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BRAIN AND MATHEMATICS

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The fundamental connecting link between mathematics and theoretical physics is the pattern recognition capabilities of the human brain. George Chapline, Physics Reports 315 1999 pp. 95-105

It sometimes appears that the resistance to accepting the evidence that cortical cells are responding to the two dimensional Fourier components of stimuli [is due] to a general unease about positing that a complex mathematical operation similar to Fourier analysis might take place in a biological structure like cortical cells. It is almost as if this evoked for some, a specter of a little man sitting in a corner of the cell huddled over a calculator. Nothing of the sort is of course implied: the cells carry out their processing by summation and inhibition and other physiological interactions within their receptive fields. There is no more contradiction between a functional description of some electronic component being a multiplier and its being made up of transistors and wired in a certain fashion. The one level describes the process, the other states the mechanism. DeValois & DeValois, 1988 p 288

The fact that the formalism describing the brain microprocess is identical with the physical microprocess allows two interpretations: (a) The neural microprocess is in fact based on relations among microphysical quantum events, and (b) that the laws describing quantum physics are applicable to certain macrophysical interactions when these attain some special characteristics" (p. 270). The formalism referred to describes the receptive fields of sensory neurons in the brain cortex. These were mapped in terms of Gabor wavelets or more generally, "four dimensional information hyperspaces based on Jacobi functions (Atick and Redlich, 1989) or Wigner distributions (Wechsler, 1991). Pribram, 1991 Epilogue

A PERSONAL ROAD OF DISCOVERY

The story of how, as a non-mathematician, my interest was engaged in Gabor-like mathematics is worthwhile repeating. Why would I follow such a path, when so many neurophysiologists and experimental psychologists shun, with the exception of statistical analyses, mathematical expressions (one could say, mathematical metaphors) in attempts to understand brain/mind transactions?

The story begins in the late 1930s, working in Ralph Gerard's laboratory at the University of Chicago. Gerard showed us that a cut separating two parts of the brain cortex did not abolish transmission of an electrical stimulus across the separation as long as the parts were in some sort of contact. Meanwhile, I discussed these observations with my physics professor. I argued with both Gerard and the physicist that such large scale phenomena could not account for the brain processes that allowed us to perceive, think and act. Gerard, of course, agreed but insisted that more than simple neuronal connections were important in understanding brain function. My physics professor also agreed but had nothing to offer. He may have mentioned quantum physics but was not versed in it.

At about the same time, Walter Miles, Lloyd Beck and I were pondering the neural substrate of vision. I was writing an undergraduate thesis on retinal processing in color sensation under the supervision of Polyak, making the point that beyond the receptors, the bipolar cells seemed to differentiate the three color bands to which the receptors were sensitive into a greater number of more restricted bandwidths. We bemoaned our inability to come up with some similar understanding for form vision. I distinctly recall saying: “wouldn’t it be wonderful if we had a spectral explanation for brain processing of black and white patterns.”

By 1948 I had my own laboratory at Yale University and began a collaboration with Wolfgang Koehler told me of his Direct Current hypothesis as the basis for cortical processing in vision and demonstrated to me and my laboratory PhD students, Mort Mishkin and Larry Weiskrantz just how the anatomy of the auditory system would explain how the scalp auditory at the apex of the skull was transmitted by the brain’s tissue: no neural connections needed. Shades of my experience with Gerard.

This time I set to work to test Koehler’s hypothesis. We worked together with monkeys and humans displaying a white cardboard in front of their eyes and recorded from their visual cortex. (It was easy in those days to do such experiments with awake humans with their permission. Surgery had been done for clinical purposes with local anesthesia of the scalp – touching the brain itself is not felt by the patient.) Indeed we found a Direct Current (DC) shift during the display. One of my students and I then repeated the experiment using auditory stimulation in monkeys and obtained the same result in recording from the auditory cortex. (See Pribram 1971 Lecture 6 for review.)

In addition, I created minute epileptogenic foci in the visual cortex of monkeys and tested for their ability to distinguish very fine horizontal from vertical lines. Once electrical seizures commenced as shown by electrical recordings from their visual cortex I expected their ability to distinguish the lines to be impaired and even totally lost. The recordings showed large slow waves and total disruption of the normally patterned electroencephalogram (EEG).

Contrary to expectation, the monkeys performed the task without any deficiency. Koehler exclaimed: “Now that you have disproved not only my theory of cortical function in perception but everyone else’s, as well, what are you going to do?” I answered: “I’ll keep my mouth shut”. In fact, I refused to teach a course on brain mechanisms in sensation and perception when I transferred to Stanford University (in 1958) shortly thereafter.

I did not come up empty-handed, however. What did occur was that the epileptic seizures delayed the monkeys' learning of the task some seven fold. This led to another series of experiments in which we imposed a DC current across the cortex from surface to depth and found that a cathodal current delayed learning while an anodal current enhanced it. There is more to this story but that has to wait for another occasion.

Once at Stanford I turned to other experiments that demonstrated cortical control of sensory input in the visual and auditory systems, feedback processes that were important to the conceptions Miller, Galanter and I had put forward in "Plans and the Structure of Behavior" (1960).

Some years into my tenure at Stanford, Ernest Hilgard and I were discussing an update of his introductory psychology text when he asked me about the status of our knowledge regarding brain physiology in perception. I answered that I was dissatisfied with what we knew: I and others had disproved Koehler's (1958) suggestion that perception could be ascribed to direct current brain electrical fields shaped like (isomorphic with) envisioned patterns. Hubel and Wiesel (1968) had just shown that elongated stimuli such as lines and edges were the best shapes to stimulate neurons in the primary visual receiving cortex – and that perception followed from putting together something like stick figures from these elementary sensitivities. As much of our perception depends on shadings and texture, the stick figure approach failed for this and other reasons to be a satisfactory. I was stumped. Hilgard, ordinarily a very kind and patient person seemed peeved and declared on a second encounter, that he did not have the luxury of procrastination as he had to have something to say in the text. So he asked once again to come up with some viable alternative to the ones I had so summarily dismissed.

