PEMFC state and parameter estimation through a high-gain based adaptive observer

Andreu Cecilia [*], **Maria Serra** [*], **Ramon Costa-Castelló** [*]

[*] *Institut de Robòtica i Informàtica Industrial, CSIC-UPC, Llorens i Artigas 4-6, 08028 Barcelona, Spain (e-mail: {andreu.cecilia; maria.serra; ramon.costa}@upc.edu)*

Abstract:
This work presents the design of an adaptive observer to estimate the cathode catalytic layer's water content of a polymer electrolyte membrane fuel cell and some of the parameters related to its water dynamics. However, in general, existing adaptive observer algorithms require a certain relative degree condition which is not satisfied in the concerned fuel cell system. This conflict is solved by modifying the adaptive observer strategy with an auxiliary signal that does satisfy the relative degree condition. This signal is estimated through a high-gain observer. The viability of the presented observer is validated through numerical simulations.

Keywords: Observers; Estimation parameters; High-gain observer; Fuel cells; Nonlinear systems

1. INTRODUCTION

A fuel cell is an electrochemical device which transforms the chemical energy of its fuel (usually hydrogen) to electrical energy without generating hazardous pollutants as CO_2, NO_x or SO_x. From all the different types of fuel cells, the polymer electrolyte membrane fuel cell (PEMFC) stands out for its high energy density and low operating temperature. By virtue of these properties, PEMFCs have proved to be a potential substitute for some classical sources of energy as the internal combustion engines in the transport sector. However, due to its low working temperature, there is a generation and transport of liquid water during the operation of the PEMFC. This water is not just a sub-product that has to be evacuated, but a variable that plays an important role for an efficient fuel cell management (Ijaodola et al. (2019)). However, one of the conflicts related to developing adequate water control algorithms is the non-existence of sensors that can measure online the fuel cell's water content. For this reason, some researchers have focused on developing observer techniques that can estimate this quantity through common measured variables as the stack voltage or the stack temperature (Görgün et al. (2006)). These observer algorithms present adequate performance, nonetheless, its accuracy relies on a proper identification of a model, which is not always a feasible task. Specifically, the estimation of parameters related to the cathode catalytic layer's (CCL) water dynamics is still an open problem (Strahl et al. (2011)). As the water content cannot be measured online, the estimation of these parameters requires an algorithm that can estimate simultaneously the water content in the CCL and the parameters related to its dynamics. In order to achieve such algorithm, this work proposes an adaptive observer scheme based on the measurements of the stack temperature. Some authors have shown that the unknown parameters of the CCL's water dynamics can be linearly parametrized without inducing a too harsh over-parametrization (Strahl et al. (2014)). In the literature, there exists multiple adaptive observer strategies to solve linear parametrized problems (Zhang (2002)), (Besançon (2000)), (Cho and Rajamani (1997)). However, these strategies require a certain relative degree condition between the measured output and the unknown parameters, which is not satisfied between the measured stack temperature and the unknown water dynamics parameters. This restrictive relative degree condition can be relaxed by applying the existing adaptive algorithms with an auxiliary signal which fulfils the required relative degree condition. As this auxiliary signal will generally be based on unmeasured states, it has to be estimated through an observer algorithm. This work proposes achieving such estimation by the design of a high gain observer.

This paper is organized as follows, section 2 presents the concerned PEMFC model, section 3 presents an adaptive observer scheme and the relative degree conflict, in section 4 an auxiliary signal with adequate relative degree is presented, section 5 introduces the high-gain observer concept, in section 6 it is presented the proposed adaptive observer with high-gain estimation, in section 7 the observer is validated numerically and some conclusions are drawn in section 8.

2. PEM FUEL CELL MODEL

This work is based on an existing open-cathode PEMFC model (Strahl et al. (2014)) that depicts the behaviour

[⋆] This work has been partially funded by the Spanish State Research Agency through the María de Maeztu Seal of Excellence to IRI (MDM-2016-0656), by the project DOVELAR (ref. RTI2018-096001-B-C32) and by AGAUR of Generalitat de Catalunya through the Advanced Control Systems grant (SAC) (2017 SGR 482).

of the fuel cell through two dynamical states: the stack temperature T_{fc} and the cathode catalyst layer's (CCL) liquid water saturation s. Specifically, the model can be presented in the following state space form:

$$\dot{T}_{fc} = K_1(IV + I^2) + K_2 v_{air}(T_{amb} - T_{fc}) \quad (1)$$

$$\dot{s} = \frac{1}{K_s}(K_3 I - K_4 s f_p(T_{fc})) - \frac{K_{diff}}{K_s} f_d(s) \quad (2)$$

In relation to the model variables, the open circuit voltage, V, the stack temperature, T_{fc}, and the stack current, I, are assumed to be measurable. Moreover, the cathode air velocity, v_{air}, and the ambient temperature, T_{amb}, are assumed to be locally constant and known.

In addition, the model includes an algebraic relation between the states and the PEMFC's stack voltage:

$$V = K_5 f_a(T_{fc}, s, I) T_{fc}$$

The factors K_1, K_2, K_3, K_4, K_5, K_s and K_{diff} are model constants and $f_p(T_{fc})$, $f_d(s)$ and $f_a(T_{fc}, s, I)$ are nonlinear functions.

