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## THE ERRATA OF COMPUTER PROGRAMMING

by
Donald E. Knuth

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COMPUTER SCIENCE DEPARTMENT School of Humanities and Sciences STANFORD UNIVERSITY

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$\square$

## THE ERRATA OF COMPUTER PROGRAMMING

This report lists anl corrections and changes of Volumes 1 and 3 of The Art of Computer Programming, as of January 5, 1979. This updates the previous list in report CS551, May 1976. The second edition of Volume 2 has been delayed two years due to the fact that it was completely revised and put into the TEX typesetting language; since publication of this new edition is not far off, no changes to Volume 2 are listed here.

The present report was prepared with a typesetting system that is now obsolete; please do not wince at the typography. All cahnges and corrections henceforth will be noted in TEX form on file ERRATA.TEX[ART,DEK] at SU-AI.

In spite of inflation, the rewards to error-detectors are still $\$ 2$ for "new" mistakes in the second edition, $\$ 1$ in the first edition.

Please do not endanger the author's morale by asking him about Volume 4. Thank you for your understanding.

## 2.0 throughout the book(s)

2128178
2
when the text of these books is on a computer I will try to be consistent in hyphenating compound adjectives like doubly-linked lists and storage-allocation algorithma, etc. ... but until then, such lapses are not to be considerod errors


The preparation of this report was supported in part by National Science Foundation grant MCS-77-23738, by Office of Naval Research contract NOOO14-76-C-0330, and by IBM Corporation. Reproduction in whole or in part is permitted for any purpose of the United States government.
1． 11 displayed formula in exercise 32
1．Y\％ 419177 ..... 8line 3 after（1）：book．$\sim$ book．＊footnote for bottom of page：In fact，permutatione are so important，Vaughan Pratt hassuggested calling them＂perms．＂As soon as Pratt＇s convention is established，toxtbooks ofcomputer acience will be somewhat shorter（and perhape lese expensivo）．
1． $2 \%$ lines $-4,-5$（twice），$-7,-15,-16$ ..... 11112／76 9
．．．ヘ ．．．1，Y5 lines 3，10，11，12， 2111112／76 10
．．．ヘ ．．．
1．50 exercise 21 line 1 ..... 713117611
Faa $\sim$ Fi2．51 line 132／28／78 $\quad 12$
manner $\sim$ matter
2．52 line 6 after Table 1 ..... 8125／76 $\quad 13$Szu－yuen へ Sxu－yüan
2．56 change in Eq．（17） 11112／76 ..... 14
2.58 Eq (18)
n 20. ..... n.
1,57 line after (19) 11112176 ..... 16
-r~~
1.66 caption to Table 2, replace third line by: ..... 9121/76 $\quad 17$
see D. E. Barton, F. N. David, and M. Merrington, Biometrike 47 (1960), 439-445; 60(1963), 169-176.
10.72 line-4 $1115178 \quad 18$
$A_{n(k-1)} \leadsto A_{n-1)(k-1)}$
2.79 lines 8,9,10 6/25176 ..... 19
Kopler, ... life. Not Johann Kepler, 1611, who was musing about the numbera he sawaround him [J. Kopler, The Sis-Cornered Snowfleke (Oxford: Clarendon Prese, 1966), p.21)
2.85 line -7 11129177 ..... 20
use came style ecript $\mathbf{F}$ in this line as in line -6 (six pleces)
2.90 new generalized Eq. (29) 8125/76 ..... 21
$\left.\left.\left.\left(x /\left(0^{x}-1\right)\right)^{n}-1-(1 /(n-1))\right)_{n-1}^{n}\right] s+(1 /(n-1)(n-2)) \sum_{n-2}^{n}\right)^{2} \cdots \sum_{k>0} B_{k}(n)_{s^{k}} / k L$. ..... (29)
(convert this to usual format for displeyed equationa)1.90 update to provious correction number 25 II112176 22
to appear, へ 75-77,
2.92 replace lines 1-3 by the following new copy 8125176 ..... 23
The coefficients $\boldsymbol{B}_{\boldsymbol{k}}{ }^{(\boldsymbol{n})}$ which appear in the last formula are called "generalizod Bernoullinumbers"; Section 1.2 .11 .2 examines them further in the important special case $n=1$. Forsmall $k$, we have $B_{k}^{(n)} / k!-(-1)^{k}\left[\begin{array}{c}n-k\end{array}\right](n-k-1)!/(n-1)!$, but when $k 2 n$ this formula breaksdown since it reduces to 0 times $\infty$. An analogous situation holds for the power series$(z / \ln (1+\varepsilon)) n$, where the coefficient of $o^{k}$ for $k<n$ is $\left\{\begin{array}{l}n-k\end{array}\right\}(n-k-1)!/(n-1)!$.
1.92 line -8 ..... 7131176 ..... 24
Faa ~~ Fà̀
1.98 caption, line 2 7131176 ..... 25
2.11 ~ ..... 2.10
1.1053 line 3 7131176 ..... 26
Faa~ Fà
2.211 three lines after (12) ..... 6/25176 ..... 27
$R_{m} \leadsto$ ..... $\left|\boldsymbol{R}_{\boldsymbol{m}}\right|$
1.1112 line 8 11/15/78 ..... 28
mately $2 \sim$ mately $(-1)^{1+k / 2} 2$
2.116 line -6 1/129177 ..... 29
Analysis ~ A crude analysis
2.1216 line -6 and Eq. (22) 11129177 ..... 30
$n^{n-1 / 2}$ ..... ~n $n^{n}$
8.827 line 5 ..... $11129177 \quad 31$
throe ~ two
2.228 exercise 5 ..... $11129177 \quad 32$
$n^{n-1 / 2} \leadsto n^{n}$
1.225 line 2 ..... 111617733
is loaded. $\sim$ are londed.
2.226 line 1 ..... 1116/77 ..... 34
The contente A A portion of the contente
2.226 line 7 ..... $116177 \quad 35$
is $\sim$ are
20228 line-19 $11115178 \quad 36$
Overflow may oceur as in ADO. Same ac ADO but with -V in place of $\mathbf{V}$.
2. 1228 lines -18 through -13 ..... 11115178 ..... 37
move thise paragraph in front of the SUB definition on the preceding two linee
21.124 line-12 419177 ..... 38
MUL requiree $0 \rightarrow$ MUL, NUH, CHAR each reguire
1.135 box 05 4119177 ..... 39
1~ ..... 10
2.2150 lines $-10,-9,-8$ ..... 4119177 ..... 40
CON $\sim$ CON ( 4 times)
2.255 line 16 ..... 11129177 ..... 41
facilate $\sim$ facilitate
2.2156 styliatic corrections ..... 6114177 ..... 42
line 2: i.e. $\sim$ e.e.
line 3: ( $\boldsymbol{X}$ ~~ (Here $X$line 5: sun. © sun;line 10: $\boldsymbol{E}$ 人) (This number $\boldsymbol{E}$line 22: the year 1 that the yoar
2.298 lines 19-21 6/14177 ..... 43An illustration...See aleo the book $\leadsto$ See, for exampla, the book
2.284 line-11 ..... 6/14/77 ..... 44
F-7 ~~•9
2.2285 line -9 ..... 6125/76 45
about 1946 ~ during 1946 and 1949
2,234 line-10 ..... 12/19176 46
down an item an item down
up the stack 1 the stack up
2,2뇨d insert now paragraph after line 4 ..... 41917747Further study of Algorithm G has beon made by D. S. Wiee and D. C. Wation, BIT 16(1976), 44-450.
2.258 line 4 9/21/76 ..... 48
we exercise 30 describee a somewhat more natural alternative, and wo
3.270 new exercise 9/21/76 ..... 49
30. [17] Suppose that queues are represented as in (12), but with an empty queve represented by $F=A$ and $R$ undefined. What insertion and deletion procedures should replace (16) and (17)?
2.303 exercise 9 line 4 ..... 31 2/77 ..... 50
girls women
2.0225 line 8 4119177 ..... 51
otherwise. otherwise, making the latter node the right son of NODE (Q).
2.(2)TM new quote to insert just before Section 2.3.2 ..... 1/16/77 ..... 52
Binary or dichotomous systems, although regulated by a principle,are among the most artificial arrangements that have ever beeninvented.
--WILLIAM SWAINSON, A Treatise on the Geography and Classification ofAnimals, Sec. 250 (1835)
2.25y line 13 6/25/76 ..... 53
In all $\sim$ Furthermore TYPE $(W)$ is set appropriately, depending on $x$. In all
13.585 line 2 12/19176 ..... 54
there is a man now living having $\sim$ somebody now living has
3.598 line-1 5127178 ..... 55
with
than
2.エロด line-2 ..... $1 / 16177$ ..... 56
as informally as
2. Y06 line 11 ..... 5/27178 ..... 57
-types ~ -tuples
2.206 line 18 11115178 ..... 58
Polya Pólya