I took the problem to my laboratory group and told them about Hilgard's problem and my dissatisfaction with the two extant proposals. I added that there was one other suggestion that had been offered which had the advantage that neither I nor anyone else knew how it might work either neurologically or with regard to perception: Lashley (1942) had proposed that interference patterns among wave fronts in brain electrical activity could serve as the substrate of perception and memory as well. This suited my earlier intuitions, but Lashley and I had discussed this alternative repeatedly, without coming up with any idea what wave fronts would look like in the brain. Nor could we figure out how, if they were there, how they could account for anything at the behavioral level. These discussions taking place between 1946 and 1948 became somewhat uncomfortable in regard to Don Hebb's book (1948) that he was writing at the time we were all together in the Yerkes Laboratory for Primate Biology in Florida. Lashley didn't like Hebb's formulation but could not express his reasons for this opinion: "Hebb is correct in all his details but he's just oh so wrong".

Within a few days of my second encounter with Hilgard, Nico Spinelli a postdoctoral fellow in my laboratory, brought in a paper written by John Eccles (Scientific American, 1958) in which he stated that although we could only examine synapses one by one, presynaptic branching axons set up synaptic wavefronts. Functionally it is these wavefronts that must be taken into consideration. I immediately realized (see Fig. 1-14, Languages of the Brain 1971) that axons entering the synaptic domain from different directions would set up interference patterns. (It was one of these occasions when one feels an utter fool. The answer to Lashly's and my first question as to where were the waves in the brain, had been staring us in the face and we did not have the wit to see it during all those years of discussion.)

Within another few days I received my current edition of Scientific American in which Emmet Leith and J. Upatnicks (1965) describe how recording of interference patterns on film tremendously enhanced storage and processing capability. Images could readily be recovered from the store by appropriate procedures that had been described by Dennis Gabor (1946) almost two decades earlier. Gabor called his mathematical formulation a hologram.

Using the mathematical holographic process as a metaphor seemed like a miraculous answer to Hilgard's question. Shading, detail, texture, everything in a pattern that we perceive can be accomplished with ease. Russell and Karen DeValois (1988) book on "Spatial Vision" and my (1991) book "Brain and Perception" provide detailed reviews of experimental results that support the conjecture that holography is a useful metaphor in coming to understand the brain/mind relation with regard to perception. Here I want to explore some further thoughts engendered by this use of a mathematical formulation to understand the brain/mind relation.

Some years later, in Paris, during a conference sponsored by UNESCO where both Gabor and I were speakers, we had a wonderful dinner together. I told him about the holographic metaphor for brain processing and we discussed its Fourier basis. Gabor was pleased in general but stated that "brain processing [of the kind we were discussing] was Fourier-like but not exactly Fourier." I asked, what then might such a relation look like and Gabor had no answer. Rather we got onto a step-wise process that could compose the Fourier - an explanation that I later used to trace the development of the brain process from retina to cortex. Gabor never then nor later told me about his 1946 contribution to communication theory and practice: that he had developed a formalism to determine the maximum compressibility of a telephone message that renders it still intelligible. He used the same mathematics that Heisenberg had used to describe processes in quantum physics and therefore called his "unit" a quantum of information. It took me several years to locate this contribution which is referred to in Likleiter's article on acoustics in Stevens 1951 Handbook of Experimental Psychology.

Does this application indicate that the formalism of quantum physics applies more generally to other scales of inquiry? Alternatively, for brain function, at what scale do actual quantum physical processing take place? At what anatomical scale(s) do we find quantum coherence and at what scale does decoherence occur? What relevance does this scale have for our experience and behavior?

To summarize: The formalisms that describe the holographic process and those that describe quanta of information apparently DO extend to scales other than the quantum. Today we use quantum holography to produce images with the technique of functional Magnetic Resonance (fMRI). The quantities described by terms of the formalisms such as Planck's constant will, of course, vary but the formulations will to a large extent be self-similar. The important philosophical implications for the brain/mind issue have been addressed in depth by Henry Stapp on several occasions (e.g 2003, "The Mindful Universe") as well as by many others including myself (e.g. Pribram, 1997, What is mind that the brain may order it?).

SCALE

Deep and surface processing scales:

Brain, being material, has at some scale a quantum physical composition. The issue is whether the grain of this scale is pertinent to providing insights into those brain processes that organize experience and behavior. In my book "Languages of the Brain" (1971) I identify two very different scales at which brain systems operate. One such scale, familiar to most students of the nervous system, is composed of circuits made up of large fibers usually called axons. These circuits operate by virtue of nerve impulses that are propagated along the fibers by neighborhood depolarization of their membranes.

But other, less well popularized, operations take place in the fine branches of neurons. The connections between neurons (synapses) take place for the most part within these fine fibers. Pre-synaptically, the fine fibers are the terminal branches of axons that used to be called teledendrons. Both their existence and their name have more recently been largely ignored. Postsynaptically, the fine fibers are dendrites that compose a feltwork within which connections (synapses and electrical ephapses) are made in every direction. This feltwork acts as a processing web.

The mathematical descriptions of processing in the brain's circuits needs to be different from the descriptions that describe processing in fine fibers. The problem that needs to be addressed with regards to circuits is that the connecting fibers are of different lengths and diameters that can distort the conduction of a pattern. The problem that needs addressing with regards to fine fiber processing is that, practically speaking, there are no propagated impulses within them so conduction has to be accomplished passively. Roberto Llinas (2000; Pellionitz and Llinas 1979; 1985) has provided a tensor theory that addresses the propagation in circuits and my holonomic (quantum holographic) theory models processing in the fine fibered web.