One of the main concerns working with this model is the identification of its parameters. Some parameters, as the fuel cell's stack mass or the cross-sectional area of the inlet manifold, which appear in the computation of K_1, K_2 and K_3, can be directly measured. Other parameters, as the fuel cell's specific heat capacity, have already been estimated experimentally (Strahl et al. (2011)). However, the estimation of parameters in equation (2) is still an open problem. Specifically, the liquid water sorption/desorption time constant (K_s) and the liquid water diffusion constant (K_{diff}) cannot be directly measured by any sensor and to the author's knowledge, the problem of developing algorithms that can estimate *online* these parameters has not been addressed before.

3. AN ADAPTIVE OBSERVER

The adaptive observer's objective is to generate an estimation of the states, $\hat{\mathbf{x}}$, and the unknown parameters, $\hat{\theta}$, such that, $\|\hat{\mathbf{x}} - \mathbf{x}\| \to 0$ and $\|\hat{\theta} - \theta\| \to 0$ as $t \to \infty$. Specifically, let a nonlinear system be depicted by the following expression:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{f}(\mathbf{x}, \mathbf{u}) + \mathbf{B}\phi(\mathbf{x}, \mathbf{u})\theta + \mathbf{g}(y, \mathbf{u}) \quad (3)$$

$$y = \mathbf{C}\mathbf{x} \quad (4)$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^q$, $y \in \mathbb{R}$. Moreover, $\theta \in \mathbb{R}^m$ is a vector of unknown constant parameters.

A system depicted by (3)-(4) is in the *adaptive observer form* iff the following conditions are satisfied (Besançon (2000)), (Cho and Rajamani (1997)):

- **Condition 1**: The concerned system is *minimum-phase* and the pair (A, C) is observable.
- **Condition 2**: The following output-parameter relative degree condition is satisfied

  $$Rank(\mathbf{CB}) = Rank(\mathbf{B}).$$

  which roughly means that the parameters, θ, appear in the first derivative of the output function, y.
- **Condition 3**: The norm of the unknown parameter vector is bounded

  $$\|\theta\| < \gamma_\theta$$

  where $\|\cdot\|$ denotes the 2-norm.
- **Condition 4**: The nonlinear functions $\mathbf{f}$ and ϕ are Lipschitz with $\gamma_{\mathbf{f}}$ and γ_ϕ as the Lipschitz constant

$$\|\phi(\mathbf{x}, \mathbf{u}) - \phi(\mathbf{z}, \mathbf{u})\| \leq \gamma_\phi \|\mathbf{x} - \mathbf{z}\|, \quad (5)$$

$$\|\mathbf{f}(\mathbf{x}, \mathbf{u}) - \mathbf{f}(\mathbf{z}, \mathbf{u})\| \leq \gamma_{\mathbf{f}} \|\mathbf{x} - \mathbf{z}\|. \quad (6)$$

Provided that a system is in adaptive observer form, then, there exists some vector $\mathbf{L} \in \mathbb{R}^{n \times 1}$, some symmetric matrices $\mathbf{Q} > 0$, $\mathbf{P} > 0$ and a matrix $\mathbf{M} > 0$ such that

$$(\mathbf{A} - \mathbf{LC})^T \mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{LC}) = -\mathbf{Q} \quad (7)$$

$$\mathbf{B}^T \mathbf{P} = \mathbf{MC} \quad (8)$$

$$(\gamma_{\mathbf{f}} + \gamma_\phi \gamma_\theta \|\mathbf{B}\|) < \frac{\lambda_{min}(\mathbf{Q})}{2\lambda_{max}(\mathbf{P})}. \quad (9)$$

where λ_{max} and λ_{min} are the maximum and minimum eigenvalue, respectively.

If one can find these matrices, then the following system

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{f}(\hat{\mathbf{x}}, \mathbf{u}) + \mathbf{B}\phi(\hat{\mathbf{x}}, \mathbf{u})\hat{\theta} + \mathbf{g}(y, \mathbf{u}) + \mathbf{L}(y - \mathbf{C}\hat{\mathbf{x}}) \quad (10)$$

$$\dot{\hat{\theta}} = \gamma \phi(\hat{\mathbf{x}}, \mathbf{u})^T \mathbf{M}(y - \mathbf{C}\hat{\mathbf{x}}); \quad \gamma > 0 \quad (11)$$

is an adaptive observer for (3)-(4) provided that the vector function $\mathbf{B}\phi(\mathbf{x}, \mathbf{u})$ is persistently exciting (Cho and Rajamani (1997)).

Definition 1. A vector field ϕ is said to be persistently exciting if there exist some constants α_1, α_2 and T_0 such that (Narendra and Annaswamy (1987)):

$$\alpha_1 \mathbf{I} \geq \int_t^{t+T_0} \phi(\tau)^T \phi(\tau) d\tau \geq \alpha_2 \mathbf{I} \quad \forall t > t_0$$

□

Taking into account the presented adaptive observer form, there is a natural parametrization of the concerned fuel cell model:

$$\mathbf{x} = [T_{fc}, s], \quad \mathbf{u} = [I, v_{air}], \quad y = T_{fc} \quad (12)$$

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\phi(\mathbf{x}, \mathbf{u}) = [K_3 I - K_4 s f_p(T_{fc}), \quad -f_d(s)]$$

$$\mathbf{g}(y, \mathbf{u}) = \begin{bmatrix} K_2 v_{air} T_{amb} + K_1(IV + I^2) \\ 0 \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{K_s} \\ \frac{K_{diff}}{K_s} \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} -K_2 v_{air} & 0 \\ 0 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad \mathbf{C} = [1 \quad 0] \quad (13)$$

Notice that, if the measured output is the fuel cell temperature, T_{fc}, then, **Condition 2** is not satisfied, as $\mathbf{CB} = 0$. However, it may be possible to compute an *auxiliary signal* z that does satisfy **Condition 2**. In such case, the adaptive observer (3)-(4) may be implemented with z as the new measured output.

