Homework week of Nov. 9: due Tues. Nov. 17

November 11, 2009

1. Prove that the Lebesgue measure of a countable set is zero. (Pay attention to the measurability issues.)

2. A G_δ set is a countable intersection of open sets. An F_σ set is a countable union of closed sets. Such sets form the second level in the Borel hierarchy.
(a) Show that any closed set in R^n is a G_δ set.
(b) Give an example of an F_σ sets which is not a G_δ set. (Think about countable dense sets.)

3. Show that an open disc in R^n is not the countable union of disjoint open rectangles. (A rectangle is a product of intervals in each variable separately.)

4. Here is an alternative definition of measurability of a set of reals: E is measurable if for every ϵ there exists a closed set F contained in E such that the outer measure of $E \setminus F$ is less than ϵ. Show that this is equivalent to the usual definition of measurability.

5. Suppose that $A \subset E \subset B$, and A and B are measurable sets of reals of finite measure, with $m(A) = m(B)$. Show that E is measurable.

6. Let N be the nonmeasurable set constructed in class. Suppose that E is a measurable set.
(a) If $E \subset N$, then $m(E) = 0$.
(b) If $m(E) > 0$, then E contains a nonmeasurable set.

7. page 247, #1,2,3,4.