For me it has been useful to compare Llinas theory with mine to be able to detail their complementarity. The primary difference between the theories rests on the difference between the neural basis each refers to: Llinas is modeling neural circuits, what I (Pribram, 1997; Pribram and Bradley,1998) have called a surface processing structure. Holonomic theory models what is going on in the fine fibered parts of these circuits, what I have referred to as deep processing. (The terms were borrowed from Noam Chomsky's analysis of linguistic structure and may, perhaps be able to provide a neurological account of these aspects of linguistic processing).

Despite the different scales of these anatomical substrates, both Llinas and I emphasize that the processing spacetime in the brain is not the same as the spacetime within which we ordinarily get about. Llinas developed a tensor theory that begins, as does holonomic theory with oscillators made up of groups of neurons or their fine fibered parts. Next both theories delineate frames of reference that can be described in terms of vectors. Llinas uses the covariance (and contravarience) among vectors to describe tensor matrices where the holonomic theory uses vectors in Hilbert phase space to express the covariance. Llinas' tensor metric is not limited to orthogonal coordinates as is holonomic theory. (Llinas indicates that if the frame of reference is thought to be orthogonal, proof must be provided. I have provided such evidence in "Brain and Perception" and indicated when orthogonality must be abandoned in favor of non-linearity).

In keeping with his caveat, Llinas does use the Fourier transform to describe covariation for the input, that is the sensory driven vectors: "[There are] two different kinds of vectorial expressions both assigned to one and the same physical location P, an invariant. The components v/i of the input vector are covariant (they are obtained by the orthogonal projection method) while the components v\j of the output vector are contravariant (obtained by the parallelogram method)" (Pellionitz and Llinas 1985, p 2953). As in the holonomic theory, the tensor theory needs to establish entities and targets and it does this (as in the holonomic theory (see Pribram 1991, Lectures 5 and 6) by using the <u>motor</u> output to create contravariant vectors. The covariant-contravariant relationship is combined into a higher level invariant tensor metric.

Thus Llinas states that "sensory systems in the CNS are using expressions of covariant type while motor systems use components of a contravariant type" (p2953). This is similar to the use of motor systems in "Brain and Perception" to form Lie groups to produce the perception of invariants basic to object perception. Llinas' theory is more specific in that it spells out contravariant properties of the motor process. On the other hand, Holonomic theory is more specific in specifying the neural substrate produced by nystagmoid and other such oscillating movements (that result in co-ordination of pixels moving together against a background of more randomly moving pixels).

Another advantage of the holonomic theory is that it can explain the fact that the processes that form the experiencing of objects, project them away from the processing medium. "Projection" can be experienced by viewing a transmission hologram. Georg von Bekesy (1967) demonstrated this attribute of visual and auditory processing by arranging a set of vibrators on the skin of the forearm. Changing the phase relations among the vibrators resulted in feeling a point stimulus moving up and down the skin. Bekesy then placed two such arrays of vibrators, one on each forearm. Now, with appropriate adjustments of phase, the sensation obtained was a point in space in front of and between the arms. A similar phenomenon occurs in stereophonic sound: adjusting the phase of the sound coming out of the two or more speakers projects the sound away from the speakers (and, of course the receiver where the processing is actually occurring).

There is more to the rich yield obtained by comparing the Tensor theory to the Holonomic theory. For instance, Pellionisz and Llinas develop a look-ahead module via Taylor-assemblies that are practically the same as the anticipatory functions based on Fourier series (Pribram 1997).

The two theories also converge as Tensor Theory is based on "a coincidence of events in which both the target and interceptor merge into a single event point. This is an invariant known in physical sciences as a four dimensional Minkowski-point or world-point." (Pellionitz and Llinas, p. 2950). Holonomic Theory also requires a high-dimensional position-time manifold. "As originally implied by Hoffman (1996) and elaborated by Caelli, et al. (1978), the perceptual representation of motion should be subject to laws resembling the Lorenz transformations of relativity theory." This means that the Poincare group (Dirac, 1930; Wigner, 1939) is relevant, requiring a manifold of as many as ten dimensions. In the context of modeling the brain process involved in the perception of Shepard figures, what needs to be accomplished "is replacing the Euclidian group [that ordinarily describes geodesics] with the Poincare group of space time isometries, the relativistic analogues of geodesics --." (Pribram 1991, p.117)

Both theories handle the fundamental issue as to "how can coordinates be assigned to an entity which is, by its nature, invariant to coordinate systems" (Pellionez and Llinas p. 2950). The very term "holonomy" was chosen to portray this issue.

It is fitting that surface structure tensor circuit theory uses insights from relativity theory while deep structure holonomy regards quantum –like processing. As physicists struggle to tie together relativity and quantum field theory in terms of quantum gravity, perhaps further insights will be obtained for understanding brain processing. (Hameroff and Penrose,1995; Smolin 2004; Ostriker and Steinhardt 2001).

The main practical difference between the theories is that In the Tensor Theory, time synchrony among brain systems (which means correlation of their amplitudes) is all that is required. Holonomic theory indicates that a richer yield is obtained when phase coherence is manifest. Principle component analysis will get you correlations but it takes Independent Component Analysis (equivalent to 4^{th} order statistics) to capture the detail (e.g. texture) represented in the phase of a signal. (King, Xie, Zheng, and Pribram 2000).

Some of the relationships between the theories are being implemented in the production of functional Magnetic Resonance Imaging (fMRI). Heisenberg matrices (representations of the Heisenberg group) are used and combine in what is called quantum holography (that is, holonomy) with the tensor geometry of relativity. (Schempp 2000)

Llinas, in a book called the "i of the vortex" (2001) spells out in detail the primacy of the Motor Systems not only in generating behavior but also in thinking (conceptualized as internal movement) and the experience of the self. This is an important perspective for the psychological and neurosciences (see e.g. Pribram in press) but addresses issues beyond the scope of this essay.

