

FROM FORMAL LOGIC TO FORMAL ONTOLOGY: THE NEW DUAL PARADIGM IN NATURAL SCIENCES

Gianfranco Basti

Faculty of Philosophy – Pontifical Lateran University, Rome

ABSTRACT

In a visionary paper published almost forty years ago J.A. Wheeler posed the provocative question: “is physics legislated by cosmogony?” in front of the “quantum information revolution”, related to the theory of “quantum computing” in fundamental physics and cosmology, a theory originally developed by the Nobel Laureate R. Feynman – the most famous of Wheeler students. The positive answer to such a question implies a deep revision of the ontology underlying the Newtonian physics of which best formal version is certainly R. Carnap’s *Logical Atomism* (LA). The present work has thus a double, related issue. On one side, we present a first formal treatment of the *Natural Realism* (NR), as the proper formal ontology of the actual evolutionary cosmology. I.e., an ontology of the *causal foundation* of the same mathematical laws of physics, given that they evolve with the universe they rule. An issue for which some theoretical physicists and mathematicians tried to develop, at the foundation level, the theory of “arboreal causal sets”. NR is thus systematically, formally different, despite several phenomenological contact points, from the analogue proposal of a naturalistic alternative to LA: the *Conceptual Natural Realism* (CNR), recently proposed by my colleague and friend, N. B. Cocchiarella. NR ontology, is based, indeed, on the logic of the *converse implication* ($q \leftarrow p$) and of its modal version ($\neg\Diamond(q \wedge \neg p)$), as the logic of the *formal causality*, according to an Aristotle and Aquinas suggestion. In it, the truth in the inferential chain is not conserved, and hence it is the proper logic of the unpredictable emergence of *coherent behaviors* in which the individuality of the elements composing the system at the beginning of the process disappears, so to justify the emergence of collective behaviors, and hence of ever more complex structures. We demonstrate thus that the proper Modal Logic (ML) of NR is **KD45**, or **secondary S5**, and its Quantified ML (QML) is a *possibilist* version (because of the axiom **D**) of the “objectual” **Q1R** system. In such a way, it is possible to formalize in NR an “arboreal” *unraveling* procedure of *causal constitution* (ancestor-descendants) – effectively a non-actualist version of R. Hayaki’s “stipulation principle” - of *nested domains/sub-domains* of possible worlds, implementing a principle of “iterated modality” and of “stratified rigidity”. In it, each level of the “unraveling” of equivalent domains has a **KD45** structure, and the whole system has a **nested KD45** structure, of growing complexity. NR seems thus an optimal candidate as *formal ontology* of an evolutionary cosmology based on the Quantum Field Theory (QFT), as irreducible to Quantum Mechanics (QM) because in the former, differently from the latter, the Stone-Von Neumann theorem of the finitely many unitarily equivalent commutation relations does not hold. So – and this is the second main issue of this work – the emerging *Dual Paradigm* (DP) in contemporary quantum physics, because considering *information* as another fundamental physical magnitude like *energy*, is giving, certainly, a positive answer to Wheeler’s first question – synthesized in another Wheeler’s famous dictum: “it from bit”. However, since it is well established that “information” in QM is the Shannon information measure and content, it is difficult to justify how such a purely syntactic notion of information can deal with the process of “cosmological construction” of ever more complex systems. On the contrary, in QFT the Boltzmann-Schroedinger notion of information as *negentropy*, acquires a precise sense. In QFT any quantum system has to be considered as an “open” system, because always interacting with the background fluctuations of the quantum vacuum. I.e., the Hamiltonian in QFT is always including the quantum system and its inseparable thermal bath. The “duality” form-matter acquires thus in QFT a physical precise sense together with the notions of negentropy and its equivalent of *free-energy*. Effectively, an amount of energy is “free” for performing a work, iff it is *properly channeled*. In QFT such a “channeling” – effectively an “ordering”, i.e. the *formal* component of the notion of information as negentropy – is related with the principle of the *infinitely many spontaneously breakdowns* of the quantum vacuum symmetry at the ground state. Each of them corresponds to the establishment in the system of a *coherence domain*, propagating with a phase velocity c^2/v in the quantum realm, and hence giving to the energetic signal, propagating with a velocity v , an instantaneous optimal “channeling”, through the phase coherence of the oscillating fields. Each fluctuating coherence domain, corresponds thus to an “emerging” *macroscopic*, “ordered state”, of which information measurement corresponds to the *variation* of the *density distribution* of the so-called “Nambu-Goldstone bosons” (NGB) - “phonons” in the crystal state of matter, “magnons” in the ferromagnetic state, DWQ in the living state of matter. These “quanta of form” are indeed *suddenly vanishing* with the ordered dynamic system of which they represent the “order parameter” amount. Hence, NGB’s are distinct from the “quanta of energy”, the “gauge bosons” (γ , W^\pm , Z^0 , g), all vectors of energy exchanges, despite NGB’s are always “interacting” with them, and with their reciprocal energy/mass transformations as the physics of “Higgs-boson” exemplifies.

Summary

FROM FORMAL LOGIC TO FORMAL ONTOLOGY: THE NEW DUAL PARADIGM IN NATURAL SCIENCES.....	1
GIANFRANCO BASTI FACULTY OF PHILOSOPHY – PONTIFICAL LATERAN UNIVERSITY, ROME	1
SUMMARY	2
1 INTRODUCTION: THE SCIENCE-ONTOLOGY RELATIONSHIP.....	2
1.1 THE SCHEME OF THE PRESENT PAPER	2
1.2 PHILOSOPHICAL LOGIC AND FORMAL ONTOLOGY	6
1.2.1 <i>A taxonomy of the different ontologies</i>	6
1.2.2 <i>A taxonomy of the different senses of being/existence in the natural languages</i>	7
2 FROM THE CNR TO THE NR FORMAL ONTOLOGY	8
2.1 CNR FORMAL ONTOLOGY: ITS CONCEPTUALIST BIAS	8
2.1.1 <i>Conceptual and natural predication in CNR and NR ontologies</i>	8
2.1.2 <i>the alleged cosmological relevance of CNR ontology and its conceptualist limit</i>	10
3 A CHANGE OF PARADIGM IN FUNDAMENTAL PHYSICS: FROM QM TO QFT	11
3.1 “IS PHYSICS LEGISLATED BY COSMOGONY?”	11
3.2 FROM QM TO QFT IN FUNDAMENTAL PHYSICS	12
3.3 QUANTUM SYSTEMS AS “OPEN SYSTEMS” IN QFT AND THE NOTION OF INFORMATION	13
3.4 THE DOUBLING OF DEGREES OF FREEDOM IN QFT AND IN COGNITIVE NEUROSCIENCE	15
3.5 A CHANGE OF PARADIGM IN THE ONTOLOGY OF THE NATURAL SCIENCES	16
4 A SEMI-FORMAL PRESENTATION OF THE NR FORMAL ONTOLOGY	18
4.1 PREMISE: THE DISTINCTION BETWEEN LOGICAL AND CAUSAL NECESSITY	18
4.1.1 <i>Quine’s criticism to C. I. Lewis’ modalization of the logical implication</i>	18
4.1.2 <i>A Medieval suggestion</i>	18
4.2 THE LOGIC OF THE CONVERSE IMPLICATION AS THE LOGIC OF THE CAUSAL NECESSITY	21
4.3 THE NR FORMAL ONTOLOGY	24
4.4 APPLICATIONS OF NR TO A FORMAL ONTOLOGY OF THE EVOLUTIONARY COSMOLOGY.....	31
4.4.1 <i>The Ontology of time and of complexity in the NR formal ontology</i>	31
4.4.2 <i>The ontology of natural kinds in NR</i>	32
4.4.3 <i>The ontology of the conceptual realism in the NR formal ontology</i>	33
4.5 THE NR LOGIC IS A PARACONSISTENT LOGIC	34
4.6 GENERAL CONCLUSIONS AND FURTHER PERSPECTIVES	36
5 BIBLIOGRAPHY	37

1 Introduction: the science-ontology relationship

1.1 *The scheme of the present paper*

In this paper, we offer for the first time a semi-formal presentation of the formal ontology of the *Natural Realism*¹ (NR), as the proper ontology of the actual fundamental physics and evolutionary cosmology. Four are the main topics of this contribution:

- 1) Firstly, in this introductory chapter, we offer an essential *methodological contribution* to our discussion. Effectively, when we discuss topics such as “the ontology of science” there is often a disastrous confusion between the scientific language and perspective, and the philosophical

¹ A more complete version of the Natural Realism formal ontology can be found in the book “The formal ontology of the natural realism”, in press within the collection, *Coleção História da Matemática par Professores*, cured by F. M. Bertato and I. M. L. D’Ottaviano (Basti, 2014).

language and perspective. Fortunately, in our present age, it is possible to avoid such confusions, because in the realm of logical sciences a clear distinction is today available between the so-called *mathematical logic*, with its symbolic and axiomatic apparatus, and the so-called *philosophical logic*, with its own symbolic and axiomatic apparatus. These developments are related with the renewed interest for ontology and its relationship with science (Putnam, 2012), in the context of the actual change of paradigm in physics and natural sciences, essentially related with the so-called *Information Paradigm* (IP) in fundamental physics – i.e., in Quantum Mechanics (QM) and in Quantum Field Theory (QFT).

- 2) Secondly, we emphasize the main differences between the *Conceptual Natural Realism*, CNR, formal ontology of N. B. Cocchiarella, and our NR, criticizing the CNR pretension of being an adequate ontology for the actual evolutionary cosmology, and for the quantum, fundamental physics.
- 3) Thirdly, before the presentation of the NR formal ontology, we sketch briefly the change of paradigm in fundamental physics from QM to QFT that recently received an essential support through the empirical confirmation of the existence of the Higgs boson and of the connected “standard model” in quantum physics. To illustrate the main differences between the two paradigms, we illustrate before:
 - a. The different interpretation between the particle-wave duality in QM as to QFT making “natural” the occurrence of the straightforward phenomena of quantum “entanglement” and quantum “non-locality” because in QFT, as field phenomena, they do not involve any absurd particle interaction like in QM.
 - b. This depends on the fact that QFT cannot be interpreted as a “second quantization” as to QM, because the classical Stone-Von Neumann theorem (Von Neumann, 1955) does not hold in QFT. This theorem states that, for system with a *finite* number of degrees of freedom, which is always the case in QM, the representations of the canonical commutation relations are all *unitarily equivalent to each other*. On the contrary, in QFT systems, the number of the degrees of freedom is not finite, “so that infinitely many unitarily inequivalent representations of the canonical commutation (bosons) and anti-commutation (fermions) relations exist”.
 - c. The emerging picture for the naturalistic ontology of QFT is thus deeply different from the atomism of the Newtonian physics of the origins of modernity, as much as the notion of “quantum vacuum” in QFT is different from the notion of “mechanical vacuum” in the Newtonian physics. The ontological paradigm of physical systems is, indeed, no longer the *isolated particle* in the mechanical vacuum (= atomism) of which the *Logical Atomism* (LA) constitutes its formal ontology counterpart. In QFT no microscopic physical system is conceivable as completely isolated (closed), since *it is always in interaction with the background fluctuations* (i.e., the quantum vacuum condition, including in itself all universe(s)). In this sense, “QFT can be recognized as an *intrinsically thermal* quantum theory” (Blasone, Jizba, & Vitiello, 2011, p. ix). Of course, because of the intrinsic character of the thermal bath in QFT, only the whole system, including its thermal bath, can recover the classical Hamiltonian character of any quantum system.
 - d. Finally, because of the presence of long-range correlations related to the essential notion of “coherence domains” in QFT systems, a lot of *macroscopic* quantum phenomena, both in many-body physics of the so-called “condensed state” of matter (crystallization, superconductivity, ferromagnetism), and in bio-chemistry, from genetics to neuroscience, have their fundamental explanation at the *microscopic* level of QFT physics. In short, because of the intrinsic “change of scale” related to the presence of long-range correlations in QFT phenomena, the ambiguous notion of “emergence” of *complex phenomena* in natural science could have at last a rigorous explanation in fundamental physics.

- 4) Fourthly, we present here our *Natural Realism* (NR) formal ontology. NR results to be in continuity with a particular extension of the Aristotelian natural metaphysics, the Aquinas metaphysics, of which NR shares the underlying logic. At the same time NR solves two main difficulties of Nino B. Cocchiarella’s formal ontology, he named *Conceptual Realism* (CR), that emerge from our analysis, and emphasized by Cocchiarella himself (Cocchiarella, 2007), namely:
- a. Despite CR is inchoately “naturalistic” in its effort to formalize the core of a naturalistic ontology by giving a *causal foundation* of the notion of *natural kind*, as distinguished from the correspondent notion of *conceptual kind* and hence of *logical class* – Cocchiarella speaks effectively on this regard about a “Conceptual Natural Realism” (CNR) (Cocchiarella, 2007, p. 273ff.) –, he recognizes, nevertheless, a primacy to the “conceptual” as to the “natural”². This conceptualist bias of Cocchiarella ontology corresponds to the absence in CNR of a sufficient formal (axiomatic) justification in his formal ontology of the fundamental distinction between the *causal* and the *logical* necessity, symbolized, respectively by the distinction in CNR between causal (\Box^c/\Diamond^c) and logical (\Box/\Diamond) modal operators. NR is different from CNR before all because it is able to give an axiomatic foundation of such a distinction.
 - b. This theoretical lack is reflected in a structural weakness of CNR metaphysics expressed in the impossibility of deciding, “Whether the appropriate modal logic of the conceptual natural realism is **S4** or **S5**” (Cocchiarella, 2007, p. 279). Effectively, a causal relation supposes a *transitive and serial but non-symmetric* accessibility relation between possible worlds, and this seems to make **S4** the proper Modal Logic (ML) of the causal necessity. On the other hand, Cocchiarella continues, the metaphysics of natural necessity has **S5** as its appropriate ML, since, “to express the matter in model-theoretic terms, the possible worlds in the multiverse that have the same laws of nature constitute an equivalence class”. In other terms, using **S4** as the proper logic of CNR, does not preclude of assuming **S5** “because **S4** is a proper part of **S5**”. This is not, however, a solution of the problem. Namely, it leaves undecided how is it possible that a natural kind, constituting on its own an equivalence class of “possible worlds” (the different species of the same genus)³ – formally characterized by a set of transitive, symmetric and reflexive accessibility relation among all of them – can be constituted through a given causal matrix. The “nested character” of such a structure supposes indeed, as in the theory of causal sets, an *arboreal* structure – and hence an *intransitive, asymmetric and irreflexive* relation – that is incompatible with an **S4** structure, and much more with an **S5** structure, defining all the universe of possible worlds as one only

² The core of CNR is indeed – as Cocchiarella himself rightly emphasizes (Cocchiarella, 2007, p. 275, n.3) kindly quoting myself (Basti, 2004) –, Aquinas’ theory of the “double signification” of the *very same* predicate Fx in categorical statements. I.e., as signifying a natural property/relation (*in intentio prima*), and as signifying a conceptual property/relation representing the first one (*in intentio secunda*). A distinction signified in CNR through a double indexation of the relative quantifiers, respectively for the *natural* $\langle \forall^n F / \exists^n F \rangle$, and the *conceptual* $\langle \forall F / \exists F \rangle$ predication of a given property/relation. This theory is opposed to Peter Abelard theory of the *double existence* of universals in the natural reality and in human mind, in the Middle Age, as well as it is opposed to the modern ontological dichotomy between the *intentional* (conscious) realm and the *natural* (realm). On the contrary, I do not agree with my friend Cocchiarella when he affirms that also the primacy of the “conceptual” signification of the predicate onto the “natural” one goes back to Aquinas, like his denoting them, respectively, as *secunda* (second) and *prima* (first) *intentio* (signification) emphasizes. We see in the rest of this paper that this is not only a terminological question, making Aquinas a precursor of the NR formal ontology and not of the CNR one, as Cocchiarella pretends.

³ It is important to emphasize that such a distinction is not today limited to biology but to the whole cosmology. In fact, quantum physics re-introduces in modern physics the distinction among several species of fundamental particles (e.g., electrons, protons neutrons) on their turn collected in different genera (e.g., leptons, barions). They are able to give an ultimate physical foundation, at the different “emergent” levels of matter organization. That is, at the different species/genera of atoms and of molecular compounds in chemistry, and at the different species/genera of organisms in biology, as well as (by dissipative QFT) is able to give a rigorous foundation of the same notion of “emergence” (with the connected scale change), till now ambiguous because ill-defined.

equivalence class. Namely, it excludes that a unique set of *fixed* physical laws can justify the evolution of the whole universe – and still less of a supposed multiverse. In other terms, in the contemporary evolutionary cosmology it has to be justified what Cocchiarella, on the contrary, considers as supposed. Namely, that, despite their differences, all the possible worlds “are determined by laws of nature that are invariant across the worlds in those equivalence classes”, so to justify the **S5** structure of the natural realism. In the actual cosmology, on the contrary, a unique set of invariant physical laws does not *actually* exist, but the physical laws evolve with the universe itself. Effectively, they *emerge* from the unique causal matrix “containing” all the history of the uni-(multi-)verse, just as the equivalence classes of the natural kinds of physical things they rule. All this makes the **KD45** (or **secondary S5**) and the possibility it offers of a nested structure of **KD45** systems (or **stratified secondary S5**) as the proper ML of the evolutionary cosmology. We illustrate, in the fourth section of this work, which are the deep consequences of such an *ontic interpretation* of the **KD45** modal systems, generally used in logic literature for its *deontic* and *epistemic* interpretations⁴.

To sum up, the deep change of paradigm in fundamental physics, involving cosmology, nuclear and atomic physics, thermodynamics, chemistry, biology and the same neuroscience, makes inadequate not only the LA ontology, but also the CR ontology that can fit, at last, with some conceptualist interpretations of QM measurement theory. Both, indeed, suppose the immutability of the mathematical laws of physics. This does not fit with an evolutionary approach in fundamental physics, where the emerging laws of physics, for different levels of matter organization, require a particular ML, a **nested KD45** structure. In it, the universality of *fundamental causal relations* substitutes the universality of some fundamental laws “including”, at the explanatory level, the whole universe. As we see, the NR formal ontology, is thus based on a modal formalization of the *converse implication* ($p \leftarrow q$), i.e., $(\neg \diamond (q \wedge \neg p))$, as characteristic of the notion of *causal implication* (Panizzoli, 2013). Just as the modal formalization of the *material implication* ($p \rightarrow q$) by C. I. Lewis, i.e., $(\neg \diamond (p \wedge \neg q))$, is characteristic of the notion of the *logical implication* (“strict implication” in Lewis terms: see (Huges & Cresswell, 1996, p. 193ff.)). We see how this formalization of the causal necessity can support a causal foundation of the *existence* of the objects constituting the universe **V** of the theory, as well as a causal foundation of the *soundness* of beliefs (where, “sound belief” = “science”) in epistemic contexts. Finally, we see the intrinsic relation between the converse implication of causal necessity and the *paraconsistent negation* both in extensional (truth-functional) and intensional (non truth-functional) contexts. Moreover, such a formalization allows avoiding, in an intriguing, “metaphysical” way, the paradox of the contradictory relation between the vertexes A (*necessary*, “ \square ”) and O (the *non-named vertex*, “ $\neg \square$ ”) of the modal square of opposition (Béziau, 2005; 2012).

