	16,4
18	38	12·1+16·8+18·18+20·9+22·2=688	18,1
20	19	16·1+18·6+20·9+22·3=370	19,5
22	14	18·[illegible]+20·9+22·2=278	19,9
24	26	20·15+22·10+24·1=544	20,9
26	18	20·5+22·12+26·1=390	21,7
28	21	20·4+22·8+24·6+26·3=484	23,1
30	12	22·1+24·3+26·8=302	25,2
32	9	24·1+26·8=232	25,8
34	4	26·2+28·2=108	27,0
36	7	24·1+26·4+28·1+30·1=186	28,8
38	0	0	0
40	3	28·1+32·2=92	30,7

$$H' = \frac{H - H_0}{\Delta H};$$

$$d' = \frac{d - d_0}{\Delta d}; \qquad (57)$$

The values of H' and d' are given in Table 38. As a result of these simplifications, we obtain new values for the intervals, that is, H' and d' instead of H and d.

The entry of the new values H' gives rise to the need to calculate new conventional mean $\bar{H}'_d$ values for each new d' value from the following expression:

$$\bar{H}'_d = [illegible] \qquad (58)$$

Calculations of $\bar{H}'_d$ for each d' interval, using formula (58), are listed in Table 41.

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Table 41

№	d'	m_d	$\Sigma H'm$	$\overline{H}'_d$
1	−9	[illegible]	−7·5 = −[illegible]	7
2	−8	12	−6·10−(−5)·2 = [illegible]	[illegible]
3	−7	[illegible]	−5·7+(−4)·[illegible] ·2 = −4[illegible]	4,[illegible]
[illegible]	−6	33	−6·2−5·[illegible] 4·1[illegible] ·12 = −1[illegible]	[illegible]85
[illegible]	−5	21	−4·2−3·[illegible]	[illegible]
6	−[illegible]	2[illegible]	−3·3−2·14−1[illegible]	−1,[illegible]
7	−3	20	−2·3−1·[illegible]−0·[illegible]	0,[illegible]
[illegible]	−2	[illegible]	−3·1 1·8−0·[illegible] 2	0,[illegible]2
9	−1	19	−1·1·0 1·[illegible] 14	[illegible]0,[illegible]7
10	0	14	0·3+1 [illegible] 2[illegible]	[illegible]0,[illegible]
11	1	[illegible]	1·15+2·[illegible]	1,[illegible]
12	[illegible]	[illegible]	1·5+2·1[illegible]	1,[illegible]
13	[illegible]	[illegible]	1·4+2·8 [illegible]	2,[illegible]
14	4	[illegible]	2 [illegible]	[illegible]
[illegible]	5	[illegible]	3·1+[illegible] 5	[illegible]89
[illegible]	6	[illegible]	[illegible]18	+4,[illegible]
17	[illegible]	[illegible]	[illegible]	+4,[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 42 gives calculated values of $\overline{H}'_d$, d', and frequency m_d from which we will now solve an equation of the (59) type, but with small values of $\overline{H}'_d$ and d' instead of large d and $\overline{H}_d$.

Table 42

$\overline{H}'_d$	−7	−5,[illegible]	[illegible]	−3,8	−2,5	−1,[illegible]	−0,8	0,[illegible]	0,7	+0,9
d'	−9	[illegible]	[illegible]	−6	−5	−4	−3	[illegible]	[illegible]	0
m_d	5	[illegible]	[illegible]	33	21	24	[illegible]	[illegible]	19	[illegible]
$\overline{H}'_d$	+1,5	[illegible]	[illegible]	+3,6	+3,9	+4,5	[illegible]	0	+6,3	
d'	1	2	[illegible]	4	5	6	[illegible]	[illegible]	[illegible]	
m_d	26	18	[illegible]	12	9	4	7	0	3	

According to familiar rules, to obtain parameters a, b, and c of the correlation equation

$$\overline{H}_d = a + bd' + cd'^2 \qquad (59)$$

it is necessary that

$$P = \sum m_d (\overline{H}_d - a + bd' - cd'^2)^2 = \min, \qquad (60)$$

and the partial derivative

$$\frac{dP}{da} = 0; \quad \frac{dP}{db} = 0; \quad \frac{dP}{dc} = 0. \qquad (61)$$

After differentiation and transformation, we obtain a system of equations (62), from which we then will determine the coefficients a, b, and c.

$$a\sum m_d + b\sum m_d d' + c\sum m_d d'^2 = \sum m_d \overline{H}'_d;$$
$$a\sum m_d d' + b\sum m_d d'^2 + c\sum m_d d'^3 = \sum m_d d' \overline{H}'_d;$$
$$a\sum m_d d'^2 + b\sum m_d d'^3 + c\sum m_d d'^4 = \sum m_d d'^2 \overline{H}'_d. \qquad (62)$$

We know the variables m_d, d', and $\overline{H}'_d$ from Table 42. Therefore, to solve the system of equations (62) it is required only to calculate d'^2, d'^3, and d'^4, and we obtain the remaining values via multiplication by m_d, d', and $\overline{H}'_d$ and subsequent summing up of the products. Table 42 gives the corresponding calculations on the basis of which the system of equations (63) with unknown coefficients a, b, and c was obtained:

$$297a - 454b + 5594c = -95;$$
$$-454a + 5\,594b - 17\,632c = 3545;$$
$$5\,594a - 17\,632b + 225\,038c = -10\,547. \qquad (63)$$

Solving equations (63) in the usual way, we find the coefficients a, b, and c.

Dividing all the equations by the coefficients of a, we get

$$a - 1{,}53b + 18{,}83c = -0{,}32;$$
$$-a + 12{,}32b - 38{,}84c = 7{,}82;$$
$$a - 3{,}15b + 40{,}25c = -1{,}89. \qquad (64)$$

Let us subtract the first equation from the second and third and arrive at two equations with two unknowns

$$-10{,}79b + 20{,}01c = -7{,}5;$$
$$-1{,}62b + 21{,}42c = -1.57. \qquad (65)$$

Table 43

N	m_d	d'	$m_d d'$	d'^2	$m_d d'^2$	d'^3	$m_d d'^3$	d'^4	[illegible]	[illegible]	$m_d \bar{H}_d$	$d'(m_d \bar{H}_d)$	$d'^2(m_d \bar{H}_d)$
1	5	−9	−45	81	405	−729	−3645	6561	32 805	−7	−35	+315	−28[illegible]
2	12	−8	−96	64	768	−512	−6144	4096	49 152	−5.[illegible]	−70	+560	−4480
3	11	−7	−77	49	539	−343	−3773	2401	26 411	−4.[illegible]	−49	+343	−2401
4	33	−6	−198	36	1188	−216	−7128	1296	[illegible]	[illegible]	127	+762	−4572
5	21	−5	−105	25	525	−125	−2625	625	[illegible]	[illegible]	−52	+260	−1300
6	24	−4	−96	16	384	−64	−1536	256	[illegible]	−1.[illegible]	−44	+176	−704
7	20	−3	−60	9	180	−27	−540	81	[illegible]	−0.[illegible]	−16	+48	−144
8	38	−2	−76	4	152	−8	−304	16	[illegible]	−0.0[illegible]	+2	−4	+8
9	19	−1	−19	1	19	−1	19	1	[illegible]	[illegible]	11	11	14
10	14	0	0	0	0	0	0	0	[illegible]	−0.9	+13	0	0
11	26	1	+26	1	26	1	26	1	[illegible]	−1.[illegible]	+38	+38	+38
12	18	2	+36	4	72	8	144	16	[illegible]	−1.[illegible]	+33	+66	+132
13	21	3	+63	9	189	27	567	81	[illegible]	[illegible]	+53	+159	+477
14	12	4	+48	[illegible]	[illegible]	64	768	256	[illegible]	[illegible]	+43	+172	+688
15	9	5	+45	[illegible]	225	125	1125	625	[illegible]	[illegible]	+35	[illegible]	+875
16	4	6	+24	36	144	216	864	1296	[illegible]	+4.[illegible]	18	+108	+648
17	7	7	+49	49	343	343	2401	2401	[illegible]	+4.[illegible]	+70	+210	+1470
18	0	8	0	64	0	512	0	4096	0	0	0	0	0
19	3	9	+27	81	243	729	2187	6561	19 683	+6.[illegible]	+19	+171	+1539
	297		−772 +318		5701		−25714 +8082		22[illegible]		−393 +298	+2563 −18	−16436 +5889
			−454				−17632				−95	+2545	−10547

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Dividing equations (65) by the coefficients of b, we get

$$-11{,}37c = +0{,}27,$$

from whence

$$c = -\frac{0{,}27}{11{,}37} = -0{,}023. \qquad (66)$$

Substituting the numerical value of c in equation (65), we find

$$-1{,}62b + 21{,}42(-0{,}023) = -1{,}57,$$

from whence

$$b = \frac{1{,}09}{1{,}62} = 0{,}67. \qquad (67)$$

Substituting numerical values of c and b in equation (64), we get

$$a - 1{,}63(0{,}67) + 18{,}83(-0{,}023) = -0{,}32,$$

from whence

$$a = 1{,}45 - 0{,}32 = 1{,}13. \qquad (68)$$

Substituting the numerical values of the coefficients a, b, and c into equation (59), we will determine the sought-for correlation equation relating height and thickness of trees, that is,

$$H' = 1{,}13 + 0{,}67d' - 0{,}023d'^2. \qquad (69)$$

We now convert from H' and d' to H and d from equations (57), that is,

$$H' = \frac{H - H_0}{\Delta H} = \frac{H - 18}{2}; \quad d' = \frac{d - d_0}{\Delta d} = \frac{d - 22}{2}.$$

Then equation (69) takes on the following form:

$$\frac{H - 18}{2} = 1{,}13 + 0{,}67\left(\frac{d - 22}{2}\right) - 0{,}023\left(\frac{d - 22}{2}\right)^2. \qquad (70)$$

We can use equation (70) to calculate the table of the relationship between H and d_{tree}. For example, when $d_{tree} = 22$, we obtain H = 20.26, and when $d_{tree} = 10$, H = 10.56. However, it is best to find a correlation equation relating H and d_{tree} in a final form. After the appropriate transformations, we find the following equation:

$$H = 1{,}176d - 0{,}0115d^2. \qquad (71)$$

Table 44 of the correlation between h and d_{tree} has been calculated from equation (71).

The curve of the correlation between h and d_{tree}, based on data in Table 44, is shown in Figure 23. The theoretical and empirical curves almost fully agree. Since out of 296 forest plots we only have three plots with H greater than 28 and d_{tree} greater than 36, then naturally the values of H and d_{tree} calculated from equation (71) exceeding these values will essentially be extrapolated.

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Table 44

d_m	4	6	[illegible]	10	12	[illegible]	[illegible]	[illegible]	[illegible]	22	24	[illegible]	28
h	4,6	6,[illegible]	[illegible]	10,5	12[illegible]	[illegible],0	[illegible]	17,[illegible]	[illegible]	[illegible]0,1	21,1	[illegible]2.6	[illegible]3,7
[illegible]	[illegible]0	32	[illegible]	[illegible]	3[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]8	50		
[illegible]	21[illegible]	[illegible]	[illegible]	[illegible],4	2[illegible]	[illegible]	[illegible]	1[illegible]	[illegible]	[illegible]0	30,[illegible]		

Legend: m = [subscript] tree

Equation (71) and Table 44 reveal an almost complete cessation of increase in height in an old forest, while the thickness still continues to rise. This phenomenon has in fact been observed in actual forests, which is accountable biologically, since further increase in height produces unstable tree condition, while an increase in thickness promotes stability. However, we can anticipate for forests in the northern and Siberian zones where the habitat conditions differ sharply from the central-European zones that the extrapolated H and d_{tree} will according to equation (71) be close to the actual ratios of tree height and thickness, for under those conditions on attainment of a specific H at an advanced age height remains almost constant, but thickness continues to rise, which is in fact expressed by the equation (71) H $>$ 29 meters and d $>$ 40 cm.

The correlation equation (71) and Table 44 have been determined for a single zone of small-leaf and coniferous forests of the Moscow and Tul'skaya oblasts. Correlational equations can be derived by the method here described for any other zones and tree stand compositions.

23. Determination of the Correlation as a Measure of Relatedness

To appraise the closeness of the relationship of the rectilinear function, we calculate the correlation coefficient, and to estimate the relatedness of the curvilinear function we determine the following correlation ratio

$$\eta = \frac{\sigma_{\bar{H}_d}}{\sigma_{H_d}}, \qquad (72)$$

If $\eta = 0$, then there is no correlation between H and d_{tree}.

If $\eta = 1$, then a functional relationship does exist between H and d_{tree}.

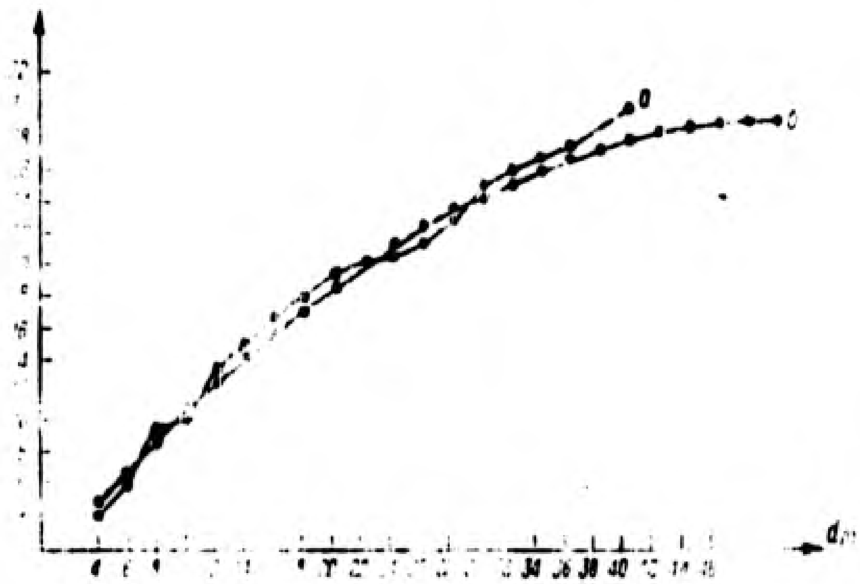

Figure 23. Theoretical (b) and empirical (a) curves of the correlation between tree height and thickness

If η is close to unity, then this evidences that there is a satisfactory and close curvilinear relationship between tree height and thickness. To appraise the correlation equation (71) we have derived, we have to determine what the value of the correlation ratio is. For this purpose, we will calculate the mean square deviation of conventional mean height $\bar{H}_d$ and the complete mean square deviation of heights H, that is,

$$\sigma_{\bar{H}_d} = \sqrt{\frac{1}{N}\sum m_d (\bar{H}_d - \bar{H}_0)^2}, \qquad (73)$$

$$\sigma_{\bar{H}_0} = \sqrt{\frac{1}{N}\sum m_H (H - \bar{H}_0)^2}, \qquad (74)$$

where

$$\bar{H}_0 = \frac{\sum m_H H}{N} = H_0 + \frac{\sum H' m_H}{\sum m_H}\Delta H. \qquad (75)$$

Then

$$\sigma_{\bar{H}_0} = \Delta H \sqrt{\frac{\sum H'^2 m_H}{\sum m_H} - \left(\frac{\sum H' m_H}{\sum m_H}\right)^2}. \qquad (76)$$

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Formulas (75) and (76) are very convenient in simplifying calculations, since they are expressed by small values of the intervals $H' = (H - H_0)/\Delta H$. We already have the values of H' and m_{ac} in Table 38.

Table 45 lists calculations of σ_{H_0} from formula (76).

Table 45

№	H'	(A)	$H'm_{ac}$ (B)	H'^2	H'^2m_{ac} (C)
1	−7	[illegible]	−35	49	246
2	−6	[illegible]	[illegible]	36	432
3	−5	[illegible]	−60	25	300
4	−4	[illegible]	−80	16	320
5	−3	[illegible]	[illegible]	9	225
6	−2	[illegible]	[illegible]	[illegible]	112
7	−1	[illegible]	−27	1	27
8	0	[illegible]	0	0	0
9	1	[illegible]	51	1	51
10	2	[illegible]	76	4	152
11	3	[illegible]	36	9	108
12	4	[illegible]	92	16	368
13	[illegible]	[illegible]	[illegible]	25	175
14	6		6	36	36
15	7		14	49	93
		[illegible]	−95		2649

Legend: A -- m_{ac}; B -- $H'm_{acc}$; C -- H'^2m_{ac}

We find from formula (75) and Table 45

$$\bar{H}_0 = 18 + \frac{95}{297} \cdot 2 = 17{,}36,$$

since $H_0 = 18$, and $\Delta H = 2$.

We find the following from formula (76) and Table 45:

$$\sigma_{\bar{H}_0} = 2\sqrt{\frac{2649}{297} - \left(\frac{95}{297}\right)^2} = 5{,}94. \qquad (77)$$

We calculate the variable σ_{H_d} from the following formula:

$$\sigma_{H_d} = \sqrt{\frac{1}{N}\sum m_d \bar{H}_d^2 - \bar{H}_0^2}\,. \qquad (78)$$

We know the value of $\bar{H}_0$, 17.36, and the values of $\bar{H}_d$ have been calculated earlier and are given in Table 39. Table 46 lists calculations of the remaining indexes in formula (78).

GRAPHICS NOT REPRODUCIBLE

Table 46

№	m_d	$\overline{H}_d$	$\overline{H}_d^2$	$m_d\overline{H}_d^2$
1	5	4,0	16,0	80,0
2	12	6,3	39,7	476,4
3	11	9,1	82,8	910,8
4	[illegible]	10,3	106,[illegible]	3 501,3
5	[illegible]	13,0	169,0	3 549,0
6	24	14,3	204,5	4 908,0
7	20	16,4	269,0	5 380,0
8	38	18,1	327,6	12 448,8
9	19	19,5	380,2	7 223,8
10	[illegible]	19,9	396,0	5 544,0
11	26	20,9	436,8	11 356,8
12	18	21,7	470,[illegible]	8 476,2
13	21	23,1	533,6	11 205,6
14	[illegible]	25,2	635,0	7 620,0
15	[illegible]	25,8	665,0	5 990,4
16	4	27,0	729,0	2 916,0
17	7	26,8	718,[illegible]	5 027,1
18	[illegible]	0	[illegible]	0,0
19	[illegible]	30,7	942,5	2 827,5
	[illegible]			99 442

$\overline{H}_0 = 17.36$, $\overline{H}_0^2 = 301.4$, $N = 297$, and $\sum m_d \overline{H}_d^2 = 99{,}442$ (from Table 46), therefore

$$\frac{\sum m_d \overline{H}_d^2}{N} = \frac{99\,442}{297} = 334,$$

and

$$\sigma_{\overline{H}_d} = \sqrt{334 - 301{,}4} = 5{,}7. \qquad (79)$$

From formulas (72), (77), and (79) we find the correlation ratio $\eta \approx 0{,}9$.

Legend: 1 -- H = [subscript] accumulated [cumulative]

The value of η is close to unity, therefore the relationship between tree height and thickness we have found in the form of equation (71) is marked by a close and high degree of correlation.

24. Multiple Correlation Between Height, Thickness, and Crown Diameter of Trees

Tree height and thickness for normal stands made up of a single species and with different site classes, are given in tables of stand growth pattern in [54]. Since forest management and forest research are conducted by relying on aerial photographs, the need to discover interrelationships between crown diameter, height, and thickness of trees has long since arisen, since from aerial photographs we can measure the first two variables, and determine tree thickness from the correlation table.

Earlier we determined the correlation equation of the function relating two variables -- h and d_{tree}. In practical terms, it is important to establish a relationship between three variables -- h, d_{tree}, and D_c, that is, to study multiple correlation.

Professor G. G. Samoylovich determined the correlation equation relating d_{tree} and D_c for pine groves in the mossy pine grove forest type on gradual hummocks for occupancy rates of 1.0, site class I, and ages 80, 90, 100, and 110 years [46]. Data on the ratio of D_c and d_{tree} are available in studies [4, 6, 54].

Professor Spurr [US] has indicated that Nash has investigated a relationship of D_c with d_{tree}, but equations of this relationship are not to be found in the book [81].

We studied experimental materials, and as a result of processing these it turned out that as h is increased by 3 meters the thickness d_{tree} increases by approximately 4 cm, while the crown diameter D_c increases by 0.5 meter. A Table of the values h, d_{tree}, and D_c, published in the study [8], has been compiled from these data. This approximational table of the interrelationship between the three variables has been verified by field data of h and d_{tree} in 111 forest tracts on the island of Sakhalin.

We took as the starting point d_{tree} values, since it can be anticipated that tree thickness has been determined by the topographers with high precision.

Verification of this table with experimental data revealed that 86.5 percent of the deviations Δh did not exceed 3 meters, and only 14.5 percent of the deviations varied from 3 to 5 meters. It can be assumed that this table is more appropriate for forests in which a slowdown in growth increment with ordinary increase in thickness has been observed, that is, for tree stands in which the increase in thickness outstrips the increase in height for mature and old stands owing to special climatic and geographic conditions of the habitat. Verification of the first table by field data of h and d_{tree} in the forests of Moscow Oblast gave satisfactory convergence of h and d_{tree} up to h_0 = 18 meters, but when $h_0 > 18$ meters appreciable deviations in thickness showed up.

Later, considerably more material of field height, thickness and crown diameter measurements were tapped.

When deriving the correlation equation (71), out of 297 forest plots there were few plots with large h and d_{tree} values (with $h > 32$ and $d_{tree} > 38$). Therefore, additional materials were resorted to, on the basis of which necessary refinements were introduced. Based on all these materials, an approximational relationship of D_c with h and d_{tree} was established [10, 11, 13, 14].

With small refinements, the multiple correlation between height, thickness, and crown diameter of trees determined by the author is listed in Table 47.

Table 47

h, m	d_m, cm	D_k, m	h, m	d_m, cm	D_k, m	h, m	d_m, cm	D_k, m
[illegible]	4.0	0,8	16	16,0	3,0	28	34	5,4
[illegible]	4.0	1.0	17	18,0	3,2	29	35	5,6
6	5.5	1.2	18	19,0	3,4	30	37	5,7
7	6.5	1,4	19	20,0	3,6	31	39	5,8
8	7,5	1,6	20	21,5	3,8	32	41	6,0
9	8,5	1,7	21	22,5	4,0	33	43	6,2
10	9,5	1,9	22	24	4,2	34	45	6,4
11	10,5	2,1	23	25,5	4,5	35	46	6,6
12	11,5	2,2	24	27	4,7	36	48	6,8
13	12,5	2,4	25	29	4,8	37	49	7,0
14	14,0	2,6	26	30	5,0	38	50	7,5
15	15,0	2,8	27	32	5,2			

Legend: m = [subscript] tree; k = [subscript] crown

Table 47 was verified by field data measurements of h and d_{tree} in 176 forest plots of Moscow and Tul'skaya oblasts.

Table 48 lists deviations Δd_{tree} of field values from table values in determination of d_{tree} from h.

It is clear from Table 48 that the correlation function in almost 80 cases out of 100 gives wholly satisfactory precision in the determination of mean tree thickness from measured height. Deviations of more than 4 cm amount to 10 percent and, as was shown, they refer to the largest h and dtree. This is evidently associated with the fact that Table 47 gives somewhat of an understatement of d_{tree} values in stands with sizable h values and requires some refinement in this part, that is, an increase in d_{tree} when $d_{tree} >$ 28 by 1-2 cm in each interval. However, by virtue of the statistical nature of the phenomena and the effect of a set of factors, we obviously must always anticipate a small number of deviations greater than 4 cm.

In determining d_{tree} from field D_c values and Table 47, 72 percent of deviations ranging from 0 to 4 cm were obtained, which evidences lesser intimacy of the relationship between tree thickness and crown diameter compared to the intimacy of the relationship between tree thickness and tree height. Therefore, in determining d_{tree} from D_c we get lesser precision.

When the height h was determined from field d_{tree} values and from Table 47, we obtained 86 percent of deviations ranging from 0 to 2 meters, 10 percent ranging from 2 to 3 meters, and one deviation of 4 meters.

The correlation Table 47 was obtained from data of pine, spruce, and small-leaf stands of Moscow and Tul'skaya oblasts, but it does not take into account the effect of the species composition of stands.

Therefore, we compiled Table 50 in which approximate values of h and d_{tree} are given for ten main tree species. Actually, the stands often consist of two to three species. In general there were no tables of stand growth pattern for mixed forests. Only now have drafts of growth pattern tables for mixed stands based on the concept of N. V. Tret'yakov begun to appear.

For mixed forests, we have to take the value of d_{tree} for single h value from Table 50 for each species, and then to obtain the mean thickness of trees in the mixed forest for the given species ratio. Table 50 also gives approximate values of h and d_{tree}. For topographic purposes, such tables are best compiled for boundary zones (regions), but for forest appraisal -- in each area of forest management work with stand growth under local conditions taken account of.

As investigations of foresters have shown, including the German Professor Weise, the Hungarian Professor Fekete, the Austrian forester Schiffel, and others, correlations of tree distribution by thickness and height in the forests of Western Europe are very similar to those indicated correlations for forests in the European part of the USSR. This has been confirmed by studies of Professor A. V. Tyurin [57].

Works of Professor N. V. Tret'yakov [54, 55] are vital in the study of correlations of the structure of tree stands of forest elements.

By way of experimentation, Table 47 was verified with field data from appraisers, given by V. F. Kozlovskiy in the study [33] for pine groves aged 100 years located in the most varied areas (Arkhangel'skaya Oblast, Komi ASSR, Khakasskaya Autonomous Oblast, and the Altay).