Quantum Brain Dynamics:

Henry Stapp in two excellent articles (Stapp 1997a and b) reviews the development of quantum theory and outlines how it is essential to understanding the mind/brain relationship. Stapp sets up the issue as follows. "Brain process is essentially a search process: the brain, conditioned by earlier experience, searches for a satisfactory response to the new situation that the organism faces. It is reasonable to suppose that a satisfactory response will be programmed by a template for action that will be implemented by a carefully tuned pattern of firings of some collection of neurons. The executive pattern would be a quasi-stable vibration that would commandeer certain energy resources, and then dissipate its energy into the initiation of the action that it represents." Patterns of firings and quasi-stable vibrations are, what I have termed the surface and deep structures of processing that are represented by Llinas' Tensor and my Holonomic Theories respectively.

Stapp goes on to note that "If the programmed action is complex and refined then this executive pattern must contain a great deal of information and must, accordingly, be confined to a small region of phase space." Holonomic theory indicates that spread functions such as those that compose holography, do indeed make it possible to contain a great deal of information within a small region (patches of dendritic receptive fields) of phase space. Stapp further notes that "the relative timing of the impulses moving along the various neurons, or groups of neurons, will have to conform to certain ideals to within very fine levels of tolerance. How does the hot, wet brain, which is being buffeted around by all sorts of thermal and chaotic disturbances find its way to such a tiny region in a timely manner?" Llinas' Tensor Theory deals with the timing issue.

Further: "How in 3n dimensional space (where n represents some huge number of degrees of freedom of the brain) does a point that is moving in a potential well that blocks out those brain states that are not good solutions to the problem --- but does not block the way to good solutions find its way in a short time to a good solution under chaotic initial conditions?" Stapp notes that classical solutions to this problem won't work and that "the quantum system [will work as it] has the advantage of being able to explore simultaneously (because the quantum state corresponds to a superposition of) all allowed possibilities." Stapp provides a viable metaphor in a glob or cloud of water acting together rather than as a collection of independently moving droplets. "The motion of each point in the cloud is influenced by its neighbors."

However classical holography will also do just this. But the advantage of holonomy, that is quantum holography, is that it windows the holographic space providing a "cellular" phase space structure, in patches of dendritic fields thus enhancing the alternatives and speed with which the process can operate. In short, though the information within a patch is entangled, cooperative processing between patches can continue to cohere or de-coherence can "localize" the process.

With regard to evidence regarding the scale at which quantum processes are actually occurring, a number of publications have reported that quantum coherence characterizes the oscillations of ions within neural tissue channels. (e.g. See Stapp 1997; and Jibu et al 1994; Jibu and Yasue in this volume). The question immediately arises as to whether decoherence occurs when the channels communicate with each other and if so, how. Stapp notes that "phase relationships, which are essential to interference phenomena, get diffused into the environment, and are difficult to retrieve. These decoherence effects will have a tendency to reduce, in a system such as the brain, the distances over which the idea of a simple quantum system holds. "

Hameroff and Penrose (1995) have also dealt with the limited range over which quantum coherence can operate. These authors suggest that excitation at the microtubular scale follows quantum principles but that decoherence self-organizes towards the end of an axon or dendrite. Davydoff (see Jibu, this Vol.) Ross Adey (1987) and I (1991) have independently proposed that microtubular quantum coherence provides saltatory coupling between dendritic spines and saltatory conduction in between nodes of Ranvier in axons via soliton waves. (See also, Jibu and Yasue, this volume). Soliton waves would thus provide a longer range over which coherence can be maintained.

An additional mechanism for coherent channel interaction has been proposed by Jibu, Pribram and Yasue (1996). This proposal focuses on the phospholipd bilayer that composes the membrane within which the channels occur. The phosphate parts of the molecule are hydrophilic capturing water as in a swamp. The water in such a region can become ordered into a super-liquid form that, by way of boson condensation, can act as a superconductor. Channels become connected over a limited distance by a transitional process that is quantum-like at a somewhat larger scale than the channels per se.

Thus, at the neural systems scale, there are two quantum-like fields, one pre-synaptic composed by the fine branching of axons as they approach the synapse; the other post-synaptic composed of the fine branches of dendrites. Hiroomi Umezawa and his collaborators (Stuart, Takahashi and Umezawa 1979) pointed out that not only quantum but “classical” processing can be derived from quantum field theory. The relevance of all this to the brain/mind issue is that both Umezawa and Giuseppe Vitiello (2004) have, on the basis of mathematical insights, proposed that interactions among these two quantum brain fields is necessary for self-reflective consciousness to occur. Hiley notes: “this is part of a bigger mathematical structure of bi-algebras that Umezawa and Vitiello are exploring. The doubling arises from a natural duality.” I add, could this doubling arise from the nature of the Fourier relationship? The Fourier transformation results in a complex number that represents both a real and a virtual line, a built-in duality.

My question is not an idle one. Our optical system performs a Fourier transform that results in the dual of real and virtual. One of these must be repressed in getting about in the space-time world. But the repression is incomplete. Experiments using glasses that invert the optical image to make the world look upside down, have shown that actively moving about re-inverts the image so that the world again looks “normal”. Re-reversal takes place over time when the glasses are removed. Vitiello’s “double” is thus twice unveiled.