⁴ Starting from the pioneering works of A. N. Prior (Prior, 1955) and of J. Hintikka (Hintikka, Knowledge and belief: an introduction to the logic of the two notions, 1962), there exists today a boundless literature on the epistemic and deontic interpretations of modal logic. **KD45** system, interpreted in epistemic context is generally interpreted as the logic of the individual *belief*, given that **S5** is the *standard* epistemic logic of the individual *knowledge*, even though this interpretation lives the epistemic logic exposed to the famous “paradox of the omniscience” for the individuals (Hendricks & Symons, 2009). On the contrary, in deontic logic, **KD45** supports a reinforcement of **KD** (McNamara, 2010), interpreted as the *standard* deontic logic system, because it formally supports the “opening” of the individual moral agents to the contribution of the community (Lomuscio & Sergot, 2003). The same holds for epistemic context, where **KD45** formally supports a particular version of the “causal” theory of reference, opening the epistemic individual agent to its environment (natural and not only social). This justifies, in epistemic logic, the passage from the *belief* to the *sound* belief, i.e., the *knowledge*, without any danger of running into the trap of the so-called “omniscience paradoxes”.

1.2 Philosophical logic and formal ontology

From the standpoint of the linguistic analysis, any ordinary language can be considered as an *implicit ontology* of the human community using it. Any ordinary language, indeed, makes able its users to communicate efficiently - and hence to interact effectively - among them, and with the particular sector of the natural, cultural and social reality, all of them share. The philosophical ontologies of the different peoples and cultures are thus only a manifestation of the implicit ontologies hidden in their own ordinary languages.

This brings us immediately to understand the notion of *formal ontology*. Effectively, the contemporary notion of “formal ontology”, as distinguished from “formal logic”, is derived from Edmund Husserl research and teaching. Indeed, in his “Third Logical Research” (Husserl, 1913/21), he distinguishes between:

- 1) *Ontology* as a discipline studying relationships between *things* (like “objects and properties”, “parts and wholes”, “relations and collections, etc.); and
- 2) *Logic* as a discipline studying relationships among *truths* (come “consistency”, “validity”, “conjunction”, “disjunction”, etc.).

On the other hand, Husserl continues, both disciplines are *formal* in the sense that they are “domain independent”. So, for instance, for the formal structure “part-whole” in ontology, there are no limitations for the type of objects that might satisfy such a relation (sets, classes, collections, etc.), just like for the formal relation of “conjunction” in logic, there are no limitations for the type of propositions that can be connected in such a way (Smith, 2005). Husserl and his school developed the formal ontology analysis using the *phenomenological* method. Today, however, in the scientific and philosophical realms, when we speak about “formal ontology”, we intend generally the “formalized ontology”, i.e., the formal ontology developed according to the *axiomatic* method, using the formal means of modal and philosophical logic.

1.2.1 A TAXONOMY OF THE DIFFERENT ONTOLOGIES

In fact, the main ontologies of whichever philosophy can be formalized like as many *theories of predication* — *nominalism, conceptualism, realism* —, and/or like as many *theories of universals*, where by “universal” — as distinguished from “class” or “set” — we intend “what can be predicated of a name”, according to Aristotle’s classical definition (*De Interpretatione*, 17a39).

So, from the standpoint of the predicate logic supposed by each ontology, it is evident that all the nominalist ontologies suppose only a *first order* predicate logic, since in such ontologies it is forbidden quantifying over predicate symbols. The predicates, indeed, in nominalism, cannot denote anything: the “universals” do not exist at all in such ontologies. There exist only individuals: universals are only linguistic conventions. Then, they cannot be proper arguments of any higher order predicate symbols. If in some cases nominalism admits higher order predicate symbols, this is only in a *substitutional* sense —, i.e., in the sense of a linguistic, conventional, shortened second order formula instead of many first order true propositions —, without any extra-linguistic referential meaning. In this sense, nominalist ontologies are very similar to empirical sciences, because both share some form of exclusiveness to the only first order predicate calculus. Effectively, indeed, the absolutization of the empirical sciences, i.e., the *empiricism*, is a sort of nominalism.

On the contrary, the other types of possible ontologies admit higher order predicates, that is quantifying over predicate symbols, because they admit, even though in different senses, the existence of the *universals*, so to make possible the quantification on predicate variables.

To sum up, following Cocchiarella (Cocchiarella, 2007) and other my papers on the same argument (Basti, 2007; 2011), we can thus distinguish among at least *three types of ontology*, with the last one subdivided into two others:

- 1) **Nominalism:** the predicable universals are reduced to the predicative expressions of a given language that, *by its conventional rules*, determines the truth conditions of the ontological propositions (Sophists, Quine, ...).

- 2) *Conceptualism*: the predicable universals are expressions of *mental concepts*, so that the laws of thought determine the truth conditions of the ontological propositions (Kant, Husserl, ...).
- 3) *Realism*: the predicable universals are expressions of *properties and relations* existing independently of the linguistic and/or mental capacities in:
 - a. *The logical realm*, we have thus the ontologies of the so-called *logical realism*, where the *logical relations* determine the truth conditions of the ontological propositions (Plato, Frege, ...) ⁵;
 - b. *The physical realm*, we have thus the ontologies of the so-called *natural realism*, or “naturalism”. On its turn, naturalism can be of two types:
 - *Atomism*: without natural kinds, where the absolute *mathematical laws of physics* are ultimately determining the truth conditions of the ontological propositions (Democritus, Wittengstein’s *Tractatus*, Carnap, ...)
 - *Essentialism*: with natural kinds, where the *real relations* (causes) among things ultimately determine the truth conditions of the ontological propositions (Aristotle, Aquinas, Kripke,...).

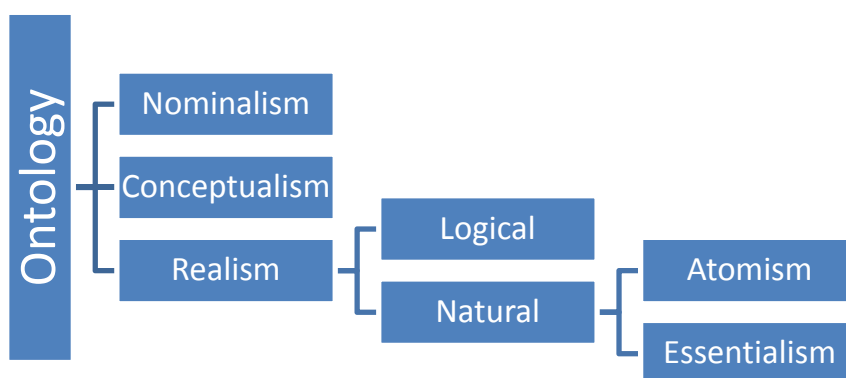


Table 1. Scheme of the primary ontologies in the history of thought

The principal difference between these two types of naturalism is that essentialism admits the *general reference*, i.e., the ability of referring of common names to natural kinds (e.g., “animal”, “mammal”, “horse”, etc.), and not only of proper names to individuals (e.g., “that horse, Fury”), while the atomism does not. Consequently, the essentialism is able to distinguish between different *de re* modalities — *actual* and *virtual* — either of natural individuals, or of natural properties and relations, both *causally* (not logically) founded. On the contrary, the atomism reduces the *de re* modalities to the *de dicto* modalities, and hence reduces the causal necessity to the logical necessity (from which its connotation as “logical atomism”, LA), as we see below.

The “essentialism” is thus able to distinguish also, not only in *logical* but also in *natural* predication, between different modalities (*possible* (i.e., true only for some possible worlds) and *necessary* (i.e., true for all possible worlds)) according to the extension of the causal relations determining them. In this way, it is able to distinguishing, respectively, between, predication of natural *properties* and *relations*, and predication of natural *kinds*.

1.2.2 A TAXONOMY OF THE DIFFERENT SENSES OF BEING/EXISTENCE IN THE NATURAL LANGUAGES

The mathematical logic reduces the notion of being/existence to the only copula of a predicative expression. Namely, it reduces the being to the simple *relation of membership*. In fact, the “existing of x ” of the existential quantification of x , $\langle \exists x \rangle$, in a predicative formula $\langle Px \rangle$, denotes the non-contradictory membership of x to a non-empty class constituting the extension of the predicate P , i.e.: $\langle \exists x Px \Leftrightarrow x \in \mathbf{P} \rangle$.

⁵ A recent example of a formal ontology of the logical realism can be found in the book of Uwe Meixner (Meixner, 2010). Of the same author, see also (Meixner, 2007).

On the contrary, the philosophical logic maintains the ordinary language distinction between, the being of the predicative relation (the *being of the essence* in the classical ontology), and the *being of the existence*, the being of the *existence predicate*, $E!(a)$. Moreover, in Quantified ML (QML), it is possible to distinguish between the *possibilist* and the *actualist* quantification.

Therefore, following for the sake of simplicity Cocchiarella's symbolism that we justify in the next section, within a *possibilist* formal ontology, we can synthesize the main senses of being/existence in the philosophical logic in the following way:

- 1) $\langle \exists x, \exists F; \forall x, \forall F \rangle$: what *can exist* (*potentia esse*, “being potentially”), but does not exist *actually* (*actu esse*, “being actually”), both as to an individual x and as to a (nominalized) property F , either in the *conceptual* or in the *natural* realm;
- 2) $\langle \exists^e x, \forall^e x; \exists^e F, \forall^e F \rangle$: what exists *actually*, both as a generic individual x , and as a property F , either in the *conceptual* or in the *natural* realm;
- 3) $\langle E!(a) := (\exists^e y) (y = a) \rangle$: what *is existing* as a concrete individual a in the *natural* realm, and never as a property F , that is, $\langle (\forall^e F) \rightarrow E!(F) \rangle$ ⁶.

2 From the CNR to the NR formal ontology

2.1 CNR formal ontology: its conceptualist bias

2.1.1 CONCEPTUAL AND NATURAL PREDICATION IN CNR AND NR ONTOLOGIES

In a famous paper on the Quantified Modal Logic (QML) J. W. Garson stated:

One of the most significant points of difference between semantical treatments of QML concerns the domain of quantification. Some systems quantify over objects, while others quantify over what Carnap (1947) called individual concepts. The second approach is more general, but it is also more abstract and more difficult to motivate (Garson, 2001, p. 271).

The generality of conceptualism is because from the Renaissance on – that is after the abandon of the Aristotelian naturalism – the *possibilism* in ontology supposes the *conceptualism* in QML semantics. Hence, to be *objectual* in QML semantics, implies to be *actualist* in ontology. So, immediately after, Garson recalls us which is the deep motivation of the choice of free logics in the *objectual*, interpretations of QML like in Hayaki's *actualist* ontology we discuss below, that is,

The basic assumption, made in the semantics for quantificational logic, is that every constant (such as g) refers to an object in the domain of quantification. (...) From the provable identity $\langle g = g \rangle$ we may thus derive, $\langle \exists x (x = g) \rangle$ by Existential Generalization. If g abbreviates “God”, then $\langle \exists x (x = g) \rangle$ reads “God exists” [but also “the unicorn exists”, if g denotes the unicorn] (Garson, 2001, p. 267). [Square parenthesis is mine, even though it shortens the sequel of Garson's argument].

We see below that we can use in NR a *possibilist* ontology without being *conceptualist*, and hence we can use an *objectual* QML semantics without any need to use the free logic. In NR ontology, indeed, we do not use the *self-identity* as a necessary and sufficient condition for the existence of all the elements of \mathbf{V} (see below, in §4.3, the axiom of foundation **OAF**₁₋₂ in the semi-formal presentation of NR).

In the light of the precedent discussion, it is more evident where is the richness (a *possibilist* ontology) and simultaneously the *weakness* (its *conceptualist* interpretation) of Cocchiarella's CNR ontology. Effectively, the strategy of redefining consistently different types of predication, and hence of quantifiers with the related modality operators, on different “cut-down's” of the possibilist space gives a great expressive richness to the semantics of CNR formal ontology. The general strategy has been recently summarized by Cocchiarella in the following passage:

⁶ In fact, the existential quantifier, at the first order, can have as its argument only a generic individual x , that, as such, does not concretely exist as a *unique* individual (see the linguistic distinction between “sortal” and “proper” names). On the contrary, the existence predicate can have as its argument only an individual, a , concretely existing in its *uniqueness*, so to make problematic the identification with its generic occurrence y , $\langle y = a \rangle$, in Cocchiarella' definition 3. We discuss below this essential topic.

Just as a predicate can be taken to stand in double way both for a concept and a natural property or relation, so too a predicate variable can be taken in a double way to have both predicable concepts and natural properties or relations as its values. The difference between the universals in the one order and the universals in the other is reflected not in a difference between two types of predicate constants or variables—where the one type stands for concepts and the other stands for natural properties and relations—but in the kind of (higher-order) reference that is made by means of predicate quantifiers, i.e., the quantifiers that can be affixed to predicate variables and that determine the conditions under which a predicate constant can be substituted for a predicate variable. In this way, the difference is reflected not in a difference of types of predicate variables to which predicate quantifiers can be affixed, but in a difference between the predicate quantifiers themselves, i.e., in the types of referential concepts the quantifiers stand for (Cocchiarella, 2013, p. 317).

It is thus possible to distinguish in CNR a *double signification*, “natural”, on one side, and “conceptual”, on the other side, of the same predicate, that, following Cocchiarella, can be symbolically translated into a *double indexing* of the predicative quantifiers — i.e., of the quantifiers having as their arguments the same predicate variables, respectively in their conceptual or natural use:

1) $(\forall F^j)(\exists x_1), \dots, (\exists x_j) F(x_1, \dots, x_j)$: *conceptual* meaning (i.e., the predicate F means a concept).

The quantifiers are without indexes, because the conceptual meaning is the normal case in CNR.

2) $(\forall^n F^j) \diamond^C (\exists^e x_1), \dots, (\exists^e x_j) F(x_1, \dots, x_j)$: *natural* meaning (i.e., the predicate F means a natural property/relation. Where: \forall^n means that the predicative variable, argument of the quantifier, is denoting a natural property/relation F . \exists^e means that the finite set of individual variables, x_1, \dots, x_j , arguments of the quantifier, are denoting a set of natural beings *actually existing*. \diamond^C means that the modal operator of possibility has to be intended in an *alethic-ontic* sense of *causal possibility*, C , “real” and not “logical”.

In other terms, it depends on the proper causal concomitance, whether the predicate F is satisfied by actually existent individuals. For instance, if F is for the predicate “being dinosaur”, it is evident that it cannot be satisfied by any actual existent individual. On the contrary, at the time in which it was satisfied, some million years ago, no existing individual could satisfy the predicate “being lizard”, while today they do. This does not mean at all that, “in the past” the lizards, like “today” the dinosaurs have no value of *biological reality*, given that they (the lizards) were, or (the dinosaurs) are, *potentially feasible in the biological reality, through the proper causal concurrence*. In other terms, they both are different *species* of the same *genus* of reptiles, actually existing in nature at different times, under different species. I.e., they belongs to different species because are products of different causal concourses, even though they belong to the same genus because they share a same “ancestor”, i.e., a same causal concourse before its branching into two different species.

On the contrary, this is not the case of mythological animals like “the phoenix”, always reviving from its ashes. It never – neither in the past, nor at the present, nor in the future – could be implemented in a matrix of biological causality, because of its *fictional* and not *biological* nature (essence)⁷. In naturalistic terms, “the phoenix”, or “the unicorn”, like whichever mythological animal, all are the products of the *mental* and not of the *biological* causality.

At this point, however, we have to introduce the deep difference between CNR and NR logical symbolism referring to the most important difference between the two approaches. Indeed, also in NR we maintain like in CNR the theory of the double indexation for the “actualist”, $\langle \forall^e F - \exists^e F / \forall^e x - \exists^e x \rangle$, and the “possibilist”, $\langle \forall F - \exists F / \forall x - \exists x \rangle$, quantifiers, but with a radical difference. The possibilist quantification in NR refers to a *natural* possibility, i.e., it refers to a *causal power*, and not to a *conceptual* faculty like in CNR, because of the two different foundation axioms – the “comprehension axiom” in CNR vs. the “ontological axiom of foundation” in NR – on which the relative QML’s are based. Consequently, both CNR and NR maintain the theory of the double reference, “natural” and “conceptual” of the very same predicate, with the double “indexation” of the relative

⁷ Cocchiarella developed a conceptualist formal ontology for *fictional* objects that we cannot discuss here (Cocchiarella, 2007, p. 163-166), also because it is deeply different from the formal ontology of them we might develop in NR.

quantifiers, since this idea, as I suggested to Cocchiarella, has a common ancestor in Aquinas semantics (see note 2, above). However, in NR –like in Aquinas – the “normal case” is the *natural* predication, not the *conceptual* predication. So, the quantification without index, $\langle \forall F - \exists F / \forall x - \exists x \rangle$, refers to the *natural* predication, while it is necessary an appropriate indexation m – for “mental”, i.e. $\langle \forall^m F - \exists^m F / \forall^m x - \exists^m x \rangle$ – for the relative *conceptual* predication, based, of course, on a particular case of natural causality, the causality characterizing some cognitive agents like humans.

2.1.2 THE ALLEGED COSMOLOGICAL RELEVANCE OF CNR ONTOLOGY AND ITS CONCEPTUALIST LIMIT

What makes interesting the CNR is its pretension of giving a formal ontology suitable of the *change of paradigm* related to the contemporary evolutionary cosmology, because of its pretension that its possibilist ontology can reckon with natural kinds of physical and/or biological objects that are only *potentially existing* in the cosmological causal, nested texture, within one of the many possible universes. In this way, such a natural kind (genus/species) and its members can be made actually existing, and/or actually extinct (i.e., made newly only potentially existing), in different spatio-temporal cells, within a given universe of the overall multiverse evolution. This is only an alleged pretension in CNR, however, because of the conceptualist bias weakening intrinsically the CNR ontology, and its use of causal modal operators \Box^C/\Diamond^C , as arbitrary (hypothetical) cut down’s of its possibilist QML. This intrinsic limitation is perfectly expressed by Cocchiarella himself in the following passage:

There is no general comprehension principle that is valid in natural realism, incidentally, the way that the comprehension principle (\mathbf{CP}_λ^) [i.e., the classical unrestricted comprehension principle of ZFC “enriched” with Church’s variable binding operator λ] is valid for conceptual realism. Natural properties and relations are not formed, or constructed out of other properties and relations by logical operations. But this does not mean that no natural property or relation can be specified in terms of a complex formula, i.e., a formula in which logical constants occur. What it does mean is that such a specification cannot be validated on logical grounds alone, but must be taken as a contingent hypothesis about the world (Cocchiarella, 2007, p. 280), italics and square parentheses are mine).*

It is hard to accept that a formal ontology of the natural realism can be based on such a *posit*, overall when it is extended to the logic of the natural kinds in it, concerning not abstract collections as the natural properties and relations (classes as ones), but plural objects such as the natural kinds, i.e., the classes-as-many of actually (or potentially) *existing* physical beings. How is it possible, in other terms, interpreting as “hypotheses” the *variables* of a logical calculus? Hypotheses can be at last the *models*, i.e., the interpretations on a given domain of such variables!