Table 49 gives deviations Δd_{tree} of field d_{tree} values from table values in the determination of d_{tree} from h and from Table 47.

Table 49 shows the satisfactory precision of correlation Table 47. Still, we must underscore that we must not be limited to one table for all areas. Multiple correlation study must be conducted for several subzones (areas) of USSR forests, for the mountainous, southern, broadleaved, and other forests of the country.

Table 48

Δd_m, см	m	f%
0--2	139	78,0
2--4	21	12,0
4--6	11	6,2
Более 6 (A)	5	3,8

Legend: A -- greater than 6

Table 49

Δd_m, см	m	f%
0—2	22	76,0
2—4	6	20,7
4—6	1	3,3

GRAPHICS NOT REPRODUCIBLE

Table 50

[illegible]	d_m, cm A	B	C	D	E	F	G	H	I	J
4								4.0		
6	[illegible]		5.0	[illegible]	5.5			5.9		
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7.6	6.8	6.7	7.5	[illegible]	—
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9.7	8.8	9.2	9.5	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11.8	[illegible]	11.7	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	12.8	[illegible]	13.9	[illegible]	11.7
[illegible]	15.0	[illegible]	[illegible]	[illegible]	[illegible]	14.9	[illegible]	15.5		[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	17.2	[illegible]	18.0	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22.8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	29.0	[illegible]		[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	29.0		[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	29.0	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	35.0	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]
[illegible]	—	[illegible]	[illegible]	[illegible]	[illegible]	—	[illegible]	40.7		
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]		50.0	41.5		
[illegible]							[illegible]	18.0		

Legend: A -- birch; B -- beech; C -- oak;
D -- spruce; E -- linden; F -- aspen; G -- fir;
H -- pine; I -- European alder; J -- ash

25. Classification of Stands by Their Density

Development of the most applicable classification of stands by their density is in practical terms essential in compiling state topographic maps, and forest and geobotanical maps of the country. This problem is no less important for forest management and selection of appropriate methods in appraisal and measurement interpretation of aerial photographs.

It is of interest to discover those critical indexes of mean tree spacing which in quantitative form underscore qualitative changes in stands at any given age biologically caused by development of phytocenoses at the time and space of habitat. Such critical indexes of forest density in most cases predetermine its timber value as well.

As far as we know, geobotanical, forestry, topographic, and geographic literatures do not contain an agreed-upon and valid enough classification of forests by density, which has an adverse effect in the practice of cartography of forested localities. Thus, for example, the absence of quantitatively specific indexes to classify forest plots in a particular density class has led to unreconciled recommendations in handbooks on states topographic operations and those in textbooks. Recommendations have been offered whose execution could lead to considerable distortion of information about the country's forests on state topographic maps. The diversity and contradictoriness of concepts about forest density has led to unreconciled data on the forest station of certain territories; and the quantitative indeterminacy of concepts on sparse forests has hampered their delineation from aerial photographs and in localities.

Let us look at several classifications of stands by density to be found in handbooks, textbooks, and manuals. On topographic maps forests are subdivided into three groups: dense forests, sparse forests, and free-standing trees. In actual practice, instructions on topographic work give the definition of only the sparse forest type: a sparse forest consists of trees growing so infrequently that they do not constitute a continuous forest, but still they cannot be indicated as free-standing trees, but later it is added that sparse forests do not present obstacles to movement and in the absence of thickets visibility is possible for considerable distances. Quantitative indeterminacy of these recommendations is self-evident.

In another set of instructions the sparse forest symbol is recommended to be used as denoting sections covered by trees not less than 4 meters in height and standing apart from each other by such distance that motor vehicles can pass between them.

We know that motor vehicles come in different sizes, therefore tree spacings required for free travel of vehicles into a forest will differ. Thus, a passenger car with a load can enter a forest when the mean tree spacing is l = 3-4 meters, but a truck requires l = 6-7 meters. For these l values the forest density N will be, respectively, 1340, 765, 340, and 247 trees per hectare. Thus, we again run into indeterminacy as to what tree density must be adopted as defining a sparse forest.

The study [22] recommends that we classify as sparse forests plots in which the mean tree spacing is 4 meters and greater.

The economic value of any particular forest tract under otherwise equal conditions is determined by the thickness, height, and density of trees. We know that young forests differ in very high density, but contain thin and short trees in practical terms unsuitable for processing into valuable building lumber. In contrast, mature forests usually have low density and high and thick stems that afford the highest qualitative construction materials for the national economy. Thus, a site class I pine forest aged 20 years can have a density of 3,970 stems per hectare, but the lumber yield will be only 90 cubic meters. The same pine forest at age 140 will have a density of only 353 trees per hectare, but the construction material yield will be 400 cubic meters. Thus, an old pine forest is approximately 10 times less dense than a young forest, but in yield of construction materials the former is 4 times more productive than the latter, without even referring to the incomparably higher quality of the wood.

We must also note that with increase in forest age canopy closure is reduced, where the most widespread stands have a closure of 0.4-0.8, but in old deciduous and pine forests of the Siberian Taiga closure amounts to only 0.3-0.4 or a usual forest density of 150-200 trees per hectare. When $l = 4$ meters and $N = 765$, closure is equal to approximately 0.7-0.8.

If we define a sparse forest to be one in which $l = 4$ meters or $N = 765$, then obviously a mature pine grove with $N = 353$ must unreservedly be designated on state maps with the sparse forest symbol. Since when $l = 4$ meters closure $C = 0.7-0.8$, and the most widespread stands have $C = 0.4-0.8$, most valuable forests have to be designated as sparse forests. As a result of this definition of a sparse forest we would be forced to designate mature forest tracts that are the most important and valuable for the national economy on state topographic maps with the sparse forest symbol, and depict mainly only young forests by green-colored areas. Consequently, the above-presented definition of a sparse forest cannot be recommended for actual use. Some specialists suggest that we consider a sparse forest to be one in which tree spacing is equal to tree height. This is the advice given by the author of the study [22], but only for larch forests of Siberia and the Far East.

Let us assume that two forest tracts have, respectively, height $h = 5$ meters ($l = 5$ meters) and $h = 35$ meters ($l = 35$ meters). In full accord with the definition, both plots must be designated on the map as sparse forests, but forest density in the first is N = 492, and in the second only 12 stems per hectare. Just as invalid is the recommendation to symbolize on maps of Siberia as sparse forests sections which have a density ranging from 12 to 492 trees per hectare, but on maps of the European part of the USSR -- the same symbol is to be used to designate sections with N = 765. With this approach consistency and unambiguity of symbols is violated -- elementary rules in any cartographic representation.

It has also been proposed to regard as sparse forests those in which crown spacing is equal to five or more crown diameters. If we take D_c = 1 and 6 meters, then for $5D_c$ tree spacing will range from 6 to 36 meters, but we obviously cannot accept as useful in delineating sparse forests N values ranging from 340 to 12 per hectare.

Professor V. V. Alekhin in the studies [1, 2] characterizes sparse forests by the presence of 100-150 trees or less per hectare. Under this definition, several authors have begun to define sparse forests as plots in which one tree is encountered in each 100-150 square meters, where such a forest, as they propose, can be sighted through at ground level for 500-600 meters.

However, Professor V. V. Alekhin has a different description of sparse forests, namely 300-500 trees per hectare, which sharply disagrees with the first definition. As far as visibility or, more properly, visibility range of 500-600 meters is concerned, it depends not only on density, but also on tree thickness. It is clear from Table 5 that a visibility range of 500-600 meters can obtain when the density N ranges from 340 to 56 trees per hectare, depending on mean d_{tree}.

In addition to sparse forests, use is also made of the term "rediny". Professor V. V. Alekhin writes that close to the northern limit of tree cover near interfluves one encounters only free-standing trees often in the form of a polustlanik [transliterated] hundreds of meters apart. Formations containing such free-standing trees among the tundra have also been called "rediny". However, in forest appraisal forest tracts that have an occupancy of the order of 0.1-0.2 are classified as rediny, which of course departs from the concept of rediny given above.

All this evidences that there is now a real practical need to develop an applicable classification of forests by their density.

Obviously, this classification must take into account the geobotanical characteristics of a forest as a plant community, its timber value, and other qualities.

A forest, just as any plant community, possesses a definite structure. In the general case the structure of a plant community (including forests) can be characterized by the following features: species of plants, living forms of plants, overall density of stems (or stalks), tree spacing, frequency of species and living forms, canopy closure, story status, height and thickness of stems (or stalks), external appearance, species makeup, age, yield, and the like.

Differences in structural features give rise to differences between plant communities. Not all features of a community are the same in importance. Under a certain ratio some features become determining, of prime importance, or dominant, while others are secondary, apparently subordinate. The meaningfulness of given features depends not only on their natural ranking in the structure, but also on the problems and points of view from which the community is studied and regarded in the interests of the national economy and science.

First of all it is useful to establish even an approximate limit between what is a forest and what is not, that is, to determine what must be classified as a forested locality and what must be regarded as free-standing trees or areas not covered by forest.

In forestry and geobotany, a forest is defined as a plant community or a population of woody plants which influence each other and the environment and are correspondingly changed in external appearance and structure during the course of their development. It must be assumed that the more or less pronounced effect of trees on each other and on the environment is possible only when there is a specific tree density, canopy closure, and tree spacing. If, for example, tree spacing amounts to 100 meters and more, then obviously no-one will call this tract a forest, these are more correctly free-standing trees that have practically no effect on each other.

As the distance between trees is reduced and with a rise in density the effect of trees on each other and on the environment gradually rises. Thus, accumulation of a certain number of trees per unit area produces a new quality of the locality -- it becomes a forest [41].

Under what density, closure, or occupancy do individual trees form a forest? Professor V. G. Nesterov has observed that stands of 0.1-0.2 occupancy actually are not yet forests, since in them the effect of the tree population on the environment and tree interaction are extremely weakly pronounced. In forest appraisal, areas uncovered with forest include plots that have been brought under continuous felling, young forests of occupancy 0.3 and less, moderate-aged and more mature forests with occupancy of 0.2 and less (rediny). It would assume that by following these instructions we can regard as non-forest or free-standing trees plots that have an occupancy of 0.3-0.2-0.1 and smaller, and as forests only those in which the occupancy is greater than 0.3-0.2. But would this solution be correct from the general-geographic, geobotanical, and cartographic points of view?

Let us take, for example, a tract of forest of occupancy 0.3 with h = 23 meters, d_{tree} = 30 cm, and yield Z = 130 cubic meters. Given this occupancy, the approximate density N = 200. It is obvious that this plot cannot be shown on maps as either forest or forested area. Let us now look at what an occupancy of 0.3 brings us. From the appraisal description we have taken two plots with occupancies 0.2 and 0.6, but with different values of h and d_{tree}. The lumber yield when p = 0.2 is Z = 90 cubic meters, but when p = 0.6 only 80 cubic meters. In accordance with the above-given data, the first plot must be indicated on a map as a non-forest, and the second as a forest. But the first plot is more valuable in yield (building materials) and would be unreasonable to designate it on the map as a non-forest. When p = 0.2, the density N = 120. This is not a dense forest, but nonetheless a forest. Consequently, for an occupancy of 0.2 there are no weighty geobotanical and cartographic grounds for denoting such sections as non-forest.

When there is an occupancy of 0.1, we must consider the species and age in order to establish the approximate forest density below which the section of the locality can be deemed as non-forest. Of the main forest species, oak stands aged 160 years at occupancy p = 1.0 have the lowest density, equal to approximately 140 individuals per hectare. Obviously, for occupancy p = 0.1 we get the most sparsely standing trees, since in this case there will be only 14 stems per hectare. When N = 14, mean tree spacing l = 30 meters. Obviously, a locality with N = 14 and l = 30 meters is on the borderline of forest and non-forest, that is, singly standing trees, since the effect of trees on each other and on the environment will practically be absent even for their greatest height h = 30 meters. We also must stress that Glavlesookhrana [Main Administration of Forestry and Forest Conservation] recommends in timber felling to leave not less than 15-20 trees per hectare as seed carriers. Thus, the scattering of seeds from a tree is equal to approximately one-half h, so that when h = 30 meters the most permissible distance between trees l must not be greater than 30 meters. Thus, in felling areas not less than 15 trees per hectare must be left standing in order to provide conditions for reforestation, while the tree spacing l must not be less than 30 meters.

Consequently, plots of a locality in which the number of trees is 15 and less per hectare and l = 30 meters and more, can with full justification be classified as part of an open, unforested locality with free-standing trees. From the timbering point of view, the plots are not of any lumber value. In traversability they do not differ from ordinary, non-forested localities. In cover properties the plots are equivalent to an open locality. From this it is clear that the category of a forest with N = 15 and l = 30 meters must be retained in the overall classification of forest by density, since it can serve for an approximate delimitation of forest and non-forest, sparse forest and free-standing trees.

We will now attempt to establish a second limit of a sparse forest, that is, to solve the question of the density that marks the uppermost limit of a sparse forest. As we know, forestry organizations appraise forests from the economic standpoint and subdivide them by yield of building materials per hectare and by levels of merchantability.

Instructions on forest appraisal stipulate the classification of forest plots as independent when there is a yield difference of 30 cubic meters, and the first yield group includes plots with Z values up to 50 cubic meters. Forests with occupancy of 0.2 are so classified as a matter of course, since they can for large h and d_{tree} values have yields of about 90 cubic meters and, consequently, enter the second yield group. But here for an occupancy of 0.1 even mature forests are regarded as so sparse that they cannot be represented as plots economically profitable for exploitation. Additionally, determination of occupancy is carried out with a precision up to 0.1. It is obvious that from the lumbering viewpoint as well, for all species and forest ages plots of 0.1 occupancy can be regarded as critical for establishing the second limit of sparse forests. Under average conditions mature forests with occupancy 0.1 will have density N equal to about 60 trees per hectare, but the mean tree spacing will be about 15 meters (Table 1). The visibility range in such forests in most cases is more than 400 meters in a level locality and in the absence of underbrush (Table 5).

Forest plots of 0.1 and less closure with $l = 15$ meters and more have unsatisfactory conditions of concealment from aerial and ground observation.

When $l = 15$ meters and more universal traversability of the forest between trees is ensured for vehicles of any size.

The following gradation of forests by density could be usefully associated, from the geobotanical and forestry points of view, with the differentiation of the most widespread thinned-out forests, since they, even though distinguished by low density, still are frequently encountered in localities, especially in northern geobotanical zones. From foresters' data, such forests have a density of the order of $N = 100-200$ trees per hectare, a closure of about 0.3-0.4, and a yield of up to 150 cubic meters. To secure a yield of 150 cubic meters per hectare, 150 trees with a volume of one cubic meter each are required. Trees attain a stem volume of one cubic meter in a mature age with large thickness and height. Thus, pine will be one cubic meter in volume for $h = 27.5$ meters and $d_{tree} = 32$ cm, and larch when $h = 26$ and $d_{tree} = 32$ cm.

When $N = 150$, mean tree spacing $l = 9$ meters and occupancy is approximately 0.25-0.3. Occupancy rates of 0.3-0.4 are considered very low. Obviously, plots with density N from 60 to 150 and l from 15 to 9 meters can be classified as low-density forests.

Such forests are traversable between trees by most vehicles and have satisfactory conditions for concealment from observation, since crown spacing is equal to approximately one mean crown diameter.

It is important to retain gradation of forests by density when $l = 5.5$ meters, since this l value is practically critical for forest traversability.

Also vital in importance is crown closure $C = 0.5$, for at this C value reliable natural concealment from observation is ensured. In forest appraisal closure or occupancy close to 0.5-0.6 is regarded as average. Thus, the limit of mean-density forests from all points of view must be taken as $l = 5.5$ meters or $C = 0.5$.

Finally, it is worth while classifying as dense forests those that have crown closure close to 1.0 and mean tree spacing l = 3.5 meters, which value characterizes forests impassable (between trees) for all vehicles.

Bearing the foregoing in mind, we can propose a classification of stands by their density in the form of Table 51.

Table 51

[illegible] леса (1)	N [illegible] (2)	Среднее расстояние между [illegible], м (3)
Лес очень [illegible] (4)	Более 1000 (9)	Менее 3.5 (14)
Лес густой (5)	[illegible] 1,000 [illegible] 400 (10)	От [illegible] 5.5 (15)
Лес средней густоты (6)	От 400 [illegible] 150 (11)	От 5.[illegible] до 9 (16)
Лес малой густоты (7)	От 150 до 60 (12)	От 9 [illegible] (17)
Лес редкий [illegible] (8)	[illegible] 60 до 15 (13)	От 15 [illegible] 30 (18)

Legend: 1 -- forest type; 2 -- number of trees per hectare; 3 -- mean tree distance; 4 -- very dense forest; 5 -- dense forest; 6 -- moderate-density forest; 7 -- low-density forest; 8 -- sparse forest; 9 -- greater than 1,000; 10 -- from 1,000 to 400; 11 -- from 400 to 150; 12 -- from 150 to 60; 13 -- from 60 to 15; 14 -- less than 3.5; 15 -- from 3.5 to 5.5; 16 -- from 5.5 to 9; 17 -- from 9 to 15; 18 -- from 15 to 30

The forest density D and the mean tree spacing l are given in the table with rounded values.

Direct features characterizing density, that is, N and l, underlie classification of forests. Mean tree spacing introduces specificity and simplicity in the effort to differentiate sparse forests and other gradations of stand density in the field and on aerial photographs. We could characterize gradations of forest density by other features, for example, by canopy closure c or by crown spacing Δd expressed in units of crown diameter D_c, which bear an approximate relationship with l and N (Table 4). It is clear from Table 4 that when l = 9, C = 0.25, then $\Delta d = 1D_c$, but when l = 10, C = 0.2, and $\Delta d = 1.2D_c$. But, first of all, C and Δd are already indirect, and not direct characteristics of forest density and, secondly, for the same canopy closure forest density can differ, for example, when C = 0.2 density

N can take on values from 407 to 72, and l from 5.5 to 13 meters depending on the variation in the value of D_c in the forest plot.

It appears worth while adopting the classification of stands by their density in characterizing forests in all maps, in textbooks, and in the literature on geobotany, sylviculture, topography, and cartography.

CHAPTER 6

METHODS OF DETERMINING FOREST DENSITY AND MEAN TREE SPACING

26. Some Problems in the Theory and Methodology of Measurement Methods of Determining Appraisal and Topographic Information from Aerial Photographs and Field Work

Information about forests (tree stand density, mean tree spacing, height, thickness, and crown diameter of trees, canopy closure, and yield) is essential for appraisal, forest management, and compiling state topographic maps. In the USSR and abroad, major studies on interpretation of aerial photographs have been carried out since 1920 through 1962. However, as many appraisers and forest management specialists have noted, the theory and methodology of measurement interpretation still has not been fully formulated and requires further improvement. It is not by chance that the International Photogrammetric Society adopted in 1956 recommendations on the need to conduct scientific research in all countries with the aim of developing in the immediate future the theory and methods of determining forest characteristics from aerial photographs, and that in 1961 the Eighth All-Union Conference on the Theory and Practice of Interpretation of Aerial Photographs took note of the underdevelopment of theory and adopted as the top-priority problem the formulation of general theoretical fundamentals and objective methods of interpreting aerial photographs. Today everyone admits that the empirical approach cannot yield a satisfactory solution to practical problems, but that the scientific solution of these problems is possible only as we endeavor to understand correlations and to formulate suitable theory and methods.

We know that in topography and geodesy theory, procedures of measurement, and methods of mathematical treatment of information about the planar-height position of points in a locality are well developed. The situation is altogether different with information about forests. In practical terms, this information has remained outside the field of interest of the topographic and geodesic sciences. But they still have not been able to resolve these problems with classical methods of geodesy and topography, since a forest is in the class of phenomena whose correlations are apprehended by other methods, mainly mathematical-statistical methods of study. Forests occupy about 30 percent of the land surface of the earth, and about 25 percent of the world's forests lie within the USSR, and so information about forests bears the same national-economic and defense importance as does any other kind of information about a locality. Therefore, it has become self-evident in our time that information about forests must also have its own theory, measurement procedures, methods of mathematical treatment, and substantiated principles on precision and reliability of securing this information both from aerial photographs as well as from field work.

As we see it, at least seven principles must underlie the theory and methods of obtaining information about forests.

1. Discovering and using statistical correlations of the distribution of stand indexes for the purpose of applying them to secure information about forests from aerial photographs and from field work. These correlations of distribution have been described in the foregoing chapters.

2. Determination of mean values of appraisal indexes as parameters of distribution series. The heart of the matter is that we always estimate not a single object -- a tree, but a population of objects -- trees of stands. A complete idea about a tree population is yielded only by a distribution series, and the latter, if it is known, is characterized by specific parameters. Therefore, determination of tree stand indexes means determination of parameters of a distribution series. In most cases the mean values emerge as such parameters, that is, mean distance between trees, mean height, mean diameters of crown and stem, etc. Recognition of the statistical nature of appraisal indexes requires a stochastic approach to setting up operations and appraising measurement results. From habit we sometimes attempt to apply ordinary estimation criteria to statistical phenomena which differ sharply from functional relationships. We can never require from statistical phenomena what they by their nature do not exhibit, however they are marked by specific correlations which must be known and properly employed.

We must also clearly see a fundamental difference between precision in measuring a single object and precision in determining mean variables as functions of parameters and distribution series. Joint studies conducted by the Alabama Polytechnic Institute in the United States and by the Committee of Forest Photogrammetry of Sweden can serve as a fresh example of research on precision in measuring individual trees. These investigations are narrow and specialized, since precision in measuring the height of a single tree from aerial photographs characterizes the precision of the instrument and the properties of the aerial photographs, but says nothing about and gives no idea as to the precision of determining mean height of a population of trees in a forest plot analogous to the situation in which the precision of measuring the height of an individual point in a terrain has nothing to say about the precision of determining the mean height in a section of a locality. If we measure quite precisely the distance between two given trees, this distance gives us no idea about forest density. The mean distance is another matter altogether, for by it we get data about density and about the percentage ratio of different distances in the given forest tract.

3. Discovery and use of correlations between indexes of stands, especially multiple and curvilinear correlations.

4. Use of the sampling method in formulas for calculating the number of measurements for a given level of precision and reliability of information about trees we have secured. Compared with exhaustive measurements, the sampling method considerably reduces the laboriousness of operations and affords the required precision when the suitable methods of measurements are applied for aerial photographs and in the field. In using sampling method formulas, knowing the variance coefficients of tree-stand indexes and the mandatory use of a priori information given us by aerial photographs and visual observation of tree stands at the locality are of decisive importance.

5. Study of tree stand composition at the locality and the structure of the photographic image of the upper forest canopy in aerial photographs of different scales with the aim of discovering the reasons for discrepancies between field and office measurements and taking them into account in determining forest characteristics from aerial photographs.

6. Use of the general method of regionalizing a forest using aerial photographs, based on the regionalized method of sampling and on the structural types of photo imaging

of different tree stands. Depending on the size and nature of crown, density, and age of stands reference standard aerial photographs are prepared with precise quantitative indexes of tree stands. These reference standards in conjunction with ocular and stereoscopic inspection of aerial photographs afford a more detailed and precise delimitation of forest plots differing in appraisal indexes.

7. Development and verification of all methods of determining indexes by experimental measurements in forests and on aerial photographs of different scales with calculation and analysis of distribution series of deviations, mean square deviations, relative errors, and systematic corrections. As in any measurement work, we must provide for independent control of field and office measurements by different methods of determining forest characteristics with the aim of preventing gross errors even at the moment when operations are carried out.

Let us now look at several principles relating to operational methods. An aerial photograph bears considerable information about forests; however, all this information is transmitted in graphic form in the aerial photograph. To secure information about forests in quantitative digital form it is necessary to carry out measurement operations using methods ensuring satisfactory precision in determining tree stand indexes. With this purpose, we must first of all carry out regionalization of forest tracts in aerial photographs. This regionalization is best conducted in three stages.

The primary regionalization has the aim of delimiting homogeneous tracts within the forest expanse; it is carried out ocularly based on clearly enough distinguishable structures of the photographic imaging of the upper forest canopy.

Second regionalization is carried out within tracts with the aim of delimiting plots that are even more homogeneous as to density and crown size in the interests of ensuring higher precision in determining appraisal indexes. Here reference standards are used and when necessary stereoscopic inspection of the tracts.

The third regionalization is carried out within the limits of tracts for the purpose of determining the habitat of sample plots, in which indexes of tree stands will be subsequently measured by methods described in the following sections. The singled-out plots are inspected stereoscopically. The total number of square or rectilinear plots, straight lines, polygons, and their size is calculated according to sampling method formulas to the specified precision and reliability of determining forest density, mean distance, thickness, height, and crown diameter of trees, as well as canopy closure.

To boost reliability of information obtained, control over measurements is indispensable. It is possible to use repeated measurements by different persons or measurements of the same variable by two different methods, which of course is preferable, as control measurements. The criteria of inacceptable disparities is best taken as deviations exceeding the tripled value of the mean square deviation, and for the latter it is best to take the specified precision of the determination of the tree stand index.

Since in forest appraisal the main job boils down to determining the yield, it appears worth while to establish the precision of determination of individual appraisal indexes depending on the specified precision in determination of stand yield. However, in determining yield wide use is made of occupancy, which is determined by an indirect procedure via canopy closure, where the very concept of occupancy has no quantitatively precise and unambiguous expression and gives rise to doubts about the concept of normal stands. Thus, determination of yield via occupancy hampers discovery of the causes of errors and establishment of strict precision for measurements of all variables. Accordingly, it appears sensible to determine yield from actual measured variables (forest density, mean tree spacing, height and thickness of stems) and from tables of stem volumes [14, 56]. Under this method precision in determining yield can be checked by the precision of directly measured variables and by the precision of tables of stem volumes.

