Exploring hadron interactions and bound states with femtoscopy in heavy-ion collisions

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On-line seminar series IV
on "RHIC Beam Energy Scan: Theory and Experiment", 2022.

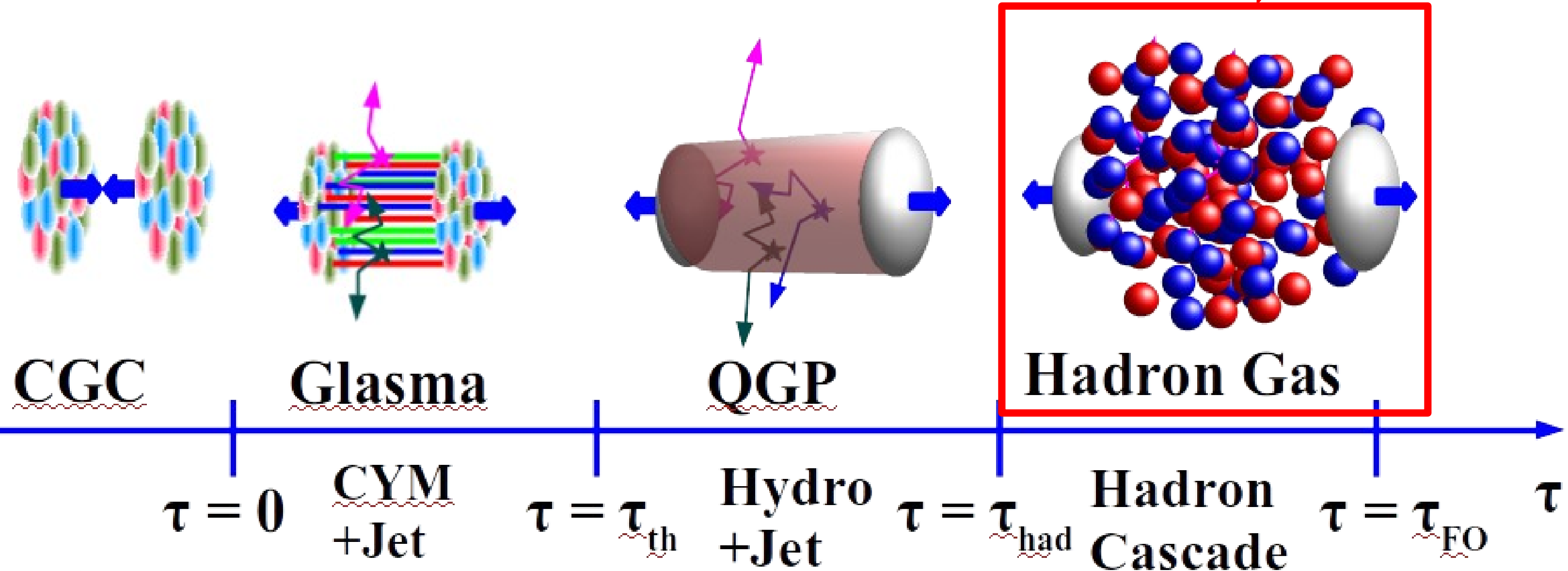
- **Introduction**
- **Interaction dependence of correlation function**
- **Bound state diagnosis by femtoscopy**
- **Recently observed / studied correlation functions, Homeworks, and Perspectives**
  - $D^- p, D\pi, DK, DD^*, D\bar{D}^*, ppp, pp\Lambda, \ldots$
  - **Three-body correlation function**
- **Summary**

High-Energy Heavy-Ion Collisions

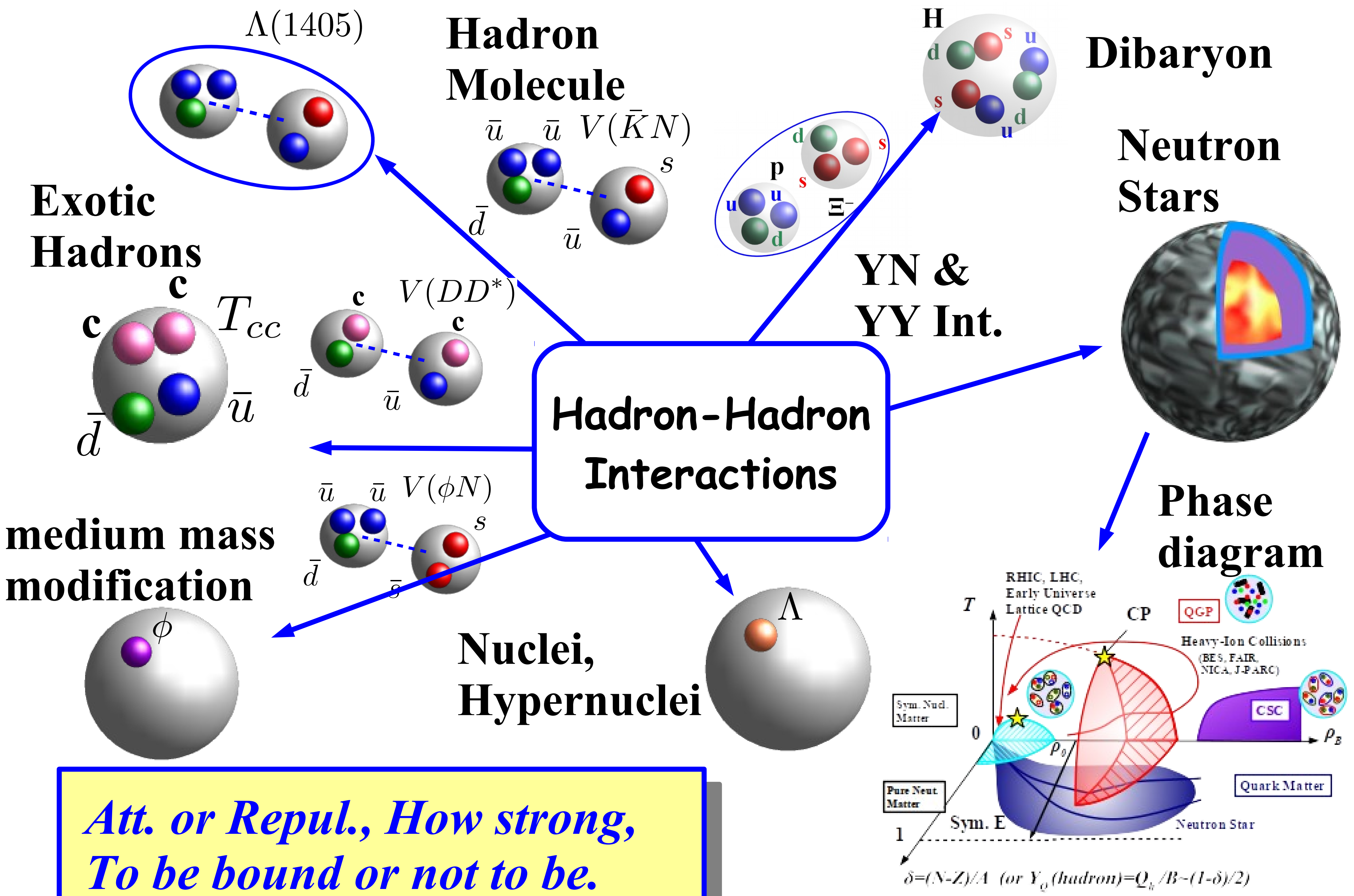
- **Main Goal of HIC physics = Discovery and Properties of QGP**
- **HIC as a playground / tool**
  - **Development of dynamical models**
  - **Physics of extreme conditions and/or strong field**
  - **Hadron physics**

Hadron Physics using HIC as Hadron Factories

- ***Simultaneous Prod. of many hadrons statistically***
- ***Nearly 4π detectors & Vertex detectors***



Hadron-Hadron Interactions



Hadron-Hadron Interactions

Hadron Molecule

Exotic Hadrons

medium mass modification

Nuclei, Hypernuclei

Dibaryon

YN & YY Int.

Neutron Stars

Phase diagram

Att. or Repul., How strong,
To be bound or not to be.

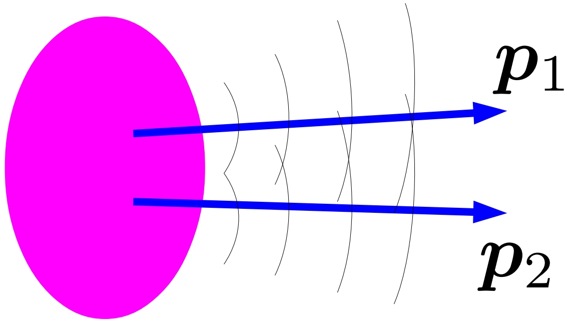
Femtoscopy

Correlation Function

Koonin('77), Pratt+('86), Lednicky+('82)

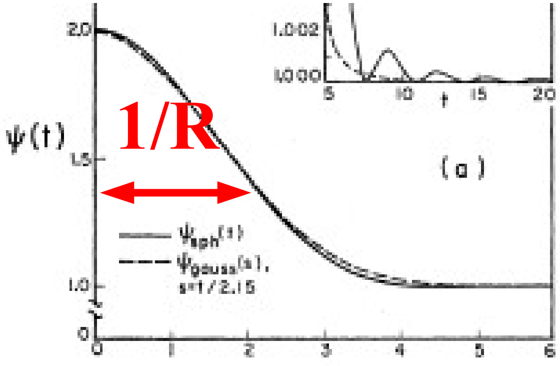
$$C(\boldsymbol{q}) = \int d\boldsymbol{r} S(\boldsymbol{r}) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2$$

$$\underline{S(\boldsymbol{r}) = \text{source function}}\,,\ \underline{\varphi_{\boldsymbol{q}}(\boldsymbol{r}) = \text{relative w.f.}}$$



Source size (HBT)

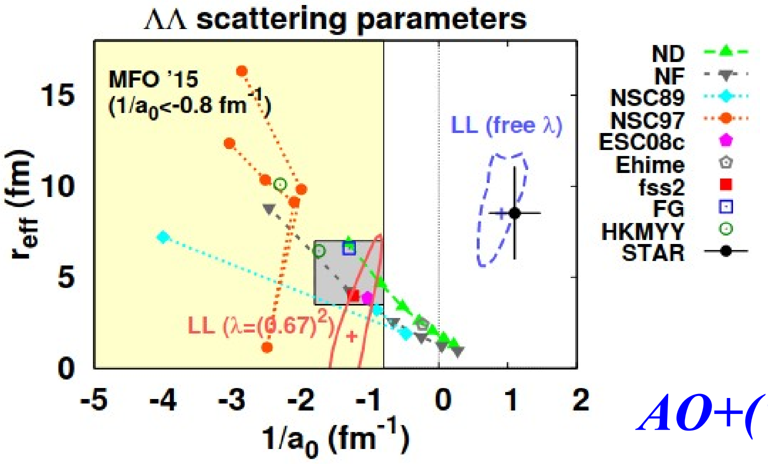
Hanbury Brown & Twiss,
Nature 10 (1956), 1047;
Goldhaber, Goldhaber, Lee, Pais,
Phys. Rev. 120 (1960), 300.



q (relative momentum)

Hadron-Hadron Interaction

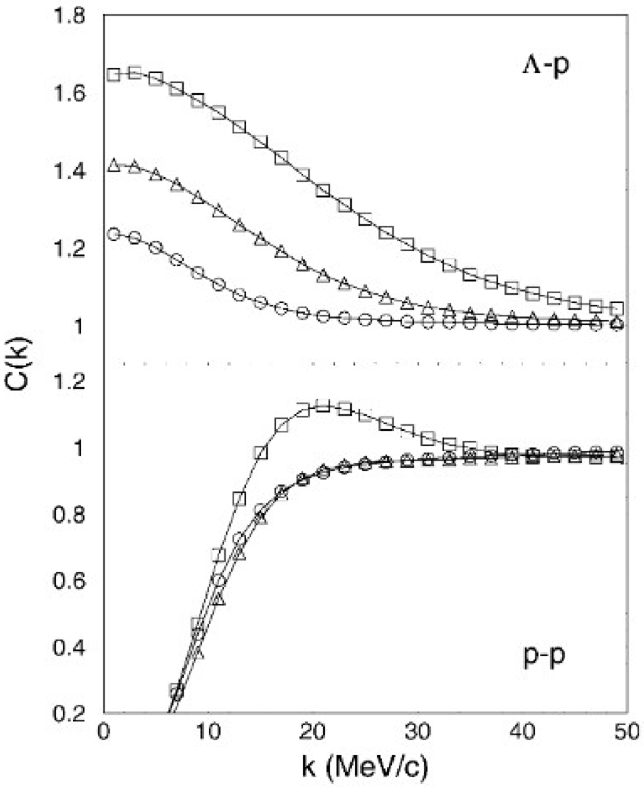
Lednickey, Lyuboshits ('82); Lednicky, Lyuboshits, Lyuboshits ('98); Heidenbauer ('19); C. Greiner, B. Muller, PLB219('89)199; AO+ ('00); Morita+ ('15~); Kamiya+('20~); STAR ('15~); ALICE ('19~)



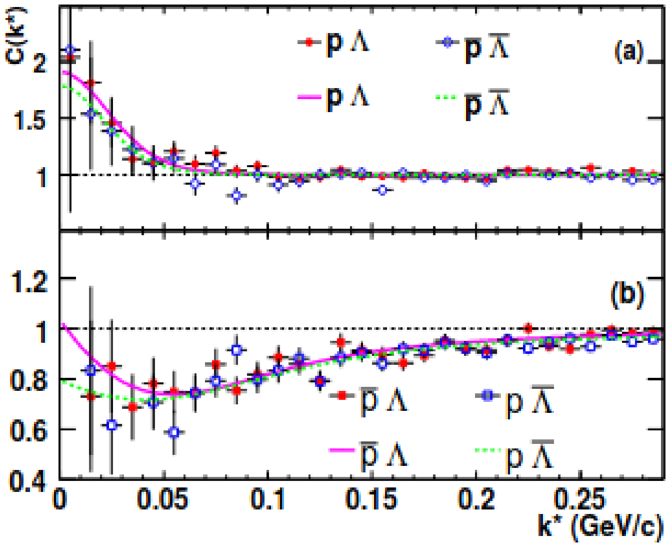
AO+('16)

… and it works

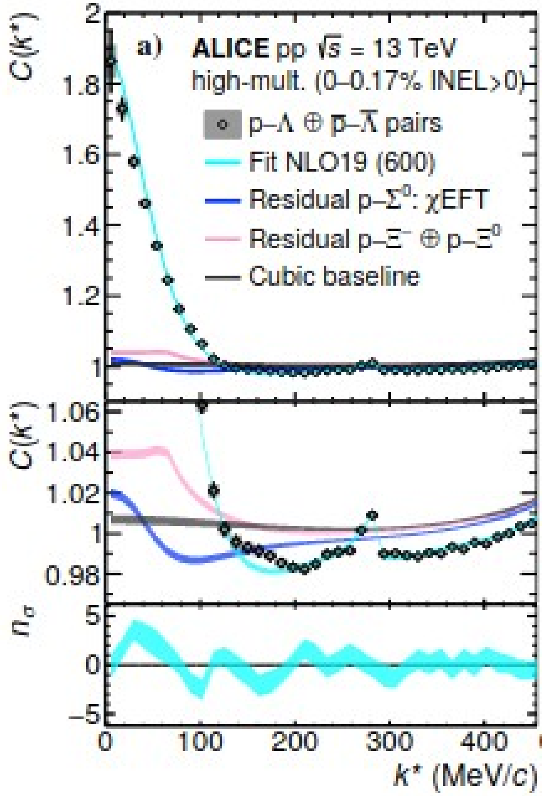
Prediction ⇨ **Measurements**



Wang, Pratt ('99, PRL, nucl-th/9907019)



STAR('06) (nucl-ex/0511003)



ALICE (2104.04427)

pΛ correlation function is well described by known NΛ interactions.
Extracted scattering parameters from data are consistent
with those in known interactions.

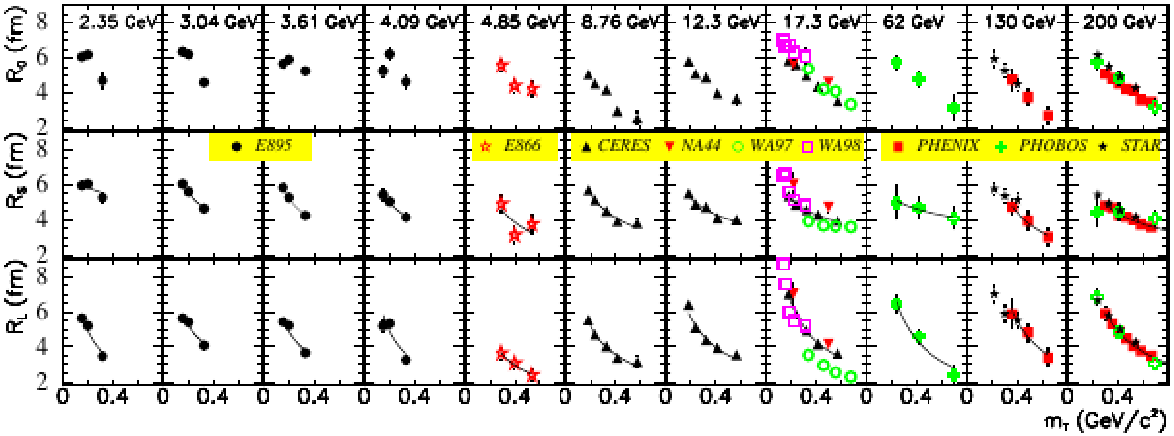
Correlation function contains information
on hadron-hadron interaction.

It is also possible to guess
the existence of a bound state
by using the correlation function.

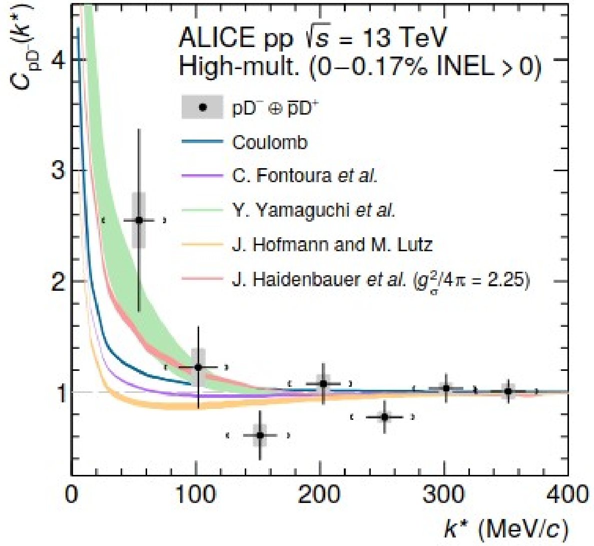
In addition to confirmation of proposed hh int.,
recent data seem to distinguish models
and to require improvements in present hadron theory.

Reservation (Excuse)

- **State-of-the-art femtoscopy of source size and shape**
  → Systematic measurement of 3D HBT radii (side, out, long)
  M. A. Lisa, S. Pratt, R. Soltz, U. Wiedemann, Ann.Rev.Nucl.Part.Sci. 55 (2005) 357-402.
- **But please forget 3D source functions, serious flow effects and so on for 1 hour. (Let us go back to 60 years ago, where no Yano-Koonin-Podgoretski existed. After the talk, we should discuss.)**
- **We will mainly use a spherical Gaussian source function.**
  - **Statistics is low for identified flavored hadron pairs, and only 1 dimensional correlation function is shown.**



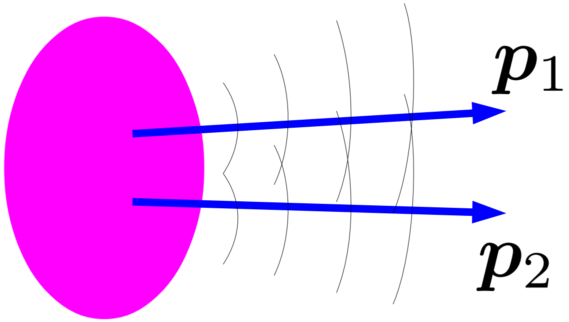
VS



ALICE('22)

Interaction Dependence of Correlation Functions

Two particle momentum correlation function



- **Single particle emission function**

$$N_i(\boldsymbol{p}) = \int d^4x S_i(x,\boldsymbol{p})$$

- **Two-particle momentum correlation function**
  - **Two particles are produced independently, and correlation is generated in the final state. (Koonin-Pratt formula)**

Koonin('77), Pratt+('86), Lednicky+('82)

2 body w.f.

$$C(\boldsymbol{q}) = \frac{N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} \simeq \frac{\int d^4x d^4y S_1(x,\boldsymbol{p}_1)S_2(y,\boldsymbol{p}_2)|\boxed{\Phi_{\boldsymbol{p}_1,\boldsymbol{p}_2}(x,y)}|^2}{\int d^4x d^4y S_1(x,\boldsymbol{p}_1)S_2(x,\boldsymbol{p}_2)}$$

$$= \int d\boldsymbol{r}\,\underline{S(\boldsymbol{r})}\,|\underline{\varphi(\boldsymbol{r};\boldsymbol{q})}|^2 = 1 + \int d\boldsymbol{r}\,S(r)\left[|\underline{\varphi_0(r;q)}|^2 - |j_0(qr)|^2\right]$$

CM var. int. **Source fn.** **relative w.f. (q=relative momentum)** **s-wave**

Spherical static source, non-identical particles, s-wave, No Coulomb

Note: k is more popular instead of q in experiment papers.*

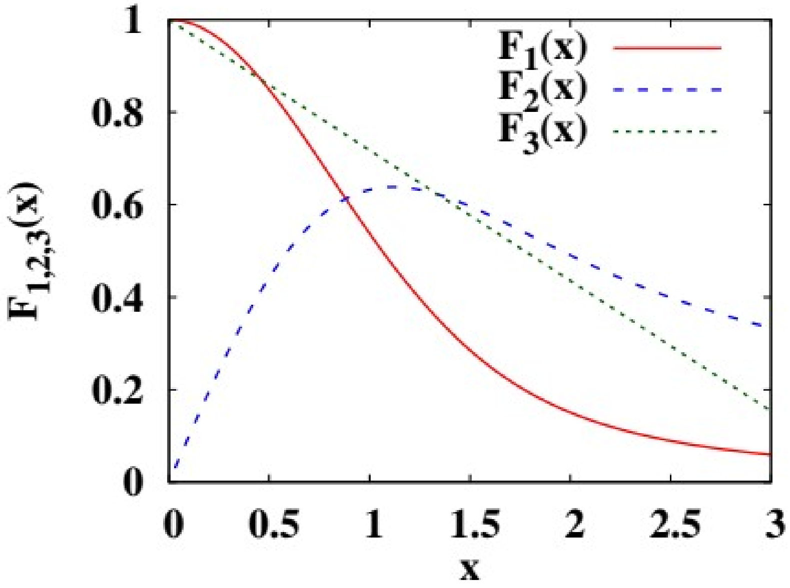
Analytic model of correlation function

- **Correlation function in Lednicky-Lyuboshits (LL) formula**
  (asymptotic w.f., non-identical particle pair, short range int. (only s-wave is modified), single channel, no Coulomb pot., static Gaussian source, real δ) *(Lednickey, Lyuboshits ('82))*

$$\varphi_0^{(-)}(r;q) \simeq \frac{e^{-i\delta}\sin(qr+\delta)}{qr}$$

$$\boxed{C_{\rm LL}(q) = 1 + \frac{2{\rm Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{{\rm Im}\, f(q)}{R} F_2(2qR) + \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\rm eff}}{R}\right)}$$

$$\left[f(q) = (q\cot\delta - iq)^{-1},\ F_1(x) = \frac{1}{x}\int_0^x dt e^{t^2-x^2},\ F_2(x) = (1-e^{-x^2})/x,\ F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}\right]$$



If you have a_0, r_{eff} and R, you can draw C(q) !

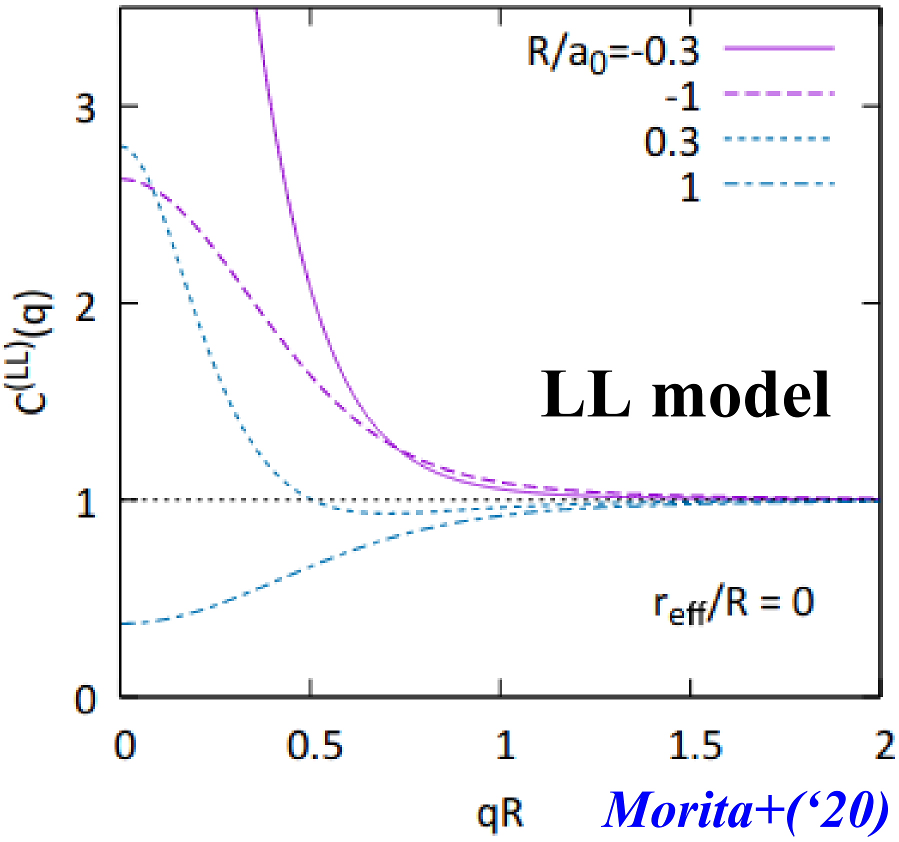
$$F_1(x) \simeq \frac{1 + c_1x^2 + c_2x^4 + c_3x^6}{1 + (c_1+2/3)x^2 + c_4x^4 + c_5x^6 + c_3x^8} \quad (0 \le x < 20)$$

$$(c_1, c_2, c_3, c_4, c_5) = (0.123, 0.0376, 0.0107, 0.304, 0.0617)$$

AO,Morita,Mihayara,Hyodo, NPA 954 ('16)294.

Interaction Dependence of C(q)

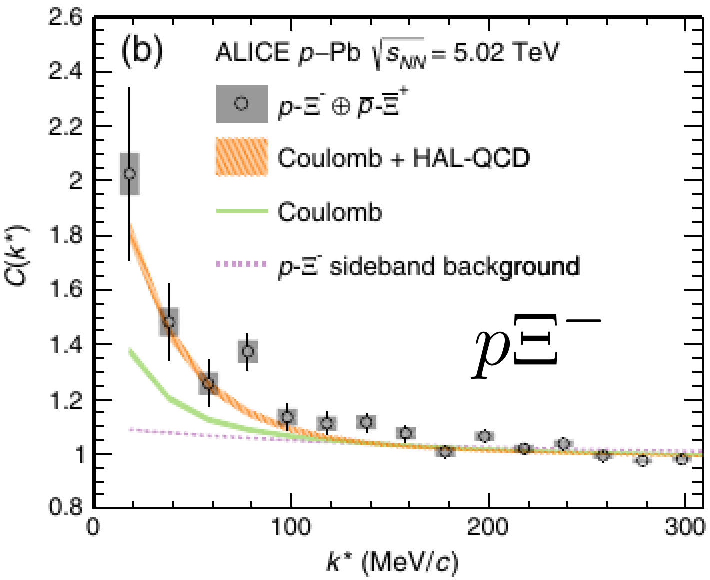
- **Repulsive interaction → C(q) is suppressed.**
- **Attractive interaction**
  - **Wave function grows rapidly at small r with attraction.
→ C(q) is enhanced for small source.**



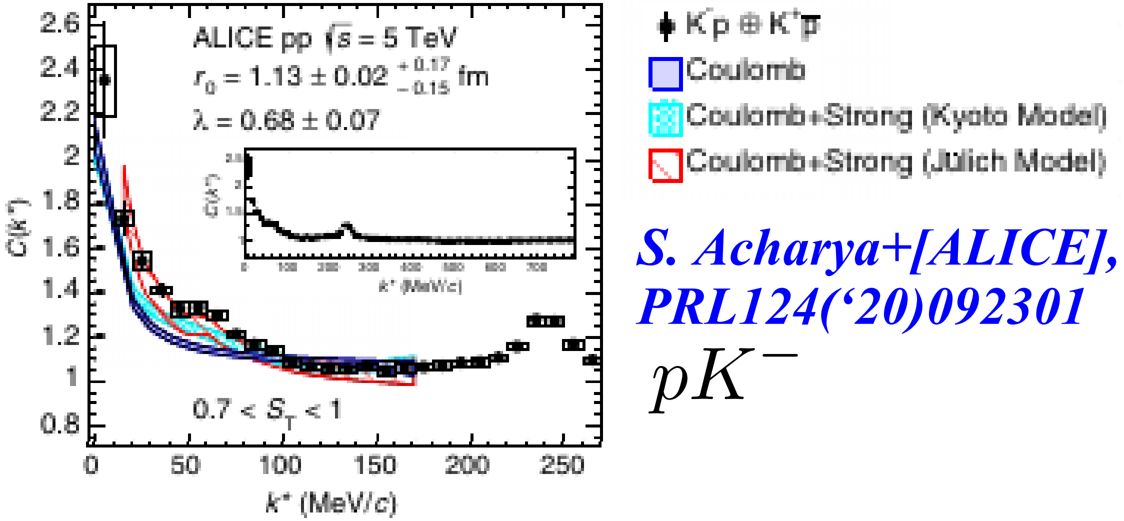
Morita+('20)

Examples of Enhanced C(q) from small source

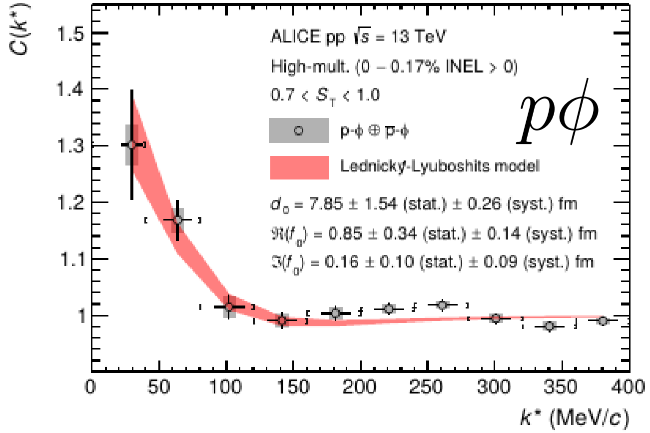
High-Multiplicity events from pp



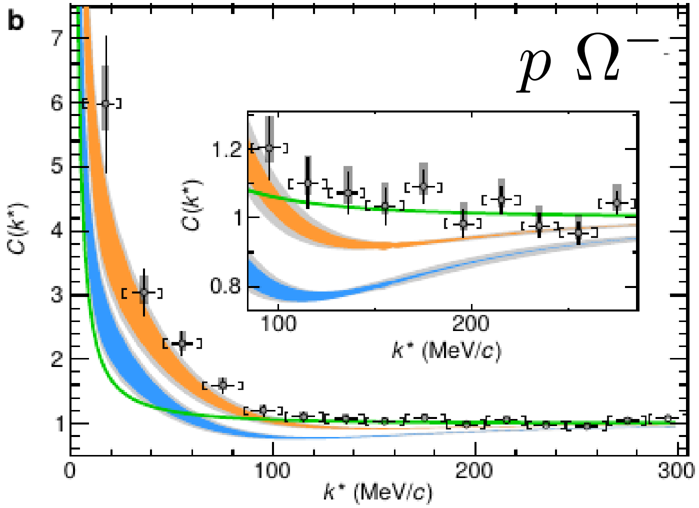
S. Acharya+[ALICE], PRL123('19)112002.