Finally, what is lacking in CNR is a general principle of *construction* of natural properties and relations out of other, more fundamental, natural properties and relations, i.e., a sort of “ontological calculus”, effectively an *ontological (causal) principle of construction*, of which a naturalistic formal ontology might define the proper logic. Such a limitation is consistent also with the other *posit* typical of CR and of CNR according to which, differently from logical realism, no *principle of rigidity* (**PR**), i.e. of trans-world identity, is allowed in CR, and hence in CNR. Where **PR** can be defined as the *stipulation* – as rightly Cocchiarella himself defines it – that a given property F is co-extensive in any possible world, i.e.:

$$\mathbf{PR} : \Box(\forall F)(\exists G)(Rigid(G) \wedge (\forall x_1) \dots (\forall x_j)[F(x_1 \dots x_j) \leftrightarrow G(x_1 \dots x_j)]) \quad (2-1)$$

This impossibility in CNR is another consequence of the impossible coexistence in CNR between the **S4** logic of the causal modal operators with the **S5** logic supposed by **PR**. In other terms, a suitable formal ontology of a modal natural realism, able to reckon with an evolutionary cosmology extended to the same physical laws and of the equivalence classes of natural objects they found, needs a *causal version* of **PR**, i.e., **PR^C**, based on a *causal stipulation principle*, founded, on its turn, onto an accessibility relation R among possible worlds, with the characters of the stipulation relation S

(see below), allowing a nested character of the causal modal operators and of the relative quantifiers, i.e., \Box_n^C/\Diamond_n^C and \forall_n/\exists_n , with $n \geq 0$, and hence allowing too a nested hierarchy of **secondary S5** and of the related “stratified” \mathbf{PR}^C , i.e., **PSR**, evidently using a “nested” **KD45** logic in an ontological, and not deontic, interpretation (see 4.4.2). In such a case, the \mathbf{PR}^C that is equivalent, in this ontological modal natural realism, to the **PR** of the logical realism, is the primary **0 level** or, properly, the primary *metaphysical* level, of the *summa genera* or metaphysical categories. They causally (not logically!) include, through a *proper foundation axiom* we define below in the NR formal ontology, all the other ontological further levels of necessity/universality so allowed.

Because both **PR** and \mathbf{PR}^C are not realizable in the socio-biological conceptualism of CNR, Cocchiarella rightly concludes that “the metaphysical necessity and possibility are not the same as conceptual necessity and possibility” (Cocchiarella, 2007, p. 133).

Where we disagree is in the connected statement that *only* the logical realism can give metaphysics a suitable **S5** logic. Indeed, it is evident that, logical realism, and its naturalist version of the LA, is the formal ontology of the Galilean-Newtonian physics as far as, this physics can be considered as a sort of “empirical confirmation” of the Platonic metaphysics against the Aristotelian naturalism, as A. V. Koyré suggested in his famous historical reconstruction of the beginning of modern science (Koyré, 1939; 1968). It is as much evident, however, that the evolutionary cosmology constitutes a change precisely in the Newtonian paradigm of modern science, and our thesis is that NR formal ontology, by the constructive principle of the nested “causal stipulation” of a stratified uni-(multi-)verse of possible worlds can be the proper *metaphysics* of such a new paradigm of modern cosmology.

3 A change of paradigm in fundamental physics: from QM to QFT

3.1 “Is physics legislated by cosmogony?”

Perhaps, the better synthesis of the actual change of paradigm in fundamental physics is the positive answer that it seems necessary to give to the following question: “Is physics legislated by cosmogony?” Such a question is the title of a visionary paper wrote in 1975 by J. A. Wheeler and C. M. Patton and published in the first volume of a fortunate series of the Oxford University about the quantum gravity (Patton & Wheeler, 1975).

Such a revolution originally amounts to the so-called *information theoretic approach* in quantum physics as the natural science counterpart of a dual ontology taking information and energy as two fundamental magnitudes in basic physics and cosmology. This approach started from Richard Feynman’s influential speculation that all of physics could be simulated by a quantum computer (Feynman, 1982), and from the famous “it from bit” ontological principle stated by his teacher J. A. Wheeler universe (Wheeler, 1990, p. 75). The cornerstones of this reinterpretation are, however, D. Deutsch’s demonstration of the universality of the Quantum Universal Turing Machine (QTM) (Deutsch, 1985), and overall C. Rovelli’s development of a *relational* QM (Rovelli, 1996). An updated survey of such an informational approach to fundamental physics is in the recent collective book edited by H. Zenil and with contributions, among the others, of R. Penrose, C. Hewitt, G. J. Chaitin, F. A. Doria, E. Fredkin, M. Hutter, S. Wolfram, S. Lloyd, besides the same D. Deutsch (Zenil, 2013).

There are, however, several theoretical versions of the information theoretic approach to quantum physics. It is not important to discuss all of them here (for an updated list in QM, see, for instance (Fields, 2012)), even though all can be reduced to essentially two.

The first one is related to a classical “infinitistic” approach to the *mathematical physics* of information in QM. Typical of this approach is the notion of the *unitary evolution* of the *wave function*, with the connected, supposed *infinite* amount of information it “contains”, “made available” in different spatio-temporal cells via the mechanism of the “decoherence” of the wave function. Finally, essential for this approach is the necessity of supposing *an external observer* (“information for

whom?” (Fields, 2012)) for the foundation of the notion and of the measure of information, reduced to the only Shannon’s, purely syntactic, measure and notion of information in QM (Rovelli, 1996). Among the most prominent representatives of such an approach, we can quote the German physicist H. D. Zeh (Zeh, 2004; 2010) and the Swedish physicist at the Boston MIT, M. Tegmark (Tegmark, 2011).

The second approach, the emergent one today, is related to a “finitistic” approach to the *physical mathematics* of information, taken as a fundamental physical magnitude together with energy. It is related to QFT, because of the possibility it gives of spanning the microphysical, macrophysical, and even the cosmological realms, within one only quantum theoretical framework, differently from QM (Blasone, Jizba, & Vitiello, 2011).

3.2 From QM to QFT in fundamental physics

The theoretical, core difference between the two approaches can be essentially reduced to the criticism of the classical interpretation of the QFT as a “second quantization” as to the QM. In QFT, indeed, the classical Stone-Von Neumann theorem (Von Neumann, 1955) does not hold. This theorem states that, for system with a *finite* number of degrees of freedom, which is always the case in QM, the representations of the canonical commutation relations are all *unitarily equivalent to each other*, so to justify the exclusive use of Shannon information in QM.

On the contrary, in QFT systems, the number of the degrees of freedom is not finite, “so that infinitely many unitarily inequivalent representations of the canonical commutation (bosons) and anti-commutation (fermions) relations exist”. Indeed, through the principle of the *Spontaneous Symmetry Breaking* (SSB) in the ground state, infinitely (not denumerable) many, quantum vacua conditions, compatible with the ground state, there exist. Moreover, this holds not only in the relativistic (microscopic) domain, but also it applies to non-relativistic many-body systems in condensed matter physics, i.e., in the macroscopic domain, and even on the cosmological scale (Blasone, Jizba, & Vitiello, 2011, p. 18. 53-96).

Indeed, starting from the discovery, during the 60’s of the last century, of the dynamically generated long-range correlations mediated by the Nambu-Goldstone bosons (Goldstone J. , 1961; Goldstone, Salam, & Weinberg, 1962), and hence of their role in the local gauge theory by the Higgs field, the discovery of these collective modes changed deeply the fundamental physics. Before all, it appears as an effective, alternative method to the classically Newtonian paradigm of the perturbation theory, and hence to its postulate of the asymptotic condition. In this way, the spontaneous breakdown of quantum vacuum symmetry; the thermal field theory; the phase transitions in a variety of problems at any scale; the process of defect formation during the process of non-equilibrium symmetry breaking in the phase transitions, characterized by an order parameter; all these phenomena and many others, can be studied by using the same approach of the inequivalent representations in QFT.

The emerging picture for the naturalistic ontology is thus deeply different from the atomism of the Newtonian one, as much the notion of mechanical vacuum is different from the notion of quantum vacuum. The ontological paradigm of physical system is no longer the isolated particle in the mechanical vacuum (= atomism) of which Carnap’s LA constitutes its formal ontology counterpart. In QFT no microscopic physical system is conceivable as completely isolated (closed), since *it is always in interaction with the background fluctuations* (quantum vacuum condition, including in itself all the universes). In this sense, “QFT can be recognized as an *intrinsically thermal* quantum theory” (Blasone, Jizba, & Vitiello, 2011, p. ix).

Of course, because of the intrinsic character of the thermal bath, the whole QFT system can recover the classical Hamiltonian character, because of the necessity of anyway satisfying the energy balance condition of each QFT (sub-)system with its thermal bath ($\Delta E = 0$), mathematically formalized by the “algebra doubling”, between an algebra and its co-algebra (Hopf algebras) (Vitiello, 2007). The more evident difference between QM and QFT is thus the deeply different physical interpretation of the Heisenberg uncertainty principle and of the related particle-wave duality. While in QM the Heisenberg uncertainty reads:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

where x is the position p the momentum of the particle and \hbar is the normalized Planck constant, in QFT the same relation reads:

$$\Delta n \Delta \varphi \geq \hbar$$

Where n is the number of quanta of the force field, and φ is the field phase. If ($\Delta n = 0$), φ is undefined so that it makes sense to neglect the waveform aspect in favor of the individual, particle-like behavior. On the contrary if ($\Delta \varphi = 0$), n is undefined because an extremely high number of quanta are oscillating together according to a well-defined phase, i.e., within a given coherence domain. In this way, it would be nonsensical to describe the phenomenon in terms of individual particle behavior, since the collective modes of the force field prevail.

In QM the uncertainty and hence the wave-particle duality relationship is between two representations, particle-like and wave-like, and accordingly the uncertainty is, respectively, on the momentum or on the position of the particle. In any case, the Schrödinger wave function in QM is not the expression of some dynamic entity like a force field, but simply the expression of different way of representing the quantum phenomenon. Hence, a conceptualist approach like CNR to the formal ontology of QM makes sense, just as a conceptualist ontology like the Kantian one makes sense for the Newtonian classical mechanics.

On the contrary, in QFT the duality is between two dynamic entities: the fundamental force field and the associated quantum particles that are simply the quanta of the associated field, different for different types of particles. In such a way, the quantum entanglement does not imply any relationship between particles like in QM, but simply it is an expression of the unitary character of a force field. In other terms, Schrödinger wave function of QM is only a rough statistical coverage of a finest structure of the dynamic nature of reality.

3.3 Quantum systems as “open systems” in QFT and the notion of information

Therefore, because of the intrinsic openness to the quantum vacuum fluctuations of any QFT system, and because of the associated thermal bath, it is possible to define in QFT, thermodynamic operators such as “entropy” and “free energy”, as well as the dynamic role they play in the different QFT systems. From the ontological standpoint, the notion of dynamically generated long-range correlations, mediated by different condensations of Nambu-Goldstone bosons, and the related notion of phase transition in terms of the dynamic constitution of different *phase coherence domains*, like as many SSB conditions of the quantum vacuum ground state, gives a new light to the Schrödinger notion of information as *neghentropy* in fundamental physics.

Indeed, at the relativistic microscopic level, a phase coherence propagate with a phase velocity of the order c^2/v , where c is the light velocity, and $v \ll c$ is the velocity of propagation of the (energy) signal. Therefore, the dynamic constitution of a coherence domain, by the SSB of the quantum vacuum (=long-distance correlations) in the ground state, corresponds to the definition of an optimal *dynamic channeling* for the successive propagation of the energy added to the system from the thermal bath. This is traveling only with velocity $v \ll c$, bringing the system out of the ground state (out of equilibrium stability condition).

All this emphasizes the logical and ontological relevance of the following passage, synthesizing the widespread applicability of QFT in the whole domain of fundamental physics, from cosmology, to the physics of condensed matter, living and neural systems included. This is particularly true nowadays, after that the empirical confirmation of the so-called “Higgs mechanism” in QFT and hence of the Standard Model in quantum physics, awarded with the Nobel Prize P. Higgs and F. Englert.

Quantum dynamics underlies macroscopic systems exhibiting some kind of ordering, such as superconductors, ferromagnets or crystals. Even the large-scale structures in the Universe, as well as the ordering in the biological systems, appear to be the manifestation of the microscopic dynamics ruling the elementary components of these systems. Therefore, in our discussion of the spontaneous breakdown of symmetry and collective modes, we

stress that one crucial achievement has been recognizing that quantum field dynamics is not confined to the microscopic world: crystals, ferromagnets, superconductors, etc. are macroscopic quantum systems. They are quantum systems not in the trivial sense that they are made by quantum components (like any physical system), but in the sense that their macroscopic properties, accounted for by the order parameter field, cannot be explained without recourse to the underlying quantum dynamics (Blasone, Jizba, & Vitiello, 2011, p. ix).

Very recently, for instance, the QFT approach is obtaining several significant successes in the precise comprehension of superconductivity mechanisms (Comin & al., 2014; da Silva Neto & al., 2014); at the same time giving an essential contribute for “lifting the fog” actually covering the notion of “complexity” both in fundamental physics and in fundamental biology (Morr, 2014). As to the cosmological application of QFT, among the many versions today available, I like to quote only the proposal of another Nobel Laureate, George W. Smoot, who recently proposed a cosmological picture based on QFT and the related “holographic principle”, as well as of the interpretation of gravity as an entropic force⁸.

Gravity is a macroscopic manifestation of a microscopic quantum theory of space-time, just as the theories of elasticity and hydrodynamics are the macroscopic manifestation of the underlying quantum theory of atoms. The connection of gravitation and thermodynamics is long and deep⁹. The observation that space-time has a temperature for accelerating observers and horizons is direct evidence that there are underlying microscopic degrees of freedom. The equipartition of energy, meaning of temperature, in these modes leads one to anticipate that there is also an entropy associated. When this entropy is maximized on a volume of space-time, then one retrieves the metric of space-time (i.e. the equations of gravity, e.g. GR [General Relativity]). Since the metric satisfies the extremum in entropy on the volume, then the volume integral of the entropy can readily be converted to surface integral, via Gauss’s Theorem. This surface integral is simply an integral of the macroscopic entropy flow producing the mean entropy holographic principle (Smoot, 2010, p. 2247). [Square parentheses are mine].

What is here to be emphasized is that in QFT the Wigner function (WF), on which the probabilities of the physical states are calculated, are deeply different from the Schrödinger wave function of QM, not only because the former, differently from the latter, is defined on the phase space of the system. Indeed, it measures a physical entity – the force field – and not a conceptual representation of a physical particle, related to a measure operation, like the wave function in QM. What is much more fundamental is that the WF uses the notion of quasi-probability (Cahill & Glauber, 1969), and not the notion of probability of the classical Kolmogorov axiomatic theory of probability (Kolmogorov, 1956).

It is well-known that the notion of quasi-probability, not only violates the third axiom of the classical theory, because *negative probabilities* are allowed. It also violates the fifth axiom, because regions integrated under given expectation values do not represent *mutually exclusive states* – i.e., the separation of variables in such distributions is not fixed, but in the case of phase transitions, can evolve dynamically. From the computability theory standpoint, this means that a physical system in

⁸ On this regard, Smoot quotes two recent contributions, of the Indian physicist T. Padmanabhan, and of the Dutch physicist E. Verlinde, who showed that gravity emerges naturally like an entropic force from the cosmological holographic principle, in relationship with the information associated to the body position. The core of Verlinde’s heuristic argument consists in the evidence that “only a finite number of degrees of freedom are associated with a given spatial volume, as dictated, by the holographic principle. The energy, that is equivalent to the matter, is distributed evenly over the degrees of freedom, and thus leads to a temperature. The product of the temperature and the change in entropy due to the displacement of matter is shown to be equal to the work done by the gravitational force. In this way we find that Newton’s law of gravity emerges in a surprisingly simple fashion” (Verlinde, 2011, p. 1107). See (Padmanabhan, 2010), quoted by Verlinde himself, for the application of the same idea to the quantum gravitation in cosmology.

⁹ Effectively, it starts with Aristotle who explained the gravitation effect with the common sense equivalent of the energy equipartition principle. That is, the principle according to which “heat prefers the centrifugal direction” for filling all the available space, so to make anisotropic (and hence non-geometrical) the physical space of mechanics. In fact, Galilei and Newton after Aristotle, like Democritus before him, made geometrical the space of mechanics by abstracting from heat. So, in the Aristotelian cosmology, because the heater bodies are obviously also the lighter ones, they naturally tend to go toward the higher places (effectively, they take the centrifugal direction in an “earth-centered” universe). The colder and heavier ones, on the contrary, take the opposite direction (centripetal) toward the lower places, i.e., toward the “earth”, the cold center of the Aristotelian universe, so to create the gravitational effect.

QFT, against the Turing Machine paradigm, is able to change dynamically “the basic symbols” of its computations, since new collective behaviors can emerge from individual ones, or vice versa. In this way, this justifies the definition of the information associated with a Wigner distribution as a “semantic information content”. In formal logic an inference process, based on such a probability calculus, in which the basic symbols (and hence “truth”) between the antecedent and the consequent *are not conserved*, cannot satisfy the logical connective of the material implication ($p \rightarrow q$ (1011)). On the contrary, it satisfies the logical connective of the converse implication ($p \leftarrow q$ (1101)), i.e., the connective of all the “form generation” processes, as we see in the Section 4. However, it is also the logic of an *inductive inference*, not as a logic of the (empirical) *corroboration* of true propositions already given, as usual after Hume, Stuart-Mill and Carnap induction theory, but as the logic of the Aristotelian (*onto*-logical) *constitution* of new true propositions.

The semantic information in QFT computations hence satisfies, the notion of “*contingent (not logical) truth*”, so to escape the Bar-Hillel & Carnap paradoxes (Carnap & Bar-Hillel, 1964), just like the “Strong Semantic Information” does in Floridi’s theory, with which it shares the same Wigner probability distribution (Floridi, 2011).

Physically, I repeat, this is the logic of the “collective behavior” characterizing the formation, on dynamical basis, of “phase coherence domains”. In cognitive neuroscience it includes a representation and its object, in the case of cognitive agents like human intentional brains, as Vitiello & Freeman demonstrated (= theory of the “extended mind”).

3.4 *The Doubling of Degrees of Freedom in QFT and in cognitive neuroscience*

We know that the relevant quantum variables in biological system are the electrical dipole vibrational modes in the water molecules, constituting the oscillatory “dynamic matrix” in which also neurons, glia cells, and the other mesoscopic units of the brain are embedded. The condensation of Goldstone massless bosons (named, in the biological case, Dipole Wave Quanta, DWQ) — corresponding, at the mesoscopic level, to the long-range correlation waves observed in brain dynamics — depends on the triggering action of the external stimulus for the symmetry breakdown of the quantum vacuum of the corresponding brain state. In such a case, the “memory state” corresponds to a coherent state for the basic quantum variables, whose mesoscopic order parameter displays itself at the mesoscopic level, by the amplitude and phase modulation of the carrier signal.