If the volume of a single stem

$$V_0 = f(d_m, h), \qquad (80)$$

and the yield per hectare

$$Z_0 = F(N, V_0), \qquad (81)$$

then the precision of yield determination

$$m_Z = \sqrt{m_N^2 + m_V^2}, \qquad (82)$$

and the precision of stem volume determination

$$m_V = \sqrt{m_D^2 + m_H^2}. \qquad (83)$$

Let us assume that m_C = 15 percent, and $m_N = m_D$, then $m_N \approx$ 10 percent. In determining N from l, our investigations showed that

$$m_l \approx \frac{m_N}{1.58} \approx 0.63 m_N, \tag{84}$$

but when m_D = 10 percent precision in determining mean diameters and tree heights, respectively, is $m_d \approx 5$ percent and $m_h \approx 4$ percent, if we use the existing stem volume tables.

We took by way of example the precision of yield determination m_Z = 15 percent and obtained the required precision in determination of all the remaining appraisal indexes (m_N = 10 percent, m_D = 10 percent, m_d = 5 percent, m_h = 4 percent, and $m_l = 0.63 m_N$). In a similar way we can compile precision classes for estimation of yield (for example, each 10 percent) and the precision classes in appraisal index determination corresponding to them. In our view, this method of establishing precision classes in appraisal work bears certain advantages.

These precision levels of appraisal indexes are then used as criteria for control of measurements using different methods and for calculating the number of measurements based on sampling method formulas.

Precision in yield determination by the described method can be boosted by increasing the precision of existing stem value tables and by using distribution series of trees based on thickness determined by sampling in the stands under appraisal.

Yield is determined by several procedures. The procedures of determining yield by stories and species with calculation of total cross-sectional area for the corresponding thickness classes and height categories are too complex. Procedures of yield determination from stand growth pattern tables require information about occupancy, site, age, and species composition.

Simpler is a procedure of determining yield from stem volume tables, whose use requires information about thickness classes, height categories, species composition, and forest density per hectare.

The nomographic method of yield determination proposed by Professor N. P. Anuchin requires for the dominant species preliminary determination of occupancy, and occupancy has to be determined from mean tree spacing, mean height, and mean diameter.

The formulas of Professor N. V. Tret'yakov afford a simple method of determining yield when there is information available about mean height, occupancy, and dominant species. However, here as well we must take into account occupancy, and to replace occupancy by canopy closure evidently leads to a drop in precision of yield estimation.

The simplest method of yield determination from aerial photographs can be regarded as the method that affords estimation of the yield even though in approximate terms from measured mean heights, mean tree spacing, canopy closure, and mean thickness obtained from tables of its correlation with height and crown diameter.

We obtain from h and d_{tree} in stem volume tables the volume of an average tree in a stand. For 1, we get from Table 1 N, the number of trees per hectare. Multiplying these variables, we find the yield in cubic meters per hectare. We measure the forest plot area from the aerial photograph or a map. The product of the area in hectares by the yield per hectare gives us the overall yield for the forest plot. When information is available on the dominant species, this variant of yield determination is simplest of all. In the absence of information on species composition, this method will yield lower precision of yield estimation.

Precision in ground ocular determination of yield in sample plots is customarily taken as ±10-12 percent. The yield from scale 1:10,000 aerial photographs for the dominant species is estimated to be approximately 20 percent less than the overall yield determined by ground methods.

For practical verification of the precision in the approximational method of yield determination from h, d_{tree}, and 1 and from stem volume tables, limited field work has been carried out.

In the first sample plot ground methods of appraisal were used to determine the yield by two procedures. Based on forest elements (stories and species) the yield of first-story pine Z_1 = 217 cubic meters, yield of first-story spruce Z_2 = 136 cubic meters, and yield of second-story spruce Z_3 = 19 cubic meters. The overall yield per hectare Z = 372 cubic meters. From tables of stem volume, thickness class and height categories the overall yield Z = 389 cubic meters.

Consequently, determination of yield by the two procedures gives a difference Δ Z = 17.5 cubic meters, or 5 percent.

We will give data from a determination of yield by an approximational procedure relying on l, h, and d_{tree}, and on stem volume tables.

In the first sample plot l_0 = 4.4 meters (obtained by exhaustive measurements), which from Table 1 gives N = 624 stems per hectare.

h_0 = 22.2 meters and d^o_{tree} = 29.6 cm. Based on h_0 and d^o_{tree}, we obtain from the stem volume table for pine the volume of a single mean tree V_0 = 0.620 cubic meter. Then the overall yield Z = 0.620·624 = 387 cubic meters, which gives an error Δ Z = 387 - 389 = -2 cubic meters, or 0.5 percent. It is clear from this that determination of total yield based on l_0, h^o, and d^o_{tree} gives good precision.

Let us now determine yield based on mean distance obtained by the sampling method, that is, l_{samp} = 4.34 (polygon method). When l = 4.34, N = 645. Then Z_{samp} = 0.620·645 = 400 cubic meters, which gives an error Δ Z = 400 - 398 = +11 cubic meters, or 3 percent.

In the second sample plot the yield, determined by two precise methods, Z_1 = 525 cubic meters and Z_2 = 512 cubic meters. From h_0 = 27.6, d^o_{tree} = 38.1, and l_0 = 5, or N = 488, the yield Z_{tree} = 585 cubic meters, and the error Δ Z = 73 cubic meters, or 15 percent. Based on the sampling determined l_{samp} = 4.9, or N = 51[illegible], the yield Z_{samp} = 622 cubic meters for an error Δ Z = 92 cubic meters, or 18 percent.

For the third sampling plot the yield determined by the appraiser from stem volume tables is Z_1 = 297 cubic meters, but from standard tables via occupancy Z_2 = 294 cubic meters.

Based on h_0 = 24.5, d^o_{tree} = 31.2, and l_0 = 5.9, or N = 356, yield Z_3 = 315 cubic meters, which gives an error Δ Z = 23 cubic meters, or 9 percent.

For the fourth sample plot the yield determined by an appraiser from occupancy and from tables, Z_1 = 246 cubic meters, from model trees Z_2 = 247 cubic meters, and from stem volume tables Z_3 = 247 cubic meters.

Based on l_0 = 4.3, N = 657, h_0 = 25, and d^o_{tree} = 24, the yield Z_4 = 326 cubic meters, which gives an error Δ Z = 79 cubic meters, or 30 percent.

Based on the sampling l_{samp} = 4.45 and N = 603, the yield Z_{samp} = 299 cubic meters, which gives ΔZ = 52 cubic meters, or a 20-percent error. Here we must bear in mind that the appraiser determined the yield only for the dominant species without taking into account the second-story yield.

In the fifth sample plot the yield (based on standard tables) Z_1 = 325 cubic meters, and from volume tables Z_2 = 330 cubic meters. Based on l_o = 5.07, N = 482, h_o = 24.8, and d^o_{tree} = 31.1, the yield Z_3 = 398, but based on sampling l_{samp} = 5 and N = 492, the yield Z_{samp} = 406 cubic meters, which gives in the first case a 20-percent error, and in the second 22 percent.

Table 52 lists composite data on precision of yield determination based on sampling-determined mean tree spacing, height, thickness, and on stem volume tables, where Z_{samp} is taken for l_{samp} arrived at by the sampling method.

Table 52

Участок (A)	Z полевое (B)	$Z_в$ выборочное (C)	ΔZ, м³	ΔZ, %
1	389	400	+11	3
2	[illegible]	[illegible]	+97	18
3	[illegible]	[illegible]	−23	9
[illegible]	[illegible]	[illegible]	[illegible]	20
5	[illegible]	[illegible]	[illegible]	22

Legend: A -- plot; B -- field; C -- Z_{samp}, sampling

It is clear from Table 52 that the simplified method of determining yield from mean tree spacing gives on the average a 15-percent error, toward the overstatement side.

Procedures and precision in determining yields from aerial photographs have been described in detail in the works of many Soviet appraisers.

In 1941 the Experimental Station of the U.S. Forest Service made an attempt to determine yield from aerial photographs with field control.

Since 1942 serious articles and works on forest interpretation have begun to appear in the United States and

Canada. Following 1945, more major studies on measuring height, crown diameter, and canopy closure from aerial photographs have been begun in Canada. The question of compiling tables suitable for describing yields from aerial photographs has also been raised. In 1946 Spurr (United States) drew up the first yield table specially adapted for determination of yield based on crown diameters, closure, and tree heights measured from aerial photographs.

In Canada workers have begun to determine yield in the field by using aerial photographs and then have commenced efforts to find methods of determining yield directly from aerial photographs. Aerial photo-tables of yields (Photo-volumes) per acre (0.4 hectare) have been compiled for this purpose.

Essentially, these are reference standard aerial photographs in which forest plots with known yields are specified. Comparison of these reference standards with the aerial photographs of other forest tracts has made it possible to determine yield. In the United States, in 1948 Nash, in 1950 Pope, and in 1952 Spurr, and in Canada in 1951 Moessner compiled tables for yield determination based on D_c, C, and h measured off of aerial photographs. In 1958 Spurr noted that the low precision of tables compiled in the United States had impeded their wide use. Therefore (in 1963) the principal method of forest appraisal in the United States was the method of combining field work with use of aerial photographs. In Canada use of special yield tables adapted for determination of C from crown diameter, mean height, and canopy closure measured from aerial photographs began to be used. The first type of yield table gave the volume of the single tree which was determined from two input data -- crown diameter and mean tree height. These tables are used only in mature forests (they are not employed in dense forests). Table 53 is an example of a table giving the volumes of a single tree.

The second type of yield table is called standard volume table. The tables give the volume per acre in cubic meters and have three input variables -- the mean crown diameter, or the measurement visible in aerial photographs, the standard (mean) height, and canopy closure (crown cover) in percentages. An example of the second type of yield table is Table 54.

The variable h is determined from the shadows and measurements of parallax in the aerial photographs. The variable D_c is measured on the aerial photographs using an incline

GRAPHICS NOT REPRODUCIBLE

Table 53

A d_c	B Средняя высота h_o					
	20	30	4[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	3,2	[illegible]	[illegible]	[illegible]	
11	2,8	[illegible]	[illegible]	[illegible]		
12	3,4	[illegible]	6,7			
13		[illegible]	8,0	[illegible]	[illegible]	[illegible]
14		[illegible]	9,3	[illegible]	[illegible]	[illegible]

Legend: A -- d_c, in feet; B -- mean height h_o in feet

Table 54

A	B, C				
	[illegible]	3[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]		[illegible]	[illegible]
[illegible]	[illegible]	[illegible]		[illegible]	[illegible]
[illegible]	[illegible]	500	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	550			[illegible]
[illegible]	[illegible]	[illegible]	[illegible]		[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Legend: A -- h in feet; B -- D_c = 10-14 feet; C -- C in percent

level, or else a template consisting of circles with previously calculated diameters. Templates of circles are printed on transparent film. Under the stereoscope, D_c is determined by visual selection of the appropriate template.

Canopy closure is determined also from scales, consisting of a series of squares with black circles covering from 5 to 95 percent of the square area. By visual comparison of the scale with the forest tract on the aerial photograph C is determined (the value of C is imprinted along the borders of the squares) under the stereoscope, thus excluding the effect of shadow cast by crowns. As Moessner has noted, this procedure of yield determination gives satisfactory precision. The Canadians determine the mean tree thickness in the field on sample plots.

In 1958 most appraisers in Canada used the combination of determining appraisal indexes from photographs with field control in sample plots. Use of the two earlier-described tables of interpretation yields considerably reduces the number of sample plots processed under field conditions.

In order to study possibilities of considerably reducing field operational work by determining appraisal characteristics from aerial photographs, Losee conducted studies on aerial photographs of scales 1:7200 and 1:1200. Field work was carried out on 0.1 acre sample plots with determination of mean h, C, and D_c values. The yield tables were also compiled from field data for h, C, and D_c. Using 1:7200 scale aerial photographs mean height was determined with an error of ±2.1 with systematic error of +0.6, and 1:1200 scale aerial photographs were used for determination of mean height with an error of ±0.5 and a systematic error of +2.1. Canopy closure was measured at the locality with a precision of ±1.7 percent. Canopy closure was determined from aerial photographs, as Losee emphasized, by an original method developed on the basis of extended study. The count in each section was conducted 12 times. C was determined by this method on 1:1200 scale aerial photographs with an error of ±5.5 percent and a systematic error of -0.3 percent, and on 1:7200 scale aerial photographs with an error of ±9.9 percent and a systematic error of -1.3 percent. We note that this method is too laborious (12 counts per area) and yielded the specified precision only because 12 counts were made. With a smaller number of counts the method will yield a considerable error, since it is not associated with direct measurement of canopy closure and gives only an indirect idea of the variable measured. Mean D_c was determined by measurement under the stereoscope of 30 crowns for each area. D_c was measured on 1:1200 scale aerial photographs with an error of ±0.33 and a systematic error of -0.09.

The yield was determined from measured h, C, and D_c and from yield tables compiled from field data h, C, and D_c.

Z per acre was obtained from the table, and the overall yield for the entire area was determined by multiplying Z by the total area of the forest section. The error in determining total yield from 1:1200 scale aerial photographs was ±7.6 percent, but from 1:7200 scale aerial photographs ±4.3 percent. Losee viewed the results as highly satisfactory, demonstrating the practical possibility of excluding all field work in forest appraisal, with the exception of collecting data for compiling yield tables (based on input data -- h, C, and D_c).

Director Garver of the Forest Service of the U.S. Department of Agriculture noted in 1953 in an article "Interpretation of Aerial Photographs in the Forest Service" that aerial photographs are widely used in the U.S. in forest appraisal. Plots of one-fifth acre size serve as standard sample plots in which all field measurements are made. Some efforts have been made to determine yields directly from aerial photographs, but thus far this has not become standard practice. Efforts are underway to develop volume tables for a single tree based on h and D_c measured from aerial photographs. In other cases use is made of standard yield tables based on mean heights, crown diameters, and canopy closure. In appraisal 1:15,840 scale aerial photographs are mainly used.

Rogers (U.S. Forest Service) wrote in 1956 that use of aerial photographs for yield determination where high precision is not required affords bypassing altogether or in large part of field work. But even where precision is required and is important, use of aerial photographs reduces the volume of field work.

There are two directions in yield determination in the United States. The first consists of estimating the yield directly from aerial photographs without verification in the field. This method is supported by a minority of appraisers. Most specialists endorse the combination of work with aerial photographs and in the field. Rogers noted that precision in determining yield from aerial photographs differs for different researchers, but forest interpretation is a new and promising effort. Therefore, studies in this direction must be continued.

Precision in determining tree heights from aerial photographs has been investigated by Professor Sammi of New York University [80]. In the University of Pennsylvania Worly and Landis [83] are studying precision in measuring heights, crown diameters, and canopy closure from aerial photographs. Yang has been engaged in determining tree counts from aerial photographs. He has concluded that the number of counted trees is reduced by 20 percent as one turns from 1:3500 scale aerial photographs to those of 1:15,000.

Hendrich and Meyer assume that yield can be determined from aerial photographs with an error of 25 percent.

Meyer and Worly [71] hold that estimation of yield by using standard tables and measurements made from aerial photographs can be carried out with an error up to 50 percent.

Rogers [78], in characterizing scientific research in aerial photography of forests and interpretation in the United States, Great Britian, Canada, France, the FRG, Sweden, Norway, the Netherlands, Thailand, and Czechoslovakia, believed that precision of yield determination for large forest tracts is close to 10 percent. Rogers has suggested that there is a possibility of estimating yields from aerial photographs without using yield tables.

A characteristic feature in the progress made in interpretation of aerial photographs of forests and other plant cover in the United States and Canada is that many military and intelligence establishments have dealt with this problem in wartime and especially in the postwar period.

Thus, Churchill (Military Intelligence Board of the U.S. Army) prepared the study, "Types of Vegetation for Interpretation of Aerial Photographs and Use of Vegetation as an Indicator of Habitat Conditions," in 1953.

O'Neill, a PhD of the Catholic University in Washington, has developed a method of preparing reference standards -- keys for interpreting vegetation from aerial photographs [75].

We know that without theory and without a grasp of correlations of distribution and statistical relationships of tree stands as natural phenomena, it is impossible to formulate new and more improved methods of counting yields. This has been emphasized in our literature and abroad. The Austrian specialist Akkerl has noted that efforts to find and use more improved methods of determining yield are impossible owing to the lack of relationships established between forest characteristics in the actual locality and on aerial photographs. The Austrian Forest Service has outlined a program of scientific research in this area.

Axelson (Sweden) has studied the question of precision in determining tree height from aerial photographs of different scales and has concluded that scale does not affect precision of measurement of h. Error in determining h from aerial photographs amounts to 1.5-3 meters. He has asserted that large-scale aerial photographs do not afford advantages in determining canopy closure and yield, where the canopy measured from aerial photographs is generally overstated. Axelson recommends use of 1:33,000 scale aerial photographs, but with a subsequent increase in scale to 1:15,000 for interpretation.

In 1960 the American Society of Photogrammetry, with participation of American and Canadian specialists, published Handbook on Interpretation of Aerial Photographs, in volume about 50 author's sheets [each sheet containing 3,000 square centimeters of printed material] [72]. The handbook includes 16 chapters in which different forms of aerial photograph interpretation are set forth (geological, soil, forestry, agriculture, engineering, geomorphological, archaeological, hydrological, zoological, etc.).

This handbook repeats and reinforces several findings arrived at by Soviet specialists in forest interpretation of aerial photographs. To increase precision in measurement interpretation of dense forests winter photographs are recommended; it is noted that yield correlates more closely with height, and not with crown diameter and canopy closure; a point method of measuring areas is proposed. This method has been developed and advanced for use in the U.S.S.R. [13].

In spite of the existence of large studies conducted by specialists of many countries of the world, the theory and method of interpretation of aerial photographs of forests are in need of further advances.

Underlying ocular appraisal is visual comparison of the given tree stand with a tree stand of a sample training plot. Information in the form of the visual image of the tree stand in the sample plot is retained in the appraiser's memory. Accumulation of such visual images is acquired by experience. However, remembering, preserving in one's memory, and reproducing visual images of complex tree stands is extremely difficult; they are forgotten and essential details are lost, which calls for constant replenishment of the images by training sessions obligatory for each appraiser before work begins.

Accordingly, a suggestion about the possibility and value of replacing complex images of tree stands in all their completeness by simpler images of individual appraisal indexes which can be easily remembered, compared, and controlled by the measurements with desired precision has been advanced. It is very difficult to control the precision of ocular appraisal, since the appraiser and the control individual perceive and reproduce the same visual images of tree stands even in the sample plots dissimilarly. Therefore, methods of determining yields by relying on direct-measured l, N, d^{o}tree, and h_0 bear purely practical advantages as well.

In conclusion, we cannot fail to note that the theory and practice of the use of aerial photography and aerial photographs in forest management in the U.S.S.R. in many respects has been better advanced than in the United States and Canada.

In fact, in the study and understanding of statistical correlations of tree stand structure the U.S.S.R. doubtless leads all other countries.

A well-argued and scientifically substantiated program of scientific research and solution of practical problems dealing with methods of appraisal and forest management has been set forth in the study [31] by Docent B. A. Kozlovskiy, Director of the All-Union Amalgamation Lesproyekt.

27. Methods of Determining Mean Tree Spacing and Forest Density in Fieldwork

Seven procedures of the sampling method have been used, tested, and verified for determination of density and mean tree spacing.

1. Procedure of measuring distances in a specified direction. Exhaustive measurements of all distances in five forest plots were conducted following the method set forth in sections 4 and 5. Sampling determination of mean spacing l_{samp} was carried out by measuring distances between trees encountered along a route following a selected direction. Sampling l_{samp} was obtained as the arithmetic mean of all measurements. The true mean distance l_0 was determined from data of exhaustive measurements.

Table 55 gives the characteristics of the precision of determination made in five plots.

Table 55

(A)	Число направлений (B)	Число расстояний (C)	(D)	l_0	(D)	[illegible]
1	2	[illegible]	[illegible]	4,4	−1,2[illegible]	−28,0
1	2	[illegible]	[illegible]	4,4	−0,7[illegible]	−16,5
2	3	[illegible]	[illegible]	5,17	−0,97	18,7
3	3	[illegible]	[illegible]	[illegible]	+2,00	34,0
4	2	[illegible]	[illegible]	[illegible]	−0,05	−1,0
5	3	19	5,[illegible]	5,1	+0,08	+1,5

Legend: A -- plot number; B -- number of directions; C -- number of distances; D -- l_{samp}

The data in Table 55 evidence the instability of the results, for the mean l_0 exhibits decided deviations both toward the overstatement side (34 percent) as well as to the understatement side (-28 percent) depending on the practitioner, the choice of direction, and the number of directions. This procedure is subjective and cannot ensure satisfactory precision.

In 1952 N. P. Anuchin proposed a nomographic method of determining occupancy and yield based on measurement of mean tree spacings.

However, his recommended method of determining mean spacings by measuring l between 30 trees encountered in a given direction cannot afford any satisfactory precision.

2. Procedure of measuring distances in traversing an external contour of the forest plot. In the course of experimental studies distances are measured only between those trees that stand close to the boundary of the plot, while trees along the bends of the contour are not counted. The mean l_{samp} determined by this method proved to be 3.4 meters greater than the true l_0. For supplementary verification of the method, measurements were made of distances between all trees standing along the boundary of a tortuous contour of the forest plot. In all, 90 tree distances were measured as compared to 36 in the first case.

The mean l_{samp} proved to be 14 percent less than the true value. Thus, this procedure is too laborious and not fully reliable, although it is important for appraisers and topographers who, in moving through felling areas and along roads, will be able to determine the mean distance without traversing within the forest.

3. Procedure of measuring all distances in sampling plots 20 x 20 meters in size affords satisfactory precision, but is too laborious.

4. Procedure of counting trees in sampling plots.

All trees in typical plots 20 x 20 meters in size (or 10 x 10 meters) were counted, and the mean distance was calculated from N taken from Table 1.

Table 56 gives the results of l_{samp} determination.

Table 56

№ участка (A)	Число площадок (B)	[illegible]	$l_в$ (C)	l истинное (D)	M_l, м	M_l, %
1	[illegible]	622	4,3	4,[illegible]	-0,10	2,[illegible]
[illegible]	[illegible]	188	5,05	5,17	-0,1[illegible]	[illegible]
[illegible]	[illegible]	358	5,9	5,[illegible]	0,00	[illegible]
1	[illegible]	48[illegible]	5,0	4,3	0,70	15,[illegible]
[illegible]	1	[illegible]	[illegible],7	5,1	0,60	10,[illegible]

Legend: A -- plot number; B -- number of plots; C -- lsamp; D -- true value

We can plainly see from Table 56 that the procedure of counting trees in plots 20 x 20 meters in size gives satisfactory precision when determining mean tree spacing and stand density.

In dense forests we can take smaller areas, that is, 10 x 10 meters, which reduces the volume of work by four to eight times. Reducing area size and increasing the number of plots involved affords fuller coverage of the diversity of distances within the section, that is, obtaining a more representative sampling population. However, further decrease in section dimensions is not worthwhile.

The required number of sampling plots can be determined from the following formula:

$$S_n = \frac{t^2 v_n^2}{\Delta_n^2} \qquad ([illegible])$$

Legend: 1 -- Ssamp

Literature on forest management contains no information about the variance of tree stand density. From our investigations, the variance of tree stand density in areas 20 x 20 meters in size is $v_n \approx$ 15-18 percent.

The approximate value of the variance of stand density was determined from the data of field measurements and counts made in two sections of pine-spruce mature tree stand covering an area of 10-15 square plots 20 x 20 meters in size each. Calculations of the variance of density for the two sections are shown in Table 57.

Table 57

Section no 1

№ (A)	n	[illegible]	Δ_n^2
1	28	[illegible]	9
2	27	[illegible]	[illegible]
3	27	[illegible]	4
4	22	[illegible]	[illegible]
5	23	[illegible]	4
6	32	[illegible]	49
7	[illegible]	[illegible]	9
8	1	[illegible]	1
9	31	[illegible]	36
10	22	[illegible]	9
	251	[illegible]	1?7

$n_0 =$ [illegible]

$\sigma_n = 1.$[illegible]

$c_v = 1$[illegible]

Section no 2

[illegible]

Legend: A -- plot number

If we take as standard plots squares $4 \cdot l_0 \times 4 \cdot l_0$ in size, where l_0 = mean tree spacing, then the number of trees n in these plots will be a constant, invariant relative to l_0, that is,

[illegible] (86)

where N = number of trees per hectare.

Formula (86) is based on the reciprocal function relating l and N in point systems (which well characterizes the curve in Figure 24) and the approximate correlation of the function $\omega = l^2 N \approx$ const remarked on by the author. If the square areas are taken as $5 \cdot l_0 \times 5 \cdot l_0$ in size, the number of trees in these plots will also be a constant, but equal to

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$$\cdots = \frac{[illegible]}{1000} \approx 29.$$

Formula (86) can be used for control of fieldwork using the sampling method of determining tree stand density and mean tree spacing.

5. Procedure of measuring sides of polygons. A polygon is formed by a group of neighboring trees. The distance between trees is measured along the smallest sides (straight lines), where none of the sides must intersect any other side (Figure 25).