1. $11 \%$ step $A 2$ lines 2-4 ..... 2/28178 ..... 59
unmarked, mark it, and if unmarked: mark it and, if (twice)
2. 2 E® lines 14-15 ..... 9/21/76 ..... 60
[Sce the ... 372.] $\sim$ An elaborate system which does this, and which also includes a mechanism for postponing operations on reference counts in order to achieve further efficiency, has been described by L. P. Deutech and D. C. Bobrow in CACM 19 (1976), 522-526.
3. 250 line 171112917761
see $\sim$ see N. E. Wiseman and J. O. Hiles, Comp. J. 10 (1968), 338-343,
4. $\mathfrak{y y}$ line 18 6/25/76 ..... 62
For these reasons the $\mathcal{\sim}$ A contrary example appears in exercise 7 ; the point is thatneither method clearly dominates the other, hence the simple
line 11$1116 / 77$63
each with a random lifetime, $\sim$ each equally likely to be the next one deleted,
2.ษฯ® new paragraph after line 6 1116177 ..... 64
Our assumption that each deletion applies to a random reserved block will be valid if the lifetime of a block is an exponentially-distributed random variable. On the other hand, if all blocks have roughly the same lifotime, this assumption is falso; John E. Shore has pointed out that type A blocks tend to be "older" than type C blocks when allocations and deletions tond to have a somewhat first-in-first-out character, since a sequence of adjacent reserved blocks tends to be in order from youngest to oldest and since the most recently allocated block is almost never type A. This tends to produce a smallor number of available blocke, giving even better performance than the fifty-percent rule would predict. [Cf. CACM 20 (1977), 812-820.]
5. ฯฯ8 line-9 11115/78 ..... 65
areas areas of the same size
6. 251 line 7 ..... 1/16/77 ..... 66
. ; John E. Shore, CACM 18 (1976), 433-440.
1.251 yet another addition after line 7 ..... 2128178 ..... 67
. ~ ; Norman R. Nielsen, CACM 20 (1977), 864-873.
7. 155 exercise 28 ..... 4/19177 ..... 68
line 2: 5; for ~ 5. For line 4: " $\sim$ The execution time is $2 \mu . "$
1.ఝ56 line 8 ..... 6/25/76 ..... 69V-1.] ~ V-1; and see especially also the work of Konrad Zuse, Berichte derCesollschaft für Math. und Datenv. 63 (Bonn, 1972), written in 1945. Zuse was the firstto develop nontrivial algorithme that worked with liste of dynamically varying lengthe.]
1.256 line -712/1917670
is divisible ..... is not divisible
lincs - 15 thru -13: The A-1 ... code; $\sim$ The machine language for keveral early computers used a three-address code to represent the computation of arithmetic expressions;
lines -11 and -10 : the $\mathbf{A}-1$ compiler language $\sim$ an oxtended three-address code
1.대ํ line 2

The latter $\sim$ Weizenbaum's

1. 2 Kibl several changes

12/19176

## line 1:. ~ ,

line 4: older $\sim$ other
now paragraph to be inserted after line 4:
A related model of computation was proposed by A. N. Kolmogorov as early as 1952. His machine essentially operates on graphs $C$, having a specially designated starting vertex $\boldsymbol{v}_{0}$. The action at each step depends only on the subgraph $C$ 'consisting of all vertices at
 includes $v_{0}$ and the vertices $v$ at distance exactly $n$ (rom $v_{0}$, and possibly other vertices; the remainder of groph $\boldsymbol{C}$ is left unaltcred, its components are attuched to the vertices $\boldsymbol{v}$ at distance $n$ as before. Here $n$ is a fixed number specified in advance for any particular algorithm, but it can be arbitrarily large. A symbol from a finits alphabet is attached to each vertex, and restrictions are made so that no two vertices with the same symbol can be adjacent to a common vertex. (See A. N. Kolmogorov, Uspakhi Mat. Nauk 8,4 (1953), 175176; Kolmogorov and Uspenskií, Uspelhi Mat. Nauk 13,4 (1958), 3-28, Amer. Math. Soc. Translations, scrics 2, 29 (1963), 217-245.) Such graph machines can casily simulate the linking automata defined above, taking one graph step per linking step; conversely, linking automata can simulate graph machines, taking at most a bounded number of steps per graph step when $n$ and the alphabet size are fixed. The linking model is, of course, quite close to the operations available to programmers on real machines, while the graph model is not.

1. 2773 exercise 44 line 2
$x_{k}{ }^{+} y_{i} \leadsto x_{j}{ }^{+} y_{k}$
1.278 line 8 1/16/77 75

### 2.9826 line 1

For oxample, Eq. (6) hoide for all complox $k$ and $n$, exoept in certain casos whon $n$ is a negative intogor; Eqg. 17), (9), (20) are novor falke, although thoy may occuationally take indetorminate forms auch as 0.00 or $\infty 00 \infty$. We can oven oxtend the binomial theorem (13) and Vandermondo's convolution (21), obtaining $\Sigma_{k}\left(r_{a+k}\right) y^{a+k}-(1+k)^{Y}$ and
 the series converge, provided that complex powers are proporly defined. [Soe In Ramshaw, Inf. Proc. Lattiers 6 (1977), 223-226.)
2. 288 new answer

11/12/76 78
42. $1 /(r+1) B(k+3, r-k+1)$, if this is dofinod according to oxercies al(b). In senoral it appears best to define ( $)$ - 0 when $k$ is a negative intoger, otherwise ( $k$ - $\lim _{s \rightarrow r}$ $\Gamma(e+1) / \Gamma(k+1) \Gamma(a-k+1)$, since this preservos most of the importans identities.
2.994 line 9

1115/78
79

Polya $\sim$ Pólya
2.999 exercise 7

11115178
80
(ft is "Glasher's constant" 1.2824271...) To $\sim \rightarrow T o$
This formula ... $n=4$. (The constant $A$ is "Glaishor's constant" 1.2824271..., which R. W. Gosper has proved equal to ( $2 \mathrm{me} \gamma-\zeta^{\prime}(2) / \rho^{\prime}(2) 1 / 12$.)

2,500
exarcise 5
11129177
81
line 1: $2 n-1 \sim 2 n+1$




2,512 more changes to answer $14 \quad 6 / 14177 \quad 87$
delete the bottom line and replace lines 1-31 by: SRAX 5

2OMINUSN
ENN1 *
inca 67,1
SAAX 5
DIV $-7-$
SLAX 5
DECA $-4,1$
JAN $1 F$
DECA 31
CHAR
LOA MARCH
JMP $2 F$
CHAR APRIL
$E 7$.
$r X=D+N$
rA-31-N
E8.
1.513 naw answer 611417788
16. The firat such year is A.D. 10317 , although the error almost leads to failure in A.D. $10108+19 k$ for $0 \leq k \leq 10$.
1.515 still more changes to answor 14replace lines 1-6 by:BEGIN ENTX 1950ENTX 1950
ENTG 1950-2000"driver"routine,JMP EASTERINCE 1ENTX 2000, 6uses theaboveJGNP EASTER+1
2.514 line 18$11129177 \quad 90$
time. (ime. (It would be faster to calculate $r_{n}(1 / m)$ directly when $m$ is amall, andthen to apply the suggested procedure.)
2.515 bottom line ..... 11/29/77 91
Berk’ly $\sim$ Berkeloy
1.516 lines $-4,-3$ 411917 ..... 92
$3)+7 \sim 7.5)+16$
2.517 exercise 12 lines 7-10 ..... 5/27178 93
delete "Thus, ...(b)."
1.518 line 5 ..... $3127178 \quad 94$
19-27. N 19-27; E. G. Cate and D. W. Twigg, ACM Trana. Meth. Sofiwere 3 (1977),104-110.
1.546 new answer 9/21/76 ..... 95
30. To insert, set $P \neq$ AVAIL, $\operatorname{INFO}(P)+Y$, LINK $(P) \leftarrow A$, if $F \cdot A$ then $F \leftarrow P$else LINK (R) $\leftarrow P$, and $R \leftarrow P$. To delete, do (9) with $F$ roplacing $T$.