To return to the topic of “scale”: In the brain, at what scale does decoherence initially occur? There are two types of processes that are excellent candidates. The local chemical activities, constituted of neuro-transmitters, neuro-modulators and neuro-regulators appear, at present, not to share properties that are best described in quantum terms. Their operation transforms the entangled quantum processes into larger scale influences on neural circuitry especially at synaptic sites. A second locus for decoherence is the region of the axon hillock. It is here that the passive conduction of dendritic activity influences the spontaneous generation of the discrete impulses that transmit the results of processing at one location to another location via neural circuitry.

FORMALISMS:

The Quantum formalism:

The initial quotation introducing this essay is from the ending of an excellent paper by George Chapline (1999) entitled "Is theoretical physics the same thing as mathematics". Chapline's provocative title employs a bit of poetic license. Nonetheless the paper provides considerable insight as to the applicability of the quantum formalism to other scales of inquiry. Chapline shows that quantum theory "can be interpreted as a canonical method for solving pattern recognition problems" (p95). In the paper he relates pattern recognition to the Wigner-Moyal formulation of quantum theory stating that this "would be a good place to start looking for a far reaching interpretation of quantum mechanics as a theory of pattern recognition" (p97). In a generalization of the Wigner- Moyal phase space he gives the physical dimensions as the Weyl quantization of a complete holographic representation of the surface. He replaces the classical variable of position within an electromagnetic field with ordinary creation and annihilation operators. He shows that "representing a Riemann surface holographically amounts to a pedestrian version of a mathematically elegant characterization of a Riemann surface in terms of its Jacobian variety and associated theta functions" (p.98). This representation is equivalent "to using the well known generalized coherent states for an SU(n) Lie algebra" (p.98). This is the formalism employed in "Brain and Percption" (Pribram, 1991) to handle the formation of invariances that describe entities and objects.

There is much more in Chaplin's paper that resonates with the holonomic, quantum holographic formulations that describe the data presented in "Brain and Perception". These formulations are based on quantum-like wavelets, Gabor and Wigner phase spaces. Whether these particular formulations will be found to be the most accurate is not the issue: rather it is that such formalisms can be attempted due to the fact that the "fundamental connecting link between mathematics and physics is the pattern recognition of the human brain" (p.104).

As an example of the utility of these insights, Chapline indicates how we might map the co-ordination of processing in the central nervous system. He notes that "the general idea [is] that a quantum mechanical theory of information flow can be looked upon as a model for the type of distributed information processing carried out in the brain." He continues, "one of the fundamental heuristics of distributed information processing networks is that minimization of energy consumption requires the use of time division multiplexing for communication between processors, and it would be natural to identify the local internal time in such networks as quantum phase" (p.104). The caveat is, as noted, that quantum phase is fragile in extent and must be supplemented by the processes described in comparing holonomic (quantum holographic) theory with the tensor theory of Llinas (which applies to neural circuitry rather than to the fine fibered quantum holographic processing per se).

Bohm and Hiley (1981) had also undertaken a topological approach to quantum mechanics based on a Wigner-Moyal cellular structure of phase space. In the current volume, Hiley (this volume) carries the approach further by relating it to Gabor's handling of signal transmission (communication) with what we now call a Gabor function (he called it a quantum of information) which is the centerpiece of the Holonomic Brain theory presented in "Brain and Perception" (1991). Hiley is able to introduce a phase space distribution function that allows calculation of quantum probabilities without having to resort to non-commuting dynamic variables. This makes easier the transition to the commuting aspects of groups. It thus shows the intimate connection with the Heisenberg group as used by Schempp in describing the fMRI process.

Hiley goes on to note that underlying the Wigner-Moyal distribution is the simplectic group. (Note that Chapline has focused on an SU(n) Lie group. The simplectic group is mathematically the more general). "The simplectic group is in turn covered by a metaplectic group that underlies Schroedinger's equation, as well as Hamilton's equation of motion and the classical ray formulation of optics. The metaplectic 'double' covers the symplectic group in the same way that the spin group SU(2) double covers the rotation group SU(3).

The importance of these insights is that "we have a mathematical structure that is basic to both classical mechanics and quantum mechanics. At this level there is no basic difference between the dynamical equations of classical and quantum mechanics. The difference arises once one asserts there is a minimum value for this action and equates this value to Planck's constant h (Hiley, this volume).

"What this means is that certain results - may look as if they are quantum in origin but in fact have nothing to do with quantum mechanics per se but arise from the group structure that is common to both forms of mechanics. For example the Fourier transformation is common to both classical and quantum situations. Indeed the Fourier transformation is at the heart of of Gabor's discussion of information transfer. Thus any results that emerge from an analysis of either the Wigner-Moyal approach or the Bohm approach may not necessarily have to do with quantum phenomena per se, and for that reason I would like to call the emerging dynamics that I will discuss below 'information dynamics'." (Hiley, this volume)

Observables, Observations, and Measurement:

Just what is the specific role of the brain in helping to organize our conscious relatedness? A historical approach helps sort out the issues. The Matter/Mind relationship has been formulated in terms of cuts. In the 17th century the initial cut was made by Renee Descartes (1662/1972) who argued for a basic difference in kind between the material substance composing the body and its brain and conscious processes such as thinking. With the advent of quantum physics in the 20th century Descartes' cut became untenable. Werner Heisenberg (1930) noted a limitation in simultaneously measuring the moment (rotational momentum) and location of a (material) mass. Dennis Gabor (1946) found a similar limit to our understanding of communication, that is, minding, because of a limitation in simultaneously measuring the spectral composition of the communication and its duration.

These indeterminacies place limits to our observations of both matter and mind and thus the location of the matter/mind cut. Heisenberg (1930) and also Wigner (1972) argued that the cut should come between our conscious observations and the elusive “matter” we are trying to observe. Niels Bohr (1961) argued more practically that the cut should come between the instruments of observations and the data that result from their use.