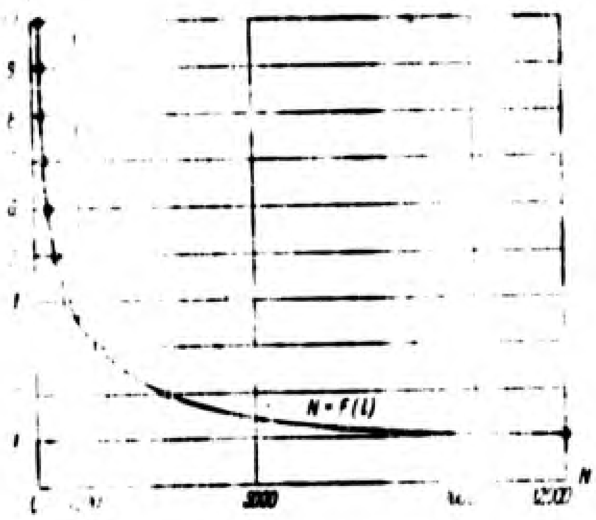

Figure 24. Tree spacing as a function of forest density

Polygons are selected in typical sites of the forest plot for the purpose of discovering precisely the mean tree spacing. Polygons must not be selected in thinned-out areas that are becoming clearings, or in sites of intense clumping of stems. Precision in determination of l_{samp} depends on the correctness in choice of polygon location in the plot. Here principles of the regionalized method of sampling studies are fully applied.

Table 58 gives the results of a determination of mean spacing l_{samp} by the polygon method with approximately 20-24 distances provided in two polygons. The true mean distance l_0 was obtained from data of exhaustive measurements of distances or counts of the number of stems in the forest plot.

It is clear from Table 58 that the error in determining mean tree spacing by this method does not exceed 6 percent, or 0.3-0.4 meter. This precision must be deemed satisfactory, since most practitioners have used this method the first time and have become acquainted with it from a brief explanatory note. The sampling method of determining l_0 by the polygon procedure with the condition of regionalized sampling method complied with can give satisfactory precision.

Table 58

№ участка (A)	(C)	l_0	Δl, м	Δl, %	(B) [illegible]
1	4,32	4,40	–0,08	2,0	2
1	4,36	4,40	–0,04	1,0	2
2	4,86	5,17	–0,31	6,0	2
2	[illegible]98	5,1[illegible]	–0,19	3,6	2
3	[illegible],27	5,90	[illegible]0,37	5,7	2
4	4,[illegible]	4,[illegible]	0,15	3,5	2
5	[illegible],00	[illegible]	–0,07	1,0	2

Legend: A -- plot number; B -- number of polygons; C -- lsamp

6. Procedure of counting trees along straight lines. This procedure consists in drawing lines in the selected plots for measurements and counting trees, whose crowns intersect or are tangent to a straight line, where only those trees that are projected onto the straight line along the perpendicular are counted. The mean tree spacing is obtained by dividing the length of the line by the number of trees counted.

Figure 26 shows a copy of a forest plot 20 x 20 meters in size on the 1:100 scale with point location of each tree and point projection of crowns. The straight lines are designated No 1 and No 2, and the crowns intersecting the lines are designated by numbers. The true l_0 = 4.4 meters. From the two mutually perpendicular straight lines the sampling lsamp = 4.58 meters. The error Δl = +0.18 meters, or 5 percent.

In the second forest section two pairs of straight lines 20 meters in length were selected in the two sampling plots, which gave an error Δl = -0.07 meter, or 1.5 percent.

The straight line procedure considerably simplifies and speeds up work, since time-consuming measurements of

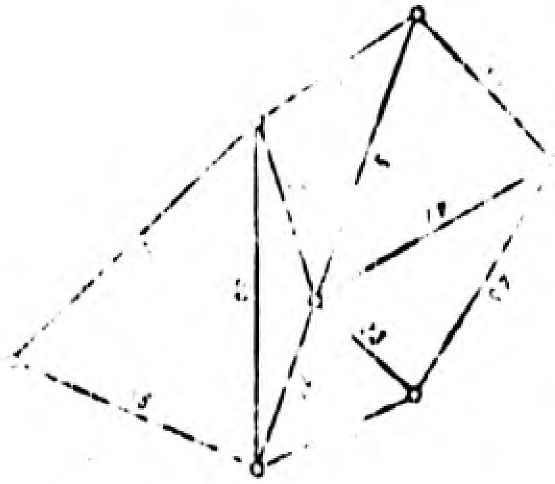

Figure 25. Polygon for determination of mean tree spacing

distances are replaced by a simple count of stems whose crowns intersect or touch a straight line. Whoever counts the trees, the results will always be identical. Therefore, the procedure does not depend on the practitioner, while in the procedure of determining distances in a specific direction each practitioner decides on his own what trees to include and what not to include in measuring tree spacing.

From experimental data, the straight line method can give satisfactory precision for four pairs of straight lines in a section, however in choosing the placement of the straight lines we must be strictly governed by the rules of the sampling method. The necessary number of lines and the length of the lines can be calculated from the author's formulas presented below.

Let us assume that n points separating r mean spacings lo will lie on the line L.

Then

$$l_0 = \frac{L}{n}, \tag{87}$$

$$r = n - 1. \tag{88}$$

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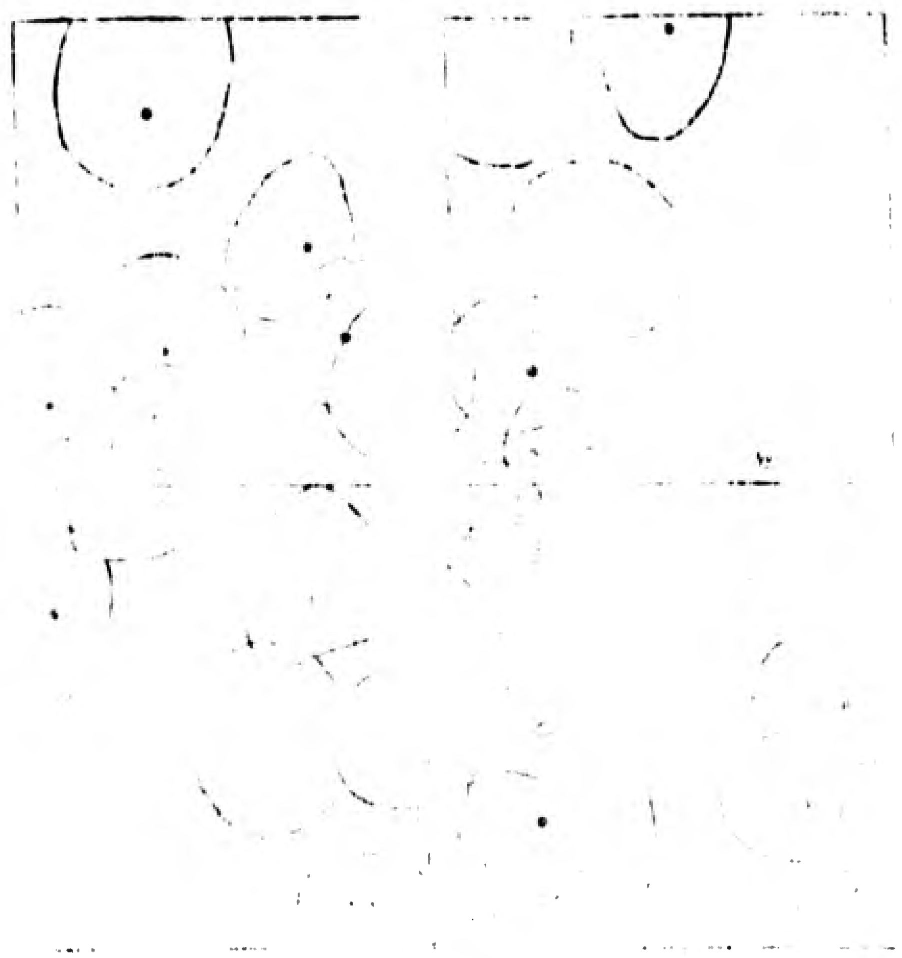
Section no 1



(A)

Legend: projections of tree crowns

Figure 26. Copy of forest plot on the scale 1:100 for determination of mean tree spacing by the straight line procedure

but the number of straight lines

[illegible] (89)

(A)

Legend: A -- Sstr

where the number of measured spacings ssp in determination of l_0 with a desired precision Δ and confidence t is calculated from the formula

(A) [illegible] (90)

Legend: A -- ssp

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Consequently, the number of lines can be determined from the following approximate formula:

Ⓐ

Legend: A -- S_{st}

We assume that we will limit ourselves to a single straight line. Then when $S_{st} = 1$ from formula (91), we obtain the required length of the straight line from the following approximate formula:

The length of the straight lines must satisfy the condition $L > 4 \cdot l_o$. It is best to round off the calculated line length to a whole number of tens of meters. It is useful to divide the total length of the line into four to five segments. These new straight lines $A = L/4$ are placed in the plot being appraised in typical locations. Use of the straight line procedure when $l_o = >$ 7-8 meters is not worth while in forests that have trees bearing small crowns.

7. Procedure of selecting the most typical spacings in the forest section. The ocular choice of mean spacing requires inspection of the forest section. This procedure is typical in the sampling method. Its use requires special training like the kind that appraisers always undergo before carrying out ground ocular appraisal. For successful use of this method one must be well acquainted with the law of spacing distribution and the theory of sampling method. Typical spacings selected are measured with a tape measure. According to experimental data, the procedure gave an error of 5-11 percent for two forest sections.

These data evidence that with suitable experience the procedure can afford satisfactory precision.

Experimental verification of the seven procedures of determining the mean spacings and tree stand density under field conditions have shown that the following procedures give satisfactory precision as well as stability of results: the polygon procedure, the procedure of counting stems in plots, and the straight line procedure. The simplest procedures are: the procedure of selecting the typical spacing and the straight line procedure. The polygon procedure and the procedure of counting stems in plots are approximately the same in their workload.

Studies have shown that the sampling method of determining mean spacing given requisite compliance with its rules does give satisfactory precision.

Control of field measurements is best carried out by two independent procedures of determining stand density and mean tree spacing.

28. Procedures of Determining Forest Density and Mean Tree Spacing From Aerial Photographs

If separate imaging of the crowns of all trees located in the forest tract shows up on aerial photographs, then determination of tree stand density and mean spacing from aerial photographs does not differ in precision from the precision attained by procedures proposed and verified under experimental conditions (section 5) and in fieldwork (section 27). But, as has been pointed out, the crowns of all trees do not always show up on aerial photographs. Therefore, it becomes necessary to establish the nature and extent of reduction in the number of imaged crowns on aerial photographs of different scales compared to the total number of trees in the locality.

Experimental studies on counting crowns on aerial photographs of the following scales: 1:4000, 1:6000, 1:8000, 1:9000, 1:12,000, 1:15,000, 1:19,000, 1:20,000, and 1:21,000 were conducted by the author beginning in 1944.

In 1951-1954, a continuous and sampling count of trees in the field and from aerial photographs was conducted by the author jointly with engineer N. A. Kornilov. As a result, the number of trees, as to be expected, counted from the number of crowns imaged in aerial photographs in most cases proved to be less than actually found on the spot. Mean tree spacing determined from aerial photographs was somewhat larger than the mean spacing between all trees in the locality. An exception to this rule relates to sparse forests and forests in which tree spacing amounts to 7-12 meters and more, as a consequence of which their crowns in most cases were imaged on moderate-scale aerial photographs.

Investigations aimed at determining the number of crowns of trees imaged and those not imaged on aerial photographs of different scales have been conducted by Russian specialists in forest management: Professor G. G. Samoylovich, I. I. Mazhugin, A. M. Berezin, I. A. Trunov, and A. Ya. Zhukov.

As studies have shown, the number of trees standing under the crowns of neighboring trees and not participating in forming the projection of the upper canopy crown increases with increase in occupancy and decreases with greater age, and in most cases thin-stem trees prove to be located under the crowns of adjoining trees. This problem has been detailed in section 2.

The deviation of mean spacings on aerial photographs and in the locality can be accounted for by the following main causes. Some of the low-standing trees are under the crowns of their neighboring, higher stems or stand in their shadow at the moment of aerial photography, especially along the borders of aerial photographs, which leads to a reduction in the number of imaged crowns.

Arrangement of trees in the form of a curtain or group side by side with standing trees, and also the growth of two stems from the same root in thickets leads to merging on aerial photographs of the images of several crowns, which reduces the number of crowns counted, if no correction is made in the value of the large merging "patches" of crowns on aerial photographs when making measurements.

The number of crowns on aerial photographs of young, usually very dense forests containing crown peaks small in size is considerably reduced; the broader sections of crowns are concealed by the crowns of trees closely overlapping them.

With increase in age and reduction in forest density the deviation of mean spacings on aerial photographs and in the locality is reduced.

The strongly pronounced double-story state of stands (for high occupancy rates of the upper and lower stories) and high undergrowth or underbrush density have an appreciable effect on reducing the number of crowns counted on aerial photographs.

As scale is reduced, some of the small crowns do not show up in the scale of aerial photographs or else their image is so small that they are not visually discernible, which also leads to a reduction in visible crown images. In this case preparation of magnified aerial photographs for measurement purposes can be of help. The quality and freshness of the aerial photograph is of great importance. Convergence of spacings on aerial photographs and in the locality depends on the year the aerial photography was done and the year the spacings were measured. Old, especially small-scale aerial photographs can give perceptible disparity between mean spacings, which often is bound up both with artificial as well as with the natural change in tree stands during the years elapsing since the aerial photography.

The effect of aerial photographic scale, forest density, and crown size is well illustrated by the experimentally-built models of different density types and tree crown sizes on different scales shown in Figure 14 (section 5).

Consequently, many factors not easily allowed for and expressed numerically have a bearing on precision of determination of stand density and mean spacing from aerial photographs.

Precision of mean tree spacing determination that is satisfactory for actual use can be obtained from aerial photographs of the large scales 1:5000-1:10,000.

If moderate-scale aerial photographs are used, it is necessary to determine the corrective coefficients for introducing them into measured spacings from aerial photographs of different scales taken of forests differing in density and age with allowance for the effect of species composition, number of stories, undergrowth and underbrush, characteristics of tree arrangement in different areas, and forest expanse types.

The author has proposed a method by which the mean spacing is determined not between all trees at the locality, but only between the principal trees that are of chief importance in appraising the tree stand.

In this case we must establish the trees that we can neglect.

Traversability, camouflaging and protective properties of a forest are generally determined by the thicker and higher trees in the upper canopy that have broader crowns, that is, by those trees which are more commonly imaged in aerial photographs. Yield also depends on counting the principal trees of moderate and high thickness classes producing images in aerial photographs in most cases.

Thus, for example, yield calculated from aerial photographs proves to be 10-20 percent closer to the actual yield than the number of counted crowns on the same aerial photographs with respect to the total number of stems in the locality. This is explained by the small effect on yield of thin and low-standing trees which usually do not show up on aerial photographs.

Overall yield of stands is produced by trees of moderate and thick classes of thickness which are more generally imaged on aerial photographs. The thickness gauge usually leads to a discrepancy, and its inclusion must be made, in approximate terms, from tables of stand growth pattern.

In the Instructions [26] the main story is considered to be the story that constitutes the part of the stands largest in yield and has the highest economic value.

The second story is differentiated and appraised only when the mean tree stand thickness exceeds 8 cm, and the difference in mean heights amounts to more than 20 percent, which for a height variance v_h = 8-10 percent is an infrequent phenomenon.

As far as undergrowth and underbrush are concerned, in appraisal they are generally not taken as stories and are separately accounted for in the overall picture.

The experience of Canadian and American appraisers who determine yield directly from aerial photographs by using specially-prepared yield tables based on heights, canopy closure, and the so-called visible crown diameters measured from aerial photographs is of some interest.

The principal and most important part of trees is imaged on aerial photographs (for more detail see Chapter 1).

Based on the foregoing, we outline an approach in determining mean spacings not between all, but only between the most important trees that in fact determine economic and other importance of the given forest tract.

This principle has in fact been adopted in further efforts to find approximational methods of determining mean tree spacing and stand density from aerial photographs. To do this, experimental studies have been carried out counting trees of different thicknesses at the locality and from aerial photographs in order to arrive at an approximate idea about the stand structure that shows up in aerial photographs of different scales.

For example, take a forest section which is a non-mature forest with a large amount of undergrowth. Based on aerial photographs of scale 1:6000, 100 percent of trees more than 6 cm thick were counted; on 1:8000 scale -- 100 percent with dtree > 8 cm were counted; on a 1:9000 scale -- 90 percent with dtree > 10 cm; on a 1:10,000 scale -- about 86-90 percent with dtree > 10 cm; and on a 1:10,000 scale -- about 60 percent.

Investigations on determination of the number (percentage) of trees standing under the crowns of neighboring trees, in relation to species composition, occupancy, and age are described in section 2. These studies, begun on the initiative of Professor G. G. Samoylovich, allow us to determine the size of the correction that has to be introduced into the variables N and l measured from aerial photographs. These corrections are best found on the basis of experimental field and office work in mathematical statistical treatment of information collected. As a result of this treatment of the information, we can obtain general and differentiated corrections for pure and mixed (containing three to five gradations in species composition) stands, for two to four gradations in canopy closure (or occupancy), and for three to five gradations in age and density of tree stands.

Also of interest is a theoretical approach to solving this problem based on an understanding of correlations of stand structure. Since correlations of tree distribution by thickness, height, and crown diameter and by tree spacing is known, for homogeneous forests we can obtain from distribution series the approximate percentage of thin, low-standing, and small-crowned trees that do not show up on aerial photographs.

Below are listed the values of mean thickness d^{0}_{tree} and thickness d_{tree} of trees constituting 5 percent of the total number of stems in a forest for a given d^{0}_{tree} and the distribution corresponding to it.

d_m	10	15	20	25	30
d_m	5	8	10	12	15

Consequently, if in a forest section d^{0}_{tree} = 20 cm, then trees with d_{tree} = 10 cm and less will be about 5 percent. This means that we can state in advance that trees of which thickness d_{tree} must not be counted for a given d^{0}_{tree}, and what percent they represent of the total number of trees at the locality. This percentage is best introduced as a correction into mean spacings measured from aerial photographs, which is easily done by relying on tables relating 1 and Y.

In forests deviating very decidedly from homogeneous stands, thin trees will evidently be greater than 5 percent. Similar calculations can be made also in the determination of the percentage of small crowns and small tree heights which most probably will not be imaged on aerial photographs for a given mean ho and $D^o{}_c$. Distribution series of heights and crowns give an idea about the size of the percentage of these trees (cf sections 19 and 20).

Experimental studies in the field and with aerial photographs aimed at verifying and appraising the principle of determining mean spacings between the main trees in stands set forth here were carried out in two forest areas of the Moscow and Tul'skaya oblasts in 153 forest plots from aerial photographs of the following scales: 1:8000, 1:9000, 1:10,-000, 1:19,000, and 1:21,000. Studies were made on aerial photographs of the scales 1:5000-1:25,000 following the same method in 1954-1955 [10]. Altogether, more than 2,000 measurements were made. Stems growing from the same root were taken as one tree.

Field determinations of mean spacings were conducted by the polygon procedure, the procedure of counting stems in plots, the straight line procedure, and the procedure of choosing the most typical spacing. Field measurements were controlled by exhaustive measurements or were carried out by two practitioners. The most experienced practitioners made the measurements in forests of Moscow Oblast. Specially trained, but poorly qualified practitioners made the measurements in the forests of Tul'skaya Oblast with the aim of verifying the measurement procedures.

Procedures of determining mean spacing from aerial photographs (following the method presented in section 26) described below were used.

In all cases, ocular regionalization of the forest tracts into plots differing in tree density was the first step in working with aerial photographs. Regionalization was carried out based on photo-imaging structure visible on individual aerial photographs. The photo-imaging structure depended on age, density, height, crown size, and scale of the aerial photographs. In accordance with this approach, seven different structures were differentiated: merged, fine-grain, moderate-grain, coarse-grain, small-patch, moderate-patch, and coarse-patch.

Mature and maturing forests usually have high height, low density, large thickness, and bushy crowns, which produce coarse- and moderate-patchy photo-imaging structure on aerial photographs.

Moderate-aged forests have moderate height and thickness, adequate density, and crowns of moderate size, which produce small-patchy and coarse-grained structure on aerial photographs. A young forest (zherdnyak, molodnyak) differs in high density, low height, and thin trees with small crowns, which produce moderate- and fine-grained structure intergrading into merged structure.

Forest regionalization on aerial photographs was in fact based on these features of forest photo-imaging structure. To simplify regionalization, typical specimens (reference standards) of aerial photographs were used for forests of different structure and density. Typical specimens (reference standards) were prepared by field and office measurements of all forest characteristics whose variables were appended to the reference standards [14]. Visual comparison of type specimens and aerial photographs (best done under the stereoscope) made possible preliminary regionalization of the forests by density and other features. Within the limits of differentiated sections typical plots were selected, on which measurements were then made with different procedures. A similar methodology was adopted in determining all other forest characteristics.

The number of differentiated plots depends on the diversity of forest photo-structures and on the amount of detail necessary in the actual practice of securing information about forests. Based on classification of forests by density developed by estimating the economic value, geobotanical factors, and other forest properties, we can, for example, limit ourselves to five gradations of forest density and the photo-structural gradations corresponding to them (cf section 25). The book [14] lists typical specimens (reference standards) of aerial photo images for 13 forest photo-structures.

Four procedures were used in measuring spacings in the selected plots within forest sections.

1. The procedure of measuring the sides of a polygon consisted in selecting a typical group of neighboring crowns under the stereoscope with subsequent measurement of spacings between crown centers of 5 to 7 trees along straight lines chosen in accordance with the rules in sections 3 and 4. The

number of polygons (groups) must not be less than 2 to 3. Measurement of spacings was carried out by using parallax-measuring stereoscopes. The mean of all the measured spacings was taken as l_0 of the forest section. This method can be used in all forests.

2. The procedure of counting crowns on straight lines consisted in drawing straight lines 0.5-1.0 cm (depending on the scale of the photographs and forest density, the length of the lines can be increased) and 0.2 mm thick across the plots. The straight lines were drawn at an angle in approximately two mutually perpendicular directions for the purpose of embracing different variants of crown placement. All the crowns that intersected or touched a line were counted along the straight lines. For control, the count was made twice under the stereoscope. Merging patches of crowns were counted as two to three crowns depending on the size of the image of the average-sized crowns in the forest section. The quotient obtained from dividing the length of straight lines by the total number of counted crowns gave the mean spacing between trees. The number of pairs of lines must not be less than three to four, and by increasing the length of the straight lines the number of pairs can be reduced to two to three. The count of the number of measured spacings and the length of the lines was conducted according to formulas in sections 8 and 27.

To get more precise results, it is necessary to determine as precisely as possible the scale of the aerial photographs, and to make the measurements on working areas of the aerial photographs. This procedure is best used for dense and moderately dense forests and on moderate-scale aerial photographs.

3. Procedure of counting crowns on plots ranging in size from 0.5 x 0.5 to 2 x 2 cm (depending on forest thickness and aerial photographic scale; in sparse forests the plots were expanded). The number of crowns counted on the plots was converted into forest thickness N per hectare, and from the value of N the mean spacing l_0 was found in Table 1. This procedure is laborious, it is best used in sparse forests, but sometimes in forests of moderate density.

4. The procedure of type specimens (reference standards) of forest density on aerial photographs. Type specimens (reference standards of density) were prepared in advance on aerial photographs with several successive values of l_0 by careful measurements (best done with field control) in the most widely used aerial photographic scales.

By visual comparison of several specimens with the data of a forest section, the closest reference standard was determined from aerial photographs, and subsequently used to arrive at the mean spacing.

As experience has shown, errors in determining mean spacing by using specimens (reference standards) prove to be the same as those resulting from measuring 15-20 spacings. This procedure considerably cuts down work volume. The procedure is simple and easily executed even by persons lacking adequate training [10].

The reference standards procedure is especially convenient for regionalization and differentiation of sections by gradations in forest density adopted on topographic and special maps.

Since typical specimens can be prepared with data on all forest categories, including yield, then they are best used also in determining yield when special precision is not called for.

In making measurements on aerial photographs, existing instruments (stereoscopes, parallax sheets, measurement magnifiers, rulers, gauges, styluses) are used that have not been adapted for work in measuring forest characteristics, and so cut down on precision and increase time spent. Therefore, it is best to develop a special set of instruments for forest interpretation. Based on experience in making measurements, we can recommend manufacturing special thin and transparent rulers, with openings for pricking of crowns and with divisions along the lower edge at intervals of 1 cm, 1 mm, 0.2, and 0.1 mm colored red. Such rulers and also stereoscopes with magnifiers replace the magnifiers unsuitable for measurements. For the straight line procedure, it is useful to make standard rulers (strips) on transparent base material, which precludes drawing these lines with styluses.

29. Experimental Data on Determination of Mean Tree Spacing From Aerial Photographs

Results of determining mean spacings from 1:8,000 aerial photographs in the forests of Tul'skaya Oblast are presented in Table 59.

It is clear from Table 59 that almost 88 percent of the examples give an error less than 0.1 meter. The mean square error in determining the mean spacing $\sigma = \pm 0.68$ meter, but if we exclude four coarse errors, then $\sigma = \pm 0.5$ meter.

Table 59

Ⓐ

Legend: A -- greater than 1.5

The size of relative errors in percentages of different spacings was of interest. Table 60 lists σ values and their expression in percentages f percent with respect to l_0 ranging from 1.5 to 10 meters.

Systematic correction $\Pi = - 0.5$ meter was introduced into measured l_0. It is clear from the table that the greatest error in percent pertains to l_0 values, that is, pertains to very dense and young forests, which was to be expected. Since these forests do not have a substantial bearing in estimating yields, their mean spacing is best indicated on the map in the form of gradations, that is, indicated simply that in the given forests, $l_0 < 2$, or 1.5, etc.

Table 60

On the average, however, the mean spacing in forests ranging from $l_0 = 2$ meters to $l_0 = 10$ meters was determined with an error of 11 percent.