4. AUXILIARY SIGNAL

Condition 2 implies that the concerned system is relative degree 1 from the unknown parameters, θ to the output, y.

Definition 4.1. Let a θ-affine single-output system be depicted by:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) + \phi(\mathbf{x},\mathbf{u})\theta \tag{14}$$
$$y = h(\mathbf{x}) \tag{15}$$

A system depicted by (14)-(15) has relative degree p from the unknown parameters, θ, to the output, y, if

$$L_\phi L_f^k h(\mathbf{x}) = 0 \quad \forall k < p-1 \tag{16}$$
$$L_\phi L_f^{p-1} h(\mathbf{x}) \neq 0 \tag{17}$$

where $L_f h(\mathbf{x})$ operation denotes the **Lie derivative** of the function $h(\mathbf{x})$ along the vector field $\mathbf{f}(\mathbf{x},\mathbf{u})$, and is computed as $L_f h(\mathbf{x}) = \frac{\partial h(\mathbf{x})}{\partial \mathbf{x}}\mathbf{f}(\mathbf{x},\mathbf{u})$. L_ϕ depicts the Lie derivative along $\phi(\mathbf{x},\mathbf{u})$. □

In the concerned fuel cell model, parametrized as (12)-(13), the relative degree of the system is higher than one, as it can be seen by the following computation.

$$L_\phi h(\mathbf{x}) = \mathbf{CB}\phi(\mathbf{x},\mathbf{u})\theta = 0$$

The idea is to design an auxiliary function $z = h_2(\mathbf{x})$ such that the system is relative degree 1 from this new function, z, to the parameters, i.e. $L_\phi h_2(\mathbf{x}) \neq 0$. Moreover, it is of interest that this auxiliary function is linear, i.e. $h_2(\mathbf{x}) = \mathbf{Hx}$, otherwise, the presented adaptive strategy cannot be implemented.

The problem of designing a relative degree 1 auxiliary signal for system (3)-(4) can be converted to the problem of finding a matrix $\mathbf{H}$ such that

$$rank(\mathbf{HB}) = rank(\mathbf{B})$$

Moreover, the auxiliary signal should be designed such that the pair (A, H) is observable.

A candidate for the presented fuel cell model could be

$$\mathbf{H} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T$$

The main concern of this auxiliary signal is that the state s is unknown, thus, the signal has to be estimated, $\hat{z} = \mathbf{H}\hat{\mathbf{x}} = \hat{T}_{fc} + \hat{s}$. This work proposes achieving such estimation through a high-gain observer. The general scheme of this adaptive observer structure is depicted in Fig. 1

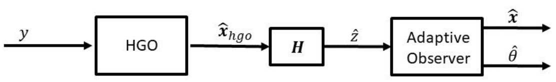

Fig. 1. Scheme of the adaptive observer with high gain estimation

5. HIGH-GAIN OBSERVER

Let a nonlinear MISO system be depicted by the following triangular structure:

$$\dot{\xi} = \mathbf{A}\xi + \Psi(\xi,\mathbf{u}) + \varphi(\xi,\theta), \tag{18}$$
$$y = \mathbf{C}\xi \tag{19}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T$$

$$\Psi(\xi,\mathbf{u}) = \begin{bmatrix} \psi_1(\xi_1,\mathbf{u}) \\ \psi_2(\xi_1,\xi_2,\mathbf{u}) \\ \vdots \\ \psi_n(\xi,\mathbf{u}) \end{bmatrix}, \quad \varphi(\xi,\theta) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \phi_2(\xi)\theta \end{bmatrix} \tag{20}$$

where θ is a vector of unknown parameters and ψ_i, ϕ_2 are known Lipschitz functions with L_i and L_ϕ as the Lipschitz constants.

$$\|\psi_i(\xi_1,\ldots,\xi_i,\mathbf{u}) - \psi_i(z_1,\ldots,z_i,\mathbf{u}))\| \leq L_i \sum_{k=1}^{i} \|\xi_k - z_k\| \tag{21}$$
$$\|\phi_2(\xi) - \phi_2(\mathbf{z})\| \leq L_\phi \|\xi - \mathbf{z}\|. \tag{22}$$