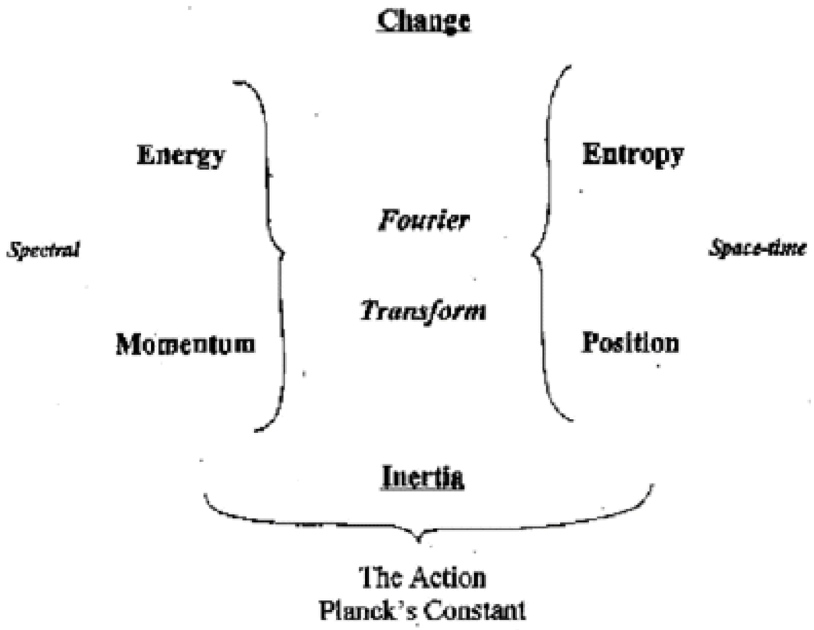
In keeping with Bohr's view, these differences in interpretion come about as a consequence of differences in focus provided by instrumentation (telescopes, microscopes, atom smashers, and chemical analyzers). Measurements made with these instruments render a synopsis of aspects of our experience as we observe the world we live in.

The Fourier Relationship:

The formalisms found to be important in quantum measurement as it relates to the brain/mind issue is the Fourier (1802) relationship. This relationship states that any space-time pattern can be transformed into the spectral domain characterized by a set of waveforms that encode amplitude, frequency and phase. Inverting the transform realizes the original space-time configuration. The transform domain is “spectral” not just “frequency” because the Fourier transformation encodes both the cosine and sine of a waveform allowing the interference between the 90 degree phase separation to be encoded discretely as coefficients.

The advantage gained by transforming into the spectral domain is that a great variety of transformed patterns can be readily convolved (multiplied) so that by performing the inverse transform the patterns have become correlated. This advantage is enhanced in quantum holography (which I have called Holonomy). Chapline (2002) in a paper entitled: “Entangled states, holography, and quantum surfaces” argues that the simplest way to encode “fundamental objects --- may be as multi-qubit entangled states” (p. 809). I suggest that, impractical as it may currently seem, it would be more productive to encode “qulets”, wavelet transformations, to preserve phase. As noted, Lie group theory can be used to describe how, by way of co-variation, various perspectives (images) of an object can form an invariant entity. (Pribram 1991) Image processing as in tomography such as PET scans and fMRI are prime examples of the utility of such encoding.

The diagram below provides one summary of what these measurements indicate both at the quantum and cosmic scale. The diagram is based on a presentation made by Jeff Chew at a conference sponsored by a Buddhist enclave in the San Francisco Bay area. I had known about the Fourier transformation in terms of its role in holography. But I had never appreciated the Fourier-based fundamental conceptualizations portrayed below. I asked Chew where I might find more about this and he noted that he'd got it from his colleague Henry Stapp who in turn had obtained it from Dirac. (Eloise Carlton a mathematician working with me and I had had monthly meetings with Chew and Stapp for almost a decade and I am indebted to them and to David Bohm and Basil Hiley for guiding me through the labyrinth of quantum thinking.)



The wave/particle dichotomy is orthogonal to the above distinction.

The diagram has two axes, a top-down and a right-left. The top-down axis distinguishes change from inertia. Change is defined in terms of energy and entropy. Energy is measured as the amount of actual or potential work necessary to change a structured system and entropy is a measure of how efficiently that change is brought about. These measurements are made in terms of numbers. Inertia is defined as moment, the rotational analogue of mass. Location is indicated by its spatial coordinates described in terms of geometry.

The right-left axis distinguishes between measurements made in the spectral domain and those made in spacetime. Spectra are composed of interference patterns where fluctuations intersect to reinforce or cancel. Holograms are examples of the spectral domain. I have called this pre-spacetime domain a potential reality because we navigate the actually experienced reality in spacetime.

The up-down axis relates mind to matter by way of sampling theory (Barrett 1993). Choices need to be made as to what aspect of matter we are to “attend”. The brain systems coordinate with sampling have been delineated and brain systems that impose contextual constraints on sampling have been identified (Pribram 1959; 1971). The down-up axis describes the emergence of mental patterns from material patterns.

My claim is that the basis function from which both matter and mind are “formed“ is the potential reality, the flux (or holo-flux, see Hiley 1996). This flux provides the ontological roots from which conscious experiences regarding matter as well as mind (psychological processes) become actualized in spacetime. To Illuminate this claim, let me begin with a story I experienced: Once, Eugene Wigner remarked that in quantum physics we no longer have observables, (invariants) but only observations. Tongue in cheek I asked whether that meant that quantum physics is really psychology, expecting a gruff reply to my sassiness. Instead, Wigner beamed a happy smile of understanding and replied, “yes, yes, that’s exactly correct”. If indeed one wants to take the reductive path, one ends up with psychology, not particles. In fact, it is a psychological process, mathematics, that describes the relationships that organize matter. In a non-trivial sense current physics is rooted in both matter and mind. (Chapline 2000, “Is physics and mathematics the same thing?”).