GRAPHICS NOT REPRODUCIBLE

Field and office measurements of l_0 on 1:9,000 aerial photographs taken in the forests of Moscow Oblast are shown in Table 61.

Table 61

Δl, м	m	[illegible]
0,0—0,5	[illegible]	[illegible]
0,6—1,0	2	[illegible]
1,1—1,5	0	[illegible]
Ⓐ Более 1,5	1	
	[illegible]	[illegible]

Legend: A -- greater than 1.5

The mean square error $\sigma = \pm 0.49$ meter with systematic correction $\Pi = -0.3$ meter. If we drop one coarse measurement, then $\sigma = \pm 0.34$ meter. For the most important and widespread tree stands with spacings from 4 to 6 meters, the average $\sigma = \pm 0.35$ meter, or only 7 percent. The higher precision is reached here owing to the experience of the practitioners taking measurements in the field and from aerial photographs.

The results of field and office measurements on 1:10,000 aerial photographs are shown in Table 62.

Table 62

Δl, м	[illegible]	[illegible]
0,0—0,5	[illegible]	[illegible]
0,6—1,0	[illegible]	[illegible]
1,1—1,5	[illegible]	[illegible]
Ⓐ Более 1,5	[illegible]	[illegible]
	[illegible]	100,0

Legend: A -- greater than 1.5

The mean square error $\sigma = \pm 0.66$ meter with systematic correction $\Pi = -0.5$ meter. For spacings of 4-6 meters, the error $\sigma = \pm 0.63$ meter, or 12.5 percent.

Distribution of deviations in measuring mean spacings on 1:19,000 aerial photographs is shown in Table 63.

Table 63

Δ, m	m	f%
0,0–0,5	13	43,5
0,6–1,0	11	36,8
1,1–1,5	4	13,3
(A) более 1,5	2	6,4
	30	100,0

Legend: A -- greater than 1.5

The mean square error $\sigma = \pm 0.92$ meter with systematic correction $\Pi = -0.8$ meter. For spacings within the range 4-6 meters, $\sigma = \pm 0.96$ meter, or 19.2 percent, but when four coarse measurements are dropped, $\sigma = \pm 0.70$, or 14 percent.

These data evidence that determination of l_0 from 1:19,000 aerial photographs gives greater errors, about twice as great as the errors gotten when measuring aerial photographs within the range of scales 1:9,000 - 1:10,000. To cut down on the size of errors it is necessary to magnify small-scale aerial photographs by two to three times, but only those sections for which measurements are projected. Thus far there are no other ways of boosting measurement precision (if we exclude use of winter aerial photographs), since we cannot alter the distribution of random deviations and the systematic corrections have been incorporated.

The distribution of deviations when measuring mean spacings on 1:21,000 aerial photographs is given in Table 64.

The mean square error $\sigma = \pm 0.48$ meter, with systematic correction $\Pi = -1.2$ meters. For distances in the range of 4-6 meters, the mean $\sigma = \pm 0.5$ meter, or 10 percent. The higher precision compared with 1:19,000 aerial photographs is realized owing to the experience of practitioners and as a consequence of the work mainly involving measurement of large spacings, and the fact that there were no sections with dense forests and small l_0 ($l_0 < 3$ meters) in the experiments.

GRAPHICS
NOT REPRODUCIBLE

Table 64

(A)

Legend: A -- greater than 1.5

Methods of determining mean tree spacing in the principal canopy of a stand yield, of course, approximate, but still quite satisfactory results.

However, experiments have shown that in this variant as well mean spacings determined from aerial photographs often prove less than field values of l_0. Therefore, we must introduce a correction into the measured l_0^v of approximately -0.5 meter on 1:10,000 aerial photographs, and -1.0 to -1.3 meters on 1:20,000 aerial photographs.

When used with aerial photographs in range of scales 1:10,000 - 1:15,000, for forests with $l_0 > 2$ meters the proposed procedures will obviously give satisfactory precision in mean spacing determination.

When used with aerial photographs in the range of scales 1:18,000 - 1:25,000, we can get precision that is applicable in practice for forests with $l_0 >$ 9-10 meters, but when magnifying aerial photographs by two to three times, there are grounds to anticipate realizing satisfactory precision even in forests with $l_0 > 5$ meters.

We must also bear in mind that mean spacing, let us say, of 5 or 6 meters is only a parameter in a spacing distribution series from which we get an idea about the percentage ratio of different spacings.

By way of example, let us present two values of mean spacings, $l_0 = 5$ meters and $l_0 = 6$ meters. We will assume the mean spacings were determined with an error of 1.0 meter, that is, much greater than the errors that we get on 1:10,000 aerial photographs.

We will now write out the spacing distribution series when l_o = 5 meters and l_o = 6 meters in Table 65.

Table 65

It is clear from the table than an error of 1 meter in determining mean spacing leads to a situation where instead of 19 percent of spacings l = 6 meters, we get 16.9 percent, that is, 2.1 percent less than the actual.

Small-scale aerial photographs used in determining mean spacings can be employed, given the condition that there is an improvement in quality of photographic film and choice of the most suitable time for aerial photography.

In estimating precision of determinations made of spacings and stand density from aerial photographs, we must bear in mind that in forestry inspection and management of forests in the U.S.S.R. at all levels is carried out by the ocular method, but in topographic work, prior to use of the author's method, such primitive procedures in mean tree spacing determination were recommended as had no theoretical foundation for reliability and precision of results produced.

CHAPTER 7

METHODS OF DETERMINING MEAN HEIGHTS, CROWN DIAMETERS, AND CANOPY CLOSURE OF STANDS

30. Methods of Determining Mean Crown Diameters

The tree distribution series in crown diameters is expressed by the normal distribution curve (section 20). Therefore, the mean crown diameter in forest sections will be determined as a parameter of the distribution series. This formulation of investigations differs fundamentally from studies of the precision of measuring crowns of individual trees.

In the U.S.S.R. detailed study on precision of crowns measured from aerial photographs of different scales has been conducted by Professor G. G. Samoylovich, N. I. Baranov, and others.

Worly and Meyer (University of Pennsylvania, United States) have investigated precision in measuring crowns of individual trees on 1:12,000 aerial photographs. They measured the crowns of 36 trees three times using different methods, which required taking 432 measurements. The error in determining an individual crown proved to be 3-4 feet, or 0.9-1.2 meters with systematic correction of 1-2 feet, or 0.3-0.6 meter. Moessner (Central Forest Experimental Station of Canada) obtained an error equal to ± 0.33 foot, or $\pm$ 0.1 meter with systematic correction of -0.3 meter. These measurements do not afford data for estimating the precision in determination of the mean diameter of the entire population of crowns in a forest section.

To determine the mean crown diameter by the sampling method with a precision of $\Delta = 10$ percent at a significance level $\alpha = 5$ percent and variance $v_c = 30$ percent, we have to measure $s = 36$ crowns.

In making measurements of mean D_c from aerial photographs, it is important to know characteristics of crown photo-imaging structure. It is obvious that crowns of tall and moderate-sized trees and some smaller trees will show up for the most part on aerial photographs. These trees exhibit larger crowns, therefore the measured crowns will give some overstatement of mean D_c^o. But, on the other hand, the widest part of the crown usually is located below the peak and is partially covered by the branches of neighboring trees, therefore it is not the widest part that will be imaged on the aerial photographs, but a narrower part, which leads to some understatement of mean D_c^o on aerial photographs. This simultaneous overstatement and understatement will mutually cancel out, but not entirely and some of the neighboring crowns will blend into one image, which can lead to an overstatement of mean D_c^o. As the scale of aerial photographs is reduced, the number of small crowns that will not show up at the scale used and will not enter into measurement will rise, which will lead again to some overstatement of mean D_c^o.

All these factors listed that have a bearing on crown structure imaged on aerial photographs in totality then determine the value of mean D_c^o. However, the mean crown diameter depends also on procedures used in measuring crowns on aerial photographs. In our investigations we proposed and used two main methods of measurement. The first method consisted of, after preliminary regionalization of the forest tract into sections and choice of plots within the sections for conducting measurements, the plots were examined under the stereoscope to discover the actual size of crown images.

In the plots, a group of trees was selected, and the membership of this group is best taken as trees in the polygons already used in measuring mean spacings. Each crown was measured with magnifiers at a 10-12-fold magnification in two mutually perpendicular directions in order to discover the actual mean diameter of each crown. A group of crowns usually consists of five to seven trees. The number of groups must not be less than three.

If we have to obtain the mean diameter with high precision, we must then appropriately enlarge the size of the sampling population.

The second method of determining crowns is based on measuring the widths of crown images and openings along a straight line. Just as in measuring mean spacing, a straight line is drawn with a stylus. All crown images intersecting or touching this line are measured and counted. Then the overall value of the measured crown images, divided by the number of crowns, gives the mean crown diameter. In the plot two mutually perpendicular lines are always drawn and the mean value of D_c is taken along these two lines. The number of pairs in the section must not be less than two to three. As the length of the straight line is increased, the number of pairs can be reduced to two. This procedure is very simple, and as a result measurements do not depend on subjective perceptions of the persons taking the measurements.

Experimental studies on measuring crowns in the field and on aerial photographs were carried out for 121 sections using aerial photographs of scales 1:8,000, 1:10,000, and 1:19,000. A total of more than 800 measurements were taken with the testing of the two main methods described earlier.

Table 66 lists the results of the field and office measurements of crowns on 1:8,000 aerial photographs using the first method.

Table 66

(A)

(B)

Legend: A -- ΔD_c; B -- greater than 1.5

The mean square error of mean crown diameter determination $\sigma = \pm 1.0$ meter, but when coarse measurements are dropped, $\sigma = \pm 0.8$ meter with systematic correction $\Pi = +0.1$ meter.

Table 67 gives the results of measuring crowns on 1:8,000 aerial photographs using the second method (measurement and count along straight lines).

The mean square error $\sigma = \pm 0.68$ meter, with systematic correction $\Pi = -0.1$ meter. For mean diameters from 3 to 5 meters, the mean $\sigma = \pm 0.51$ meter, or 12.7 percent, which can be taken as satisfactory precision.

Table 67

(A)

(A)

Legend: A -- ΔD_c; B -- greater than 1.5

Table 68 gives the results of measuring crowns on 1:8,000 aerial photographs.

Mean square error $\sigma = \pm 0.64$ meter with systematic correction Π = -0.2 meter.

Table 68

(A)

(B)

Legend: A -- ΔD_c; B -- greater than 1.5

For crowns ranging in size from 3 to 5 meters, $\sigma = \pm 0.64$ meter, or 16 percent.

Table 69 gives measurements of crowns on 1:10,000 aerial photographs.

The mean square error $\sigma = \pm 0.77$ meter, with systematic correction Π = -0.4 meter.

For crowns from 3 to 5 meters in width, the mean $\sigma = \pm 0.78$ meter, or 19.5 percent.

Table 69

ΔD_к, м (A)	[illegible]	[illegible]
0,0—0,5	9	[illegible]
0,6—1,0	15	[illegible]
1,1—1,5	3	10 7
(B) Более 1,5	[illegible]	[illegible]
		[illegible]

Legend: A -- ΔD_C; B -- greater than 1.5

Investigations showed that in determining crowns by the method of counting them and making measurements along straght lines on 1:8,000 aerial photographs, overstatement of crowns is small and requires introduction of systematic correction into measured D_C^{av} values on aerial photographs of only Π = -0.1 meter, on 1:10,000 aerial photographs -- about -0.2 meter, and on 1:19,000 photographs the correction Π = -0.4 meter.

31. Methods of Determining Mean Height of Tree Stands From Aerial Photographs

Determination of heights of trees from photographs by the method of measuring shadow length was conducted by Mikhayev in 1917, by Martsinskovskiy in 1925, by G. G. Samoylovich in 1927, A. K. Pronin in 1931, and N. I. Baranov in 1933.

Determination of mean heights of tree stands by the ocular method using stereoscopic height was investigated by the scientific research laboratory of the forest aviation trust in the Forest Engineering Academy imeni S. M. Kirov. Determination of heights by distances and longitudinal parallaxes was conducted by G. G. Samoylovich, Andrews (United States), Losee (Canada), and others. Precision of tree height measurement using the D-5 stereoscope and the PL-2 parallax sheets was investigated in the studies [48, 49, 52]. D. M. Kireyev investigated precision in determination of tree stand heights by the method of measuring the parallax shift of the tree apex relative to its base (the method does require use of stereomeasuring instruments) and by the method of measuring heights using the STD-1 stereometer.

In ground appraisal, precision in determining mean height is taken as equal to ±10 percent. If h_0 is determined by the sampling method with a precision $\Delta = 10$ percent, then at a confidence level $p = 0.99$ and variance $v_h = 10$ percent, it is required to measure the heights of 9 trees. Separate sections are marked out when the difference in mean heights is greater than 2 meters.

A great many studies on determination of precision in measuring heights of individual trees have been conducted in recent years in our country and abroad. However, the main issue does not lie in studying precision of measurements of individual tree heights, but in determining the mean height of a tree stand, for these are two distinct problems. The following problem was resolved in the experimental studies described below. Measurement of tree stand heights in the field and from aerial photographs was carried out on 158 forest sections. The measurements were taken independently by three practitioners on aerial photographs of scales 1:10,000, and 1:19,000 in three areas using parallax sheets and a stereometer.

Precision in determining mean tree stand height depends a great deal on how trees are distributed by height at the locality and on the structure of those trees that show up on the aerial photograph. In field measurements, sometimes mean height is overstated, since unwittingly one's attention is drawn to the principal stems of the upper canopy of the tree stand.

On aerial photographs it is mainly trees that are higher than the average and close to the average height that are imaged, while lower-standing trees participate to a lesser extent in forming the upper canopy that is imaged on aerial photographs. In stereoscopic measurements, we must bring the crosshair of the instrument precisely at the level of the tree stand canopy. Aligning the crosshair on the apex of trees projecting above the main canopy will lead to overstating the mean h_0 values, but placing the crosshair below the main canopy will bring about an understatement of mean height. Deviations above and below the main canopy have a very sensitive effect on precision of mean height determination, since the variance of height is very small. When $v_h = 10$ percent and $h_0 = 20$ meters, the limits of deviation of the main caopy will lie within the limits $\Delta h = \pm 2$ meters. If we express Δh in stereoscopic height, then when $f_c = 200$ mm and the stereoscopic magnification $k = 1.5$, the doubled value h will be 0.8-0.9 mm on aerial photographs of scale 1:10,000.

Given these data, a 0.2 mm error in crosshair alignment can give a deviation of ±1 meter. Therefore, careful choice of the surface level of the principal canopy of the stand is of great importance in precision of determining the mean tree stand height from aerial photographs. In actual practice, in measuring heights individual trees are selected, but it is best if there is an opportunity to conduct measurements at the surface level of the main canopy by selecting the most typical locations for this. When measuring mean heights based on individual trees, the latter must be selected according to their closest approximation to the mean height of the principal canopy of the stand. If however we take any outstanding tree, standing alone and convenient to measure, in making our measurements, then this tree can have a height as distinct as we wish from the mean tree stand height. When there is high forest density, the principal canopy surface on aerial photographs is formed by the higher trees with $h = h_0$ and higher. So, overstating mean height is probable in this case. We must also keep in mind that it is not the very uppermost parts of the crown that participate in forming the surface of the principal canopy, but the lower crown cross-sections, which reduces the visible surface of the canopy, especially in small-scale aerial photographs in which the narrow and fine tips of the crowns can be imaged not at all or else be hard to differentiate if they are. This phenomenon, most likely, will lead to understating mean heights measured on small-scale aerial photographs.

In addition, the stereoscopic height of a forest on 1:20,000 aerial photographs (or smaller in scale) is very decidedly reduced, therefore a 0.1 mm error in crosshair alignment on poorly differentiable crown apexes will give a deviation of 0.5 meter. Density, shape, and illumination of crowns affect precision of height determination [51,52]. For reliable determination of mean height, the number of trees to be measured is calculated from formulas in section 8, but the number of alignments on the upper canopy of a forest plot must not be less than three.

All these features of mean height measurements were taken into account in the experimental work.

Table 70 lists the results of measuring mean heights of trees from 1:10,000 aerial photographs.

The mean square deviation $\sigma = \pm 1.3$ meters with systematic correction $\Pi = +0.5$ meter. For heights from 14 to 22 meters, that is, on the average for 18 meter, the deviation $\sigma = \pm 1.3$ meters, or 7.2 percent, which can be deemed permissible error.

Table 71 lists the results of measuring mean heights from 1:10,000 aerial photographs in another area.

When two coarse measurements are dropped, the mean square deviation $\sigma = \pm 1.26$ meters, with systematic correction $= +0.4$ meter, which can be accounted for owing to tree growth, since field measurements of heights were conducted after the aerial photography session. When two coarse measurements are allowed for, $\sigma = \pm 1.67$ meters, or 10.8 percent. For heights in the range 14-22 meters, or averaging for $h_o = 18$ meters, the error was 8.1 percent. In this experiment, the deviation did not exceed 2 meters in 92 percent of all cases, that is, precisely the difference in heights that is taken in delimiting appraisal sections.

Table 72 lists the measurements of heights on 1:10,000 aerial photographs in the third area.

The mean square deviation $\sigma = \pm 1.7$ meters with systematic correction $\Pi = -0.1$ meter. For heights h = 18 meters, $\sigma = \pm 1.64$ meters, or 9 percent.

Table 73 gives measurements of mean heights from 1:19,000 aerial photographs.

When one coarse measurement is excluded, the mean square deviation $\sigma = \pm 1.64$ meters, with systematic correction $\Pi = +2.0$ meters. The clear understatement of heights measured from these aerial photographs can be explained by two factors. The first is that field measurements of h_o were made three years after the aerial photography had been completed. In three years the mean growth increment of the tree stand was approximately 0.7 meter. The rest of the 1.3 meters can be explained as stemming from the structure of imaging and shape of crowns on small-scale aerial photographs. Based on these facts, we cannot agree with the findings of Johnsons (United States), who on the basis of combined studies conducted by the Committee of Forest Photogrammetry of Sweden and the Alabama Polytechnic Institute in the United States asserted that precision in determination of tree heights does not depend on the scale of aerial photographs. This conclusion was arrived at on the grounds of studies made of precision in measuring heights of individual trees, but he is not justified in determination of the precision of measuring mean tree stand heights. Studies conducted by D. M. Kireyev (U.S.S.R.) and by Losee (Canada) also bear this out.

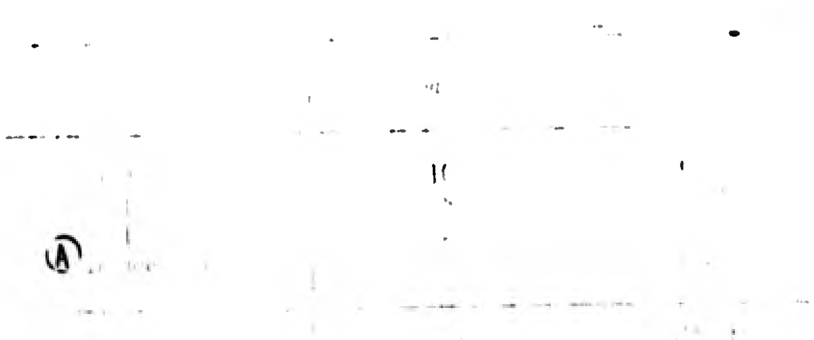
Table 70

Legend: A -- greater than 3

Table 71

Legend: A -- greater than 3.0

Table 72



Legend: A -- greater than 3.0

Table 73

Legend: A -- greater than 3.0

As experience has shown, determination of mean height is possible with 1:19,000 aerial photographs, but this requires introduction of corrections, while on the average the error can be close to 10 percent. Worly and Meyer (United States) obtained an error of from 8 to 10 feet on 1:12,000 aerial photographs, that is 2.4 meters to 3 meters. Moessner (Canada) got an error of 10 feet on 1:20,000 aerial photographs. As a whole, however, based on data of foreign researchers the error in determining tree heights is close to 12 percent. From the experimental studies presented earlier, it is clear that mean tree stand heights from aerial photographs of scale range 1:10,000-1:15,000 are determined with a precision of 7-10 percent, but when 1:20,000 aerial photographs are used -- with an error of 10-12 percent. We must, of course, bear in mind that one can encounter individual deviations even greater in size, but we can guarantee the deviations given above with a probability close to unity, if all conditions of the sampling method and the measure-techniques are correctly met.

32. Methods of Determining Canopy Closure From Aerial Photographs

In forest appraisal, canopy closure is used to determine occupancy of tree stands with allowance for interaction between tree stands and occupancy in turn is used to calculate stand yield, that is, to solve the main problem in inventorying the forest resources of the country. Canopy closure is necessary also for canopy representation on maps.

Accordingly, deriving methods of determining canopy closure from aerial photographs and in fieldwork is of practical interest in securing appraisal and topographic information about forests.

In 1939, it was recommended in studies of the All-Union Trust of Forest Aviation to divide forest sections, by canopy closure, into four groups: $C_1 = 1.0$; $C_2 = 0.75$; $C_3 = 0.5$; and $C_4 = 0.25$.

Determination of occupancy from canopy closure is vital in forest interpretation, but at the same time, according to A. K. Pronin, "this problem has not been at all fully dealt with."

Canopy closure is determined either ocularly or by superimposition in the plane of openings in the crown canopy with subsequent measurement of the areas occupied by the openings. Abroad, chiefly in Canada and the United States, following the Second World War appraisal and inventory of forests from aerial photographs were advanced based on determination of crown canopy, for the latter was adopted as one of the chief variables in compiling standard yield tables (Standard Volume Tables) from which yields of tree stands were estimated.

Studies by the author on determination of canopy closure were conducted in 1947-1950 based on small samplings, but broader experimental work in the field and on aerial photographs was conducted in 1951-1959.

In determining canopy closure, just as in determining other forest characteristics, the sampling method was used, on the basis of which several measurement procedures in estimating canopy closure in the field and from aerial photographs were proposed.

It is necessary to know the variance of canopy closure when using the sampling method. In the literature available to us in the U.S.S.R. and abroad we have been unable to find data on investigations of crown canopy variance. The lack of information about variance has compelled, for example, Losee (Canada), regarded as one of the top specialists in the United States and Canada, to conduct investigations on the precision of canopy closure determinations without theoretical calculations and to conduct extremely laborious empirical studies in order to arrive at data satisfying a significance level of 5 percent, that is, a reliability of measurement equal to the probability $p = 0.95$

To determine the variance of canopy closure, experimental studies have been conducted in two forest sections. On a 1:100 sketch, all trees and point projections of their crowns,

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totalling 1514 stems, were plotted with instruments. Areas occupied by the crowns of 1514 trees were measured on 102 plots each 10 x 10 meters in area. From these data, the actual canopy closure was calculated in each of the 102 plots. Data of 42 plots were used in sampling determination of canopy closure variance.

Based on these measurements, the author calculated the variance, dispersion, and mean canopy closure.

The variance of canopy closure proved to be equal to

(A)

The precision of canopy closure determination needed in practice can be established indirectly, since there is no data on this problem in the literature. In forest appraisal, the precision in determining occupancy is taken as 0.1 (ground ocular appraisal). For topographic purposes, it is quite adequate to determine canopy closure with a precision of 10-15 percent. Thus, we can assume that canopy closure in the field and from aerial photographs is permissibly determined with an error of the order of 10 percent or even 10-15 percent. If we take the precision in canopy closure determinations as equal to Δ = 15 percent, then with a confidence level t = 2 (p = 0.95), the sampling population is

plots

In the straight line procedure (20 meters in length), four 10 x 10 meter plots correspond approximately to a pair of straight lines. Then the number of straight lines required to determine canopy closure with a precision of 10 percent at a confidence level p = 0.95, will be

(A) (B)

Knowing the variance v_{cl}, the desired precision Δ , and the desired confidence t, it is easy to theoretically calculate the required sampling population of measurements in any methods of canopy closure determination.

The following methods of determining canopy closure in the field and on aerial photographs have been proposed and tested:

the method of determining canopy closure on typical polygons; the method of determining canopy closure on square plots 20 x 20 meters and 10 x 10 meters in size; the method of determining width of crown and intercrown openings along straight lines; the method of measuring mean crown diameter and mean tree spacing; the method of typical specimens (reference standards) of canopy closure on aerial photographs.

Exhaustive measurements of canopy closure were taken on two forest sections.

True canopy closure in the first section C = 0.535. Mean crown diameter was determined based on these same measurements. The total area occupied by the crowns in the section was P_C = 2486.6 square meters, and the total number of stems N - 262, so the area of the average crown was

$$(A)\ p_к = \frac{2486,6}{262} \approx 9,5\ м^2.$$

Legend: A -- p_c

Assuming that $p_c = \pi/4\ (D_c^2)$, we get

$$(A)\ D_к = 1,13\sqrt{p_к}. \qquad (93)$$

Legend: A -- D_c

From expression (93) and P_c, the mean crown diameter $D_c^o = 1.13 \times \sqrt{9.5} = 3.5$ meters.

The actual mean tree spacing l_o = 4.4 meters, while the relative clearance between crowns was

$$\Delta d = \frac{l - D_к^{(A)}}{(A)\ D_к} = \frac{1}{3,8}\ D_к\ (A)$$

Legend: A -- D_c

Based on the approximational relationship between C, 1, and d (from the graph in Figure 15 or Table 4), closure C = 0.58 was determined, giving an error Δ C = 0.58 - 0.53 = 0.05, or only 5 percent.