There is ... exist. $\sim$ When $n>1$, the number of series-parallel networks with $n$ edges is $2 c_{n}$ [soe P. A. MacMahon, Proc. London Mach. Soc. 22 (1891), 330-339].
2.588 fourth line before exercise 33

5/27178
minimal. N minimal. [This argument in the case of binary trees was apparently first $^{\text {p }}$ discovered by C. S. Peirce in an unpublished manuscript; soe his New Elements of Mashemasics 4 (The Hague: Mouton, 1976), 303-304.]
2,594 updates to previous change number 150 $9121 / 76 \quad 103$
to appear, $\sim$ 491-500,
(see also the important new contribution by H. G. Baker, Jr., CACM 81 (1978), 200-294, for which I will probably want to revise Section 2.3 .5 entirelyl)
1.594 update to provious change number 151 11129177104
Clark's list-copying algorithm appeared in CACM 21 (1978), 351-357, and Robeon's in CACM 20 (1977), 431-433
2.597 last line of answer 6 1/16/77 ..... 105
list. ~ list. For an alternative improvement to Algorithm A, see exercise 6.2.3-30.
1.597 exercise 8 ..... 6/25176 ..... 106
line 1: also set $R$ also set $M+\infty, R$ line 3: If R = A or M $\sim$ II M
2.601 exercise 26 line 3 ..... 2128178 ..... 107
two. $\sim$ two, with blocks in decreasing order of sise.
$P \geq M \sim P \geq M-2^{k}$.
2,601 program line number 12 4/19177 ..... 108
$j \leadsto j$
2.602 new answer 2128178 ..... 109
31. Seo David L. Ruseell, SIAM J. Computing 6 (1977), 607-621.
2,603 addition to previous change 153 4119177 ..... 110
.] $\sim$; Lars-Erik Thorelli, BIT 16 (1976), 426-441.
2.BO6 exercise 41, numerator in value of a[5] 6114177 ..... III
19559 ~ ..... 18535
1.607 ..... 6/25/76 ..... 112
delete A-1 compiler, 458.
2.618L Aardenno-... ..... $11129177 \quad 113$
Taniana ~ Tatyana
2.6276 ..... 12/19176 ..... 114
AMM AMM
1.618L 5/27178 ..... 115
Baker, Henry Givone, Jr., 594.
2,608( 4/19177 ..... 116
add p487 to antry for Binomial theorem, gemeralimations of
1.B191 Bobrow entry ..... 9121/76 ..... 117
add p420
2,619 5/27178 ..... 118
Cate, Eako George, 518.
2.629 11129177 ..... 119
Cheney, Christopher John, 480.
1.B20B new definition entry 1219176 ..... 120
Data organization: A way to reprosent information in a date etructure, together with algorithme that accose and/or modify this structure.
2.321ㄴ$2128178 \quad 121$
Derangemente, 177.
2.B21ㄴ Doutech entry ..... $9121 / 76$ ..... 122
add $\mathbf{p} 420$
2.622B End of file entry ..... $312177 \quad 123$
224 ~ ..... 223
2.623D Garwick entry ..... $11115178 \quad 124$
$244 \sim$ ..... 245
2.62MB Hopper entry 6/25176 ..... 125
255,458. ..... 225.
1.6ఆM
11129/77 ..... 126
Hiles, John Owen, 420.
2,6ひษは$312177 \quad 127$Invert a linked list, 266, 276.
2.BEMR INT entry
6/14/77 ..... 128
225. 224-225.
2,625® 5/27/78 ..... 129
Laibnitz (- Loibnis) ~ Laibnis (- Loibnits)
2,62551 12/19176 ..... 130
Kolmogorov, Andrei Nikoleevich, 463.
2.62BL MacMahon entry 115179 ..... 131
add p. 583
1,3276 9121/76 ..... 132
Merrington, Maxine, 66.
2.6286 ..... $2128178 \quad 133$
Nielsen, Norman Ruscell, 451.
2,6288 ..... 5127178134
Peirce, Charles Santiago Sandera, 588.
2.629 4119177 ..... 135
add p 44 to Pratt entry
2.6296 ..... 6114177 ..... 136
Petolino, Joseph Anthony, Jr., 511.
2.6290 ..... 5127178 197
Prüfor, Heina $\leadsto$ Prüfer, Ernst Paul Heins
1.6290Pring, Dietrich G.
2.650L
4/19177 ..... 139
Ramshaw, Lyle Harold, 487.
1.6300312177140
Reversing a list, 266, 276.
1.631L new entry ..... 1| 5179 141
Series-paraliol networke, 583.
2,65LL ..... $1 / 16 / 77 \quad 142$
Shore, John E., 446, 45.
2.6316
2128178 ..... 143
Ruscell, David Lowie, 602.
1.635ㄴ
1116177 ..... 144
Swainson, William, 332.
1.632L Stirling numbers entry ..... 8/25/76 145
90 , $20-91$,
2,6326:
4119177 ..... 146
add p630 to Thorelli entry
1，6381 4119177 ..... 147
Watson，Dan Caldwell， 248.
1，6353® ..... 4119177148
add p487 to Vandermonde entry
1，6353 ..... 5127178 ..... 149
Twisg，David William， 518.
1．65311129／77150
Taniana Tatyana
1，63꺠12／19176151
Uspenskii，Vladimir Andreovich， 463.
4119177152
add p248 to Wiee entry
1．65TM ..... 6／25176 153
Windley，Peter F．
2．6591L Weizenbaum entry ..... 9121／76 ..... 154delete p4201．B乌゙凡11129177 155
Wireman，Neil Ernest， 420.
2,63Mは ..... 6125176 156
Young Tanner, Rocolind Coeilia Hildogard, 75.
2.6S6 (namely the endpapers of the book) ..... $4119177 \quad 157$
also make any changee apocified for pages 136-137
3.028 quotation for bottom of page ..... $5127178 \quad 158$
Two hours' datiy exarcise. . will be enough--M H. MAHDN, The Mandy Norse Book (Edinburgh, 1865)
\$.8B line 21$312177 \quad 159$
mideln $\uparrow$ Müdeln
2. 8 R line 26 ..... $312177 \quad 160$
Weiner $\sim$ Wiener
3.24 line 13 ..... $228178 \quad 161$
(1965 ..... (1965)
3.34 bottom line of determinant on line 12 ..... 5/27178 162