Conversely, communication ordinarily occurs by way of a material medium Bertrand Russell (1948) addressed the issue that the form of the medium is largely irrelevant to the form of the communication. In terms of today’s functionalism it is the communicated sample of a pattern that is of concern, not whether it is conveyed by a cell phone, a computer or a brain and human body. But not to be ignored is the fact that communication depends on being embodied, instantiated in some sort of material medium. This convergence of matter on mind, and of mind on matter, gives credence to their common ontological root. (Pribram 1986; 1998). My claim is that this root, though constrained by measures in spacetime, needs a more fundamental order, a potential that underlies and transcends spacetime. The spectral basis of the quantal nature of both matter and of communication portray this claim.

Of Matter and Mind:

One way of interpreting the “Fourier” diagram is that it indicates matter to be an “ex-formation”, an externalized (extruded, palpable, concentrated) form of flux.

By contrast, thinking and its communication (minding) are the consequence of an "internalized" (neg-entropic) forming of flux, its in-formation.
Hiley (this volume) comes to a similar perspective in that he stresses the formative aspect of in-formation. As noted, in discussing Bohm's quantum potential, Hiley begins with the Wigner-Moyal approach to the Schroedinger wave function. The real part of the equation describes what, in my formulation, is ex-formation. The virtual part of the equation describes the quantum potential: "it has no external source in the sense that the electric field has its source in a distribution of charges. Thus it does not emerge from an interaction Hamiltonian as does classical force. - - - In this sense it cannot be thought to act as an efficient cause. It is more like a formative cause that shapes the development of the process. - - - Thus we can think of the information as active from within giving shape to the whole process and this shape depends on the environment [the material context] in key ways." In the Fourier diagram this formative cause is labeled action (after Feinman).

Flux, measured as spectral density, is here defined (see Pribram and Bradley 1998) as change or lack thereof, basic to both energy (the amount of actual or potential work involved in altering structural patterns) and inertia (measured as the rotational momentum of mass). David Bohm (1973) had a concept similar to flux in mind which he called a holomovement. He felt that my use of the term "flux" had connotations for him that he did not want to buy into. I, on the other hand, felt holomovement to be vague in the sense of asking "what is moving?" We are dealing with fluctuations, and in the nervous system with oscillating hyper- and depolarizations characterized by the field potentials we can map from the fine fibered parts of the system.

Quantum physics is a science of matter. In quantum physics the Fourier transformation is primarily applied in relating the position in space of a mass to its rotational momentum (spin). Much has been written regarding the indeterminacy of this relationship at the lower limit of measurement, that is, that at the limit it is impossible to accurately measure both position and moment. This is also known as Heisenberg's uncertainty principle.

In the physics of matter the terms moment and position refer to a stable status: "moment" to the inertia of a mass and "position" to its location. By contrast, "energy" and "entropy" in thermodynamics refer to change measured as a quantitative amount of work necessary to effect the change and the efficiency with which the change is carried out. Both moment (rotational momentum) and energy are measured in terms of frequency (or spectral density) (times Planck's constant). Position is measured with respect to location, entropy as it evolves over duration for instance as power, the amount of work per unit time).

The Fourier relation envisions the waveforms involved in measuring frequency not as a linear continuum but rather as a clock-face-like circle – thus one can triangulate and obtain the cosine and sine of the waveform to produce their interference and measure phase in the spectral domain. This was Fourier's definitive insight (or was it that of the mathematicians in Egypt with whom he discoursed during Napoleon's expedition?) that has made his theorem "probably the most far reaching principle of mathematical physics" as Feynman has declared it. Thus, the Fourier energy-time relation becomes, in a sense, "spatialized".

In quantum physics very little has been made of the uncertainty involved in relating energy and time. Dirac and especially Wigner (1972) called attention to this indeterminacy in discussing the delta function, but for the most part quantum physicists (e.g. Bohr) have focused on the relationship between energy and mass as in Einstein's equation: E=mc*. By squaring c, the constant representing the speed of light, a linear measure of time becomes "spatialized" into an area-like concept, Minkowsky's space-time. I will return to a discussion of this version of time when considering brain processes. In short, much of the thinking that has permeated theories describing matter has been grounded in space-time, not the spectral aspects addressed by the Fourier transformation. For quantum physicists interested in the composition of matter, the Einstein/Minkowsky spatialization of time and energy comes naturally.

For brain function, Dirac's and Wigner's indeterminacy in the relation between energy and time is the more cogent. As noted, during the 1970s and 1980s the maps of dendritic receptive fields of neurons in the primary visual and other sensory cortexes were described by a space-time constrained Fourier relation, the Gabor elementary function, a windowed Fourier transform, essentially a sinc function, a kind of wavelet in phase (Hilbert) space. Gabor had used the same mathematics that Heisenberg had used; he therefore called his unit a "quantum of information" warning that by this he meant only to indicate the formal identity of the formulation, not a substantive one.

Gabor had undertaken his mathematical enterprise to determine the minimum uncertainty, the maximum compressibility, with which a telephone message could be transmitted across the Atlantic cable without any loss in intelligibility. He later (1954) related this minimum uncertainty to Shannon's BIT, the measure of a reduction of uncertainty. In turn, Shannon had related his measure of uncertainty to Gibbs' and Boltzman's measure of entropy. The stage was set for the issues of current concern in this part of the essay: a set of identical formalisms that refer to widely different substantive and theoretical bodies of knowledge.

Thermodynamics:

Contrast the referents of the formulations in classical, relativity and quantum physics to those in thermodynamics: First there are no references to the momentum and position of a mass. Second, the emphasis is on energy as measured not as a pseudo-spatial quantity but as dynamic, often "free" energy.