Based on data of continuous measurements in the second section, the actual C = 0.572, and the mean crown area was

$$(A)\ p_к = \frac{3906,9}{293} \approx 13,3\ м^2.$$

Legend: A -- p_c

mean spacing l_o = 5.17, and mean crown diameter D_c^o = 4.1 meters, while Δd = 1/3.3 (D_c^o). Based on Δd and l_o from Table 4 or from Figure 15 we get C = 0.50, which gives an error ΔC = 0.07, or 7 percent. These examples show that canopy closure can be determined approximately from Table 4 or from Figure 15 if we know D_c^o and l_o, where the error in determination proved to be equal to 5-7 percent, of course, given the condition that there are precise values of D_c^o and l_o. Since l_o and D_c will be determined by the earlier described methods, then they can be used to determine also C from Table 4 or from Figure 15.

Determination of canopy closure by the polygon method (using the same polygons on which sampling determination of mean spacing was conducted) was carried out under field conditions.

In the first section, the actual C_o = 0.53, but the sampling (based on six polygons) C_{samp} = 0.53. In the second section, C_o = 0.57, but C_{samp} = 0.53.

We will cite measurements of canopy closure in the forest on plots 20 x 20 meters in size. In three plots of the first section C_{samp} = 0.43, which gives an error ΔC = 0.1 or 10 percent. In the second section, based on three plots C_{samp} = 0.48, which gives an error ΔC = 0.09, or 9 percent.

In the third section, canopy closure was determined from two 20 x 20 meter plots, which gave C_{samp} = 0.67 with C_o = 0.70 and an error of 3 percent. Still we have to recognize that determination of canopy closure from 20 x 20 meter plots is too laborious. It is simpler to determine C from 10 x 10 meter plots with the same precision.

The simplest measurement method in determining canopy closure must be deemed the method of measuring crown width and inter-crown openings along straight lines. Lines (as in the method of straight lines when determining mean tree spacings) are drawn along the boundaries of a homogeneous forest section. The width of crowns intersecting or touching a line 0.1 - 0.2 mm thick (on aerial photographs) as well as the length of intervals between crowns along the line are measured, using a magnifier along these lines. The overall length of these values is equal to the length of the line. Then the overall crown width in percentages of the straight line length gives canopy closure in percentages or in fractions of unity. Line segments are best drawn in two mutually perpendicular directions for more accurate allowance for crown size in different orientations.

Figures 26 and 27 show copies of forest plots 20 x 20 meter each in size on a 1:100 scale with point projections of the crowns of all trees in the locality. The method of determining canopy closure based on two mutually perpendicular straight lines 20 meters in length at the locality or 20 cm on the sketch gave the following results.

Figure 27 shows that the overall width of crowns along the first line l_1 = 13.5 cm, and along the second l_2 = 6.2 cm, which correspondingly gives C_1 = 13.5/20 = 0.62 and C_2 = 0.31, and the mean C_{samp} = 0.46. With the true C_0 = 0.53, the error was 7 percent.

To boost the precision of determining canopy closure it is necessary to take not less than two pairs of mutually perpendicular straight lines, placing them along the most typical forest sections. The straight line method, along with its simplicity and precision, also exhibits objectivity, since the results of measurements of crowns and inter-crown openings along straight lines do not depend on subjective perceptions of the practitioner. Precision will depend on the choice of the placement of straight lines in the section, and this requires preliminary inspection of the aerial photographs under the stereoscope. Magnification of the straight lines leads to a greater allowance for differences in canopy, and, consequently, to higher precision in determining the mean closure value.

The thickness of the lines drawn on the aerial photograph has an effect on precision of canopy determination. Straight lines 0.2 mm in thickness and 1.0 cm long are drawn on Figure 27. Based on the pair of straight lines 0.2 mm thick, the closure C_1 = 0.46, but along the straight lines 1.0 cm in thickness (cross-hatched) C_2 = 0.58, while the true C_0 = 0.53. So, increasing the thickness of lines leads to an increase in C. As the scale of the aerial photographs is reduced, straight lines of the same thickness (for example, 0.1-0.2 mm) will cover increasingly broader strips and this under otherwise equal conditions will lead to an increase in C, which must also be borne in mind when determining canopy closure by the straight line method on aerial photographs of different scales. Precision in determination of C by the straight line method also depends on forest density. In sparse forests, the straight line method will yield gross errors and in such forests it is best to determine C from reference standards (on aerial photographs) by ocular comparison.

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Section No 1

(A)

Figure 27. Copy of forest plot on 1:100 scale for determination of canopy closure by the straight line method. Legend: A -- projections of tree crowns

The method of type specimens (reference standards or closure scales) of canopy closure on aerial photographs with previously measured values of C can be used for any forests. In those cases when we have to regionalize forest sections by three closure gradations (0.5, 0.25, and 0.1) indicated in the forest classification scheme (section 25), it is required to prepare only three reference standard aerial photographs, corresponding to $C_1 = 0.5$, $C_2 = 0.25$, and $C_3 = 0.1$. Visual comparison of standards with forest sections on aerial photographs of the same scale allows us to determine the approximate closure without any measurement calculations. In actual practice, the reference standard method is

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the simplest and the speediest. The method of determining canopy closure based on specially prepared artificial transparencies on which circles of different diameters have been drawn is used in Canada and the United States. The blacked-in area of the circles expresses the canopy closure. Choice of such transparencies is made for closures from 0.5 to 0.95. Comparing these artificial "reference standards" with forest sections on aerial photographs, the closest reference standard is determined, and this is used to estimate canopy closure.

Experimental studies on determining canopy closure have been conducted by us on 1:10,000 aerial photographs in 36 forest plots by the straight line method.

Table 74 lists the results of office measurements of canopy closure.

Table 74

	m	[illegible]
[illegible]	14	[illegible]
[illegible]	15	[illegible]
[illegible]	5	[illegible]
(A) [illegible]	2	5.4
	36	100

Legend: A -- greater than 15

The mean square deviation $\sigma_{cl} = \pm 0.08$, or 8 percent, with systematic correction $\Pi = +0.06$, or 6 percent. It follows from this that the straight line method gives satisfactory precision in determining canopy closure from aerial photographs. Still, it was found that the straight line method gives an approximate 6 percent understatement of closure. To eliminate this understatement, it is best to take lines not 0.1 mm thick on 1:10,000 aerial photographs, but 0.2-0.3 mm thick, but not thicker, since this would lead to systematic overstatement of closure.

It is clear from these studies that the sampling method of determining canopy closure using the straight line procedure ensures a precision under field conditions of 5-7 percent, and 8-12 percent when using aerial photographs.

Canopy closure (by methods used in the United States and Canada) is determined on 1:12,000 aerial photographs with an error of 10-20 percent in Canada and the United States, based on data of Coleman and Rogers given by them at the congress of the International Society of Photogrammetry. According to Losee's data (Canada), the mean square error in determining closure from 1:7,200 aerial photographs is $\sigma_{cl} = \pm 9.9$ percent, with systematic correction of -1.3 percent, which approximately agrees with the precision of determining canopy closure in our experiments. From the data of Worly and Meyer (United States), they have obtained an error in determining canopy closure from 1:12,000 aerial photographs of about 10 percent (with systematic correction of 5-10 percent), which is to be expected, since the method of transparencies is based on ocular data on not on instrumental measurements. As we can see from comparison of studies conducted in the U.S.S.R. and abroad, methods we have proposed will be productive of more precise results, since they are based on objective measurements on the sampling method theory.

CHAPTER 8

METHODS OF DETERMINING MEAN THICKNESS OF TREES IN A STAND FROM AERIAL PHOTOGRAPHS

Calculation of the number of measurements proceeds according to the formulas under the sampling method (section 6). The diameters of 25 trees must be measured to determine the mean tree diameter in the locality to a precision of Δ = 10 percent for a confidence level p = 0.95 and variance v_d = 25 percent.

In actual practice, mean diameter is determined by ocular sampling of seven to ten trees that are of average thickness.

For control in determining mean diameter, its ratios with the thinnest and the thickest trees in a homogeneous stand can be used. Field determinations of mean diameter are made more simply than determinations of mean height and crown diameter. Precision of mean diameter determination in ground appraisal is set at ± 10 percent; delimiting of forest plots into independent areas is carried out when there is a difference of more than 4 cm in the thickness of the average tree. In topographic work, determination of mean thickness precision is not stipulated in instructions.

While height, crown diameter, tree spacing, and canopy closure can be measured with the same degree of precision directly from aerial photographs, it is not possible to measure tree diameter at a height of 1.3 meters from the ground using flat aerial photographs. Therefore, the only method of determining mean thickness from aerial photographs is preliminary derivation and mathematical expression of the correlation between diameter and other tree characteristics. The correlation equation and the correlation table of the ratio between

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thickness, height, and crown diameter, calculated on the basis of the correlation equation, are given in sections 22 and 24. This last-named section gives the results of experimental studies on determination of mean thickness by relying on the correlation Table 47 based on crown diameters, heights measured from aerial photographs, and on these two variables considered in combination.

Determination of stem thickness under office conditions from aerial photographs is of practical importance in resuming topographic maps and in forest appraisal. So searching for even approximate and indirect methods of determining mean thickness from aerial photographs is of theoretical and practical interest.

The experimental studies described below were conducted gradually, beginning from 1948, from aerial photographs of scales 1:8,000, 1:10,000, and 1:19,000 in four areas, totalling 262 forest sections, which required more than 2500 measurements in the field and from aerial photographs.

As the result of these studies, the precision of correlation equations relating forest characteristics was verified as well as the precision of various methods of measuring and determining these characteristics from aerial photographs of various scales covering tree stands differing in density, height, and composition.

33. Determination of Mean Tree Thickness From Crown Diameters and Correlation Table

In the experiments, the goal of discovering the precision of determination of $d^{?}_{tree}$ from D8 measured from aerial photographs as well as from the correlation Table 47 was posed.

Table 75 lists the results of field and office measurements from 1:8,000 aerial photographs.

Table 75

[illegible]	m	[illegible]
[illegible]	[illegible]	73,8
[illegible] – [illegible]	[illegible]	14,6
(A) [illegible] 6,0	[illegible]	11,6
		100,0

Legend: A -- more than 6.0. m = [subscript] tree

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The mean square deviation $\sigma_d = \pm 3.6$ cm with systematic correction $\Pi = -0.3$ cm.

Table 76 lists results of the determination of d^{o}_{tree} from 1:10,000 aerial photographs.

Table 76

Δd_m, см	m	%
0,0—4,0	17	[illegible]
4,1—6,0	[illegible]	[illegible]
(A) Более 6,0	[illegible]	[illegible]
	34	[illegible]

Legend: A -- greater than 6.0; m = [subscript] tree

The mean square deviation $\sigma_d = \pm 4.6$ cm with systematic correction $\Pi = -1.2$ cm, the emergence of which is accounted for by the +0.2 meter overstatement of the measured crowns.

Table 77 lists measurements from 1:19,000 aerial photographs.

Table 77

Δd_m, см	m	%
0,0—4,0	12	40
4,1—6,0	10	33
(A) Более 6,0	8	27
	30	100

Legend: A -- greater than 6.0; m = [subscript] tree

The mean square deviation $\sigma_d = \pm 4.67$ cm with systematic correction $\Pi = -3.1$ cm, which is accounted for by the approximately +0.4 meter systematic overstatement of measured crowns using data of aerial photographs.

It is clear from the data given that as the scale of the aerial photographs is reduced the error of determination of mean tree thickness from D_c^{av} and from Table 47 rises. On 1:8,000 aerial photographs the mean square deviation $\sigma_d = \pm 3.6$ cm lies within the bounds of permissible error, but deviations on 1:10,000 and 1:19,000 aerial photographs exceed the possible allowances. As a whole, however, precision in determining thickness from crown size is not high, which evidences the not wholly intimate correlation between these variables and that the correlation between D_c and d_{tree} requires further investigation and refinement. This in fact is also true of crown measurements from aerial photographs. Still, experience shows that there is a possibility of approximation or estimation of tree thickness from crowns measured on aerial photographs and efforts in this direction must be kept up.

34. Determination of Mean Tree Thickness From Mean Tree Stand Height and From the Correlation Table

Table 78 lists the results of determination of d^0_{tree} from h_0 and correlation Table 47. The mean heights are measured from 1:10,000 aerial photographs.

The mean square deviation $\sigma = \pm 2.5$ cm. If we count deviations for trees from 18 to 26 cm in thickness, that is, on the average for d_{tree} = 22 cm, then we get an error of mean tree diameter determination of roughly 12 percent, that is, a precision close to the precision of mean thickness determination in ground appraisal of forests (10 percent).

Table 78

Δd_m, см	m	f%
0,0—4,0	26	84,0
4,1—6,0	3	9,7
(A) Более 6,0	2	6,3
	31	100,0

Legend: A -- greater than 6.0

Table 79 lists data of the determination of d^0_{tree} based on h values measured on 1:10,000 aerial photographs of the second area.

The mean square deviation $\sigma = \pm 3.0$ cm, but when four coarse measurements are excluded, $\sigma = \pm 2.5$ cm with systematic correction Π = -1.7 cm, which is accounted for by a +0.1 meter systematic overstatement of h^0. The deviation in percentages for the mean thickness d_{tree} = 22 cm is 10.9 percent, which actually does not exceed the permissible error of 10 percent. We must note that large deviations relate mainly to slender and low-standing trees. If we take the most important and the most widespread thickness values from 18 to 26 cm, the mean deviation and distribution of deviations for these are more satisfactory, for example, in the given area 80 percent of the deviations are less than 4 cm and only 20 percent lie within the limits 4 to 6 cm.

Table 79

Ⓐ

Legend: A -- greater than 6.0

Table 80 lists the results of determination of d^o_{tree} based on h measured on 1:10,000 aerial photographs in the third area.

Table 80

Ⓐ

Legend: A -- greater than 6.0

The mean square deviation $\sigma_d = \pm 3.4$ cm with systematic correction $\Pi = +1.1$ cm, which is accounted for by a -0.5 meter systematic understatement of measured h_0 values and by other factors. If we drop coarse measurements, $\sigma_d = \pm 2.6$ cm, and the error in percentages will be close to 12.6 percent for mean thickness values $d^o_{tree} = 22$ cm.

Table 81 lists data of the determination of d^o_{tree} made from 1:19,000 aerial photographs.

The mean square deviation $\sigma_d = \pm 2.5$ cm with systematic correction $\Pi = +1.1$ cm. For trees with $d_{tree} = 22$ cm, the error is close to 12 percent. In this case, distribution of deviations is somewhat poorer than on 1:10,000 aerial photographs. But owing to the fact that systematic understatement of heights amounts to -1.5 meters and the correlation between h and d_{tree} gave understated values of d^{o}_{tree}, fairly good results were obtained when mean thickness was determined even from 1:19,000 aerial photographs.

Table 81

Ⓐ

Legend: A -- greater than 6.0

As a whole, it can be held that determination of mean thickness from heights measured on aerial photographs and by use of the correlation Table 47 is greater in precision than determination of d^{o}_{tree} that rely on D^{o}_{c}.

35. Determination of Mean Tree Thickness Simultaneously From Height, Crown Diameter, and Correlation Table

Under this method, in determining mean thickness multiple correlation between three characteristics of tree stands is used. Mean tree thickness is taken from Table 47 based on two input data -- mean height and mean crown diameter, and for the final value the average of two determinations is adopted. It is presumed that this method affords mutual compensation of deviations obtained in independent determination of d^{o}_{tree} from h_0 and from D^{o}_{c} and thus securing a more satisfactory precision of office determinations of mean thickness.

Table 82 lists the results of determination of d^{o}_{tree} from h_0 and D^{o}_{c} measured on 1:10,000 aerial photographs.

The mean square deviation $\sigma_d = \pm 2.4$ cm with systematic correction $\Pi = -0.4$ cm. The small systematic correction results from mutual extinction of systematic overstatements of D_c and understatement of h measured on aerial photographs (+1.2 and -1.1 meters, respectively).

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Table 82

$\Delta d_{н}$, см	m	%
0,0–4,0	[illegible]	96,6
4,1–6,0	[illegible]	[illegible]
(A) Более 6,0	[illegible]	0,0
	31	100,0

Legend: A -- greater than 6.0

Distribution of deviations is also considerably improved, since almost 97 percent of all deviations are less than 4 cm. The mean square deviation for trees ranging in thickness from 18 to 26 cm is equal to ± 2.2 cm, or only 10 percent of the mean thickness $d_{tree} = 22$ cm.

The resulting precision of mean thickness determination using this method can be held satisfactory, since in general it is difficult to anticipate getting more satisfactory precision for variables that are statistical and that have been subjected to numerous natural factors, and also owing to causes associated with the techniques of making measurements on aerial photographs and the approximational correlation between h, d_{tree}, and D_c.

If so great a diversity of factors still affords satisfactory precision in mean thickness determination, then this above all evidences that in nature a statistical correlation which shows up ultimately among the mass of random events does actually exist and takes on the force of necessity.

Table 83 lists the results of the determination of d^{o}_{tree} based on h_o and D^{o}_{c} measured from 1:19,000 aerial photographs.

Table 83

Δd_{m}, см	m	%
0,0–4,0	19	86
4,1–6,0	2	9,1
(A) Более 6,0	1	4,9
	22	100,0

Legend: A -- greater than 6.0

The mean square deviation $\sigma_d = \pm 2.0$ cm with systematic correction $\Pi = -1.4$ cm, which is accounted for by the large systematic overstatements of D_c (+3.1 meters) and understatements of h_0 (-1.1 meters). For average trees 22 cm in thikness (from 18 to 26 cm), the error is about 10 percent.

The results obtained for 1:19,000 aerial photographs must be held as satisfactory, but this precision is a consequence of the fact that the measurements were taken mainly on tree stands with trees more than 17-18 cm in thickness. Determination of d_{tree} for tree stands with lower thickness classes will obviously have lower precision.

As a whole, determination of mean tree thickness simultaneously from h_0 and D_c^o gives higher precision, and in the experiments run the error of 10 percent corresponds to a precision of field appraisal. This precision can be viewed as wholly adequate for topographic and other purposes.

Determination of mean tree thickness from height and correlation Table 50. This table affords the opportunity of allowing for the effect of species composition of tree stands on the precision of mean thickness determination. It is approximational. However, its use afforded some increase in precision of estimation of d^o_{tree}. In mixed stands, d_{tree} is taken from the table for the same height for different species, but the mean d^o_{tree} is obtained by allowing for the percentage ratio of tree species. This method advanced by the author has proven quite laborious, but it nonetheless has meant a reduction in deviations. This denotes that stand composition does have an effect on the precision of d_{tree} determination.

Since this experiment was verified for a small number of plots, we do not present the results of the test here. Table 50 requires refinement and inclusion of tree species widespread in Siberia and other parts of the U.S.S.R. Completion of studies aimed at discovering multiple correlation between d_{tree}, h, and D_c with provision made for composition of mixed and pure stands in different parts of the U.S.S.R. is a separate undertaking.

The following conclusions can be drawn from the results of the studies conducted.

Determination of mean thickness based on h_0 and D_c measured on aerial photographs and taken from the correlation tables give approximational information about forests. However, until other, more precise methods are found, we will have to use existing methods and be satisfied with the precision they can give us.

The simplest method must be viewed as the method of determining mean thickness from tree stand height. The method of determining d^{o}_{tree} from crowns affords low precision as a consequence of the low intimacy of the relationship between D_c and d_{tree}. A more intimate and more stable correlation has been found between h and d_{tree}, ensuring in most cases determination of d^{o}_{tree} based on h_o values with an error of the order of 12 percent. The method of determining d^{o}_{tree} simultaneously from h_o and D^{o}_{c} gives higher precision and on the average affords determination of d^{o}_{tree} with an error of 10 percent.

Analysis of deviations greater than 4 cm affords grounds to assert that in most cases they pertain to those forest plots for which too great a violation in the ratio of h, d_{tree}, and D_c has been observed, for example, h = 18.5 meters, d_{tree} = 40 cm, and D_c = 6.8 meters. In plots differing sharply from normal plots in low height and in large thickness of trees, or in contrast, in large h and small d_{tree} values, crown diameter often proves to be very large in the first case and too small in the second. It is best in these plots to determine d^{o}_{tree} simultaneously from h_o and D^{o}_{c}, which affords more satisfactory d^{o}_{tree} values.

However, we also can encounter cases of other relationships between d_{tree}, D_c, and h, and here this procedure will not bring us desired results. Studies described earlier have been conducted by the author to secure topographic information about forests, but the theory, methods, and procedures set forth in this work can be used (transformed and developed) also for forest-inventory work of different precision classes.

CHAPTER 9

APPLICATIONS OF MEASUREMENT INTERPRETATION OF AERIAL PHOTOGRAPHS IN FOREST MANAGEMENT AND AERIAL APPRAISAL OF FORESTS

36. Use of Measurement Interpretation of Aerial Photographs in Forest Management

Interpretation of aerial photographs is used in forest management at different levels. However, the extent of its application, content, and scope differ. Only one principle is wholly obvious: thus far, recognition and measurement properties of aerial photographs or, as the term now is, the information capacity of aerial photographs, have not been fully utilized. The quality of forest management and its profitability depend entirely on this use.

Materials of aerial photography and office interpretation using measurement procedures will be of the greatest importance in primary forest management both in the first, as well as in the second year of operations. Success in forest management above all will depend on the quality of the aerial photographic materials.

Aerial photographs in scales from 1:5,000 to 1:15,000 are the most suitable for measurement interpretation. In the latter, the number of details is considerably reduced and crown sizes are minute. However, it is possible to use a two or three-fold magnification of aerial photographs which can compensate for some of their drawbacks. As far as 1:25,000 aerial photographs are concerned, although their magnification leads to a relative increase in crown sizes and openings between crowns, this does not introduce substantial changes into the nature of the imaging of the stand canopy. Aerial photographs of this scale can be used under reserve zone conditions not intended for exploitation in the immediate future.

Spectrozonal aerial photographs are the most suitable as well as aerial photographs on panchromatic and orthochromatic aerial films obtained during the period of seasonal changes in the forest tracts. If the aerial photography is not performed in a stereotyped way, but with allowance taken of the condition of stands, then the quality of the aerial photographs, their measurement properties and ultimately savings of funds can be boosted.

In all cases the aerial photography must be performed a year ahead of forest management. During this time measurement properties of aerial photography must be studied and preparatory work carried out. Aerial photographs not more than two-three years old are most suitable for interpretation. In mature and overmature stands of the near-tundra and reserve zones, no substantial changes must be introduced into the stand themselves during this period, though in felling areas, in young forests, and in moderate-aged stands owing to the intensity of ongoing processes changes in the nature of photographic images can be substantial.

Preliminary aerial-visual observations will assist in singling out those sections that will be subjected to repeated aerial photography, therefore a new sampling session of aerial photography is possible, even when there are five-year old aerial photographs available.

In the first season of forest management work (after obtaining aerial photographs for the area), before entering the forest it is essential to study the nature of the stands from materials of the early aerial-appraisal inspection of forests or other available material. Information summaries are drawn up in which the scope of stand diversity is shown for the main appraisal indexes of the stands (in composition, growth classes, occupancy rates, site classes, forest types, etc.)

Then aerial photographs are studied aiming at finding the extent of differentiation of different categories of stands, and the possibility of using these to measure appraisal indexes, and tests are made of the suitability of measurement instruments, templates, and scales.

On this basis, premises are developed on the use of measurement methods of interpreting aerial photographs in determining the main appraisal indexes of stands.

And, finally, by using reproductions of superimposition montage, photo-layouts, one or several flight routes are plotted for the purpose of familiarization with the nature and condition of stands in the given area. These flights are of importance for the final choice of the appraiser training locations, sampling plots, and number and extent of paths for ground training appraisal, bearing in mind the covering of the most diverse stands and the compact layout of training areas.

Later, all work with aerial photographs must be conducted in conjunction with ground operations, with office and measurement interpretation of aerial photographs, or with aerial appraisal of forests in given combinations and scope depending on local conditions and the problems dealt with. After delimiting the sampling plots, the latter must be carefully coordinated at the actual locality and drawn on the aerial photographs based on easily discernable markers.

Appraisal-interpretation description and a count of crown participation in stand canopy are carried out at the sampling plots, in addition to ordinary work stipulated in the forest management instructions, as applied to the method set forth in our study [50].

In addition to necessary measurements of trees and their crowns, training is conducted at the same sampling plots on visual determination of tree crown diameters, stand canopy closure, composition of stand canopy, and the mean tree spacing, as well as determination of average height of stands and average height of trees making up the horizontal projection of stand canopy.

If color aerial photographs are available, additional training is conducted on recognition of tree species, species composition of stands, and their age based on differences in color tones.

The number of samples must be found in accordance with the diversity of the stands and must not be less than 20-30 per object of operations.

Besides sampling plots, in this same field season appraisal sections are delineated on aerial photographs along pathlines, they are subjected to preliminary examination in the stereoscope, appraisal indexes are measured, and there is office comparison of appraisal descriptions with those entered into the appraisal log.

After this preliminary preparation, on-the-scene appraisal of stands is carried out with entry into the appraisal log of the indexes of crown shape and size, composition of stand canopy closure, and features of canopy structure that have a bearing from the point of view of measurement interpretation of aerial photographs.

In parallel with on-the-scene appraisal, after the aerial photographs have been inspected, the characteristic features for interpreting them and for aerial appraisal of forests are entered. Along with ocular appraisal, the following necessary measurement are taken: height of tree stands, mean breast diameter, mean tree spacing, and crown diameters by species.

This work on appraisal paths laid down along the most diverse and representative stands serves as a reliable basis for subsequent training of appraisers. Here the appraisal station locations must be marked (by pricking) on the aerial photographs. All known methods and measurement instruments are used for appraisal characterization of stands in these localities to formulate the most objective and reliable characterization of the tree stands.