Moreover, the solutions of (18) are assumed to be bounded and the function $\phi_2(\xi)$ is assumed to have a maximal

$$\|\phi_2(\xi)\| \leq \phi_{2,max}. \tag{23}$$

In such form, a high-gain observer is a copy of this triangular structure with a high-gain feedback term proportional to the output estimation error, $\frac{\alpha_i}{\varepsilon^i}(y - \hat{\xi}_1)$, (Khalil (2017)):

$$\begin{cases} \dot{\hat{\xi}}_1 = \hat{\xi}_2 + \psi_1(\hat{\xi}_1,\mathbf{u}) + \dfrac{\alpha_1}{\varepsilon}(y - \hat{\xi}_1) \\ \vdots \\ \dot{\hat{\xi}}_i = \hat{\xi}_{i+1} + \psi_i(\hat{\xi}_1,\ldots,\hat{\xi}_i,\mathbf{u}) + \dfrac{\alpha_i}{\varepsilon^i}(y - \hat{\xi}_1) \\ \vdots \\ \dot{\hat{\xi}}_n = \psi_n(\hat{\xi},\mathbf{u}) + \phi_2(\hat{\xi})\bar{\theta} + \dfrac{\alpha_n}{\varepsilon^n}(y - \hat{\xi}_1) \end{cases} \tag{24}$$

This observer presents three elements that require a proper tuning: the parameters α_i, ε and the factor $\bar{\theta}$.

First, the factors α_i have to be chosen so the polynomial

$$s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1}s + \alpha_n \tag{25}$$

has all the roots in the left half plane. The roots will determine the behaviour of the estimation's error transient response. In general, there is no direct method to find the roots with the "best" transient response performance, nevertheless, they can be optimised by applying loop-shaping criteria (Khalil (2017)).

Second, the vector $\bar{\theta}$ is a nominal value of the unknown parameters. By means of this factor, the HGO will be independent from the estimated parameters, $\hat{\theta}$. Moreover, as the vector fields ψ_n and ϕ_2 are Lipschitz and the function ϕ_2 presents a maximal, the following bound can be defined:

$$\|\psi_n(\xi,\mathbf{u}) + \phi_2(\xi)\theta - \psi_n(\hat{\xi},\mathbf{u}) - \phi_2(\hat{\xi})\bar{\theta}\| \leq L_{n2}\|\xi - \hat{\xi}\| + M \tag{26}$$

where $L_{n2} = L_n + L_\phi\gamma_\theta$ and $M = \phi_{2,max}\|\theta - \bar{\theta}\|$.

Finally, the parameter ε is a positive constant in $\mathbb{R}_{<1}$, that has to be chosen sufficiently small in order to ensure the convergence of the state estimation error. If a system presents the triangular structure (18) and the parameters α are designed so the polynomial (25) is Hurwitz, there exists a positive constant $\varepsilon^* < 1$ such that for $0 < \varepsilon < \varepsilon^*$, the estimation error of the high gain observer (24) converges to a bounded error proportional to ε with a convergence rate proportional to ε^{-1} (Khalil (2017)).

Specifically, the estimation error converges to a bounded region of the form:

$$\|\xi - \hat{\xi}\| \leq \beta \tag{27}$$

This bound can be reduced by decreasing the value of ε. However, the parameter ε cannot be designed arbitrarily small, as excessively low values of ε will enhance the well-known peaking phenomena (Esfandiari and Khalil (1992)) and increase the observer's noise sensibility (Astolfi et al. (2016)). Nevertheless, the noise sensibility could be reduced by applying a dynamic dead-zone modification (Cecilia and Costa-Castelló (2020)).

5.1 Model Transformation

The concerned fuel cell model (1)-(2) does not present the triangular structure (18). For this reason, it is necessary to define a θ-independent diffeomorphism that relates both structures. The second order fuel cell model presents the following properties

- The model is θ affine.
- The model is **uniformly observable**, in the sense of being observable for all the control actions (Bornard et al. (1995)).
- The autonomous model, i.e. $\mathbf{u} = 0$, is **strong differential observable of order 2** (Gauthier and Kupka (2001)).
- The model is relative degree 2 between the output, $y = T_{fc}$, and the unknown parameters, θ (see definition 4.1).

under such properties, the following map

$$\Phi(\mathbf{x}) = \begin{bmatrix} T_{fc} \\ K_1(IV + I^2) \end{bmatrix} \tag{28}$$

is a θ-independent diffeomorphism between the fuel cell model and the presented triangular structure. In consequence, the high gain observer can be depicted in the original coordinates, $\mathbf{x}$. Specifically,

$$\dot{\hat{\mathbf{x}}} = \mathbf{g}(\hat{\mathbf{x}}, \mathbf{u}) + \left(\frac{\partial \Phi}{\partial \mathbf{x}}(\hat{\mathbf{x}})\right)^{-1} \begin{bmatrix} (\alpha_1/\varepsilon)(y - \hat{T}_{fc}) \\ (\alpha_2/\varepsilon^2)(y - \hat{T}_{fc}) \end{bmatrix} \tag{29}$$

where the function $\mathbf{g}$ is the following one

$$\mathbf{g}(\hat{\mathbf{x}}, \mathbf{u}) = \begin{bmatrix} K_1(IV + I^2) + K_2 v_{air}(T_{amb} - T_{fc}) \\ \dfrac{1}{\bar{K}_s}(K_3 I - K_4 s f_p(T_{fc})) - \dfrac{\bar{K}_{diff}}{\bar{K}_s} f_d(s) \end{bmatrix}$$

and the factors $\bar{K}_s$ and $\bar{K}_{diff}$ depict the nominal value of the unknown parameters.