In the classical Umezawa’s model (Umezawa, 1995), however, the system suffered in an “intrinsic limit of memory capacity”. Namely, each new stimulus produces the associated DWQ condensation, by cancelling the precedent one, for a sort of “overprinting”. *This limit is systematically overcome in dissipative QFT where the many-body model predicts the coexistence of physically distinct amplitude modulated and phase modulated patterns*, as it is observed in the brain. That is, by considering the brain as it is, namely an “open”, “dissipative” system continuously interacting with its environment, there not exists one only ground (quantum vacuum) state, like in thermal field theory of Umezawa where the system is studied at equilibrium. On the contrary, in principle, there exists infinitely many ground states (quantum vacuum’s), so to give the system a potentially infinite capacity of memory. To sum up, the solution of the overprinting problem relies on three facts (Vitiello, 2004):

- 1) In a dissipative (non-equilibrium) quantum system, there are (in principle) infinitely many quantum vacuum’s (ground or zero-energy) states, on each of which a whole set of non-zero energy states (or “state space” or “representation states”) can be built.
- 2) Each input triggers one possible irreversible time-evolution of the system, by inducing a “symmetry breakdown” in one quantum vacuum, i.e., by inducing in it an ordered state, a coherent behavior, effectively “freezing” some possible degrees of freedom of the constituting elements behaviors (e.g., by “constraining” them to oscillate on a given frequency). At the same time, the input “labels” dynamically the induced coherent state, as an “unitary non-equivalent state” of the system dynamics. In fact, such a coherent state persists in time as a ground state (DWQ are not energetic bosons, are Goldstone bosons) so to constitute a specific “long-term” memory

state for such a specific coupling between the brain dynamics and its environment. On the other hand, a brain no longer dynamically coupled with its environment is, either in a pathological state (schizophrenia), or it is directly dead.

- 3) At this point emerges the DDF principle as a both physical and mathematical necessity of the model. Physical, because a dissipative system, even though in non-equilibrium, must anyway satisfy the *energy balance*. Mathematical, because the 0 energy balance requires a “doubling of the system degrees of freedom”. The *doubled* degrees of freedom, say \tilde{A} (the tilde quanta, where the non-tilde quanta A denote the brain degrees of freedom), thus represent the environment to which the brain is coupled. The environment (state) is thus represented as the “time-reversed *double*” of the brain (state) on which it is impinging. The environment is hence “modeled on the brain”, according to the finite set of degrees of freedom the environment itself elicited. Anyway, which are the available degrees of freedom to be elicited for that input depends on the brain itself that, for this reason, is effectively a *self-organizing* system.

Of course, the point 3 represents the essential idea of the “doubling algebra” (algebra/co-algebra) formalism, constituting the mathematical core of the dissipative QFT model that we cannot illustrate here, and for which we refer to (Celeghini, Rasetti, & Vitiello, 1992), and to the wide literature quoted in (Vitiello, 2009). Of the DDF we have illustrated elsewhere its logical relevance, for an original solution of the reference problem (see (Basti, 2013a; 2013b) and below.

The huge amount of such an experimental evidence found, during the last ten years, its proper mathematical modeling in the dissipative QFT approach of Vitiello and his collaborators, so to justify the publication during the last years of several joint papers on these topics (see, for a synthesis, (Freeman & Vitiello, 2006; Freeman & Vitiello, 2008)).

To sum up (Vitiello, 2009), Freeman and his group used several advanced brain imaging techniques such as multi-electrode EEG, electro-corticograms (ECoG), and magneto-encephalogram (MEG) for studying what neurophysiologist generally consider as the *background activity* of the brain, often filtering it as “noise” with respect to the synaptic activity of neurons they are exclusively interested in. By studying these data with computational tools of signal analysis to which physicists, differently from neurophysiologists, are acquainted, they discovered the massive presence of patterns of AM/FM phase-locked oscillations. They are intermittently present in resting and/or awake subjects, as well as in the same subject actively engaged in cognitive tasks requiring interaction with the environment. In this way, we can describe them as features of the background activity of brains, modulated in amplitude and/or in frequency by the “active engagement” of a brain with its surround. These “wave packets” extend over coherence domains covering much of the hemisphere in rabbits and cats (Freeman W. J., 2004; Freeman W. J., 2004; Freeman W. J., 2005; Freeman W. J., 2006), and regions of linear size of about 19 cm in human cortex (Freeman, Burke, Holmes, & Vanhatalo, 2003), with near zero phase-dispersion (Freeman, Ga'al, & Jornten, 2003). Synchronized oscillations of large-scale neuron arrays in the β and γ ranges are observed by MEG imaging in the resting state and in the motor-task related states of the human brain (Freeman & Rogers, 2003).

3.5 A change of paradigm in the ontology of the natural sciences

In this same QFT framework, P. Davies recently edited a book, together with the Danish philosopher N. H. Gregersen, *Information and the nature of reality. From physics to metaphysics* (Davies & Gregersen, 2010), in which he announces with enthusiasm the birth of a new *scientific ontology* with an empirical and even *experimental* outcome¹⁰, deeply different from the Platonic-Newtonian

¹⁰ Effectively, Davies does not speak about duality. His paper is aimed at vindicating simply the *ontic* nature of information as a fundamental physical magnitude in cosmology and hence in all nature, against the subjective interpretation of it by Leo Szilard, dominating in the last half of the XX century, before the actual development of the information physics and cosmology.

paradigm. Davies introduces this change of paradigm in the evolutionary cosmology in the following provocative way,

In a universe limited in resources and time – for example, in a universe subject to the *cosmic information bound*¹¹ - concepts such as real numbers, infinitely precise parameter values, differentiable functions and the unitary evolution of the wave function are a fiction: a useful fiction to be sure, but a fiction nevertheless (Davies, 2010, p. 82).

In other terms, the change of paradigm consists in the turnaround of the “Platonic” relationship, characterizing the Galilean-Newtonian beginning of the modern science:

Mathematics → *Physical Laws* → *Information*

Into the other one, much more powerful for its heuristic power:

Information → *Mathematics* → *Physical Laws*

The specific problem on this regard, as we have anticipated, but we cannot develop here, is about the notion and measure of “information” in QFT. This is based, indeed, on the notion and measure of “quasi-probability”, typical of the Wigner function, and not on the classical notion of probability proper, both of the wave functions of QM, and of the Shannon information notion and measure as well.

To sum up, what contemporary fundamental physics and fundamental biology need is a formal ontology of the *causal foundation of the natural laws*. Namely, it is necessary a deep change of paradigm in ontology as to the LA of the Newtonian physics and of the Kantian epistemology, which imply, on the contrary, a *logical foundation of causality*, and hence an atomistic and reductionist approach to complex phenomena (Mazzocchi, 2008).

Of course, for formally justifying a causal foundation of the natural laws the “causal indexation” of modal operators (and of logical connectives), by an arbitrary cut-down of the relative quantifiers, among the non-denumerable many types allowed as in Cocchiarella’s CNR, it is not the solution (see above, §2.1.2). It is like to try to cure an infection with a placebo.

On the contrary, the attempt of the so-called “causal-set theory” goes in the right direction, as both L. Smolin and R. Penrose also recently emphasized (Smolin, 2013; Penrose, 2013, p. xxiv; Kronheimer & Penrose, 1967). Even though it is not sufficient, because, in set theory, it is in principle impossible to deal formally with a constructive approach to arboreal structures, without supposing the König lemma and the infinity of its paths.

Let us see now how NR formal ontology can suggest an original solution of such problems because, from one side it can support a non-reductionist approach to the constitution of the different types of “wholeness”, either characterizing the complexities in natural sciences, or the “sethood” in logic and mathematics. On the other side, NR is able, to support a constructive approach for demonstrating how “arboreal” structures (i.e., based on non-transitive, non-symmetric, non-reflexive relations) can justify the existence of equivalence structures (i.e., based on transitive, symmetrical and reflexive relations), like classes or sets, the logical laws ruling them included. That is, what the causal set theory is searching for, without having the formal means for solving the problem.

¹¹ It is a fundamental parameter of the physical universe that can be obtained in many ways. Seth Lloyd, for example, first calculated it starting from the quantum physics hypothesis that the states of matter are fundamentally discrete and form an enumerable set. It is thus possible to calculate approximately how many bits of information whichever volume of the universe can *actually* contain. Because the universe is expanding, but it is anyway *finite*, an event horizon can be defined within the universe itself. Therefore, for the actual universe inside this horizon at the actual time, the *cosmic information bound* is $\approx 10^{122} \approx 2^{400}$ bit. This number has a very elegant physical interpretation, because it is defined by the area of the whole horizon at a given time, divided for the smallest area allowed by quantum discretization, the so-called “Planck Area” $\approx 10^{-65}$ cm², given that in such an area 1 bit at last can be implemented. As Davies rightly suggests such a number is not new at all in physics. It corresponds to $N^{3/2}$, where N is the “Eddington-Dirac number”. Moreover, it corresponds also to the actual age of the universe calculated in atomic units. What is significant is that we obtain the same quantity also starting from the hypothesis of the “holographic universe” of t’Hooft e Susskind. In this way, this magnitude is an optimal candidate for becoming a new constant of nature like c or h . A constant, however, concerning not *energy* but *information*.

4.1 Premise: the distinction between logical and causal necessity

4.1.1 QUINE'S CRITICISM TO C. I. LEWIS' MODALIZATION OF THE LOGICAL IMPLICATION

In the first chapter of one of his masterpieces, *Mathematical Logic*, W. V. O. Quine, rightly emphasizes the difference between the semantic sense of the term “implies”, strictly related with the notion of “truth”, and the syntactic sense of the logical connective “if... then” and of its symbol “ \supset ”. Based on these considerations too often neglected, Quine criticizes, on one side, Whitehead and Russell, who blurred in their *Principia* such fundamental distinctions, and, on the other side, Lewis and Smith who, trying to solve such a misunderstanding, essentially missed the point.

What Quine rightly rebukes to C. I. Lewis' notion of “strict implication” is that it is a long way from representing a satisfactory theory of implication. At last, it offers a theory of “modes of statement composition” according to conditionals “of a non-truth-functional sort”, typical of the so-called intensional, and not extensional interpretations of modal logic.

On the contrary, according to Quine, a theory of implication that is satisfactory for metaphysical uses as Lewis originally pretended, has to take “statements as names of some sort of entities”, so “to take implication as a relation between those entities, rather than between the statements themselves”. Finally, it must be a theory able to justify also the “difference” or the “identity” of the entities designated by these statements, since the problem concerns not only relations such as “implication”, but also like “equivalence”, “compatibility” and similar (Quine, 1983, p. 31-32).

With these words, it is evident that Quine is saying us that a satisfactory theory of ontological implication has to be:

- 1) A theory of *metaphysical* and not *logical* implication, and hence of *causal* and not *logical* necessity, because it has to deal with relations among existing *entities* and not only among the statements referring to them.
- 2) A theory able to justify on a causal basis either the “differences”, or the “identities” among the denoted entities.
- 3) A theory able to illuminate, on the same causal basis, the alleged “obscurity” of such referential entities that are, in the light of Quine's examples and of the precedent discussion, “natural kinds”¹² or, if we want to use a word banned from the modern philosophical jargon, “natural essences”, or in short, “natures”.
- 4) Finally, a theory able to give also an ontological foundation of the notion of *truth*, given the strict relation existing between the notions of “implication” and of “truth”.

Let us see how NR ontology is able to satisfy all these requirements.

4.1.2 A MEDIEVAL SUGGESTION

The suggestion for a possible solution of the problem is coming from the Middle Age, by Thomas Aquinas who was interested like us in the foundation of a naturalistic ontology based on the *causal necessity*, as opposed to the logical necessity of the Platonic metaphysics. His aim was, indeed, to make the Christian metaphysics and theology compatible with the emergent naturalism of the Aristotelian ontology in the newborn Universities of the beginning of the second millennium.

Our aim, at the beginning of the third millennium, is similar, not only because we share the same theological convictions, but also because this is today in continuity with the necessity of substituting the logical realism of the Newtonian physical ontology (=LA) with the naturalism of the evolutionary approach in actual cosmology.

¹² Even though this is not in continuity with Quine's teaching, who always criticized the notion of “natural kind” as “objects” to which the common names refer. “Natural kinds”, however, both in NR and CNR, are not “objects” at all.

In this light, there is a fundamental passage from Aquinas *Commentary on Aristotle's Physics*, in which he explains what is proper of the demonstrative procedures in physical sciences, as based, differently from mathematical sciences, on the causal necessity, and not on the logical necessity. In the “Lecture 15” of this *Commentary* on the Second Book of *Physics*, with the title: “How necessity is found in natural things” (see, for the English translation of this text: (Blackwell, Spath, & Thirlkel, 1999, p. 135ff.)), Aquinas comments an Aristotelian passage [*Physics*, II, 199b,34 - 200b,9] very significant for our aims. In it Aristotle starts, by asking himself “whether the necessity (in physics) is ‘hypothetical’ (*ex ypothéseos*) or ‘simple’ (*aplôs*)”, that is “assertoric”, like in metaphysics and in logic. Aristotle chooses the first alternative, namely, the physical demonstrations are hypothetical, because of the contingent nature of the physical beings, but with an important difference, according to the *a priori* or *a posteriori* character of the causes involved¹³.

The Aquinas commentary on this passage is very interesting, because he makes a logical analysis of these two types of demonstration, showing a deep knowledge of the Stoic sentential logic of the hypothetical demonstrations, and of their two fundamental laws of the *modus ponens* and of the *modus tollens*, effectively not yet discovered by Aristotle. Moreover, this passage demonstrates as well his deep knowledge of the logic of the “converse implication”, of which Aquinas suggests in this case a modal development that is original, both with respect to the Aristotelian modal syllogism (Hintikka, 1972; van Rijen, 1989; Nortmann, 2002; Malink, 2006), and with respect to Lewis’ modal version of the material implication.

Effectively, Aquinas is able to distinguish in his *Commentary* of the quoted passage of Aristotle two types of hypothetical demonstrations in physics, corresponding in terms of modern logics, respectively, to the logic of the *direct* and of the *converse* implication. The first one, Aquinas says, is indeed similar to the “simple” character of the assertoric demonstrations of mathematics, which is the case when the demonstration is by causes that are *prior* as to the physical process, i.e. the *initial causes* from which a given process starts: the *material* and the *efficient* causes. In the LA of the modern Newtonian physics, these correspond to the *initial conditions* — respectively, the *position* and the *momentum* — of the classical mechanics.

Effectively, in another passage of his *Commentary* to Aristotle’s *Physics* (*In Phys.*, II, 11, 1-9), Aquinas says that this is the case of Democritus, mechanistic approach to physics. In it, no “formal cause” is necessary, since it supposes that the final state of a physical process is completely determined by its material and efficient, *initial* causes. In this way, he states, the initial causes are like the postulates of a geometric demonstration, while the final state is like a theorem deduced by such postulates, so that physics demonstrations are at all similar to geometry demonstrations.

However, Aquinas annotates, we cannot apply this logic to the *generation processes* of a new accidental or substantial form in the matter substratum, which correspond to what in physics we denote as *phase transitions*. Namely, the generation of a new accidental form occurs when a new state of a given substance occurs, without changing its nature (e.g., the passage of water between the icy and

¹³ According to the translation offered by Blackwell *et al.* in their version of the Aquinas *Commentary*, the core of the Aristotelian argument [*Physics*, II, 200a 15-33] reads as follows: “Necessity in mathematics is in a way similar to necessity in things which come to be through the operation of nature. Since a straight line is what it is, it is necessary that the angles of a triangle should equal two right angles. But not conversely; though if the angles are not equal to two right angles, then the straight line is not what it is either. However, in things which come to be for an effect, the reverse is true. If the effect is to exist or does exist, that also which precedes it will exist or does exist; otherwise just as there, if the conclusion is not true, the premise will not be true, so here the effect or ‘that for the sake of which’ will not exist. For this too is itself a starting-point, but of the reasoning, not of the action; while in mathematics the starting-point is the starting-point of the reasoning only, because there is no action. If then there is to be a house, such-and-such things must be made or be there already or exist, or generally the matter relative to the end, bricks and stones if it is a house. The end, however, is not due to these except as the matter, nor will it come to exist because of them. Yet if they do not exist at all, neither will the house, or the saw—the former in the absence of stones, the latter in the absence of iron—just as in the other case the premises will not be true, if the angles of the triangle are not equal to two right angles. The necessary in nature, then, is plainly what we call by the name of matter, and the changes in it. Both causes must be stated by the physicist, but especially the end; for that is the cause of the matter, not vice versa”.

the liquid state, in both senses). On the contrary, the generation of a new substantial form occurs when the transformation changes the nature of the substance (e.g., like in chemical reactions, and/or in biological generation of new individuals). In both cases, indeed, the initial causes are not able at all to determine completely the final state of the dynamics, so to make it fully predictable starting from them. We have thus to consider the physical process as a whole, the final state included. In such cases, indeed, the *formal causality* is involved¹⁴.

To sum up, in this case of the form-generation processes, Thomas continues, the logic of this type of inference is like when we reason about something that “ought to be” (*debeat esse*), as far as this is the intended final state of a given action by an intentional agent, so to be in its *active power*. With a fundamental difference, however. In the case of a physical process of a form generation, there is no *intentionality* involved, and so *no finalism*, so that we are not in the semantic realm of the *deontic* logic, but in the realm of the *alethic* logic, even though the syntax of the two forms of demonstration is the same. Practically, it is like in our case, when we use the syntax of **KD45** ML not in a deontic intensional interpretation, but in an alethic, ontological, intensional interpretation.

What Aquinas is suggesting us, indeed, is that we are only reckoning with a *non-mechanistic, but dual ontology of the physical causality*, because involving not only a change in *matter* but also in the *form* of the process – the coherence domain of a phase transition, in QFT terms. That is, a physical causality where the end-state of the physical process does not depend only on the initial conditions, like in Newtonian (and Democritus) mechanics, because the initial components of the process lose their individuality, so that *a new collective behavior of the system emerges*. Hence a *new natural form* emerges, even though Aquinas does not use the term “emergence”, but the term *eductio* (*eductio*) of the new form, evidently as opposed to the term *deduction*, valid for the logic of the mechanics demonstrations¹⁵.

In both cases – the *intentional* case and the *physical* case of the form emergence –, Aquinas says, it is like if the final state – either if it is intended by some intentional agent (psychology), or if it is not (physics) –, plays the role of the premise of the inference. This means that formally, we are faced with a *converse implication*, $\langle p \leftarrow q \rangle$ and no longer with a *direct implication* $\langle p \rightarrow q \rangle$ like in mechanics.

To sum up, Aquinas is affirming here an *analogy* – formally: the same *syntactic* structure, interpreted semantically onto two different *domains* – between *the causal realm of ontology*, when we deal with a physical process, in which a new level of matter organization emerges, as unpredictable from the initial material and efficient causes, and *the deontic realm of intentionality*. Without confusing them, however, that is, without confusing the “to be” (alethic), with the “ought to be” (deontic) at the semantic level, so to fit perfectly with the requirements of the so-called “Hume law”. In both cases, indeed, their logic, their syntactic structure, is the logic of the *converse implication*, according to the **KD45** ML, interpreted as a **secondary S5** and not as a **deontic S5**, following the two more diffused, ordinary ways of connoting **KD45** in current modal logic literature.