The extent of the training paths must be such that for each area of operations a quite specific familiarization with stand growth conditions and the correlations in their canopy structure is secured. By way of a guideline, we note that the extent of paths must not be less than 10 kilometers with the description along the paths of not less than 50 appraisal sections. This same number of sections can be obtained by random selection from the forest tract.

Later, data of the sampling plots and data obtained along appraisal routes undergo processing to establish true relationships essential for further production and measurement interpretation of aerial photographs, between the following:

crown diameters and diameters at breast height;

stereoscopic heights of tree stands of forest elements and calculated mean tree stand heights;

mean tree stand heights and mean diameters;

composition of horizontal stand canopy projection and actual composition of stands calculated from yields;

extent of stand canopy closure and stand occupancy rates;

mean distance between crowns visible on the photograph and mean tree spacing in the actual locality.

As a result of establishing these relationships between these indexes, a composite table is drawn up specifying the size of the corrections that are necessary to be inserted for proper determination of the compositions of mixed stands, occupancy via canopy closure, mean tree spacings, mean tree stand heights, and from this listing indexes essential for determining stand yields are refined.

If color photographs are available, a table is drawn up for determination based on color tones of species composition of stands, tree stand age, and habitat conditions (site class).

At the conclusion of this period, by using a variety of instruments, templates, and scales and the above-proposed methods, appraisal indexes are measured for all the objectives of forest appraisal. As a result, the methods of operations are refined for subsequent regular interpretation of aerial photographs, the suitability of particular equipment is determined, and the indexes of confidence with which appraisal features of stands discovered after statistical treatment of data are calculated.

Contour and appraisal-measurement interpretation of aerial photographs is undertaken in the office period by stereoscope analysis.

Based on fieldwork with aerial photographs, features of the interpretation of different categories of forests or forest types are refined. The appraisal interpreter working under field conditions must have suitable tables of relationships between D_c and d_{tree}; h and d_{tree}; the extent of crown closure and occupancy rates; canopy composition based on the participation therein of various species and stand composition by yield levels, as well as tables of corrections to be inserted into appraisal indexes of stands and color characteristics for species, forest generations (ages), and habitat conditions.

Prior to the onset of regular and measurement interpretation of aerial photographs, it is recommended that training be conducted in the determination and measurement of appraisal indexes of stands in sampling plots and along appraisal routes using available tables. By combining these with true data, analysis must be made of errors of interpretation and measurement.

The results of measuring tree heights or heights of tree stands as well as other appraisal indexes that require calculations are written up in individual reports that are later compiled into log form.

Tree stand heights are measured by difference in parallax if the ground surface in the actual plot or close to it is visible in the photograph. After suitable training, the measurement of heights by the ocular-stereoscopic method is possible [47, 54].

Determination of diameters at breast height d_{tree} is accomplished based on measurement of crown diameters and mean tree heights. The number of measurements is determined by relying on field determination of variants. The values of d_{tree} are obtained from correlation equations or tables previously compiled on the basis of such equations.

Stand composition is measured by using templates made for the given aerial photographic scale, with the insertion of corrections into the stand makeup coefficient if there are mixed or complex stands.

Stand occupancy is determined from crown closure by inserting required corrections in the event of pronounced discrepancy therein. Mean tree spacing and the number of trees are determined by the methods described above.

Forest type, stand age, and site class are determined by ocular interpretation using previously calculated data that serve as the starting-point for obtaining appraisal characteristics of the stands. Auxiliary tables published in the work [54] can be used in determining site class.

The following must be verified under local conditions (in sampling plots) for determination of stand yields:

applicability of standard tables of the sums of cross-sectional areas [basal areas] and stand yields for an occupancyof 1.0, using the above-calculated mean heights and occupancies by stand stories for those determined from aerial photographs;

the method described in section 26;

or by use of other methods of determining stand reserves (according to I. A. Trunov and G. G. Samoylovich).

Methodologically, it is above all desirable to determine the forest type, species composition of stands, height by individual species and then to proceed to measurement and determination of other appraisal indexes of stands that are interrelated.

If summer panchromatic aerial photos are available when the difference in species is poorly pronounced, the methodological recommendations set forth by the author in Geograficheskiy Sbornik [Geographic Collection], No 5, 1955, must be borne in mind.

In summing up a balanced analysis of the opportunities of appraisal-measurement interpretation, we determine the properties of aerial photographs and the necessity for those supplements that must be carried out during the field period of the second year of operations, as well as refine work methods. To be specific: the question of whether it is best to use office interpretation or to engage in aerial appraisal over part or all of the tract is being resolved, as is the issue of which of the appraisal indexes should be determined by measuring aerial photographs and which by supplementing as a result of aerial appraisal by helicopters or fixed-wing aircraft.

For instance, if spectrozonal photographs of mixed and complex stands are available, the following can be determined in the office: stand composition, occupancy, and site class and age -- by aerial appraisal. If panchromatic aerial photographs are available for the same region -- composition of stands and age can be determined by aerial appraisal, and the rest of the indexes -- by interpretation of aerial photographs. This is an approximate scheme that must be corrected depending on the nature of the stands predominating in different parts of the forest tract under appraisal.

Finally, the volume of essential ground operations that make it possible to coordinate into a single whole all the most rational suggestions in forest inventory for a particular forest-plant cover area are determined.

In summing work done during the first year, methods of subsequent operations and preparation for second-year field appraisal are undertaken. These other operations must include contour and measurement-appraisal interpretation of aerial photographs within the bounds of each appraisal plot. This work is done by specialists who have undergone suitable preparation and training.

Forest-management instructions at level IV of primary forest management recommend ground appraisal only for block clearings, but if spectrozonal photographs of a scale not smaller than 1:15,000 and not more than five years are available, appraisal-measurement stereoscopic interpretation of aerial photography must be conducted instead of aerial appraisal from helicopters.

In particular cases, it is permissible to engage in aerial appraisal or stereoscopic interpretation without ground appraisal in block clearings. Ground routes are pursued only in those localities where diversity in appraisal characteristics is pronounced.

At level III of forest management, both appraisal-measurement interpretation of inter-clearing expanses as well as their aerial appraisal are conducted in economically low-value plots in individual sparsely inhabited sections of the tree farm that has been established (forest industry area).

This recommendation greatly limits the possibilities of materials of aerial photography and must be re-examined with characteristics of the forest tract taken into account.

Appraisal-measurement interpretation will be of autonomous importance for refining characteristics of group III forests in which forest exploitation will proceed in the long run. In areas of the near-tundra zone, the northern reaches of Siberia, and the Far East previously subjected to aerial appraisal, it is quite possible with a small volume of ground appraisal to use widely measurement methods of interpretation and their automation based on interpretation by forest categories. By way of example, we can cite the work done in Gornaya Shoriya, the results of which have been published in the study [44].

37. Combined Method of Forest Inventory via Aerial Appraisal With Measurement Interpretation of Aerial Photographs

Aerial appraisal of forests (in conjunction with fieldwork in forest management levels III and IV) is carried out in sparsely-settled forest regions of the North, the Urals, Siberia, and the Far East.

At present, aerial appraisal of forests by helicopter is carried out at a flight velocity of 40-50 km/hr and from a flight altitude of 50-100 meters. Under this flight regime, appraisal description of stands proceeds, in which, in addition to stand composition, occupancy, site class, and age, mean tree stand height, and diameter at breast height are indicated.

Aerial photographs serve chiefly to distinguish plot contours, and photo-layouts -- to project flight routes, orientation during flight, superimposition of plot contours, and within their limits entering of appraisal characteristics of stands.

It is quite understandable that the operational methods employed are not based on the full-fledged use of recognition and measurement properties of aerial photographs. These methods must be improved both as to preparatory and appraisal-training work, as well as in the use of measurement interpretation of aerial photographs.

Aerial photography for purposes of subsequent primary forest management is carried out one year ahead of operations.

For all kinds of aerial appraisal, color spectrozonal aerial photographs of a scale not smaller than 1:15,000 will be the best for operations. Preliminary calculations have shown that some of the costly flight and field operations will be cut back by relying on quality of aerial photographs and the possibility of using them to secure greater information.

Ground training of aerial appraisers essentially must go hand in hand with the training of appraisers in the same scope and content as described earlier.

The aerial appraiser, just as an ordinary appraiser, must engage not only in contour, but also in appraisal-measurement interpretation. Additionally, during ground training aerial photographs must be used for simultaneous characterization both of interpretation as well as aerial-visual features of stands side by side with determination of appraisal indexes of stands. These features must include above all those that determine the nature of stand canopy structure (shape and size of tree crowns, composition of the visible part of the stand canopy, canopy closure, etc.) and also account for the relationship between appraisal and interpretation indexes of stands.

In compiling appraisal characteristics of stands, one must simultaneously analyze both the interpretation qualities of aerial photographs as well as aerial-visual features of stands.

In conducting training together with appraisers and in measurement interpretation of aerial photographs, aerial appraisers will be wholly trained for independent work with aerial photographs.

Preliminary study of natural interrelationships between appraisal indexes of stands must precede aerial appraisal of forests and must be a constituent part of the whole technological process.

The mission of aerial appraisers in preparing for flights, along with contour-topographic interpretation of 1:15,000 aerial photographs, will embrace measuring the heights of tree stands of forest elements and other appraisal indexes of stands (composition, canopy closure, crown diameter, tree stand density, etc.). If panchromatic, autumn panchromatic, or spectrozonal aerial photographs are available covering the bounds of the plot contours, those appraisal indexes that can be differentiated with full certitude in stereoscopic interpretation are written down in the course of interpretation. For example, the numerator gives the composition and height of tree stands by species and the denominator -- canopy closure and site class. The rest of the missing appraisal indexes are written up during flight over these same plots.

Given black-and-white summer panchromatic aerial photographs, the stand coefficients must be written down during flight time, and height by species or forest generation (if the latter is readily distinguishable) from data of measurement-interpretation of aerial photographs.

Thanks to the opportunities afforded by aerial photographs for a complete survey of the entire appraisal tract, determination both of stand composition as well as occupancy will be more objective and precise than when flying over the plot during a single, or on rare occasions several, minute. The same can be said for tree stand height. Its measurement with the use of stereo-instruments in different plots of the tract will be more precise than when scanning a narrow strip from a helicopter. Introduction into ocular aerial appraisal of measurement indexes increases its objectivity and quality of operations and leads to refinement of the indexes calculated, in particular, stand yields.

Parallax-measuring stereoscopes, parallax sheets, templates, and scales can be used in measurement characterization of stands as well as stereo-measurement instruments.

Thus, depending on stand characteristics, quality, and interpretation properties of aerial photographs, for each tract only those appraisal indexes that can be interpreted confidently and free from doubt are determined.

Composition coefficients, if they are not written down during the course of interpretation of aerial photographs, age by species and forest generations, site class, forest type, merchantability of tree stand, and percentage of windfall and dead standing timber are written up by the aerial appraiser in helicopter flight from a specified altitude.

Under this combined method of operations, interpretation properties of aerial photographs and the advantages given by aerial appraisal when inspecting stands from shallow flight altitude are more fully used.

Incorporation into operational use of the proposals recommended, taking into account also the possibility of landing the helicopter when necessary close to or in the middle of stands, will make it possible to secure, in conjunction with a limited scope of fieldwork, quite reliable materials in forest inventory, especially in near-tundra and semibog areas of the north and homogeneous forests of Siberia and the Far East applicable to the requirements of forest management levels IV and even III.

If forests are located only along streams, scattered among bogs, are homogeneous in appraisal indexes, and will not be exploited for several decades, office appraisal-measurement interpretation based on a small volume of field appraisal for several flights by helicopter for landing to become familiar with stand character will fully replace exhaustive aerial appraisal of forests and cut the costs of operations down to one-fourth or one-fifth.

CHAPTER 10

PROSPECTS FOR AUTOMATING INTERPRETATION OF AERIAL PHOTOGRAPHS OF FORESTS AND ELECTRONIC COMPUTERS IN FOREST MANAGEMENT

38. Hypothesis on Types of Tree Arrangement and Cybernetic Principle of Stand Study

The method of point sets and systems and the geometric analogues of tree arrangement with their characteristics dervic from the main appraisal indexes (chapter 2), constructed on the basis of this method, can be used as the mathematical framework for modeling.

In principle, this method can be applied to the study of any phytocenosis. The method of mathematical modeling is best combined with the cybernetic principle of stand study in the course of stand development in time and space. successes in cybernetics, information theory, electronics, and mathematics strongly affect progress in many branches of knowledge.

At the present time, cybernetics and mathematics have also penetrating the science studying animate organisms. They assist in solving problems of mathematical statistica modeling of biological phenomena and in exploring and modeling certain functions of the activity of the brain and nervous system.

One of the main principles adopted in cybernetics is the principle of feedback, which allows us to retain the stable existence, self-organization, and self-regulation of animate organisms and certain automata. The common ground of the feedback control principle in control of processes in animate and inanimate nature has been confirmed. There are no grounds either to refute the operation of this principle in developmental processes of phytocenoses, including forests.

From this viewpoint, the stand can be represented as a biologically self-regulating system developing in time and space on the principle of multilateral feedback. The successful survival and development of this system is based on those interrelationships and stable quantitative ratios of stand indexes that have been formed in the course of adaptation to given havitat conditions.

If self-regulation is violated in this system (as a consequence of violation of multilateral feedback), then the system and the natural quantitative ratios inherent in it (correlations of arrangement and distribution, correlational ties, etc.) are also violated. Discovering the reasons for the development and disturbance of this system is a problem for foresters and geobotanists. Accordingly, we must recognize as most vitalthe research and concepts of Professor V. G. Nesterov[1], who obtained by mathematical statistical methods of meteorological criterion of plant moisture content and the variable "decorrelation coefficient" as an indicator of the perfection of plant organisms.

Without taking up these involved problems, it appears to us to be useful to advance a working hypothesis about the types of tree arrangements in stands and the types of tree spacing distribution functions corresponding to them in the dynamics of stand development as self-regulating systems with feedback.

For differentiating and mathematical description of the most probable types of tree arrangements in stands we use the method of point sets and systems. Tree arrangement in areas (plants in general, points, and discrete objects) can be divided into two main classes: regular and non-regular.

The Swedish scientist Svedberg adopted as a criterion of regularity of individual plant arrangement in a plot of a locality to be the coefficient of plant density dispersion per unit area. The number of plants per unit area he called "profusion," however in the more exact sense this corresponds

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[1] V. G. Nesterov, "Experience in Determining the Automatism of Organisms and Its Applications," Biologicheskiye Aspekty Kibernetiki (Biological Aspects of Cybernetics), U.S.S.R. Academy of Sciences Pub. House, Moscow, 1962.

to plant density. Svedberg made an error when he characterized arrangement types as plant density in areas, since arrangement of plants in a given area at the same density can be most varied in regularity. Therefore, we cannot deem density to be a satisfactory criterion of regularity or nonregularity of plant arrangement in an area.

Accordingly, the author proposes that we consider as the meaning of mathematically precise tree (plants or points) arrangement in the form of any regular point system investigated in chapter 2 to correspond to the term "regular arrangements." As noted earlier, the regular system of points placed at the apexes of adjoining equilateral triangles is a most remarkable system. In this system, the distances between all neighboring points are the same, and this arrangement gives the largest number of points for the area covered. Therefore, we must accept as the first class and first arrangement type the regular placement of points at apexes of adjoining equilateral triangles.

The second class -- the class of nonregular arrangement -- is best divided into a series of types which differ in degree of nonregularity.

We can propose the following arrangement types as the first variant:

type I -- regular arrangement of trees in the area;

type II -- weak nonregularity of arrangement;

type III -- optimal nonregularity of arrangement;

type IV -- severe nonregularity of arrangement.

What then are the characteristics and mathematical features of the four arrangement types?

Type I -- regular arrangement. This arrangement type has precise mathematical characteristics. Points (trees) are located at the apexes of adjoining equilateral triangles or in the centers of circles touching each other. Distances between points (trees) are the same. Therefore, $l_0 = l_1 = l_2 = \ldots = l_n$; $\sigma = 0$, $v = 0$.

The spacing distribution type is expressed by a single vertical straight line equal to 100 percent of the spacings (Figure 28).

The distribution function f_1 = const. It can be represented as the maximum case of normal distribution in the form of a series out of the same interval l_0 when $\sigma = 0$, v = 0, and frequency fraction $w_0 = 1.0$. Then, $f_1 = w_0$ and f percent = 100 (Figure 29). The relationship between spacing l and density N is determined by formulas (4) and (5).

Is such an ideal type of plant arrangement met in nature? By artificial plantings (seedings) we can get this type of tree (plant) arrangement. This arrangement has several advantages over all other arrangement types, including those with any regular systems.

The most favorable spatial conditions for development of each tree (plant) are provided under this system. Each stem is given the same feeding area, the best conditions for spreading of the root system to all sides, and the largest number of directions for access to air and sun (when the row planting layout is used there is just one direction, in the square-nested arrangement -- two directions, but in the triangular arrangement -- there are four directions). The greatest number of plants per hectare is provided for.

It is obvious that under these arrangement conditions we can anticipate the greatest harvest yield, the highest forest site class, and the fullest use of the area (locality). And it is approximately to this ideal arrangement type that we can classify the "normal" stand.

In our view, the first type of arrangement is of practical importance for application in farm work as the most profitable type of planting (sowing) system for many farm crops. In addition to the above-noted propitious conditions for plant development, the proposed system of planting (sowing) can yield the greatest harvest (under otherwise equal conditions), since this system makes possible within the same area the placement of the greatest number of plants (considerably greater than in the row and square planting systems). As an example, let us take an area of one hectare and locate plants in it at distances of 0.5 meter. Under the square planting system, one hectare can accommodate 40,401 plants, but under the triangular system -- 46,717 plants, that is, 13.5 percent more, which is tantamount to increasing the yield by 13.5 percent from each hectare. If the planting (sowing) of corn, for example, is carried out on 100,000 hectare and each hectare gives a harvest increment of about 14 percent, and each percentage is equal to even one pood [36 pounds], the increment would be reckoned by the enormous figure of 1,400,000 poods.

However, in nature in natural forests the type I arrangement is not encountered. Nonregular tree arrangement, as we know, is observed in forests. The reason is to be found in the numerous factors of forest habitat and development which in totality govern the nonregularity of stem arrangement in a section of a locality.

Type II -- weak nonregularity of arrangement (Figure 28). Mathematical characteristics of this arrangement type can as a premise be formulated as follows.

Tree spacings are not identical. Therefore, a mean distance l_2 is arrived at. When there is weak irregularity, most tree spacings have small deviations from l_2, which leads to σ_2 small in value. Then the spacing distribution type f_2, retaining the general correlation of normal distribution, will be characterized by a distribution curve swelling upward with a sharp decline toward the X-axis. Table 84 lists calculations of the function f_2 for $\sigma_2 = 0.1$, $l_2 = 0.44$ meter, and $v_2 = 10$ percent, with the parameters $l_2 = 4.4$ and $\Delta l = 0.9$.

Figure 28. Arrangement types for trees and their approximational mathematical characteristics: type I -- regular; type II -- weakly nonregular; type III -- optimally nonregular; type IV -- severely nonregular.

Legend: A -- type I; B -- type II; C -- type III; D -- type IV

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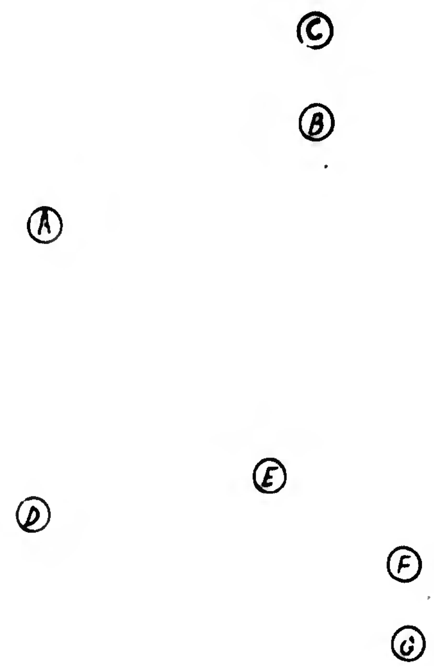

Figure 29. Types of tree spacing distribution curves

Legend: A -- when $\sigma = 0.44$, $v = 10\%$; B -- type II; C -- type I; D -- when $\sigma = 1.3$, $v - 43.2\%$; E -- $\sigma = 3.96$, $v = 50\%$; F -- type III; G -- type IV

Figure 29 shows the type II distribution curve constructed from the function $f_2(1)$.

If we consider the arrangement type II as applied to forests, then as an assumption this type in schematic form as well can apply to the youngest forest, starting with sprouts and up to the point when the trees can exist only in an intimate community of high density as similar individuals and with very small stem distances.

Thus, for example, a vast number of sprouts of young pine and spruce are observed in felling areas -- up to several tens of thousands per hectare; they give the appearance of a thick brush. Nonregularity of arrangement in this period is caused by the randomness of natural scattering of seeds in the plot. Under the given conditions, we can expect weak nonregularity of arrangement and small deviations in individual spacings from the mean plant spacing, the mean spacing itself being very small. Here it is most probable to expect also very small values of σ.

Table 84

l_i	$l_i - l$	t	z_i	$f(t)$*
0,9	[illegible]	7,9[illegible] / 0,8[illegible]	[illegible]	[illegible] / 0,061
1,8	[illegible]6	[illegible] / [illegible],657	[illegible] / [illegible]32.	0,000 / 0,0[illegible]
2,6	[illegible]8	4,08 / [illegible],478	[illegible],000[illegible] / [illegible]	0,[illegible]002 / [illegible]
3,5	[illegible]	2,05 / 0,228	0,04[illegible] / [illegible]	[illegible]081 / [illegible]
4,4	[illegible]0,0	0,0 / 0,0	[illegible] / [illegible]	0,[illegible] / 0,091
5,[illegible]	[illegible]	2,05 / 0,228	[illegible] / [illegible]	0,081 / 0,0[illegible]
6,2	[illegible],8	4,08 / 0,478	[illegible] / [illegible]	0,00002 / [illegible],081
7,0	2,6	5,91 / 0,657	[illegible] / [illegible]	0,000 / [illegible],073
7,[illegible]	[illegible]	7,[illegible]5 / 0,[illegible]83	[illegible] / [illegible]	0,00[illegible] / 0,[illegible]

*f_2 -- upper line when $\sigma_2 = 0.44$, $v_2 = 10\%$, $\Delta l/\sigma_2 = 2.05$;

f_4 -- lower line when $\sigma_4 = 3.96$, $v_4 = 90\%$, and $\Delta l/\sigma_4 = 0.227$.

This arrangement will be preserved until the young saplings acquire stability adequate for independent existence. Until that moment, all plants can successfully develop solely on account of intimate joint existence of the entire plant community. If this dense stable community did not exist, each young plant considered separately could survive only with great labor.

Later, with growth the young plants begin to displace each other, their root systems intermingle, and capture of nutrient and moisture takes places; the negligibly small feeding area becomes inadequate for normal development of all this dense plant mass and upon root interweaving the process of self-thinning sets in. Particular individuals outpace others in growth. Securing greater amounts of light, they develop a stronger root system and crown. Individuals lagging in growth, falling under the canopy of other plants, become feeble and gradually expire.

This is approximately how the process of natural thinning of a young forest takes place [1,2]. An increase in the feeding area of each plant affords surviving plants an opportunity to acquire individual stability, and this stable community is consolidated, successfully developing under normal habitat conditions.

It can be assumed that from this moment the type II arrangement intergrades into the type III arrangement, that is, to a more nonregular plant arrangement. Gradual thinning of plants becomes random.

In one way or another this plant community acquires the optimal nonregular arrangement in the plot that is produced by nature itself for successful development of the community. Thinning will occur even later, but more weakly, which is mainly reflected in an increase in mean tree spacing and not in the arrangement type, which by this time has been consolidated and become stable and most favorable for the development of each plant and for the entire plant community in the given plot of the locality.

Type III -- optimal nonregularity of arrangment (Figure 28). Mathematical characteristics of the type III arrangement are as follows: mean distance l_3 is variable; $\sigma_3 \approx 0.43 \cdot l_3$; $v_3 \approx 43.2\%$. The spacing distribution type is determined by a statistical function (47) and by the distribution series shown in Table 26; the type III distribution curve f_3 (Figure 29). It can be assumed that the nonregular type III arrangement is the most widespread, the most stable and the most optimal for the natural development for the plant community under ordinary habitat conditions and during the period of life that is characteristic generally of most stands. During this period all trees affect each other and form an intimately interacting and successfully developing plant community in which those changes in tree indexes occur (height, thickness, and mean spacing) with development of the community, but the entire community continues its development as a mutually related complex adapting to given habitat conditions, with a stable arrangement type induced by nature itself and with a spacing distribution type which is induced by the correlation of the latter. Therefore, this type of arrangement and distribution can be called optimal.

If the forest has taken hold, adapted to given habitat conditions, and constituted a stable community, this state and type of arrangement and distribution can be retained even in old forests, where different habitat conditions, it appears, will be reflected to some extent only in the value of l_0, but not in the arrangement type and spacing distribution. But only under especially habitat conditions (soil-climatic factors or artificial measures) can intense nonregularity of tree arrangement in the area take hold, which of course will lead to a change in the stem spacing distribution type. In this case, a transition to the type IV arrangement is probable.

Type IV -- severely nonregular arrangement (Figure 28). Approximate mathematical characteristics of the arrangement type are as sollows: variable and usually large mean spacing l_4; $\sigma \approx 0.9 \cdot l_4$ and $v_4 \approx 90\%$. The distribution curve f_4 is shown in Figure 29. It is a gradual, low normal distribution curve extended along the X-axis.