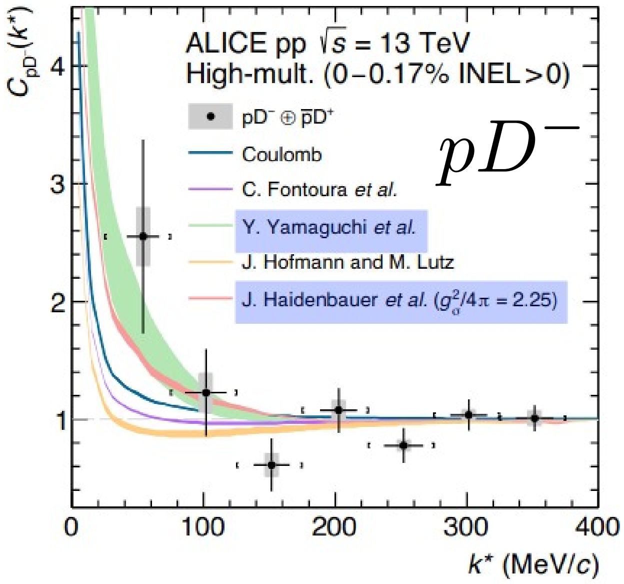
S. Acharya+[ALICE], PRL124('20)092301



ALICE, 2105.05578

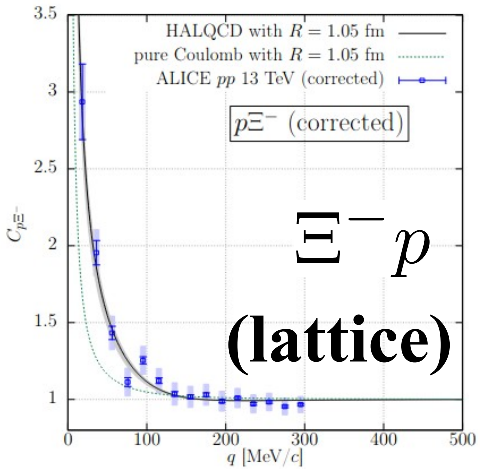


S. Acharya+[ALICE], 2005.11495 [nucl-ex] (pp 13 TeV)



Acharya+[ALICE] (2201.05352)

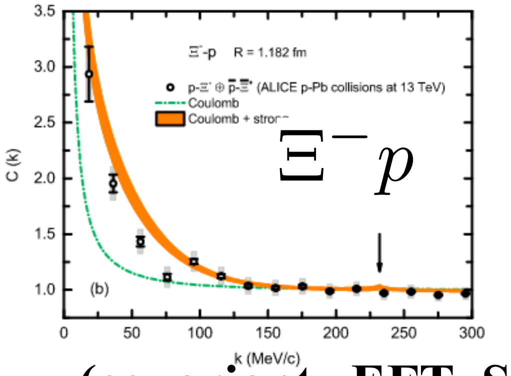
Theoretical femtoscopic study of hh int. (examples)



$\Xi^- p$

(lattice)

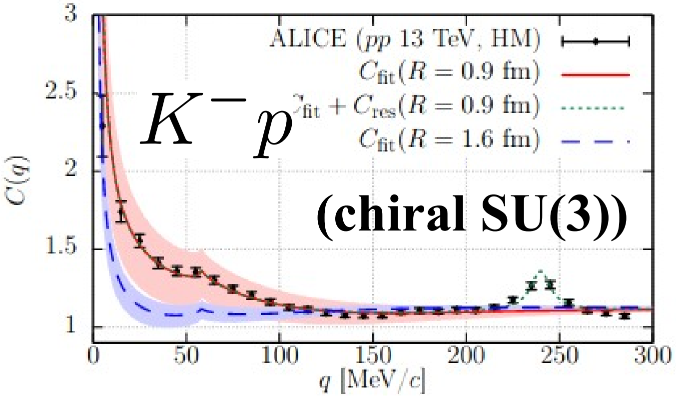
Y.Kamiya, K.Sasaki, et al., (2108.09644)



$\Xi^- p$

(covariant χEFT, S=-2)

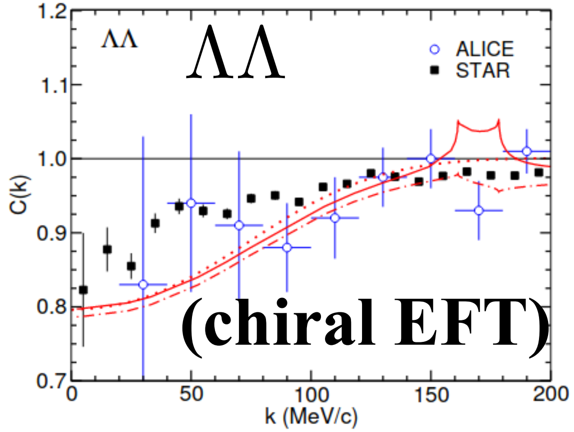
Z.-W. Liu, K.-W. Li, L.-S. Geng (2201.04997)



$K^- p$

(chiral SU(3))

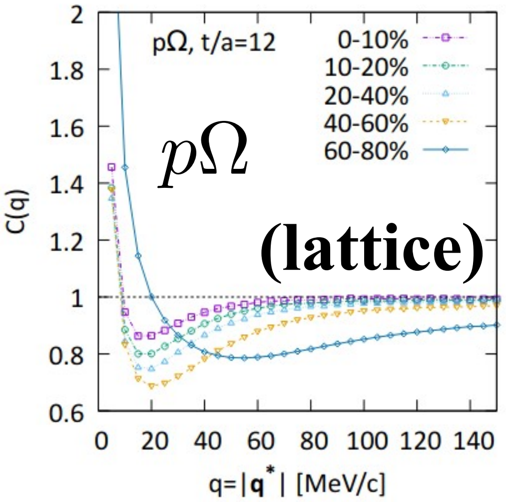
Kamiya+(1911.01041)



ΛΛ

(chiral EFT)

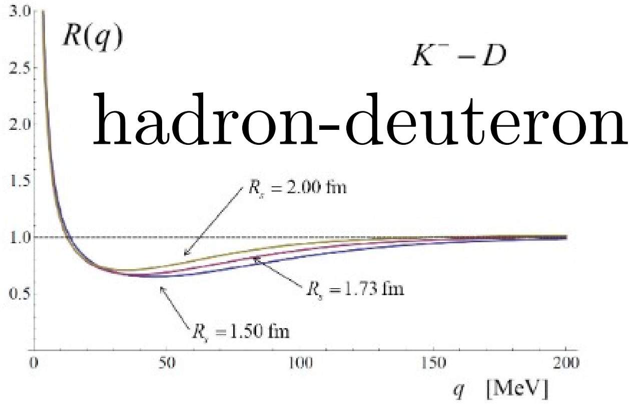
Haidenbauer(1808.05049), Morita+(1408.6682)



$p\Omega$

(lattice)

Morita, Gongyo et al., (1908.05414), Morita, AO, Etminan, Hatsuda (1605.06765)



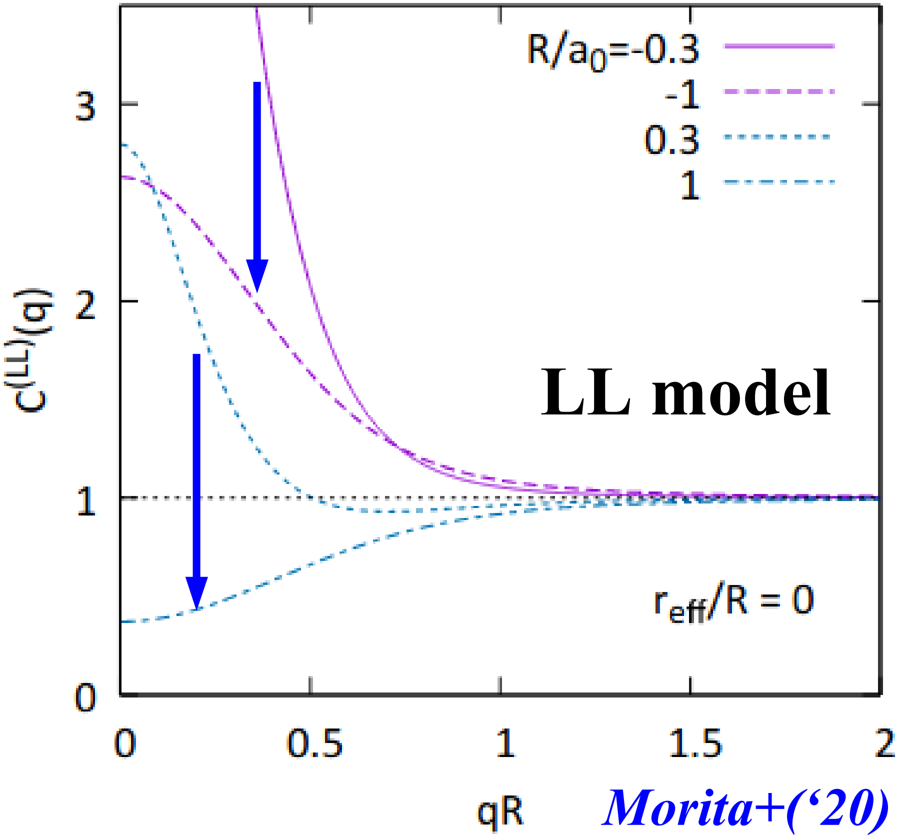
hadron-deuteron

Mrówczyński, Słón (1904.08320, K⁻d), Haidenbauer (2005.05012, Λd), Etminan, Firoozabadi (1908.11484, Ωd), K.Ogata+ (Ξ⁻ d,2103.00100)

Interaction Dependence of C(q)

- **Repulsive interaction → C(q) is suppressed.**
- **Attractive interaction**
  - **Wave function grows rapidly at small r with attraction.**
    → C(q) is enhanced for small source.
  - **Without a bound state ($a_0 < 0$)**
    → C(q) > 1
  - **With a bound state ($a_0 > 0$)**
    → Region with C(q) < 1 appears

Why is C(q) suppressed
when there is a bound state ?
Do we really see
enhanced C(q) for small R
and suppressed C(q) for large R
when there is a bound state ?



Morita+('20)

Bound state diagnosis by femtoscopy

Wave function around threshold (S-wave, attraction)

a_0 =scatt. length
r_{eff}=eff. range

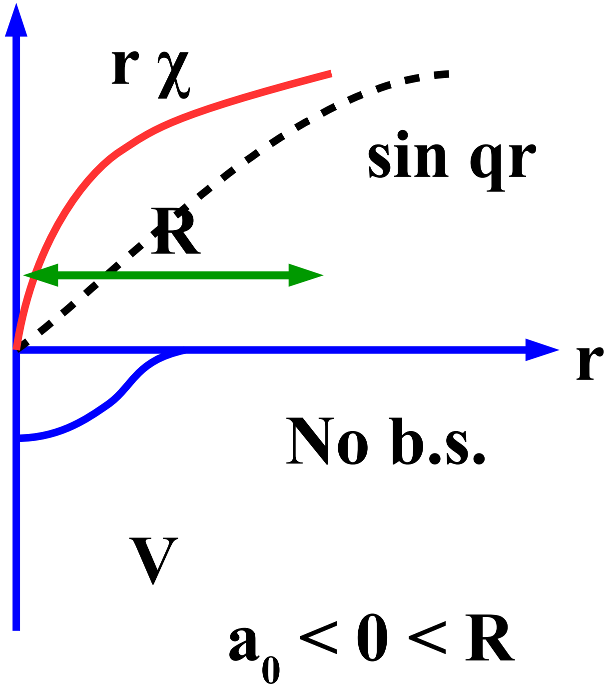
- **Low energy w.f. and phase shift**

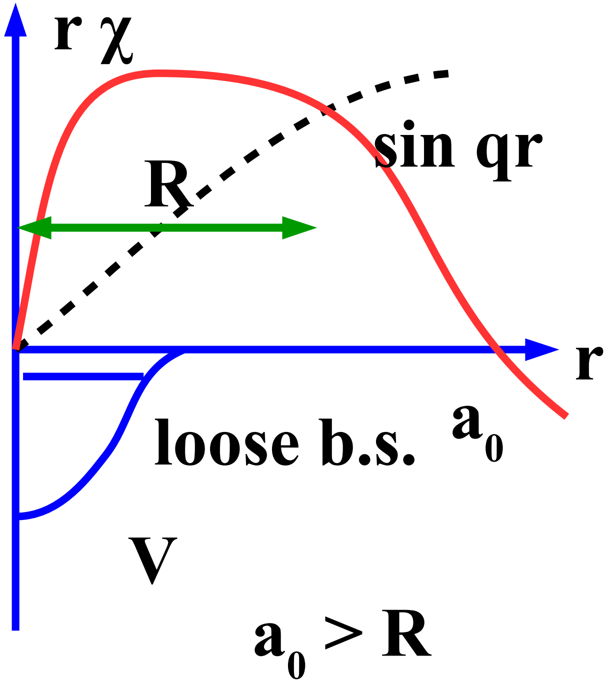
$$u(r) = qr\chi_q(r) \to \sin(qr+\delta(q)) \sim \sin(q(r-a_0))$$

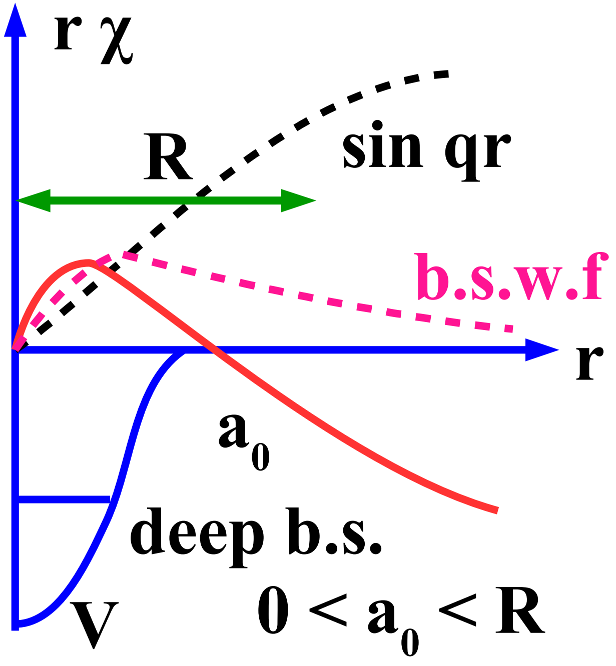
$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 + \mathcal{O}(q^4)\ (\delta \sim -a_0 q)$$

Nucl. and Atomic Phys. convention

  - **Wave function grows rapidly at small r with attraction.**
  - **With a bound state ($a_0>0$), a node appears around $r=a_0$**
    → Suppressed $|\text{w.f.}|^2$ on average







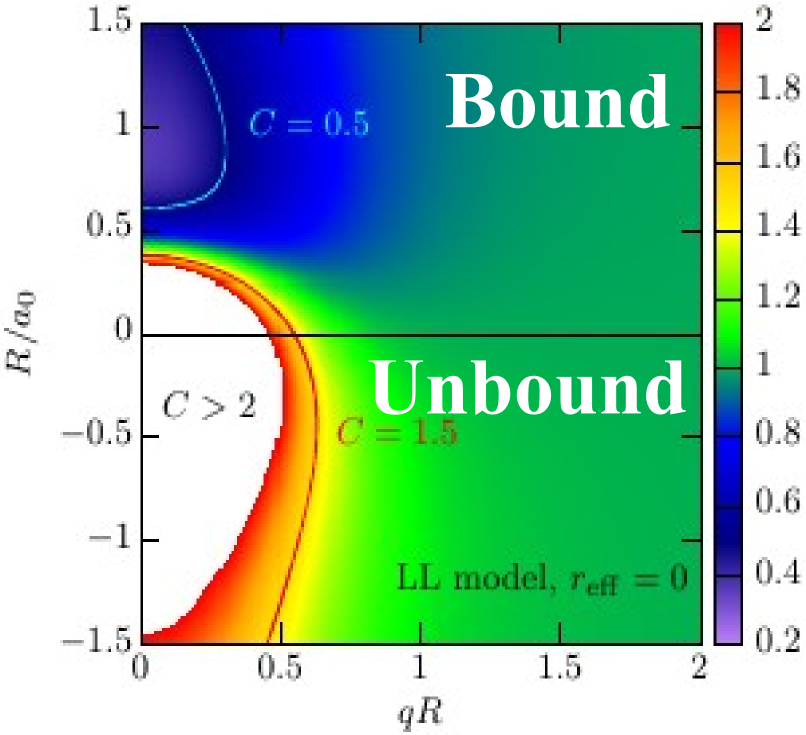
R Dependence of Correlation Function

- **Source size (R) dependence of C(q) is helpful to deduce the existence of a bound state.**

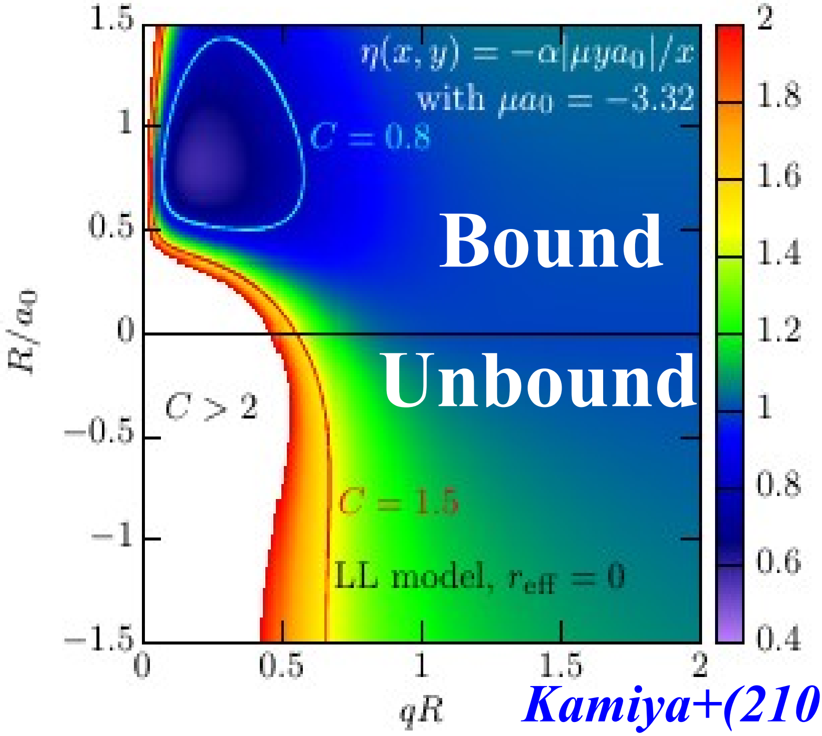
  Morita+('16, '20), Kamiya+('20), Kamiya+(2108.09644)

- **Bird's-eye view of C(q) using the Lednicky-Lyuboshits formula with the zero range approx. (r_{eff}=0)** *[Lednickey, Lyuboshits ('82)]*
  - **Universal function, C(q)=C(qR, R/a_0) (r_{eff}=0, w/o Coulomb)**

w/o Coulomb



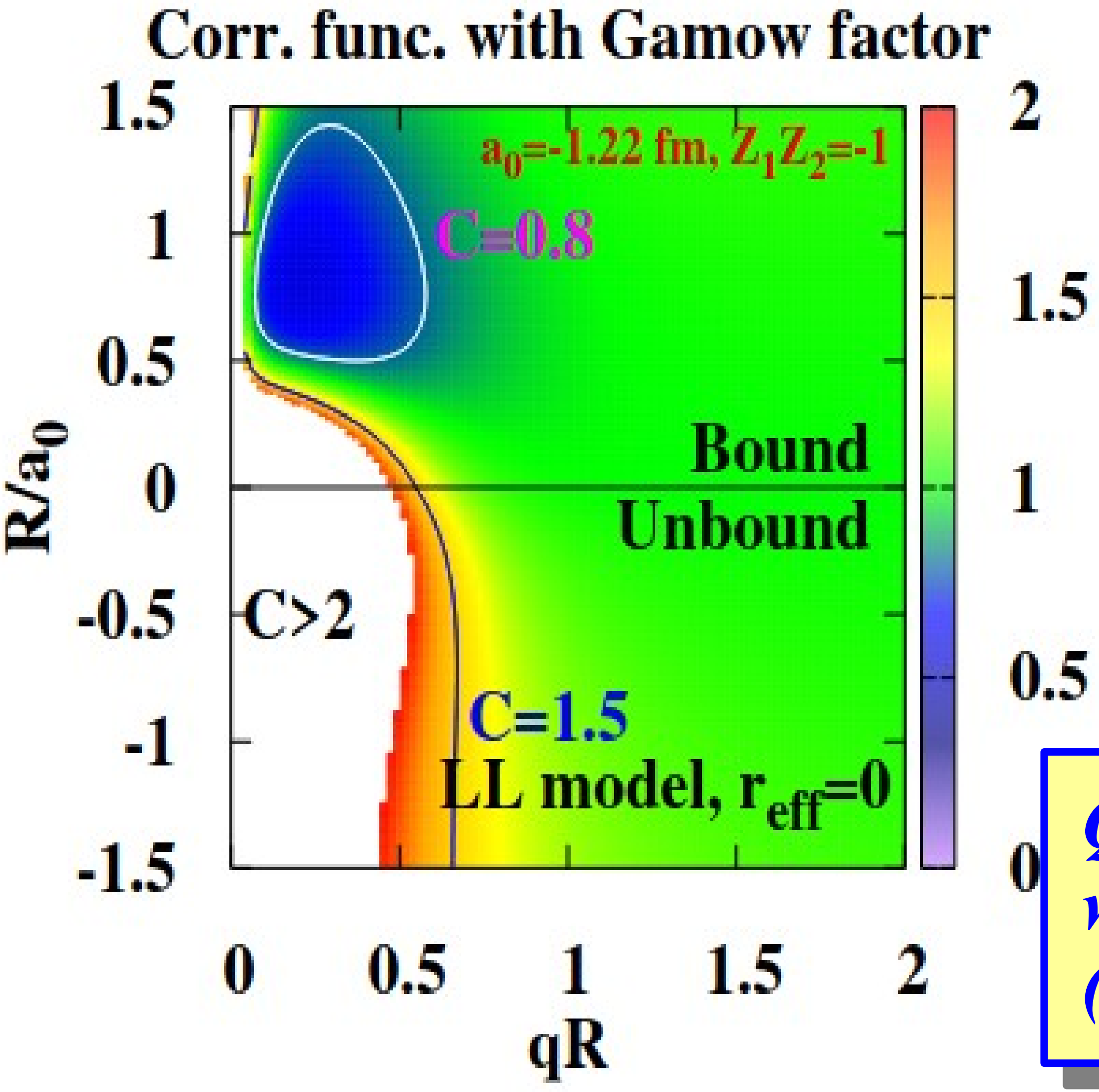
With Coulomb



Kamiya+(2108.09644)

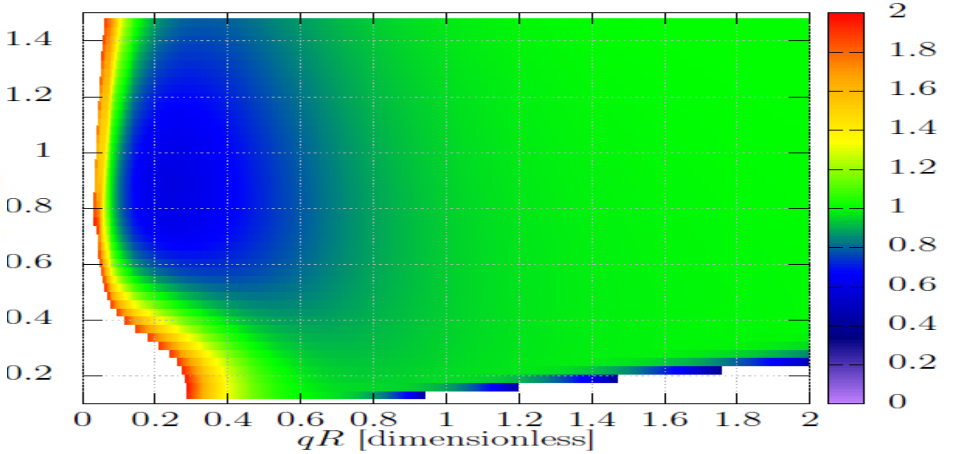
R Dependence of Correlation Function

LL model with Coulomb (r_{eff}=0)



Realistic NΩ potential (J=2, HAL QCD, a_0=3.4 fm) + Coulomb, Coupled-channel

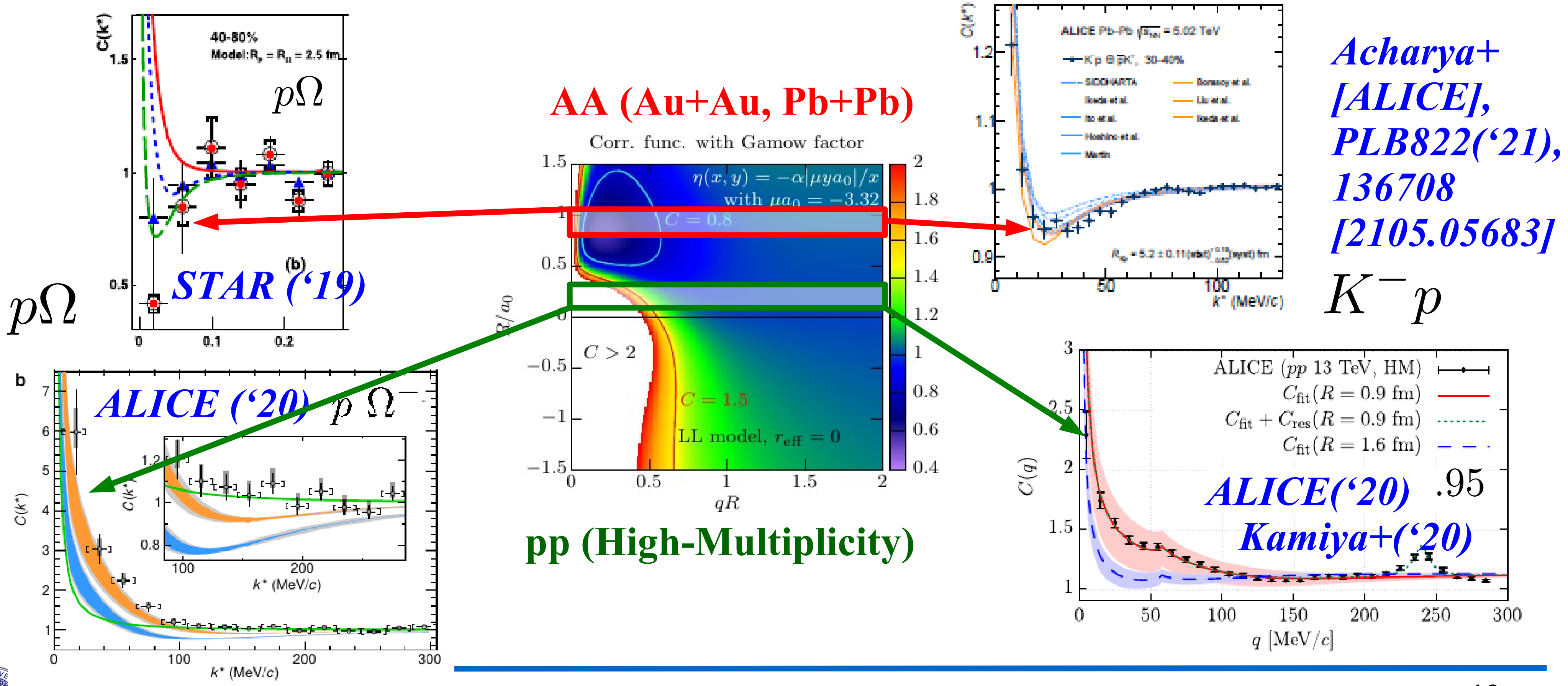
Courtesy of Y. Kamiya



Qualitative feature remains with realistic interactions (and coupled-channel effects)

Bound State Dip

- **With a bound state, C(q) is expected to show a dip for R ~ $|a_0|$.**
- **KN, ΩN → Bound states are expected, and dip is observed in AA**
  Goldman+('87); Oka ('88); Etminan+[HAL QCD] ('14); Iritani+[HAL QCD]('19); Dalitz, Tuan ('59); Akaishi, Yamazamki ('02); Jido+('03); Hyodo, Jido ('12); Morita+('16,'20); Kamiya+('20); Haidenbauer('18).
- **a_0(ΩN)=3.4 fm (Iritani+('19, HAL QCD)), a_0(K^- p)=0.65-0.80i fm (SIDDHARTA)**



AA (Au+Au, Pb+Pb)

pp (High-Multiplicity)

STAR ('19)

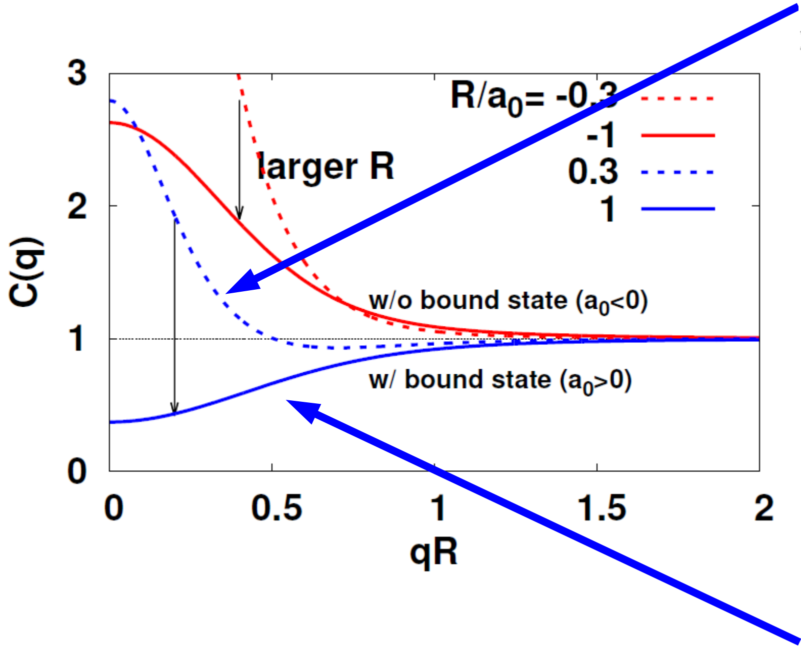
ALICE ('20)