Thomas states explicitly all this in a successive passage that we report here completely:

Next where he [Aristotle] says, ‘Necessity in mathematics ...’ (200 a 15), he compares the necessity which is in the generation of natural things to the necessity which is in the demonstrative sciences. (...)

Indeed, an ‘a priori’ necessity is found in the demonstrative sciences, as when we say that since the definition of a right angle is such, it is necessary that a triangle be such and so, i.e., that it have three angles equal to two right angles. Therefore, from that which is first assumed as a principle, the conclusion arises by necessity [i.e., the logical necessity of the *modus ponens*].

¹⁴ Recently, the German philosopher U. Meixner developed a semi-formal analysis of the “formal necessity” in the Aristotelian theory of causality similar to ours, even though from the standpoint of the ontology of the logical realism (Plato), and not of natural realism (Aristotle) like ours (Meixner, 2003).

¹⁵ It is evident the similarity with the ontology underlying QFT and its interpretation of the particle-field duality principle, we discussed in §3, $\langle \Delta n \Delta \varphi \geq \hbar \rangle$, whereas the collective modes of the force field prevail over the individuality of the particles.

The converse, however, does not follow, i.e., if the conclusion is, then the principle is. Because, *sometimes*, a true conclusion can be drawn from false propositions. On the contrary, it does follow that if the conclusion is not true, then, neither is the given premise true. Because, a false conclusion can be drawn only from a false premise [i.e., the logical necessity of the *modus tollens*].

On the contrary, in things which happen for the sake of something (*quae fiunt propter aliquid*), either according to technique, or according to nature, this converse does obtain [i.e., according to the connective of the *converse implication*]. For, if the final state (*finis*) either will be or is, then *it is necessary* that what is prior to the final state either will have been, or is [i.e., it is not question of time]¹⁶. If, however, that which is prior to the final state is not, then the final state will not be, just as in demonstrative sciences, if the conclusion is not true, the premise will not be true [i.e., both in direct and converse implication if the antecedent is false, the consequent is false too].

It is clear, therefore, that in things that come to be for the sake of a final state, the final state holds the same order that the premise holds in demonstrative sciences. This is so because the final state also is a principle, not indeed of action, but of reasoning. For, from the conclusion we begin to reason about those things that are the means for reaching such a conclusion. In demonstrative sciences, however, we do not consider a principle of action, but only a principle of reasoning, because there are no actions in demonstrative sciences, but only demonstrations. Hence, in things that happen because of reaching a final state, this properly holds the place that the premise holds in demonstrative sciences. Hence, there is a similarity on both sides, even though they seem related conversely because of the fact that the end is last in action, which does not pertain to demonstration (Aquinas, *In Phys.*, II, 15, 5) [Square parentheses are mine].

Aquinas suggestion is thus double:

- 1) The logic of the emergent complexities in physics (form generation), and/or of the spontaneous symmetry breakdown of the infinitely many quantum vacuum conditions in QFT, is the logic of the converse implication, i.e., the logic of the *causal necessity* (= formal causality) as irreducible to the logic of the *logical necessity*;
- 2) If we want to have a proper formal ontology of the causal necessity, as far as – against Leibniz posit – it is not reducible to the logical necessity, we need to give a *modal version of the converse implication* as the proper logic of the *causal entailment*.

In other terms, just as the modal version of the material implication, i.e., the so-called “strict implication” of C. I. Lewis gives a definition of the *logical entailment*, i.e., “*q* follows logically from *p*” (Huges & Cresswell, 1996, p. 203)¹⁷, the opposite holds for the *causal entailment*, i.e., “*p* precedes causally *q*”. This is the proper logic of the formal ontology of NR, as opposed to the LA of classical mechanics, or to the CNR of QM.

4.2 *The logic of the converse implication as the logic of the causal necessity*¹⁸

To sum up, the double, convergent suggestions from Aquinas and Quine invite us to a deep re-consideration of the axiomatic theory of modal logic inherited by the pioneering work of C. I. Lewis at the beginning of the last century.

As we know, C. I. Lewis defined the notion of strict implication for avoiding the well-known paradoxes of implication related to the notion of the truth-functional conditional “if-then”, interpreted as material *implication* of the mathematical logic¹⁹. I.e., given the truth table of the material implication:

¹⁶ Namely, by using the modal operator “it is necessary”, what Aquinas is here suggesting is the necessity of a modalization of the converse implication for being the proper modal logic of the causal implication. See below.

¹⁷ As they explain, “to say that a proposition, *p*, entails a proposition, *q*, is simply an alternative way of saying that *q* follows logically from *p*, or that the inference from *p* to *q* is logically valid”.

¹⁸ For this sub-section and the following one I am deeply indebted with the doctoral work (Panizzoli, 2013) of my former student and now my collaborator Dr. Francesco Panizzoli.

¹⁹ See the deep reflections of Quine on this regard, summarized in § 4.1.1.

	p	q	$p \rightarrow q$
1.	1	1	1
2.	1	0	0
3.	0	1	1
4.	0	0	1

Several paradoxes, the so-called “paradoxes of the material implication” (Huges & Cresswell, 1996, p. 194), follow from this truth table, such as:

- 1) $p \rightarrow (q \rightarrow p)$
- 2) $\neg p \rightarrow (p \rightarrow q)$

I.e.: (1) given a true proposition, any proposition, either true or false, can imply it; (2) if a proposition is false, it implies any proposition whatsoever. Moreover, since for any proposition p , either the antecedent of (1), or the antecedent of (2) must be true, also the following paradox holds:

- 3) $(p \rightarrow q) \vee (q \rightarrow p)$.

For avoiding such paradoxes it is sufficient, Lewis suggests, to make “stronger” the notion of “implication”, so to distinguish between implications that hold materially, and implications that hold *necessarily* or *strictly*, namely, it is necessary that if p is true, so is q . From this the definition of the “strict implication”(\rightarrow) follows:

$$\text{Def.: } (\alpha \rightarrow \beta) := (\Box(\alpha \rightarrow \beta)) \leftrightarrow (\neg \Diamond(\alpha \wedge \neg \beta)) \quad (4-1)$$

Where α and β are propositional meta-variables.

Practically, it is like if we eliminate from the truth table of the material implication the 2nd row, so to grant the fundamental law of logical semantics that *truth is always preserved in any valid inference*, that is:

	p	q	$p \rightarrow q$
1.	1	1	1
2.	1	0	0
3.	0	1	1
4.	0	0	1

The intrinsic relationship between logical semantics and strict implication forces us to interpret the strict implication as “entailment”, that is, as a relation between *true propositions* and not between wff’s. Namely, $\langle p \rightarrow q \rangle$ properly means “ p entails q ” as the converse of “ q follows *logically* from p ”, or, in other terms, it properly means that “the inference from p to q is *logically* valid”. This semantics, however, originates the so-called “paradoxes of the strict implication”. They, unfortunately, are as many very strong ways for asserting that the so-called “principle of Pseudo-Scotus” or the “principle of explosion” (EP) (*ex contradictione sequitur quodlibet*) is a valid inference in logic (see paradox (1) below). According to (Huges & Cresswell, 1996, p. 203) a list of such paradoxes is, indeed, the following:

- 1) $(p \wedge \neg p) \rightarrow q$
- 2) $q \rightarrow (p \vee \neg p)$
- 3) $\neg \Diamond p \rightarrow (p \rightarrow q)$
- 4) $\Box q \rightarrow (p \rightarrow q)$

Now, Lewis himself stated that, if we want to avoid (1) and the other related paradoxes, we have to exclude other intuitively valid principles, before all the so-called “principle of the disjunctive syllogism”:

$$((p \vee q) \wedge \neg p) \rightarrow q \quad (4-2)$$

However, for excluding this principle, it is necessary to refer to the so-called *relevance logics* (Hughes & Cresswell, 1996, p. 205), i.e., it is necessary to define a valid criterion of *relevance* of a premise as to a given conclusion, that means using the notion of *paraconsistent* negation, refusing the general validity of the *same extensionality* between a proposition and its negation (Béziau, 2000). After our semi-formal presentation of the NR formal ontology we see that the logic of NR is precisely a *relevance logic*, introducing a *formal* ontological criterion of relevance of a given premise as to a given conclusion.

As a first step, following Aquinas suggestion, let us introduce now the notion of *converse implication* and of its “strict”, modal version. The truth table of the converse implication is the following:

	p	q	$p \leftarrow q$
1.	1	1	1
2.	1	0	1
3.	0	1	0
4.	0	0	1

Anyway, if we interpret the converse relation as a *syntactic* relation among wff, it has no relevance for an ontology that, as such, is simply an interpretation of a modal calculus. On the contrary, if we want to use the converse implication for justifying a formal ontology of the *causal* necessity as complementary of the *logical* necessity, we have to interpret also it semantically, as a *strict* converse implication relating statements denoting *things causally related*, as Quine required for justifying a notion of *ontological implication* (see above 4.1.1). In such a case, it makes sense to define the notion of *causal* necessity, as eliminating the possibility that an *effect* (denoted by q) *exists* without its *cause* (denoted by p)²⁰. In other term, we have to eliminate the 3rd row of the converse implication truth table. I.e.:

	p	q	$p \leftarrow q$
1.	1	1	1
2.	1	0	1
3.	0	1	0
4.	0	0	1

From this truth table, the semantic interpretation of the “strict converse implication” ($p \leftarrow q$) derives, with the meaning “ q entails p ”, i.e., ontologically, “(the effect connoted by) q entails (its cause connoted by) p ”, which is the converse of “ p precedes causally q ”. This reading of an *ontological entailment* is the opposite of “ q follows logically from p ”, expressing the *logical entailment* of C. I. Lewis’ strict implication just discussed, because of the reversal of the connective between the causal and the logical realm. In this way, we can write the definition of the strict converse implication, as the key-notion of the logic of the causal necessity, \Box^C .

$$\mathbf{Def.}: (\alpha \leftarrow \beta) := (\Box^C (a \leftarrow \beta)) \leftrightarrow (\neg \diamond (\neg \alpha \wedge \beta)) \quad (4-3)$$

Because of the relationship between implication and inclusion, and because, in this case, the necessity condition is given in the antecedent of the conditional, we can define the notion of the *causal*

²⁰ It is evident that we must interpret both p and q as denoting as many *events/beings* in the ontological causal-effect relation between them. In other terms, they correspond semantically to “definite descriptions” of the respective referents. We see thereafter that in NR it is possible to justify a theory of the *direct, causal* reference, not between names and things, but between categorical propositions, like p and q , and their referents, based on a *natural* and not *conceptual* theory of the “double saturation” between the unary predicate and its only argument of each definite descriptions. The “natural” double saturation founds the “conceptual” one, and not vice versa like in CNR, as a true *naturalist* ontology requires. Also for this reason, we connote our ontology as NR and not CNR, like the Cocchiarella one.

inclusion ($p \supseteq_c q$) as complementary of the usual *logical inclusion* ($p \subseteq q$). Consequently, the semantic notion of “ p precedes causally q ”, or shortly, “ p causes q ”, is the ontological interpretation of the strict converse implication. That is, ($p \rightarrow_c q$) is the *ontological* counterpart in the natural realm of the *semantical* reading of ($p \leftarrow q$), as “the effect (connoted by) q entails its cause (connoted by) p ” in the logical realm.

This is the inversion of the verse of the inference between the *ordo essendi* and *ordo cognoscendi* (“what is first in being, is last in knowing”) of the Aristotelian epistemology. We met already with it in Aquinas passage on the converse implication quoted in §4.1.2, and we discuss again in §4.4.1 about this inversion, for “lifting the fog on complexity” about the related and very ambiguous notions of “downward” and “backward” causation.

Of course, the collection of the objects included in the domain of the same causal relation do not constitute properly a *class*, so that no class membership predicate $\langle \in \rangle$ holds for them, otherwise we should fall in what Quine defines as the confusion between “distributive” (based on “membership”) and “cumulative” predication (based on the simple “inclusion”)²¹.

Because of the strict or “intrinsic” relationship between the notion of “implication” and the notion of “truth”, both on the ontological and on the logical sides, we can define on this basis an *ontological* and not *logical* condition of membership to the Universal Class **V**.

We can suppose, indeed, that through a common dependence (causal inclusion) on a causal relation – effectively an “ontological entailment” – of each element of the Universal Class **V** with one only “primary generator” $\langle \Gamma \rangle$, a “secondary” transitive-symmetrical-reflexive relation among these dependent elements could be constituted, and hence, an equivalence domain among them.

In this way, not only the *necessary*, but also the *sufficient* condition for the full membership to **V** – and hence for the “full (actual) existence” of each of its member – is given, according to a proper *Ontological Axiom of Foundation (OAF)* of such a formal ontology, the NR formal ontology, as we see.

4.3 The NR formal ontology

We can now give a semi-formal, synthetic presentation of the NR formal ontology and of its logic:

VARIABLES

x, y, \dots : individual variables
 α, β, \dots : individual meta-variables
 P, Q, \dots : predicate variables
 ξ, ζ, \dots : predicate meta-variables

CONSTANTS

a, b, \dots : individual constants
 μ, ν, \dots : individual meta-constants
 P, Q, \dots : predicate constants
 φ, ψ, \dots : predicate meta-constants
 $E!$ ___ existence predicate
 Γ : primary generator (**PG**)
 γ_n : secondary generators (**SG**)

²¹As Quine emphasizes, medieval logicians knew very well this fallacy, like in the famous paralogism: “The Apostles are twelve, Peter is Apostle, then Peter is twelve”. For this reason, both in Platonic and Thomistic tradition the term used for the *formal causality*, is “participation”, of the individual to the species, and of the species to the genus, by which the reversal of the direction both of the implication and of the inclusion relations as to the logical ones is perfectly justified also in the natural language. Aquinas extended the notion of participation also to the *existence* with the notion of “participation of being”.

COLLECTIONS

- $\overset{\subseteq}{\mathbf{V}}$: universal collection [including only individuals, as no (sub-)domain defined in it]
 \mathbf{V} : universal class
 Λ : empty class
 A, B, C, \dots : natural kinds (genera, species...)
 $\mathbf{P}, \mathbf{Q}, \mathbf{R}, \dots$: logical classes

CONNECTIVES

- $\neg, \wedge, \rightarrow, \leftarrow, \leftrightarrow$: propositional connectives
 $non, et, \Rightarrow, \Leftarrow, \Leftrightarrow$: propositional meta-connectives
 $(\alpha \twoheadrightarrow \beta) := (\Box(\alpha \rightarrow \beta)) \leftrightarrow (\neg\Diamond(\alpha \wedge \neg\beta))$: strict logical implication
 $\models (p \twoheadrightarrow q)$: logical entailment, i.e., “ p entails q ”, or “ q follows *logically* from p ”, or “the inference from p to q is *logically* valid”
 $(\alpha \leftarrow \beta) := (\Box^c(a \leftarrow \beta)) \leftrightarrow (\neg\Diamond(\neg\alpha \wedge \beta))$: strict converse implication
 $\models (p \leftarrow q) = (p \rightarrow_c q)$: causal entailment, i.e., “ q entails p ”, or “ p precedes *causally* q ”, or “the inference from p to q is *causally (ontologically)* valid”
 $(p \supset_c q)$ vs. $(p \subseteq q)$: causal vs. logical inclusion, i.e., “ p includes causally q ” vs. “ q includes logically p ”

QUANTIFICATION

- \forall, \exists : binding variables denoting what potentially is
 \forall^e, \exists^e : binding variables denoting what actually exists
 \forall^m, \exists^m : binding variables denoting what mentally exists (concepts)
 $\hat{x} \dots$: the collection/class of x such that...

MAIN MODAL AXIOMS OF NR

- N**: $\langle \mathbf{X} \rightarrow \alpha \Rightarrow (\Box \mathbf{X} \rightarrow \Box \alpha) \rangle$
K: $\langle \mathbf{k} + \mathbf{N} \rangle$, fundamental modal system, where \mathbf{k} is the ordinary propositional calculus.
D: $\langle \Box \alpha \rightarrow \Diamond \alpha \rangle$
4: $\langle \Box \alpha \rightarrow \Box \Box \alpha \rangle$
5 or E: $\langle \Diamond \alpha \rightarrow \Box \Diamond \alpha \rangle$

NEW FOUNDATION AXIOMS (ONTOLOGICAL AXIOMS OF FOUNDATION, OAF) FOR NR

$\forall x \left(x \subset \overset{\subseteq}{\mathbf{V}} \right) \leftrightarrow \hat{x}(\Gamma \rightarrow_c x)$: Principle of inclusion in the *collection* $\overset{\subseteq}{\mathbf{V}}$, i.e.:

$$(\mathbf{OAF}_1) : \forall x \begin{cases} \text{either } \left(((\Gamma \rightarrow_c x) = 1) \rightarrow x \subset \overset{\subseteq}{\mathbf{V}} \right) \leftrightarrow \exists x \\ \text{or } \left(((\Gamma \rightarrow_c x) = 0) \rightarrow x \not\subset \overset{\subseteq}{\mathbf{V}} \right) \leftrightarrow \neg \exists x \end{cases}$$

Where $\left(\left(\Gamma \supset_c \overset{\subseteq}{\mathbf{V}} \right) \wedge \neg(\Gamma \subseteq \mathbf{V}) \right)$ i.e.:

Definition of the Primary Generator (**PG**) $\langle \Gamma \rangle$:

$$\Gamma := \left(\left(\Gamma \supset_c \overset{\subseteq}{\mathbf{V}} \right) \wedge \neg(\Gamma \subseteq \mathbf{V}) \right)$$

Principle of Primary Causation **PC**:

(**PC**): $R_{\Gamma}^{\rightarrow_c} := \forall x (\Gamma \rightarrow_c x)$ i.e., “Everything exists potentially in the active power of Γ ”

For justifying the existence of differences among the elements of $\bar{\mathbf{V}}$ and hence the self-identity of each of them, so to grant their membership to the universal class \mathbf{V} it is necessary to demonstrate a Lemma of Secondary Causation (**LSC**), namely:

$$\mathbf{LSC} : \forall x, y \left[\left((\Gamma \rightarrow_c x) \wedge (\Gamma \rightarrow_c y) \right) \wedge \left((y \neq x) \Rightarrow (x \rightarrow_c y) \right) \right] \quad (4-4)$$

Demonsration: It follows immediately from **OAF**₁ and from the Euclidean rule applied to the relation (\rightarrow_c), i.e.: $\langle (xRy) \wedge (xRz) \rightarrow (yRz) \rangle$. \square

Remark 1: On this regard, we answer by **LSC** the deep notation of Quine quoted in §4.1.1. Over there he said that a satisfactory ontological theory of implication would require the rigorous definition of the conditions according to which the “obscure” entities designated by two statements, put into the relation of ontological implication, can be said “to be the same or different entities”. In parentheses, **OAF**₁ gives us also a first “enlightenment” onto the “obscurity” of such entities. They are simply the beings, which “were”, “is”, or “will be” existing in the universe(s), as far as their common *existence* is the product of a fundamental or “primary” causality embracing the whole history of the universe(s), and as far as their different *species* are the product of “secondary” causes, at different stages of (each) universe history.