Notice that the bounded error (27) can only ensure the bound $\|\Phi(\mathbf{x}) - \Phi(\hat{\mathbf{x}})\| \leq \beta$. However, as the map Φ is a diffeomorphism, therefore, has a differentiable inverse, and there exists a constant δ such that

$$\|\Phi(\mathbf{x}) - \Phi(\hat{\mathbf{x}})\| \leq \beta \implies \|\mathbf{x} - \hat{\mathbf{x}}\| \leq \delta. \tag{30}$$

6. HIGH-GAIN BASED ADAPTIVE OBSERVER

Taking into account the details presented in past sections, the proposed adaptive observer takes the following form:

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= \mathbf{A}\hat{\mathbf{x}} + \mathbf{f}(\hat{\mathbf{x}}, \mathbf{u}) + \mathbf{B}\phi(\hat{\mathbf{x}}, \mathbf{u})\hat{\theta} + \mathbf{g}(y, \mathbf{u}) \\ &\quad + \mathbf{L}(\hat{z} - \mathbf{H}\hat{\mathbf{x}}) \end{aligned} \tag{31}$$

$$\dot{\hat{\theta}} = \gamma\phi(\hat{\mathbf{x}}, \mathbf{u})^T \mathbf{M}(\hat{z} - \mathbf{H}\hat{\mathbf{x}}) \tag{32}$$

$$\hat{z} = \mathbf{H}\hat{\mathbf{x}}_{hgo} \tag{33}$$

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_{hgo} &= \mathbf{g}(\hat{\mathbf{x}}_{hgo}, \mathbf{u}) \\ &\quad + \left(\frac{\partial \Phi}{\partial \mathbf{x}}(\hat{\mathbf{x}}_{hgo})\right)^{-1} \begin{bmatrix} (\alpha_1/\varepsilon)(y - \hat{y}_{hgo}) \\ (\alpha_2/\varepsilon^2)(y - \hat{y}_{hgo}) \end{bmatrix} \end{aligned} \tag{34}$$

Theorem 2. If the parameters $\mathbf{L}$, γ and $\mathbf{M}$ are designed as presented in section 3, the parameters α_i and ε are tuned as in section 5 and the vector field $\mathbf{B}\phi(\hat{\mathbf{x}}, \mathbf{u})$ is persistently exciting (see definition 1) and has a maximal ϕ_{max}, then, the estimation errors $\|\mathbf{x} - \hat{\mathbf{x}}\|$ and $\|\theta - \hat{\theta}\|$ of the proposed adaptive observer (31)-(34) will converge to a bounded region.

Proof. The state estimation error dynamics between the observer (31) and the system (3), $\mathbf{e}_x = \mathbf{x} - \hat{\mathbf{x}}$, are depicted by the following expression

$$\begin{aligned} \dot{\mathbf{e}}_x =& (\mathbf{A} - \mathbf{L}\mathbf{H})\mathbf{e}_x + \mathbf{f}(\mathbf{x}, \mathbf{u}) - \mathbf{f}(\hat{\mathbf{x}}, \mathbf{u}) \\ &+ \mathbf{B}\phi(\mathbf{x}, \mathbf{u})\theta - \mathbf{B}\phi(\hat{\mathbf{x}}, \mathbf{u})\hat{\theta} + \mathbf{L}\mathbf{H}(\mathbf{x} - \hat{\mathbf{x}}_{hgo}) \end{aligned} \tag{35}$$

Similarly, the parameter estimation error dynamics, $\mathbf{e}_\theta = \theta - \hat{\theta}$ is described by

$$\dot{\mathbf{e}}_\theta = -\gamma\phi(\hat{\mathbf{x}}, \mathbf{u})^T \mathbf{M}\mathbf{H}\mathbf{e}_x + \gamma\phi(\hat{\mathbf{x}}, \mathbf{u})^T \mathbf{M}\mathbf{H}(\mathbf{x} - \hat{\mathbf{x}}_{hgo})$$

Consider the Lyapunov function candidate $V = \mathbf{e}_x^T \mathbf{P}\mathbf{e}_x + \frac{1}{\gamma}\mathbf{e}_\theta^T \mathbf{e}_\theta$. Then

$$\begin{aligned} \dot{V} =& \mathbf{e}_x^T((\mathbf{A} - \mathbf{L}\mathbf{H})^T\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{L}\mathbf{H}))\mathbf{e}_x \\ &+ 2\mathbf{e}_x^T\mathbf{P}(\mathbf{f}(\mathbf{x}, \mathbf{u}) - \mathbf{f}(\hat{\mathbf{x}}, \mathbf{u})) \\ &+ 2(\mathbf{B}\phi(\mathbf{x}, \mathbf{u})\theta - \mathbf{B}\phi(\hat{\mathbf{x}}, \mathbf{u})\hat{\theta})^T\mathbf{P}\mathbf{e}_x \\ &+ 2\mathbf{e}_x^T\mathbf{P}\mathbf{L}\mathbf{H}(\mathbf{x} - \hat{\mathbf{x}}_{hgo}) + \frac{2}{\gamma}\mathbf{e}_\theta^T\dot{\mathbf{e}}_\theta \\ \leq& \mathbf{e}_x^T((\mathbf{A} - \mathbf{L}\mathbf{H})^T\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{L}\mathbf{H}))\mathbf{e}_x \\ &+ 2\gamma_{\mathbf{f}}\|\mathbf{e}_x^T\mathbf{P}\|\|\mathbf{e}_x\| + 2\gamma_\phi\gamma_\theta\|\mathbf{B}\|\|\mathbf{e}_x\|\|\mathbf{P}\mathbf{e}_x\| \\ &+ 2\mathbf{e}_\theta(\mathbf{B}\phi(\hat{\mathbf{x}}, \mathbf{u}))^T\mathbf{P}\mathbf{e}_x \\ &+ 2\mathbf{e}_x^T\mathbf{P}\mathbf{L}\mathbf{H}(\mathbf{x} - \hat{\mathbf{x}}_{hgo}) + \frac{2}{\gamma}\mathbf{e}_\theta^T\dot{\mathbf{e}}_\theta \end{aligned} \tag{36}$$