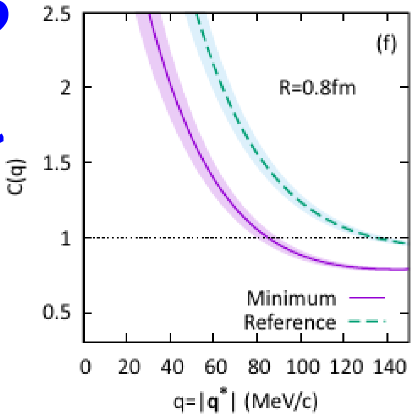
Acharya+ [ALICE], PLB822('21), 136708 [2105.05683]

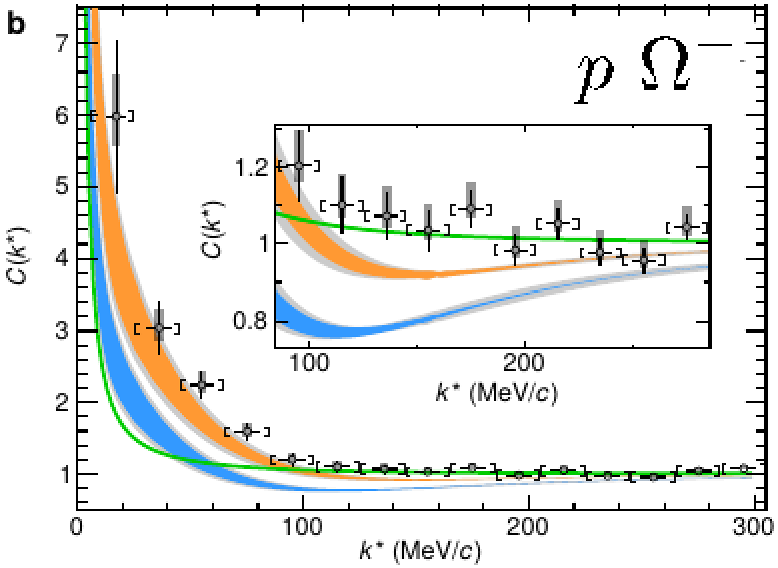
ALICE('20) Kamiya+('20)

STAR+ALICE suggests a $N\Omega$ dibaryon state

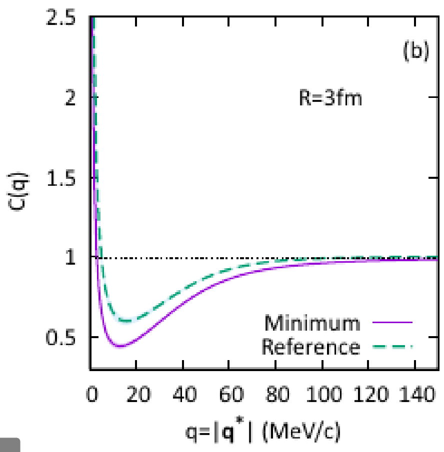
Morita+, PRC101('20)015201
[1908.0414] (Gaussian source)
Lattice BB pot. from HAL QCD
[Iritani+('19)]

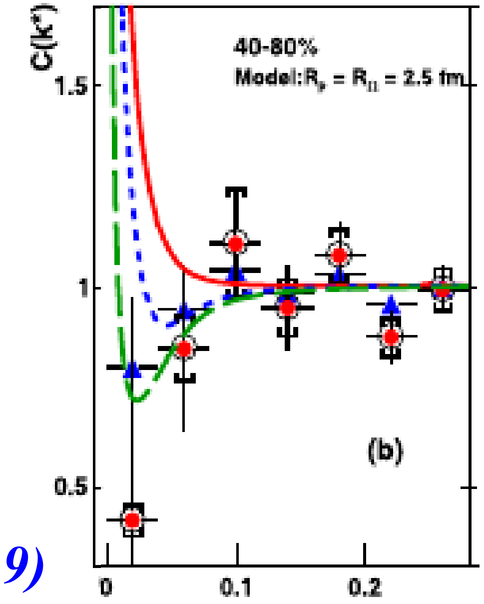






ALICE, Nature588('20)232 [2005.11495]





Reference: $V_{J=1}=V_{J=2}$

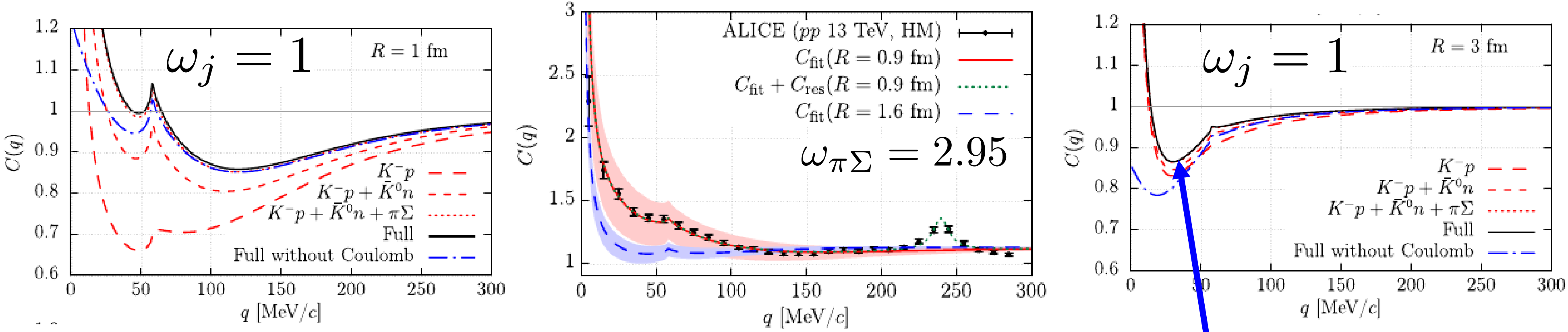
Minimum: $\varphi_{J=1}=0$

Dip from a bound state survives Coulomb.

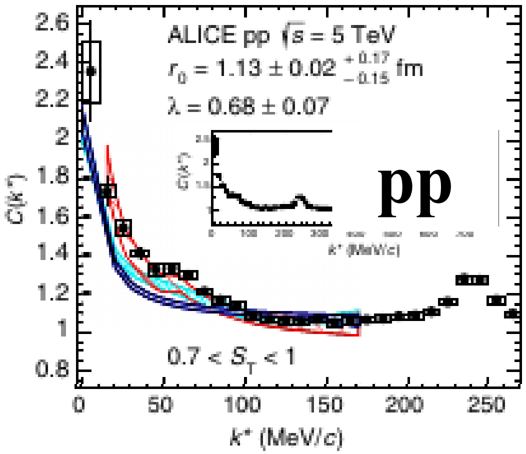
STAR, PLB790 ('19) 490 [1808.02511].

Source Size Dependence of $C(pK^-)$

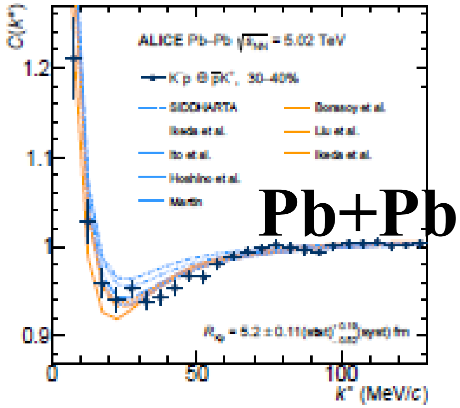
- Coupled-channel effects are suppressed when R is large, and “pure” pK^- wave function may be observed in HIC.



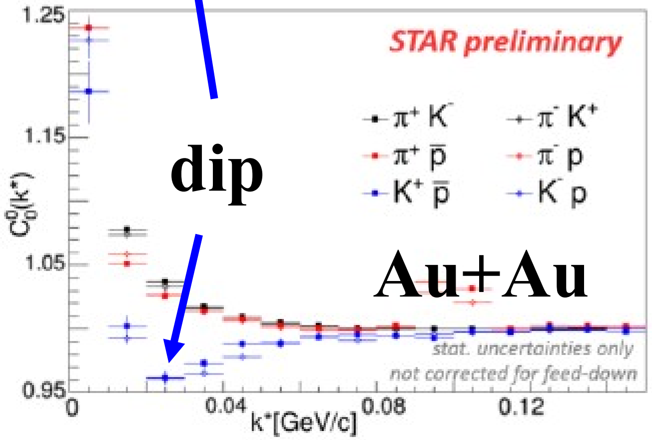
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124(‘20)132501.



S. Acharya+[ALICE], PRL124(‘20)092301



S. Acharya+[ALICE], 2105.05683



dip

Siejka+[STAR, preliminary], NPA982 (‘19)359.

STAR(prel.) & new ALICE data show a dip at small q.

Scattering length from K^-p correlation function

- **LL model fit (w/ Coulomb) to the correlation function data**

  S. Acharya+[ALICE], PLB 822 ('21) 136708 [2105.05683] ($\delta \sim +a_0 q$, HEP convention)

  $$a_0 = -0.91 \pm 0.03(\text{stat})^{+0.17}_{-0.03}(\text{syst}) + i[0.92 \pm 0.05(\text{stat})^{+0.12}_{-0.33}(\text{syst})]\ \text{fm}$$

- **Consistent with SIDDHARTA (kaonic atom) data, and errors are comparable to previous dedicated experiments.**

  M. Bassi et al. [SIDDHARTA], NPA 881 ('12) 88 [1201.4635]

  $$a_0 = -0.65 \pm 0.10 + i[0.81 \pm 0.15]\ \text{fm}$$

- **Femtoscopy reconfirmed $\bar{K}N$ bound state nature of Λ(1405)**

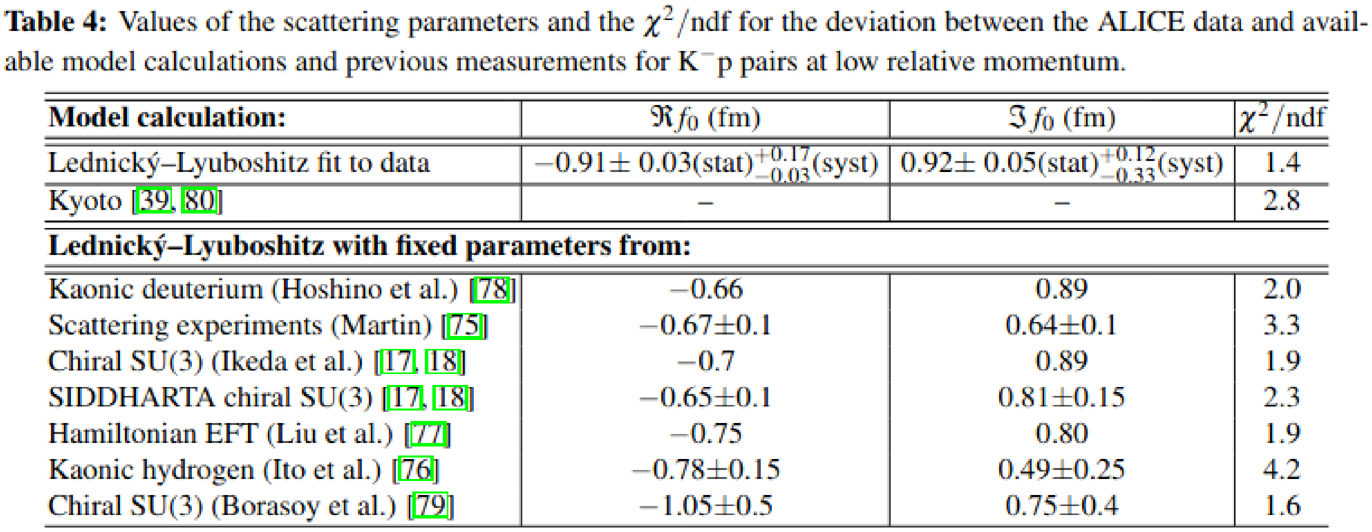
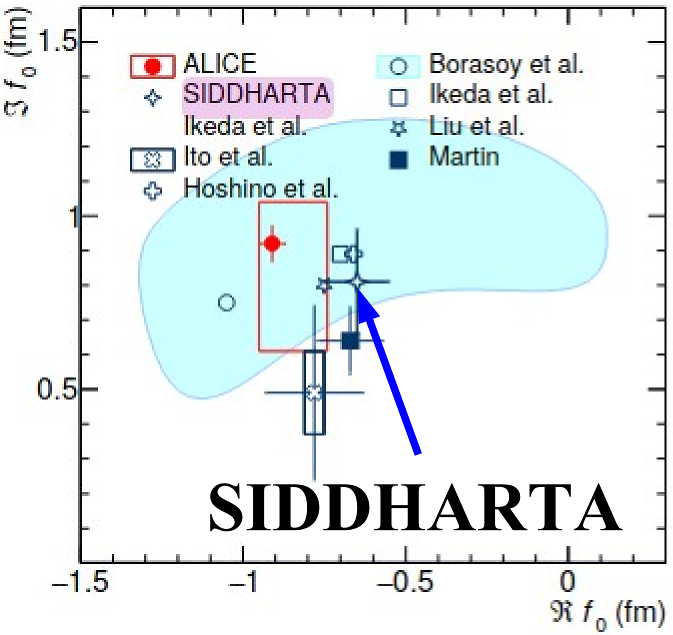


Table 4: Values of the scattering parameters and the χ^2/ndf for the deviation between the ALICE data and available model calculations and previous measurements for K^-p pairs at low relative momentum.

Model calculation:	$\Re f_0$ (fm)	$\Im f_0$ (fm)	χ^2/ndf
Lednický–Lyuboshitz fit to data	$-0.91 \pm 0.03(\text{stat})^{+0.17}_{-0.03}(\text{syst})$	$0.92 \pm 0.05(\text{stat})^{+0.12}_{-0.33}(\text{syst})$	1.4
Kyoto [39, 80]	–	–	2.8
Lednický–Lyuboshitz with fixed parameters from:			
Kaonic deuterium (Hoshino et al.) [78]	−0.66	0.89	2.0
Scattering experiments (Martin) [75]	−0.67±0.1	0.64±0.1	3.3
Chiral SU(3) (Ikeda et al.) [17, 18]	−0.7	0.89	1.9
SIDDHARTA chiral SU(3) [17, 18]	−0.65±0.1	0.81±0.15	2.3
Hamiltonian EFT (Liu et al.) [77]	−0.75	0.80	1.9
Kaonic hydrogen (Ito et al.) [76]	−0.78±0.15	0.49±0.25	4.2
Chiral SU(3) (Borasoy et al.) [79]	−1.05±0.5	0.75±0.4	1.6

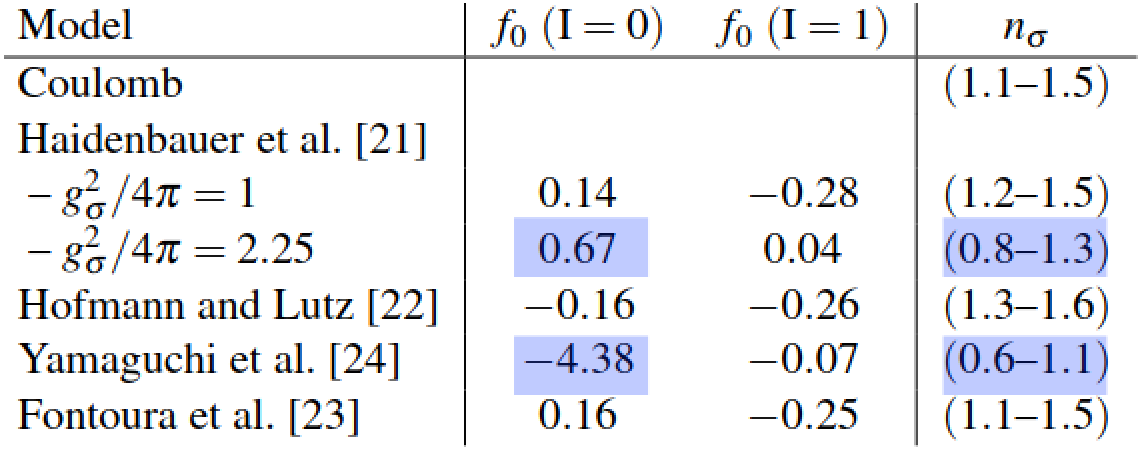


Marginal case: D⁻ p correlation function

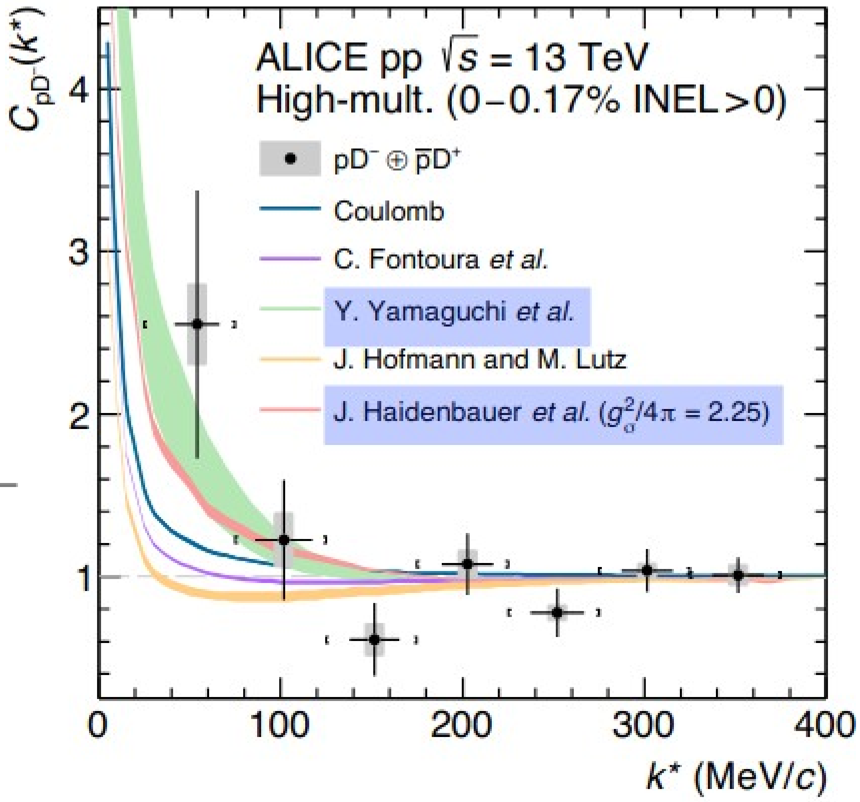
- **“First study of the two-body scattering involving charm hadrons”**

  Acharya+[ALICE] (2201.05352)

  - **D⁻ p corr. func. is measured.**
  - **Enhanced CF from Coulomb.**
  - **One range gaussian potential with strength fitted to the I=0 scattering length of the model → attractive potentials are favored**



Model	f_0 (I = 0)	f_0 (I = 1)	n_σ
Coulomb			(1.1–1.5)
Haidenbauer et al. [21]			
– $g_\sigma^2/4\pi = 1$	0.14	−0.28	(1.2–1.5)
– $g_\sigma^2/4\pi = 2.25$	0.67	0.04	(0.8–1.3)
Hofmann and Lutz [22]	−0.16	−0.26	(1.3–1.6)
Yamaguchi et al. [24]	−4.38	−0.07	(0.6–1.1)
Fontoura et al. [23]	0.16	−0.25	(1.1–1.5)



[21] Haidenbauer+(0704.3668) (weakly / mildly attractive (I=0))
[22] Hofmann, Lutz (hep-ph/0507071) (repulsive (I=0))
[23] Fontoura+(1208.4058) (weakly attractive (I=0))
[24] Yamaguchi, Ohkoda, Yasui, Hosaka (1105.0734) (att., w/ bound state (I=0))

To be bound or not to be bound

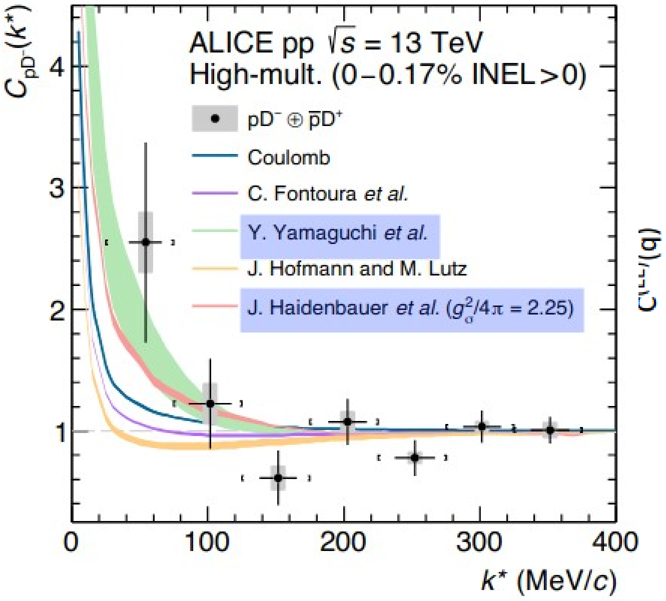
- **When there is a bound state, CF shows interesting dependence on the source size and relative momentum.**
- **D^- p corr. func. shows the behavior with a bound state, and the best fit parameter set (R, a_0) is in the bound region. (If bound, it is the first weakly decaying pentaquark state.)**

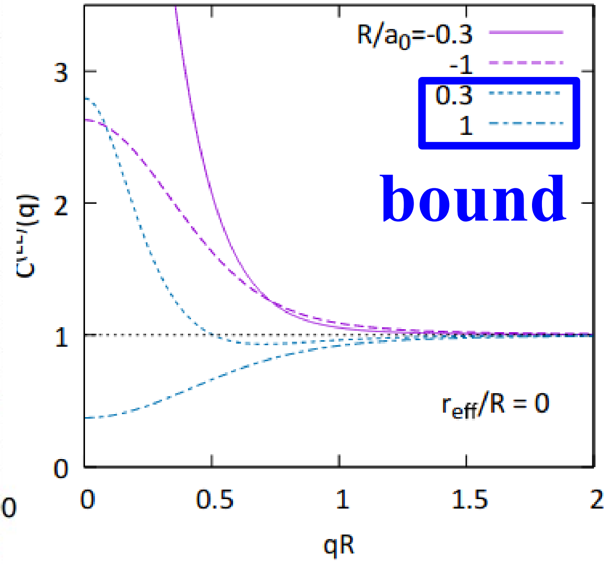
$$k \cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} k^2 + \mathcal{O}(k^3)$$

(Nuclear and atomic phys. convention.)

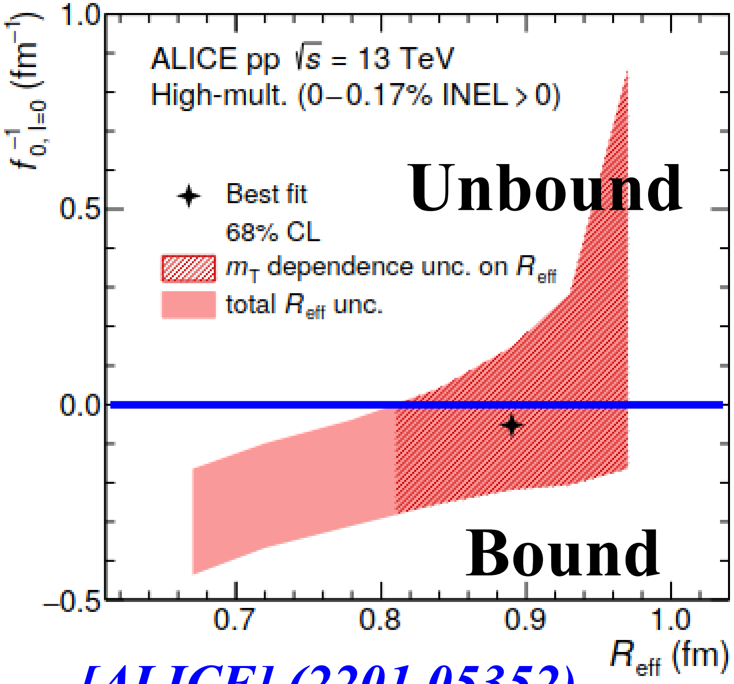
$$k \cot\delta = +\frac{1}{f_0} + \frac{1}{2} r_{\rm eff} k^2 + \mathcal{O}(k^3)$$

(High-E. phys. convention.)





Morita+(1908.05414)

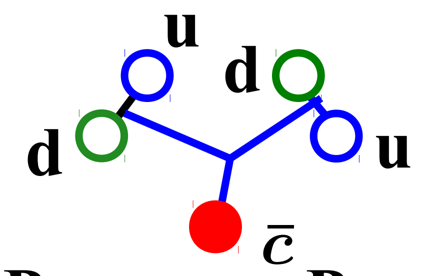


[ALICE] (2201.05352)

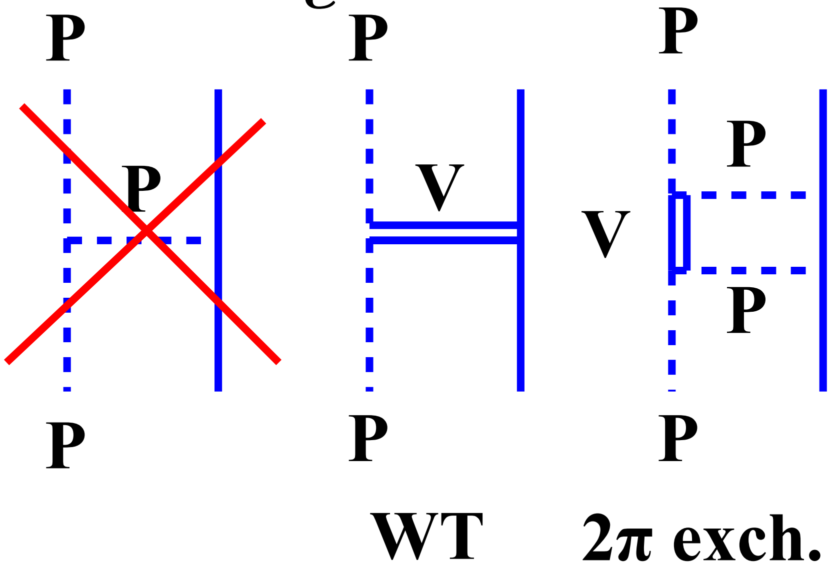
Is it interesting ? Yes !

- **Chiral symmetry → PS-B int. is dominated by vector exch.**
  - **Weinberg-Tomozawa interaction (vector coupling) appears in the leading order in the chiral quark model.**
  - **WT int. is generally repulsive in exotic channels.**
- **With heavy-quarks, PS and V meson masses becomes closer (heavy-quark sym.), then (two) PS meson exch. can be important. (higer-order in chiral perturbation)**
  Yasui, Sudoh (0906.1452)
- **Charmed pentaquark (Θ_c) may exist.**

  $$D^-(\bar{c}d)-p(uud) \to \bar{c}-ud-ud\ \text{(pentaquark)}$$



- **Attraction btw PS-B suggests importance of higher-order term(s) in chiral perturbation theory.**

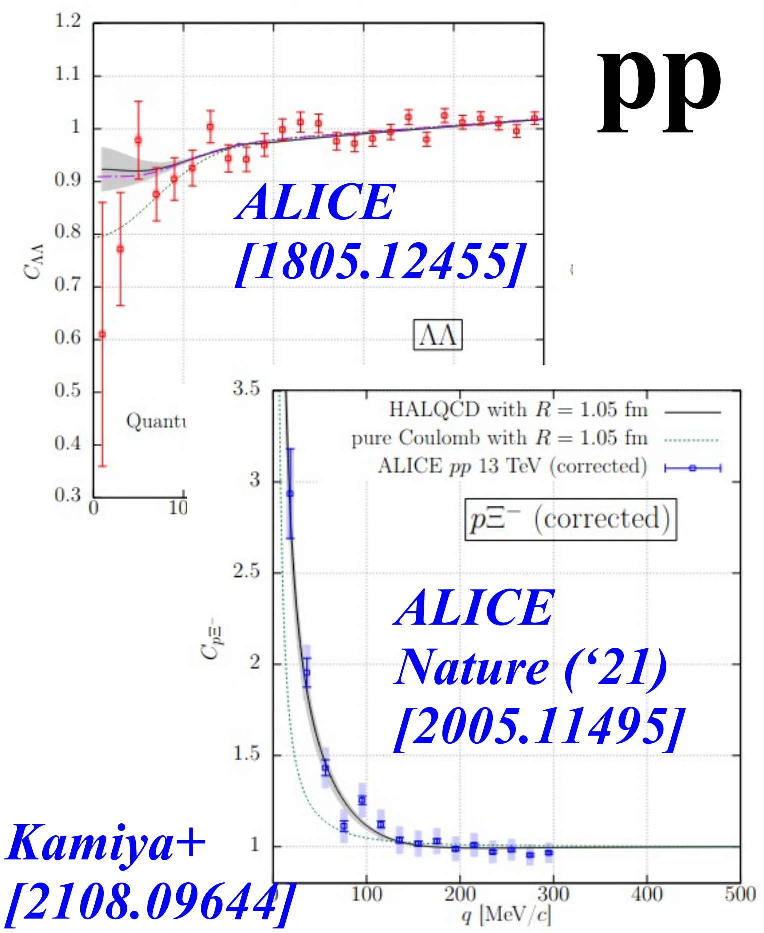


Femtoscopy may cause change of paradigm in hadron physics !

Case without a bound state

- **ΛΛ and NΞ seem to be unbound from lattice QCD calculation !**
  Sasaki+ [HAL], NPA998 (‘20)121737 [1912.08630]
- **Source size dependence of ΛΛ and pΞ⁻ correlation functions**
  → No dip or suppressed behavior in AA collisions.

pp

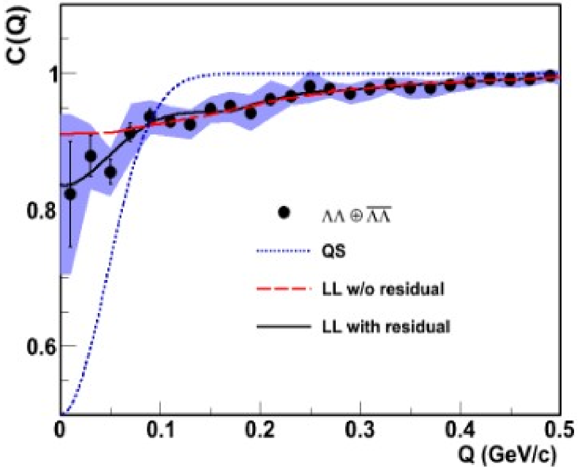


ALICE [1805.12455]

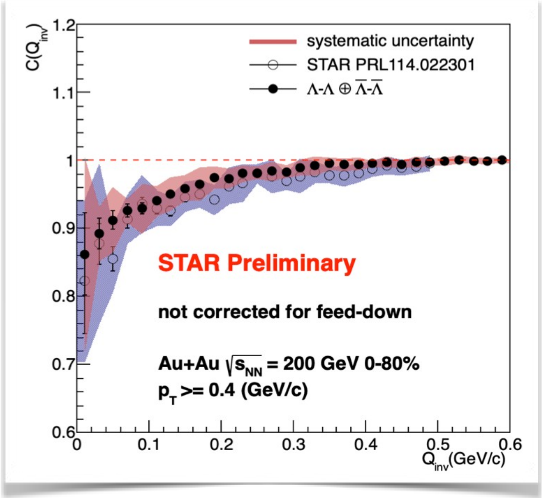
ALICE Nature (‘21) [2005.11495]

Kamiya+ [2108.09644]

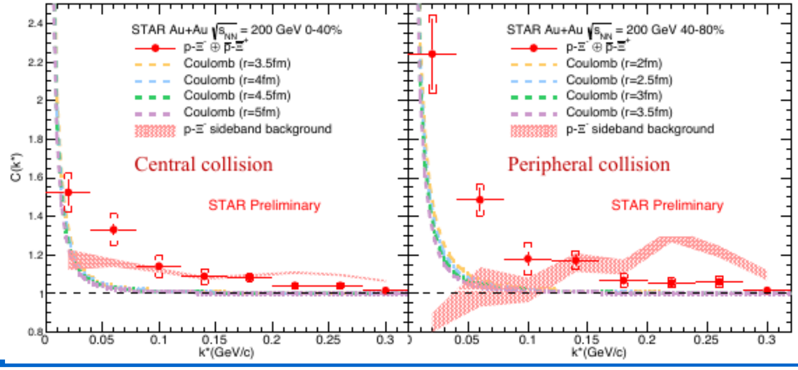
Au+Au



STAR, PRL(‘15) [1408.4360]



Moe Isshiki+ (STAR, prelim., 2109.10953).