3.90 Eq. (26) $2128178 \quad 163$
the $j$ in of should be in amoller (euperacript sise) font
3.57 line 2 of atep 53 ..... $2128178 \quad 164$
right $\sim$ right of
3.58 line 4 $2 / 28178$ ..... 165
$a_{1} a_{2}, \sim a_{1}, a_{2}$,
3.63 line -4 5/27178 ..... 166
St, ..... X's
X's ~ ..... S's
3.65 line -8 2128178 ..... 167
to better understand $\left.i_{n} \sim t\right)$ understand $t_{\boldsymbol{n}}$ better
3.67 following (50) 5/27178 ..... 168
lines 2-4: we find...Euler's $\sim$ Euler's
line 5: in this case, since $\sim$ sinc
lines 7-8 (the two lines following (51)): $n$; this...we have proved that $\sim$
$n$. The derivative $\varepsilon^{(m)}(y)$ is a polynomial in $y$ times $e^{-2 y^{2}}$, hence $\boldsymbol{R}_{m}=O\left(n^{(t+1-m) / 4}\right)$$\int_{-\infty}{ }^{+\infty}\left|g^{(m)}(y)\right| d y=O\left(n^{(\ell+1-m) / 4}\right)$. Furthermore if we replace $\alpha$ and $\beta$ by $-\infty$ and $+\infty$ inthe right-hand side of (50), we make an error of at most $O\left(\exp \left(-2 n^{6}\right)\right)$ in each term. Thus
3.69 exercise 8 6114177 ..... 169
accent over 0 in Erdös should be " not "
3.72 new copy for exercise 28 ..... $11115178 \quad 170$
28. [M43] Prove that the average length of the longest increasing subsequence of arandom permutation on $\{1,2, \ldots, n\}$ is asymptotically $2 \sqrt{n}$. (This is the average length ofrow 1 in the correspondence of Theorem A.)
3.79 last line before exercises ..... $9121 / 76$ ..... 171
Feurzig $1 \sim$ Feurzeig
3.85 lines 7 and 12 11129177 ..... 172
$\log _{2} \sim$ ..... $1 g$
3.98 line 4 II/29177 ..... 173
$\log _{2} \leadsto 1 g$
3.104 line-2 6/14177 ..... 174
inversions. $\leadsto$ inversions. Discuses corresponding improvemente to Program $\mathbf{S}$.
3. 217 simplifications of step Q2 ..... 12/19176 $\quad 175$
line 3: $K \leftarrow K_{l}, R+R_{l}$. $\sim+K+K_{l}$.
line 4: $K$ and $R \leadsto \boldsymbol{K}$
3.118 comment to program line 05 ..... 12/19176 ..... 176
$K+K_{l}, R+R_{l} \leadsto K+K_{l}$
5025 line-3 ..... 6/14/77 177
$s_{N} \leadsto s_{N}$
3.226 line - 6 12/19176 ..... 178
instructions " $\boldsymbol{K}+\boldsymbol{K}_{\boldsymbol{l}}^{\boldsymbol{l}} \boldsymbol{R}+\boldsymbol{R}_{\boldsymbol{l}}$ " $\sim$ instruction $" K+\boldsymbol{K}_{\boldsymbol{l}}^{\boldsymbol{n}}$
3228 line-3 ..... $4119177 \quad 179$
v. v- v. Yihsiao Wang has suggested that the mean of throe key values such as (28) beused as the threshold for partitioning; he has proved that the number of comparisonsrequired to sort uniformly distributed random data will than be asymptotic to 1.082 n lg n .
3.13510 lines after (42) $5 / 27178$ ..... 180
$(N / x)^{l} \leadsto$ $(N / x e)^{6}$
3.2527 lines after (42) 5/27178 ..... 181
$O\left(N^{t-1 / 2} e^{-\pi N / 2}\right) \sim O\left(\mid t+i N^{t-1 / 2} e^{-t-\pi N / 2}\right)$
3.233 in the discussion following (45) $5 / 27178$ ..... 182
line 3: $N^{*} \sim \mid M \cdot 1 M^{r}$
line 4: negligible. neg negligible, when $\boldsymbol{N}$ and $\boldsymbol{N}$ are mueh larger then $\boldsymbol{M}$.
W.134 Eq. (46) and the line following ..... $2128178 \quad 183$
, $\sim \cdot O\left(n^{-M}\right)$,
where $\sim$ for arbitrarily large $M$, where
2, 23 d ..... 2128178 ..... 184
$f(n) \leadsto|f(n)|$1725 ヘ 173
2, 125 exercise 16 11129177 ..... 185
HM46 HM42
2.2888 exercise 46 lower limit of integral 6114177 ..... 186
$a+i \infty \sim a-i \infty$
3.138 exercise $\mathbf{5 2}$ binomial coefficient in the sum 6/14/77 ..... 187
remove apurious (raction line between $2 n$ and $n+\ell$
2,14\% line 10 $2 / 28178$ ..... 188
Langhize, $\sim$ Language
$3.15 \$$ 11112176 ..... 189
about here I will somoday insert material about the new "binomial queuc" algorithme to be discuseed in papors by Vuillemin and Brown, since they appoar to outperform lofties trees
2.2158 line-5 5/27178 ..... 190
$a_{i} \wedge a_{1}$
2.264 line 21 of program ..... 5/27178 ..... 191
$L_{q} \leadsto L_{p}$
3.176 line - 12 5/27178 192
$M=b \sim M-b^{r}$
3.157 lines 25-27 9121176 ..... 193
that the multiplicity ... Algorithm R, even 1that it ultimately apende too much time fuscing with very amall piles. Algorithm R isrelatively efficient, even
3.192 line -7 ..... $5127178 \quad 194$
Well's $\sim$ Wells's
3. 293 line -15 ..... $5127178 \quad 195$
less $\sim$ fewer
3.199 Eq. (4) ..... $2128178 \quad 196$$\lg 「 \sim \mathrm{rlg}$
3,208 replacement for exercise 14 ..... 11129177197
14. [f1] (F. K. Hwang.) Let $h_{3 k}=\left((43 / 28) \cdot 2^{k}\right\rfloor-1, h_{3 k+1}=h_{3 k} \cdot 3 \cdot 2^{k-3}, h_{3 k+2}=$$L(17 / 7) \cdot 2^{k}-6 / 7$ f for $k \geq 3$, and lot the initial values be defined co that $\left(h_{0}, h_{1}, h_{2}, \ldots\right)$ -(1, 1, 2, 2, 3, 4,5, 7, 9, 11, 14, 18, 23, 29, 38, 48, 60, 76,97, 121, 154, ...). Prove that $M\left(3, h_{i}\right)$ )$t$ and $M\left(3, h_{t}-1\right) \leq$ for all $t$, thereby establishing the exact values of $M(3, n)$ for all $n$.

```
17 ~N 16** (twice)
add footnote:
** See K. Noshita, Tranc. of the ISCE of Japan, E69, 12 (Dec. 1976), 17-18.
```

3.215 line 4 after second illustration ..... $312 / 77199$
the values listed in the table for $n \geq 8 \sim$ the values shown for $V_{4}(9), V_{5}(10)$ and their duale $V_{6}(9), V_{6}(10)$
line 17: A. Schönhage [to appear] ~ A. Schönhage, M. Paterson, and N. Pippenger [J. Comp. Sys. Sci. 13 (1976), 184-199]
line 18: asymptotic $\sim$
lines 19-20: $3 n$, and ... 1.25n. $\sim 3 n+O(n \log n)^{3 / 4}$. On the other hand, Vaughan Pratt has obtained on asymptotic lower bound of $1.75 n$ for this problem [cf. Proc. IEEE Conf. Switching and Automate Thoory 14 (1973), 70-81); a generalization of his result appears in exercise 25.

### 3.219 exercise 14

12/19176
201

Show that ... comparisons. $\sim$ Let $U_{t}(n)$ be the minimum number of comparicone noeded to find the $t$ largest of $n$ elements, without necessarily knowing their rolative order. Show that $U_{2}(5) \leq 5$.

## 3,250

25. [M32] (A. Schönhage, 1974.) (a) In the notation of exercise 14, prove that $U_{8}(n) \geq$ $\min \left(2+U_{i}(n-1), 2+U_{t-1}(n-1)\right)$ for $n \geq 3$. Hint: Construct an adversary by reducing from $n$ to $n-1$ as soon as the current partial ordering is not composed of components or or (b) Similarly, prove that $U_{t}(n) \geq \min \left(2+U_{t}(n-1), 3+U_{t-1}(n-1), 3+U_{t}(n-2)\right)$ for $n \geq 5$, by constructing an adversary which deals with components $\bullet, \cdots, 0,(c)$ Therefore we have $U_{t}(n) \geq n+t+\min (L(n-t) / 2 \downarrow, t)-3$ for $1 \leq 1 \leq n / 2$ (d) The inequalities in (a) and (b) apply also when $V$ or $W$ replaces $U$, thereby eatablishing the optimality of several entries in Table 1.
8.2525 line 1 5／27178 ..... 203
Lm／2」～2 $2(m / 2\rfloor$
Ln／2J ヘ～2Ln／2」
3.229 remarks about current best known sorting networks ..... 1／16／77
line 19：D．Van Voorhis in 1974．～～R．L．Drysdale III in his undergraduate honors projeot at Xnox College in 1973.
lines 20－21：© $n$ Ig $n \cdot O(n)$ comparators，．．．3651，～ $(371 / 960) n$ ig $n+O(n)$ comparators；in particular，his construction yiolds $\mathbf{S}(256) \leq 3657$ ， line 22：［To be published．］～［SIAM J．Compusing 4 （1975），264－270．］
3,252 update to provious change number 250 8125176 ..... 205
［JACM，to appear］～［JACM 23 （1976），566－571］
23．233 line 9 ..... 5127178206
！］～］）
3．2Y3 rating of exercise 48 1／16／77 ..... 207
／IM49～ NM46
3,2599 lines 4，5，6， 7 ..... 9／21／76 ..... 208
has not yet ．．．$m=8$ ．This increase $\sim$is difficult to analyze precisely，but T．O．Espelid has shown how to oxtend the snowplowanalogy to obtain an approximate formula for the behavior［BIT 16 （1976），133－142］．According to his formula，which agrees well with empirical teste，the run length will beabout $2 P+b(m-1.5)(2 P+b(m-2)) /(2 P+b(2 m-3))$ ，when $b$ is the block size and $m \geqslant 2$ ．Such anincrease
3．26D insert new paragraph before Table 2 2／28178 ..... 209
The ideas of delayed run－reconstitution and natural seloction can be combined，as discucsed by T．C．Ting and Y．W．Wang in Comp．J． 20 （1977），298－301．
\$.262 line 7 ..... 5127178 ..... 210
ahould be the square roet of (to-10)P
5.2659 line -1 ..... 5127178211
boinge $\sim \sim$ begins
\$3.279 line 10 after Table 4 ..... $6114 / 77212$
JACM (to appear) ~ S/AM J. Computing 6 (1977), 1-39
3,2882 line before the big tableau ..... 5127178 ..... 213
" $R$," へ~" $R^{\prime \prime}$,
8.284 line 22 ..... || 5179 ..... 214
143 ..... 145
5,589 lines 4, 13, 20 ..... $115179 \quad 215$
$25 \sim 27$
3,305 line-4 8125176 ..... 216