The utility of energy for structured work (as in a steam engine) is of concern in thermodynamics; its efficiency in structured use or rather, its inefficiency as dissipation into unstructured heat is measured as entropy. In the diagram of the Fourier relation, thermodynamics focuses on the upper part of the relationship (the dynamics of energy and time) just as physics focuses on the lower part (the statics of momentum and location of a mass or particle).

The distinction devolves on the conception of time. As noted, time in relativistic and quantum physics has been spatialized as clock time, the Kronos of the ancient Greeks. Time in thermodynamics is a measure of process, how quickly energy is expended. This amount of time, its duration, may vary with circumstance. It is the "Duree" of Bergson, the Kairos of an "Algebraic Deformation in Inequivalent Vacuum States" (Correlations, ed. K.G. Bowden, Proc. ANPA 23, 104-134, 2001).

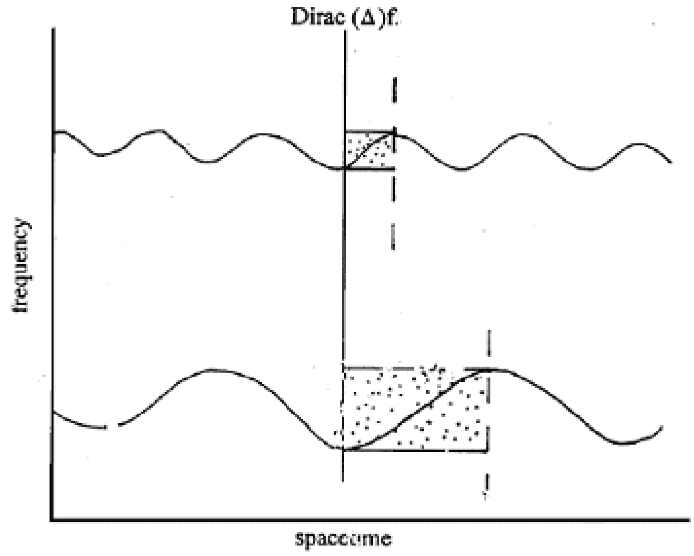
Brain processes partake of both aspects of time. In the posterior parts of the brain, the processes described by the Fourier transform domain, by virtue of movement, form symmetry groups that describe invariance, that is, objects in space and in Kronos, clock time. Alternatively, in the frontal and limbic portions of the brain the processes described result in the experience of Kairos, the duration of an episode. The evidence for these statements is reviewed in detail in Lecture 10, "Brain and Perception".

Meaning:

Shannon (1948; Shannon and Weaver 1949) insisted that his measure of the amount of information as the amount of reduction of uncertainty did not provide a measure of meaning: "One has the vague feeling that information and meaning may prove to be something like a pair of conjugate variables in quantum theory, they being subject to some joint restriction that condemns a person to the sacrifice of the one as he insists on having much of the other". Looking at the Fourier diagram, we can ask, which of the conjugate relationships are appropriate to serving Shannon's intuition with regard to meaning? My answer is that it is the relationship between Shannon's and Gabor's measures of information as negentropy and the location (the placement, the sampling) of a mass on the right side of the diagram.

Meaning is, in a nontrivial sense, the instantiation in matter of information. We might say, meaning matters. Bohm noted that his "active information" did something, had an influence on the course of the quantum material relationship. Charles Pearce stated: "What I mean by meaning is what I mean to do." Doing acts on the material world we live in.

This returns us to the statements made by Stapp: "Brain process is essentially a search process – the brain searches for a satisfactory response – and then dissipates [increases the entropy of] its energy in the initiation of the action that it represents". Llinas also emphasizes the primacy of the motor systems in implementing thought and in the experiencing of the self. A "satisfactory" response is a meaningful one. "Implementation" involves acting on the world we live in.



Logons, Gabor Elementary Functions:
Quanta of Information

With regard to language, meaning is the semantic relationship between linguistic "informative" patterns that ultimately lead to the deictic, "the pointing to the lived-in material world" to which that pattern refers (Pribram, 1975).

But there is another meaning to meaning, the meaning in music and in the pragmatics (the rhetoric) of language (Pribram, 1982). This meaning of meaning does not involve doing. Rather it is evocative, it engages not the striped muscular system of the body but the smooth muscles and endocrines. What is needed to account for this form of meaning is an addition to Pearce's "what I mean to do". This addition is: "What I mean by meaning is what I mean to experience." When I walk into a concert hall I am prepared to experience a familiar or not so familiar rendition of a repertoire. When Marc Antony addressed the crowd at Caesar's funeral he proclaimed: "I come to *bury* Caesar, not to praise him". The prosodics of this declamation as well as the semantics play into the expected experience of the audience. Prosody is a right hemisphere, semantics a left hemisphere process.

The time is ripe for untangling patterns of information from patterns of meaning. The proposal presented here stems directly from the other analyses undertaken. I continue to be amazed and awed by the power of mathematical conceptualizations in understanding the roots of brain function. These roots grow in the soil of the pattern processing of the brain, patterns we call information and meaning.

To summarize: The formal, mathematical descriptions of our subjective experiences (our theories) of observations in the quantum, thermodynamic and communications domains are non-trivially coordinate with each other. They are also coordinate with brain processes that, by way of projection, unify the experiential with the physical. By this I mean that the experiences of observations (measurements) in quantum physics, in thermodynamics and in communication appear to us to be "real", that is, extra-personal. Adaptation to living in the world makes it likely that this coordination of mathematical descriptions thus represents the useful reality within which we operate.

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As noted in the text, I am deeply indebted to David Bohm and to Basil Hiley for inspiration and corrective management of my course of theorizing. Additionally I have learned much in my association with Henry Stapp and Geoffrey Chew and more recently from Sisir Roy. I hope this manuscript will challenge them to continue to critique what often I feel they think of as my wayward ways.

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