K. Mi+ (STAR, preliminary), Au+Au, APS2021.

Source size dependence of C(q) seems to be useful to deduce the existence / non-existence of a bound state.

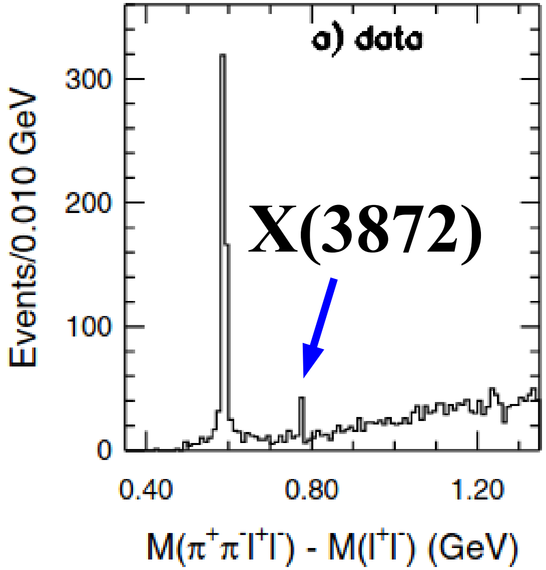
This provides a good motivation, but it is not a PROOF, I'm afraid.

For the confirmation, invariant mass peak with significance of 5-7 σ needs to be found.

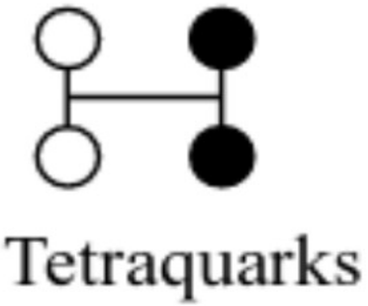
Recently observed / studied correlation functions, Homeworks, and perspectives

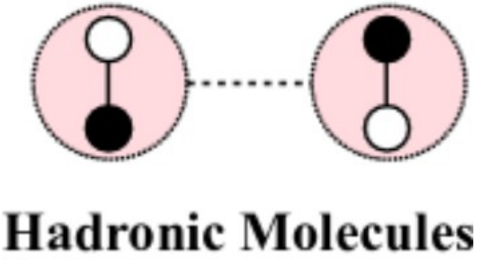
Exotic Hadrons including $c\bar{c}$ / cc / $\bar{c}\bar{c}$

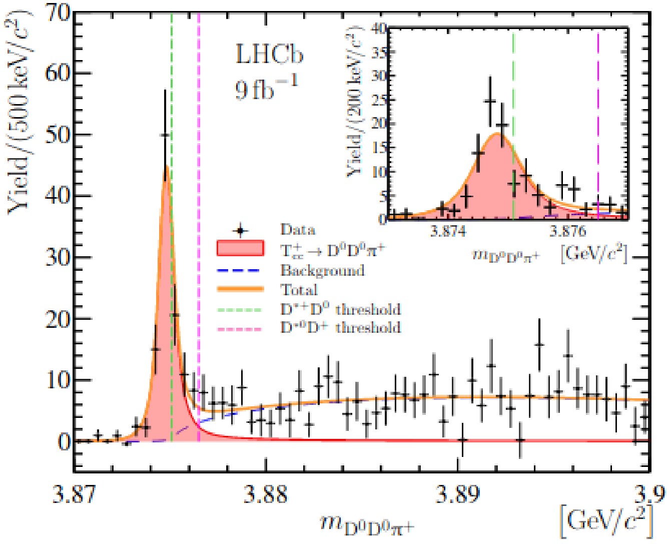
- **Main play ground of exotic hadron physics**
  - **X(3872)** ***Belle ('03)*** $c\bar{c}q\bar{q}$
  - **Many X,Y,Z states**
    Belle, CDF, BaBar, LHCb, CMS, BESIII, ...
    Beijing Spectrometer
  - **Charmed pentaquark Pc** ***LHCb ('15, '19)***
  - **Doubly charmed tetraquark state Tcc**
    LHCb ('21) $cc\bar{q}\bar{q}$
- **Structure of exotic hadrons**
  - **Compact multiquark states**
    → "good" [ud] diquark gains energy
  - **Hadronic molecules**
    → Many exotic states around thresholds
  - **Their mixture...**



S.K.Choi+[Belle], PRL91, 262001 ('03)





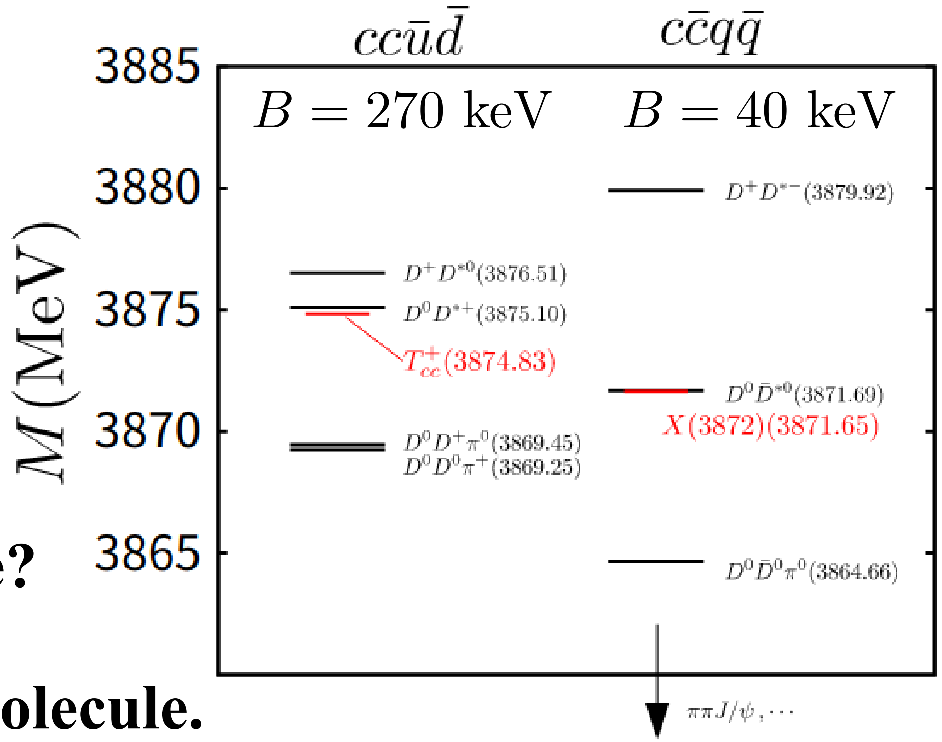


R. Aaji+ [LHCb], 2109.01038, 2109.01056

Compact Tetraquarks or Hadronic Molecules

- **Tcc = Compact Tetraquark ?**
  Good $[\bar{u}\bar{d}]$ diquark gains energy
  S. Zouzou+('86), ZPC30,457.

- **X(3872)**
  - $c\bar{c}$ **component ? production cross section** *Bignamini+ (0906.0882)*
  - **Large yield in Pb+Pb → Molecule?**
    Sirunyan+ [CMS] (2102.13048)
    c.f. Δr/Δp is similar in HIC and molecule.
    ExHIC ('11,'11,'17)



- **Hadronic Molecule Conditions**
  - **Appears around the threshold → OK**
  - **Have large size** $R \simeq 1/\sqrt{2\mu B}$ **→ Yield**
  - **Described by the *hh* interaction**

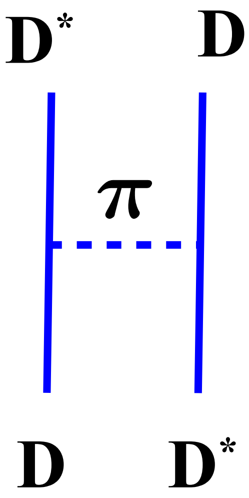
How can we access hh int. with charm ?
→ Femtoscopy

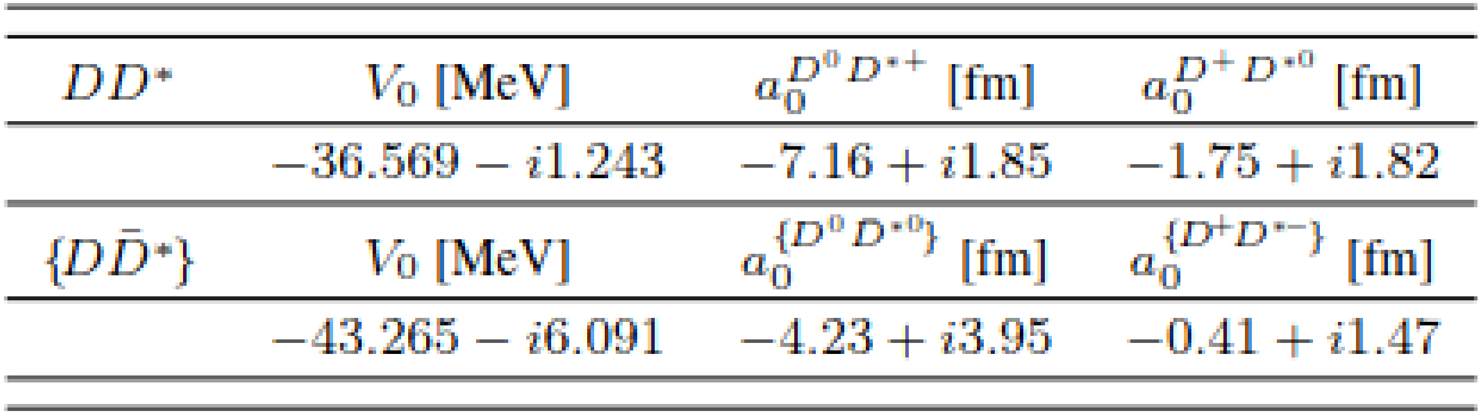
Femtoscopic study of charmed hadron int.

- DD^* and $D\bar{D}^*$ correlation functions. *Kamiya, Hyodo, AO (2203.13814)*
  - Related with Tcc and X(3872)
  - *ALICE run3 can measure the correlation functions.*
- Model interaction
  - Range = one pion exchange *Yasui, Sudoh (0906.1452)*
  - Strength is fitted to the pole mass.
  - Isospin dep.
    - I=0: One range gaussian, strength fitted to the mass
    - I=1: ignored

$$\{D^0\bar{D}^{*0}\} = (D^0\bar{D}^{*0} + \bar{D}^0 D^{*0})/\sqrt{2}\ (C=+1)$$

$$\{D^+ D^{*-}\} = (D^+ D^{*-} + D^- D^{*+})/\sqrt{2}\ (C=+1)$$

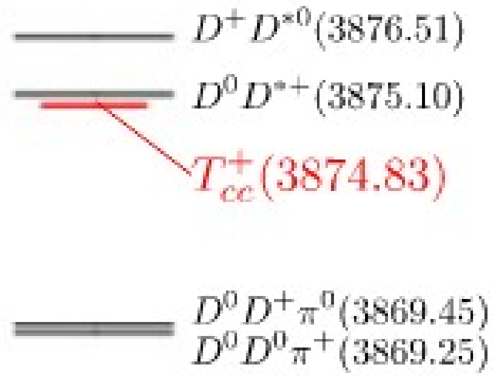


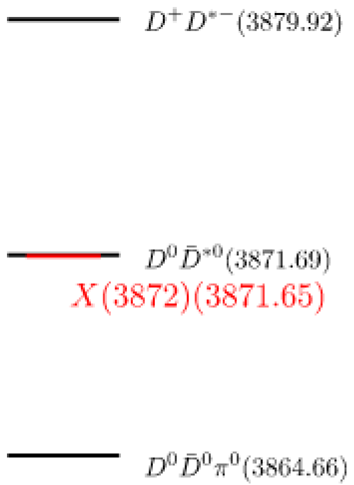


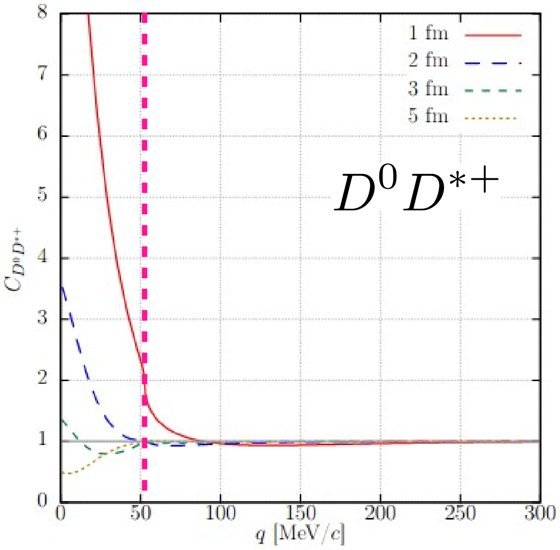
DD^*	V_0 [MeV]	$a_0^{D^0 D^{*+}}$ [fm]	$a_0^{D^+ D^{*0}}$ [fm]
	$-36.569 - i1.243$	$-7.16 + i1.85$	$-1.75 + i1.82$
$\{D\bar{D}^*\}$	V_0 [MeV]	$a_0^{\{D^0\bar{D}^{*0}\}}$ [fm]	$a_0^{\{D^+ D^{*-}\}}$ [fm]
	$-43.265 - i6.091$	$-4.23 + i3.95$	$-0.41 + i1.47$

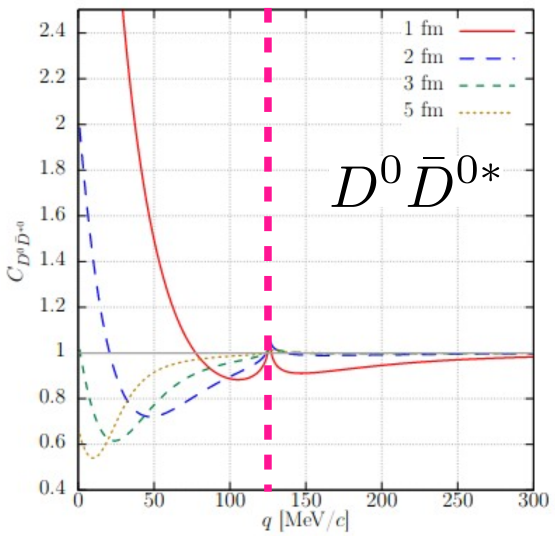
$D^0 D^{*+}$ and $D^+ \bar{D}^{*0}$ *Correlation Functions*

- **Features of C(q) with a bound state**
  - **Enhancement at small source, Dip at large source.**
  - **Modification of potential (Changing the range, V(I=1)=0 or ± V(I=0)/3) does not change C(q) significantly. (dominated by the pole)**
  - **Measurement in Run3 is awaited.**









Tcc and X(3872) structure

- **Hadronic molecule structure is assumed**
  → Eigenmomentum $k \simeq -i/a_0\,,\ a_0 \simeq R = 1/\sqrt{2\mu B}$
- **What happens when multiquark state mixes ?**
  → Deviation from weak binding relation (X=compositeness)
  Weinberg, Phys. Rev. 137, B672 (1965), Hyodo, Jido, Hosaka (1108.5524), Kunigawa, Hyodo (2112.00249)

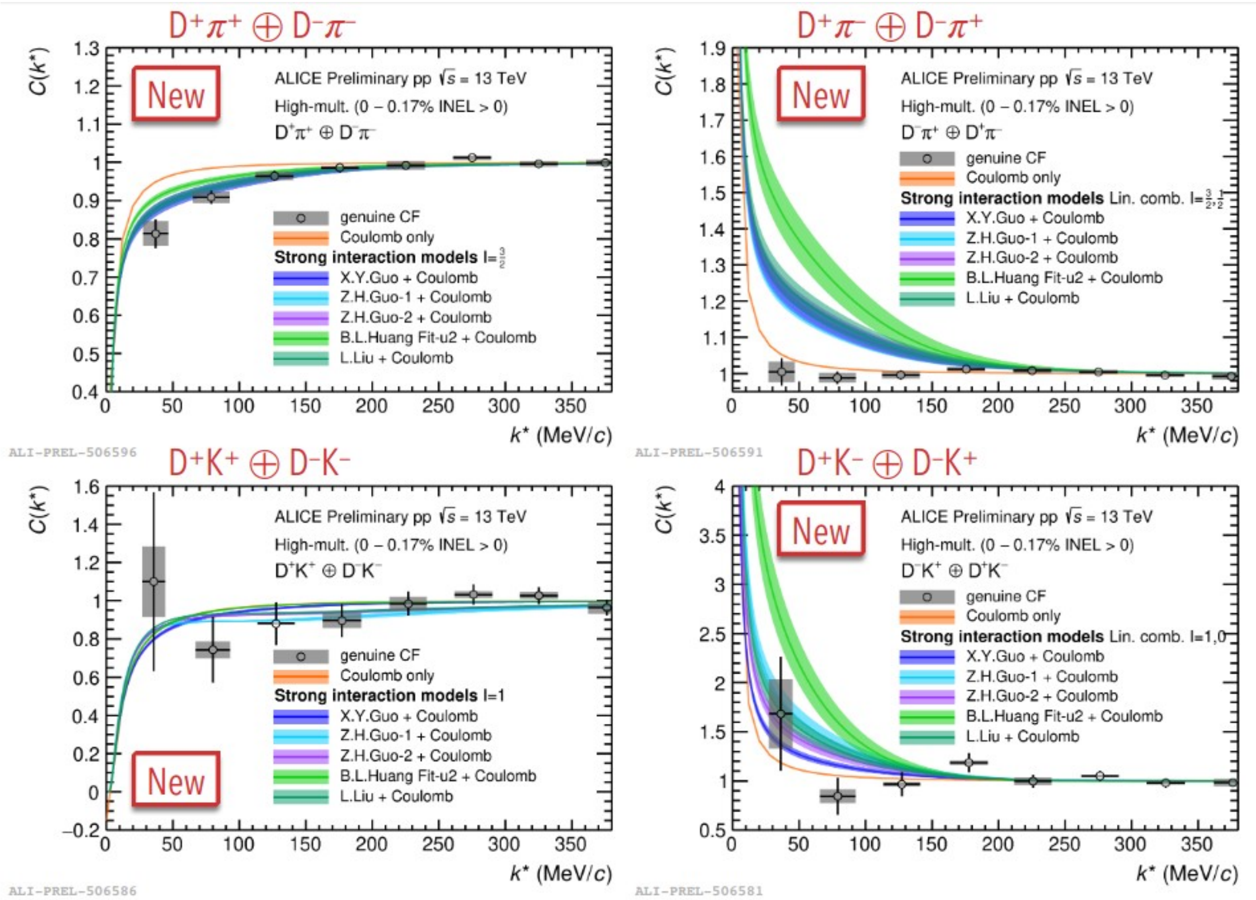
$$a_0 = R\left[\frac{2X}{1+X}\right] + \mathcal{O}(R_{\rm typ})$$

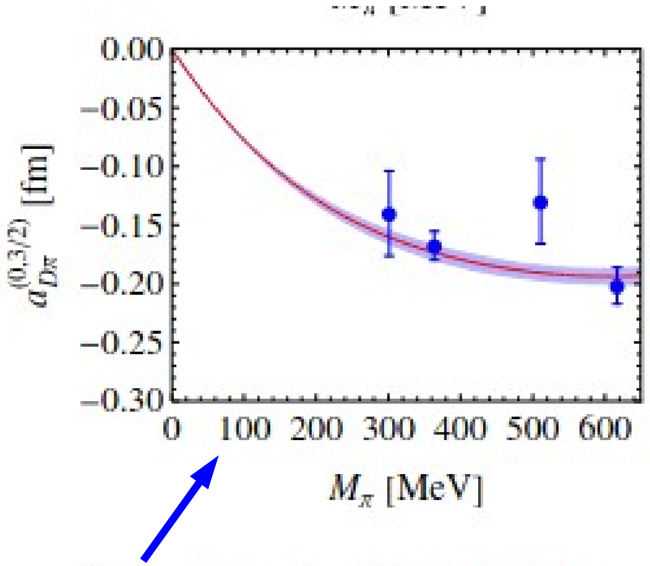
$$\left[R_{\rm typ} = \max(m_\pi^{-1}, r_{\rm eff}), R = 1/\sqrt{2\mu B}\right]$$

- **Hadronic molecule assumption → X=1**
  Pure multiquark state → X=0
- **Smaller scattering length in DD* may signal the *genuine* tetraquark nature of Tcc.**

Homework to Hadron Physics (1)

- **Present chiral models do not explain $D\pi$ and $D\bar{K}$ correlation.**
  - **Overestimate $C(D^+\pi^-)$ → Mystery ? Extrapolation to phys. mass ?**
    Leading order = Weinberg-Tomozawa (vector exch., repulsive)
    Further repulsive interaction ?
  - **Overestimate $C(D^+K^-)$ → Further repulsion or bound state ?**





L. Liu et al, Phys. Rev. D87 (2013) 014508
X.-Y. Guo et al, Phys. Rev. D 98 (2018) 014510
B.-L. Huang et al, Phys. Rev. D 105 (2022) 036016
Z.-H. Guo et al Eur. Phys. J. C 79 (2019) 13

Fabrizio Grosa@QM2022

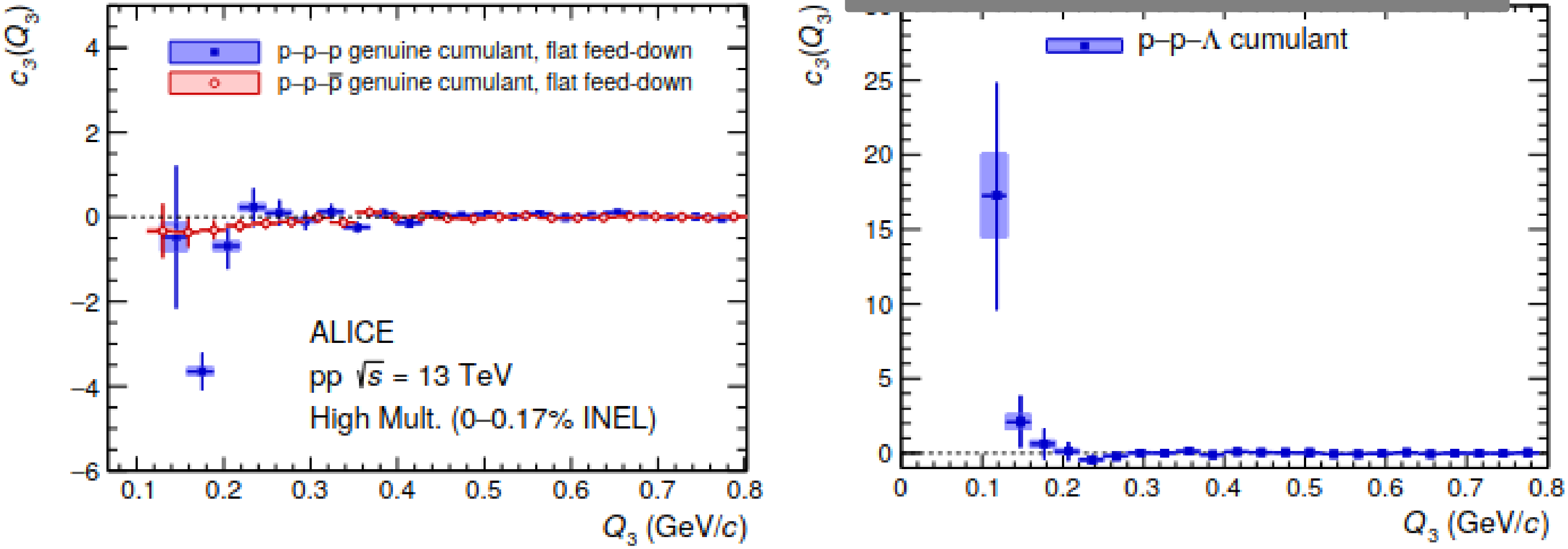
Homework to Hadron (Nuclear) Physics (2)

- **Three-body correlation function (ppp, ppΛ)**
  - **Cumulant**

$$c_3 = C_{123} - C_{12} - C_{23} - C_{31} + 2$$

  - **Can we extract three-baryon repulsion ?**
    (important to solve the hyperon puzzle)
    → One needs to solve continuum three-body w.f.
    with Coulmb potential.

Theoretical challenge



ALICE [2206.03344] (Raffaele Del Grande @QM2022)

Homework to Hadron (Nuclear) Physics (3)

- **Correlation function including vector mesons**
  - **Femtoscopy** *ALICE (PRL, 2105.05578)*

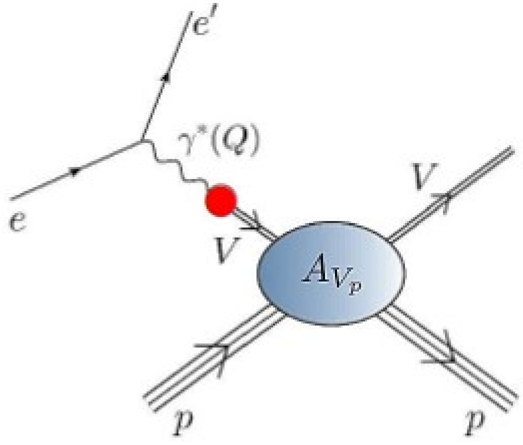
$$a_0(\phi p) = 0.85 + i0.16 \text{ fm}$$

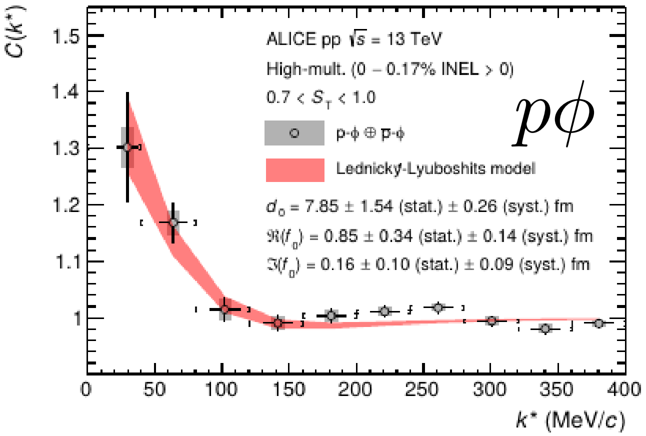
  - **Contradiction with the photo production ? scattering length is O(0.1 fm)**
    E.g. Strakovsky, Pentchev, Titov (2001.08851)

$$|a_0(\phi p)| = (0.063 \pm 0.010) \text{ fm}$$

  - **Smaller than lattice QCD result (J=3/2) ?**
    Lyu, Doi, Hatsuda, Ikeda (2205.10544)

$$a_0(\phi p, J = 3/2) = 1.43 \text{ fm}$$



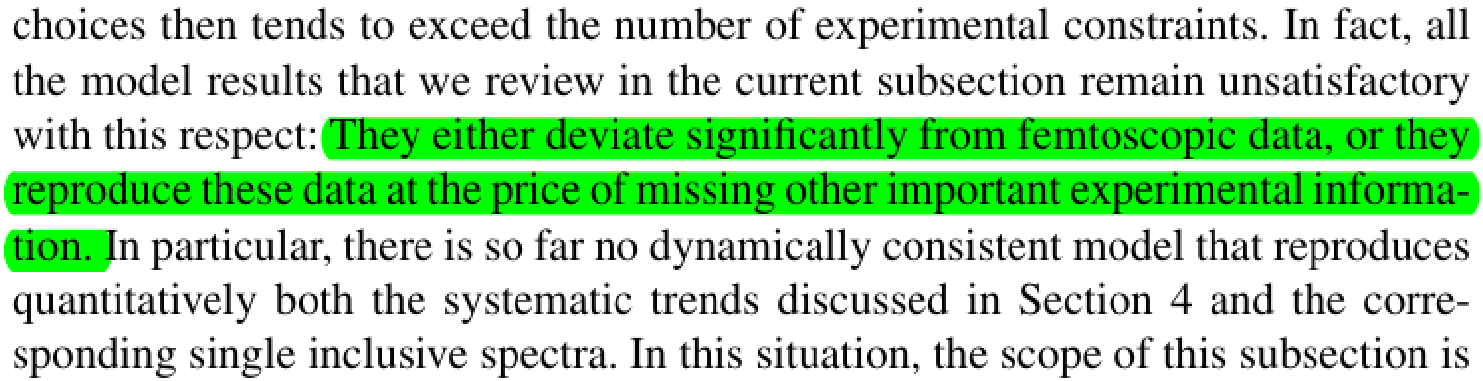


ALICE, 2105.05578

What's wrong ?

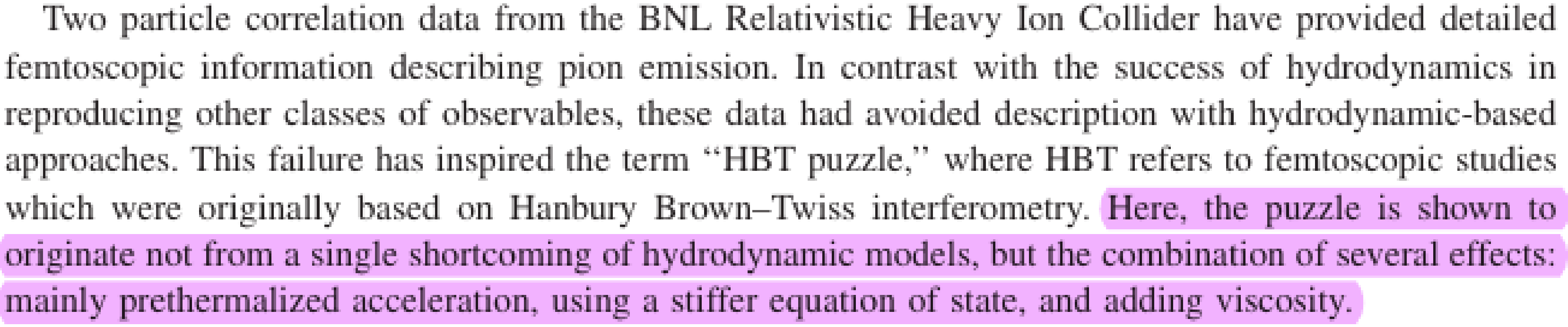
Toward dynamical source

- **Calculating HBT radius in dynamical models is not easy (HBT puzzle).**
  M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014];



choices then tends to exceed the number of experimental constraints. In fact, all the model results that we review in the current subsection remain unsatisfactory with this respect: They either deviate significantly from femtoscopic data, or they reproduce these data at the price of missing other important experimental information. In particular, there is so far no dynamically consistent model that reproduces quantitatively both the systematic trends discussed in Section 4 and the corresponding single inclusive spectra. In this situation, the scope of this subsection is

- **But carefully constructed hydrodynamic model may answer.**
  S. Pratt, PRL102('09)232301 [0811.3363].