If the parameter adaptive dynamics, $\dot{\mathbf{e}}_\theta$, depicted by (32), is implemented in the stated Lyapunov function and the unknown parameters are assumed to be constant, i.e. $\dot{\theta} = 0$, the following result is obtained

$$\begin{aligned} \dot{V} \leq& \mathbf{e}_x^T((\mathbf{A} - \mathbf{L}\mathbf{H})^T\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{L}\mathbf{H}))\mathbf{e}_x \\ &+ 2(\gamma_{\mathbf{f}} + \gamma_\phi\gamma_\theta\|\mathbf{B}\|)\|\mathbf{P}\|\|\mathbf{e}_x\|^2 \\ &+ 2\mathbf{e}_x^T\mathbf{P}\mathbf{L}\mathbf{H}(\mathbf{x} - \hat{\mathbf{x}}_{hgo}) \\ &+ 2\mathbf{e}_\theta^T\phi(\hat{\mathbf{x}}, \mathbf{u})^T\mathbf{M}\mathbf{H}(\mathbf{x} - \hat{\mathbf{x}}_{hgo}) \end{aligned} \tag{37}$$

If the matrices $\mathbf{M}$, $\mathbf{L}$ and $\mathbf{P}$ satisfy the conditions (7)-(9), with $\mathbf{C} = \mathbf{H}$, then, there is a positive constant d such that (Besançon (2000)):

$$\begin{aligned} \dot{V} \leq& -\mathbf{e}_x^T d\mathbf{e}_x + 2\mathbf{e}_x^T\mathbf{P}\mathbf{L}\mathbf{H}(\mathbf{x} - \hat{\mathbf{x}}_{hgo}) \\ &+ 2\mathbf{e}_\theta^T\phi(\hat{\mathbf{x}}, u)^T\mathbf{M}\mathbf{H}(\mathbf{x} - \hat{\mathbf{x}}_{hgo}) \\ \leq& -d\|\mathbf{e}_x\|^2 + 2\|\mathbf{e}_x\|\|\mathbf{P}\mathbf{L}\mathbf{H}\|\delta \\ &+ 2\|\mathbf{e}_\theta\|\|\phi(\hat{\mathbf{x}}, \mathbf{u})\|\|\mathbf{M}\mathbf{H}\|\delta \end{aligned} \tag{38}$$

where δ is defined in (30).

If the vector field $\phi(\hat{\mathbf{x}}, \mathbf{u})$ presents a maximal ϕ_{max}, then, the Lyapunov function is bounded as follows

$$\dot{V} \leq -d\|\mathbf{e}_x\|^2 + 2\|\mathbf{e}_x\|\|\mathbf{PLH}\|\delta + 4\gamma_\theta \phi_{max}\|\mathbf{MH}\|\delta. \quad (39)$$

Moreover, if the following constants are defined

$$k_1 = d \quad (40)$$

$$k_2 = 2\|\mathbf{PLH}\|\delta \quad (41)$$

$$k_3 = 4\gamma_\theta \phi_{max}\|\mathbf{MH}\|\delta, \quad (42)$$

it can be shown that the Lyapunov function's derivative (39) is strictly negative outside the region defined by

$$\frac{k_2 - \sqrt{k_2^2 + 4k_1k_3}}{2k_1} \leq \|\mathbf{e}_x\| \leq \frac{k_2 + \sqrt{k_2^2 + 4k_1k_3}}{2k_1}. \quad (43)$$

Notice that no conclusion has been drawn about the parameter estimation error, $\|\theta - \hat{\theta}\|$. As the state estimation error converges to a bounded region, only the following bound can be deduced from the state error dynamics (35)

$$\|\mathbf{B}\phi(\mathbf{x}, \mathbf{u})\theta - \mathbf{B}\phi(\hat{\mathbf{x}}, \mathbf{u})\hat{\theta}\| \to \delta_\theta$$

where δ_θ is some positive constant.

However, since $\|\mathbf{x} - \hat{\mathbf{x}}\|$ converges to a bounded region and the vector field ϕ is Lipschitz, it is easy to show that $\|\mathbf{B}\phi(\mathbf{x}, \mathbf{u})(\theta - \hat{\theta})\|$ will also converge to a bounded region. In consequence, if the vector ϕ is persistently exciting (see definition 1), then, the parameter estimation error, $\|\theta - \hat{\theta}\|$, will also converge to a bounded region (Narendra and Annaswamy (1987)). □

7. NUMERICAL SIMULATIONS

The observer performance has been validated through a set of simulations. In the simulations, the PEMFC model is excited with changes in the stack current, Fig. 2. This current signal has been designed in order to provide sufficient excitation for the parameter estimation method (see definition 1), and induces a stack temperature profile which is used by the adaptive observer (31)-(34) to estimate the fuel cell model states and water dynamics parameters.