3,326 line-7 11129177 ..... 217
$L[p] \leadsto L[m]$
Enswic lines 1 and 7 ..... 6114177218
1 ~
\$. WCl the foldout illustration
in the bottom example (010) look at line 4 of the six linees where there is a longish black har as the seventh activity (the sixth activity is a chortor black barh...and lines $1,2,3$, and 5 have a blank bar just above and bolow this longish black bar; actually linoes 1,2,3, and 5 should have parallol upward-sianting diagonal linee lthe aymbol for "roading in forward direction") inside theso blank bars

| 3.548 line 9 after the first illustration | 9127178 | 220 |
| :---: | :---: | :---: |
| tape $C$ ~ tape $A$ tape $D$ ~ tape $B$ |  |  |
| 23.352 line-9 | 6114177 | 221 |
| is $\sim \sim$ in |  |  |
| 33,352 exarcise 3 | 11129177 | 222 |
| merge $\sim$ radix cort |  |  |
| 2.356 line-11 | 5/27178 | 223 |
| T3 へ~ Track 3 |  |  |

3.358 line-20 ..... 12/19/76 224artifically 4 tificially
3.351 Equation (8) ..... 8125176 ..... 225
$\mathrm{H}_{2}{ }^{2} \leadsto \mathrm{~B}_{1}{ }^{2}$
35326/25/76 226
about here I should mention C. MeCulloch's now approsech to oxternal disk sorting (embodied in the KA Sort on Honeywall 200)
3.324 stylistic improvements 1116177 ..... 227
line 17: large, and ... unthinkable! $\sim$ largej it is, however, so large that $\mathbf{N}$ seoks areunthinkable.
line 24: But $\sim$ On the other hand,line 84: ! ~ .
3.382 table entries for Straight insertion 6114177 ..... 228
Length: $12 \sim$ ..... 10
Spere: N~N+1
Average: $2 N^{2}+9 N \sim 1.5 N^{2}+9.5 N$
Maximum: $4 \sim 3$
$N=16: 494$ ~ 412
N-1000: 1985574 ~ 1491928
3.384 insert new paragraph before line -1 ..... 6/25176 229In Germany, K. Zuse independently constructed a program for etraight insertionsorting in 1945, as one of the simplest examples of linear list operations in his "Plankalkul"language. (This pioneering work remained unpublishod for nearly 30 yeare; see Berichteder Cosollsechaft für Math. und Datonu. 63 (1972), part 4, 84-85.)
3.387 line 2 ..... 8125176 ..... 230
near-optional $\sim$ near-optimal
W.394 caption to Fig. 1 ..... $312 / 77$ ..... 231
scarch. $\sim$ or "house-to-houce" search.
3.594 Fig. 1 4119177 ..... 232
label the downward branch coming out of box $\mathbf{S 8}$ with an - sign
3.MOD lines 12 and -5 ..... 2128178 ..... 233
running time $\sim$ average running time
3. 2112 correction to previous change 263 ..... 4/19177 ..... 234
delete this change, the book was right the first time
3. 413 lines $-4,-3$ 4119177 ..... 235
and $N>2^{k}$, we ~ we
$\left.\operatorname{llg}\left(N-2^{k}\right) \mathrm{J}+1 \sim \operatorname{rig}\left(\mathrm{~N}+1-2^{k}\right)\right\rceil$
5.cำ lines 13-14 $312 / 77$ ..... 236
H. Bottenbruch ... $\mathrm{He} \sim$ D. H. Lehmer [Proc. Symp. Appl. Math. 10 (1960), 180-181]was apparently the first to publish a binary search algorithm which works for all $\mathbf{N}$. Thenext step was taken by H. Bottenbruch [JACM 9 (1962), 814], who
3.219 line 30 11112/76 ..... 237
, but his flowchart and analysis were incorrect. $\sim$.
23. 258 line 7 (append to step D1) ..... 5127178 ..... 238
(For example, if $Q \cdot \operatorname{RLINK}(P)$ for some $P$, this means we would set RLINK $(P) \leftarrow$LLINK (T), etc.)
3.258 Fig. 16 ..... 6114177239insert "a)" and "b)" to the left of the roots of the trees, and change circles to squares inthe right descendants of nodes AN and AS in the upper tree
53. 4339 Update to previous change 276 ..... 11/15/78 240
the Garsia-Wachs algorithm appeared in SIAM J. Computing, Dec. 1977, pp. 622ff; but now it seems an even better way has beon found by Hu, Klejtman, and Tamaki (UCSD report 78-CS-016)
line 6: optimum. Cf. ~ optimum; ef.
line 7: .) $\sim$. On machines which cannot make three-way comparisons at once, a program for Algorithm $T$ will have to make two comparicons in step $T 2$, one for equality and one for less-than; B. Sheil and V. R. Pratt have observed that these comparisone need not involve the same key, and it may well be best to have a binary tree whose internal nodes specify an equality test or a less-than test but not always both. This situation would be interesting to explore as an alternative to the stated problem.)
3.259 line -3

$$
312177 \quad 242
$$

$$
\text { put a small inverted } U \text { over the ie in Akadomiia }
$$

### 3.2156 Fig. 22

 $9121 / 76 \quad 243$the arrows botwoen bozes 12 and A3 should be reverced (downward arrow on left, upward arrow on right); aleo delete "P - A" below boxes A3 and A4 and insert the words "Leaf found" between the two arrows loading to AS

### 3.255 line 15

2/28178
244
necessary. $\sim$ necessary. Essentially the came method can be used if the tree is threaded (ef. exercise 6.2.2-2), since the balancing act never needs to make difficult changes to thread links.
3.255 line after (4) 11/29/77 ..... 245
$K \leadsto K$
3.EీB1 Table 1 11129177 ..... 246
I will recompute this table, since .144 should be .143 ; also will modify the discussion on page 462 accordingly and will refer to exercise 11
\$. ఇB1 line 2 after ception 11129177 ..... 247
change - and - to typewriter-style type (+ and -)

I will rewrite this, as these trees have been atudied almost too thoroughly by now

### 3.970 exercise 10 <br> 11129177 <br> 249

Does ... e? What is the asymptotic average number of comparisons made by Algorithm A when inserting the Nih item, assuming that iteme are inserted in random order?
3.920 exercise 16

11129177250
the root node $F$ were $\sim$ node $E$ and the root node $F$ were both
3. \%7 new exercise $11 \quad 11 / 29177 \quad 251$
[M24] (Mark R. Brown.) Prove that when n?6 the average number of oxtornal nodes of cach of the types $+A,-A,++B,+-B_{1}++B,--B$ is exactly $(n+1) / 14$, in a random balanced tree of $n$ internal nodes constructed by Algorithm $\mathbf{A}$.
3.978 near the bottom $\quad 1115 / 78252$

```
lines \(-7,-5,-4: \log \sim 18\)
line -3: \(350 \leadsto 307\)
```

3. 278 update to provious change 293 II15/78 253
, to appear 0 ( $\mathbf{~ ( 1 9 7 8 ) , ~ 1 7 1 - 1 8 1 ~}$
3.§79 new paragraph before the exercises $\quad 12119 / 76 \quad 254$

It is possible for many independent users to be accessing and updating different parts of a large B-tree file simultaneously without "deadlock," if the algorithme are implemented properly; see B. Samadi, Inf. Proc. Lettors 5 (1976), 107-112.
3.2183 line $25 \quad 713176 \quad 255$
$55 \sim 49$
3.986 lines 6 and -2 ..... 5127178256lose $\sim$ fowor
3.991 line-14 ..... 5/27178 257
text, ag. (tertia...
3.505 line-14 ..... 5127178258
$t 0$ uniquely identify them 10 identify thom uniquoly
3.507 line 13, add new sentence ..... $2128178 \quad 259$
See R. Sprugnoli, CACM 20 (1977) 841-850, for a diecussion of suitable techniquee.
3.509 line 3 ..... 5127178260
superimpose a / over the ${ }^{\circ}$ sign
5.518 lines 5-7 $4 / 19177$ ..... 261
using circular ... complicated. Nt hashing FIRE and cearching down its list,suggested by D. E. Ferguson, since the lists are short.
3. 526 new paragraph after line 19 11129177 ..... 262
E. C. Mallach [Comp. J. 20 (1977), 137-140] has experimented with refinements of Brent's variation, and further recent work on this topic hes been done by G. Gonnet and I. Munro [Proc. ACM Symp. Thoory Comp. 9 (1977), 113-181]

Algorithm R may move some of the table entries, and this is undesirable if they are being pointed to from elsewhere. Another approach to deletions is possible by adapting some the ideas used in garbage collection (cf. Section 2.3.5): We might keep a "reference count" with each key telling how many other keys collide with it; then it is possible to convert unoccupied cells to empty status when their reforence count is zero. Alternatively we might go through the entire table whenever too many deleted entries have accumulated, changing all the unoccupied positions to empty and then looking up all remaining keys, in order to sce which unoccupied positions really require 'deloted' status. This procedure, which avoids relocation and works with any hash technique, was originally suggested by T. Gunji and E. Goto [to appear].
23.558 update to previous change 307 11115/78
[ [To appear.] へ J. Comp. Syat. Sci. 16 (1978), 226-274.