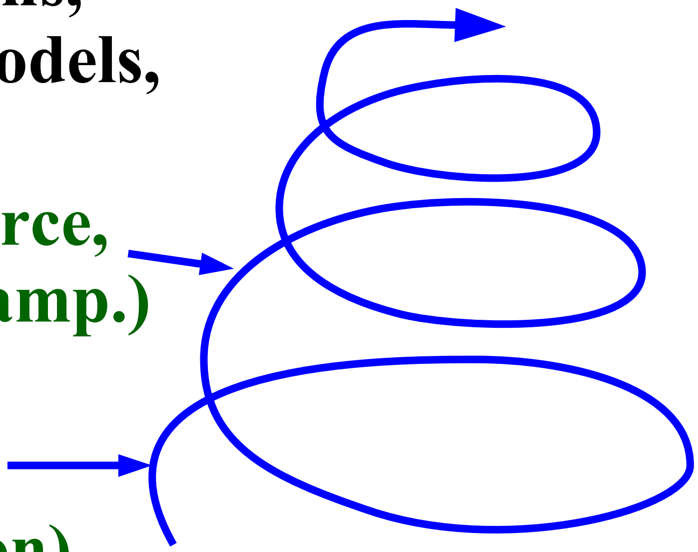


Two particle correlation data from the BNL Relativistic Heavy Ion Collider have provided detailed femtoscopic information describing pion emission. In contrast with the success of hydrodynamics in reproducing other classes of observables, these data had avoided description with hydrodynamic-based approaches. This failure has inspired the term "HBT puzzle," where HBT refers to femtoscopic studies which were originally based on Hanbury Brown–Twiss interferometry. Here, the puzzle is shown to originate not from a single shortcoming of hydrodynamic models, but the combination of several effects: mainly prethermalized acceleration, using a stiffer equation of state, and adding viscosity.

- **How about afterburner effects ?**

Summary

- **Femtoscopy (study using correlation functions) is useful to explore various hadron-hadron interactions (in the s-wave).**
    - **Correlation functions of many pairs have been measured in these 7 years, 2015-2022.**
    - **Some of the hh interactions have been constrained.**
    - **Source size dependence suggests the existence / non-existence of a bound state.**
    - **Recent data start to explore charm hadron interactions and three-body correlation functions.**
- **For more realistic estimate of hh interactions, we need reliable interactions and source models, together with more data.**



State-of-the-art

2nd round
(dynamical source,
C(q) → scatt. amp.)

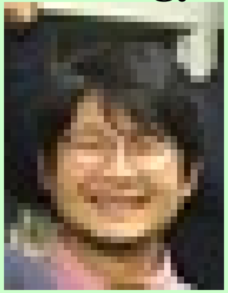
1st round
(simple source,
existing interaction)

Thank you for your attention !

I would like to thank my coauthors and colleagues on femtoscopic study of hadron-hadron interactions.

K. Morita

S. Gongyo



T. Hatsuda

T. Hyodo

K. Ogata

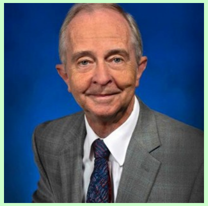
T. Fukui

F. Etminan

K.Sasaki **Y. Kamiya** **(ALICE)**

W. Weise

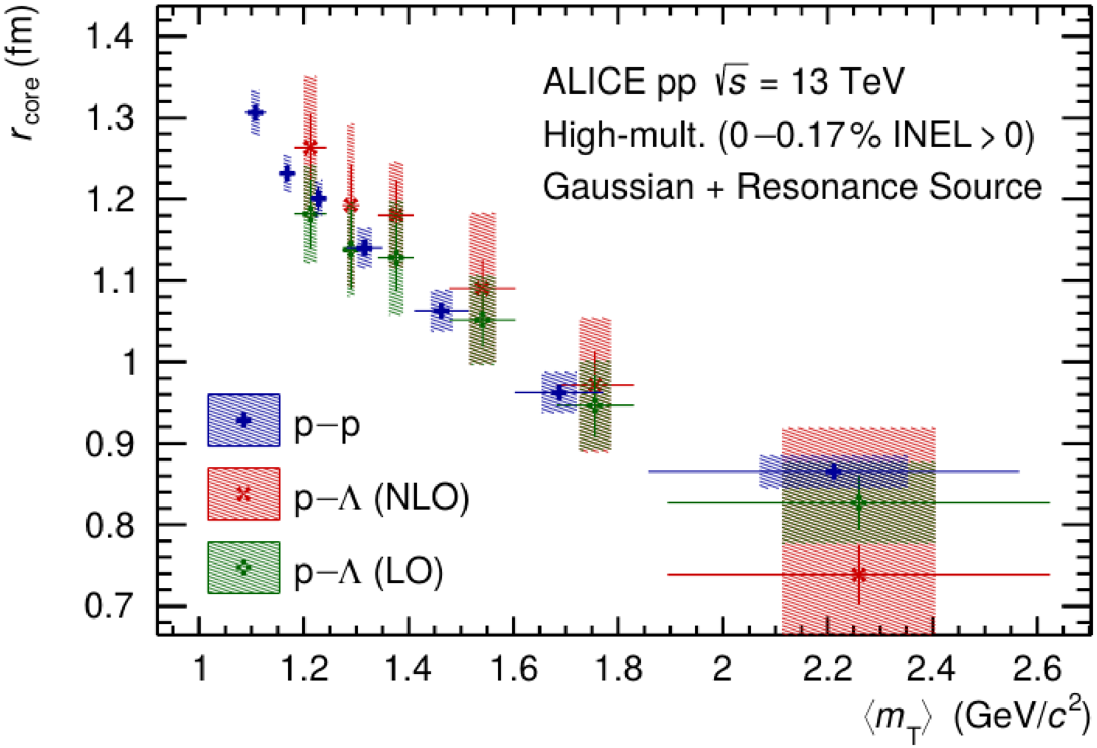
J. Haidenbauer



ALICE members (Laura, Valentina, Oton, Ramona, Emma, Raffaele, …) (Huan, Neha, Prof. Lednicky, Berndt, Jinhui, …),

We are sorry, but we use a Gaussian Source !

- **Calculating HBT radius in dynamical models is not easy**
  M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014]; S. Pratt, PRL102('09)232301 [0811.3363].
- **and a Gaussian source seems to work at the current precision of hh interaction studies.**
  S. Acharya+[ALICE], PLB811('20)135849.
  - **primary (universal ?)+ decay of short-lived resonances ~ eff. Gaussian**
  - **Flow and source geometry effects are seen in CF, but the uncertainty of hh int. is the largest.**

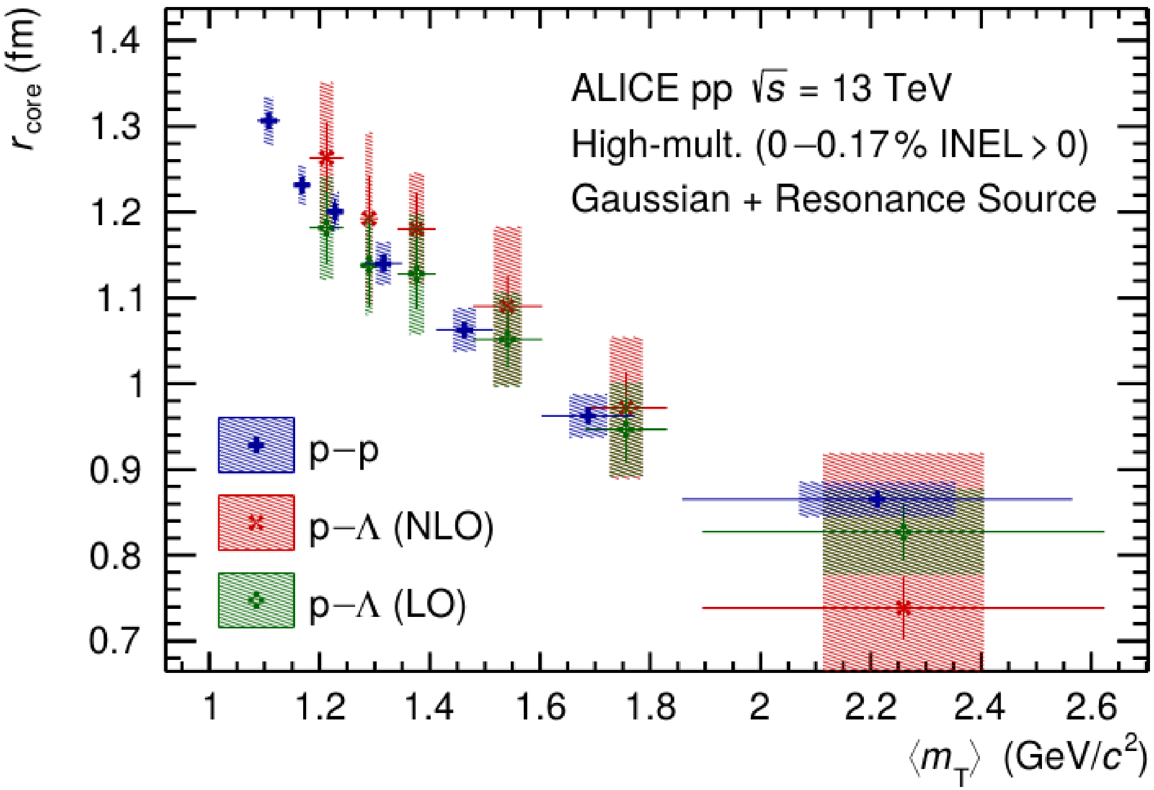


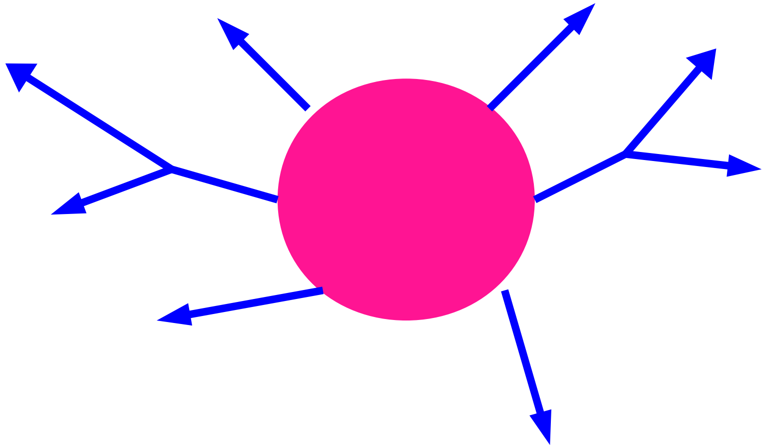
Source Size

- "Universal" source model

  S. Acharya+[ALICE], PLB811('20)135849

  - Fit pp and pΛ correlation function with Gaussian source (core) and decay of resonances.
  - Then the core size (r_{core}) seems to be universal as a function of m_T
  - Universal core + decay gives effective size
  - Good as the first guess.
  - (We need to allow 20-30 % uncertainty.)





S. Acharya+[ALICE], PLB811('20)135849

How can we measure the radius of a star ?

- **Two photon intensity correlation**
  Hanbury Brown & Twiss, Nature 10 (1956), 1047.
  - **Simultaneous two photon observation probability is enhanced from independent emission cases**
    → angular diameter of Sirius=0.0063 sec

Recent measurement (Wikipedia)
5.936±0.016 msec

A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN
Jodrell Bank Experimental Station, University of Manchester
AND
Dr. R. Q. TWISS
Services Electronics Research Laboratory, Baldock

NATURE November 10, 1956 Vol. 178

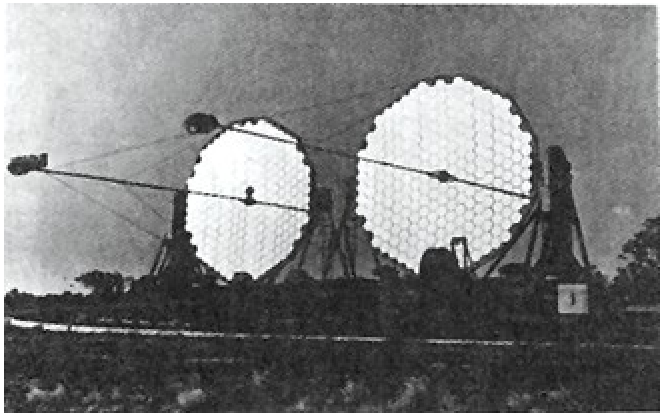
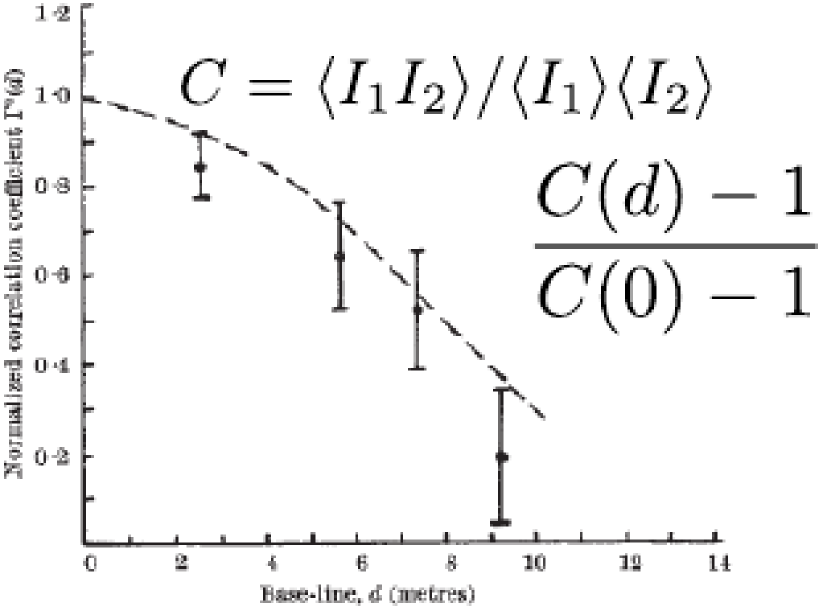


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1].

HBP telescope (from Goldhaber, ('91))



$$C = \langle I_1 I_2 \rangle / \langle I_1 \rangle \langle I_2 \rangle$$

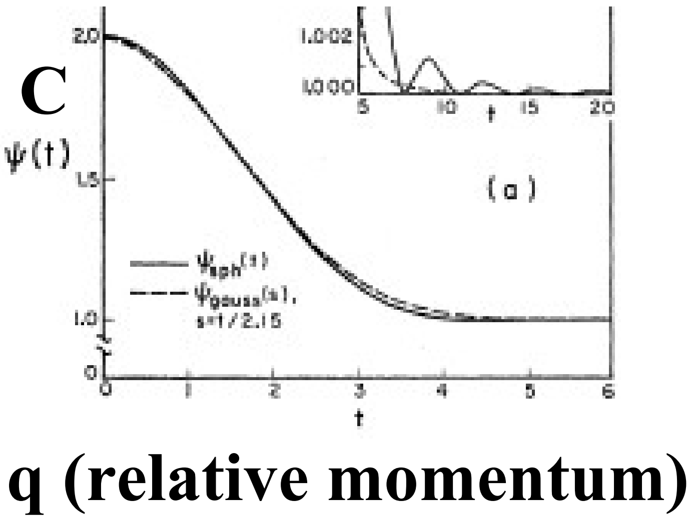
$$\frac{C(d) - 1}{C(0) - 1}$$

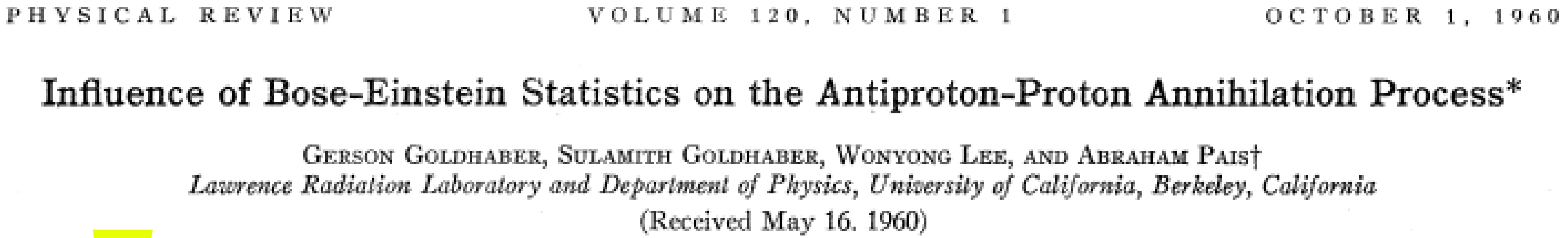
Fig. 2. Comparison between the values of the normalized correlation coefficient Γ²(d) observed from Sirius and the theoretical values for a star of angular diameter 0·0063″. The errors shown are the probable errors of the observations

HBT ('56)

How can we measure source size in nuclear reactions ?

- **Two pion interferometry**
  G. Goldhaber, S. Goldhaber, W. Lee, A. Pais, Phys. Rev. 120 (1960), 300
  - **Two pion emission probability is enhanced at small relative momenta**
    → Pion source size ~ 0.75 ℏ / μc

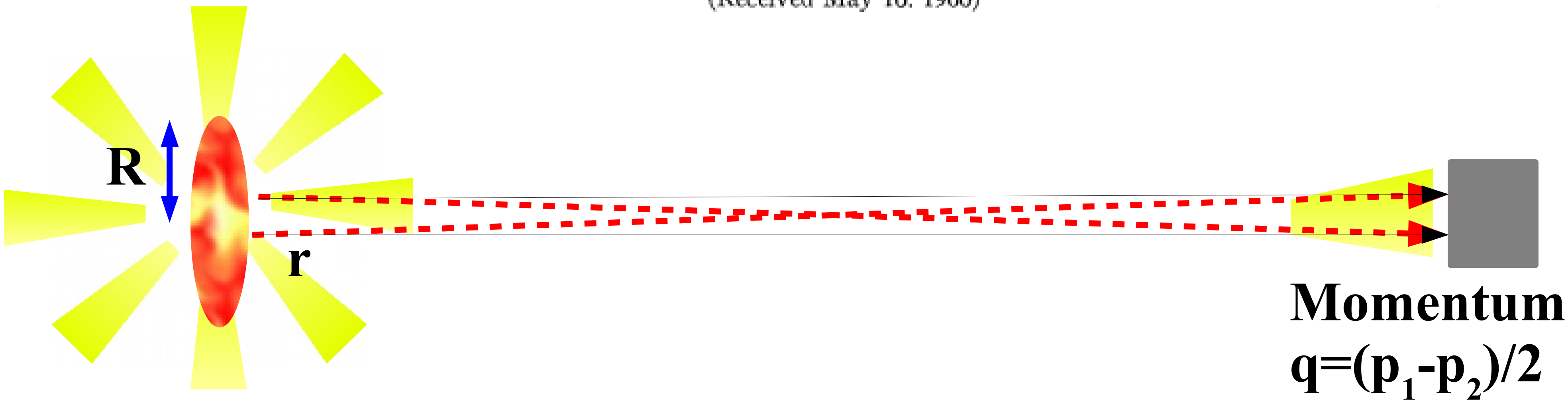




PHYSICAL REVIEW VOLUME 120, NUMBER 1 OCTOBER 1, 1960

Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process*

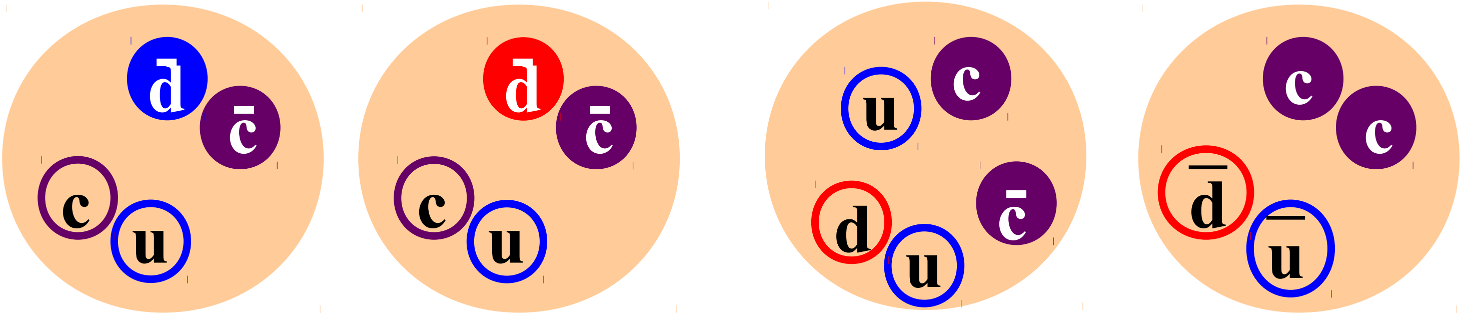
Gerson Goldhaber, Sulamith Goldhaber, Wonyong Lee, and Abraham Pais†
Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California
(Received May 16. 1960)



Momentum
$q=(p_1-p_2)/2$

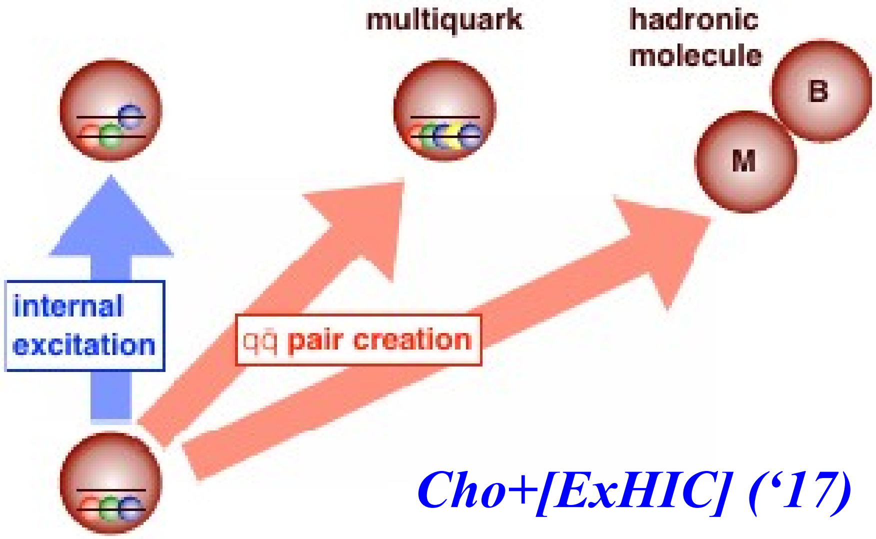
Exotic Hadrons

- Exotic hadrons (X, Y, Z, Pc, Tcc, ….)
  → Discovered/Proposed at LEPS, Belle, BaBar, BES, LHCb, ...



X(3872)　Z(4430)　Pc　T_{cc}

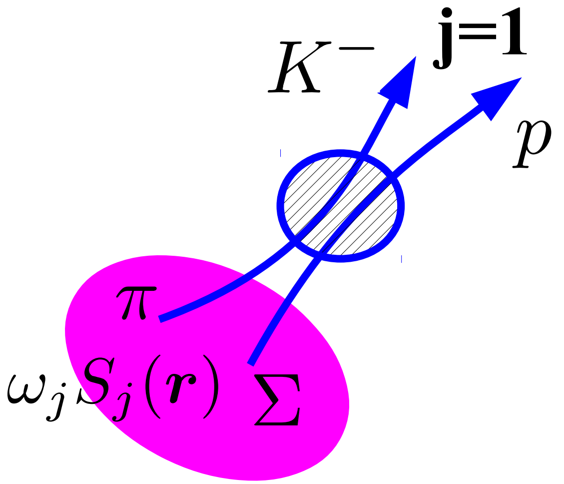
- Various pictures
  - Compact multiquark state (with di-quark component)
  - Hadronic molecule
  - (Triangle) Singularity
  - …



Cho+[ExHIC] ('17)

What is the structure of exotic hadrons ?
Can we access h-h interactions with heavy quarks ?

Correlation function with coupled-channel effects



- **KPLLL formula = CC Schrodinger eq.**
  under $\Psi^{(-)}$ boundary cond. + channel source
  Koonin('77), Pratt+('86), Lednicky-Lyuboshits-Lyuboshits ('98), Heidenbauer ('19), Kamiya, Hyodo, Morita, AO, Weise ('20).

$$\Psi^{(-)}(\boldsymbol{q};\boldsymbol{r}) = \left[\phi(\boldsymbol{q};\boldsymbol{r}) - \phi_0(q;r)\right]\delta_{1j} + \psi^{(-)}(q;r)$$

$$\psi_j^{(-)}(q;r) \to \frac{1}{2iq_j}\left[\frac{u_j^{(+)}(q_jr)}{r}\delta_{1j} - A_j(q)\frac{u_j^{(-)}(q_jr)}{r}\right]$$

$$\boxed{C(q) = \int d\boldsymbol{r} S_1(r)\left[|\phi(\boldsymbol{q};\boldsymbol{r})|^2 - |\phi_0(q;r)|^2\right] + \sum_j \int d\boldsymbol{r}\omega_j S_j(r)|\psi_j^{(-)}(q;r)|^2}$$

- **No Coulomb** $\phi(\boldsymbol{q};\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}}, \phi_0(q;r) = j_0(qr), u_j^{(\pm)}(qr) = e^{\pm iqr},$

$$A_j(q) = \sqrt{(\mu_j q_j)/(\mu_1 q_1)}S_{1j}^\dagger(q_1)\ (S_{ji} = i \to j \text{ S-matrix})$$

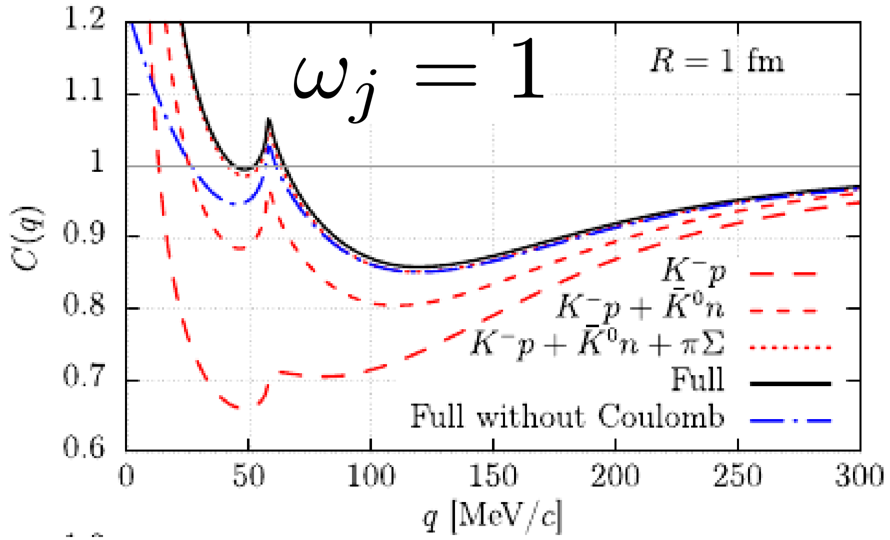
- **With Coulomb**

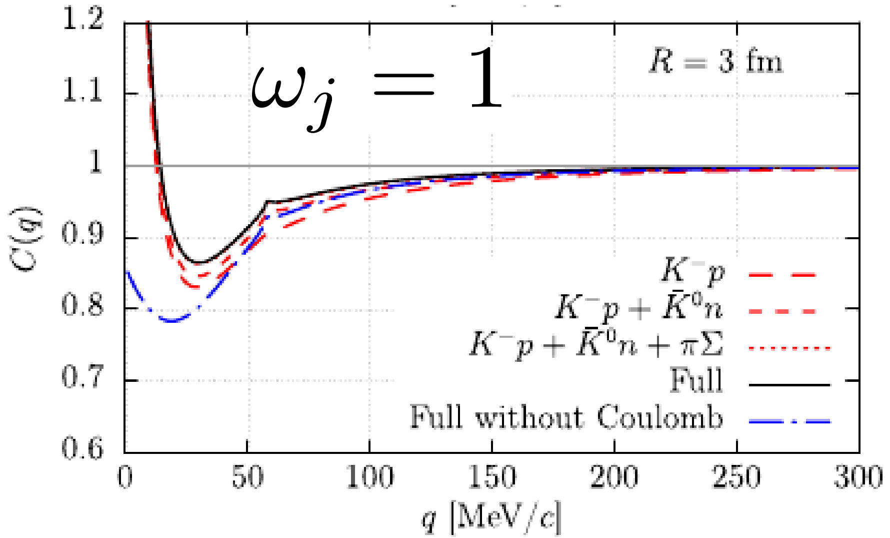
$$\phi(\boldsymbol{q};\boldsymbol{r}) = \text{Full Coulomb w.f.}, \phi_0(q;r) = \text{s-wave Coulomb w.f.},$$

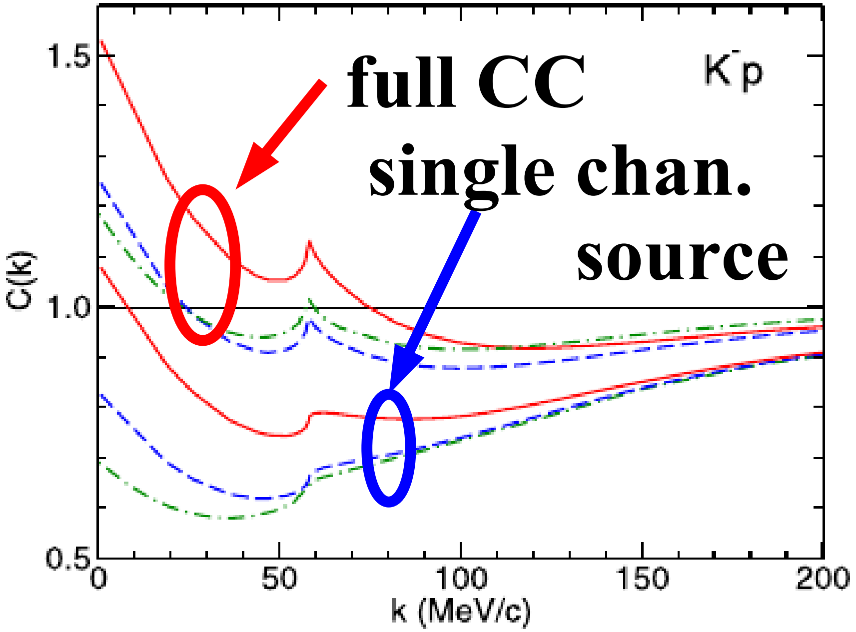
$$u_j^{(\pm)}(qr) = \pm e^{\mp i\sigma_j}\left[iF(qr) \pm G(qr)\right](F,G = \text{regular (irregular) Coulomb fn.})$$

Coupled-channel effects in K^-p correlation function

- **K^-p – $\bar{K}^0 n$ ($\bar{K}N$-$\pi\Sigma$) coupling is decisive (visible) at R=1 fm.**
- **Source effects of $\bar{K}^0 n$ and $\pi\Sigma$ are not large at R=3 fm.**
  (Solving CC Eq. is still important.)







full CC

single chan.