The distribution function f_4 has been calculated in Table 84 with the parameters $\sigma_4 = 3.96$; $v_4 = 90$; $\Delta l/\sigma_4 = 0.227$.

Earlier it was noted hypothetical under which conditions the existence of type IV arrangement in nature and the type f_4 spacing distribution corresponding to it are possible. Under these conditions, unstable, severely nonregular tree arrangement in a plot of the locality appears and a violation of the interrelationships between plants essentially sets in, as a result of which the plant community gradually disintegrates. At first they break down into individual parts (groups) within which proceeds a weak interrelationship with individuals can still be retained, but subsequently stem spacings become so great that within the groups a breakdown in relationships sets in and the forest as a plant community ceases to exist for stem spacings attain such values that any relationships between trees are lost and free-standing trees are formed at large spacings from each other.

Tree arrangement types and tree spacing distribution functions described are viewed by the author as a first hypothesis, which needs to be verified by experimental measurements for different stand types and ages.

For type III it is useful to investigate the stability limits of the function to either side from $V \approx 40$ percent within the limits $v = 40 \pm 20$ percent in order to determine the boundary regions between the distribution types II and III, and between types IV and III.

Some remarks can be made about normal stands as related to the arrangement types considered here. It appears more worthwhile to view as "normal" (more correctly, optimal stands) not stands that exhibit occupancy close to unity, but stands close to type III arrangement and the distribution correlations corresponding to them, for in nature this arrangement type predominates and is most stable during the greater part of the life of each forest.

Foresters and appraisers have long since arrived at these findings about normal stands. The difference here is only that our assumptions are based on mathematical investigations of arrangement types and tree spacing distribution types in stands.

The correlation and stability of type III can be accounted for by the presence of biologically self-regulating systems in the form of a plant community with multilateral feedback.

In conclusion, we must once again emphasize that the suggestions advanced here about arrangement types must be viewed as a first hypothesis, which requires experimental verification for tree cover. In the geobotanical method, it appears to be worthwhile to use as indicators first of all mean spacing, the most powerful indicator of principal differences in development of a plant community, and then distribution types and curves with variable parameters σ and v, which taken together are secondary indicators of other factors of plant habitat.

39. Principles for Automating Aerial Photo-Interpretation of Forests

What we by tradition have become accustomed to regarding as interpretation of aerial photographs appears best to call the theory and methods of obtaining information about a locality from aerial photographs. An aerial photograph is a source and carrier of diverse information about a locality, including forests. A large amount of information from an aerial photograph can be obtained by simple visual inspection without any interpretation or decoding. The advantage of an aerial photograph here lies precisely in the fact that it passes on its information in graphic form and not in especially coded or enciphered form requiring some keys or code for deciphering or decoding information visible to the eye. Also, the photograph bears that information about objects that is difficult or impossible to obtain by direct visual observation with required precision and in a quantitative numerical expression. In these cases it becomes necessary to develop a theory, methods, procedures, instruments, and techniques.

Advances in interpreting aerial photographs have come in successive stages from ocular estimation to measurement methods, from simple visually observed information to the securing of more precise and diversified information about various objects and phenomena. At the first stage, interpretation of aerial photographs is limited to ocular estimation, and at the second stage simple measurements and reference standards begin to be used. The third stage is characterized by use of complex photogrammetric instruments for precise measurements of a stereo model of the aerial photographs and by the elaboration of theory, methods, and procedures of measurement interpretation based on an understanding and use of statistical correlations of distribution, correlations, and the sampling method. Thus, interpretation of aerial photographs of forests gradually acquire all the features of a scientific solution of a problem. The essential prerequisites for moving on to the fourth stage -- the stage of serching for methods of mechanization and automation -- have also been provided.

Given today's stage of science and technology, it is not at all too early and not wishful thinking to start searching for methods of automation. While the third stage is involved mainly with opticomechanical photogrammetric instruments, the fourth stage invariably will be associated with the use of electronic instruments, which in turn will require application of advances in such sciences as cybernetics, information theory, electronics, the theory of algorithmization and programming, and also electronic computers.

Right now it appears possible to use three principles of the automation of securing and processing about forests from aerial photographs: the principle of comparison, the principle of measurement, and the principle of comparison and measurement simultaneously.

The principle of comparison is used also in ordinary ocular interpretation of aerial photographs that rely on reference standards. An aerial photograph of the forest plot under interpretation is successively compared with a series of reference standard aerial photographs, whose appraisal indexes were known in advance. The human labor is reduced to visual comparison of the structure of the photo-image of the aerial photograph pair and to detection of the greatest similarity of the given structure to one of the given reference standard photographs. Use of this principle in automatic interpretation amounts to performing the comparison and detection of similarity of images by machine and not by man. If we are able to model the process of comparison in the form of logic operations, then machines can do this work.

What then are the theoretical and technical possibilities today for solving this problem?

The problem of comparison and discovery of similarity between photoimages of a forest from aerial photographs is closely bound up with the cybernetic problem of recognition or reading of images by automatic devices. A number of automatic devices have been developed in the U.S.S.R. and abroad in recent years for reading (recognition) of alphabeticonumeric information, and some successes have been recorded in developing devices for recognition automatically of such images as figures and other similar representations. Underlying the reading automatic devices is the principle of comparison and identification of images and objects by the human brain and the principle of self-teaching of machines to recognize visual images. Thus, the theory of automatic reading devices is founded on physical modeling of the proposed process of recognizing images and objects by man, on the use of bionics.

This process in systematic form can be represented as follows. Visual apperception of objects is subjected to processing and transformation, as a result of which images of the object -- reference standards -- pile up in the human brain. Recognition of objects upon their repeated visual apperception occurs by comparison of the image received with the reference images stored in the brain. The agreement of these two images is recorded and understood by the individual as the fact of the recognition of the given object. In recognizing images, machines employ this same principle. In actual practice, the images themselves, binary codes of images, geometric features of figures (letters, digits, arbitrary signs), and point subsets of images can in fact be compared. In these cases, logic rules of forming figures from simple elements (straight lines, curved lines, angles of intersection, their arrangement and mutual relationships), counting of the number of elements and the corresponding algorithm describing the figure of each sign are used. Reference standard images prepared in advance in program form are stored in the machine's memory bank. The images to be recognized are perceived and compared with reference standards stored in the machine's memory. This principle can be used also in automating the interpretation of aerial photographs of forests. However, here we immediately encounter the problem of what to compare and what to take as reference standard images?

Photographic representation of a forest on aerial photographs can be depicted in the form of a point set (analogous to the screen imaging of half-tone originals) and then point subsets of various structures of the photographic image of tree stands on aerial photographs can be taken as the object of comparison. But this compels us to find a method of transforming a half-tone image into a discrete point set and to express the latter in digital form, that is, to carry out binary coding of an image. All images can be divided into two groups based on the complexity of binary coding: hatched (discrete) and half-tone (continuous).

In hatched images the boundary of the transition from black to white is expressed in jumplike fashion and corresponds to the logical principle -- Yes -- No, which in binary representation is identical to -- One -- Zero. If we designate the black elements by one's and the white with zero's, then by using the opticomechanical line scanner, we can carry out binary coding of the hatched images and thus convert the graphic image into digital form in the form of binary numbers, and the latter can be entered into punched and inserted into the electronic computer's memory for subsequent processing and comparison of binary codes of images with the codes of images to be interpreted (recognized).

The process of transforming half-tone images of an aerial photograph into digital binary code. This requires first the transformation of the continuous image into a discrete image, that is, quantification of the image on the aerial photograph or film. D. S. Lebedev [36] proposed a device for entering a motion picture film onto punched cards by determining and quantifying the transparency coefficients (ratio of light intensity for light passing through the film to the intensity of impinging light). This method evidently can be used also for entering aerial photographs of forests onto punched cards. Now we can propose a possible technological scheme of automatic interpretation of aerial photographs. Its basis will be the principle of comparing binary codes of images of the aerial photograph to be deciphered with reference standard aerial photographs:

preparation of reference standard (identical in area of square sections) of aerial photographs of forests of different density, height, and thickness of trees, canopy closure, species composition, age, and yield; determination of precise values of this information about tree stands by measurements at the locality;

quantifying and entering of binary codes of the images on reference standard photographs onto punched cards with the insertion of the latter into the electronic computer's memory (with a program of hunting for the reference numbers of the reference standard sections);

entering binary codes of point values of appraisal indexes for the stands of each reference standard area onto punched cards and input of the latter into the electronic computer's memory;

quantifying and insertion of the binary code of the image of the standard forest area being interpreted onto punched cards and input of the latter into the electronic computer's memory;

compilation and input into the electronic computer of a program for comparing codes of the images of the reference standards and aerial photographs undergoing interpretation with logic operations of estimating the greatest agreement between two codes and a command to release for printing digital values of appraisal indexes.

In this method, the greatest agreement of binary codes will serve as the criterion of reliability of interpretation. Actually, we estimate the confergence of subsets of zero's and one's. We can determine the size of the permissible deviations in codes by experimental mathematical statistical studies. For this purpose, it is worthwhile using the method of statistical tests, which sometimes is called in the scientific literature the Monte-Carlo method, or the method of trial and error. The basis of this method is modeling of tests in a scheme of random events and corresponding agreement criteria described by us in chapter 3.

The principle of combining comparison and measurement has been proposed and described in the studies [15,16]. In this method of automation, codes of the numerical values of appraisal indexes of reference standard aerial photographs are compared with the codes of the numerical values of these indexes measured electronically on the aerial photographs (films) being interpreted. The basis for measurements consists of methods developed and described earlier for measurement-interpretation of aerial photos of forests, but transformed appropriately for processing binary codes based on the appropriate algorithm. Schematically, this method is as follows:

preparation of reference standards (standard square areas) of aerial photographs of a forest with precise values of canopy closure, mean tree spacing, mean crown diameter, mean height and thickness of trees, yield, species composition, and age;

binary coding of precise numerical values of these appraisal indexes of reference standard aerial photographs and their input via punched cards into the electronic computer's memory;

quantifying, binary coding, and measurement of canopy closure, mean tree spacing, and mean crown diameter on the standard areas of the aerial photographs being interpreted; binary coding of the numerical values of three indexes and their input into the computer's memory;

compilation and input into the computer of a program for comparing codes of measured (three) indexes of the aerial photographs being interpreted with codes of the precise values of these indexes for reference standard aerial photographs and logic operations of the estimation of greatest agreement between codes of indexes of the aerial photograph under interpretation with one of the codes of the reference standard aerial photographs; issuance of a command to print appraisal indexes of the aerial photographs undergoing interpretation.

Execution of the scheme described requires an algorithm for determining l_0, C_0, and D_c^0 in binary code. We can use opticomechanical scanning of the forest photo-imaging as the technical means of determining and measuring these indexes in binary code.

An algorithm for determining three appraisal indexes of a stand in binary code has been proposed in the studies

[15,16]. Essentially, this algorithm consists of the following: let us assume that in scanning of an aerial photograph covering a standard area, corresponding to the crown image are one's, and to the openings -- zero's. Then canopy closure can be represented in the form of a ratio of the total of one's to the total of one's and zero's of the standard area. The C_0 obtained in this way is compared with the reference standard C_{st} stored in the memory of the computer. We can adopt the logic rule $C_0 - C_{st} \leqslant 0.1$ as a criterion of convergence.

The mean tree spacing in the line l_i equals the ratio of the length of the line L to the number of discrete groups of one's in the line N. The length of the line of the standard area is known to us. The number of groups of one's expresses the number of crowns in the line n. The ratio of the total of mean areas over all lines to the number of lines in the standard area is taken as the mean tree spacing l_0 of the aerial photograph being interpreted, that is, $l_0 = \sum l_i/n$. The criterion of convergence l_0 with reference standard l_0 values is determined by the logic rule $l_0 - l_{st} \leqslant 0.1$ meter.

The mean crown diameter is determined by the following method. For the length of the images of all crowns along the line we can take the number of one's in the line m and multiply it by the linear size of the scanning unit, that is, $D \approx 0.2$ meter. The crown diameter in the line is equal to the ratio of the length of all crowns in the line to the number of discrete groups of one's in the line, that is, $D_c^1 = D/n$. The ratio of the total of crown diameters over all lines to the total number of lines in the standard reference area is taken as the mean diameter of crowns in the aerial photograph undergoing interpretation, that is, $D_c^0 = \sum D_c^i/l$. The criterion of convergence of crown diameters 0.1 meter $\geqslant D_c^0 - D_c^{st}$, where D_c^{st} is the mean crown diameter of the reference standard areas.

The method described includes three criteria of interpretation reliability, where the program of comparison and selection of the reference standard aerial photograph closest in its appraisal indexes can be refined by resorting to correlations and inserting corrective coefficients obtained via experimental statistical tests. The precision of appraisal index determination can be affected by shadows cast by crowns, but since this effect is systematic in character, it can be allowed for by a statistical method. In the future, however, it is possible to use a method of automatic determination of mean heights from a stereo-model of the forest, which doubtless will increase precision of automatic interpretation of aerial photographs of forests.

The methods of automating interpretation that we have described doubtless must be viewed as the first approximation and as one of several possible approaches to solving this complex problem.

40. Prospects for Electronic Computers in Forest Management

Use of electronic computers for solving scientific and production problems in forest management is altogether possible and at the present time does not represent any fundamental difficulties. Formulating algorithms and programs for solving problems in forest management by use of computers does present some complications, but these problems are resolvable in practical terms.

The advantages of computers is that they afford complete automation of labor-consuming calculations and processing of diverse information about forests, where the machines do this work thousands of times faster than a large number of highly qualified engineers and technicians. Statistical correlations of distribution, correlational functions, and compilation of stand growth pattern tables require for their study fairly complex formulas, processing a vast amount of starting data, and carrying out laborious calculations requiring large outlays of time and efforts, for which even scientific personnel are not always enthusiastic. Use of electronic computers opens up here unbounded vistas for science and production. Only mastery of the working principles required in preparing problems for computer solution is required.

We know that computers can perform simple arithmetical and logical operations. Therefore, in preparing for solutions of problems on the computer, preliminary breakdown of formulas or systems of equations into specified elementary operations which the machine will execute in strict sequence following a man-formulated program is required. High-speed electronic computers perform tens of thousands of such elementary operations per second.

To use the electronic computer, we must first of all have starting information on forests for which we have resolved to obtain the entire set of appraisal indexes of tree stands. We will conventionally call such information we wish to get after processing starting information "processed information." Appraisal indexes measured and determined by methods of field and office appraisal and entered on forms of appraisal descriptions of tract, card entries of sampling plots and model trees can serve as the starting data. The following can be the processed information obtained by using electronic computers:

table of classes of age, site, occupancy, merchantability, and yield for dominant and other species; this information can be provided by the machine in summary form for each quarter, economic accounting unit, forestry station, tree farm, oblast, kray [territory], and republic;

statistical correlations on the distribution of appraisal indexes of stands in the form of theoretical functions, series, and tables for any species (pure and mixed), ages, and areas of U.S.S.R. forests with data about variance, dispersion, and mean values of appraisal indexes;

correlation equations and tables of the function relating appraisal indexes of stands of any species composition, age, and density (for any territory of the country's forests) with curvilinear and multiple correlation taken into account;

tables of stand growth pattern, pure and mixed, for which fractionalizing of areas occupied by forests is suitable;

optimal variants of the system of plantings, organization, and planning of tree plantations, location, and construction of forest roads.

Solution of the listed problems using computers can be carried out separately, at different times, without any rigorous system of organization and storage of starting and processed information about the country's forests. This approach to the overall problem area is not rational and economically expeditious. As it appears to us, use of the electronic computer in forest management from the very outset must be organized most advantageously from the scientific and production points of view. This requires compiling a new type of map adapted for combined use with electronic computers. Right now many maps and layouts of tree farms and other territorial units are being compiled. However, ordinary maps are not suitable for securing all essential information about forests. This information must be obtained by ocular and manual methods, but obtaining processed information involves great outlays of time and effort.

For successful use of electronic computers in forest management, it is best to prepare special maps bearing information about forests, used in conjunction with electronic computers. Compiling such maps does not however present any fundamental difficulties. Essentially, they amount to the following. The boundaries of the tracts, blocks, forestry stations, tree farms, and other territorial units are written on the sheet of the map of a suitably selected scale and projection. Each of these units are given an ordinal number, which then is written on the map in the center of the area that images this unit. For guidance and coordination of forest areas with the rest of the objective and locality, roads, inhabited localities, hydro-sites, and their names are also specified on the map. Thus, all the starting information about stands of tracts will be linked by numbers to the information map.

The starting information is inserted in standard tables giving the number of the tract indicated on the map. Information about standard tables is added to the external memory of the electronic computer via preliminary binary coding and punching. Data programs are drawn up for processing original information for the above-listed problems. The programs are inserted into the operational memory of the computer. With the information card in front of him, the engineer chooses the necessary unit numbers for which it is required to secure processed information out of the machine. Thus, the computer can be fed any version of problem solution and any territorial units. Extremely convenient and economical methods of storage and replenishment of all the information about the country's forests are being formulated, since this information will be entered on magnetic tapes and as necessary any starting or processed information in printed form without any human participation, that is, automatically, can be rapidly secured from the tapes.

As a whole, organization and methods of working with computers in forest management can be presented, in our opinion, in the following order:

compilation of information maps adapted for work with electronic computers;

preparation of tables containing starting information, punching them onto cards (binary coding), and insertion into the computer's external memory on magnetic tapes;

Formulation of algorithms and compilation of programs for the solution of scientific and production problems in forest management using computers, and input of programs into the operational memory of computers;

the practical solution of problems in forest management with the aid of information maps and computers relying on original information and programs inserted into the machine.

What then are the chief features of the execution of the second and third stages of methods for working with computers in forest management?

1. Tables of starting information must be prepared and the latter must be fed to computers. Starting information (appraisal indexes) about stands within the bounds of tracts is determined by methods of field and office appraisal and is entered onto standard forms. The forms of original information tables are best designed anew, since forms of appraisal descriptions of tracts, sampling plots, and model trees are unsuitable for punched card work (binary coding) and subsequent feed of their information content into computers.

Since modern computers are digital machines, all the starting information to be fed into the computer must be expressed in digital form. In the computer's memory cells all information is entered and stored in binary digital form. Therefore, all data must be expressed in the starting information tables in decimal or other counting systems suitable for transformation (punched card) into binary form, that is, into digits of binary counting.

Quantitative indexes are easily and simply transmitted into information tables by decimal digits. It is also necessary to express the qualitative characteristics of stands that now are transmitted into appraisal descriptions in alphabetic(word) form, for example, species composition. For transmitting alphabetic information into digital form, arbitrary coding can be used, which must be retained unchanged in all information tables. A method of encoding, using the example of digital transmission of information about ten tree species, is shown in Table 85.

In accordance with this table, the usual inscription of species composition 5C3E2K [5 Pine 3 Spruce 2 Cedar] in decimal code must be entered in the starting information table as -- 503124, and in binary code -- 101 0 11 1 10 100.

GRAPHICS
NOT REPRODUCIBLE

Table 85

(2)

(1) (3) (4) (5)

(6)
(7) (8)
(9)
(10) (11)
(12)
(13) (14)
(15)

Legend:

1 --	species	9 --	larch
2 --	digital code	10 --	cedar
3 --	decimal	11 --	birch
4 --	binary	12 --	aspen
5 --	octuple	13 --	ash
6 --	pine	14 --	poplar
7 --	spruce	15 --	oak
8 --	fir		

Owing to the economy of the computer's memory, the most widespread and the most frequently encountered species are best assigned the smallest code digits. Such data as age class, site class, and forest type should be entered in starting information tables in a standardized decimal code, and Roman numerals, for example, must never be used.

It is not necessary to list information that cannot be used by the electronic computer in the starting information tables, and there is no requirement to provide the information that can be obtained by arithmetic and other operations on the starting information, since the machine does this itself more rapidly and better than man and man does not have to engage in this work. In a word, the tables must include only the information that has been obtained by the appraiser by direct measurements and determinations.

It is clear from this table that all the information has been expressed in decimal digital form. Similar tables are compiled by appraisers for each tract.

GRAPHICS
NOT REPRODUCIBLE

As an example, we present part of a starting information table:

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ

Legend: A -- block number; D -- species composition
B -- tract number; E -- age
C -- area of tract; F -- mean indexes
m = [subscript] tree; k = [subscript] crown

The position system of entering information into tables economizes the memory capacity of computers. Feeding information from tables into the computer makes use of punched cards. Information is entered onto the punched cards from tables by perforators that automatically the decimal code to binary code in the form of punches. This part of the work is done by ordinary methods and does not present any difficulties. It appeared rational that starting information tables about each section be entered onto a separate punched card. From the cards the starting information is fed in strictly specific order into the external memory of the computer, and best of all, onto magnetic tape. Thus, the Strela electronic computer has a 43-column code cell. The external memory consists of magnetic tapes. A single tape has 511 zones, and each zone contains 2048 numbers (cells). It is best that all information for a block or a forestry station be entered onto the same zone, and information about forests of an oblasts stored on the same magnetic tape. Thus, all starting information will be entered into the memory of the computer.

2. Algorithms must be developed, programs for processing starting information compiled, and programs fed into the operational memory of the computer.

This stage of computer use in forest management is the most complex. First of all, a list must be compiled and the problems that we wish to solve on the electronic computer formulated, and then the algorithms for their solution drawn up. Simple sequences of arithmetic operations of addition, subtraction, multiplication, and division can serve as such algorithms: for example, counting the area of sections by blocks, forestry stations, etc.; formulas for calculating yields from starting information (for example, from N_0, d^0_{tree}, and h_0, and from stem volume tables, the latter being best replaced by formulas for calculating the volume of a single tree by relying on d^0_{tree} and h_0); formulas of theoretical functions of the distribution of appraisal indexes proposed in chapters 3 and 4; systems of equations for calculating the correlation between tree height and thickness given in chapter 5; forest management project-planning, etc.

All these algorithms must have a strict sequence of arithmetic and logic operations, as a result of the execution of which the person or the machine receives the final solution of the problem posed. By relying on the algorithms thus developed, a program is drawn up in alphabeticonumeric form, which ensures solution of the bulk of closely related problems under the same, but with different starting information for different forested areas of the U.S.S.R. The programs are fed in the usual way into the computer.

To clarify the second stage of the undertaking, let us give one example. A problem is posed: calculate the table of the correlation between tree height and tree thickness. We use as the algorithm the correlation equation (94) similar to the equation we derived in section 22, that is,

$$h = 1.157d_m - 0.011d_m^2 \qquad (94)$$

Now we will compile the basic program for calculating the correlation table based on formula (94) and on starting information fed into the computer's memory. To do this, we break down the right-hand side of the formula into several successive arithmetic operations, as the result of the execution of which the machine will issue forth the sought-for correlation table.

Let us designate the constants in formula (94) by letters, that is, a = 1.157 and b =0.011. Then, to calculate h we have to carry out the following four operations:

$$\begin{aligned} A_1 &= a \cdot d_m, \\ A_2 &= d_m \cdot d_m \\ A_3 &= b \cdot A_2 \\ A_4 = h &= A_1 - A_3. \end{aligned} \qquad (95)$$

GRAPHICS NOT REPRODUCIBLE

Each of the elementary operations A is conducted by an electronic computer on a separate command, therefore successive command numbers must be stipulated in the program -- we will call them A_1, A_2, A_3, and A_4.

Then we must indicate in the program the nature of the operation-command to be carried out (addition, subtraction, multiplication). At this point, then, the numbers of the cells from which we must take the starting information to carry out the stipulated operation on it are entered into the program, and the results of the operation are placed in the cell whose address has been entered into the successive section of the program.

Let us call the addresses (numbers) of the cells in which starting information is placed $\alpha_1 = a$, $\alpha_2 = b$, and $\alpha_3 = d_{tree}$, and the addresses of the cells in which intermediate and final results of the operations are placed $\omega_1 = A_1$, $\omega_2 = A_2$, $\omega_3 = A_3$, and $\omega_4 = A_4$. The fundamental diagram of the program drawn up following the stipulated rules is shown in Table 86.

Table 86

Номер команды (1)	Код операции (2)	Исходная информация (3)		Результат операции (4)	Пояснения (5)
		1	2	3	
A_1	×	α_1	α_3	ω_1	$A_1 = a \cdot d_m = 1{,}157 d_m$
A_2	×	α_3	α_3	ω_2	$A_2 = d_m \cdot d_m = d_m^2$
A_3	×	α_2	ω_2	ω_3	$A_3 = b \cdot A_2 = 0{,}011 d_m^2$
A_4	—	ω_1	ω_3	ω_4	$A_4 = h = A_1 - A_3 = 1{,}157 d_m - 0{,}011 d_m^2$

Legend: 1 -- command number; 2 -- operation code; 3 -- starting information; 4-- operation results; 5 -- remarks

Starting information from the external memory is conveyed into the cells of the operational memory. The program stipulates other operations-commands as well which are not described here. A similar program is compiled for each problem to be solved. A set of programs is fed in the ordinary way into the computer's memory, which automatically performs all operations and issues the results of solving the problem in printed form.

Such are the principles for using computers in forest management. It is clear from what has been said that here we do not have any basic difficulties and the concern is only that we enter this extremely promising innovation for science and practice in forest management in a practical way.

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13. ABSTRACT

The present status of photo-interpretation of forests is discussed along with results of theoretical and experimental studies on use of mathematical modeling in investigating tree stands.

The use of computers in forest management is feasible and requires starting information and the preparation of special maps bearing information essential for electronic computer operation.

Mathematical modeling expands the theory of timber stand study and leads to new methods of measurement and determination of appraisal indexes concerning timber stands.

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