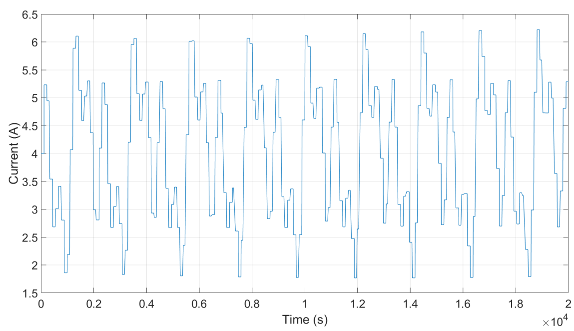

Fig. 2. Current profile used in the simulations.

In the simulation, the proposed adaptive observer is compared with different design values of ε, the other design parameters are summarized in table 1. Moreover, the state and parameter estimation values have been properly scaled in order to avoid computational issues.

Furthermore, these simulations consider the case in which no information is available about the unknown parameters,

Table 1. Observer parameters

Parameter	Value
α_1	0.1047
α_2	0.0012
l_1	$-1.074E^{-6}$
l_2	0.009
M	25
γ	1

in consequence, the nominal values $\bar{K}_s$ and $\bar{K}_{diff}$ are assumed to be 0.

In Fig. 3, it is depicted the fuel cell model's liquid water saturation, s, and the high gain observer estimation (34), $\hat{s}_{HGO}$, for the design values $\varepsilon = 0.1$ and $\varepsilon = 0.01$. It is clear, that a small value of ε ensures the convergence of the state estimation error to a lower bounded region δ, $\|\mathbf{x} - \hat{\mathbf{x}}_{hgo}\| \leq \delta$, which is a well-known result in high-gain observers (see section 5).

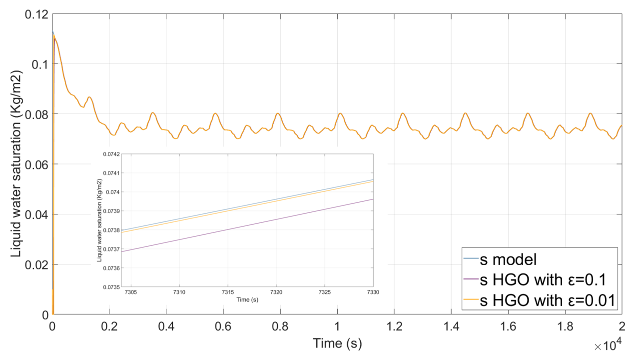

Fig. 3. Model's liquid water saturation (blue), HGO estimation with $\varepsilon = 0.1$ (purple) and HGO estimation with $\varepsilon = 0.01$ (yellow).

In Fig. 4, it is depicted the fuel cell model's liquid water saturation, s, and the adaptive observer estimation (31), $\hat{s}$, for the design values $\varepsilon = 0.1$ and $\varepsilon = 0.01$. The smaller value of ε presents a convergence of the state estimation error, $\|\mathbf{x} - \hat{\mathbf{x}}\|$, to a smaller bounded region. This result is coherent with the proof presented in section 6, as a smaller value of δ (30) induces a lower bound in the state estimation error (43).

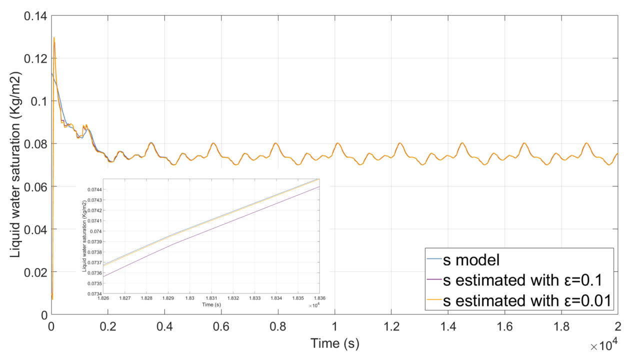

Fig. 4. Model's liquid water saturation (blue), adaptive observer estimation with $\varepsilon = 0.1$ (purple) and adaptive observer estimation with $\varepsilon = 0.01$ (yellow).

In Fig. 5 and Fig. 6, it is depicted the parameter estimation of the proposed adaptive observer and the model's value of said parameters. It can be seen that in the case of $\varepsilon = 0.1$, the parameter estimation error converges to a bounded relative error of the order of 5%, and the case $\varepsilon = 0.01$ converges to a relative error of the order of 0.3%.

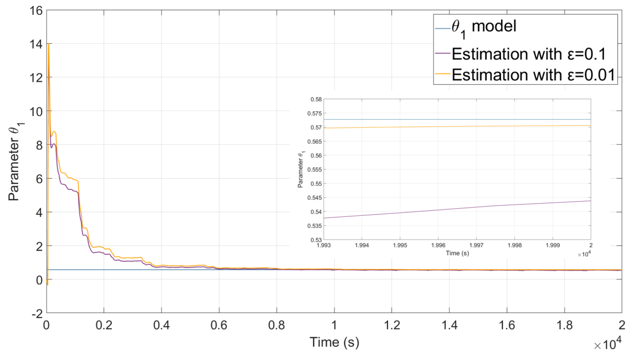

Fig. 5. Model's θ_1 (blue), adaptive observer estimation with $\varepsilon = 0.1$ (purple) and adaptive observer estimation with $\varepsilon = 0.01$ (yellow).