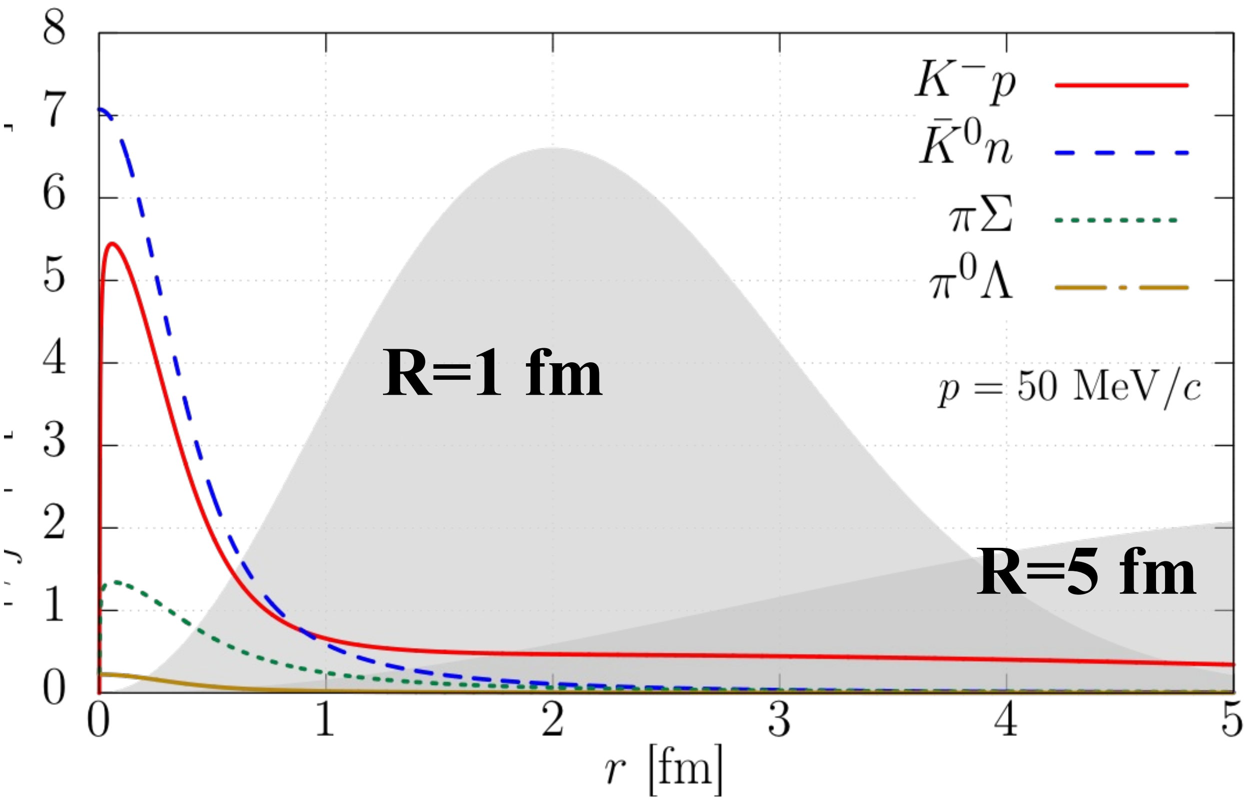
source

J. Haidenbauer, NPA981('19)1.
(Julich, NLO30, w/ CC effects, w/o Coulomb)

Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.

Discriminating Coupled-Channel Effects

- Source size dependence again !
  - Unmeasured coupled-channel wave functions disappear soon.
    → CFs with large source is dominated
    by the measured channel wave function !
  - Scattering parameters from CFs with large source
    Coupled-channel effects from CFs with small source.



R=1 fm

R=5 fm

w.f. Kamiya+,arXiv:1911.01041v1

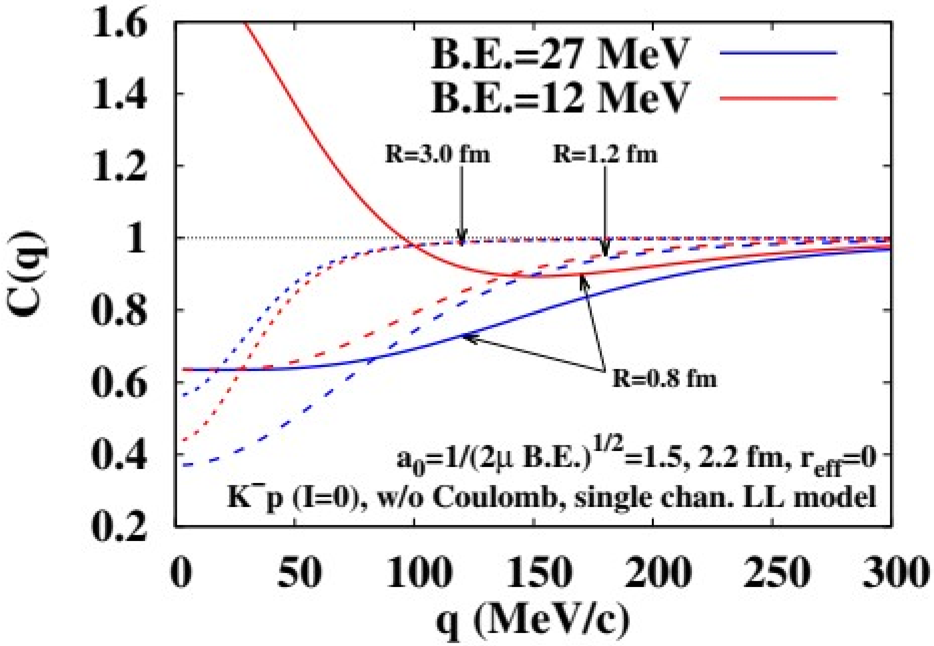
Another example: binding energy dependence of C(q)

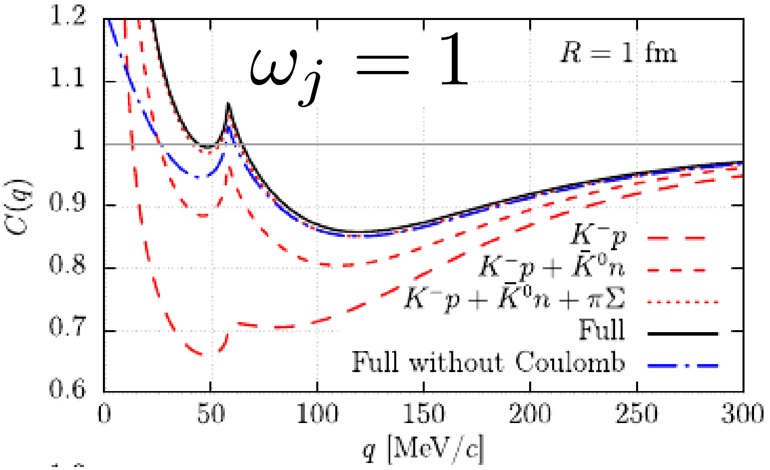
- **A frequently asked question**

  Can we guess the binding energy from C(q) ?

- **Λ(1405) ~ $\bar{\text{K}}$N (I=0) bound state**
  - **M=1405 MeV (B.E.=27 MeV) or 1420 MeV (B.E.=12 MeV) ?**
  - **A toy model: zero r_{eff}, single channel LL model w/o Coulomb, I=0**

    $$a_0 = \hbar / \sqrt{2\mu \times \text{B.E.}}$$
  - **C(q) depends on B.E. at small R. (Do not be serious!)**



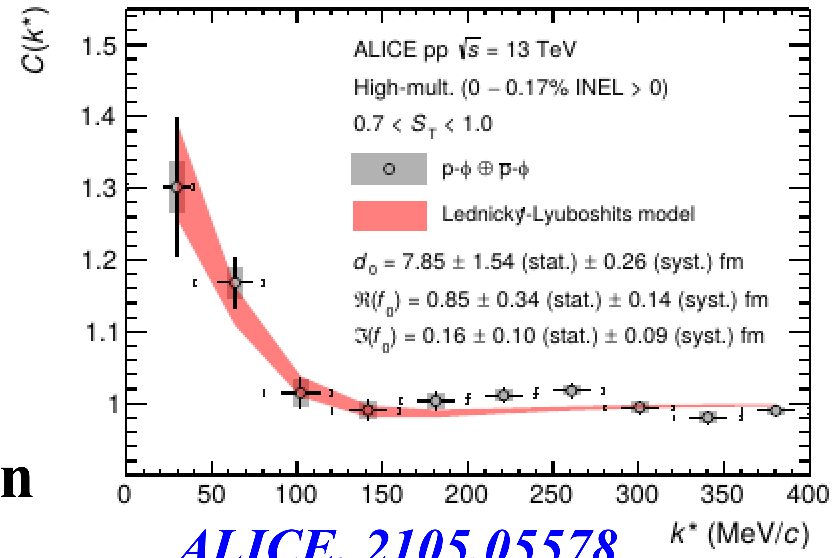


More serious calculation
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.

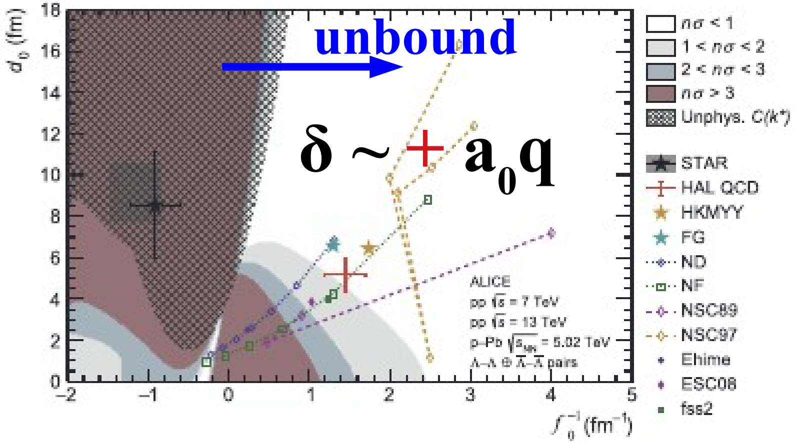
Lednicky-Lyuboshits formula application examples

- pϕ correlation function
  - Re(a_0)=0.85±0.34 (stat.)±0.14(syst.) fm (q cot δ ~ 1/a_0, high-energy physics convention)
- ΛΛ correlation function
  - Quantum statistics + strong interaction
  - Weakly attractive potential

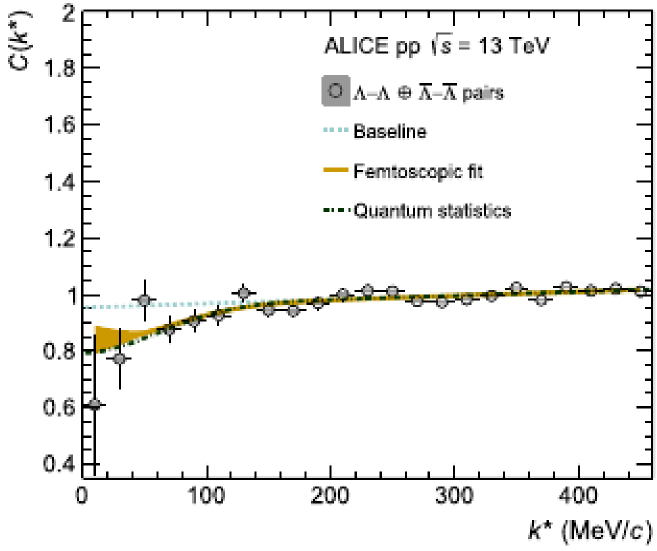
$$C(\boldsymbol{q}) = 1 - \frac{\lambda}{2} e^{-4q^2R^2} + \frac{\lambda}{2}\int d\boldsymbol{r}\, S(r)\left\{|\varphi_0(r)|^2 - |j_0(qr)|^2\right\}$$



ALICE, 2105.05578

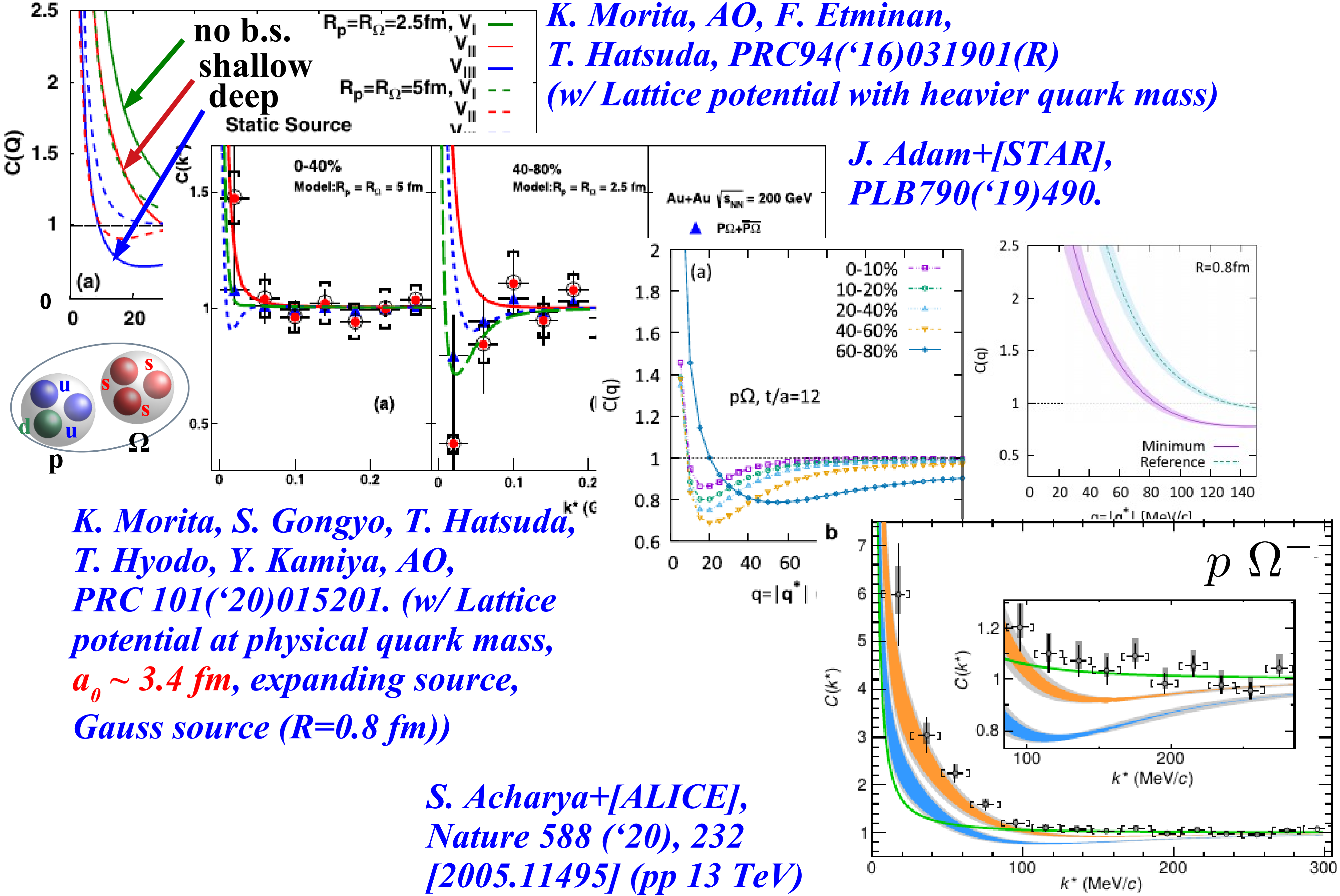


S. Acharya+[ALICE], PLB797('19)134822



ALICE, PLB797 ('19) 134822 [1905.07209]

$p\Omega^-$ correlation function



K. Morita, AO, F. Etminan,
T. Hatsuda, PRC94('16)031901(R)
(w/ Lattice potential with heavier quark mass)

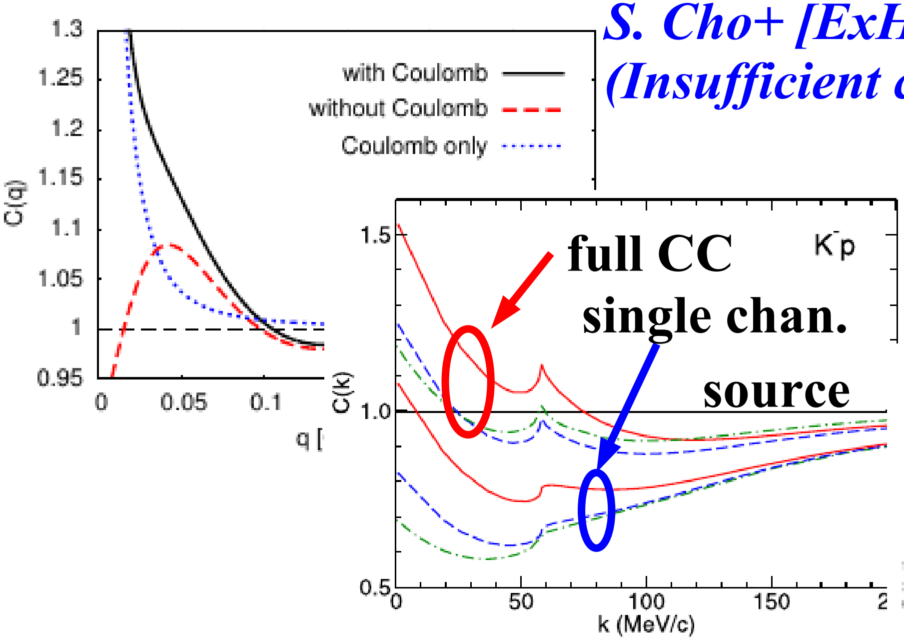
J. Adam+[STAR],
PLB790('19)490.

K. Morita, S. Gongyo, T. Hatsuda,
T. Hyodo, Y. Kamiya, AO,
PRC 101('20)015201. (w/ Lattice
potential at physical quark mass,
a_0 ~ 3.4 fm, expanding source,
Gauss source (R=0.8 fm))

S. Acharya+[ALICE],
Nature 588 ('20), 232
[2005.11495] (pp 13 TeV)

pK⁻ correlation

S. Cho+ [ExHIC], PPNP95('17)279.
(Insufficient coupled-channel effects)



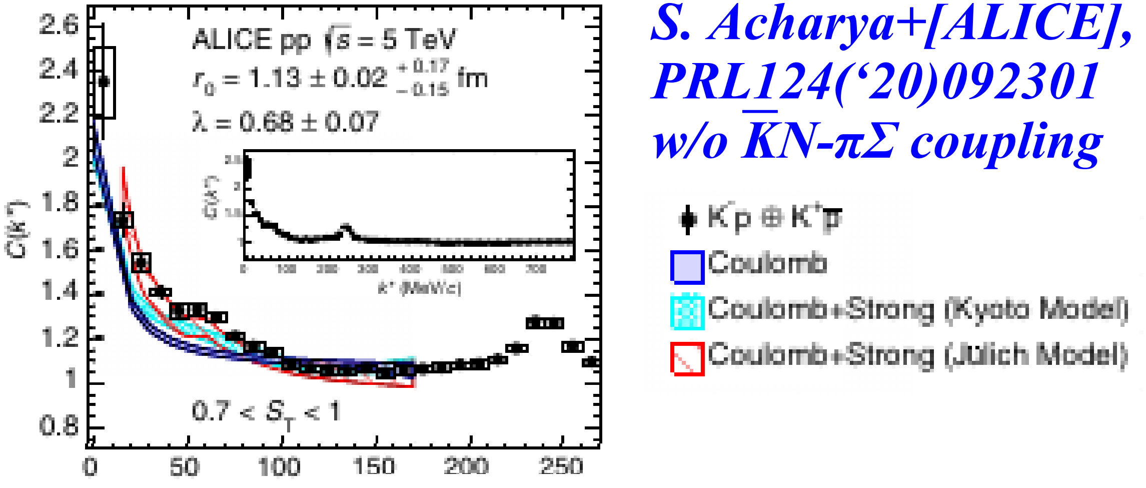
full CC

single chan.

source

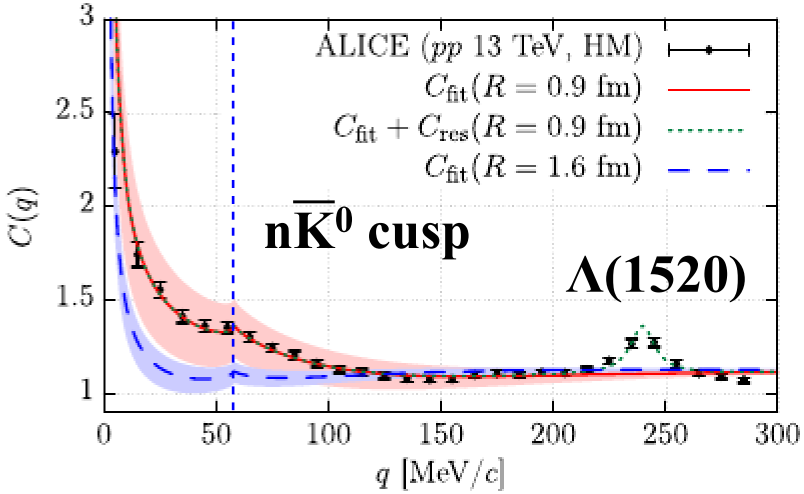
J. Haidenbauer, NPA981('19)1.
(Julich, NLO30, w/ CC effects,
w/o Coulomb)

S. Acharya+[ALICE],
PRL124('20)092301
w/o $\overline{K}N$-πΣ coupling



CF with small source is explained ! Source size dep. may clarify bound state nature of Λ(1405).

Y. Kamiya, T. Hyodo, K. Morita, AO,
W. Weise, PRL124('20)132501
[1911.01041] (Chiral SU(3) dynamics).



$n\overline{K}^0$ cusp

Λ(1520)

Parameters in correlation function data

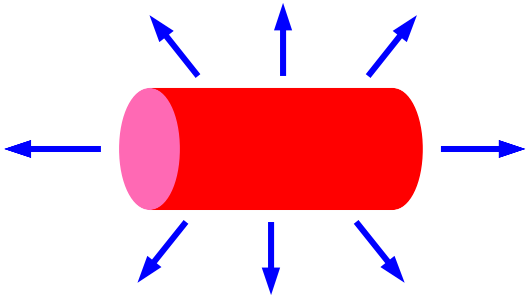
- **Actual data contains non-primary and misidentified particles, particles from jets, and the source size and weights are not fully known.**

$$C_{\rm exp}(q;R,\lambda,N,\omega) = N(q)\left[1+\lambda(C_{\rm theory}(q;R,\omega)-1)\right]$$

- **R = Source size (length of homogeneity)**
  - **Guess based on systematics (m_T scaling) or dynamical models.**
  - **Flow and source shape are also important for identical pairs.**
- **λ = chaoticity parameter → pair purity**
  - **λ = ("primary" pair) / (accepted pair)**
  - **In the best case of ΛΛ → $\lambda = [(\text{primary } \Lambda) / (\text{primary } \Lambda + \Sigma^0)]^2$**
- **N(q)=a + bq, Normalization + Jet effects**
- **ω_j = Source weight**
  - **$\omega_j \propto$ product of particle number at around the emission time.**
  - **Statistical model, blast-wave, MC simulation, …**

Semi-Realistic Source Function

- **"Cylindrical shape + blast wave" model**
  S. Chapman, P. Scotto, U. Heinz, Heavy Ion Phys.1,1('95);
  K. Morita, T. Furumoto, AO, PRC91, 024916 ('15);
  K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC101('20), 015201.



$$S_{\rm cyl}(x,\boldsymbol{k}) = \frac{2J+1}{(2\pi)^3} m_T \cosh(y-\eta_s) \underline{n_F(u\cdot k/T)}\, \underline{e^{-\boldsymbol{r}_T^2/2R_T^2}}\, f_\tau(\tau)$$

Fermi dist.

(Gaussian in r_T)

$$f_\tau(\tau) = e^{-(\tau-\tau_0)^2/2(\Delta\tau)^2}/\sqrt{2\pi(\Delta\tau)^2} \to \delta(\tau-\tau_0)$$

$$u^\mu = (\underline{\cosh y_T}\, \underline{\cosh\eta_s}, \underline{\sinh y_T} \cos\phi, \underline{\sinh y_T} \sin\phi, \underline{\cosh y_T}\, \underline{\sinh\eta_s})$$

$$y_T = \alpha\rho^\beta \ \ (\rho = r_T/R_T)$$

Bjorken+radial flow

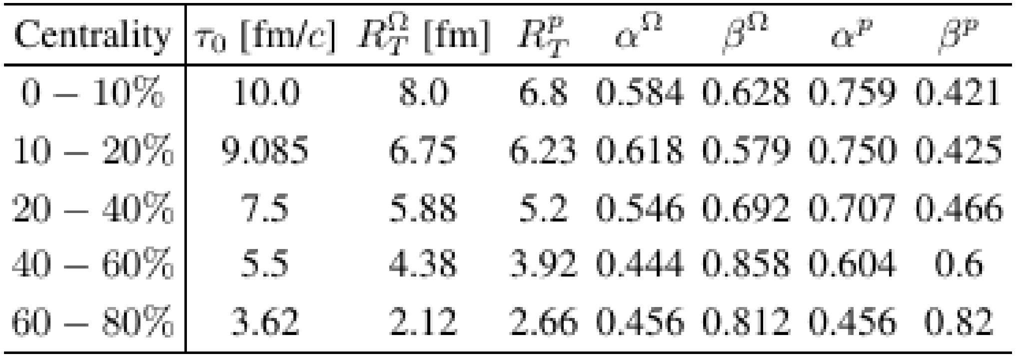
$$E\frac{dN}{d\boldsymbol{k}} = \int d^4x S_{\rm cyl}(x,\boldsymbol{k})$$

$$= \frac{2J+1}{(2\pi)^3} 2m_T V \int_0^\infty \rho d\rho e^{-\rho^2/2} I_0\left(\frac{p_T}{T}\sinh y_T\right) K_1\left(\frac{m_T}{T}\cosh y_T\right)$$

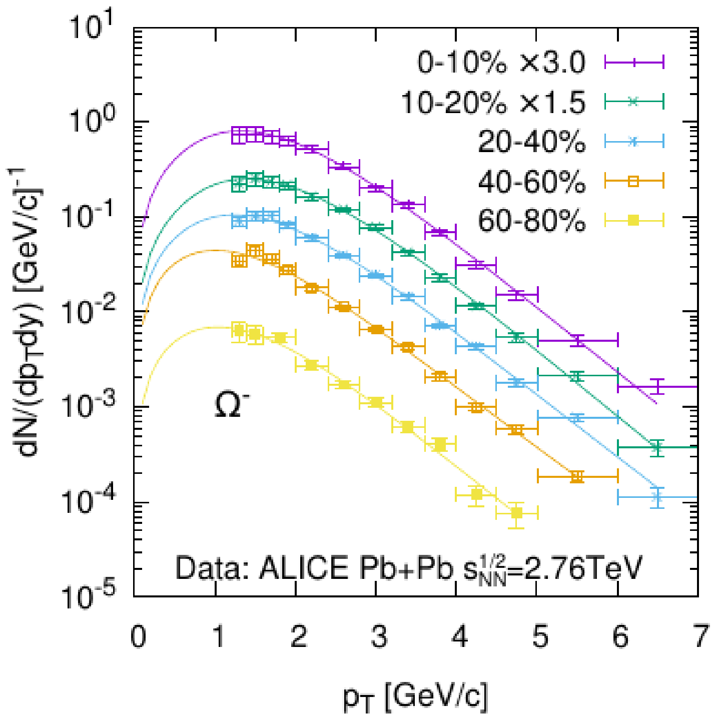
modified Bessel $I_0(z) = \frac{1}{2\pi}\int_0^{2\pi} e^{z\cos\theta}\, d\theta,\ K_1(z) = \frac{1}{2}\int_{-\infty}^{\infty} e^{-z\cosh\eta} \cosh\eta\, d\eta$

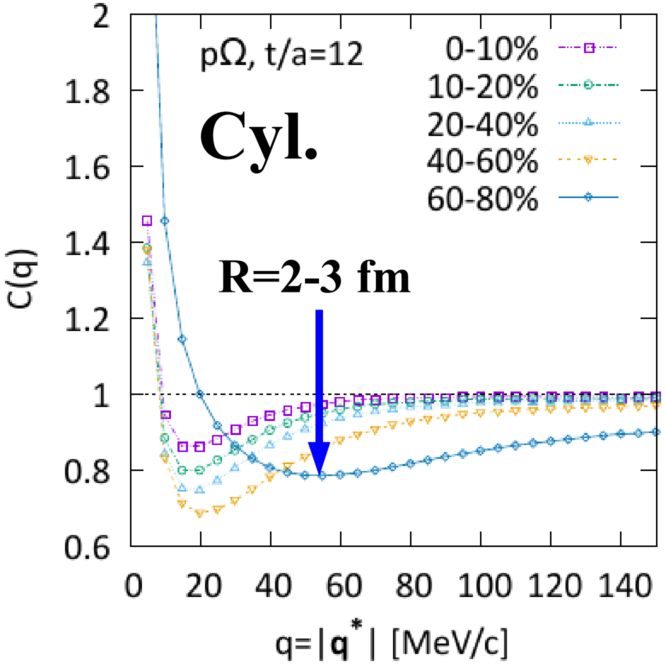
Semi-Realistic Source Function

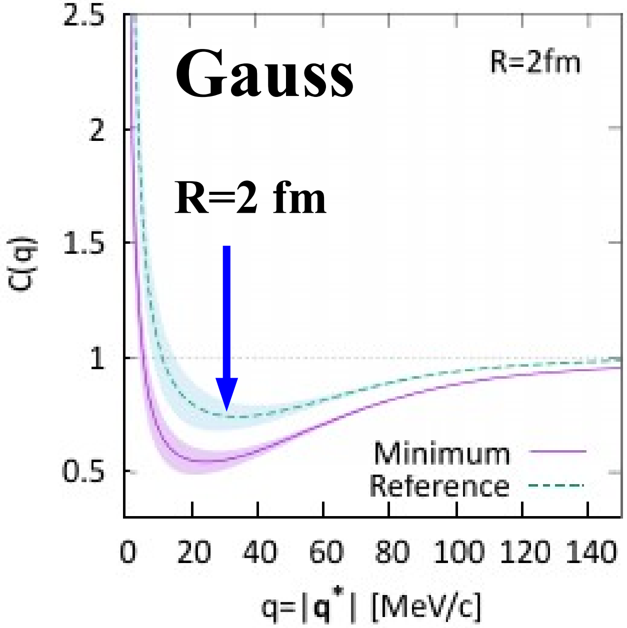
- **Correlation function from cylindrical source**
  - **Production spectra are well described.**
  - **Dip momentum at the similar size is shifted upwards by the flow.**
  - **Problem: 9D integral**
  - **R= homogeneity length**
    ≠ actual source size
    Correction factor ?