Remark 2: In other words, any difference in NR ontology has a *causal* justification, by a “texture” of secondary causes *inside* a given universe²², not outside it. Such a principle is perfectly fitting with the “energetically open” character of any quantum system in QFT, within an “energetically closed” universe²³.

So, on the basis of **LSC**, we can the define the notion of Secondary Generator (**SG**) $\langle \gamma \rangle$:

$$(\mathbf{SG}) \gamma := (\exists \gamma) (\forall y, z) \left[\left((\gamma \rightarrow_c y) \wedge (\gamma \rightarrow_c z) \right) \wedge \left((y \rightarrow_c z) \wedge (z \rightarrow_c y) \right) \right] \Rightarrow (y = z) \wedge (\gamma \neq (y, z))$$

We can thus define also the notion of Secondary Causation **SC**:

(**SC**) $\bar{R}_{\gamma}^{\rightarrow_c} := \left(\forall x, y (x = y) \Rightarrow (\gamma \rightarrow_c (x, y)) \right) \wedge \left(\forall w, z (w \neq z) \Rightarrow \left((\gamma' \rightarrow_c w) \wedge (\gamma'' \rightarrow_c z) \right) \right)$ i.e., “Any difference as has a causal explanation in its proper generator $\langle \gamma \rangle$ ”.

²² This formal ontology fits with the Aristotelian one, where “qualities” are properly “actions/passions” (*actiones/passiones*), e.g. “being white” is properly a “whitening”, “being black” is properly a “blackening”, etc., just as, conversely, in logic, any predicate verb can be translated into its participial form plus the copula “is” (e.g., “Mark loves” corresponds to “Mark is loving”, etc.). More deeply, all the “sensible qualities” (e.g., colors) are only the causal effects on the five senses of the four, more fundamental “active/passive” qualities. They are effectively, four fundamental dynamical forces, from which all the other “qualities” (forces, *virtutes* in Latin) derive, all related with heat (“hot” and “cold”, active; “humid” and “dry”, passive), by which the four elements (“water”, “earth”, “air”, and “fire”) interact (thermo-)dynamically among them, for the ultimate material constitution of all physical bodies. Indeed, the “water” is “cold/humid”, the “earth” is “cold/dry”, the “air” is “hot/humid” and the “fire” is “hot/dry”. For this reason, a sufficient amount of water can extinguish, for instance, the fire, or, conversely, an insufficient amount of water makes it transformed by fire into air (vapor).

²³ For this reason the causal action from the first generator $\langle \Gamma \rangle$ cannot be interpreted at all as the ultimate “energy reservoir” of the universe, like if it was a “boundary condition” for the universe dynamics. This is, for instance, the mistake of Descartes metaphysical theology. This is not, however, the mistake of Aquinas one, as erroneously Hawking states in many passages of his last book on the ontology of cosmology (Hawking & Mlodinow, 2010).

As **SG** emphasizes, $\langle \vec{R}_\gamma \rangle$ is nested inside $\langle \vec{R}_\Gamma \rangle$. Namely, the causal power on the *natures* (essences, genera/species) of the different things, $\langle \vec{R}_\gamma \rangle$, depends ultimately on the universal, all including, causal power on the *existences* of all things, $\langle \vec{R}_\Gamma \rangle$.

In other terms, the “secondary generator(s)” is (are) element(s) of the collection $\vec{\mathbf{V}}$ acting causally on other elements of $\vec{\mathbf{V}}$, by which the differences/identities among them ultimately depend. In such a way, it is possible to distinguish between a complementary, double, *com-posite* relation of “causal entailment” from $\langle \Gamma \rangle$ and $\langle \gamma \rangle$, i.e. $\langle \left(\begin{smallmatrix} \vec{R}_\Gamma \circ \vec{R}_\gamma \end{smallmatrix} \right) \rangle$ – or, more simply, the related foundational scheme: $\langle (\Gamma \circ \gamma) \rightarrow_c _ \rangle$ – defining, respectively, the *necessary* and the *sufficient* condition for the *membership* to the universal *class* \mathbf{V} , and hence for the *actual existence* of whichever thing.

Indeed, based on **SG** and **SC**, we can define also the notion of *ontological self-identity*, **OSI**, that is of “causally founded self-identity” as a formal version of the “being for itself and in itself” of any individual substance or “subsistent being” of classical metaphysics. Namely, instead of having like in classical logic:

$$(x = x) := (\forall x, w)(x \in w) \leftrightarrow (x \in w)$$

We have:

$$\mathbf{OSI}: (x = x) := (\forall^e x)((\Gamma \circ \gamma) \rightarrow_c x) \leftrightarrow ((\Gamma \circ \gamma) \rightarrow_c x)$$

In this way, we can define also a principle of membership to the universal class \mathbf{V}

$$\forall^e x(x \in \mathbf{V}) \leftrightarrow \hat{x}((\Gamma \circ \gamma) \rightarrow_c x),$$

That is, we can complete **OAF₁** with its second part, i.e.:

$$\mathbf{OAF}_2: \forall^e x \begin{cases} \text{either } (((\Gamma \circ \gamma) \rightarrow_c x) = 1) \rightarrow x \in \mathbf{V} \leftrightarrow \exists^e x \\ \text{or } (((\Gamma \rightarrow_c x) = 0) \rightarrow \neg x \in \mathbf{V}) \leftrightarrow \neg \exists^e x \end{cases}$$

In such a way, the existence of both \mathbf{V} and of its complement $\mathbf{\Lambda}$ is granted, since, in the causal entailment $\langle (p \rightarrow_c q) \rangle$, for $p \equiv 1$, both $q \equiv 1$ and $q \equiv 0$ are allowed.

Two consequences deriving immediately from **OAF₁₋₂** double foundation axiom, are:

C1: NR gives an ontological and not conceptual like in CNR, justification of the difference between the *possibilist* $\langle \forall/\exists \rangle$ and the *actualist* quantifiers $\langle \forall^e/\exists^e \rangle$.

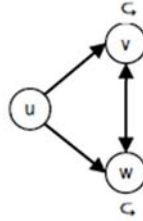
C2: NR introduces with the quantifiers, also the connected modal causal operators $\langle \Box^c/\Diamond^c \rangle$, not through arbitrary Henkin-like “cuts” based on “hypotheses”, on which no logical calculus can be founded, like in CNR, but through a given set of axioms, rules and definitions, on which an ontological modal calculus of propositions and predicates can be founded.

ML OF THE PROPOSITIONAL CALCULUS OF NR

If the introduction of the necessity operator $\langle \Box^c \rangle$ grants that we can assume in NR logic the modal axiom **K**, we can assume also that the causal relation, both primary and secondary, satisfies an *Euclidean* (not transitive!) relation between the nested generators $\langle \Gamma, \gamma \rangle$ and all

the elements of \mathbf{V} they constitute. In such a way, the modal, Euclidean, axiom **5** holds too. Of course, we can assume also a *serial relation*, so that also the modal axiom **D** holds, and, finally, a *secondary reflexive* and *transitive* relation, holding exclusively among the elements of \mathbf{V} (without including the primary generator $\langle \Gamma \rangle$). In this way, the modal axiom **4** holds among all the elements of \mathbf{V} connected among them through their common relation with the primary generator $\langle \Gamma \rangle$. Hence a *secondary* transitive-symmetrical-reflexive relation R is given among all the elements of \mathbf{V} (with the exclusion of $\langle \Gamma \rangle$), so that \mathbf{V} constitutes a proper class generated by $\langle \Gamma \rangle$ at this fundamental level $\langle 0 \rangle$ - and hence at each further level n of the consequent world unraveling.

To sum up, in relational semantics, in an over-simplified universe $\{\mathbf{W}\}$ with only three worlds $\{u, v, w\}$, and where the world u represents the first generator $\langle \Gamma \rangle$, and the subset $\{v, w\}$, represents the elements of \mathbf{V} , we have the following. For the Euclidean property $\langle \forall u, v, w (uRv \wedge uRw) \rightarrow vRw \rangle$; hence, for seriality, $\langle \forall u, v (uRv \rightarrow vRv) \rangle$; finally, $\langle \forall u, v, w (uRv \wedge uRw) \rightarrow (vRw \wedge wRv \wedge vRv \wedge wRw) \rangle$. I.e.,



We can then state that **KD45**, or **secondary S5**, is the ML of the NR ontology.

QML OF THE PREDICATE CALCULUS OF NR

The QML of the predicate calculus of NR is a higher order predicate calculus characterizing a modal relational semantics in which there exists a nested structure of possible worlds, at different orders of growing complexity.

Effectively, several other domains and sub-domains, among the elements of \mathbf{V} can be progressively “constructed”, or *causally entailed*, by a nested structure of natural kind generation – formally a hierarchical structure of nested **KD45** structures –, as a procedure of *iterating modality* through nested *world-stories*. The notion of **SG** exemplifies the fundamental, starting, level of such a nesting procedure, i.e., the 0 level. Of course, because of **OAF₂**, each level $n \geq 0$ constitutes a new level of actualization of the whole uni-(multi-)verse \mathbf{W} , each actualizing progressively at level n the causal power of $\langle R_{\Gamma \circ \gamma_{n-1}} \rangle$, synthetically $\langle R^* \rangle$, in a subset of actually existing objects.

Such a constructive procedure is a process of worlds/objects *unraveling*, by which new sets of equivalence relations among worlds/objects – and hence a new sub-domains of \mathbf{V} – is *ontologically stipulated* via $\langle R^* \rangle$, as the actual outcome of a given world-story. *Generally*, the unraveling procedure, **UP**, in propositional ML (model theory included), and in computer science, is the following (Blackburn, De Rijke, & Venema, 2002, p. 218):

UP: Let (W, R) be a frame generated by some point $w \in W$, where $\{W\}$ is some subset of the whole universe \mathbf{W} , at some step k of its evolution. The unraveling of (W, R) around w is the frame (\vec{w}, \vec{R}) where:

(I) \vec{w} is the set of all finite sequences (w, w_1, \dots, w_n) such that $w, w_1, \dots, w_n \in W$

and $Rww_1, \dots, w_{n-1}w_n$;

(II) If $\vec{s}_1, \vec{s}_2 \in \vec{W}$, then $\vec{R}\vec{s}_1\vec{s}_2$ if there is some $v \in W$ such that $\vec{s}_1 + (v) = \vec{s}_2$, where $+$ denotes sequence concatenation.

If $\mathfrak{M} = (W, R, V)$ is a model and (\vec{W}, \vec{R}) is the unraveling of (W, R) around w and p is a propositional variable, then we define the valuation

\vec{V} on (\vec{W}, \vec{R}) as follows:

$$\vec{V}(p) = \{(w, w_1, \dots, w_n) \in \vec{W} \mid w_n \in V(p)\}$$

The model $\vec{\mathfrak{M}} = (\vec{W}, \vec{R}, \vec{V})$ is called the unraveling of \mathfrak{M} around w .

By unraveling, any set of formulas is thus satisfiable on an *irreflexive, intransitive* and *asymmetric* tree.

At this point we can define the notion of the Ontological Generation Procedure, **OGP**, by which the composite relation R^* might construct suitable world-stories, by unraveling (making actual) new *domains* of worlds/objects, potentially existing in the causal power of the preceding secondary generator(s), all included (causally entailed) in the causal power of the first generator.

Remark: We could define **OGP** also as an “ontological *stipulation* procedure”, so to justify Hayaki’s remark that in Kripke relational semantics worlds and objects are to be *stipulated* not “observed”, that is “constructed” and not “supposed”. Indeed, the notion of “ontological stipulation” in NR has to be intended like in constructive geometry as a procedure for constructing new objects satisfying the geometry rules and axioms – in NR, satisfying its ontological (causal) rules and axioms. And not like in Hayaki’s “free logic” as a procedure for defining according to rules “non-existing” objects as referents of “dummy names”, like when we say, in a geometrical demonstration: “let us suppose that there exist the triangle $ABC\dots$ ”.

OGP: For justifying the construction of world-stories, the binary accessibility relation R^* (effectively $\langle R_{\Gamma \circ \gamma_n} \rangle$), must satisfy the following conditions²⁴:

1. R^* forms a *tree*, that is:
 - a. R^* is *generated*. It has an origin point $\langle \gamma_n \rangle$, in one of the actual worlds at the state W_n of the universe evolution. $\langle \gamma_n \rangle$, is thus R^* -related to other worlds at the successive level deep $n+1$. The original point is at level 0. The immediately stipulated worlds from level 0 are at level 1, from the level 1 are at level 2, and so on. We define thus as *secondary generators* $\langle \gamma_n \rangle$, with the index $n \geq 0$ denoting the world-story level, all the worlds (“ancestors”), progressively originating a world-story of new levels of stipulated worlds/objects in a nested way,.
 - b. R^* is *antisymmetric*. For no two distinct elements at two different levels, R^* holds in both directions. Nevertheless, the symmetric relation holds between each pair of worlds of the same level that have the same origin, i.e., the same generator $\langle \gamma_n \rangle$,

²⁴ Effectively, the conditions we are here attributing to our composite causal relation R^* are the same that Hayaki attributed to the stipulation relation S on non-actual objects (Hayaki, 2003). This depends, we repeat, on the fact that because of our foundation axioms we can be *possibilist* without being *conceptualist*.

worlds having a direct common ancestor. This confirms that the modal axiom **5** holds for such a logic²⁵.

- c. R^* is anticonvergent. I.e., the branches, once separated do not rejoin.
2. R^* is irreflexive. For granting this other important property of R^* , we have to recall that generally the reflexive relation is linked in ML to the axiom **T**. On the contrary, the causal relation is in R^* related to the axiom **D** granting, by seriality, that the chain of the derivations is always closed. This is a new confirmation that R^* within our modal system **KD45** perfectly fits with the logic of the R^* relation.
3. R^* is intransitive. Such a condition grants that any R^* -generated world has a direct ancestor, a condition that a transitive relation cannot grant in principle. On the contrary, the “weak” transitivity of the Euclidean relation perfectly satisfies such a condition, confirming our interpretation. Newly, R^* in the modal system **KD45** perfectly fits also with this third condition.
4. Between every pair of unraveled worlds/objects at the level n , each of them having a binary relation R^* with the common ancestor $\langle \gamma_{n-1} \rangle$, it is possible to develop a Euclidean relation, such that a secondary transitive-symmetrical-reflexive (=equivalence) binary relation R can be constituted between each of pairs of the worlds/objects. The totality of these pairs constitute thus a new domain of predication – a new natural kind of actually existing things –, at the state n of the whole universe story-sequence. This confirms that each stipulation procedure of nested world-stories has a nested **KD45** structure.

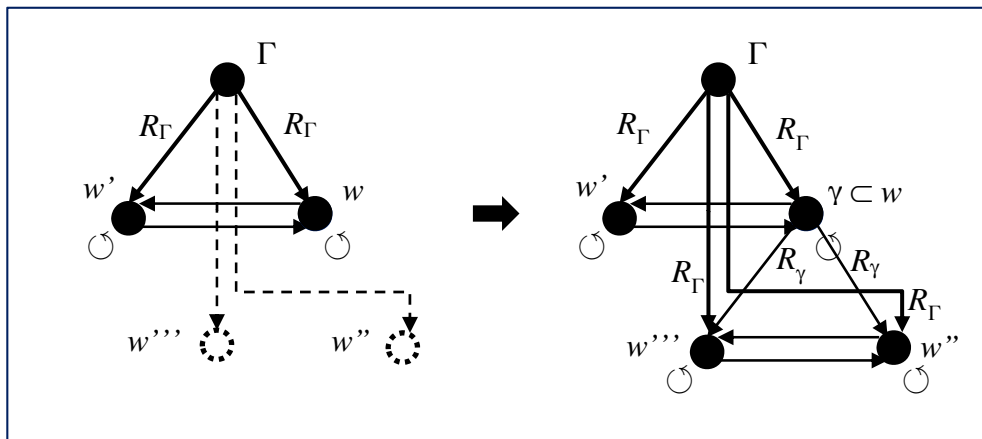


Figure 4-1. Scheme of OGP in the NR ontology. (Left) The worlds w'' and w''' are potentially existing in the active power of the primary generator $\langle \Gamma \rangle$. (Right) After being “unravelled” by the secondary generator $\langle \gamma \rangle$ within the world w , the worlds w'' and w''' become actually existing with all their elements. It is evident the iterated, nested **KD45 structure of the system.**

Finally, the objectual QML of the NR formal ontology is a modified version, **Q1R***, of the **Q1R** system. Indeed, because of our **OAF₁₋₂** we can use **Q1R** as the QML of NR, without supposing a free logic. Namely, all names, either proper (for individuals) or common (for natural kinds), have a referent, which is possibly existing, because of **OAF₁**, or actually existing, because of **OAF₁₋₂**.

Q1R-model: Generally, it is an ordered quintuple $\langle W, R, D, Q, V \rangle$, where:

²⁵ Effectively, Hayaki says that in such a way the axiom **B** ($\alpha \rightarrow \Box \Diamond \alpha$), and not **5** ($\Diamond \alpha \rightarrow \Box \Diamond \alpha$), holds, because of the actualist character of her ontology. Without such an actualist posit, **B** (the so-called “Browerian axiom”) and **5** (or **E**, the so-called “Euclidean axiom”) are effectively equivalent, and generally so are considered in many modal logic handbooks.

- W is a set of worlds, effectively a sub-set of the universe $\{\mathbf{W}\}$;
- R is the binary accessibility relation;
- D is the domain;
- Q is a function assigning to each world $w \in W$ a subset $D(w) \subseteq D$, i.e. the domain of quantification of w ;
- V is a valuation assigning for each world $w \in W$:
 - an object in D to each term t , variables included;
 - a set of ordered n -tuples of elements of D to each n -ary predicate, and
 - the set $D(w)$ to the existence predicate E .
- V must satisfy the rigidity condition on terms **VRT**:
VRT: $V(t, w) = V(t, w')$ for all w, w' in W

Q1R*-model: for applying to NR the **Q1R** logic, the model is not the quintuple $\langle W, R, D, Q, V \rangle$, as above, but a sextuple $\langle \gamma, W, R^*, D, Q, V \rangle$, where the difference is given by:

1. γ is an element of W ; and
2. R^* is our **OSG**, i.e., an arboreal, irreflexive and intransitive relation on W ; with γ being the point of origin.

Finally, in NR, a **Q1R*-model** must satisfy the rigidity condition (**VRT**) and the arboreal **SB** condition so re-defined:

SB*: If a non-actual object z appears for the first time in a world w (i.e., is not present in any ancestors of w , including γ), then z can appear only in w or its descendants.

4.4 Applications of NR to a Formal Ontology of the Evolutionary Cosmology

4.4.1 THE ONTOLOGY OF TIME AND OF COMPLEXITY IN THE NR FORMAL ONTOLOGY

The *ontology of time* in NR supposes clearly an *Aristotelian* tense-logic and not a *Diodorean* one. Indeed, this latter is very poor and, overall, no ontology of *history* can be founded one the Diodorean ML semantics of time, where the “possible in time”, i.e. $\langle \diamond' \rangle$, is limited to the “present” (n), and the “future” ($\mathcal{F} > n$), but not to the “past” ($\mathcal{P} < n$).