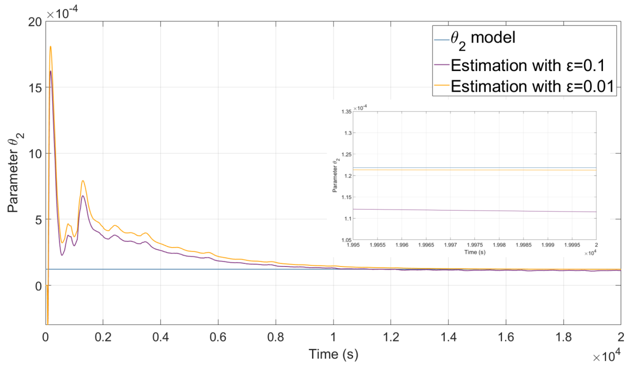

Fig. 6. Model's θ_2 (blue), adaptive observer estimation with $\varepsilon = 0.1$ (purple) and adaptive observer estimation with $\varepsilon = 0.01$ (yellow).

8. CONCLUSION

In this work, the problem of estimating simultaneously the liquid water saturation of a PEMFC and some of the parameters related to its water dynamics has been addressed through an adaptive observer strategy. In the literature, the available adaptive observers require a certain relative degree condition, which is not satisfied in the concerned PEMFC model. In order to solve this problem, an auxiliary signal which does satisfy the condition has been designed and its value has been estimated through a high gain observer.

The proposed observer has been validated in a simulation environment, in which it is shown that the state and parameter estimation converges to a bounded error. This estimation error can be reduced by increasing the gain of the HGO with the drawback of enhancing the peaking phenomena and the noise sensibility. This drawback could be potentially reduced by a feedback of the estimated parameter, $\hat{\theta}$, to the high high-gain observer. However, further study is required to improve the transient behaviour of such adaptive observer. Moreover, the input signal required in order to satisfy the persistence of excitation condition may be too harsh for some fuel cell systems. For this reason, it could be interesting to study a modification of the presented observer that relaxes the excitation condition.

REFERENCES

Astolfi, D., Marconi, L., Praly, L., and Teel, A. (2016). Sensitivity to High-Frequency Measurement Noise of Nonlinear High-Gain Observers. *IFAC-PapersOnLine*. doi:10.1016/j.ifacol.2016.10.274.

Besançon, G. (2000). Remarks on nonlinear adaptive observer design. *Systems and Control Letters*. doi:10.1016/S0167-6911(00)00065-7.

Bornard, G., Celle-Couenne, F., and Gilles, G. (1995). Observability and Observers. In *Nonlinear Systems-T.1, Modeling and Estimation*, 173–217. Chapman & Hall, London.

Cecilia, A. and Costa-Castelló, R. (2020). Observador de alta ganancia con zona muerta ajustable para estimar la saturación de agua líquida en pilas de combustible tipo pem. *Revista Iberoamericana de Automática e Informática industrial*, 17(2), 169–180.

Cho, Y.M. and Rajamani, R. (1997). A systematic approach to adaptive observer synthesis for nonlinear systems. *IEEE Transactions on Automatic Control*. doi:10.1109/9.566664.

Esfandiari, F. and Khalil, H.K. (1992). Output feedback stabilization of fully linearizable systems. *International Journal of Control*. doi:10.1080/00207179208934355.

Gauthier, J.P. and Kupka, I. (2001). Observability concepts. In *Deterministic Observation Theory and Applications*, 9–19. Cambridge University Press.

Görgün, H., Arcak, M., and Barbir, F. (2006). An algorithm for estimation of membrane water content in PEM fuel cells. *Journal of Power Sources*. doi:10.1016/j.jpowsour.2005.07.053.

Ijaodola, O.S., El- Hassan, Z., Ogungbemi, E., Khatib, F.N., Wilberforce, T., Thompson, J., and Olabi, A.G. (2019). Energy efficiency improvements by investigating the water flooding management on proton exchange membrane fuel cell (PEMFC). *Energy*. doi:10.1016/j.energy.2019.04.074.

Khalil, H.K. (2017). High-Gain Observers. In *High-Gain Observers in Nonlinear Feedback Control*, 17–30. Society of Industrial and Applied Mathematics, Philadelphia.

Narendra, K.S. and Annaswamy, A.M. (1987). Persistent excitation in adaptive systems. *International Journal of Control*. doi:10.1080/00207178708933715.

Strahl, S., Husar, A., Puleston, P., and Riera, J. (2014). Performance improvement by temperature control of an open-cathode PEM fuel cell system. In *Fuel Cells*. doi:10.1002/fuce.201300211.

Strahl, S., Husar, A., and Serra, M. (2011). Development and experimental validation of a dynamic thermal and water distribution model of an open cathode proton exchange membrane fuel cell. In *Journal of Power Sources*. doi:10.1016/j.jpowsour.2010.10.074.

Zhang, Q. (2002). Adaptive observer for multiple-input-multiple-output (mimo) linear time-varying systems. *IEEE Transactions on Automatic Control*, 47(3), 525–529. doi:10.1109/9.989154.