| 3.535 line after (48) | $2128178 \quad 265$ |
| :---: | :---: |
| likely we, $\sim \rightarrow$ likely, we |  |
| 3.53ㄴT line-5 | 312177266 |
| buckets $\sim$ pages or buckets |  |
| 3,53V line -8 | 4119177267 |

$3.5 \%$ line 16 $6 / 14177$ ..... 268
change one of $\sim$ change
3.5凡9 exercise 60 ..... 11 $5 / 79$ ..... 269
M48 ..... HM41
3.599 another quote, put above the other
She made a hash of the proper names, to be sure. --GRANT ALLEN, The Tents of Sham, Ch. 26 (1889)$1116 / 77$270
3.561 new paragraph to insert after line 18 31 2177 ..... 271
If carefully selected nonrandom codes are used, it is possible to use superimposed coding without having any false drops, as shown by W. H. Kautz and R. C. Singleton, IEEE Transactions IT-10 (1964), 363-377; see exercise 16 for one of their constructions.
3.563 line 11 5127178 ..... 272
the $N * * D * E \sim$ the form $N * * D * E$
3.564 line 9 8125176 ..... 273
his Ph. D. thesis (Stanford University, 1973).] ~ SIAM J. Computing 6 (1976), 19-50.]
$3.566 \mathrm{Eq}$. (11) 31 2177 ..... 274
this is all wrong, it should be the 31 sextuples shown in the first printing of vol. 3 on page565
3.566 line -7 11115178 ..... 275
Pfefferneuse Pfefferneusce
3.570 line 6 ..... 312177 ..... 276
systems or

$\qquad$
systems on
53. 57 [ new exercise 312177 ..... 277
16. [25] (W. H. Kautz and R. C. Singleton.) Show that a Steiner triple system of order v can be used to construct $v(v-1) / 6$ codewords of $v$ bits each such that no codeword is contained in the superposition of any two others.
3.586 new paragraph after answer 19 11／12176 ..... 278
A similar algorithm can be used to find $\max \left\{x_{i}+x_{j} \mid x_{i}+x_{j} \leq c\right\}$ ；or ..... e．g．，$\min \left\{x_{i}+y_{j}\left|x_{i}+y_{j}\right\rangle t\right\}$ given $t$ and two sorted files $x_{1} \leq \cdots \leq x_{m}, y_{l} \leq \cdots \leq y_{n}$ ．
3．5V1 line－612／19176279
junctions；$\leadsto$ junctions；STELA，an alternative spelling of＇stele＇；
3.579 answer 7，line 3
5127178 ..... 280
$>\boldsymbol{B}_{k}$ and append $\left(\boldsymbol{B}_{\boldsymbol{k}}+1\right) \sim \geq k-B_{k}$ and append $k-B_{k}$
2585 new paragraph for answer 88／25／76281A simple $O\left(n^{2}\right)$ algorithm to count the number of permutations of $\{1, \ldots, n\}$ havingrespective run lengths $l_{1}, \ldots, l_{k}$ has been given by N．G．de Bruijn，Nieuwo Archief voorWiskunde（3） 18 （1970），61－65．
3.594 new answer
11／15／78 ..... 282
28．This result is due to A．M．Yershik and S．V．Kerov，Dokl．Akad．Nauk SSSR 233（1977），1024－1028．See also B．F．Logan and L．A．Shepp，Advances in Math． 26 （1977），206－222．
3.599 exercise 14 line 7 11／29177 ..... 283
$\stackrel{13) ;}{=k ;}$ 13），and still another by the identity in the answer to exercise 5．2．2－16 with $f(k)$
3 Bly exercise 33，comments to program 7／31／76 ..... 284
line 07：ri2 へ ..... rI3
rl3 へ～ri2
lines 09 and 15：To L4 へ $\uparrow$ To L4 with $q \leftrightarrow p$

## 8,60 <br> replace lines 3 and 4 by the following new copy <br> 6/14177 285

The $\infty$ triok aleo apoeds up Program $\mathrm{S}_{1}$ the following oode auggeated by J. H. Halperin uses this idea and the MOVE inatruction to reduce the running time to ( $6 B+11 N-10) \mathrm{m}$, asauming that location INPUT $+N+1$ already contains tho largest possible one-word value:

| 01 | START | ENT2 N-1 | 1 |
| :--- | :--- | :--- | :--- |
| 08 | $2 H$ | LDA INPUT, 2 | $N-1$ |
| 03 |  | ENT1 INPUT, 2 | $N-1$ |
| 04 |  | JMP 3F | $N-1$ |
| 05 | $4 H$ | MOVE $1,1(1)$ | $B$ |
| 06 | $3 H$ | CMPA 1,1 | $B+N-1$ |
| 07 |  | JG $4 B$ | $B+N-1$ |
| 08 | $5 H$ | STA 0,2 | $N-1$ |
| 09 |  | DEC2 1 | $N$ |
| 10 |  | J2P 2B | $N-1$ |

Doubling up the inner loop would save an additional $\boldsymbol{B} / \mathbf{8}$ or so unite of time.

### 3.605 exercise 4 <br> 2128178 <br> 286

lower the $\Sigma$ sign and the relation below it
3.B66 line 10 of the program $2128178 \quad 287$
$r A \sim r A$
3.CDG answer 11 11/29177 288

In seneral, ... eloments. $\sim$ The situation becomes more complicatod when $N=64$ K. Sodgewick has shown how to compute the worst-case permutations, and he has proved that the maximum number of exchanges is $1-\lg \lg N / \lg N+O(1 / \log N)$ times the number of compericons [SIAM J. Computing, to appear].
16. Consider the $\left(\begin{array}{l}2 n\end{array}\right)$ lattice patha from $(0,0)$ to $(n, n)$ as in Fige. 11 and 18 , and attach weighte $\int(i-j)$ if $i \geqslant j, \int(j-i-1)+1$ if $i<j$, to the line from $(i, j)$ to $(i+1, j)_{3}$ here $N(k)$ is the number of bite $b_{r} b_{r+1}$ in the binary expantion $k$ - $\left(\ldots b_{2} b_{1} b_{0}\right)_{2}$. The total number of exchanges on the final merge when $N \cdot \mathbf{8 n}$ is $\sum_{0<j<j<n}(2 N(j)+1)\left(\frac{2 j-j}{i-j}\right)\binom{2 n-2 i+j-1}{n-j-1}$.
R. Sedgewick has simplified this sum to
$(1 / 2)_{n}\left(\frac{2 n}{n}\right)+2 \Sigma_{k \geqslant 1}\left(2_{n-k}^{2 n}\right) \Sigma_{0 \leqslant j<k} f(j)$ and unod the gamma function mothod to obtain the asymptotic formula $\left(i_{n} n\right)\left((1 / 4) n \lg n+\left(\lg \left(T(1 / 4)^{2} / 2 \pi\right)+1 / 4-(\gamma+2) /(4 \ln 2)+8(n)\right) n\right.$ * $O(\sqrt{ } n \log n))$, where $(f(n)$ is a periodic function of Ig $n$ with magnitude bounded by 0005 ; hence about $1 / 4$ of the comparisona lead to exchanges, on the average, as $n \rightarrow \infty$. [SIAM] J. Computing, to appear.]
3.62U second lins of answer 31 $11129 / 77$ ..... 290
step ~ stop
\$3.612 last line of answer 37 2/28/78 ..... 291
.]
3.625 exercise 48 line 4 in limite to the integral$1 / 2 \sim-1 / 2 \sim$ (iwico)
3.626 line 26 of the program$r$ A M ra
5,628 answer 20 line 25/27178294
$0 \leq \boldsymbol{q}<\boldsymbol{k} \sim 0$ O<qsk
\$.629 answer 27 ne 1 3127178 ..... 295
$d \backslash n \leadsto d \backslash$

### 3.627 line 16

1/5179 296
See also $\sim$ See aleo P. A. MacMahon, Proc. London Meth. Sec. (1891), 341-344;
20.627 bottom of page, new paragraph for answor $6 \quad 8125176297$
M. Paterson observes that if the multiplicities of keys are $\left\{n_{1}, \ldots, n_{m}\right\}$, the number of comparisona can be reduced to $n$ ig $n-\Sigma n_{i}$ Ig $n_{i}+O(n) ;$ ece SJAM J. Compucing 6 (1976), 2.
3.630 answer 20 ..... 5127/78 298
line 5: l-1 ~ $1+1$
line 6: $2^{-l+1}$ ~ $2^{-l}$
line 6: $2^{-1} \sim 2^{-l-1}$
line 6: $2^{l}$ ~~ $2^{l+1}$ (twico)
line 7: LIENJ. $1 \leadsto \mathrm{LE}_{\mathrm{E}} \mathrm{NJ}$