Centrality	τ_0 [fm/c]	R_T^Ω [fm]	R_T^p	α^Ω	β^Ω	α^p	β^p
0 – 10%	10.0	8.0	6.8	0.584	0.628	0.759	0.421
10 – 20%	9.085	6.75	6.23	0.618	0.579	0.750	0.425
20 – 40%	7.5	5.88	5.2	0.546	0.692	0.707	0.466
40 – 60%	5.5	4.38	3.92	0.444	0.858	0.604	0.6
60 – 80%	3.62	2.12	2.66	0.456	0.812	0.456	0.82







λ (chaoticity parameter → pair purity)

- **λ = chaoticity parameter → pair purity**
  - **λ = (“primary” pair) / (accepted pair)** $C_{\mathrm{exp}}(q) = N\left[1 + \lambda(C_{\mathrm{theory}}(q) - 1)\right]$
  - **In the best case of ΛΛ → λ = [(primary Λ) / (primary Λ + Σ^0)]^2**
  - **MC simulations seem to be useful.**

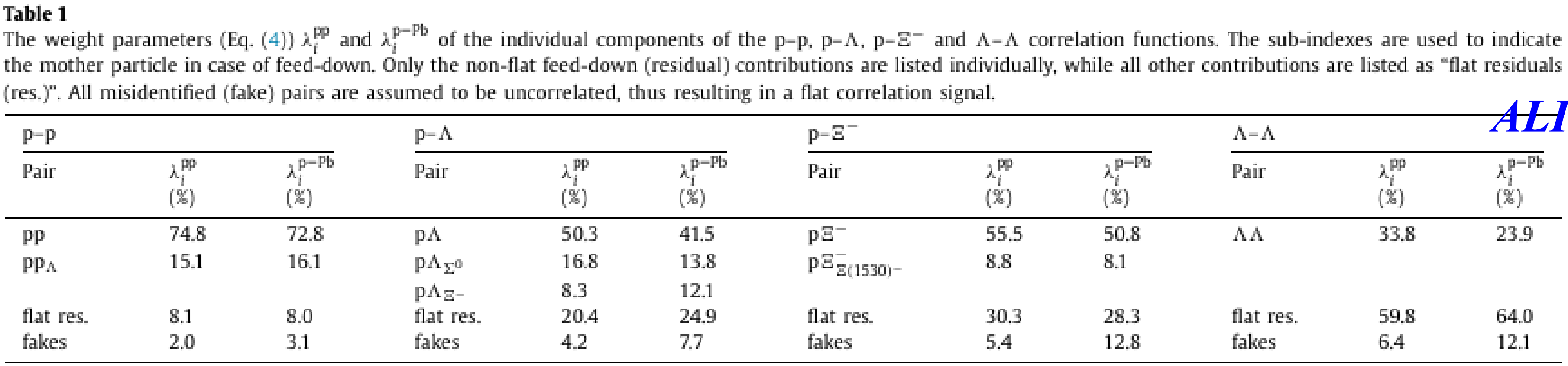
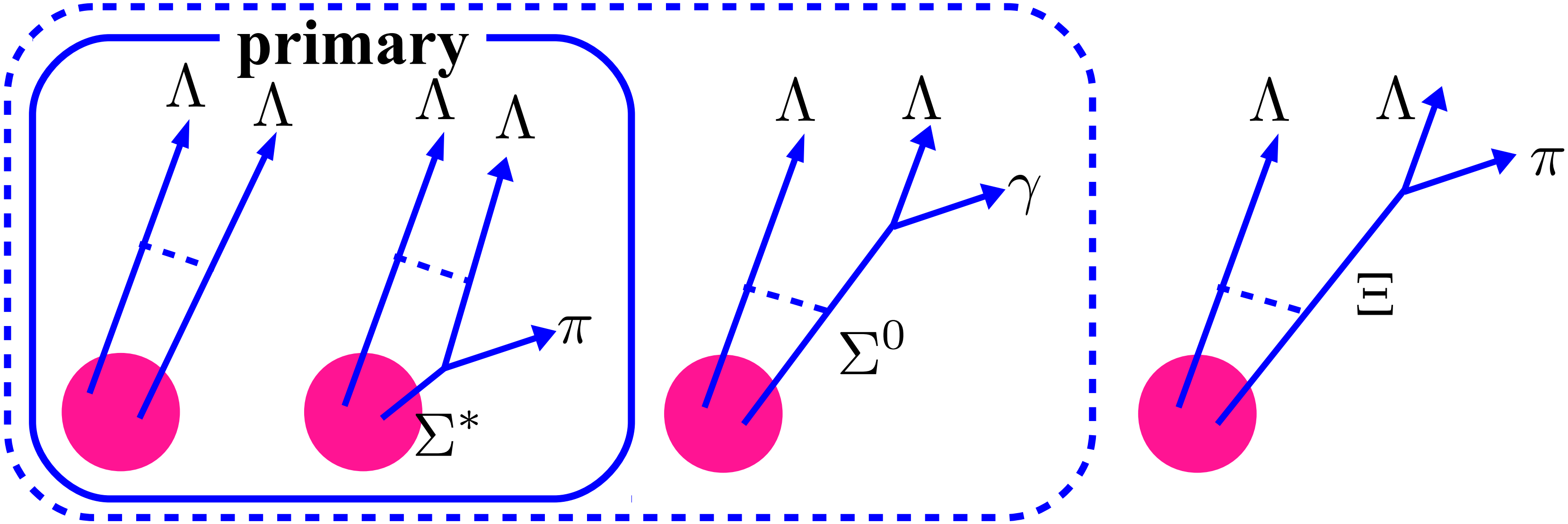


Table 1

The weight parameters (Eq. (4)) λ_i^{pp} and $\lambda_i^{\mathrm{p-Pb}}$ of the individual components of the p–p, p–Λ, p–Ξ^- and Λ–Λ correlation functions. The sub-indexes are used to indicate the mother particle in case of feed-down. Only the non-flat feed-down (residual) contributions are listed individually, while all other contributions are listed as “flat residuals (res.)”. All misidentified (fake) pairs are assumed to be uncorrelated, thus resulting in a flat correlation signal.

p–p			p–Λ			p–Ξ^-			Λ–Λ		
Pair	λ_i^{pp} (%)	$\lambda_i^{\mathrm{p-Pb}}$ (%)	Pair	λ_i^{pp} (%)	$\lambda_i^{\mathrm{p-Pb}}$ (%)	Pair	λ_i^{pp} (%)	$\lambda_i^{\mathrm{p-Pb}}$ (%)	Pair	λ_i^{pp} (%)	$\lambda_i^{\mathrm{p-Pb}}$ (%)
pp	74.8	72.8	pΛ	50.3	41.5	pΞ^-	55.5	50.8	ΛΛ	33.8	23.9
pp_{Λ}	15.1	16.1	$\mathrm{p}\Lambda_{\Sigma^0}$	16.8	13.8	$\mathrm{p}\Xi^-_{\Xi(1530)^-}$	8.8	8.1			
			$\mathrm{p}\Lambda_{\Xi^-}$	8.3	12.1						
flat res.	8.1	8.0	flat res.	20.4	24.9	flat res.	30.3	28.3	flat res.	59.8	64.0
fakes	2.0	3.1	fakes	4.2	7.7	fakes	5.4	12.8	fakes	6.4	12.1

ALICE (‘20)



Lorentz invariant representation of C(q)

- **d^3p is not Lorentz invariant, but d^3p/E is invariant.**

$$C(\boldsymbol{q},\boldsymbol{P}) = \frac{E_1 E_2 dN_{12}/d\boldsymbol{p}_1 d\boldsymbol{p}_2}{(E_1 dN_1/d\boldsymbol{p}_1)(E_2 dN_2/d\boldsymbol{p}_2)}$$

$$P \equiv p_1 + p_2, q^\mu \equiv \frac{1}{2}\left[(p_1-p_2)^\mu - \frac{(p_1-p_2)\cdot P}{p^2}P^\mu\right] = \frac{E'_2 p_1^\mu - E'_1 p_2^\mu}{M_{\rm inv}}$$

$$(E'_i = E_i \text{ in the pair rest frame})$$

- **Free two-body wave function**

$$\exp(-ip_1x_1 - ip_2x_2) = \exp(-iPX - iq(x_1-x_2)) = \exp(-iPX + i\boldsymbol{q}\cdot\boldsymbol{r})$$

$$X = \frac{E'_1x_1 + E'_2x_2}{M_{\rm inv}}, \boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2 - \boldsymbol{v}(t_1-t_2), \boldsymbol{v} = \boldsymbol{P}/\sqrt{M_{\rm inv}^2 + \boldsymbol{P}^2}$$

$$(p_1 = E'_1 P/M_{\rm inv} + q, p_2 = E'_2 P/M_{\rm inv} - q)$$

- **Correlation function (w.f. is defined in the pair rest frame)**

$$C(\boldsymbol{q},\boldsymbol{P}) = \frac{\int d^4x_1 d^4x_2 S_1(x_1,\boldsymbol{p}_1)S_2(x_2,\boldsymbol{p}_2)|\varphi^{(-)}(\boldsymbol{r},\boldsymbol{q})|^2}{\int d^4x_1 S_1(x_1,\boldsymbol{p}_1)\int d^4x_2 S_2(x_2,\boldsymbol{p}_2)} = \int d\boldsymbol{r} S(\boldsymbol{r};\boldsymbol{q},\boldsymbol{P})|\varphi^{(-)}(\boldsymbol{r},\boldsymbol{q})|^2$$

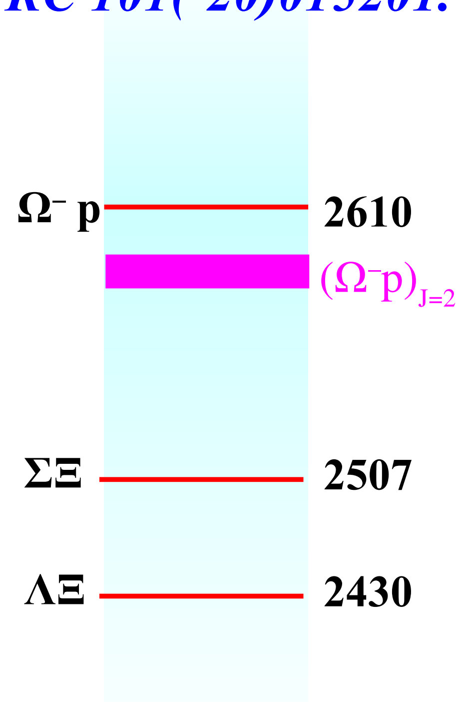
$$S(\boldsymbol{r};\boldsymbol{q},\boldsymbol{P}) = \frac{\int dt d^4X S_1(X + E'_2x/M_{\rm inv},\boldsymbol{p}_1)S_2(X + E'_1x/M_{\rm inv},\boldsymbol{p}_2)}{\int d^4x_1 S_1(x_1,\boldsymbol{p}_1)\int d^4x_2 S_2(x_2,\boldsymbol{p}_2)} \quad [x = x_1 - x_2 = (t,\boldsymbol{r})]$$

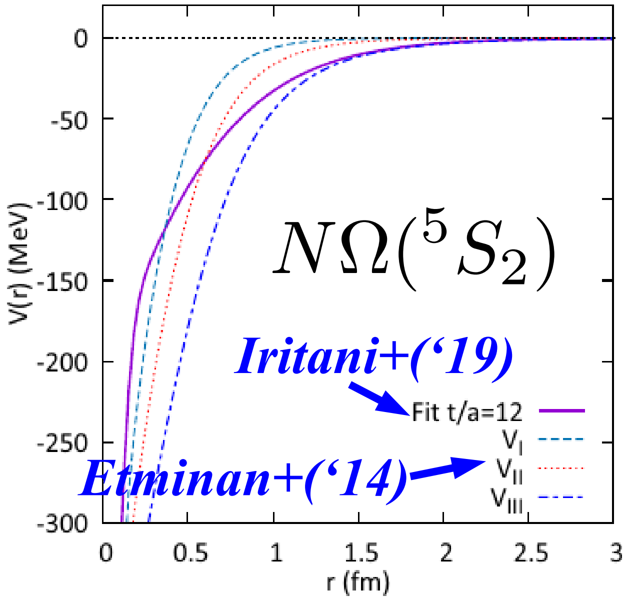
(Source function can depend on q and P.)

NΩ interaction and NΩ bound state

K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC 101('20)015201.

- **Ω^- (sss): J^π=3/2+, M=1672 MeV**
- **Ω^- p bound state as a S= –3 dibaryon ?**
  - **No quark Pauli blocking in ΩN, H=uuddss, and d*=ΔΔ channels.** *Oka ('88), Gal ('16)*
  - **J=2 state ($^5S_2$) couples to Octet-Octet baryon pair only with L ≥ 2 → Small width is expected.**
    T. Goldman+, PRL59('87),627;
    F. Etminan+[HAL], NPA928('14)89;
    Iritani+[HAL], PLB792('19)284;
    Sekihara,Kamiya,Hyodo, PRC98('18)015205.
  - **Correlation has been measured at RHIC & LHC !** *STAR ('19); ALICE ('20)*





Let us try to discover the first S<0 dibaryon !

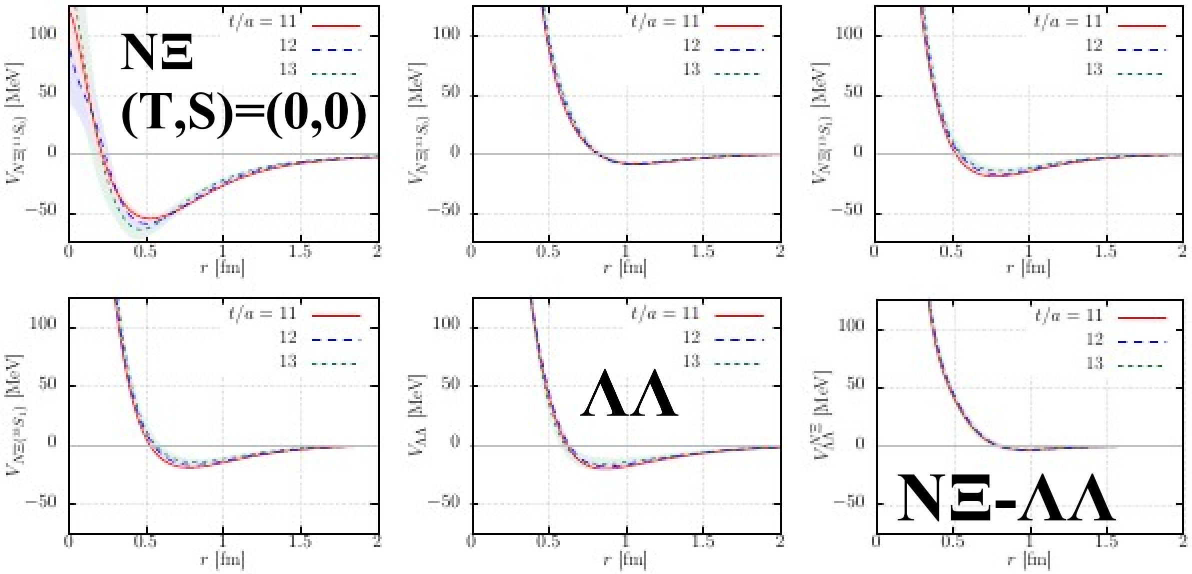
NΞ-ΛΛ potential from Lattice QCD

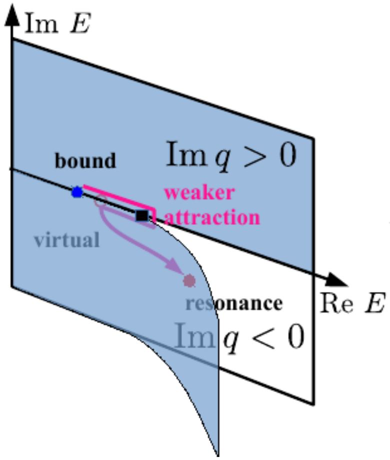
- **NΞ-ΛΛ potential at almost physical quark mass (m_π=146 MeV) by HAL QCD Collaboration**

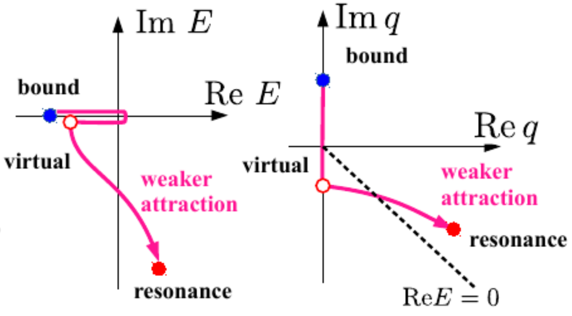
  K. Sasaki et al. [HAL QCD Collab.], NPA 998 ('20) 121737 (1912.08630)

  - **Strong attraction in (T,S)=(0,0) of NΞ**
  - **Weak attraction in ΛΛ (Coupling with NΞ causes ΛΛ attraction)**
  - **There is no bound state in NΞ-ΛΛ system (except for Ξ^- atom), but there is a virtual pole around the NΞ threshold (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, (+, –, +) [relevant=(–,+,+)]**

sign of Im(eignen momentum)





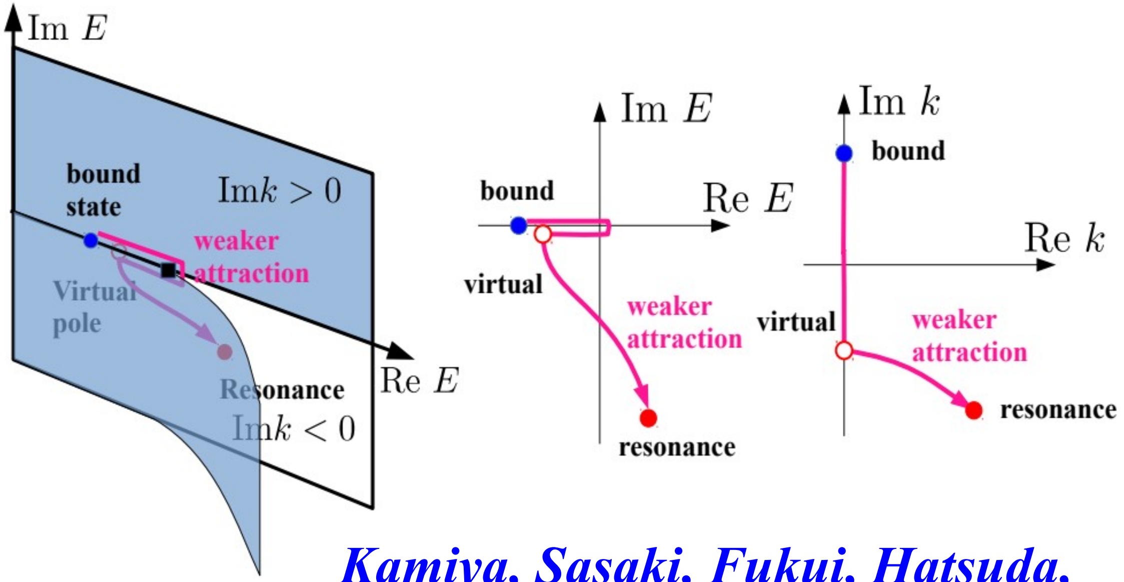


Fate of H dibaryon state ~ Virtual Pole ?

- **Recent HAL QCD results at almost physical quark mass**
  - **There is no bound state in NΞ-ΛΛ system (except for Ξ^- atom), but there is a virtual pole around the NΞ threshold (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, (+, –, +) [channels = 1(ΛΛ), 2($n\Xi^0$), 3($p\Xi^-$)]**
  - **Wave function in $n\Xi^0$ channel diverges while the Re(energy) is lower than the threshold → Virtual pole**

$$u_i(r) \propto \exp(iq_i r) = \exp(i\mathrm{Re}(q_i)r)\exp(-\mathrm{Im}(q_i)r)$$

  - **If it appears in the (–,+,+) Riemann sheet, it is a ΛΛ resonance (a NΞ bound state).**



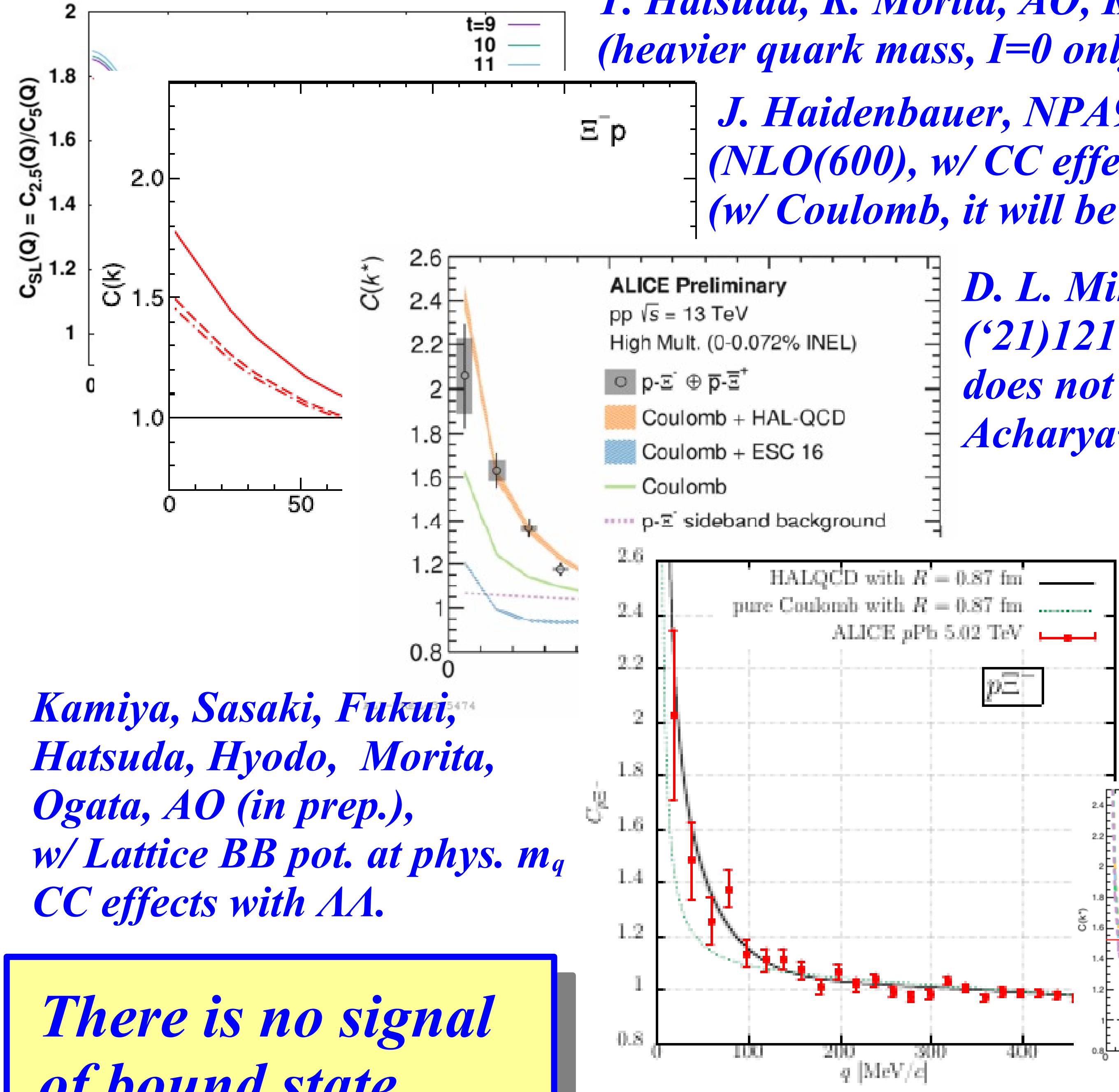
Kamiya, Sasaki, Fukui, Hatsuda, Hyodo, Morita, Ogata, AO, in prep.

$p\Xi^-$ correlation function

T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856.
(heavier quark mass, I=0 only, w/o CC effects)

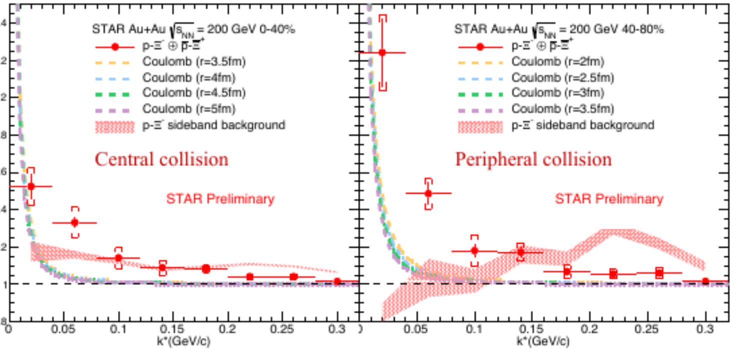
J. Haidenbauer, NPA981('19)1.
(NLO(600), w/ CC effects, w/o Coulomb)
(w/ Coulomb, it will be comparable with data.)

D. L. Mihairov+[ALICE], NPA 1005 ('21)121760 (QM2019). (Nijmegen pot. does not explain the data. w/o CC)
Acharya+(ALICE), Nature ('20)



Kamiya, Sasaki, Fukui, Hatsuda, Hyodo, Morita, Ogata, AO (in prep.), w/ Lattice BB pot. at phys. m_q CC effects with ΛΛ.

K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021.
(No Dip at larger R)



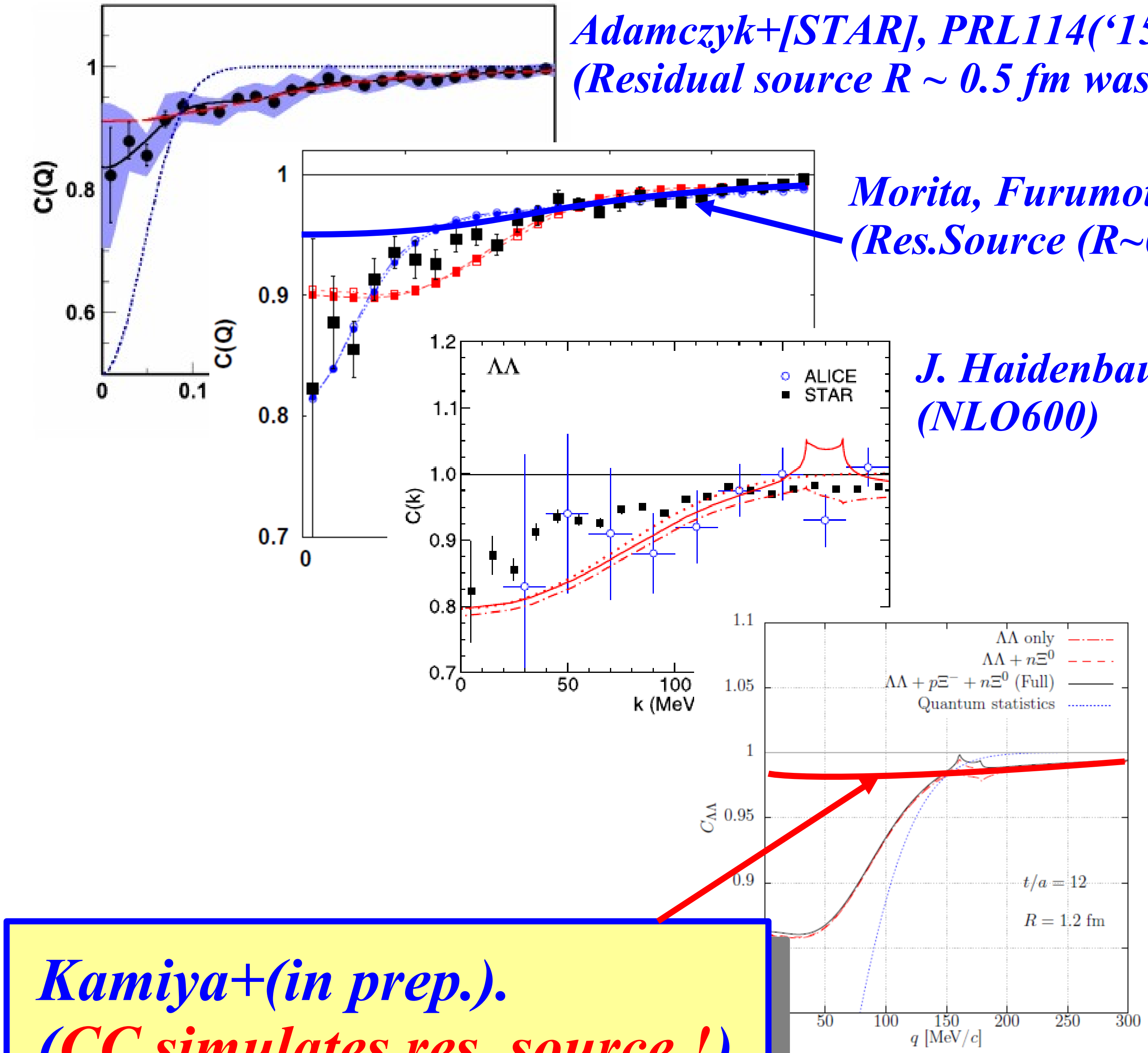
There is no signal of bound state.

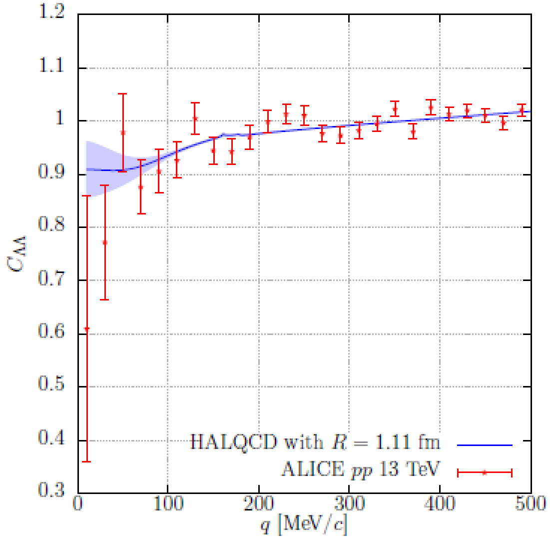
ΛΛ correlation function

Adamczyk+[STAR], PRL114('15)022301
(Residual source R ~ 0.5 fm was assumed.)