On the contrary, the notion of “history” suppose an *active power* of the past as to the present, just as the present has an *active power* as to the future, and not vice versa. In other terms, we need the so-called *Aristotelian theory of time* whose ML is, of course **S_t5**. I.e., it is sound as to **S5** (Cocchiarella, 2007, p. 45), and not only as to **S4**, as it is for the temporal ML, **KT_t4.3**, usually associated to untravelling (Blackburn, De Rijke, & Venema, 2002, p. 219).

Effectively the **S_t5** in NR is a nested **KD_t45**, structure, compatible also with a multi-verse cosmological hypothesis, but where, differently from Aristotle, there is no necessity of supposing an infinity of time as to the past *within each universe*. Indeed, because of **OAF_{1,2}**, there exists, within each universe, a time $n = 0$, corresponding to the level of the first secondary generators from which all the world-stories of that universe start (see **OAF₁**). In this way, we can define a Principle of Time Ontology (**PTO**), where “what is always the case”, $\langle \Box' \varphi \rangle$, can be justified as follows:

$$\begin{array}{l}
 \diamond'_k \varphi := (\mathcal{P}\varphi \vee \varphi \vee \mathcal{F}\varphi) \\
 \text{PTO: } \Box'_n \varphi := (\neg \mathcal{P}\neg\varphi \vee \varphi \vee \neg \mathcal{F}\neg\varphi) \\
 \therefore \Box' \varphi \leftrightarrow (\neg \diamond' \neg \varphi)
 \end{array}
 \quad \text{where: } \begin{cases} \mathcal{P}\varphi & \text{for } k < n \\ \varphi & \text{for } k = n \\ \mathcal{F}\varphi & \text{for } k > n \end{cases}$$

Of course, an “arrow of time” is modally defined, because there exists an ontological difference between the two temporal possibility operators, $\langle \mathcal{P}, \mathcal{F} \rangle$ in NR. Indeed, the “past” is made of things already “unraveled” at levels $\langle n-k \rangle$ of the universe becoming, when they actually *existed* as “individuals”. Afterward, they continue to exist in their world-story no-longer as actual individuals, but *virtually* as casually active “parts” in the new “wholes” actually existing, as *more complex things*, at the new actual level n of their world-story. On the contrary, the “future” is made of things not “yet” unraveled, and hence not yet determined in their proper nature, so that they cannot exert any “backward” causality.

Remark: In other terms, the direction of “the arrow of time” in the time ontology of NR coincides with the direction of the arrow of the causal implication, of the “causal entailment”, i.e., “the cause precedes always its effect” (\rightarrow_c). It precedes “ontologically” and hence also “temporally” the effect. In this sense, the fact that behind the direction of the *ontological* “causal entailment” there is the opposite direction of the *logical* and hence *epistemological* “strict converse implication” (\leftarrow), suggests a *definitive clarification* of the ambiguous notions of “backward” (from the “effect” as to the “cause”) and/or of “downward” causation (from the “whole” as to its “parts”), often associated to the notions of “emergence” and/or of “complexity” in natural sciences (De Haan, 2006; Mazzocchi, 2008). Only if we suppose the “atomism” in physics, and the “logical atomism” in logic and ontology, we confuse the ontology of causation with its logic and epistemology. “Backward” and “downward” are the verses of the *inferential* process between propositions, not of the *ontological*, causal process that is the *referent* of the propositional inference (see Quine’s precious criticism to Russell’s and Lewis’ confusions on this regard: §4.1.1, and Aquinas’ suggestion of the inversion between the logical and the ontological verses of the implication: §4.1.2). The emerging complex system, as far as it is related with the QFT notions of phase coherence domain and hence with its “duality principle” (“force field/its quanta” of QFT, vs. “wave function/particle” of QM: see §3.5) supposes that the “parts” (e.g., the electrons in an atom) are no longer actually existing “individuals”, with whom it is possible to “interact” as causes and/or as effects. The new individual, *actually* existing is the “whole” (e.g., the atom), which “is” its parts (e.g., the component electronic force fields). It does not “interact” with them, but “determines” their quantitative properties, so to satisfy the principle of phase coherence, being “entangled” with them as a “whole”. The parts are no longer actually existing individuals, they *were* individuals, before their composition in the new whole. When the new, more complex, individual comes to the *actual* existence, it is because its components passed to the *virtual* existence in it, and *vice versa*. When the “parts” were actually existing individuals (e.g., the electrons were “free particles” oscillating with their own frequencies), the “whole” existed only *virtually* in the causal power of its future parts (indeed, it existed as an ion, not as an atom). *Virtus* in Latin means literally *force*!

4.4.2 THE ONTOLOGY OF NATURAL KINDS IN NR

From all the preceding relations, we can define a formal ontology of natural kinds in NR. It is based on the notion of nesting of the physical causality, and so it is deeply different from the CNR ontology of natural kinds (Cocchiarella, 2007), where the causal explanation of natural kinds and of their hierarchy is purely *hypothetical*. Evidently confusing the natural and mathematical science inferential method, on one side, and the inferential method of formal semantics and formal ontology, on the other side. The main elements of NR formal ontology of the natural kinds can be synthesized in the following principles:

- A principle of Stratification for Natural Kinds (**SNK**) holds in NR, i.e.:

$$\mathbf{SNK} : (\forall_n^k A)(\forall yA)\Box_n^c(E!(a) \rightarrow ((a = y) \wedge ((a \wedge y) \in_k A)))$$

Where A is a natural kind (genus-species) of physical objects (e.g., quarks, or, at higher n , protons, neutrons, or at even higher n , different species of atoms, etc.), and the apex k to the quantifier signifies that the quantifier is ranging over natural kind predicate variables, all causally constituted at some level n of the universe evolution. The same k , as an index to the membership predicate \in , means that we are speaking about membership to a natural kind (“plural object” or “class-as-many”), as a collection of objects sharing the same ancestor $\langle \gamma_{n-1} \rangle$.

- A principle of Stratification for Natural Properties (**SNP**), shared by objects belonging either to the same or to different natural kinds (e.g., the electromagnetic charge), holds in NR, i.e.:

$$\mathbf{SNP} : (\forall_n^e F^j) \diamond_n^c (\exists_n^e x_1, \dots, x_j) F(x_1, \dots, x_j)$$

Where we recall that the index e signifies that the quantifiers are ranging over actually existing predicates and individual variables.

- A Principle of Stratified Rigidity (**PSR**) holds in NR. Such a condition occurs for a predicate F , at the level of the universe evolution n in which it is generated within a given world-story, and holds for all the successive $n+k$ levels (with $k \geq 0$) of the universe evolution, i.e.:

$$\mathbf{PSR} : \Box_n^c (\forall_n F) (\exists_{n+k} G) (Rigid(G) \wedge (\forall x_1, \dots, x_j) [F(x_1 \dots x_j) \leftrightarrow G(x_1 \dots x_j)])$$

- From all the precedent definitions, a Principle of concrete Existence (**PCE**) for physical individuals derives, defined as follows:

$$\mathbf{PCE} : (\forall^k A)(\forall yA)\Box_n^c(E!(a) \rightarrow (y = a))$$

From which it emerges that any physical being exists as an individual a *only* as a member y of a natural kind, and hence as the outcome of a world-story, shared by other individuals of the same species, inside the history of the universe.

- Finally, a Principle of Biological Individualization (**PBI**), different from (**PCE**), holds in NR for the concrete existence of biological individuals as formalizing the *epigenetic*, self-organizing, factors by which the organism “individualizes” for itself (e.g., by activating/de-activating some sequences of DNA) the *genetic* factors of its specific DNA, i.e.:

$$\mathbf{PBI} : (\forall^{k_l} A)(\forall yA)\Box_n^c(E!(a) \rightarrow (y \rightleftharpoons a))$$

Where the index l affixed to the apex k of the natural kind quantifier means that this is a natural kind of living beings.

4.4.3 THE ONTOLOGY OF THE CONCEPTUAL REALISM IN THE NR FORMAL ONTOLOGY

Finally, for justifying that such an ontology is a “natural *conceptual* realism”, NCR, and not only a “natural” realism, NR, we have to justify the passage from the natural to the conceptual realm, since it is not justified, like in CNR by an axiom such that the Fregean “comprehension axiom” on which the calculus of CNR ultimately depends.

In the case of the most complex among the living beings, i.e., the animals and finally the humans, the nesting of **KD45** can arrive until the inner generation, inside the animal individuality, of *cognitive simulations* of the outer world. This representational faculty can be generally characterized as the ability of redefining the generic argument of a predicate on the singular occurrence of an individual referent. In this way, there exists at the cognitive level a *mirroring* of the same individualization principle acting at the biological level. In other terms, it is possible to reply at the cognitive

level the same principle of “double saturation” between an individual and its genetic factors (= epigenesis) of the biological generation process, as a “double saturation” between a predicate and its argument for justifying an inductive procedure. In this way, the etymological relationship between “concept” in psychology and “conceived” in biology would have a straightforward explanation.

- That is, a Principle of Cognitive Induction (**PCI**) holds in NR, i.e.:

$$\mathbf{PCI}: (\forall^m F^j)(\forall^m x)\Box\Diamond_{n+1}^C \left(\exists! a \left((x \rightleftharpoons a) \wedge (F(x_1, \dots, x_{j-1}, a)) \right) \right)$$

Where, the index m (mental) emphasizes we changed from the natural to the *conceptual* realm, at the $(n+1)$ *abstract* level as to the n level of the actual existence. Consequently, x is a mental variable denoting a generic individual, a is a mental constant denoting a given existing individual, and the double modal operator, $\langle \Box\Diamond_{n+1}^C \rangle$ emphasizes that we are speaking about a “causally necessary possibility”, that is a *faculty* of the cognitive agent as such.

- Finally, in the case of the singular denotation, it is proper of the human mind to re-define onto a singular individual, also the predicate and not only its argument. This means that we can define in NR a Principle of Generalizing Abstraction, **PGA** – the realistic substitute in NR of the Fregean Comprehension Principle of CNR –, by which it is possible to construct conceptual domains, and that reads as follows:

$$\mathbf{PGA}: \left[(\exists \varphi, \alpha) (E! \mu ((\mu \rightarrow_c \varphi) \wedge (\mu \rightarrow_c \alpha) \wedge (\varphi \rightleftharpoons \alpha)) \Leftrightarrow \varphi \alpha) \right] \wedge ((\varphi \alpha \Rightarrow \forall^m x (\varphi x)) \equiv \mathbf{A})$$

Where, $\langle \varphi \alpha \rangle$ denotes meta-logically a “definite description”, characterized by the identity between the predicate and its argument, and that is causally constructed, $\langle \rightarrow_c \rangle$ by its referential individual object $\langle \mu \rangle$. The right term of such a conjunction is thus a formulation of the “principle of universal generalization” of the classical predicate calculus, valid only for mental objects, and $\langle \mathbf{A} \rangle$ is a symbol denoting an abstract class (a “class-as-one”), corresponding to the extension of the predicate $\langle \varphi \rangle$.

4.5 The NR logic is a paraconsistent logic

It is well known that there exist consistent elements of para-consistent logic both in Ancient and Middle-Age logic (Gomes & D'Ottaviano, 2013). From this historical point of view, we can now add also Aquinas to such a list.

Anyway, as we saw in §4.2, the notion of “logical entailment” – i.e., “ p entails q ”, that is, “ q follows logically from p ” – as the proper semantics of Lewis’ strict implication, $\langle p \rightarrow q \rangle$, is ultimately a very strong way – effectively the strongest – for affirming the logical truth of the so-called “principle of Pseudo-Scotus”. That is, the “principle of explosion”, **PE** (*ex contradictione sequitur quodlibet*), expressed in the formula (1) below. As we already recalled, according to (Huges & Cresswell, 1996, p. 203) a list of paradoxes deriving from the notion of logical entailment is, indeed, the following:

- 5) $(p \wedge \neg p) \rightarrow q$
- 6) $q \rightarrow (p \vee \neg p)$
- 7) $\neg \Diamond p \rightarrow (p \rightarrow q)$
- 8) $\Box q \rightarrow (p \rightarrow q)$

Hence, following Lewis himself, we saw that if we want to avoid (1) and the other related paradoxes, we have to exclude before all the so-called “principle of the disjunctive syllogism”:

$$((p \vee q) \wedge \neg p) \rightarrow q$$

Namely, we have to refer to the so-called *relevance logics* (Huges & Cresswell, 1996, p. 205), i.e., it is necessary to define a valid criterion of *relevance* of a premise as to a given conclusion, and

hence to refer to the notion of *paraconsistent negation* (Béziau, What is a paraconsistent logic?, 2000).

It is evident that, as we have already anticipated, the ML of NR avoids all the paradoxes related to the notion of the “logical entailment”. Before all, the truth-table of the converse implication, $\langle p \leftarrow q \rangle$, states that from the false only the false can be inferred. So, the modalization of the converse implication, – i.e., $\langle p \rightarrow_c q \rangle$, or, $\langle \neg \diamond (q \wedge \neg p) \rangle$, implies that the semantics of the “causal entailment” – i.e., “ q entails p ”, that is, “ p precedes causally q ” – makes false the paradox (1), that is $\langle \neg [(p \wedge \neg p) \rightarrow_c q] \rangle$. In other terms, neither the “principle of pseudo-Scotus” nor the “principle of the disjunctive syllogism” – i.e., $\langle \neg (((p \vee q) \wedge \neg p) \rightarrow_c q) \rangle$ – hold in NR logic.

On the other hand, it is evident that a logic of causality based on the “converse implication” is naturally a *relevance* logic, since only a true premise (denoting the cause) can imply a true conclusion (denoting the effect).

It is evident too, moreover, that the principle of iterated modality and the consequent “stratified” nature of the necessity operator in the NR objectified QML, opens NR logic to the possibility of an original interpretation in it of the *paraconsistent* negation. This true, as far as both are based, not only on the refusal of **PE** and of its trivial consequence, but also on the principle of the non-coextensive character of an affirmation with its negation in contradictory statements (Béziau, What is a paraconsistent logic?, 2000). The stratified, nested character of the necessity operator in NR logic opens thus the possibility for an original version of the *constructive* use of the contradiction, typical of the paraconsistent logics. Indeed, because of the nested character of the causal necessity operator, while the negation $\langle \neg p \rangle$, contradicting the affirmative $\langle p \rangle$, negates $\langle p \rangle$ at its proper necessity level, the affirmation $\langle p \rangle$ potentially includes all the other propositions not yet unraveled by the iterated modality procedure illustrated in §4.3. From the ontological standpoint, this ultimately depends on the axioms **OAF**_{1,2}, from which the action of the primary generator $\langle \Gamma \rangle$ emerges as including all the levels and all the branches of the iterated modality hierarchy – both the levels already unraveled, and the levels not yet unraveled – in a *causal*, not *logical* way. That is, in which the contradiction does not propagate itself to the lower levels of the argumentation tree. Indeed, in the causal implication is valid, i.e. $\langle \diamond^c ((p \leftarrow q) \equiv 1) \rangle$, also for $\langle p \equiv 1, q \equiv 0 \rangle$, and not only for $\langle p \equiv 1, q \equiv 1 \rangle$, like in the logical deduction.

In other terms, the information (truth) in the causal implication is not conserved between the antecedent(s) and the consequent(s). In fact, at each level of the unraveling procedure the *actual* information *increases*, since a new structure emerges, as absolutely, logically, *unpredictable* from the precedent ones. Such an emergence however, is at the cost of a decrease of the *potential* information included in the precedent levels, since at each of them the procedure chose one only of the two possible branches $\langle 1, 0 \rangle$ that were available.

Of course, much more analysis is needed, for deepening the relationship between the **KD45** modal logic, and its iterated, nested structure in our NR formal ontology, and the paraconsistent negation, by expanding the analysis between modal logics – particularly, the “un-named vertex $\langle \neg \square \rangle$ ” of the modal square of oppositions – and the paraconsistent negation. Such an analysis has already been developed, for instance in (Béziau, 2005), but limited to **S5** and **S4** systems and to a four-valued modal system such as **M4**. On these topics, much more intriguing, and hence requiring a much deeper examination, are the most recent (Béziau, 2012; 2013), extending the analysis to the more powerful hexagon, and the modal hexagon, of oppositions, used also at the meta-logical level. Moreover, this initial survey of the relationships between NR logic and paraconsistent logics, displays a possible connection with the hierarchy of logical systems, and of the related algebras, proper of Da Costa’s paraconsistent logic, C_ω (Da Costa & Alves, 1977). Both approaches indeed are developed on the basis of positive intuitionistic logic, so that both admit for systems of level n (where $1 \leq n < \omega$), that consistent formulas of the type $\langle p^n \wedge \neg p^{n+1} \rangle$ hold in a theory. In addition, we can

say that the core of NR formal ontology is precisely for supporting such a possibility in a *constructive* way, not only from the logical, but overall from the *ontological* standpoint, so to finish definitely the cloying, ideological debate between the supporters of classical metaphysics and the supporters of the evolutionary theories in natural sciences.

Finally, a consideration about the relationship with dialectical logics and more generally with *dialetheism*. Both the logic of NR and many paraconsistent logics, the C_ω included, validate the law of non-contradiction, i.e., $\langle \Vdash \neg(\alpha \wedge \neg\alpha) \rangle$, even though both invalidate **EP**. In this sense, both of them are deeply different from the “dialetheism”, i.e., the metaphysical position, typical, for instance, of the Hegelian *Science of Logic*, according to which there exist logically *true* (ontologically *real*) contradictions. Hence, both NR and paraconsistent logics can give a contribution to the overcoming of an ideological approach to this problem, characterizing the philosophy of logic during the XIX, and a large part of the XX centuries (Da Costa, Béziau, & Bueno, Paraconsistent logic in a historical perspective, 1995, p. 112-113).

4.6 General conclusions and further perspectives

In this paper we offered for the first time a semi-formal general presentation of the *Natural Realism* (NR) formal ontology, according to the double perspective of:

- 1) Providing a formal ontology for the change of paradigm actually involving the fundamental physics, based on QFT, as far as it is irreducible to QM, and it is related to an evolutionary approach to cosmology. Such an evolutionary vision involves also the same foundations of the mathematical laws of physics. In other terms, “physics is legislated by cosmogony”, according to the visionary expression of John Archibald Wheeler.
- 2) Providing a formal ontology for the *natural* realism endowed with a *suitable* logical calculus, able to justify the construction of arboreal structures of quantification domains of growing complexity, based on an *objectual* and not *conceptual* QML. Such a logical calculus is a possibilist version of Hayaki’s *nested stipulation principle*, because based on a modal version of the logic of *the converse implication*. Namely, the logic of the Aristotelian *formal causality*, as dynamically generated by an acting causality on the matter indeterminacy, determining the *emergence* (*eductio* in Latin as opposed to *deductio*) of a new form in it. That is, the logic of the dynamical emergence of collective behaviors such as the “phase coherence domains”, according to the QFT interpretation of the QM particle-wave duality (uncertainty relation) in terms of particle-field duality (indeterminacy relation), $\langle \Delta n \Delta \varphi \geq \hbar \rangle$, where n is the number of the field quanta and φ is the field phase. Such a modal calculus, originally developed in an actualist context, makes possible the construction of arboreal structures of quantification domains of growing complexity.