3.655 answer $10 \quad 312177 \quad 300$
[Inf. Proc. Letters ~
〕.
3.635 supplement to now answer 22
$9121176 \quad 301$
[Seo C. K. Yap, CACM 10 (1976), 501-508, for a furthor improvomont.]
26. (a) Let the vertices of the two types of components be designated $a ; b<c$. The adversary acts as follows on nonredundant comparisons: Case 1, a:a ${ }^{-}$, make an arbitrary decision. Case $2, x: b$, say that $x>b$; all future comparisons gib with this particular $b$ will result in $\boldsymbol{y}>b$, otherwise the comparisons are decided by an adversary for $U_{t}(n-1)$, yielding $\geq 2+U_{t}(n-1)$ comparisons in all. This reduction will be abbroviated "let $b=\boldsymbol{m i n} ; 2+U_{t}(n-1)$." Case 3, $x: c$, let $c=\max ; 2+U_{t-1}(n-1)$.
(b) Let the new types of vertices be designated $d_{1}, d_{2}<e ; f<c<h>i$. Case 1 , a:á or c:c; arbitrary decision. Case 2 , a:c, say that $a<c$. Case 3 , $z: b$, let $b=m i n ; 2+U_{8}(n-1)$. Case 4, $x: d$, let $d=\min ; 2+U_{t}(n-1)$. Case 5 , $x: 0$, let $0=\max ; 3+U_{t-1}(n-1)$. Case $6, x: f$, let $f=\mathbf{m i n} ;$ $2+U_{t}(n-1)$. Case 7, $x: \varepsilon$, let $f$ and $a=\min ; 3+U_{t}(n-2)$. Case $8, x: h$, let $h=\max ; 3+U_{t-1}(n-1)$. Case $9, x: i$, let $i=\min ; 2+U_{i}(n-1)$.
(c) For $t=1$ we have $U_{t}(n)=n-1$, so the inequality holds. For $1<t \leq n / 2-1$, use induction and (b). For $t=(n-1) / 2$, use induction and (a). For $t=n / 2, U_{6}(n-1)=U_{6-1}(n-1)$; use induction and (a). Exercise 14 now yields the following lower bound for the median: $\left.V_{t}(2 t-1) \geq 3 t+L t / 2\right\rfloor-4$.

33 update to previous correction number $345 \quad 2 / 28178 \quad 303$
(To appear.) $\sim$ IEEE Trans. C-27 (1978), 84-87.
3.GY1 line -2 1116177304

Pollard.] $\sim$ Pollard.] All such identitics can be obtained from a system of four axioms and a rule of inference for mulsivalued logic due to tukasiowicz; soe Rose and Rosser, Trana. Amor. Math. Soc. 87 (1958), 1-53.
3.641
exercise 43
$312177 \quad 305$
A. Waksman and $M$. Green have proved that $\sim$ By slightly extending a construction due to L. J. Goldatein and S. W. Leibholz, IEEE Trans. EC-16 (1967), 637-641, one can show that $P(n) \leq P(\ln / 2 \mathrm{~J})+P([n / 27)+n-1$, hence
Eq. 5.3.1-3, cf. ... Groen also has proved $\sim$ Eq. 5.3.i-3; M. W. Green has proved (unpublished)
3.645 line 14

One might complain that the algorithm compares KEY values that haven't been initialized. If such behavior is too shocking, it can be avoided by setting all KEYE to 0 , say, in step R1.

## 5. 658 line 7 <br> $5127178 \quad 308$

increase $l$ by 1 , set $\ldots$, and return $\sim$ set $\ldots$, increase $l$ by $l$, and return

## 3.B65 exercise 3 line 7

11112/76 309
Trabb-Pardo ~ Trabb Pardo
3.671 exercise 2 2/28178 310
line 1: RTAG $\sim$ RTAG ( 0 )
line 2: RLINK $(P)$. $\sim$ RLINK $(P)$ and RTAG $(P) \leftarrow+$. In step T4, change the test RLINK (P) A to RTAG(P) +.
last line: .] $\sim$. Similar remarks apply with simultanoous left and right threading.]
3.673 tree illustration in answer 23 11115/78 311
$5 \sim 9$

### 3.675 new answer 11 11129177 312

11. Clearly there are as many $+A^{\prime}$ 's as $--B^{\prime}$ 's and $+-B^{\prime} 6$, when $n \geq 2$, and there is symmetry between + and - . If there are $M$ nodes of types $+A$ and $-A$, consideration of all possible cases when $n \geq 1$ shows that the next random insertion produces $M-1$ such nodes with probability $3 M /(n+1)$, otherwise it produces exactly $M+1$ such nodes. The result follows. [To be published.]

[^0]the line following the tree should become the following finstead of what was stated in the former corroction number 355):
It is perhaps most difficult to insert a new node at the extreme left of a tree like this. An insertion algorithm taking at most $O(\log n)^{2}$ steps has bese presented by D. S. Hirschberg, CACM 19 (1976), 471-473.
3.678 update to previous change $678 \quad 1115178315$
, to appear 9 (1978), 171-181
20.679 changes to answer $5 \quad 6114177 \quad 316$
450. The worst ... chars.

Interpretation 1, trying to maximize the stated minimum: 450. (The worat ... chare.)
Interpretation 2, trying to equalize the number of keys after splitting, in order to keep branching factors high: 155 ( 15 short koys followed by 16 long ones).
2.B8D bottom, new paragraph for answer $4 \quad 7131176 \quad 317$

A more versatile way to economize on trie storage hee been proposed by Kurt Maly, CACM 19 (1976), 409-415.
2.68C line -8 $2 / 28178 \quad 318$
$n \sim N$
3.688 exercise 1 2128178 319
$-38 \sim \rightarrow-37$
3.687 answer 4 6/14177 320
change line 1 to: Consider cases with $\&$ pairs. The smallest n such that in line 2 (the displayed formula), interchange $m$ and $n$ everwhere, then add ", for $m=365$,"
3.687 update to previous change number 3656/14/77321
Computing, to appear. $\sim$ Computing 6 (1977), 201-234.
3.688 new answer 12/19/76 ..... 32210. See F. R. K. Chung and R. I. Graham, Ars Combinatoria 1 (1976), 57-76.
3.689exercise 146/14/77323
line 2: keys $\sim$ all keysline 12: until $\sim$ until TAG $(P)=1$ andline 12: points $\sim$ points (perhaps indirectly through words with TAG - 2)
3,698 replace all but first line of answer 37 by:12/19176 324
$M^{N} N_{N}=\frac{f}{2} \Sigma\left(N_{k_{1}, \ldots, k_{M}}\right)\left(k_{1}\left(k_{1}-\frac{d}{2}\left(k_{1}-1\right)+\cdots+k_{M}\left(k_{M}-\frac{f}{2}\left(k_{M}-1\right)\right)\right.\right.$$-\frac{1}{3} M \Sigma\binom{N}{k}(M-1)^{N-k} k\left(k-\frac{f}{2}\right)(k-1)$

$$
=\frac{1}{3} M N(N-1)(N-2) \sum(N-3)(M-1)^{N-k}+\frac{1}{2} M N(N-1) \sum\left(N_{k-2}-2\right)(M-1)^{N-k}
$$