Morita, Furumoto, AO, PRC91('15) 024916.
(Res.Source (R~0.5 fm) + flow)

J. Haidenbauer, NPA981('19)1.
(NLO600)

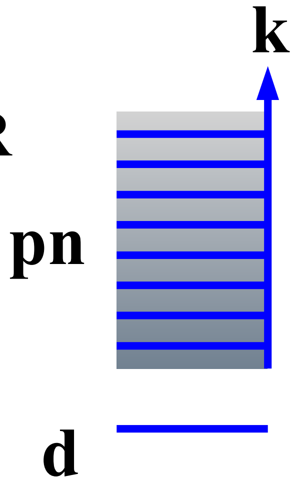


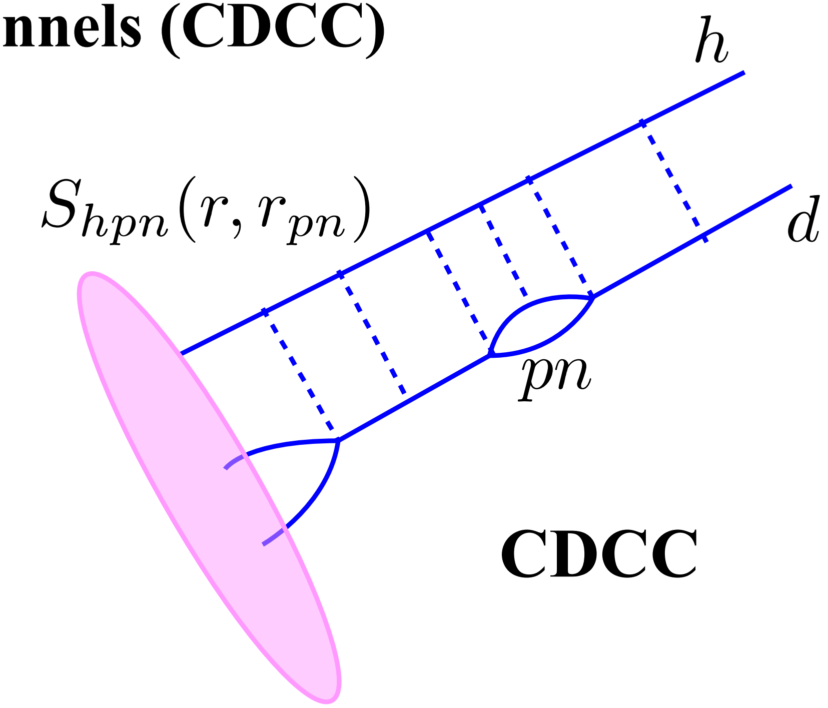


Kamiya+(in prep.).
(CC simulates res. source !)

Hadron-Deuteron correlation function

- **Hadron-deuteron correlation (Λd, K$^-$d, Ξ^-d, Ω^-d, …)**
  S.Mrówczyński, Patrycja Słoń, Acta Phys.Polon.B51('20),1739 [1904.08320](K-d,pd); J.Haidenbauer, PRC102('20)034001[2005.05012](Λd); F.Etminan+[2006.12771](Ωd).
  - **Scattering length data of these are important to evaluate**
    - **binding energy and lifetime of hyper triton (Λd)**
    - **I=1 $\bar{\mathrm{K}}$N interaction (K$^-$d, Ξ^-d)**
    - **and the existence of a bound state.**
  - **Problem: *Breakup and Dynamical Formation of d* (d ↔ pn)**
    → Continuum-discretized coupled-channels (CDCC)
    M.Kamimura+('86); N. Austern+('87);
    M.Yahiro, K.Ogata, T.Matsumoto, K.Minomo, PTEP 2012 (2012) 01A206.
  - **Measurable at LHC-ALICE and (probably) RHIC-STAR**





$\Xi^- d$ C(q) using CDCC

- **Three-body wave functions (s-wave)**

$$\psi^{(-)}(r,\rho;q) = \sum_n \sum_k A_{kn}\underline{\varphi_k(\rho)}\,\underline{\chi_{nk}(r;q_{nk})}$$

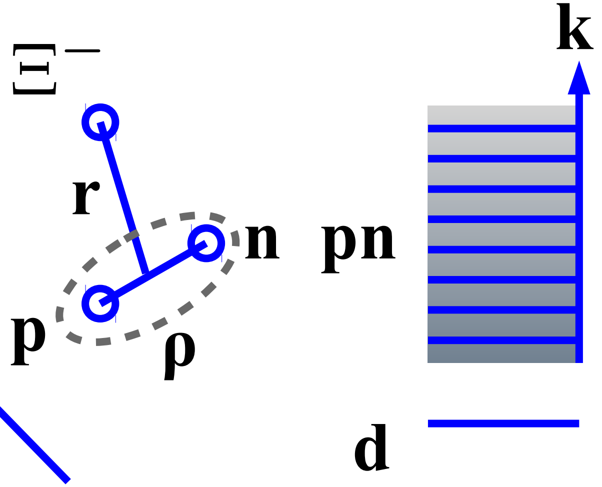
J, spin, isospin, ...

intrinsic momentum bin

kinematic factor

normalized pn w.f. in k-th bin

Ξ^--pn w.f.



- **$\Xi^- d$ Correlation function**

$$C(q) = \underline{C^{\mathrm{C}}_{\ell>0}(q)} + \underline{\frac{1}{2\cdot 3}}\int d\boldsymbol{r}\, S(r)\sum_{nk}\left|\chi_{nk}(r;q_{nk})\right|^2$$

pure Coulomb

$1/(2J_1+1)/(2J_2+1)$

"$\Xi^- d$" source fn.

- **Potential = HAL QCD potential at almost physical quark masses**

K. Sasaki et al. [HAL QCD Collab.], NPA 998 ('20) 121737 (1912.08630)

(coupling with ΛΛ is ignored).

Ξ^-d correlation function: Result

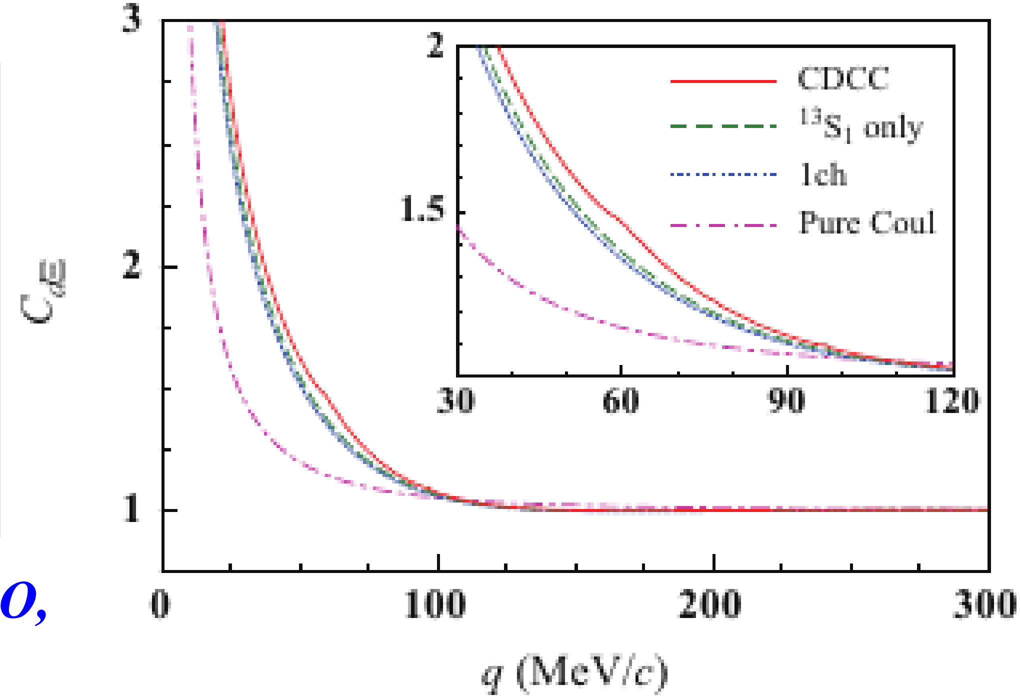
- **CDCC results of Ξ^-d correlation function**
  - **Enhancement from pure Coulomb C(q) by ΞN interaction from HAL QCD potential.**
  - **Breakup & Reformation effects ~ 10 % (Barely measurable)**
  - **Dynamical formation of deuteron is (maximally) included.**

$$\text{Implicit assumption:} \int d\boldsymbol{\rho} S(\rho)|\varphi_k(\rho)|^2 \simeq \text{const.}$$

  - **Threshold cusp at d → pn threshold is seen, but not prominent.**

Single channel description may not be bad.
→ Bound or Unbound in Ξd from Experimental data (if measured).

K. Ogata, T. Fukui, Y. Kamiya, and AO, PRC, to appear (arXiv:2103.00100).



Correlation function from T-matrix

- **s-wave w.f. using the half-off-shell T-matrix (T_0)**

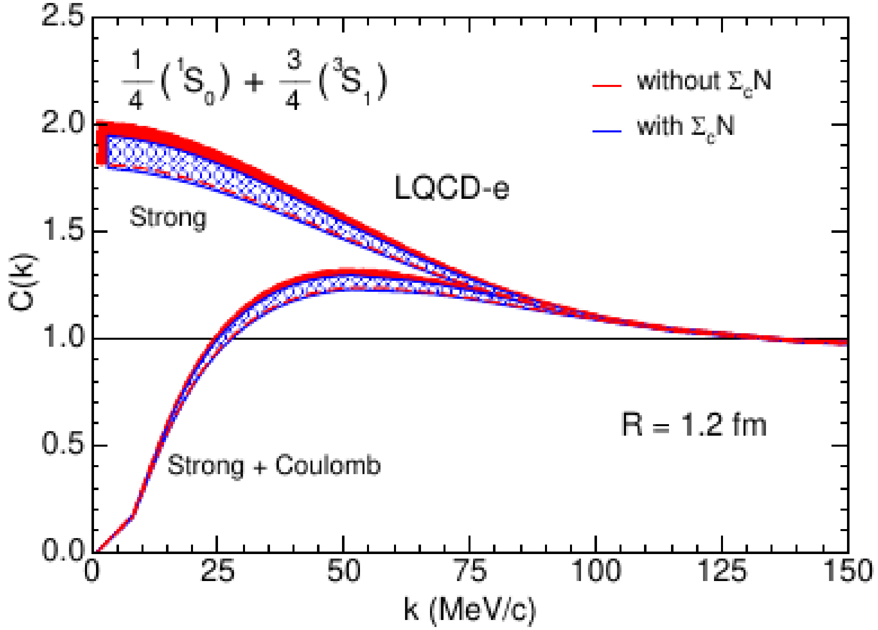
J. Haidenbauer, NPA 981('19)1.

$$\widetilde{\psi}_0(k,r) = j_0(kr) + \frac{1}{\pi}\int dq\, q^2 j_0(qr)\, \frac{1}{E - E_1(q) - E_2(q) + i\varepsilon} T_0(q,k;E)$$

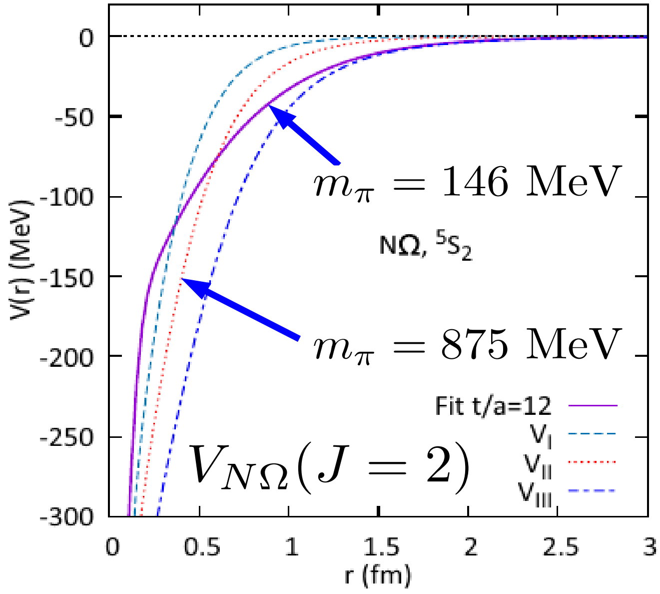
$$\psi_0^{(-)}(k,r) = e^{-2i\delta_0}\widetilde{\psi}_0(k,r) \to \frac{e^{-i\delta_0}}{kr}\sin(kr+\delta_0) = \frac{1}{2ikr}\left(e^{ikr} - e^{-2i\delta_0}e^{-ikr}\right)$$

- **Strong T-matrix + Coulomb potential**

J. Haidenbauer, G. Krein, and T. C. Peixoto, EPJA 56 ('20)184;
using the Vincent-Phatak method
[C.M. Vincent and S.C. Phatak, PRC10('74)391;
B. Holzenkamp, K. Holinde and J. Speth,
NPA 500('89)485 (1989)]



Modern Hadron-Hadron Interactions



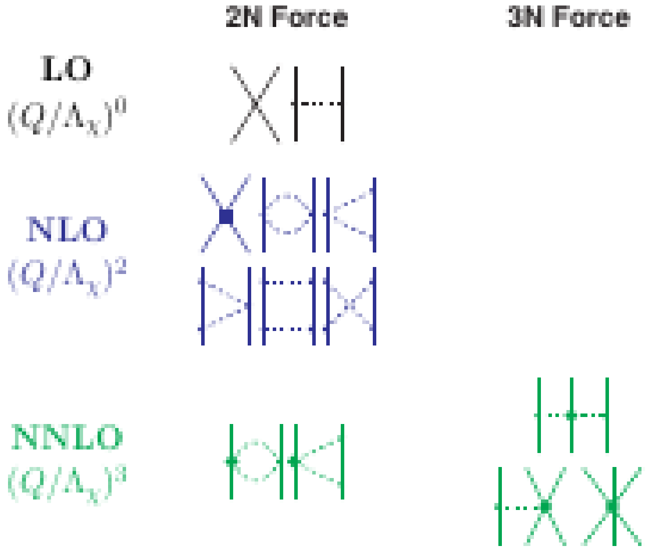
- Lattice QCD ***hh*** potential
  - V_{hh} is obtained from the Schrödinger eq. for the Nambu-Bethe-Salpeter (NBS) amplitude.

    N. Ishii, S. Aoki, T. Hatsuda, PRL99('07)022001.

    → ΩΩ, NΩ, ΛΛ-NΞ potentials at phys. quark mass are published
- Chiral EFT / Chiral SU(3) dynamics
  - V_{hh} at low E. can be expanded systematically in powers of Q/Λ.

    S. Weinberg ('79);R. Machleidt, F. Sammarruca ('16); Y. Ikeda, T. Hyodo, W. Weise ('12).

    → NN, NY, YY, $\bar{K}$N-πΣ-πΛ, ...
- Quark cluster models,
  Meson exchange models,
  More phenomenological models, …



Let us examine modern hh interactions !

C(q) in the low momentum limit

- **Correlation function at small q (and r_{eff}=0) → F_1=1, F_2=0, F_3=1**

$$\Delta C_{\mathrm{LL}}(q) \to \frac{|f(0)|^2}{2R^2} + \frac{2\mathrm{Re} f(0)}{\sqrt{\pi}R} \quad (q \to 0)$$

$$f(q) = (q\cot\delta - iq)^{-1} \simeq \left(-\frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} q^2 - iq\right)^{-1} \to -a_0$$

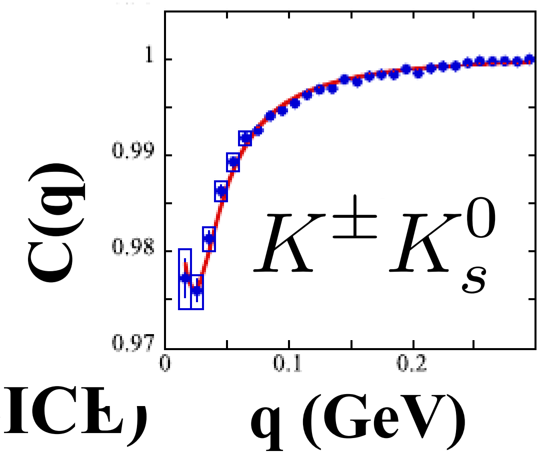
$$C_{\mathrm{LL}}(q \to 0) = 1 + \frac{a_0^2}{2R^2} - \frac{2a_0}{\sqrt{\pi}R} = 1 - \frac{2}{\pi} + \frac{1}{2}\left(\frac{a_0}{R} - \frac{2}{\sqrt{\pi}}\right)^2$$

$$1 - 2/\pi \simeq 0.36, \quad \sqrt{\pi}/2 \simeq 0.89$$

C(q → 0) takes a minimum of 0.36 at R/a_0 = 0.89 in the LL model with r_{eff}=0.

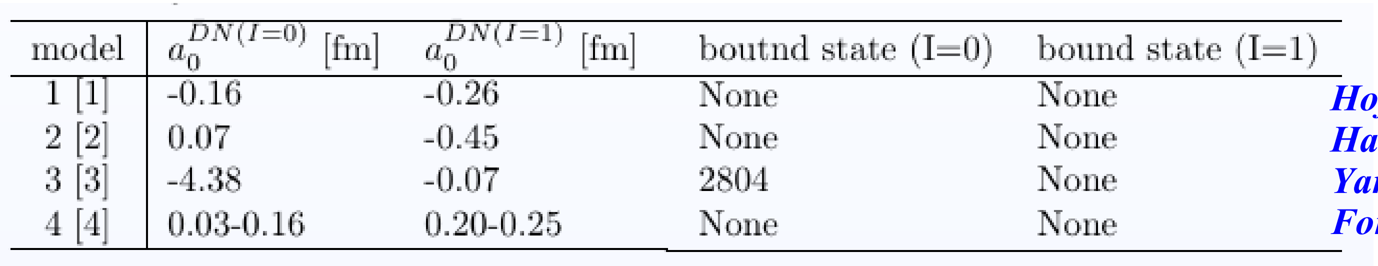
Recent & Near-Future Correlation Functions

- **$\bar{p}p$, $p\bar{\Lambda}$** *E.g. A. Kisiel [ALICE], Acta Phys.Polon.Supp. 6 ('13)519*
- **$K^{\pm}K^0_s$** *S.Acharya+ [ALICE], PLB774 ('17)64 [1705.04929]*
  → Slightly suppressed at low q
  Tetraquark component of a_0 meson



- **pΛ [2104.04427], pφ [2105.05578],**
  $p\bar{\Lambda}$, $\Lambda\bar{\Lambda}$ [2105.05190], $p\Sigma^0$ ['20 [1910.14407]] (ALICE)
- **$pD^{\pm}$ (in prog.) Scatt. length is strongly model dependent.**
  → To be discriminated by experiment !

$\bar{D}p$



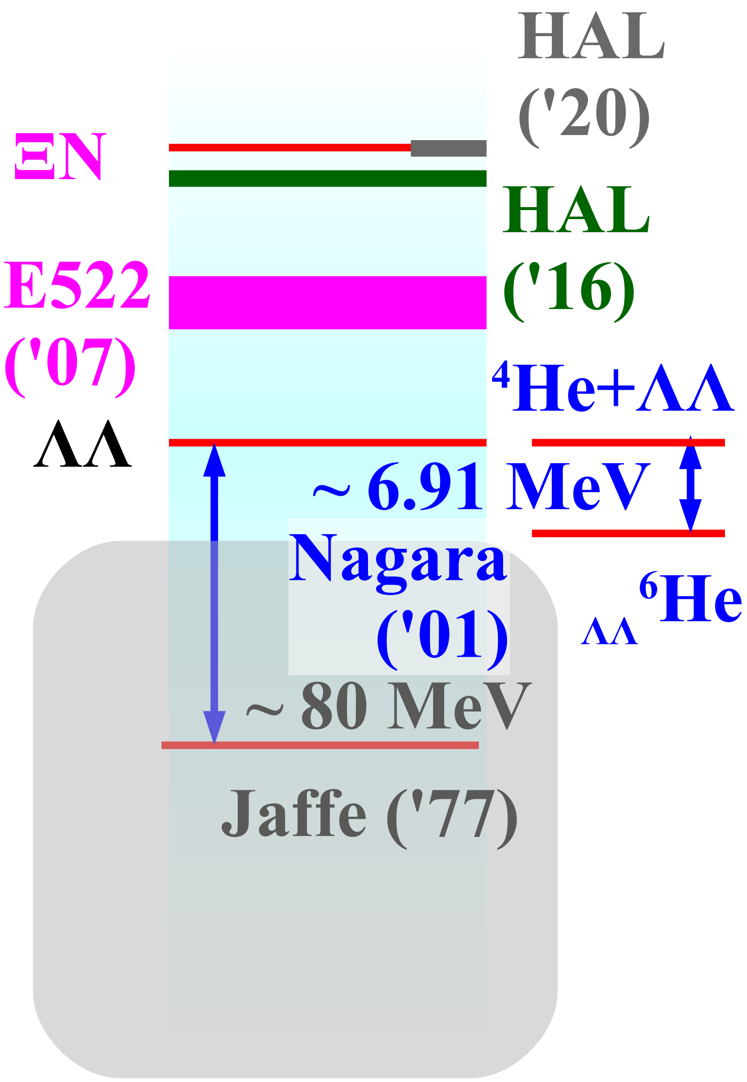
model	$a_0^{DN(I=0)}$ [fm]	$a_0^{DN(I=1)}$ [fm]	boutnd state (I=0)	bound state (I=1)
1 [1]	-0.16	-0.26	None	None
2 [2]	0.07	-0.45	None	None
3 [3]	-4.38	-0.07	2804	None
4 [4]	0.03-0.16	0.20-0.25	None	None

Hofmann+('05)
Haidenbauer+('07)
Yamaguchi+('11)
Fontoura+('13)

- **deuteron-hadron CF**
  S. Mrówczyński and P. Słoń, Acta Phys.Polon.B51('20)1739 [1904.08320]; F. Etminan, M. M. Firoozabadi, [1908.11484]; J. Haidenbauer, PRC102('20)034001 [2005.05012]; K.Ogata, T.Fukui, Y.Kamiya, AO [2103.00100].

H dibaryon state, to be bound or not to be bound ?

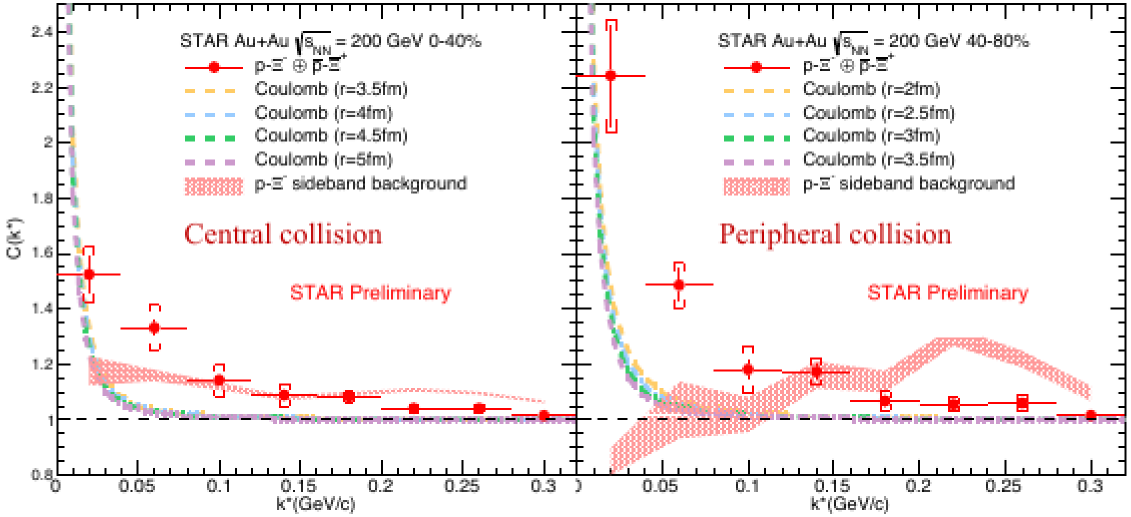
- **H-dibaryon: 6-quark state (uuddss)**
  - **Prediction:** *R.L.Jaffe, PRL38(1977)195*
  - **Ruled-out by double Λ hypernucleus**
    Takahashi et al.,PRL87('01) 212502
  - **Resonance or Bound "H" ?**
    Yoon et al.(KEK-E522)+AO ('07)
  - **Discovery of Ξ^- nucleus**
    Nakazawa et al. PTEP2015('15),033D02
- **Lattice QCD results**
  - **Bound (below ΛΛ threshold):**
    HALQCD('11), NPLQCD('11,'13), Mainz('19)
    (heavier quark mass or SU(3) limit)
  - **Resonance (Bound state of NΞ):**
    HAL QCD ('16,18) **(HAL preliminary)**
  - **Virtual Pole (around NΞ threshold)**
    HAL QCD ('20) **(almost physical m_q)**



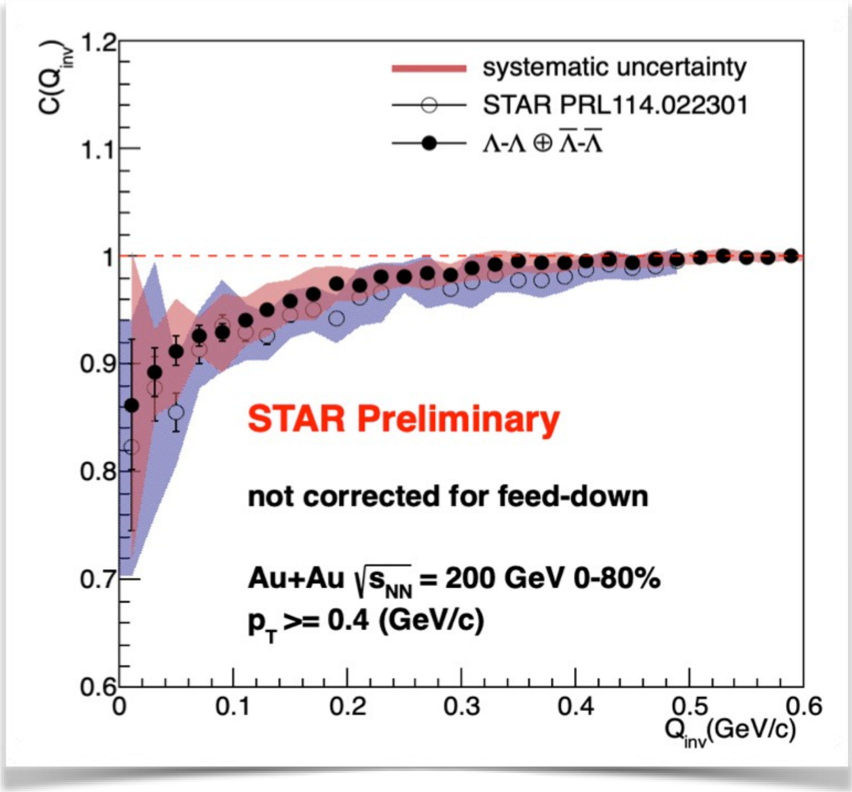
We examine LQCD NΞ-ΛΛ potential and discuss H using CF !

Ξ^-p & ΛΛ correlation functions (AA)

- **Correlation function data from AA collisions**
  [c.f. Shah, Mon., Isshiki, Tue.]
  K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021.
  Moe Isshiki+ (STAR, preliminary), Strangeness physics workshop, 2021.
  - **We do not see a dip in C(Ξ^- p) from Au+Au.**
    → There will be no bound state of Ξ^- p.
  - **Much higher statistics data of C(ΛΛ) from Au+Au are obtained.**
    → LL formula fit will be possible.



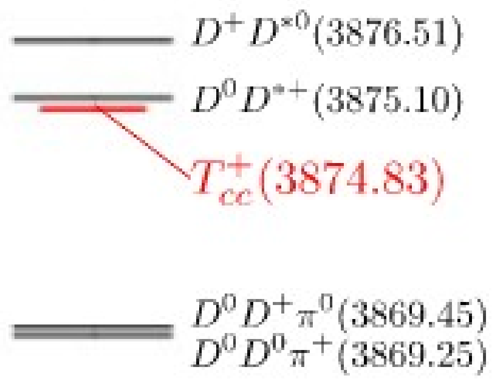
K. Mi+(STAR, preliminary),
Au+Au 200 AGeV, APS2021.
(No Dip at larger R)

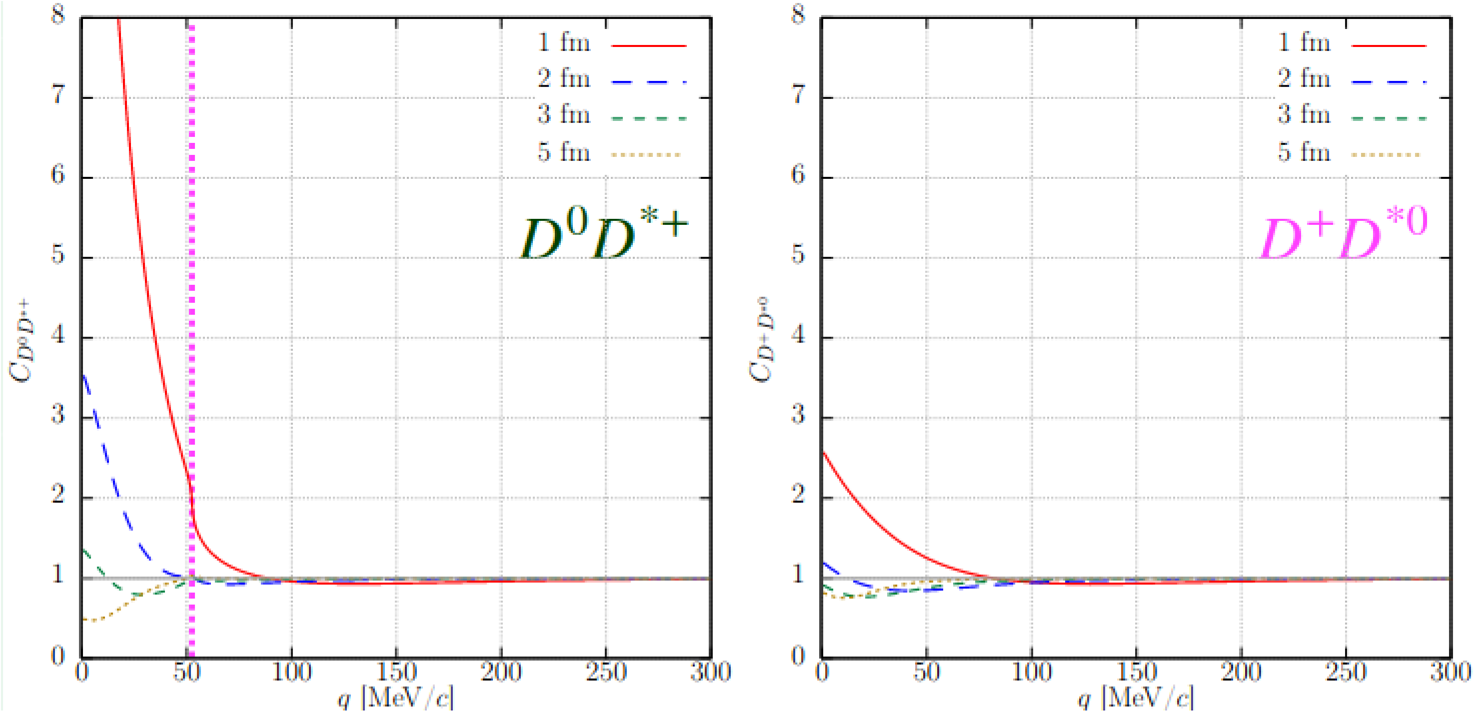


Moe Isshiki+ (STAR, preliminary).

$D^0 D^{*+}$ and $D^+ D^{*0}$ *Correlation Functions*

- **For small source (R= 1 fm)**
  C(q) > 8 for the lower channel (D^0D^{*+}) (Very strong)
  C(q) ~ 2.5 for upper channel (D^+D^{*0}) (strong)
- **For large source (R=5 fm), CF show a dip**
- **Strong enhancement for small source, dip for large source**
  → Characteristic dependence with a bound state (Tcc)
- **Cusp is not significant**





$D^0\bar{D}^{*0}$ and D^+D^{*-} Correlation Functions

- $C(D^0\bar{D}^{*0})$: **Strong enh. for small source, dip for large source** → **Characteristic dependence with a bound state (X(3872))**
- $C(D^+D^{*-})$: **Coulomb dominant**
- **Cusp may be observed for small size**