The connection between these two main perspectives emerges compellingly in the *Second Section* of this work when we discussed the logical-philosophical background of Cocchiarella’s CNR. A possibilist ontology, as required by the evolutionary cosmology, is indeed generally considered as compatible only with a “conceptual” QML, having its foundation in the Fregean “comprehension principle” (**CP**) of ZFC set theory, extended via the “unrestricted” **CP** for allowing the presence of higher order classes inside it. This is the deep reason for which Cocchiarella developed a natural realism on a *conceptualist* basis, the CNR, because he is considering otherwise as impossible a logical calculus for the natural realism of an evolutionary cosmology. That is, a realism giving a causal foundation to the emergence of new levels of matter organization (symmetries) and hence of new physical laws. It is thus evident, in such a way, why for CNR the natural realism, because of its conceptualist roots, can suggest only mere *hypotheses* about the emergence of new levels of matter organization and of natural kinds of physical beings in it, so to miss the target of a suitable formal ontology for the *natural realism*. On the other hand, this lack of a suitable foundation axiom for the

causal realm, as distinguished from the *logical* realm, is the deep logical reason for which the theory of “causal sets” (Reid, 2001; Penrose, 2013) failed in its pretension of defining a logical calculus for the natural realism, the emergence of new physical laws included.

Henceforth, in the *Third Section* of this work, after a sketchy presentation of the QFT approach to fundamental physics, from cosmology, to the standard model of the elementary particles, to the physics of the condensed matter, living matter and neuropile included, we present in the fourth section a summary of the NR formal ontology. Its principal merit, according to us, is that it is able to satisfy, from the ontological side, the requirement for an ontology of the natural realism, able to cope with the change of paradigm that quantum physics and, more specifically, QFT, as irreducible to QM, is imposing to the Newtonian approach to modern physics.

The formal ontology of such a new paradigm is illustrated in the *Fourth Section* of this work, in which we give a first semi-formal presentation of NR formal ontology. It is based on Aquinas’ suggestion that the logic of the causal necessity for the *emergence* (in Latin, *eductio*, “eduction”) of new natural forms of matter organization from the potentiality of the matter, is the (modal) logic of the *converse implication*.

Finally, there exists a straightforward evidence that the logic of NR is ultimately a paraconsistent logic, because it satisfies the NC principle but not EP. Because of **OAF**₁₋₂ and the hierarchical nested semantics it generates, indeed, it is evident that a paraconsistent negation holds in NR logic, because of the non-coextensive character of affirmation and negation in it, and the consequent stratified character of the rigidity principle.

Such a relationship with a paraconsistent logic and with a hierarchical paraconsistent logic as Da Costa’s C_n logic (Da Costa, Krause, & Bueno, 2007), and, more generally, the relationship between NR formal ontology with its causal reference theory, and Da Costa’s notion of *pragmatic truth* (Da Costa, Bueno, & French, 1998) requires a specific further inquiry.

5 Bibliography

- Basti, G. (2004). Analogia, ontologia e problema dei fondamenti. In G. Basti, & C. Testi, *Aanalogia e autoreferenza* (pp. 159-236). Milano-Genova: Marietti 1820.
- Basti, G. (2007). Ontologia formale: per una metafisica post-moderna. In A. Strumia (Ed.), *Il problema dei fondamenti. Da Aristotele, a Tommaso d’Aquino, all’ontologia formale* (pp. 193-228). Siena: Cantagalli.
- Basti, G. (2011). Ontologia formale. Tommaso d’Aquino ed Edith Stein. In A. Ales-Bello, F. Alfieri, & M. Shahid (Eds.), *Edith Stein, Hedwig Conrad-Martius, Gerda Walter. Fenomenologia della persona, della vita e della comunità* (pp. 107-388). Bari: Laterza.
- Basti, G. (2013a). Intelligence and reference. Formal ontology of the natural computation. In G. Dodig-Crnkovic, & R. Giovagnoli (Eds.), *Computing Nature. Turing Centenary Perspective* (pp. 139-159). Berlin-Heidelberg: Springer-Verlag.
- Basti, G. (2013b). A change of paradigm in cognitive neurosciences Comment on: "Dissipation of 'dark energy' by cortex in knowledge retrieval" by Capolupo, Freeman and Vitiello. *Physics of life reviews*, 5(10), 97-98.
- Basti, G. (2014). *The formal ontology of the natural realism*. Campinas: Sociedade Brasileira de Historia de Matematica.
- Béziau, J.-Y. (2000). What is a paraconsistent logic? In D. Batens, & al. (Eds.), *Frontiers of paraconsistent logic* (pp. 95-111). Baldock: Research Studies Press.

- Béziau, J.-Y. (2005). Paraconsistent logic from a modal viewpoint. *Journal of applied logic*, 3, 7-14.
- Béziau, J.-Y. (2012). The power of the exagon. *Logica Universalis*. doi:10.1007/s11787-012-0046-9
- Béziau, J.-Y. (2013). The metalogical exagon of opposition. *Preprint*.
- Blackburn, P., De Rijke, M., & Venema, Y. (2002). *Modal logic. Cambridge tracts in theoretical computer science*. Cambridge, UK: Cambridge UP.
- Blackwell, R. J., Spath, R. J., & Thirlkel, E. W. (Eds.). (1999). *Thomas Aquinas "Commentary on Aristotle's Physics" [Aristotelian Commentary Series]*. South Bend, IN: St. Augustine Press.
- Blasone, M., Jizba, P., & Vitiello, G. (2011). *Quantum field theory and its macroscopic manifestations. Boson condensation, ordered patterns and topological defects*. London: Imperial College Press.
- Cahill, K. E., & Glauber, R. J. (1969). Density operators and quasiprobability distributions. *Physical Review*, 177(5), 1882-1902.
- Carnap, R. (1946). Modality and quantification. *Journal of symbolic logic*, 11, 33-64.
- Carnap, R. (1947). *Meaning and necessity*. Chicago: Chicago UP.
- Carnap, R., & Bar-Hillel, Y. (1964). An outline of a theory of semantic information. In Y. Bar-Hillel, *Language and information: selected essays on their theory and application* (pp. 221-274). Reading, Ma & London, UK: Addison-Wesley.
- Celeghini, E., Rasetti, M., & Vitiello, G. (1992). Quantum dissipation. *Annals of Phys.*, 215, 156-170.
- Cocchiarella, N. B. (2007). *Formal Ontology and Conceptual Realism*. Berlin-New York: Springer Verlag.
- Cocchiarella, N. B. (2013). Predication in conceptual realism. *Axiomathes*, 23(2), 301-21. doi:10.1007/s10516-010-9140-x
- Comin, R., & al. (2014). Charge order driven by Fermi-arc instability in Bi₂Sr₂-xLa_xCuO_{6+d}. *Science*, 343, 390-392.
- Da Costa, N. C., & Alves, E. H. (1977). Semantical analysis of the calculi Cn. *Notre Dame Journal of Formal Logic*, 18, 621-630.
- Da Costa, N. C., Béziau, J.-Y., & Bueno, O. (1995). Paraconsistent logic in a historical perspective. *Logique & Analyse*, 150-151-152, 111-125.
- Da Costa, N. C., Bueno, O., & French, S. (1998). The logic of pragmatic truth. *Journal of philosophical logic*, 27(6), 603-620.
- Da Costa, N. C., Krause, D., & Bueno, O. (2007). Paraconsistent logic and paraconsistency. In J. Dale (Ed.), *Philosophy of Logic* (pp. 791-911). Amsterdam: Elsevier.
- da Silva Neto, E. H., & al. (2014). Ubiquitous Interplay between charge ordering and high-temperature superconductivity in cuprates. *Science*, 343, 393-396.
- Davies, P. (2010). Universe from bit. In P. Davies, & N. H. Gregersen (Eds.), *Information and the nature of reality. From physics to metaphysics*. (pp. 65-91). Cambridge, UK: Cambridge UP.
- Davies, P., & Gregersen, N. H. (A cura di). (2010). *Information and the nature of reality. From physics to metaphysics*. Cambridge, UK: Cambridge UP.
- De Haan, J. (2006). How emergence arises. *Ecol. Compl.*, 3, 293-301.
- Deutsch, D. (1985). Quantum theory, the Church-Turing principle and the universal quantum computer. *Proc. R. Soc. Lond. A*, 400, 97-117.
- Feynman, R. (1982). Simulating physics with computers. *Int. J. Theor. Phys.*, 21, 467-488.
- Fields, C. (2012). If Physics Is an Information Science, What Is an Observer? *Information*, 3(1), 92-123. doi:10.3390/info3010092
- Floridi, L. (2011). Semantic conceptions of information. In E. N. Zalta (Ed.), *Stanford Encyclopedia of Philosophy. Spring 2011 Ed.* (pp. 1-70). Retrieved September 9, 2012, from <http://plato.stanford.edu/archives/spr2011/entries/information-semantic/>

- Freeman, W. J., Ga'al, G., & Jornten, R. (2003). A neurobiological theory of meaning in perception. Part 3. Multiple cortical areas synchronize without loss of local autonomy. *Intern. J. Bifurc. Chaos*, *13*, 2845–2856.
- Freeman, W. J. (2004). Origin, structure, and role of background EEG activity. Part 1. Analytic amplitude. *Clin. Neurophysiol.*, *115*, 2077–2088.
- Freeman, W. J. (2004). Origin, structure, and role of background EEG activity. Part 2. Analytic phase. *Clin. Neurophysiol.*, *115*, 2089–2107.
- Freeman, W. J. (2005). Origin, structure, and role of background EEG activity. Part 3. Neural frame classification. *Clin. Neurophysiol.*, *116*, 111–1129.
- Freeman, W. J. (2006). Origin, structure, and role of background EEG activity. Part 4. Neural frame simulation. *Clin. Neurophysiol.*, *117*, 572–589.
- Freeman, W. J., & Rogers, L. J. (2003). A neurobiological theory of meaning in perception. Part 5. Multicortical patterns of phase modulation in gamma EEG. *Int. J. Bifurc. Chaos*, *13*, 2867–2887.
- Freeman, W. J., & Vitiello, G. (2006). Nonlinear brain dynamics as macroscopic manifestation of underlying many-body field dynamics. *Physics of Life Reviews*, *3*(2), 93–118.
- Freeman, W. J., & Vitiello, G. (2008). Dissipation and spontaneous symmetry breaking in brain dynamics. *Journal of Physics A: Mathematical and Theoretical*, *41*(30), 304042. doi:10.1088/1751-8113/41/30/304042
- Freeman, W. J., Burke, B. C., Holmes, M. D., & Vanhatalo, S. (2003). Spatial spectra of scalp EEG and EMG from awake humans. *Clin. Neurophysiol.*, *114*, 1055–1060.
- Frölich, H. (1968). Long range coherence and energy storage in biological systems. *Int. J. of Quantum Chemistry*, *2*, 641ff.
- Frölich, H. (Ed.). (1988). *Biological coherence and response to external stimuli*. Berlin: Springer.
- Garson, J. W. (2001). Quantification in modal logic. In D. Gabbay, & F. Guenther (Eds.), *Handbook of Philosophical Logic. Second Edition, Vol. III* (pp. 267–324). Berlin-New York: Springer.
- Goldstone, J. (1961). Goldstone, J (1961). "Field Theories with Superconductor Solutions". 19:.. *Nuovo Cimento*, *19*, 154–164. doi:10.1007/BF02812722
- Goldstone, J., Salam, A., & Weinberg, S. (1962). Broken Symmetries. *Physical Review*, *127*, 965–970. doi:doi:10.1103/PhysRev.127.965
- Gomes, E. L., & D'Ottaviano, I. M. (2013). *Uma história concisa da lógica paraconsistente*. Campinas: Sociedade Brasileira de História da Matemática.
- Hawking, S., & Mlodinow, L. (2010). *The grand design. New answers to the ultimate questions of life*. London: Bantam Press.
- Hayaki, R. (2003). Actualism and higher-order worlds. *Philosophical studies*, *115*(2), 149–178.
- Hendricks, V., & Symons, J. (2009). Epistemic logic. In E. Zalta (Ed.), *Stanford Encyclopedia of Philosophy* (Spring 2009 ed.). Retrieved from <http://plato.stanford.edu/archives/spr2009/entries/logic-epistemic/>
- Hill, J. W., & Kolb, D. K. (2001). *Chemistry for Changing Times, 9th Ed.* Upper Saddle River, NJ: Prentice Hall.
- Hintikka, J. (1962). *Knowledge and belief: an introduction to the logic of the two notions*. Cornell: Cornell UP.
- Hintikka, J. (1972). *Time and Necessity; Studies in Aristotle's Theory of Modality*. Oxford, UK: Clarendon Press.
- Huges, G. E., & Cresswell, M. J. (1996). *A new introduction to modal logic*. London: Routledge.
- Husserl, E. (1913/21). *Logische Untersuchungen, Halle: Niemeyer, 2nd edition*. (J. N. Findlay, Trans.) London: Routledge and Kegan Paul, 1970.
- Kolmogorov, A. N. (1956). *Foundations of the theory of probability. Second English edition*. (N. Morrison, Trans.) New York: Chelsea Publishing.
- Koyré, A. V. (1939). *Études Galiléenne*. Paris: Hermann.

- Koyré, A. V. (1968). *Newtonian studies*. Chicago: Chicago UP.
- Kronheimer, E. H., & Penrose, R. (1967). On the structure of causal space. *Proc. Camb. Phil. Soc.*, 63, 481-501.
- Lomuscio, A., & Sergot, M. (2003). Deontic interpreted systems. (W. Van der Hoek, & M. Wooldridge, Eds.) *Studia Logica (Special Issue on the Dynamics of Knowledge)*, 75(3), 63-92.
- Malink, M. (2006). A Reconstruction of Aristotle's Modal Syllogistic 27 (2):. *History and Philosophy of Logic*, 27(2), 95–141.
- Mazzocchi, F. (2008). Complexity in biology. Exceeding the limits of reductionism and determinism using complexity theory. *EMBO Reports*, 9(1), 10-15.
- McNamara, P. (2010). Deontic logic. In E. Zalta (Ed.), *Stanford Encyclopedia of Philosophy* (Fall 2010 ed.). Retrieved from <http://plato.stanford.edu/archives/fall2010/entries/logic-deontic/>
- Meixner, U. (2003). Der Begriff der Notwendigkeit in der Antike und in der Gegenwart. In H. Rott, & V. Horak (Eds.), *Possibility and Reality* (pp. 13-50). Frankfurt: Ontos Verlag.
- Meixner, U. (2007). *The theory of ontic modalities*. Frankfurt: Ontos Verlag.
- Meixner, U. (2010). *Axiomatic formal ontology (Synthese Library)*. Berlin-New York: Springer Verlag.
- Morr, D. K. (2014). Lifting the fox of complexity. *Science*, 343, 382-383.
- Nortmann, U. (2002). The Logic of Necessity in Aristotle: An Outline of Approaches to the Modal Syllogistic, Together with a General Account of de dicto- and de re-Necessity. *History and Philosophy of Logic*, 23, 253–265.
- Padmanabhan, T. (2010). Equipartition of energy in the horizon degrees of freedom and the emergence of gravity. *Mod.Phys.Letters A*, 25, 1129-36.
- Panizzoli, F. (2013). *Metafisica della partecipazione e ontologia formale (Published Doctoral Thesis)*. Rome: Lateran UP.
- Patton, C. M., & Wheeler, J. A. (1975). Is physics legislated by cosmogony? In C. J. Isham, R. Penrose, & D. W. Sciama (Eds.), *Quantum gravity* (pp. 538-605). Oxford, UK: Clarendon Press.
- Penrose, R. (2013). Foreword. In H. Zenil (Ed.), *A computable universe. Understanding and exploring nature as computation. Foreword by Sir Roger Penrose* (pp. i-xxxvi). Singapore-Hackensack, NJ-London: World Scientific Publishing.
- Prior, A. N. (1955). *Formal logic*. Oxford: Oxford UP, 1962.
- Putnam, H. (2012). *Philosophy in an Age of Science: Physics, Mathematics, and Skepticism*. (M. De Caro, & D. McArthur, Eds.) Boston, MA: Harvard UP (In Press).
- Quine, W. V. (1983). *Mathematical logic. Revised edition*. Cambridge, MA: Harvard UP.
- Reid, D. D. (2001). Discrete Quantum Gravity and Causal Sets. *Canadian Journal of Physics*, 79, 1-16.
- Rovelli, C. (1996). Relational quantum mechanics. *Int. J. Theor. Phys.*, 35, 1637–1678.
- Smith, B. (2005). Against Fantology. In J. C. Marek, & M. E. Reicher (Eds.), *Experience and Analysis* (pp. 153-170). Wien: HPT&ÖBV.
- Smolin, L. (2013). *Time reborn. From the crisis of physics to the future of universe*. Boston-New York: Houghton Mifflin Harcourt Publishing.
- Smoot, G. F. (2010). Go with the Flow, Average Holographic Universe. *Int.Journ.of Modern Physics D*, 19, 2247-58.
- Tegmark, M. (2011). *How unitary cosmology generalizes thermodynamics and solves the inflationary entropy problem*. Retrieved March 16, 2012, from Arxiv.org: <http://arxiv.org/pdf/1108.3080.pdf>
- Umezawa, H. (1995). H. Umezawa, Development in concepts in quantum field theory in half century. *Math. Japonica*, 41, 109–124.
- van Rijen, J. (1989). *Aspects of Aristotle's Logic of Modalities*. Dordrecht: Reidel.

- Verlinde, E. (2011). The origin of gravity and the laws of Newton. *Journal of High Energy Physics*, 29(4), 1104-33.
- Vitiello, G. (2004). The dissipative brain. In G. G. Globus, K. H. Pribram, & G. Vitiello (Eds.), *Brain and Being - At the boundary between science, philosophy, language and arts* (pp. 317-330). Amsterdam: John Benjamins Pub. Co.
- Vitiello, G. (2007). Links. Relating different physical systems through the common QFT algebraic structure. *Lecture Notes in Physics*, 718, 165-205.
- Vitiello, G. (2009). Coherent states, fractals and brain waves. *New Mathematics and Natural Computing*, 5(1), 245-264.
- Von Neumann, J. (1955). *Mathematical foundations of quantum mechanics*. Princeton, NJ: Princeton UP.
- Wheeler, J. A. (1990). Information, physics, quantum: The search for links. In W. H. Zurek (Ed.), *Complexity, entropy, and the physics of information*. Redwood City, CA: Addison-Wesley.
- Zeh, H. D. (2004). Wave function: 'it' or 'bit'? In J. D. Barrow, P. C. Davies, & C. L. Harper Jr. (Eds.), *Science and Ultimate Reality* (pp. 103-120). Cambridge, MA: Cambridge UP.
- Zeh, H. D. (2010). Quantum discreteness is an illusion. *Foundations of Physics*, 40, 1476-1493.
- Zenil, H. (Ed.). (2013). *A computable universe. Understanding and exploring nature as computation. Foreword by Sir Roger Penrose*. Singapore-Hackensack, NJ-London: World Scientific Publishing.