$$
=\frac{1}{3} M N(N-1)(N-2) M^{N-3}+\frac{1}{2} M N(N-1) M^{N-2} .
$$

The variance is $S_{N}-((N-1) / 2 M)^{2}=(N-1)(N+6 M-5) / 12 M^{2} \approx \frac{d}{2} \times \frac{1}{122^{2}}$.
3,698 new answer ..... 11 5179 325
60. No; see M. Ajtai, J. Komlós, and E. Szemerédi, Inf. Proc. Lotters 7 (1978), 270-273.
3.700new answer$312 / 77 \quad 326$
16. Let each triple correspond to a codeword, where each codeword has exactly three 1 bits, identifying the elements of the corresonding triple. If $u, v, v$ are distinct codewords, $u$ has at most two 1 bits in common with the superposition of $v$ and $v$, since it had at most one in common with $v$ or wolone. [Similarly, from quadruple systems of order $v$ we can construct $v(v-1) / 12$ codewords, none of which is contained in the superposition of any three others, etc.]
3.700~ ..... 1| $5179 \quad 328$
Ajtai, Mikloe, 698.
3.7206 ..... 1116177329
Allon, Charles Grant Bloirfindie, 549.
3. $210 น ~$ ..... 4119177330
add p576 to AND entry
3.821 ..... 11115178 331
delete index entries for R. M. Beer and P. Brock
3. $5210 ~ 1 ~$ 11/29177 ..... 332
Brown, Mark Robbin, 470.
\$.722L ..... 4119177333
delete Circular liets entry
5.5226 12/19176 ..... 334
Chung, Fan Rong King, 688.
3.712R de Bruijn entry ..... 8125176 335
add p. 585
3.71288 ..... 12/19176 336
Deadlock, 479.
8.723 6114177 ..... 337accent over 0 in Erdös chould be＂not＂
3．725亿 $116 / 77$ ..... 398
Dryedale，Robert Lowis（Scoth III， 239.
2．7120 ..... 4119177 ..... 339
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2.71218 ..... 11112／76 ..... 340
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3．ปロฯฉ ..... 4119177341
add p518 to Ferguson ontry
3．714ட line 5 ..... $9121 / 76$ ..... 342
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5．21ณは 2128178 ..... 343
Gonnet Hass，Gaston Hienry， 526.
3．714ณ 312177 ..... 344
Goldstein，Larry Jool， 641.
3.71918 6114177 ..... 345
Halperin，John Harria，604．
3．52ฯは 6／14177 ..... 346
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3．51ฯは 11129177 ..... 347
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\＄． $2 \boxed{14}$ ..... 12119176 348
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3．71ヶは ..... 12119176 349
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$3.725 \square$ ..... $4119177 \quad 350$
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$3.725 \square$ ..... 9121176 351
Hirschberg，Daniol Syna Moeoce， 677.
53．725B new entry 5127178 ..... 352
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3.7266 1／ 5179 ..... 353
Komlóe，Jánoc， 698.
3.716 BL Kieitman entry ..... $2128178 \quad 354$
640 ..... 639
3．7ロ6LLehmer，Derrick Henry， 419.
3．716亿 ..... $312177 \quad 356$
add pp．561， 570 to Kauts entry
3．706亿 ..... 11115178 957
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3.7268 ..... 116177358
add p641 to Lukasiewies entry
3．706R ..... $312177 \quad 359$
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3.7264 ..... 6／25176 360
Lozinskiǐ，Elieser Loonid Solomonovich， 681.
3．727L MacMahon entry ..... 115179361
add p． 627
\＄．727 ..... 7131176 362
Maly，Kurt， 680.
$8.727 \square$ 11129177 ..... 363Mallach，Efrem Gerchon，546．
$\$ .7271$ ..... 12/19176 364
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5. 81810 ..... 2128178365
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3.81810 ..... $6114177 \quad 367$MOVE, 604.
3.7286 ..... $312177 \quad 368$
add p. 215 to Noshita ontry
$\$ .7280$ ..... 4119177369
delete Newell entry
3.7186
1219176 ..... 370
Nitty gritty Nitty-gritty3.7181
4119177 ..... 371
Packed data, 401.
\$. 5 28® now entry ..... $3 / 27178 \quad 372$
Pardo, seo Trabb Parda
3．7188 Paterson entry ..... $8125176 \quad 373$
add p． 627
3.7196 11115178 ..... 374
add p． 576 to Pollard entry
3．7291 111617 ..... 375
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3.72918 ..... 312177 ..... 376
Rearrangeable network，ece Permutation network．
3．729R now entry ..... 5127178 ..... 377
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3．720L 11129177 ..... 378add pp．606， 607 to Sedgowick entry
3．720L12／19176 379
Samadi，Behrokh， 479.
3．720亿 12／19176 ..... 380
add p． 220 to Schönhage entry
\＄．フ2＠は ..... 312177 ..... 381
3．720R entry for SLB ..... $8125176 \quad 382$
add p． 509
3．720は12119176 383
Sheil，Beaumont Alfred， 450.
3．721ロ ..... $2128178 \quad 384$
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3．721D replacement for previous change 416 ..... $115179 \quad 385$
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3． 52010 ..... $1116177 \quad 386$
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3． 7826 Threaded tree entry ..... $2128178 \quad 388$add p457
3．72以 ..... 11112176 389
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3．722は ..... 1116177390
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3．782は ..... 2128178 ..... 391
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3．722は ..... 31277 ..... 392
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3．722® ..... $312 / 77393$
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3．722は 4／19177 ..... 394
Wang，Yihsieo， 128.
3．722以 new names ..... 6／25176 395
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Windley，Peter F．
3．722飞1 ..... 11112176 396
Yap，Chee－Keng， 637.
3.722818 11115178 ..... 397
Vershik，Anatoliǐ Moiseevich， 594.
3． 528.1 6114177 ..... 398
2－ordered，87，103，112， 135.
3．726（namely the endpapers of the book） ..... 4119177399
also make any changes specified for pager 136－137 of volume 1

### 3.2496

add p. 450 to Vaughan Pratt entry
3.765 addendum to provious change 324

11115178 401
John M. Pollard has discovered an clegant method for index computation in about $O(\checkmark p)$ operations mod $p$, requiring very little memory, based on the theory of random mappinge. See Mash. Comp. 32 (1978), 918-924, where he also suggests another method based on numbers $n_{j}=r{ }^{j} \bmod p$ that have only amall prime factore.
9.(D) changes for the book Mariages Stables
p12 line 18: Ac ~ Aa
p14 line 4: Ab~~Bb
p18 line -5: $B_{i} \sim B_{j}$ and $A_{i} \leadsto A_{j}$ (four changes)
p18 line -4: $b_{i} \sim b_{j}$ and $a_{i} \sim a_{j}$ (four changes)
p18 line -3: $a_{n}$ ~ $a_{k}$

p32 line 6: exercises $\sim$ exercices
p32 line - 5 exercise $\sim$ exercice
p35 illustration: delete are from 4 of clubs to 8 of hearts
p38 line -11: $C$ ~ B
p47 line 2: Chebyahov $\sim$ Tchöbichov
p50 lines $-12,-10,-3$ and p51 line 5: Chobyshor $\sim$ Tchebicher
p52 line -6: $c \leadsto C$
p65 line -4: m ~ $m$
p66 line -10, denominator of third term in sum: $n+1 \sim n-1$
p71 line 8: que $\boldsymbol{R}_{\boldsymbol{A}}$ * $\sim$ que
p74 line -1: $X \sim x$
p78 line-7: $X \leadsto x$
p78 line -4: $O[\Lambda] \sim O(1]$
p86 line 10: femmes. $\sim \rightarrow$ femmes?

p92 line -8: exercise $\sim$ exercice
p93 line 4: et ( $\mathrm{Aa}, \mathrm{Bb}, \mathrm{Ce} \sim$ et ( $\mathrm{Aa}, \mathrm{Bc}, \mathrm{Ch}$
p93 lines $-6,-3,-2$ : crossed-out eshould be crossed-out $c$
p95 line 3: $n!P_{n} \sim n!p_{n}$
p95 line 9: $\Sigma \sim \boldsymbol{\Sigma}_{\boldsymbol{i}}$
p95 line -2: formula should be preceded by (3)
$p 95$ line -2: $d x_{2} \ldots, ., d x_{n} d y_{1} d y_{2} \ldots, \ldots d y_{n} \sim d x_{2} \ldots d x_{n} d y_{1} d y_{2} . d y_{n}$
p86 lines 13-14 should say: II $\left(y_{1}, X_{L s}\right), I I\left(Y_{R}, s, s\right)$.
p86 line -2 , change final comma to a period p86 line -1 , delete this line
pll2 line -5: p. The $\sim$ p. [See his incredible book On Numbers and Gemen, publiched by Academic Press in 1976.) The pll3 Mechomatik $\sim$ Amelyaio

## 5 Jan 1979 TEXNOW[ 1,LTP] PAGE 2-1

THE TEX/METAFONT PROJECT.
WHAT HAS BEEN DONE:
Don Knuth has finished (and frozen) the implementation of TEX (the typesetting system) and is currently involved in the implementation of METAFONT (the font generator).

WHAT WE WANT TO DO:
We want to complement TEX / METAFONT with a suitable hardware environment, namely:

* An XGP type device that will provide hardcopy capabilities both for proofreading and for (medium quality) originals.
* A high resolution typesetting device for high quality originals.
* A high resolution CRT terminal, capable of displaying TEX output.

We also want to make the system widely available, thus it is needed to implement it in a more widespread language (PASCAL).

And finally we would like to try our hand in making TEX more interactive than what it is now. (This one is a tougher cookie.)

IF YOU ARE INTERESTED:
There are many things to be done. There are learning oportunities. There are academic goodies (units, CS293 projects, etc). And there is also monies.

FOR MORE INFO:
Send a message to LTP, or call 74425 or 74377.


[^0]:    3,676 new answer to exercise 16
    11129177313
    Delete $E$; Case 3 rebalancing at $D$. Delete $G$; replace $F$ by $G$; Case 2 rebalancing at $H$; balance factor adjusted at $K$.
    (a new illustration, in the same style as before, must be supplied now)

